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HMM-based H_∞ filtering for Markov jump systems with partial information and sensor nonlinearities

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Summary
This work examines the H_∞ filtering issue for Markov jump systems in the circumstances of partial information on Markov chain and randomly occurring sensor nonlinearities. The partial information considered in this work includes partial information on the Markov state, on transition probabilities and on detection probabilities. A hidden Markov model with partially known transition probabilities and detection probabilities is introduced to describe the above partial information phenomenon. The randomly occurring sensor nonlinearities considered in this work depend on the system operating mode. Based on the Lyapunov methodology and the introduced hidden Markov model, some effective H_∞ performance analysis criteria are derived for the filtering error system under the circumstances of partial information and sensor nonlinearities. In addition, the design procedure of the desired hidden Markov model-based filter is established, and finally two examples are used to verify the theoretical results.

KEYWORDS
H_∞ filtering, hidden Markov model, Markov jump systems, partial information, sensor nonlinearities

1 | INTRODUCTION

The study of Markov jump systems (MJSs) has been a lively research focus during the past decades. This is mainly motivated by the fact that many engineering systems can be described by such kind of systems and examples can be found in aircraft control systems, manufacturing systems, economic systems, and so on.[1] Up to now, numerous results about the stability analysis and essential control/filtering problems have been reported in the literature, for example, References 2-8 and their references.

In MJSs, generally speaking, the information of the Markov state (also called system operating mode) plays a significant role in the controller/filter design. One frequently used setting in many existing results is that such information can be intactly acquired and in this scenario, the candidate controller/filter can be chosen as the mode-dependent one. It should be noted that this setting may not be always met in practical engineering owing, in part, to various equipment constraints or unbearable high expense. To circumvent the drawback of the above setting, some efforts have been applied towards dealing with the unknown or partial information of the Markov state and some methods have been put forward. One method is the mode-independent design method, in which the information of the Markov state does not need to be known.[9,10] This method is a blind design method because it completely ignores the information of the Markov state and leads to conservatism. To counter this problem, the other available method is the clustering method. The fundamental idea of this one is that notwithstanding the Markov state is unknown, it is likely to estimate which cluster it belongs to by utilizing the partial information of the Markov state.[11] More recently, a versatile approach, the detector-based approach (also called hidden Markov model [HMM]-based approach), was presented for discrete-time MJSs[12,13] and continuous-time MJSs[14,15] to handle the partial information of the Markov state, which unifies the mode-dependent method, the clustering method, and the mode-independent method. The characteristic of this approach is that the detector can give an estimated state of the Markov state with certain detection probabilities (also called observation probabilities or conditional probabilities in some papers). HMMs have been successfully applied to many practical problems, for example, speech recognition[16] and credit card fraud detection.[17] Based on the HMM-based approach, for MJSs with partial information of the Markov state, the H_∞ control problems were treated in References 18-20; the mixed H_2/H_∞ control problem was considered in Reference 21; the dissipativity control problems by using the state feedback control strategy and the output feedback control strategy were investigated in References 22 and 23, respectively; the result for the H_2 filtering problem was presented in Reference 24; the H_∞ filtering problem was studied in Reference 25; the l_2-l_∞ filtering problem was addressed in Reference 26; the extended dissipativity filtering problem was explored in Reference 27; the sliding mode control issue was addressed in Reference 28 and the dissipativity-based fault detection problem for two-dimensional MJSs was resolved in Reference 29.

Although the above theoretical results are meaningful, there are some potential obstructions to applying them to actual engineering. The first obstruction is that the information of the transition probabilities is presupposed to be known. As pointed out in Reference 30, the objective of acquiring the full information of the transition probabilities may be unlikely achieved or the expenditure of achieving this objective in actual engineering is unaffordable. The second one is about the detection probabilities. The information of the detection probabilities in HMM is also presupposed to be known such that the dilemmas of acquiring the whole information of the transition probabilities also inevitably exist in acquiring the whole information of the detection probabilities. Therefore, when using the HMM-based approach to handle the partial information of the Markov state in MJSs, a challenging problem appears: *how to overcome difficulties for acquiring the whole information of the transition probabilities and detection probabilities?* An effective method is considering the partially known transition probabilities and detection probabilities. In Reference 31, the sliding mode control issue was studied for MJSs via the HMM-based approach, in which transition probabilities were set completely known and the detection probabilities were set partially known. Whereafter, using a similar setting, the synchronization control problem was addressed in Reference 32 for Markov jump neural networks. The situation with both the transition probabilities and the detection probabilities being partially known was considered for the stabilization issue of MJSs in Reference 33 and for the H_∞ or passivity-based control issue of Markov jump singularly perturbed systems in References 34,35. However, when using the HMM-based approach to handle the partial information of the Markov state, to our knowledge, the partially known transition probabilities and detection probabilities have not been taken into account simultaneously for the filtering problem in MJSs, which partly inspires the current work.

On the other hand, the peculiarity of sensors has a tremendous influence on filtering issues. The setting of a linear sensor is convenient for the filter design, however, it may not be applicable in practice due partly to the effects of complex working environments on the sensor and partly to inherent saturation characteristics of the sensor itself. In this context, filtering issues subject to sensor nonlinearities have aroused the attention of many scholars.[36,37] When taking Markov jump parameters into account, the l_2–l_∞ filtering problem with randomly occurring sensor nonlinearities (ROSNs) was considered in Reference 38; and the finite-time filtering issues with ROSNs were handled in References 39,40. It is worth pointing out that the ROSNs in the above-mentioned results do not depend on the system operating mode. In some cases, the probability of ROSNs may rely on the system operating mode.[41] However, the mode-dependent ROSNs have not been brought into proper focus for filtering issues of the MJSs with partial information of the Markov state, especially also with partially known transition probabilities and detection probabilities, which further inspires our present work.

Contributions: This work addresses the HMM-based H_∞ filtering issue for MJSs with the partial information on Markov chain and ROSNs. The main contributions of this work are as such. (a) The partial information problems are fully considered in the investigation of the H_∞ filtering issue for MJSs. The partial information problems contain three parts: (i) The Markov state, that is, the system operating mode signal, cannot be accessed for the filter design. To crack this nut, a detector is employed to detect the Markov state with detection probabilities; (ii) The knowledge of transition probabilities of the system operating mode signal is fragmentary and some transition probabilities are partially known; (iii) The knowledge of detection probabilities of the detector is also fragmentary and some unknown detection probabilities are existent as well. (b) An HMM with partially known transition probabilities and detection probabilities is introduced to well model the above-described three partial information problems. Compared with the HMM with complete information of transition probabilities and detection probabilities, the one introduced in this work is more practical. (c)

With the help of the introduced HMM, the effective H_∞ performance analysis criteria and the design procedure of the desired HMM-based filter for MJSs subject to the partial information and ROSNs are established.

Organization: The system and the concerned problem are formulated in Section 2. The effective H_∞ performance analysis criteria and the design procedure of the desired HMM-based filter are presented in Section 3. Simulation results are given in Section 4. Conclusion and some suggestions for the forthcoming works are given in Section 5.

Notations: $(\Omega, \mathcal{F}, \Pr)$: The complete probability space, in which Ω is the sample space, $\mathcal{F}$ is the σ-algebra of subsets of Ω, and $\Pr$ is the probability measure on $\mathcal{F}$. $\mathbb{E}\{\cdot\}$: the mathematical expectation. $\mathfrak{R}^{n_1}$: the n_1-dimensional Euclidean space. $\mathfrak{R}^{n_1\times n_2}$: the set of $n_1 \times n_2$ real matrices. X^T/X^{-1}: the transpose/inverse of the matrix X. $\star$: a symmetric term in a matrix.

2 | PROBLEM FORMULATION

2.1 | Plant description

Consider the following MJS (Σ_s) in discrete-time domain on the probability space $(\Omega, \mathcal{F}, \Pr)$:

$$\Sigma_s : \begin{cases} x(k+1) = A_{\zeta(k)}x(k) + C_{\zeta(k)}\nu(k), \\ z_s(k) = D_{\zeta(k)}x(k) + G_{\zeta(k)}\nu(k), \end{cases} \tag{1}$$

where $x(k) \in \mathfrak{R}^{n_x}$ and $z_s(k) \in \mathfrak{R}^{n_z}$ are, respectively, the state vector and the objective signal to be estimated; $\nu(k) \in \mathfrak{R}^{n_\nu}$ is an exogenous disturbance; $A_{\zeta(k)} \in \mathfrak{R}^{n_x\times n_x}$, $C_{\zeta(k)} \in \mathfrak{R}^{n_x\times n_\nu}$, $D_{\zeta(k)} \in \mathfrak{R}^{n_z\times n_x}$, and $G_{\zeta(k)} \in \mathfrak{R}^{n_z\times n_\nu}$ are known matrices; $\{\zeta(k), k\}$ is a discrete-time homogeneous Markov chain and takes values in a finite set $\mathscr{R} = \{1, 2, \ldots, r\}$ with a transition probability matrix $\Xi \triangleq [\alpha_{ij}]$ given by

$$\begin{cases} \alpha_{ij} \triangleq \Pr\{\zeta(k+1) = j \mid \zeta(k) = i\}, \\ \alpha_{ij} \geq 0, \quad \forall i,\ j \in \mathscr{R}, \\ \sum_{j=1}^{r} \alpha_{ij} = 1, \quad \forall i \in \mathscr{R}. \end{cases}$$

For system Σ_s, the measured output $y(k) \in \mathfrak{R}^{n_y}$ considered in this paper is subjected to the ROSN

$$y(k) = \lambda_{\zeta(k)}(k)H_{\zeta(k)}x(k) + (1 - \lambda_{\zeta(k)}(k))\sigma(H_{\zeta(k)}x(k)) + M_{\zeta(k)}\nu(k), \tag{2}$$

where $H_{\zeta(k)} \in \mathfrak{R}^{n_y\times n_x}$ and $M_{\zeta(k)} \in \mathfrak{R}^{n_y\times n_\nu}$ are known matrices; $\sigma(\cdot)$ represents the sensor nonlinearity; $\lambda_{\zeta(k)}(k)$ is a stochastic variable and subjects to the Bernoulli distribution

$$\Pr\{\lambda_{\zeta(k)}(k) = 1\} = \overline{\lambda}_{\zeta(k)}, \quad \Pr\{\lambda_{\zeta(k)}(k) = 0\} = 1 - \overline{\lambda}_{\zeta(k)},$$

where $0 \leq \overline{\lambda}_{\zeta(k)} \leq 1, \forall \zeta(k) \in \mathscr{R}$. For $\zeta(k) = i \in \mathscr{R}$, $A_{\zeta(k)}$, $C_{\zeta(k)}$, $D_{\zeta(k)}$, $G_{\zeta(k)}$, $H_{\zeta(k)}$, $M_{\zeta(k)}$, $\lambda_{\zeta(k)}(k)$ and $\overline{\lambda}_{\zeta(k)}$ are denoted as A_i, C_i, D_i, G_i, H_i, M_i, $\lambda_i(k)$ and $\overline{\lambda}_i$, respectively.

Assumption 1 (41). A nonlinear function $\sigma(\cdot)$ is said to satisfy a sector condition if

$$(\sigma(\eta) - O_1\eta)^T(\sigma(\eta) - O_2\eta) \leq 0, \quad \forall \eta \in \mathbb{R}^{n_u}, \tag{3}$$

where $O_2 > O_1 \geq 0$ are given constant diagonal matrices with compatible dimensions.

The nonlinear function $\sigma(H_ix(k))$ is supposed to be decomposed into a linear part $O_1H_ix(k)$ and a nonlinear part $\sigma_s(H_ix(k))$ as

$$\sigma(H_ix(k)) = O_1H_ix(k) + \sigma_s(H_ix(k)). \tag{4}$$

Then from Assumption 1, the nonlinear part $\sigma_s(H_ix(k))$ belongs to the set Λ_s, where

$$\Lambda_s = \{\sigma_s(H_ix(k)) : \sigma_s^T(H_ix(k))[\sigma_s(H_ix(k)) - (O_2 - O_1)H_ix(k)] \leq 0\}. \tag{5}$$

2.2 | Partial information problems

In practice, the whole information of Markov chain may not be acquired because of various equipment constraints or high expense. Thus, this section will discuss the partial information problems on Markov chain including the partial information on the Markov state, on transition probabilities and on detection probabilities.

2.2.1 | Partial information on Markov state

In this subsection, we consider that the Markov state $\zeta(k)$ is not always directly accessible. The information about the Markov state $\zeta(k)$ we have is only obtained via a detector based on an HMM $(\zeta(k), \xi(k))$. The variable $\xi(k)$ is the output of the detector, which takes values in a finite set $\mathscr{F} = \{1, 2, \dots, f\}$ and subjects to a detection matrix $\Omega = [\beta_{im}]$ satisfying

$$\begin{cases} \beta_{im} \triangleq \Pr\{\xi(k) = m | \zeta(k) = i\}, \\ \beta_{im} \geq 0, \quad \forall i \in \mathscr{R},\ m \in \mathscr{F}, \\ \sum_{m=1}^{f} \beta_{im} = 1, \quad \forall i \in \mathscr{R}. \end{cases} \tag{6}$$

Remark 1. For MJSs, it is significant to consider that whether the Markov state (also called the system mode) is available or not in the process of designing a filter/controller. In the case that the Markov state is available all the time, the candidate filter/controller can be chosen as a mode-dependent one. In the circumstance that the Markov state cannot be acquired, the mode-independent filter/controller would be a good choice. In this paper, the issue of the partial information on the Markov state is explored, which leads to the scenario that the Markov state cannot be directly acquired in the design process of a filter. To deal with this problem, a detector is utilized to detect the Markov state with a detection matrix Ω. If $\mathscr{R} = \mathscr{F}$ and $\beta_{im} = 1, \forall i = m$, then it means that the Markov state $\zeta(k)$ can be accurately detected by the detector and this case corresponds to the mode-dependent case. If $\mathscr{F} = 1$, then it means that no matter which the state of the Markov state $\zeta(k)$ is in, the detector is always in a single mode, which corresponds to the mode-independent case.

Remark 2. In some previous literature, the piecewise homogeneous Markov model was usually utilized for describing the asynchronous mode phenomenon in MJSs, in which the current controller/filter mode is dependent on both the current system mode and the last controller/filter mode.[38-41] In the HMM setting, the current controller/filter mode only relies on the current system mode.

Remark 3. Recently, there have appeared many works that utilize the HMM to handle the partial information problem of the Markov state.[12,13,18-27,42] However, those works premise that all the information of transition probabilities and detection probabilities is acquirable. In practice, acquiring only the complete information of transition probabilities may not be an easy job or may need unbearable costs, not to mention obtaining the whole information of detection probabilities simultaneously. This work considers the filtering issue for MJSs and aims to relax restrictions on the requirements of the complete information of transition probabilities and detection probabilities. In the following two subsections, we respectively formulate the partial information on transition probabilities of $\zeta(k)$ and on detection probabilities of $\xi(k)$ in the HMM $(\zeta(k), \xi(k))$.

2.2.2 | Partial information on transition probabilities

The partially known transition probabilities of the Markov state $\zeta(k)$ are considered in this work. For example, when $r = 4$, Ξ may be as Ξ_{example}:

$$\Xi_{\text{example}} = \begin{bmatrix} \alpha_{11} & ? & \alpha_{13} & ? \\ ? & ? & \alpha_{23} & \alpha_{24} \\ ? & \alpha_{32} & ? & ? \\ ? & ? & ? & \alpha_{44} \end{bmatrix},$$

where "?" represents the unknown element. For notation clarity, the following notations are given:

$$\begin{cases} \mathscr{O} = \mathscr{O}^i_{\mathscr{K}} \cup \mathscr{O}^i_{\mathscr{U}}, \\ \mathscr{O}^i_{\mathscr{K}} \triangleq \{j | \alpha_{ij} \text{ is known}\}, \\ \mathscr{O}^i_{\mathscr{U}} \triangleq \{j | \alpha_{ij} \text{ is unknown}\}. \end{cases}$$

Moreover, if $\mathscr{O}^i_{\mathscr{K}} \neq \emptyset$, then it is further described as

$$\mathscr{O}^i_{\mathscr{K}} = \{o^i_1, o^i_2, \dots, o^i_{\ell}\},$$

where o^i_{ℓ} is the column number of the ℓ-th known element in the i-th row in the transition probabilities matrix Ξ.

2.2.3 | Partial information on detection probabilities

In this part, the detection probabilities in the detection matrix Ω are considered to be partially known. When $r = 4$ and $f = 4$, for example, Ω may be as Ω_{example}:

$$\Omega_{\text{example}} = \begin{bmatrix} \beta_{11} & ? & ? & \beta_{14} \\ ? & ? & ? & ? \\ \beta_{31} & ? & \beta_{33} & ? \\ ? & \beta_{42} & ? & ? \end{bmatrix},$$

where "?" represents the unknown elements. For notation clarity, the following notations are given:

$$\begin{cases} \mathscr{W} = \mathscr{W}^i_{\mathscr{K}} \cup \mathscr{W}^i_{\mathscr{U}}, \\ \mathscr{W}^i_{\mathscr{K}} \triangleq \{m | \beta_{im} \text{ is known}\}, \\ \mathscr{W}^i_{\mathscr{U}} \triangleq \{m | \beta_{im} \text{ is unknown}\}. \end{cases}$$

Moreover, if $\mathscr{W}^i_{\mathscr{K}} \neq \emptyset$, then it is further described as

$$\mathscr{W}^i_{\mathscr{K}} = \{w^i_1, w^i_2, \dots, w^i_{\jmath}\},$$

where $w^i_{\jmath}$ is the column number of the $\jmath$-th known element in the i-th row in the detection matrix Ω.

Remark 4. It is worth pointing out that the partial information problems considered in this work contain two special cases which were studied in some existing literature. Case (a): The partial information problem is encountered only in transition probabilities but the Markov state is completely available for controller/filter design.[30] This case corresponds to the setting of partially known transition probabilities in Section 2.2.2 and the case that $\mathscr{R} = \mathscr{F}$ and $\beta_{im} = 1, \forall i = m$ in this work. Case (b): The partial information problem is encountered only in detection probabilities but the transition probabilities are completely known.[31,32] This case corresponds to the case that $\mathscr{O}^i_{\mathscr{U}} = \emptyset$ in Section 2.2.2 and the setting of partially known detection probabilities in Section 2.2.3 in this work.

Remark 5. In some works, for example, Reference 21 and section VI in Reference 12, the polytopic uncertainty in the transition and/or detection probabilities was considered and it was designated to treat the case where the transition and/or detection probabilities are not exactly known but are subjected to a polytopic domain constraint. On the other hand, the partially unknown setting investigated in this paper aims to deal with the case that there may exist unknown elements in the transition and/or detection probabilities. The polytopic uncertainty setting and the partially unknown setting in the transition and/or detection probabilities in the HMM are applicable to different situations.

2.3 | Hidden Markov model-based filter

From the analysis of Section 2.2.1, one can see that using the Markov state $\zeta(k)$ for filter design is not appropriate while the output of the detector $\xi(k)$ is always available for the filter design. Thus, in this work, the following HMM-based filter (Σ_f) is constructed, which only relies on the output of the detector $\xi(k)$:

$$\Sigma_f : \begin{cases} \psi(k+1) = \mathfrak{A}_{\xi(k)}\psi(k) + \mathfrak{B}_{\xi(k)}y(k), \\ z_f(k) = \mathfrak{C}_{\xi(k)}\psi(k), \end{cases} \tag{7}$$

where $\psi(k) \in \mathfrak{R}^{n_x}$ is the filter state vector; $z_f(k) \in \mathfrak{R}^{n_z}$ is the output vector; $\mathfrak{A}_{\xi(k)} \in \mathfrak{R}^{n_x \times n_x}$, $\mathfrak{B}_{\xi(k)} \in \mathfrak{R}^{n_x \times n_v}$ and $\mathfrak{C}_{\xi(k)} \in \mathfrak{R}^{n_z \times n_x}$ are filter gains to be designed. $\forall \xi(k) = m \in \mathscr{F}$, $\mathfrak{A}_{\xi(k)}$, $\mathfrak{B}_{\xi(k)}$, and $\mathfrak{C}_{\xi(k)}$ are denoted as $\mathfrak{A}_m$, $\mathfrak{B}_m$, and $\mathfrak{C}_m$, respectively.

Let $\vartheta(k) \triangleq \begin{bmatrix} x^T(k) & \psi^T(k) \end{bmatrix}^T$ and $z_e(k) \triangleq z_s(k) - z_f(k)$. The following filtering error system (Σ_e) can be derived:

$$\Sigma_e : \begin{cases} \vartheta(k+1) = \mathbb{A}_{i,m}\vartheta(k) + \mathbb{B}_{i,m}\sigma_s(H_i x(k)) + \mathbb{C}_{i,m}v(k) + (\lambda_i(k) - \bar{\lambda}_i)\mathbb{Y}_{i,m}(k), \\ z_e(k) = \mathbb{D}_{i,m}\vartheta(k) + G_i v(k), \end{cases} \tag{8}$$

where

$$\mathbb{A}_{i,m} \triangleq \begin{bmatrix} A_i & 0 \\ \bar{\lambda}_i \mathfrak{B}_m H_i + (1-\bar{\lambda}_i)\mathfrak{B}_m O_1 H_i & \mathfrak{A}_m \end{bmatrix}, \quad \mathbb{B}_{i,m} \triangleq (1-\bar{\lambda}_i)\mathcal{I}_1^T \mathfrak{B}_m,$$

$$\mathbb{C}_{i,m} \triangleq \begin{bmatrix} C_i \\ \mathfrak{B}_m M_i \end{bmatrix}, \quad \mathbb{D}_{i,m} \triangleq \begin{bmatrix} D_i & -\mathfrak{C}_m \end{bmatrix}, \quad \mathcal{I}_1 \triangleq \begin{bmatrix} 0 & I \end{bmatrix}, \quad \mathcal{I}_2 \triangleq \begin{bmatrix} I & 0 \end{bmatrix},$$

$$\mathbb{Y}_{i,m}(k) \triangleq \mathcal{I}_1^T \mathfrak{B}_m (H_i - O_1 H_i)\mathcal{I}_2 \vartheta(k) - \mathcal{I}_1^T \mathfrak{B}_m \sigma_s(H_i x(k)).$$

Definition 1 (41). System (8) is said to be stochastically stable if for $v(k) \equiv 0$ and any initial condition $\vartheta(0) \in \mathfrak{R}^{2n_x}$, $\forall \zeta(0) \in \mathscr{R}$, the following inequality holds:

$$\mathbb{E}\left\{ \sum_{k=0}^{\infty} \|\vartheta(k)\|^2 \, |\vartheta(0), \zeta(0) \right\} < \infty.$$

The objective of this work: Considering the randomly occurring sensor nonlinearity in the measured output (2) and the partial information problems formulated in Section 2.2, design the HMM-based filter (7) for system (1) such that the stochastic stability of the filtering error system (8) can be guaranteed and an H_∞ disturbance attenuation performance index γ of the filtering error system (8) can be satisfied, that is, under the zero initial condition ($\vartheta(0) = 0$ and $\forall \zeta(0) \in \mathscr{R}$),

$$\mathbb{E}\left\{ \sum_{k=0}^{\infty} z_e^T(k) z_e(k) \right\} < \gamma^2 \mathbb{E}\left\{ \sum_{k=0}^{\infty} v^T(k) v(k) \right\}, \tag{9}$$

holds for all nonzero $v(k) \in l_2[0, \infty)$.

3 | MAIN RESULTS

This section presents the criteria ensuring the stochastic stability of the filtering error system (8) with an H_∞ disturbance attenuation performance index γ and also gives an effective solution for the HMM-based filter (7) design issue.

3.1 | Stochastic stability and H_∞ disturbance attenuation level analysis

Theorem 1. *Consider the filtering error system (8). Given scalars $\gamma > 0$, $0 \leq \bar{\lambda}_i \leq 1$, if there exist a scalar $\delta_{i,m} > 0$ and a matrix $P_i \in \mathfrak{R}^{2n_x \times 2n_x} > 0$ for any $i \in \mathscr{R}$, satisfying*

$$\sum_{m=1}^{f} \beta_{im}\{ Y_{1i,m}^T \mathscr{P}_i Y_{1i,m} + \bar{\lambda}_i(1-\bar{\lambda}_i) Y_{2i,m}^T \mathscr{P}_i Y_{2i,m} + Y_{3i,m}^T Y_{3i,m} + Y_{4i,m} \} + \text{diag}\{-P_i, 0, 0\} < 0, \tag{10}$$

where $\mathscr{P}_i \triangleq \sum_{j=1}^{r} \alpha_{ij} P_j$ *and*

$$Y_{1i,m} \triangleq \begin{bmatrix} \mathbb{A}_{i,m} & \mathbb{C}_{i,m} & \mathbb{B}_{i,m} \end{bmatrix}, \quad Y_{2i,m} \triangleq \begin{bmatrix} \mathcal{I}_1^T \mathfrak{B}_m (H_i - O_1 H_i) \mathcal{I}_2 & 0 & -\mathcal{I}_1^T \mathfrak{B}_m \end{bmatrix},$$

$$Y_{3i,m} \triangleq \begin{bmatrix} \mathbb{D}_{i,m} & G_i & 0 \end{bmatrix}, \quad Y_{4i,m} \triangleq \begin{bmatrix} 0 & 0 & \delta_{i,m} \mathcal{I}_2^T H_i^T (O_2 - O_1)^T \\ \star & -\gamma^2 I & 0 \\ \star & \star & -2\delta_{i,m} I \end{bmatrix},$$

then the filtering error system (8) is stochastically stable with a prescribed H_∞ *disturbance attenuation level bound* γ*.*

Proof. Consider the Lyapunov function as follows:

$$V(k) = \vartheta^T(k) P_{\zeta(k)} \vartheta(k).$$

Let $\mathbb{E}\{\Delta V(k)\} = \mathbb{E}\{ V(k+1, \vartheta(k+1), \zeta(k+1) = j) | \zeta(k) = i\} - V(k, \vartheta(k), \zeta(k))$. Then

$$\begin{aligned} \mathbb{E}\{\Delta V(k)\} &= \mathbb{E}\{\vartheta^T(k+1) \mathscr{P}_i \vartheta(k+1)\} - \vartheta^T(k) P_i \vartheta(k) \\ &= \Theta(k) \left\{ \sum_{m=1}^{f} \beta_{im} [Y_{1i,m}^T \mathscr{P}_i Y_{1i,m} + \bar{\lambda}_i (1 - \bar{\lambda}_i) Y_{2i,m}^T \mathscr{P}_i Y_{2i,m}] - \mathrm{diag}\{P_i, 0, 0\} \right\} \Theta^T(k), \end{aligned} \tag{11}$$

where $\Theta(k) \triangleq \begin{bmatrix} \vartheta^T(k) & v^T(k) & \sigma_s^T(H_i x(k)) \end{bmatrix}$.

From (5), one can see that the following condition holds for any scalar $\delta_{i,m} > 0, \forall i \in \mathscr{R}, m \in \mathscr{F}$:

$$2\delta_{i,m} \sigma_s^T (H_i x(k)) [\sigma_s(H_i x(k)) - (O_1 - O_2) H_i x(k)] \leq 0. \tag{12}$$

When $v(k) \equiv 0$, combining (11) with (12), one has

$$\begin{aligned} \mathbb{E}\{\Delta V(k)\} \leq& \begin{bmatrix} \vartheta^T(k) & \sigma_s^T(H_i x(k)) \end{bmatrix} (Y_{5i,m} - \mathrm{diag}\{P_i, 0\}) \begin{bmatrix} \vartheta^T(k) & \sigma_s^T(H_i x(k)) \end{bmatrix}^T \\ &- 2\delta_{i,m} \sigma_s^T (H_i x(k)) [\sigma_s(H_i x(k)) - (O_1 - O_2) H_i x(k)] \\ =& \begin{bmatrix} \vartheta(k) \\ \sigma_s(H_i x(k)) \end{bmatrix}^T \left\{ Y_{5i,m} + \sum_{m=1}^{f} \beta_{im} \begin{bmatrix} 0 & \delta_{i,m} \mathcal{I}_2^T H_i^T (O_2 - O_1)^T \\ \star & -2\delta_{i,m} I \end{bmatrix} + \mathrm{diag}\{-P_i, 0\} \right\} \begin{bmatrix} \vartheta(k) \\ \sigma_s(H_i x(k)) \end{bmatrix}, \end{aligned}$$

where

$$Y_{5i,m} \triangleq \sum_{m=1}^{f} \beta_{im} \left\{ \begin{bmatrix} \mathbb{A}_{i,m}^T \\ \mathbb{B}_{i,m}^T \end{bmatrix} \mathscr{P}_i \begin{bmatrix} \mathbb{A}_{i,m}^T \\ \mathbb{B}_{i,m}^T \end{bmatrix}^T + \bar{\lambda}_i (1 - \bar{\lambda}_i) \begin{bmatrix} \mathcal{I}_2^T (H_i^T - H_i^T O_1^T) \mathfrak{B}_m^T \mathcal{I}_1 \\ -\mathfrak{B}_m^T \mathcal{I}_1 \end{bmatrix} \mathscr{P}_i \begin{bmatrix} \mathcal{I}_2^T (H_i^T - H_i^T O_1^T) \mathfrak{B}_m^T \mathcal{I}_1 \\ -\mathfrak{B}_m^T \mathcal{I}_1 \end{bmatrix}^T \right\}.$$

It should be noted that $Y_{5i,m} + \sum_{m=1}^{f} \beta_{im} \begin{bmatrix} 0 & \delta_{i,m} \mathcal{I}_2^T H_i^T (O_2 - O_1)^T \\ \star & -2\delta_{i,m} I \end{bmatrix} + \mathrm{diag}\{-P_i, 0\} < 0$ is guaranteed by (10). Thus, one has $\mathbb{E}\{\Delta V(k)\} < 0$. Following the similar proof of theorem 1 in Reference 41, it can be shown that the filtering error system (8) is stochastically stable.

Next, let $J \triangleq \mathbb{E}\left\{ \sum_{k=0}^{\infty} [z_e^T(k) z_e(k) - \gamma^2 v^T(k) v(k)] \right\}$. Under the zero initial condition and for all nonzero $v(k) \in l_2[0, \infty)$, one has

$$\begin{aligned} J &\leq \mathbb{E}\left\{ \sum_{k=0}^{\infty} [z_e^T(k) z_e(k) - \gamma^2 v^T(k) v(k) + \Delta V(k)] \right\} \\ &\leq \sum_{k=0}^{\infty} \Theta(k) \left\{ \sum_{m=1}^{f} \beta_{im} \{ Y_{1i,m}^T \mathscr{P}_i Y_{1i,m} + \bar{\lambda}_i (1 - \bar{\lambda}_i) Y_{2i,m}^T \mathscr{P}_i Y_{2i,m} + Y_{3i,m}^T Y_{3i,m} + Y_{4i,m} \} + \mathrm{diag}\{-P_i, 0, 0\} \right\} \Theta^T(k), \end{aligned}$$

which, combining with (10), implies (9) holds. This completes the proof. ■

Remark 6. In the above theorem, α_{ij}, $\forall i,j \in \mathscr{R}$ and β_{im}, $\forall i \in \mathscr{R}$, $m \in \mathscr{F}$ are coupled. Thus, only when all α_{ij}, $\forall i,j \in \mathscr{R}$ and β_{im}, $\forall i \in \mathscr{R}$, $m \in \mathscr{F}$ are known, the above theorem is available for the H_∞ performance analysis. In this work, we consider that both α_{ij} and β_{im} may be partially unknown. Therefore, α_{ij} and β_{im} should be separated and the unknown α_{ij} and β_{im} should be disposed. For this consideration, Theorem 2 is given.

Theorem 2. *Consider the filtering error system (8). Given scalars $\gamma > 0$, $0 \le \overline{\lambda}_i \le 1$, if there exist a scalar $\delta_{i,m} > 0$ and matrices $P_i \in \mathfrak{R}^{2n_x \times 2n_x} > 0$, $X_{i,m} \in \mathfrak{R}^{2n_x \times 2n_x} > 0$ for any $i \in \mathscr{R}$, $m \in \mathscr{F}$ satisfying*

$$\begin{bmatrix} -P_i & \Pi_i^{(1,2)} \\ \star & \Pi_i^{(2,2)} \end{bmatrix} < 0, \tag{13}$$

$$\begin{bmatrix} -X_{i,m} & 0 & \delta_{i,m}\mathcal{I}_2^T H_i^T (O_2 - O_1)^T & \mathbb{D}_{i,m}^T & \Psi_{i,m}^{(1)} & \Psi_{i,m}^{(4)} \\ \star & -\gamma^2 I & 0 & G_i^T & \Psi_{i,m}^{(2)} & 0 \\ \star & \star & -2\delta_{i,m} I & 0 & \Psi_{i,m}^{(3)} & \Psi_{i,m}^{(5)} \\ \star & \star & \star & -I & 0 & 0 \\ \star & \star & \star & \star & \Psi_{i,m}^{(0)} & 0 \\ \star & \star & \star & \star & \star & \Psi_{i,m}^{(0)} \end{bmatrix} < 0, \tag{14}$$

where

$$\Pi_i^{(1,2)} \triangleq \begin{cases} \begin{bmatrix} \sqrt{\beta_{i1}} X_{i,1} & \sqrt{\beta_{i2}} X_{i,2} & \dots & \sqrt{\beta_{iq}} X_{i,q} \end{bmatrix}, & \text{if } \mathscr{W}_{\mathscr{U}}^i = \emptyset; \\ \begin{bmatrix} \sqrt{\beta_{iw_1^i}} X_{i,w_1^i} & \sqrt{\beta_{iw_2^i}} X_{i,w_2^i} & \dots & \sqrt{\beta_{iw_j^i}} X_{i,w_j^i} & \tilde{\beta}_i X_{i,\mu} \end{bmatrix}, & \forall \mu \in \mathscr{W}_{\mathscr{U}}^i, \quad \text{if } \mathscr{W}_{\mathscr{U}}^i \neq \emptyset \text{ and } \mathscr{W}_{\mathscr{K}}^i \neq \emptyset; \\ X_{i,\mu}, \quad \forall \mu \in \mathscr{W}_{\mathscr{U}}^i, & \text{if } \mathscr{W}_{\mathscr{K}}^i = \emptyset; \end{cases}$$

$$\Pi_i^{(2,2)} \triangleq \begin{cases} \mathrm{diag}\{-X_{i,1}, -X_{i,2}, \dots, -X_{i,q}\}, & \text{if } \mathscr{W}_{\mathscr{U}}^i = \emptyset; \\ \mathrm{diag}\{-X_{i,w_1^i}, -X_{i,w_2^i}, \dots, -X_{i,w_j^i}, -X_{i,\mu}\}, & \forall \mu \in \mathscr{W}_{\mathscr{U}}^i, \quad \text{if } \mathscr{W}_{\mathscr{U}}^i \neq \emptyset \text{ and } \mathscr{W}_{\mathscr{K}}^i \neq \emptyset; \\ -X_{i,\mu}, \quad \forall \mu \in \mathscr{W}_{\mathscr{U}}^i, & \text{if } \mathscr{W}_{\mathscr{K}}^i = \emptyset; \end{cases}$$

$$\Psi_{i,m}^{(0)} \triangleq \begin{cases} -\sum_{j=1}^{r} \alpha_{ij} P_j, & \text{if } \mathscr{O}_{\mathscr{U}}^i = \emptyset; \\ \mathrm{diag}\left\{ -\sum_{j \in \mathscr{O}_{\mathscr{K}}^i} \alpha_{ij} P_j, -\left(1 - \sum_{j \in \mathscr{O}_{\mathscr{K}}^i} \alpha_{ij}\right) P_\eta \right\}, & \forall \eta \in \mathscr{O}_{\mathscr{U}}^i, \quad \text{if } \mathscr{O}_{\mathscr{U}}^i \neq \emptyset \text{ and } \mathscr{O}_{\mathscr{K}}^i \neq \emptyset; \\ -P_\eta, \quad \forall \eta \in \mathscr{O}_{\mathscr{U}}^i, & \text{if } \mathscr{O}_{\mathscr{K}}^i = \emptyset; \end{cases}$$

$$\Psi_{i,m}^{(l)} \triangleq \begin{cases} \Gamma_{i,m}^l \left(\sum_{j=1}^{r} \alpha_{ij} P_j \right), & \text{if } \mathscr{O}_{\mathscr{U}}^i = \emptyset; \\ \begin{bmatrix} \Gamma_{i,m}^l \left(\sum_{j \in \mathscr{O}_{\mathscr{K}}^i} \alpha_{ij} P_j \right) & \Gamma_{i,m}^l \left(1 - \sum_{j \in \mathscr{O}_{\mathscr{K}}^i} \alpha_{ij} \right) P_\eta \end{bmatrix}, & \text{if } \mathscr{O}_{\mathscr{U}}^i \neq \emptyset \text{ and } \mathscr{O}_{\mathscr{K}}^i \neq \emptyset; \\ \Gamma_{i,m}^l P_\eta, \quad \forall \eta \in \mathscr{O}_{\mathscr{U}}^i, & \text{if } \mathscr{O}_{\mathscr{K}}^i = \emptyset; \end{cases} \quad l = 1, 2, 3, 4, 5,$$

with

$$\tilde{\beta}_i \triangleq \sqrt{1 - \sum_{m \in \mathscr{W}_{\mathscr{K}}^i} \beta_{im}}, \quad \Gamma_{i,m}^1 \triangleq \mathbb{A}_{i,m}^T, \quad \Gamma_{i,m}^2 \triangleq \mathbb{C}_{i,m}^T, \quad \Gamma_{i,m}^3 \triangleq (1 - \overline{\lambda}_i)\mathfrak{B}_m^T \mathcal{I}_1,$$

$$\Gamma_{i,m}^4 \triangleq \sqrt{\overline{\lambda}_i (1 - \overline{\lambda}_i)} \mathcal{I}_2^T (H_i^T - H_i^T O_1^T) \mathfrak{B}_m^T \mathcal{I}_1, \quad \Gamma_{i,m}^5 \triangleq -\sqrt{\overline{\lambda}_i (1 - \overline{\lambda}_i)} \mathfrak{B}_m^T \mathcal{I}_1,$$

then the filtering error system (8) is stochastically stable with a prescribed H_∞ disturbance attenuation level bound γ.

Proof. First, we prove that (13) can derive

$$-P_i < -\sum_{m=1}^{f} \beta_{im} X_{i,m}. \tag{15}$$

When $\mathscr{W}_{\mathcal{U}}^{i} = \emptyset$, according to the Schur complement, (13) is equivalent to (15).

When $\mathscr{W}_{\mathcal{U}}^{i} \neq \emptyset$ and $\mathscr{W}_{\mathcal{K}}^{i} \neq \emptyset$, according to the Schur complement, (13) is equivalent to

$$-P_i + \sum_{m\in\mathscr{W}_{\mathcal{K}}^{i}} \beta_{im} X_{i,m} + \left(1 - \sum_{m\in\mathscr{W}_{\mathcal{K}}^{i}} \beta_{im}\right) X_{i,\mu} < 0. \tag{16}$$

Since $\frac{\beta_{i\mu}}{1-\sum_{m\in\mathscr{W}_{\mathcal{K}}^{i}} \beta_{im}} > 0, \forall \mu \in \mathscr{O}_{\mathcal{U}}^{i}$ and $\frac{\sum_{\mu\in\mathscr{W}_{\mathcal{U}}^{i}} \beta_{i\mu}}{1-\sum_{m\in\mathscr{W}_{\mathcal{K}}^{i}} \beta_{im}} = 1$, one can obtain

$$-P_i + \sum_{m\in\mathscr{W}_{\mathcal{K}}^{i}} \beta_{im} X_{i,m} + \sum_{\mu\in\mathscr{W}_{\mathcal{U}}^{i}} \beta_{i\mu} X_{i,\mu} < 0,$$

which is equivalent to (15).

When $\mathscr{W}_{\mathcal{K}}^{i} = \emptyset$, according to the Schur complement, one can get from (13) that

$$-P_i + X_{i,\mu} < 0. \tag{17}$$

Since $\beta_{i\mu} \geq 0$ and $\sum_{\mu=1}^{f} \beta_{i\mu} = 1$, (15) can be guaranteed by (17). Thus, one can see that (13) $\Rightarrow$ (15) for any $\mathscr{W}_{\mathcal{U}}^{i}$ and $\mathscr{W}_{\mathcal{K}}^{i}$.

Next, we aim to prove that (14) and (15) can derive (10).

When $\mathscr{O}_{\mathcal{U}}^{i} = \emptyset$, according to the Schur complement, (14) is equivalent to

$$Y_{1i,m}^{T} \mathscr{P}_i Y_{1i,m} + \overline{\lambda}_i(1-\overline{\lambda}_i) Y_{2i,m}^{T} \mathscr{P}_i Y_{2i,m} + Y_{3i,m}^{T} Y_{3i,m} + \begin{bmatrix} -X_{i,m} & 0 & \delta_{i,m}\mathcal{I}_2^{T} H_i^{T}(O_2 - O_1)^{T} \\ \star & -\gamma^2 I & 0 \\ \star & \star & -2\delta_{i,m} I \end{bmatrix} < 0, \tag{18}$$

which, combining with (15), can easily lead to (10).

When $\mathscr{O}_{\mathcal{U}}^{i} \neq \emptyset$ and $\mathscr{O}_{\mathcal{K}}^{i} \neq \emptyset$, according to the Schur complement, it can be obtained from (14) that

$$\begin{aligned} &Y_{1i,m}^{T}\left[\sum_{j\in\mathscr{O}_{\mathcal{K}}^{i}} \alpha_{ij} P_j + \left(1 - \sum_{j\in\mathscr{O}_{\mathcal{K}}^{i}} \alpha_{ij}\right) P_\mu\right] Y_{1i,m} + \overline{\lambda}_i(1-\overline{\lambda}_i) Y_{2i,m}^{T}\left[\sum_{j\in\mathscr{O}_{\mathcal{K}}^{i}} \alpha_{ij} P_j + \left(1 - \sum_{j\in\mathscr{O}_{\mathcal{K}}^{i}} \alpha_{ij}\right) P_\mu\right] Y_{2i,m} \\ &+ Y_{3i,m}^{T} Y_{3i,m} + \begin{bmatrix} -X_{i,m} & 0 & \delta_{i,m}\mathcal{I}_2^{T} H_i^{T}(O_2 - O_1)^{T} \\ \star & -\gamma^2 I & 0 \\ \star & \star & -2\delta_{i,m} I \end{bmatrix} < 0. \end{aligned} \tag{19}$$

Since $\frac{\alpha_{i\eta}}{1-\sum_{j\in\mathscr{O}_{\mathcal{K}}^{i}} \alpha_{ij}} \geq 0, \forall \eta \in \mathscr{O}_{\mathcal{U}}^{i}$ and $\frac{\sum_{\eta\in\mathscr{O}_{\mathcal{U}}^{i}} \alpha_{i\eta}}{1-\sum_{j\in\mathscr{O}_{\mathcal{K}}^{i}} \alpha_{ij}} = 1$, one can get (18) from (19), which can further produce (10).

When $\mathscr{O}_{\mathcal{K}}^{i} = \emptyset$, by using the Schur complement, one can attain from (14) that

$$Y_{1i,m}^{T} P_\eta Y_{1i,m} + \overline{\lambda}_i(1-\overline{\lambda}_i) Y_{2i,m}^{T} P_\eta Y_{2i,m} + Y_{3i,m}^{T} Y_{3i,m} + \begin{bmatrix} -X_{i,m} & 0 & \delta_{i,m}\mathcal{I}_2^{T} H_i^{T}(O_2 - O_1)^{T} \\ * & -\gamma^2 I & 0 \\ * & * & -2\delta_{i,m} I \end{bmatrix} < 0. \tag{20}$$

Since $\alpha_{i\eta} \geq 0$ and $\sum_{\eta=1}^{r} \alpha_{i\eta} = 1$, (18) can be guaranteed by (20), which can subsequently generate (10). Thus, one can see that for any $\mathcal{O}_{\mathcal{K}}^{i}$ and $\mathcal{O}_{\mathcal{U}}^{i}$, (14) and (15) can derive (10). Utilizing Theorem 1, the proof is complete. ■

Remark 7. Theorem 2 establishes the criterion of the stochastic stability together with an H_∞ disturbance attenuation level analysis for the filtering error system (8). The criterion is available when partial information on transition probabilities, on detection probabilities, or yet both of them. Thus, a unified framework is provided for the filtering issue of MJSs with partial information.

3.2 | HMM-based filter design

Based on the criterion given in Theorem 2, the effective solution for the HMM-based filter (7) design issue is derived in this subsection.

Theorem 3. *Consider the filtering error system (8). Given scalars $\gamma > 0$, $0 \leq \overline{\lambda}_i \leq 1$, d_1, d_2, if there exist a scalar $\delta_{i,m} > 0$ and matrices $P_i \in \mathfrak{R}^{2n_x\times 2n_x} > 0$, $X_{i,m} \in \mathfrak{R}^{2n_x\times 2n_x} > 0$, $S_{1i,m} \in \mathfrak{R}^{n_x\times n_x}$, $S_{2i,m} \in \mathfrak{R}^{n_x\times n_x}$, $S_m \in \mathfrak{R}^{n_x\times n_x}$, $\mathscr{A}_m \in \mathfrak{R}^{n_x\times n_x}$, $\mathscr{B}_m \in \mathfrak{R}^{n_x\times n_v}$ and $\mathscr{C}_m \in \mathfrak{R}^{n_z\times n_x}$ for any $i \in \mathscr{R}$, $m \in \mathscr{F}$ satisfying (13) and*

$$\begin{bmatrix} -X_{i,m} & 0 & \delta_{i,m}\mathcal{I}_2^T H_i^T (O_2 - O_1)^T & \overline{\mathbb{D}}_{i,m}^T & \overline{\Psi}_{i,m}^{(1)} & \overline{\Psi}_{i,m}^{(4)} \\ \star & -\gamma^2 I & 0 & G_i^T & \overline{\Psi}_{i,m}^{(2)} & 0 \\ \star & \star & -2\delta_{i,m} I & 0 & \overline{\Psi}_{i,m}^{(3)} & \overline{\Psi}_{i,m}^{(5)} \\ \star & \star & \star & -I & 0 & 0 \\ \star & \star & \star & \star & \overline{\Psi}_{i,m}^{(0)} & 0 \\ \star & \star & \star & \star & \star & \overline{\Psi}_{i,m}^{(0)} \end{bmatrix} < 0, \tag{21}$$

where

$$\overline{\Psi}_{i,m}^{(0)} \triangleq \begin{cases} \sum_{j=1}^{r} \alpha_{ij} P_j - \mathbb{S}_{i,m} - \mathbb{S}_{i,m}^T, \quad \text{if } \mathcal{O}_{\mathcal{U}}^{i} = \emptyset; \\ \text{diag}\left\{ \sum_{j\in\mathcal{O}_{\mathcal{K}}^{i}} \alpha_{ij} P_j - \mathbb{S}_{i,m} - \mathbb{S}_{i,m}^T, \left(1 - \sum_{j\in\mathcal{O}_{\mathcal{K}}^{i}} \alpha_{ij}\right) P_\eta - \mathbb{S}_{i,m} - \mathbb{S}_{i,m}^T \right\}, \ \forall \eta \in \mathcal{O}_{\mathcal{U}}^{i}, \ \text{if } \mathcal{O}_{\mathcal{U}}^{i} \neq \emptyset \text{ and } \mathcal{O}_{\mathcal{K}}^{i} \neq \emptyset; \\ P_\eta - \mathbb{S}_{i,m} - \mathbb{S}_{i,m}^T, \quad \forall \eta \in \mathcal{O}_{\mathcal{U}}^{i}, \quad \text{if } \mathcal{O}_{\mathcal{K}}^{i} = \emptyset; \end{cases}$$

$$\overline{\Psi}_{i,m}^{(l)} \triangleq \begin{cases} \overline{\Gamma}_{i,m}^{l}, \quad \text{if } \mathcal{O}_{\mathcal{U}}^{i} = \emptyset \text{ or } \mathcal{O}_{\mathcal{K}}^{i} = \emptyset; \\ \begin{bmatrix} \overline{\Gamma}_{i,m}^{l} & \overline{\Gamma}_{i,m}^{l} \end{bmatrix}, \quad \text{if } \mathcal{O}_{\mathcal{U}}^{i} \neq \emptyset \text{ and } \mathcal{O}_{\mathcal{K}}^{i} \neq \emptyset; \end{cases} \quad l = 1, 2, 3, 4, 5,$$

$$\overline{\mathbb{D}}_{i,m} \triangleq D_i \mathcal{I}_2 - \mathscr{C}_m \mathcal{I}_1,$$

with $\mathbb{S}_{i,m} \triangleq \begin{bmatrix} S_{1i,m} & d_1 S_m \\ S_{2i,m} & d_2 S_m \end{bmatrix}$ *and*

$$\begin{aligned} \overline{\Gamma}_{i,m}^{1} \triangleq\ & \mathcal{I}_2^T A_i^T (S_{1i,m}^T \mathcal{I}_2 + S_{2i,m}^T \mathcal{I}_1) + \overline{\lambda}_i \mathcal{I}_2^T H_i^T \mathscr{B}_m^T (d_1 \mathcal{I}_2 + d_2 \mathcal{I}_1) + (1 - \overline{\lambda}_i) \mathcal{I}_2^T H_i^T O_1^T \mathscr{B}_m^T (d_1 \mathcal{I}_2 + d_2 \mathcal{I}_1) \\ & + \mathcal{I}_1^T \mathscr{A}_m^T (d_1 \mathcal{I}_2 + d_2 \mathcal{I}_1), \\ \overline{\Gamma}_{i,m}^{2} \triangleq\ & C_i^T S_{1i,m}^T \mathcal{I}_2 + C_i^T S_{2i,m}^T \mathcal{I}_1 + M_i^T \mathscr{B}_m^T (d_1 \mathcal{I}_2 + d_2 \mathcal{I}_1), \quad \overline{\Gamma}_{i,m}^{3} \triangleq (1 - \overline{\lambda}_i) \mathscr{B}_m^T (d_1 \mathcal{I}_2 + d_2 \mathcal{I}_1), \\ \overline{\Gamma}_{i,m}^{4} \triangleq\ & \sqrt{\overline{\lambda}_i (1 - \overline{\lambda}_i)} \mathcal{I}_2^T (H_i^T - H_i^T O_1^T) \mathscr{B}_m^T (d_1 \mathcal{I}_2 + d_2 \mathcal{I}_1), \quad \overline{\Gamma}_{i,m}^{5} \triangleq -\sqrt{\overline{\lambda}_i (1 - \overline{\lambda}_i)} \mathscr{B}_m^T (d_1 \mathcal{I}_2 + d_2 \mathcal{I}_1), \end{aligned}$$

then the filtering error system (8) is stochastically stable with a prescribed H_∞ disturbance attenuation level bound γ and the filter gains are obtained as

$$\mathfrak{A}_m = S_m^{-1} \mathscr{A}_m, \quad \mathfrak{B}_m = S_m^{-1} \mathscr{B}_m, \quad \mathfrak{C}_m = \mathscr{C}_m.$$

Proof. From the inequality $-ZP^{-1}Z^T \leq P - Z - Z^T$, it is not hard to see from (21) that

$$\begin{bmatrix} -X_{i,m} & 0 & \delta_{i,m}\mathcal{I}_2^T H_i^T (O_2 - O_1)^T & \overline{\mathbb{D}}_{i,m}^T & \overline{\Psi}_{i,m}^{(1)} & \overline{\Psi}_{i,m}^{(4)} \\ \star & -\gamma^2 I & 0 & G_i^T & \overline{\Psi}_{i,m}^{(2)} & 0 \\ \star & \star & -2\delta_{i,m} I & 0 & \overline{\Psi}_{i,m}^{(3)} & \overline{\Psi}_{i,m}^{(5)} \\ \star & \star & \star & -I & 0 & 0 \\ \star & \star & \star & \star & \Pi_{i,m} & 0 \\ \star & \star & \star & \star & \star & \Pi_{i,m} \end{bmatrix} < 0, \tag{22}$$

where

$$\Pi_{i,m} \triangleq \begin{cases} -\mathbb{S}_{i,m}\left(\sum_{j=1}^{r}\alpha_{ij}P_j\right)^{-1}\mathbb{S}_{i,m}^T, & \text{if } \mathcal{O}_{\mathcal{U}}^i = \emptyset; \\ \operatorname{diag}\left\{-\mathbb{S}_{i,m}\left(\sum_{j\in\mathcal{O}_{\mathcal{K}}^i}\alpha_{ij}P_j\right)^{-1}\mathbb{S}_{i,m}^T, -\mathbb{S}_{i,m}\left(\left(1-\sum_{j\in\mathcal{O}_{\mathcal{K}}^i}\alpha_{ij}\right)P_\eta\right)^{-1}\mathbb{S}_{i,m}^T\right\}, \quad \forall \eta \in \mathcal{O}_{\mathcal{U}}^i, & \text{if } \mathcal{O}_{\mathcal{U}}^i \neq \emptyset \text{ and } \mathcal{O}_{\mathcal{K}}^i \neq \emptyset; \\ -\mathbb{S}_{i,m}(P_\eta)^{-1}\mathbb{S}_{i,m}^T, \quad \forall \eta \in \mathcal{O}_{\mathcal{U}}^i, & \text{if } \mathcal{O}_{\mathcal{K}}^i = \emptyset. \end{cases}$$

Let $\mathscr{A}_m = S_m\mathfrak{A}_m$, $\mathscr{B}_m = S_m\mathfrak{B}_m$, $\mathscr{C}_m = \mathfrak{C}_m$, $\mathbb{S}_{i,m} \triangleq \begin{bmatrix} S_{1i,m} & d_1 S_m \\ S_{2i,m} & d_2 S_m \end{bmatrix}$ and

$$\Phi_{i,m} \triangleq \begin{cases} \operatorname{diag}\left\{ I, I, I, I, \left(\sum_{j=1}^{r}\alpha_{ij}P_j\right)\mathbb{S}_{i,m}^{-1}, \left(\sum_{j=1}^{r}\alpha_{ij}P_j\right)\mathbb{S}_{i,m}^{-1} \right\}, \quad \text{if } \mathcal{O}_{\mathcal{U}}^i = \emptyset; \\ \operatorname{diag}\left\{ I, I, I, I, \left(\sum_{j\in\mathcal{O}_{\mathcal{K}}^i}\alpha_{ij}P_j\right)\mathbb{S}_{i,m}^{-1}, \left(1-\sum_{j\in\mathcal{O}_{\mathcal{K}}^i}\alpha_{ij}\right)P_\eta\mathbb{S}_{i,m}^{-1}, \left(\sum_{j\in\mathcal{O}_{\mathcal{K}}^i}\alpha_{ij}P_j\right)\mathbb{S}_{i,m}^{-1}, \left(1-\sum_{j\in\mathcal{O}_{\mathcal{K}}^i}\alpha_{ij}\right)P_\eta\mathbb{S}_{i,m}^{-1} \right\}, \\ \text{if } \mathcal{O}_{\mathcal{U}}^i \neq \emptyset \text{ and } \mathcal{O}_{\mathcal{K}}^i \neq \emptyset; \\ \operatorname{diag}\{I, I, I, I, P_\eta\mathbb{S}_{i,m}^{-1}, P_\eta\mathbb{S}_{i,m}^{-1}\}, \quad \text{if } \mathcal{O}_{\mathcal{K}}^i = \emptyset. \end{cases}$$

Thus, premultiplying (22) by $\Phi_{i,m}$ and postmultiplying (22) by $\Phi_{i,m}^T$, one can get (14). This completes the proof. ∎

Here, the objectives to derive the criteria ensuring the stochastic stability of the filtering error system (8) with an H_∞ disturbance attenuation performance index γ and giving the effective solution for HMM-based filter (7) design issue have both been achieved. In the next section, the obtained results will be verified through some examples.

4 | NUMERICAL EXAMPLES

In this section, two examples are used to validate the theoretical results.

Example 1. In this example, the following parameters are presented for the considered system Σ_s with $\mathscr{R} = \{1, 2, 3, 4\}$:

$$\text{Mode 1}: A_1 = \begin{bmatrix} -0.16 & 0.05 \\ 0.38 & 0.68 \end{bmatrix}, \; C_1 = \begin{bmatrix} -1.4 \\ -0.3 \end{bmatrix}, \; H_1 = \begin{bmatrix} 0.12 & 0.67 \end{bmatrix}, \; M_1 = 0.4, \; D_1 = \begin{bmatrix} 0.7 & 0.3 \end{bmatrix}, \; G_1 = 0.19,$$

$$\text{Mode 2}: A_2 = \begin{bmatrix} 0.63 & 0.23 \\ 0.55 & -0.68 \end{bmatrix}, \; C_2 = \begin{bmatrix} 0.2 \\ -0.5 \end{bmatrix}, \; H_2 = \begin{bmatrix} 0.23 & 0.14 \end{bmatrix}, \; M_2 = 0.5, \; D_2 = \begin{bmatrix} 0.6 & 0.2 \end{bmatrix}, \; G_2 = 0.87,$$

$$\text{Mode 3}: A_3 = \begin{bmatrix} 0.75 & -0.15 \\ 0.15 & 0.8 \end{bmatrix}, \; C_3 = \begin{bmatrix} 0.6 \\ 0.35 \end{bmatrix}, \; H_3 = \begin{bmatrix} 0.26 & 0.14 \end{bmatrix}, \; M_3 = 0.2, \; D_3 = \begin{bmatrix} 0.4 & 0.5 \end{bmatrix}, \; G_3 = 0.43,$$

$$\text{Mode 4}: A_4 = \begin{bmatrix} -0.64 & 0.45 \\ 0.73 & 0.76 \end{bmatrix}, \; C_4 = \begin{bmatrix} -0.41 \\ -0.23 \end{bmatrix}, \; H_4 = \begin{bmatrix} 0.45 & 0.65 \end{bmatrix}, \; M_4 = 0.3, \; D_4 = \begin{bmatrix} 0.1 & 0.8 \end{bmatrix}, \; G_4 = 0.36,$$

and $\overline{\lambda}_l = 0.6$ $(l = 1, 2, 3, 4)$, $d_1 = d_2 = 1$. The nonlinear function $\sigma(\cdot)$ is assumed as $\sigma(\kappa) = \frac{O_2+O_1}{2}\kappa + \frac{O_2-O_1}{2}\sin(\kappa)$ with $O_2 = 0.9$ and $O_1 = 0.6$.

The first objective of this example is to explore the influence of different detection cases on the H_∞ performance index γ. The transition probability matrix Ξ of the Markov state $\zeta(k)$ is assumed to be known as:

$$\Xi = \begin{bmatrix} 0.15 & 0.25 & 0.5 & 0.1 \\ 0.35 & 0.05 & 0.2 & 0.4 \\ 0.1 & 0.4 & 0.2 & 0.3 \\ 0.45 & 0.15 & 0.3 & 0.1 \end{bmatrix}.$$

For clear description, the different detection matrix Ω is given in Table 1, which includes the following three types. Type I: $\mathscr{F} = \mathscr{R}$ and $\Omega = \Omega_1$. This case means that the Markov state $\zeta(k)$ can be accurately detected by the detector, that is, $\xi(k) = \zeta(k)$ all the time, which corresponds to the mode-dependent case. Type II: $\mathscr{F} = \mathscr{R}$ and $\Omega = \Omega_l$, $(l = 2, 3, 4, 5)$. These cases mean that the Markov state $\zeta(k)$ cannot be accurately detected for all modes and it can be seen as the asynchronous case. For example, $\Omega = \Omega_2$ indicates that when $\zeta(k) = 1, 2, 3$, the Markov state $\zeta(k)$ can be accurately detected by the detector, i.e., $\xi(k) = \zeta(k)$ when $\zeta(k) = 1, 2, 3$. When $\zeta(k) = 4$, and the probabilities of the output of the detector $\xi(k) = 1$, $\xi(k) = 2$, $\xi(k) = 3$ and $\xi(k) = 4$ are 0.1, 0.1, 0.2 and 0.6, respectively. Type III: $\mathscr{F} = \{1\}$ and $\Omega = \Omega_6$. This case means no matter which state the Markov state $\zeta(k)$ is in, the output of the detector $\xi(k)$ is always $\xi(k) \equiv 1$ and it can be seen as the mode-independent case. By using Theorem 3, the calculation results are shown in Table 2.

Observations and conclusions: Table 2 shows that the optimized H_∞ performance index $\gamma_{\min}$ increases with changing the detection matrix Ω from Type I to Type III. It indicates that the detection matrix Ω has a significant impact on the H_∞ performance of system and the H_∞ performance deteriorates with the decrease of the detection accuracy.

The second objective of this example is to investigate the influence of the partially known transition probabilities in Ξ and detection probabilities in Ω on the H_∞ performance index γ. The different transition matrix Ξ and detection matrix Ω are given in Table 3 and the calculation results are displayed in Table 4.

Observations and conclusions: From Table 4, it can be concluded that the optimized H_∞ performance index $\gamma_{\min}$ increases as there are more unavailable transition probabilities and detection probabilities, which implies that for acquiring better H_∞ performance index, more information of transition probabilities and detection probabilities should be obtained. Therefore, it is necessary to strike a balance between the cost of acquiring the knowledge of transition probabilities and detection probabilities and the importance of the system H_∞ performance index.

Finally, let $\Omega = \Omega_8$, $\Xi = \Xi_3$, and the optimized H_∞ performance index $\gamma_{\min} = 1.8416$. The filter gains can be obtained as

$$A_{f1} = \begin{bmatrix} -0.4376 & 0.3702 \\ 0.1815 & -0.1323 \end{bmatrix}, \quad B_{f1} = \begin{bmatrix} 0.3728 \\ -1.4437 \end{bmatrix}, \quad C_{f1} = \begin{bmatrix} -0.1589 & -0.3579 \end{bmatrix},$$

$$A_{f2} = \begin{bmatrix} 0.3025 & 0.4661 \\ -0.1186 & -0.3043 \end{bmatrix}, \quad B_{f2} = \begin{bmatrix} 0.3797 \\ -1.4660 \end{bmatrix}, \quad C_{f2} = \begin{bmatrix} -0.1919 & -0.3579 \end{bmatrix},$$

$$A_{f3} = \begin{bmatrix} -0.6188 & 0.2967 \\ 0.2808 & -0.2334 \end{bmatrix}, \quad B_{f3} = \begin{bmatrix} 0.5682 \\ -1.5505 \end{bmatrix}, \quad C_{f3} = \begin{bmatrix} -0.2448 & -0.3919 \end{bmatrix},$$

$$A_{f4} = \begin{bmatrix} -0.4607 & 0.3995 \\ 0.2214 & -0.2687 \end{bmatrix}, \quad B_{f4} = \begin{bmatrix} 0.4794 \\ -1.5281 \end{bmatrix}, \quad C_{f4} = \begin{bmatrix} -0.1596 & -0.3732 \end{bmatrix}.$$

TABLE 1 Different detection probabilities matrix Ω in Example 1

$\Omega_1 = \begin{bmatrix} 10 & 00 \\ 01 & 00 \\ 00 & 10 \\ 00 & 01 \end{bmatrix}$	$\Omega_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0.1 & 0.1 & 0.2 & 0.6 \end{bmatrix}$	$\Omega_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0.2 & 0.2 & 0.5 & 0.1 \\ 0.1 & 0.1 & 0.2 & 0.6 \end{bmatrix}$
$\Omega_4 = \begin{bmatrix} 10 & 00 \\ 0.30.1 & 0.40.2 \\ 0.20.2 & 0.50.1 \\ 0.10.1 & 0.20.6 \end{bmatrix}$	$\Omega_5 = \begin{bmatrix} 0.6 & 0.1 & 0.2 & 0.1 \\ 0.3 & 0.1 & 0.4 & 0.2 \\ 0.2 & 0.2 & 0.5 & 0.1 \\ 0.1 & 0.1 & 0.2 & 0.6 \end{bmatrix}$	$\Omega_6 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

TABLE 2 Optimized H_∞ performance index γ for different detection probabilities matrix Ω in Example 1

	Type I	**Type II**				**Type III**
	(Mode-Dependent Case)	**(Asynchronous Case)**				**(Mode-Independent Case)**
	$\Omega = \Omega_1$	$\Omega = \Omega_2$	$\Omega = \Omega_3$	$\Omega = \Omega_4$	$\Omega = \Omega_5$	$\Omega = \Omega_6$
$\gamma_{\min}$	1.1750	1.2117	1.2790	1.3909	1.3973	1.4022

TABLE 3 Different partially known transition probabilities matrix Ξ and detection probabilities matrix Ω in Example 1

$\Xi_1 = \begin{bmatrix} \mathbf{0.10.3} & \mathbf{0.50.1} \\ \mathbf{0.30.1} & \mathbf{0.20.4} \\ \mathbf{0.10.4} & \mathbf{0.40.1} \\ \mathbf{0.40.2} & \mathbf{0.30.1} \end{bmatrix}$	$\Xi_2 = \begin{bmatrix} \mathbf{0.1} & \mathbf{?} & \mathbf{0.5} & \mathbf{?} \\ \mathbf{0.3} & \mathbf{0.1} & \mathbf{0.2} & \mathbf{0.4} \\ \mathbf{?} & \mathbf{0.4} & \mathbf{?} & \mathbf{0.1} \\ \mathbf{0.4} & \mathbf{0.2} & \mathbf{0.3} & \mathbf{0.1} \end{bmatrix}$	$\Xi_3 = \begin{bmatrix} \mathbf{0.1} & \mathbf{?} & \mathbf{0.5} & \mathbf{?} \\ \mathbf{?} & \mathbf{0.1} & \mathbf{0.2} & \mathbf{?} \\ \mathbf{?} & \mathbf{0.4} & \mathbf{?} & \mathbf{0.1} \\ \mathbf{0.4} & \mathbf{?} & \mathbf{0.3} & \mathbf{?} \end{bmatrix}$	$\Xi_4 = \begin{bmatrix} \mathbf{?} & \mathbf{?} & \mathbf{?} & \mathbf{?} \\ \mathbf{?} & \mathbf{?} & \mathbf{?} & \mathbf{?} \\ \mathbf{?} & \mathbf{?} & \mathbf{?} & \mathbf{?} \\ \mathbf{?} & \mathbf{?} & \mathbf{?} & \mathbf{?} \end{bmatrix}$
$\Omega_5 = \begin{bmatrix} 0.60.1 & 0.20.1 \\ 0.30.1 & 0.40.2 \\ 0.20.2 & 0.50.1 \\ 0.10.1 & 0.20.6 \end{bmatrix}$	$\Omega_7 = \begin{bmatrix} 0.6 & 0.1 & 0.2 & 0.1 \\ 0.3 & ? & ? & 0.2 \\ 0.2 & 0.2 & 0.5 & 0.1 \\ ? & 0.1 & 0.2 & ? \end{bmatrix}$	$\Omega_8 = \begin{bmatrix} 0.6 & ? & 0.2 & ? \\ 0.3 & ? & ? & 0.2 \\ 0.2 & ? & 0.5 & ? \\ ? & 0.1 & 0.2 & ? \end{bmatrix}$	$\Omega_9 = \begin{bmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{bmatrix}$

TABLE 4 Optimized H_∞ performance index γ for different transition probabilities matrix Ξ and detection probabilities matrix Ω in Example 1

$\gamma_{\min}$	$\Xi = \Xi_1$	$\Xi = \Xi_2$	$\Xi = \Xi_3$	$\Xi = \Xi_4$
$\Omega = \Omega_5$	1.4114	1.5477	1.8134	2.4630
$\Omega = \Omega_7$	1.4149	1.5571	1.8365	2.5481
$\Omega = \Omega_8$	1.4157	1.5602	1.8416	2.5571
$\Omega = \Omega_9$	1.4161	1.5622	1.8452	2.5911

Further, let $v(k) = 0.65\exp(-0.3k)$ and the initial condition $\vartheta(k) = \begin{bmatrix} 1.5 & -3 & 0 & 0 \end{bmatrix}^T$. The sequences of $\lambda_l(k)$ $(l = 1,2,3,4)$ are randomly given in Figure 1. Then, under the obtained filter gains, the possible evolutions of the Markov state and the output of the detector $\xi(k)$ (filter mode) are plotted in Figure 2. The filtering error response is shown in Figure 3, which indicates the availability of the filter design method.

Example 2. In this example, we consider a single-link robot system:[43]

$$\begin{cases} \dot{\theta} = \psi, \\ J\dot{\psi} = -D\psi - MgL\sin(\theta). \end{cases}$$

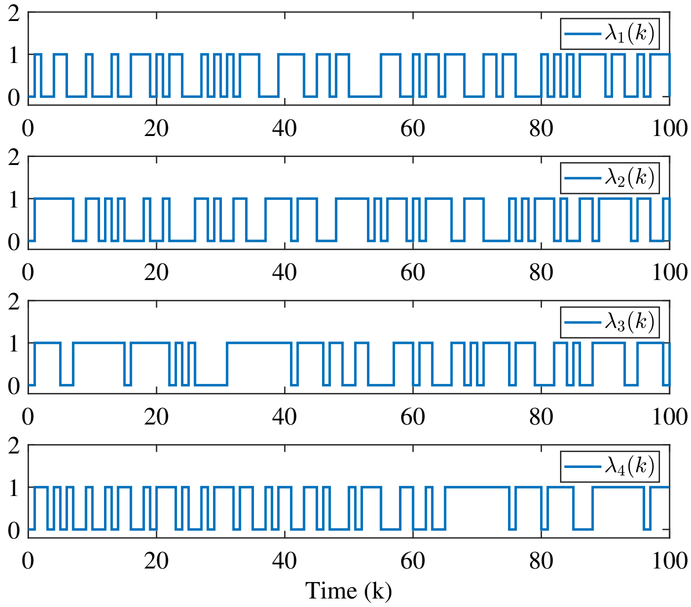

FIGURE 1 The sequences of $\lambda_l(k)$ ($l = 1,2,3,4$) in Example 1 [Colour figure can be viewed at wileyonlinelibrary.com]

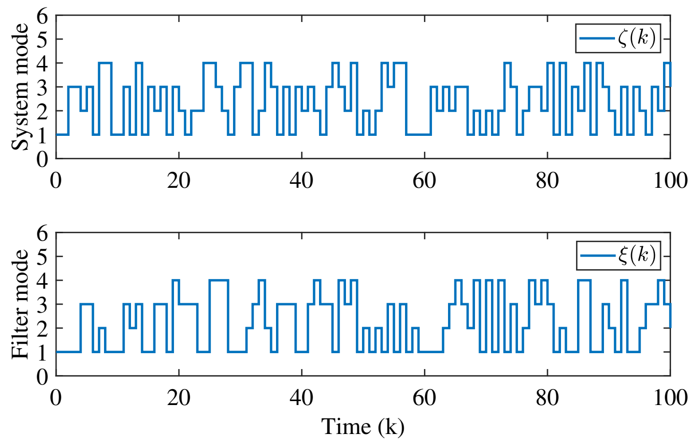

FIGURE 2 The possible evolutions of the Markov state and the output of the detector $\xi(k)$ (filter mode) in Example 1 [Colour figure can be viewed at wileyonlinelibrary.com]

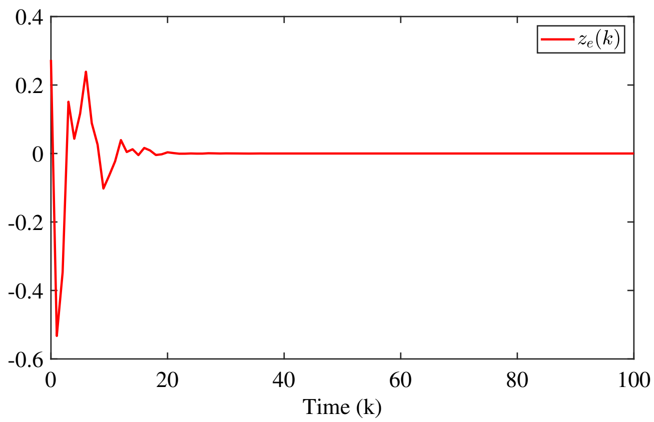

FIGURE 3 The filtering error response in Example 1 [Colour figure can be viewed at wileyonlinelibrary.com]

The physical meaning of θ, ψ, J, D, M, g, and L can be found in Reference 43. As in Reference 43, $L = 0.5$, $D = 2$, $g = 9.81$. M and J have three modes: Mode 1 $M = J = 1$; Mode 2 $M = J = 5$; Mode 3 $M = J = 10$. According to Reference 43, the discrete-time model system parameters can be obtained as

$$A_1 = \begin{bmatrix} 0.59 & 0.26 \\ -1.26 & 0.08 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.48 & 0.37 \\ -1.79 & 0.34 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 0.47 & 0.38 \\ -1.88 & 0.39 \end{bmatrix},$$

$$C_l = \begin{bmatrix} 0.1 \\ -0.1 \end{bmatrix}, \quad H_l = D_l = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad M_l = G_l = 0.5, \quad l = 1, 2, 3.$$

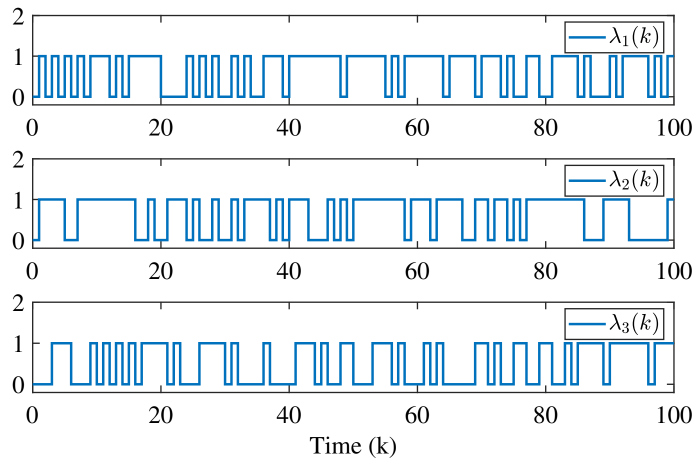

FIGURE 4 The sequences of $\lambda_l(k)$ $(l=1,2,3)$ in Example 2 [Colour figure can be viewed at wileyonlinelibrary.com]

Here, the transition probabilities matrix Ξ is with some partially unknown elements:

$$\Xi = \begin{bmatrix} 0.8 & ? & ? \\ ? & 0.8 & ? \\ ? & ? & 0.8 \end{bmatrix}.$$

The operation mode of the system is obtained via a detector, which meets the detection probabilities matrix Ω:

$$\Omega = \begin{bmatrix} 0.45 & ? & ? \\ ? & 0.78 & ? \\ ? & ? & 0.68 \end{bmatrix}.$$

Then, let $\bar{\lambda}_1 = 0.56$, $\bar{\lambda}_2 = 0.83$, $\bar{\lambda}_3 = 0.67$, $d_1 = d_2 = 1$, the nonlinear function $\sigma(\kappa) = \frac{O_2+O_1}{2}\kappa + \frac{O_2-O_1}{2}\sin(\kappa)$ with $O_2 = 0.9$ and $O_1 = 0.6$, and the exogenous disturbance $v(k) = 0.5\exp(-0.1k)$. We aim to design a filter such that the disturbance attenuation level of the resulting filtering error system is $\gamma = 0.6$. By using Theorem 3, the filter gains can be obtained as:

$$A_{f1} = \begin{bmatrix} 0.2165 & 0.3051 \\ -1.2761 & 0.2850 \end{bmatrix}, \quad B_{f1} = \begin{bmatrix} -0.2687 \\ 0.2904 \end{bmatrix}, \quad C_{f1} = \begin{bmatrix} -1.5116 & -0.1850 \end{bmatrix},$$
$$A_{f2} = \begin{bmatrix} 0.2303 & 0.3411 \\ -1.4118 & 0.3077 \end{bmatrix}, \quad B_{f2} = \begin{bmatrix} -0.2614 \\ 0.2922 \end{bmatrix}, \quad C_{f2} = \begin{bmatrix} -1.5168 & -0.2009 \end{bmatrix},$$
$$A_{f3} = \begin{bmatrix} 0.2271 & 0.3471 \\ -1.4500 & 0.3185 \end{bmatrix}, \quad B_{f3} = \begin{bmatrix} -0.2703 \\ 0.2967 \end{bmatrix}, \quad C_{f3} = \begin{bmatrix} -1.6056 & -0.2367 \end{bmatrix}.$$

Given the initial condition $\vartheta(0) = \begin{bmatrix} 0.5\pi & -2 & 0 & 0 \end{bmatrix}^T$, Figure 4 shows the sequences of $\lambda_i(k)$ $(i=1,2,3)$ and Figure 5 depicts the possible evolutions of the Markov state $\zeta(k)$ and the output of the detector $\xi(k)$. The filtering error response is shown in Figure 6. From Figure 6, one can see that the obtained filter is effective.

5 | CONCLUSION

In this paper, the H_∞ filtering issue for MJSs in the circumstances of the partial information on the Markov chain and ROSNs has been explored, in which the partial information is well modeled by an HMM with partially known transition probabilities and detection probabilities. With the help of the HMM and Lyapunov methodology, the effective H_∞ performance analysis criteria and the desired HMM-based filter design procedure have been derived. In the end,

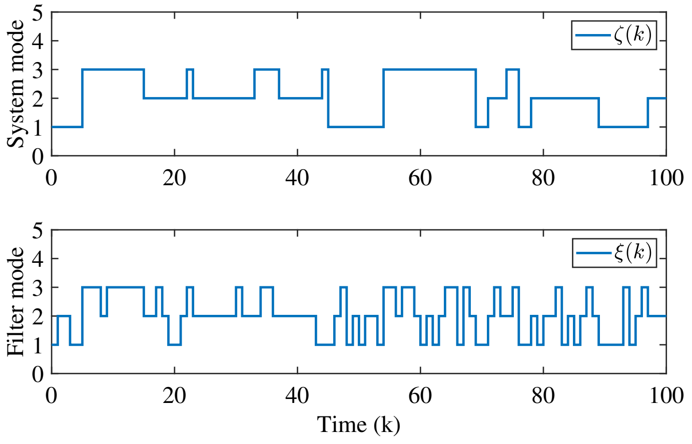

FIGURE 5 The possible evolutions of the Markov state and the output of the detector $\xi(k)$ (filter mode) in Example 2 [Colour figure can be viewed at wileyonlinelibrary.com]

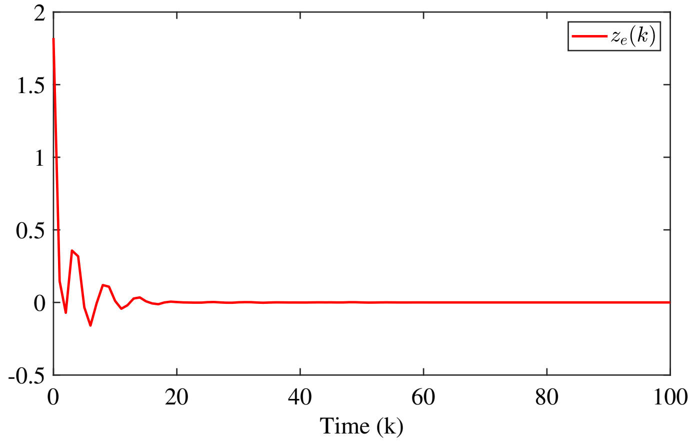

FIGURE 6 The filtering error response in Example 2 [Colour figure can be viewed at wileyonlinelibrary.com]

to manifest the practicability of obtained theoretical results, two illustrative examples have been given in the examples section. Lastly, on the one hand, expanding our results to handle the partial information problems encountered in semi-MJSs and continuous-time MJSs will be the next research tasks. On the other hand, considering the filtering issues for networked MJSs with packet loss but without packet acknowledgment is another interesting topic deserving further investigation.[44]

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CONFLICT OF INTEREST

The authors declare no potential conflict of interests.

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