Jump Linear Quadratic Gaussian Control in Continuous Time

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***Abstract*—This paper is concerned with the optimal quadratic control of continuous-time linear systems that possess randomly jumping parameters which can be described by finite-state Markov processes. The systems are also subject to Gaussian input and measurement noise. In this paper, the optimal solution for the jump linear quadratic Gaussian (JLQG) problem is given. This solution is based on a separation theorem. The optimal state estimator is sample path dependent (it may depend upon past as well as the current values of the jump parameter). If the plant parameters are constant in each value of the underlying jumping process, then the controller portion of the compensator converges to a time invariant control law. However, the filter portion of the optimal infinite time horizon JLQG compensator is not time invariant. Thus, a suboptimal filter which does converge to a steady-state solution (under certain conditions) is derived and a time-invariant compensator is obtained.**

I. Introduction Problem Formulation

This paper extends the jump linear quadratic control problem results of [21] to the case of additive Gaussian white noise on the input and output. We consider jump linear systems described by

$$\begin{aligned}\dot{x}(t) &= A(t,r(t))x(t) + B(t,r(t))u(t) + F(t,r(t))w(t)\\ y(t) &= C(t,r(t))x(t) + G(t,r(t))v(t) \qquad (1)\end{aligned}$$

where $x(t) \in \mathbb{R}^n$ is the x-process (plant) state, $u(t) \in \mathbb{R}^m$ is the x-process input, $y(t) \in \mathbb{R}^p$ is the x-process output (observation), $w(t) \in \mathbb{R}^{n1}$ and $v(t) \in \mathbb{R}^{n2}$ are independent Gaussian white noises. The parameter $r(t)$ represents a continuous-time discrete-state Markov process taking values in a finite set $\mathbb{S} = \{1, 2, \cdots, s\}$ with transition probability $Pr(r(t+\Delta) = j | r(t) = i)$ given by

$$Pr(r(t+\Delta) = j | r(t) = i) = \begin{cases}\lambda_{ij}(t)\Delta + o(\Delta) & \text{if } i \neq j\\ 1 + \lambda_{ii}(t)\Delta + o(\Delta) & \text{if } i = j.\end{cases} \qquad (2)$$

Here $\lambda_{ij}(t) \geq 0$ is the transition probability rate from i to j $(i \neq j)$, and

$$\lambda_i(t) \triangleq -\lambda_{ii}(t) = \sum_{j=1, j\neq i}^{s} \lambda_{ij}(t)$$

as in [44]. Note that $|\lambda_{ij}(t)| < \infty$ $(\forall t)$ because $r(t)$ is a finite state Markov process [12]. Also note that (2) can be replaced by the transition probability matrix $Pr(r(t+\Delta) = j | r(t) = i)\} = \exp(\{\lambda_{ij}(t)\Delta\})$ ([44]) because here $\mathbb{S}$ is a finite set. The functions $r(t)$ are constant except for a finite number of simple jumps in any finite subinterval of $[t_0, T]$. We call the elements in the set $\mathbb{S}$ the "forms" of this dynamic system as in [11]. For systems (1) and (2), the underlying probability space $(\Omega, \mathscr{F}, P)$ can be constructed. $A(t, r(t))$, $B(t, r(t))$, $C(t, r(t))$, $F(t, r(t))$, and $G(t, r(t))$ are appropriately dimensioned matrices. These matrices are deterministic time functions for given $r(t)$ value. The matrices $A(t,i)$, $B(t,i)$, $C(t,i)$, $F(t,i)$, $G(t,i)$, $R(t,i)$, and $Q(t,i)$ are continuous in t for any fixed $r(t) = i \in \mathbb{S}$. We assume $\lambda_{ij}(t)$ are continuous on $t \in [t_0, T]$. Note that the jumps of $r(t)$ occur at a finite number points on any finite subinterval of $[t_0, T]$. Thus, the discontinuous points of $r(t)$ on $t \in [t_0, T]$ constitute, at most, a zero measure set. It is assumed that $x(t_0)$ $(-\infty < t_0 < +\infty)$ is Gaussian, with mean and covariance

$$E\{x(t_0)\} = \bar{x}_0 \qquad E\{[x(t_0) - \bar{x}_0][x(t_0) - \bar{x}_0]'\} = \bar{P}_0. \qquad (3.a)$$

The noises $w(t), v(t)$ are zero mean, white Gaussian processes with

$$\begin{aligned}E\{w(t)w'(\tau)\} &= \delta(t-\tau)I\\ E\{v(t)v'(\tau)\} &= \delta(t-\tau)I \qquad (3.b)\end{aligned}$$

where $\delta(t)$ is the Dirac delta function. We further assume that $x(t_0)$, $w(t)$, and $v(t)$ are mutually independent. Consider $(\mathscr{F}_t, S_t)$, the σ subalgebra induced by the observations $\{y(\tau), r(\tau)\colon \tau \in [t_0, t]\}$ where $\mathscr{F}_t$ is the σ algebra induced by the values of the x-process $\{x(\tau)\colon \tau \in [t_0, t]\}$ and S_t is the algebra induced by the values of the form process $\{r(\tau)\colon r \in [t_0, t]\}$. Subject to (1)–(3), we want to minimize

$$J = E\left\{x'(T)K_T(r(T))x(T) + \int_{t_0}^{T}[x'(t)Q(t,r(t))x(t) + u'(t)R(t,r(t))u(t)]\,dt\right\} \qquad (4)$$

by choosing control $u(t)$ over admissible (form dependent, output feedback) control laws

$$u(t) = \psi(t, y(\tau), r(\tau)\colon \tau \leq t)$$
$$\psi\colon [t_0, T] \times \mathbb{R}^p \times \mathbb{S} \to \mathbb{R}^m$$

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where $\psi \in \Psi$, the class of measurable functions on $(\mathscr{F}_t, S_t)$, satisfies the following local Lipschitz and linear growth conditions as in [44]: for some constant k (depending on ψ)

$$\|\psi(t,y,r) - \psi(t,\tilde{y},r)\| < k\|y - \tilde{y}\|,$$
$$\|\psi(t,y,r)\| < k(1 + \|y\|)$$

for all t, y, $\tilde{y}$, and r. Here $\|x\| = (x_1^2 + x_2^2 + \cdots + x_n^2)^{1/2}$ is the Euclidean norm of vector x and $\|M\|$ = (the square root of the largest eigenvalue of $M'M$) is the corresponding operator norm of a matrix M. The cost weighting matrices R, K_T and Q are real-valued and symmetric, with R positive definite and K_T and Q positive semidefinite for each $t \in [t_0, T]$. They are continuous for each value of $r(t)$ on $[t_0, T]$. The optimal control problem (1)–(4) is the *jump linear quadratic Gaussian* (JLQG) problem.

We assume here that the values of $r(t)$ are available at each time t. This *perfect form observation* assumption is crucial, since it allows us to avoid the more difficult and generally unsolved "dual control" problem (see, e.g., [9] and [16]). With this assumption, if there is no noise (i.e., $w = v = 0$) in (1) then the system is linear. However, it is not deterministic. Feedback using the observed values of $r(t)$ will be crucial.

As in other engineering literature, we use the "white noise" (w, v terms) description in (1). It can be implemented to achieve an analog continuous-time simulation ([42], page 163). To derive the separation theorem, we will also use the more rigorous representation

$$dx = A(t,r(t))x(t)\,dt + B(t,r(t))u(t)\,dt + F(t,r(t))\,d\xi$$
$$dz = C(t,r(t))x(t)\,dt + G(t,r(t))\,d\zeta \qquad (1.a)$$

where $\xi \in \mathbb{R}^{n1}$ and $\zeta \in \mathbb{R}^{n2}$ are standard Weiner processes. Formally, if we take $d\xi(t)/dt = w(t)$, $d\zeta(t)/dt = v(t)$, and $dz(t)/dt = y(t)$, then (1) is equivalent to (1.a) because F and G are not dependent on x.

Note that the Markov process $\{x, r\}$ in problems (1)–(4) is a joint process, consisting of an n-dimensional diffusion and a step (jump) process. The problem formulations (1)–(4) are special cases of the (generally unsolved) problem of optimal control of Markov process.

Special cases of this general Markov process control problem have been solved. A general treatment of the optimal control of the diffusion process appears in Fleming and Rishel [14]. The solution of the most famous "special case" of the optimal control problem for diffusion processes, the linear quadratic Gaussian problem is well known (e.g., [3]). Menaldi and Robin [31] consider another special case of control problems for general Markov processes where the cost does not depend on control u, and jumps are additive to the x process. This is different from our case here, where the jumps are in the system parameters.

The study of jump linear systems can be traced back at least to [23]. Sworder [36] and Wonham [44] solved the jump linear Gaussian (JLQ) problem for finite time horizons. Wonham also solved the infinite time horizon problem (as $T \to \infty$), and derived a set of sufficient conditions for the existence of unique, finite steady-state solutions. Sworder and Robinson [38] considered problems with u- and x-dependent form transition probability rates, obtaining the nonlinear partial differential equation related to the optimal solution. Rishel [33] and [34] considered the optimal control of jump nonlinear systems where the cost may be nonquadratic. Kushner [24] applied an almost sure stability concept to the jump linear system. Examples of applications of jump linear systems (sometimes called "hybrid systems" because the x-state is continuous ($x \in \mathbb{R}^n$) while the value of r is discrete ($r \in \mathbb{S}$) are described in [1], [37] for manufacturing systems, [4] for aircraft control, [32] for target tracking, [35] for robotics, [39] for solar receiver control, and in [41] for power systems. Athans [5] suggested that this jump linear system model could potentially be used in *battle management command, control and communications* (BM/C^3) systems. Stochastic stabilizability, controllability, observability, and detectability properties of the continuous-time jump linear systems were developed in [21]. Based on these properties, necessary and sufficient conditions for the existence of a steady state, stabilizing solution of the infinite time JLQ problem were then obtained. Somewhat different recent results in the theory of continuous time jump linear systems appeared in [26], [27], and [28]. Results for optimal control of discrete time jump linear systems can be found in [7], [6], [11], and [17]–[20]. Blom [8] reviewed the jump linear system theory and studied the estimation problem. The recent monograph of Mariton [30] addresses some of these topics.

In Section II, we consider a separation theorem which allows us to design the optimal x-state estimator and optimal JLQ controller separately. Their combination is the optimal JLQG compensator. In Section III, time invariant (for each $r(t)$ value) jump linear Gaussian systems are considered. The equivalence between the optimal x-state estimation problem and the optimal quadratic control problem of an appropriately-defined algebraic dual system is established. Based on the duality properties and stabilizability and observability results, the infinite time horizon JLQG problem for time invariant systems is solved. Unlike the infinite time horizon controller, the optimal infinite time horizon filter generally will not converge to a steady-state version. Therefore, a suboptimal time invariant filter is derived in Section IV. It, together with the optimal infinite time horizon JLQ controller, yield a time invariant (suboptimal) compensator.

II. Problem Solution

In the JLQG problems (1)–(4), the exact value of the x state is not known. The first question is whether the control $u(t)$ can be expressed in such a way that it only depends on the expected value of the current $x(t)$, given

$(\mathscr{F}_t, S_t)$. If this is true, then we can design the optimal estimator (filter) and the optimal controller separately. The filter will give the optimal estimate of $x(t)$, based on $(\mathscr{F}_t, S_t)$, and the controller will give $u(t)$, based on the x estimate which is provided by the filter. For the JLQG problems (1)–(4) under the assumption that the value of $r(t)$ is available at each time t, this separation property holds.

Theorem 1 (Separation Theorem): Subject to an additional assumption that

$$G(t,i)G'(t,i) > 0 \qquad \forall\, i \in \mathbb{S}, \qquad \forall\, t \in [t_0, T] \quad (5)$$

the JLQG problems (1)–(4) have an optimal feedback control law $\psi \in \Psi$

$$u(t) = \psi(t, \hat{x}(t), r(t)) \qquad t \in [t_0, T]$$

which is a function of the observed $r(t)$ and

$$\hat{x}(t) = E\{x(t)|\mathscr{F}_t, S_t\}. \quad (6)$$

Moreover, the conditional expectation $\hat{x}(t)$ minimizes the conditional expected square error

$$E\{[x(t) - \hat{x}(t)]'[x(t) - \hat{x}(t)]|\mathscr{F}_t, S_t\}. \quad (7)$$

Sketch of Proof: To prove this separation theorem, the approach of Wonham ([43, theorem 2.1]) is followed. However, the result in [43] pertains to a diffusion process; here a more general type of Markov process is considered. In [14, page 154] a dynamic programming equation for general Markov process is formally established. We can show that it holds for problems (1)–(5) (a special case of the general Markov process). As in [14], the optimal cost-to-go $V(t,x,r)$ from state $(x(t), r(t))$ at time t is the solution of the following partial differential equation:

$$V_t(t,x,r) + \min_{u \in \Psi}\{\mathscr{A}^u(t)V(t,x,r) + \mathscr{L}(t,x,r,u)\} = 0. \quad (8)$$

Here $\mathscr{L}$ is the cost function [i.e., the integrand on the right-hand side of (4)] and $\mathscr{A}^u(t)$ is the differential generator for this Markov process, related to control u. The derivation of (8) in [14] is only formal. However, for (1)–(5), these equations can be properly established. First, for our system, let the differential generator $\mathscr{A}^u(t)$ related to a feedback law $u = -L(t, r(t))x(t)$ act on $V(t,x,r)$ [where $L(t, r(t))$ is continuous in t for any given $r(t) = i \in \mathbb{S}$). Then we have the following ([44, equation (2.29)]):

$$\begin{aligned}\mathscr{A}^u(t)&V(t,x,r(t)=i)\\ &= \tfrac{1}{2}\operatorname{tr}[F'(t,i)V_{xx}(t,x,i)F(t,x,i)]\\ &\quad + [A(t,i) - B(t,i)L(t,i)]'V_x(t,x,i)\\ &\quad + \sum_{j=1}^{1} \lambda_{ij}(t)V(t,x,j). \end{aligned} \quad (9)$$

Thus, we can say V is in the domain of $\mathscr{A}^u(t)$. In [14], formal arguments were used to obtain (8). These involved dividing an equation like (10) by $(\tau - t)$ and then letting $\tau \to t^+$ as follows:

$$0 \le E\left\{\int_t^\tau [V_t + \mathscr{A}V(\theta, r(\theta), x(\theta)) + x'(\theta)Q(\theta, r(\theta)) \cdot x(\theta) + u'R(\theta, r(\theta))u]\, d\theta | t, x(t)\right\}. \quad (10)$$

For the problem under consideration here, if we assume that V is the class of functions with continuous derivatives of first order in t on $[t_0, T]$ and first and second orders in x on $\mathbb{R}^n$, almost everywhere. Then by the approach of [14, theorem 4.1], (8) follows from the measurability properties of the integrand in (10). The integrand in (10) is continuous, except on a zero measure subset of $[t_0, T]$. Therefore, it is measurable ([15, page 42, (2.2)]) and thus (8) is valid almost surely ([15, page 49, (2.15)]). Note that for our problem, since $x(t)$ is only continuous with probability one ([44, page 143]) the separation property, like the optimality of JLQ solutions, almost surely holds.

Given this use of dynamic programming, the result of [43] can be applied analogously to establish separation. Note that (1)–(5) are LQG problems for a time varying system, with the exception that here our parameters A, B, C, F, G are unanticipative because of their dependence on $r(t)$. The basic Kalman filter result is valid for this case (as shown by Kalman and Bucy [22, section XII]. Thus, the estimator portion of the solution, a Kalman filter, is optimal almost surely because $r(t)$ has piecewise constant trajectory. Note that (5) is condition (A.2) of [43, theorem 2.1], which is needed to ensure the existence of the Kalman filter. From [43, section VII], we have (6) and (7). □

The following corollary is immediate from Theorem 1.

Corollary 1 (Optimal JLQG Compensator): For the JLQG problems (1)–(5), the optimal control is given by

$$u(t) = -L(t,i)\hat{x}(t) \quad (11)$$

where the control laws in each form $i \in \mathbb{S}$ are

$$L(t,i) = R^{-1}(t,i)B'(t,i)K(t,i) \quad (12)$$

for $t \in [t_0, T]$. The controller gains $\{K(t,i)\colon i \in \mathbb{S}\}$ are the solutions of the following coupled matrix differential Riccati equations:

$$\begin{aligned}\dot{K}(t,i) = &-A'(t,i)K(t,i) - K(t,i)A(t,i)\\ &- Q(t,i) - \sum_{j=1}^{s} \lambda_{ij}(t)K(t,j)\\ &+ K(t,i)B(t,i)R^{-1}(t,i)B'(t,i)K(t,i) \end{aligned} \quad (13)$$

with terminal conditions

$$K(T,i) = K_T(i) \qquad (i \in \mathbb{S}). \quad (14)$$

The estimate $\hat{x}(t)$ is generated by the following Kalman–Bucy filter

$$\frac{d\hat{x}(t)}{dt} = A(t, r(t))\hat{x}(t) + B(t, r(t))u(t) + \Gamma(t)[y(t) - C(t, r(t))\hat{x}(t)] \quad (15)$$

with initial condition

$$\hat{x}(t_0) = \bar{x}_0 \quad (16)$$

where the filter gain matrix Γ is given by

$$\Gamma(t) = P(t)C'(t,r(t))[G(t,r(t))G'(t,r(t))]^{-1} \quad (17)$$

and the conditional covariance $P(t)$ is obtained from the matrix differential Riccati equation

$$\begin{aligned}\dot{P}(t) = &A(t,r(t))P(t) + P(t)A'(t,r(t)) \\ &+ F(t,r(t))F'(t,r(t)) \\ &- P(t)C'(t,r(t))[G(t,r(t))G'(t,r(t))]^{-1} \\ &\cdot C(t,r(t))P(t) \end{aligned} \quad (18)$$

with initial condition

$$P(t_0) = \bar{P}_0. \quad (19)$$

□

Note that in (15)–(18), the instantaneously observed value of the form $r(t)$ is a known parameter. Also note that the filtering equation obtained for (1.a) is

$$\begin{aligned} d\hat{x}(t) = &A(t,r(t))\hat{x}(t)\,dt + B(t,r(t))u(t)\,dt \\ &+ \Gamma(t)[dz(t) - C(t,r(t))\hat{x}(t)\,dt] \end{aligned}$$

which yields (15).

For $\Gamma(t)$ and $P(t)$) calculations, two matrices are solved *forward in time* from t_0 to T. For the controller ($K(t,i)$: $i \in \mathbb{S}$) calculations, a set of s matrices are obtained by integrating *backwards in time* from T to t_0. The on-line filter calculations (17)–(19) involve a single vector equation for P, because $r(t)$ is known at each time t (although it is not known in advance). The unanticipative nature of $\{r(t)\}$ does not enter the filter parameter calculations. Thus, $P(t)$ and $\Gamma(t)$ depend on the sample path of $\{r(t)\}$ which has actually occurred during the system operation [not only the current value of $r(t)$]. Note that in the optimal filter, the transition probability rate $\lambda_{ij}(t)$ does not appear. This is unlike the suboptimal filters described in [29, section IV]. The control law parameters in (12)–(14) must be calculated for each form $i \in \mathbb{S}$. Since the values of the form process $\{r(t)\}$ are not known in advance, expected values are used in calculating the cost-to-go. Thus, the transition probability rate $\lambda_{ij}(t)$ enters into (13).

III. Infinite Time Horizon Problem

We next consider the infinite time horizon case. Our motivation is to obtain a steady state [i.e., constant for each value of $r(t)$] compensator that optimally controls the system. We now consider only the time invariant cases of (1)–(5), where the transition rates λ_{ij} are time invariant and the matrices A, B, C, F, G, Q, R are not explicitly dependent on t. For notational convenience let $A_i = A(r(t) = i)$, etc.

Because of the influence of noise, the cost J in (4) will grow without bound as T tends to infinity. Instead of (4), we consider the following cost

$$\bar{J} = \lim_{T\to\infty} E\left\{\frac{1}{T-t_0}\int_{t_0}^{T}[x'(t)Q(r(t))x(t) + u'(t)R(r(t))u(t)]\,dt\right\}. \quad (4a)$$

The desired result is the JLQG compensator which minimizes (4a). The infinite time horizon JLQG result is obtained by the following steps:

i) The infinite time JLQ problem is solved as in [21].

ii) Using duality arguments, the corresponding filter problem is solved for infinite time. This will be done below.

iii) The separation result (established in Section II) is then applied to obtain the overall compensator.

In the following discussion, the stabilizability and observability definitions that are used pertain to the "no noise" system

$$\dot{x}(t) = A(r(t))x(t) + B(r(t))u(t) \quad (20)$$

$$y(t) = C(r(t))x(t). \quad (21)$$

For notational simplicity, we say that $\{A(r(t)), B(r(t))\}$ is mean square stabilizable if system (2, 20, 21) is mean square stabilizable (the definition of stochastic stabilizability is given in [21]; in [13] it is extended to define stability and show that it is equivalent to mean square stability). This is, in general, different from the stabilizability of the pair (A_i, B_i) for each $i \in \mathbb{S}$. We say that $\{C(r(t)), A(r(t))\}$ is observable if system (2, 20, 21) is observable ([21]). Note, that the observability of $\{C(r(t)), A(r(t))\}$ is equivalent to the observability of the pair (C_i, A_i) for each $i \in \mathbb{S}$ ([21]).

In the literature, the equivalence of the state estimation problem and the deterministic linear quadratic regulator (LQR) problem for a dual system is used to obtain the steady-state properties of the estimator. For the JLQG problem, the same approach can be followed, except that the influence of the jump process $r(t)$ cannot be ignored. Here we show that the optimal x-state estimation problem of systems (1) and (2) are equivalent to the optimal quadratic control problem of the following system

$$\dot{x}^*(\tau) = A'(r^*(\tau))x^*(\tau) - C'(r^*(\tau))u^*(\tau) \quad (22)$$

$$y^*(\tau) = B'(r^*(\tau))x^*(\tau) \quad (23)$$

where $x^* \in \mathbb{R}^n$, $y^* \in \mathbb{R}^m$, $u^* \in \mathbb{R}^p$ and $r^* \in \mathbb{S}$. The new running variable $\tau \triangleq t_f - t$ is the "reverse" time, where t_f is an arbitrarily given fixed time; (22) and (23) are the dual of systems (20) and (21) with respect to time t_f ([25]). The underlying process $\{r^*(\tau)\}$, where $r^*(\tau) \triangleq r(t_f - \tau)$, is memoryless and anticipative under τ. That is, at $\tau = \tau_1$, we know the values of $r^*(\tau)$ for all $\tau \geq \tau_1$, but do not know the values of $r^*(\tau)$ for $\tau < \tau_1$.

Because the "future" (under τ) $r^*(\tau)$ values are known, it is easy to see that the following duality property between (20, 21) and (22, 23) holds:

$$\{C(r(t)), A(r(t))\} \text{ is observable} \Leftrightarrow \{A'(r^*(\tau)), C'(r^*(\tau))\} \text{ is controllable}.$$

Next we will show that the state estimation problems of (1) and (2) are a linear quadratic regulator problem (22) and (23). Consider the filtering problem of systems (1) and (2) for the time invariant case. Assume that $u \equiv 0$ on $[t_0, T]$. We seek a minimum variance estimate on x. As in [2], at any given time $T < \infty$, we want to construct from $\{S_T, \mathscr{F}_T\}$ a certain number β, such that

$$E\{[b'x(T) - \beta]^2\} \tag{24}$$

is minimized, where b is an arbitrary constant vector. From [2], β can be expressed as a linear function

$$\beta = \int_{t_0}^{T} u^*(T-t) y(t)\, dt. \tag{25}$$

The problem is to find $u^*(T-t)$ for $t \in [t_0, T]$ such that (24) is minimized. We have the following result.

Lemma 1: The above filtering problem can be converted to the control problem of finding $u^*(\tau)$ for the dual systems (22) and (23) with $t_f = T$, thus minimizing the cost function

$$\begin{aligned}(x^*(\tau_f))' P^*(r^*(\tau_f)) x^*(\tau_f) &\\ + \int_{\tau_0}^{\tau_f} [(x^*(\tau))' Q^*(r^*(\tau)) x^*(\tau) &\\ + (u^*(\tau))' R^*(r^*(\tau)) u^*(\tau)]\, d\tau &\end{aligned} \tag{26}$$

where

$$\begin{aligned} Q^*(r^*(\tau)) &= F(r^*(\tau)) F'(r^*(\tau)) \\ &= F(r(T-\tau)) F'(r(T-\tau)) \end{aligned} \tag{27.1}$$

$$\begin{aligned} R^*(r^*(\tau)) &= G(r^*(\tau)) G'(r^*(\tau)) \\ &= G(r(T-\tau)) G'(r(T-\tau)) \end{aligned} \tag{27.2}$$

$$P^*(r^*(\tau_f)) = \bar{P}_0 \tag{27.3}$$

and

$$\tau_0 = t_f - T = 0 \qquad \tau_f = T - t_0 \qquad \square$$

Note that here $Q^* \geq 0$, and from (5), $R^* > 0$.

Sketch of Proof: We temporarily assume that $u = 0$, $x_0 = 0$ and $t_f = T$ is finite, as in [2]. Then for $x(\cdot)$ subject to (1) and $x^*(\cdot)$ subject to (22), with $r(t) = r^*(T-t)$, we have

$$\begin{aligned} \frac{d}{dt}([x^*(T-t)]' x(t)) &= -\frac{d}{d(T-t)}([x^*(T-t)]') x(t) \\ &\quad + [x^*(T-t)]' \frac{d}{dt} x(t) \\ &= -[A'(r^*(T-t)) x^*(T-t) - C'(r^*(T-t)) \\ &\quad \cdot u^*(T-t)]' x(t) + [x^*(T-t)]' \\ &\quad \cdot [A(r(t)) x(t) + F(r(t)) w(t)] \\ &= [u^*(T-t)]' C(r(t)) x(t) \\ &\quad + [x^*(T-t)]' F(r(t)) w(t) \\ &= [u^*(T-t)]' y(t) - [u^*(T-t)]' G(r(t)) v(t) \\ &\quad + [x^*(T-t)]' F(r(t)) w(t). \end{aligned} \tag{28}$$

Integrating both sides of (28) from t_0 to T, we have

$$\begin{aligned} [x^*(0)]' x(T) &- [x^*(T-t_0)]' x(T-t_0) \\ &= \int_{t_0}^{T} [u^*(T-t)]' y(t)\, dt \\ &\quad - \int_{t_0}^{T} [u^*(T-t)]' G(r(t)) v(t)\, dt \\ &\quad + \int_{t_0}^{T} [x^*(T-t)]' F(r(t)) w(t)\, dt. \end{aligned}$$

Taking $b = x^*(0)$ and recalling that $\beta = \int_{t_0}^{T} [u^*(T-t)]' y(t)\, dt$ we have

$$\begin{aligned} b'x(T) - \beta &= [x^*(T-t_0)]' x(t_0) \\ &\quad - \int_{t_0}^{T} [u^*(T-t)]' G(r(t)) v(t)\, dt \\ &\quad + \int_{t_0}^{T} [x^*(T-t)]' F(r(t)) w(t)\, dt. \end{aligned} \tag{29}$$

Since $\tau = T - t$, then $\tau_0 = T - T = 0$ and $\tau_f = T - t_0$. Following [2, page 175], we square each side of (29) and take the expectation. Because of the independence of $x(t_0)$, $w(t)$, and $v(t)$, and the fact that the values of $r^*(T-t)$ $(= r(t))$ on $t \in [t_0, T]$ are known at time t for fixed T, we have (26) and (27).

Following the arguments in [2], under the condition that $\{C(r(t)), A(r(t))\}$ is observable, the temporary assumption that T is finite can be removed. The condition that $u = 0$ and $\bar{x}_0 = 0$ can also be removed by modifying the filter equation (under the assumption that the closed-loop systems (1)–(3) is mean square stable; Theorem 2-i) guarantees this). $\square$

Based on the above result, we have the following theorem.

Theorem 2 (Infinite Time Horizon Optimal JLQG Compensator): For the time-invariant case of the JLQG problem with (4a) the following applies:

i) If the pair $(Q_i^{1/2}, A_i)$ is observable for each $i \in \mathbb{S}$ and $\{A(r(t)), B(r(t))\}$ is mean square stabilizable, then as $T \to \infty$, we have the following optimal control law

$$u(t) = -L_i \hat{x}(t) \qquad \text{if } r(t) = i \in \mathbb{S} \tag{30}$$

with

$$L_i = R_i^{-1} B_i' K_i \qquad \text{for } r(t) = i \tag{31}$$

where the K_i are the unique, positive definite solutions of the following set of coupled algebraic Riccati equations:

$$A_i' K_i + K_i A_i + \sum_{j=1}^{s} \lambda_{ij} K_j + Q_i - K_i B_i R_i^{-1} B_i' K_i = 0. \tag{32}$$

ii) The optimal estimate $\hat{x}(t)$ is given by

$$\frac{d\hat{x}(t)}{dt} = A(r(t))\hat{x}(t) + B(r(t))u(t) + \Gamma(t)[y(t) - C(r(t))\hat{x}(t)] \quad (33)$$

where

$$\hat{x}(t_0) = x_0. \quad (34)$$

Here the filter gain matrix Γ is given by

$$\Gamma(t) = P(t)C'(r(t))[G(r(t))G'(r(t))]^{-1} \quad (35)$$

and the conditional covariance $P(t)$ is obtained from the matrix Riccati equation

$$\begin{aligned}\dot{P}(t) &= A(r(t))P(t) + P(t)A'(r(t)) + F(r(t))F'(r(t)) \\ &\quad - P(t)C'(r(t))[G(r(t))G'(r(t))]^{-1} \\ &\quad \cdot C(r(t))P(t)\end{aligned} \quad (36)$$

with initial condition

$$P(t_0) = \bar{P}_0 \quad (37)$$

where, at each time t, the values of A_i, B_i, C_i correspond to the current form $r(t) = i$.

iii) If the pair (C_i, A_i) is observable for each $i \in \mathbb{S}$, and the matrix F_iF_i' is positive definite for each $i \in \mathbb{S}$, then $P(t)$ and $\Gamma(t)$ are bounded above (in mean square) as $t \to \infty$. That is, the filter is mean square stable.

Sketch of Proof: Part i) is immediate from the combination of the separation property (Theorem 1) with [21, theorem 5] [note that the necessity portion of [21, theorem 5] does not apply, due to the use of cost (4a)]. Part ii) is immediate from Corollary 1. Part iii) can be established using duality arguments of Lemma 1 as follows: mean square stability of the state estimator can be obtained from the mean square stability of this closed-loop dual systems (22) and (23). The observability of the pair (C_i, A_i) $(i \in \mathbb{S})$ ensures the observability of $(\{C(r(t), A(r(t)\}$ ([21]). This implies the controllability of $\{A'(r^*(\tau)), C'(r^*(\tau))\}$. With the positive definiteness of F_iF_i', a time varying Lyapunov function can thus be obtained for (36) to serve as an upper bound for $P(t)$, which then can be used to prove the stability of the filter. This proves Part iii). This argument is only valid, however, under the assumption of that the closed-loop systems (1)–(3) are mean square stable; this is true because of i). □

Note that in some applications, the initial value $(\bar{x}_0, \bar{P}_0)$ may not be available. In such a case, we can take $\hat{x}(t_0) = 0$ and $P(t_0) = \alpha I$, where α is a positive number. As in [10], it can be shown (with the assumptions of this theorem) that when $\alpha \to \infty$, as $(t - t_0) \to \infty$, the estimate $\hat{x}(t)$ thus obtained converges to the optimal estimate.

IV. Suboptimal Steady-State JLQG Solution

The optimal compensator of the infinite time horizon problem given by Theorem 2 is, unfortunately, time varying since the filter parameters P and Γ depend on the sample path of $r(t)$. Thus, the gains of this optimal JLQG compensator cannot be obtained in advance of the system operation. In order to obtain a compensator with constant parameters, we must get around this form sample path dependence. In this section, a suboptimal steady-state filter and consequently a compensator are obtained by reducing the information available to the filter from $[\mathscr{F}_t, S_t$ to $\mathscr{F}_t, r(t)]$. That is, we "throw away" information of past values of $r(t)$ and replace them with estimated values, based on the following posterior probabilities:

$$Pr(r(t-\Delta)=j|r(t)=i) \triangleq \begin{cases}\lambda_{ij}^p\Delta + o(\Delta) & \text{if } i \neq j \\ 1 + \lambda_{ii}^p\Delta + o(\Delta) & \text{if } i = j\end{cases} \quad (38)$$

where λ_{ij}^p is the posterior transition rate and the superscript ^{p} denotes "posterior." By Bayes' rule and the assumption that we have no information about $(r(\tau)\colon \tau < t)$, thus the events $\{r(t-\Delta) = k, k = 1, 2, \cdots, s\}$ are with equal probabilities, we have

$$\begin{aligned}Pr(r(t-\Delta)=j|r(t)=i) &= \frac{Pr(r(t) = i|r(t-\Delta) = j)Pr(r(t-\Delta) = j)}{\sum_{k=1}^{s} Pr(r(t) = i|r(t-\Delta) = k)Pr(r(t-\Delta) = k)} \\ &= \frac{Pr(r(t) = i|r(t-\Delta) = j)}{\sum_{k=1}^{s} Pr(r(t) = i|r(t-\Delta) = k)} \\ &= \frac{Pr(r(t+\Delta) = i|r(t) = j)}{\sum_{k=1}^{s} Pr(r(t+\Delta) = i|r(t) = k)}.\end{aligned} \quad (39)$$

Thus, we can find λ_{ij}^p by ([12, page 239, (1.5)]). Substituting (2) into (39), for $j \neq i$ we have

$$\begin{aligned}\lambda_{ij}^p &= \lim_{\Delta\to 0} \frac{Pr(r(t-\Delta) = j|r(t) = i)}{\Delta} \\ &= \lim_{\Delta\to 0} \frac{1}{\Delta} \frac{\lambda_{ji}\Delta + o(\Delta)}{1 + \sum_{k=1}^{s} \lambda_{ki}\Delta + o(\Delta)} = \lambda_{ji}.\end{aligned} \quad (40.a)$$

For $j = i$, we have

$$\begin{aligned}\lambda_{ii}^p &= -\lim_{\Delta\to 0} \frac{1 - Pr(r(t-\Delta) = i|r(t) = i)}{\Delta} \\ &= -\lim_{\Delta\to 0} \frac{1 - \left[1 - \sum_{k=1, k\neq i}^{s} Pr(r(t-\Delta) = k|r(t) = i)\right]}{\Delta} \\ &= -\lim_{\Delta\to 0} \frac{\sum_{k=1, k\neq i}^{s} \lambda_{ki}\Delta + o(\Delta)}{\Delta} = -\sum_{k=1, k\neq i}^{s} \lambda_{ki}.\end{aligned} \quad (40.b)$$

Thus, we have the following suboptimal filter

$$\frac{d}{dt}\tilde{x}(t) = A_i\tilde{x}(t) + B_i u(t) + \Gamma(t,i)[y(t) - C_i\tilde{x}(t)] \tag{41}$$

$$\tilde{x}(t_0) = \bar{x}_0 \tag{42}$$

$$\Gamma(t,i) = P(t,i)C_i'[G_iG_i']^{-1} \tag{43}$$

$$\frac{d}{dt}\tilde{P}(t,i) = A_i\tilde{P}(t,i) + \tilde{P}(t,i)A_i' + F_iF_i' + \sum_{j=1}^{s}\lambda_{ij}^{p}\tilde{P}(t,j) - \tilde{P}(t,i)C_i'[G_iG_i']^{-1} \cdot C_i\tilde{P}(t,i) \tag{44}$$

$$\tilde{P}(t_0,i) = \bar{P}_0 \tag{45}$$

where $\{\lambda_{ij}^{p}: i, j \in \mathbb{S}\}$ is given by (40). Under certain conditions described below, this filter will converge to a constant version, with gains that can be obtained by solving a set of coupled algebraic Riccati equations. This provides a more easily implementable (but suboptimal) JLQG compensator.

Theorem 3 (Suboptimal Steady-State JLQG Compensator): Consider the time-invariant case of JLQG problem with cost (4a). Let us use filters (41)–(45) in the compensator, if:

i) $\{A(r(t)), B(r(t))\}$ is mean square stabilizable and the pair $(Q_i^{1/2}, A_i)$ is observable for each $i \in \mathbb{S}$;

ii) $\{A'(r^*(\tau)), C'(r^*(\tau))\}$ is mean square stabilizable and $\{A(r(t)), (F(r(t)F'(r(t))^{1/2}\}$ is stochastically controllable [21] [here r^* is assumed to be unanticipative but with forward (in τ) transition rate λ_{ij}^{p} as given by (40)].

Then as $T \to \infty$, we have the following steady-state compensator:

$$u(t) = -L_i\tilde{x}(t) \qquad \text{for } r(t) = i \tag{46}$$

where

$$L_i = R_i^{-1}B_i'K_i. \tag{47}$$

Here $\{K_i: i \in \mathbb{S}]$ are the unique symmetric positive definite solutions of the set of s coupled algebraic Riccati equations:

$$A_i'K_i + K_iA_i + \sum_{j=1}^{s}\lambda_{ij}K_j + Q_i - K_iB_iR_i^{-1}B_i'K_i = 0. \tag{48}$$

The suboptimal estimate $\tilde{x}(t)$ in (46) is given by the solution of

$$\frac{d}{dt}\tilde{x}(t) = A_i\tilde{x}(t) + B_iu(t) + \Gamma_i[y(t) - C_i\tilde{x}(t)] \qquad \text{for } r(t) = i \tag{49}$$

with initial condition

$$\tilde{x}(t_0) = \bar{x}_0. \tag{50}$$

The filter gain matrices Γ_i are given by

$$\Gamma_i = \tilde{P}_iC_i'[G_iG_i']^{-1}. \tag{51}$$

The conditional covariances $\tilde{P}_i$ ($i \in \mathbb{S}$) are the set of unique positive definite symmetric solutions of the set of s coupled equations

$$A_i\tilde{P}_i + \tilde{P}_iA_i' + F_iF_i' - \tilde{P}_iC_i'[G_iG_i']^{-1}C_i\tilde{P}_i + \sum_{j=1}^{s}\lambda_{ij}^{p}\tilde{P}_j = 0. \tag{52}$$

Sketch of Proof: The controller results are immediate from [21]. The filter portion is obtained by considering the equivalent regulator problem for the dual system; with the information pattern $(\mathscr{F}_t, r(t))$ it becomes a JLQ problem. The mean square stabilizability of $\{A'(r^*(\tau)), C'(r^*(\tau))\}$ and the stochastic observability of $\{(F(r^*(\tau)F'(r^*(\tau))^{1/2}, A'(r^*(\tau))\}$ [which is implied by the stochastic controllability of the pair $\{A(r(t), F(r(t)F'(r(t))^{1/2}\}]$ satisfies the condition of [21, theorem 5]. Convergence then follows. □

Note that the filter gains Γ_i and $\tilde{P}_i$, like the controller gains K_i and L_i, only depend on the current $r(t)$ value. Thus, they can be computed in advance of the system operation.

For the systems that have absorbing forms (i.e., $\exists i \in \mathbb{S}$, such that $\lambda_{ij} = 0\ \forall j \in \mathbb{S}$), the suboptimal steady-state filters (49)–(52) can be improved as follows: for absorbing form i, we apply the usual steady-state Kalman filter:

$$A_i\tilde{P}_i + \tilde{P}_iA_i' + F_iF_i' - \tilde{P}_iC_i'[G_iG_i']^{-1}C_i\tilde{P}_i = 0. \tag{53}$$

After r enters an absorbing form i, it cannot leave i and the system will behave like a constant parameter system thus we use (53).

V. Example

Consider the infinite time horizon JLQG problem of four form jump linear systems (1)–(5), with transition rate matrix

$$\Lambda = \begin{bmatrix} -1.01 & 1 & 0.01 & 0 \\ 1 & -1.01 & 0 & 0.01 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Here, forms 1 and 2 are communicating transient forms They might represent two kinds of operating conditions; 3 and 4 are absorbing forms that might represent partial failure conditions of the system. The system parameters are

$$A_1 = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}\ B_1 = \begin{bmatrix} 0 \\ 2 \end{bmatrix}\ A_2 = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}\ B_2 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} -1 & 2 \\ 2 & 5 \end{bmatrix}\ B_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}\ A_4 = \begin{bmatrix} 5 & 2 \\ 2 & -1 \end{bmatrix}\ B_4 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$C_1 = C_2 = [1 \quad 1] \quad C_3 = [0 \quad 1] \quad C_4 = [1 \quad 0]$$

$$F_1 = F_2 = F_3 = F_4 = 0.4I \quad G_1 = G_2 = G_3 = G_4 = 0.5$$

and the cost weighting matrices are

$$Q_1 = Q_2 = I \qquad Q_3 = Q_4 = 2I \qquad R_1 = R_2 = 1 \qquad R_3 = R_4 = 0.5$$

with

$$\bar{x}_0' = [5 \quad 7] \qquad \bar{P}_0 = 4I.$$

Now we consider the infinite time horizon JLQG problem. By Theorem 3, the suboptimal steady-state compensator is given by

$$u(t) = -L_i \bar{x}(t) \qquad \text{if } r(t) = i \tag{54}$$

where

$$L_1 = [17.84428 \quad 18.927532] \qquad L_2 = [18.927532 \quad 17.84428]$$
$$L_3 = [3.71218 \quad 11.62232] \qquad L_4 = [11.62232 \quad 3.7128] \tag{55}$$

and the suboptimal steady-state filter is

$$\frac{d}{dt}\bar{x}(t) = H_i \bar{x}(t) + F_i y(t) \qquad \text{if } r(t) = i \tag{56}$$

where

$$\Gamma_1 = \begin{bmatrix} -2.882744 \\ 10.552972 \end{bmatrix} \qquad \Gamma_2 = \begin{bmatrix} 10.552972 \\ -2.882744 \end{bmatrix}$$
$$\Gamma_3 = \begin{bmatrix} 4.8048848 \\ 14.731184 \end{bmatrix} \qquad \Gamma_4 = \begin{bmatrix} 14.731184 \\ 4.8048848 \end{bmatrix}$$

and $H_i = A_i - B_i L_i - \Gamma_i C_i$. Note (56) is (49) and here L_i and Γ_i can be obtained from the solution K_i of (48) and $\tilde{P}_i$ of (52), respectively. The set of s coupled matrix differential equations (52) can be solved off-line and the result saved and used during the system operation. The optimal JLQG compensator is given by Theorem 2:

$$u(t) = -L_i \hat{x}(t) \qquad \text{if } r(t) = i \tag{57}$$

where L_i is given by (55). The optimal estimate $\hat{x}(t)$ generated by (33) and (34) which have time varying coefficients from the solutions of (35)–(37). To obtain the conditional covariance $P(t)$, we must solve (36), which is a single matrix differential equation, on-line, along with the system operation, because we need to know the value of $r(t)$ at time t.

In the computer simulation of this example system controlled by optimal compensator (57) and the suboptimal compensator of (54)–(56), over the ensemble of form sample paths, the optimal JLQG compensator will, of course, perform better. However, for some examples, the two compensators have similar performance. An example of this is shown in [45].

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