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Adaptive tracking control for a class of Markovian jump systems with time-varying delay and actuator faults

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Abstract

This paper is concerned with the output tracking control problem for a class of continuous-time Markovian jump systems with time-varying delay and actuator faults. The transition probabilities considered are more general, which include completely known ones, completely unknown ones and the unknown but with known bounds. In order to attenuate the effect of actuator degradation, a virtual tracking model is constructed using the normal control law. Then, an adaptive control method is employed to make the states of fault augmented model asymptotically track those of a virtual tracking model, which means that the fault-tolerant tracking controller can be designed to achieve the goal of system output tracking the reference signal asymptotically. In the proposed approach, the adaptive laws involving in the connections among various system modes are constructed with such a novel structure that the fault estimation is fast enough and the high-frequency oscillations can be reduced effectively. Finally, the numerical example is presented to show the effectiveness of the proposed method.

1. Introduction

With the rapid development of modern industrial systems, the structure of system is becoming more and more complex, which means that the actual system is more vulnerable to components failure resulted from the abrupt variations of external environment or unexpected changes in signals. If there is any component failure, system may end up suffering performance degradation and even it

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is difficult to guarantee the stability. Therefore, safety and reliability are crucial factors for practical system. In addition to taking protective measures to keep normal operation of system components, it is necessary to design effective control strategies to ensure the performance of system with faults. These types of control methodologies are usually known as fault-tolerant control (FTC). With the rising demand for safety and reliability in the modern industries, the research on FTC problem for dynamic systems with component failure has received more and more attention during the past two decades [1–5]. Generally, FTC can be achieved in two ways: passive and active approaches. The passive FTC approach usually presents controller with simple structure, which may have limited fault-tolerant ability. Compared with passive approaches, the controller designed with active approach has better self-regulation, because the fault detection or estimation is usually needed and the active fault tolerant controller can be modified online according to the detected or estimated fault information (see [2,6,7]). A new finite-horizon H_∞ fault estimation problem with delayed measurements is addressed in [6], which is an effective method to estimate faults for discrete time-varying system. In [7], nonlinear sliding mode observers are proposed to detect and to isolate sensors and actuators faults. Among the active approaches, adaptive control is an effective method and has been widely employed to solve FTC problem [8–11]. Ref. [9] uses sliding-mode observers to achieve fault detection and isolation, then an accommodation scheme is proposed to compensate for the effect of the fault based on a novel adaptive fault-estimation observer. In this context, our works are concerned with adaptive fault tolerant control.

For industrial systems, abrupt environmental disturbances, sudden changes in the operating point of a nonlinear plant and even random failures may lead to random changes of system parameters. In recent years, Markov jump system (MJS) has been widely investigated as the fact that many practical systems suffering random changes can be modeled with MJSs, such as power systems, manufactory processes and networked control systems [12–14]. The sensor-to-controller and controller-to-actuator random network-induced delays are modeled as Markov chains in [14]. MJS is a kind of stochastic hybrid systems including two components which are the continuous states and the finite system modes. The states are determined by differential equation, while the system modes are governed by a Markov chain. For MJS, the transition probabilities of the modes evolution are critical parameters, which are usually assumed completely known in many previous works. The deterministic filter for continuous-time MJS and the adaptive backstepping controller are designed in [15,16] respectively, where the transition probabilities are assumed to be exactly known. In fact, it is often different to get precise values of all the transition probabilities in the presence of inevitable measurement error. So, it is necessary to study MJSs in which the transition probabilities are partly known, see [17–20]. In some cases, the accurate values of the unknown transition probabilities cannot be obtained, but their bounds can be measured. Refs. [21,22] consider the more general transition probabilities, which include the exactly known, the unknown, and the unknown but with known bounds. A lot of works about MJSs have been completed. There are also many results about the FTC problem of MJSs. In [23], the stochastic stability of fault tolerant control systems with noise has been studied using the Lyapunov function approach. Considering sensor fault, actuator fault and input disturbances simultaneously in [24], a novel sliding mode observer approach is developed to solve the FTC problem of MJSs. The general partly unknown transition probabilities considered in this paper are not investigated in the two works above.

In practical engineering, a stabilizing controller is often sought to force the output of system to track the desired signal on the premise of guaranteeing the stability of system. Since the output tracking problem is of great practical significance, a lot of researchers have attempted to solve it. In [25], the integral tracking method is proposed based on the input–output approach for linear systems. Adaptive control method in [26] is employed to solve fault-tolerant tracking problems in linear time-invariant system with actuator faults. In recent years, the problems of stability and stabilization about time-delay MJSs with the partially unknown transition probabilities have been widely studied [27,28], while there is little work to investigate the fault-tolerant tracking control problem of this kind of system with time delay. In [29], the state tracking problem is considered for Markov jump nonlinear system which is constructed into linear models with the gradient linearization procedure. Although the time delay is investigated, the transition probabilities include only completely known case and unknown case, and the actuator failure is not considered.

Motivated by the statements above, this paper investigates the output tracking control problem for a class of MJSs with time-varying delay and actuator faults. The problems involved here are actuator faults, time-varying delay, nonlinear uncertainty and the partly unknown transition probabilities. Firstly, based on the internal model principle, the tracking error is utilized as the input to construct an additional dynamic, which combined with the original system is regarded as the tracking model. Using a method with less conservativeness to deal with the partly unknown transition probabilities, the robust tracking controller for time-delay MJSs without actuator fault is presented in the form of LMIs. Then, considering the influence of the actuator degradation, a virtual tracking model is designed with the estimation of fault parameter which is generated online by a novel adaptive law. The structure of the adaptive laws involving in the connections among various system modes is modified so that the fault estimation is fast enough and the high-frequency oscillations can be reduced effectively. Based on the error dynamic between the actual tracking model and the virtual one, the adaptive fault-tolerant controller can be designed to achieve the goal of the output of fault system tracking the reference signal asymptotically. Finally, the numerical example is given to demonstrate the effectiveness of the proposed method.

Notation: The superscript T stands for matrix transposition, R^n denotes the n dimensional Euclidean space. $He(X)$ represents the sum of X and X^T. In addition, in symmetric block matrices or long matrix expressions, $*$ is used as an ellipsis for the terms that are introduced by symmetry. The notation $P>0$ (≥ 0) means P is real symmetric positive (semi-positive) definite. $\varepsilon[\cdot]$ stands for the expectation operator with respect to the given probability. $\mathrm{diag}(X_1, X_2, \ldots, X_n)$ is used to denote a block diagonal matrix whose main diagonal is $X_1, X_2, \ldots, X_n$. For a vector $a=(a_1, a_2, \ldots, a_k)^T$, $\Lambda(a)$ denotes $\mathrm{diag}(a_1, a_2, \ldots, a_k)$.

2. Problem statement

Consider a class of nonlinear MJS with time-varying delay and actuator faults:

$$\dot{x}(t)=A(r(t))x(t)+A_d(r(t))x(t-h(t))+B(r(t))\Lambda(\rho(t))u(t)+B_w(r(t))\omega(t)+f(t,x(t),r(t))$$

$$y(t)=C(r(t))x(t)+D(r(t))\Lambda(\rho(t))u(t)$$

$$x(t)=\psi(t), \quad -\overline{h}\leq t\leq 0 \tag{1}$$

where $x(t)\in R^n$ is the state variable, $A(r(t))\in R^{n\times n}$, $A_d(r(t))\in R^{n\times n}$, $B(r(t))\in R^{n\times m}$ and $B_w(r(t))\in R^{n\times m_0}$ are known matrix functions of the Markov jump process $r(t)$. $h(t)$ is the known time-varying delay with $h(t)\in(0,\overline{h}]$, $\dot{h}(t)\leq\overline{\mu}<1$, $\psi(t)$ is a vector-valued initial continuous function defined on the interval $[-\overline{h},0]$. $\omega(t)\in L_2^{m_0}[0,\infty)$ is the external disturbance signal. $r(t)$ is a time-homogeneous Markov process with right continuous trajectories and takes values on the finite set $\mathbb{L}=\{1,2,\ldots,\mathbb{N}\}$ with the following mode transition probabilities [32]:

$$\Pr\{r(t+dt)=j|r(t)=i\}=\begin{cases}\pi_{ij}dt+o(dt), & i\neq j\\ 1+\pi_{ii}dt+o(dt), & i=j\end{cases}$$

where $dt>0$, $\lim_{dt\to 0}o(dt)/dt=0$. $\pi_{ij}\geq 0$, for $i\neq j$, is the transition rate from mode i to mode j, and

$$\sum_{j=1,i\neq j}^{\mathbb{N}}\pi_{ij}=-\pi_{ii},\quad i=(1,\ldots,\mathbb{N}).$$

This paper considers more general transition probabilities in which some elements are exactly known, some ones are merely known with lower and upper bounds, and others may have no information to use. For instance, the transition probability matrix might be described by matrix

$$\begin{bmatrix}\pi_{11} & ? & \pi_{13} & \cdots & \pi_{1\mathbb{N}}\\ ? & ? & \alpha & \cdots & \pi_{2\mathbb{N}}\\ \vdots & \cdots & \cdots & \ddots & \vdots\\ \pi_{\mathbb{N}1} & \beta & \cdots & \cdots & \pi_{\mathbb{N}\mathbb{N}}\end{bmatrix}\tag{2}$$

where "?" denotes the unaccessible elements, α and β have known lower and upper bounds ($\underline{\alpha}\leq\alpha\leq\overline{\alpha}$ and $\underline{\beta}\leq\beta\leq\overline{\beta}$) and π_{ij} is exactly known.

For notation clarity, $\forall i,j\in\mathbb{L}$, it can be further rewritten that $\mathbb{L}=\mathbb{L}_k^i\cup\mathbb{L}_{uk}^i$ with

$$\mathbb{L}_k^i\triangleq\{j:\underline{\pi}_{ij}\leq\pi_{ij}\leq\overline{\pi}_{ij}\},\quad \mathbb{L}_{uk}^i\triangleq\{j:j\notin\mathbb{L}_k^i\},$$

$$\mathcal{L}_k^i\triangleq\{m|m\in\mathbb{L}_k^i \text{ and } m\neq i\},\quad \mathcal{L}_{uk}^i\triangleq\{m|m\in\mathbb{L}_{uk}^i \text{ and } m\neq i\}.$$

$$\lambda_k^i=\sum_{m\in\mathcal{L}_k^i}\pi_{im},\quad \underline{\lambda}_k^i=\sum_{m\in\mathcal{L}_k^i}\underline{\pi}_{im},\quad \lambda_{uk}^i=\sum_{m\in\mathcal{L}_{uk}^i}\pi_{im},\quad \underline{\lambda}_*=-\underline{\pi}_{ii}-\underline{\lambda}_k^i$$

$$\mathcal{P}_k^i=\sum_{m\in\mathcal{L}_k^i}\pi_{ij}P_m,\quad \overline{\mathcal{P}}_k^i=\sum_{m\in\mathcal{L}_k^i}\overline{\pi}_{ij}P_m$$

where $\underline{\pi}_{ij}$ and $\overline{\pi}_{ij}$ are known lower and upper bounds of π_{ij}, respectively.

Remark 1. In order to make the result more general, the elements π_{ij} are unknown but with upper and lower known bounds are also included in $\mathbb{L}_k^i$. When π_{ij} is exactly known, it can be considered that the upper bound is equal to the lower bound.

Throughout this paper, the relevant matrices and the nonlinear functions associated with the ith ($r(t)=i$) mode are simplistically denoted by

$$A_i=A(r(t)),\quad A_{di}=A_d(r(t)),\quad B_i=B(r(t)),\quad B_{wi}=B_w(r(t))$$

$$f_i(x(t))=f(t,x(t),r(t)),\quad C_i=C(r(t)),\quad D_i=D(r(t))$$

In system (1), the nonlinear function $f_i(x(t))$ is assumed to satisfy the following conditions:

$$f_i(x(t))=\Delta(r(t),t)x(t)$$

$$\Delta(r(t),t) = M(r(t))\Phi(r(t),t)N(r(t)) \tag{3}$$

where M_i, N_i are known constant matrices when $r(t) = i$. $\Phi(r(t),t)$ is time-varying matrix satisfying $\|\Phi(r(t),t)\| \le 1$.

The actuator degradation is considered in this paper, which can be described as follows:

$$u_k^F(t) = \rho_k(t)u_k(t), \quad 0 < \underline{\rho}_k \le \rho_k(t) \le \overline{\rho}_k \le 1, \ \ k = 1, \dots, m \tag{4}$$

where $\rho_k(t)$ is a degradation parameter, which like in [8] is assumed to be the piecewise constant, that is, $\dot{\rho}_k(t) = 0$. $u_k^F(t)$ denotes the kth actuator that has failed. $\underline{\rho}_k$ and $\overline{\rho}_k$ are the lower and upper bounds of $\rho_k(t)$, respectively. When $\underline{\rho}_k = \overline{\rho}_k = 1$, there is no fault for the kth actuator. When $0 < \underline{\rho}_k \le \rho_k(t) \le \overline{\rho}_k < 1$, the kth actuator is loss of effectiveness. Choose $\rho(t) = (\rho_1(t), \rho_2(t), \dots, \rho_m(t))^T$, the MJS with actuator faults and time-varying delay is presented as Eq. (1).

It is known that $\{x(t), r(t)\}$ is a Markov process with an initial state (x_0, r_0). The solution $x(t, x_0, r_0)$ of the system (1) with $r_0 \in \mathbb{L}$ is denoted by $x(t)$, and the weak infinitesimal generator acting on function V is defined as follows:

$$\xi V(x(t), r(t), t) = \lim_{\epsilon \to 0^+} \frac{1}{\epsilon}\left[\varepsilon\left(V(x(t+\epsilon), t+\epsilon, r(t+\epsilon))\big| x(t), r(t) = i\right) - V(x(t), r(t), t)\right].$$

Throughout this paper, the following definition is used.

Definition 1. The system (1) with $\omega(t) = 0$ and $u(t) = 0$ is said to be stochastically stable if for finite $\psi(t) \in R^n$ defined on $[-\overline{h}, 0]$ and $r(0) \in \mathbb{L}$, the following inequality is satisfied:

$$\lim_{t \to \infty} \varepsilon\left\{\int_0^t x^T(s, \psi, r(0))x(s, \psi, r(0))\, ds\right\} < \infty \tag{5}$$

where $x(s, \psi, r(0))$ denotes the solution to system at time t under the initial conditions $\psi(t)$ and $r(0)$.

The objective of paper is to design a suitable tracking controller for system (1) so that a reference signal $y_r(t)$ can be tracked correctly, where $y_r(t) \in L_2^m[0, \infty]$. The tracking error signal is defined $e(t) = y(t) - y_r(t)$. In [29], the problem of state $x(t)$ tracking reference signal is studied, which is a spacial case of this paper when $C_i = I$, $D_i = 0$.

Based on internal model principle, an additional dynamic is designed as follows:

$$\dot{\zeta}(t) = A_a\zeta(t) + B_a e(t)$$

where A_a and B_a can be chosen considering the feature of the reference signal and external disturbance, which is more general than the case $A_a = 0$, $B_a = I$ used in [29].

Then, an augmented state is selected as $\overline{x} = [x^T(t)\zeta^T(t)]^T$, so $x(t) = I_N\overline{x}$, $I_N = [I 0]$ with appropriate dimension. The augmented system can be written as

$$\dot{\overline{x}}(t) = \overline{A}_i\overline{x}(t) + \overline{A}_{di}\overline{x}(t-h(t)) + \overline{B}_i\Lambda(\rho)u(t) + \overline{B}_{wi}\overline{\omega}(t) + \overline{B}_f f_i(I_N\overline{x}) \tag{6}$$

where

$$\overline{A}_i = \begin{bmatrix} A_i & 0 \\ B_aC_i & A_{ai} \end{bmatrix}, \quad \overline{A}_{di} = \begin{bmatrix} A_{di} & 0 \\ 0 & 0 \end{bmatrix}, \quad \overline{B}_i = \begin{bmatrix} B_i \\ B_aD_i \end{bmatrix},$$

$$\overline{B}_{wi} = \begin{bmatrix} B_{wi} & 0 \\ 0 & -B_{ai} \end{bmatrix}, \quad \overline{\omega}(t) = \begin{bmatrix} \omega(t) \\ y_r(t) \end{bmatrix}, \quad \overline{B}_{fi} = \begin{bmatrix} I \\ 0 \end{bmatrix}$$

Then, as the statement above, the main purpose of this paper is to design a robust adaptive fault-tolerant controller to reduce the effect of actuator fault and achieve the objective of the output of MJSs tracking the reference signal asymptotically in the case where external disturbance and nonlinear term have effects on the system dynamics.

Remark 2. For the problem of actuator and sensor faults, a novel sliding mode observer is designed to attenuate the effect of system fault in [24], while the transition probabilities are assumed to be completely known. Considering linear time-invariant model, adaptive fault-tolerant tracking problem is studied in [26]. In [29], the continuous PI tracking controller is designed for a class of MJSs subject to the partly unknown transition probabilities, while the actuator failure is not considered and the transition probabilities considered includes only completely known cases and unknown cases. As the authors know, there is almost no research work to investigate the problem of adaptive fault-tolerant tracking control for MJSs with actuator fault and time-varying delay.

3. Adaptive fault-tolerant tracking controller design

In this section, fault-tolerant tracking controller can be designed and a novel adaptive approach is utilized. Before presenting the main results, the robust tracking controller for system (1) without fault will be designed firstly.

If there is no fault in the system, the augmented tracking control system can be expressed as

$$\dot{\overline{x}}(t) = \overline{A}_i\overline{x}(t) + \overline{A}_{di}\overline{x}(t-h(t)) + \overline{B}_iu(t) + \overline{B}_{wi}\overline{\varpi}(t) + \overline{B}_f f_i(I_N\overline{x}) \tag{7}$$

Consider the following controller for system (7) as

$$u(t) = K_{Ni}\overline{x}(t) = [K_{ai} \quad K_{bi}]\begin{bmatrix} x(t) \\ \zeta(t) \end{bmatrix} \tag{8}$$

Then, the closed-loop tracking control system without faults is given by

$$\dot{\overline{x}}(t) = \overline{A}_{ci}\overline{x}(t) + \overline{A}_{di}\overline{x}(t-h(t)) + \overline{B}_{wi}\overline{\varpi}(t) + \overline{B}_f f_i(I_N\overline{x}) \tag{9}$$

where $\overline{A}_{ci} = \overline{A}_i + \overline{B}_iK_{Ni}$.

Firstly, the normal robust controller design for Markov jump system (7) will be presented in the following lemma.

Lemma 1. *The Markov jump system (7) is robustly stochastically stable if there exist scalars* $\gamma > 0$, $\eta_i > 0$, *matrices* $W_i > 0$, $S^* > 0$ *and* Y_i *such that the following conditions hold*:

$$i \in \mathbb{L}_k^i, \quad l \in \mathbb{L}_{uk}^i$$

$$\begin{bmatrix} \Psi_{1i} & W_i & W_i & Y_i & \overline{B}_{wi} & W_i I_N^T N_i^T & \mathcal{F}_i & \sqrt{\underline{\lambda}_*} W_i \\ * & -Q^{-1} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -S^* & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -R^{-1} & 0 & 0 & 0 & 0 \\ * & * & * & * & -\gamma^2 I & 0 & 0 & 0 \\ * & * & * & * & * & -\eta_i I & 0 & 0 \\ * & * & * & * & * & * & -\mathcal{G}_i & 0 \\ * & * & * & * & * & * & * & -W_l \end{bmatrix} < 0 \tag{10}$$

$$i \in \mathbb{L}_{uk}^i \begin{cases} \begin{bmatrix} \Psi_{2i} & W_i & W_i & Y_i & \overline{B}_{wi} & W_i I_N^T N_i^T & \mathcal{F}_i \\ * & -Q^{-1} & 0 & 0 & 0 & 0 & 0 \\ * & * & -S^* & 0 & 0 & 0 & 0 \\ * & * & * & -R^{-1} & 0 & 0 & 0 \\ * & * & * & * & -\gamma^2 I & 0 & 0 \\ * & * & * & * & * & -\eta_i I & 0 \\ * & * & * & * & * & * & -\mathcal{G}_i \end{bmatrix} < 0 \\ W_m \geq W_i, m \in \mathbb{L}_{uk}^i \end{cases} \tag{11}$$

$$\begin{bmatrix} T & I \\ * & W_i \end{bmatrix} > 0 \tag{12}$$

where

$$\Psi_{1i} = He(\overline{A}_i W_i + \overline{B}_i Y_i) + \overline{\pi}_{ii} W_i + (1-\overline{\mu})^{-1} \overline{A}_{di} S^* \overline{A}_{di}^T + \eta_i \overline{B}_f M_i M_i^T \overline{B}_f^T$$

$$\Psi_{2i} = He(\overline{A}_i W_i + \overline{B}_i Y_i) - \underline{\lambda}_k^i W_i + (1-\overline{\mu})^{-1} \overline{A}_{di} S^* \overline{A}_{di}^T + \eta_i \overline{B}_f M_i M_i^T \overline{B}_f^T$$

$$\mathcal{F}_i = [\underbrace{\sqrt{\overline{\pi}_{ij_1}} W_i \quad \cdots \quad \sqrt{\overline{\pi}_{ij_\sigma}} W_i}_{j_\sigma \in \mathcal{L}_k^i}]$$

$$\mathcal{G}_i = \text{diag}\underbrace{\{W_{j_1} \quad W_{j_2} \quad \cdots \quad W_{j_\sigma}\}}_{j_\sigma \in \mathcal{L}_k^i}$$

then the tracking controller is

$$u(t) = K_{Ni} \overline{x}(t), \quad K_{Ni} = Y_i W_i^{-1} \tag{13}$$

With the constrain (12), *the following robust performance can be achieved*:

$$\int_0^t [x^T(t) Q_1 x(t) + \zeta^T(t) Q_2 \zeta(t) + u^T(t) R u(t)]\, dt \leq \gamma^2 \int_0^t \overline{\varpi}^T(t) \overline{\varpi}(t)\, dt + \overline{x}^T(0) T \overline{x}(0) \tag{14}$$

where $Q = \text{diag}[Q_1, Q_2] > 0$, γ *can be regarded as* H_∞*−like performance index, the regulated output with* $R > 0$ *is defined as* $z(t) = [Q^{1/2}, 0]^T \overline{x}(t) + [0, R^{1/2}]^T u(t)$.

Proof. The stochastic Lyapunov–Krasovskii functional is chosen as follows:

$$V(\overline{x}(t),r(t),t)=\overline{x}^T(t)P(r(t))\overline{x}(t)+\int_{t-h(t)}^{t}\overline{x}^T(\tau)S\overline{x}(\tau)\,d\tau \tag{15}$$

The weak infinitesimal generator acting on $V(\cdot)$ from the point $r(t)=i(i\in\mathbb{L})$ is given as

$$\begin{aligned}\xi V(\overline{x}_t,r(t),t)&=2\overline{x}^T(t)P_i\dot{\overline{x}}(t)+\overline{x}^T(t)\sum_{j=1}^{\mathbb{N}}\pi_{ij}P_j\overline{x}(t)\\&\quad+\overline{x}^T(t)S\overline{x}(t)-(1-\dot{h}(t))\overline{x}^T(t-h(t))S\overline{x}(t-h(t))\\&=2\overline{x}^T(t)P_i[\overline{A}_{ci}\overline{x}(t)+\overline{A}_{di}\overline{x}(t-h(t))+\overline{B}_{wi}\overline{\varpi}(t)+\overline{B}_ff_i(I_N\overline{x})]\\&\quad+\overline{x}^T(t)\sum_{j=1}^{\mathbb{N}}\pi_{ij}P_j\overline{x}(t)+\overline{x}^T(t)S\overline{x}(t)-(1-\dot{h}(t))\overline{x}^T(t-h(t))S\overline{x}(t-h(t))\end{aligned} \tag{16}$$

It follows

$$\begin{aligned}&\xi V(\overline{x}(t),r(t),t)+z^T(t)z(t)-\gamma^2\overline{\varpi}^T(t)\overline{\varpi}(t)\\&\quad=\overline{x}^T(t)P_i[\overline{A}_{ci}\overline{x}(t)+\overline{A}_{di}\overline{x}(t-h(t))+\overline{B}_{wi}\overline{\varpi}(t)+\overline{B}_ff_i(I_N\overline{x})]\\&\qquad+[\overline{A}_{ci}\overline{x}(t)+\overline{A}_{di}\overline{x}(t-h(t))+\overline{B}_{wi}\overline{\varpi}(t)+\overline{B}_ff_i(I_N\overline{x})]^TP_i\overline{x}(t)\\&\qquad+\overline{x}^T(t)S\overline{x}(t)-(1-\dot{h}(t))\overline{x}^T(t-h(t))S\overline{x}(t-h(t))+\overline{x}^T(t)Q\overline{x}(t)\\&\qquad+\overline{x}^T(t)K_{Ni}^TRK_{Ni}\overline{x}(t)-\gamma^2\overline{\varpi}^T(t)\overline{\varpi}(t)+\overline{x}^T(t)\sum_{j=1}^{\mathbb{N}}\pi_{ij}P_j\overline{x}(t)\end{aligned} \tag{17}$$

Considering $f_i=\Delta_ix(t)$ and $\dot{h}(t)\le\overline{\mu}$, we have

$$\begin{aligned}&\xi V(\overline{x}(t),r(t),t)+z^T(t)z(t)-\gamma^2\overline{\varpi}^T(t)\overline{\varpi}(t)\\&\quad\le\overline{x}^T(t)P_i[\overline{A}_{ci}\overline{x}(t)+\overline{A}_{di}\overline{x}(t-h(t))+\overline{B}_{wi}\overline{\varpi}(t)]\\&\quad+[\overline{A}_{ci}\overline{x}(t)+\overline{A}_{di}\overline{x}(t-h(t))+\overline{B}_{wi}\overline{\varpi}(t)]^TP_i\overline{x}(t)\\&\quad+\overline{x}^T(t)S\overline{x}(t)-(1-\overline{\mu})\overline{x}^T(t-h(t))S\overline{x}(t-h(t))+\overline{x}^T(t)Q\overline{x}(t)\\&\quad+\overline{x}^T(t)K_{Ni}^TRK_{Ni}\overline{x}(t)-\gamma^2\overline{\varpi}^T(t)\overline{\varpi}(t)+\overline{x}^T(t)\sum_{j=1}^{\mathbb{N}}\pi_{ij}P_j\overline{x}(t)\\&\quad+\eta_i\overline{x}^T(t)P_i\overline{B}_fM_iM_i^T\overline{B}_f^TP_i\overline{x}(t)+\eta_i^{-1}\overline{x}^T(t)I_N^TN_i^TN_iI_N\overline{x}(t)\end{aligned} \tag{18}$$

So,

$$\begin{aligned}&\xi V(\overline{x}(t),r(t),t)+z^T(t)z(t)-\gamma^2\overline{\varpi}^T(t)\overline{\varpi}(t)\\&\quad\le\chi^T(t)\Pi_i\chi(t)\end{aligned} \tag{19}$$

where $\chi(t)=[\overline{x}^T(t),\overline{x}^T(t-h(t)),\overline{\varpi}^T(t)]^T$,

$$\Pi_i=\begin{bmatrix}\Omega_i+\sum_{j=1}^{\mathbb{N}}\pi_{ij}P_j & P_i\overline{A}_{di} & P_i\overline{B}_{wi}\\ * & -(1-\overline{\mu})S & 0\\ * & * & -\gamma^2I\end{bmatrix}$$

$$\Omega_i=P_i\overline{A}_{ci}+\overline{A}_{ci}^TP_i+Q+S+K_{Ni}^TRK_{Ni}$$

$$+\eta_i P_i \overline{B}_f M_i M_i^T \overline{B}_f^T P_i + \eta_i^{-1} I_N^T N_i^T N_i I_N$$

Since $\pi_{ii} < 0$ which may be unknown, the proof is separated into two cases, $i \in \mathbb{L}_k^i$ and $i \in \mathbb{L}_{uk}^i$.

Case 1: $i \in \mathbb{L}_k^i$.

In this case, π_{ii} is bounded or exactly known. Based on $\sum_{j=1}^{\mathbb{N}} \pi_{ij} = \lambda_k^i + \pi_{ii} + \lambda_{uk}^i = 0$ and $-\sum_{l \in \mathcal{L}_{uk}^i} \pi_{il}/(\pi_{ii} + \lambda_k^i) = 1$, it is easy to obtain that

$$\begin{aligned}\Pi_i &= \begin{bmatrix} \Omega_i + \mathcal{P}_k^i + \pi_{ii} P_i + \sum_{l \in \mathcal{L}_{uk}^i} \pi_{il} P_l & P_i \overline{A}_{di} & P_i \overline{B}_{wi} \\ * & -(1-\overline{\mu})S & 0 \\ * & * & -\gamma^2 I \end{bmatrix} \\ &= -\frac{\sum_{l \in \mathcal{L}_{uk}^i} \pi_{il}}{\pi_{ii} + \lambda_k^i} \Pi_{il}\end{aligned}$$

where

$$\Pi_{il} = \begin{bmatrix} \Omega_i + \mathcal{P}_k^i + \pi_{ii} P_i - (\pi_{ii} + \lambda_k^i) P_l & P_i \overline{A}_{di} & P_i \overline{B}_{wi} \\ * & -(1-\overline{\mu})S & 0 \\ * & * & -\gamma^2 I \end{bmatrix}$$

Considering the fact that $0 \le -\pi_{il}/(\pi_{ii} + \lambda_k^i) \le 1$, it is easy to obtain that $\Pi_{il} \le 0$ can guarantee $\Pi_i \le 0$.

Since $\underline{\pi}_{ij} \le \pi_{ij} \le \overline{\pi}_{ij}$, the following inequality can be obtained:

$$\begin{aligned}\Pi_{il} &\le \tilde{\Pi}_{il} \\ &= \begin{bmatrix} \Omega_i + \overline{\mathcal{P}}_k^i + \overline{\pi}_{ii} P_i - (\underline{\pi}_{ii} + \underline{\lambda}_k^i) P_l & P_i \overline{A}_{di} & P_i \overline{B}_{wi} \\ * & -(1-\overline{\mu})S & 0 \\ * & * & -\gamma^2 I \end{bmatrix}\end{aligned}$$

So $\tilde{\Pi}_{il} < 0$ can make $\Pi_i \le 0$ hold.

Choose $W_i = P_i^{-1}$, $S^* = S^{-1}$. Pre- and post- multiply the left sides of $\tilde{\Pi}_{il} < 0$ by $\mathrm{diag}(W_i, I, I)$, respectively. Apply Schur complement and introduce new variable $Y_i = K_{Ni} W_i$.

It can be obtained that Eq. (10) makes $\Pi_i \le 0$.

Case 2: $i \in \mathbb{L}_{uk}^i$.

Because $\sum_{j=1}^{\mathbb{N}} \pi_{ij} = 0$, there exists

$$x^T(t) \sum_{j=1}^{\mathbb{N}} \pi_{ij} P_i x(t) = 0 \tag{20}$$

$$\pi_{ii} = -(\lambda_k^i + \lambda_{uk}^i) \tag{21}$$

Then, Π_i can be written as

$$\Pi_i = \begin{bmatrix} \Omega_i + \sum_{j \in \mathcal{L}_k^i} \pi_{ij}(P_j - P_i) & P_i\overline{A}_{di} & P_i\overline{B}_{wi} \\ * & -(1-\overline{\mu})S & 0 \\ * & * & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} \sum_{m \in \mathcal{L}_{uk}^i} \pi_{im}(P_m - P_i) & 0 & 0 \\ * & 0 & 0 \\ * & * & 0 \end{bmatrix} \tag{22}$$

Considering $\underline{\pi}_{ij} \le \pi_{ij} \le \overline{\pi}_{ij}$, the following inequality can be obtained:

$$\Pi_i \le \begin{bmatrix} \Omega_i + \sum_{j \in \mathcal{L}_k^i} \overline{\pi}_{ij} P_j - \sum_{j \in \mathcal{L}_k^i} \underline{\pi}_{ij} P_i & P_i\overline{A}_{di} & P_i\overline{B}_{wi} \\ * & -(1-\overline{\mu})S & 0 \\ * & * & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} \sum_{m \in \mathcal{L}_{uk}^i} \pi_{im}(P_m - P_i) & 0 & 0 \\ * & 0 & 0 \\ * & * & 0 \end{bmatrix} \tag{23}$$

So, $\Pi_i < 0$ if the following inequalities hold:

$$\begin{bmatrix} \Omega_i + \sum_{j \in \mathcal{L}_k^i} \overline{\pi}_{ij} P_j - \sum_{j \in \mathcal{L}_k^i} \underline{\pi}_{ij} P_i & P_i\overline{A}_{di} & P_i\overline{B}_{wi} \\ * & -(1-\overline{\mu})S & 0 \\ * & * & -\gamma^2 I \end{bmatrix} < 0 \tag{24}$$

$$\begin{bmatrix} \sum_{m \in \mathcal{L}_{uk}^i} \pi_{im}(P_m - P_i) & 0 & 0 \\ * & 0 & 0 \\ * & * & 0 \end{bmatrix} < 0 \tag{25}$$

Choose $W_i = P_i^{-1}$, $S^* = S^{-1}$. It is known that $W_m > W_i$ is equivalent to $P_m < P_i$. Pre- and post-multiply the left sides of Eq. (24) by $\mathrm{diag}(W_i, I, I)$, respectively. Applying Schur complement and introducing new variable $Y_i = K_{Ni} W_i$, it is easy to prove that inequalities in Eq. (11) guarantee Eqs. (24) and (25) hold.

So, Eqs. (10) and (11) can make sure that $\Pi_i < 0$, which means

$$\xi V(\overline{x}(t), r(t), t) + z^T(t)z(t) - \gamma^2 \overline{\omega}^T(t)\overline{\omega}(t) < 0 \tag{26}$$

Integrating both sides of the above inequality, we have

$$\varepsilon \int_0^\infty \xi[V(\overline{x}(t), r(t), t)]\, dt + \varepsilon \int_0^\infty [z^T(t)z(t) - \gamma^2 \overline{\omega}^T(t)\overline{\omega}(t)]\, dt < 0 \tag{27}$$

With inequality (12) and $P_i = W_i^{-1}$, it can be obtained that

$$\int_0^\infty z^T(t)z(t)\, dt < \int_0^\infty \gamma^2 \overline{\omega}^T(t)\overline{\omega}(t)]\, dt + \overline{x}^T(0) T \overline{x}(0) \tag{28}$$

If there does not exist disturbance, it is obtained that $\xi V(\overline{x}_t, r(t), t) < 0$, which means that system (7) is stochastically stable. The proof is completed. □

Remark 3. In order to obtain a proper normal control gain K_{Ni}, the following optimization problem with suitable scalars $\nu > 0$ and $\varpi > 0$ can be solved:

$$\min \nu \mathrm{Tr}(T) + \varpi\gamma \quad \text{s.t. Eqs.(10)–(12)}$$

The closed loop tracking model without faults is considered as Eq. (9) which is the same as the one of [29]. However, the state tracking problem and the structure of PI tracking controller in [29] are only two special cases of this paper. When dealing with the partly unknown transition probabilities, the approach used in this paper is less conservative, which can be seen in the proof of Lemma 1. Moreover, the actuator fault is not considered in [29].

Remark 4. In this paper, the method used in Lemma 1 to deal with the partly unknown transition probabilities is less conservative than that of Proposition 3.1 in [29]. In order to demonstrate this conclusion, consider the following simple Markov jump linear system:

$$\dot{x}(t) = A_i x(t) \tag{29}$$

The partly unknown transition probabilities considered include only two cases, that is the known elements and the unknown ones. Based on Proposition 3.1 in [29], the conditions to guarantee system (29) stochastically stable are that there exists a set of positive-definite symmetric matrices P_i such that

$$\begin{aligned} &\left(1 + \sum_{l \in \mathbb{L}_k^i} \pi_{il}\right)(P_i A_i + A_i^T P_i) + \sum_{j \in \mathbb{L}_k^i} \pi_{ij} P_j < 0 \\ &P_i A_i + A_i^T P_i + P_j < 0 j \neq i, \quad j \in \mathbb{L}_{uk}^i \\ &P_i A_i + A_i^T P_i + P_j > 0 j = i, \quad j \in \mathbb{L}_{uk}^i \end{aligned} \tag{30}$$

However, based on the proof of Lemma 1 of this paper, the conditions to guarantee system (29) stochastically stable are

$$\begin{aligned} &i \in \mathbb{L}_k^i \left\{ P_i A_i + A_i^T P_i + \sum_{j \in \mathbb{L}_k^i} \pi_{ij} P_j - (\pi_{ii} + \lambda_k^i) P_l < 0, \quad l \in \mathbb{L}_{uk}^i \right. \\ &i \in \mathbb{L}_{uk}^i \begin{cases} P_i A_i + A_i^T P_i + \sum_{j \in \mathcal{L}_k^i} \pi_{ij}(P_j - P_i) < 0, \\ P_m - P_i \leq 0, \quad m \in \mathbb{L}_{uk}^i \end{cases} \end{aligned} \tag{31}$$

Different methods to deal with the partly unknown transition probabilities lead to the differences between Eqs. (30) and (31). Consider the two cases $i \in \mathbb{L}_k^i$ and $i \in \mathbb{L}_{uk}^i$, it is easy to prove that if the conditions (30) hold, then the conditions (31) hold. Therefore, it demonstrates that the method used in this paper to deal with the partly unknown transition probabilities is less conservative.

If there are actuator faults, the augmented tracking system is presented as Eq. (6). In this paper, nonlinear function $f_i(\cdot)$ and the time delay $h(t)$ are assumed to be known.

In order to design the adaptive fault-tolerant tracking controller, a virtual tracking model is firstly constructed as follows:

$$\dot{\tilde{x}}(t)=\overline{A}_i\tilde{x}(t)+\overline{A}_{di}\tilde{x}(t-h(t))+\overline{B}_i\Lambda(\hat{\rho}(t))u_0(t)+\overline{B}_{wi}[0y_r^T(t)]^T+\overline{B}_f f_i(I_N\tilde{x}(t)) \tag{32}$$

where $u_0(t)$ is the virtual controller to be designed, $\hat{\rho}(t)$ is the estimation of $\rho(t)$, which can be generated online with a novel adaptive law.

If $u_0(t)=\Lambda(\hat{\rho}(t))^{-1}K_{Ni}\tilde{x}(t)$, the virtual model can be rewritten as

$$\dot{\tilde{x}}(t)=\overline{A}_{ci}\tilde{x}(t)+\overline{A}_{di}\tilde{x}(t-h(t))+\overline{B}_{wi}[0y_r^T(t)]^T+\overline{B}_f f_i(I_N\tilde{x}(t)) \tag{33}$$

which matches normal case (no fault) of the closed-loop augmented system (9) with $\omega(t)=0$ and is robustly stochastically stable.

With the above results, the fault-tolerant output tracking problem can be solved if system (6) can track (32) with $u_0(t)=\Lambda(\hat{\rho}(t))^{-1}K_{Ni}\tilde{x}(t)$, $\lim_{t\to\infty}(\overline{x}(t)-\tilde{x}(t))=0$ and a proper adaptive controller $u(t)$.

In this paper, the adaptive controller is designed as $u(t)=u_0(t)+F_ie(t)$. Define $e(t)=\overline{x}(t)-\tilde{x}(t)$, the error dynamic equation between system (6) and virtual model (32) is obtained as

$$\begin{aligned}\dot{e}(t)=&(\overline{A}_i+\overline{B}_i\Lambda(\rho(t))F_i)e(t)+\overline{A}_{di}e(t-h(t))+\tilde{B}_{wi}\omega(t)+\overline{B}_i\Lambda(\tilde{\rho}(t))u_0(t)\\&+\overline{B}_f f_i(I_Ne(t))\end{aligned} \tag{34}$$

where $\tilde{\rho}(t)=\rho(t)-\hat{\rho}(t)$, and $\tilde{B}_{wi}=[B_{wi}^T\ 0]^T$.

System (34) is also a MJS. Therefore, as long as proper F_i can be obtained, the fault-tolerant controller can be designed to guarantee system (34) robust stochastically stable. The design procedure of F_i is given in the following theorem.

Theorem 1. *If there exist symmetric matrix $X_i>0$, diagonal matrix $\Upsilon>0$ and $\Upsilon_f>0$, matrices L_i and S_e^* with appropriate dimensions, and positive scalars τ, ϕ_{1i} and ϕ_{2i}, such that the following conditions hold*:

$$i\in\mathbb{L}_k^i,\quad l\in\mathbb{L}_{uk}^i$$

$$\begin{bmatrix}\Psi_{1i}^e & \tilde{B}_{wi} & X_i & X_i & X_iI_N^TN_i^T & \mathcal{F}_i^e & \sqrt{\lambda_*}X_i\\ * & -\phi_1^2I & 0 & 0 & 0 & 0 & 0\\ * & * & -I & 0 & 0 & 0 & 0\\ * & * & * & -S_e^* & 0 & 0 & 0\\ * & * & * & * & -\phi_{2i}I & 0 & 0\\ * & * & * & * & * & -\mathcal{G}_i^e & 0\\ * & * & * & * & * & * & -X_l\end{bmatrix}<0 \tag{35}$$

$$i\in\mathbb{L}_{uk}^i\begin{cases}\begin{bmatrix}\Psi_{2i}^e & \tilde{B}_{wi} & X_i & X_i & X_iI_N^TN_i^T & \mathcal{F}_i^e\\ * & -\phi_1^2I & 0 & 0 & 0 & 0\\ * & * & -I & 0 & 0 & 0\\ * & * & * & -S_e^* & 0 & 0\\ * & * & * & * & -\phi_{2i}I & 0\\ * & * & * & * & * & -\mathcal{G}_i^e\end{bmatrix}<0\\ X_m>X_i,\quad m\in\mathbb{L}_{uk}^i\end{cases} \tag{36}$$

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$$\dot{\hat{\rho}}_k(t) = \mathrm{Proj}\{\Gamma_{ik}\} = \begin{cases} 0, & \hat{\rho}_k(t) = \overline{\rho}_k,\ \Gamma_{ik} > 0 \ or\ \hat{\rho}_k(t) = \underline{\rho}_k,\ \Gamma_{ik} < 0 \\ \Gamma_{ik} & otherwise,\ k = 1, 2, \ldots, m \end{cases} \tag{37}$$

$$\dot{\hat{\rho}}_{fk}(t) = \mathrm{Proj}\{\Gamma^*_{ik}\} = \begin{cases} 0, & \hat{\rho}_{fk}(t) = \overline{\rho}_k,\ and\ \Gamma^*_{ik} > 0 \ or\ \hat{\rho}_{fk}(t) = \underline{\rho}_k \ and\ \Gamma^*_{ik} < 0 \\ \Gamma^*_{ik} & otherwise, k = 1, 2, \ldots, m \end{cases} \tag{38}$$

where

$$\Psi^e_{1i} = He(\overline{A}_i X_i + \overline{B}_i \Lambda(\rho(t)) L_i) + \overline{\pi}_{ii} X_i + (1-\overline{\mu})^{-1} \overline{A}_{di} S^*_e \overline{A}^T_{di} + \phi_{2i} \overline{B}_f M_i M_i^T \overline{B}^T_f$$

$$\Psi^e_{2i} = He(\overline{A}_i X_i + \overline{B}_i \Lambda(\rho(t)) L_i) - \underline{\lambda}^i_k X_i + (1-\overline{\mu})^{-1} \overline{A}_{di} S^*_e \overline{A}^T_{di} + \phi_{2i} \overline{B}_f M_i M_i^T \overline{B}^T_f + \phi_{1i} \tilde{B}_{wi} \tilde{B}^T_{wi}$$

$$\mathcal{F}^e_i = \underbrace{[\sqrt{\pi_{ij_1}} X_i \quad \cdots \quad \sqrt{\pi_{ij_\sigma}} X_i]}_{j_\sigma \in \mathcal{L}^i_k}$$

$$\mathcal{G}^e_i = \mathrm{diag}\underbrace{\{X_{j_1} \quad X_{j_2} \quad \cdots \quad X_{j_\sigma}\}}_{j_\sigma \in \mathcal{L}^i_k}$$

$\Gamma_i = \Upsilon[\Lambda(u_0(t)) \overline{B}^T_i X_i^{-1} e(t) - \tau(\hat{\rho}(t) - \hat{\rho}_f(t))]$, $\Gamma^*_i = \Upsilon_f(\hat{\rho}(t) - \hat{\rho}_f(t))$, *$\Gamma_{ik}$ and Γ^*_{ik} are the kth row of Γ_i and Γ^*_i, respectively. Then system (34) is robust stochastic stable, and the feedback gain $F_i = L_i X_i^{-1}$.*

Proof. The stochastic Lyapunov–Krasovskii functional is chosen as follows:

$$V_e(e(t), r(t), t) = e^T(t) V(r(t)) e(t) + \int_{t-h(t)}^{t} e^T(v) S_e e(v)\, dv + \tilde{\rho}^T(t) \Upsilon^{-1} \tilde{\rho}(t) + \tau \tilde{\rho}^T_f(t) \Upsilon_f^{-1} \tilde{\rho}_f(t) \tag{39}$$

where V_i, S_e, Υ and Υ_f are symmetric positive matrices and $\tau > 0$, $\tilde{\rho}_f(t) = \rho(t) - \hat{\rho}_f(t)$.

The weak infinitesimal generator acting on $V(\cdot)$ from the point $r(t) = i (i \in \mathbb{L})$ is presented as

$$\begin{aligned} \xi V_e(e(t), r(t), t) &= 2e^T(t) V_i \dot{e}(t) + e^T(t) \sum_{j=1}^{\mathbb{N}} \pi_{ij} V_j e(t) + e^T(t) S_e e(t) \\ &\quad - (1 - \dot{h}(t)) e^T(t - h(t)) S_e e(t - h(t)) - 2\tilde{\rho}^T(t) \Upsilon^{-1} \dot{\hat{\rho}}(t) \\ &\quad - 2\tau \tilde{\rho}^T_f(t) \Upsilon_f^{-1} \dot{\hat{\rho}}_f(t) \\ &= 2e^T(t) V_i [(\overline{A}_i + \overline{B}_i \Lambda(\rho(t)) F_i) e(t) + \overline{A}_{di} e(t - h(t)) + \tilde{B}_{wi} \omega(t) \\ &\quad + \overline{B}_i \Lambda(\tilde{\rho}(t)) u_0(t) + \overline{B}_f f_i(I_N e(t))] + e^T(t) \sum_{j=1}^{\mathbb{N}} \pi_{ij} V_j e(t) \\ &\quad + e^T(t) S_e e(t) - (1 - \dot{h}(t)) e^T(t - h(t)) S_e e(t - h(t)) \\ &\quad - 2\tilde{\rho}^T(t) \Upsilon^{-1} \dot{\hat{\rho}}(t) - 2\tau \tilde{\rho}^T_f(t) \Upsilon_f^{-1} \dot{\hat{\rho}}_f(t) \\ &= 2e^T(t) V_i [(\overline{A}_i + \overline{B}_i \Lambda(\rho(t)) F_i) e(t) + \overline{A}_{di} e(t - h(t)) + \tilde{B}_{wi} \omega(t) \\ &\quad + \overline{B}_f f_i(I_N e(t))] + e^T(t) \sum_{j=1}^{\mathbb{N}} \pi_{ij} V_j e(t) + e^T(t) S_e e(t) \end{aligned}$$

$$-(1-\dot{h}(t))e^T(t-h(t))S_e e(t-h(t)) + \mathcal{R}_i \tag{40}$$

where $\mathcal{R}_i = 2e^T(t)V_i\overline{B}_i\Lambda(u_0(t))\tilde{\rho}(t) - 2\tilde{\rho}^T(t)\Upsilon^{-1}\dot{\hat{\rho}}(t) - 2\tau\tilde{\rho}_f^T(t)\Upsilon_f^{-1}\dot{\hat{\rho}}_f(t)$

Define $\mathcal{H}_i = \Lambda(u_0(t))\overline{B}_i^T V_i e(t)$, $\mathcal{H}_{ik}$ is the kth row of $\mathcal{H}_i$. Υ_k and Υ_{fk} are the kth diagonal element of Υ and Υ_f, respectively.

So, $\mathcal{R}_i = 2\sum_{k=1}^m \mathcal{R}_{ik}$, where $\mathcal{R}_{ik} = \mathcal{H}_{ik}\tilde{\rho}_k(t) - \tilde{\rho}_k(t)\Upsilon_k^{-1}\dot{\hat{\rho}}_k(t) - \tau\tilde{\rho}_{fk}^T(t)\Upsilon_{fk}^{-1}\dot{\hat{\rho}}_{fk}(t)$.

When $\dot{\hat{\rho}}_k(t) = \Gamma_{ik}$ and $\dot{\hat{\rho}}_{fk}(t) = \Gamma_{ik}^*$, it is easy to prove that $\mathcal{R}_{ik} = 0$.

When $\hat{\rho}_k(t) = \overline{\rho}_k$, $\Gamma_{ik} > 0$ and $\dot{\hat{\rho}}_{fk}(t) = \Gamma_{ik}^*$. We have $\tilde{\rho}_k(t) \le 0$, and $\hat{\rho}_k(t) - \hat{\rho}_{fk}(t) > 0$

$$\begin{aligned}
\mathcal{R}_{ik} &= \mathcal{H}_{ik}\tilde{\rho}_k(t) - \tau\tilde{\rho}_{fk}(t)\Upsilon_{fk}^{-1}\dot{\hat{\rho}}_{fk}(t) \\
&= \mathcal{H}_{ik}\tilde{\rho}_k(t) - \tau\tilde{\rho}_{fk}(t)(\hat{\rho}_k(t) - \hat{\rho}_{fk}(t)) \\
&< \mathcal{H}_{ik}\tilde{\rho}_k(t) - \tau\tilde{\rho}_k(t)(\hat{\rho}_k(t) - \hat{\rho}_{fk}(t)) \\
&= \tilde{\rho}_k(t)(\mathcal{H}_{ik} - \tau(\hat{\rho}_k(t) - \hat{\rho}_{fk}(t))) = \Upsilon_k^{-1}\tilde{\rho}_k(t)\Gamma_{ik} \le 0
\end{aligned} \tag{41}$$

For the other cases, as presented in Eqs. (37) and (38) with the projection function, $\mathcal{R}_{ik} \le 0$ can also be guaranteed.

So, adaptive laws (37) and (38) can guarantee that $\mathcal{R}_i = 2\sum_{i=1}^m \mathcal{R}_{ik} \le 0$.

With adaptive laws as Eqs. (37) and (38) where $X_i = V_i^{-1}$. It can be obtained that

$$\begin{aligned}
&\xi V_e(e(t), r(t), t) + e^T(t)e(t) - \phi_1^2\omega^T(t)\omega(t) \\
&\quad \le 2e^T(t)V_i[(\overline{A}_i + \overline{B}_i\Lambda(\rho(t))F_i)e(t) + \overline{A}_{di}e(t-h(t)) + \tilde{B}_{wi}\omega(t) \\
&\quad + \overline{B}_f f_i(e(t))] + e^T(t)\sum_{j=1}^{\mathbb{N}}\pi_{ij}V_j e(t) + e^T(t)S_e e(t) \\
&\quad - (1-\dot{h}(t))e^T(t-h(t))S_e e(t-h(t)) + e^T(t)e(t) - \phi_1^2\omega^T(t)\omega(t) \\
&\quad \le 2e^T(t)V_i[(\overline{A}_i + \overline{B}_i\Lambda(\rho(t))F_i)e(t) + \overline{A}_{di}e(t-h(t))] \\
&\quad + e^T(t)\sum_{j=1}^{\mathbb{N}}\pi_{ij}V_j e(t) + e^T(t)S_e e(t) - (1-\overline{\mu})e^T(t-h(t))S_e e(t-h(t)) \\
&\quad + 2e^T(t)V_i\tilde{B}_{wi}\omega(t) + e^T(t)e(t) - \phi_1^2\omega^T(t)\omega(t) \\
&\quad + \phi_{2i}e^T(t)V_i\overline{B}_f M_i M_i^T\overline{B}_f^T V_i e(t) + \phi_{2i}^{-1}e^T(t)I_N^T N_i^T N_i I_N e(t)
\end{aligned} \tag{42}$$

Thus, it follows that

$$\xi V_e(e_t, r(t), t) + e^T(t)e(t) - \phi_1^2\omega^T(t)\omega(t) \le \chi_e^T(t)\Pi_{ei}\chi_e(t) \tag{43}$$

where

$$\chi_e(t) = [e^T(t), e^T(t-h(t)), \omega^T(t)]^T$$

$$\Pi_{ei} = \begin{bmatrix} \Omega_{ei} + \sum_{j=1}^{\mathbb{N}}\pi_{ij}V_j & V_i\overline{A}_{di} & V_i\tilde{B}_{wi} \\ * & -(1-\overline{\mu})S_e & 0 \\ * & * & -\phi_1^2 I \end{bmatrix} \tag{44}$$

$$\Omega_{ei} = He(V_i(\overline{A}_i + \overline{B}_i\Lambda(\rho(t))F_i)) + S_e + I + \phi_{2i}V_i\overline{B}_f M_i M_i^T\overline{B}_f^T V_i + \phi_{2i}^{-1}I_N^T N_i^T N_i I_N$$

In this paper, some transition probabilities are unknown.

Since $\pi_{ii} < 0$ which may be unknown, the proof is separated into two cases, $i \in \mathbb{L}_k^i$ and $i \in \mathbb{L}_{uk}^i$.

Case 1: $i \in \mathbb{L}_k^i$.

In this case, π_{ii} is bounded or exactly known. Based on $\sum_{j=1}^{\mathbb{N}} \pi_{ij} = \lambda_k^i + \pi_{ii} + \lambda_{uk}^i = 0$ and $-\sum_{l \in \mathcal{L}_{uk}^i} \pi_{il}/(\pi_{ii} + \lambda_k^i) = 1$, it is easy to obtain that

$$\Pi_{ei} = \begin{bmatrix} \Omega_{ei} + \sum\limits_{l \in \mathcal{L}_k^i} \pi_{il} V_l + \pi_{ii} V_i + \sum\limits_{l \in \mathcal{L}_{uk}^i} \pi_{il} V_l & V_i \overline{A}_{di} & V_i \tilde{B}_{wi} \\ * & -(1-\overline{\mu}) S_e & 0 \\ * & * & -\phi_1^2 I \end{bmatrix} = -\frac{\sum_{l \in \mathcal{L}_{uk}^i} \pi_{il}}{\pi_{ii} + \lambda_k^i} \Pi_{eil}$$

where

$$\Pi_{eil} = \begin{bmatrix} \Omega_{ei} + \sum\limits_{l \in \mathcal{L}_k^i} \pi_{il} V_l + \pi_{ii} V_i - (\pi_{ii} + \lambda_k^i) V_l & V_i \overline{A}_{di} & V_i \tilde{B}_{wi} \\ * & -(1-\overline{\mu}) S_e & 0 \\ * & * & -\phi_1^2 I \end{bmatrix}$$

Considering the fact that $0 \leq -\pi_{il}/(\pi_{ii} + \lambda_k^i) \leq 1$, it is easy to obtain that $\Pi_{eil} \leq 0$ can guarantee $\Pi_{ei} \leq 0$.

Since $\underline{\pi}_{ij} \leq \pi_{ij} \leq \overline{\pi}_{ij}$, the following inequality can be obtained:

$$\Pi_{eil} \leq \tilde{\Pi}_{eil} = \begin{bmatrix} \Omega_{ei} + \sum\limits_{l \in \mathcal{L}_k^i} \overline{\pi}_{il} V_l + \overline{\pi}_{ii} V_i - (\underline{\pi}_{ii} + \underline{\lambda}_k^i) V_l & V_i \overline{A}_{di} & V_i \tilde{B}_{wi} \\ * & -(1-\overline{\mu}) S_e & 0 \\ * & * & -\phi_1^2 I \end{bmatrix}$$

So $\tilde{\Pi}_{eil} < 0$ can make $\Pi_{ei} \leq 0$ hold.

Choose $X_i = V_i^{-1}$, $S_e^* = S_e^{-1}$. Pre- and post- multiply the left sides of $\tilde{\Pi}_{eil} < 0$ by $\mathrm{diag}(X_i, I, I)$, respectively. Apply Schur complement and introduce new variable $L_i = F_i X_i$.

It can be obtained that Eq. (35) makes $\Pi_{ei} \leq 0$.

Case 2: $i \in \mathbb{L}_{uk}^i$.

Because $\sum_{j=1}^{\mathbb{N}} \pi_{ij} = 0$, there exists

$$x^T(t) \sum_{j=1}^{\mathbb{N}} \pi_{ij} V_i x(t) = 0, \pi_{ii} = -(\lambda_k^i + \lambda_{uk}^i) \tag{45}$$

Then, Π_{ei} can be written as

$$\Pi_{ei} = \begin{bmatrix} \Omega_{ei} + \sum_{j \in \mathcal{L}_k^i} \pi_{ij} (V_j - V_i) & V_i \overline{A}_{di} & V_i \tilde{B}_{wi} \\ * & -(1-\overline{\mu}) S_e & 0 \\ * & * & -\phi_1^2 I \end{bmatrix} + \begin{bmatrix} \sum_{m \in \mathcal{L}_{uk}^i} \pi_{im} (V_m - V_i) & 0 & 0 \\ * & 0 & 0 \\ * & * & 0 \end{bmatrix} \tag{46}$$

Considering $\underline{\pi}_{ij} \le \pi_{ij} \le \overline{\pi}_{ij}$, the following inequality can be obtained:

$$\Pi_{ei} \le \begin{bmatrix} \Omega_i + \sum_{j \in \mathcal{L}_k^i} \overline{\pi}_{ij} V_j - \sum_{j \in \mathcal{L}_k^i} \underline{\pi}_{ij} V_i & V_i \overline{A}_{di} & V_i \tilde{B}_{wi} \\ * & -(1-\overline{\mu}) S_e & 0 \\ * & * & -\phi_1^2 I \end{bmatrix} + \begin{bmatrix} \sum_{m \in \mathcal{L}_{uk}^i} \pi_{im}(P_m - P_i) & 0 & 0 \\ * & 0 & 0 \\ * & * & 0 \end{bmatrix} \tag{47}$$

So, $\Pi_{ei} < 0$ if the following inequalities hold:

$$\begin{bmatrix} \sum_{m \in \mathcal{L}_{uk}^i} \pi_{im}(V_m - V_i) & 0 & 0 \\ * & 0 & 0 \\ * & * & 0 \end{bmatrix} < 0 \tag{48}$$

$$\begin{bmatrix} \Omega_{ei} + \sum_{j \in \mathcal{L}_k^i} \overline{\pi}_{ij} V_j - \sum\limits_{j \in \mathcal{L}_k^i} \underline{\pi}_{ij} V_i & V_i \overline{A}_{di} & V_i \tilde{B}_{wi} \\ * & -(1-\overline{\mu}) S_e & 0 \\ * & * & -\phi_1^2 I \end{bmatrix} < 0 \tag{49}$$

Choose $X_i = V_i^{-1}$, $S_e^* = S_e^{-1}$. It is known that $X_m > X_i$ is equivalent to $V_m < V_i$. Pre- and post-multiply the left sides of Eq. (49) by $\mathrm{diag}(X_i, I, I)$, respectively. Apply Schur complement and introduce new variable $L_i = F_i X_i$. It can be obtained that inequalities in Eq. (36) make Eqs. (49) and (48) hold.

So, Eqs. (35) and (36) can make sure that $\Pi_{ei} < 0$, which means

$$\xi V_e(e_t, r(t), t) + e^T(t) e(t) - \phi_1^2 \omega^T(t) \omega(t) < 0 \tag{50}$$

Integrating both sides of the above inequality, we have

$$\varepsilon \int_0^\infty \xi [V_e(e_t, r(t), t)]\, dt + \varepsilon \int_0^\infty [e^T(t) e(t) - \phi_1^2 \omega^T(t) \omega(t)]\, dt < 0 \tag{51}$$

With $V_i = X_i^{-1}$, the adaptive H_∞ performance [4] can be obtained that

$$\int_0^\infty e^T(t) e(t)\, dt < \int_0^\infty \phi_1^2 \omega^T(t) \omega(t)\, dt + \mathcal{Z}(0) \tag{52}$$

where $\mathcal{Z}(0) = e^T(0) T_e e(0) + \tilde{\rho}^T(0) \Upsilon^{-1} \tilde{\rho}(0) + \tau \tilde{\rho}_f^T(0) \Upsilon_f^{-1} \tilde{\rho}_f(0)$, T_e is some positive define matrix which satisfying

$$\begin{bmatrix} T_e & I \\ * & X_i \end{bmatrix} > 0 \tag{53}$$

Actually Eq. (50) also implies that $\xi V_e(e_t, r(t), t) < 0$ for $e(t) \ne 0$ and $\omega(t) = 0$, which means by Definition 1 that the error system (34) is stochastically stable. So, Theorem 1 can be proved. □

Remark 5. The adaptive estimation method is used in [9,10,26], where the adaptive law is usually constructed as the following standard structure:

$$\dot{\hat{\rho}} = \Upsilon\Lambda(u_0(t))\overline{B}_i^T X_i^{-1} e(t) \tag{54}$$

It makes $\hat{\rho}$ susceptible to mode changes and external disturbance. In order to reduce the effect, the modification term $\tau(\hat{\rho}(t) - \hat{\rho}_f(t))$ is added into adaptive law (37). In fact, modifying the relevant parameters in standard adaptive law can achieve faster fault estimation, which usually means increasing the eigenvalues of matrix Υ. However, the high-gain method can cause high-frequency oscillations resulting in bad performance [30,31]. Note that $\hat{\rho}_f(t)$ is a low-pass filter of $\hat{\rho}(t)$. The output of the filter is $\hat{\rho}_f(t)$, which is regarded as a feedback to modify the main adaptive law (37). So, the new architecture is presented by adding modification term and (38), which can provide a critical trade-off between the rate of estimation and the system performance. Different from the adaptive laws in [30,31], the projection function is used to limit the boundary of estimation value and the adaptive laws proposed in this paper are related to system jump modes. The simulation results in this paper may further demonstrate this point.

Based on Theorem 1, the adaptive fault tolerant controller can be designed, which guarantees that the augmented system (6) can track the virtual model (32) which is stochastically stable with $u_0(t) = \Lambda(\hat{\rho}(t))^{-1} K_{Ni}\tilde{x}(t)$. It means that the objective of fault tolerant tracking control can be achieved.

4. Numerical example

In this section, a numerical example is given to demonstrate the benefits and effectiveness of the proposed methods.

Example. Consider a continuous-time MJS (1) whose parameters are given as

$$A_1 = \begin{bmatrix} 0.703 & 0.694 \\ 1.75 & -0.203 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.812 & 0.524 \\ 1.81 & -0.221 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 0.901 & 0.504 \\ 1.83 & -0.261 \end{bmatrix}, \quad A_4 = \begin{bmatrix} 0.543 & 0.204 \\ 1.31 & -0.302 \end{bmatrix},$$

$$A_{d1} = A_{d2} = A_{d3} = A_{d4} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.22 & 0.07 \\ 0.18 & 0.3 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.18 & 0.06 \\ -0.12 & 0.3 \end{bmatrix},$$

$$B_3 = \begin{bmatrix} 0.21 & 0.05 \\ 0.21 & 0.32 \end{bmatrix}, \quad B_4 = \begin{bmatrix} 0.17 & 0.08 \\ -0.2 & 0.25 \end{bmatrix},$$

$$B_{\omega 1} = B_{\omega 2} = B_{\omega 3} = B_{\omega 4} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}.$$

Choose $C_i = I, D_i = 0$, which means that the controller needs to be designed to make both states of the system track the reference signals.

Then, for the transition probability matrix, two different cases are considered.

Case 1: The transition probability matrix is chosen as follows:

$$\begin{bmatrix} -1.2 & 0.1 & ? & ? \\ 0.3 & ? & ? & 0.1 \\ 0.2 & ? & -0.9 & ? \\ ? & 0.6 & ? & ? \end{bmatrix}, \tag{55}$$

where ? denotes the unknown transition probabilities.

In the first case, the actuator fault is not considered and the partly known transition probabilities only contain two cases: the known elements and the unknown ones, which is studied in [29].

Considering the above time-delay MJS and the partly known transition probability matrix (55), it can be found that Theorem 3.2 in [29] is suited for designing the tracking controller. However, using the LMI toolbox in MATLAB, the conditions in Theorem 3.2 of [29] are not feasible for the MJS above.

Using Lemma 1 in this paper, the state tracking problem can be solved. Set $Q = \mathrm{diag}\{1, 1, 1, 1\}$, $R = \mathrm{diag}\{1, 1\}$, the controller can be obtained as

$$K_1 = \begin{bmatrix} -19.6939 & -1.2366 & -8.8262 & 2.6532 \\ -3.1339 & -8.2877 & -0.4271 & -6.1237 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} -25.1762 & -1.2582 & -10.2925 & 2.8670 \\ -16.5619 & -13.3114 & -2.5314 & -5.8272 \end{bmatrix},$$

$$K_3 = \begin{bmatrix} -21.0251 & -1.9143 & -8.9434 & 1.3840 \\ -3.0619 & -9.2802 & 0.7958 & -6.3490 \end{bmatrix},$$

$$K_4 = \begin{bmatrix} -35.9472 & 8.2421 & -15.9236 & 12.8517 \\ -10.5048 & -9.6349 & -3.0311 & -6.1648 \end{bmatrix}.$$

As the analysis in Remark 4, it means that Lemma 1 of this paper is less conservative than Theorem 3.2 of [29], which is mainly attributed to the better method dealing with the unknown transition probabilities in this paper.

Case 2: In this case, except for the exactly known and unknown transition probabilities, there are some ones which are merely known with lower and upper bounds. The new transition probability matrix is chosen as

$$\begin{bmatrix} -1.2 & 0.1 & ? & ? \\ 0.3 & ? & ? & 0.1 \\ \alpha_1 & ? & \alpha_2 & ? \\ ? & 0.6 & ? & ? \end{bmatrix},$$

where $0.13 < \alpha_1 < 0.25$, $-0.95 < \alpha_2 < -0.85$.

The reference signal is

$$y_{r1} = \begin{cases} 6, & 0 \le t \le 20 \\ 0, & t > 20, \end{cases} \qquad y_{r2} = \begin{cases} 4, & 10 \le t \le 20 \\ 0, & t > 20 \end{cases}$$

The energy-bounded external disturbance is

$$\omega(t) = \begin{cases} 1 + 0.6 \sin(100t), & 0 \le t \le 15 \\ 0, & t > 15 \end{cases}$$

For the uncertainty, $\Phi(r(t), t) = 0.1 \sin(t)$. The time-varying delay is chosen as $h(t) = 2 + 0.7 \sin(t)$ which is bounded by $\overline{h}(t) = 2.7$. In this example, a possible modes evolution is presented in Fig. 1.

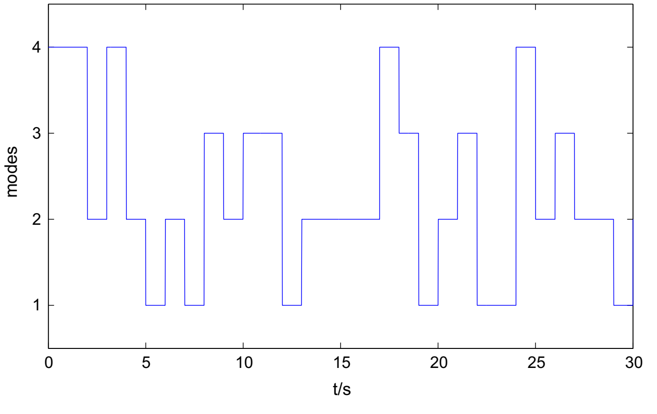

Fig. 1. A possible modes evolution.

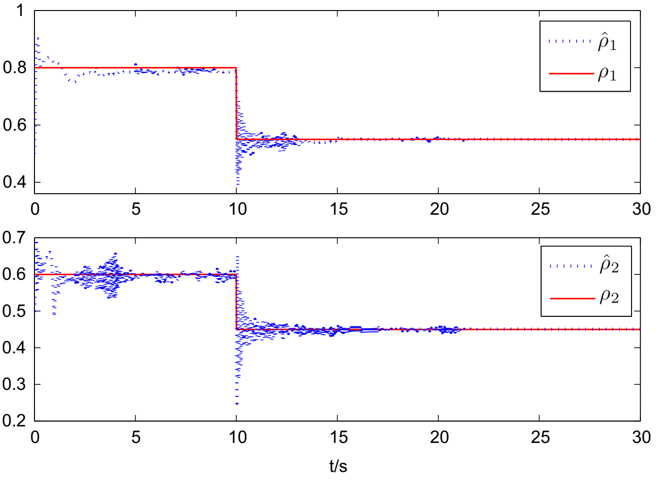

Fig. 2. The fault estimation $\hat{\rho}_i$ (dash line) of the fault ρ_i (solid line) with standard adaptive law (54).

For the system without actuator faults, the tracking controller can be calculated with the conditions in Lemma 1:

$$K_1 = \begin{bmatrix} -233.4467 & 105.1673 & -172.0837 & 141.7351 \\ 104.7859 & -79.5141 & 69.9061 & -104.0543 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} -155.7632 & 34.9239 & -131.6957 & 45.4488 \\ -41.5831 & -33.8032 & -38.4442 & -35.3493 \end{bmatrix},$$

$$K_3 = \begin{bmatrix} -109.9556 & 26.6761 & -88.6786 & 35.1628 \\ 32.1699 & -33.7919 & 22.4750 & -40.5060 \end{bmatrix},$$

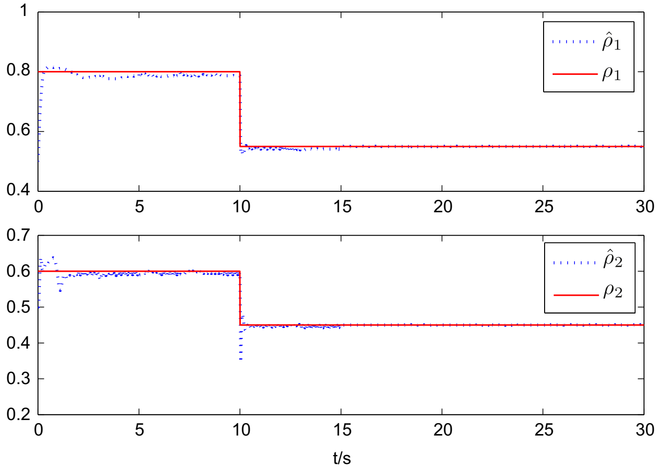

Fig. 3. The fault estimation $\hat{\rho}_i$ (dash line) of the fault ρ_i (solid line) with the proposed adaptive law (37).

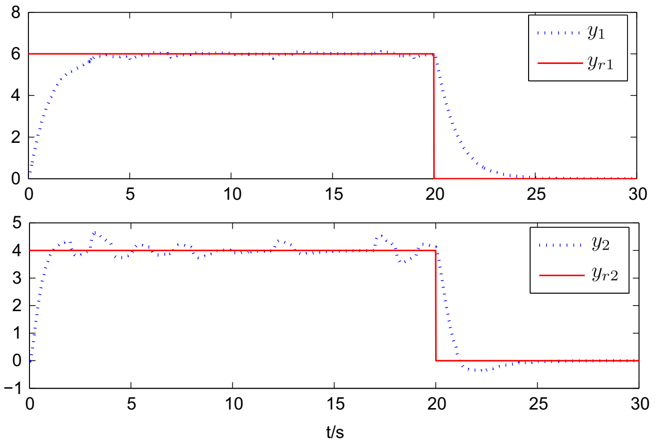

Fig. 4. The reference output y_{ri} (solid line) and the output y_i (dash line).

$$K_4 = 1000 * \begin{bmatrix} -1.7441 & 0.6541 & -1.3788 & 0.8431 \\ 0.0940 & -0.0802 & 0.0614 & -0.1037 \end{bmatrix}.$$

In this case, the actuator faults are considered as

$$u_k^F(t) = \rho_k(t) u_k(t), \quad k = 1, 2$$

$$\rho_1(t) = \begin{cases} 0.8, & 0 \le t \le 10 \\ 0.55, & t > 10, \end{cases} \qquad \rho_2(t) = \begin{cases} 0.6, & 0 \le t \le 10 \\ 0.45, & t > 10 \end{cases}$$

The bound of the faults are $0.2 \le \rho_k(t) \le 0.9,\ k = 1, 2$.

Then, using the conditions (35) and (36) in Theorem 1, F_i and X_i can be obtained.

Set $\Gamma_i = \mathrm{diag}\{50, 50\}$. If the adaptive law is chosen as the standard one (54), the robust stochastic stability of the error system (34) can also be guaranteed, and the estimations $\hat{\rho}_i$ of the fault parameters ρ_i are shown as the dash lines in Fig. 2.

In this paper, the adaptive law is reconstructed by adding a modification term, which is expected to generate better estimations of the fault parameters ρ_i. Choose $\Gamma_i^* = \mathrm{diag}\{50, 50\}$ and $\tau = 4$. Γ_i is given with the same value $\mathrm{diag}\{50, 50\}$. Then, the estimations $\hat{\rho}_i$ of the fault parameters ρ_i are shown as the dash lines in Fig. 3. Compared with Fig. 2, it is easy to find that the estimations in Fig. 3 are better, which indicates the effectiveness of the adaptive law (37). On the other hand, the controller is designed as $u(t) = \Lambda(\hat{\rho}(t))^{-1} K_{Ni} \tilde{x}(t) + F_i e(t)$, which needs to use the fault estimation $\hat{\rho}_i$. If the standard adaptive law is used to estimate the fault parameter, the controller can be more vulnerable to be effected by high-frequency oscillations. Therefore, it is reasonable to use the adaptive law with new structure to construct the fault tolerant controller in this paper.

Then, based on the fault estimation, the controller can work well to deal with the tracking problem. The result is shown in Fig. 4. It can be found from Fig. 4 that the output can track the reference signal with small error which is the result of mode jumping and the extra disturbance. Finally, the tracking error is back to zero when there is not extra disturbance any more. These results demonstrate that the method proposed in this paper is effective.

5. Conclusion

This paper considers the problem of the output tracking control for a class of MJSs with time-varying delay and actuator faults. Firstly, the tracking error is utilized as the input to construct an additional dynamic, which combined with the original system is regarded as the tracking model. In the process of designing the robust tracking controller for time-delay MJSs without actuator fault, a less conservative method is used to deal with the partly unknown transition probabilities. Then, considering the influence of the actuator degradation, a virtual tracking model is designed with the estimation of fault parameter which is generated online by a novel adaptive law. Based on the error dynamic between the actual tracking model and the virtual one, the adaptive fault-tolerant controller can be designed to achieve the goal of the output of fault system tracking the reference signal asymptotically. Finally, the numerical example is given to demonstrate the effectiveness of the proposed method.

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