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Feasibility and Convergence Analysis of Discrete-Time H_∞ A Priori Filters

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Abstract

The performance and convergence analysis of discrete-time H_∞ *a priori* filters is addressed in this paper. We deal with a more general term of Riccati equations arising from standard discrete-time H_∞ *a priori* filters than the existing results ([1]). A sufficient condition for feasibility and convergence of H_∞ *a priori* filters is given in terms of certain matrix inequality of initial condition.

1 Introduction

It is well known that algebraic and differential (difference) Riccati equations play an important role in H_∞ filtering and control theory. For example, the existence of solution of a sign indefinite algebraic Riccati equation (ARE) is crucial in the design of controllers and filters ensuring certain H_∞ performance (see e.g. [2] and references cited therein). In the continuous time case, the non-existence of a robust filter over finite horizon is associated with the occurrence of finite escape phenomena of the solution of the relevant differential Riccati equation (see e.g. [3]). But in the discrete-time case, the non-existence of a robust filter over finite horizon is no longer associated with the solution of the difference Riccati equation (DRE) becoming unbounded. Rather, the existence of the filter requires the fulfillment at each step of a suitable matrix inequality (feasibility condition). This requirement is stricter than that of the continuous time case, because feasibility can be lost even when the solution of the corresponding DRE remains bounded. Thus, it is important to find conditions under which we can ensure feasibility of the solutions of DREs over an arbitrarily long time interval, and convergence towards the steady-state stable H_∞ filter.

The pioneering work of convergence analysis of H_∞ Riccati difference equation can be found in Bolzern *et al.* [4] and [1]. In [4], the performance of H_∞ *a posteriori* filters was studied, and a sufficient condition ensuring feasibility and convergence to the steady-state stabilizing solution was given. The feasible conditions arising from H_∞ *a posteriori* filters is different from that of H_∞ *a priori* filters. Actually, the feasibility constraint of H_∞ *a priori* filters is tighter than that of *a posteriori* filters from some point of view. Moreover, *a priori* filters are very widely used in the context of H_∞ filtering (see e.g. [5] and references cited therein). Thus it is of significant interest to study the performance of H_∞ *a priori* filters. In [1], feasible and convergent problems of H_∞ *a priori* filters were investigated but only for a variant version of standard difference Riccati equation.

In this paper, we shall examine the performance analysis of H_∞ *a priori* filters. The convergence analysis of Riccati equations arising from standard H_∞ *a priori* filter will be studied without the assumption $D[B^T \ \ D^T] = [0 \ \ I]$. Our result is neither tighter nor weaker than that of [1]. It can be viewed as a nice alternative to the result in [1].

Notations: Most of the notations used in this note are standard. $\Re^n$ denotes the n-dimensional Euclidean space and $||\cdot||$ refers to the Euclidean vector norm. $l_2[0,N]$ stands for the space of square summable vector sequence over $[0,N]$, and $||\cdot||_2$ is the $l_2[0,N]$ norm defined by $||\cdot||_2 := \left(\sum_0^N ||\cdot||^2\right)^{\frac{1}{2}}$. Finally, we introduce the decomposition of a symmetric matrix M into its positive part and negative part, i.e. $M = M^+ + M^-$, where $M^+ \geq 0$ ($M^- \leq 0$). In addition, the nonzero eigenvalues of M^+ (the nonzero eigenvalues of M^-) are the positive (negative) eigenvalues of M.

2 Preliminaries

Consider the following linear discrete-time system

$$x_{k+1} = Ax_k + B\omega_k \tag{2.1}$$
$$y_k = Cx_k + D\omega_k \tag{2.2}$$
$$z_k = Lx_k \tag{2.3}$$

where $x_k \in \Re^n$ is the system state, $\omega_k \in \Re^q$ is the noise, $y_k \in \Re^m$ is the output measurements, $z_k \in \Re^p$ is a linear combination of the state variables to be estimated, and A, B, C, D, and L are known real constant matrices with appropriate dimensions.

We shall use following performance measures:

$J_N = \sup_{0 \neq (x_0, \omega) \in \Re^n \times l_2[0,N]} \frac{\|z_k - \hat{z}_k\|_2^2}{x_0^T R_0 x_0 + \|\omega_k\|_2^2}$ and $J_\infty = \sup_{0 \neq \omega \in l_2[0,\infty)} \frac{\|z_k - \hat{z}_k\|_2^2}{\|\omega_k\|_2^2}$, where the vector $\hat{z}_k$ is an estimate of z, γ is the prescribed level of attenuation and $R_0 = R_0^T > 0$ is a given weight matrix for the initial state x_0. It is worth noting that an estimator of z_k is called *a priori* filter if $\hat{z}_k$ is obtained based on the output measurements $\{y_0, y_1, \cdots, y_{k-1}\}$, while $\hat{z}_k$ is referred to as *a posteriori* filter if it is obtained by using the measurements $\{y_0, y_1, \cdots, y_k\}$.

Next, let us introduce following two Riccati equations which play key roles in this paper: the difference Riccati equation (DRE)

$$\begin{aligned} P_{k+1} &= AP_kA^T - (AP_kC_1^T + BD_1^T)(C_1P_kC_1^T + R_1)^{-1} \\ &\quad (C_1P_kA^T + D_1B^T) + BB^T \end{aligned} \tag{2.4}$$

and the algebraic Riccati equation (ARE)

$$\begin{aligned} P &= APA^T - (APC_1^T + BD_1^T)(C_1PC_1^T + R_1)^{-1} \\ &\quad (C_1PA^T + D_1B^T) + BB^T \end{aligned} \tag{2.5}$$

where $C_1 = \begin{bmatrix} C^T & \frac{1}{\gamma}L^T \end{bmatrix}^T$, $D_1 = \begin{bmatrix} D^T & 0 \end{bmatrix}^T$, $R_1 = diag\{DD^T, -I\}$.

Now we shall give the following definition.

Definition 2.1. (Stabilizing solution)([5]) *A real symmetric matrix P_s is said to be a stabilizing solution to ARE (2.5) if P_s satisfies (2.5) and the matrix $\hat{A} = A - (AP_sC_1^T + BD_1^T)(C_1P_sC_1^T + R_1)^{-1}C_1$ is Schur stable.*

Throughout this paper, we make the following assumptions:

Assumption 2.1.
(a) (A, B) is reachable and (A, C) is detectable;
(b) $DD^T > 0$;
(c) The ARE (2.5) admits a stabilizing solution P_s.

It should be noted that Assumption 2.1(a) implies that $P_s > 0$ and is necessary for the existence of an exponentially stable filter, and Assumption 2.1(b) means that all the components of the measurement vector are noisy. It should be pointed out that P_s is unique if it exists ([6]).

The following two theorems provide solutions to the finite-horizon and infinite-horizon H_∞ filtering problems for the system (2.1)-(2.3) (see e.g. [5] and [7]).They are dependent on the existence of positive definite solutions of the associated difference (algebraic) Riccati equation.

Theorem 2.1. (Finite-horizon H_∞ *a priori* filter) *Consider the system (2.1)-(2.3) and let $R_0 = R_0^T > 0$ be a given initial state weighting matrix. Then there exists an H_∞ a priori filter such that $J_N < \gamma^2$ if and only if there exists a solution $P_k = P_k^T > 0$ over $[0, N]$ to the DRE (2.4) with $P_0 = R_0^{-1}$ such that $P_k^{-1} - \gamma^{-2}L^TL > 0$. Moreover, if the above conditions are satisfied, a suitable filter is given by*

$$\begin{aligned} \hat{x}_{k+1} &= A\hat{x}_k + K_k(y_k - C\hat{x}_k), \quad \hat{x}_0 = 0 \\ \hat{z}_k &= L\hat{x}_k \end{aligned}$$

where $K_k = (A\Phi_kC^T + BD^T)(C\Phi_kC^T + DD^T)^{-1}$ and $\Phi_k = P_k + \frac{1}{\gamma^2}P_kL^T(I - \frac{1}{\gamma^2}LP_k^{-1}L^T)^{-1}LP_k$.

Theorem 2.2. (Infinite-horizon H_∞ *a priori* filter) *Consider the system (2.1)-(2.3) satisfying Assumption 2.1. Then there exists an H_∞ a priori filter such that $J_\infty < \gamma^2$ if and only if there exists a stabilizing solution $P_s = P_s^T > 0$ to the ARE (2.5) such that $P_s^{-1} - \gamma^{-2}L^TL > 0$. In such case, a suitable filter is obtained by replacing P_k with P_s in equalities (2.6-2.6). Moreover, this filter is stable.*

It is clear that the existence of H_∞ *a priori* filter is related to the DRE (2.4) (over finite-time horizon) or ARE (2.5) (over infinite-time horizon), and the fulfillment of a suitable matrix inequality (feasibility condition). It is worth stressing that the main difference between this paper and [1] is that the results of [1] are based on a variant version of standard H_∞ Riccati equation (2.4) and with the assumption $D[B^T \;\; D^T] = [0 \;\; I]$.

Next, we shall adopt the following definition of feasible solution in the remainder of the paper.

Definition 2.2. (Feasible solution of H_∞ DRE) *A real positive definite solution P_k of (2.4) is termed a "feasible solution" if it satisfies*

$$P_k^{-1} - \gamma^{-2}L^TL > 0$$

at every step $k \in [0, N]$.

The feasibility and convergence analysis problem studied in this paper can be stated as follows: *Given an arbitrarily large N, find suitable conditions on the initial state P_0 such that the solution P_k is feasible at every step $k \in [0, N]$ and converges to the stabilizing solution P_s as $N \to \infty$.*

We end this section by giving two preliminary results which play important roles in deriving the main results of this paper. The first is an extension of a comparison result of DRE in [8].

Lemma 2.1. *Consider the following difference Riccati equation*

$$\begin{aligned} P_{k+1} &= AP_kA^T - (AP_kC^T + BD^T)(CP_kC^T + R)^{-1} \\ &\quad (CP_kA^T + DB^T) + BB^T \end{aligned}$$

Let P_k^1 and P_k^2 be solutions of (2.6) with different initial conditions $P_0^1 = \bar{P}_0^1 \geq 0$ and $P_0^2 = \bar{P}_0^2 \geq 0$, respectively. Then the difference between the two solutions $\tilde{P}_k = P_k^2 - P_k^1$ satisfies the following equation

$$\tilde{P}_{k+1} = \tilde{A}_k\tilde{P}_k\tilde{A}_k^T - \tilde{A}_k\tilde{P}_kC^T(C\tilde{P}_kC^T + \tilde{R}_k)^{-1}C\tilde{P}_k\tilde{A}_k^T$$

where $\tilde{A}_k = A - (AP_k^1C^T + BD^T)(CP_k^1C^T + R)^{-1}C$ and $\tilde{R}_k = CP_k^1C^T + R$.

In order to extend Lemma 1 of [4], we need the following assumption:

Assumption 2.2. *The matrix* $\bar{A} = A - BD^T(DD^T)^{-1}C$ *is invertible.*

Lemma 2.2. *Consider difference Riccati equation (2.4). Let* P_k^1 *and* P_k^2 *be two solutions of (2.4) with different initial conditions* $P_0^2 \geq P_0^1 > 0$. *Then, under Assumptions 2.1 and 2.2, when* P_k^2 *is feasible, it results that* $P_k^2 \geq P_k^1 > 0$ *and* P_k^1 *is feasible too. Furthermore, if* $P_0^2 > P_0^1$, *then* $P_k^2 > P_k^1$.

3 Convergence Analysis of H_∞ Difference Riccati Equation

Before presenting main results of this section, we shall introduce the following Lyapunov equation:

$$\hat{A}^T Y \hat{A} - Y = -M_- \tag{3.1}$$

where $\hat{A}$ is the same as in Definition 2.1 and

$$\begin{aligned} M &= \hat{A}^{-T}(G + C_1^T \hat{R}^{-1} C_1)\hat{A}^{-1} - G && (3.2) \\ G &= -P_s^{-1} - P_s^{-1}(\gamma^{-2}L^T L - P_s^{-1})^{-1}P_s^{-1} && (3.3) \\ \hat{R} &= C_1 P_s C_1^T + R_1 && (3.4) \end{aligned}$$

The following Theorem establishes a relationship between the initial state P_0 and feasibility of the solution of the DRE (2.4).

Theorem 3.1. *Consider the difference Riccati equation (2.4). Let Assumptions 2.1 and 2.2 hold, and let* Y *be the solution of the Lyapunov equation (3.1).*

Then the solution P_k *of DRE (2.4) is feasible over* $[0, \infty)$ *if for some sufficiently small* $\epsilon > 0$, *the initial condition satisfies*

$$0 < P_0 < (G - Y + M_- + \epsilon I)^{-1} + P_s \tag{3.5}$$

Proof: The procedure of the proof is similar to that in [1]. There are three cases.

(i) *The case that* $P_0 < P_s$. Because P_s is a constant feasible solution of (2.4), the feasibility of P_k follows from Lemma 2.2 directly.

(ii) *The case that* $P_0 > P_s$. Let's define $X_k = P_k - P_s$. Then, applying Lemma 2.1 to (2.4) and (2.5), we readily obtain that X_k satisfies

$$\begin{aligned} X_{k+1} &= \hat{A}X_k\hat{A}^T - \hat{A}X_k C_1^T(C_1 X_k C_1^T + \hat{R})^{-1}C_1 X_k \hat{A}^T \\ &= \hat{A}(X_k^{-1} + C_1^T \hat{R}^{-1} C_1)^{-1}\hat{A}^T && (3.6) \end{aligned}$$

where $X_0 = P_0 - P_s$, $\hat{A} = A - (AP_sC_1^T + BD_1^T)(C_1P_sC_1^T + R_1)^{-1}C_1$ and $\hat{R} = C_1P_sC_1^T + R_1$.

Now letting $Z_k = X_k^{-1} - G$, where G is defined by (3.3). It is worth noting that $G \geq 0$ since P_s is feasible.

Note that $\hat{A}$ is invertible as $\bar{A}$ is invertible and P_s is feasible. Then by (3.6), we have $Z_{k+1} = \hat{A}^{-T}Z_k\hat{A}^{-1} + M$, where M is defined by (3.2) and $Z_0 = (P_0 - P_s)^{-1} - G$.

Since P_s is feasible and $X_k > 0$ according to Lemma 2.2, it is clear that the feasibility of P_k is equivalent to the positive definiteness of Z_k, which follows from $Z_k = P_s^{-1}[(P_s^{-1} - P_k^{-1})^{-1} - (P_s^{-1} - \gamma^{-2}L^TL)^{-1}]P_s^{-1}$.

Now consider the following Lyapunov equation

$$\hat{Z}_{k+1} = \hat{A}^{-T}\hat{Z}_k\hat{A}^{-1} + M_- \tag{3.7}$$

with $\hat{Z}_0 = Z_0$. By definition $M \geq M_-$, so that $Z_k \geq \hat{Z}_k$. Then $\hat{Z}_k > 0$ is sufficient to guarantee the positivity of Z_k. Now we compute (3.7) as follows

$$\begin{aligned} Z_k \geq \hat{Z}_k &= (\hat{A}^{-k})^T\Big(Z_0 + \sum_{j=1}^{k}(\hat{A}^j)^T M_- \hat{A}^j\Big)\hat{A}^{-k} \\ &\geq (\hat{A}^{-k})^T\Big(Z_0 + \sum_{j=1}^{\infty}(\hat{A}^j)^T M_- \hat{A}^j\Big)\hat{A}^{-k} && (3.8) \end{aligned}$$

Next, refer to Lemma 21.6 of [9], and from (3.1), we have

$$Y = \sum_{j=0}^{\infty}(\hat{A}^j)^T M_- \hat{A}^j = M_- + \sum_{j=1}^{\infty}(\hat{A}^j)^T M_- \hat{A}^j \tag{3.9}$$

Now comparing (3.8) and (3.9), we have

$$Z_k \geq \hat{Z}_k \geq (\hat{A}^{-k})^T(Z_0 + Y - M_-)\hat{A}^{-k} \tag{3.10}$$

So, if $Z_0 + Y - M_- > 0$, then $\hat{Z}_k > 0$, and, in turn $Z_k > 0$. Here $Z_0 + Y - M_- > 0$ can be rewritten as

$$(P_0 - P_s)^{-1} - G + Y - M_- > 0 \tag{3.11}$$

Since $-Y + M_- \geq 0$ and $G \geq 0$, then (3.5) implies (3.11). Thus the proof of feasibility for the case of $P_0 > P_s$ is completed.

(iii) *The case that* $P_0 - P_s$ *is not sign definite.* There always exists $\bar{P}_0$ satisfying (3.5) and such that $\bar{P}_0 > P_0$ and $\bar{P}_0 > P_s$. Then in view of subcase (ii), the solution $\bar{P}_k$ of RDE (2.4) starting from $\bar{P}_0$ is feasible. Finally, the feasibility of P_k readily comes from Lemma 2.2. $\nabla\nabla\nabla$

We shall now study the convergence of the solution of the DRE (2.4). It is easy to know that (2.4) satisfies the following matrix recursions

$$\begin{aligned} P_{k+1} &= \bar{A}S_k^{-1}\bar{A}^T + B[I - D^T(DD^T)^{-1}D]B^T \\ S_k &= P_k^{-1} + C_1^T R_1^{-1} C_1 && (3.12) \end{aligned}$$

so S_k satisfies the following DRE,

$$\begin{aligned} S_k &= \{\bar{A}S_k^{-1}\bar{A}^T + B[I - D^T(DD^T)^{-1}D]B^T\}^{-1} \\ &\quad + C_1^T R_1^{-1} C_1 && (3.13) \end{aligned}$$

and the associated ARE is

$$S = \{\bar{A}S^{-1}\bar{A}^T + B[I - D^T(DD^T)^{-1}D]B^T\}^{-1} + C_1^T R_1^{-1} C_1 \quad (3.14)$$

Under Assumption 2.1 and 2.2, in view of the result in [6], (3.14) admits both the stabilizing solution S_s and antistabilizing solution S_a, and furthermore, $S_s - S_a > 0$.

The following main result provides a sufficient condition for ensuring convergence as well as feasibility of the solution of the DRE (2.4) over $[0, \infty)$.

Theorem 3.2. *Consider the difference Riccati equation (2.4). Let Assumptions 2.1 and 2.2 hold, then the solution P_k of DRE (2.4) is feasible over $[0,\infty)$ and converges to the stabilizing solution P_s of ARE (2.5) as $k \to \infty$ if P_s is feasible and for some sufficiently small $\epsilon > 0$, the initial condition satisfies*

$$0 < P_0 < (G - Y + M_- + \epsilon I)^{-1} + P_s \quad (3.15)$$

where G, Y and M are defined as in Theorem 3.1.

Proof: At first, it shall be noted that P_k is feasible over $[0, \infty)$ from Theorem 3.1.

The proof of convergence is carried out by using similar technique in [4]. Consider (2.4), (2.5), (3.13) and (3.14), the study of the convergence of P_k is equivalent to the study of the convergence of S_k to S_s. So in what follows, we focus on the proof of the convergence of S_k.

Let $U = (S_s - S_a)^{-1}$, then from (3.14), we have

$$\begin{aligned} U &= \Big\{\left[\bar{A}S_s^{-1}\bar{A}^T + B[I - D^T(DD^T)^{-1}D]B^T\right]^{-1} \\ &\quad + \left[\bar{A}S_a^{-1}\bar{A}^T + B[I - D^T(DD^T)^{-1}D]B^T\right]^{-1}\Big\}^{-1} \\ &= \tilde{A}^T U \tilde{A} + S_s^{-1} - P_s \hat{A}^T P_s^{-1} \hat{A} P_s \quad (3.16) \end{aligned}$$

where $\tilde{A} = P_s^{-1}\hat{A}P_s$.

Next, let $W = P_s[G - Y + M_- + P_s^{-1}]P_s$, then from (3.1), we have

$$W = \tilde{A}^T U \tilde{A} + S_s^{-1} - P_s \hat{A}^T P_s^{-1} \hat{A} P_s + N \quad (3.17)$$

where

$$\begin{aligned} N &= P_s C_1^T \hat{R}^{-1} C_1 P_s + P_s(G - \hat{A}^T G \hat{A})P_s \\ &\quad - P_s \hat{A}^T M_- \hat{A} P_s \\ &= P_s \hat{A}^T M_+ \hat{A} P_s \geq 0 \end{aligned}$$

Comparing (3.16) and (3.17), we have $W \geq U$.

Now cosider (3.12) and (3.15), we obtain

$$\begin{aligned} S_0 &= P_0^{-1} + C_1^T R_1^{-1} C_1 \\ &> [(G - Y + M_- + \epsilon I)^{-1} + P_s]^{-1} + C_1^T R_1^{-1} C_1 \\ &= S_s - P_s^{-1}[G - Y + M_- + \epsilon I + P_s^{-1}]^{-1} P_s^{-1} \\ &\geq S_s - W^{-1} \geq S_s - U^{-1} = S_a \quad (3.18) \end{aligned}$$

From (3.18), we have $S_0 > S_a$. Then refer to Lemma 7 of [6], it is clear that $\lim_{k\to\infty} S_k = S_s$. ▽▽▽

Theorem 3.2 gives a sufficient condition for feasibility and convergence of standard H_∞ *a priori* filters. It can be viewed as a nice alternative to the result in [1] where only a variant version of the standard H_∞ filtering solution was considered. Besides, we don't require the assumption $D[B^T \quad D^T] = [0 \quad I]$ as in [1]. Generally speaking, the above feasible and convergent conditions are only sufficient, but it can be easily shown that they are also necessary in the two special cases that $\gamma \to \infty$ and first order systems.

It is worth noting that the technique employed in this paper cannot be extended to study the performance analysis problem arising from standard discrete-time H_∞ *a posteriori* filters ([4]). On the other hand, the methodology of [4] cannot be used to study the performance analysis problem, which arises from standard discrete-time H_∞ *a priori* filters, addressed in this paper. Although both problems are mentioned in [1], it is based on a variant version of standard H_∞ *a priori* and *a posteriori* filters.

In the case that $D[B^T \quad D^T] = [0 \quad I]$, then Assumption 2.2 can be replaced by the assumption that A is invertible. The invertibility assumption of $\bar{A}$ or A seems difficult to eliminate.

4 Numerical Examples

In this section, we shall study two examples. The first one is ([1]): $A = \begin{bmatrix} 1.5 & -0.5 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} -0.4 & 0 \\ 0.6 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$, $D = \begin{bmatrix} 0 & 1 \end{bmatrix}$, $L = \begin{bmatrix} 1 & 1 \end{bmatrix}$.

By using the stationary filter design procedure (Theorem 2.2), we get $\gamma_{min} = 2.15$ with $P_s = \begin{bmatrix} 1.5213 & 0.7746 \\ 0.7746 & 1.3205 \end{bmatrix}$. According to Theorem 3.2, we can compute if the initial state P_0 satisfies $0 < P_0 < \begin{bmatrix} 1.6303 & 0.7221 \\ 0.7221 & 1.5410 \end{bmatrix} = \bar{P}$, then P_k due to the H_∞ DRE will be feasible for $\forall k \geq 0$ and converge to the stabilizing solution P_s as $k \to \infty$.

Let $Q_k = P_k^{-1} - \gamma^{-2} L^T L$. Then, the feasibility condition is equivalent to that $Q_k > 0$. P_k, Q_k, $\lambda_{min}(P_k)$ and $\lambda_{min}(Q_k)$ are displayed in Figure 1 (a)-(b). It shows that P_k converges to P_s and is feasible at every step.

An initial condition can also be computed with the method in [1]: $0 < P_0' < \begin{bmatrix} 1.6872 & 0.7286 \\ 0.7286 & 1.4707 \end{bmatrix} = \bar{P}'$. Since $\bar{P} - \bar{P}'$ is sign indefinite, it means that the two bounds $\bar{P}$ and $\bar{P}'$ cannot be compared.

Next, we consider another discrete-time system, which is slightly different from the one in Example 1, with $D[B^T \quad D^T] \neq [0 \quad I]$. Here we let $D = \begin{bmatrix} 1 & 1 \end{bmatrix}$.

Then by applying standard H_∞ filtering results, a steady-state filter is obtained with $\gamma_{min} = 3.10$ and $P_s = \begin{bmatrix} 4.3403 & 1.8855 \\ 1.8855 & 1.1430 \end{bmatrix}$, and the suitable scope of initial state is $0 < P_0 < \begin{bmatrix} 4.6503 & 1.6131 \\ 1.6131 & 1.7332 \end{bmatrix}$. Similarly, P_k, Q_k, $\lambda_{min}(P_k)$, $\lambda_{min}(Q_k)$ are displayed in Figure 1 (c)-(d). It shows that P_k converges to P_s while remains feasible at every step.

5 Conclusion

In this paper, we have studied convergence analysis of Riccati equations arising from standard discrete-time H_∞ *a priori* filters. The result can be viewed as a nice alternative to the results in [1] which based on a variant version of H_∞ DREs.

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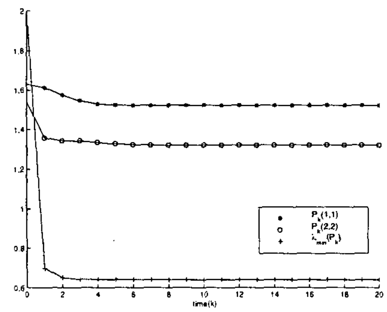
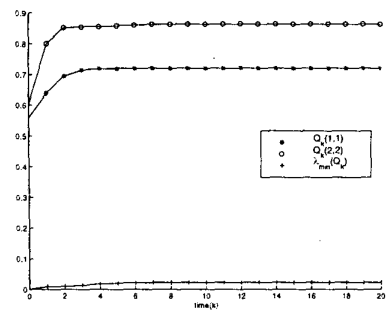
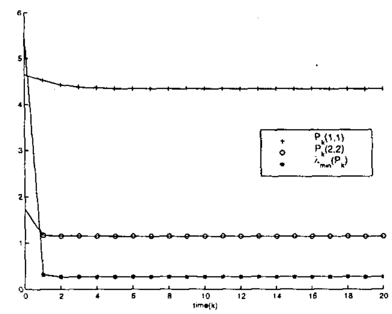


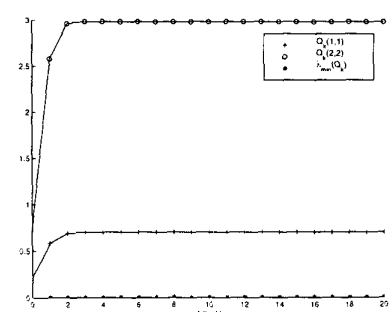
(a) Example 1: P_k



(b) Example 1: Q_k



(c) Example 2: P_k



(d) Example 2: Q_k

Figure 1: Results of Numerical Examples.