The approach introduced in the present note answers a fundamental question, namely how to evaluate in a precise way to what extent the state estimation error dynamics can be assigned. On the other side, in Eising's paper only the rate of convergence of the horizontal state estimation error can be freely assigned, while no precise statement is given concerning the dynamics of the vertical state estimation error, which cannot be arbitrarily assigned.

Eising's procedure is based on some restrictive conditions on the matrices entering in the Roesser's model. These conditions need not be satisfied for applying our design procedure, where less restrictive necessary and sufficient conditions are used.

APPENDIX

Theorem 3 (Instability Criterion): Assume that a 2D system Σ is stable with a degree of stability ρ, let

$$\Delta_\Sigma(z_1,\ z_2)=\sum_{i,j} a_{ij}z_1^i z_2^j$$

and n = deg Δ_Σ. Then

$$|a_{ij}|<\binom{n}{i+j}\frac{1}{\rho^{i+j}} \qquad (i,\ j)\neq(0,\ 0). \tag{17}$$

In order to prove the theorem above, we need the following lemmas.

Lemma 1: Assume that the polynomial

$$p(x)=1+p_1x+p_2x^2+\cdots+p_nx^n, \qquad p_k\in\mathbb{C}$$

is devoid of zeros in $\{z\in\mathbb{C}:|z|\le\rho\}$. Then the coefficients satisfy the following bounds

$$|p_k|<\binom{n}{k}\frac{1}{\rho^k}, \qquad k=1,\ 2,\ \cdots,\ n.$$

Lemma 2: Assume that the polynomial

$$p(x)=p_o+p_1x+\cdots+p_{n-1}x^{n-1}, \qquad p_k\in\mathbb{C}$$

satisfies the inequality

$$|p(e^{j\omega})|<M, \qquad \forall\omega\in\mathbb{R}.$$

Then $|p_k|<M$, $k=0,1,\cdots,n-1$. The proofs of Lemmas 1 and 2 are omitted and can be derived from Mansour's paper [5].

Proof of Theorem 3: Since Σ is stable with a degree of stability ρ, there are no roots of $\Delta_\Sigma(z_1, z_2) = 1 + \Sigma_{1ij}a_{ij}z_1^iz_2^j$ in the closed polydisk $\mathcal{P}_\rho$. Let λ denote any complex number of unitary module. The polynomial

$$p(x)\triangleq\Delta_\Sigma(x,\ \lambda x)=1+p_1(\lambda)x+\cdots+p_n(\lambda)x^n$$

is devoid of zeros for any x, with $|x|\le\rho$, so that by Lemma 1, we have

$$|p_k(\lambda)|<\binom{n}{k}\frac{1}{\rho^k}, \qquad k=1,\ 2,\ \cdots,\ n.$$

It is easy to check that

$$p_k(\lambda)=a_{ko}+a_{k-1\,1}\lambda+\cdots+a_{ok}\lambda^k$$

and application of Lemma 2 gives

$$|a_{k-r\,r}|<\binom{n}{k}\frac{1}{\rho^k}, \qquad r=0,\ 1,\ 2,\ \cdots,\ k$$

that is equivalent to (17).

REFERENCES

[1] E. Fornasini and G. Marchesini, "Stability analysis of 2D systems," *IEEE Trans. Circuits Syst.*, vol. CAS-27, pp. 1210–17, 1980.

[2] ——, "Double indexed dynamical systems: State space models and structural properties," *Math. Syst. Theory*, vol. 12, pp. 59–72, 1978.

[3] O. Zariski and P. Samuel, *Commutative Algebra*. New York: Van Nostrand, 1958.

[4] M. Bisiacco, "On the state reconstruction of 2D systems," *Syst. Contr. Lett.*, vol. 5, pp. 347–353, 1985.

[5] M. Mansour, "Instability criteria of linear discrete systems," *Automatica*, vol. 2, pp. 167–78, 1965.

[6] M. Hantus and E. Sontag, "An approach to detectability and observers," in *Algebraic Methods in Linear Systems*, C. I. Byrnes and C. F. Martin, Eds. (AMS Lectures in Appl. Math., vol. 18). 1980, pp. 99–136.

[7] R. Eising, "Observer based 2D filter design," *IEEE Trans. Circuits Syst.*, vol. CAS-30, pp. 760–62, 1983.

On Controllability of Linear Systems with Stochastic Jump Parameters

M. MARITON

Abstract—**This note studies the notion of stochastic ϵ-controllability in probability for linear systems subject to sudden jumps in parameter values. By using a Lyapunov-like approach, sufficient conditions are obtained in terms of a set of coupled Bernoulli equations. The potential use of the results is illustrated by an example.**

I. INTRODUCTION

Fault-tolerance is the new challenge faced by control system theory: aircraft with reduced static stability should carry out their mission with damaged control surfaces; large flexible structures for the space station should be designed to operate with failed actuators/sensors.

Dynamical systems with jumps in parameter values provide a model to study these issues. They are hybrid systems: to the usual continuous state variables, one appends a discrete variable, called the mode, or regime, to describe sudden changes such as failures. Systems of this type first appeared in the works of Krasovskii and Lidskii [1] and Florentin [2] around 1960 and more recently they were proposed to deal with various applications, from solar thermal receivers [3] to electric load modeling [4].

Most of the research focused on jump linear quadratic (JLQ) systems: the state satisfies a linear differential equation and the mode behaves like a Markov chain with finitely many values. An optimal control law is sought which minimizes the mathematical expectation of a quadratic cost functional. For this problem several results were obtained, including the full state feedback JLQ regulator [5], [6] and the output feedback regulator [7].

However, it is surprising to note that such a basic notion as controllability has so far not been defined for this class of systems. It is the purpose of the present note to fill this gap.

The basic assumption here is that both the plant state and the plant mode are perfectly observed.

Jump linear systems are first described and the notion of stochastic ϵ-controllability in probability is introduced. The main result then consists of sufficient conditions for controllability of jump linear systems. They are expressed in terms of the solutions of a set of coupled Bernoulli equations. Finally, a simple example illustrates the way the results may be used.

II. PROBLEM FORMULATION

In this section linear systems with Markovian jumps in parameter values are introduced together with the notion of stochastic controllability to be worked with.

Manuscript received October 23, 1985; revised January 21, 1986.

IEEE Log Number 8608741.

The author is with the Laboratoire des Signaux et Systèmes, C.N.R.S.-E.S.E., Gif-sur-Yvette, France.

0018-9286/86/0700-0680$01.00

The *plant state* $x(t)$, a vector in R^n, satisfies a linear differential equation

$$\dot{x}(t)=A(r(t))x(t)+B(r(t))u(t) \tag{1}$$

where $u(t)$, a vector in R^m, represents control actions. Matrices $A(r(t))$ and $B(r(t))$ of appropriate dimensions have random entries since they depend on the *plant mode* $r(t)$ which is a discrete random variable with values in $S = \{1, 2, \cdots, N\}$. The dynamics of the plant mode are described by the transition probabilities of an underlying Markov chain

$$\text{prob } \{r(t+\Delta)=j|r(t)=i\}=\begin{cases}\pi_{ij}\Delta+o(\Delta) & \text{if } i\neq j\\ 1+\pi_{ii}\Delta+o(\Delta) & \text{if } i=j.\end{cases} \tag{2}$$

It will often be convenient to note the current mode by some index i, i.e., $[A_i, B_i]$ will stand for $[A(r(t), B(r(t))]$ when $r(t) = i$.

The concept of controllability, introduced by Kalman [8], is now well established for deterministic systems. However, in the stochastic case it is more difficult to obtain analogous results. After initial studies in discrete time [9], [10], research developed along two main directions. Local controllability notions, tied to stability, were first studied [11]–[13] while global notions [14], [15], based on corresponding concepts in stochastic process theory [16], appeared more recently.

Here the local notion of Sunahara [11] and Klamka [12], stochastic ϵ-controllability, is considered.

Definition 1: An initial state x_o of the system is said to be stochastically controllable with probability ρ to a target domain with the norm $\sqrt{\epsilon}$, in the time interval $[t_o, t_f]$, if there exists a control $u(t, x)$ such that

$$\text{prob } \{\|x(t_f)\|^2 \geqq \epsilon|x(t_o)=x_o\} \leqq 1-\rho. \tag{3}$$

Definition 2: The controllability is said *complete* if the above condition holds for every initial state $x_o \in R^n$.

III. Main Result

Using a Lyapunov-like approach, sufficient conditions for stochastic controllability in the above sense for the jump linear system (1), (2) are now obtained.

Theorem: The initial state x_o of the system (1), (2) is stochastically ϵ-controllable with probability ρ within the time interval $[t_o, t_f]$ if the following conditions are satisfied:

1) There exist N bounded, symmetric, and positive definite matrices $S_i(t)$ defined on $[t_o, t_f]$ that satisfy the following set of coupled Bernoulli equations:

$$\left.\begin{array}{l}\dfrac{dS_i}{dt}+A_i^TS_i+S_iA_i-S_iB_iB_i^TS_i+\displaystyle\sum_{j=1}^{N}\pi_{ij}S_j=0\\ \text{with } S_i(t_f)=\dfrac{I_n}{\alpha_i},\end{array}\right\} i=1 \text{ to } N \tag{4}$$

where $0 < \alpha_i \ll \epsilon$.

2) The given initial state $x(t_o) = x_o$ satisfies

$$x_o^TS_i(t_o)x_o \leqq (1-\rho)\frac{\epsilon}{\alpha_i}, \qquad i=1 \text{ to } N. \tag{5}$$

Proof: Define the scalar function $V(t, x, r)$ on $[t_o, t_f]$ as

$$V(t, x, i)=x^TS_i(t)x, \qquad \text{when } r(t)=i \tag{6}$$

where the S_i matrices are defined by (4).

This Lyapunov function has bounded continuous first and second derivatives with respect to x and a first derivative with respect to t, $t \neq t_f$. Define the control action as

$$u(t, x, i)=-\frac{1}{2}B_i^TS_i(t)x, \qquad \text{when } r(t)=i. \tag{7}$$

Note that a control action of the form (7) explicitly uses the assumption of plant state and plant mode availability.

Along the trajectories of (1), (2) controlled by (7), the generalized derivative of V is, according to [17],

$$\frac{dE\{V|r(t)=i\}}{dt}=x^T\left(\frac{dS_i}{dt}+A_i^TS_i+S_iA_i-S_iB_iB_i^TS_i+\sum_{j=1}^{N}\pi_{ij}S_j\right)x \tag{8}$$

and condition 1) implies

$$\frac{dE\{V|r(t)=i\}}{dt}=0. \tag{9}$$

Let F_t be the information pattern (σ-algebra) generated by the observations $\{x(s), r(s), t_o \leqq s \leqq t\}$. Then $\{V(t, x, r), F_t\}$ is a nonnegative super-Martingale and, by the Martingale probability inequality [18], it follows that

$$\text{prob } \{\sup V(t, x, r) \geqq \lambda|x(t_o)=x_o\} \leqq \frac{V(t_o, x_o, r(t_o))}{\lambda}. \tag{9}$$

From the terminal condition of (4) and condition 2) this implies

$$\text{prob }\left\{\frac{1}{\alpha_i}\|x(t_f)\|^2 \geqq \lambda|x(t_o)=x_o\right\} \leqq (1-\rho)\frac{\epsilon}{\lambda\alpha_i}. \tag{10}$$

Now choose λ as $\lambda = \epsilon/\alpha_i$ and (10) becomes

$$\text{prob } \{\|x(t_f)\|^2 \geqq \epsilon|x(t_o)=x_o\} \leqq 1-\rho \tag{11}$$

which completes the proof. □

Some remarks may help explain this result.

Remark 1: Condition 2) should be interpreted as defining the domain of controllable initial states for fixed α_i, ϵ, and ρ. In fact, defining the hitting probability as

$$\rho_H=\text{prob } \{\|x(t_f)\|^2 \leqq \epsilon|x(t_o)=x_o\}$$

one can deduce from (5)

$$\rho_H=1-\frac{\alpha_i}{\epsilon}x_o^TS_i(t_o)x_o \qquad \text{when } r(t_o)=i. \tag{12}$$

See the example below for an illustration of this point.

The upper bound given by (3) is significant only if $(1 - \rho)$ is smaller than 1. Hence, the conditions $\alpha_i \ll \epsilon$, $i = 1$ to N, merely serve to ensure that this is satisfied.

Remark 2: Complete controllability is obtained when (5) holds for every x_o. Since this is always possible by choosing a smaller α_i, it is clear that controllability and complete controllability are equivalent for jump linear systems. The proportional feedback form of the control law (7) is then not surprising. As α_i becomes smaller, the gain $B_i^TS_i$ becomes larger: the choice of α_i reflects the usual control accuracy/control energy tradeoff.

Remark 3: The deterministic controllability of each pair $[A_i, B_i]$, $i = 1$ to N, taken separately implies the existence of N bounded, symmetric, and positive definite solutions to the decoupled Riccati equations

$$\left.\begin{array}{l}\dfrac{dS_i}{dt}+A_i^TS_i+S_iA_i-S_iB_iB_i^TS_i+Q_i=0\\ \text{with } S_i(t_f)=0\end{array}\right\} i=1 \text{ to } N \tag{13}$$

for arbitrary positive definite matrices Q_i, $i = 1$ to N.

Replacing (4) by (13), the theorem and its proof also hold: (9) becomes

$$\frac{dE\{V|r(t)=i\}}{dt}=x^T\left(\sum_{j=1}^{N}\pi_{ij}S_j-Q_i\right)x. \tag{14}$$

Next choose Q_i as

$$Q_i=\sum_{\substack{j=1\\ j\neq i}}^{N}\pi_{ij}S_j. \tag{15}$$

Recalling that π_{ii} is nonpositive, (14) and (15) yields

$$\frac{dE\{V|r(t)=i\}}{dt}=\pi_{ii}x^TS_ix\leq 0 \tag{16}$$

and V still is a nonnegative super-Martingale. Hence, *deterministic controllability of each mode implies stochastic controllability* of the overall system (1), (2). *The converse is false.*

IV. Example

To illustrate the use of the above results, a simple example is considered. It is not significant from the point of view of controllability itself, but it allows us complete hand computations of the quantities involved in the above theorem.

Consider a scalar jump linear process with two modes

$$\text{mode 1}\ \ \dot{x}=a_1x+b_1u$$

$$\text{mode 2}\ \ \dot{x}=a_2x+b_2u.$$

The matrix of transition rates is $\Pi = \begin{pmatrix} -\pi_1 & \pi_1 \\ \pi_2 & -\pi_2\end{pmatrix}$. For this system, the coupled Bernoulli equations (4) can be written

$$\begin{cases} \dot{s}_1+2a_1s_1-s_1^2b_1^2-\pi_1s_1+\pi_1s_2=0, & s_1(t_f)=\dfrac{1}{\alpha_1} \\ \dot{s}_2+2a_2s_2-s_2^2b_1^2-\pi_2s_1+\pi_2s_2=0, & s_2(t_f)=\dfrac{1}{\alpha_2}. \end{cases}$$

To obtain a trivial integration, values are assigned to the problem parameters

$$a_1=-1,\ a_2=0,\ b_1=-1,\ b_2=\frac{1}{\sqrt{2}},\ \pi_1=1,\ \pi_2=2$$

$$\alpha_2=\frac{\alpha_1}{2}=\frac{\alpha}{2}.$$

This choice gives

$$s_1(t)=\frac{e^{t-t_f}}{1+\alpha-e^{t-t_f}},\ s_2(t)=2s_1(t).$$

If the initial mode is uniformly distributed (Prob $\{r(t_o)=1\}$ = Prob $\{r(t_o)=2\}=1/2$), the value at $t=t_o=0$ of the Lyapunov function is

$$V_o=\frac{x_o^2}{2}(s_1(o)+s_2(o))=\frac{3}{2}x_o^2\frac{e^{t-t_f}}{1+\alpha-e^{t-t_f}}.$$

So that the minimal value of $(1-\rho)$ is from (12)

$$(1-\rho)_{\min}=\frac{3\alpha x_o^2e^{-t_f}}{2\epsilon(1+\alpha-e^{-t_f})}.$$

Fig. 1 gives a 2D representation of $(1-\rho)_{\min}$ as a function of t_f and α.

As t_f increases, it becomes easier to attain the target and the bound becomes tighter as ρ tends to 1. A smaller α makes it possible to obtain a desired probability level. Of course, very small values of α result in large feedback gains, and this can cause problems in the implementation of the control law (7).

Another way to use the above theorem is illustrated in Fig. 2 in terms of the controllable domain

$$x_{o\,\max}=\left(\frac{2\epsilon(1-\rho)}{3\alpha}\cdot\frac{1+\alpha-e^{-t_f}}{e^{-t_f}}\right)^{1/2}$$

as a function of t_f and ϵ for three values of ρ. Larger values of t_f and ϵ increase the controllable domain, while as the probability of attaining the target comes closer to one the domain shrinks to the point $x_o=0$.

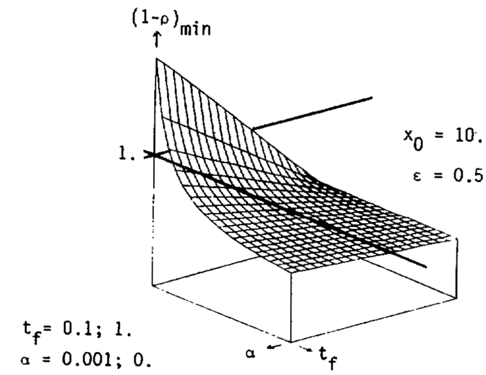

Fig. 1. Influence of t_f (final time) and α (free parameter) on $(1-\rho)_{\min}$.

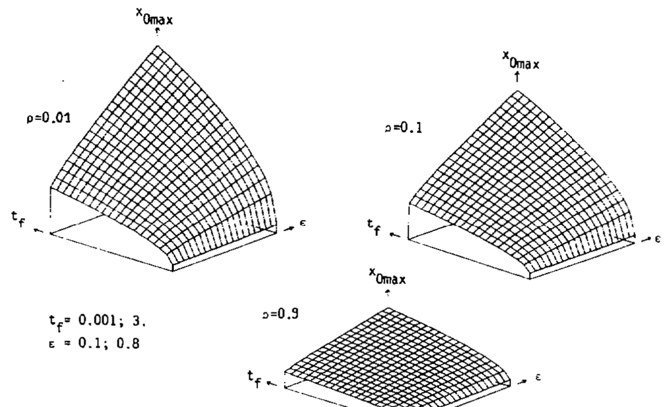

Fig. 2. Influence of t_f (final time) and ϵ (size of the target) on the controllable domain $x_{o\max}$.

V. Conclusion

For a class of linear systems with stochastic jumps in parameter values, a notion of controllability was introduced. Sufficient conditions for ϵ-controllability in probability were obtained through a Lyapunov-like approach. Central to the proposed analysis is a set of coupled Bernoulli equations from which an explicit feedback law is deduced that steers, in probability, the initial state to the target.

It must be noted that the obtained theorem does not provide an algebraic test for controllability as in the deterministic case. Of course, from Remark 4, the familiar rank conditions (rank $(B_i, A_iB_i, \cdots, A_i^{n-1}B_i)=n,\ i=1$ to N), are more demanding sufficient conditions, and are clearly conservative. The global approach to stochastic controllability, e.g., [14] is now under study. It should lead to necessary and sufficient conditions, involving the strong connexity of the Markov chain and an algebraic rank test.

Acknowledgment

The author gratefully acknowledges the reviewer's comments that helped to improve a first version of this note.

References

[1] N. N. Krasovskii and E. A. Lidskii, "Analytical design of controllers in systems with random attributes," *Automat. Remote Contr.*, Parts I–III, pp. 1021–1025, pp. 1141–1146, pp. 1289–1294, 1961.

[2] J. J. Florentin, "Optimal control of continuous time Markov stochastic systems," *J. Electron. Contr.*, vol. 10, pp. 473–488, 1961.

[3] D. D. Sworder and R. O. Rogers, "An LQ-solution to a control problem associated with a solar thermal central receiver," *IEEE Trans. Automat. Contr.*, vol. AC-28, pp. 971–978, Oct. 1983.

[4] R. Malhame and C. Y. Chong, "Electric load model synthesis by diffusion approximation of a high-order hybrid-state stochastic system," *IEEE Trans. Automat. Contr.*, vol. AC-30, pp. 854–860, Sept. 1985.

[5] D. D. Sworder, "Feedback control of a class of linear systems with jump parameters," *IEEE Trans. Automat. Contr.*, vol. AC-14, pp. 9–14, Feb. 1969.

[6] W. M. Wonham, "Random differential equations in control theory," in *Probabilistic Methods in Applied Mathematics, Vol. 2,* A. T. Bharucha-Reid Ed. New York: Academic, 1971, pp. 131–212.

[7] M. Mariton and P. Bertrand, "Output feedback for a class of linear systems with

stochastic jump parameters," *IEEE Trans. Automat. Contr.*, vol. AC-30, pp. 898–900, Sept. 1985.
[8] R. E. Kalman, "Contributions to the theory of optimal control," *Bol. Soc. Math. Mexicana*, vol. 5, pp. 102–119, 1960.
[9] ——, "Control of randomly varying linear dynamical systems," *Symp. Appl. Math.*, vol. 13, New York: Amer. Math. Society, 1962, pp. 287–298.
[10] M. M. Connors, "Controllability of discrete linear random dynamical systems," *SIAM J. Contr.*, vol. 5, no. 2, pp. 183–209, 1967.
[11] Y. Sunahara, T. Kabeuchi, Y. Asada, S. Aihara, and K. Kishino, "On stochastic controllability for nonlinear systems," *IEEE Trans. Automat. Contr.*, vol. AC-19, pp. 49–54, Feb. 1974.
[12] J. Klamka and L. Socha, "Some remarks about stochastic controllability," *IEEE Trans. Automat. Contr.*, vol. AC-22, pp. 880–881, Oct. 1977.
[13] L. Socha, "Some remarks about stochastic controllability for linear composite systems," *IEEE Trans. Automat. Contr.*, vol. AC-29, pp. 60–63, Jan. 1984.
[14] J. Zabczyk, "Controllability of stochastic linear systems," *Syst. Contr. Lett.*, vol. 1, pp. 25–31, July 1981.
[15] M. Ehrhardt and W. Kliemann, "Controllability of linear stochastic systems," *Syst. Contr. Lett.*, vol. 2, pp. 145–153, Oct. 1982.
[16] L. Arnold and W. Kliemann, "Qualitative theory of stochastic systems," in *Probabilistic Analysis and Related Topics*, Vol. 3, A. T. Bharucha-Reid Ed. New York: Academic, 1982, pp. 1–79.
[17] I. I. Kats and N. N. Krasovskii, "On the stability of systems with random parameters," *J. Appl. Math. Mech.*, vol. 24, pp. 1225–1246, 1960.
[18] H. J. Kushner, *Stochastic Stability and Control*. New York: Academic, 1967.

On the Existence of an Optimal Observer in Singular Measurement Systems

Z. J. PALMOR AND Y. HALEVI

***Abstract*—An essential stage in each of the existing methods for singular optimal estimation is to obtain a nonsingular measurement vector which is a linear combination of the original measurement vector and a finite number of its derivatives. A necessary and sufficient condition for the existence of such a nonsingular measurement is presented. If this condition is satisfied, then the existence of an optimal observer is guaranteed. The new condition is shown to be considerably simpler and of much wider applicability than a recently published condition which is just a sufficient one. The new condition requires just a rank check of a single composite matrix consisting of the system and noise matrices. Unlike the other condition, the existence of an optimal estimator is determined here without carrying out the complex procedures for singular optimal estimation.**

I. INTRODUCTION

We consider the system

$$\dot{x}(t) = Ax(t) + Bw(t) \tag{1.1}$$

$$y(t) = Cx(t) + \eta(t) \tag{1.2}$$

$$x \in R^n, \ w \in R^p, \ y \in R^m, \ \eta \in R^m$$

$w(t)$ and $\eta(t)$ are uncorrelated, zero mean white noise processes with intensities I_p and R, respectively. We further assume that (C, A) is detectable. If rank $R = r < m$, then optimal estimation of the state cannot be accomplished via the standard Kalman filter and the measurement is called singular.

Several methods have been suggested for optimal estimation in singular observation cases. The first among these methods was given by Bryson and Johansen [1]. Two other methods are the stochastic observer of Tse and Athans [10] and the limiting procedure of Friedland [2]. While the Bryson method may be applied to the general case, the other two methods share the common assumption that the first derivative of the noise-free measurements leads to a nonsingular measurement equation, and therefore are restricted to the class of systems having this property. See also the recent monograph of O'Reilly [8, ch. 6]. Halevi and Palmor [5] have presented an extended limiting procedure which is applicable to the most general cases.[1] In their method, closed-form expressions for the optimal observer are obtained and various properties of the mechanisms of singular optimal estimation are clarified.

In two recent papers [3], [4] an observer for singular measurement systems was suggested. In what follows, we shall refer to this observer as the Haas observer. The approach taken there is similar to that of the Bryson–Johansen method, especially in its step by step representation [6]. In two aspects, however, the Haas observer deviates from that of Bryson and Johansen. First, not all the noise-free measurements arising from the successive differentiations are utilized to reduce the order of the dynamic observer, which remains of order $n - m + r$. This resembles the observation in [7] concerning the stochastic observer [10]. The second deviation, which is more significant, is that in the Haas observer the output transformation after each differentiation is applied only to the new measurements rather than to the extended vector containing all the accumulated measurements up to and including this stage. In the following it will be shown that this difference restricts the class of systems that the Haas observer can handle.

A common assumption in all the aforementioned singular estimation methods is that differentiations and output transformations of the noise-free component of the measurement will eventually lead to a new measurement vector of order $m - r$, which is fully corrupted by white noise with a nonsingular covariance matrix. This nonsingular measurement is an essential stage in the observers of Bryson and Haas and appears implicitly in all the other methods. In [4] it is pointed out that there are cases for which this assumption is erroneous, and a sufficient condition for the existence of such a nonsingular measurement is presented. (There is some confusion in [4] whether this condition is a necessary one, too. Our results show that the latter is merely a sufficient condition.) We shall refer to this condition in the sequel as the Haas condition.

In this note a necessary and sufficient condition for the existence of a nonsingular measurement vector, which is a linear combination of the original measurement vector and a finite number of its derivatives, is presented. The proposed condition involves just the *a priori* known matrices A, B, C, and R. Its application is considerably simpler than that of the Haas condition which actually requires going through all stages of differentiations and output transformations.

It should be noted that although the existence of a nonsingular measurement is mandatory for the use of the existing estimation methods, it is not a necessary condition for the existence of an optimal estimator. An immediate counterexample is the totally singular case where there are more measurements than stochastic inputs, i.e., $m > p$. In this situation one can never obtain a nonsingular measurement yet a completely accurate estimation may sometimes be available. Such an estimation is certainly optimal, although generally nonunique.

The note is organized as follows. In Section II a necessary and sufficient condition for the existence of a nonsingular measurement is derived. In Section III this condition is compared to the Haas condition. An example in Section IV demonstrates the use of the suggested condition and its advantage over the Haas condition. We summarize and discuss our results in Section V.

II. AN EXISTENCE CONDITION FOR NONSINGULAR MEASUREMENTS

We start with some definitions and results from the theory of rational matrices useful for subsequent development. These results are analogous to the relatively more known theory of polynomial matrices.

Manuscript received November 21, 1985; revised December 2, 1985, December 12, 1985, and March 11, 1986.
The authors are with the Faculty of Mechanical Engineering, Technion—Israel Institute of Technology, Haifa, Israel.
IEEE Log Number 8608742.

[1] It has been brought to our attention by the reviewers that similar results using geometric theory have very recently been presented [9].

0018-9286/86/0700-0683$01.00 © 1986 IEEE