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# Sliding mode control for Markovian jump repeated scalar nonlinear systems with packet dropouts: The uncertain occurrence probabilities case☆

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ABSTRACT

This paper studies the sliding-mode-based control problem for a class of delayed Markovian jump repeated scalar nonlinear systems (RSNSs) subject to packet dropouts (PDs) and randomly occurring uncertainties (ROUs). Here, the phenomenon of the Markovian PDs is modeled by utilizing a random variable obeying Markov process. By using the Bernoulli distributed random variables, the ROUs under uncertain occurrence probabilities (UOPs) are characterized from the mathematical perspective. In addition, the time-delay considered is bounded time-varying with known upper and lower bounds. The attention is on designing a sliding-mode-based control method for Markovian jump RSNSs in the simultaneous presence of Markovian PDs, ROUs and time-varying delays. Specifically, the robustly asymptotically mean-square stability is ensured for corresponding sliding mode dynamics by proposing new sufficient criterion. Moreover, the reachability in discrete setting is analyzed, i.e., the state trajectories of the system from any initial condition are globally driven onto a small region nearby the pre-designed sliding surface in mean square sense by synthesizing a robust sliding mode control (SMC) law. Finally, the effectiveness of developed control method is verified by some simulations.



## 1. Introduction

Over the past few decades, the sliding mode control (SMC) technique has been widely studied because of its successful applications, see e.g. [1–4]. Generally speaking, the basic principle of SMC is to drive the state trajectories of system onto

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the pre-defined sliding surface by synthesizing a discontinuous control law, and then the resultant system in the specified sliding surface has attractive invariant properties, such as insensitiveness to unknown time-varying disturbances. In addition, it should be noted that the SMC has the advantage with respect to the convenient implementation. According to different indices, a great number of methods have been reported for various dynamical systems via SMC approaches, see e.g. [5–9]. Notice that the control technique have been digitally implemented for many realistic systems, such as networks control systems [10,11]. Therefore, some SMC problems have been tackled for discrete systems, see e.g. [12–14]. For instance, the conception of new quasi-sliding mode has been introduced in [12], which has been commonly employed by many researchers, such as [13,15,16]. In particular, a new robust SMC scheme has been given in [13] for discrete-time delayed singular systems with external disturbances, where a delay-dependent criterion has been developed to ensure the admissibility for addressed systems based on the free weighting matrix technique.

Markovian jump systems (MJSs) can be seen as a special class of stochastic dynamical systems with the transitions between different systems governed by a Markov process [17–21]. Such MJSs have received increasing investigation attention in recent years due to the convenience characterization of the practical systems with structures changed randomly [22,23]. Accordingly, a large number of interesting results for MJSs have been reported to meet various requirements, such as robust stabilization and stability analysis [24,25], filter and controller design [14,26,27]. Among them, in [14], the stabilization problem of the discrete-time MJSs with mixed time-delays has been analyzed by using SMC approach, where the stochastic stability condition of the sliding motion has been presented by means of the matrix inequality technique. On the other hand, the repeated scalar nonlinearities (RSNs) can be applied to approximate a large class of nonlinear dynamical systems such as recurrent neural networks [28,29]. Therefore, the synthesis problems for systems with RSNs have been increasingly addressed [30–32]. For example, the output feedback controller has been synthesized in [31] for MJSs with RSNs, and new sufficient criterion has been obtained to ensure the stability.

As it is well known, it becomes increasingly prevalent that the system information is transmitted via a communication network medium since the network technology develops rapidly. Compared with traditional cases, the insertion of the communication networks could bring great convenience, but it also induces some inevitable phenomena, such as data packet dropouts/losses [33,34], actuator failures [35] and transmission delays [36]. It should be mentioned that most of the existing SMC methods have relied on an assumption that the data packet can be always successfully transmitted through the network, which might be difficult to be guaranteed in some cases [37]. Hence, in order to attenuate the effects caused by the data packet dropouts, some useful algorithms have been developed for systems based on the SMC technique, see e.g. [10,38,39]. Specifically, the designs of SMC scheme subject to packet losses have been presented respectively for uncertain discrete systems [10] and discrete stochastic systems [39], where the PDs have been characterized by Bernoulli distributed variables and new update schemes have been proposed with help to compensate the impacts of imprecise data. In [40], the SMC scheme with successive packet losses has been designed for MJSs, and a robust controller has been constructed to guarantee satisfactory performance of the addressed system irrespective of the packet losses as well as Markovian jump parameters. It should be noted that another modelling way for PDs has been given recently, where the phenomenon of PDs has been modeled by a two-state Markov chain. To be more specific, a novel SMC scheme has been designed in [41] for a class of discrete systems subject to Markovian PDs. Up to now, few SMC methods subject to Markovian PDs can be available for the analysis of Markovian jump RSNSs, not to mention that the randomly occurring uncertainties (ROUs) under uncertain occurrence probabilities (UOPs) are met simultaneously. In fact, additional attention should be made for the modelling way with respect to the ROUs under UOPs, which can provide more flexibility and robustness analysis for the synthesis issue of networked control systems.

Motivated by above inspired discussions, the problem of SMC with PDs and ROUs is tackled for Markovian jump RSNSs with time-delays. Firstly, the PDs are depicted by employing a random variable obeying the Markov process. And then, a linear switching surface is designed based on the available probability information of data dropouts. In the sequel, sufficient condition is given to guarantee that the resulted sliding mode dynamics in the pre-set switching surface is robustly asymptotically mean-square stable by utilizing the stochastic analysis method. Moreover, a minimization algorithm is introduced to deal with the original non-convex problem. The addressed problem has two challenges/difficulties as follows: 1) How to establish a proper model which can reflect the behaviours of Markovian jump characteristics, nonlinear disturbances, random modelling errors under imprecise occurrence probabilities, PDs as well as time-varying bounded delays comprehensively? and 2) how to propose an efficient robust control method to evaluate the effects from the above phenomena onto the whole control performance? In summary, the advantages/novelties of the paper can be stated as follows: (i) The robust SMC problem is discussed for addressed delayed Markovian jump RSNSs with Markovian PDs and ROUs for the first time; (ii) a new SMC synthesis method is provided by combining the positive definite diagonally dominant idea with stochastic analysis method, which is capable of attenuating the multiple effects from PDs, time-varying bounded delays, RSNs and ROUs with imprecise occurrence probabilities; and (iii) a minimization algorithm is employed to enhance the feasibility of proposed SMC method, which involves the linear indices and can be easily tested by using standard software. Finally, the effectiveness of new SMC method is verified by some simulations.

*Notations*: The following notations will be utilized throughout the paper. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ are the $n$-dimensional Euclidean space and the set of all $n\times m$ real matrices, respectively. The superscript "$\mathcal{T}$" represents transition probability matrix and the superscript "$T$" represents matrix transposition. $(\Omega, \mathcal{F}, \mathcal{P})$ is a probability space, where $\Omega$ denotes a sample space, $\mathcal{F}$ denotes the $\sigma$-algebra of subsets of $\Omega$, and $\mathcal{P}$ denotes the probability measure on $\mathcal{F}$. $P>0$ $(P\geq 0)$ means that $P$ is a symmetric and positive definite (positive semi-definite) matrix, and $I$, 0 denote the identity matrix and the zero matrix with proper dimensions, respectively. $\mathbb{E}\{x\}$ is the expectation of variable $x$. diag$\{Y_1, Y_2, \ldots, Y_n\}$ stands for a block diagonal matrix where diagonal blocks are matrices $Y_1, Y_2, \ldots, Y_n$. The star '*' in a matrix denotes a term that can be induced by symmetry. $\|\cdot\|$ represents Euclidean norm of a vector and its induced norm of a matrix. Matrices, if not explicitly stated, are assumed to be compatible for algebraic computations.

## 2. Problem formulation and brief preliminaries

Let the parameter $r_k$ $(k \in Z^+)$ be a discrete homogeneous Markovian chain, which takes values from $\mathcal{N} = \{1, 2, \ldots, N\}$ with the transition probability matrix $\Pi \triangleq [\pi_{ij}]_{i,j\in\mathcal{N}}$. Here,

$$\pi_{ij} \triangleq \Pr(r_{k+1} = j | r_k = i) \geq 0, \quad \forall i, j \in \mathcal{N}, k \in Z^+,$$

and $\sum_{j=1}^{N} \pi_{ij} = 1$ for each $i \in \mathcal{N}$.

In this paper, we consider the following class of discrete delayed Markovian jump systems with RSNs:

$$x_{k+1} = (A(r_k) + \alpha_{k,r_k}\Delta A(r_k))g(x_k) + A_d(r_k)h(x_{k-d_k}) + B(r_k)(u_k + f(x_k, k, r_k)) + E(r_k)g(x_k)\omega_k, \tag{1}$$

where $x_k \in \mathbb{R}^n$ is the vector of system state; $u_k \in \mathbb{R}^t$ is the control input; $d_k \in [d_m, d_M]$ denotes the time-varying delay, and scalars $d_M$, $d_m$ are known upper and lower bounds of the time-delay. $f_{i,k} \triangleq f(x_k, k, r_k)$ $(r_k = i \in \mathcal{N})$ denotes an unknown bounded nonlinearity, $g(x_k) \in \mathbb{R}^n$ and $h(x_k) \in \mathbb{R}^n$ are the RSNs. $A_i \triangleq A(r_k)$, $A_{di} \triangleq A_d(r_k)$, $B_i \triangleq B(r_k)$ and $E_i \triangleq E(r_k)$ stand for known matrices. $\omega_k \in \mathbb{R}^1$ is a zero-mean Wiener process on a probability space $(\Omega, \mathcal{F}, \mathcal{P})$ with $\mathbb{E}\{\omega_k^2\} = 1$ and $\mathbb{E}\{\omega_k\omega_j\} = 0$ $(k \neq j)$.

The matrix $\Delta A(r_k) \triangleq \Delta A_i$ stands for the bounded parameter uncertainty with the form:

$$\Delta A_i = M_i F_i N_i, \tag{2}$$

where $M_i$ and $N_i$ $(i \in \mathcal{N})$ are constant matrices with appropriate dimensions, and the unknown matrix $F_i$ satisfies $F_i^T F_i \leq I$. The random variable $\alpha_{i,k} \triangleq \alpha_{k,r_k}$ $(i \in \mathcal{N})$ is a $0-1$ distributed one and satisfies

$$\begin{aligned}
&\text{Prob}\{\alpha_{i,k} = 1\} = \mathbb{E}\{\alpha_{i,k}\} = \bar{\alpha}_i + \Delta\alpha_i, \\
&\text{Prob}\{\alpha_{i,k} = 0\} = 1 - \mathbb{E}\{\alpha_{i,k}\} = 1 - (\bar{\alpha}_i + \Delta\alpha_i),
\end{aligned} \tag{3}$$

where $\bar{\alpha}_i + \Delta\alpha_i \in [0, 1]$ and $|\Delta\alpha_i| \leq \epsilon_i$ with $\bar{\alpha}_i$ and $\epsilon_i$ being known scalars. Further, we know $0 \leq \epsilon_i \leq \min\{\bar{\alpha}_i, 1 - \bar{\alpha}_i\}$.

**Remark 1.** In (3), the random variables $\alpha_{i,k}$ $(i \in \mathcal{N})$ are utilized to describe the ROUs firstly and the scalars $\Delta\alpha_i$ are introduced to depict the UOPs. From (3), we can see that the occurrence probabilities are allowed to be uncertain ones within certain interval. To be more specific, this modelling way represents an extension compared with the traditional ROUs under deterministic probabilities is extended here, and the traditional modelling ways as in [42] can be recovered via setting $\Delta\alpha_i = 0$ simply. Besides, the ROUs under UOPs mentioned here can enrich the description way of modelling errors and related control/filtering methods can be expected to deal with this phenomena. Generally, the nominal expectation $\bar{\alpha}_i$ and the corresponding bound $\epsilon_i$ can be obtained in terms of the practical circumstances [43,44].

The following random variable $\theta_k$, which is used to indicate the phenomenon of PDs, obeys two-state stochastic process. The corresponding probability matrix is shown as follows:

$$\mathcal{T} = \begin{bmatrix} 1-q & q \\ p & 1-p \end{bmatrix}, \tag{4}$$

where $p := \Pr\{\theta_{k+1} = 0 | \theta_k = 1\}$ and $q := \Pr\{\theta_{k+1} = 1 | \theta_k = 0\}$.

**Remark 2.** As in [41], the Gilbert–Elliott model is employed to characterize the phenomenon of PDs. In such a model, the random variable $\theta_k$ is indeed a two-state Markov chain to describe the phenomenon of Markovian PDs. To be specific, the case $\theta_k = 1$ means that data packet at time $k$ has been received via the controller. Conversely, the case $\theta_k = 0$ means that the phenomenon of Markovian PDs has occurred during the data transmission.

To handle the packet loss, the following update compensation scheme is selected:

$$x_{c,k} = \theta_k x_k + (1 - \theta_k)x_{c,k-1}. \tag{5}$$

From the above updating rule, we can see that the data packet is perfectly received at time $k$ if $\theta_k = 1$, and otherwise the data packet dropout occurs.

**Remark 3.** According to Assumption 1, as discussed in [41], the Markov chain described by (4) has a unique stationary distribution $\pi := [\pi_0 \;\; \pi_1]$ with $\pi\mathcal{T} = \pi$ and $\pi_0 + \pi_1 = 1$. After the derivations, one has $\pi = [\bar{\theta} \;\; 1 - \bar{\theta}]$, where $\bar{\theta} = \frac{p}{p+q}$.

For the purpose of subsequent illustrations, we make the following assumptions throughout this paper.

**Assumption 1.** The Markov chain employed to depict the phenomenon of the PDs has ergodicity.

**Assumption 2.** The matrix $B_i$ $(i \in \mathcal{N})$ is of column full rank.

**Assumption 3.** The nonlinear function $g(\cdot):\mathbb{R}\to\mathbb{R}$ satisfies

$$|g(a)+g(b)|\le|a+b|,\quad \forall a,b\in\mathbb{R}.$$

For $x_k=\begin{bmatrix}x_k^1 & x_k^2 & \dots & x_k^n\end{bmatrix}^T$, the $n$-dimensional RSNs $g(x_k)$ and $h(x_k)$ are described by

$$g(x_k)=\begin{bmatrix}g(x_k^1) & g(x_k^2) & \dots & g(x_k^n)\end{bmatrix}^T,\ \ h(x_k)=\begin{bmatrix}h(x_k^1) & h(x_k^2) & \dots & h(x_k^n)\end{bmatrix}^T.$$

To proceed, the following helpful lemmas need to be recalled.

**Lemma 1** [14]. *For $a$, $b$ and $P>0$ of compatible dimensions, one obtains:*

$$a^Tb+b^Ta\le a^TPa+b^TP^{-1}b.$$

**Lemma 2** [14]. *Given constant matrices $\mathcal{U}_1$, $\mathcal{U}_2$, $\mathcal{U}_3$, where $\mathcal{U}_i=\mathcal{U}_i^T$ $(i=1,2)$ and $\mathcal{U}_2>0$, then $\mathcal{U}_1+\mathcal{U}_3^T\mathcal{U}_2^{-1}\mathcal{U}_3<0$ if and only if*

$$\begin{bmatrix}\mathcal{U}_1 & \mathcal{U}_3^T\\ * & -\mathcal{U}_2\end{bmatrix}<0\ \text{ or }\ \begin{bmatrix}-\mathcal{U}_2 & \mathcal{U}_3\\ * & \mathcal{U}_1\end{bmatrix}<0.$$

**Lemma 3** [14]. *Assume that $Q=Q^T$, $M$, $F$ and $N$ are properly dimensioned matrices with $F^TF\le I$. Then $Q+MFN+N^TF^TM^T<0$ holds, if and only if there exists $\varepsilon>0$ such that $Q+\varepsilon MM^T+\varepsilon^{-1}N^TN<0$ or,*

$$\begin{bmatrix}Q & \varepsilon M & N^T\\ \varepsilon M^T & -\varepsilon I & 0\\ N & 0 & -\varepsilon I\end{bmatrix}<0.$$

**Lemma 4** [45]. *Let $P$ be a positive semi-definite diagonally dominant matrix. Then, for the RSN $g(x)$, we have*

$$g^T(x)Pg(x)\le x^TPx,\quad \forall x\in\mathbb{R}^n.$$

## 3. Design of discrete SMC

This section aims to tackle the robust SMC problem of Markovian jump RSNSs subject to PDs and ROUs. To start with, the switching surface involved the probability information of data packet dropouts is appropriately proposed. And then, the robust mean-square asymptotic stability of related sliding mode dynamics in the switching surface is guaranteed by proposing the sufficient criterion, which is derived by means of the diagonally dominant Lyapunov method. In what follows, a desired discrete sliding mode controller is designed to ensure the reachability condition in the discrete-time setting. Moreover, an LMIs-based minimization computational algorithm is presented to check the feasibility of obtained method.

### *3.1. Switching surface*

Firstly, the following linear switching surface is proposed:

$$s_k=(1-\bar\theta)G_ix_k+\bar\theta G_iA_ix_{c,k-2},\tag{6}$$

where $G_i\in\mathbb{R}^{m\times n}$ is a matrix to be determined later such that, $G_iB_i$ is non-singular and $G_iJ_i:=G_i[A_{di}\ E_i]=0$. In this paper, we choose $G_i=B_i^T\widetilde{P}_i$ to ensure the non-singularity of $G_iB_i$. Here, $\widetilde{P}_i=\sum_{j=1}^N\pi_{ij}P_j$ and $P_j$ $(j\in\mathcal{N})$ is a positive diagonally dominant matrix.

According to the feature of the ideal quasi-sliding mode

$$s_{k+1}=s_k=0,\tag{7}$$

together with (1) and (6), the equivalent control can be given by

$$u_{eq}=-(G_iB_i)^{-1}G_i\left\{[A_i+(\bar\alpha_i+\Delta\alpha_i)\Delta A_i]g(x_k)+\frac{p}{q}A_ix_{c,k-1}\right\}-f_{i,k}.\tag{8}$$

Additionally, by substituting (8) into (1), the following sliding motion equation can be obtained:

$$x_{k+1}=\mathcal{A}_{i,k}g(x_k)-\frac{p}{q}B_i(G_iB_i)^{-1}G_iA_ix_{c,k-1}+A_{di}h(x_{k-d_k})+E_ig(x_k)\omega_k,\tag{9}$$

where $\mathcal{A}_{i,k}=(A_i+\alpha_{i,k}\Delta A_i)-B_i(G_iB_i)^{-1}G_i[A_i+(\bar\alpha_i+\Delta\alpha_i)\Delta A_i]$.

Now, based on the Lyapunov stability theorem, a new sufficient criterion is given to guarantee the robustly asymptotic stability of the sliding motion (9).

**Theorem 1.** *Given scalars $\delta_i\in(0,1)$ $(i\in\mathcal{N})$ and $\delta\in(0,1)$, if there exist positive diagonally dominant matrices $P_i>0$ and $R>0$, positive-definite matrices $Q_i(0)>0$ and $Q_i(1)>0$, and scalar $\varepsilon_i>0$ satisfying*

$$\Psi^i:=\begin{bmatrix}\Psi_{11}^i & * & *\\ \Psi_{21}^i & \Psi_{22}^i & *\\ \Psi_{31}^i & 0 & \Psi_{33}^i\end{bmatrix}<0,\tag{10}$$

$$\Phi^i := \begin{bmatrix} \Phi^i_{11} & * & * \\ \Phi^i_{21} & \Phi^i_{22} & * \\ \Phi^i_{31} & 0 & \Phi^i_{33} \end{bmatrix} < 0, \tag{11}$$

$$B_i^T \widetilde{P}_i J_i = 0, \tag{12}$$

*where*

$$\begin{aligned}
\Psi^i_{11} &= \mathrm{diag}\{-\delta_i P_i, -\delta R, -Q_i(1), -(1-\delta_i)P_i + \varepsilon_i N_i^T N_i, -(1-\delta)R\},\\
\Psi^i_{21} &= \begin{bmatrix} p\widetilde{Q}_i(0) & 0 & 0 & 0 & 0 \\ (1-p)\widetilde{Q}_i(1) & 0 & 0 & 0 & 0 \\ (d_M - d_m + 1)R & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2p}{q}G_iA_i & 0 & 0 \end{bmatrix},\\
\Psi^i_{22} &= \mathrm{diag}\{-p\widetilde{Q}_i(0), -(1-p)\widetilde{Q}_i(1), -(d_M - d_m + 1)R, -2G_iB_i\},\\
\Psi^i_{31} &= \begin{bmatrix} 0 & 0 & 0 & 12G_iA_i & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 12\widetilde{P}_iA_i & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \widetilde{P}_iE_i & 0 \\ 0 & 0 & 0 & 0 & 2\widetilde{P}_iA_{di} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},\\
\Psi^i_{33} &= \begin{bmatrix} -12G_iB_i & * & * & * & * & * & * \\ 0 & -12G_iB_i & * & * & * & * & * \\ 0 & 0 & -12\widetilde{P}_i & * & * & * & * \\ 0 & 0 & 0 & -12(\bar{\alpha}_i + \epsilon_i)\widetilde{P}_i & * & * & * \\ 0 & 0 & 0 & 0 & -\widetilde{P}_i & * & * \\ 0 & 0 & 0 & 0 & 0 & -2\widetilde{P}_i & * \\ 0 & 12(\bar{\alpha}_i + \epsilon_i)M_i^T G_i^T & 0 & 12(\bar{\alpha}_i + \epsilon_i)M_i^T \widetilde{P}_i & 0 & 0 & -\varepsilon_i I \end{bmatrix},\\
\Phi^i_{11} &= \mathrm{diag}\{-\delta_i P_i, -\delta R, -Q_i(0), -(1-\delta_i)P_i + \varepsilon_i N_i^T N_i, -(1-\delta)R\},\\
\Phi^i_{21} &= \begin{bmatrix} 0 & 0 & (1-q)\widetilde{Q}_i(0) & 0 & 0 \\ 0 & 0 & q\widetilde{Q}_i(1) & 0 & 0 \\ (d_M - d_m + 1)R & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2p}{q}G_iA_i & 0 & 0 \end{bmatrix},\\
\Phi^i_{22} &= \mathrm{diag}\{-(1-q)\widetilde{Q}_i(0), -q\widetilde{Q}_i(1), -(d_M - d_m + 1)R, -2G_iB_i\},\\
\Phi^i_{31} &= \begin{bmatrix} 0 & 0 & 0 & 12G_iA_i & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 12\widetilde{P}_iA_i & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \widetilde{P}_iE_i & 0 \\ 0 & 0 & 0 & 0 & 2\widetilde{P}_iA_{di} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},\\
\Phi^i_{33} &= \begin{bmatrix} -12G_iB_i & * & * & * & * & * & * \\ 0 & -12G_iB_i & * & * & * & * & * \\ 0 & 0 & -12\widetilde{P}_i & * & * & * & * \\ 0 & 0 & 0 & -12(\bar{\alpha}_i + \epsilon_i)\widetilde{P}_i & * & * & * \\ 0 & 0 & 0 & 0 & -\widetilde{P}_i & * & * \\ 0 & 0 & 0 & 0 & 0 & -2\widetilde{P}_i & * \\ 0 & 12(\bar{\alpha}_i + \epsilon_i)M_i^T G_i^T & 0 & 12(\bar{\alpha}_i + \epsilon_i)M_i^T \widetilde{P}_i & 0 & 0 & -\varepsilon_i I \end{bmatrix}
\end{aligned}$$

*with* $\widetilde{P}_i = \sum_{j=1}^N \pi_{ij} P_j$, $\widetilde{Q}_i(0) = \sum_{j=1}^N \pi_{ij} Q_j(0)$ *and* $\widetilde{Q}_i(1) = \sum_{j=1}^N \pi_{ij} Q_j(1)$, *then the sliding motion (9) is robustly asymptotically mean-square stable.*

**Proof.** To start with, the following Lyapunov–Krasovskii functional is selected for the sliding motion (9)

$$V(\xi_k, i, \theta_k) = V_1(\xi_k, i, \theta_k) + V_2(\xi_k, i, \theta_k) + V_3(\xi_k, i, \theta_k), \tag{13}$$

with

$$
\begin{aligned}
V_1(\xi_k, i, \theta_k) &= x_k^T P_i x_k, \\
V_2(\xi_k, i, \theta_k) &= x_{c,k-1}^T Q_i(\theta_k) x_{c,k-1}, \\
V_3(\xi_k, i, \theta_k) &= \sum_{l=k-d_k}^{k-1} x_l^T R x_l + \sum_{j=-d_M+1}^{-d_m} \sum_{l=k+j}^{k-1} x_l^T R x_l, \\
\xi_k &= [x_k^T \; x_{k-d_k}^T \; x_{c,k-1}^T \; g^T(x_k) \; h^T(x_{k-d_k})]^T.
\end{aligned}
$$

Here, the diagonally dominant matrices $P_i > 0$ $(i \in \mathcal{N})$ as well as $R > 0$, and the matrix $Q_i(\theta_k) > 0$ $(i \in \mathcal{N})$ are matrices to be determined. Then, take the difference calculation of $V_k$ along (9)

$$
\mathbb{E}\{\Delta V(\xi_k, i, \theta_k)\} = \mathbb{E}\{\Delta V_1(\xi_k, i, \theta_k)\} + \mathbb{E}\{\Delta V_2(\xi_k, i, \theta_k)\} + \mathbb{E}\{\Delta V_3(\xi_k, i, \theta_k)\},
$$

where

$$
\begin{aligned}
\mathbb{E}\{\Delta V_1(\xi_k, i, \theta_k)\} &= \mathbb{E}\{V_1(\xi_{k+1}, r_{k+1}, \theta_{k+1}) | \xi_k, i, \theta_k\} - V_1(\xi_k, i, \theta_k) \\
&= \mathbb{E}\left\{x_{k+1}^T \widetilde{P}_i x_{k+1} | \xi_k, i, \theta_k\right\} - x_k^T P_i x_k \\
&= \mathbb{E}\{g^T(x_k)\mathcal{A}_{i,k}^T \widetilde{P}_i \mathcal{A}_{i,k} g(x_k) + \left(\frac{p}{q}\right)^2 x_{c,k-1}^T A_i^T G_i^T (G_i B_i)^{-1} G_i A_i x_{c,k-1} \\
&\quad + h^T(x_{k-d_k}) A_{di}^T \widetilde{P}_i A_{di} h(x_{k-d_k}) + g^T(x_k) E_i^T \widetilde{P}_i E_i g(x_k) \\
&\quad - \frac{2p}{q} g^T(x_k) \mathcal{A}_{i,k}^T \widetilde{P}_i B_i (G_i B_i)^{-1} G_i A_i x_{c,k-1} + 2 g^T(x_k) \mathcal{A}_{i,k}^T \widetilde{P}_i A_{di} h(x_{k-d_k}) - x_k^T P_i x_k \Big\}
\end{aligned} \tag{14}
$$

with $\mathcal{A}_{i,k}$ being defined below (9). By utilizing Lemma 1, it follows that

$$
\begin{aligned}
& -\frac{2p}{q} g^T(x_k) \mathcal{A}_{i,k}^T \widetilde{P}_i B_i (G_i B_i)^{-1} G_i A_i x_{c,k-1} \\
& \leq g^T(x_k) \mathcal{A}_{i,k}^T \widetilde{P}_i \mathcal{A}_{i,k} g(x_k) + \left(\frac{p}{q}\right)^2 x_{c,k-1}^T A_i^T G_i^T (G_i B_i)^{-1} G_i A_i x_{c,k-1},
\end{aligned} \tag{15}
$$

$$
\begin{aligned}
& 2 g^T(x_k) \mathcal{A}_{i,k}^T \widetilde{P}_i A_{di} h(x_{k-d_k}) \\
& \leq g^T(x_k) \mathcal{A}_{i,k}^T \widetilde{P}_i \mathcal{A}_{i,k} g(x_k) + h^T(x_{k-d_k}) A_{di}^T \widetilde{P}_i A_{di} h(x_{k-d_k}),
\end{aligned} \tag{16}
$$

$$
\begin{aligned}
& g^T(x_k) \mathcal{A}_{i,k}^T \widetilde{P}_i \mathcal{A}_{i,k} g(x_k) \\
& \leq 4 g^T(x_k) A_i^T \widetilde{P}_i A_i g(x_k) + 4\alpha_{i,k}^2 g^T(x_k) \Delta A_i^T \widetilde{P}_i \Delta A_i g(x_k) + 4 g^T(x_k) A_i^T G_i^T (G_i B_i)^{-1} G_i A_i g(x_k) \\
& \quad + 4(\bar{\alpha}_i + \Delta\alpha_i)^2 g^T(x_k) \Delta A_i^T G_i^T (G_i B_i)^{-1} G_i \Delta A_i g(x_k).
\end{aligned} \tag{17}
$$

Further, substituting inequalities (15)–(17) into (14) obtains

$$
\begin{aligned}
& \mathbb{E}\{\Delta V_1(\xi_k, i, \theta_k)\} \\
& = \mathbb{E}\left\{x_{k+1}^T \widetilde{P}_i x_{k+1} | \xi_k, i, \theta_k\right\} - x_k^T P_i x_k \\
& \leq \mathbb{E}\Big\{ - x_k^T P_i x_k + 12 g^T(x_k) A_i^T \widetilde{P}_i A_i g(x_k) + 12(\bar{\alpha}_i + \epsilon_i) g^T(x_k) \Delta A_i^T \widetilde{P}_i \Delta A_i g(x_k) \\
& \quad + 12 g^T(x_k) A_i^T G_i^T (G_i B_i)^{-1} G_i A_i g(x_k) + 12(\bar{\alpha}_i + \epsilon_i)^2 g^T(x_k) \Delta A_i^T G_i^T (G_i B_i)^{-1} G_i \Delta A_i g(x_k) \\
& \quad + g^T(x_k) E_i^T \widetilde{P}_i E_i g(x_k) + 2\left(\frac{p}{q}\right)^2 x_{c,k-1}^T A_i^T G_i^T (G_i B_i)^{-1} G_i A_i x_{c,k-1} + 2 h^T(x_{k-d_k}) A_{di}^T \widetilde{P}_i A_{di} h(x_{k-d_k}) \Big\}.
\end{aligned} \tag{18}
$$

Similarly, one has

$$
\begin{aligned}
\mathbb{E}\{\Delta V_3(\xi_k, i, \theta_k)\} &= \mathbb{E}\{V_3(\xi_{k+1}, r_{k+1}, \theta_{k+1}) | \xi_k, i, \theta_k\} - V_3(\xi_k, i, \theta_k) \\
&= \mathbb{E}\left\{(d_M - d_m + 1) x_k^T R x_k - x_{k-d_k}^T R x_{k-d_k}\right\}.
\end{aligned} \tag{19}
$$

Besides, in view of the stability requirement for (9), there is a need to prove that

$$
\begin{aligned}
& \mathbb{E}\{V(\xi_{k+1}, r_{k+1}, \theta_{k+1}) | \xi_k, i, \theta_k = 1\} - V(\xi_k, i, \theta_k = 1) < 0, \\
& \mathbb{E}\{V(\xi_{k+1}, r_{k+1}, \theta_{k+1}) | \xi_k, i, \theta_k = 0\} - V(\xi_k, i, \theta_k = 0) < 0.
\end{aligned}
$$

For simplification purpose, the derivations for $\theta_k = 1$ are provided. To be specific,

$$\begin{aligned}\mathbb{E}\{\Delta V_2(\xi_k, i, \theta_k)\} &= \mathbb{E}\Big\{x_{c,k}^T \widetilde{Q}_i(\theta_{k+1}) x_{c,k} | \xi_k, i, \theta_k = 1\Big\} - x_{c,k-1}^T Q_i(1) x_{c,k-1} \\ &= \mathbb{E}\Big\{p x_k^T \widetilde{Q}_i(0) x_k + (1-p) x_k^T \widetilde{Q}_i(1) x_k - x_{c,k-1}^T Q_i(1) x_{c,k-1}\Big\}, \end{aligned} \tag{20}$$

where $\widetilde{Q}_i(0)$ and $\widetilde{Q}_i(1)$ can be found below (12). Then, it follows from (18)–(20) that

$$\begin{aligned}&\mathbb{E}\{V(\xi_{k+1}, r_{k+1}, \theta_{k+1}) | \xi_k, i, \theta_k = 1\} - V(\xi_k, i, \theta_k = 1) \\ &\le \mathbb{E}\Big\{ - x_k^T P_i x_k + (d_M - d_m + 1) x_k^T R x_k + p x_k^T \widetilde{Q}_i(0) x_k + (1-p) x_k^T \widetilde{Q}_i(1) x_k \\ &\quad + 12 g^T(x_k) A_i^T \widetilde{P}_i A_i g(x_k) + 12(\bar{\alpha}_i + \epsilon_i) g^T(x_k) \Delta A_i^T \widetilde{P}_i \Delta A_i g(x_k) \\ &\quad + 12 g^T(x_k) A_i^T G_i^T (G_i B_i)^{-1} G_i A_i g(x_k) + 12(\bar{\alpha}_i + \epsilon_i)^2 g^T(x_k) \Delta A_i^T G_i^T (G_i B_i)^{-1} G_i \Delta A_i g(x_k) \\ &\quad + g^T(x_k) E_i^T \widetilde{P}_i E_i g(x_k) + 2(\frac{p}{q})^2 x_{c,k-1}^T A_i^T G_i^T (G_i B_i)^{-1} G_i A_i x_{c,k-1} \\ &\quad - x_{c,k-1}^T Q_i(1) x_{c,k-1} + 2 h^T(x_{k-d_k}) A_{di}^T \widetilde{P}_i A_{di} h(x_{k-d_k}) - x_{k-d_k}^T R x_{k-d_k} \Big\}. \end{aligned} \tag{21}$$

From Lemma 4, one has

$$\begin{aligned} &-(1-\delta_i) x_k^T P_i x_k \le -(1-\delta_i) g^T(x_k) P_i g(x_k), \\ &-(1-\delta) x_{k-d_k}^T R x_{k-d_k} \le -(1-\delta) h^T(x_{k-d_k}) R h(x_{k-d_k}) \end{aligned}$$

with scalars $\delta_i \in (0, 1)$ $(i \in \mathcal{N})$ and $\delta \in (0, 1)$. Furthermore, it follows from the above inequalities that

$$-x_k^T P_i x_k \le -\delta_i x_k^T P_i x_k - (1-\delta_i) g^T(x_k) P_i g(x_k), \tag{22}$$

$$-x_{k-d_k}^T R x_{k-d_k} \le -\delta x_{k-d_k}^T R x_{k-d_k} - (1-\delta) h^T(x_{k-d_k}) R h(x_{k-d_k}). \tag{23}$$

By combining (22), (23) with (21), we arrive at

$$\begin{aligned}&\mathbb{E}\Big\{V(\xi_{k+1}, r_{k+1}, \theta_{k+1}) | \xi_k, i, \theta_k = 1\Big\} - V(\xi_k, i, \theta_k = 1) \\ &\quad \le \mathbb{E}\Big\{ - \delta_i x_k^T P_i x_k + (d_M - d_m + 1) x_k^T R x_k + p x_k^T \widetilde{Q}_i(0) x_k + (1-p) x_k^T \widetilde{Q}_i(1) x_k \\ &\qquad - (1-\delta_i) g^T(x_k) P_i g(x_k) + 12 g^T(x_k) A_i^T \widetilde{P}_i A_i g(x_k) + 12(\bar{\alpha}_i + \epsilon_i) g^T(x_k) \Delta A_i^T \widetilde{P}_i \Delta A_i g(x_k) \\ &\qquad + 12 g^T(x_k) A_i^T G_i^T (G_i B_i)^{-1} G_i A_i g(x_k) + 12(\bar{\alpha}_i + \epsilon_i)^2 g^T(x_k) \Delta A_i^T G_i^T (G_i B_i)^{-1} G_i \Delta A_i g(x_k) \\ &\qquad + g^T(x_k) E_i^T \widetilde{P}_i E_i g(x_k) + 2\left(\frac{p}{q}\right)^2 x_{c,k-1}^T A_i^T G_i^T (G_i B_i)^{-1} G_i A_i x_{c,k-1} - x_{c,k-1}^T Q_i(1) x_{c,k-1} \\ &\qquad + 2 h^T(x_{k-d_k}) A_{di}^T \widetilde{P}_i A_{di} h(x_{k-d_k}) - (1-\delta) h^T(x_{k-d_k}) R h(x_{k-d_k}) - \delta x_{k-d_k}^T R x_{k-d_k} \Big\} \\ &\quad = \mathbb{E}\Big\{\xi_k^T \Xi_i \xi_k\Big\}, \end{aligned}$$

where

$$\begin{aligned} \xi_k &= [x_k^T \; x_{k-d_k}^T \; x_{c,k-1}^T \; g^T(x_k) \; h^T(x_{k-d_k})]^T, \\ \Xi_i &= \begin{bmatrix} L_1 & L_2 & L_3 & L_4 & L_5 \end{bmatrix}^T \Pi^{-1} \begin{bmatrix} L_1 & L_2 & L_3 & L_4 & L_5 \end{bmatrix} \\ &\quad + \mathrm{diag}\{-\delta_i P_i, -\delta R, -Q_i(1), -(1-\delta_i) P_i, -(1-\delta) R\}, \\ \Pi &= \mathrm{diag}\Big\{p\widetilde{Q}_i(0), (1-p)\widetilde{Q}_i(1), (d_M - d_m + 1)R, 2G_iB_i, 12G_iB_i, 12G_iB_i, 12\widetilde{P}_i, 12(\bar{\alpha}_i + \epsilon_i)\widetilde{P}_i, \widetilde{P}_i, 12\widetilde{P}_i\Big\}, \\ L_1 &= \begin{bmatrix} p\widetilde{Q}_i(0) & (1-p)\widetilde{Q}_i(1) & (d_M - d_m + 1)R & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T, \\ L_2 &= 0, \;\; L_3 = \begin{bmatrix} 0 & 0 & 0 & \frac{2p}{q} A_i^T G_i^T & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T, \\ L_4 &= \begin{bmatrix} 0 & 0 & 0 & 0 & 12 A_i^T G_i^T & 12(\bar{\alpha}_i + \epsilon_i) \Delta A_i^T G_i^T & 12 A_i^T \widetilde{P}_i & 12(\bar{\alpha}_i + \epsilon_i) \Delta A_i^T \widetilde{P}_i & E_i^T \widetilde{P}_i & 0 \end{bmatrix}^T, \\ L_5 &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 A_{di}^T \widetilde{P}_i \end{bmatrix}^T. \end{aligned}$$

In view of Lemma 2, we can see that $\Xi_i < 0$ is equivalent to the following form

$$\begin{bmatrix} -\delta_i P_i & 0 & 0 & 0 & 0 & L_1^T \\ 0 & -\delta R & 0 & 0 & 0 & L_2^T \\ 0 & 0 & -Q_i(1) & 0 & 0 & L_3^T \\ 0 & 0 & 0 & -(1-\delta_i)P_i & 0 & L_4^T \\ 0 & 0 & 0 & 0 & -(1-\delta)R & L_5^T \\ L_1 & L_2 & L_3 & L_4 & L_5 & -\Pi \end{bmatrix} < 0.$$

Then, rewrite the above inequality as

$$\Sigma^i + \widetilde{M}_i F_i \widetilde{N}_i + (\widetilde{M}_i F_i \widetilde{N}_i)^T < 0,$$

where

$$\Sigma^i = \begin{bmatrix} \Sigma_{11}^i & * & * \\ \Sigma_{21}^i & \Sigma_{22}^i & * \\ \Sigma_{31}^i & 0 & \Sigma_{33}^i \end{bmatrix},$$

$$\Sigma_{11}^i = \mathrm{diag}\{-\delta_i P_i, -\delta R, -Q_i(1), -(1-\delta_i)P_i, -(1-\delta)R\},$$

$$\Sigma_{21}^i = \begin{bmatrix} p\widetilde{Q}_i(0) & 0 & 0 & 0 & 0 \\ (1-p)\widetilde{Q}_i(1) & 0 & 0 & 0 & 0 \\ (d_M - d_m + 1)R & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2p}{q} G_i A_i & 0 & 0 \end{bmatrix},$$

$$\Sigma_{22}^i = \mathrm{diag}\{-p\widetilde{Q}_i(0), -(1-p)\widetilde{Q}_i(1), -(d_M - d_m + 1)R, -2G_i B_i\},$$

$$\Sigma_{31}^i = \begin{bmatrix} 0 & 0 & 0 & 12G_iA_i & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 12\widetilde{P}_i A_i & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \widetilde{P}_i E_i & 0 \\ 0 & 0 & 0 & 0 & 2\widetilde{P}_i A_{di} \end{bmatrix},$$

$$\Sigma_{33}^i = \mathrm{diag}\{-12G_iB_i, -12G_iB_i, -12\widetilde{P}_i, -12(\bar{\alpha}_i + \epsilon_i)\widetilde{P}_i, -\widetilde{P}_i, -2\widetilde{P}_i\},$$

$$\widetilde{M}_i^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 12(\bar{\alpha}_i + \epsilon_i)M_i^T G_i^T & 0 & 12(\bar{\alpha}_i + \epsilon_i)M_i^T \widetilde{P}_i & 0 & 0 \end{bmatrix},$$

$$\widetilde{N}_i = \begin{bmatrix} 0 & 0 & 0 & N_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Subsequently, according to the Lemma 3, we conclude that $\mathbb{E}\{\Delta V_k^i | \xi_k, i, \theta_k = 1\} < 0$ can be guaranteed by (10). Similarly, (11) in Theorem 1 is given subject to $\theta_k = 0$. Consequently, $\mathbb{E}\{\Delta V(\xi_k, i, \theta_k)\} < 0$ holds. Therefore, the resultant sliding motion (9) is robustly asymptotically mean-square stable. □

**Remark 4.** So far, we have examined the effects from RSNs, PDs and UOPs onto the resultant sliding motion in discrete-time setting. In particular, a new stability condition has been provided to assure the robustly asymptotically mean-square stability of the resultant sliding motion, under which the related information of above factors has been explicitly reflected in the main results.

### 3.2. Design of robust sliding mode controller

In this subsection, the design of robust sliding mode controller is given firstly and then the reachability analysis in discrete case is discussed.

By combining (1) with (6), the following equation is obtained:

$$s_{k+1} = (1-\bar{\theta})G_iA_ix_k + \bar{\theta}G_iA_ix_{c,k-1} + (1-\bar{\theta})G_iB_iu_k + \Delta_a(k) + \Delta_f(k) + \Delta_g(k), \tag{24}$$

where $\Delta_a(k) = \alpha_{i,k}(1-\bar{\theta})G_i\Delta A_i g(x_k)$, $\Delta_f(k) = (1-\bar{\theta})G_iB_if_{i,k}$ and $\Delta_g(k) = (1-\bar{\theta})G_iA_i[g(x_k) - x_k]$. Then, recalling the boundedness of $\Delta A_i$ and $f_{i,k}$, we know that $\Delta_a(k)$, $\Delta_f(k)$ and $\Delta_g(k)$ are also bounded. Generally, there exist the bounds $\underline{\delta}_{i,a}^j$, $\bar{\delta}_{i,a}^j$, $\underline{\delta}_{i,f}^j$, $\bar{\delta}_{i,f}^j$, $\underline{\delta}_{i,g}^j$ and $\bar{\delta}_{i,g}^j$ $(j = 1, 2, \dots, t)$ satisfying

$$\underline{\delta}_{i,a}^j \le \delta_{i,a}^j(k) \le \bar{\delta}_{i,a}^j, \quad \underline{\delta}_{i,f}^j \le \delta_{i,f}^j(k) \le \bar{\delta}_{i,f}^j, \quad \underline{\delta}_{i,g}^j \le \delta_{i,g}^j(k) \le \bar{\delta}_{i,g}^j$$

with $\delta_{i,a}^j(k)$, $\delta_{i,f}^j(k)$ and $\delta_{i,g}^j(k)$ $(j = 1, 2, \dots, t)$ being $j$-th elements of the vectors $\Delta_a(k)$, $\Delta_f(k)$ and $\Delta_g(k)$ defined below (24). Next, set

$$\Delta_{ao} = \begin{bmatrix} \delta_{ao}^1 & \delta_{ao}^2 & \dots & \delta_{ao}^t \end{bmatrix}^T, \quad \delta_{ao}^j = \frac{\underline{\delta}_{i,a}^j + \bar{\delta}_{i,a}^j}{2},$$

$$\Delta_{as} = \text{diag}\left\{\delta_{as}^1, \delta_{as}^2, \ldots, \delta_{as}^t\right\}, \quad \delta_{as}^j = \frac{\bar{\delta}_{i,a}^j - \underline{\delta}_{i,a}^j}{2},$$

$$\Delta_{fo} = \begin{bmatrix}\delta_{fo}^1 & \delta_{fo}^2 & \cdots & \delta_{fo}^t\end{bmatrix}^T, \quad \delta_{fo}^j = \frac{\underline{\delta}_{i,f}^j + \bar{\delta}_{i,f}^j}{2},$$

$$\Delta_{fs} = \text{diag}\left\{\delta_{fs}^1, \delta_{fs}^2, \ldots, \delta_{fs}^t\right\}, \quad \delta_{fs}^j = \frac{\bar{\delta}_{i,f}^j - \underline{\delta}_{i,f}^j}{2},$$

$$\Delta_{go} = \begin{bmatrix}\delta_{go}^1 & \delta_{go}^2 & \cdots & \delta_{go}^t\end{bmatrix}^T, \quad \delta_{go}^j = \frac{\underline{\delta}_{i,g}^j + \bar{\delta}_{i,g}^j}{2},$$

$$\Delta_{gs} = \text{diag}\left\{\delta_{gs}^1, \delta_{gs}^2, \ldots, \delta_{gs}^t\right\}, \quad \delta_{gs}^j = \frac{\bar{\delta}_{i,g}^j - \underline{\delta}_{i,g}^j}{2}.$$

In view of above discussions, the following SMC law is designed:

$$u_k = -\frac{1}{1-\bar{\theta}}(G_iB_i)^{-1}[G_iA_ix_{c,k} + \Delta_o + \Delta_s \text{sgn}(s_{c,k})], \tag{25}$$

where $\Delta_o = \Delta_{ao} + \Delta_{fo} + \Delta_{go}$, $\Delta_s = \Delta_{as} + \Delta_{fs} + \Delta_{gs}$, $s_{c,k}$ is the switching function (6) with $x_k$ being replaced by $x_{c,k}$.

Next, we are in a position to discuss the reachability via the stochastic analysis technique and Lyapunov stability theorem.

**Theorem 2.** *Given scalars $\delta_i \in (0, 1)$ $(i \in \mathcal{N})$ and $\delta \in (0, 1)$, assume that there exist diagonally dominant matrices $P_i > 0$ and $R > 0$, matrices $Q_i(0) > 0$ and $Q_i(1) > 0$, and scalars $\varepsilon_i > 0$ satisfying (12) and following inequalities*

$$\widetilde{\Psi}^i := \begin{bmatrix} \widetilde{\Psi}_{11}^i & * & * \\ \widetilde{\Psi}_{21}^i & \widetilde{\Psi}_{22}^i & * \\ \widetilde{\Psi}_{31}^i & 0 & \widetilde{\Psi}_{33}^i \end{bmatrix} < 0, \tag{26}$$

$$\widetilde{\Phi}^i := \begin{bmatrix} \widetilde{\Phi}_{11}^i & * & * \\ \widetilde{\Phi}_{21}^i & \widetilde{\Phi}_{22}^i & * \\ \widetilde{\Phi}_{31}^i & 0 & \widetilde{\Phi}_{33}^i \end{bmatrix} < 0, \tag{27}$$

*where*

$$\widetilde{\Psi}_{11}^i = \text{diag}\{-\delta_iP_i, -\delta R, -Q_i(1), -(1-\delta_i)P_i + \varepsilon_iN_i^TN_i, -(1-\delta)R, -\gamma I\},$$

$$\widetilde{\Psi}_{22}^i = \text{diag}\{-p\widetilde{Q}_i(0), -(1-p)\widetilde{Q}_i(1), -(d_M - d_m + 1)R, -3I, -3G_iB_i\},$$

$$\widetilde{\Psi}_{21}^i = \begin{bmatrix} p\widetilde{Q}_i(0) & 0 & 0 & 0 & 0 & 0 \\ (1-p)\widetilde{Q}_i(1) & 0 & 0 & 0 & 0 & 0 \\ (d_M - d_m + 1)R & 0 & 0 & 0 & 0 & 0 \\ 3\bar{\theta}G_iA_i & 0 & 0 & 0 & 0 & 0 \\ \frac{3\bar{\theta}}{1-\bar{\theta}}G_iA_i & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \widetilde{\Psi}_{31}^i = \begin{bmatrix} 0 & 0 & 3\bar{\theta}G_iA_i & 0 & 0 & 0 \\ 0 & 0 & 0 & 8\widetilde{P}_iA_i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \widetilde{P}_iE_i & 0 & 0 \\ 0 & 0 & 0 & 0 & 2\widetilde{P}_iA_{di} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\widetilde{\Psi}_{33}^i = \begin{bmatrix} -3I & * & * & * & * & * \\ 0 & -8\widetilde{P}_i & * & * & * & * \\ 0 & 0 & -8(\bar{\alpha}_i + \epsilon_i)\widetilde{P}_i & * & * & * \\ 0 & 0 & 0 & -\widetilde{P}_i & * & * \\ 0 & 0 & 0 & 0 & -2\widetilde{P}_i & * \\ 0 & 0 & 8(\bar{\alpha}_i + \epsilon_i)M_i^T\widetilde{P}_i & 0 & 0 & -\varepsilon_iI \end{bmatrix},$$

$$\widetilde{\Phi}_{11}^i = \text{diag}\{-\delta_iP_i, -\delta R, -Q_i(0), -(1-\delta_i)P_i + \varepsilon_iN_i^TN_i, -(1-\delta)R, -\gamma I\},$$

$$\widetilde{\Phi}_{22}^i = \text{diag}\{-4G_iB_i, -3I, -(d_M - d_m + 1)R, -4G_iB_i, -(1-q)\widetilde{Q}_i(0), -q\widetilde{Q}_i(1), -3I\},$$

$$\widetilde{\Phi}_{21}^i = \begin{bmatrix} 4G_iA_i & 0 & 0 & 0 & 0 & 0 \\ 3(1-\bar{\theta})G_iA_i & 0 & 0 & 0 & 0 & 0 \\ (d_M - d_m + 1)R & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{4}{1-\bar{\theta}}G_iA_i & 0 & 0 & 0 \\ 0 & 0 & (1-q)\widetilde{Q}_i(0) & 0 & 0 & 0 \\ 0 & 0 & q\widetilde{Q}_i(1) & 0 & 0 & 0 \\ 0 & 0 & 3(1-\bar{\theta})G_iA_i & 0 & 0 & 0 \end{bmatrix},$$

$$\widetilde{\Phi}_{31}^{i}=\begin{bmatrix}0&0&0&10\widetilde{P}_iA_i&0&0\\0&0&0&0&0&0\\0&0&0&\widetilde{P}_iE_i&0&0\\0&0&0&0&2\widetilde{P}_iA_{di}&0\\0&0&0&0&0&0\end{bmatrix},\quad \widetilde{\Phi}_{33}^{i}=\begin{bmatrix}-10\widetilde{P}_i&*&*&*&*\\0&-10(\bar{\alpha}_i+\epsilon_i)\widetilde{P}_i&*&*&*\\0&0&-\widetilde{P}_i&*&*\\0&0&0&-2\widetilde{P}_i&*\\0&10(\bar{\alpha}_i+\epsilon_i)M_i^T\widetilde{P}_i&0&0&-\varepsilon_iI\end{bmatrix},$$

*$\widetilde{P}_i=\sum_{j=1}^{N}\pi_{ij}P_j$, $\widetilde{Q}_i(0)=\sum_{j=1}^{N}\pi_{ij}Q_j(0)$ and $\widetilde{Q}_i(1)=\sum_{j=1}^{N}\pi_{ij}Q_j(1)$. Then, the system (1) can be driven (in mean square sense) onto a neighborhood of specified sliding surface (6) by applying (25).*

**Proof.** Firstly, it follows from (1) and (25) that

$$\begin{aligned}x_{k+1}&=[I-B_i(G_iB_i)^{-1}G_i](A_i+\alpha_{i,k}\Delta A_i)g(x_k)+\frac{1-\bar{\theta}-\theta_k}{1-\bar{\theta}}\bar{A}_ix_k+A_{di}h(x_{k-d_k})\\&\quad-\frac{1-\theta_k}{1-\bar{\theta}}\bar{A}_ix_{c,k-1}+E_ig(x_k)\omega_k+\frac{1}{1-\bar{\theta}}B_i(G_iB_i)^{-1}[\Delta(k)-\Delta_o-\Delta_s\mathrm{sgn}(s_{c,k})],\end{aligned}\tag{28}$$

where $\bar{A}_i=B_i(G_iB_i)^{-1}G_iA_i$ and $\Delta(k)=\Delta_a(k)+\Delta_f(k)+\Delta_g(k)$. Then, substituting (25) into (24), $s_{k+1}$ can be further expressed by

$$s_{k+1}=(1-\bar{\theta}-\theta_k)G_iA_ix_k+[\theta_k-(1-\bar{\theta})]G_iA_ix_{c,k-1}+[\Delta(k)-\Delta_o-\Delta_s\mathrm{sgn}(s_{c,k})].$$

To proceed with the reachability analysis, we choose the Lyapunov–Krasovskii functional as follows:

$$V(\eta_k,i,\theta_k)=V_1(\eta_k,i,\theta_k)+V_2(\eta_k,i,\theta_k)+V_3(\eta_k,i,\theta_k)+V_4(\eta_k,i,\theta_k),\tag{29}$$

where

$$\begin{aligned}V_1(\eta_k,i,\theta_k)&=x_k^TP_ix_k,\quad V_2(\eta_k,i,\theta_k)=x_{c,k-1}^TQ_i(\theta_k)x_{c,k-1},\\V_3(\eta_k,i,\theta_k)&=\sum_{l=k-d_k}^{k-1}x_l^TRx_l+\sum_{j=-d_M+1}^{-d_m}\sum_{l=k+j}^{k-1}x_l^TRx_l,\\V_4(\eta_k,i,\theta_k)&=s_k^Ts_k,\quad \eta_k=[x_k^T\ x_{k-d_k}^T\ x_{c,k-1}^T\ g^T(x_k)\ h^T(x_{k-d_k})\ s_k]^T\end{aligned}$$

with diagonally dominant matrices $P_i>0$ $(i\in\mathcal{N})$ and $R>0$, matrix $Q_i(\theta_k)>0$ $(i\in\mathcal{N})$ being matrices to be determined. Subsequently, taking the difference of $V(\eta_k,\ i,\ \theta_k)$ along (28) yields

$$\begin{aligned}&es\mathbb{E}\Big\{\Delta V(\eta_k,i,\theta_k)\Big\}\\&=\mathbb{E}\Big\{V(\eta_{k+1},r_{k+1},\theta_{k+1})|\eta_k,i,\theta_k\Big\}-V(\eta_k,i,\theta_k)\\&=\mathbb{E}\Big\{x_{k+1}^T\widetilde{P}_ix_{k+1}+x_{c,k}^T\widetilde{Q}_i(\theta_{k+1})x_{c,k}+s_{k+1}^Ts_{k+1}|\eta_k,i,\theta_k\Big\}-x_k^TP_ix_k\\&\quad-x_{c,k-1}^TQ_i(\theta_k)x_{c,k-1}-s_k^Ts_k+(d_M-d_m+1)x_k^TRx_k-x_{k-d_k}^TRx_{k-d_k}.\end{aligned}\tag{30}$$

Along the same line in Theorem 1, there is a need to ensure the following conditions

$$\begin{aligned}&\mathbb{E}\Big\{V(\eta_{k+1},r_{k+1},\theta_{k+1})|\eta_k,i,\theta_k=1\Big\}-V(\eta_k,i,\theta_k=1)<0,\\&\mathbb{E}\Big\{V(\eta_{k+1},r_{k+1},\theta_{k+1})|\eta_k,i,\theta_k=0\Big\}-V(\eta_k,i,\theta_k=0)<0.\end{aligned}$$

The proof for the case $\theta_k=1$ is presented here only. For $\theta_k=1$, one has

$$\begin{aligned}x_{k+1}&=[I-B_i(G_iB_i)^{-1}G_i](A_i+\alpha_{i,k}\Delta A_i)g(x_k)+\frac{-\bar{\theta}}{1-\bar{\theta}}\bar{A}_ix_k+A_{di}h(x_{k-d_k})\\&\qquad+E_ig(x_k)\omega_k+\frac{1}{1-\bar{\theta}}B_i(G_iB_i)^{-1}[\Delta(k)-\Delta_o-\Delta_s\mathrm{sgn}(s_{c,k})],\\s_{k+1}&=-\bar{\theta}G_iA_ix_k+\bar{\theta}G_iA_ix_{c,k-1}+[\Delta(k)-\Delta_o-\Delta_s\mathrm{sgn}(s_{c,k})].\end{aligned}$$

Then, it is not difficult to derive that

$$\begin{aligned}&\mathbb{E}\Big\{x_{k+1}^T\widetilde{P}_ix_{k+1}|\eta_k,i,\theta_k=1\Big\}\\&=\mathbb{E}\Big\{g^T(x_k)(A_i+\alpha_{i,k}\Delta A_i)^T[I-B_i(G_iB_i)^{-1}G_i]^T\widetilde{P}_i[I-B_i(G_iB_i)^{-1}G_i](A_i+\alpha_{i,k}\Delta A_i)g(x_k)\end{aligned}$$

$$
\begin{aligned}
&+\left(\frac{\bar{\theta}}{1-\bar{\theta}}\right)^2 x_k^T A_i^T G_i^T (G_i B_i)^{-1} G_i A_i x_k + h^T(x_{k-d_k}) A_{di}^T \widetilde{P}_i A_{di} h(x_{k-d_k}) + g^T(x_k) E_i^T \widetilde{P}_i E_i g(x_k)\\
&+\frac{1}{(1-\bar{\theta})^2}[\Delta(k)-\Delta_o-\Delta_s \operatorname{sgn}(s_{c,k})]^T (G_i B_i)^{-1}[\Delta(k)-\Delta_o-\Delta_s \operatorname{sgn}(s_{c,k})]\\
&-2\Big([I-B_i(G_iB_i)^{-1}G_i](A_i+\alpha_{i,k}\Delta A_i)g(x_k)\Big)^T \widetilde{P}_i\Big(\frac{\bar{\theta}}{1-\bar{\theta}}\bar{A}_i x_k\Big)\\
&+2\Big([I-B_i(G_iB_i)^{-1}G_i](A_i+\alpha_{i,k}\Delta A_i)g(x_k)\Big)^T \widetilde{P}_i\Big(A_{di}h(x_{k-d_k})\Big)\\
&+2\Big([I-B_i(G_iB_i)^{-1}G_i](A_i+\alpha_{i,k}\Delta A_i)g(x_k)\Big)^T \widetilde{P}_i\Big(\frac{1}{1-\bar{\theta}}B_i(G_iB_i)^{-1}[\Delta(k)-\Delta_o-\Delta_s \operatorname{sgn}(s_{c,k})]\Big)\\
&-2\Big(\frac{\bar{\theta}}{1-\bar{\theta}}\bar{A}_i x_k\Big)^T \widetilde{P}_i\Big(\frac{1}{1-\bar{\theta}}B_i(G_iB_i)^{-1}[\Delta(k)-\Delta_o-\Delta_s \operatorname{sgn}(s_{c,k})]\Big),
\end{aligned}
\tag{31}
$$

where $\bar{A}_i$ is defined below (28). Utilizing Lemma 1 and noticing $G_i = B_i^T \widetilde{P}_i$, we can obtain

$$
\begin{aligned}
&-2\Big([I-B_i(G_iB_i)^{-1}G_i](A_i+\alpha_{i,k}\Delta A_i)g(x_k)\Big)^T \widetilde{P}_i\Big(\frac{\bar{\theta}}{1-\bar{\theta}}\bar{A}_i x_k\Big)\\
&\quad \le g^T(x_k)(A_i+\alpha_{i,k}\Delta A_i)^T[I-B_i(G_iB_i)^{-1}G_i]^T \widetilde{P}_i[I-B_i(G_iB_i)^{-1}G_i](A_i+\alpha_{i,k}\Delta A_i)g(x_k)\\
&\quad\quad +\left(\frac{\bar{\theta}}{1-\bar{\theta}}\right)^2 x_k^T A_i^T G_i^T (G_iB_i)^{-1} G_i A_i x_k,
\end{aligned}
\tag{32}
$$

$$
\begin{aligned}
&2\Big([I-B_i(G_iB_i)^{-1}G_i](A_i+\alpha_{i,k}\Delta A_i)g(x_k)\Big)^T \widetilde{P}_i\Big(A_{di}h(x_{k-d_k})\Big)\\
&\quad \le g^T(x_k)(A_i+\alpha_{i,k}\Delta A_i)^T[I-B_i(G_iB_i)^{-1}G_i]^T \widetilde{P}_i[I-B_i(G_iB_i)^{-1}G_i](A_i+\alpha_{i,k}\Delta A_i)g(x_k)\\
&\quad\quad +h^T(x_{k-d_k})A_{di}^T \widetilde{P}_i A_{di} h(x_{k-d_k}),
\end{aligned}
\tag{33}
$$

$$
\begin{aligned}
&2\Big([I-B_i(G_iB_i)^{-1}G_i](A_i+\alpha_{i,k}\Delta A_i)g(x_k)\Big)^T \widetilde{P}_i\Big(\frac{1}{1-\bar{\theta}}B_i(G_iB_i)^{-1}[\Delta(k)-\Delta_o-\Delta_s \operatorname{sgn}(s_{c,k})]\Big)\\
&\quad \le g^T(x_k)(A_i+\alpha_{i,k}\Delta A_i)^T[I-B_i(G_iB_i)^{-1}G_i]^T \widetilde{P}_i[I-B_i(G_iB_i)^{-1}G_i](A_i+\alpha_{i,k}\Delta A_i)g(x_k)\\
&\quad\quad +\frac{1}{(1-\bar{\theta})^2}[\Delta(k)-\Delta_o-\Delta_s \operatorname{sgn}(s_{c,k})]^T (G_iB_i)^{-1}[\Delta(k)-\Delta_o-\Delta_s \operatorname{sgn}(s_{c,k})],
\end{aligned}
\tag{34}
$$

$$
\begin{aligned}
&-2\Big(\frac{\bar{\theta}}{1-\bar{\theta}}\bar{A}_i x_k\Big)^T \widetilde{P}_i\Big(\frac{1}{1-\bar{\theta}}B_i(G_iB_i)^{-1}[\Delta(k)-\Delta_o-\Delta_s \operatorname{sgn}(s_{c,k})]\Big)\\
&\le \left(\frac{\bar{\theta}}{1-\bar{\theta}}\right)^2 x_k^T A_i^T G_i^T (G_iB_i)^{-1} G_i A_i x_k + \frac{1}{(1-\bar{\theta})^2}[\Delta(k)-\Delta_o-\Delta_s \operatorname{sgn}(s_{c,k})]^T (G_iB_i)^{-1}\\
&\times[\Delta(k)-\Delta_o-\Delta_s \operatorname{sgn}(s_{c,k})],
\end{aligned}
\tag{35}
$$

$$
\begin{aligned}
&g^T(x_k)(A_i+\alpha_{i,k}\Delta A_i)^T[I-B_i(G_iB_i)^{-1}G_i]^T \widetilde{P}_i[I-B_i(G_iB_i)^{-1}G_i](A_i+\alpha_{i,k}\Delta A_i)g(x_k)\\
&\quad \le g^T(x_k)(A_i+\alpha_{i,k}\Delta A_i)^T \widetilde{P}_i(A_i+\alpha_{i,k}\Delta A_i)g(x_k) - g^T(x_k)(A_i+\alpha_{i,k}\Delta A_i)^T G_i^T (G_iB_i)^{-1} G_i(A_i+\alpha_{i,k}\Delta A_i)g(x_k)\\
&\quad \le g^T(x_k)(A_i+\alpha_{i,k}\Delta A_i)^T \widetilde{P}_i(A_i+\alpha_{i,k}\Delta A_i)g(x_k)\\
&\quad \le 2g^T(x_k)A_i^T \widetilde{P}_i A_i g(x_k) + 2\alpha_{i,k}^2 g^T(x_k)\Delta A_i^T \widetilde{P}_i \Delta A_i g(x_k).
\end{aligned}
\tag{36}
$$

Substituting (32)–(36) into (31) yields

$$
\begin{aligned}
&\mathbb{E}\Big\{x_{k+1}^T \widetilde{P}_i x_{k+1} | \eta_k, i, \theta_k = 1\Big\}\\
&\quad \le \mathbb{E}\Big\{8g^T(x_k)A_i^T \widetilde{P}_i A_i g(x_k) + 8\alpha_{i,k}^2 g^T(x_k)\Delta A_i^T \widetilde{P}_i \Delta A_i g(x_k) + 3\left(\frac{\bar{\theta}}{1-\bar{\theta}}\right)^2 x_k^T A_i^T G_i^T (G_iB_i)^{-1} G_i A_i x_k
\end{aligned}
$$

$$+2h^T(x_{k-d_k})A_{di}^T\widetilde{P}_iA_{di}h(x_{k-d_k})+g^T(x_k)E_i^T\widetilde{P}_iE_ig(x_k)+3\frac{1}{(1-\bar{\theta})^2}[\Delta(k)-\Delta_o-\Delta_s\mathrm{sgn}(s_{c,k})]^T$$
$$\times(G_iB_i)^{-1}[\Delta(k)-\Delta_o-\Delta_s\mathrm{sgn}(s_{c,k})]\Big\}. \tag{37}$$

Additionally, noticing $\|\Delta(k)-\Delta_o-\Delta_s\mathrm{sgn}(s_c(k))\|\le 2\|\Delta_s\|$, we arrive at

$$\begin{aligned}
&\mathbb{E}\Big\{x_{k+1}^T\widetilde{P}_ix_{k+1}|\eta_k,i,\theta_k=1\Big\}\\
&\le\mathbb{E}\Big\{8g^T(x_k)A_i^T\widetilde{P}_iA_ig(x_k)+8(\bar{\alpha}_i+\epsilon_i)g^T(x_k)\Delta A_i^T\widetilde{P}_i\Delta A_ig(x_k)+3\left(\frac{\bar{\theta}}{1-\bar{\theta}}\right)^2x_k^TA_i^TG_i^T(G_iB_i)^{-1}G_iA_ix_k\\
&+2h^T(x_{k-d_k})A_{di}^T\widetilde{P}_iA_{di}h(x_{k-d_k})+g^T(x_k)E_i^T\widetilde{P}_iE_ig(x_k)+\frac{12}{(1-\bar{\theta})^2}\|(G_iB_i)^{-1}\|\|\Delta_s\|^2\Big\}.
\end{aligned}\tag{38}$$

Similarly, we have

$$\begin{aligned}
&\mathbb{E}\Big\{s_{k+1}^Ts_{k+1}\Big|\eta_k,i,\theta_k=1\Big\}\\
&=\mathbb{E}\Big\{\bar{\theta}^2x_k^TA_i^TG_i^TG_iA_ix_k+\bar{\theta}^2x_{c,k-1}^TA_i^TG_i^TG_iA_ix_{c,k-1}+2\big(-\bar{\theta}G_iA_ix_k\big)^T\big[\bar{\theta}G_iA_ix_{c,k-1}\big]\\
&+[\Delta(k)-\Delta_o-\Delta_s\mathrm{sgn}(s_{c,k})]^T[\Delta(k)-\Delta_o-\Delta_s\mathrm{sgn}(s_{c,k})]+2(-\bar{\theta}G_iA_ix_k)^T[\Delta(k)-\Delta_o-\Delta_s\mathrm{sgn}(s_{c,k})]\\
&+2\big[\bar{\theta}G_iA_ix_{c,k-1}\big]^T[\Delta(k)-\Delta_o-\Delta_s\mathrm{sgn}(s_{c,k})]\Big\}\\
&\le\mathbb{E}\Big\{3\bar{\theta}^2x_k^TA_i^TG_i^TG_iA_ix_k+3\bar{\theta}^2x_{c,k-1}^TA_i^TG_i^TG_iA_ix_{c,k-1}+12\|\Delta_s\|^2\Big\}.
\end{aligned}\tag{39}$$

Besides, together with the inequalities (38), (39) and the following equality

$$\begin{aligned}
\mathbb{E}\Big\{x_{c,k}^T\widetilde{Q}_i(\theta_{k+1})x_{c,k}\Big|\eta_k,i,\theta_k=1\Big\}&=\mathbb{E}\Big\{x_k^T\widetilde{Q}_i(\theta_{k+1})x_k\Big|\eta_k,i,\theta_k=1\Big\}\\
&=\mathbb{E}\Big\{x_k^T\big[p\widetilde{Q}_i(0)+(1-p)\widetilde{Q}_i(1)\big]x_k\Big\},
\end{aligned}\tag{40}$$

it can be obtained that

$$\begin{aligned}
&\mathbb{E}\Big\{V(\eta_{k+1},r_{k+1},\theta_{k+1})|\eta_k,i,\theta_k=1\Big\}-V(\eta_k,i,\theta_k=1)\\
&\le\mathbb{E}\Big\{8g^T(x_k)A_i^T\widetilde{P}_iA_ig(x_k)+8(\bar{\alpha}_i+\epsilon_i)g^T(x_k)\Delta A_i^T\widetilde{P}_i\Delta A_ig(x_k)\\
&+3\left(\frac{\bar{\theta}}{1-\bar{\theta}}\right)^2x_k^TA_i^TG_i^T(G_iB_i)^{-1}G_iA_ix_k+2h^T(x_{k-d_k})A_{di}^T\widetilde{P}_iA_{di}h(x_{k-d_k})+g^T(x_k)E_i^T\widetilde{P}_iE_ig(x_k)\\
&+\frac{12}{(1-\bar{\theta})^2}\|(G_iB_i)^{-1}\|\|\Delta_s\|^2+px_k^T\widetilde{Q}_i(0)x_k+(1-p)x_k^T\widetilde{Q}_i(1)x_k+3\bar{\theta}^2x_k^TA_i^TG_i^TG_iA_ix_k\\
&+3\bar{\theta}^2x_{c,k-1}^TA_i^TG_i^TG_iA_ix_{c,k-1}+12\|\Delta_s\|^2-x_k^TP_ix_k-x_{c,k-1}^TQ_i(1)x_{c,k-1}-s_k^Ts_k\\
&+(d_M-d_m+1)x_k^TRx_k-x_{k-d_k}^TRx_{k-d_k}\Big\}.
\end{aligned}\tag{41}$$

Using Lemma 4 to inequality (41) as in Theorem 1, we can conclude

$$\begin{aligned}
&\mathbb{E}\Big\{V(\eta_{k+1},r_{k+1},\theta_{k+1})|\eta_k,i,\theta_k=1\Big\}-V(\eta_k,i,\theta_k=1)\\
&\le\mathbb{E}\Big\{x_k^T\Big[p\widetilde{Q}_i(0)+(1-p)\widetilde{Q}_i(1)+(d_M-d_m+1)R+3\bar{\theta}^2A_i^TG_i^TG_iA_i+3\left(\frac{\bar{\theta}}{1-\bar{\theta}}\right)^2A_i^TG_i^T(G_iB_i)^{-1}G_iA_i\\
&-\delta_iP_i\Big]x_k-\delta x_{k-d_k}^TRx_{k-d_k}+x_{c,k-1}^T\Big[3\bar{\theta}^2A_i^TG_i^TG_iA_i-Q_i(1)\Big]x_{c,k-1}+8g^T(x_k)A_i^T\widetilde{P}_iA_ig(x_k)\\
&+8(\bar{\alpha}_i+\epsilon_i)g^T(x_k)\Delta A_i^T\widetilde{P}_i\Delta A_ig(x_k)+g^T(x_k)E_i^T\widetilde{P}_iE_ig(x_k)-(1-\delta_i)g^T(x_k)P_ig(x_k)\\
&+2h^T(x_{k-d_k})A_{di}^T\widetilde{P}_iA_{di}h(x_{k-d_k})-(1-\delta)h^T(x_{k-d_k})Rh(x_{k-d_k})+\beta\|\Delta_s\|^2-s_k^Ts_k\Big\}
\end{aligned}$$

with scalars $\delta_i \in (0,\ 1)$, $\delta \in (0,\ 1)$ and $\beta = \frac{12}{(1-\bar{\theta})^2}\|(G_iB_i)^{-1}\| + 12$. When the system states escape a specified region of the sliding surface, i.e., $\|s_k\| \geq \sqrt{\frac{\beta}{1-\gamma}}\|\Delta_s\|$ with $\gamma \in (0,\ 1)$, we have

$$\begin{aligned}
\mathbb{E}\{\Delta V(\eta_k, i, \theta_k = 1)\} &= \mathbb{E}\Big\{V(\eta_{k+1}, r_{k+1}, \theta_{k+1})|\eta_k, i, \theta_k = 1\Big\} - V(\eta_k, i, \theta_k = 1) \\
&\leq \mathbb{E}\Big\{x_k^T\Big[p\widetilde{Q}_i(0) + (1-p)\widetilde{Q}_i(1) + (d_M - d_m + 1)R + 3\bar{\theta}^2 A_i^T G_i^T G_i A_i \\
&\quad + 3\left(\frac{\bar{\theta}}{1-\bar{\theta}}\right)^2 A_i^T G_i^T (G_iB_i)^{-1} G_i A_i - \delta_i P_i\Big]x_k - \delta x_{k-d_k}^T R x_{k-d_k} \\
&\quad + x_{c,k-1}^T\Big[3\bar{\theta}^2 A_i^T G_i^T G_i A_i - Q_i(1)\Big]x_{c,k-1} + 8g^T(x_k)A_i^T\widetilde{P}_i A_i g(x_k) \\
&\quad + 8(\bar{\alpha}_i + \epsilon_i)g^T(x_k)\Delta A_i^T\widetilde{P}_i\Delta A_i g(x_k) + g^T(x_k)E_i^T\widetilde{P}_i E_i g(x_k) - (1-\delta_i)g^T(x_k)P_i g(x_k) \\
&\quad + 2h^T(x_{k-d_k})A_{di}^T\widetilde{P}_i A_{di} h(x_{k-d_k}) - (1-\delta)h^T(x_{k-d_k})Rh(x_{k-d_k}) - \gamma s_k^T s_k\Big\} \\
&= \mathbb{E}\Big\{\eta_k^T\widetilde{\Xi}^i\eta_k\Big\}, \qquad (42)
\end{aligned}$$

where

$$\begin{aligned}
\widetilde{\Xi}^i &= \mathrm{diag}\{\widetilde{\Xi}_{11}^i, -\delta R, 3\bar{\theta}^2 A_i^T G_i^T G_i A_i - Q_i(1), \widetilde{\Xi}_{44}^i, 2A_{di}^T\widetilde{P}_i A_{di} - (1-\delta)R, -\gamma I\}, \\
\widetilde{\Xi}_{11}^i &= p\widetilde{Q}_i(0) + (1-p)\widetilde{Q}_i(1) + (d_M - d_m + 1)R + 3\bar{\theta}^2 A_i^T G_i^T G_i A_i + 3\left(\frac{\bar{\theta}}{1-\bar{\theta}}\right)^2 A_i^T G_i^T (G_iB_i)^{-1} G_i A_i - \delta_i P_i, \\
\widetilde{\Xi}_{44}^i &= 8A_i^T\widetilde{P}_i A_i + 8(\bar{\alpha}_i + \epsilon_i)\Delta A_i^T\widetilde{P}_i\Delta A_i + E_i^T\widetilde{P}_i E_i - (1-\delta_i)P_i,
\end{aligned}$$

and $\eta_k$ is defined below (29).

Next, the matrix $\widetilde{\Xi}^i$ in (42) can be rewritten by the following form:

$$\begin{aligned}
\widetilde{\Xi}^i = {} & \begin{bmatrix} K_1 & 0 & K_3 & K_4 & K_5 & 0 \end{bmatrix}^T \widetilde{\Pi}^{-1} \begin{bmatrix} K_1 & 0 & K_3 & K_4 & K_5 & 0 \end{bmatrix} \\
& + \mathrm{diag}\{-\delta_i P_i, -\delta R, -Q_i(1), -(1-\delta_i)P_i, -(1-\delta)R, -\gamma I\},
\end{aligned}$$

where

$$\begin{aligned}
\widetilde{\Pi} &= \mathrm{diag}\Big\{p\widetilde{Q}_i(0), (1-p)\widetilde{Q}_i(1), (d_M - d_m + 1)R, 3I, 3G_iB_i, 3I, 8\widetilde{P}_i, 8(\bar{\alpha}_i + \epsilon_i)\widetilde{P}_i, \widetilde{P}_i, 2\widetilde{P}_i\Big\} \\
K_1 &= \begin{bmatrix} p\widetilde{Q}_i(0) & (1-p)\widetilde{Q}_i(1) & (d_M - d_m + 1)R & 3\bar{\theta}A_i^T G_i^T & \dfrac{3\bar{\theta}}{1-\bar{\theta}}A_i^T G_i^T & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T, \\
K_3 &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 3\bar{\theta}A_i^T G_i^T & 0 & 0 & 0 & 0 \end{bmatrix}^T, \\
K_4 &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 8A_i^T\widetilde{P}_i & 8(\bar{\alpha}_i + \epsilon_i)\Delta A_i^T\widetilde{P}_i & E_i^T\widetilde{P}_i & 0 \end{bmatrix}^T, \\
K_5 &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2A_{di}^T\widetilde{P}_i \end{bmatrix}^T.
\end{aligned}$$

It follows from Lemma 2 that $\widetilde{\Xi}^i < 0$ is equivalent to

$$\Lambda^i = \begin{bmatrix} \Lambda_{11}^i & * & * \\ \Lambda_{21}^i & \Lambda_{22}^i & * \\ \Lambda_{31}^i & 0 & \Lambda_{33}^i \end{bmatrix} < 0,$$

where

$$\begin{aligned}
\Lambda_{11}^i &= \mathrm{diag}\{-\delta_i P_i, -\delta R, -Q_i(1), -(1-\delta_i)P_i, -(1-\delta)R, -\gamma I\}, \\
\Lambda_{21}^i &= \begin{bmatrix} p\widetilde{Q}_i(0) & 0 & 0 & 0 & 0 & 0 \\ (1-p)\widetilde{Q}_i(1) & 0 & 0 & 0 & 0 & 0 \\ (d_M - d_m + 1)R & 0 & 0 & 0 & 0 & 0 \\ 3\bar{\theta}G_iA_i & 0 & 0 & 0 & 0 & 0 \\ \dfrac{3\bar{\theta}}{1-\bar{\theta}}G_iA_i & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\
\Lambda_{22}^i &= \mathrm{diag}\{-p\widetilde{Q}_i(0), -(1-p)\widetilde{Q}_i(1), -(d_M - d_m + 1)R, -3I, -3G_iB_i\},
\end{aligned}$$

$$\Lambda_{31}^{i}=\begin{bmatrix}0&0&3\bar{\theta}G_iA_i&0&0&0\\0&0&0&8\widetilde{P}_iA_i&0&0\\0&0&0&8(\bar{\alpha}_i+\epsilon_i)\widetilde{P}_i\Delta A_i&0&0\\0&0&0&\widetilde{P}_iE_i&0&0\\0&0&0&0&2\widetilde{P}_iA_{di}&0\end{bmatrix},$$
$$\Lambda_{33}^{i}=\mathrm{diag}\{-3I,-8\widetilde{P}_i,-8(\bar{\alpha}_i+\epsilon_i)\widetilde{P}_i,-\widetilde{P}_i,-2\widetilde{P}_i\}.$$

Further, we rewrite the above inequality by

$$\widetilde{\Lambda}^{i}+\widetilde{M}_iF_i\widetilde{N}_i+(\widetilde{M}_iF_i\widetilde{N}_i)^{T}<0,$$

where

$$\widetilde{\Lambda}^{i}=\begin{bmatrix}\widetilde{\Lambda}_{11}^{i}&*&*\\\widetilde{\Lambda}_{21}^{i}&\widetilde{\Lambda}_{22}^{i}&*\\\widetilde{\Lambda}_{31}^{i}&0&\widetilde{\Lambda}_{33}^{i}\end{bmatrix},$$
$$\widetilde{\Lambda}_{11}^{i}=\mathrm{diag}\{-\delta_iP_i,-\delta R,-Q_i(1),-(1-\delta_i)P_i,-(1-\delta)R,-\gamma I\},$$
$$\widetilde{\Lambda}_{21}^{i}=\begin{bmatrix}p\widetilde{Q}_i(0)&0&0&0&0&0\\(1-p)\widetilde{Q}_i(1)&0&0&0&0&0\\(d_M-d_m+1)R&0&0&0&0&0\\3\bar{\theta}G_iA_i&0&0&0&0&0\\\dfrac{3\bar{\theta}}{1-\bar{\theta}}G_iA_i&0&0&0&0&0\end{bmatrix},$$
$$\widetilde{\Lambda}_{22}^{i}=\mathrm{diag}\{-p\widetilde{Q}_i(0),-(1-p)\widetilde{Q}_i(1),-(d_M-d_m+1)R,-3I,-3G_iB_i\},$$
$$\widetilde{\Lambda}_{31}^{i}=\begin{bmatrix}0&0&3\bar{\theta}G_iA_i&0&0&0\\0&0&0&8\widetilde{P}_iA_i&0&0\\0&0&0&0&0&0\\0&0&0&\widetilde{P}_iE_i&0&0\\0&0&0&0&2\widetilde{P}_iA_{di}&0\end{bmatrix},$$
$$\widetilde{\Lambda}_{33}^{i}=\mathrm{diag}\{-3I,-8\widetilde{P}_i,-8(\bar{\alpha}_i+\epsilon_i)\widetilde{P}_i,-\widetilde{P}_i,-2\widetilde{P}_i\},$$
$$\widetilde{M}_i^{T}=\begin{bmatrix}0&0&0&0&0&0&0&0&0&0&0&0&0&8(\bar{\alpha}_i+\epsilon_i)M_i^{T}\widetilde{P}_i&0&0\end{bmatrix},$$
$$\widetilde{N}_i=\begin{bmatrix}0&0&0&N_i&0&0&0&0&0&0&0&0&0&0&0&0\end{bmatrix}.$$

It follows from Lemma 3 that

$$\widetilde{\Lambda}^{i}+\varepsilon_i^{-1}\widetilde{M}_i\widetilde{M}_i^{T}+\varepsilon_i\widetilde{N}_i^{T}\widetilde{N}_i<0.$$

Finally, according to the Lemma 2, we conclude that $\mathbb{E}\{\Delta V_k^i|\xi_k,i,\theta_k=1\}<0$ can be guaranteed by (26) when $\|s_k\|\geq\sqrt{\frac{\beta}{1-\gamma}}\|\Delta_s\|$. As in Theorem 1, the condition (27) is given subject to $\theta_k=0$. In conclusion, $\mathbb{E}\{\Delta V_k^i|\xi_k,i,\theta_k=1\}<0$ holds. Thus, the states of system (1) can be are driven (in mean square sense) onto a specified neighborhood of sliding surface (6) via the SMC law (25). □

**Remark 5.** In Theorem 2, a mode-dependent sufficient criterion is proposed to ensure the desirable performance of the addressed robust SMC problem. However, in view of the existences of the diagonally dominant matrices, the feasibility of above proposed criterion is not easy to be checked. As in [45], this inconvenient constrained condition can be removed via adding some alternative inequalities, which is addressed in the following theorem.

**Theorem 3.** *Given scalars $\delta_i\in(0,1)$ $(i\in\mathcal{N})$ and $\delta\in(0,1)$, if there exist positive-definite matrices $0<P_i:=[p_{\mu\nu}(i)]\in\mathbb{R}^{n\times n}$, $0<R:=[r_{\mu\nu}]\in\mathbb{R}^{n\times n}$, $Q_i(0)>0$ and $Q_i(1)>0$, symmetric matrices $H_i:=[h_{\mu\nu}(i)]\in\mathbb{R}^{n\times n}$ and $K:=[k_{\mu\nu}]\in\mathbb{R}^{n\times n}$, scalar $\varepsilon_i>0$ satisfying (12), (26), (27) and the following conditions*

$$\forall\mu\neq\nu,\quad h_{\mu\nu}(i)\geq 0,\quad k_{\mu\nu}\geq 0,\tag{43}$$

$$\forall\mu\neq\nu,\quad p_{\mu\nu}(i)+h_{\mu\nu}(i)\geq 0,\quad r_{\mu\nu}+k_{\mu\nu}\geq 0,\tag{44}$$

$$\forall\mu,\quad p_{\mu\mu}(i)-\sum_{\mu\neq\nu}(p_{\mu\nu}(i)+2h_{\mu\nu}(i))\geq 0,\quad r_{\mu\mu}-\sum_{\mu\neq\nu}(r_{\mu\nu}+2k_{\mu\nu})\geq 0,\tag{45}$$

*then the states of system (1) can be driven (in mean square sense) onto a specified neighborhood of sliding surface by SMC law (25).*

**Remark 6.** In Theorem 3, the diagonally dominant condition is completely eliminated, and the stability criterion of resultant sliding motion is obtained. It is necessary to note that the equation $B_i^T \widetilde{P}_i J_i = 0$ in (12) also leads to certain inconvenience of examining the feasibility of obtained method. In order to address this difficulty, an LMIs-based minimization algorithm will be proposed later.

*3.3. A Computational Method*

It is worthwhile to point out that the results obtained in Theorem 3 are not strict LMIs due to the existence of the constraint $B_i^T \widetilde{P}_i J_i = 0$ $(i \in \mathcal{N})$. According to the technique as in [46], the equation constraint $B_i^T \widetilde{P}_i J_i = 0$ is expressed equivalently by $\mathrm{tr}[(B_i^T \widetilde{P}_i J_i)^T B_i^T \widetilde{P}_i J_i] = 0$. Next, the inequality $(B_i^T \widetilde{P}_i J_i)^T B_i^T \widetilde{P}_i J_i \le \zeta I$ is introduced with $\zeta > 0$. From Lemma 2, we have

$$\begin{bmatrix} -\zeta I & J_i^T \widetilde{P}_i B_i \\ B_i^T \widetilde{P}_i J_i & -I \end{bmatrix} \le 0. \tag{46}$$

Then, we can obtain the following equivalent minimization problem:

$$\begin{aligned} &\min \quad \zeta \\ &\text{s.t.} \quad (26), (27) \text{ and } (43)-(46). \end{aligned} \tag{47}$$

**Remark 7.** It should be noticed that the solvability of the equation constraint condition $B_i^T \widetilde{P}_i J_i = 0$ involved in the main results is converted to the feasibility of the inequality (46) by introducing a scalar $\zeta$. When the scalar approaches 0, then the equation constraint condition is seen to be achieved approximately. Thus, the fore-mentioned difficulty induced by $B_i^T \widetilde{P}_i J_i = 0$ is overcome. Obviously, the minimization problem (47) is presented in terms of LMIs, which is involved some linear indices/parameters and can be easily testified by using standard software. Consequently, it can be concluded that the original non-convex problem is ultimately solved now.

**Remark 8.** Up to now, the sliding-mode-based control problem has been solved for a class of delayed Markovian jump RSNSs with PDs and ROUs under UOPs. Here, the time-varying delays with known upper and lower bounds have been discussed, and the delay-dependent criteria have been proposed to ensure the stability and reachability condition in discrete-time setting. It is noted that new Lyapunov–Krasovskii functional incorporated with the delay information has been introduced when deriving main results. In order to reduce the conservativeness caused by the time-delays, the delay-fractioning method as in [14,42] can be utilized, where the delay-fractioning idea can be introduced when constructing the Lyapunov–Krasovskii functional and then the corresponding SMC method can be obtained readily. On the other hand, the major difficulties we met include: a) A proper model has been established to comprehensive characterize the behaviours of Markovian jump characteristics, nonlinear disturbances, random modelling errors under imprecise occurrence probabilities, PDs as well as time-varying bounded delays; and b) an efficient robust SMC method has been given to examine the impacts of mentioned phenomena onto the whole control performance, where the related information has been explicitly reflected in main results. In particular, the parameters $M_i$, $N_i$, $\bar{\alpha}_i$ and $\epsilon_i$ refer to the ROUs under UOPs, the scalars $p$ and $q$ correspond to the PDs, the scalars $d_m$ and $d_M$ reflect the information of time-varying bounded delays. Compared with the existing SMC methods, this paper represents one of first attempt to develop a sliding-mode-based control technique for a class of Markovian jump RSNSs with time-varying delays, PDs and ROUs under UOPs, which can enrich the SMC methodologies regarding to the discrete-time Markovian jump systems in the networked environments.

## 4. A simulation example

In this section, the following numerical simulations are utilized to illustrate the effectiveness of obtained sliding-mode-based control scheme.

The related parameters of system (1) are given by:

$$\begin{aligned}
&A_1 = \begin{bmatrix} 0.15 & -0.25 & 0 \\ 0 & 0.13 & 0.01 \\ 0.03 & 0 & -0.05 \end{bmatrix}, \quad A_{d1} = \begin{bmatrix} -0.03 & 0 & 0.01 \\ 0.02 & 0.03 & 0 \\ 0.04 & 0.05 & -0.01 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -0.01 & 0.023 \\ 0.05 & 0.05 \\ 0.04 & 0.03 \end{bmatrix}, \\
&E_1 = \begin{bmatrix} 0.015 & 0 & -0.01 \\ 0.017 & 0.015 & 0 \\ 0.02 & 0.025 & -0.01 \end{bmatrix}, \quad M_1 = \begin{bmatrix} 0.1 \\ 0.2 \\ 0.05 \end{bmatrix}, \quad N_1 = \begin{bmatrix} 0.3 \\ 0.1 \\ 0.5 \end{bmatrix}, \\
&A_2 = \begin{bmatrix} 0.12 & -0.06 & 0.17 \\ 0.06 & 0.04 & -0.13 \\ 0.01 & -0.02 & 0.14 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} -0.03 & 0 & 0.01 \\ 0.02 & 0.03 & 0 \\ 0.04 & 0.05 & -0.01 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.08 & -0.017 \\ 0.01 & 0.04 \\ 0.03 & 0.026 \end{bmatrix}, \\
&E_2 = \begin{bmatrix} 0.015 & 0 & -0.01 \\ 0.017 & 0.015 & 0 \\ 0.02 & 0.025 & -0.01 \end{bmatrix}, \quad M_2 = \begin{bmatrix} 0.2 \\ 0.1 \\ 0.3 \end{bmatrix}, \quad N_2 = \begin{bmatrix} 0.01 \\ 0.02 \\ 0.04 \end{bmatrix}, \\
&F_1 = \sin(0.2k), \quad F_2 = \sin(0.6k).
\end{aligned}$$

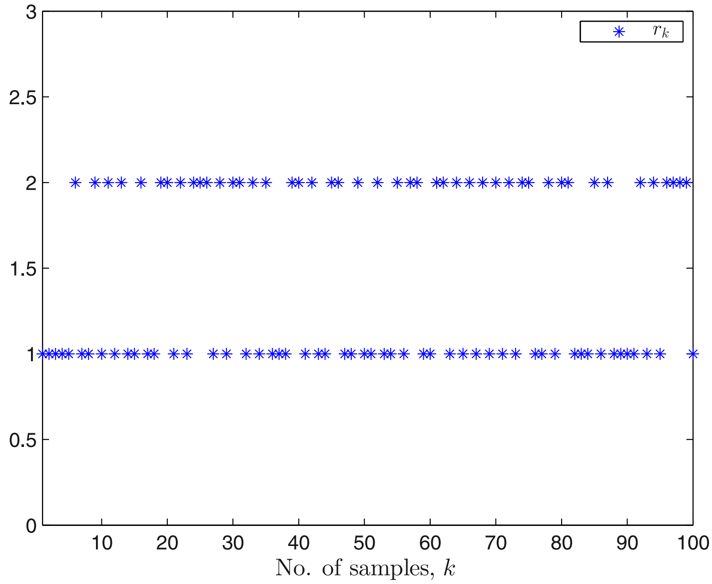


**Fig. 1.** Random mode $r_k$ (Case 1).

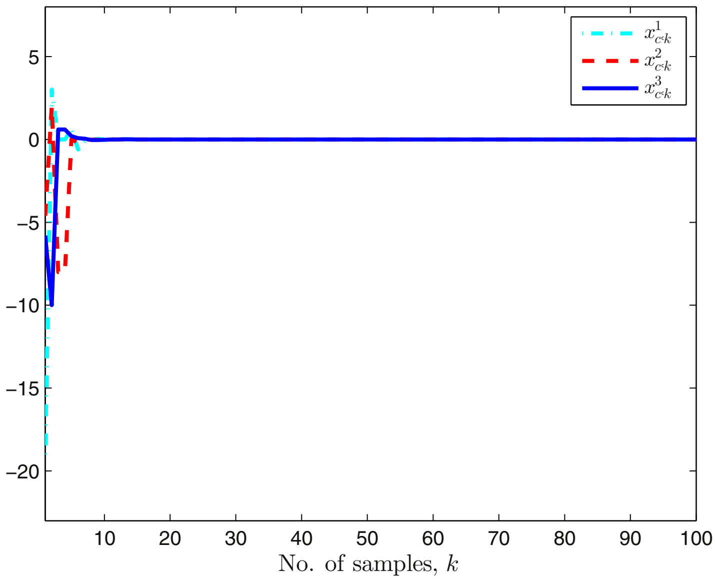


**Fig. 2.** The response of state $x_{c,k}$ (Case 1).

The nonlinearities are set as

$$g(x_k) = h(x_k) = \sin(x_k), \quad f_1 = f_2 = \begin{bmatrix} 0.49\sin(x_k^1 x_k^3) & 0.13\sin(x_k^2) \end{bmatrix}^T,$$

with $x_k^l$ $(l = 1, 2, 3)$ being the $l$-th element of $x_k$. Let $\bar{\alpha}_1 = 0.7$, $\bar{\alpha}_2 = 0.75$, $\epsilon_1 = \epsilon_2 = 0.2$. We set the upper/lower bounds of $d_k$ as $d_m = 2$ and $d_M = 5$. In addition, the bounds of $\delta^j_{i,a}(k)$, $\delta^j_{i,f}(k)$ and $\delta^j_{i,g}(k)$ are given by

$$\begin{aligned}
&\bar{\delta}^j_{i,a} = \frac{q}{p+q} \parallel G_i M_i \parallel \parallel N_i g(x_k) \parallel, \quad \underline{\delta}^j_{i,a} = -\frac{q}{p+q} \parallel G_i M_i \parallel \parallel N_i g(x_k) \parallel, \\
&\bar{\delta}^j_{i,f} = \frac{q}{p+q} \parallel G_i \parallel \parallel B_i f_{i,k} \parallel, \quad \underline{\delta}^j_{i,f} = -\frac{q}{p+q} \parallel G_i \parallel \parallel B_i f_{i,k} \parallel, \quad \bar{\delta}^j_{i,g} = \frac{q}{p+q} \parallel G_i A_i \parallel \parallel g(x_k) - x_k \parallel, \\
&\underline{\delta}^j_{i,g} = -\frac{q}{p+q} \parallel G_i A_i \parallel \parallel g(x_k) - x_k \parallel \quad (i, j = 1, 2).
\end{aligned}$$

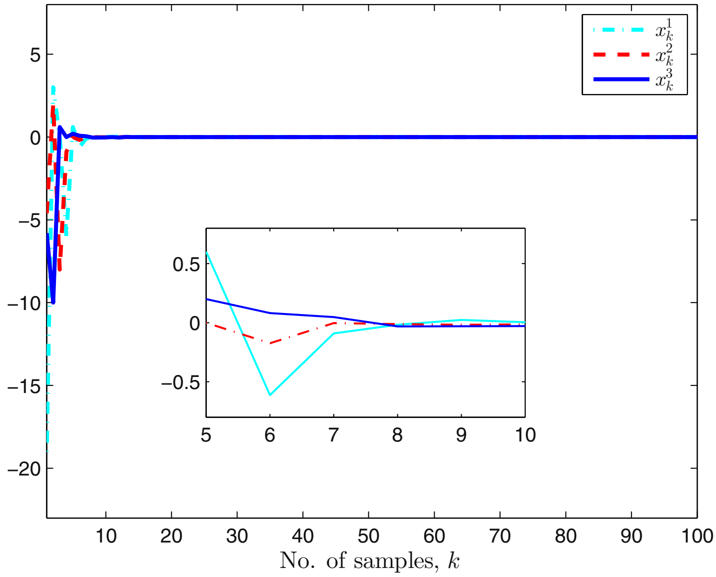


**Fig. 3.** The response of state $x_k$ (Case 1).

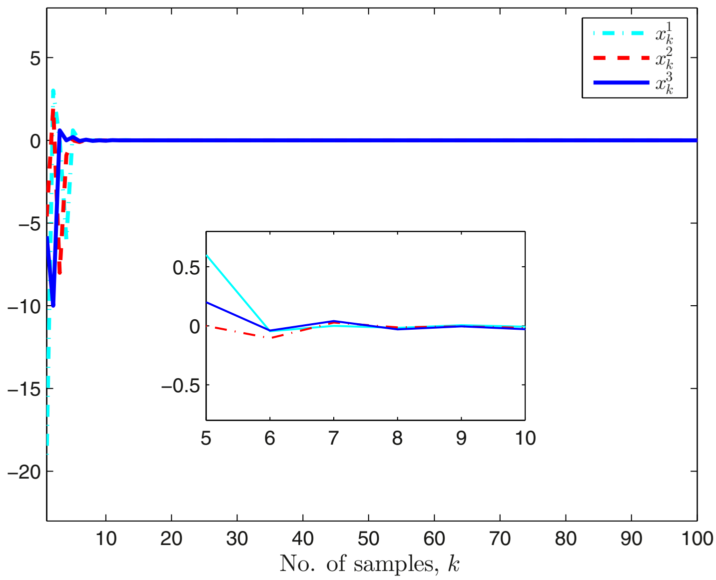


**Fig. 4.** The response of state $x_k$ (Case 2).

Then, for prescribed scalars $\delta_1 = 0.3$, $\delta_2 = 0.6$, $\delta = 0.2$ and $\gamma = 0.6$, solving the minimization problem (47) leads to

$$P_1 = \begin{bmatrix} 0.8853 & -0.4949 & -0.1373 \\ -0.4949 & 2.4617 & -0.2697 \\ -0.1373 & -0.2697 & 2.0662 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 0.6655 & -0.0201 & 0.5961 \\ -0.0201 & 2.8536 & -2.8205 \\ 0.5961 & -2.8205 & 9.2380 \end{bmatrix},$$

$$G_1 = \begin{bmatrix} -0.0391 & 0.1172 & 0.0705 \\ -0.0085 & 0.1036 & 0.0453 \end{bmatrix}, \quad G_2 = \begin{bmatrix} 0.0709 & -0.0577 & 0.2966 \\ 0.0034 & 0.0412 & 0.1172 \end{bmatrix},$$

$$\varepsilon_1 = 1.7156, \quad \varepsilon_2 = 7.9650, \quad \zeta = 3.3134 \times 10^{-4}.$$

In the simulation, by applying the newly proposed SMC law as in (25), the corresponding simulating results are shown in Figs. 1–3, 5 and Fig. 7 under Case 1 ($p = 0.3$, $q = 0.5$). Among them, Fig. 1 illustrates one possible realization of the mode $r_k$. It is easy to see from Figs. 2 and 3 that the signal $x_{c,k}$ and system state response $x_k$ rapidly approach the small neighborhood of the equilibrium point. The switching surface $s_k$ is plotted in Fig. 5. In addition, Fig. 7 represents the control

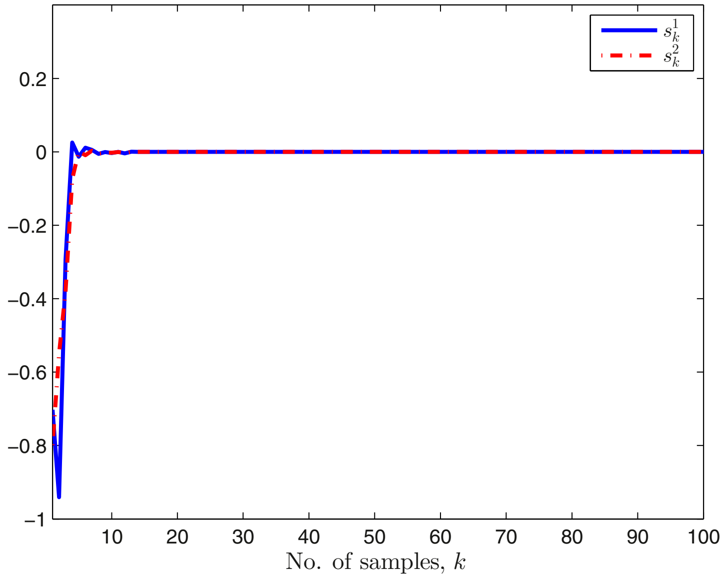


**Fig. 5.** The response of sliding mode variable $s_k$ (Case 1).

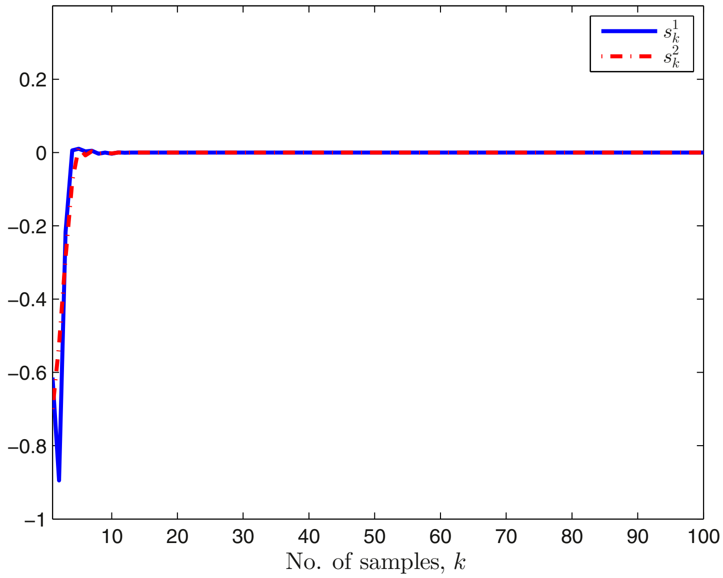


**Fig. 6.** The response of sliding mode variable $s_k$ (Case 2).

input $u_k$. Then, the following conclusion can be drawn from above simulations that the newly developed sliding-mode-based control method performs a satisfactory performance.

Furthermore, in order to further show the effectiveness and advantages of the proposed robust SMC scheme, we conduct additional simulation experiments for different PDs case. For more details, another case of the PDs is considered, i.e. Case 2 ($p = 0.1$ and $q = 0.8$). Accordingly, the comparison results can be obtained based on the proposed robust SMC method, see Figs. 4, 6 and 8 for more details. By comparing the Fig. 3 with Fig. 4, it can be found that the system response $x_k$ can reach the stable state faster under Case 2 as plotted in Fig. 4. In addition, it also can be observed from Figs. 7 and 8 that less control energy is needed for the Case 2. The reason is that the PDs in Case 2 are not severe and thus the more valid state information for the robust controller design can be obtained than the one in Case 1. As such, the control performance is improved in Case 2. Similar conclusions can be revealed by observing Figs. 5–8, which further demonstrate that better control performance can be achieved when the PDs are not severe (Case 2).

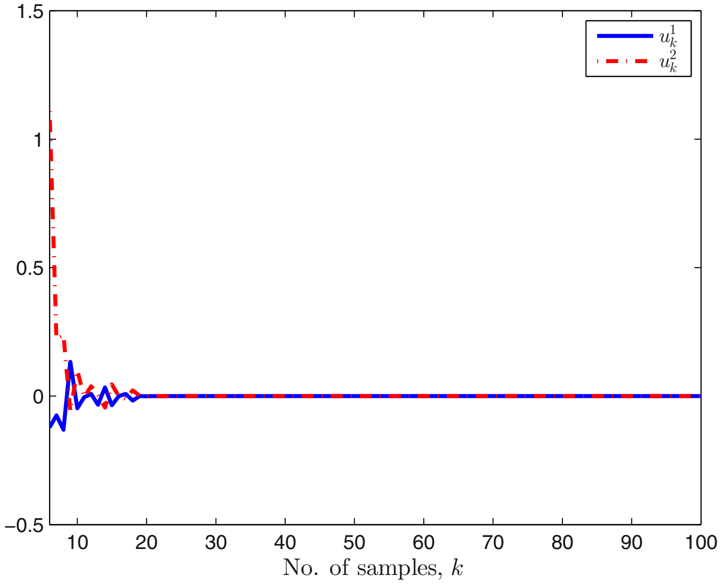


**Fig. 7.** The control input $u_k$ (Case 1).

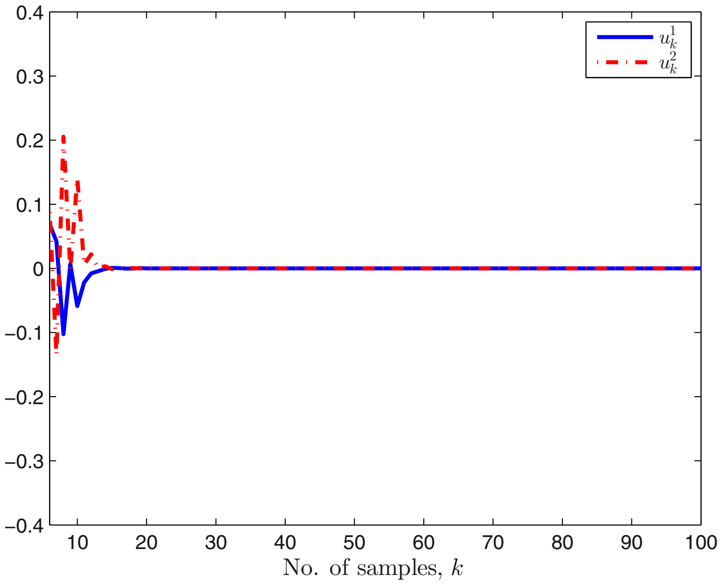


**Fig. 8.** The control input $u_k$ (Case 2).

## 5. Conclusions

In this paper, the problem of sliding-mode-based control has been addressed for discrete delayed Markovian jump RSNSs with Markovian PDs and ROUs. The key task is to design a mode-delay-dependent SMC method to reduce the effects induced by PDs and ROUs under UOPs. Firstly, a sliding surface dependent on the related dropouts information has been constructed. Then, by combining the diagonally dominant Lyapunov method, a mode-delay-dependent sufficient criterion has been presented such that the asymptotic stability of the sliding motion is guaranteed in mean-square sense. Moreover, a sliding-mode-based controller has been designed in order to ensure the reachability criterion. Finally, the numerical example has been given to show the effectiveness of obtained sliding-mode-based control scheme. It is worthwhile to point out that the main features include that a) a new robust SMC methodology has been developed for discrete delayed Markovian RSNSs, which has the advantages to handle the Markovian PDs and ROUs under UOPs; and b) the complexity of addressed topic is effective tackled by proposed control scheme. Based on the proposed results, further research topics include the robust SMC design of networked systems with input delay as in [47]. Moreover, the output-feedback-based SMC tolerant problem can be addressed for MJSs with time-varying delays and sensor failures when the state information is immeasurable.

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