

# Yairon Cid-Ruiz

Nationality: Spanish | Cuban; Date of Birth: 26/08/1990

E-mail: [ycidrui@ncsu.edu](mailto:ycidrui@ncsu.edu)

Website: <https://ycid.github.io>

## CONTACT INFORMATION

Professional Address: Department of Mathematics, SAS Hall 4214, North Carolina State University,  
Box 8205 Raleigh, NC 27695 USA.

## WORK EXPERIENCE

- 01/2024 – **Assistant Professor**, North Carolina State University.
- 02/2024 – 03/2024 **Research Member**, SLMath (MSRI).
- 08/2022 – 12/2023 **Postdoctoral researcher**, KU Leuven.
- 10/2020 – 07/2022 **Postdoctoral researcher**, Ghent University.
- 09/2019 – 09/2020 **Postdoctoral researcher**, Max Planck Institute for Mathematics in the Sciences.
- 09/2016 – 08/2019 **Early stage researcher**, Universitat de Barcelona.  
PhD candidate under a Marie Skłodowska-Curie grant.
- 09/2014 – 07/2015 **Graduate Teaching Assistant**, Universidad Central “Marta Abreu” de las Villas.

## EDUCATION

- 09/2016 – 08/2019 **Ph.D. in Mathematics**, *summa cum laude*.  
Center: Universitat de Barcelona.  
Advisor: Carlos D’Andrea.
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- 09/2015 – 08/2016 **Postgraduate Diploma Programme in Mathematics**, *top student prize*.  
Center: Abdus Salam International Centre for Theoretical Physics (ICTP).  
Advisor: Lothar Göttsche.
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- 09/2009 – 07/2014 **Bachelor in Computer Science**, *summa cum laude*.  
Center: Universidad Central “Marta Abreu” de las Villas (UCLV).
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## PUBLICATIONS & PREPRINTS

1. “Bounding the degrees of a minimal  $\mu$ -basis for a rational surface parametrization”, JOURNAL OF SYMBOLIC COMPUTATION 95 (2019), 134–150,  
[arXiv:1611.07506](#).
2. “A  $D$ -module approach on the equations of the Rees algebra”, JOURNAL OF COMMUTATIVE ALGEBRA Vol 14 (2022), No. 2, 155–176,  
[arXiv:1706.06215](#).
3. “Regularity and Gröbner bases of the Rees algebra of edge ideals of bipartite graphs”, LE MATEMATICHE Vol 73 No 2 (2018), pp. 279–296,  
[arXiv:1801.06731](#).
4. (with Sepehr Jafari, Beatrice Picone and Navid Nemati), “Regularity of bicyclic graphs and their powers”, JOURNAL OF ALGEBRA AND ITS APPLICATIONS Vol. 19, No. 03, 2050057 (2020),  
[arXiv:1802.07202](#).
5. (with Laurent Busé and Carlos D’Andrea), “Degree and birationality of multi-graded rational maps”, PROCEEDINGS OF THE LONDON MATHEMATICAL SOCIETY (3) 121 (2020) 743–787,  
[arXiv:1805.05180](#).
6. “Multiplicity of the saturated special fiber ring of height two perfect ideals”, PROCEEDINGS OF THE AMERICAN MATHEMATICAL SOCIETY 148 (2020), no. 1, 59–70,  
[arXiv:1807.03189](#).
7. (with Aron Simis), “Degree of rational maps via specialization”, INTERNATIONAL MATHEMATICS RESEARCH NOTICES, Volume 2022, Issue 6, March 2022, Pages 4451–4502,  
[arXiv:1901.06599](#).

8. “Noetherian operators, primary submodules and symbolic powers”, COLLECTANEA MATHEMATICA 72, p. 175–202 (2021),  
arXiv:1909.07253.
9. (with Vivek Mukundan), “Multiplicity of the saturated special fiber ring of height three Gorenstein ideals”, ACTA MATHEMATICA VIETNAMICA 46, pp. 663–674 (2021),  
arXiv:1909.13633.
10. “Mixed multiplicities and projective degrees of rational maps”, JOURNAL OF ALGEBRA 566 (2021), 136–162,  
arXiv:2001.00547.
11. (with Roser Homs and Bernd Sturmfels), “Primary ideals and their differential equations”, FOUNDATIONS OF COMPUTATIONAL MATHEMATICS 21, pp. 1363–1399 (2021),  
arXiv:2001.04700.
12. (with Marc Chardin and Aron Simis), “Generic freeness of local cohomology and graded specialization”, TRANSACTIONS OF THE AMERICAN MATHEMATICAL SOCIETY 375 (2022), no. 1, 87–109,  
arXiv:2002.12053.
13. (with Federico Castillo, Binglin Li, Jonathan Montaño and Naizhen Zhang), “When are multidegrees positive?”, ADVANCES IN MATHEMATICS 374 (2020), 107382,  
arXiv:2005.07808.
14. (with Federico Castillo, Binglin Li, Jonathan Montaño and Naizhen Zhang), “When are multidegrees positive?”, FPSAC 2021 Proceedings, SÉMINAIRE LOTHARINGIEN DE COMBINATOIRE 85B (2021), Art. 46, 12 pp.
15. (with Jonathan Montaño), “Convex bodies and graded families of monomial ideals”, REVISTA MATEMÁTICA IBEROAMERICANA 38 (2022), no. 6, 2033–2056,  
arXiv:2010.07918.
16. (with Jonathan Montaño), “Mixed multiplicities of graded families of ideals”, JOURNAL OF ALGEBRA 590 (2022) 394–412,  
arXiv:2010.11862.
17. “Equations and multidegrees for inverse symmetric matrix pairs”, LE MATEMATICHE Vol. LXXVI (2021) - Issue II, pp. 369–381,  
arXiv:2011.04616.
18. (with Justin Chen, Marc Härkönen, Robert Krone and Anton Leykin), “Noetherian Operators in Macaulay2”, JOURNAL OF SOFTWARE FOR ALGEBRA AND GEOMETRY, Vol. 12 (2022), 33–41.  
arXiv:2101.01002.
19. (with Bernd Sturmfels), “Primary decomposition with differential operators”, INTERNATIONAL MATHEMATICS RESEARCH NOTICES, rnac178, 2022,  
arXiv:2101.03643.
20. (with Justin Chen), “Primary decomposition of modules: a computational differential approach”, JOURNAL OF PURE AND APPLIED ALGEBRA, 226, 2022,  
arXiv:2104.03385.
21. (with Fatemeh Mohammadi and Leonid Monin), “Multigraded algebras and multigraded linear series”, JOURNAL OF THE LONDON MATHEMATICAL SOCIETY, Volume 109, Issue 3, 2024,  
arXiv:2104.05397.
22. “Fiber-full modules and a local freeness criterion for local cohomology modules”, MATHEMATISCHE ZEITSCHRIFT 303, 30 (2023),  
arXiv:2106.07777.
23. (with Ritvik Ramkumar), “The fiber-full scheme”, to appear in JOURNAL OF PURE AND APPLIED ALGEBRA,  
arXiv:2108.13986.
24. (with Federico Castillo, Fatemeh Mohammadi and Jonathan Montaño), “Double Schubert polynomials do have saturated Newton polytopes”, FORUM OF MATHEMATICS, SIGMA Vol. 11:e100 1–9,  
arXiv:2109.10299.
25. (with Oliver Clarke and Fatemeh Mohammadi), “A study of nonlinear multiview varieties”, JOURNAL OF ALGEBRA 620 (2023) 363–391,  
arXiv:2112.06216.

26. (with Ritvik Ramkumar), “*A local study of the fiber-full scheme*”, JOURNAL OF ALGEBRA 636 (2023) 248–278,  
arXiv:2202.06652.
27. (with Claudia Polini and Bernd Ulrich), “*Generalized Jouanolou duality, weakly Gorenstein rings, and applications to blowup algebras*”, to appear in JOURNAL FÜR DIE REINE UND ANGEWANDTE MATHEMATIK (CRELLE’S JOURNAL),  
arXiv:2205.03837.
28. (with Alessio Caminata and Aldo Conca), “*Multidegrees, prime ideals, and non-standard gradings*”, ADVANCES IN MATHEMATICS 435 (2023) 109361,  
arXiv:2208.07238.
29. (with Federico Castillo, Fatemeh Mohammadi and Jonathan Montaña), “*K-polynomials of multiplicity-free varieties*”,  
arXiv:2212.13091.
30. (with Ilya Smirnov), “*Effective generic freeness and applications to local cohomology*”, JOURNAL OF THE LONDON MATHEMATICAL SOCIETY, Volume 110, Issue 4, October 2024,  
arXiv:2302.08196.
31. “*Relative mixed multiplicities and mixed Buchsbaum-Rim multiplicities*”,  
arXiv:2311.15105.
32. “*Polar multiplicities and integral dependence*”, INTERNATIONAL MATHEMATICS RESEARCH NOTICES 2024, 00(0), 1–18,  
arXiv:2401.10198.
33. (with Jack Jeffries), “*Uniformity in nonreduced rings via Noetherian operators*”, to appear in INTERNATIONAL MATHEMATICS RESEARCH NOTICES,  
arXiv:2404.02057.
34. (with Claudia Polini and Bernd Ulrich), “*Multidegrees, families, and integral dependence*”,  
arXiv:2405.07000.
35. (with Yupeng Li and Jacob P. Matherne), “*Log-concavity of polynomials arising from equivariant cohomology*”,  
arXiv:2411.17572.
36. (with Yupeng Li and Jacob P. Matherne), “*Log-concavity of polynomials arising from equivariant cohomology*”, FPSAC 2025 Proceedings, SÉMINAIRE LOTHARINGIEN DE COMBINATOIRE 93B (2025) Art. 68, 12 pp.
37. “*Mixed Segre zeta functions and their log-concavity*”,  
arXiv:2507.06424.
38. (with Jacob P. Matherne and Anna Shapiro), “*Syzygies of polymatroidal ideals*”,  
arXiv:2507.13153.
39. “*Segre classes and integral dependence*”,  
arXiv:2512.08863.

## AWARDS, GRANTS & FELLOWSHIPS

- 09/2025–08/2027: NSF Grant DMS #2502321, “*Multigraded structures in Algebra, Geometry, and Combinatorics*”, (\$119,093).
- 09/2025–08/2030: Simons Foundation, *Travel Support for Mathematicians*, (\$37,635).
- 06/02/2025–06/06/2025: Participation in a *Collaborate@ICERM* Group, Institute for Computational and Experimental Research in Mathematics, Brown University, Providence.
- 03/2023: Awarded the *Heisenberg Programme* from the Deutsche Forschungsgemeinschaft (DFG). Declined in favor of an Assistant Professor position at North Carolina State University.
- 11/2022 – 12/2022: *Oberwolfach Research Fellow*, MFO, Oberwolfach Research Institute for Mathematics.
- 06/2022 – 07/2022: *Professore Visitatore in Italia (Visiting Professor)*, supported by Istituto Nazionale di Alta Matematica “Francesco Severi” – Gruppo Nazionale per le Strutture Algebriche, Geometriche e le loro Applicazioni (INdAM-GNSAGA), University of Genova.

- 10/2021 – 09/2024: *Postdoctoral Fellowship Research Foundation Flanders (FWO)*, Ghent University & KU Leuven, Belgium.
- 09/2016 – 08/2019: *Marie Skłodowska-Curie Fellowship*, Universitat de Barcelona, Barcelona, Spain.
- 09/2015 – 08/2016: Awarded a *fully funded scholarship to pursue the Postgraduate Diploma in Mathematics at ICTP*, Trieste, Italy.
- Extraordinary Doctorate Award, Universitat de Barcelona, 2019.
- Winner of the top student prize in Mathematics in International Centre for Theoretical Physics, 2016.
- One of the 10 selected students in the Postgraduate Diploma Programme in Mathematics in International Centre for Theoretical Physics, 2015.
- Best graduate student in 2014 in the Universidad Central “Marta Abreu” de Las Villas, 2014.
- SILVER MEDAL in the XV Iberoamerican Mathematical Olympiad, 2012.
- BRONZE MEDAL in the XIII Iberoamerican Mathematical Olympiad, 2010.
- BRONZE MEDAL in the Caribbean Regional Final of the Latin American programming ACM-ICPC, 2012.

## RESEARCH VISITS

- Basque Center for Applied Mathematics, 2 weeks, 16/10/2023 – 27/10/2023, host: Ilya Smirnov.
- Basque Center for Applied Mathematics, 1 week, 10/10/2022 – 15/10/2022, host: Ilya Smirnov.
- University of Notre Dame & Purdue University, 1 month, 03/2022, hosts: Claudia Polini & Bernd Ulrich.
- University of Genova, 1 month, 07/2022, host: Aldo Conca.
- Universidad de Sevilla, 1 week, 22/04/2019 – 26/04/2019, host: Francisco Jesús Castro Jiménez.
- International Centre for Theoretical Physics, 1 week, 11/02/2019 – 15/02/2019, host: Tarig Abdelgadir.
- Politecnico di Torino, 1 week, 05/11/2018 – 09/11/2018, host: Aron Simis.
- Johannes Kepler University Linz, 2 months, 10/2018 – 11/2018, host: Josef Schicho.
- INRIA Sophia Antipolis, 6 months, 10/2017 – 03/2018, host: Lurent Busé.

## INVITED TALKS

- “*Mixed Segre zeta functions and their log-concavity*”, Algebra and Combinatorics Seminar, Tulane University, New Orleans, December 2025.
- “*Mixed Segre zeta functions and their log-concavity*”, Commutative Algebra Seminar, University of Utah, Salt Lake City, November 2025.
- “*Mixed Segre zeta functions and their log-concavity*”, Algebraic Geometry/Commutative Algebra Seminar, University of Notre Dame, Notre Dame, November 2025.
- “*Matroid Schubert varieties*”, Arbeitsgemeinschaft “Combinatorial Hodge Theory”, Mathematisches Forschungsinstitut Oberwolfach (MFO), Germany, October 2025.
- “*Noetherian operators in Commutative Algebra*”, 50th Annual Spring Lecture Series, University of Arkansas, Fayetteville, May, 2025.
- “*Log-concavity of polynomials arising from equivariant cohomology*”, Commutative Algebra in The South, Conference in Georgia State University, Atlanta, April 2025.
- “*Log-concavity of polynomials arising from equivariant cohomology*”, Special Year Seminar, Institute for Advanced Study, Princeton, March 2025.
- “*Multiplicities and integral dependence*”, Algebra and Combinatorics seminar, North Carolina State University, September 2024.
- “*Numerical criteria for integral dependence and their behavior in families*”, Seminar on Commutative Algebra and Algebraic Geometry, UC Berkeley, March 2024.
- “*Duality and blow-up algebras*”, Algebra and Combinatorics seminar, North Carolina State University, January 2024.

- “*Multidegrees, polymatroids and Schubert polynomials*”, Complex Geometry Seminar, Institut de Mathématiques de Toulouse Université Paul Sabatier, November 2023.
- “*Some lectures on multidegrees*”, 28th National School on Algebra Interactions between Algebra and Geometry, Bucharest, Romania, June 2023.
- “*Duality and blow-up algebras*”, Foundations of Computational Mathematics, Sorbonne Université, June 2023.
- “*Duality and blow-up algebras*”, Algebra Seminar, Institute of Mathematics, Osnabrück University, April 2023.
- “*Multidegrees at the crossroads of Algebra, Geometry, and Combinatorics*”, Colloquium, Department of Mathematics, Auburn University, January 2023.
- “*Multidegrees at the crossroads of Algebra, Geometry, and Combinatorics*”, Colloquium (online), Department of Mathematics, New Mexico State University, January 2023.
- “*Multidegrees at the crossroads of Algebra, Geometry, and Combinatorics*”, Colloquium, Department of Mathematics, North Carolina State University, January 2023.
- “*K-polynomials of multiplicity-free varieties*”, Conference: “Written Geometry: Commutative Algebra”, CIRM, Luminy, France, January 2023.
- “*Double Schubert polynomials do have saturated Newton polytopes*”, Geometry meets Combinatorics in Bielefeld, Germany, September 2022.
- “*The fiber-full scheme and applications*”, Algebra & Geometry Seminar, University of Genova, Italy, July 2022.
- “*Describing non-reduced schemes with differential operators*”, Workshop on Differential Algebra, MPI Leipzig, Germany, June 2022.
- “*Double Schubert polynomials do have saturated Newton polytopes*”, AMS 2022 Spring Western Virtual Sectional Meeting (online), May 2022.
- “*A local study of the fiber-full scheme*”, AMS Spring Central Virtual Sectional Meeting, Purdue University, USA, March 2022.
- “*Differential primary decomposition of modules*”, Congreso Bienal de la Real Sociedad Matemática Española 2022, Universidad de Castilla - La Mancha, Ciudad Real, Spain, January 2022.
- “*Double Schubert polynomials do have saturated Newton polytopes*”, Congreso Bienal de la Real Sociedad Matemática Española 2022, Universidad de Castilla - La Mancha, Ciudad Real, Spain, January 2022.
- “*An invitation to the fiber-full scheme*”, Seminar Number Theory and Algebraic Geometry, KU Leuven, Leuven, Belgium, November 2021.
- “*An invitation to the fiber-full scheme*”, Jornada de Jóvenes Doctores en Geometría Algebraica, Universitat de Barcelona, Barcelona, Spain, November 2021.
- “*The fiber-full scheme*”, Fellowship of the Ring seminar (online), October 2021.
- “*Convex bodies and graded families of ideals*”, Society for Industrial and Applied Mathematics, Applied Algebraic Geometry Conference (online), August 2021.
- “*Primary decomposition with differential operators*”, ICERM conference (online): D-modules, Group Actions, and Frobenius: Computing on Singularities, August 2021.
- “*Primary decomposition with differential operators*”, Simon Fraser University Number Theory and Algebraic Geometry Seminar (online), July 2021.
- “*Primary decomposition with differential operators*”, Mathematical Congress of the Americas (online), July 2021.
- “*Convex bodies and graded families of monomial ideals*”, IIT Bombay Virtual Commutative Algebra Seminar (online), July 2021.
- “*Multigraded algebras and multigraded linear series*”, Technische Universität Berlin (online), June 2021.
- “*Equations and multidegrees for inverse symmetric matrix pairs*”, Effective Methods in Algebraic Geometry MEGA 2021 (online), May 2021.
- “*Primary decomposition with differential operators*”, Seminari de Geometria Algebraica de Barcelona (online), April 2021.

- “*Primary ideals and differential operators*”, The MAX computer algebra seminar (online), February 2021.
- “*When are multidegrees positive?*”, Ghent Algebra and Geometry Seminars Weekly Research Seminars, Ghent, Belgium, November 2020.
- “*Ehrenpreis-Palamodov theorem in commutative algebra*”, Differential Operators in Commutative Algebra Seminar (online), July 2020.
- “*Specialization of graded modules and generic freeness of local cohomology*”, Nonlinear Algebra Seminar Online, March 2020.
- “*Noetherian operators, primary submodules and symbolic powers*”, V congreso de jóvenes investigadores RSME, Castellón, Spain, January 2020.
- “*Noetherian operators, primary submodules and symbolic powers*”, Algebra Seminar, Osnabrück, Germany, November 2019.
- “*Rational maps, syzygies and specialization*”, Seminar on Nonlinear Algebra, Leipzig, Germany, September 2019.
- “*Ehrenpreis – Palamodov Theorem*”, Second edition of the conference: Computing with D-modules, Leipzig, Germany, September 2019.
- “*Specialization of rational maps*”, Society for Industrial and Applied Mathematics, Applied Algebraic Geometry Conference, Bern, Switzerland, July 2019.
- “*Rational maps and syzygies*”, Seminario de Álgebra de la Universidad de Sevilla, Sevilla, Spain, April 2019.
- “*Saturated special fiber ring and rational maps*”, Seminari de Geometria Algebraica de Barcelona, Barcelona, Spain, March 2019.
- “*Saturated special fiber ring and rational maps*”, Mathematics Seminar, International Centre for Theoretical Physics, Italy, February 2019.
- “*Rational maps and the saturated special fiber ring*”, Joint Meeting of the Czech, Slovenian, Austrian, Slovak and Catalan Mathematical Societies, Bratislava, Slovakia, September 2018.
- “*Degree and birationality of multi-graded rational maps*”, ARCADES Doctoral School II and ESR Days in Barcelona, Barcelona, Spain, September 2018.
- “*Regularity and Gröbner bases of the Rees algebra of edge ideals of bipartite graphs*”, XVI Encuentro de Álgebra Computacional y Aplicaciones (EACA), Zaragoza, Spain, July 2018.
- “*Regularity of bicyclic graphs and their powers*”, IPPI Workshop 2018 (post-Pragmatic 2017 event), Turin, Italy, March 2018.
- “*Regularity and Gröbner bases of the Rees algebra of edge ideals of bipartite graphs*”, Journées Nationales de Calcul Formel, CIRM, Luminy, France, January 2018.
- “*A D-module approach on the equations of the Rees algebra*”, Séminaire d’algèbre, topologie et géométrie, University of Nice, France, November 2017.
- “*Bounding the degrees of a minimal  $\mu$ -basis for a rational surface parametrization*”, Society for Industrial and Applied Mathematics, Applied Algebraic Geometry Conference, Atlanta, United States, August 2017.
- “*Bounding the degrees of a minimal  $\mu$ -basis for a rational surface parametrization*”, Effective Methods in Algebraic Geometry MEGA 2017, Nice, France, June 2017.
- “*Bounding the degrees of a minimal  $\mu$ -basis for a rational surface parametrization*”, Seminari de Geometria Algebraica de Barcelona, Barcelona, Spain, March 2017.

## CONFERENCES & WORKSHOPS

- Arbeitsgemeinschaft “Combinatorial Hodge Theory”, Mathematisches Forschungsinstitut Oberwolfach (MFO), Germany, October 2025.
- Conference: “Written Geometry: Commutative Algebra”, CIRM, Luminy, France, January 2023.
- Workshop on Differential Algebra, MPI Leipzig, Germany, June 2022.
- Mathematical Congress of the Americas 2021 (online), July 2021.
- Summer School on Randomness and Learning in Non-Linear Algebra, Leipzig, Germany, July 2019.
- Thematic Program in Commutative Algebra and its Interaction with Algebraic Geometry. In Honor of Bernd Ulrich. Notre Dame University, United States, June 2019.
- Learning Week III in ARCADES, Nice, France, March 2019.
- Second ARCADES Software & Industrial Workshop, Cambridge, UK, January 2019.
- Frobenius Action in Commutative Algebra: Recent Developments, Barcelona, Spain, January 2019.
- Macaulay2 Workshop, Leipzig, Germany, June 2018.
- Learning Week II in ARCADES, Nice, France, March 2018.
- Workshop on Commutative Algebra, Syzygies and Singularities, Nice, France, December 2017.
- First Software & Industrial Workshop, and Midterm Review ARCADES, Athens, Greece, November 2017.
- Research school in Algebraic Geometry and Commutative Algebra, Pragmatic 2017, Catania, Italy, June 2017.
- Learning Week I in Algebraic Representations in Computer-Aided Design for complEx Shapes (ARCADES), Nice, France, March 2017.
- 1st Doctoral School ARCADES, Oslo, Norway, December 2016.
- Workshop in Algebra, Algebraic Geometry, Algebraic Topology and Applications sponsored by Centro de Investigaciones en Matemáticas(CIMAT, Guanajato, México) in Ciego de Avila, Cuba, 2014.

## ORGANIZATIONAL ACTIVITIES

- Program Committee of the conference “*Computer Algebra in Scientific Computing 2024*”, Rennes, France, September 2024.
- Member of the Organizing Committee of the minisymposia “*Differential Equations in Algebraic Geometry and Beyond*”, 2021 SIAM Conference on Applied Algebraic Geometry (online), August 2021.
- Member of the Organizing Committee of the minisymposia “*Syzygies and applications to Geometry*”, 2019 SIAM Conference on Applied Algebraic Geometry, Bern, Switzerland, July 2019.

## TEACHING EXPERIENCE

<b>North Carolina State University</b>	01/2024 –
Algebraic Geometry	Spring 2026
Introduction to Toric Varieties	Fall 2025
Computer Algebra II	Spring 2025
Computer Algebra I	Fall 2024
<b>KU Leuven</b>	08/2022 – 12/2023
Supervisor of three Bachelor Projects in Pure Mathematics	Fall 2022
<b>Ghent University</b>	10/2020 – 07/2022
Calculus	Spring 2021
<b>Universidad Central “Marta Abreu” de las Villas</b>	09/2014 – 07/2015
Differential Geometry of Curves and Surfaces	Spring 2015
Topology	Fall 2014

## LANGUAGES

*Spanish:* Mother tongue.  
*English:* Fluent.  
*Dutch:* Intermediate (level B1).

**COMPUTER SKILLS**

C/C++, JAVA,  $\text{\LaTeX}$ , Linux, Wolfram Mathematica, MACAULAY2, SINGULAR.