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Short Paper

JUMP LINEAR H^∞ CONTROL: THE DISCRETE-TIME CASE*

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Abstract. This paper deals with the problem of H^∞ control via state feedback for a class of discrete-time linear systems with Markovian jumping parameters. Our approach relies heavily on the solution of a certain set of interconnected Riccati difference equations. The results obtained here are, to some extent, the discrete-time counterpart of those obtained in de Souza and Fragoso (1993).

Key Words— H^∞ control, linear systems, jumping parameters, discrete-time.

1. Introduction

事实 既成

It is a *fait accompli* that the standard H^∞ problem sets a generic framework for the linear-quadratic worst-case design. Besides being an attractive formulation for the control problem when exogenous 外源 signal uncertainty is considered, important issues such as stabilization of uncertain systems, tracking and model matching can be recast as a standard problem (see, for instance, Petersen, 1987). Furthermore, the H^∞ formulation appears naturally as the worst-case counterpart of the LQG problem. Initially, efforts to solve the standard H^∞ problem were phrased in terms of frequency domain concepts (see, e.g., Francis, 1987). These methods seem attractive when the minimum H^∞ norm of the closed-loop system needs to be achieved, although in many instances, the computation of the controller is rather involved.

In recent years, interest in the H^∞ control problem has shifted to the state-space setting. For linear time-invariant systems, it has been shown that the state feedback H^∞ control can be tackled via a sign-indefinite algebraic Riccati equation (see, e.g., Khargonekar et al., 1988; Petersen, 1987; 1989). For the output feedback case, it is now known that a solution to the problem can be found from the solution of two sign-indefinite algebraic Riccati equations (see, e.g., Doyle et al., 1989; Glover and Doyle, 1988; Green et al., 1990). More recently, the problem of H^∞ control for finite horizon, linear time-varying systems has been solved in Limebeer et al. (1992) and Tadmor (1990) in both the state and output feedback cases.

In this paper, we are mainly interested in the H^∞ control problem for a class of linear discrete-time systems whose structures are subject to abrupt parameter changes, modeled here by a discrete-time finite-state Markovian chain. These

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changes may be the consequence of abrupt phenomena, such as component and/or interconnection failures. (There are sometimes also called multiple regime behavior.) This is to be found, for instance, in aircraft control problems, large-scale flexible structures for space stations (such as antenna, mirrors, solar arrays, etc.), situations in which an actuator or a sensor failure is a common occurrence. Due, in part, to some successful applications, the interest in this class has increased steadly, witness the fairly extensive associated literature (see, for instance, Blair and Sworder (1975), Chizeck et al. (1986), Costa and Fragoso (1993), Fragoso (1989), Fragoso and Costa (1993 a; b), Ji and Chizeck (1988; 1989; 1990), Ji et al. (1991), and the references therein).

The approach here relies on the solution of a certain set of interconnected Riccati difference equations. The result obtained in this paper is, to some extent, the discrete-time counterpart of that obtained in de Souza and Fragoso (1993). As far as the authors are aware, these are the only two works dealing with the H^∞ control problem for the jump linear class depicted above (discrete and continuous version).

2. Problem Formulation

Fix a probability space $(\Omega, \mathscr{F}, P)$ and consider the following class of dynamical systems:

$$(\Sigma_1)\begin{cases} x_{k+1} = A(k, \theta_k)x_k + B_1(k, \theta_k)w_k + B_2(k, \theta_k)u_k, \\ x_0 = 0, \quad \theta_0 = i, \qquad 0 \le k \le N-1 \end{cases}$$

with r-dimensional controlled output

$$z_k = C(k, \theta_k)x_k + D(k, \theta_k)u_k, \tag{1}$$

and assume that

$$D^T(k, \theta_k)[C(k, \theta_k) \;\; D(k, \theta_k)] = [0 \;\; R(k, \theta_k)], \tag{2}$$

$$R(k, \theta_k) = R^T(k, \theta_k) > 0, \tag{3}$$

where $x_k \in \mathscr{R}^n$ denotes the state vector, $w_k \in \mathscr{R}^q$ stands for the disturbance with $\{w_k\} \in l_2[0, N-1]$, the space of square summable sequences on $[0, N-1]$ and $\{u_k\}$ is a sequence of m-dimensional control functions. Furthermore, $\{\theta_k\}$ is a Markov chain with stationary transitions and finite state space $\mathscr{S} = \{1, 2, \cdots, N^*\}$, such that

$$P(\theta_{n+1} = j \mid \theta_0, \cdots, \theta_n = i) = P(\theta_{n+1} = j \mid \theta_n = i) \triangleq p_{ij}$$

and $\{A(k, i), B_1(k, i), B_2(k, i), C(k, i), D(k, i);\ i \in \mathscr{S}\}$ are real matrices of suitable dimensions for each k, $k = 0, 1, \cdots, N$.

Henceforth, we assume that $\{x_k\}$ and $\{\theta_k\}$ are directly accessible to the controller. Furthermore, in order to put the H^∞ control problem in a stochastic setting, we bring to bear the space $l_2([\Omega, \mathscr{F}, P], [0, N-1])$ of sequences, $\{z_k\}$, for which

$$\|z\|_2 = \left\{ E\left[\sum_{k=0}^{N-1} z_k^T z_k \right] \right\}^{\frac{1}{2}} < \infty,$$

where $E[\cdot]$ stands for the mathematical expectation. Moreover, from now on, we shall use indistinctly $\|\cdot\|_2$ to the norm either in $l_2[0, N-1]$ or in $l_2([\Omega, \mathscr{F}, P], [0, N-1])$, whenever the context makes it clear to which one we are referring.

This paper is mainly concerned with the problem of state feedback H^∞ control for the system (Σ_1). We consider the problem of designing a state feedback control law,

$$\boxed{u_k = u(k, x_k, \theta_k) = -K(k, \theta_k)x_k,}$$

such that for all $w \in l_2[0, N-1]$, $w \neq 0$,

$$\boxed{\|z\|_2 \leq \gamma \|w\|_2,}$$

where $\{z_k\}$ is the controlled output defined by (1), and $\gamma > 0$ is the prescribed level of disturbance attenuation to be achieved.

The H^∞ control problem for the infinite horizon case will also be addressed. In this situation, it will be assumed that matrices in (Σ_1) and (1) will be constant while the Markov chain remains in a certain state $j \in \mathscr{S}$. We consider the following class of Markovian jumping parameter systems:

$$\boxed{(\Sigma_2) \begin{cases} x_{k+1} = A(\theta_k)x_k + B_1(\theta_k)w_k + B_2(\theta_k)u_k, \\ x_0 = 0, \quad \theta_0 = i, \qquad k = 0, 1, 2, \cdots \end{cases}}$$

with controlled output

$$z_k = C(\theta_k)x_k + D(\theta_k)u_k, \tag{4}$$

where it is assumed that

$$D^T(\theta_k)[C(\theta_k) \;\; D(\theta_k)] = [0 \;\; R(\theta_k)], \tag{5}$$

$$R(\theta_k) = R^T(\theta_k) > 0. \tag{6}$$

The problem now is stated as follows. Given a prescribed level of disturbance attenuation $\gamma > 0$, design a control law,

$$u_k = u(x_k, \theta_k) = -K(\theta_k)x_k, \tag{7}$$

such that the closed-loop system (Σ_2) is *mean square stable* (see Definition 3.1), where the norm $\|\cdot\|_2$ is now given by

$$\|z\|_2 = \left\{ E\left[\sum_{k=0}^{\infty} z_k^T z_k \right] \right\}^{\frac{1}{2}}. \tag{8}$$

3. The H^∞ Control Problem

In this section, we exhibit the theorems that solve the state feedback H^∞ control problem for the Markovian jumping parameter systems described in Sec. 2. The Riccati equation approach has been adopted, and both the finite and infinite-horizon case have been considered.

3.1 The finite horizon case Following recent results in H^∞ control for time-varying systems without jumping parameters, the approach used here is based on matrix Riccati difference equations. The main difference here, *prima facie*, is that a solution to the control problem is given in terms of a set of interconnected Riccati difference equations, instead of a Riccati difference equation, as is the case in the context of linear time-varying systems without jumping parameters.

The main theorem in this subsection reads now as follows.

Theorem 3.1. Consider the system (Σ_1), with controlled output given by (1), and let $\gamma>0$ be a prescribed level of disturbance attenuation. Then, a state feedback controller exists such that

$$\|z\|_2 \leq \gamma \|w\|_2$$

for all $w\in l_2[0, N-1]$, $w\neq 0$, provided that $\{S(j, i);\ i\in\mathscr{S};\ j=0,1,\cdots,N-1\}$ are symmetric $(n\times n)$ matrices computed backward in time by the following set of interconnected discrete-time Riccati equations:

$$\begin{aligned}
S(j, i) = {} & C^T(j, i)C(j, i) + A^T(j, i)E_i[S(j+1, \theta_{j+1})]A(j, i) \\
& + A^T(j, i)E_i[S(j+1, \theta_{j+1})] \\
& \times B_1(j, i)[\gamma^2 I - B_1^T(j, i)E_i[S(j+1, \theta_{j+1})]B_1(j, i)]^{-1} \\
& \times B_1^T(j, i)E_i[S(j+1, \theta_{j+1})]A(j, i) \\
& - \{A^T(j, i)E_i[S(j+1, \theta_{j+1})] \\
& \times B_1(j, i)[\gamma^2 I - B_1^T(j, i)E_i[S(j+1, \theta_{j+1})]B_1(j, i)]^{-1} \\
& \times B_1^T(j, i)E_i[S(j+1, \theta_{j+1})]B_2(j, i) \\
& + A^T(j, i)E_i[S(j+1, \theta_{j+1})]B_2(j, i)\} \\
& \times H^{-1}(j, i)\{B_2^T(j, i)E_i[S(j+1, \theta_{j+1})]A(j, i) \\
& + B_2^T(j, i)E_i[S(j+1, \theta_{j+1})] \\
& \times B_1(j, i)[\gamma^2 I - B_1^T(j, i)E_i[S(j+1, \theta_{j+1})]B_1(j, i)]^{-1} \\
& \times B_1^T(j, i)E_i[S(j+1, \theta_{j+1})]A(j, i)\},
\end{aligned} \tag{9}$$

where

$$S(N, i) = 0, \quad i\in\mathscr{S} \text{ and } j = N-1, N-2, \cdots, 0$$

with

$$\gamma^2 I - B_1^T(j, i) E_i[S(j+1, \theta_{j+1})] B_1(j, i) > 0. \tag{10}$$

Moreover, a suitable control law is given by

$$u_j^*(x_j, i) = -H^{-1}(j, i) L(j, i) x_j \qquad \text{for} \quad \theta_j = i \in \mathscr{S} \tag{11}$$

with

$$\begin{aligned} H(j, i) \triangleq\ & R(j, i) + B_2^T(j, i) E_i[S(j+1, \theta_{j+1})] B_2(j, i) \\ & + B_2^T(j, i) E_i[S(j+1, \theta_{j+1})] \\ & \times B_1(j, i)[\gamma^2 I - B_1^T(j, i) E_i[S(j+1, \theta_{j+1})] B_1(j, i)]^{-1} \\ & \times B_1^T(j, i) E_i[S(j+1, \theta_{j+1})] B_2(j, i) \end{aligned} \tag{12}$$

and

$$\begin{aligned} L(j, i) \triangleq\ & B_2^T(j, i) E_i[S(j+1, \theta_{j+1})] A(j, i) \\ & + B_2^T(j, i) E_i[S(j+1, \theta_{j+1})] \\ & \times B_1(j, i)[\gamma^2 I - B_1^T(j, i) E_i[S(j+1, \theta_{j+1})] B_1(j, i)]^{-1} \\ & \times B_1^T(j, i) E_i[S(j+1, \theta_{j+1})] B_2(j, i), \end{aligned} \tag{13}$$

where

$$\boxed{\begin{aligned} E_i[S(j+1, \theta_{j+1})] & \triangleq E[S(j+1, \theta_{j+1}) | x_j, \theta_j = i] \\ & = \sum_{l=1}^{N^*} S(j+1, l) p_{il}, \end{aligned}}$$

and p_{il} is the probability of transition from state i $(\theta_j = i)$ to state l.

Proof. We begin by defining the following functional:

$$J(u) \triangleq E\left[\sum_{j=0}^{N-1} (z_j^T z_j - \gamma^2 w_j^T w_j)\right]. \tag{14}$$

The proof is carried out now by showing that $J(u^*) \leq 0$, where $u^*(\cdot)$ is given by expression (11). First, define the summation,

$$E\left[\sum_{j=0}^{N-1} (x_{j+1}^T S(j+1, \theta_{j+1}) x_{j+1} - x_j^T S(j, \theta_j) x_j)\right],$$

and notice that it is identically null, stemming from the fact that $S(N, \cdot) = 0$ and $x_0 = 0$. Adding this to $J(u)$, given by (14), we have

$$J(u) = E\left[\sum_{j=0}^{N-1} (z_j^T z_j + x_{j+1}^T S(j+1, \theta_{j+1}) x_{j+1} - x_j^T S(j, \theta_j) x_j - \gamma^2 w_j^T w_j)\right].$$

Fix now an arbitrary Markovian feedback control policy $\{u_j(x_j, \theta_j)\}$ with $\theta_j = i$, $i \in \mathscr{S}$, and consider, via equation (Σ_1), the corresponding sequence $\{x_j\}$.

Then,

$$\begin{aligned} J(u) &= E\left\{ \sum_{j=0}^{N-1} E[z_j^T z_j + x_{j+1}^T S(j+1, \theta_{j+1}) x_{j+1} \right. \\ &\qquad \left. - x_j^T S(j, \theta_j) x_j - \gamma^2 w_j^T w_j \,|\, x_j, \theta_j] \right\} \\ &= E\left\{ \sum_{j=0}^{N-1} [z_j^T z_j + E_i[x_{j+1}^T S(j+1, \theta_{j+1}) x_{j+1}] \right. \\ &\qquad \left. - x_j^T S(j, \theta_j) x_j - \gamma^2 w_j^T w_j] \right\}. \end{aligned}$$

Using again (Σ_1) and (1), (2), we have

$$\begin{aligned} J(u) = E\Bigg\{ & \sum_{j=0}^{N-1} \{x_j^T [C^T(j, \theta_j) C(j, \theta_j) - S(j, \theta_j)] x_j + u_j^T R(j, \theta_j) u_j \\ & + [w_j^T B_1^T(j, \theta_j) + u_j^T B_2^T(j, \theta_j) + x_j^T A^T(j, \theta_j)] E_i[S(j+1, \theta_{j+1})] \\ & \times [A(j, \theta_j) x_j + B_2(j, \theta_j) u_j + B_1(j, \theta_j) w_j] - \gamma^2 w_j^T w_j \} \Bigg\}, \end{aligned}$$

or

$$\begin{aligned} J(u) = E\Bigg\{ & \sum_{j=0}^{N-1} \{x_j^T \{C^T(j, \theta_j) C(j, \theta_j) - S(j, \theta_j) \\ & + A^T(j, \theta_j) E_i[S(j+1, \theta_{j+1})] A(j, \theta_j)\} x_j \\ & + u_j^T \{R(j, \theta_j) + B_2^T(j, \theta_j) E_i[S(j+1, \theta_{j+1})] B_2(j, \theta_j)\} u_j \\ & + w_j^T \{B_1^T(j, \theta_j) E_i[S(j+1, \theta_{j+1})] B_1(j, \theta_j) - \gamma^2 I\} w_j \\ & + u_j^T B_2^T(j, \theta_j) E_i[S(j+1, \theta_{j+1})] A(j, \theta_j) x_j \\ & + x_j^T A^T(j, \theta_j) E_i[S(j+1, \theta_{j+1})] B_2(j, \theta_j) u_j \\ & + w_j^T B_1^T(j, \theta_j) E_i[S(j+1, \theta_{j+1})] A(j, \theta_j) x_j \\ & + x_j^T A^T(j, \theta_j) E_i[S(j+1, \theta_{j+1})] B_1(j, \theta_j) w_j \\ & + w_j^T B_1^T(j, \theta_j) E_i[S(j+1, \theta_{j+1})] B_2(j, \theta_j) u_j \\ & + u_j^T B_2^T(j, \theta_j) E_i[S(j+1, \theta_{j+1})] B_1(j, \theta_j) w_j \} \Bigg\}, \end{aligned}$$

or yet

$$\begin{aligned} J(u) = E\Bigg\{ & \sum_{j=0}^{N-1} \{x_j^T \{C^T(j, \theta_j) C(j, \theta_j) - S(j, \theta_j) \\ & + A^T(j, \theta_j) E_i[S(j+1, \theta_{j+1})] A(j, \theta_j)\} x_j \\ & + u_j^T \{R(j, \theta_j) + B_2^T(j, \theta_j) E_i[S(j+1, \theta_{j+1})] B_2(j, \theta_j)\} u_j \\ & + u_j^T B_2^T(j, \theta_j) E_i[S(j+1, \theta_{j+1})] A(j, \theta_j) x_j \\ & + x_j^T A^T(j, \theta_j) E_i[S(j+1, \theta_{j+1})] B_2(j, \theta_j) u_j \\ & - w_j^T \{\gamma^2 I - B_1^T(j, \theta_j) E_i[S(j+1, \theta_{j+1})] B_1(j, \theta_j)\} w_j \\ & + w_j^T \{B_1^T(j, \theta_j) E_i[S(j+1, \theta_{j+1})] A(j, \theta_j) x_j \end{aligned}$$

$$
\begin{aligned}
&+B_1^T(j,\theta_j)E_i[S(j+1,\theta_{j+1})]B_2(j,\theta_j)u_j\}\\
&+\{x_j^TA^T(j,\theta_j)E_i[S(j+1,\theta_{j+1})]B_1(j,\theta_j)\\
&+u_j^TB_2^T(j,\theta_j)E_i[S(j+1,\theta_{j+1})]B_1(j,\theta_j)\}w_j\}\Big\}.
\end{aligned}
$$

For the sake of simplicity, define now

$$
\begin{aligned}
&Q_j(\theta_j,x_j,u_j)\\
&\quad \triangleq B_1^T(j,\theta_j)E_i[S(j+1,\theta_{j+1})]A(j,\theta_j)x_j\\
&\qquad + B_1^T(j,\theta_j)E_i[S(j+1,\theta_{j+1})]B_2(j,\theta_j)u_j,
\end{aligned}
\tag{15}
$$

$$
\begin{aligned}
&\Delta_j(\theta_j)\\
&\quad \triangleq \gamma^2I - B_1^T(j,\theta_j)E_i[S(j+1,\theta_{j+1})]B_1(j,\theta_j)>0,
\end{aligned}
\tag{16}
$$

$$
\begin{aligned}
&\Gamma_j(\theta_j,x_j,u_j)\\
&\quad \triangleq \Delta_j(\theta_j)^{-1}Q_j(\theta_j,x_j,u_j).
\end{aligned}
\tag{17}
$$

Then,

$$
\begin{aligned}
J(u)=E\Big\{&\sum_{j=0}^{N-1}\{x_j^T\{C^T(j,\theta_j)C(j,\theta_j)-S(j,\theta_j)\\
&+A^T(j,\theta_j)E_i[S(j+1,\theta_{j+1})]A(j,\theta_j)\}x_j\\
&+u_j^T\{R(j,\theta_j)+B_2(j,\theta_j)E_i[S(j+1,\theta_{j+1})]B_2(j,\theta_j)\}u_j\\
&+u_j^TB_2^T(j,\theta_j)E_i[S(j+1,\theta_{j+1})]A(j,\theta_j)x_j\\
&+x_j^TA^T(j,\theta_j)E_i[S(j+1,\theta_{j+1})]B_2(j,\theta_j)u_j\\
&-[w_j-\Gamma_j(\theta_j,x_j,u_j)]^T\Delta_j(\theta_j)[w_j-\Gamma_j(\theta_j,x_j,u_j)]\\
&+Q_j^T(\theta_j,x_j,u_j)\Delta_j^{-1}(\theta_j)Q_j(\theta_j,x_j,u_j)\}\Big\}.
\end{aligned}
\tag{18}
$$

Now, bearing in mind the definition of $Q_j(\cdot)$ in (15) and making $Q_j \triangleq Q_j(\theta_j,x_j,u_j)$ for easiness of notation, we have

$$
\begin{aligned}
&Q_j^T\Delta_j^{-1}(\theta_j)Q_j\\
&\quad = x_j^TA^T(j,\theta_j)E_i[S(j+1,\theta_{j+1})]\\
&\qquad \times B_1(j,\theta_j)\Delta_j^{-1}(\theta_j)B_1^T(j,\theta_j)E_i[S(j+1,\theta_{j+1})]\\
&\qquad \times A(j,\theta_j)x_j + u_j^TB_2^T(j,\theta_j)E_i[S(j+1,\theta_{j+1})]\\
&\qquad \times B_1(j,\theta_j)\Delta_j^{-1}(\theta_j)B_1^T(j,\theta_j)E_i[S(j+1,\theta_{j+1})]\\
&\qquad \times B_2(j,\theta_j)u_j + x_j^TA^T(j,\theta_j)E_i[S(j+1,\theta_{j+1})]\\
&\qquad \times B_1(j,\theta_j)\Delta_j^{-1}(\theta_j)B_1^T(j,\theta_j)E_i[S(j+1,\theta_{j+1})]B_2(j,\theta_j)u_j\\
&\qquad + u_j^TB_2^T(j,\theta_j)E_i[S(j+1,\theta_{j+1})]B_1(j,\theta_j)\Delta_j^{-1}(\theta_j)B_1^T(j,\theta_j)\\
&\qquad \times E_i[S(j+1,\theta_{j+1})]A(j,\theta_j)x_j.
\end{aligned}
\tag{19}
$$

Substituting now (19) into (18) and rearranging the terms, we get

$$
\begin{aligned}
J(u)& \\
&= E\left\{ \sum_{j=0}^{N-1} \{x_j^T\{C^T(j,\theta_j)C(j,\theta_j)-S(j,\theta_j)\right.\\
&\quad +A^T(j,\theta_j)E_i[S(j+1,\theta_{j+1})]A(j,\theta_j)\\
&\quad +A^T(j,\theta_j)E_i[S(j+1,\theta_{j+1})]\\
&\quad \times B_1(j,\theta_j)\Delta_j^{-1}(\theta_j)B_1^T(j,\theta_j)\\
&\quad \times E_i[S(j+1,\theta_{j+1})]A(j,\theta_j)\}x_j\\
&\quad +u_j^T\{R(j,\theta_j)+B_2^T(j,\theta_j)\\
&\quad \times E_i[S(j+1,\theta_{j+1})]B_2(j,\theta_j)\\
&\quad +B_2^T(j,\theta_j)E_i[S(j+1,\theta_{j+1})]B_1(j,\theta_j)\\
&\quad \times \Delta_j^{-1}(\theta_j)B_1^T(j,\theta_j)E_i[S(j+1,\theta_{j+1})]B_2(j,\theta_j)\}u_j\\
&\quad +u_j^T\{B_2^T(j,\theta_j)E_i[S(j+1,\theta_{j+1})]A(j,\theta_j)\\
&\quad +B_2^T(j,\theta_j)E_i[S(j+1,\theta_{j+1})]B_1(j,\theta_j)\\
&\quad \times \Delta_j^{-1}(\theta_j)B_1^T(j,\theta_j)E_i[S(j+1,\theta_{j+1})]A(j,\theta_j)\}x_j\\
&\quad +x_j^T\{A^T(j,\theta_j)E_i[S(j+1,\theta_{j+1})]B_2(j,\theta_j)\\
&\quad +A^T(j,\theta_j)E_i[S(j+1,\theta_{j+1})]B_1(j,\theta_j)\\
&\quad \times \Delta_j^{-1}(\theta_j)B_1^T(j,\theta_j)E_i[S(j+1,\theta_{j+1})]B_2(j,\theta_j)\}u_j\\
&\quad \left. -[w_j-\Gamma_j(\theta_j,x_j,u_j)]^T\Delta_j(\theta_j)[w_j-\Gamma_j(\theta_j,x_j,u_j)]\}\right\},
\end{aligned}
$$

or yet, by using (12) and (13), we get

$$
\begin{aligned}
J(u)& \\
&= E\left\{ \sum_{j=0}^{N-1} \{x_j^T\{C^T(j,\theta_j)C(j,\theta_j)-S(j,\theta_j)\right.\\
&\quad +A^T(j,\theta_j)E_i[S(j+1,\theta_{j+1})]A(j,\theta_j)\\
&\quad +A^T(j,\theta_j)E_i[S(j+1,\theta_{j+1})]B_1(j,\theta_j)\Delta_j^{-1}(\theta_j)\\
&\quad \times B_1^T(j,\theta_j)E_i[S(j+1,\theta_{j+1})]A(j,\theta_j)\}x_j\\
&\quad +[u_j+H^{-1}(j,\theta_j)L(j,\theta_j)x_j]^T\\
&\quad \times H(j,\theta_j)[u_j+H^{-1}(j,\theta_j)L(j,\theta_j)x_j]\\
&\quad -[w_j-\Gamma_j(\theta_j,x_j,u_j)]^T\Delta_j(\theta_j)[w_j-\Gamma_j(\theta_j,x_j,u_j)]\\
&\quad \left. -x_j^TL^T(j,\theta_j)H^{-1}(j,\theta_j)L(j,\theta_j)x_j\}\right\}.
\end{aligned}
\tag{20}
$$

Furthermore, rearranging the terms in (20) we obtain

$$
\begin{aligned}
&J(u)\\
&= E\Bigg\{\sum_{j=0}^{N-1}\{x_j^T\{C^T(j,\theta_j)C(j,\theta_j)-S(j,\theta_j)\\
&\quad +A^T(j,\theta_j)E_i[S(j+1,\theta_{j+1})]A(j,\theta_j)\\
&\quad +A^T(j,\theta_j)E_i[S(j+1,\theta_{j+1})]B_1(j,\theta_j)\Delta_j^{-1}(\theta_j)\\
&\quad \times B_1^T(j,\theta_j)E_i[S(j+1,\theta_{j+1})]A(j,\theta_j)\\
&\quad -L^T(j,\theta_j)H^{-1}(j,\theta_j)L(j,\theta_j)\}x_j\\
&\quad +[u_j+H^{-1}(j,\theta_j)L(j,\theta_j)x_j]^T H(j,\theta_j)[u_j+H^{-1}(j,\theta_j)L(j,\theta_j)x_j]\\
&\quad -[w_j-\Gamma_j(\theta_j,x_j,u_j)]^T\Delta_j(\theta_j)[w_j-\Gamma_j(\theta_j,x_j,u_j)]\}\Bigg\}. \qquad (21)
\end{aligned}
$$

Therefore, using (9)–(11), bearing in mind (12), (13), and assuming that $\theta_j = i$, we get from (21) that

$$
J(u^*) = -E\left\{\sum_{j=0}^{N-1}[w_j-\Gamma_j(\theta_j,x_j,u_j^*)]^T\Delta_j(\theta_j)[w_j-\Gamma_j(\theta_j,x_j,u_j^*)]\right\}\le 0,
$$

and the result follows.

Remark 1: Notice that when $\gamma\to\infty$, Eq. (9) becomes

$$
\begin{aligned}
S(j,i) = {}& C^T(j,i)C(j,i) + A^T(j,i)E_i[S(j+1,\theta_{j+1})]A(j,i)\\
& - A^T(j,i)E_i[S(j+1,\theta_{j+1})]B_2(j,i)M^{-1}(j,i)\\
& \times B_2^T(j,i)E_i[S(j+1,\theta_{j+1})]A(j,i)
\end{aligned}
$$

with

$$
M(j,i) \triangleq R(j,i) + B_2^T(j,i)E_i[S(j+1,\theta_{j+1})]B_2(j,i),
$$

and the control policy, Eq. (11), becomes

$$
u_j^*(x_j,i) = -M^{-1}(j,i)B_2^T(j,i)E_i[S(j+1,\theta_{j+1})]A(j,i)x_j,
$$

which are the interconnected set of Riccati equations and the control policy, respectively, for the Jump-Linear-Quadratic-Gaussian OPTIMAL CONTROL problem, as in Fragoso (1989), i.e., the H^∞ solution converges to the optimal dual solution.

3.2 The infinite horizon case In this situation, recall that feedback control is required to guarantee the mean square stability of the closed-loop system while achieving a prescribed level of disturbance attenuation $\gamma>0$. We consider the problem formulation as described in Sec. 2 ((Σ_2) and Eqs. (4)–(7)).

Before presenting the main result of this subsection, we state the definition of *internal mean square stabilizability* and recall the notion of *mean square stabilizability* and *mean square detectability* (inspired by Costa and Fragoso (1993)).

First consider

1. $A \triangleq (A(1), A(2), \cdots, A(N^*)) \in \mathcal{M}(\mathcal{R}^{nN^*}, \mathcal{R}^n)$,
2. $B_2 \triangleq (B_2(1), B_2(2), \cdots, B_2(N^*) \in \mathcal{M}(\mathcal{R}^{mN^*}, \mathcal{R}^n)$,
3. $C \triangleq (C(1), C(2), \cdots, C(N^*)) \in \mathcal{M}(\mathcal{R}^{nN^*}, \mathcal{R}^r)$,

where $\mathcal{M}(\mathcal{R}^m, \mathcal{R}^n)$ denotes the normed linear space of all n by m real matrices.

Definition 3.1. System (Σ_2), is said to be *internally mean square stable* (IMSS), if the solution of

$$x_{k+1} = A(\theta_k)x_k$$

is such that $E(\|x_k\|^2) \to 0$, as $k \to \infty$ for arbitrary initial state (x_0, θ_0).

Definition 3.2. System (Σ_2), with $u_k \equiv 0$ is *mean square stable* (MSS), if there exist $q \in \mathcal{R}$, such that for any initial state (x_0, θ_0), we have

$$q(k) \underset{k \to \infty}{\to} q,$$

where $q(k) = E\{\|x(k)\|^2\}$. Furthermore, (A, B_2) is *mean square stabilizable* (MSS) if there exist feedback gains $\{K(i); i \in \mathcal{S}\}$, such that the control law

$$u_j = -K(i)x_j, \qquad \theta(j) = i \in \mathcal{S},$$

ensures that the closed-loop system is mean square stable.

Definition 3.3. We say that (C, A) is *mean square detectable* (MSD) if there exists $K \triangleq (K(1), K(2), \cdots, K(N^*)) \in \mathcal{M}(\mathcal{R}^{rN^*}, \mathcal{R}^n)$, such that for any initial condition (x_0, θ_0), we have that

$$x_{k+1} = (A(\theta_k) - K(\theta_k)C(\theta_k))x_k$$

is MSS.

Remark 3.1: Notice that MSD, in the sense of Definition 3.3, is not equivalent to mean square detectability for each mode $(C(i), A(i))$.

The following auxiliary result is required in the proof of the main result.

Lemma 1. If (Σ_2) is IMSS, then it is with $u_k \equiv 0$, MSS.

Proof. See Appendix.

Finally consider $X \in \mathcal{M}(\mathcal{R}^n, \mathcal{R}^n)$, such that

$$\Delta_i(X) \triangleq \gamma^2 I - B_1^T(i)XB_1(i) > 0, \tag{22}$$

$$\mathcal{T}_i(X) \triangleq XB_1(i)\Delta_i^{-1}(X)B_1^T(i)X, \tag{23}$$

$$\mathcal{L}_i(X) \triangleq \mathcal{T}_i(X) + X, \tag{24}$$

$$\mathcal{Y}_i(X) \triangleq (R + B_2^T(i)\mathcal{L}_i(X)B_2(i)), \tag{25}$$

$$\mathcal{G}_i(X) \triangleq \mathcal{Y}_i^{-1}(X)B_2^T(i)\mathcal{L}_i(X)A(i). \tag{26}$$

The main result reads now as follows.

Theorem 3.2. Consider the system (Σ_2), with controlled output given by (4), and assume that the pair (C, A) is MSD in the sense of Definition 3.3. Let $\gamma > 0$ be a prescribed level of disturbance attenuation. Then, there exists a state feedback control policy, such that the resulting closed-loop system is MSS and $\|z\|_2 \leq \gamma \|w\|_2$ for all $\{w_k\} \in l^2[0, \infty)$, $w \not\equiv 0$, provided that $\{S(i); i \in \mathscr{S}\}$, a set of symmetric matrices, is the solution of the following set of interconnected algebraic Riccati equations:

$$
\begin{aligned}
S(i) = {} & C^T(i)C(i) + A^T(i)E_i[S(j)]A(i) \\
& + A^T(i)E_i[S(j)]B_1(i)[\gamma^2 I - B_1^T(i)E_i[S(j)]B_1(i)]^{-1} \\
& \times B_1^T(i)E_i[S(j)]A(i) - \{B_2^T(i)E_i[S(j)]A(i) + B_2^T(i)E_i[S(j)] \\
& \times B_1(i)[\gamma^2 I - B_1^T(i)E_i[S(j)]B_1(i)]^{-1}B_1^T(i)E_i[S(j)]A(i)\}^T \\
& \times \mathscr{Y}_i^{-1}(E_i[S(j)])\{B_2^T(i)E_i[S(j)]A(i) + B_2^T(i)E_i[S(j)] \\
& \times B_1(i)[\gamma^2 I - B_1^T(i)E_i[S(j)]B_1(i)]^{-1}B_1^T(i)E_i[S(j)]A(i)\}
\end{aligned}
\tag{27}
$$

with

$$\gamma^2 I - B_1^T(i)E_i[S(j)]B_1(i) > 0$$

for $i \in \mathscr{S}$, where

$$\boxed{E_i[S(j)] = \sum_{j=1}^{N^*} p_{ij} S(j),}$$

and $\mathscr{Y}_i(\cdot)$ is defined by (25). Moreover, a suitable control law is given by

$$
\begin{aligned}
u_j(x_j, i) &= -\mathscr{Y}_i^{-1}(E_i[S(j)])\{B_2^T(i)E_i[S(j)]A(i) \\
&\quad + B_2^T(i)E_i[S(j)]B_1(i)[\gamma^2 I - B_1^T(i)E_i[S(j)]B_1(i)]^{-1} \\
&\quad \times B_1^T(i)E_i[S(j)]A(i)\}x_j \\
&= -\mathscr{G}_i(E_i[S(j)])x_j, \qquad \theta_j = i \in \mathscr{S}.
\end{aligned}
\tag{28}
$$

Proof. From expressions (22)–(26), we can rewrite (27) as

$$
\begin{aligned}
S(i) = {} & C^T(i)C(i) + A^T(i)E_i[S(j)]A(i) + A^T(i)\mathscr{T}_i(E_i[S(j)])A(i) \\
& - \mathscr{G}_i^T(E_i[S(j)])\mathscr{Y}_i(E_i[S(j)])\mathscr{G}_i(E_i[S(j)]),
\end{aligned}
$$

or yet,

$$
\begin{aligned}
S(i) = {} & C_f^T(i)C_f(i) + A^T(i)E_i[S(j)]A(i) \\
& - \mathscr{G}_i^T(E_i[S(j)])\mathscr{Y}_i(E_i[S(j)])\mathscr{G}_i(E_i[S(j)])
\end{aligned}
\tag{29}
$$

with

$$C_f^T(i) \triangleq [C^T(i) \quad A^T(i)\mathscr{T}_i^{\frac{1}{2}}(E_i[S(j)])]. \tag{30}$$

Now, from Lemma A. 3 (see Appendix) we have that (C_f, A) is MSD. It follows then from Propositions 7 and 8 in Costa and Fragoso (1993) and Theorem 1 in Fragoso and Costa (1993 b), that $\{S(i); i \in \mathscr{S}\}$ is the unique nonnegative definite mean square stabilizing solution of (29) in the sense that (28) ensures that the closed-loop system is mean square stable.

Finally, as the closed-loop system is mean square stable, it can be easily shown, *mutatis-mutandis*, via the same argument as in the finite horizon case, that $\|z\|_2 < \gamma \|w\|_2$ for all $\{w_k\} \in l^2[0, \infty)$, $w \neq 0$.

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Appendix

Before we proceed into the proof of Lemma 1, consider (Σ_2) with $u_k \equiv 0$, i.e.,

$$\left.\begin{aligned} x_{k+1} &= A(\theta_k)x_k + B_1(\theta_k)w_k \\ x_0 &= x^0, \qquad \theta_0 = i \end{aligned}\right\} \tag{A.1}$$

and define

(D.1) $\mathscr{L}(H)=(\mathscr{L}_1(H), \cdots, \mathscr{L}_{N*}(H))$ with $H=(H_1, H_2, \cdots, H_{N*})$, $H_j \in \mathscr{M}(\mathscr{R}^n, \mathscr{R}^n)$ and $\mathscr{L}_j(H) \triangleq \sum_{i=1}^{N*} p_{ij} A(i) H_i A^T(i)$.

(D.2) $P(k)=(P_1(k), \cdots, P_{N*}(k))$ with $P_i(k)=E\{x(k)x^T(k)1_{\{\theta(k)=i\}}\} \in \mathscr{M}(\mathscr{R}^n, \mathscr{R}^n)$, where $1_{\{\cdot\}}$ stands for the indicator function.

(D.3) $F(q)=(F_1(q), \cdots, F_{N*}(q))$ with $q=(q_1, \cdots, q_{N*})$ and $F_j(q) \triangleq \sum_{i=1}^{N*} p_{ij} A(i) q_i$.

(D.4) $q(k)=(q_1(k), \cdots, q_{N*}(k))$ with $q_i(k) \triangleq E(x(k)1_{\{\theta(k)=i\}})$.

(D.5) $v(k)=(v_1(k), \cdots, v_{N*}(k))$ with $v_j(k) \triangleq \sum_{i=1}^{N*} p_{ij} B_1(i) w(k) P(\theta(k)=i)$.

(D.6) $V(k)=(V_1(k), \cdots, V_{N*}(k))$, with

$$\begin{aligned} V_j(k) \triangleq \sum_{i=1}^{N*} p_{ij} [&B_1(i)w(k)w^T(k)P(\theta(k)=i) \\ &+ A(i)q_i(k)w^T(k)B_1^T(i) + B_1(i)w(k)q_i^T(k)A^T(i)]. \end{aligned}$$

Finally, define also the norm $\|H\|_1 = \sum_{i=1}^{N*} \|H_i\|$ for $H=(H_1, \cdots, H_{N*})$, $H_i \in \mathscr{M}(\mathscr{R}^n, \mathscr{R}^n)$, where $\|\cdot\|$ stands for the usual uniform induced norm in $\mathscr{M}(\mathscr{R}^n, \mathscr{R}^n)$.

Lemma A.1. For system (A.1), we have

$$q(k+1) = Fq(k) + v(k),$$

$$P(k+1) = \mathscr{L}(P(k)) + V(k),$$

where $q(k)$ is as defined in (D.4) and $P(k)$ as in (D.2).

Proof. The proof is easily carried out, via direct calculation, by using Eq. (A. 1) and the definitions above.

Lemma A.2. System (Σ_2) is IMSS, if and only if $r_\sigma(\mathscr{L})<1$, where $\mathscr{L}$ is defined as in (D.1), and $r_\sigma(\cdot)$ denotes the usual spectral radius of an operator.

Proof. Follows from a combination of Theorem 1 in Fragoso and Costa (1993 b) and Lemma 1 in Costa and Fragoso (1993).

Proof of Lemma 1. If (Σ_2) is IMSS, then by Lemma A.2 $r_\sigma(\mathscr{L})<1$, where $\mathscr{L}$ is defined in (D.1). It follows then that there exist $0<\xi<1$ and $\beta\geq 0$ (cf. Costa and Fragoso, 1993), such that

$$\|\mathscr{L}^k\|_1 \leq \beta\xi^k, \qquad k\in\mathscr{X}. \tag{A.2}$$

Furthermore, for the homogeneous part of Eq. (A.1), we have that $q(k+1)=Fq(k)$ and

$$\begin{aligned}\|q(k)\|_1 &= \|F^k q(0)\|_1 = \sum_{i=1}^{N^*}\|E(x(k)1_{\{\theta(k)=i\}})\| \\ &\leq E(\|x(k)\|) \leq (E(\|x(k)\|^2))^{\frac{1}{2}} \underset{k\to\infty}{\to} 0,\end{aligned}$$

since we are assuming that (Σ_2) is IMSS. As this holds for any $(x(0), \theta(0))$, we get that $r_\sigma(F)<1$, and, therefore, there exist $\gamma\geq 0$ and $\lambda\in(0,1)$, such that (cf. Costa and Fragoso, 1993)

$$\|F^k\|_1 \leq \gamma\lambda^k. \tag{A.3}$$

Now, from Lemma A.1, we have

$$q(k) = F^k q(0) + \sum_{l=0}^{k-1} F^{k-1-l}v(l), \tag{A.4}$$

$$P(k) = \mathscr{L}^k P(0) + \sum_{l=0}^{k-1}\mathscr{L}^{k-1-l}V(l), \tag{A.5}$$

and consequently, for any $k=0, 1, \cdots,$

$$\|q(k)\|_1 \leq \gamma\lambda^k\|q(0)\|_1 + \sum_{l=0}^{k-1}\gamma\lambda^{k-1-l}\|v(l)\|_1 \leq c,$$

where $c=\gamma\|q(0)\|_1+(\mu\gamma/1-\lambda)$ (depends only on $\|q(0)\|_1$) with μ denoting an upper bound for $\|v(l)\|_1$. Furthermore, from (A.2) and (A.5), we have

$$\|P(k)\|_1 \leq \beta\xi^k\|P(0)\|_1 + \sum_{l=0}^{k-1}\beta\xi^{k-1}\|V(l)\|_1. \tag{A.6}$$

Now, from the fact that $\|q(k)\|_1\leq c$ for any $k=0, 1, \cdots,$ we get easily from (D.6) that for any $l=0, 1, \cdots,$ $\|V(l)\|_1\leq a$ for some $a\geq 0$ (which depends only on $\|q(0)\|_1$). Thus, from (A.6), we get

$$\|P(k)\|_1 \leq \beta\xi^k\|P(0)\|_1 + a\beta\sum_{l=0}^{k-1}\xi^{k-1-l}. \tag{A.7}$$

On the other hand,

$$E[\|x(k)\|^2] = \operatorname{tr}\left(\sum_{i=1}^{N^*} P_i(k)\right) \le n\left\|\sum_{i=1}^{N^*} P_i(k)\right\|$$
$$\le n\sum_{i=1}^{N^*}\|P_i(k)\| = n\|P(k)\|_1, \tag{A.8}$$

where n is the dimension of $x(k)$, and $\operatorname{tr}(\cdot)$ stands for the usual trace of a matrix.

Finally, from (A.7) and (A.8), we have that

$$E[\|x(k)\|^2] \le n\beta\xi^k\|P(0)\|_1 + a\beta\sum_{l=0}^{k-1}\xi^{k-1-l}$$

or

$$\lim_{k\uparrow\infty} E[\|x(k)\|^2] \le \frac{a\beta}{1-\xi},$$

and the result follows.

Lemma A.3. If (C, A) is MSD, then (C_f, A) is MSD.

Proof. If (C, A) is MSD, then, by Definition 3.3, we can find $K=(K(1), K(2), \cdots, K(N^*))$, such that for any initial condition (x_0, θ_0), we have that

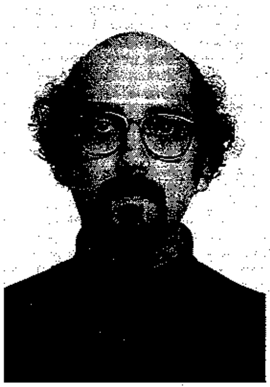
$$x_{k+1} = (A(\theta_k) - K(\theta_k)C(\theta_k))x_k$$

is MSS.

Defining now $\bar{K}(i)=[K(i)\ \ 0]$, we have

$$\begin{aligned} x_{k+1} &= (A(\theta_k) - \bar{K}C_f(\theta_k))x_k \\ &= \left(A(\theta_k) - [K(\theta_k)\ \ 0]\begin{bmatrix} C(\theta_k) \\ l_{\theta_k}^{\frac{1}{2}}(E_{\theta_k}[\Sigma(j)])A(\theta_k)\end{bmatrix}\right)x_k \\ &= (A(\theta_k) - K(\theta_k)C(\theta_k))x_k, \end{aligned}$$

and the result follows.



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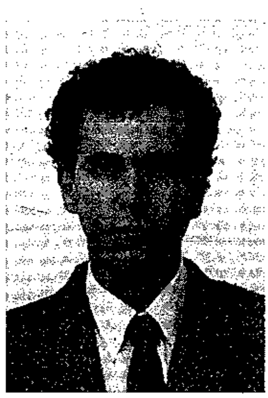

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