An LQ-Solution to a Control Problem Associated with a Solar Thermal Central Receiver

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Abstract—**The linearized process dynamics of the steam boiler in a solar-powered central receiver change abruptly when clouds interfere with the sun's rays. The steam temperature regulator used to maintain proper exit steam conditions must control a system with variable structure and discontinuous state trajectories. This paper investigates the quadratic-optimal control of such a system, and gives the design equations for the optimal regulator.**

Introduction

THE mathematical models which are used in analytical procedures for control system design are often too limited to show the full range of possible system behavior. If the analyst feels that there is significant uncertainty surrounding the way in which the system dynamics will evolve in time, he may choose to work with a stochastic model which gives a quantitative indication of the relative likelihoods of various possible scenarios. For example, if the system changes its internal dynamics in response to discrete changes in the external environment, the relevant system model may consist of a set of submodels. The variable nature of the system dynamics is represented by transitions between these various submodels.

If the submodels are linear, the criterion functional is quadratic; and if the modal transitions have a Markov property, it is possible to derive an explicit expression for the optimal regulator. This regulator has a linear state feedback form with gains related to the solution of a set of ordinary differential equations. An implicit assumption in the development of this regulator is that the state trajectories are continuous at discontinuity points of the dynamic matrices.

This continuity assumption is not always satisfied in applications of interest. To illustrate this, consider the problem which motivated this study. A solar 10 MWe electrical generating system has been constructed in the California desert (see Fig. 1). A field of movable mirrors (heliostats) is used to focus the sun's energy on a central boiler. One of the most important control loops in the boiler is the steam temperature regulator which controls the feedwater flow rate in such a way as to maintian the proper outlet steam temperature. The steam temperature regulator has been designed on the basis of a linear dynamic model which has evolved from both analytical and empirical studies of the dynamic behavior of a solar-powered central receiver.

A difficulty which soon presents itself in this design is that the linear perturbation model for the boiler is dependent upon the output power of the boiler. The reason for this is fairly obvious. The system lags tend to be flow rate dependent, and the boiler flow rate is strongly dependent upon the received insolation. As a result of this system variability, several models are required to characterize the evolution of the boiler as it moves from low power at the beginning of the operating day to maximum power near noon, and then again to low power at dusk. This diurnal variation in the system behavior is slow and predictable and can be easily accommodated by a regulator which changes its gains at certain quantized levels of insolation.

A more difficult problem is posed by the motion of clouds over the heliostats. On a partly cloudy day, the clouds tend to cover the heliostats in a time that is quite short when compared to the dominant system time constants, and they move away just as fast. These sudden changes in insolation may be frequent and are essentially unpredictable. The relevant system model, dependent as it is upon the insolation level, thus changes in a discrete and apparently random fashion.

Another anomaly is created by the cloud action. The perturbation variables in the system are referenced to a set of nominal operating points of the nonlinear equation of system motion. Most of the dynamic system variables are temperatures, pressures, and flows which are continuous across discontinuities in insolation. When the insolation level changes discretely, however, the quiescent operating points of these variables may change, thus producing an apparent discontinuity in the error variables.

This paper considers the linear-quadratic (LQ) regulation problem for a class of systems of this type. While motivated by the study of the solar-powered boiler, the results of this investigation are not limited to this particular application. The example presented in Section IV studies an inner loop of the steam temperature controller for a single boiler panel. Using linearized models, a comparison is made of the performance of the LQ regulator and an alternative regulator which was used at the U. S. Department of Energy Central Receiver Test Facility (CRTF) during some early tests on the dynamic response of a prototype receiver panel.

Problem Description

The global model of the system to be controlled will be assumed to be given by an ordinary differential equation of the form

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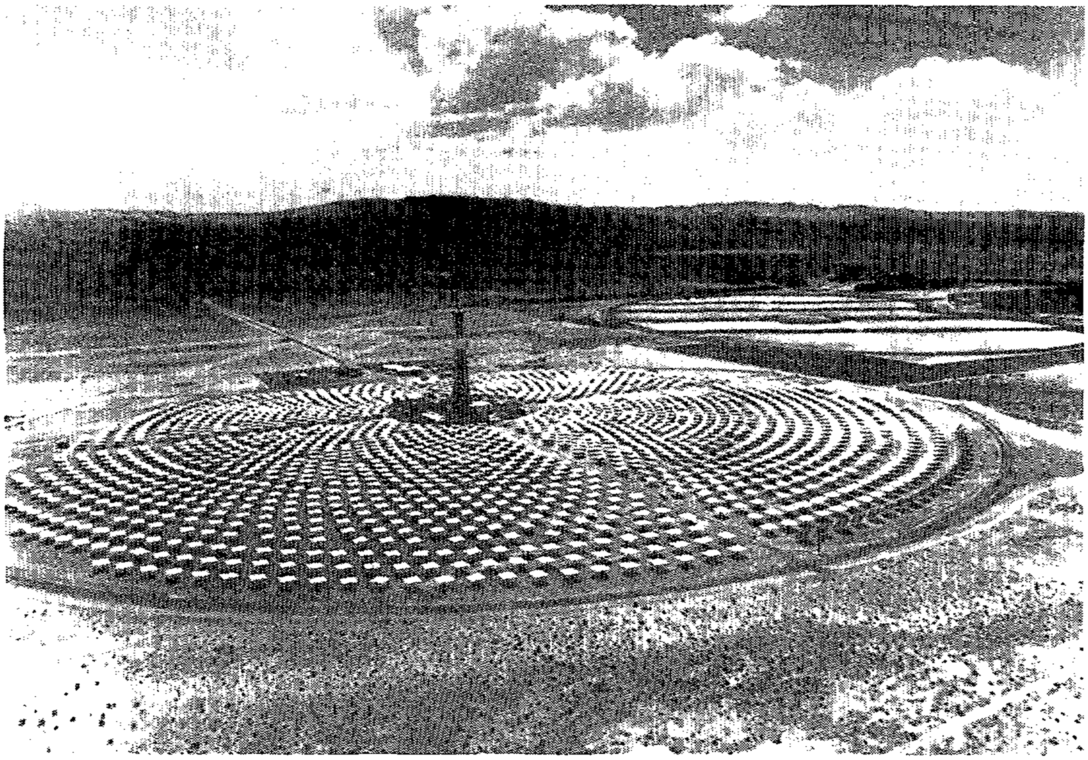

Fig. 1. Daggett, CA, site of the 10 MWe solar thermal central receiver.

$$\dot{\xi} = f(\xi, \omega, t, r); \qquad t \geqslant t_o$$
$$\xi(t_o) = \xi_o \tag{1}$$

where ξ is the vector system state, ω is the vector actuating signal, and r is a scalar indicator variable representing the exogenous influences on the system. In the case of the solar-powered boiler, (1) would represent a lumped mass approximation to the partial differential equations which more accurately describe the boiler.

Control system synthesis procedures are often based upon the linear perturbation model associated with (1). Suppose the exogenous variable r is a constant r_o, and suppose that (x_n, v_n) satisfies the equation

$$\dot{x}_n = f(x_n, v_n, t, r_o); \qquad t \geqslant 0$$
$$x_n(t_o) = x_o$$

and represents the desired operating condition of the system corresponding to the given value of r. The actual system response will be subject to a variety of external influences and will be denoted by (x_p, v_p). The difference between these two sets of functions gives the error variables (x, v):

$$x(t) = x_p(t) - x_n(t)$$
$$v(t) = v_p(t) - v_n(t).$$

If f is sufficiently smooth and (x, v) sufficiently small, then a linear perturbation model can be used to approximate the equation of evolution of (x, v).

$$\dot{x} = F_{r_o}x + G_{r_o}v; \qquad t \geqslant 0$$
$$x(t_o) = x_o$$

where

$$F_{r_o} = \frac{\partial f}{\partial \xi}(x_n, v_n, t, r_o)$$

$$G_{r_o} = \frac{\partial f}{\partial \omega}(x_n, v_n, t, r_o).$$

A good example of the above modeling procedure is provided in [1] where the nonlinear model and the associated perturbation equation are presented for a fossil-fired power plant. The variation in the eigenvalues of the dynamic matrix of the linear model is displayed graphically. A study with similar objectives for a solar-powered boiler was reported in [2]. A combination of analysis and empirical data from the CRTF gave rise to a family of models like that given in (2) with the addition of a time delay factor. For the purposes of this study, it will be assumed that any time delays have been incorporated into (2) with a Padé approximation, or, alternatively, that a Smith predictor [3] implementation will be used. This latter approach will permit the time delay to be handled separately.

Thus far, the model is of the classical sort, and if r were constant, a regulator design utilizing (2) could be carried out. Unfortunately, in the class of problems of concern here, r models the unpredictable variation in the external environment. It may, for example, represent the passage of a cloud over the heliostats. When a cloud is thus situated, the nominal operating point of the boiler is changed significantly from that which obtains when full sun is reflected toward the receiver. Suppose the values of r are discretized, and that $r(t) \in S = \{1, \cdots, s\}$. Then, a sudden change in insolation is modeled by a change in the value of r.

When r changes, two changes immediately occur in (2). First, since the model coefficients depend upon r, the perturbation dynamics change. Second, the error state changes discontinuously. To illustrate this, suppose that $r(t) = j \neq r(t^-) = i$. The error state $x(t)$ is given by

$$x(t) = x_p(t) - x_n(t; r(t) = j)$$
$$x(t) = x_p(t) - x_n(t^-; r(t^-) = i) + x_n(t^-; r(t^-) = i) - x_n(t; r(t) = j). \quad (3)$$

In the applications of interest, the components of x_n will be temperatures, enthalpies, pressures, etc., which are, for a given value of r, continuous functions of time. For the same reason, x_p is continuous. Hence,

$$x(t) = x(t^-) + \lambda \quad (4)$$

where λ is the change in the nominal operating point occasioned by the change in r. Note that (4) differs from the discontinuity models used in other related studies. In [4], for example, $x(t)$ was assumed to be a linear function of $x(t^-)$. Here, it is the offset term λ which gives the problem its distinctive character.

The system model is given by (2) at continuity points of r and by (4) at discontinuity points. It is convenient to generalize (3) slightly and assume that λ is a random variable whose probability distribution depends only on $r(t^-)$ and $r(t)$, with finite first and second moments given by

$$E(\lambda | r(t) = j, r(t^-) = i) = \hat{\lambda}_{ij} \quad (5)$$
$$E(\lambda\lambda' | r(t) = j, r(t^-) = i) = \Gamma_{ij}. \quad (6)$$

Certainly the behavior described by (3) fits within this class of characterizations. For convenience, the definition of λ can be extended slightly by the equation

$$\text{prob}(\lambda = 0 | r(t) = r(t^-)) = 1.$$

With the system model given by (2) and (4), only the statistical behavior of r is required to complete the description of the system. It will be assumed that r is a Markov process with a finite state space and transition matrix $Q = [q_{ij}]$ given by

$$\text{prob}(r(t+\Delta) = j | r(t) = i) = \begin{cases} 1 + q_{ii}\Delta + o(\Delta); & i = j \\ q_{ij}\Delta + o(\Delta); & i \neq j;\ i, j \in S. \end{cases}$$

The elements of the Q matrix have a simple physical relationship to the r process when Q is independent of time. For example, if Q is a cloud model, then $(-q_{ii})^{-1}$ is the mean interval over which the insolation is of level i. The other elements of Q have a related interpretation. As part of thermal cycling studies, data giving the instantaneous insolation level at the construction site are available over a one-year period. Such data can be used to select a representative Q matrix.

The regulator will be chosen to minimize a generalized quadratic function of the state error

$$J = E\left\{x'(t_f)Px(t_f) + \int_{t_o}^{t_f}(x'Mx + v'Nv)\,dt\right\} \quad (7)$$

where $[t_o, t_f]$ is the operating interval. In the case of the solar-powered boiler, $[t_o, t_f]$ is the interval from just before sunrise to shortly before sunset. The regulator can make use of state feedback and a measurement of r, i.e.,

$$v(t) = u(t, x(t), r(t)).$$

In the case of the solar boiler, this latter feedback is accomplished through the use of solar flux sensors on the receiver panels.

The feedback regulator which minimizes (7) is known for a system described by (2) at continuity points of r and continuous error state trajectories ($\lambda \equiv 0$). For this case, u is linear in $x(t)$ and stochastically stable under fairly weak conditions on the system equation [5]. It is reasonable to assume that such properties would carry over to this application; and as a consequence, J is a useful intermediary for finding an attractive control rule of simple structure.

Synthesis of the Optimal Regulator

The method of choice in a Markov decision problem of the type described in the previous section is well known [6].[1] The optimal regulator is the minimizing u in Bellman's equation

$$0 = V_t + \min_u [A^u V + x'Mx + v'Nv]$$
$$V(t_f, x, i) = x'Px; \qquad i \in S. \quad (8)$$

The function V has the usual interpretation as the minimum-cost-to-go functional

$$V(t, x, i) = E\left\{x'(t_f)Px(t_f) + \int_t^{t_f}(x'Mx + v'Nv)\,dt \,|\, x(t) = x, r(t) = i\right\} \quad (9)$$

and A^u is the infinitesimal operator of the (x, r) process

$$A^u f(x, i) = \lim_{\Delta \to o} \frac{1}{\Delta} E\{f(x(t+\Delta), r(t+\Delta)) - f(x(t), r(t)) | x(t) = x, r(t) = i\}. \quad (10)$$

From (2) and (4), A^u can be evaluated:

$$A^u f(x, i) = f_x(F_i x + G_i v) + \sum_j q_{ij} E\{f(x+\lambda, j) | r(t) = j, r(t^-) = i\}. \quad (11)$$

Substituting (11) into (8) and evaluating the optimal regulator, it follows that

$$v = -\tfrac{1}{2}N^{-1}G_i'V_x' \qquad \text{if } r(t) = i,\ x(t) = x \quad (12)$$
$$0 = V_t + V_x F_i x - \frac{1}{4}V_x G_i N^{-1} G_i' V_x' + x'Mx + \sum_j q_{ij} E(V(t, x+\lambda, j) | r(t^-) = i). \quad (13)$$

Equation (13) is of a somewhat unusual form because of the nonlocal character of the last term.

The explicit expression for v and V can be obtained by the well-known procedure of assuming that V is a quadratic form in x with coefficients which depend upon i. The result of this calculation is

$$v = -N^{-1}G_i'P_i(x + \alpha_i) \qquad \text{if } r(t) = i,\ x(t) = x \quad (14)$$
$$V(t, x, i) = (x + \alpha_i)'P_i(x + \alpha_i) + \beta_i \quad (15)$$

[1] See also, [7] in which Rishel considers the nonlinear jump parameter problem with continuous state trajectories. The effect of the discontinuities is evident from a comparison of [7, eq. (17)] to (11) above.

where

$$\dot{P}_i = -P_iF_i - F_i'P_i + P_iG_iN^{-1}G_i'P_i - M - \sum_j q_{ij}P_j$$

$$P_i(t_f) = P; \quad i \in S \tag{16}$$

$$\dot{\alpha}_i = (F_i + P_i^{-1}M)\alpha_i + \sum_j q_{ij}P_i^{-1}P_j(\alpha_i - \alpha_j - \hat{\lambda}_{ij})$$

$$\alpha_i(t_f) = 0; \quad i \in S \tag{17}$$

$$\begin{aligned}\dot{\beta}_i = &-\sum_j q_{ij}\beta_j - \alpha_i'M\alpha_i \\ &-\sum_j q_{ij}\Big[(\alpha_i - \alpha_j - \hat{\lambda}_{ij})'P_j(\alpha_i - \alpha_j - \hat{\lambda}_{ij}) \\ &+ \mathrm{tr}\big(P_j(\Gamma_{ij} - \hat{\lambda}_{ij}\hat{\lambda}_{ij}')\big)\Big]\end{aligned}$$

$$\beta_i(t_f) = 0; \quad i \in S. \tag{18}$$

Equation (14) gives the optimal regulator in terms of a gain $N^{-1}G_iP_i$, and a bias α_i. The gain is precisely what would be appropriate if there were no state error discontinuities. The equation for $\{P_i;\ i \in S\}$ is that obtained for the case in which $\lambda \equiv 0$ in [5], and it can be solved without regard to $\{(\alpha_i, \beta_i);\ i \in S\}$.

The bias term can be found from (17) once (16) is solved. From (15), it is apparent that when $r(t) = i$, $x(t) = -\alpha_i(t)$ is a favorable state error. It is here that the return function V is minimized, and it is here that the actuating signal is equal to zero. To provide a rationale for the role of α_i, consider the solar receiver. Suppose for simplicity that $S = \{1,2\}$ with

$$r(t) = \begin{cases} 1 & \text{clear sky} \\ 2 & \text{dense cloud.} \end{cases} \tag{19}$$

When moving from clear to cloudy conditions, some of the flow rates, pressures, etc., will have significant errors. The regulator given by (14) will attempt to remove these errors using (9) as a penalty function. Still, the controller must be cognizant of the fact that it will eventually return to clear sky operating conditions. It must simultaneously satisfy the exigencies imposed by the instantaneous rate of cost increase in mode 2 and by the desire that the system be in a desirable dynamic state if a modal transition should occur. Consequently, a state error of zero in mode 2 may not be desirable if a shift to mode 1 is envisioned momentarily with the concomitant jump in error.

It is interesting to note that the bias in $\boldsymbol{u}$ depends only on the expected value of the jump in state error. Indeed, if $\hat{\lambda}_{ij} \equiv 0$

$$\alpha_i(t) \equiv 0$$

$$\dot{\beta}_i = -\sum_j q_{ij}\beta_j + \sum_j q_{ij}\mathrm{tr}(P_j\Gamma_{ij}); \quad i \in S.$$

Thus, if the mean jump in x is zero for all modal variations, then there is no bias in the regulator, and the constant term in V is given by a linear differential driven by the variance of λ.

Another special case is that in which λ has a specific value for each modal transition, i.e.,

$$\mathrm{prob}(\lambda = \hat{\lambda}_{ij}) = 1 \quad \text{if} \quad r(t) = j; \quad r(t^-) = i.$$

The equation for $\{\alpha_i;\ i \in S\}$ remains unchanged, but (18) is slightly simplified;

$$\dot{\beta}_i = -\alpha_i'M\alpha_i - \sum_j q_{ij}\Big[\beta_j + (\alpha_i - \alpha_j - \hat{\lambda}_{ij})'P_j(\alpha_i - \alpha_j - \hat{\lambda}_{ij})\Big]$$

$$\beta_i(t_f) = 0; \quad i \in S.$$

An Example

To gain insight into the transient behavior of the LQ regulator, consider the control of the steam temperature at the outlet of a single boiler panel. Fig. 2 shows a simplified functional block diagram of the control loop used at CRTF as described in [9] and [10]. To clarify the role of the various blocks and feedback links, a qualitative description of the operation of the closed loop system would be helpful. A panel of the receiver is shown by the two blocks which relate W (feedwater flow rate), T_s (outlet steam temperature), and T_m (metal temperature). Changes in feedwater flow rate act to change both T_m and T_s, and the input heat flow rate acts directly on T_m. In the absence of exogenous disturbances, the feedwater flow rate should be proportional to the incident heat flow rate in order that a fixed value for the outlet steam temperature be maintained. Hence, the output of the proportional path from the heat flux sensors on the panel to W is labeled as the nominal flow rate (W_{nom}).

With the nominal flow rate command provided by the forward path, the feedback link can be thought of as generating a trim correction to the flow rate (δW). The outer loop compares the actual steam temperature to a temperature reference (960°F), and generates a difference signal which is used to vary the feedwater flow rate. The compensation gain is made flow rate dependent because the panel dynamics change significantly with flow rate.

There are sensors on the panel to measure both the temperature of the metal structure (T_m) and the incident flux (Q). There are two sets of six individual sensors distributed over the face of the panel that can be used singly or in combination to provide an indication of the value of each of the associated variables. In truth, metal temperature and incident flux are distributed processes, and the suggested measurement of these quantities entails an approximation. For example, a small error in measuring the effective insolation will yield a small bias in W_{nom}. Such errors are rendered negligible in steady state through the use of an integral path in the compensation. Because this is a low frequency effect, these errors of misclassification will not be included in this dynamic analysis.

The controller shown in Fig. 2 uses feedforward compensation for the primary disturbance through the link from Q. The efficacy of the feedforward path could clearly be enhanced by measuring the wind direction and speed of the cloud cover some distance from the collector, and then using this information to minimize cloud induced disturbances. Such a solution to the disturbance problem, while potentially quite useful from the point of view of the dynamic response of the system, is far too expensive to be considered in this application. Unless the performance penalty associated with the myopic regulator proposed here is extreme, further investment in regulator equipment would not be warranted.

The third sensor in the regulator shown in Fig. 2 is the steam temperature thermocouple. In contrast with the panel mounted sensors which respond quickly to changes in their measured variables, the output of the steam temperature sensor is slow to reflect changes in the actual steam temperature. Cost and reliability considerations require that this thermocouple be placed in a protective well within the primary steam pipe. Consequently, the measurement of T_s lags the actual value by a significant amount.

For the reason outlined above, sole use of the outer loop does not provide satisfactory response, particularly on partly cloudy days during which the panel must respond quickly to frequent changes in insolation. To enhance loop stability, it has been found expedient to include the metal temperature loop shown in Fig. 2. The thermocouples used to sense metal temperature operate in a much more expeditious manner than does their steam temperature counterpart because they rely on a metal-to-metal contact.

Because of the separation in response times of the metal and steam temperature thermocouples, the closed loop system can be thought of as consisting of two parts: a high-frequency metal

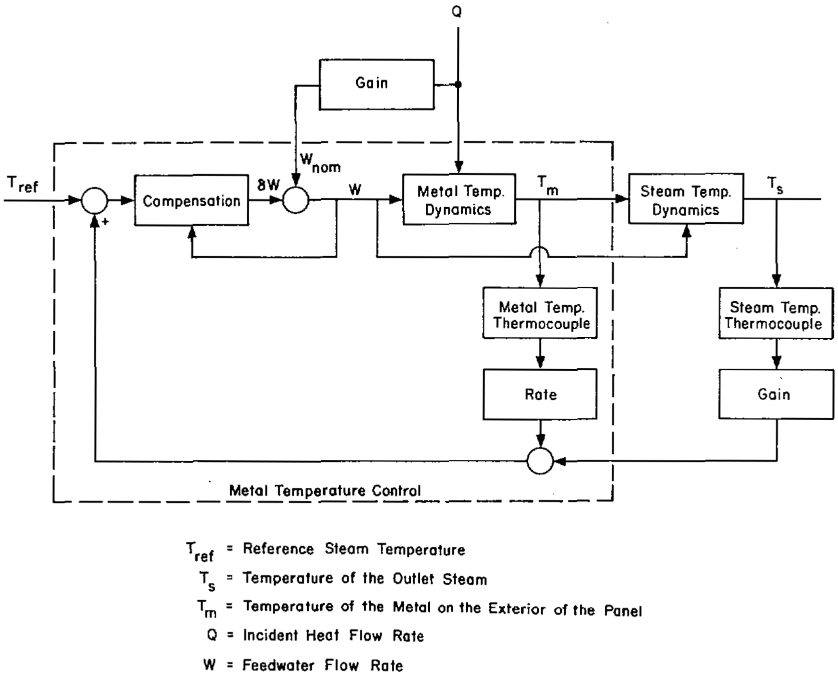

Fig. 2. A simplified steam temperature control loop (CRTF).

temperature inner control loop (shown within the dotted lines on Fig. 2), and a low-frequency steam temperature outer loop. In this section, the response characteristics of this inner loop will be of primary interest. If the changes in insolation are frequent, the gain of the T_s loop is too low to be effective in stabilizing the system. For this reason, a responsive inner loop is required if the overall system is to perform in a satisfactory manner.

To test the utility of the LQ regulator, the behavior of the CRTF metal temperature loop will be compared to the same loop designed using (14). Before making this comparison it would be well to observe that while both design approaches seek to solve the same problem, each has structural advantages and disadvantages. The CRTF design used a pure rate inner loop with time variable dynamics. A constant coefficient system with compensation zero at the origin of the s-plane tends to respond more slowly when the feedback link is closed because the open loop poles are drawn to the right and toward lower frequencies. This application is obviously not that of a constant coefficient system, but the closed loop system should be rendered slower by the rate feedback.

The LQ design obviates this difficulty. This loop has tabular data giving the required nominal temperature and feedwater flow rate for each insolation level. The loop is truly a digital loop in the sense that the equations for implementation are too sophisticated to be easily mechanized on a completely analog system. It makes better use of the available sensors on the panel, and since the actual controller will be implemented on a digital computer, it avoids any difficulties associated with generating the metal temperature rate. On the other hand, the performance of the LQ loop is probably more model dependent than is the CRTF loop.

To make a specific numerical comparison of the two regulators, consider a single receiver panel operating on a partly cloudy day. The panel will be assumed to operate alternatively in two modes [see (19)]. During sunny conditions it has a high feedwater flow rate, and when one of a sequence of clouds with similar opacity passes over the heliostats, the flow rate is reduced. Reference [2] provides linear receiver models of various complexity for a CRTF test panel and for different operating conditions. Restricting attention to the heated section of the panel and the simple situation suggested by (19), the models for two typical conditions can be written

$$\frac{\delta T_m}{\delta W}=\begin{cases} -\dfrac{8.8}{1+s/1.8}\ {}^\circ\mathrm{F/lb/min}; & r=1, \quad Q=1.75\times10^5\ \mathrm{BTU/min}, \quad W_{\mathrm{nom}}=186\ \mathrm{lb/min} \\ -\dfrac{52.8}{1+s/0.36}\ {}^\circ\mathrm{F/lb/min}; & r=2, \quad Q=2.7\times10^4\ \mathrm{BTU/min}, \quad W_{\mathrm{nom}}=30\ \mathrm{lb/min} \end{cases} \tag{20}$$

where δT_m is the metal temperature error and δW is the deviation from nominal flow rate.

Equation (20) gives the system model at continuity points of r. When the insolation changes, the quiescent metal temperature changes in concert, taking on a higher value during periods with unobscured sun than it does during cloudy intervals. Again [2] provides a model for this phenomenon, and for the operating levels given in (20)

$$\hat{\lambda}_{ij}=\begin{cases} 17.2^\circ\mathrm{F} & i=1,\ j=2 \\ -17.2^\circ\mathrm{F} & i=2,\ j=1 \\ 0 & i=j \end{cases}$$

$$\Gamma_{ij}-\hat{\lambda}^2_{ij}=0 \qquad \text{for all } i,j. \tag{21}$$

The model provided by (20) and (21) is a very simple description of the metal temperature dynamics of the boiler. The pole associated with the heated portion of the panel is shown in [2] to be the lowest frequency factor in the transfer function. Transfer function poles associated with the uppermost unheated portion of the panel and the outlet pipe follow with spacing of an octave or so. There is a time delay factor a decade beyond the first pole. For simplicity these effects have not been included in this austere example. Once the exact transfer functions of the panels used in the actual 10 MWe system are better known, the analysis of a more comprehensive panel model will be warranted. Cloud data are available from the Daggett, CA, site of the receiver [11]. For the purposes of this example, consideration will be given to a partly cloudy day in May of 1979. Near noon on the day in question it became partly cloudy. The mean interval during which a cloud obscured the heliostats was approximately 2.3 min. The mean duration of a clear interval was 4.3 min. A simple Markov cloud model matching the sample means is

$$Q=\begin{bmatrix} -0.23 & 0.23 \\ 0.43 & -0.43 \end{bmatrix}. \tag{22}$$

Equation (22) describes an idealized cloud structure; i.e., at time t there is either a dense cloud or no cloud at all. A Markov model can be made to conform to more realistic cloud patterns by introducing more modes into S [see (19)]. In any event a Markov model should be considered to be only a first-order approximation to the actual evolution of the cloud process.

The choice of this time interval also illustrates one of the limitations of this synthesis approach. During the morning of the day in question there had been few clouds, and had the system been operating it would have experienced few transients. The change in cloud conditions was fairly abrupt, and some intervention on the part of the operator would be required to utilize the model given in (22). While additional instrumentation at a distance from the heliostat field or adaptive algorithms within the controller would permit the system to prepare for or accommodate itself to the disturbances, the present system has no such flexibility. Thus, external assistance is required for proper operation. During periods with either few clouds or very many clouds the LQ regulator may provide very limited performance improvement because the transients are either very infrequent or too frequent to follow, respectively.

The performance index to be used in the example is

$$\hat{V}(\delta T_{m_o}, i)=\lim_{T\to\infty} E\left\{\frac{1}{T}\int_0^T\left(0.0355(\delta T_m)^2 + (\delta W)^2\right)dt \,\middle|\, \delta T_m(0)=\delta T_{m_o}, r(0)=i\right\}. \tag{23}$$

The motivation for the form of the criterion function is discussed in [8]. Suffice it to say that since there are an infinite number of state discountinuities in an infinite horizon problem, the integral measure of performance given in (7) is infinite for all control rules. The value of $\hat{V}$ measures the limiting rate of cost growth.

The basic description of the LQ problem is now complete. Equation (20) gives the linear dynamic equation appropriate to each of the two modes of operation. Equation (21) gives the jump condition in nominal metal temperature occasioned by transitions between levels of insolation. Equation (22) provides the parameters of the cloud model, and (23) furnishes a performance index which can be used to compare the response of alternative regulators. It will be assumed that the metal temperature thermocouple is so fast that its dynamics can be neglected.

The LQ regulator is delineated by (14), and is shown pictorially in Fig. 3. Because of the infinite horizon in (23), the stationary solutions to (16) and (17) can be used. Direct substitution yields

$$\delta W_{\text{LQ}}=\begin{cases} 0.1079(\delta T_m+1.26); & r=1 \\ 0.1679(\delta T_m-1.59); & r=2. \end{cases} \tag{24}$$

It was pointed out in [8] that if the equation for β [see (18)] is written as

$$\dot{\beta}_i=-\sum_j q_{ij}\beta_j-\gamma_i$$

then

$$\hat{V}_{\text{LQ}}=\boldsymbol{p}(\infty)'\boldsymbol{\gamma}$$

where $\boldsymbol{p}(\infty)$ is the vector of equilibrium probabilities for r (see [8, eq. (31)]) and $\boldsymbol{\gamma}$ is the vector with ith component γ_i. Again, direct substitution yields

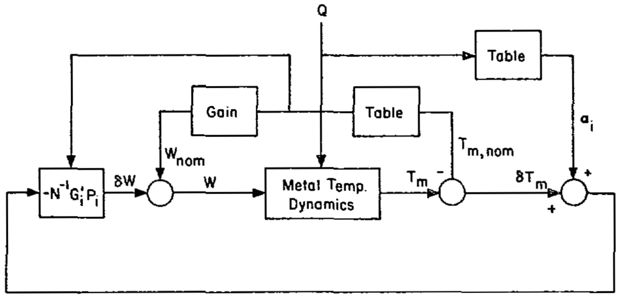

Fig. 3. Block diagram of the LQ metal temperature regulator.

$$\hat{V}_{\text{LQ}}=0.553. \tag{25}$$

The LQ regulator pictures the cloudy mode as being disadvantageous ($P_1=6.8\times10^{-3}<P_2=8.8\times10^{-3}$). Despite the fact that the open loop gain in the cloudy mode is high, the LQ-gain is 60 percent higher during intervals of clouds. This high gain is the regulator's attempt to speed up a rather recalcitrant mode.

When $r=1$, the panel is operating in direct sun. Equation (24) implies that a metal temperature error of approximately $-1.26°$F is attractive to it. The reason for this can be better understood in terms of the relative system dynamics in the two operational modes. Clearly the regulator prefers sunny conditions, but even during the time in which $r(t)=1$, the regulator is apprehensive about the unavoidable transfer to $r(t)=2$. The regulator would like to have as small an initial error as possible after the transition to the cloudy mode while still maintaining a small error in the clear sky mode. The regulator cannot satisfy these exigencies simultaneously, but it can reduce somewhat the operating temperature during intervals of high insolation in order that the initial condition induced by a passing cloud will be somewhat reduced. The bias for $r=2$ has a similar rationale.

Consider next the CRTF regulator. As mentioned earlier, this regulator uses a rate loop that is feedwater flow rate dependent. Reference [9, pp. 4–34] provides the compensation which was employed in this loop during some of the frequency response tests that were used to verify the model given in (20). If the PID compensation is replaced by its midband gain equivalent, the feedback link is given by

$$\delta W=K_{\text{CRTF}}\frac{d}{dt}(\delta T_m) \tag{26}$$

where

$$K_{\text{CRTF}}=\begin{cases} 179.4\ \text{lb/min/°F/min}; & r=1 \\ 72.0\ \text{lb/min/°F/min}; & r=2. \end{cases} \tag{27}$$

It was intended that the gain variation indicated by (27) would partially compensate for the change in forward gain of the panel [see (20)].

The metal temperature loop using (26) is highly damped. Standard methods can be used to deduce the performance characteristics of the CRTF loop

$$\hat{V}_{\text{CRTF}}=2.77. \tag{28}$$

To gain insight into the time response of the steam temperature regulator, a simulation study was initiated. The metal temperature dynamics were those indicated in (20) and (21). The insolation pattern was the same function which led to the estimate of the Q matrix given in (22).

Two types of tests were performed. In the first, the metal loop was tested alone with the LQ-regulator and the CRTF regulator in turn. Both the metal temperature and steam temperature transients were deduced, the latter using again the linearized steam temperature model presented in [2].

In the second test, the steam temperature loop was closed through the steam temperature thermocouple proposed for this application. Structural constraints gave rise to a slow acting temperature sensor

$$\frac{(T_s)_{\text{measured}}}{T_s} = \frac{1}{1+s/2}.$$

As shown in Fig. 2, a signal proportional to the measured steam temperature is compared to the reference steam temperature. This gain is one if $(T_s)_{\text{measured}}$ is given in °F. No attempt has been made to optimize the gains for this second test. Rather, this test gives an indication of how relative inner loop dynamics are reflected in overall system performance.

Fig. 4 shows a plot of metal temperature versus time during a partly cloudy period. There had been a long interval of unobscured sun preceding the arrival of the clouds, and the panel had attained the quiescent operating point associated with $r=1$. With the steam link open, the CRTF loop is highly damped. During the 20 min period of alternating sun and clouds, the metal temperature hardly moved from its initial condition. The response time of the LQ loop is much shorter, and the metal temperature attempts to track the varying insolation level.

The dynamics of the inner loop are reflected in steam temperature. Fig. 5 shows steam temperature versus time for the same cloud function with the steam temperature link still open. Because the CRTF regulator maintains a nearly constant temperature, the outlet steam temperature is correct during sunny conditions. When a cloud causes a reduction in insolation, the nominal feedwater flow rate is reduced in concert. However, the metal temperature is too high for this reduced flow and a temperature error occurs ($T_s \cong 976$°F). The LQ loop follows the cloud conditions with greater fidelity and the T_s error is smaller. The initial transient after each change in insolation is well damped, and the offset level is attained.

Although the steam temperature loop was neglected in the analysis leading to (24), it is of interest to see how the metal temperature dynamics influence steam temperature. Closing the outer loop, as shown in Fig. 2, leads to the results shown in Fig. 6. The CRTF loop is highly oscillatory. This inherent behavior of the loop is accentuated by the cloud dynamics which are near the resonant frequency of the closed loop system. Inopportune transition times in $r(t)$ (see, for example, $t=1.5$ min or $t=15$ min) result in large steam temperature errors.

The LQ regulator has a faster and better damped response. The nonzero value of the quiescent operating point helps to reduce the size of the errors (see $t=8.6$ min). Of course, unfavorable transition times again lead to large transients (see $t=1.5$), but these disturbances are smaller than those associated with the CRTF design. The simulation study tends to confirm the relative measures of performance given by (25) and (28). To gain more confidence in the validity of this comparison, a simulation using a nonlinear model like that given in [2] is required.

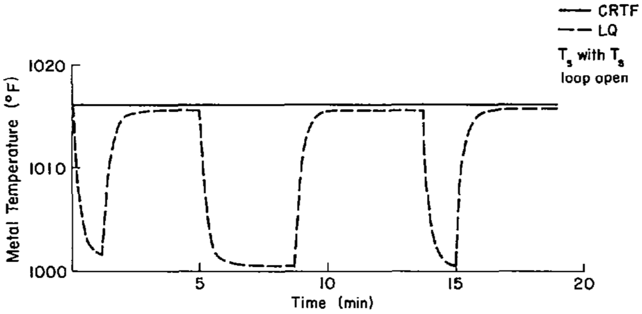

Fig. 4. Metal temperature with steam loop open.

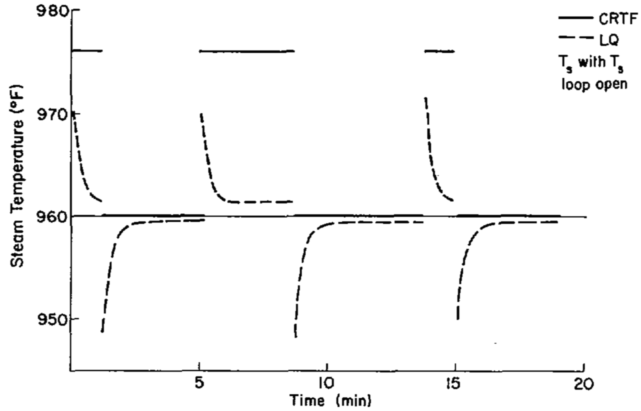

Fig. 5. Steam temperature with steam loop open.

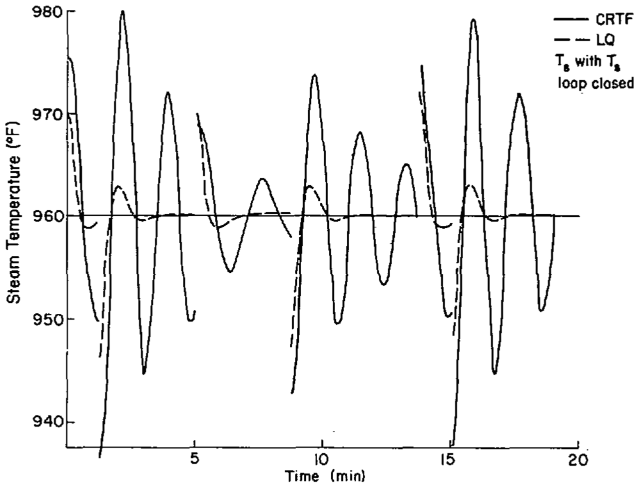

Fig. 6. Steam temperature with steam loop closed.

Conclusions

This paper presents the quadratic-optimal regulator for a linear system subject to state discontinuities and parameter variations at isolated points in time. The design equations are sets of ordinary differential equations which can be integrated in sequence. The regulator gain is the same as that encountered in the study of systems without discontinuities in state error. The equation for the bias terms $\{\alpha_i\}$ has within its forcing term factors of the form $\alpha_j - \alpha_i - \hat{\lambda}_{ij}$. If an $i \to j$ transition occurs at time t, $\alpha_i - \alpha_j$ is the associated change in regulator bias. For this transition, $\hat{\lambda}_{ij}$ is the mean discontinuity in x. Thus, the driving factor is simply the difference between the bias change and the mean state discontinuity. This difference is weighted by the relative penalty of being in mode j compared to that associated with mode i, i.e., by the factor $P_i^{-1}P_j$. Hence, this difference in easy-to-control modes creates little impact on the equation for α_i.

The utility of the regulator given in (14) in applications must be carefully investigated. There are certain approximations used in the analytical development which may not be apropos. For example, the perturbation model is a linearization of the nonlinear system equations (1) about the nominal state associated with a particular value of r. Although the state error after a modal transfer may be large, the regulator given by (14) is expected to act to reduce the error expeditiously. After the initial transient, then, the system state should be in a near neighborhood of the desired level. Of course, if the elements of Q are large, it may happen that the transients are so frequent that the system never has the opportunity to properly damp out one before another occurs. Indeed, if α_i is too large, the steady-state system operation may be displaced from that assumed. In the case of the system temperature regulator discussed earlier, these considerations do not seem to be of great concern, but they may arise in other applications.

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Technical Notes and Correspondence

Decoupled Control of Robots Via Asymptotic Regulators

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Abstract—**A procedure for industrial robot control synthesis based on asymptotic regulator properties is presented. It involves computer linearization of the dynamic manipulator model along a given nominal trajectory and an efficient algorithm for synthesis of a robust linear regulator ensuring the decoupled control of the system.**

I. Introduction

An industrial robot is composed of a multidegree-of-freedom mechanism, actuator subsystems, and a computer based control system including various sensors. Mathematical models of mechanical parts of manipulation systems are nonlinear and increase in complexity with the number of mechanism members [1]. Since the influence of the inertial forces in mechanical parts causes the interaction between links, it results in strong coupling among the actuator subsystems. In order to overcome this problem we shall utilize the two-stage procedure for control synthesis [2] incorporating the stage of nominal dynamics and the stage of perturbed regimes.

The stage of nominal dynamics is associated with feedforward calculation of expected actuator forces (or torques) and the calculation of programmed inputs of the actuators, taking into account the coupling between subsystems. Thus, on the stage of perturbed regimes, where tracking of the nominal trajectory should be provided for, the coupling among subsystems is decreased. The tracking problem, i.e., the problem of achieving a prespecified closeness of the instantaneous states to that at nominal trajectories, is now reduced to the problem of ensuring the *a priori* imposed practical stability of the system around the state space origin. The model of a manipulator system in such a state space is nonstationary, i.e., changes its characteristics along nominal trajectories. Besides, the analytical form of such a model is very complex and differs depending upon the manipulator kinematic chain structure. Thus, application of the optimal regulator synthesis is rather complex and the obtained results are not always applicable. Apart from that, the weighting matrices

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