Event-Triggered Resilient Filtering With the Interval Type Uncertainty for Markov Jump Systems

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***Abstract*—The problem of event-triggered resilient filtering for Markov jump systems is investigated in this article. The hidden Markov model is used to characterize asynchronous constraints between the filters and the systems. Gain uncertainties of the resilient filter are the interval type in this article, which is more accurate than the norm-bounded type to model the uncertain phenomenon. The number of linear matrix inequalities constraints can be decreased significantly by separating the vertices of the uncertain interval, so that the difficulty of calculation and calculation time can be reduced. Moreover, the event-triggered scheme is applied to depress the consumption of network resources. In order to find a balance between reducing bandwidth consumed and improving system performance, the threshold parameter is designed as a diagonal matrix in the event-triggered scheme. Utilizing the convex optimization method, the sufficient conditions are derived to guarantee that the filtering error systems are stochastically stable and satisfy the extended dissipation performance. Finally, a single-link robot arm system is delivered to certify the effectiveness and advantages of the proposed method.**

***Index Terms*—Asynchronous filtering, event-triggered scheme, hidden Markov model, interval gain uncertainty, Markov jump systems (MJSs).**

I. Introduction

In realistic systems, the stochastic mode switching commonly occurs due to the sudden change in the external environment and internal parameters or structure. The Markov jump systems (MJSs) are introduced to deal with the above problem [1], [2], [3], [4]. As a prominent tool of modeling jump systems, MJSs have attracted extensive attention in many areas, such as power system [5], communication networks [6], economic models [7], and networks control [8]. Gorynin et al. [9] have employed a Markov switching process to approximate the nonlinear system. In [10], the issues of H_∞ control and filtering for MJSs have been investigated with the piecewise-homogeneous transition probabilities. Besides, a lot of noteworthy studies on control or filtering of MJSs have been published in [11], [12], [13], and [14]. It is worth noting that, most existing researches on control or filtering of MJSs follow the assumption that the mode signals of systems are fully accessible such that the synchronization between the filler/controller modes and the system modes can be guaranteed [15], [16], [17]. However, considering the adverse effects of time delays, data dropouts, and so on, such an ideal assumption is hard to be satisfied in the actual systems. To overcome this shortage, the hidden Markov model has been established to characterize the asynchrony between filters and systems in [18]. In [19], an asynchronous filter has been designed for Markov jump neural networks by adopting a hidden Markov model.

Most of the above results comply with a hypothesis that the filter/controller gains are invariable. However, due to the aging of components, time delays, etc., inaccuracy or uncertainty is unavoidable in the design of filter/controller. In order to ensure reliable control and provide engineers with a safe-tuning margin, it is necessary to design the resilient filter/controller which is insensitive to its gain fluctuation. Thus, a resilient filter with great capability of anti-jamming have been constructed in [20] for discrete-time Markov jump neural networks. In order to deal with the influence of long-time delays, a nonfragile controller has been designed for uncertain nonlinear networked control systems in [21]. Note that the gain uncertainties considered in the study of the above nonfragile problem are all norm-bounded type. However, the interval type of uncertainty introduced in [22] is more precise than the norm-bounded type to illustrate the uncertain gains. In [23], a structured vertex separator of the uncertainty interval has been designed to improve the computation efficiency considerably. Although many effective resilient filters have been designed in [24], [25], and [26], the issue of resilient filter taking account of interval gain uncertainties still need further study, which is one of the primary motivations for this article.

At the same time, the problem of dissipativity [27] and $l_2 - l_\infty$ performance [28] has received considerable attention from researchers. Extended dissipativity, as a more comprehensive performance, was introduced for the first time in [29] to research the filter design for MJSs with time-varying delays. By adjusting the dissipativity parameters, the extended dissipativity can degrade into some special performances, such as dissipativity, H_∞ performance [30], passivity [31], and $l_2 - l_\infty$ performance. The work in [32] has provided the design method of a sliding-mode controller with extended dissipativity. In [33], the extended dissipativity-based

Manuscript received 31 May 2022; revised 29 July 2022; accepted 4 December 2022. Date of publication 19 December 2022; date of current version 22 November 2023. This work was supported in part by the National Natural Science Foundation of China under Grant 61903093, Grant 62121004, and Grant 62141606; in part by the Science and Technology Program of Guangzhou under Grant 202201010337; in part by the Natural Science Foundation of Guangdong Province, China, under Grant 2022A1515010271; and in part by the Key Area Research and Development Program of Guangdong Province under Grant 2021B0101410005. This article was recommended by Associate Editor T. Huang. *(Corresponding author: Hongxia Rao.)*

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Color versions of one or more figures in this article are available at https://doi.org/10.1109/TCYB.2022.3227446.

Digital Object Identifier 10.1109/TCYB.2022.3227446

state estimation issue of MJSs has been studied. Thus, this advanced performance measure will be applied to examine the performance of the event-triggered asynchronous filtering in this article.

In addition, the event-triggered scheme is also the focus of most research at present due to its remarkable ability in saving network resources. It is well known that actual systems are always constrained by limited bandwidth and its information process capability. Different from the time-triggered scheme, which updates signals of the systems during each sampling period, the event-triggered scheme transmits the sampled data only when the triggering condition is satisfied [34], [35], [36], [37]. The event-triggered scheme based on sampled-data has been investigated in [34] for networked systems where the positive minimum interevent time can be ensured. An adaptive event-triggered strategy has been presented in [35] where the threshold parameters are calculated online according to an adaptive law, rather than as a predefined constant. In [37], an event-triggered control system has been modeled as an impulsive system and meanwhile, the minimum interevent times has been extended. However, relevant issues have not been fully discussed, for example, how to deal with the dilemma between reducing bandwidth consumption and improving system performance.

Motivated by the above discussions, this article investigates the issue of event-triggered asynchronous filtering with interval gain uncertainty for MJSs. Finally, a practical example is presented to illustrate the applicability and effectiveness of the proposed method. And, the impact of interval gain uncertainty on dissipation performance is examined in this work. The major contributions are summarized as follows.

1) Extended dissipativity, as a more comprehensive performance, is adopted to investigate the problem of event-triggered asynchronous filtering in this article, which can degrade into other performance, such as strict dissipativity, $l_2 - l_\infty$ performance, etc.
2) Inspired by [23], the parameter uncertainty of the resilient filter is set by an interval type, which is closer to the actual situation than the norm-bounded type. Furthermore, to decrease the computational complexity, an effective approach is introduced to separate the set of uncertain parameters.
3) Different from the traditional event-triggered scheme, a special event-triggered scheme with a threshold matrix is proposed in this article. And, the impact of the threshold on saving network resources and improving system performance is analyzed.

Notations: $\Pr\{\cdot\}$ means the probability and $l_2[0, \infty)$ stands for the space of square-summable vector functions over $[0, \infty)$. $\mathrm{diag}\{\cdots\}$ stands for a block-diagonal matrix. Besides, $\mathcal{E}\{x\}$ stands for expectation of x.

II. Problem Formulation

A. Problem Statement

In this section, the mode of the systems will be built up by the MJSs. Let $\mathcal{M} = \{1, 2, 3, \ldots, M\}$ be a positive integer set, and the values ψ_k ($\psi_k \in \mathcal{M}, k \geq 0$) represent a discrete-time Markov chain. Furthermore, the mode transition probability Π is given as follows:

$$\begin{cases} \Pi = \{\pi_{\mu\nu}\} \quad \forall \mu, \nu \in \mathcal{M} \\ \pi_{\mu\nu} = \Pr\{\psi_{k+1} = \nu | \psi_k = \mu\} \\ \sum_{\nu=1}^{M} \pi_{\mu\nu} = 1 \quad \forall \pi_{\mu\nu} \in [0, 1]. \end{cases} \tag{1}$$

In order to facilitate the formulation, we denote ψ_k, ψ_{k+1} as μ, ν, respectively. Then, the mode of the discrete-time MJSs can be designed as follows:

$$\begin{cases} x(k+1) = A_\mu x(k) + B_{1\mu}\rho(k) \\ z(k) = C_{1\mu}x(k) \\ y(k) = C_{2\mu}x(k) + B_{2\mu}\rho(k) \end{cases} \tag{2}$$

where $x(k) \triangleq [x_1(k), x_2(k), \ldots, x_{n_x}(k)]^T \in \mathbb{R}^{n_x}$is the state of the system, $\rho(k) \in \mathbb{R}^{n_w}$ stands for the external disturbance signal belonging to $l_2(0, \infty)$. $z(k) \in \mathbb{R}^{n_z}$ denotes the regulated output and $y(k) \in \mathbb{R}^{n_y}$ is the measured output. The systems matrices $A_\mu, B_{1\mu}, C_{1\mu}, C_{2\mu}$, and $B_{2\mu}$ are alterant depending on the mode of the systems with appropriate dimensions. When the systems are in a certain mode (for instance, $\mu = 1$), the systems matrices $A_1, B_{11}, B_{21}, C_{11}$, and C_{21} are constant matrices.

B. Event-Triggered Scheme

Before further statement, considering the asynchronization of the mode between the system and the filter, the parameter ϕ_k is introduced to represent the filter mode. And, $\mathcal{N} = \{1, 2, \ldots, N\}$ is a positive integer set of ϕ_k. Accordingly, for a given conditional probability matrix Ξ, ϕ_k has the following forms:

$$\begin{cases} \Xi = \{\xi_{\mu\omega}\} \quad \forall \mu \in \mathcal{M} \quad \forall \omega \in \mathcal{N} \\ \xi_{\mu\omega} = \Pr\{\phi_k = \omega | \psi_k = \mu\} \\ \sum_{\omega=1}^{N} \xi_{\mu\omega} = 1 \quad \forall \xi_{\mu\omega} \in [0, 1] \end{cases} \tag{3}$$

where ω presents the mode of filter ϕ_k.

In this part, by determining whether the measured output $y(k)$ needs to be sent to the filter, the event-triggered scheme can reduce both the transmissions of the data and consumption of network resource. Its trigger function is given as follows:

$$f(\delta(k), \sigma_\omega) = \delta^T(k)\varphi_\omega\delta(k) - y^T(k)\sigma_\omega\varphi_\omega\sigma_\omega y(k) \tag{4}$$

where $\delta(k) \triangleq y(k) - y(l_i)$, $y(k)$ represents the current measured output and $y(l_i)$ is the latest measured output to be transmitted. $\sigma_\omega \triangleq \mathrm{diag}\{\sigma_{1\omega}, \sigma_{2\omega}, \ldots, \sigma_{n_y\omega}\}$ is the threshold matrix with each scalar $\sigma_{i\omega} \in [0, 1]$, and the weighting matrix φ_ω is symmetric and positive-definite matrix.

Next, the measured output will be transmitted once the triggering condition is satisfied

$$f(\delta(k), \sigma_\omega) \geq 0. \tag{5}$$

Correspondingly, the next triggered moment l_{i+1} is decided by the following criteria:

$$l_{i+1} = \inf_k\{k > l_i | f(\delta(k), \sigma_\omega) \geq 0\}. \tag{6}$$

Remark 1: According to the conditional probability matrix and the modes of systems, a hidden Markov chain was obtained to illustrate the filter modes. Different cases of asynchronization can be modeled as various conditional probability matrices. Specifically, the system and the filter are completely synchronized if the conditional probability matrices $\Xi = I$, that is, the closer the conditional probability matrix is to the identity matrix, the higher the degree of synchronization between the system and the filter is.

Remark 2: Different from the transmission strategy in [38], the threshold parameter of the event-triggered scheme is framed as a matrix σ_ω with a diagonal structure, which is used to adjust the transmission frequency of $y(k)$. When the subsystem i is volatile, a smaller threshold element $\sigma_{i\omega}$ can help to improve system performance. Conversely, a larger threshold element $\sigma_{i\omega}$ may effectively reduce the transmission task. It means that a matrix threshold σ_ω, whose elements can be flexibly adjusted according to subsystem requirements, has greater potential to effectively reduce bandwidth occupation while maintaining the desired system performance. In addition, the event-triggered scheme is equivalent to the one in [38] when all elements of the matrix threshold σ_ω take the same value. (e.g., $\sigma_\omega = \varrho I$ with a scalar ϱ).

C. Event-Based Asynchronous Filter

In conformity with the hidden Markov model and the event-triggered scheme proposed in the previous part, it is feasible to describe the filter model as the following structure:

$$\begin{cases} x_F(k+1) = (A_{F\omega} + \Delta A_{F\omega})x_F(k) \\ \quad + (B_{F\omega} + \Delta B_{F\omega})y(l_i) \\ z_F(k) = (C_{F\omega} + \Delta C_{F\omega})x_F(k) \end{cases} \tag{7}$$

where $x_F(k) \triangleq [x_{F1}(k), x_{F2}(k), \ldots, x_{Fn_x}(k)] \in \mathbb{R}^{n_x}$ stands for the filter state vector, and $k \in [l_i, l_{i+1} - 1]$. $z_F(k)$ represents the filter output vector. The filter gains matrices $A_{F\omega}$, $B_{F\omega}$, and $C_{F\omega}$ will be determined later, which are alterant depending on the filter mode with appropriate demission. Furthermore, the uncertain matrices $\Delta A_{F\omega}$, $\Delta B_{F\omega}$, and $\Delta C_{F\omega}$ are interval type with the following form:

$$\begin{aligned} \Delta A_{F\omega} &\triangleq [u_{1ij}]_{n_x \times n_x}, \quad |u_{1ij}| \le u \\ \Delta B_{F\omega} &\triangleq [u_{2ij}]_{n_x \times n_y}, \quad |u_{2ij}| \le u \\ \Delta C_{F\omega} &\triangleq [u_{3ij}]_{n_z \times n_x}, \quad |u_{3ij}| \le u. \end{aligned} \tag{8}$$

For further analysis, let $\alpha_i \in \mathbb{R}^{n_x}$, $\beta_i \in \mathbb{R}^{n_y}$, and $\gamma_i \in \mathbb{R}^{n_z}$ denote the unit column vector in which the subscript means that the ith element is 1 and the others are 0. Accordingly, the above-mentioned uncertain terms can be rewritten as follows:

$$\begin{aligned} \Delta A_{F\omega} &= \sum_{i=1}^{n_x}\sum_{j=1}^{n_x} u_{1ij}\alpha_i\alpha_j^T \\ \Delta B_{F\omega} &= \sum_{i=1}^{n_x}\sum_{j=1}^{n_y} u_{2ij}\alpha_i\beta_j^T \\ \Delta C_{F\omega} &= \sum_{i=1}^{n_z}\sum_{j=1}^{n_x} u_{3ij}\gamma_i\alpha_j^T. \end{aligned} \tag{9}$$

D. Filtering Error System

Integrating system (2) and filter system (7), then the following error system can be obtained easily:

$$\begin{cases} \tilde{x}(k+1) = \tilde{A}_{\mu\omega}\tilde{x}(k) + \tilde{B}_{\mu\omega}\rho(k) - \tilde{E}_{\mu\omega}\delta(k) \\ \tilde{z}(k) = \tilde{C}_{\mu\omega}\tilde{x}(k) \end{cases} \tag{10}$$

where $\tilde{x}(k) = \begin{bmatrix} x^T(k) & x_F^T(k) \end{bmatrix}^T$, $\tilde{z}(k) = z(k) - z_F(k)$, and

$$\begin{aligned} \tilde{A}_{\mu\omega} &= \begin{bmatrix} A_\mu & 0 \\ (B_{F\omega} + \Delta B_{F\omega})C_{2\mu} & A_{F\omega} + \Delta A_{F\omega} \end{bmatrix} \\ \tilde{B}_{\mu\omega} &= \begin{bmatrix} B_{1\mu} \\ (B_{F\omega} + \Delta B_{F\omega})B_{2\mu} \end{bmatrix}, \; \tilde{E}_{\mu\omega} = \begin{bmatrix} 0 \\ B_{F\omega} + \Delta B_{F\omega} \end{bmatrix} \\ \tilde{C}_{\mu\omega} &= \begin{bmatrix} C_{1\mu} & -(C_{F\omega} + \Delta C_{F\omega}) \end{bmatrix}. \end{aligned}$$

E. Preliminaries

In this section, there are some necessary preliminaries and lemmas, which will be used to accomplish the central results of this article.

Definition 1: Without the external disturbance signal $\rho(k)$, the discrete-time system (10) is stochastically stable if the following inequality holds:

$$\mathcal{E}\left\{\sum_{k=0}^{+\infty} \|\tilde{x}(k)\|^2\right\} < \infty. \tag{11}$$

Definition 2: Given matrices $\mathcal{G}_1 = \mathcal{G}_1^T \le 0$, $\mathcal{G}_2$, $\mathcal{G}_3 = \mathcal{G}_3^T > 0$, and $\mathcal{G}_4 = \mathcal{G}_4^T \ge 0$, the filtering error system (7) are extended dissipative under the zero-initial condition when the following criteria satisfy:

$$\begin{aligned} &\sum_{t=0}^{K} \mathcal{E}\left\{\tilde{z}^T(t)\mathcal{G}_1\tilde{z}(t) + 2\tilde{z}^T(t)\mathcal{G}_2\rho(t) + \rho^T(t)\mathcal{G}_3\rho(t)\right\} \\ &\quad \ge \sup_{0 \le k \le K} \mathcal{E}\left\{\tilde{z}^T(k)\mathcal{G}_4\tilde{z}(k)\right\}. \end{aligned} \tag{12}$$

Without loss of generality, there are two assumptions for Definition 2 to facilitate the latter derivation.

Assumption 1: The following equalities hold:

$$\begin{aligned} \mathcal{G}_1 &= \mathcal{G}_1^T = -\left(\mathcal{G}_1^+\right)^2 \le 0, \mathcal{G}_1^+ \ge 0 \\ \mathcal{G}_4 &= \mathcal{G}_4^T = \left(\mathcal{G}_4^+\right)^2 \ge 0, \mathcal{G}_4^+ \ge 0. \end{aligned}$$

Assumption 2: $(\|\mathcal{G}_1\| + \|\mathcal{G}_2\|)\|\mathcal{G}_4\| = 0$.

Remark 3: By tuning the dissipativity parameters, the extended dissipativity can degrade into some common performance indexes, for instance: 1) strictly $(\mathcal{G}_1, \mathcal{G}_2, \mathcal{G}_3)$-$\varpi$-dissipativity when $\mathcal{G}_3 = \bar{\mathcal{G}}_3 - \varpi I$, $\varpi > 0$, and $\mathcal{G}_4 = 0$; 2) H_∞ performance when $\mathcal{G}_1 = -I$, $\mathcal{G}_2 = 0$, $\mathcal{G}_3 = \varpi^2 I$, and $\mathcal{G}_4 = 0$; 3) passivity with $\mathcal{G}_1 = 0$, $\mathcal{G}_2 = I$, $\mathcal{G}_3 = \varpi I$, and $\mathcal{G}_4 = 0$; and 4) $l_2 - l_\infty$ performance when $\mathcal{G}_1 = 0$, $\mathcal{G}_2 = 0$, $\mathcal{G}_3 = \varpi^2 I$, and $\mathcal{G}_4 = I$.

Lemma 1 [23]: Given the constant matrices with appropriate dimensions S, E_1, and E_2, the following three conditions are equivalent:

1)

$$S + E_1U_1E_2 + (E_1U_1E_2)^T < 0$$

where $U_1 \in \mathbb{R}^{l\times l}$ is a diagonal matrix which all the elements $|u_i| \le u$.

2)

$$S + E_1UE_2 + (E_1UE_2)^T < 0$$

where U is a subset of U_1, and the diagonal elements take value in the boundary values of the gain variations $\{-u, u\}$.

3) For a given symmetric matrix $\Upsilon \in R^{2l\times 2l}$, the following inequalities are satisfied:

$$\begin{bmatrix} S & E_1 \\ E_1^T & 0 \end{bmatrix} + \begin{bmatrix} E_2 & 0 \\ 0 & I \end{bmatrix}^T \Upsilon \begin{bmatrix} E_2 & 0 \\ 0 & I \end{bmatrix} < 0 \quad (13)$$

$$\begin{bmatrix} I \\ U \end{bmatrix}^T \Upsilon \begin{bmatrix} I \\ U \end{bmatrix} \ge 0. \quad (14)$$

Lemma 2 [39]: For the positive-definite matrix P_μ and any matrix R, we can easily obtain the following inequality:

$$(P_\mu - R)P_\mu^{-1}(P_\mu - R)^T > 0. \quad (15)$$

And, (15) can be converted into the following inequality:

$$-P_\mu^{-1} < R^{-1}(P_\mu - R - R^T)R^{-T}. \quad (16)$$

III. Main Results

In this part, the asynchronous and resilient filter of the discrete-time systems (2) are devised. By giving some sufficient conditions for linear matrix inequalities (LMIs), we can ensure that the error systems (10) are stochastically stable and satisfy the extended dissipation performance index.

To facilitate the following derivations of the results, we denote:

$$S_0 \triangleq \begin{bmatrix} -\bar{P}_\mu^{-1} & 0 & \Omega_{1\mu\omega} \\ * & -I & \mathcal{G}_1^+\Omega_{2\mu\omega} \\ * & * & Q_3 \end{bmatrix}$$

with $\Omega_{1\mu\omega} = [\tilde{A}_{\mu\omega} \quad -\tilde{E}_{\mu\omega} \quad \tilde{B}_{\mu\omega}]$, $\Omega_{2\mu\omega} = [\tilde{C}_{\mu\omega} \quad 0 \quad 0]$, and

$$\bar{P}_\mu \triangleq \sum_{\nu=1}^{M} \pi_{\mu\nu}P_\nu$$

$$Q_3 \triangleq \begin{bmatrix} \Phi_1 - \zeta_{\mu\omega} & 0 & \Phi_2 - \widetilde{C}_{\mu\omega}^T\mathcal{G}_2 \\ * & -\varphi_\omega & 0 \\ * & * & \Phi_3 - \mathcal{G}_3 \end{bmatrix}$$

where $\Phi_1 \triangleq \begin{bmatrix} C_{2\mu}^T\sigma_\omega\varphi_\omega\sigma_\omega C_{2\mu} & 0 \\ 0 & 0 \end{bmatrix}$, $\Phi_2 \triangleq \begin{bmatrix} C_{2\mu}^T\sigma_\omega\varphi_\omega\sigma_\omega B_{2\mu} \\ 0 \end{bmatrix}$, and $\Phi_3 \triangleq B_{2\mu}^T\sigma_\omega\varphi_\omega\sigma_\omega B_{2\mu}$.

Theorem 1: Consider the error systems (10). Let constant matrices $\mathcal{G}_1$, $\mathcal{G}_2$, $\mathcal{G}_3$, and $\mathcal{G}_4$ satisfy the forms in Definition 2. If there exist positive-definite matrices $P_\mu > 0$, $\zeta_{\mu\omega} > 0$ and $\varphi_\omega > 0$, for any $\mu \in \mathcal{M}$ and $\omega \in \mathcal{N}$, such that the following LMIs satisfy:

$$\sum_{\omega=1}^{N} \xi_{\mu\omega}\zeta_{\mu\omega} - P_\mu < 0 \quad (17)$$

$$S_0 < 0 \quad (18)$$

$$\hat{S}_1 = \begin{bmatrix} -I & \mathcal{G}_4^+\tilde{C}_{\mu\omega} \\ * & -P_\mu \end{bmatrix} < 0. \quad (19)$$

Then, the filtering error systems (10) are stochastically stable and satisfy the extended dissipative.

Proof: First of all, a Lyapunov function was construct as the following model:

$$V_\mu(k) = \tilde{x}^T(k)P_\mu\tilde{x} \quad \forall \mu \in \mathcal{M} \quad (20)$$

and define

$$\eta(k) \triangleq [\tilde{x}^T(k)\delta^T(k)\rho^T(k)]^T$$
$$\hat{\eta}(k) \triangleq [\tilde{x}^T(k)\delta^T(k)]^T$$
$$\hat{\Omega}_{1\mu\omega} \triangleq [\tilde{A}_{\mu\omega} - \tilde{E}_{\mu\omega}]^T$$
$$\hat{\Omega}_{2\mu\omega} \triangleq [\tilde{C}_{\mu\omega}0]^T.$$

Given the above, we can prove that the filtering error systems (10) are stochastically stable first. According to Definition 1, in the case of the external disturbance $\rho(k) \equiv 0$ and considering $f(\delta(k), \sigma_\omega) < 0$, it follows that:

$$\begin{aligned}
\mathcal{E}\{\Delta V(k)\} &< \mathcal{E}\{V_\nu(k+1) - V_\mu(k)\} - \mathcal{E}\{f(\delta(k), \sigma_\omega)\} \\
&= \mathcal{E}\left\{\hat{\eta}^T(k)\hat{\Omega}_{1\mu\omega}\bar{P}_\mu\hat{\Omega}_{1\mu\omega}\hat{\eta}(k)\right\} - \mathcal{E}\{\tilde{x}^T(k)P_\mu x(k)\} \\
&\quad - \mathcal{E}\{\delta^T(k)\varphi_\omega\delta(k) - y^T(k)\sigma_\omega\varphi_\omega\sigma_\omega y(k)\} \\
&< \hat{\eta}^T(k)\left[\sum_{\omega=1}^{N}\xi_{\mu\omega}\hat{\Omega}_{1\mu\omega}\bar{P}_\mu\hat{\Omega}_{1\mu\omega}\right]\hat{\eta}(k) \\
&\quad - \tilde{x}^T(k)\left[\sum_{\omega=1}^{N}\xi_{\mu\omega}\zeta_{\mu\omega}\right]x(k) \\
&\quad - \mathcal{E}\{\delta^T(k)\varphi_\omega\delta(k) - y^T(k)\sigma_\omega\varphi_\omega\sigma_\omega y(k)\} \\
&= \hat{\eta}^T(k)\sum_{\omega=1}^{N}\xi_{\mu\omega}\left(\hat{Q}_1 + \hat{Q}_3\right)\hat{\eta}(k)
\end{aligned} \quad (21)$$

where $\hat{Q}_1 \triangleq \hat{\Omega}_{1\mu\omega}^T\bar{P}_\mu\hat{\Omega}_{1\mu\omega}$, $\hat{Q}_3 \triangleq \begin{bmatrix} \Phi_1 - \zeta_{\mu\omega} & 0 \\ * & -\varphi_\omega \end{bmatrix}$, and the last inequality satisfies due to inequality (17).

By applying the Schur complement, one can show the following inequality from the condition (18):

$$\begin{bmatrix} -\bar{P}_\mu^{-1} & \hat{\Omega}_{1\mu\omega} \\ * & \hat{Q}_3 \end{bmatrix} < 0.$$

Further, it is equivalent to $\hat{Q}_1 + \hat{Q}_3 < 0$, which means that $\mathcal{E}\{\Delta V(k)\} < 0$. Thus, there exists a scalar $e > 0$ small enough and satisfies the following constraint:

$$\mathcal{E}\{\Delta V(k)\} = -e\mathcal{E}\left\{\|\tilde{x}(k)\|^2\right\}. \quad (22)$$

Next, summing up the left and right sides of (22) from 0 to $K = +\infty$, and by sorting, one can hold that

$$\begin{aligned}
\mathcal{E}\left\{\sum_{k=0}^{K}\|\tilde{x}(k)\|^2\right\} &= \frac{1}{e}\mathcal{E}\{V_{\psi_0}(0) - V_{\psi_{K+1}}(K+1)\} \\
&< \frac{1}{e}V_{\psi_0}(0) < +\infty
\end{aligned} \quad (23)$$

and with Definition 1, it is proved that the systems (10) are stochastically stable.

Second, we will prove the extended dissipative performance of the systems (10). To facilitate our analysis, we denote that

$$Z(k) \triangleq \tilde{z}^T(k)\mathcal{G}_1\tilde{z}(k) + 2\tilde{z}^T(k)\mathcal{G}_2\rho(k) + \rho^T(k)\mathcal{G}_3\rho(k) = \eta^T(k)\begin{bmatrix} \widetilde{C}_{\mu\omega}^T\mathcal{G}_1\widetilde{C}_{\mu\omega} & 0 & \widetilde{C}_{\mu\omega}^T\mathcal{G}_2 \\ * & 0 & 0 \\ * & * & \mathcal{G}_3 \end{bmatrix}\eta(k). \tag{24}$$

Similar to the inequality (21), it follows that with (24):

$$\mathcal{E}\{\Delta V(k) - f(\delta(k), \sigma_\omega) - Z(k)\} < \eta^T(k)\sum_{\omega=1}^{N}\xi_{\mu\omega}(Q_1 - Q_2 + Q_3)\eta(k) \tag{25}$$

where

$$Q_1 \triangleq \Omega_{1\mu\omega}^T\bar{P}_\mu\Omega_{1\mu\omega}, \quad Q_2 \triangleq \Omega_{2\mu\omega}^T\mathcal{G}_1\Omega_{2\mu\omega}$$
$$Q_3 \triangleq \begin{bmatrix} \Phi_1 - \zeta_{\mu\omega} & 0 & \Phi_2 - \tilde{C}_{\mu\omega}\mathcal{G}_2 \\ * & -\varphi_\omega & 0 \\ * & * & \Phi_3 - \mathcal{G}_3 \end{bmatrix}.$$

According to the Schur complement, it is not difficult to conclude that $Q_1 - Q_2 + Q_3 < 0$ from the condition (18), which means that $\mathcal{E}\{\Delta V(k)\} < \mathcal{E}\{Z(k)\}$, then summing up both sides from 0 to $k-1$, we can obtain

$$\mathcal{E}\{V_\mu(k) - V_{\psi_0}(0)\} < \sum_{t=0}^{k-1}\mathcal{E}\{Z(k)\}. \tag{26}$$

Based on the zero-initial condition and the definition of the Lyapunov function, one can obtain easily

$$\mathcal{E}\{V_\mu(k)\} < \sum_{t=0}^{k-1}\mathcal{E}\{Z(k)\}. \tag{27}$$

Hence, by combining (19) and (27), it yields that

$$\begin{aligned}\mathcal{E}\{\tilde{z}^T(k)\ \mathcal{G}_4\tilde{z}(k)\} &= \mathcal{E}\{\tilde{x}^T(k)\widetilde{C}_{\mu\omega}^T\mathcal{G}_4\widetilde{C}_{\mu\omega}\tilde{x}(k)\} \\ &< \mathcal{E}\{\tilde{x}^T(k)P_\mu\tilde{x}(k)\} \\ &< \sum_{t=0}^{k-1}\mathcal{E}\{Z(t)\}.\end{aligned} \tag{28}$$

According to Definition 2, our purpose is to derive the following condition holds for any $\mathcal{G}_1$, $\mathcal{G}_2$, $\mathcal{G}_3$, $\mathcal{G}_4$:

$$\sup_{0<k\leq K}\mathcal{E}\{\tilde{z}^T(k)\ \mathcal{G}_4\tilde{z}(k)\} < \sum_{t=0}^{K}\mathcal{E}\{Z(t)\}. \tag{29}$$

To this end, we consider the following two cases.

When $\mathcal{G}_4 = 0$, it is obvious that $\sum_{t=0}^{k-1}\mathcal{E}\{Z(k)\} > 0$ from (28). Further, $\sum_{t=0}^{K}\mathcal{E}\{Z(k)\} > 0$ holds for all $K > 0$, so that (29) holds.

When $\mathcal{G}_4 \neq 0$, according to Definition 2, $\mathcal{G}_1 = \mathcal{G}_2 = 0$ and $\mathcal{G}_3 > 0$, it means that $Z(t) = \rho^T(t)\mathcal{G}_3\rho(t) \geq 0$. Then from (28), the following inequality holds for any $K \geq k$:

$$\mathcal{E}\{\tilde{z}^T(k)\ \mathcal{G}_4\tilde{z}(k)\} < \sum_{t=0}^{k-1}\mathcal{E}\{Z(t)\} < \sum_{t=0}^{K}\mathcal{E}\{Z(t)\}. \tag{30}$$

Therefore, we can conclude that (29) holds by taking the supremum over $K \geq k \geq 0$.

The above discussions show that the systems (10) are extended dissipative. ■

Up to now, we have proved that the systems (10) are stochastically stable and extended dissipative in Theorem 1. But it is important to note that the inequalities in Theorem 1 cannot be figured out by LMI Control Toolbox. Hence, to obtain classic LMI solvable inequality, the following theorem is given.

Before giving Theorem 2, we denote

$$E_1 = \begin{bmatrix} E_{11} & E_{12} & \cdots & E_{1l}\end{bmatrix}, E_2 = \begin{bmatrix} E_{21}^T & E_{22}^T & \cdots & E_{2l}^T\end{bmatrix}^T$$
$$\hat{E}_1 = \begin{bmatrix} \hat{E}_{11} & \hat{E}_{12} & \cdots & \hat{E}_{1\hat{l}}\end{bmatrix}, \hat{E}_2 = \begin{bmatrix} \hat{E}_{21}^T & \hat{E}_{22}^T & \cdots & \hat{E}_{2\hat{l}}^T\end{bmatrix}^T$$

where $\hat{l} = n_z n_x$, $\hat{E}_{1k} = \begin{bmatrix} -(\mathcal{G}_4^+\gamma_i)^T & 0 & 0\end{bmatrix}^T$, $\hat{E}_{2k} = \begin{bmatrix} 0 & 0 & \alpha_j^T\end{bmatrix}$, for $k = (i-1)n_x + j$, $i = 1, \ldots, n_z$, $j = 1, \ldots, n_x$, similarly, $l = n_x^2 + n_x n_y + n_z n_x$, and E_{1k}, E_{2k} have the following three cases.

1) When $k = (i-1)n_x + j$, for $i, j = 1, \ldots, n_x$
$$E_{1k} = \begin{bmatrix} (R_{3\omega}\alpha_i)^T(R_{3\omega}\alpha_i)^T & 0 & 0 & 0 & 0 & 0\end{bmatrix}^T$$
$$E_{2k} = \begin{bmatrix} 0 & 0 & 0 & 0 & \alpha_j^T & 0 & 0\end{bmatrix}.$$

2) When $k = n_x^2 + (i-1)n_y + j$, for $i = 1, \ldots, n_x$, $j = 1, \ldots, n_y$
$$E_{1k} = \begin{bmatrix} (R_{3\omega}\alpha_i)^T(R_{3\omega}\alpha_i)^T & 0 & 0 & 0 & 0 & 0\end{bmatrix}^T$$
$$E_{2k} = \begin{bmatrix} 0 & 0 & 0 & \beta_j^T C_{2\mu} & 0 & -\beta_j^T & \beta_j^T B_{2\mu}\end{bmatrix}.$$

3) When $k = n_x^2 + n_x n_y + (i-1)n_x + j$, for $i = 1, \ldots, n_z$, $j = 1, \ldots, n_x$
$$E_{1k} = \begin{bmatrix} 0 & 0 & (-\mathcal{G}_1^+\gamma_i)^T & 0 & 0 & 0 & (\mathcal{G}_2^+\gamma_i)^T\end{bmatrix}^T$$
$$E_{2k} = \begin{bmatrix} 0 & 0 & 0 & 0 & \alpha_j^T & 0 & 0\end{bmatrix}.$$

Theorem 2: Given matrices $\mathcal{G}_1$, $\mathcal{G}_2$, $\mathcal{G}_3$, and $\mathcal{G}_4$ satisfy the forms in Definition 2. If there exist positive-definite matrices $P_\mu = \begin{bmatrix} P_{1\mu} & P_{2\mu} \\ * & P_{3\mu}\end{bmatrix} > 0$, $\zeta_{\mu\omega} = \begin{bmatrix} \zeta_{1\mu\omega} & \zeta_{2\mu\omega} \\ * & \zeta_{3\mu\omega}\end{bmatrix} > 0$, $\varphi_\omega > 0$, and $R_{\mu\omega} = \begin{bmatrix} R_{1\mu\omega} & R_{3\omega} \\ R_{2\mu\omega} & R_{3\omega}\end{bmatrix}$, for any $\mu \in \mathcal{M}$ and $\omega \in \mathcal{N}$, such that (17) satisfies and

$$\begin{bmatrix} S_2 & E_1 \\ E_1^T & 0\end{bmatrix} + \begin{bmatrix} E_2 & 0 \\ 0 & I\end{bmatrix}^T\Upsilon\begin{bmatrix} E_2 & 0 \\ 0 & I\end{bmatrix} < 0 \tag{31}$$
$$\begin{bmatrix} \hat{S}_2 & \hat{E}_1 \\ \hat{E}_1^T & 0\end{bmatrix} + \begin{bmatrix} \hat{E}_2 & 0 \\ 0 & I\end{bmatrix}^T\hat{\Upsilon}\begin{bmatrix} \hat{E}_2 & 0 \\ 0 & I\end{bmatrix} < 0 \tag{32}$$
$$\begin{bmatrix} I & U\end{bmatrix}\Upsilon\begin{bmatrix} I & U\end{bmatrix}^T \geq 0 \tag{33}$$
$$\begin{bmatrix} I & \hat{U}\end{bmatrix}\hat{\Upsilon}\begin{bmatrix} I & \hat{U}\end{bmatrix}^T \geq 0 \tag{34}$$

where $\Upsilon \in \mathbb{R}^{2l\times 2l}$ and $\hat{\Upsilon} \in \mathbb{R}^{2\hat{l}\times 2\hat{l}}$ are symmetric matrices, $U = \mathrm{diag}\{u_1, u_2, \ldots, u_l\}$, for all $u_i \in \{-u, u\}$, $\hat{U} = \mathrm{diag}\{\hat{u}_1, \hat{u}_2, \ldots, \hat{u}_l\}$, for all $\hat{u}_i \in \{-u, u\}$ and

$$S_2 = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} \\ * & \Xi_{22} & \Xi_{23} \\ * & * & \Xi_{33}\end{bmatrix}, \hat{S}_2 = \begin{bmatrix} -I & \mathcal{G}_4^+C_{1\mu} & -\mathcal{G}_4^+C_{FR} \\ * & -P_{1\mu} & -P_{2\mu} \\ * & * & -P_{3\mu}\end{bmatrix}$$
$$\Xi_{11} = \begin{bmatrix} \bar{P}_{1\mu} - R_{1\mu\omega} - R_{1\mu\omega}^T & \bar{P}_{2\mu} - R_{3\omega} - R_{2\mu\omega}^T & 0 \\ * & \bar{P}_{3\mu} - R_{3\omega} - R_{3\omega}^T & 0 \\ * & * & -I\end{bmatrix}$$

$$\Xi_{12} = \begin{bmatrix} R_{1\mu\omega}A_\mu + B_{FR}C_{2\mu} & A_{FR} \\ R_{2\mu\omega}A_\mu + B_{FR}C_{2\mu} & A_{FR} \\ \mathcal{G}_1^+ C_{1\mu} & -\mathcal{G}_1^+ C_{FR} \end{bmatrix}$$
$$\Xi_{13} = \begin{bmatrix} -B_{FR} & R_{1\mu\omega}B_{1\mu} + B_{FR}B_{2\mu} \\ -B_{FR} & R_{2\mu\omega}B_{1\mu} + B_{FR}B_{2\mu} \\ 0 & 0 \end{bmatrix}$$
$$\Xi_{22} = \begin{bmatrix} C_{2\mu}^T \sigma_\omega \varphi_\omega \sigma_\omega C_{2\mu} - \zeta_{1\mu\omega} & -\zeta_{2\mu\omega} \\ * & -\zeta_{3\mu\omega} \end{bmatrix}$$
$$\Xi_{23} = \begin{bmatrix} 0 & C_{2\mu}^T \sigma_\omega \varphi_\omega \sigma_\omega B_{2\mu} - C_{1\mu}^T \mathcal{G}_2 \\ 0 & -C_{FR}^T \mathcal{G}_2 \end{bmatrix}$$
$$\Xi_{33} = \begin{bmatrix} -\varphi_\omega & 0 \\ * & B_{2\mu}^T \sigma_\omega \varphi_\omega \sigma_\omega B_{2\mu} - \mathcal{G}_3 \end{bmatrix}.$$

Furthermore, the gains of the filter (7) are determined by

$$A_{F\omega} = R_{3\omega}^{-1} A_{FR}, B_{F\omega} = R_{3\omega}^{-1} B_{FR}, C_{F\omega} = C_{FR}$$
$$\Delta A_{F\omega} = R_{3\omega}^{-1} \Delta A_{FR}, \Delta B_{F\omega} = R_{3\omega}^{-1} \Delta B_{FR}, \Delta C_{F\omega} = \Delta C_{FR}.$$

Then, the filtering error systems (10) are said to be stochastically stable and extended dissipative.

Proof: First, we denote $S_1 = \begin{bmatrix} \Xi_{11} & \Xi_{12}' & \Xi_{13}' \\ * & \Xi_{22} & \Xi_{23}' \\ * & * & \Xi_{33} \end{bmatrix}$ and

$$\Delta S = \begin{bmatrix} \mathbf{0}_{2n_x(2n_x+n_z)} & \Gamma_{12} & \Gamma_{13} \\ \mathbf{0}_{n_z(2n_x+n_z)} & \Gamma_{22} & \mathbf{0}_{n_z(n_y+n_w)} \\ \mathbf{0}_{(2n_x+n_y)(2n_x+n_z)} & \mathbf{0}_{(2n_x+n_y)2n_x} & \mathbf{0}_{(2n_x+n_y)(n_y+n_w)} \\ \mathbf{0}_{n_w(2n_x+n_z)} & \Gamma_{42} & \mathbf{0}_{n_w(n_y+n_y)} \end{bmatrix}$$

where

$$\Xi_{12}' = \begin{bmatrix} R_{1\mu\omega}A_\mu + (B_{FR} + \Delta B_{FR})C_{2\mu} & A_{FR} + \Delta A_{FR} \\ R_{2\mu\omega}A_\mu + (B_{FR} + \Delta B_{FR})C_{2\mu} & A_{FR} + \Delta A_{FR} \\ \mathcal{G}_1^+ C_{1\mu} & -\mathcal{G}_1^+ (C_{FR} + \Delta C_{FR}) \end{bmatrix}$$
$$\Xi_{13}' = \begin{bmatrix} -B_{FR} - \Delta B_{FR} & R_{1\mu\omega}B_{1\mu} + (B_{FR} + \Delta B_{FR})B_{2\mu} \\ -B_{FR} - \Delta B_{FR} & R_{2\mu\omega}B_{1\mu} + (B_{FR} + \Delta B_{FR})B_{2\mu} \\ 0 & 0 \end{bmatrix}$$
$$\Xi_{23}' = \begin{bmatrix} 0 & C_{2\mu}^T \sigma_\omega \varphi_\omega \sigma_\omega B_{2\mu} - C_{1\mu}^T \mathcal{G}_2 \\ 0 & -(C_{FR} + \Delta C_{FR})^T \mathcal{G}_2 \end{bmatrix}$$
$$\Gamma_{12} = \begin{bmatrix} \Delta B_{FR} C_{2\mu} & \Delta A_{FR} \\ \Delta B_{FR} C_{2\mu} & \Delta A_{FR} \end{bmatrix}, \Gamma_{13} = \begin{bmatrix} -\Delta B_{FR} & \Delta B_{FR} B_{2\mu} \\ -\Delta B_{FR} & \Delta B_{FR} B_{2\mu} \end{bmatrix}$$
$$\Gamma_{22} = \begin{bmatrix} 0 & -\mathcal{G}_1^+ \Delta C_{FR} \end{bmatrix}, \Gamma_{42} = \begin{bmatrix} 0 & -\mathcal{G}_2^T \Delta C_{FR} \end{bmatrix}.$$

According to Lemma 1, the following equation can be obtained from (31) and (33):

$$\begin{aligned} S_1 &= S_2 + \Delta S + (\Delta S)^T \\ &= S_2 + \sum_{i=1}^{l} u_i E_{1i} E_{2i} + \left(\sum_{i=1}^{l} u_i E_{1i} E_{2i} \right)^T \\ &= S_2 + E_1 U E_2 + (E_1 U E_2)^T < 0. \end{aligned} \tag{35}$$

Then we denote $\mathbf{R} = \text{diag}\{R_{\mu\omega}^{-1}, I, I, I, I, I\}$, accordingly, by premultiplying and post-multiplying (35) with $\mathbf{R}$ and its transpose, respectively, and applying Lemma 2, the following form can be obtain $S_0 < \mathbf{R} S_1 \mathbf{R}^T < 0$ which means that the condition (18) of Theorem 1 holds.

Similarly, we can obtain condition (19) from the inequalities (32) and (34). And according to the method of Theorem 1, the error systems (10) are said to be stochastically stable and extended dissipative. ∎

Remark 4: In Theorem 2, some sufficient conditions for the filter error systems (10) to be stochastically and extended dissipative have been given, which can be figured out by LMI Control Toolbox. However, according to Lemma 1, all the boundary values of uncertain parameters demand to be checked, so that the number of LMIs included in (33) and (34) are growing exponentially with $l = n_x^2 + n_x n_y + n_z n_x$ and $\hat{l} = n_z n_x$, respectively. Obviously, the results obtained in Theorem 2 are difficult to apply due to the large computational complexity, especially for high-dimensional systems. In order to deal with this problem, Theorem 3 is given.

Before diving into the further deduction, we divide l and $\hat{l}$ parameters into m and $\hat{m}$ segments, respectively. Thus, the sequence numbers $s_0, s_1, \ldots, s_m$ and $\hat{s}_0, \hat{s}_1, \ldots, \hat{s}_{\hat{m}}$ satisfy that $0 = s_0 < s_1 < \ldots < s_m = l$ and $0 = \hat{s}_0 < \hat{s}_1 < \ldots < \hat{s}_{\hat{m}} = \hat{l}$, and the symmetric matrices Υ and $\hat{\Upsilon}$ have the following structure:

$$\Upsilon = \begin{bmatrix} \text{diag}\{\Upsilon_{11}^1, \ldots, \Upsilon_{11}^m\} & \text{diag}\{\Upsilon_{12}^1, \ldots, \Upsilon_{12}^m\} \\ * & \text{diag}\{\Upsilon_{22}^1, \ldots, \Upsilon_{22}^m\} \end{bmatrix} \tag{36}$$
$$\hat{\Upsilon} = \begin{bmatrix} \text{diag}\left\{\hat{\Upsilon}_{11}^1, \ldots, \hat{\Upsilon}_{11}^{\hat{m}}\right\} & \text{diag}\left\{\hat{\Upsilon}_{12}^1, \ldots, \hat{\Upsilon}_{12}^{\hat{m}}\right\} \\ * & \text{diag}\left\{\hat{\Upsilon}_{22}^1, \ldots, \hat{\Upsilon}_{22}^{\hat{m}}\right\} \end{bmatrix} \tag{37}$$

where Υ_{11}^i, Υ_{12}^i, and $\Upsilon_{22}^i \in \mathbb{R}^{(s_i - s_{i-1}) \times (s_i - s_{i-1})}$ for $i \in \{1, 2, \ldots, m\}$, $\hat{\Upsilon}_{11}^j$, $\hat{\Upsilon}_{12}^j$, and $\hat{\Upsilon}_{22}^j \in \mathbb{R}^{(\hat{s}_j - \hat{s}_{j-1}) \times (\hat{s}_j - \hat{s}_{j-1})}$ for $j \in \{1, 2, \ldots, \hat{m}\}$.

Theorem 3: Given matrices $\mathcal{G}_1$, $\mathcal{G}_2$, $\mathcal{G}_3$, and $\mathcal{G}_4$ satisfy the forms in Definition 2. For any $\mu \in \mathcal{M}$ and $\omega \in \mathcal{N}$, if there exist positive-definite matrices $P_\mu = \begin{bmatrix} P_{1\mu} & P_{2\mu} \\ * & P_{3\mu} \end{bmatrix} > 0$, $\zeta_{\mu\omega} = \begin{bmatrix} \zeta_{1\mu\omega} & \zeta_{2\mu\omega} \\ * & \zeta_{3\mu\omega} \end{bmatrix} > 0$, $\varphi_\omega > 0$, $R_{\mu\omega} = \begin{bmatrix} R_{1\mu\omega} & R_{3\omega} \\ R_{2\mu\omega} & R_{3\omega} \end{bmatrix} > 0$, symmetric matrixes Υ with structure (36) and $\hat{\Upsilon}$ with structure (37), such that (17), (31), and (32) satisfy, and

$$\Omega^i = \begin{bmatrix} I \\ U^i \end{bmatrix}^T \begin{bmatrix} \Upsilon_{11}^i & \Upsilon_{12}^i \\ * & \Upsilon_{22}^i \end{bmatrix} \begin{bmatrix} I \\ U^i \end{bmatrix} \geq 0 \quad \forall i \in \{1, 2, \ldots, m\} \tag{38}$$
$$\hat{\Omega}^j = \begin{bmatrix} I \\ \hat{U}^j \end{bmatrix}^T \begin{bmatrix} \hat{\Upsilon}_{11}^j & \hat{\Upsilon}_{12}^j \\ * & \hat{\Upsilon}_{22}^j \end{bmatrix} \begin{bmatrix} I \\ \hat{U}^j \end{bmatrix} \geq 0 \quad \forall j \in \left\{1, 2, \ldots, \hat{m}\right\} \tag{39}$$

where $U^i = \text{diag}\{u_{s_{i-1}+p}, \ldots, u_{s_i}\}$, for all $u_{s_{i-1}+p} \in \{-u, u\}$, $i \in \{1, \ldots, m\}$, $p \in \{1, \ldots, s_i - s_{i-1}\}$, and $\hat{U}^j = \text{diag}\{\hat{u}_{\hat{s}_{j-1}+q}, \ldots, \hat{u}_{\hat{s}_j}\}$, for all $\hat{u}_{\hat{s}_{j-1}+q} \in \{-u, u\}$, $j \in \{1, \ldots, \hat{m}\}$, $q \in \{1, \ldots, \hat{s}_j - \hat{s}_{j-1}\}$.

Then, the filtering error systems (10) are said to be stochastically stable and extended dissipative.

Proof: First, we denote

$$\Omega = \text{diag}\left\{\Omega^1, \Omega^2, \ldots, \Omega^m\right\}, \ \hat{\Omega} = \text{diag}\left\{\hat{\Omega}^1, \hat{\Omega}^1, \ldots, \hat{\Omega}^{\hat{m}}\right\}$$

where

$$\Omega^i = \Upsilon_{11}^i + \Upsilon_{12}^i U^i + \left(\Upsilon_{12}^i U^i\right)^T + U^i \Upsilon_{22}^i U^i$$
$$\hat{\Omega}^j = \hat{\Upsilon}_{11}^j + \hat{\Upsilon}_{12}^j \hat{U}^j + \left(\hat{\Upsilon}_{12}^j \hat{U}^j\right)^T + \hat{U}^j \hat{\Upsilon}_{22}^j \hat{U}^j.$$

Via using the Schur complement, it follows that $\Omega > 0$, $\hat{\Omega} > 0$ from (38) and (39), which are equivalent to (33) and (34). Then according to Theorem 2, the proof is completed. ■

Remark 5: In Theorem 3, by dividing the matrices Υ and $\hat{\Upsilon}$, the numbers of LMIs involved in (38) and (39) are reduced to $L_m = \sum_{i=1}^{m} 2^{s_i - s_{i-1}}$ and $L_{\hat{m}} = \sum_{j=1}^{\hat{m}} 2^{\hat{s}_j - \hat{s}_{j-1}}$, respectively. In this case, the number of LMIs can be effectively reduced by increasing the value of m and $\hat{m}$, and decreasing the max value of $s_i - s_{i-1}$ and $\hat{s}_j - \hat{s}_{j-1}$. Nevertheless, compare with the algorithms obtained in Theorem 2, the results in Theorem 3 may be more conservative due to the structure constraints on matrices Υ and $\hat{\Upsilon}$ [23]. A more detailed discussion is presented in the simulation.

IV. Examples

In order to confirm the advantage and usefulness of the proposed method, in this part, a single-link robot arm system [40] is simulated, and the model can be formulated as follows:

$$\ddot{\theta}(t) = -\frac{gM_\mu L}{J_\mu}\sin(\theta(t)) - \frac{K_\mu}{J_\mu}\dot{\theta}(t) + \frac{1}{J_\mu}u(t) \tag{40}$$

where $\theta(t)$ is the angle of the arm and $u(t)$ is the control signal. And g stands for the acceleration of gravity, M_μ, L, J_μ, and K_μ represent the mass, length, inertia, and damping of the robot arm, respectively.

Then, denote $x_1(t) = \theta(t)$ and $x_2(t) = \dot{\theta}(t)$ and consider the external disturbance $\rho(t)$ and the control signal $u(t) \equiv 0$. To obtain the discrete-time system (2), performing the similar operation in [40]. Corresponding to the system (2), the parameters of system matrices are set as follows:

$$\begin{aligned}
A_1 &= \begin{bmatrix} 1 & 0.1 \\ -0.46 & 0.7 \end{bmatrix}, A_2 = \begin{bmatrix} 1 & 0.1 \\ -0.34 & 0.66 \end{bmatrix} \\
A_3 &= \begin{bmatrix} 1 & 0.1 \\ -0.42 & 0.68 \end{bmatrix}, C_{21} = \begin{bmatrix} 1.4 & 0.25 \\ 1.1 & 0.26 \end{bmatrix} \\
C_{22} &= \begin{bmatrix} 0.9 & 0.31 \\ 0.9 & 0.3 \end{bmatrix}, C_{23} = \begin{bmatrix} 1.3 & 0.3 \\ 1.2 & 0.15 \end{bmatrix} \\
B_{11} &= \begin{bmatrix} -0.03 \\ 0.16 \end{bmatrix}, B_{12} = \begin{bmatrix} -0.01 \\ 0.14 \end{bmatrix}, B_{13} = \begin{bmatrix} -0.02 \\ 0.16 \end{bmatrix} \\
C_{11} &= \begin{bmatrix} 1.1 \\ 2.5 \end{bmatrix}^T, C_{12} = \begin{bmatrix} 1.2 \\ 1.9 \end{bmatrix}^T, C_{13} = \begin{bmatrix} 1.3 \\ 1.5 \end{bmatrix}^T \\
B_{21} &= \begin{bmatrix} -0.03 \\ 0.18 \end{bmatrix}, B_{22} = \begin{bmatrix} -0.07 \\ 0.21 \end{bmatrix}, B_{23} = \begin{bmatrix} -0.06 \\ 0.25 \end{bmatrix}.
\end{aligned}$$

On the other hand, the external disturbance signal $\rho(k)$ is $\rho(k) = e^{-0.05k}\sin(0.1\pi k)$, and the mode transition probability matrix Π is set by

$$\Pi = \begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.3 & 0.6 & 0.1 \\ 0.4 & 0.4 & 0.2 \end{bmatrix}. \tag{41}$$

Furthermore, Table I presents four conditional probability matrices for varying asynchronous degrees. The dissipative coefficients $\mathcal{G}_1$, $\mathcal{G}_2$, $\mathcal{G}_3$, and $\mathcal{G}_4$ are selected as $\mathcal{G}_1 = -0.85$, $\mathcal{G}_2 = -0.25$, $\mathcal{G}_3 = 5 - \varpi$, and $\mathcal{G}_4 = 0$, respectively. Correspondingly, the strictly dissipation performance index ϖ is obtained.

TABLE I
Conditional Probability Matrices Ξ

Case I (fully synchronous)	Case II (high synchronous)
$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0.9 & 0.1 & 0 \\ 0 & 1 & 0 \\ 0.1 & 0.1 & 0.8 \end{bmatrix}$
Case III (middle synchronous)	**Case IV (low synchronous)**
$\begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0.2 & 0.6 & 0.2 \\ 0.1 & 0.1 & 0.8 \end{bmatrix}$	$\begin{bmatrix} 0.4 & 0.3 & 0.3 \\ 0.3 & 0.4 & 0.3 \\ 0.3 & 0.3 & 0.4 \end{bmatrix}$

According to Theorem 3, ten parameters of U $(l = 10)$ are divided into five segments $(m = 5)$, resulting in $s_{i+1} - s_i = 2$, $i = 1, \ldots, m$. And, the threshold matrix σ_ω of the event-triggered scheme has the following form:

$$\sigma_\omega = \mathrm{diag}\{0.60, 0.35\}, \omega = 1, 2, 3. \tag{42}$$

For Case III and $u = 0.02$, the optimal strictly dissipative performance is 3.1168. Via solving the LMIs in Theorem 3, the matrices of filter (7) can be worked out as follows:

$$\begin{aligned}
A_{F1} &= \begin{bmatrix} 0.2892 & -0.3275 \\ -0.5570 & 0.4070 \end{bmatrix}, A_{F2} = \begin{bmatrix} 0.4485 & -0.1057 \\ -0.8094 & 0.0998 \end{bmatrix} \\
A_{F3} &= \begin{bmatrix} 0.4112 & -0.1628 \\ -0.7336 & 0.1835 \end{bmatrix}, B_{F1} = \begin{bmatrix} -0.1467 & -0.3985 \\ 0.8292 & -1.4170 \end{bmatrix} \\
B_{F2} &= \begin{bmatrix} -0.1883 & -0.3486 \\ 0.8227 & -1.4194 \end{bmatrix}, B_{F3} = \begin{bmatrix} -0.1813 & -0.3527 \\ 0.8352 & -1.4353 \end{bmatrix} \\
C_{F1} &= \begin{bmatrix} -1.8315 \\ -2.2508 \end{bmatrix}^T, C_{F2} = \begin{bmatrix} -1.1823 \\ -1.5048 \end{bmatrix}^T, C_{F3} = \begin{bmatrix} -1.1472 \\ -1.6358 \end{bmatrix}^T.
\end{aligned}$$

Considering the above condition, we can obtain the following figures under the initial $x(0) = \begin{bmatrix} -0.1 & 0.1 \end{bmatrix}^T$. Fig. 1 shows the regulated outputs $z(k)$ and filter outputs $z_F(k)$, which represents the filter outputs follow the trace of the regulated outputs effectively by applying the proposed method. To further illustrate the error between $z(k)$ and $z_F(k)$, Fig. 2 is given. Furthermore, two modes chains are shown in Fig. 3, which represents that the modes of the system (2) and the filter (7) are low synchronous. The triggered interval and the triggered instants are displayed in Fig. 4, and one can show that the frequency of information transmission has decreased significantly by employing the event-triggered scheme.

The above analysis preliminarily demonstrates the validity of the proposed approach, thus, a variety of comparisons are used to further certify the effectiveness of the method in this work. First, we compare three uncertain parameters u for various conditional probability matrices. The results are reported in Table II. Obviously, the optimal performance index $\varpi_{\max}$ decreases monotonically as u (or the asynchronous degree) increases. Correspondingly, by changing the values of $\mathcal{G}_1$, $\mathcal{G}_2$, $\mathcal{G}_3$, and $\mathcal{G}_4$ to $\mathcal{G}_1 = 0$, $\mathcal{G}_2 = 0$, $\mathcal{G}_3 = \varpi^2 I$, and $\mathcal{G}_4 = I$, respectively, we can obtain the optimal $l_2 - l_\infty$ performance indices $\varpi_{\min}$, which has been discussed in Remark 3. Then, the similar monotonously of $\varpi_{\min}$ are displayed in Table III.

In addition, a comparative verification is executed to analyze the influence of the matrix Υ with different segmentation

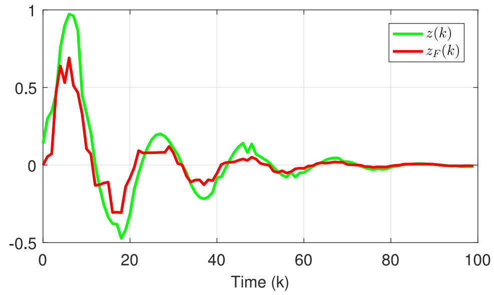

Fig. 1. Regulated output $z(k)$ and filter output $z_F(k)$.

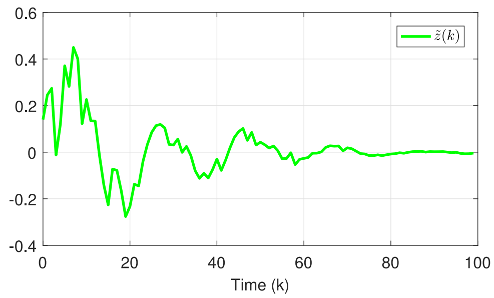

Fig. 2. Estimation error $\tilde{z}(k)$.

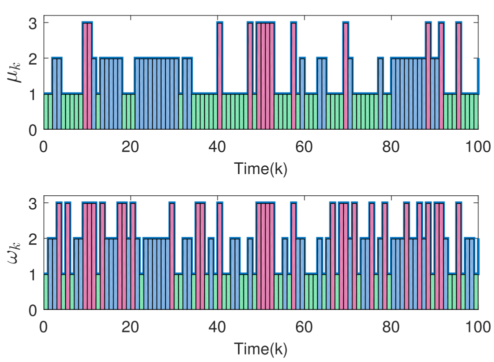

Fig. 3. Chains of system modes and filter modes.

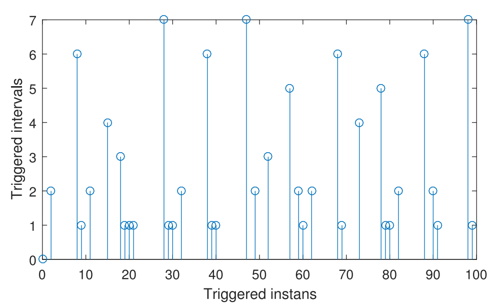

Fig. 4. Triggered intervals and instants.

according to some indices, including the optimal dissipative performance indices $\varpi_{\max}$, the number of LMIs included in (38) and the computing time. As seen from Table IV, the number of LMIs (L_m) decreases significantly as the amount of matrix blocks m increases, and this trend also appears in the evolution of computing time. In other words, by properly constructing the matrix Υ with more blocks, the computational complexity of the proposed algorithm can be effectively

TABLE II
OPTIMAL PERFORMANCE INDICES $\varpi_{\max}$ FOR VARIOUS CASE OF ASYNCHRONY AND VARIOUS DEGREE OF PARAMETERS UNCERTAINTY WHEN $m = 5$

ϖ_{max}	$u = 0$	$u = 0.01$	$u = 0.02$
Case I	3.4332	3.2772	3.2199
Case II	3.4330	3.2699	3.2103
Case III	3.3998	3.1893	3.1168
Case IV	3.3376	3.1060	3.0296

TABLE III
OPTIMAL $l_2 - l_\infty$ PERFORMANCE INDICES $\varpi_{\min}$ FOR VARIOUS CASE OF ASYNCHRONY AND VARIOUS DEGREE OF PARAMETERS UNCERTAINTY WHEN $m = 5$

ϖ_{min}	$u = 0$	$u = 0.01$	$u = 0.02$
Case I	0.3240	0.3837	0.4269
Case II	0.3339	0.3909	0.4325
Case III	0.3465	0.4036	0.4468
Case IV	0.3576	0.4120	0.4531

TABLE IV
COMPARISON BETWEEN DIFFERENT SEGMENTATION OF Υ WITH $u = 0.02$

m	ϖ_{max}	L_m	computing time/(s)
1	3.1196	2^{10}	1009.1970
2	3.1177	2×2^5	12.8592
5	3.1168	5×2^2	4.3729

reduced. Besides, in terms of the strict dissipative performance $\varpi_{\max}$, comparing with the case of $m = 1$, the obtained performance $\varpi_{\max}$ degrades by 0.06% when constructing the matrix Υ with $m = 2$ and decreases by 0.09% with $m = 5$. Another point worth noting is that excessive pursuit of low computational complexity will deteriorate the performance of the resulting system. Fortunately, through properly setting the segmentation dimension m of the matrix Υ, the computational complexity of the resulting algorithm can be reduced effectively while the desired system properties can be maintained.

In order to verify the superiority of the proposed event-triggering scheme, a performance analysis of using different thresholds is carried out and the relevant results are listed in Table V where T_r represents the transmission ratio with the following definition:

$$T_r = \frac{\text{The amount of data transferred}}{\text{The total of data}} \times 100\%.$$

In Table V, an obvious phenomenon is that a better optimal performance $\varpi_{\max}$ can be obtained by using the matrix threshold in (42) rather than adopting the scalar one ($\sigma_\omega = 0.50I$) when maintaining the same transmission ratio. Besides, compare with utilizing the matrix threshold, employing the scalar threshold ($\sigma_\omega = 0.35I$) needs to consume more communication resources to obtain similar optimal performance. It implies that the proposed event-triggered transmission scheme with a matrix threshold has greater potential to deal with the contradiction between effectively reducing bandwidth occupation and improving system performance.

TABLE V
COMPARISON BETWEEN DIFFERENT TRIGGERING THRESHOLD σ_ω

		$\sigma_\omega = 0.50I$	σ_ω in (42)	$\sigma_\omega = 0.35I$
ϖ_{max}	Case I	3.0388	3.2199	3.2589
	Case II	3.0346	3.2103	3.2542
	Case III	2.9553	3.1168	3.1693
	Case IV	2.8654	3.0296	3.0740
T_r	Case I	40.6%	40.9%	45.4%
	Case II	37.1%	38.2%	44.5%
	Case III	39.3%	38.7%	46.1%
	Case IV	40.5%	39.5%	46.1%

V. CONCLUSION

This article has studied the event-triggered asynchronous filtering issue for discrete-time MJSs. To increase the anti-jamming capability, a resilient filter with the interval type uncertainty was introduced in this work. Further, we can divide the vertices of the uncertainty interval to reduce computational complexity. Sufficient conditions are derived to guarantee that the filtering error systems are stochastically stable and extended dissipative. At the end of this article, the practical example has given to prove the usefulness of the proposed approach. Besides, the threshold matrix of the proposed event-triggered strategy should be predetermined based on engineering experience and through repeatedly debugging. How to find an effective way to select the appropriate threshold matrix according to the performance requirement theoretically may be an interesting topic worthy of further discussion.

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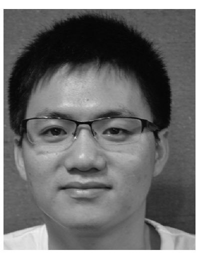

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