Adaptive Fuzzy Tracking Control for Strict-Feedback Markov Jumping Nonlinear Systems With Actuator Failures and Unmodeled Dynamics

Zheng Wang, Yuan Yuan, and Hongjiu Yang

Abstract—**In this paper, an adaptive fuzzy tracking controller is developed for a class of strict-feedback Markovian jumping systems subjected to multisource uncertainties. The unpredictable actuator failures, the unknown nonlinearities, and the unmodeled dynamics are simultaneously taken into consideration, which evolve according to the Markov chain. It is noted that the elements in the transition rate matrix of the Markov chain are not fully available. In virtue of the norm estimation approach, the challenges caused by the complex multiple uncertainties and actuator failures are effectively handled. Furthermore, to compensate for the unavailable switching nonlinearities, the fuzzy logic systems are employed as online approximators. As a result, a novel adaptive fuzzy fault-tolerant tracking control structure is constructed. The sufficient condition is provided to guarantee that the studied system is stochastically stable. Finally, a number of illustrative examples are employed to demonstrate the effectiveness of the proposed methodology.**

Index Terms—**Adaptive tracking control, backstepping control, fault-tolerant control, fuzzy control, Markovian jumping nonlinear systems.**

I. Introduction

As IS well known, the Markovian jump systems (MJSs) have been widely employed to model many practical engineering systems, such as manufacturing systems, power systems, economic systems, and communication systems. As such, it is not surprising that the control problem for the MJSs have become a tremendous hot topic [1], [2]. For the purpose of dealing with the uncertainties, a number of control schemes have been reported, such as the H_∞ control schemes [3], [4]; disturbance-observer-based control schemes [5], [6]; and sliding mode control schemes [7], [8]. However, it should be noticed that the mentioned literature restricts its attention on the MJSs without nonlinearities or with the nonlinearities obeying the Lipschitz condition. To handle this, very recently, several adaptive sliding mode control schemes have been designed for the non-Lipschitz Markovian jump nonlinear systems (MJNSs) [9]–[11]. Note that in [9]–[11], the mismatched/unmatched disturbances have not been considered which is pervasive in engineering practice. To overcome such a limitation, the so-called backstepping approach has been put forward [12]. Despite the progress, in the aforementioned literature, it has been implicitly assumed that the unknown parameters never vary with the switching modes. Clearly, for the MJNSs with randomly jumping unknown parameters and Markovian switching unknown functions, which are encountered more often in practice, the control approaches in [9]–[16] are not capable of achieving desired control performances. Moreover, it should be pointed out that, to the best of the author's knowledge, the MJNSs in strict-feedback form have never been investigated.

On the other hand, the actuator failures are also often encountered in a wealthy of realistic applications, such as robotic systems, spacecrafts, and hypersonic vehicle systems. It has been well recognized that the actuator failures are one of the main causes which could degrade the control performances [17]–[22]. To solve this problem, a lot of effective control methods have been proposed in the past few decades [23], [24]. In [25] and [26], several passive fault-tolerant control structures have been developed according to robust control theory. Furthermore, to achieve the adaptive capability for actuator and sensor faults, a number of active fault-tolerant controllers, which possess reconfigurable structures, have been proposed [27]. Note that in the aforementioned results, the randomly changing actuator failures which often exist in practice have been rarely investigated. Most recently, two fault-tolerant schemes have been developed for Markovian jumping actuator faults [28], [29]. However, the parameters and the structures of the controlled systems in [28] and [29] are never allowed to vary with the Markovian modes. Therefore, the designed controllers in [28] and [29] are not capable of achieving the desired performances for the concerned MJNSs with randomly switching actuator failures.

Furthermore, to obtain the wanted control performances, the unmodeled dynamics has to be adequately handled. As is well known, the unmodeled dynamics (also denoted as the ignored dynamics or dynamic uncertainties) widely exists

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in engineering practice, which might lead to the instability of the controlled systems. In recent years, a great deal of literature has been reported on control of systems with unmodeled dynamics [30], [31]. However, up to now, the control schemes have not been investigated for the dynamic uncertainties with Markovian switching parameters and randomly jumping structures.

In this paper, a novel adaptive fuzzy control scheme [32]–[35] is developed for a class of strict-feedback MJNSs with randomly varying uncertainties and Markovian jumping actuator failures. The main challenges stem from the immanent randomly jumping characteristics of the parameters and structures in the considered system. It is worth noting that it is difficult to estimate the unknown parameters which keep randomly changing and few results have been reported. Additionally, the mismatched and random features of the uncertainties bring about more challenges in the controller design. In this paper, by estimating the upper bounds of the unknown parameters, the difficulties caused by the Markovian jumping unknown parameters and actuator failures are circumvented. Meanwhile, the unknown nonlinearities are handled by the fuzzy logic systems (FLSs). An adaptive fuzzy control approach is finally synthesized in the context of the adaptive backstepping technique. Compared with the existing literature, the main contributions of the proposed method are highlighted as follows.

1) The established control method first solves the tracking control problem of the strict feedback MJNSs with randomly changing actuator faults and Markovian switching dynamic uncertainties.
2) As far as the authors know, it is also the first attempt to tackle the tracking control problem for the pure Markov jumping strict-feedback systems in the sense that the parameters and structures of the MJNSs are permitted to be Markovian switching.
3) By using the proposed method, the transition rate matrix is allowed to be completely unknown (Remark 4).

The structure of this paper is provided herein. Section II provides the problem description and several imperative definitions. In Section III, the adaptive fault-tolerant tracking controller is designed. The convergence analysis is also given in this section. The simulation results are shown in Section IV. In this paper, A^{T} represents the transpose of matrix A. $\|\cdot\|$ represents the Euclidean norm. The probability space is represented by $(\Omega, \mathscr{F}, \mathscr{P})$. Ω, $\mathscr{F}$, and $\mathscr{P}$ represent the sample space, σ-algebra of subsets of the sample space, and probability measure on $\mathscr{F}$, respectively. $\mathbf{E}\{\cdot\}$ represents the mathematical expectation. $\mathbf{P}\{\cdot\}$ represents the probability.

II. Problem Formulation and Preliminaries

A. Problem Statement and Basic Assumptions

A class of Markov jumping nonlinear systems can be modeled as

$$
\begin{aligned}
\dot{x}_i &= x_{i+1} + \theta^{\mathrm{T}}(r_t, t) f_i(r_t, \bar{x}_i) + \psi_i(r_t, \bar{x}_i) + \Delta_i(r_t, \omega, \bar{x}_i) \\
&\quad i = 1, 2, \ldots, n-1 \\
\dot{x}_n &= \theta^{\mathrm{T}}(r_t, t) f_n(r_t, x) + \sum_{j=1}^{m} b_j(r_t, x) u_j \\
&\quad + \psi_n(r_t, x) + \Delta_n(r_t, \omega, x) \\
y &= x_1
\end{aligned} \tag{1}
$$

where $x = [x_1, x_2, \ldots, x_n]^{\mathrm{T}} \in \mathbb{R}^n$ represents the system states, $u = [u_1, u_2, \ldots, u_m]^{\mathrm{T}} \in \mathbb{R}^m$ denotes the control inputs; $\bar{x}_i = [x_1, x_2, \ldots, x_i]^{\mathrm{T}}$. For each mode, $\theta(r_t, t) \in \mathbb{R}^s$ denotes the unknown parameter vector which is bounded and piecewise continuous. $b_j(r_t, x, t) \neq 0$. $f_i(r_t, \bar{x}_i) \in \mathbb{R}^s$ and $b_j(r_t, x, t) \in \mathbb{R}$ are both known. The unknown function $\psi_i(r_t, \bar{x}_i) \in \mathbb{R}$ is presumed to be smooth. The unmodeled states are represented by $\omega \in \mathbb{R}$

$$\dot{\omega} = q(\omega, x_1). \tag{2}$$

The dynamic disturbance caused by the unmodeled states is denoted as $\Delta_i(r_t, \omega, \bar{x}_i)$. Here, it is assumed that Δ_i and q satisfy the Lipschitz continuous condition. As a right-continuous Markov process, $\{r_t, t \geq 0\}$ takes values in a finite set $S = \{1, 2, \ldots, N\}$. The transition rate matrix is represented by $\Pi = (\pi_{k_1 k_2})_{N \times N}$ $k_1, k_2 = 1, 2, \ldots, N$, and the mode transition probabilities are given as

$$
P_r(r_{t+\Delta t} = k_2 | r_t = k_1) = \begin{cases} \pi_{k_1 k_2} \Delta t + o(\Delta t), & k_1 \neq k_2 \\ 1 + \pi_{k_1 k_1} \Delta t + o(\Delta t), & k_1 = k_2 \end{cases} \tag{3}
$$

where $\Delta t > 0$ and $\lim_{\Delta t \to 0} o(\Delta t)/\Delta t = 0$. $\pi_{k_1 k_2}$ is the transition rate from mode k_1 to k_2. $\pi_{k_1 k_2} > 0$ when $k_1 \neq k_2$ and $\pi_{k_1 k_1} = -\sum_{k_2=1, k_2 \neq k_1}^{N} \pi_{k_1 k_2}$. In this paper, the transition rates or probabilities of the jumping process are assumed to be only partly accessed. In other words, some elements in matrix Π are assumed to be unknown. For example, the transition rates or probability matrix Π of system (1) with four operation modes may be described as

$$
\Pi = \begin{bmatrix} ? & \pi_{12} & ? & ? \\ ? & \pi_{22} & ? & \pi_{24} \\ ? & ? & ? & \pi_{34} \\ \pi_{41} & \pi_{42} & ? & ? \end{bmatrix} \tag{4}
$$

where "?" denotes the inaccessible elements. $\forall k_1 \in S$, and we represent $S = S_1^{k_1} \cup S_2^{k_1}$ with

$$
\begin{aligned}
S_1^{k_1} &\triangleq \left\{ k_2 : \pi_{k_1 k_2} \text{ is known} \right\} \\
S_2^{k_1} &\triangleq \left\{ k_2 : \pi_{k_1 k_2} \text{ is unknown} \right\}.
\end{aligned} \tag{5}
$$

Denote the input of the jth actuator as $v_j(t)$, which is to be designed. The failures that may occur on the jth actuator can be modeled as

$$
\begin{aligned}
u_j(t) &= h_j(r_t) v_j(t) + \delta_j(r_t), \quad j = 1, 2, \ldots, m \\
h_j(r_t) \delta_j(r_t) &= 0
\end{aligned} \tag{6}
$$

where $\delta_j(r_t) \in \mathbb{R}$ is an unknown variable related to the Markovian variable r_t. $h_j(r_t)$ is a stochastic function which takes value in the interval $[0, 1]$, changing with Markovian variable r_t. Three types of failures can be covered by (6).

1) $0 < h_j(r_t) < 1$ and $\delta_j(r_t) = 0$. In this case, $u_j(t) = h_j(r_t)v_j(t)$. This implies partial loss of effectiveness of actuators. For example, $h_j(r_t) = 0.8$ means the jth actuator loses 20% of its effectiveness.
2) $h_j(r_t) = 0$ and $\delta_j(r_t) \neq 0$. In this case, $u_j(t) = \delta_j(r_t)$, $u_j(t)$ can no longer be influenced by $v_j(t)$ and stuck at an unknown value $\delta_j(r_t)$. It is known as total loss of effectiveness (TLOE). Especially, since $\delta_j(r_t) \neq 0$, this case corresponds to the lock-in place case of TLOE.
3) $h_j(r_t) = 0$ and $\delta_j(r_t) = 0$. Such a case can be regarded as the float type of TLOE.

For the sake of simplicity, when $r_t = k$, $k \in S$, $\psi_i(r_t, \bar{x}_i)$ can be denoted by $\psi_i(k, \bar{x}_i)$. The same setting with the other functions, such as $f_i(r_t, \bar{x}_i)$, is expressed as $f_i(k, \bar{x}_i)$ and so on.

The design objective is to develop an adaptive control scheme to maintain the desired trajectory tracking for MJNS (1) in simultaneous presence of the Markovian jumping actuator failures and multisource uncertainties.

Before proceeding, several assumptions are made.

Assumption 1: At any time instant, up to $m-1$ actuators are at the state of TLOE. Denoting $H = 1/\inf_{k\in S} \|\sum_{j=1}^{m} h_j(k)\|$, we assume that H is bounded.

Assumption 2: $\forall 1 \le i \le n$, it is assumed that the dynamic uncertainty $\Delta_i(k, \omega_i, \bar{x}_i)$ satisfies the following condition for mode k:

$$\|\Delta_i(k, \omega_i, \bar{x}_i)\| \le \varphi_{i,1,k}(\bar{x}_i) + \varphi_{i,2,k}(\omega) \tag{7}$$

where $\varphi_{i,1,k}(\bar{x}_i)$ and $\varphi_{i,2,k}(\omega)$ are unknown non-negative smooth functions.

Assumption 3: For mode k, the unmodeled states are assumed to be exponentially input-to-state practically stable. In other words, considering system (2), there exists a Lyapunov function $V_\omega(\omega)$ such that

$$\begin{aligned} \alpha_1(\omega) \le V_\omega(\omega) \le \alpha_2(\omega) \\ \frac{\partial V_\omega(\omega)}{\partial \omega} q(\omega, x_1) \le -\gamma_1 V_\omega(\omega) + \xi(x_1) + \gamma_2. \end{aligned} \tag{8}$$

α_1 and α_2 are K_∞ functions, and $\xi(\cdot)$ is a nonlinear function which is known, non-negative, and smooth. The constants $\gamma_1 > 0$ and $\gamma_2 \ge 0$ are accessible.

Lemma 1 [36]: The following inequality holds:

$$0 \le \|z\| - \frac{z^2}{\sqrt{z^2 + \rho^2}} < \rho \tag{9}$$

where $\rho > 0$ is a constant and $z \in \mathbb{R}$.

Lemma 2 [37]: Let V_ω represent a Lyapunov function satisfying (8). Given any constant γ_0 ($\gamma_1 > \gamma_0 > 0$), there exists a finite time $T_0 = T_0(\gamma_0, r_0, \omega_0)$, a non-negative signal $D(t)$ ($t \ge 0$), and a dynamic signal given by

$$\dot{r} = -\gamma_0 r + \bar{\xi}(x_1) + \gamma_2, \quad r(t_0) = r_0 > 0 \tag{10}$$

satisfying $D(t) = 0$ for $t \ge T_0$ and

$$V_\omega(\omega(t)) \le r(t) + D(t), \quad \forall t \ge T_0 \ge 0 \tag{11}$$

where t_0 is the initial time of (10). Without losing generality, we can select that $\bar{\xi}(x_1) = x_1^2 \xi_0(x_1^2)$. $\xi_0(\cdot)$ denotes a nonlinear function which is non-negative and smooth. Note that $r > 0$ holds for $t > 0$.

Lemma 3 [38]: For any $\upsilon > 0$, consider the set Ω_υ defined by $\Omega_\upsilon = \{z||z| < 0.2554\upsilon\}$. Then, for $z \notin \Omega_\upsilon$, the inequality $1 - 16\tanh^2(z/\upsilon) \le 0$ is satisfied.

Remark 1: Notice that Assumption 1 is made for the actuator failures to guarantee that the addressed system is controllable. In addition, it should be pointed out that it is often (see [10], [30], [39]) assuming that the following condition holds: $\|\Delta_i(\omega_i, \bar{x}_i)\| \le \varphi_{i,1}(\bar{x}_i) + \varphi_{i,2}(\omega_i)$. However, in this paper, the unknown parameters and the known functions of the dynamic uncertainties are both allowed to be randomly switching (Assumption 2). Because of the aforementioned dynamic Markovian uncertainties, the control problem becomes challenging and most of the current research achievements are not capable of achieving the wanted control performance. In fact, in numerals practical systems, the unmodeled dynamics often vary with system modes. For example, when the single-link manipulator switches to mode 1 from mode 2, the parameters of the unmodeled part also switch from one mode to another. Meanwhile, because of the strong interrelation and the couplings between the actuators and the mechanical systems, the parameters of the actuator failures are often changed as well. Therefore, Assumption 2 makes more practical sense. Finally, Lemmas 1–3 are introduced to provide several mathematical fundamentals for the bound function, the dynamic signal, and the hyperbolic tangent function.

B. Infinitesimal Operator and Boundedness in Probability

To begin with, we recall the definition of stochastic stability for the considered MJNSs.

Definition 1 [40]: For any function $V(t, x, r_t)$, at point $\{t, x, k\}$, the weak infinitesimal operator of $\{x(t), r_t, t \ge 0\}$ can be defined as

$$\mathscr{L}V(x(t), k) = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x}\dot{x}(t) + \sum_{k_1=1}^{N} \pi_{kk_1} V(x(t), k_1). \tag{12}$$

Definition 2 [40]: Consider a stochastic process $x(t)$, if $\|x(t)\|$ is bounded in probability uniformly in t, that is, $\lim_{R\to\infty}\sup_{t>0}\mathbf{P}\{\|x(t)\| > R\} = 0$, then $x(t)$ is said to be bounded in probability.

C. Fuzzy Logic Systems

Typically, FLS consists of four parts: 1) the knowledge base; 2) the singleton fuzzifier; 3) the product inference; and 4) the center average defuzzifier [41]. First of all, the knowledge base can be constructed by a set of IF-THEN rules as follows.

R^l: If x_1 is F_1^l and … and x_n is F_n^l.

Then, y is $G^l, i = 1, 2, \ldots, N_l$, where $x = [x_1, x_2, \ldots, x_n]^{\mathrm{T}} \in \mathbb{R}^n$ and y are the input and the output of the FLS, respectively. Fuzzy sets F_i^l and G^l are associated with the fuzzy functions $\mu_{F_i^l}$ and μ_{G^l}. N_l is the number of the rules. Then, the FLS with the singleton fuzzifier, product inference, and center average defuzzifier can be formulated as

$$y(x) = \frac{\sum_{i=1}^{N_l} \bar{y}_l \prod_{i=1}^{n} \mu_{F_i^l}(x_i)}{\sum_{i=1}^{N_l}\left[\prod_{i=1}^{n} \mu_{F_i^l}(x_i)\right]} \tag{13}$$

where $\bar{y}_l = \max_{y\in\mathbb{R}}\mu_{G^l}(y)$. Let $\phi(x) = [\phi_1(x), \phi_2(x), \ldots, \phi_{N_l}(x)]^{\mathrm{T}}$ with

$$\phi_l(x) = \frac{\prod_{i=1}^{n}\mu_{F_i^l}(x_i)}{\sum_{i=1}^{N_l}\left[\prod_{i=1}^{n}\mu_{F_i^l}(x_i)\right]}$$

and defining $W^{\mathrm{T}} = [\bar{y}_1, \bar{y}_2, \ldots, \bar{y}_{N_l}] = [w_1, w_2, \ldots, w_{N_l}]$, we can rewrite the FLS as $y(x) = W^{\mathrm{T}}\phi(x)$.

Lemma 4 [37], [42]: Let $F(x)$ be a continuous function that is defined on a compact set Ω_x. For any given positive constant ε, there always exists an FLS $y(x)$ in terms of (13) such that

$$\sup_{x\in\Omega_x}|F(x) - y(x)| = |F(x) - y(x)| < \varepsilon.$$

III. Adaptive Fuzzy FTC With Partly Unknown Transition Probabilities

In this part, the adaptive fault-tolerant control problem for MJNSs is studied under Assumptions 1–3.

The design of u is generated by the following procedures. For mode k, we introduce the error variables

$$\begin{aligned} z_1 &= y - y_r \\ z_i &= x_i - \beta_{i-1}(k) - y_r^{(i-1)}, \quad i = 2, \ldots, n \end{aligned} \tag{14}$$

where $\beta_{i-1}(k)$ is the virtual control determined at the $(i-1)$th step. With respect to the uncertainties caused by actuator failures and the unmodeled dynamics, we define

$$\begin{aligned} \vartheta &= \sup_{k\in S, t\geq 0}\|\theta(k,t)\| \\ \mu &= \max\left\{\sup_{k\in S}\|W_i(k)\| : i = 1, 2, \ldots, n\right\}. \end{aligned} \tag{15}$$

$\hat{\vartheta}$ and $\hat{\mu}$ are utilized to denote the estimates of ϑ and μ. $\tilde{\vartheta} = \hat{\vartheta} - \vartheta$, $\tilde{\mu} = \hat{\mu} - \mu$. The positive scalars $\eta_{z_i,k}, \eta_{\vartheta,k}, \eta_{\mu,k}$ are to be designed subsequently.

Step 1: Combining (1) and (14) yields

$$\begin{aligned} \dot{z}_1 &= z_2 + \beta_1(k) + \theta^{\mathrm{T}}(k,t)f_1(k,x_1) \\ &\quad + \psi_1(k,x_1) + \Delta_1(k,\omega,x_1). \end{aligned} \tag{16}$$

Then, for mode k, we consider the following Lyapunov function:

$$V_1\left(k, z_1, \tilde{\vartheta}, \tilde{\mu}, r\right) = \frac{1}{2\eta_{z_1,k}}z_1^2 + \frac{1}{2\eta_{\vartheta,k}}\tilde{\vartheta}^2 + \frac{1}{2\eta_{\mu,k}}\tilde{\mu}^2 + \frac{r}{\eta_r}. \tag{17}$$

Along (16), we can take the infinitesimal generator of $V_1(k)$ as

$$\begin{aligned} \mathscr{L}V_1(k) &= \frac{z_1}{\eta_{z_1,k}}\big[z_2 + \beta_1(k) + \theta^{\mathrm{T}}(k,t)f_1(k,x_1) \\ &\qquad + \psi_1(k,x_1) + \Delta_1(k,\omega,x_1)\big] \\ &\quad + \frac{1}{\eta_{\vartheta,k}}\tilde{\vartheta}\dot{\hat{\vartheta}} + \frac{1}{\eta_{\mu,k}}\tilde{\mu}\dot{\hat{\mu}} + \frac{\bar{\xi}(x_1)}{\eta_r} - \frac{\gamma_0 r}{\eta_r} + \frac{\gamma_2}{\eta_r} \\ &\quad + \sum_{k_1=1}^{N}\left[\frac{\pi_{kk_1}}{2\eta_{z_1,k_1}}z_1^2 + \frac{\pi_{kk_1}}{2\eta_{\vartheta,k_1}}\tilde{\vartheta}^2 + \frac{\pi_{kk_1}}{2\eta_{\mu,k_1}}\tilde{\mu}^2\right]. \end{aligned} \tag{18}$$

Using Assumption 2, we have

$$\|z_1\Delta_1(k,\omega,x_1)\| \leq \|z_1\|\varphi_{1,1,k}(x_1) + \|z_1\|\varphi_{1,2,k}(\omega). \tag{19}$$

Then from Lemma 1, we know that

$$\|z_1\|\varphi_{1,1,k}(x_1) \leq z_1\bar{\varphi}_{1,1,k}(x_1) + \varepsilon_{11}$$

where

$$\bar{\varphi}_{1,1,k}(x_1) = \frac{z_1}{\sqrt{z_1^2\varphi_{1,1,k}^2(x_1) + \varepsilon_{11}^2}}, \quad \varepsilon_{11} > 0.$$

According to (11) and using the properties of K_∞-function $\alpha_1(\cdot)$, we can obtain

$$\begin{aligned} \|z_1\|\varphi_{1,2,k}(\omega) &\leq \|z_1\|\varphi_{1,2,k}\circ\alpha_1^{-1}(2r_1) \\ &\quad + \|z_1\|\varphi_{1,2,k}\circ\alpha_1^{-1}(2D_1(t)). \end{aligned}$$

Furthermore, applying Young's inequalities yields

$$\begin{aligned} \|z_1\|\varphi_{1,2,k}(\omega) &\leq \|z_1\|\varphi_{1,2,k}\circ\alpha_1^{-1}(2r_1) \\ &\quad + \frac{1}{4}z_1^2 + \left[\varphi_{1,2,k}\circ\alpha_1^{-1}(2D_1(t))\right]^2. \end{aligned}$$

By using Lemma 1 again, the following inequality can be obtained:

$$\|z_1\|\varphi_{1,2,k}\circ\alpha_1^{-1}(2r_1) \leq z_1\bar{\varphi}_{1,2,k}(x_1,r) + \varepsilon_{12}$$

where

$$\bar{\varphi}_{1,2,k}(x_1,r) = \frac{z_1}{\sqrt{z_1^2\left[\varphi_{1,2,k}\circ\alpha_1^{-1}(2r_1)\right]^2 + \varepsilon_{12}^2}}, \quad \varepsilon_{12} > 0.$$

Therefore, we can rewrite (19) as

$$\begin{aligned} \|z_1\Delta_1(k,\omega,x_1)\| &\leq z_1\bar{\varphi}_{1,1,k}(x_1) + \varepsilon_{11} + z_1\bar{\varphi}_{1,2,k}(x_1,r) \\ &\quad + \varepsilon_{12} + \frac{1}{4}z_1^2 + d_{1,k} \end{aligned} \tag{20}$$

where $d_{1,k} = [\varphi_{1,2,k}\circ\alpha_1^{-1}(2D_1(t))]^2$. To deal with the unknown functions $\psi_1(k,x_1)$, $\bar{\varphi}_{1,1,k}(x_1)$, and $\bar{\varphi}_{1,2,k}(x_1,r)$, an FLS is introduced to approximate the sum of them for each mode. It follows from Lemma 4 that:

$$\begin{aligned} &\psi_1(k,x_1) + \bar{\varphi}_{1,1,k}(x_1) + \bar{\varphi}_{1,2,k}(x_1,r) \\ &\quad = W_1{}^{\mathrm{T}}(k)\phi_1(k,x_1,r) + \varepsilon_{\phi,1}(k,x_1,r) \end{aligned} \tag{21}$$

where $\varepsilon_{\phi,1}(k,x_1,r)$ is the FLS approximation error with supreme ε_{13}. Accordingly, by resorting to Lemma 1, we have

$$\begin{aligned} &z_1\psi_1(k,x_1) + z_1\Delta_1(k,\omega,x_1) \\ &\quad = z_1W_1{}^{\mathrm{T}}(k)\phi_1(k,x_1,r) + z_1\varepsilon_{\phi,1}(k,x_1,r) \\ &\qquad + \varepsilon_{11} + \varepsilon_{12} + \frac{1}{4}z_1^2 + d_{1,k} \\ &\quad \leq z_1\mu\bar{\phi}_1(k,x_1,r) + \mu\varepsilon_\mu + \frac{1}{2}\varepsilon_{13}^2 \\ &\qquad + \varepsilon_{11} + \varepsilon_{12} + \frac{3}{4}z_1^2 + d_{1,k} \end{aligned} \tag{22}$$

where $\bar{\phi}_1(k,x_1,r) = ([z_1\phi_1{}^{\mathrm{T}}(k,x_1,r)\phi_1(k,x_1,r)]/[\sqrt{z_1^2\phi_1{}^{\mathrm{T}}(k,x_1,r)\phi_1(k,x_1,r) + \varepsilon_\mu^2}])$. On the other hand, according to Lemma 1, given any $\varepsilon_\vartheta > 0$, we have

$$\begin{aligned} z_1\theta^{\mathrm{T}}(k,t)f_1(k,x_1) &\le \vartheta\|z_1\|\|f_1(k,x_1)\| \\ &\le \vartheta z_1\bar{f}_1(k,x_1) + \vartheta\varepsilon_\vartheta \end{aligned} \tag{23}$$

where $\bar{f}_1(k,x_1) = ([z_1 f_1{}^{\mathrm{T}}(k,x_1)f_1(k,x_1)]/[\sqrt{z_1^2 f_1{}^{\mathrm{T}}(k,x_1)f_1(k,x_1)+\varepsilon_\vartheta^2}])$. In view of (18)–(23), we can obtain that

$$\begin{aligned} \mathscr{L}V_1(k) = {} & \frac{z_1}{\eta_{z_1,k}}\Bigg[z_2 + \beta_1(k) + \vartheta\bar{f}_1(k,x_1) \\ & + \mu\bar{\phi}_1(k,x_1,r) + \frac{3}{4}z_1 + \frac{\eta_{z_1,k}\bar{\xi}(x_1)}{\eta_r z_1}\Bigg] \\ & + \frac{1}{\eta_{z_1,k}}\left[\vartheta\varepsilon_\vartheta + \mu\varepsilon_\mu + \frac{1}{2}\varepsilon_{13}^2 + \varepsilon_{11} + \varepsilon_{12} + d_{1,k}\right] \\ & + \frac{1}{\eta_{\vartheta,k}}\tilde{\vartheta}\dot{\hat{\vartheta}} + \frac{1}{\eta_{\mu,k}}\tilde{\mu}\dot{\hat{\mu}} - \frac{\gamma_0 r}{\eta_r} + \frac{\gamma_2}{\eta_r} \\ & + \sum_{k_1=1}^{N}\left[\frac{\pi_{kk_1}}{2\eta_{z_1,k_1}}z_1^2 + \frac{\pi_{kk_1}}{2\eta_{\vartheta,k_1}}\tilde{\vartheta}^2 + \frac{\pi_{kk_1}}{2\eta_{\mu,k_1}}\tilde{\mu}^2\right]. \end{aligned} \tag{24}$$

Noting that $\eta_{z_1,k}\bar{\xi}(x_1)/(\eta_r z_1)$ is discontinuous at $z_1 = 0$, we introduce the function $\tanh^2(z_1/\upsilon_1)$. Therefore, (24) can be re-expressed as

$$\begin{aligned} \mathscr{L}V_1(k) = {} & \frac{z_1}{\eta_{z_1,k}}\Bigg[z_2 + \beta_1(k) + \vartheta\bar{f}_1(k,x_1) + \frac{3}{4}z_1 \\ & + \mu\bar{\phi}_1(k,x_1,r) + \frac{16}{z_1}\tanh^2(z_1/\upsilon_1)\frac{\eta_{z_1,k}\bar{\xi}(x_1)}{\eta_r}\Bigg] \\ & + \frac{1}{\eta_{z_1,k}}\left[\vartheta\varepsilon_\vartheta + \mu\varepsilon_\mu + \frac{1}{2}\varepsilon_{13}^2 + \varepsilon_{11} + \varepsilon_{12} + d_{1,k}\right] \\ & + \frac{\bar{\xi}(x_1)}{\eta_r}\left(1 - 16\tanh^2(z_1/\upsilon_1)\right) \\ & + \frac{1}{\eta_{\vartheta,k}}\tilde{\vartheta}\dot{\hat{\vartheta}} + \frac{1}{\eta_{\mu,k}}\tilde{\mu}\dot{\hat{\mu}} - \frac{\gamma_0 r}{\eta_r} + \frac{\gamma_2}{\eta_r} \\ & + \sum_{k_1=1}^{N}\left[\frac{\pi_{kk_1}}{2\eta_{z_1,k_1}}z_1^2 + \frac{\pi_{kk_1}}{2\eta_{\vartheta,k_1}}\tilde{\vartheta}^2 + \frac{\pi_{kk_1}}{2\eta_{\mu,k_1}}\tilde{\mu}^2\right]. \end{aligned} \tag{25}$$

In view of (24), for mode k, we choose the first stabilizing function as

$$\begin{aligned} \beta_1(k) = {} & -\left(c_1 + \frac{3}{4} + c'_1(k)\right)z_1 - \hat{\vartheta}\bar{f}_1(k,x_1) - \hat{\mu}\bar{\phi}_1(k,x_1,r) \\ & - \frac{16}{z_1}\tanh^2(z_1/\upsilon_1)\frac{\eta_{z_1,k}\bar{\xi}(x_1)}{\eta_r} \end{aligned} \tag{26}$$

where

$$c_1 > 0, \quad c'_1(k) = \frac{\eta_{z_1,k}}{2}\sum_{k_1\in S_1^k}\frac{\pi_{kk_1}}{\eta_{z_1,k_1}} + \sum_{k_1\in S_2^k}\frac{\eta_{z_1,k}}{2\eta_{z_1,k_1}}. \tag{27}$$

Let

$$\begin{aligned} \tau_{\vartheta,1}(k) &= \frac{\eta_{\vartheta,k}}{\eta_{z_1,k}}\left(z_1\bar{f}_1(k,x_1) - \sigma_\vartheta(k)\hat{\vartheta}\right) \\ \tau_{\mu,1}(k) &= \frac{\eta_{\mu,k}}{\eta_{z_1,k}}\left(z_1\bar{\phi}_1(k,x_1,r) - \sigma_\mu(k)\hat{\mu}\right) \end{aligned} \tag{28}$$

where

$$\begin{aligned} \sigma_\vartheta(k) &= \sigma_{\vartheta,0} + \eta_{z_1,k}\sum_{k_1\in S_1^k}\frac{\pi_{kk_1}}{\eta_{\vartheta,k_1}} + \sum_{k_1\in S_2^k}\frac{\eta_{z_1,k}}{\eta_{\vartheta,k_1}}, \sigma_{\vartheta,0} > 0 \\ \sigma_\mu(k) &= \sigma_{\mu,0} + \eta_{z_1,k}\sum_{k_1\in S_1^k}\frac{\pi_{kk_1}}{\eta_{\mu,k_1}} + \sum_{k_1\in S_2^k}\frac{\eta_{z_1,k}}{\eta_{\mu,k_1}}, \sigma_{\mu,0} > 0. \end{aligned} \tag{29}$$

It can be obtained that

$$\sum_{k_1=1}^{N}\frac{\pi_{kk_1}}{2\eta_{z_1,k_1}} - \frac{c'_1(k)}{\eta_{z_1,k}} = \sum_{k_1\in S_2^k}\frac{\pi_{kk_1}-1}{2\eta_{z_1,k_1}} \le 0. \tag{30}$$

Therefore, (25) becomes

$$\begin{aligned} \mathscr{L}V_1(k) \le {} & \frac{1}{\eta_{z_1,k}}\left[-c_1 z_1{}^2 + z_1 z_2\right] + \frac{1}{\eta_{\vartheta,k}}\tilde{\vartheta}\left(\dot{\hat{\vartheta}} - \tau_{\vartheta,1}(k)\right) \\ & + \frac{1}{\eta_{\mu,k}}\tilde{\mu}\left(\dot{\hat{\mu}} - \tau_{\mu,1}(k)\right) \\ & - \frac{1}{\eta_{z_1,k}}\left[\sigma_\vartheta(k)\tilde{\vartheta}\hat{\vartheta} + \sigma_\mu(k)\tilde{\mu}\hat{\mu}\right] \\ & + \sum_{k_1=1}^{N}\left[\frac{\pi_{kk_1}}{2\eta_{\vartheta,k_1}}\tilde{\vartheta}^2 + \frac{\pi_{kk_1}}{2\eta_{\mu,k_1}}\tilde{\mu}^2\right] - \frac{\gamma_0 r}{\eta_r} + \frac{\gamma_2}{\eta_r} \\ & + \frac{1}{\eta_{z_1,k}}\left[\vartheta\varepsilon_\vartheta + \mu\varepsilon_\mu + \frac{1}{2}\varepsilon_{13}^2 + \varepsilon_{11} + \varepsilon_{12} + d_{1,k}\right] \\ & + \frac{\bar{\xi}(x_1)}{\eta_r}\left(1 - 16\tanh^2(z_1/\upsilon_1)\right). \end{aligned} \tag{31}$$

It follows from $-2\tilde{\vartheta}\hat{\vartheta} \le -\tilde{\vartheta}^2 + \vartheta^2$, $-2\tilde{\mu}\hat{\mu} \le -\tilde{\mu}^2 + \mu^2$ that:

$$\begin{aligned} -\frac{\sigma_\vartheta(k)}{2\eta_{z_1,k}} + \sum_{k_1=1}^{N}\frac{\pi_{kk_1}}{2\eta_{\vartheta,k_1}} &\le -\frac{\sigma_{\vartheta,0}}{2\eta_{z_1,k}} \\ -\frac{\sigma_\mu(k)}{2\eta_{z_1,k}} + \sum_{k_1=1}^{N}\frac{\pi_{kk_1}}{2\eta_{\mu,k_1}} &\le -\frac{\sigma_{\mu,0}}{2\eta_{z_1,k}}. \end{aligned} \tag{32}$$

Therefore, it can be obtained that

$$\begin{aligned} \mathscr{L}V_1(k) = {} & -\frac{c_1 z_1^2}{\eta_{z_1,k}} + \frac{z_1 z_2}{\eta_{z_1,k}} + \frac{1}{\eta_{\vartheta,k}}\tilde{\vartheta}\left(\dot{\hat{\vartheta}} - \tau_{\vartheta,1}(k)\right) \\ & + \frac{1}{\eta_{\mu,k}}\tilde{\mu}\left(\dot{\hat{\mu}} - \tau_{\mu,1}(k)\right) - \frac{1}{2\eta_{z_1,k}}\left[\sigma_{\vartheta,0}\tilde{\vartheta}^2 + \sigma_{\mu,0}\tilde{\mu}^2\right] \\ & + \frac{\bar{\xi}(x_1)}{\eta_r}\left(1 - 16\tanh^2(z_1/\upsilon_1)\right) - \frac{\gamma_0 r}{\eta_r} + \varepsilon_1 \end{aligned} \tag{33}$$

where

$$\begin{aligned} \varepsilon_1 = {} & \left[\vartheta\varepsilon_\vartheta + \mu\varepsilon_\mu + \varepsilon_{13}^2/2 + \varepsilon_{11} + \varepsilon_{12} + d_{1,k}\right]\Big/\eta_{z_1,k} \\ & + \left[\sigma_\vartheta(k)\vartheta^2 + \sigma_\mu(k)\mu^2\right]/2\eta_{z_1,k} + \gamma_2/\eta_r. \end{aligned}$$

Step i ($2 \le i \le n-1$): Similarly, combining (1) and (14) yields

$$\begin{aligned} \dot{z}_i = {} & z_{i+1} + \beta_i(k) + \theta^{\mathrm{T}}(k,t)f_i(k,\bar{x}_i) + \psi_i(k,\bar{x}_i) \\ & + \Delta_i(k,\omega,\bar{x}_i) - \varpi_i(k) \\ & - \sum_{j=1}^{i-1}\frac{\partial\beta_{i-1}(k)}{\partial x_j}\left[\theta(k,t)^{\mathrm{T}}f_j(k,\bar{x}_j) + \psi_j(k,\bar{x}_j) + \Delta_j(k,\omega,\bar{x}_j)\right] \end{aligned} \tag{34}$$

where

$$\varpi_i(k) = \sum_{j=1}^{i-1}\left[\frac{\partial\beta_{i-1}(k)}{\partial x_j}x_{j+1} + \frac{\partial\beta_{i-1}(k)}{\partial y_r^{(j-1)}}y_r^{(j)}\right] + \frac{\partial\beta_{i-1}(k)}{\partial r}\dot{r} + \frac{\partial\beta_{i-1}(k)}{\partial\hat{\vartheta}}\dot{\hat{\vartheta}} + \frac{\partial\beta_{i-1}(k)}{\partial\hat{\mu}}\dot{\hat{\mu}}.$$

For mode k, we denote $\bar{z}_i = [z_1, z_2, \ldots, z_i]^{\mathrm{T}}$ and construct the ith Lyapunov function as

$$V_i\Big(k, \bar{z}_i, \tilde{\vartheta}, \tilde{\mu}, r\Big) = V_{i-1}\Big(k, \bar{z}_{i-1}, \tilde{\vartheta}, \tilde{\mu}, r\Big) + \frac{1}{2\eta_{z_i,k}}z_i^2 \quad (35)$$

where $V_{i-1}(k)$ satisfies

$$\begin{aligned}\mathscr{L}V_{i-1}(k) = &-\sum_{j=1}^{i-1}\left[\frac{c_jz_j^2}{\eta_{z_j,k}}\right] + \frac{z_{i-1}z_i}{\eta_{z_{i-1},k}} - \frac{\gamma_0 r}{\eta_r} + \varepsilon_{i-1}\\ &+ \frac{1}{\eta_{\vartheta,k}}\tilde{\vartheta}\Big(\dot{\hat{\vartheta}} - \tau_{\vartheta,i-1}(k)\Big) + \frac{1}{\eta_{\mu,k}}\tilde{\mu}\Big(\dot{\hat{\mu}} - \tau_{\mu,i-1}(k)\Big)\\ &- \frac{1}{2\eta_{z_1,k}}\Big[\sigma_{\vartheta,0}\tilde{\vartheta}^2 + \sigma_{\mu,0}\tilde{\mu}^2\Big]\\ &+ \frac{\bar{\xi}(x_1)}{\eta_r}\Big(1 - 16\tanh^2(z_1/\upsilon_1)\Big). \end{aligned} \quad (36)$$

Then, the infinitesimal generator of Lyapunov function $V_i(k)$ can be obtained as

$$\begin{aligned}\mathscr{L}V_i(k) = &\mathscr{L}V_{i-1}(k) + \frac{z_i}{\eta_{z_i,k}}\big[z_{i+1} + \beta_i(k) - \varpi_i(k)\big]\\ &+ \frac{z_i}{\eta_{z_i,k}}\theta^{\mathrm{T}}(k,t)\left[f_i(k,\bar{x}_i) - \sum_{j=1}^{i-1}\frac{\partial\beta_{i-1}(k)}{\partial x_j}f_j\big(k,\bar{x}_j\big)\right]\\ &+ \frac{z_i}{\eta_{z_i,k}}\left[\psi_i(k,\bar{x}_i) - \sum_{j=1}^{i-1}\frac{\partial\beta_{i-1}(k)}{\partial x_j}\psi_j\big(k,\bar{x}_j\big)\right]\\ &+ \frac{z_i}{\eta_{z_i,k}}\left[\Delta_i(k,\omega,\bar{x}_i) - \sum_{j=1}^{i-1}\frac{\partial\beta_{i-1}(k)}{\partial x_j}\Delta_j\big(k,\omega,\bar{x}_j\big)\right]\\ &+ \sum_{k_1=1}^{N}\frac{\pi_{kk_1}z_i^2}{2\eta_{z_i,k_1}}. \end{aligned} \quad (37)$$

Follow a similar line with that in step 1. According to Assumption 2, Assumption 3, Lemma 1, and Lemma 2, we have

$$\begin{aligned} z_i&\left[\Delta_i(k,\omega,\bar{x}_i) - \sum_{j=1}^{i-1}\frac{\partial\beta_{i-1}(k)}{\partial x_j}\Delta_j\big(k,\omega,\bar{x}_j\big)\right]\\ &\le z_i\bar{\varphi}_{i,1}(k,\bar{x}_i) + \varepsilon_{i,1} + z_i\bar{\varphi}_{i,2}(k,\bar{x}_i,r)\\ &\quad + \varepsilon_{i,2} + \frac{z_i^2}{4}\left[1 + \sum_{j=1}^{i-1}\left(\frac{\partial\beta_{i-1}(k)}{\partial x_j}\right)^2\right] + d_{i,k} \end{aligned} \quad (38)$$

where

$$\begin{aligned}\bar{\varphi}_{i,1}(k,\bar{x}_i) &= \frac{z_i\bar{\varphi}_{i,1,0}^2(k,\bar{x}_i)}{\sqrt{z_i^2\bar{\varphi}_{i,1,0}^2(k,\bar{x}_i) + \varepsilon_{i,1}^2}}\\ \bar{\varphi}_{i,2}(k,\bar{x}_i,r) &= \frac{z_i\bar{\varphi}_{i,2,0}^2(k,\bar{x}_i,r)}{\sqrt{z_i^2\bar{\varphi}_{i,2,0}^2(k,\bar{x}_i,r) + \varepsilon_{i,2}^2}}\\ d_{i,k} &= \sum_{j=1}^{i}\Big[\varphi_{j,2}\Big(k,\alpha_1^{-1}(2D(t))\Big)\Big]^2\\ \bar{\varphi}_{i,1,0}(k,\bar{x}_i) &= \varphi_{i,1}(k,\bar{x}_i) + \sum_{j=1}^{i-1}\left\|\frac{\partial\beta_{i-1}(k)}{\partial x_j}\right\|\varphi_{j,1}\big(k,\bar{x}_j\big)\\ \bar{\varphi}_{i,2,0}(k,\bar{x}_i,r) &= \varphi_{i,2}\Big(k,\alpha_1^{-1}(2r)\Big)\\ &\quad + \sum_{j=1}^{i-1}\left\|\frac{\partial\beta_{i-1}(k)}{\partial x_j}\right\|\varphi_{i,2}\Big(k,\alpha_1^{-1}(2r)\Big). \end{aligned} \quad (39)$$

Similar to step 1, an FLS is introduced to approximate the sum of the unknown functions for each mode. Therefore, it follows from Lemma 4 that:

$$\begin{aligned}&\left[\psi_i(k,\bar{x}_i) - \sum_{j=1}^{i-1}\frac{\partial\beta_{i-1}(k)}{\partial x_j}\psi_j\big(k,\bar{x}_j\big)\right] + \bar{\varphi}_{i,1}(k,\bar{x}_i) + \bar{\varphi}_{i,2}(k,\bar{x}_i,r)\\ &\quad = W_i^{\mathrm{T}}(k)\phi_i(k,\bar{x}_i,r) + \varepsilon_{\phi,i}(k,\bar{x}_i,r) \end{aligned} \quad (40)$$

where $\varepsilon_{\phi,i}(k,\bar{x}_i,r)$ is the FLS approximation error satisfying $|\varepsilon_{\phi,i}| \le \varepsilon_{i,3}$. Then, by using (38), (40), and Lemma 1, we can obtain that

$$\begin{aligned} z_i&\left[\psi_i(k,\bar{x}_i) - \sum_{j=1}^{i-1}\frac{\partial\beta_{i-1}(k)}{\partial x_j}\psi_j\big(k,\bar{x}_j\big)\right]\\ &+ z_i\left[\Delta_i(k,\omega,\bar{x}_i) - \sum_{j=1}^{i-1}\frac{\partial\beta_{i-1}(k)}{\partial x_j}\Delta_j\big(k,\omega,\bar{x}_j\big)\right]\\ &\le z_i\mu\bar{\phi}_i(k,\bar{x}_i,r) + \mu\varepsilon_\mu + \frac{1}{2}z_i^2 + \frac{1}{2}\varepsilon_{i,3}^2 + \varepsilon_{i,1} + \varepsilon_{i,2}\\ &\quad + \frac{z_i^2}{4}\left[1 + \sum_{j=1}^{i-1}\left(\frac{\partial\beta_{i-1}(k)}{\partial x_j}\right)^2\right] + d_{i,k} \end{aligned} \quad (41)$$

where $\bar{\phi}_i(k,\bar{x}_i,r) = ([z_i\phi_i^{\mathrm{T}}(k,\bar{x}_i,r)\phi_i(k,\bar{x}_i,r)]/[\sqrt{z_i^2\phi_i^{\mathrm{T}}(k,\bar{x}_i,r)\phi_i(k,\bar{x}_i,r) + \varepsilon_\mu^2}])$, $\varepsilon_\mu > 0$. In virtue of Lemma 1, given any $\varepsilon_\vartheta > 0$, one has

$$\begin{aligned} z_i\theta^{\mathrm{T}}(k,t)&\left[f_i(k,\bar{x}_i) - \sum_{j=1}^{i-1}\frac{\partial\beta_{i-1}(k)}{\partial x_j}f_j\big(k,\bar{x}_j\big)\right]\\ &\le \vartheta z_i\bar{f}_i(k,\bar{x}_i) + \vartheta\varepsilon_\vartheta \end{aligned} \quad (42)$$

where

$$\begin{aligned}\bar{f}_i(k,\bar{x}_i) &= \frac{z_i\bar{f}_{i,0}(k,\bar{x}_i)^{\mathrm{T}}\bar{f}_{i,0}(k,\bar{x}_i)}{\sqrt{z_i^2\bar{f}_{i,0}(k,\bar{x}_i)^{\mathrm{T}}\bar{f}_{i,0}(k,\bar{x}_i) + \varepsilon_\vartheta^2}}\\ \bar{f}_{i,0}(k,\bar{x}_i) &= f_i(k,\bar{x}_i) - \sum_{j=1}^{i-1}\frac{\partial\beta_{i-1}(k)}{\partial x_j}f_j\big(k,\bar{x}_j\big). \end{aligned}$$

In view of (37)–(42), we have

$$\begin{aligned}\mathscr{L}V_i(k) &= \mathscr{L}V_{i-1}(k) + \frac{1}{\eta_{z_i,k}}\\ &\quad\times\Bigg[z_i z_{i+1} + z_i\beta_i(k) - z_i\varpi_i(k) + \vartheta z_i\bar{f}_i(k,\bar{x}_i) + \vartheta\varepsilon_\vartheta\\ &\qquad + z_i\mu\bar{\phi}_i(k,\bar{x}_i,r) + \mu\varepsilon_\mu + \frac{1}{2}z_i^2 + \frac{1}{2}\varepsilon_{i,3}^2 + \varepsilon_{i,1}\\ &\qquad + \varepsilon_{i,2} + \frac{z_i^2}{4}\left[1 + \sum_{j=1}^{i-1}\left(\frac{\partial\beta_{i-1}(k)}{\partial x_j}\right)^2\right] + d_{i,k}\Bigg]\\ &\quad + \sum_{k_1=1}^{N}\frac{\pi_{kk_1}z_i^2}{2\eta_{z_i,k_1}}. \end{aligned}\tag{43}$$

Considering (43), we select the ith virtual control signal as

$$\begin{aligned}\beta_i(k) &= -\big(c_i + c'_i(k)\big)z_i + \varpi_i(k) - \frac{z_i}{2}\\ &\quad - \frac{z_i}{4}\left[1 + \sum_{j=1}^{i-1}\left(\frac{\partial\beta_{i-1}(k)}{\partial x_j}\right)^2\right] - \frac{\eta_{z_i,k}}{\eta_{z_{i-1},k}}z_{i-1}\\ &\quad - \hat{\vartheta}\bar{f}_i(k,\bar{x}_i) - \hat{\mu}\bar{\phi}_i(k,\bar{x}_i,r)\end{aligned}\tag{44}$$

where

$$c'_i(k) = \frac{\eta_{z_i,k}}{2}\sum_{k_1\in S_1^k}\frac{\pi_{kk_1}}{\eta_{z_i,k_1}} + \sum_{k_1\in S_2^k}\frac{\eta_{z_i,k}}{2\eta_{z_i,k_1}}.$$

It is easy to know that

$$\sum_{k_1=1}^{N}\frac{\pi_{kk_1}}{2\eta_{z_i,k_1}} - \frac{c'_i(k)}{\eta_{z_i,k}} = \sum_{k_1\in S_2^k}\frac{\pi_{kk_1}-1}{2\eta_{z_i,k_1}} \le 0.\tag{45}$$

Let

$$\begin{aligned}\tau_{\vartheta,i}(k) &= \tau_{\vartheta,i-1}(k) + \frac{\eta_{\vartheta,k}}{\eta_{z_i,k}}z_i\bar{f}_i(k,\bar{x}_i)\\ \tau_{\mu,i}(k) &= \tau_{\mu,i-1}(k) + \frac{\eta_{\mu,k}}{\eta_{z_i,k}}z_i\bar{\phi}_i(k,\bar{x}_i,r).\end{aligned}\tag{46}$$

Combining (44)–(46), one has

$$\begin{aligned}\mathscr{L}V_i(k) &= -\sum_{j=1}^{i}\left[\frac{c_j z_j^2}{\eta_{z_j,k}}\right] + \frac{z_i z_{i+1}}{\eta_{z_i,k}} + \frac{1}{\eta_{\vartheta,k}}\tilde{\vartheta}\left(\dot{\hat{\vartheta}} - \tau_{\vartheta,i}(k)\right)\\ &\quad + \frac{1}{\eta_{\mu,k}}\tilde{\mu}\left(\dot{\hat{\mu}} - \tau_{\mu,i}(k)\right) - \frac{1}{2\eta_{z_1,k}}\left[\sigma_{\vartheta,0}\tilde{\vartheta}^2 + \sigma_{\mu,0}\tilde{\mu}^2\right]\\ &\quad + \frac{\bar{\xi}(x_1)}{\eta_r}\left(1 - 16\tanh^2(z_1/\upsilon_1)\right) - \frac{\gamma_0 r}{\eta_r} + \varepsilon_i\end{aligned}\tag{47}$$

where $\varepsilon_i = \varepsilon_{i-1} + [\vartheta\varepsilon_\vartheta + \mu\varepsilon_\mu + \varepsilon_{i,3}^2/2 + \varepsilon_{i,1} + \varepsilon_{i,2} + d_{i,k}]/\eta_{z_i,k}$. It should be highlighted that for mode k, the virtual control law $\beta_i(k)$ is a smooth function of $\bar{x}_i$, r, $\dot{\hat{\vartheta}}$, and $\dot{\hat{\mu}}$.

Step n: From Assumption 1, it is easy to know that there exists an upper bound for $\delta_j(k,t)$. Before proceeding, we define

$$\bar{\delta} = \sup_{1\le j\le m, k\in S, t\ge 0}\left\|\delta_j(k,t)\right\|.\tag{48}$$

Meanwhile, we employ $\hat{\bar{\delta}}$ and $\hat{H}$ to denote the estimations of $\bar{\delta}$ and H. $\tilde{\bar{\delta}} = \hat{\bar{\delta}} - \bar{\delta}$, $\tilde{H} = \hat{H} - H$. Moreover, positive scalars $\eta_{\bar{\delta},k}$ and $\eta_{H,k}$ are employed as parameters to be designed. Then, combining (1) and (14) yields

$$\begin{aligned}\dot{z}_n &= \theta^{\mathrm{T}}(k,t)f_n(k,x) - \sum_{j=1}^{n-1}\frac{\partial\beta_{n-1}(k)}{\partial x_j}\theta^{\mathrm{T}}(k,t)f_j\big(k,\bar{x}_j\big)\\ &\quad - \varpi_n(k) + \psi_n(k,x) - \sum_{j=1}^{n-1}\frac{\partial\beta_{n-1}(k)}{\partial x_j}\psi_j\big(k,\bar{x}_j\big)\\ &\quad + \Delta_n(k,\omega_n,x) - \sum_{j=1}^{n-1}\frac{\partial\beta_{n-1}(k)}{\partial x_j}\Delta_j\big(k,\omega_j,\bar{x}_j\big)\\ &\quad + \sum_{j=1}^{m}b_j(k,x)h_j(k)v_j(t) + \sum_{j=1}^{m}b_j(k,x)\delta_j(k)\end{aligned}\tag{49}$$

where

$$\begin{aligned}\varpi_n(k) &= \sum_{j=1}^{n-1}\left[\frac{\partial\beta_{n-1}(k)}{\partial x_j}x_{j+1} + \frac{\partial\beta_{n-1}(k)}{\partial y_r^{(j-1)}}y_r^{(j)}\right] + \frac{\partial\beta_{n-1}(k)}{\partial r}\dot{r}\\ &\quad + \frac{\partial\beta_{n-1}(k)}{\partial\hat{\vartheta}}\dot{\hat{\vartheta}} + \frac{\partial\beta_{n-1}(k)}{\partial\hat{\mu}}\dot{\hat{\mu}} + y_r^{(n)}.\end{aligned}$$

Accordingly, we construct the following Lyapunov function:

$$\begin{aligned}V_n\Big(k,\bar{z}_n,\tilde{\vartheta},\tilde{\mu},\tilde{\bar{\delta}},\tilde{H},r\Big) &= V_{n-1}\Big(k,\bar{z}_{n-1},\tilde{\vartheta},\tilde{\mu},r\Big) + \frac{1}{2\eta_{z_n,k}}z_n^2\\ &\quad + \frac{1}{2\eta_{\bar{\delta},k}}\tilde{\bar{\delta}}^2 + \frac{1}{2H\eta_{H,k}}\tilde{H}^2.\end{aligned}\tag{50}$$

It can be obtained that

$$\begin{aligned}\mathscr{L}V_n(k) &= \mathscr{L}V_{n-1}(k) + \frac{z_n}{\eta_{z_n,k}}\left[\sum_{j=1}^{m}b_j(k,x)h_j(k)v_j(t) - \varpi_n(k)\right]\\ &\quad + \frac{z_n}{\eta_{z_n,k}}\theta^{\mathrm{T}}(k,t)\left[f_n(k,x) - \sum_{j=1}^{n-1}\frac{\partial\beta_{n-1}(k)}{\partial x_j}f_j\big(k,\bar{x}_j\big)\right]\\ &\quad + \frac{z_n}{\eta_{z_n,k}}\left[\Delta_n(k,\omega,x) - \sum_{j=1}^{n-1}\frac{\partial\beta_{n-1}(k)}{\partial x_j}\Delta_j\big(k,\omega,\bar{x}_j\big)\right]\\ &\quad + \frac{z_n}{\eta_{z_n,k}}\left[\psi_n(k,x) - \sum_{j=1}^{n-1}\frac{\partial\beta_{n-1}(k)}{\partial x_j}\psi_j\big(k,\bar{x}_j\big)\right]\\ &\quad + \frac{z_n}{\eta_{z_n,k}}\sum_{j=1}^{m}b_j(k,x)\delta_j(k) + \frac{1}{\eta_{\bar{\delta},k}}\tilde{\bar{\delta}}\dot{\hat{\bar{\delta}}} + \frac{1}{H\eta_{H,k}}\tilde{H}\dot{\hat{H}}\\ &\quad + \sum_{k_1=1}^{N}\left[\frac{\pi_{kk_1}}{2\eta_{z_n,k_1}}z_n^2 + \frac{\pi_{kk_1}}{2\eta_{\bar{\delta},k_1}}\tilde{\bar{\delta}}^2 + \frac{\pi_{kk_1}}{2H\eta_{H,k_1}}\tilde{H}^2\right].\end{aligned}\tag{51}$$

By using the same arguments as in step 1, it follows from Assumption 2, Assumption 3, Lemma 1, and Lemma 2 that:

$$\begin{aligned}&z_n\left[\Delta_n(k,\omega,x) - \sum_{j=1}^{n-1}\frac{\partial\beta_{n-1}(k)}{\partial x_j}\Delta_j\big(k,\omega,\bar{x}_j\big)\right]\\ &\quad\le z_n\bar{\varphi}_{n,1}(k,x) + \varepsilon_{n,1} + z_n\bar{\varphi}_{n,2}(k,x,r)\\ &\qquad + \varepsilon_{n,2} + \frac{z_n^2}{4}\left[1 + \sum_{j=1}^{n-1}\left(\frac{\partial\beta_{n-1}(k)}{\partial x_j}\right)^2\right] + d_{n,k}\end{aligned}\tag{52}$$

where $\bar{\varphi}_{n,1}(k,x), \bar{\varphi}_{n,2}(k,x,r), d_{n,k}$ are defined in (39) with $i = n$. Therefore, we introduce the FLSs such that

$$\left[\psi_n(k,x) - \sum_{j=1}^{n-1}\frac{\partial\beta_{n-1}(k)}{\partial x_j}\psi_j\big(k,\bar{x}_j\big)\right] + \bar{\varphi}_{n,1}(k,x) + \bar{\varphi}_{n,2}(k,x,r) = W_n{}^{\mathrm{T}}(k)\phi_n(k,x,r) + \varepsilon_{\psi,n}(k,x,r) \tag{53}$$

where $\varepsilon_{\psi,n}(k,x,r)$ is the FLS approximation error satisfying $|\varepsilon_{\psi,n}| \le \varepsilon_{n,3}$. By using Lemma 1, it can be obtained that

$$\begin{aligned} z_n&\left[\psi_n(k,x_n) - \sum_{j=1}^{n-1}\frac{\partial\beta_{n-1}(k)}{\partial x_j}\psi_j\big(k,\bar{x}_j\big)\right] \\ &+ z_n\left[\Delta_n(k,\omega,x) - \sum_{j=1}^{n-1}\frac{\partial\beta_{n-1}(k)}{\partial x_j}\Delta_j\big(k,\omega,\bar{x}_j\big)\right] \\ &\le z_n\mu\bar{\phi}_n(k,x,r) + \mu\varepsilon_\mu + \frac{1}{2}z_n^2 + \frac{1}{2}\varepsilon_{n,3}^2 + \varepsilon_{n,1} + \varepsilon_{n,2} \\ &+ \frac{z_n^2}{4}\left[1 + \sum_{j=1}^{n-1}\left(\frac{\partial\beta_{n-1}(k)}{\partial x_j}\right)^2\right] + d_{n,k} \end{aligned} \tag{54}$$

where $\bar{\phi}_n(k,x,r)$ is defined in (39) with $i = n$, $\varepsilon_\mu > 0$. In virtue of Lemma 1, given any $\varepsilon_\vartheta > 0$, $\varepsilon_{\bar{\delta}} > 0$, we have

$$\begin{aligned} z_n\theta^{\mathrm{T}}(k,t)&\left[f_n(k,x) - \sum_{j=1}^{n-1}\frac{\partial\beta_{n-1}(k)}{\partial x_j}f_j\big(k,\bar{x}_j\big)\right] \\ &\le \vartheta z_n\bar{f}_n(k,x) + \vartheta\varepsilon_\vartheta z_n\sum_{j=1}^{m} b_j(k,x)\delta_j(k) \le \bar{\delta}z_n\bar{b}(k,x) + \bar{\delta}\varepsilon_{\bar{\delta}} \end{aligned} \tag{55}$$

where

$$\bar{f}_n(k,x) = \frac{z_n\bar{f}_{n,0}^{\mathrm{T}}(k,x)\bar{f}_{n,0}(k,x)}{\sqrt{z_n^2\bar{f}_{n,0}^{\mathrm{T}}(k,x)\bar{f}_{n,0}(k,x) + \varepsilon_\vartheta^2}}$$

$$\bar{b}(k,x) = \frac{z_n\sum\limits_{j=1}^{m} b_j^2(k,x)}{\sqrt{z_n^2\sum\limits_{j=1}^{m} b_j^2(k,x) + \varepsilon_{\bar{\delta}}^2}}$$

$$\bar{f}_{n,0}(k,x) = f_n(k,x) - \sum_{j=1}^{n-1}\frac{\partial\beta_{n-1}(k)}{\partial x_j}f_j\big(k,\bar{x}_j\big).$$

Considering (51)–(55), we have

$$\begin{aligned} \mathscr{L}V_n(k) = &\mathscr{L}V_{n-1}(k) + \frac{1}{\eta_{\bar{\delta},k}}\tilde{\bar{\delta}}\dot{\hat{\bar{\delta}}} + \frac{1}{H\eta_{H,k}}\tilde{H}\dot{\hat{H}} \\ &+ \frac{1}{\eta_{z_n,k}}\left[-z_n\beta_n(k) + z_n\sum_{j=1}^{m} b_j(k,x)h_j(k)v_j(t)\right] \\ &+ \frac{1}{\eta_{z_n,k}}\left[\begin{array}{l} \vartheta z_n\bar{f}_n(k,x) + z_n\mu\bar{\phi}_n(k,x,r) + \bar{\delta}z_n\bar{b}(k,x) \\ +\frac{1}{2}z_n^2 + \frac{z_n^2}{4}\left[1 + \sum\limits_{j=1}^{n-1}\left(\frac{\partial\beta_{n-1}(k)}{\partial x_j}\right)^2\right] \\ +z_n\beta_n(k) - z_n\varpi_n(k) \end{array}\right] \\ &+ \frac{1}{\eta_{z_n,k}}\left[\vartheta\varepsilon_\vartheta + \mu\varepsilon_\mu + \bar{\delta}\varepsilon_{\bar{\delta}} + \frac{1}{2}\varepsilon_{n,3}^2 + \varepsilon_{n,1} + \varepsilon_{n,2} + d_{n,k}\right] \\ &+ \sum_{k_1=1}^{N}\left[\frac{\pi_{kk_1}}{2\eta_{z_n,k_1}}z_n^2 + \frac{\pi_{kk_1}}{2\eta_{\bar{\delta},k_1}}\tilde{\bar{\delta}}^2 + \frac{\pi_{kk_1}}{2H\eta_{H,k_1}}\tilde{H}^2\right]. \end{aligned} \tag{56}$$

Accordingly, we select the following virtual control signal for mode k:

$$\begin{aligned} \beta_n(k) = &-\big(c_n + c_n'(k)\big)z_n + \varpi_n(k) - \frac{1}{2}z_n \\ &- \frac{1}{4}z_n\left[1 + \sum_{j=1}^{n-1}\left(\frac{\partial\beta_{n-1}(k)}{\partial x_j}\right)^2\right] - \frac{\eta_{z_n,k_1}}{\eta_{z_{n-1},k_1}}z_{n-1} \\ &- \hat{\vartheta}\bar{f}_n(k,x) - \hat{\mu}\bar{\phi}_n(k,x,r) - \hat{\bar{\delta}}\bar{b}(k,x) \end{aligned} \tag{57}$$

where

$$c_n'(k) = \frac{\eta_{z_n,k}}{2}\sum_{k_1\in S_1^k}\frac{\pi_{kk_1}}{\eta_{z_n,k_1}} + \sum_{k_1\in S_2^k}\frac{\eta_{z_n,k}}{2\eta_{z_n,k_1}}. \tag{58}$$

Let

$$\begin{aligned} \tau_{\vartheta,n}(k) &= \tau_{\vartheta,n-1}(k) + \frac{\eta_{\vartheta,k}}{\eta_{z_n,k}}z_n\bar{f}_n(k,x) \\ \tau_{\mu,n}(k) &= \tau_{\mu,n-1}(k) + \frac{\eta_{\mu,k}}{\eta_{z_n,k}}z_n\bar{\phi}_n(k,x,r) \\ \tau_{\bar{\delta}}(k) &= \frac{\eta_{\bar{\delta},k}}{\eta_{z_n,k}}\Big(z_n\bar{b}(k,x) - \sigma_{\bar{\delta}}(k)\hat{\bar{\delta}}\Big) \end{aligned} \tag{59}$$

where

$$\sigma_{\bar{\delta}}(k) = \sigma_{\bar{\delta},0} + \eta_{z_n,k}\sum_{k_1\in S_1^k}\frac{\pi_{kk_1}}{\eta_{\bar{\delta},k_1}} + \sum_{k_1\in S_2^k}\frac{\eta_{z_n,k}}{\eta_{\bar{\delta},k_1}},\ \sigma_{\bar{\delta},0} > 0. \tag{60}$$

It is not difficult to see that

$$\begin{aligned} &\sum_{k_1=1}^{N}\frac{\pi_{kk_1}}{2\eta_{z_n,k_1}} - \frac{c_n'(k)}{\eta_{z_n,k}} \le 0 \\ &-\frac{\sigma_{\bar{\delta}}(k)}{2\eta_{z_n,k}} + \sum_{k_1=1}^{N}\frac{\pi_{kk_1}}{2\eta_{\bar{\delta},k_1}} \le -\frac{\sigma_{\bar{\delta},0}}{2\eta_{z_n,k}}. \end{aligned} \tag{61}$$

Therefore, by combining (57)–(60), we have

$$\begin{aligned} \mathscr{L}V_n(k) \le &-\sum_{j=1}^{n}\left[\frac{c_jz_j^2}{\eta_{z_j,k}}\right] \\ &+ \frac{1}{\eta_{z_n,k}}\left[-z_n\beta_n(k) + z_n\sum_{j=1}^{m} b_j(k,x)h_j(k)v_j(t)\right] \\ &+ \frac{1}{\eta_{\vartheta,k}}\tilde{\vartheta}\Big(\dot{\hat{\vartheta}} - \tau_{\vartheta,n}(k)\Big) + \frac{1}{\eta_{\mu,k}}\tilde{\mu}\Big(\dot{\hat{\mu}} - \tau_{\mu,n}(k)\Big) \\ &+ \frac{1}{\eta_{\bar{\delta},k}}\tilde{\bar{\delta}}\Big(\dot{\hat{\bar{\delta}}} - \tau_{\bar{\delta}}(k)\Big) - \frac{1}{2\eta_{z_1,k}}\Big[\sigma_{\vartheta,0}\tilde{\vartheta}^2 + \sigma_{\mu,0}\tilde{\mu}^2\Big] \\ &- \frac{1}{2\eta_{z_n,k}}\Big[\sigma_{\bar{\delta},0}\tilde{\bar{\delta}}^2\Big] + \frac{1}{H\eta_{H,k}}\tilde{H}\dot{\hat{H}} + \sum_{k_1=1}^{N}\frac{\pi_{kk_1}\tilde{H}^2}{2H\eta_{H,k_1}} \\ &+ \frac{1}{\eta_{z_n,k}}\Big[\vartheta\varepsilon_\vartheta + \mu\varepsilon_\mu + \bar{\delta}\varepsilon_{\bar{\delta}} + \frac{1}{2}\varepsilon_{n,3}^2 + \varepsilon_{n,1} + \varepsilon_{n,2} \\ &\qquad + d_{n,k} + \frac{1}{2}\sigma_{\bar{\delta}}(k)\bar{\delta}^2\Big] \\ &+ \frac{\bar{\xi}(x_1)}{\eta_r}\Big(1 - 16\tanh^2(z_1/\upsilon_1)\Big) - \frac{\gamma_0 r}{\eta_r} + \varepsilon_{n-1}. \end{aligned} \tag{62}$$

The final control signal is

$$v_j(k,t) = -\frac{z_n \hat{H}^2 \beta_n^2(k)}{b_j(k,x)\sqrt{z_n^2 \hat{H}^2 \beta_n^2(k) + \varepsilon_H^2}} \tag{63}$$

where $\varepsilon_v > 0$. Considering that $h_j(k) > 0$, we have

$$\begin{aligned} z_n \sum_{j=1}^{m} b_j(k,x) h_j(k) v_j(t) &\leq -\frac{z_n^2 \hat{H}^2 \beta_n^2(k)}{H\sqrt{z_n^2 \hat{H}^2 \beta_n^2(k) + \varepsilon_H^2}} \\ &\leq \frac{-\left\| z_n \hat{H} \beta_n(k) \right\| + \varepsilon_H}{H} \\ &\leq \frac{z_n \hat{H} \beta_n(k) + \varepsilon_H}{H}. \end{aligned} \tag{64}$$

Therefore

$$z_n \left[-\beta_n(k) + \sum_{j=1}^{m} b_j(k,x) h_j(k) v_j(t) \right] \leq \frac{z_n \tilde{H} \beta_n(k) + \varepsilon_H}{H}. \tag{65}$$

Select

$$\tau_H(k) = -\frac{\eta_{H,k}}{\eta_{z_n,k}} \Big(z_n \beta_n(k) + \sigma_H(k) \hat{H} \Big) \tag{66}$$

where

$$\sigma_H(k) = \sigma_{H,0} + \eta_{z_n,k} \sum_{k_1 \in S_1^k} \frac{\pi_{kk_1}}{\eta_{H,k_1}} + \sum_{k_1 \in S_2^k} \frac{\eta_{z_n,k}}{\eta_{H,k_1}}, \sigma_{H,0} > 0. \tag{67}$$

Then, the adaptive laws are selected as

$$\begin{aligned} &\dot{\hat{\vartheta}}(k) = \tau_{\vartheta,n}(k), \dot{\hat{\mu}}(k) = \tau_{\mu,n}(k) \\ &\dot{\hat{\bar{\delta}}}(k) = \tau_{\bar{\delta}}(k), \dot{\hat{H}}(k) = \tau_H(k). \end{aligned} \tag{68}$$

Therefore, we have

$$\begin{aligned} \mathscr{L}V_n(k) \leq &-\sum_{j=1}^{n} \left[\frac{c_j z_j^2}{\eta_{z_j,k}} \right] - \frac{1}{2\eta_{z_1,k}} \Big[\sigma_{\vartheta,0} \tilde{\vartheta}^2 + \sigma_{\mu,0} \tilde{\mu}^2 \Big] \\ &- \frac{1}{2\eta_{z_n,k}} \left[\sigma_{\bar{\delta},0} \tilde{\bar{\delta}}^2 + \frac{\sigma_{H,0} \tilde{H}^2}{H} \right] \\ &+ \frac{\bar{\xi}(x_1)}{\eta_r} \Big(1 - 16\tanh^2(z_1/\upsilon_1) \Big) - \frac{\gamma_0 r}{\eta_r} + \varepsilon_n \end{aligned} \tag{69}$$

where

$$\begin{aligned} \varepsilon_n = \frac{1}{\eta_{z_n,k}} \bigg[&\vartheta \varepsilon_\vartheta + \mu \varepsilon_\mu + \bar{\delta} \varepsilon_{\bar{\delta}} + \frac{1}{2} \varepsilon_{n,3}^2 + \varepsilon_{n,1} + \varepsilon_{n,2} \\ &+ d_{n,k} + \frac{1}{2} \sigma_{\bar{\delta}}(k) \bar{\delta}^2 + \varepsilon_H / H + \frac{1}{2} \sigma_H(k) H \bigg] + \varepsilon_{n-1}. \end{aligned}$$

Remark 2: It should be noticed that this paper differs from [27] and [36] mainly due to the additional term induced by the Markovian characteristics. Let us take ϑ as an example. The additional term $\sum_{k_1=1}^{N} (\pi_{kk_1}/2\eta_{\vartheta,k_1}) \tilde{\vartheta}^2$ makes it difficult for the conventional adaptive gains to be applied. By using the redesigned gains (29), (60), and (67), this problem can be handled effectively.

Remark 3: In the considered system model, the tracking control problem is much more difficult than the tracking control problem because a nonlinear term cannot be eliminated directly. According to (24), if we utilize $-\eta_{z_1,k}\bar{\xi}(x_1)/(\eta_r z_1)$ in the virtual control law, a singular phenomenon will appear at $z_1 = 0$. To address this problem, we introduce the function $\tanh(z_1/\upsilon_1)$ satisfying $\lim_{z_1 \to 0}([\tanh(z_1/\upsilon_1)]/z_1) = 1$ such that the singular phenomenon can be avoided.

Remark 4: Note that even if all elements of the transition rate matrix are unavailable, the proposed approach is still applicable, while the results in recent works [2], [6], [11] are not. For example, see step 1 if the transition rate matrix is completely unknown, according to the proposed method, we can obtain that $c_1'(k) = \sum_{k_1=1}^{N} (\eta_{z_1,k}/2\eta_{z_1,k_1})$, $\sigma_\vartheta(k) = \sigma_{\vartheta,0} + \sum_{k_1=1}^{N} (\eta_{z_1,k}/\eta_{\vartheta,k_1})$, $\sigma_\mu(k) = \sigma_{\mu,0} + \sum_{k_1=1}^{N} (\eta_{z_1,k}/\eta_{\mu,k_1})$. It is apparent that (30) and (32) can still be fulfilled such that (33) holds. In a similar way, we have (69).

Remark 5: It should be noted that the variables can be unified for all Markovian modes by estimating their bounds. Take θ_k as an example. An additional term $\sum_{k_1=1}^{N} (\pi_{kk_1}/2\eta_{\theta,k_1}) \tilde{\theta}_{k_1}^2$ in $\mathscr{L}V(k)$ cannot be compensated when we estimate θ_k directly ($\tilde{\theta}_{k_1}$ denotes the estimation error of θ_{k_1}). By estimating the upper bounds of the parameters, this problem can be handled. (Taking θ_k and θ_{k_1} as examples, we can obtain a unified variable ϑ.)

Remark 6: It should be pointed out that the unknown parameters and the known functions in [10]–[14] are not assumed to be randomly switching. This paper makes the first attempt to deal with the tracking control problem for the nonlinear systems with the Markovian switching unknown parameters, the unmodeled dynamics, and the actuator failures.

Remark 7: Note that r is a positive signal. Since $\gamma_1 > \gamma_0 > 0$ and $r_0 > 0$, by solving (10), we know that $r > 0$ holds for $t > 0$. Next, the reason why $c_n'(k)$ can be calculated straightly is explained. In (58), $c_n'(k)$ is defined as

$$c_n'(k) = \frac{\eta_{z_n,k}}{2} \sum_{k_1 \in S_1^k} \frac{\pi_{kk_1}}{\eta_{z_n,k_1}} + \sum_{k_1 \in S_2^k} \frac{\eta_{z_n,k}}{2\eta_{z_n,k_1}}.$$

According to (5), for π_{kk_1}, we can be obtain that

$$\begin{aligned} S_1^k &\triangleq \big\{ k_1 : \pi_{kk_1} \text{ is known} \big\} \\ S_2^k &\triangleq \big\{ k_1 : \pi_{kk_1} \text{ is unknown} \big\}. \end{aligned}$$

Hence, if $k_1 \in S_1^k$, then π_{kk_1} is available, which is used in (58). If $k_1 \in S_2^k$, then π_{kk_1} is unavailable, which is not required to calculate $c_n'(k)$. By dividing the Markovian modes according to knowability, the novel gains are designed as (58), where all information is available.

Remark 8: In this paper, one of the main difficulties is the intrinsic Markovian switching characteristics of the unknown actuator faults. It is difficult to estimate the unknown parameters which keep randomly changing. Instead of estimating the Markovian switching parameters directly, several adaptive laws are employed to estimate upper bounds of the parameters for the deviation fault and the loss of effectiveness. As a result, the obstacles caused by the Markovian jumping actuator failures can be circumvented.

Remark 9: This paper differs from [2] because of the following.

1) In this paper, the fault-tolerant control problem of the Markov nonlinear system is studied, and the fault-tolerant control structure is developed. However, [2] does not take the actuator failures into consideration. Due to the random transition characteristics of the actuator faults, the problem studied in this paper is more challenging.
2) In [2], it is assumed that the transition rate matrix of the Markov system is fully known. However, in this paper, it is assumed that the elements in the transition rate matrix can be unknown, and the corresponding adaptive control law is designed.
3) It must be pointed out that, unlike the nonlinear systems without the consideration of the unmodeled dynamics, the design procedures of the stabilization problem and tracking control problem are quite different. Most of these papers, such as [30] and [31], restrict their attention on the stabilization problem. This constitutes one of the motivations for the current investigation.

Theorem 1: Under Assumptions 1–3, consider the strict-feedback MJNS (1) with control signals (63) and the parameter update laws (68). Suppose that the initial conditions are bounded. Then, all of the signals of the closed-loop system are bounded in probability and the output y can converge to any given vicinity of the target signal in probability.

Proof:

Case 1 ($z_1 \notin \Omega_{\upsilon_1}$): Based on the results obtained in Section III, according to Lemma 3, we have

$$\begin{aligned}\mathscr{L}V_n(\chi(t), r_t = k) \le &-\sum_{j=1}^{n}\left[\frac{c_j z_j^2}{\eta_{z_j,k}}\right]\\ &-\frac{1}{2\eta_{z_1,k}}\left[\sigma_{\vartheta,0}\tilde{\vartheta}^2 + \sigma_{\mu,0}\tilde{\mu}^2\right]\\ &-\frac{1}{2\eta_{z_n,k}}\left[\sigma_{\bar{\delta},0}\tilde{\bar{\delta}}^2 + \frac{\sigma_{H,0}\tilde{H}^2}{H}\right]\\ &-\frac{\gamma_0 r}{\eta_r} + \varepsilon_n\end{aligned} \tag{70}$$

where $\chi = [\bar{z}_n^{\mathrm{T}}, \tilde{\vartheta}, \tilde{\mu}, \tilde{\bar{\delta}}, \tilde{H}, r]^{\mathrm{T}}$. Define

$$c = \min_{1\le i\le n, k\in S}\left\{2c_i, \frac{\sigma_{\vartheta,0}\eta_{\vartheta,k}}{\eta_{z_1,k}}, \frac{\sigma_{\mu,0}\eta_{\mu,k}}{\eta_{z_1,k}}, \frac{\sigma_{\bar{\delta},0}\eta_{\bar{\delta},k}}{\eta_{z_n,k}}, \frac{\sigma_{H,0}\eta_{H,k}}{\eta_{z_n,k}}, \gamma_0\right\}. \tag{71}$$

Equation (70) becomes

$$\mathscr{L}V_n(\chi(t), r_t) \le -cV_n(\chi(t), r_t) + \varepsilon_n. \tag{72}$$

Then, by using Dynkin's formula, it can be proven that

$$\mathbf{E}\{V_n(\chi(t), r_t)\} - V_n(\chi(0), r_0) = \mathbf{E}\left\{\int_0^t \mathscr{L}V_n(\chi(s), r_s)ds\right\}. \tag{73}$$

Accordingly, the following inequality holds:

$$\frac{d\mathbf{E}\{V_n(\chi(t), r_t)\}}{dt} \le -c\mathbf{E}\{V_n(\chi(t), r_t)\} + \varepsilon_n. \tag{74}$$

From (74), we can obtain

$$\mathbf{E}\{V_n(\chi(t), r_t)\} \le V_n(\chi(0), r_0)e^{-ct} + \frac{\varepsilon_n}{c}. \tag{75}$$

TABLE I
PARAMETERS OF EXAMPLE 1 FOR THE FOUR MODES

	Mode 1	Mode 2	Mode 3	Mode 4
θ	2	3	2.5	2.2
f_1	$x_1^2 e^{-0.5x_1}$	$2x_1^2 e^{-0.5x_1}$	$x_1^2 e^{-x_1}$	$x_1^2 e^{-0.7x_1}$
ψ_1	$0.1x_1^2 sin(x_1)$	$0.1 sin(x_1)$	$0.1x_1$	$0.1x_1 \sin(x_1^2)$
Δ_1	$\omega \sin x_1$	$2\omega \sin x_1$	$0.5\omega \sin x_1$	$\omega \sin 2x_1$
f_2	$x_1x_2^2 + x_1^2$	$2x_1x_2^2 + x_1^2$	$1.5x_1x_2^2 + x_1^2$	$x_1x_2^2 + 0.5x_1^2$
ψ_2	$0.1(x_1 + x_2)$	$0.1x_1 \sin(x_2)$	$0.1x_2^2$	$0.1x_2 \sin(x_2^2)$
Δ_2	$\omega \cos x_1x_2$	$2\omega \cos x_1x_2$	$\omega \cos(2x_1x_2)$	$\omega \cos(x_1^2x_2)$
h	0.3	0.4	0.5	0.6
δ	0.85	-0.3	0.6	0.45

Let $V_R = \inf_{\|\chi(t) \ge R, t\ge 0\|} V_n(\chi(t), r_t)$. Then from [43, Lemma 1.4], it can be checked that

$$\begin{aligned}\mathbf{P}\{\|\chi(t) \ge R\|\} &\le \frac{\mathbf{E}\{V_n(\chi(t), r_t)\}}{V_R}\\ &\le \frac{V_n(\chi(0), r_0)e^{-ct}}{V_R} + \frac{\varepsilon_n}{cV_R}.\end{aligned} \tag{76}$$

Considering $\lim_{R\to\infty} V_R \to \infty$, we can arrive at the conclusion that $\chi(t)$ is bounded in probability. Meanwhile, it can be observed that $\lim_{t\to\infty}\mathbf{P}\{\|\chi(t) \ge R\|\} \le \varepsilon_n/(cV_R)$ for $R > 0$.

Case 2 ($z_1 \in \Omega_{\upsilon_1}$): In this case, $|z_1| \le 0.2554\upsilon_1$ and, hence, z_1 is bounded. Therefore, for bounded z_1 and x_1, it is obvious that r in (10), $\beta_1(k)$ in (26), and $\tau_{\vartheta,1}(k)$, $\tau_{\mu,1}(k)$ in (28) are bounded. That is to say, z_2 in (14) is also bounded. Then, by analyzing the boundedness of the signals step by step, we know that for $1 \le i \le n-1$, $\beta_i(k)$, $\tau_{\vartheta,i}(k)$, $\tau_{\mu,i}(k)$, $z_i(k)$, and $x_i(k)$ are all bounded signals. For step n, it is easy to know that $\dot{\hat{\vartheta}}(k)$, $\dot{\hat{\mu}}(k)$, $\dot{\hat{\bar{\delta}}}(k)$, $\dot{\hat{H}}(k)$, $\beta_n(k)$, z_n, x_n, and $v_j(k,t), j = 1, \ldots, m$ are also bounded. Note that the bound for z_1 can be made arbitrarily small by appropriately adjusting υ_1.

Therefore, from the discussions in cases 1 and 2, it can be concluded that all signals of the closed-loop system are bounded in probability, and the output signal can converge to any given vicinity of the target signal in probability. The proof is completed. ■

IV. SIMULATIONS

In this section, two simulation examples are provided to verify the effectiveness and advantages of the proposed method.

Example 1: According to (1), we consider a second-order nonlinear plant which keeps switching in four modes and contains the randomly jumping parameters and functions given by Table I. The unmodeled dynamic equation is $\dot{\omega} = -\omega + x_1^2 + 0.5$. The target signal y_r satisfies that $\ddot{y}_r + 2\dot{y}_r + y_r = \sin(2\pi t/20) + \sin(2\pi t/10)$ in the presence of the Markovian switching multiple uncertainties.

It is clear that Assumptions 1 and 2 can be satisfied. In fact, it is evident that the unmodeled states fulfill Assumption

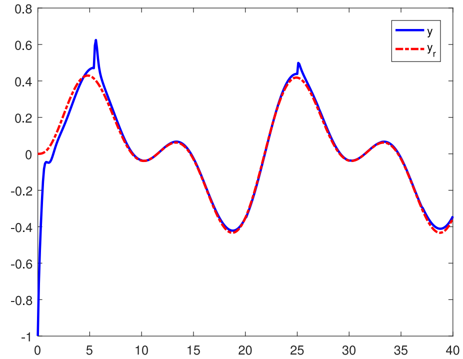

Fig. 1. Tracking performance of Example 1.

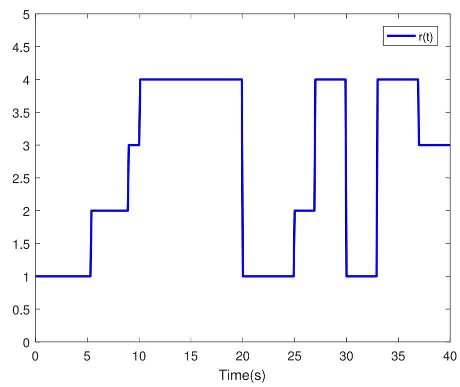

Fig. 2. Trajectory of Markovian modes of Example 1.

3. Let $V_\omega(\omega) = \omega^2$, then $\dot{V}_\omega(\omega) = -2\omega^2 + 2\omega x_1^2 + \omega$. With the aid of Young's inequalities, it can be checked that $2\omega x_1^2 \le (1/4\lambda_1)(2\omega)^2 + \lambda_1 x_1^4$, $\omega \le (1/4\lambda_2) + \lambda_2\omega^2$. We select $\lambda_1 = 2.5$, $\lambda_2 = 0.4$, then $\gamma_1 = 1.2, \bar{\xi}(x_1) = 2.5x_1^4, \gamma_2 = 0.625$. As a result, $\dot{r} = -r + 2.5x_1^4 + 0.625, r(t_0) = 0.1$. Herein, we assume the transition rate matrix is completely unknown. Based on the procedures provided in Section III, the adaptive fuzzy fault-tolerant control scheme can be constructed. The design constants are selected as

$$\begin{aligned}
&\eta_{z_1,1} = 1;\ \eta_{z_2,1} = 2;\ \eta_{\vartheta,1} = \eta_{\bar{\delta},1} = \eta_{\mu,1} = 0.9;\ \eta_{H,1} = 3\\
&\eta_{z_1,2} = 1;\ \eta_{z_2,2} = 2.2;\ \eta_{\vartheta,2} = \eta_{\mu,2} = \eta_{\bar{\delta},2} = 0.7;\ \eta_{H,2} = 2\\
&\eta_{z_1,3} = 1;\ \eta_{z_2,3} = 1.9;\ \eta_{\vartheta,3} = \eta_{\mu,3} = \eta_{\bar{\delta},3} = 0.8;\ \eta_{H,3} = 1\\
&\eta_{z_1,4} = 1;\ \eta_{z_2,4} = 1.3;\ \eta_{\vartheta,4} = \eta_{\mu,4} = \eta_{\bar{\delta},4} = 1.5;\ \eta_{H,4} = 4\\
&\quad c_1 = 25;\ c_2 = 80;\ \sigma_{\vartheta,0} = \sigma_{\bar{\delta},0} = \sigma_{\mu,0} = 0.2;\ \sigma_{H,0} = 0.02\\
&\quad \varepsilon_\vartheta = \varepsilon_{\bar{\delta}} = \varepsilon_\mu = 0.1;\ \varepsilon_H = 10;\ \eta_r = 20;\ \upsilon_1 = 1.
\end{aligned}$$

By using the aforementioned constants, the parameters $c_1'(k), c_2'(k), c_3'(k)$ and $\sigma_\vartheta(k), \sigma_{\bar{\delta}}(k), \sigma_\mu(k), \sigma_H(k)$ can be computed for each mode. The fuzzy membership functions are defined as $\mu_{F_i^l}(x_i) = e^{-0.5(x_i-6+2l)^2}$, $\mu_{F_r^l}(r) = e^{-0.5(r-6+2l)^2}$, $l = 1, 2, \ldots, 5, i = 1, 2$. The initial conditions are chosen as $x_1(0) = -1, x_2(0) = 1, \omega(0) = 0.4, \vartheta(0) = \bar{\delta}(0) = \mu(0) = 0$ and $H(0) = 0$. Figs. 1 and 3 provide the simulation results. Fig. 1 depicts the output trajectories of the considered system. The switching signal is presented by r_t in Fig. 2. Fig. 3 shows the adaptive parameters generated by the switching parameter update laws.

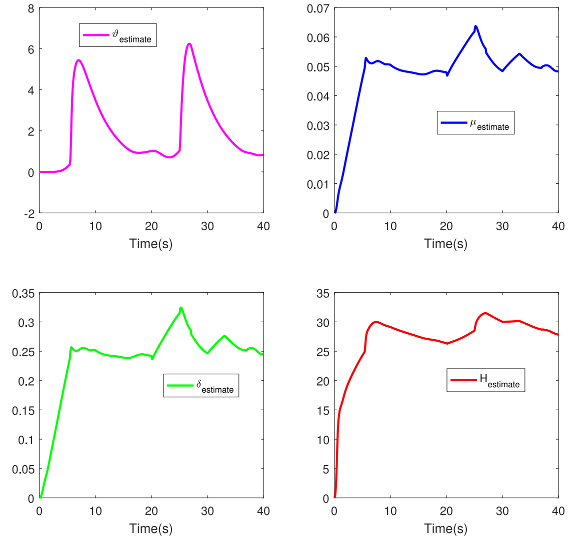

Fig. 3. Trajectories of the adaptive parameters of Example 1.

Example 2: To further show the effectiveness of the proposed method, a simulation example (a single-link manipulator including motor dynamics) [44], is considered

$$\begin{aligned}
&D\ddot{q} + B\dot{q} + N\sin(q) = \tau\\
&M\dot{\tau} + H_m\tau = u - K_m\dot{q}
\end{aligned} \tag{77}$$

where q, $\dot{q}$, and $\ddot{q}$ denote the link position, velocity, and acceleration, respectively; τ is the torque produced by the electrical subsystem; u represents the electromechanical torque; D is the mechanical inertia; B is the positive coefficient of viscous friction; N is a positive constant related to the mass of the load and the coefficient of gravity; $M = 0.1H_m$ is the armature inductance; H_m is the armature resistance; and K_m is the back electromotive-force coefficient. Let $x_1 = q$, $x_2 = \dot{q}$, and $x_3 = \tau$. Then, (77) can be considered as follows:

$$\begin{aligned}
\dot{\omega} &= -\omega + x_1^2 + 0.5\\
\dot{x}_1 &= x_2\\
\dot{x}_2 &= \frac{x_3}{D} - \frac{B}{D}x_2 - \frac{N}{D}\sin x_1 + \Delta_2(x,\omega)\\
\dot{x}_3 &= \frac{u}{M} - \frac{K_m}{M}x_2 - \frac{H_m}{M}x_3 + \Delta_3(x,\omega).
\end{aligned} \tag{78}$$

It is assumed that the plant is randomly jumping in four modes with the parameters and functions given in Table II. Furthermore, we also consider the randomly changing actuator faults, which are given in Table II.

In the controller design, we assume B, N, and K_m are unknown parameters. Meanwhile, the transition rate matrix is assumed completely unavailable. Similarly, the virtual control functions, adaptation laws, and controller are synthesized according to the backstepping design procedures in Section III. In Example 2, the design constants are chosen as

$$\begin{aligned}
&\eta_{z_1,1} = 1;\ \eta_{z_2,1} = 2;\ \eta_{z_3,1} = 1.4;\ \eta_{\vartheta,1} = \eta_{\bar{\delta},1} = \eta_{\mu,1} = 0.9\\
&\eta_{z_1,2} = 1;\ \eta_{z_2,2} = 2.2;\ \eta_{z_3,2} = 1.5;\ \eta_{\vartheta,2} = \eta_{\mu,2} = \eta_{\bar{\delta},2} = 0.7\\
&\eta_{z_1,3} = 1;\ \eta_{z_2,3} = 1.9;\ \eta_{z_3,3} = 1.45;\ \eta_{\vartheta,3} = \eta_{\mu,3} = \eta_{\bar{\delta},3} = 0.8\\
&\eta_{z_1,4} = 1;\ \eta_{z_2,4} = 1.3;\ \eta_{z_3,4} = 1.6;\ \eta_{\vartheta,4} = \eta_{\mu,4} = \eta_{\bar{\delta},4} = 1.5
\end{aligned}$$

TABLE II
PARAMETERS OF THE PLANT IN EXAMPLE 2 FOR THE FOUR MODES

	Mode 1	Mode 2	Mode 3	Mode 4
N	10	12	11	10.5
B	1	2	1.3	1.7
K_m	0.2	0.25	0.27	0.23
H_m	1	1.5	1.3	1.1
M	0.1	0.15	0.13	0.11
D	1	1	1	1
Δ_2	$\omega \sin x_2$	$2\omega \sin x_2$	$0.5\omega \sin x_2$	$\omega \sin 2x_2$
Δ_3	$\omega \cos x_1 x_2$	$2\omega \cos x_1 x_2$	$\omega \cos 2x_1 x_2$	$\omega \cos x_1^2 x_2$

TABLE III
PARAMETERS OF THE ACTUATOR FAULTS IN EXAMPLE 2 FOR THE FOUR MODES

	Mode 1	Mode 2	Mode 3	Mode 4
h	0.5	0.8	0.55	0.65
δ	2	2.2	3	1.9

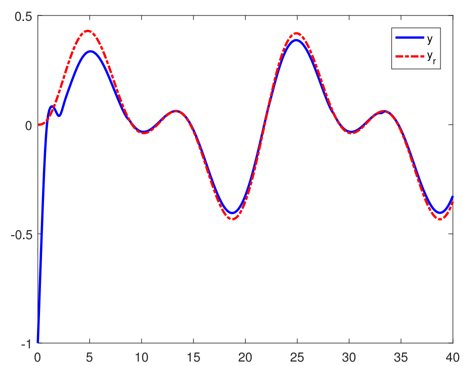

Fig. 4. Tracking performance of Example 2.

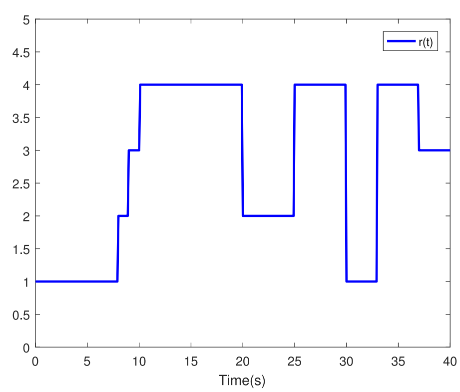

Fig. 5. Trajectory of Markovian modes of Example 2.

$$\eta_{H,1} = 0.03;\ \eta_{H,2} = 0.02;\ \eta_{H,3} = 0.01;\ \eta_{H,4} = 0.04$$
$$c_1 = 2;\ c_2 = 20;\ c_3 = 40;\ \sigma_{\vartheta,0} = \sigma_{\bar{\delta},0} = \sigma_{\mu,0} = 0.2$$
$$\sigma_{H,0} = 0.02;\ \varepsilon_\vartheta = \varepsilon_{\bar{\delta}} = \varepsilon_\mu = 0.1;\ \varepsilon_H = 1;\ \eta_r = 20;\ \upsilon_1 = 0.3.$$

The other parameters are obtained by using (27), (29), (44), (60), and (67). The fuzzy membership functions are defined in Example 1 with $1 \le i \le 3$. The initial conditions are selected as $x_1(0) = -1$, $x_2(0) = 1$, $x_3(0) = 1$, $\omega(0) = 0.4$, $\vartheta(0) = \bar{\delta}(0) = \mu(0) = 0$ and $H(0) = 0$. The simulation results are shown in Figs. 4 and 6. Fig. 4 presents the tracking performance of the closed-loop system. Fig. 5 provides the switching signal r_t. Fig. 6 shows the adaptive parameters generated by the switching parameter update algorithm.

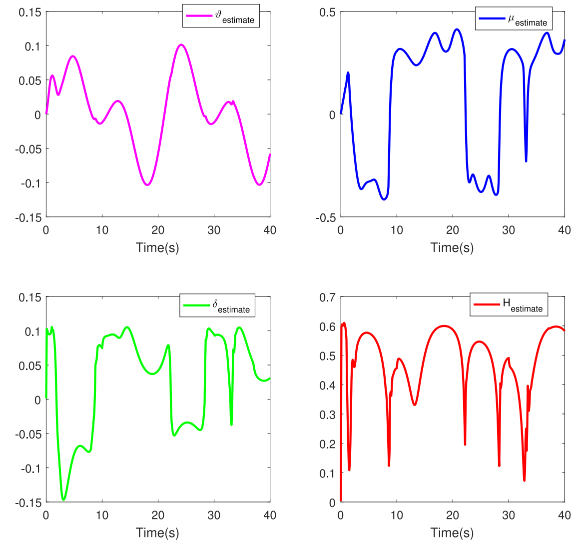

Fig. 6. Trajectories of the adaptive parameters of Example 2.

Obviously, the output of the closed-loop system can converge to any given vicinity of the desired signal with the proposed controller. Moreover, from Fig. 6, we can see that the adaptive parameters are all bounded, which demonstrates the validity of our method.

V. CONCLUSION

In this paper, a fuzzy-approximation-based adaptive control approach has been designed for a class of strict-feedback MJNSs subject to dynamic uncertainties and actuator failures. Different from most of the existing control design for the systems with unmodeled dynamics, the output tracking control problem, rather than the stabilization problem, has been adequately handled by introducing a hyperbolic tangent function. Furthermore, by using the proposed method, the transition rate matrix is allowed to be partly unknown and the computation load can be reduced. To the best of the author's knowledge, it is the first attempt to tackle the tracking control problem for the strict-feedback nonlinear systems which keep completely Markovian jumping. The effectiveness of the proposed method has been demonstrated by two practical examples. Possible future directions include the consensus control protocol design for Markovian multiagent systems and the event-trigged control for Markovian systems.

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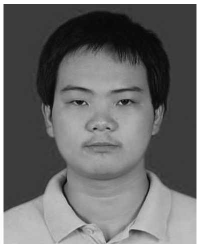

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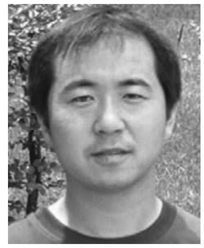

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