Proceedings of the 42nd IEEE
Conference on Decision and Control
Maui, Hawaii USA, December 2003

A Mode-Independent $\mathcal{H}_\infty$ Filter Design for Discrete-Time Markovian Jump Linear Systems[1]

Carlos E. de Souza

Deptartment of Systems and Control
Lab. Nacional de Computação Científica – LNCC/MCT
Av. Getúlio Vargas 333
25651-075 Petrópolis, RJ, Brazil
e-mail: csouza@lncc.br

Abstract

This paper addresses the problem of $\mathcal{H}_\infty$ filtering for discrete-time linear systems with Markovian jumping parameters. The main contribution of the paper is to provide a linear matrix inequality approach for designing an asymptotically stable linear time-invariant $\mathcal{H}_\infty$ filter for systems where the jumping parameter is not accessible. The cases where the transition probability matrix of the Markov chain is either exactly known, or unknown but belongs to a given polytope, are treated. The robust $\mathcal{H}_\infty$ filtering problem where the system matrices for each operating mode are unknown but belongs to a given polytope is also considered. A new internal mean square stability condition as well as a bounded real lemma for discrete-time Markovian jump linear systems are also developed.

1 Introduction

Over the last decade a lot of interest has been devoted to the problem of $\mathcal{H}_\infty$ filtering for linear discrete-time systems. In $\mathcal{H}_\infty$ filtering the noise sources are arbitrary deterministic signals with bounded energy, or average power, and a filter is sought which ensures a prescribed upper-bound on the ℓ_2-induced gain from the noise signals to the estimation error; see, e.g. [10], [13], [16] and the references therein. This filtering approach is very appropriate to applications where the statistics of the noise signals are not exactly known.

The main purpose of this paper is to study the problem of $\mathcal{H}_\infty$ filtering for discrete-time Markovian jump linear (MJL) systems. This class of systems, which has been attracting an increasing attention in the literature (see, e.g. [1]-[4], [7]-[9], [11], [12], [14], [15] and the references therein), is very appropriate to model plants whose structure is subject to random abrupt parameters changes due to, for instance, component and/or interconnections failures, sudden environment changes, change of the operating point of a linearized model of a nonlinear system, etc. Filtering problems for MJL systems in the minimum mean square error sense have been addressed in [2], [4], and [11], among others, whereas $\mathcal{H}_\infty$ filtering has been considered in [8] and [9] for the continuous-time case and in [7] for discrete-time systems. A common feature of the existing $\mathcal{H}_\infty$ filtering results is that the jumping parameter is assumed to be accessible and the filter has the system feature of having Markovian jumps, i.e. the filter is a MJL system as well. To the best of the author's knowledge, to date the problem of $\mathcal{H}_\infty$ filtering for discrete-time MJL systems with a *non-accessible* jumping parameter via a time-invariant filter has not yet been addressed.

This paper considers the problem of $\mathcal{H}_\infty$ filtering for discrete-time MJL systems where the jumping parameter is not accessible. Attention is focused on the design of an asymptotically stable linear discrete time-invariant (*mode-independent*) filter which provides a mean square stable error dynamics and a prescribed upper-bound on the ℓ_2-induced gain from the noise signals to the estimation error. Robust $\mathcal{H}_\infty$ filtering problems where either the transition probability matrix of the Markov chain, or the matrices of the system state-space model for each operating mode, are uncertain but belong to given polytopes are also treated. A linear matrix inequality (LMI) approach is developed for solving these $\mathcal{H}_\infty$ filtering problems. The proposed filter design methods can also be readily extended to the case where the jumping parameter is accessible and a mode-dependent linear filter is sought. Two examples are presented to demonstrate the potentials of the methods of this paper.

Notation. Throughout the paper the superscript 'T' stands for matrix transposition, $\Re^n$ denotes the n dimensional Euclidean space, $\Re^{n\times m}$ is the set of $n\times m$ real matrices, I_n is the $n\times n$ identity matrix, and ℓ_2 stands for the space of squared summable vector sequences of a given dimension over the non-negative integers. The notation $P > 0$ for $P \in \Re^{n\times n}$, means that P is symmetric and positive definite.

[1]This work was supported by CNPq, Brazil, under PRONEX grant No. 0331.00/00, CNPq grants 46.8652/00-0/APQ and 30.2317/02-3/PQ and IM-AGIMB.

2 Problem Formulation

Fix an underlying probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and consider the stochastic system $(\mathcal{S})$:

$$(\mathcal{S}):\ x(k+1) = A(\theta_k)x(k) + B(\theta_k)w(k) \qquad (1)$$
$$y(k) = C(\theta_k)x(k) + D(\theta_k)w(k) \qquad (2)$$
$$s(k) = L(\theta_k)x(k) \qquad (3)$$

where $x(k) \in \Re^n$ is the state, $w(k) \in \Re^{n_w}$ is the noise signal, which is assumed to be an arbitrary signal in ℓ_2, $y(k) \in \Re^{n_y}$ is the measurement, and $s(k) \in \Re^{n_s}$ is the signal to be estimated. $\{\theta_k\}$ is a discrete-time homogeneous Markov chain with finite state-space $\Xi = \{1, \ldots, N\}$ and stationary transition probability matrix $\Lambda = [\lambda_{ij}]$, where

$$\lambda_{ij} := \mathbb{P}\{\theta_{n+1} = j \mid \theta_0, \ldots, \theta_n = i\} = \mathbb{P}\{\theta_{n+1} = j \mid \theta_n = i\}.$$

The set Ξ comprises the operation modes of system $(\mathcal{S})$ and for each possible value of $\theta_k = i$, $i \in \Xi$, we denote the matrices associated with the "i-th mode" by

$$A_i := A(\theta_k),\ B_i := B(\theta_k),\ C_i := C(\theta_k),\ D_i := D(\theta_k),$$
$$L_i := L(\theta_k),\ \text{ for } \ \theta_k = i$$

where A_i, B_i, C_i, D_i and L_i are known real constant matrices for any $i \in \Xi$.

No assumption is made on the accessibility of the jumping process $\{\theta_k\}$. This is in contrast with existing approaches of $\mathcal{H}_\infty$ filtering for MJL systems, such as those of [7]-[9], which require the jumping parameter to be available for the filter implementation.

The filtering problem to be addressed is to obtain an estimate, $\hat{s}(k)$, of $s(k)$ via a causal *mode-independent* linear filter which provides a uniformly small estimation error, $\tilde{s}(k) := s(k) - \hat{s}(k)$, for all $w \in \ell_2$. Attention is focused on the design of a linear time-invariant, asymptotically stable, filter of order n with state-space realization

$$(\mathcal{F}):\ \hat{x}(k+1) = A_f\hat{x}(k) + B_f y(k), \quad \hat{x}(0) = 0$$
$$\hat{s}(k) = C_f\hat{x}(k) \qquad (4)$$

where the matrices $A_f \in \Re^{n \times n}$, $B_f \in \Re^{n \times n_y}$ and $C_f \in \Re^{n_s \times n}$ are to be found.

It follows from (1)-(4) that the dynamics of the estimation error, $\tilde{s}(k)$, can be described by the following state-space model

$$(\mathcal{S}_e):\ \xi(k+1) = \tilde{A}(\theta_k)\xi(k) + \tilde{B}_e(\theta_k)w(k) \qquad (5)$$
$$\tilde{s}(k) = \tilde{C}(\theta_k)\xi(k) \qquad (6)$$

where

$$\tilde{A}(\theta_k) = \begin{bmatrix} A(\theta_k) & 0 \\ B_f C(\theta_k) & A_f \end{bmatrix}, \quad \tilde{B}(\theta_k) = \begin{bmatrix} B(\theta_k) \\ B_f D(\theta_k) \end{bmatrix},$$
$$\tilde{C}(\theta_k) = \begin{bmatrix} L(\theta_k) & -C_f \end{bmatrix}, \ \ \xi = \begin{bmatrix} x^T & \hat{x}^T \end{bmatrix}^T.$$

In order to put the $\mathcal{H}_\infty$ filtering problem for system $(\mathcal{S})$ in a stochastic setting, let the space $\ell_2[(\Omega, \mathcal{F}, \mathbb{P})]$ of $\mathcal{F}$-measurable sequences $\{\tilde{s}(k)\}$ for which

$$\| \tilde{s} \|_2 := \left\{ \mathbf{E}\left[\sum_{k=0}^{\infty} \tilde{s}^T(k)\tilde{s}(k) \right] \right\}^{\frac{1}{2}} < \infty$$

where $\mathbf{E}[\cdot]$ stands for mathematical expectation. For the sake of simplifying the notation, $\|\cdot\|_2$ will be used to denote the norm either in $\ell_2[(\Omega, \mathcal{F}, \mathbb{P})]$ or in ℓ_2, the later defined by

$$\|w\|_2 := \left[\sum_{k=0}^{\infty} w^T(k)w(k) \right]^{\frac{1}{2}}, \quad \text{for } \ w \in \ell_2.$$

Next, we recall the notion of *internal mean square stability* and a related result.

Definition 2.1 *System $(\mathcal{S})$ is said to be internally mean square stable (IMSS), if the solution to the stochastic difference equation*

$$x(k+1) = A(\theta_k)x(k)$$

is such that $E[\|x(k)\|^2] \to 0$, as $k \to \infty$ for any finite initial condition $x(0) \in \Re^n$ and $\theta_0 \in \Xi$.

Lemma 2.1 ([14]) *System $(\mathcal{S})$ is IMSS if and only if there exist matrices $P_i > 0$, $i = 1, \ldots, N$, satisfying the following inequalities:*

$$P_i - A_i^T \left(\sum_{j=1}^{N} \lambda_{ij} P_j \right) A_i > 0, \quad i = 1, \ldots, N.$$

This paper is concerned with the following $\mathcal{H}_\infty$ filtering problem for system $(\mathcal{S})$:

Given a scalar $\gamma > 0$, design an asymptotically stable filter (4) which ensures that the estimation error system $(\mathcal{S}_e)$ is IMSS and its ℓ_2-induced gain, i.e. $\mathcal{H}_\infty$ norm, is less than γ, namely:

$$\|\mathcal{S}_e\|_\infty := \sup_{w \in \ell_2} \left\{ \frac{\|\tilde{s}\|_2}{\|w\|_2};\ w \not\equiv 0,\ \xi(0) = 0 \right\} < \gamma. \qquad (7)$$

We conclude this section by recalling a version of the bounded real lemma (BRL) for discrete-time MJL systems. To this end, let the following system:

$$(\mathcal{S}_1):\quad x(k+1) = A(\theta_k)x(k) + B(\theta_k)w(k) \qquad (8)$$
$$z(k) = C(\theta_k)x(k) \qquad (9)$$

where $x(k) \in \Re^n$, $w(k) \in \Re^{n_w}$ and $z(k) \in \Re^{n_z}$, whereas θ_k and the matrices $A(\theta_k), B(\theta_k)$ and $C(\theta_k)$ are as in system $(\mathcal{S})$. Then, we have the following LMI based BRL which can be readily derived from a result derived in [3] and given in terms of coupled algebraic Riccati equations.

Lemma 2.2 *Given system $(\mathcal{S}_1)$ and a scalar $\gamma > 0$, the following conditions are equivalent:*

(a) System $(\mathcal{S}_1)$ is IMSS and $\|\mathcal{S}_1\|_\infty < \gamma$.

(b) There exist matrices P_i, $i = 1, \dots, N$, satisfying the following LMIs:

$$\begin{bmatrix} P_i & A_i^T \bar{P}_i & 0 & C_i^T \\ \bar{P}_i A_i & \bar{P}_i & \bar{P}_i B_i & 0 \\ 0 & B_i^T \bar{P}_i & \gamma I & 0 \\ C_i & 0 & 0 & \gamma I \end{bmatrix} > 0, \ i = 1, \dots, N \quad (10)$$

where

$$\bar{P}_i = \sum_{j=1}^{N} \lambda_{ij} P_j, \quad i = 1, \dots, N.$$

Further, $V(x(k), \theta_k) = x^T(k) P(\theta_k) x(k)$, where $P(\theta_k) = P_i$ when $\theta_k = i$, $i = 1, \dots, N$, is a stochastic Lyapunov function for the unforced system of $(\mathcal{S}_1)$.

3 $\mathcal{H}_\infty$ Filter Design

First, we present an alternative version of Lemma 2.2. This new version of the BRL was motivated by the work of [5] and has the feature that the products $\bar{P}_i A_i$ and $\bar{P}_i B_i$ that appear in (10) are replaced by $G_i A_i$ and $G_i B_i$, respectively, where G_i is a slack matrix variable. The advantages of introducing such over-parameterization was first unveiled in [5] in the context of robust stability of linear discrete time-invariant systems. The novelty here is that the result is stated in the more general context of MJL systems. This BRL will be fundamental in the derivation of the mode-independent $\mathcal{H}_\infty$ filters proposed in this paper.

Theorem 3.1 *Given system $(\mathcal{S}_1)$ and a scalar $\gamma > 0$, the following conditions are equivalent:*

(a) System $(\mathcal{S}_1)$ is IMSS and $\|\mathcal{S}_1\|_\infty < \gamma$.

(b) There exist matrices P_i and G_i, $i = 1, \dots, N$, satisfying the following LMIs:

$$\begin{bmatrix} P_i & A_i^T G_i^T & 0 & C_i^T \\ G_i A_i & G_i + G_i^T - \bar{P}_i & G_i B_i & 0 \\ 0 & B_i^T G_i^T & \gamma I & 0 \\ C_i & 0 & 0 & \gamma I \end{bmatrix} > 0, \ i = 1, \dots, N \quad (11)$$

where

$$\bar{P}_i = \sum_{j=1}^{N} \lambda_{ij} P_j, \quad i = 1, \dots, N. \quad (12)$$

Further, $V(x(k), \theta_k) = x^T(k) P(\theta_k) x(k)$, where $P(\theta_k) = P_i$ when $\theta_k = i$, $i = 1, \dots, N$, is a stochastic Lyapunov function for the unforced system of $(\mathcal{S}_1)$.

Proof. $(a) \Rightarrow (b)$: By Lemma 2.2, there exist matrices $P_i > 0$, $i = 1, \dots, N$, satisfying (10). Then, it follows immediately that (11) holds with the same matrix P_i and with $G_i = \bar{P}_i$, $i = 1, \dots, N$.

$(b) \Rightarrow (a)$: Pre- and post multiplying (11) by

$$\begin{bmatrix} I & -A_i^T & 0 & 0 \\ 0 & -B_i^T & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix}$$

and its transpose, respectively leads to

$$\begin{bmatrix} P_i - A_i^T \bar{P}_i A_i & -A_i^T \bar{P}_i B_i & C_i^T \\ -B_i^T \bar{P}_i A_i & \gamma I - B_i^T \bar{P}_i B_i & 0 \\ C_i & 0 & \gamma I \end{bmatrix} > 0, \ i = 1, \dots, N$$

and the proof follows immediately by applying Schur's complement and considering Lemma 2.2. $\nabla\nabla\nabla$

Theorem 3.1 is equivalent to Lemma 2.2 and thus has no advantages over that lemma when used as a necessary and sufficient condition for ensuring an upper-bound on $\|\mathcal{S}_1\|_\infty$. However, due to the extra degrees of freedom introduced by the variables G_i, Theorem 3.1 offers potential advantages over Lemma 2.2 when used as a sufficient condition for $\|\mathcal{S}_1\|_\infty < \gamma$, including allowing for the design of a mode-independent filter.

Note that in the light of Lemma 2.1, Theorem 3.1 includes the following new result on internal mean square stability for discrete-time MJL systems.

Lemma 3.1 *System $(\mathcal{S}_1)$ is IMSS if and only if there exist matrices P_i and G_i, $i = 1, \dots, N$, satisfying the following LMIs:*

$$\begin{bmatrix} P_i & A_i^T G_i^T \\ G_i A_i & G_i + G_i^T - \sum_{j=1}^{N} \lambda_{ij} P_j \end{bmatrix} > 0, \ i = 1, \dots, N.$$

Further, $V(x(k), \theta_k) = x^T(k) P(\theta_k) x(k)$, where $P(\theta_k) = P_i$ when $\theta_k = i$, $i = 1, \dots, N$, is a stochastic Lyapunov function for the unforced system of $(\mathcal{S}_1)$.

Observe that in the case of one mode operation, Lemma 3.1 reduces to the stability result of [5] in the context of linear time-invariant discrete-time systems.

The next theorem presents the mode-independent $\mathcal{H}_\infty$ filter design, which is based on conditions (11) of Theorem 3.1 with the constraints $G_i = G$, $i = 1, \dots, N$.

Theorem 3.2 *Consider system $(\mathcal{S})$ and let $\gamma > 0$ be a given scalar. There exists an n-th order filter (4) which ensures that the estimation error system $(\mathcal{S}_e)$ is IMSS and $\|\mathcal{S}_e\|_\infty < \gamma$ if there exist matrices Q, S, Y, Z and $X_i \in \Re^{n \times n}$, $i = 1, \dots, N$, $K \in \Re^{n \times n_y}$ and $R \in \Re^{n_z \times n}$ satisfying the following LMIs:*

$$\begin{bmatrix} \mathcal{X}_i & \mathcal{A}_i^T & 0 & \mathcal{C}_i^T \\ \mathcal{A}_i & \Upsilon - \bar{\mathcal{X}}_i & \mathcal{B}_i & 0 \\ 0 & \mathcal{B}_i^T & \gamma I & 0 \\ \mathcal{C}_i & 0 & 0 & \gamma I \end{bmatrix} > 0, \ \ i = 1, \dots, N \quad (13)$$

where

$$\bar{\mathcal{X}}_i = \sum_{j=1}^{N} \lambda_{ij} \mathcal{X}_j, \quad i = 1, \dots, N, \quad (14)$$

$$\mathcal{A}_i=\begin{bmatrix} ZA_i & ZA_i \\ YA_i+KC_i+Q & YA_i+KC_i \end{bmatrix},\ \mathcal{B}_i=\begin{bmatrix} ZB_i \\ YB_i+KD_i \end{bmatrix} \quad (15)$$

$$\mathcal{C}_i=\begin{bmatrix} L_i-R & L_i \end{bmatrix},\ \Upsilon=\begin{bmatrix} Z+Z^T & Z+Y^T+S^T \\ Z^T+Y+S & Y+Y^T \end{bmatrix}. \quad (16)$$

Moreover, the transfer function matrix of a suitable filter is given by

$$H_{\hat{s}y}(z) = R(zI_n - S^{-1}Q)^{-1}S^{-1}K. \quad (17)$$

Proof. It will be shown that if the LMIs (13) hold, then the filter (17) ensures that the estimation error system $(\mathcal{S}_e)$ satisfies the inequalities (11) of Theorem 3.1 with $G_i = G$, $i = 1, \ldots, N$ to be defined as below.

First, we shall prove that the matrices S, Y and Z are nonsingular. To this end, note that (13) ensures that $X_i > 0$ and $\Upsilon > 0$, which implies that Z and Y are nonsingular matrices. Further, pre- and post-multiplying Υ by $[I \ \ -I]$ and its transpose, respectively, implies that $S+S^T < 0$, and thus S is nonsingular.

Inspired by [13], define nonsingular matrices U and V such that $S = VU^TZ^T$ and introduce the following nonsingular matrices

$$G^T=\begin{bmatrix} Y^T & \bullet \\ V^T & \bullet \end{bmatrix},\quad G^{-T}=\begin{bmatrix} Z^{-T} & \bullet \\ U^T & \bullet \end{bmatrix},\quad \mathcal{T}=\begin{bmatrix} Z^T & Y^T \\ 0 & V^T \end{bmatrix} \quad (18)$$

where the elements $\bullet$ are uniquely determined from the equalities $G^TG^{-T} = G^{-T}G^T = I$. Further, define the following state-space realization for the filter (17)

$$A_f = V^{-1}QS^{-1}V,\ \ B_f = V^{-1}K,\ \ C_f = RS^{-1}V. \quad (19)$$

Considering the matrices in (15), (16), (18) and (19), it can be readily verified that

$$\begin{aligned} &\mathcal{A}_i = \mathcal{T}^T\tilde{A}_iG^{-T}\mathcal{T},\ \ \mathcal{B}_i = \mathcal{T}^T\tilde{B}_i, \\ &\mathcal{C}_i = \tilde{C}_iG^{-T}\mathcal{T},\ \ \Upsilon = \mathcal{T}^TG^{-1}(G+G^T)G^{-T}\mathcal{T} \end{aligned} \quad (20)$$

where $\tilde{A}_i$, $\tilde{B}_i$ and $\tilde{C}_i$ denote the matrices $\tilde{A}(\theta_k)$, $\tilde{B}(\theta_k)$ and $\tilde{C}(\theta_k)$, respectively, for $\theta_k = i$.

Next, post- and pre-multiplying (13) by $\mathcal{J} = \mathrm{diag}\{\mathcal{T}^{-1}G^T, \mathcal{T}^{-1}G^T, I_{n_w}, I_{n_y}\}$ and $\mathcal{J}^T$, respectively, and considering (20), it can be established that the LMIs of (13) are equivalent to:

$$\begin{bmatrix} P_i & \tilde{A}_i^TG^T & 0 & \tilde{C}_i^T \\ G\tilde{A}_i & G+G^T-\bar{P}_i & G\tilde{B}_i & 0 \\ 0 & \tilde{B}_i^TG^T & \gamma I & 0 \\ \tilde{C}_i & 0 & 0 & \gamma I \end{bmatrix} > 0, \quad i=1,\ldots,N$$

where

$$P_i = G\mathcal{T}^{-T}X_i\mathcal{T}^{-1}G^T,\quad \bar{P}_i = \sum_{j=1}^{N}\lambda_{ij}P_j,\ i=1,\ldots,N.$$

The result then follows from Theorem 3.1. ∇∇∇

Remark 3.1 Theorem 3.2 provides an LMI method for designing a *mode-independent* $\mathcal{H}_\infty$ linear filter for discrete-time MJL systems with a non-accessible jumping parameter. This is contrast with [7] which has developed *mode-dependent* $\mathcal{H}_\infty$ filters for MJL systems under the assumption that the jumping parameter is accessible. □

Note that in the case of one mode operation, i.e. when there are no jumps in system $(\mathcal{S})$, Theorem 3.2 reduces to the $\mathcal{H}_\infty$ filtering result of [13], which is indeed necessary and sufficient for the problem solvability.

Theorem 3.2 can be easily extended to the case where the transition probability matrix $\Lambda = [\lambda_{ij}]$ is unknown, but belongs to a given polytope, namely $\Lambda \in \mathcal{P}_\Lambda$, where $\mathcal{P}_\Lambda$ is a polytope with vertices Λ_i, $i = 1, \ldots, \Gamma_\Lambda$, i.e.

$$\mathcal{P}_\Lambda := \left\{ \Lambda \mid \Lambda = \sum_{r=1}^{\Gamma_\Lambda}\alpha_r\Lambda_r;\ \ \alpha_r \geq 0,\ \sum_{r=1}^{\Gamma_\Lambda}\alpha_r = 1 \right\} \quad (21)$$

where $\Lambda_r = [\lambda_{ij}^{(r)}], i,j = 1,\ldots,N,\ r = 1,\ldots,\Gamma_\Lambda$ are given transition probability matrices. It should be noted that the convex hull of transition probability matrices is also a transition probability matrix. In this setting, and considering that the LMIs in (13) are affine in the transition probabilities λ_{ij}, the following robust $\mathcal{H}_\infty$ filtering result follows readily from Theorem 3.2.

Theorem 3.3 *Consider system $(\mathcal{S})$ with an uncertain transition probability matrix Λ belonging to the polytope $\mathcal{P}_\Lambda$ and let $\gamma > 0$ be a given scalar. There exists an n-th order filter (4) which ensures that the estimation error system $(\mathcal{S}_e)$ is IMSS and $\|\mathcal{S}_e\|_\infty < \gamma$ over the polytope $\mathcal{P}_\Lambda$ if there exist matrices Q, S, Y, Z and $X_i \in \Re^{n\times n}$, $i = 1, \ldots, N$, $K \in \Re^{n\times n_y}$ and $R \in \Re^{n_s\times n}$ satisfying the following LMIs:*

$$\begin{bmatrix} X_i & \mathcal{A}_i^T & 0 & \mathcal{C}_i^T \\ \mathcal{A}_i & \Upsilon-\bar{X}_i^{(r)} & \mathcal{B}_i & 0 \\ 0 & \mathcal{B}_i^T & \gamma I & 0 \\ \mathcal{C}_i & 0 & 0 & \gamma I \end{bmatrix} > 0,\quad \begin{cases} i = 1,\ldots,N \\ r = 1,\ldots,\Gamma_\Lambda \end{cases}$$

where

$$\bar{X}_i^{(r)} = \sum_{j=1}^{N}\lambda_{ij}^{(r)}X_j,\ \ i=1,\ldots,N,\ r=1,\ldots,\Gamma_\Lambda$$

and the matrices $\mathcal{A}_i$, $\mathcal{B}_i$, $\mathcal{C}_i$ and Υ are as defined in (15) and (16). Moreover, the transfer function matrix of a suitable filter is given by (17).

As the LMIs in (13) are affine in the matrices A_i, B_i, C_i, D_i and L_i, $i = 1, \ldots, N$, of system $(\mathcal{S})$, another extension of Theorem 3.2 is to the case where these matrices are unknown, but belong to given polytopes $\mathcal{P}_i$, $i = 1, \ldots, N$, described by:

$$\mathcal{P}_i := \Big\{ (A_i, B_i, C_i, D_i, L_i) \ \mid\ (A_i, B_i, C_i, D_i, L_i)$$

$$= \sum_{j=1}^{\Gamma_i} \alpha_{ij}\Big(A_i^{(j)}, B_i^{(j)}, C_i^{(j)}, D_i^{(j)}, L_i^{(j)}\Big);\ \alpha_{ij} \geq 0,\ \sum_{j=1}^{\Gamma_i} \alpha_{ij} = 1\Big\}$$

where $A_i^{(j)}$, $B_i^{(j)}$, $C_i^{(j)}$, $D_i^{(j)}$, $L_i^{(j)}$ are given matrices. In this situation, the following robust $\mathcal{H}_\infty$ filter result is obtained:

Theorem 3.4 *Consider system $(\mathcal{S})$ with the matrices A_i, B_i, C_i, D_i and L_i belonging to the polytope $\mathcal{P}_i$, $i = 1, \ldots, N$ and let $\gamma > 0$ be a given scalar. There exists an n-th order filter (4) which ensures that the estimation error system $(\mathcal{S}_e)$ is IMSS and $\|\mathcal{S}_e\|_\infty < \gamma$ over the polytopes $\mathcal{P}_i$, $i = 1, \ldots, N$, if there exist matrices Q, S, Y, Z and $X_i \in \Re^{n\times n}$, $i = 1, \ldots, N$, $K \in \Re^{n\times n_y}$ and $R \in \Re^{n_s \times n}$ satisfying the following LMIs:*

$$\begin{bmatrix} X_i & (\mathcal{A}_i^{(r)})^T & 0 & (\mathcal{C}_i^{(r)})^T \\ \mathcal{A}_i^{(r)} & \Upsilon - \bar{X}_i & \mathcal{B}_i^{(r)} & 0 \\ 0 & (\mathcal{B}_i^{(r)})^T & \gamma I & 0 \\ \mathcal{C}_i^{(r)} & 0 & 0 & \gamma I \end{bmatrix} > 0, \ \begin{cases} i = 1, \ldots, N \\ r = 1, \ldots, \Gamma_i \end{cases}$$

where $\bar{X}_i$ and Υ are as defined in (14) and (16), respectively, and $\mathcal{A}_i^{(r)}$, $\mathcal{B}_i^{(r)}$ and $\mathcal{C}_i^{(r)}$ are obtained from the matrices $\mathcal{A}_i$, $\mathcal{B}_i$ and $\mathcal{C}_i$ given in (15) and (16) with A_i, B_i, C_i, D_i and L_i replaced by $A_i^{(r)}$, $B_i^{(r)}$, $C_i^{(r)}$, $D_i^{(r)}$ and $L_i^{(r)}$, respectively. Moreover, the transfer function matrix of a suitable filter is given by (17).

Remark 3.2 It should be noted that when the jumping parameter is available, Theorem 3.2 can be extended to allow for the design of a MJL filter, namely, a mode-dependent filter with a state-space representation of the form:

$$(\tilde{\mathcal{F}}):\ \hat{x}(k+1) = \tilde{A}_f(\theta_k)\hat{x}(k) + \tilde{B}_f(\theta_k)y(k),\ \hat{x}(0) = 0$$
$$\hat{s}(k) = \tilde{C}_f(\theta_k)\hat{x}(k)$$

Indeed, it follows from the proof of Theorem 3.2 that this problem can be solved with the Theorem 3.2 by replacing the matrices K, Q, R, S, Y and Z appearing in the LMIs in (13) by K_i, Q_i, R_i, S_i, Y_i and Z_i, respectively. This corresponds to design the filter based on Theorem 3.1 with a matrix G_i with the same structure as G of (18). In this context, the matrices of a suitable filter are given by:

$$\tilde{A}_f(\theta_k) = S_i^{-1}Q_i, \quad \tilde{B}_f(\theta_k) = S_i^{-1}K_i,$$
$$\tilde{C}_f(\theta_k) = R_i, \quad \text{when } \theta_k = i$$

Note that in this case the conditions of Theorem 3.2 turn out to be necessary and sufficient and this theorem is equivalent to the main result of [7]. Similar remark also applies to the robust $\mathcal{H}_\infty$ filter designs of Theorems 3.3 and 3.4, except that now these theorems only provide sufficient conditions for the solvability of the underlying robust $\mathcal{H}_\infty$ filtering problems. It should be remarked that, to the best of the author's knowledge, to-date there is no method available in the literature to solve those robust filtering problems. □

4 Examples

Two examples are presented to illustrate the applicability of the $\mathcal{H}_\infty$ filter designs of Theorems 3.2–3.4, as well as their extended versions for the design of Markovian jump linear filters as described in Remark 3.2.

Example 1 Consider system $(\mathcal{S})$ of (1)-(3) with two operating modes described by:

$$A_1 = \begin{bmatrix} 0 & -0.5 \\ 1 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & -0.33 \\ 1 & 1.4 \end{bmatrix},$$
$$B_1 = B_2 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0 \end{bmatrix}, \quad C_1 = C_2 = \begin{bmatrix} 1 & 0 \end{bmatrix},$$
$$D_1 = D_2 = \begin{bmatrix} 0 & 1 \end{bmatrix}, \quad L_1 = L_2 = \begin{bmatrix} 0 & 1 \end{bmatrix}.$$

Regarding the transition probability matrix Λ, two cases are considered:

Case 1: Λ is exactly known and given by:

$$\Lambda = \begin{bmatrix} 0.8 & 0.2 \\ 0.4 & 0.6 \end{bmatrix}.$$

The minimum upper-bound γ on the $\mathcal{H}_\infty$ norm of the estimation error system obtained with Theorem 3.2 is $\gamma = 4.5955$ and the corresponding filter matrices are:

$$A_f = \begin{bmatrix} -0.21725 & 1.00163 \\ 0.25076 & 0.47498 \end{bmatrix}, \quad B_f = \begin{bmatrix} -1.14367 \\ -0.38725 \end{bmatrix},$$
$$C_f = \begin{bmatrix} 2.23286 & -3.54392 \end{bmatrix}.$$

It should be observed that if the jumping parameter is assumed to be accessible, the minimum γ that can be achieved with a mode-dependent linear filter (see Remark 3.2) is $\gamma = 3.8704$. Note that, as expected, the use of a mode-independent filter (due to the non-accessibility of the jumping parameter) increases the minimum achieved noise attenuation level γ as compared to a Markovian jump filter. For this example, the noise attenuation level has increased by 18.7%.

Case 2: Λ is unknown, but belongs to a polytope $\mathcal{P}_\Lambda$, as defined in (21), with 2 vertices given by:

$$\Lambda_1 = \begin{bmatrix} 0.75 & 0.25 \\ 0.5 & 0.5 \end{bmatrix}, \quad \Lambda_2 = \begin{bmatrix} 0.85 & 0.15 \\ 0.3 & 0.7 \end{bmatrix}.$$

The minimum γ obtained with the robust $\mathcal{H}_\infty$ filter design of Theorem 3.3 is $\gamma = 9.8247$ and the corresponding filter matrices are:

$$A_f = \begin{bmatrix} -1.17279 & 2.31615 \\ -0.35194 & 1.19390 \end{bmatrix}, \quad B_f = \begin{bmatrix} -1.86193 \\ -0.57012 \end{bmatrix},$$
$$C_f = \begin{bmatrix} 2.23975 & -4.64617 \end{bmatrix}.$$

If the jumping parameter is assumed to be accessible, the minimum γ achieved with the robust Markovian jump linear filter obtained from Theorem 3.3 as described in Remark 3.2 is $\gamma = 7.09456$.

Example 2 Let the system considered in Example 1/Case 1, except that now the $(1,2)$-elements of the matrices A_1 and A_2 are unknown, but lie in the intervals $[-0.6,\ -0.45]$ and $[-0.4,\ -0.3]$, respectively, namely A_1 belongs to a polytope $\mathcal{P}_1$ with vertices

$$A_1^{(1)} = \begin{bmatrix} 0 & -0.6 \\ 1 & 1 \end{bmatrix}, \quad A_1^{(2)} = \begin{bmatrix} 0 & -0.45 \\ 1 & 1 \end{bmatrix}$$

whereas A_2 belongs to a polytope $\mathcal{P}_2$ having the following vertices:

$$A_2^{(1)} = \begin{bmatrix} 0 & -0.4 \\ 1 & 1.4 \end{bmatrix}, \quad A_2^{(2)} = \begin{bmatrix} 0 & -0.3 \\ 1 & 1.4 \end{bmatrix}.$$

The minimum γ obtained with the robust $\mathcal{H}_\infty$ filter design of Theorem 3.4 is $\gamma = 6.9945$ and the corresponding filter matrices are:

$$A_f = \begin{bmatrix} -0.48258 & 1.14598 \\ 0.14106 & 0.59867 \end{bmatrix}, \quad B_f = \begin{bmatrix} -0.91105 \\ -0.38173 \end{bmatrix},$$

$$C_f = \begin{bmatrix} 3.22120 & -4.90868 \end{bmatrix}.$$

Note that when the jumping parameter is assumed to be accessible, the minimum γ achieved with the mode-dependent $\mathcal{H}_\infty$ filter of Theorem 3.4 as described in Remark 3.2 is $\gamma = 4.7505$.

5 Conclusions

This paper has addressed the problem of $\mathcal{H}_\infty$ filtering for a class of discrete-time MJL systems where the jumping parameter is not accessible. An LMI method is proposed for designing a *mode-independent* filter that ensures the mean square stability of the estimation error dynamics and a prescribed upper-bound on the ℓ_2-induced gain from the noise signals to the estimation error. Robust $\mathcal{H}_\infty$ filtering problems where either the transition probability matrix of the Markov chain, or the matrices of the system state-space model, are uncertain but belong to given polytopes are also treated. The proposed filter design methods are also extended to the case where the jumping parameter is accessible and a Markovian jump (mode-dependent) linear filter is sought. A new internal mean square stability condition as well as a bounded real lemma for discrete-time MJL systems are also developed. The technique of this paper has also been recently adapted for designing mode-independent $\mathcal{H}_2$ control and filter for discrete-time MJL systems with a non-accessible jumping parameter [6].

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