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Brief paper

$\mathcal{H}_\infty$ filtering for 2D Markovian jump systems[☆]

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Abstract

This paper is concerned with the problem of $\mathcal{H}_\infty$ filtering for 2D discrete Markovian jump systems. The mathematical model of 2D jump systems is established upon the well-known Roesser model. Our attention is focused on the design of a full-order filter, which guarantees the filtering error system to be mean-square asymptotically stable and has a prescribed $\mathcal{H}_\infty$ disturbance attenuation performance. Sufficient conditions for the existence of a desired filter are established in terms of linear matrix inequalities (LMIs), and the corresponding filter design is cast into a convex optimization problem which can be efficiently solved by using commercially available numerical software. A numerical example is provided to illustrate the effectiveness of the proposed design method.

Keywords: Linear matrix inequality (LMI); Markovian jump linear systems (MJLS); $\mathcal{H}_\infty$ filtering; 2D systems

1. Introduction

Over the past decades, Markovian jump linear systems (MJLS) have received considerable attention. This family of systems is modelled by a set of linear systems with the transitions between the models determined by a Markov chain taking values in a finite set. Applications of this class of systems may be found in many processes, such as target tracking problems, manufactory processes, solar thermal receivers, fault-tolerant systems and economic problems, see for example, Boukas and Yang (1999) and Cao and Lam (2000) and the references therein. From a mathematical point of view, MJLS can be regarded as a special class of stochastic systems with system matrices changed randomly at discrete time points governed by a Markov process, and remain linear time-invariant systems between random jumps. MJLS also belong to the category of hybrid systems with finite discrete operation modes, where every operation mode corresponds to some dynamic system (Xiong, Lam, Gao, & Ho, 2005). Many important results have been reported for this kind of system. For instance, controllability, stabilizability and stability analysis are investigated in Ji and Chizeck (1990), Xie, Ogai, and Inoe (2006) and Yue, Fang, and Won (2003), stabilization and control problems are solved in Aberkane, Christophe Ponsart, and Sauter (2006), Cao and Lam (2000), Shi, Boukas, and Agarwal (1999) and Xiong et al. (2005), filtering problems are studied in Shi et al. (1999), Wang, Lam, and Liu (2004), Xu, Chen, and Lam (2003) and Xu, Chen, and Lam (2004) and model reduction problem has also been reported in Zhang, Huang, and Lam (2003). Unfortunately, the aforementioned results are only concerned with 1D systems. To the best of the authors' knowledge, the corresponding problems on 2D systems have not been fully investigated yet, research in this area should be very important and useful for researchers and designers in this field, which motivate us to carry out the present work.

2D systems' model represents a wide range of practical systems, such as those in image data processing and transmission, thermal processes, gas absorption and water stream heating (Lu & Antoniou, 1992). Therefore, in recent years 2D discrete systems have been extensively studied, and many important results have been available in the literature. To mention a few, the stability problem is investigated in

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Hinamoto (1997) and Lu (1994), the control and filtering problems are considered in Gao, Lam, Wang, and Xu (2004), Gao, Lam, Xu, and Wang (2004), Hoang, Tuan, Nguyen, and Hosoe (2005), Tuan, Apkarian, and Nguyen (2002), Wu, Wang, Gao, and Wang (2007a,b), and the model approximation problem is addressed in Wu, Shi, Gao, and Wang (2006).

Filtering is an important problem in control and signal processing areas (Basin, Perez, & Martinez-Zuniga, 2006; Zhang, Basin, M, & Skliar, 2006). Enlightened by Gao, Lam, Xu et al. (2004), in this paper, we further extend the results obtained for 1D jump systems, to investigate the problems of $\mathcal{H}_\infty$ filtering for 2D systems with Markovian jump parameters. The mathematical model of 2D jump systems is established upon the well-known Roesser model. Our attention is focused on the design of a full-order filter, which guarantees the filtering error system to be mean-square asymptotically stable and has a prescribed $\mathcal{H}_\infty$ disturbance attenuation performance. Sufficient conditions for the existence of such filters are established in terms of linear matrix inequalities (LMIs), and the corresponding filter design is cast into a convex optimization problem which can be efficiently solved by using commercially available numerical software (Boyd, El Ghaoui, Feron, & Balakrishnan, 1994). A numerical example is provided to demonstrate the effectiveness of the proposed filter design procedures.

2. Problem formulation

Consider the following 2D discrete system in Roesser model with Markovian jump parameters:

$$\begin{aligned}
\mathcal{S}: \begin{bmatrix} x^h_{i+1,j} \\ x^v_{i,j+1} \end{bmatrix} &= A(r_{i,j}) \begin{bmatrix} x^h_{i,j} \\ x^v_{i,j} \end{bmatrix} + B(r_{i,j})\omega_{i,j} \\
y_{i,j} &= C(r_{i,j}) \begin{bmatrix} x^h_{i,j} \\ x^v_{i,j} \end{bmatrix} + D(r_{i,j})\omega_{i,j} \\
z_{i,j} &= L(r_{i,j}) \begin{bmatrix} x^h_{i,j} \\ x^v_{i,j} \end{bmatrix}
\end{aligned} \tag{1}$$

where $x^h_{i,j} \in \mathbb{R}^{n_1}$, $x^v_{i,j} \in \mathbb{R}^{n_2}$ represent the horizontal and vertical states, respectively. $\omega_{i,j} \in \mathbb{R}^l$ is the noise signal which belongs to $l_2\{[0,\infty),[0,\infty)\}$. $y_{i,j} \in \mathbb{R}^p$ is the measured output and $z_{i,j} \in \mathbb{R}^q$ is the signal to be estimated. $A(r_{i,j})$, $B(r_{i,j})$, $C(r_{i,j})$, $D(r_{i,j})$ and $L(r_{i,j})$ are real-valued system matrices. These matrices are functions of $r_{i,j}$, which is a discrete-time, discrete-state homogeneous Markovian process on the probability space, takes values in a finite state space $\mathcal{L} \triangleq \{1,\ldots,S\}$, and has the mode transition probabilities

$$\begin{aligned}
\mathrm{pr}\left\{r_{i+1,j} = n \mid r_{i,j} = m\right\} &= \mathrm{pr}\left\{r_{i,j+1} = n \mid r_{i,j} = m\right\} \\
&= p_{mn}
\end{aligned} \tag{2}$$

where $p_{mn} \geq 0$ and, for any $m \in \mathcal{L}$ satisfies $\sum_{n=1}^{S} p_{mn} = 1$.

To simplify the notation, when the system operates at the mth mode, that is, $r_{i,j} = m$, the matrices $A(r_{i,j})$, $B(r_{i,j})$, $C(r_{i,j})$, $D(r_{i,j})$ and $L(r_{i,j})$ are denoted as A_m, B_m, C_m, D_m and L_m respectively. Unless otherwise stated, similar simplification is also applied to other matrices in the following.

Throughout the paper, we denote the system state as $x_{i,j} \triangleq \begin{bmatrix} x^{hT}_{i,j} & x^{vT}_{i,j} \end{bmatrix}^{\mathrm{T}}$. The boundary condition (X_0, R_0) is defined as follows:

$$\begin{aligned}
X_0 &\triangleq \begin{bmatrix} x^{hT}_{0,0} & x^{hT}_{0,1} & x^{hT}_{0,2} & \cdots & x^{vT}_{0,0} & x^{vT}_{1,0} & x^{vT}_{2,0} & \ldots \end{bmatrix}^{\mathrm{T}} \\
R_0 &\triangleq \left\{ r_{0,0}, r_{0,1}, r_{0,2}, \ldots, r_{0,0}, r_{1,0}, r_{2,0}, \ldots \right\}.
\end{aligned}$$

We make the following assumptions.

Assumption 1. The boundary condition is assumed to satisfy

$$\lim_{N\to\infty} \mathbb{E}\left\{ \sum_{k=0}^{N} (|x^h_{0,k}|^2 + |x^v_{k,0}|^2) \right\} < \infty \tag{3}$$

where $\mathbb{E}\{\cdot\}$ denotes the expectation operation.

Assumption 2. System $(\mathcal{S})$ in (1) is mean-square asymptotically stable.

Here, we are interested in designing a full-order $\mathcal{H}_\infty$ filter for $(\mathcal{S})$ in (1) with the following form:

$$\begin{aligned}
\mathcal{F}: \begin{bmatrix} \hat{x}^h_{i+1,j} \\ \hat{x}^v_{i,j+1} \end{bmatrix} &= A_f(r_{i,j}) \begin{bmatrix} \hat{x}^h_{i,j} \\ \hat{x}^v_{i,j} \end{bmatrix} + B_f(r_{i,j}) y_{i,j} \\
\hat{z}_{i,j} &= C_f(r_{i,j}) \begin{bmatrix} \hat{x}^h_{i,j} \\ \hat{x}^v_{i,j} \end{bmatrix}
\end{aligned} \tag{4}$$

where $\hat{x}^h_{i,j} \in \mathbb{R}^{n_1}$, $\hat{x}^v_{i,j} \in \mathbb{R}^{n_2}$ is the filter state vector, $A_f(r_{i,j})$, $B_f(r_{i,j})$ and $C_f(r_{i,j})$ are matrices to be determined. Now, augmenting the model of $(\mathcal{S})$ to include the states of the filter $(\mathcal{F})$, we can obtain the following filtering error system $(\mathcal{E})$:

$$\begin{aligned}
\mathcal{E}: \begin{bmatrix} e^h_{i+1,j} \\ e^v_{i,j+1} \end{bmatrix} &= \bar{A}(r_{i,j}) \begin{bmatrix} e^h_{i,j} \\ e^v_{i,j} \end{bmatrix} + \bar{B}(r_{i,j})\omega_{i,j} \\
\tilde{z}_{i,j} &= \bar{C}(r_{i,j}) \begin{bmatrix} e^h_{i,j} \\ e^v_{i,j} \end{bmatrix}
\end{aligned} \tag{5}$$

where $e^h_{i,j} \triangleq \begin{bmatrix} x^{hT}_{i,j} & \hat{x}^{hT}_{i,j} \end{bmatrix}^{\mathrm{T}}$, $e^v_{i,j} \triangleq \begin{bmatrix} x^{vT}_{i,j} & \hat{x}^{vT}_{i,j} \end{bmatrix}^{\mathrm{T}}$, $\tilde{z}_{i,j} \triangleq z_{i,j} - \hat{z}_{i,j}$ and

$$\begin{aligned}
\tilde{A}(r_{i,j}) &\triangleq \begin{bmatrix} A(r_{i,j}) & 0 \\ B_f(r_{i,j})C(r_{i,j}) & A_f(r_{i,j}) \end{bmatrix}, \\
\tilde{B}(r_{i,j}) &\triangleq \begin{bmatrix} B(r_{i,j}) \\ B_f(r_{i,j})D(r_{i,j}) \end{bmatrix}, \qquad \Gamma \triangleq \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & 0 & I \end{bmatrix}, \\
\tilde{C}(r_{i,j}) &\triangleq \begin{bmatrix} L(r_{i,j}) & -C_f(r_{i,j}) \end{bmatrix}, \\
\bar{A}(r_{i,j}) &\triangleq \Gamma \tilde{A}(r_{i,j}) \Gamma, \\
\bar{B}(r_{i,j}) &\triangleq \Gamma \tilde{B}(r_{i,j}), \qquad \bar{C}(r_{i,j}) \triangleq \tilde{C}(r_{i,j}) \Gamma.
\end{aligned} \tag{6}$$

Before presenting the main objective of this paper, we first introduce the following definitions for the filtering error system $(\mathcal{E})$ in (5), which will be essential for our derivation subsequently.

Definition 1. The filtering error system $(\mathcal{E})$ in (5) with $\omega_{i,j} = 0$ is said to be mean-square asymptotically stable if

$$\lim_{i+j\to\infty} \mathbb{E}\left\{\left|e_{i,j}\right|^2\right\} = 0$$

for every boundary condition (X_0, R_0) satisfying Assumption 1.

Definition 2. Given a scalar $\gamma > 0$, the filtering error system $(\mathcal{E})$ in (5) is said to be mean-square asymptotically stable with an $\mathcal{H}_\infty$ disturbance attenuation level γ, if it is mean-square asymptotically stable and satisfies

$$\|\widetilde{z}\|_E < \gamma \|\omega\|_2$$

for all nonzero $\omega \triangleq \{\omega_{i,j}\} \in l_2\{[0,\infty),[0,\infty)\}$ and under zero initial and boundary conditions, where

$$\|\widetilde{z}\|_E \triangleq \sqrt{\mathbb{E}\left\{\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\left|\widetilde{z}_{i,j}\right|^2\right\}},$$

$$\|\omega\|_2 \triangleq \sqrt{\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\left|\omega_{i,j}\right|^2}.$$

Our objective is to develop full-order filters of the form $(\mathcal{F})$ in (4) such that the filtering error system $(\mathcal{E})$ in (5) is mean-square asymptotically stable with an $\mathcal{H}_\infty$ disturbance attenuation level $\gamma > 0$.

3. Main results

3.1. Filtering analysis

The following theorem is essential for solving the $\mathcal{H}_\infty$ filtering problem formulated in the previous section.

Theorem 1. *The filtering error system $(\mathcal{E})$ in (5) is mean-square asymptotically stable with an $\mathcal{H}_\infty$ disturbance attenuation level $\gamma > 0$ if there exist matrices $Y_m^h > 0$, $Y_m^v > 0$, $m = 1, \dots, S$ such that the following LMIs hold:*

$$\begin{bmatrix} -I & \tilde{C}_m Y_m & 0 & 0 \\ * & -Y_m & 0 & \Psi_1 \\ * & * & -\gamma^2 I & \Psi_2 \\ * & * & * & \Psi_3 \end{bmatrix} < 0, \quad m = 1, \dots, S \tag{7}$$

where $Y_m = \operatorname{diag}\{Y_m^h, Y_m^v\}$, $m = 1, \dots, S$, and

$$\Psi_1 \triangleq \begin{bmatrix} Y_m \tilde{A}_m^{\mathrm{T}} & \cdots & Y_m \tilde{A}_m^{\mathrm{T}} \end{bmatrix}$$

$$\Psi_2 \triangleq \begin{bmatrix} \tilde{B}_m^{\mathrm{T}} & \cdots & \tilde{B}_m^{\mathrm{T}} \end{bmatrix},$$

$$\Psi_3 \triangleq \operatorname{diag}\left\{-p_{m1}^{-1} Y_1, \dots, -p_{mS}^{-1} Y_S\right\}.$$

Proof. First, we establish the stochastic stability of the filtering error system $(\mathcal{E})$ in (5) with $\omega_{i,j} \equiv 0$. It will be shown that the filtering error system $(\mathcal{E})$ in (5) with $\omega_{i,j} \equiv 0$ is mean-square asymptotically stable if the following LMIs hold:

$$\begin{bmatrix} -Y_m & \Psi_1 \\ * & \Psi_3 \end{bmatrix} < 0, \quad m = 1, \dots, S. \tag{8}$$

Define $P_m \triangleq \operatorname{diag}\{P_m^h, P_m^v\} = Y_m^{-1}$, $m = 1, \dots, S$, then by performing a congruence transformation to (8) by $\operatorname{diag}\{Y_m^{-1}, I\}$, (8) is equivalent to

$$\begin{bmatrix} -P_m & \Phi_1 \\ * & \Phi_3 \end{bmatrix} < 0, \quad m = 1, \dots, S \tag{9}$$

where

$$\begin{aligned} \Phi_1 &\triangleq \begin{bmatrix} \tilde{A}_m^{\mathrm{T}} & \cdots & \tilde{A}_m^{\mathrm{T}} \end{bmatrix}, \\ \Phi_3 &\triangleq \operatorname{diag}\left\{-p_{m1}^{-1} P_1^{-1}, \dots, -p_{mS}^{-1} P_S^{-1}\right\}. \end{aligned} \tag{10}$$

By Schur complement (Boyd et al., 1994), (9) is equivalent to

$$\Upsilon_m \triangleq \tilde{A}_m^{\mathrm{T}} \tilde{P}_m \tilde{A}_m - P_m < 0, \quad m = 1, \dots, S \tag{11}$$

where $\tilde{P}_m = \sum_{n=1}^{S} p_{mn} P_n$. Now consider the following index:

$$\begin{aligned} \mathcal{I}_{i,j} \triangleq \mathbb{E}\Bigg\{\Bigg(&\begin{bmatrix} e_{i+1,j}^h \\ e_{i,j+1}^v \end{bmatrix}^{\mathrm{T}} \Gamma \begin{bmatrix} P^h(r_{i+1,j}) & 0 \\ 0 & P^v(r_{i,j+1}) \end{bmatrix} \\ &\times \Gamma \begin{bmatrix} e_{i+1,j}^h \\ e_{i,j+1}^v \end{bmatrix} - e_{i,j}^{\mathrm{T}} \Gamma P\left(r_{i,j}\right) \Gamma e_{i,j}\Bigg)\Bigg| \left(e_{i,j}, r_{i,j} = m\right)\Bigg\} \end{aligned} \tag{12}$$

where $P(r_{i,j}) \triangleq \operatorname{diag}\{P^h(r_{i,j}), P^v(r_{i,j})\}$, which is denoted as P_m when $r_{i,j} = m$. Note that P_m is constant for each m. Then along the solution of the filtering error system $(\mathcal{E})$ in (5) with $\omega_{i,j} \equiv 0$, we have

$$\begin{aligned} \mathcal{I}_{i,j} &= \sum_{n=1}^{S}\left\{\begin{bmatrix} e_{i+1,j}^h \\ e_{i,j+1}^v \end{bmatrix}^{\mathrm{T}} \Gamma \begin{bmatrix} p_{mn} P_n^h & 0 \\ 0 & p_{mn} P_n^v \end{bmatrix} \right. \\ &\quad \left. \Gamma \begin{bmatrix} e_{i+1,j}^h \\ e_{i,j+1}^v \end{bmatrix}\right\} - e_{i,j}^{\mathrm{T}} \Gamma P_m \Gamma e_{i,j} \\ &= e_{i,j}^{\mathrm{T}} \Gamma \left[\tilde{A}^{\mathrm{T}}(r_{i,j}) \tilde{P}_m \tilde{A}(r_{i,j}) - P_m\right] \Gamma e_{i,j} \\ &\triangleq e_{i,j}^{\mathrm{T}} \Gamma \Upsilon_m \Gamma e_{i,j}. \end{aligned}$$

This means that for all $e_{i,j} \neq 0$, we have the expression in Box I where $\alpha \triangleq 1 - \min_{m\in\mathcal{L}}\left(\frac{\lambda_{\min}(-\Upsilon_m)}{\lambda_{\max}(P_m)}\right)$ ($\lambda_{\min}(\cdot)$, $\lambda_{\max}(\cdot)$ denote the minimum and the maximum eigenvalues of a real symmetric matrix respectively). Since $\min_{m\in\mathcal{L}}\left(\frac{\lambda_{\min}(-\Upsilon_m)}{\lambda_{\max}(P_m)}\right) > 0$, we have $\alpha < 1$. Obviously,

$$\begin{aligned} \alpha &\geq \frac{\mathbb{E}\left\{\left(\begin{bmatrix} e_{i+1,j}^h \\ e_{i,j+1}^v \end{bmatrix}^{\mathrm{T}} \Gamma \begin{bmatrix} P^h(r_{i+1,j}) & 0 \\ 0 & P^v(r_{i,j+1}) \end{bmatrix} \Gamma \begin{bmatrix} e_{i+1,j}^h \\ e_{i,j+1}^v \end{bmatrix}\right)\Bigg| \left(e_{i,j}, r_{i,j}\right)\right\}}{e_{i,j}^{\mathrm{T}} \Gamma P\left(r_{i,j}\right) \Gamma e_{i,j}} \\ &> 0 \end{aligned}$$

that is, α belongs to (0, 1) and is independent of $e_{i,j}$. Here, letting $\bar{e}_{i,j} = \Gamma e_{i,j}$, and then we have

$$\begin{aligned} &\mathbb{E}\left\{\left[\bar{e}_{i+1,j}^{hT} P^h(r_{i+1,j}) \bar{e}_{i+1,j}^h + \bar{e}_{i,j+1}^{vT} P^v(r_{i,j+1}) \bar{e}_{i,j+1}^v\right]\right| \\ &\quad \times \left(\bar{e}_{i,j}, r_{i,j}\right)\Big\} \leq \alpha \bar{e}_{i,j}^{\mathrm{T}} P\left(r_{i,j}\right) \bar{e}_{i,j}. \end{aligned}$$

Taking the expectation of both sides, we have

$$\begin{aligned} &\mathbb{E}\left\{\bar{e}_{i+1,j}^{hT} P^h(r_{i+1,j}) \bar{e}_{i+1,j}^h + \bar{e}_{i,j+1}^{vT} P^v(r_{i,j+1}) \bar{e}_{i,j+1}^v\right\} \\ &\quad \leq \alpha \mathbb{E}\left\{\bar{e}_{i,j}^{hT} P^h(r_{i,j}) \bar{e}_{i,j}^h + \bar{e}_{i,j}^{vT} P^v(r_{i,j}) \bar{e}_{i,j}^v\right\}. \end{aligned} \tag{13}$$

$$\frac{\mathbb{E}\left\{\left(\begin{bmatrix} e^h_{i+1,j} \\ e^v_{i,j+1}\end{bmatrix}^{\mathrm{T}} \Gamma \begin{bmatrix} P^h(r_{i+1,j}) & 0 \\ 0 & P^v(r_{i,j+1})\end{bmatrix} \Gamma \begin{bmatrix} e^h_{i+1,j} \\ e^v_{i,j+1}\end{bmatrix} - e^{\mathrm{T}}_{i,j}\Gamma P\left(r_{i,j}\right)\Gamma e_{i,j}\right) \middle| \left(e_{i,j}, r_{i,j}\right)\right\}}{e^{\mathrm{T}}_{i,j}\Gamma P\left(r_{i,j}\right)\Gamma e_{i,j}}$$
$$= -\frac{e^{\mathrm{T}}_{i,j}\Gamma\left[-\Upsilon(r_{i,j})\right]\Gamma e_{i,j}}{e^{\mathrm{T}}_{i,j}\Gamma P(r_{i,j})\Gamma e_{i,j}} \leq -\min_{m\in\mathcal{L}}\left(\frac{\lambda_{\min}(-\Upsilon_m)}{\lambda_{\max}(P_m)}\right) = \alpha - 1$$

Box I.

Upon relationship (13), it can be established that

$$\mathbb{E}\left\{\bar{e}^{hT}_{0,k+1}P^h(r_{0,k+1})\bar{e}^h_{0,k+1}\right\} = \mathbb{E}\left\{\bar{e}^{hT}_{0,k+1}P^h(r_{0,k+1})\bar{e}^h_{0,k+1}\right\}$$
$$\mathbb{E}\left\{\bar{e}^{hT}_{1,k}P^h(r_{1,k})\bar{e}^h_{1,k} + \bar{e}^{vT}_{0,k+1}P^v(r_{0,k+1})\bar{e}^v_{0,k+1}\right\}$$
$$\leq \alpha\mathbb{E}\left\{\bar{e}^{hT}_{0,k}P^h(r_{0,k})\bar{e}^h_{0,k} + \bar{e}^{vT}_{0,k}P^v(r_{0,k})\bar{e}^v_{0,k}\right\}$$
$$\mathbb{E}\left\{\bar{e}^{hT}_{2,k-1}P^h(r_{2,k-1})\bar{e}^h_{2,k-1} + \bar{e}^{vT}_{1,k}P^v(r_{1,k})\bar{e}^v_{1,k}\right\}$$
$$\leq \alpha\mathbb{E}\left\{\bar{e}^{hT}_{1,k-1}P^h(r_{1,k-1})\bar{e}^h_{1,k-1} + \bar{e}^{vT}_{1,k-1}P^v(r_{1,k-1})\bar{e}^v_{1,k-1}\right\}$$
$$\vdots$$
$$\mathbb{E}\left\{\bar{e}^{hT}_{k+1,0}P^h(r_{k+1,0})\bar{e}^h_{k+1,0} + \bar{e}^{vT}_{k,1}P^v(r_{k,1})\bar{e}^v_{k,1}\right\}$$
$$\leq \alpha\mathbb{E}\left\{\bar{e}^{hT}_{k,0}P^h(r_{k,0})\bar{e}^h_{k,0} + \bar{e}^{vT}_{k,0}P^v(r_{k,0})\bar{e}^v_{k,0}\right\}$$
$$\mathbb{E}\left\{\bar{e}^{vT}_{k+1,0}P^v(r_{k+1,0})\bar{e}^v_{k+1,0}\right\} = \mathbb{E}\left\{\bar{e}^{vT}_{k+1,0}P^v(r_{k+1,0})\bar{e}^v_{k+1,0}\right\}.$$

Adding both sides of the above inequality system yields

$$\mathbb{E}\left\{\sum_{j=0}^{k+1}\left[\bar{e}^{hT}_{k+1-j,j}P^h(r_{k+1-j,j})\bar{e}^h_{k+1-j,j} + \bar{e}^{vT}_{k+1-j,j}P^v(r_{k+1-j,j})\bar{e}^v_{k+1-j,j}\right]\right\}$$
$$\leq \alpha\mathbb{E}\left\{\sum_{j=0}^{k}\left[\bar{e}^{hT}_{k-j,j}P^h(r_{k-j,j})\bar{e}^h_{k-j,j} + \bar{e}^{vT}_{k-j,j}P^v(r_{k-j,j})\bar{e}^v_{k-j,j}\right]\right\}$$
$$+ \mathbb{E}\left\{\bar{e}^{hT}_{0,k+1}P^h(r_{0,k+1})\bar{e}^h_{0,k+1} + \bar{e}^{vT}_{k+1,0}P^v(r_{k+1,0})\bar{e}^v_{k+1,0}\right\}.$$

Using this relationship iteratively, we obtain

$$\mathbb{E}\left\{\sum_{j=0}^{k+1}\left[\bar{e}^{hT}_{k+1-j,j}P^h(r_{k+1-j,j})\bar{e}^h_{k+1-j,j} + \bar{e}^{vT}_{k+1-j,j}P^v(r_{k+1-j,j})\bar{e}^v_{k+1-j,j}\right]\right\}$$
$$\leq \alpha^{k+1}\mathbb{E}\left\{\bar{e}^{hT}_{0,0}P^h(r_{0,0})\bar{e}^h_{k-j,j} + \bar{e}^{vT}_{0,0}P^v(r_{0,0})\bar{e}^v_{0,0}\right\}$$
$$+ \mathbb{E}\left\{\sum_{j=0}^{k}\alpha^j\left[\bar{e}^{hT}_{0,k+1-j}P^h(r_{0,k+1-j})\bar{e}^h_{0,k+1-j} + \bar{e}^{vT}_{k+1-j,0}P^v(r_{k+1-j,0})\bar{e}^v_{k+1-j,0}\right]\right\}$$
$$= \mathbb{E}\left\{\sum_{j=0}^{k+1}\alpha^j\left[\bar{e}^{hT}_{0,k+1-j}P^h(r_{0,k+1-j})\bar{e}^h_{0,k+1-j} + \bar{e}^{vT}_{k+1-j,0}P^v(r_{k+1-j,0})\bar{e}^v_{k+1-j,0}\right]\right\}.$$

Therefore, we have

$$\mathbb{E}\left\{\sum_{j=0}^{k+1}\left[\left|\bar{e}^h_{k+1-j,j}\right|^2 + \left|\bar{e}^v_{k+1-j,j}\right|^2\right]\right\} \leq \kappa\sum_{j=0}^{k+1}\alpha^j\mathbb{E}\left\{\left|\bar{e}^h_{0,k+1-j}\right|^2 + \left|\bar{e}^v_{k+1-j,0}\right|^2\right\} \tag{14}$$

where

$$\kappa \triangleq \frac{\max_{m\in\mathcal{L}}(\lambda_{\max}(P_m))}{\min_{m\in\mathcal{L}}(\lambda_{\min}(P_m))}.$$

Now, denote $\mathcal{X}_k \triangleq \sum_{j=0}^{k}\left[\left|\bar{e}^h_{k-j,j}\right|^2 + \left|\bar{e}^v_{k-j,j}\right|^2\right]$, then upon inequality (14) we have

$$\mathbb{E}\{\mathcal{X}_0\} \leq \kappa\mathbb{E}\left\{\left|\bar{e}^h_{0,0}\right|^2 + \left|\bar{e}^v_{0,0}\right|^2\right\}$$
$$\mathbb{E}\{\mathcal{X}_1\} \leq \kappa\left[\alpha\mathbb{E}\left\{\left|\bar{e}^h_{0,0}\right|^2 + \left|\bar{e}^v_{0,0}\right|^2\right\} + \mathbb{E}\left\{\left|\bar{e}^h_{0,1}\right|^2 + \left|\bar{e}^v_{1,0}\right|^2\right\}\right]$$
$$\mathbb{E}\{\mathcal{X}_2\} \leq \kappa\left[\alpha^2\mathbb{E}\left\{\left|\bar{e}^h_{0,0}\right|^2 + \left|\bar{e}^v_{0,0}\right|^2\right\} + \alpha\mathbb{E}\left\{\left|\bar{e}^h_{0,1}\right|^2 + \left|\bar{e}^v_{1,0}\right|^2\right\} + \mathbb{E}\left\{\left|\bar{e}^h_{0,2}\right|^2 + \left|\bar{e}^v_{2,0}\right|^2\right\}\right]$$
$$\vdots$$
$$\mathbb{E}\{\mathcal{X}_N\} \leq \kappa\left[\alpha^N\mathbb{E}\left\{\left|\bar{e}^h_{0,0}\right|^2 + \left|\bar{e}^v_{0,0}\right|^2\right\} + \alpha^{N-1} \times \mathbb{E}\left\{\left|\bar{e}^h_{0,1}\right|^2 + \left|\bar{e}^v_{1,0}\right|^2\right\} + \cdots + \mathbb{E}\left\{\left|\bar{e}^h_{0,N}\right|^2 + \left|\bar{e}^v_{N,0}\right|^2\right\}\right].$$

Adding both sides of the above inequality system yields

$$\sum_{k=0}^{N}\mathbb{E}\{\mathcal{X}_k\} \leq \kappa(1+\alpha+\cdots+\alpha^N)\mathbb{E}\left\{\left|\bar{e}^h_{0,0}\right|^2 + \left|\bar{e}^v_{0,0}\right|^2\right\}$$
$$+\kappa(1+\alpha+\cdots+\alpha^{N-1}) \times \mathbb{E}\left\{\left|\bar{e}^h_{0,1}\right|^2 + \left|\bar{e}^v_{1,0}\right|^2\right\} + \cdots + \kappa\mathbb{E}\left\{\left|\bar{e}^h_{0,N}\right|^2 + \left|\bar{e}^v_{N,0}\right|^2\right\}$$
$$\leq \kappa(1+\alpha+\cdots+\alpha^N)\mathbb{E}\left\{\left|\bar{e}^h_{0,0}\right|^2 + \left|\bar{e}^v_{0,0}\right|^2\right\}$$

$$
\begin{aligned}
&+\kappa(1+\alpha+\cdots+\alpha^N)\\
&\times \mathbb{E}\left\{\left|\bar{e}^h_{0,1}\right|^2+\left|\bar{e}^v_{1,0}\right|^2\right\}+\cdots+\kappa(1+\alpha+\cdots+\alpha^N)\\
&\times \mathbb{E}\left\{\left|\bar{e}^h_{0,N}\right|^2+\left|\bar{e}^v_{N,0}\right|^2\right\}\\
=&\ \kappa\frac{1-\alpha^N}{1-\alpha}\mathbb{E}\left\{\sum_{k=0}^{N}\left[\left|\bar{e}^h_{0,k}\right|^2+\left|\bar{e}^v_{k,0}\right|^2\right]\right\}.
\end{aligned}
$$

Then, by Assumption 1 the right-hand side of the above inequality is bounded, which means $\lim_{k\to\infty}\mathbb{E}\{\mathcal{X}_k\}=0$, that is, $\mathbb{E}\left\{\left|\bar{e}_{i,j}\right|^2\right\}\to 0$ as $i+j\to\infty$, by which $\mathbb{E}\left\{\left|e_{i,j}\right|^2\right\}\to 0$ as $i+j\to\infty$, then by Definition 1, the filtering error system $(\mathcal{E})$ in (5) with $\omega_{i,j}\equiv 0$ is mean-square asymptotically stable.

Next, we shall establish the $\mathcal{H}_\infty$ performance of the filtering error systems (5). To this end, we introduce the following index:

$$
\begin{aligned}
\mathcal{J} \triangleq \mathbb{E}\Bigg\{\Bigg(&\begin{bmatrix} e^h_{i+1,j}\\ e^v_{i,j+1}\end{bmatrix}^{\mathrm{T}}\Gamma\begin{bmatrix} P^h(r_{i+1,j}) & 0\\ 0 & P^v(r_{i,j+1})\end{bmatrix}\\
&\times\Gamma\begin{bmatrix} e^h_{i+1,j}\\ e^v_{i,j+1}\end{bmatrix}-e^{\mathrm{T}}_{i,j}\Gamma P(r_{i,j})\Gamma e_{i,j}\\
&+\tilde{z}^{\mathrm{T}}_{i,j}\tilde{z}_{i,j}-\gamma^2\omega^{\mathrm{T}}_{i,j}\omega_{i,j}\Bigg)\Bigg|\left(e_{i,j},r_{i,j}=m\right)\Bigg\}
\end{aligned}\tag{15}
$$

where $P(r_{i,j})=\mathrm{diag}\left\{P^h(r_{i,j}),P^v(r_{i,j})\right\}>0$. Then, along the solution of the filtering error system $(\mathcal{E})$ in (5), we have

$$
\begin{aligned}
\mathcal{J} &= \left[\tilde{A}_m\bar{e}_{i,j}+\tilde{B}_m\omega_{i,j}\right]^{\mathrm{T}}\tilde{P}_m\left[\tilde{A}_m\bar{e}_{i,j}+\tilde{B}_m\omega_{i,j}\right]\\
&\quad-\bar{e}^{\mathrm{T}}_{i,j}P_m\bar{e}_{i,j}+\bar{e}^{\mathrm{T}}_{i,j}\tilde{C}^{\mathrm{T}}_m\tilde{C}_m\bar{e}_{i,j}-\gamma^2\omega^{\mathrm{T}}_{i,j}\omega_{i,j}\\
&\triangleq \xi^{\mathrm{T}}\Sigma\xi
\end{aligned}
$$

where $\tilde{P}_m$ is defined in (11), $\xi\triangleq\begin{bmatrix}\bar{e}^{\mathrm{T}}_{i,j} & \omega^{\mathrm{T}}_{i,j}\end{bmatrix}^{\mathrm{T}}$ and

$$
\Sigma\triangleq\begin{bmatrix}\tilde{A}^{\mathrm{T}}_m\tilde{P}_m\tilde{A}_m+\tilde{C}^{\mathrm{T}}_m\tilde{C}_m-P_m & \tilde{A}^{\mathrm{T}}_m\tilde{P}_m\tilde{B}_m\\ * & \tilde{B}^{\mathrm{T}}_m\tilde{P}_m\tilde{B}_m-\gamma^2 I\end{bmatrix}.
$$

On the other hand, define $P_m\triangleq\mathrm{diag}\left\{P^h_m,P^v_m\right\}=Y^{-1}_m$, $m=1,\ldots,S$, then by performing a congruence transformation to (7) by $\mathrm{diag}\left\{I,Y^{-1}_m,I,I\right\}$, (7) is equivalent to

$$
\begin{bmatrix}-I & \bar{C}_m & 0 & 0\\ * & -P_m & 0 & \Phi_1\\ * & * & -\gamma^2 I & \Psi_2\\ * & * & * & \Phi_3\end{bmatrix}<0,\quad m=1,\ldots,S \tag{16}
$$

where Φ_1 and Φ_3 are defined in (10) and Ψ_2 is given in (7). By Schur complement, LMI (16) implies $\Sigma<0$, then for $\xi\neq 0$, we have $\mathcal{J}<0$, which means for every $r_{i,j}\in\mathcal{L}$, we have

$$
\begin{aligned}
&\mathbb{E}\left\{\left[\bar{e}^{hT}_{i+1,j}P^h(r_{i+1,j})\bar{e}^h_{i+1,j}+\bar{e}^{vT}_{i,j+1}P^v(r_{i,j+1})\bar{e}^v_{i,j+1}\right]\right|\\
&\qquad\times\left.\left(\bar{e}_{i,j},r_{i,j}\right)\right\}\\
&<\mathbb{E}\left\{\left[\bar{e}^{\mathrm{T}}_{i,j}P\left(r_{i,j}\right)\bar{e}_{i,j}-\tilde{z}^{\mathrm{T}}_{i,j}\tilde{z}_{i,j}+\gamma^2\omega^{\mathrm{T}}_{i,j}\omega_{i,j}\right]\right|\\
&\qquad\times\left.\left(\bar{e}_{i,j},r_{i,j}\right)\right\}.
\end{aligned}
$$

Taking the expectation of both sides yields

$$
\begin{aligned}
&\mathbb{E}\left\{\bar{e}^{hT}_{i+1,j}P^h(r_{i+1,j})\bar{e}^h_{i+1,j}+\bar{e}^{vT}_{i,j+1}P^v(r_{i,j+1})\bar{e}^v_{i,j+1}\right\}\\
&\quad<\mathbb{E}\left\{\bar{e}^{\mathrm{T}}_{i,j}P\left(r_{i,j}\right)\bar{e}_{i,j}-\tilde{z}^{\mathrm{T}}_{i,j}\tilde{z}_{i,j}\right\}+\gamma^2\omega^{\mathrm{T}}_{i,j}\omega_{i,j}.
\end{aligned}\tag{17}
$$

Upon relationship (17), it can be established that

$$
\begin{aligned}
&\mathbb{E}\left\{\bar{e}^{hT}_{0,k+1}P^h(r_{0,k+1})\bar{e}^h_{0,k+1}\right\}=\mathbb{E}\left\{\bar{e}^{hT}_{0,k+1}P^h(r_{0,k+1})\bar{e}^h_{0,k+1}\right\}\\
&\mathbb{E}\left\{\bar{e}^{hT}_{1,k}P^h(r_{1,k})\bar{e}^h_{1,k}+\bar{e}^{vT}_{0,k+1}P^v(r_{0,k+1})\bar{e}^v_{0,k+1}\right\}\\
&\quad<\mathbb{E}\left\{\bar{e}^{\mathrm{T}}_{0,k}P\left(r_{0,k}\right)\bar{e}_{0,k}-\tilde{z}^{\mathrm{T}}_{0,k}\tilde{z}_{0,k}\right\}+\gamma^2\omega^{\mathrm{T}}_{0,k}\omega_{0,k}\\
&\mathbb{E}\left\{\bar{e}^{hT}_{2,k-1}P^h(r_{2,k-1})\bar{e}^h_{2,k-1}+\bar{e}^{vT}_{1,k}P^v(r_{1,k})\bar{e}^v_{1,k}\right\}\\
&\quad<\mathbb{E}\left\{\bar{e}^{\mathrm{T}}_{1,k-1}P\left(r_{1,k-1}\right)\bar{e}_{1,k-1}-\tilde{z}^{\mathrm{T}}_{1,k-1}\tilde{z}_{1,k-1}\right\}\\
&\qquad+\gamma^2\omega^{\mathrm{T}}_{1,k-1}\omega_{1,k-1}\\
&\vdots\\
&\mathbb{E}\left\{\bar{e}^{hT}_{k+1,0}P^h(r_{k+1,0})\bar{e}^h_{k+1,0}+\bar{e}^{vT}_{k,1}P^v(r_{k,1})\bar{e}^v_{k,1}\right\}\\
&\quad<\mathbb{E}\left\{\bar{e}^{\mathrm{T}}_{k,0}P\left(r_{k,0}\right)\bar{e}_{k,0}-\tilde{z}^{\mathrm{T}}_{k,0}\tilde{z}_{k,0}\right\}+\gamma^2\omega^{\mathrm{T}}_{k,0}\omega_{k,0}\\
&\mathbb{E}\left\{\bar{e}^{vT}_{k+1,0}P^v(r_{k+1,0})\bar{e}^v_{k+1,0}\right\}=\mathbb{E}\left\{\bar{e}^{vT}_{k+1,0}P^v(r_{k+1,0})\bar{e}^v_{k+1,0}\right\}.
\end{aligned}
$$

Adding both sides of the above inequality system and considering the zero boundary condition yield

$$
\begin{aligned}
&\mathbb{E}\left\{\sum_{j=0}^{k+1}\left[\bar{e}^{\mathrm{T}}_{k+1-j,j}P(r_{k+1-j,j})\bar{e}_{k+1-j,j}\right]\right\}\\
&\quad<\mathbb{E}\left\{\sum_{j=0}^{k}\left[\bar{e}^{\mathrm{T}}_{k-j,j}P(r_{k-j,j})\bar{e}_{k-j,j}-\tilde{z}^{\mathrm{T}}_{k-j,j}\tilde{z}_{k-j,j}\right]\right\}\\
&\qquad+\gamma^2\sum_{j=0}^{k}\omega^{\mathrm{T}}_{k-j,j}\omega_{k-j,j}.
\end{aligned}
$$

Summing up both sides of the above inequality from $k=0$ to $k=N$, we have

$$
\begin{aligned}
&\mathbb{E}\left\{\sum_{k=0}^{N}\sum_{j=0}^{k}\tilde{z}^{\mathrm{T}}_{k-j,j}\tilde{z}_{k-j,j}\right\}<\gamma^2\sum_{k=0}^{N}\sum_{j=0}^{k}\omega^{\mathrm{T}}_{k-j,j}\omega_{k-j,j}\\
&\quad-\mathbb{E}\left\{\sum_{j=0}^{N+1}\bar{e}^{\mathrm{T}}_{N+1-j,j}P(r_{N+1-j,j})\bar{e}_{N+1-j,j}\right\}.
\end{aligned}
$$

Therefore, we have

$$
\mathbb{E}\left\{\sum_{k=0}^{\infty}\sum_{j=0}^{k}\tilde{z}^{\mathrm{T}}_{k-j,j}\tilde{z}_{k-j,j}\right\}<\gamma^2\sum_{k=0}^{\infty}\sum_{j=0}^{k}\omega^{\mathrm{T}}_{k-j,j}\omega_{k-j,j}
$$

that is, $\|\tilde{z}\|_E<\gamma\|\omega\|_2$ for all nonzero $\omega=\left\{\omega_{i,j}\right\}\in l_2\{[0,\infty),[0,\infty)\}$. This completes the proof. □

Remark 1. Notice that there exist product terms between the Lyapunov and system matrices in the LMI condition (7) of Theorem 1, which will bring some difficulties in the solution of filter synthesis problem. Applying the approach proposed by Apkarian, Tuan, and Bernussou (2001), in the following, we will make a decoupling between the Lyapunov and system matrices by introducing a slack matrix variable. This decoupling technique enables us to obtain a more easily tractable condition for synthesis of filter. But, some conservativeness will be introduced due to the common matrix variable X, see the following result.

Theorem 2. *The filtering error system $(\mathcal{E})$ in (5) is mean-square asymptotically stable with an $\mathcal{H}_\infty$ disturbance attenuation level $\gamma > 0$ if there exist matrices $\bar{Y}_m = \operatorname{diag}\{\bar{Y}_m^h, \bar{Y}_m^v\}$, $\bar{Y}_m^h > 0$, $\bar{Y}_m^v > 0$, $m = 1, \ldots, S$ and X such that the following LMIs hold:*

$$\begin{bmatrix} \bar{\Psi}_3 & 0 & \bar{\Psi}_1^{\mathrm{T}} & \bar{\Psi}_2^{\mathrm{T}} \\ * & -I & \tilde{C}_m & 0 \\ * & * & \bar{Y}_m - X - X^{\mathrm{T}} & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0, \quad m = 1, \ldots, S \tag{18}$$

where

$$\bar{\Psi}_1 \triangleq \begin{bmatrix} \tilde{A}_m^{\mathrm{T}} X & \cdots & \tilde{A}_m^{\mathrm{T}} X \end{bmatrix}$$
$$\bar{\Psi}_2 \triangleq \begin{bmatrix} \tilde{B}_m^{\mathrm{T}} X & \cdots & \tilde{B}_m^{\mathrm{T}} X \end{bmatrix},$$
$$\bar{\Psi}_3 \triangleq \operatorname{diag}\left\{ -p_{m1}^{-1} \bar{Y}_1, \ldots, -p_{mS}^{-1} \bar{Y}_S \right\}.$$

The desired result can be carried out by employing the same techniques as those in Theorem 2 of Gao, Lam, Wang et al. (2004).

3.2. Filter synthesis

Now, we are in a position to solve the $\mathcal{H}_\infty$ filter synthesis problem based on Theorem 2. The following theorem provides a sufficient condition for the existence of such $\mathcal{H}_\infty$ filter for system $(\mathcal{S})$.

Theorem 3. *For 2D MJLS $(\mathcal{S})$, there exists a filter in the form of (4) such that the filtering error system $(\mathcal{E})$ in (5) is mean-square asymptotically stable with an $\mathcal{H}_\infty$ disturbance attenuation level $\gamma > 0$, if there exist matrices $\tilde{Y}_m = \operatorname{diag}\left\{\tilde{Y}_m^h, \tilde{Y}_m^v\right\}$, $\tilde{Y}_m^h > 0$, $\tilde{Y}_m^v > 0$, $m = 1, \ldots, S$ and U, V, W, $\bar{A}_{fm}$, $\bar{B}_{fm}$, $\bar{C}_{fm}$ such that for $m = 1, \ldots, S$, the following LMIs hold:*

$$\begin{bmatrix} \Omega_{11} & 0 & \Omega_{13} & \Omega_{14} & \Omega_{15} \\ * & -I & L_m & -\bar{C}_{fm} & 0 \\ * & * & \tilde{Y}_m^h - U - U^{\mathrm{T}} & -V - W^{\mathrm{T}} & 0 \\ * & * & * & \tilde{Y}_m^v - W - W^{\mathrm{T}} & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix} < 0 \tag{19}$$

where

$$\Omega_{11} \triangleq \begin{bmatrix} \begin{bmatrix} -p_{m1}^{-1}\tilde{Y}_1^h & 0 \\ * & -p_{m1}^{-1}\tilde{Y}_1^v \end{bmatrix} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ * & \cdots & \begin{bmatrix} -p_{mS}^{-1}\tilde{Y}_S^h & 0 \\ * & -p_{mS}^{-1}\tilde{Y}_S^v \end{bmatrix} \end{bmatrix},$$

$$\Omega_{14} \triangleq \begin{bmatrix} \begin{bmatrix} \bar{A}_{fm} \\ \bar{A}_{fm} \end{bmatrix} \\ \vdots \\ \begin{bmatrix} \bar{A}_{fm} \\ \bar{A}_{fm} \end{bmatrix} \end{bmatrix}$$

$$\Omega_{13} \triangleq \begin{bmatrix} \begin{bmatrix} U^{\mathrm{T}} A_m + \bar{B}_{fm} C_m \\ V^{\mathrm{T}} A_m + \bar{B}_{fm} C_m \end{bmatrix} \\ \vdots \\ \begin{bmatrix} U^{\mathrm{T}} A_m + \bar{B}_{fm} C_m \\ V^{\mathrm{T}} A_m + \bar{B}_{fm} C_m \end{bmatrix} \end{bmatrix},$$

$$\Omega_{15} \triangleq \begin{bmatrix} \begin{bmatrix} U^{\mathrm{T}} B_m + \bar{B}_{fm} D_m \\ V^{\mathrm{T}} B_m + \bar{B}_{fm} D_m \end{bmatrix} \\ \vdots \\ \begin{bmatrix} U^{\mathrm{T}} B_m + \bar{B}_{fm} D_m \\ V^{\mathrm{T}} B_m + \bar{B}_{fm} D_m \end{bmatrix} \end{bmatrix}.$$

Moreover, a desired $\mathcal{H}_\infty$ filter is given in the form of (4) with parameters as follows:

$$\begin{bmatrix} A_{fm} & B_{fm} \\ C_{fm} & 0 \end{bmatrix} = \begin{bmatrix} W^{-T} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \bar{A}_{fm} & \bar{B}_{fm} \\ \bar{C}_{fm} & 0 \end{bmatrix}. \tag{20}$$

Proof. As mentioned in the proof of Theorem 2, X is nonsingular if (18) holds. Now, partition X as

$$X = \begin{bmatrix} X_1 & X_2 \\ X_4 & X_3 \end{bmatrix} \tag{21}$$

where X_1, X_2, X_3, X_4 are all $(n_1 + n_2) \times (n_1 + n_2)$ matrices. Without loss of generality, we assume that X_3 and X_4 are nonsingular. To see this, let the matrix $Z \triangleq X + \tau\Lambda$, where τ is a positive scalar and

$$\Lambda = \begin{bmatrix} 0 & I \\ I & I \end{bmatrix}, \qquad Z = \begin{bmatrix} Z_1 & Z_2 \\ Z_4 & Z_3 \end{bmatrix}.$$

Observe that Z is nonsingular for $\tau > 0$ in a neighborhood of the origin since X is nonsingular. Thus, it can be easily verified that there exists an arbitrarily small $\tau > 0$ such that Z_3 and Z_4 are nonsingular and inequality (18) is feasible with X replaced by Z. Since Z_3 and Z_4 are nonsingular, we thus conclude that there is no loss of generality to assume the matrices X_3 and X_4 to be nonsingular. Introduce the following matrices:

$$U \triangleq X_1, \qquad V \triangleq X_2 X_3^{-1} X_4,$$
$$W \triangleq X_4^{\mathrm{T}} X_3^{-T} X_4, \qquad R \triangleq \begin{bmatrix} I & 0 \\ 0 & X_3^{-1} X_4 \end{bmatrix} \tag{22}$$

$$\begin{bmatrix} \bar{A}_{fm} & \bar{B}_{fm} \\ \bar{C}_{fm} & 0 \end{bmatrix} \triangleq \begin{bmatrix} X_4^{\mathrm{T}} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} A_{fm} & B_{fm} \\ C_{fm} & 0 \end{bmatrix} \begin{bmatrix} X_3^{-1} X_4 & 0 \\ 0 & I \end{bmatrix} \tag{23}$$

$$\tilde{Y}_j \triangleq R^{\mathrm{T}} \bar{Y}_j R, \quad (j = 1, 2, \ldots, S),$$
$$\Pi \triangleq \operatorname{diag}\{\underbrace{R, R, \ldots, R}_{S}\}. \tag{24}$$

Performing congruence transformations to (18) by diagonal matrix diag$\{\Pi, I, R, I\}$, we have

$$\begin{bmatrix} \Pi^{\mathrm{T}}\bar{\Psi}_3\Pi & 0 & \Pi^{\mathrm{T}}\bar{\Psi}_1^{\mathrm{T}}R & \Pi^{\mathrm{T}}\bar{\Psi}_2^{\mathrm{T}} \\ * & -I & \tilde{C}_m R & 0 \\ * & * & R^{\mathrm{T}}\bar{Y}_m R - R^{\mathrm{T}}XR - R^{\mathrm{T}}X^{\mathrm{T}}R & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0, \quad m = 1, \ldots, S. \tag{25}$$

Considering (21)–(24), we can obtain (19) from (25). On the other hand, (23) is equivalent to

$$\begin{bmatrix} A_{fm} & B_{fm} \\ C_{fm} & 0 \end{bmatrix} = \begin{bmatrix} X_4^{-T} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \bar{A}_{fm} & \bar{B}_{fm} \\ \bar{C}_{fm} & 0 \end{bmatrix} \times \begin{bmatrix} X_4^{-1}X_3 & 0 \\ 0 & I \end{bmatrix} \tag{26}$$

and according to (4), the transfer function from measured output y_{ij} to estimated signal $\hat{z}_{ij}$ can be described by

$$T_{\hat{z}y} = C_{fm}\left(\mathrm{diag}\{z_1 I, z_2 I\} - A_{fm}\right)^{-1} B_{fm}. \tag{27}$$

Substituting (26) into (27) will supply

$$\begin{aligned} T_{\hat{z}y} &= \bar{C}_{fm}X_4^{-1}X_3\left(\mathrm{diag}\{z_1 I, z_2 I\} - X_4^{-T}\bar{A}_{fm}X_4^{-1}X_3\right)^{-1} \\ &\quad \times X_4^{-T}\bar{B}_{fm} \\ &= \bar{C}_{fm}\left(\mathrm{diag}\{z_1 I, z_2 I\} - W^{-T}\bar{A}_{fm}\right)^{-1} W^{-T}\bar{B}_{fm}. \end{aligned} \tag{28}$$

Therefore, we can conclude from (28) that the parameters of filter ($\mathcal{F}$) in (4) can be constructed by (20). This completes the proof. □

Remark 2. Note that Theorem 3 provides a sufficient condition for solvability of $\mathcal{H}_\infty$ filtering problem for 2D MJLS. Since the obtained condition is of the strict LMI framework, the desired filter can be determined by solving the following convex optimization problem:

$$\begin{aligned} &\min \delta \quad (\text{where } \delta \triangleq \gamma^2) \\ &\text{subject to } \tilde{Y}_m^h > 0,\ \tilde{Y}_m^v > 0, \quad m = 1, \ldots, S \text{ and (19)}. \end{aligned} \tag{29}$$

4. Numerical example

In a real world, some dynamical processes in gas absorption, water stream heating and air drying can be described by the Darboux equation (Marszalek, 1984):

$$\begin{aligned} \frac{\partial^2 s(x,t)}{\partial x \partial t} &= a_0(r_{x,t})s(x,t) + a_1(r_{x,t})\frac{\partial s(x,t)}{\partial t} \\ &\quad + a_2(r_{x,t})\frac{\partial s(x,t)}{\partial x} + b(r_{x,t})f(x,t) \end{aligned} \tag{30}$$

$$\begin{aligned} y(x,t) &= c_1(r_{x,t})s(x,t) + c_2(r_{x,t}) \\ &\quad \times \left[\frac{\partial s(x,t)}{\partial t} - a_2(r_{x,t})s(x,t)\right] + d(r_{x,t})f(x,t) \end{aligned} \tag{31}$$

$$\begin{aligned} z(x,t) &= l_1(r_{x,t})s(x,t) + l_2(r_{x,t}) \\ &\quad \times \left[\frac{\partial s(x,t)}{\partial t} - a_2(r_{x,t})s(x,t)\right] \end{aligned} \tag{32}$$

where $s(x,t)$ is an unknown function at x(space) $\in [0, x_f]$ and t (time) $\in [0, \infty)$, $f(x,t)$ is the input function, $y(x,t)$ is the measured output, and $z(x,t)$ is the signal to be estimated. $a_0(r_{x,t})$, $a_1(r_{x,t})$, $a_2(r_{x,t})$, $b(r_{x,t})$, $c_1(r_{x,t})$, $c_2(r_{x,t})$, $d(r_{x,t})$, $l_1(r_{x,t})$ and $l_2(r_{x,t})$ are real coefficients. These coefficients are functions of $r_{x,t}$, which is a Markovian process on the probability space, takes values in a finite state space $\mathcal{L} \triangleq \{1, \ldots, S\}$.

Note that (30)–(32) is a partial differential equation (PDE) and, in practice, it is often desired to predict the unknown signal $z(x,t)$ through the available measurement $y(x,t)$, which renders the filtering problem. Similar to the technique used in Du, Xie, and Zhang (2001), we define

$$\begin{aligned} &h(x,t) \triangleq \frac{\partial s(x,t)}{\partial t} - a_2(r_{x,t})s(x,t) \\ &x^h(i,j) \triangleq s(i,j) \triangleq s(i\Delta x, j\Delta t), \\ &x^v(i,j) \triangleq h(i,j) \triangleq h(i\Delta x, j\Delta t), \end{aligned}$$

and then the PDE model (30)–(32) can be converted into the form of a 2D Roesser model with Markovian jump parameters of the form of ($\mathcal{S}$) in (1).

As discussed in Du et al. (2001), the discrepancy between the PDE model and its 2D difference approximation depends on the step sizes Δx and Δt which may be treated as uncertainty in the difference model. Obviously, the smaller the step sizes Δx and Δt, the closer the PDE model and the difference model.

Now, subject to the selection of the parameters $a_0(r_{x,t})$, $a_1(r_{x,t})$, $a_2(r_{x,t})$, $b(r_{x,t})$, $c_1(r_{x,t})$, $c_2(r_{x,t})$, $d(r_{x,t})$, $l_1(r_{x,t})$ and $l_2(r_{x,t})$, we let the system matrices in (1) be given as follows (with two operation modes):

The first mode:

$$\begin{aligned} &A_1 = \begin{bmatrix} -2.2 & 0.5 \\ -0.2 & -1.8 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} 0.3 \\ 0.5 \end{bmatrix}, \\ &C_1 = \begin{bmatrix} 1.0 & 0.0 \\ 1.0 & 0.6 \end{bmatrix}, \\ &D_1 = \begin{bmatrix} 0.0 \\ 0.3 \end{bmatrix}, \qquad L_1 = \begin{bmatrix} 1.0 & 1.0 \\ 0.0 & -1.0 \end{bmatrix}. \end{aligned} \tag{33}$$

The second mode:

$$\begin{aligned} &A_2 = \begin{bmatrix} -1.8 & 0.6 \\ -0.3 & -1.2 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} 0.3 \\ 0.4 \end{bmatrix}, \\ &C_2 = C_1, \qquad D_2 = D_1, \qquad L_2 = L_1. \end{aligned} \tag{34}$$

Assume that the transition probability matrix is given by

$$p = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} = \begin{bmatrix} 0.3 & 0.7 \\ 0.6 & 0.4 \end{bmatrix}. \tag{35}$$

It is easy to verify by Theorem 1 in Gao, Lam, Xu et al. (2004) that the system ($\mathcal{S}$) with (33)–(35) is mean-square asymptotically stable. Solving the LMIs condition obtained in Theorem 3 by applying the well-developed LMI Tool-box in the MATLAB environment directly, we obtain that the minimized feasible γ is $\gamma^* = 1.1234$ and

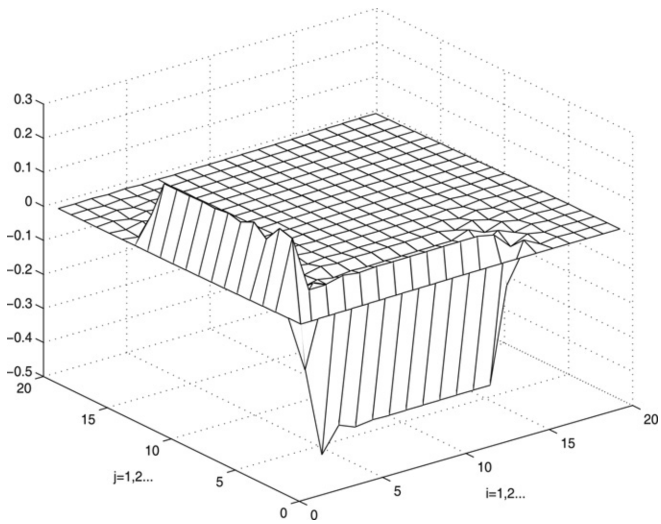

Fig. 1. Filtering error $\tilde{z}_{i,j}$ for $\omega_{i,j} = 0$: 1st component.

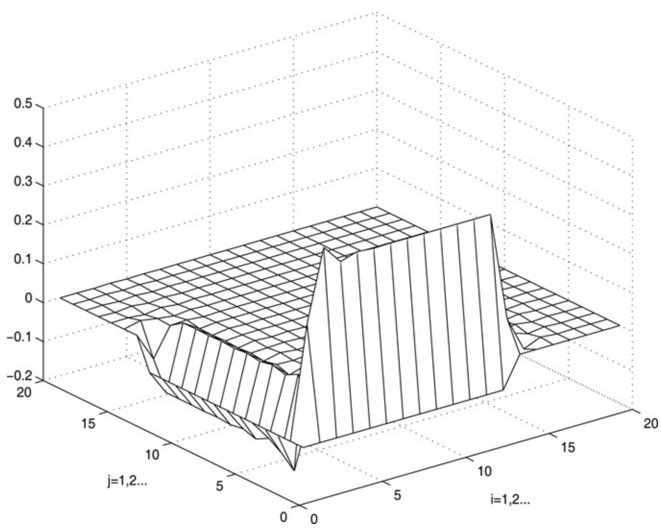

Fig. 2. Filtering error $\tilde{z}_{i,j}$ for $\omega_{i,j} = 0$: 2nd component.

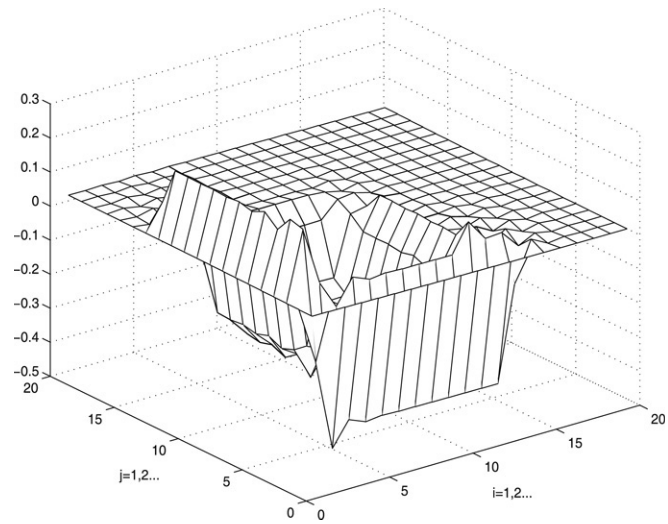

Fig. 3. Filtering error $\tilde{z}_{i,j}$ for $\omega_{i,j} \neq 0$: 1st component.

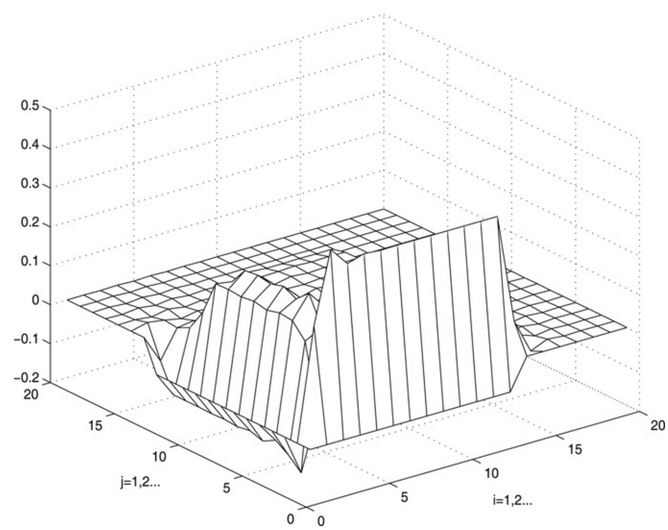

Fig. 4. Filtering error $\tilde{z}_{i,j}$ for $\omega_{i,j} \neq 0$: 2nd component.

$$
\begin{aligned}
A_{f1} &= \begin{bmatrix} -0.1769 & 0.0043 \\ 0.2207 & -0.0054 \end{bmatrix}, \\
B_{f1} &= \begin{bmatrix} 7.1689 & -2.8901 \\ -0.6122 & 0.9053 \end{bmatrix}, \\
C_{f1} &= \begin{bmatrix} -1.3894 & -0.6137 \\ 0.3519 & 0.5532 \end{bmatrix}, \\
A_{f2} &= \begin{bmatrix} -0.2213 & 0.1004 \\ 0.1581 & -0.0717 \end{bmatrix}, \\
B_{f2} &= \begin{bmatrix} 5.8326 & -2.5068 \\ -0.0899 & 0.3710 \end{bmatrix}, \\
C_{f2} &= \begin{bmatrix} -1.3894 & -0.6137 \\ 0.3519 & 0.5532 \end{bmatrix}.
\end{aligned}
\tag{36}
$$

In the following, we shall show the usefulness of the designed filter by presenting simulation results. Our simulation is based on the obtained filter matrices in (36). To show the asymptotic stability of the filtering error system, assume that $\omega_{i,j} = 0$ and let the initial and boundary conditions to be

$$
\begin{cases} x^h_{0,j} = 0.2, & 0 \le j \le 15 \\ x^v_{i,0} = 0.3, & 0 \le i \le 15 \\ x^h_{0,j} = x^v_{i,0} = 0, & i, j > 15. \end{cases}
$$

Then the obtained filtering error signal $\tilde{z}_{i,j}$ is shown in Figs. 1 and 2, from which we can see that $\tilde{z}_{i,j}$ converges to zero under the above conditions. Next, assume zero initial and boundary conditions, and let the disturbance input $\omega_{i,j}$ be

$$
\omega_{i,j} = \begin{cases} 0.1, & 3 \le i, j \le 10 \\ 0, & \text{otherwise}. \end{cases}
$$

Figs. 3 and 4 show the filtering error $\tilde{z}_{i,j}$. Now we will calculate the actual $\mathcal{H}_\infty$ performance under the above specific conditions. By calculation, we have $\|\tilde{z}\|_E = 0.7219$ and $\|\omega\|_2 = 0.8000$, which yields $\gamma = 0.9024$ (below the prescribed value $\gamma^* = 1.1234$).

5. Conclusion

In this paper, the problem of $\mathcal{H}_\infty$ filtering for 2D MJLS has been investigated. A sufficient condition has been developed for the design of general full-order filter in terms of LMIs, which guarantees mean-square asymptotic stability and a prescribed $\mathcal{H}_\infty$ performance level of the filtering error system. Then the filter design has been cast into a convex optimization problem. A numerical example has been provided to show the usefulness of the proposed filter design methods.

References

Aberkane, S., Christophe Ponsart, J., & Sauter, D. (2006). Output feedback H_∞ control of a class of stochastic hybrid systems with wiener process via convex analysis. *International Journal of Innovative Computing, Information and Control*, *6*, 1179–1196.

Apkarian, P., Tuan, H. D., & Bernussou, J. (2001). Continuous-time analysis, eigenstructure assignment, and H_2 synthesis with enhanced linear matrix inequalities (LMI) characterizations. *IEEE Transactions on Automatic Control*, *46*, 1941–1946.

Basin, M., Perez, J., & Martinez-Zuniga, R. (2006). Optimal filtering for nonlinear polynomial systems over linear observations with delay. *International Journal of Innovative Computing, Information and Control*, *2*, 863–874.

Boukas, E. K., & Yang, H. (1999). Exponential stabilizability of stochastic systems with Markovian jumping parameters. *Automatica*, *35*, 1437–1441.

Boyd, S., El Ghaoui, L., Feron, E., & Balakrishnan, V. (1994). *Linear matrix inequalities in systems and control theory*. Philadelphia: SIAM.

Cao, Y. Y., & Lam, J. (2000). Robust H_∞ control of uncertain Markovian jump systems with time delay. *IEEE Transactions on Automatic Control*, *45*, 77–83.

Du, C., Xie, L., & Zhang, C. (2001). H_∞ control and robust stabilization of two-dimensional systems in Roesser models. *Automatica*, *37*, 205–211.

Gao, H., Lam, J., Wang, C., & Xu, S. (2004). Robust H_∞ filter for uncertain 2D stochastic systems. *Circuits Systems and Signal Processing*, *23*, 479–505.

Gao, H., Lam, J., Xu, S., & Wang, C. (2004). Stabilization and H_∞ control of two-dimensional Markovian jump systems. *IMA Journal of Mathematical and Control Information*, *21*, 377–392.

Hinamoto, T. (1997). Stability of 2-D discrete systems described by the Fornasini-Marchesini second model. *IEEE Transactions on Circuits and Systems (I)*, *44*, 254–257.

Hoang, N. T., Tuan, H. D., Nguyen, T. Q., & Hosoe, S. (2005). Robust mixed generalized H_2/H_∞ filtering of 2-D nonlinear fractional transformation systems. *IEEE Transactions on Signal Processing*, *53*, 4697–4706.

Ji, Y., & Chizeck, H. J. (1990). Controllability, stabilizability, and continuous-time Markovian jump linear quadratic control. *IEEE Transactions on Automatic Control*, *35*, 777–788.

Lu, W. S. (1994). On a Lyapunov approach to stability analysis of 2-D digital filters. *IEEE Transactions on Circuits and Systems (I)*, *41*, 665–669.

Lu, W. S., & Antoniou, A. (1992). *Two-dimensional digital filters*. New York: Marcel Dekker.

Marszalek, W. (1984). Two-dimensional state-space discrete models for hyperbolic partial differential equations. *Applied Mathematical and Modeling*, *8*, 11–14.

Shi, P., Boukas, E. K., & Agarwal, R. K. (1999). Kalman filtering for continuous-time uncertain systems with Markovian jumping parameters. *IEEE Transactions on Automatic Control*, *44*, 1592–1597.

Tuan, H. D., Apkarian, P., & Nguyen, T. Q. (2002). Robust mixed H_2/H_∞ filtering of 2-D systems. *IEEE Transactions on Signal Processing*, *50*, 1759–1771.

Wang, Z., Lam, J., & Liu, X. H. (2004). Robust filtering for discrete-time Markovian jump delay systems. *IEEE Transactions on Signal Processing Letters*, *11*, 659–662.

Wu, L., Shi, P., Gao, H., & Wang, C. (2006). H_∞ mode reduction for two-dimensional discrete state-delayed systems. *IEE Proceedings Part J: Vision, Image and Signal Processing*, *153*, 769–784.

Wu, L., Wang, Z., Gao, H., & Wang, C. (2007a). H_∞ and l_2-l_∞ filtering for two-dimensional linear parametervarying systems. *International Journal of Robust & Nonlinear Control*, *17*, 1129–1154.

Wu, L., Wang, Z., Gao, H., & Wang, C. (2007b). Robust H_∞ filtering for uncertain two-dimensional discrete systems with state delays. *Signal Processing*, *87*, 2213–2230.

Xie, L., Ogai, H., & Inoe, Y. (2006). Numerical solving of hybrid dynamic switching system and its application. *International Journal of Innovative Computing, Information and Control*, *2*, 849–862.

Xiong, J., Lam, J., Gao, H., & Ho, D. W. C. (2005). On robust stabilization of Markovian jump systems with uncertain switching probabilities. *Automatica*, *41*, 897–903.

Xu, S., Chen, T., & Lam, J. (2003). Robust H_∞ filtering for uncertain Markovian jump systems with mode-dependent time-delays. *IEEE Transactions on Automatic Control*, *48*, 900–907.

Xu, S., Chen, T., & Lam, J. (2004). Robust H_∞ filtering for a class of nonlinear discrete-time Markovian jump systems. *Journal of Optimization Theory and Applications*, *122*, 651–668.

Yue, D., Fang, J., & Won, S (2003). Delay-dependent robust stability of stochastic uncertain systems with time delay and Markovian jump parameters. *Circuits, Systems and Signal Processing*, *22*, 351–365.

Zhang, H., Basin, M, & Skliar, M. (2006). Optimal state estimation for continuous, stochastic, state-space system with hybrid measurements. *International Journal of Innovative Computing, Information and Control*, *2*, 357–370.

Zhang, L., Huang, B., & Lam, J. (2003). H_∞ model reduction of Markovian jump linear systems. *Systems & Control Letters*, *50*, 103–118.

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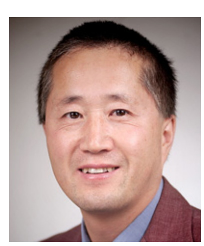

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