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Brief paper

Mode-dependent H_∞ filtering for discrete-time Markovian jump linear systems with partly unknown transition probabilities[☆]

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ABSTRACT

In this paper, the problem of H_∞ filtering for a class of discrete-time Markovian jump linear systems (MJLS) with partly unknown transition probabilities is investigated. The considered systems are more general, which cover the MJLS with completely known and completely unknown transition probabilities as two special cases. A mode-dependent full-order filter is constructed and the bounded real lemma (BRL) for the resulting filtering error system is derived via LMI formulation. Then, an improved version of the BRL is further given by introducing additional slack matrix variables to eliminate the cross coupling between system matrices and Lyapunov matrices among different operation modes. Finally, the existence criterion of the desired filter is obtained such that the corresponding filtering error system is stochastically stable with a guaranteed H_∞ performance index. A numerical example is presented to illustrate the effectiveness and potential of the developed theoretical results.

1. Introduction

As a class of stochastic hybrid systems, Markovian jump systems have been extensively studied in the past decades, see for example, Boukas (2005), Shi, Boukas, and Agarwal (1999), and Xu, Chen, and Lam (2003). By stochastic hybrid feature, we mean that the considered systems contain continuous and discrete dynamics, which are described respectively by classical differential (or difference) equations and Markov stochastic process (or Markov chain). As a crucial factor, the transition probabilities in the jumping process determine the system behavior, and many issues on Markovian jump system have been investigated assuming the complete knowledge of the transition probabilities. A recent extension is to consider the systems with uncertain transition probabilities, in which the robust methodologies are adopted to cope with the norm-bounded or polytopic types of uncertainties in the transition probabilities matrix, see for example, De Souza, Trofino, and Barbosa (2006) and Xiong, Lam, Gao, and Daniel (2005). However, in these references, the structure and "nominal" terms of the uncertain transition probabilities are still assumed to be known *a priori*.

The ideal assumptions on the transition probabilities facilitate the treatment of considered problems, but the applicability of the obtained results is inevitably limited. A typical example could be found in Networked control systems (NCS). It is well known that the time-varying delays induced by communication channels can be modeled as Markov chains, and accordingly the resulting closed-loop system can be studied by means of jump linear systems theory, see for example, Krtolica, Ozguner, Chan, Goktas, Winkelman, and Liubakka (1994) and Zhang, Shi, Chen, and Huang (2005). However, the variation of delays in all kinds of communication networks (especially Internet) can be vague and random, all or part of the elements in the expected transition probabilities matrix are probably hard or expensive to obtain. Consequently, the resulting NCS modeled by jump systems with completely known transition probabilities is actually questionable. Therefore, either in theory or in practice, it is necessary and significant to further consider more general jump systems with partly unknown transition probabilities. Some results on stability and stabilization of the systems have been obtained recently Zhang and Boukas (2009) and Zhang, Boukas, and Lam (2008).

On another research front line, state estimation is an important research issue in control field and has found many practical applications. Many useful results on estimation and filtering for all kinds of dynamic systems have been reported, and H_∞ filtering has been recognized to be one of the most popular

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approaches to deal with external noise sources with unknown statistics (Wang, Lam, & Liu, 2004b; Xu et al., 2003; Zhang, Boukas, & Shi, 2008; Zhang, Shi, Wang, & Gao, 2006). Considering Markovian jump systems with completely known or completely unknown transition probabilities, the mode-dependent and mode-independent filter design approaches have been developed, respectively, see for example, Boukas (2005), de Souza and Fragoso (1997), De Souzaet al. (2006), Liu, Sun, He, and Sun (2004), and Wang, Lam, and Liu (2004a). However, it seems more practicable and challenging to design filters, especially mode-dependent filters, for the underlying systems with partly unknown transition probabilities, which inspires us for this study.

In this paper, the H_∞ filtering problem for a class of discrete-time Markovian jump linear system (MJLS) with partly unknown transition probabilities is investigated. The considered systems are more general than the systems with completely known or completely unknown transition probabilities, which can be viewed as two special cases of the ones tackled here. A mode-dependent full-order filter is constructed and the bounded real lemma (BRL) for the resulting filtering error system is derived in terms of LMI. Also, an improved version of the BRL is given by introducing additional slack matrix variables to eliminate the cross coupling between system matrices and Lyapunov matrices among different operation modes. Furthermore, the existence condition of the desired filter is obtained such that the corresponding filtering error system is stochastically stable and has a guaranteed H_∞ performance index. A numerical example is presented to illustrate the effectiveness and potential of the developed theoretical results.

Notation: The notation used in this paper is fairly standard. The superscript "T" stands for matrix transposition, $\mathbb{R}^n$ denotes the n dimensional Euclidean space, the notation $|\cdot|$ refers to the Euclidean vector norm. $l_2[0,\infty)$ is the space of square summable infinite sequence and for $w = \{w(k)\} \in l_2[0,\infty)$, its norm is given by $\|w\|_2 = \sqrt{\sum_{k=0}^{\infty}|w(k)|^2}$. For notation $(\Omega, \mathcal{F}, \mathcal{P})$, Ω represents the sample space, $\mathcal{F}$ is the σ-algebra of subsets of the sample space and $\mathcal{P}$ is the probability measure on $\mathcal{F}$. $E[\cdot]$ stands for the mathematical expectation and for sequence $e = \{e(k)\} \in l_2((\Omega,\mathcal{F},\mathcal{P}),[0,\infty))$, its norm is given by $\|e\|_{E_2} = \sqrt{E\left[\sum_{k=0}^{\infty}|e(k)|^2\right]}$. In addition, in symmetric block matrices or long matrix expressions, we use $*$ as an ellipsis for the terms that are introduced by symmetry and $diag\{\cdots\}$ stands for a block-diagonal matrix. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations. The notation $P > 0$ (≥ 0) means P is real symmetric positive (semi-positive) definite. I and 0 represent respectively, identity matrix and zero matrix.

2. Problem formulation and preliminaries

Fix the probability space $(\Omega, \mathcal{F}, \mathcal{P})$ and consider the following class of discrete-time Markovian jump linear systems:

$$\begin{aligned} x(k+1) &= A(r_k)x(k) + B(r_k)w(k)\\ y(k) &= C(r_k)x(k) + D(r_k)w(k)\\ z(k) &= H(r_k)x(k) + L(r_k)w(k) \end{aligned} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state vector, $w(k) \in \mathbb{R}^l$ is the disturbance input which belongs to $l_2[0,\infty)$, $y(k) \in \mathbb{R}^m$ is the measurement output and $z(k) \in \mathbb{R}^v$ is the objective signal to be estimated. $\{r_k, k \geq 0\}$ is a discrete-time homogeneous Markov chain, which takes values in a finite set $\mathcal{I} \triangleq \{1,\ldots,N\}$ with a transition probabilities matrix $\Lambda = \{\pi_{ij}\}$ namely, for $r_k = i, r_{k+1} = j$, one has

$$\Pr(r_{k+1} = j | r_k = i) = \pi_{ij}$$

where $\pi_{ij} \geq 0\, \forall i,j \in \mathcal{I}$, and $\sum_{j=1}^{N}\pi_{ij} = 1$. The set $\mathcal{I}$ contains N modes of system (1) and for $r_k = i \in \mathcal{I}$, the system matrices of the ith mode are denoted by A_i, B_i, C_i, D_i, H_i and L_i, which are considered here to be real known with appropriate dimensions.

In addition, the transition probabilities of the jumping process $\{r_k, k \geq 0\}$ in this paper are assumed to be partly accessed, i.e., some elements in matrix $\boldsymbol{\Lambda}$ are unknown. For instance, for system (1) with 4 operation modes, the transition probabilities matrix may be as:

$$\begin{bmatrix} \pi_{11} & ? & \pi_{13} & ?\\ ? & ? & ? & \pi_{24}\\ \pi_{31} & \pi_{32} & ? & ?\\ ? & ? & \pi_{43} & \pi_{44} \end{bmatrix}$$

where "?" represents the unaccessible elements. For notation clarity, $\forall i \in \mathcal{I}$, we denote that

$$\mathcal{I}_{\mathcal{K}}^i \triangleq \{j : \pi_{ij} \text{ is known}\}, \qquad \mathcal{I}_{\mathcal{UK}}^i \triangleq \{j : \pi_{ij} \text{ is unknown}\}. \tag{2}$$

Also, we denote $\pi_{\mathcal{K}}^i \triangleq \sum_{j\in\mathcal{I}_k^i}\pi_{ij}$ throughout the paper.

Remark 1. The jumping process $\{r_k, k \geq 0\}$ in the existing literature is commonly assumed to be completely accessible ($\mathcal{I}_{\mathcal{UK}}^i = \emptyset, \mathcal{I}_{\mathcal{K}}^i = \mathcal{I}$) or completely unaccessible ($\mathcal{I}_{\mathcal{K}}^i = \emptyset, \mathcal{I}_{\mathcal{UK}}^i = \mathcal{I}$). Note that the transition probabilities with polytopic or norm-bounded uncertainties still need the knowledge of structure (polytopic uncertainties), bounds (norm-bounded ones) and "nominal" terms (both cases). Therefore, our transition probabilities matrix considered in the sequel is a more natural assumption to the Markovian jump systems.

Here, we are interested in designing a mode-dependent full-order filter of the form:

$$\begin{aligned} x_F(k+1) &= A_F(r_k)x_F(k) + B_F(r_k)y(k)\\ z_F(k) &= C_F(r_k)x_F(k) + D_F(r_k)y(k) \end{aligned} \tag{3}$$

where $A_F(r_k), B_F(r_k), C_F(r_k)$ and $D_F(r_k)$, $\forall r_k \in \mathcal{I}$ are filter gains to be determined. The filter with the above structure is assumed to jump synchronously with the modes in system (1), which is hereby mode-dependent.

Augmenting the model of (1) to include the states of the filter, we obtain the following dynamics:

$$\begin{aligned} \tilde{x}(k+1) &= \tilde{A}(r_k)\tilde{x}(k) + \tilde{B}(r_k)w(k)\\ e(k) &= \tilde{C}(r_k)\tilde{x}(k) + \tilde{D}(r_k)w(k) \end{aligned} \tag{4}$$

where $\tilde{x}(k) = [x^{\mathrm{T}}(k)x_F^{\mathrm{T}}(k)]^{\mathrm{T}}$, $e(k) = z(k) - z_F(k)$ and

$$\tilde{A}(r_k) = \begin{bmatrix} A(r_k) & 0\\ B_F(r_k)C(r_k) & A_F(r_k) \end{bmatrix},$$

$$\tilde{B}(r_k) = \begin{bmatrix} B(r_k)\\ B_F(r_k)D(r_k) \end{bmatrix},$$

$$\tilde{C}(r_k) = \begin{bmatrix} H(r_k) - D_F(r_k)C(r_k) & -C_F(r_k) \end{bmatrix},$$

$$\tilde{D}(r_k) = L(r_k) - D_F(r_k)D(r_k).$$

Obviously, the resulting system (4) is also a Markovian jump linear system with partly unknown transition probabilities (2). Now, to present the main objective of this paper more precisely, we also introduce the following definitions for the filtering error system (4), which are essential for the later development.

Definition 1. System (4) is said to be stochastically stable if for $w(k) \equiv 0$ and every initial condition $\tilde{x}_0 \in \mathbb{R}^n$ and $r_0 \in \mathcal{I}$, the following holds:

$$E\left\{\sum_{k=0}^{\infty}\left\|\tilde{x}(k)\right\|^2 |\tilde{x}_0, r_0\right\} < \infty$$

Definition 2. Given a scalar $\gamma > 0$, system (4) is said to be stochastically stable and has an H_∞ noise attenuation performance index γ if it is stochastically stable and under zero initial condition, $\|e\|_{E_2} < \gamma \|w\|_2$ holds for all nonzero $w(k) \in l_2[0, \infty)$.

Thus, the objective of this paper is to design a mode-dependent full-order filter with the form (3) such that the filtering error system (4) is stochastically stable and has a guaranteed H_∞ noise attenuation performance.

Remark 2. Note that if $\mathcal{I}^i_{\mathcal{K}} = \emptyset$, for all $i \in \mathcal{I}$, the corresponding system (1) or (4) will be reduced to an extreme case, i.e., a switched system under arbitrary switching, then the objective of filter design here will be to guarantee that the filter error system (4) is stable for any switching sequence with an H_∞ noise attenuation performance.

3. Main results

3.1. H_∞ filtering analysis

Let us first discuss H_∞ filtering analysis for the filtering error system (4) under given filter gains in (3). The following lemma presents a bounded H_∞ performance criterion (i.e., the so-called bounded real lemma (BRL)) for system (4) with the partly unknown transition probabilities (2).

Lemma 1. *Consider system (4) with partly unknown transition probabilities (2) and let $\gamma > 0$ be a given constant. If there exist matrix $P_i > 0$, $\forall i \in \mathcal{I}$ such that*

$$\Theta_i = \begin{bmatrix} -\mathcal{P}^i_{\mathcal{K}} & 0 & \mathcal{P}^i_{\mathcal{K}}\tilde{A}_i & \mathcal{P}^i_{\mathcal{K}}\tilde{B}_i \\ * & -\pi^i_{\mathcal{K}} I & \pi^i_{\mathcal{K}}\tilde{C}_i & \pi^i_{\mathcal{K}}\tilde{D}_i \\ * & * & -\pi^i_{\mathcal{K}} P_i & 0 \\ * & * & * & -\pi^i_{\mathcal{K}}\gamma^2 I \end{bmatrix} < 0, \quad (5)$$

$$\Theta_{ij} = \begin{bmatrix} -P_j & 0 & P_j\tilde{A}_i & P_j\tilde{B}_i \\ * & -I & \tilde{C}_i & \tilde{D}_i \\ * & * & -P_i & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0, \quad \forall j \in \mathcal{I}^i_{\mathcal{UK}} \quad (6)$$

where $\mathcal{P}^i_{\mathcal{K}} \triangleq \sum_{j\in\mathcal{I}^i_{\mathcal{K}}} \pi_{ij} P_j$, then the filtering error system (4) is stochastically stable (or stable for any switching sequence if $\mathcal{I}^i_{\mathcal{K}} = \emptyset$, for all $i \in \mathcal{I}$) with an H_∞ performance index γ.

Proof. Construct a stochastic Lyapunov function as

$$V(\tilde{x}_k, k) = \tilde{x}_k^{\mathrm{T}} P_i \tilde{x}_k, \quad \forall r_k = i \in \mathcal{I} \quad (7)$$

where P_i satisfy (5) and (6). Then, for $r_k = i$, $r_{k+1} = j$, one has

$$\begin{aligned} &E\left[\Delta V(\tilde{x}_k, k)\right] \\ &\triangleq E\left[V(\tilde{x}_{k+1}, k+1|\tilde{x}_k, r_k) - V(\tilde{x}_k, k)\right] \\ &= \tilde{x}_{k+1}^{\mathrm{T}} \sum_{j\in\mathcal{I}} \pi_{ij} P_j \tilde{x}_{k+1} - \tilde{x}_k^{\mathrm{T}} \left[\sum_{j\in\mathcal{I}^i_{\mathcal{K}}} \pi_{ij} + \sum_{j\in\mathcal{I}^i_{\mathcal{UK}}} \pi_{ij}\right] P_i \tilde{x}_k \\ &= \tilde{x}_{k+1}^{\mathrm{T}} \left[\mathcal{P}^i_{\mathcal{K}} + \sum_{j\in\mathcal{I}^i_{\mathcal{UK}}} \pi_{ij} P_j\right] \tilde{x}_{k+1} \\ &\quad - \tilde{x}_k^{\mathrm{T}} \left[\pi^i_{\mathcal{K}} P_i + \sum_{j\in\mathcal{I}^i_{\mathcal{UK}}} \pi_{ij} P_i\right] \tilde{x}_k \\ &= \tilde{x}_{k+1}^{\mathrm{T}} \mathcal{P}^i_{\mathcal{K}} \tilde{x}_{k+1} - \pi^i_{\mathcal{K}} \tilde{x}_k^{\mathrm{T}} P_i \tilde{x}_k \\ &\quad + \sum_{j\in\mathcal{I}^i_{\mathcal{UK}}} \pi_{ij} \left[\tilde{x}_{k+1}^{\mathrm{T}} P_j \tilde{x}_{k+1} - \tilde{x}_k^{\mathrm{T}} P_i \tilde{x}_k\right] \\ &= \tilde{x}_k^{\mathrm{T}} \left[\tilde{A}_i^{\mathrm{T}} \mathcal{P}^i_{\mathcal{K}} \tilde{A}_i - \pi^i_{\mathcal{K}} P_i\right] \tilde{x}_k \\ &\quad + \sum_{j\in\mathcal{I}^i_{\mathcal{UK}}} \pi_{ij} \tilde{x}_k^{\mathrm{T}} \left[\tilde{A}_i^{\mathrm{T}} P_j \tilde{A}_i - P_i\right] \tilde{x}_k. \end{aligned} \quad (8)$$

On the other hand, if (5) and (6) hold, we know from some basic matrix manipulations that

$$\begin{bmatrix} -\mathcal{P}^i_{\mathcal{K}} & \mathcal{P}^i_{\mathcal{K}}\tilde{A}_i \\ * & -\pi^i_{\mathcal{K}} P_i \end{bmatrix} < 0, \qquad \begin{bmatrix} -P_j & P_j\tilde{A}_i \\ * & -P_i \end{bmatrix} < 0, \quad j \in \mathcal{I}^i_{\mathcal{UK}}$$

Furthermore, by Schur complement, we have

$$\tilde{A}_i^{\mathrm{T}} \mathcal{P}^i_{\mathcal{K}} \tilde{A}_i - \pi^i_{\mathcal{K}} P_i < 0, \quad (9)$$

$$\tilde{A}_i^{\mathrm{T}} P_j \tilde{A}_i - P_i < 0, \quad j \in \mathcal{I}^i_{\mathcal{UK}}. \quad (10)$$

Therefore, if (9) and (10) hold, we know from (8) that

$$\begin{aligned} E[\Delta V] &\le -\lambda_{\min}\left[-\left(\tilde{A}_i^{\mathrm{T}} \mathcal{P}^i_{\mathcal{K}} \tilde{A}_i - \pi^i_{\mathcal{K}} P_i\right)\right] \tilde{x}_k^{\mathrm{T}} \tilde{x}_k \\ &\quad - \sum_{j\in\mathcal{I}^i_{\mathcal{UK}}} \pi_{ij} \min_j \left\{\lambda_{\min}\left[-\left(\tilde{A}_i^{\mathrm{T}} P_j \tilde{A}_i - P_i\right)\right]\right\} \tilde{x}_k^{\mathrm{T}} \tilde{x}_k \\ &= -\lambda_{\min}\left[-\left(\tilde{A}_i^{\mathrm{T}} \mathcal{P}^i_{\mathcal{K}} \tilde{A}_i - \pi^i_{\mathcal{K}} P_i\right)\right] \tilde{x}_k^{\mathrm{T}} \tilde{x}_k \\ &\quad - (1 - \pi^i_{\mathcal{K}}) \min_j \left\{\lambda_{\min}\left[-\left(\tilde{A}_i^{\mathrm{T}} P_j \tilde{A}_i - P_i\right)\right]\right\} \tilde{x}_k^{\mathrm{T}} \tilde{x}_k \\ &\le -(\beta_1 + \beta_2) \tilde{x}_k^{\mathrm{T}} \tilde{x}_k = -(\beta_1 + \beta_2) \|\tilde{x}_k\|^2 \end{aligned} \quad (11)$$

where $\beta_1 = \inf\{\lambda_{\min}(-(\tilde{A}_i^{\mathrm{T}} \mathcal{P}^i_{\mathcal{K}} \tilde{A}_i - \pi^i_{\mathcal{K}} P_i)), i \in I\}$ and $\beta_2 = \inf\{(1 - \pi^i_{\mathcal{K}}) \min_j [\lambda_{\min}(-(\tilde{A}_i^{\mathrm{T}} P_j \tilde{A}_i - P_i))], i \in I\}$. From (11), setting $\beta = \beta_1 + \beta_2$, we obtain that for any $T \ge 1$,

$$\begin{aligned} E\left\{\sum_{k=0}^{\mathrm{T}} \|\tilde{x}_k\|^2\right\} &\le \frac{1}{\beta} \left\{E\left[V(\tilde{x}_0, 0)\right] - E\left[V(\tilde{x}_{T+1}, T+1)\right]\right\} \\ &\le \frac{1}{\beta} E\left[V(\tilde{x}_0, 0)\right] \end{aligned}$$

which implies that $E\left\{\sum_{k=0}^{\mathrm{T}} \|\tilde{x}_k\|^2\right\} \le \frac{1}{\beta} E\left[V(\tilde{x}_0, 0)\right] < \infty$. Thus, the system is stochastically stable from Definition 1. Note that β will reduce to only β_1 (respectively, β_2) if all the transition probabilities are known (respectively, unknown).

Now, to establish the H_∞performance for the system, consider the following performance index:

$$J \triangleq E\left\{\sum_{k=0}^{\infty} \left[e^{\mathrm{T}}(k) e(k) - \gamma^2 w^{\mathrm{T}}(k) w(k)\right]\right\}$$

under zero initial condition, $V(\tilde{x}(k), r_k)\,|_{k=0} = 0$, and we have

$$\begin{aligned} J &\le E\left\{\sum_{k=0}^{\infty} \left[e^{\mathrm{T}}(k) e(k) - \gamma^2 w^{\mathrm{T}}(k) w(k) + \Delta V\right]\right\} \\ &= \sum_{k=0}^{\infty} \zeta^{\mathrm{T}}(k) \Phi_i \zeta(k) \end{aligned}$$

where $\zeta(k) \triangleq \left[\tilde{x}^{\mathrm{T}}(k) \quad w^{\mathrm{T}}(k)\right]^{\mathrm{T}}$ and

$$\Phi_i \triangleq \begin{bmatrix} \tilde{A}_i^{\mathrm{T}} \bar{\mathcal{P}}_i \tilde{A}_i - P_i + \tilde{C}_i^{\mathrm{T}} \tilde{C}_i & \tilde{A}_i^{\mathrm{T}} \bar{\mathcal{P}}_i \tilde{B}_i + \tilde{C}_i^{\mathrm{T}} \tilde{D}_i \\ * & -\gamma^2 I + \tilde{B}_i^{\mathrm{T}} \bar{\mathcal{P}}_i \tilde{B}_i + \tilde{D}_i^{\mathrm{T}} \tilde{D}_i \end{bmatrix}$$

$$\bar{\mathcal{P}}_i \triangleq \sum_{j\in\mathcal{I}^i_{\mathcal{K}}} \pi_{ij} P_j + \sum_{j\in\mathcal{I}^i_{\mathcal{UK}}} \pi_{ij} P_j = \mathcal{P}^i_{\mathcal{K}} + \sum_{j\in\mathcal{I}^i_{\mathcal{UK}}} \pi_{ij} P_j$$

By Schur complement, $\Phi_i < 0$ is equivalent to:

$$\Xi_i \triangleq \begin{bmatrix} -\bar{\mathcal{P}}_i & 0 & \bar{\mathcal{P}}_i\tilde{A}_i & \bar{\mathcal{P}}_i\tilde{B}_i \\ * & -I & \tilde{C}_i & \tilde{D}_i \\ * & * & -P_i & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0. \tag{12}$$

Note that (12) can be rewritten as

$$\begin{aligned}\Xi_i &= \begin{bmatrix} -\sum_{j\in\mathcal{I}^i_{\mathcal{K}}}\pi_{ij}P_j & 0 & \sum_{j\in\mathcal{I}^i_{\mathcal{K}}}\pi_{ij}P_j\tilde{A}_i & \sum_{j\in\mathcal{I}^i_{\mathcal{K}}}\pi_{ij}P_j\tilde{B}_i \\ * & -\pi^i_{\mathcal{K}}I & \pi^i_{\mathcal{K}}\tilde{C}_i & \pi^i_{\mathcal{K}}\tilde{D}_i \\ * & * & -P_i & 0 \\ * & * & * & -\pi^i_{\mathcal{K}}\gamma^2 I \end{bmatrix} \\ &\quad + \sum_{j\in\mathcal{I}^i_{\mathcal{UK}}}\pi_{ij}\begin{bmatrix} -P_j & 0 & P_j\tilde{A}_i & P_j\tilde{B}_i \\ * & -I & \tilde{C}_i & \tilde{D}_i \\ * & * & -P_i & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} \\ &= \Theta_i + \sum_{j\in\mathcal{I}^i_{\mathcal{UK}}}\pi_{ij}\Theta_{ij}.\end{aligned}$$

Therefore, inequalities (5) and (6) guarantee $\Xi_i < 0$, i.e., $J < 0$ which means that $\|e\|_{E_2} < \gamma \|w\|_2$, this completes the proof. □

Remark 3. In Lemma 1, if $\mathcal{I}^i_{\mathcal{K}} = \emptyset, \forall i \in \mathcal{I}$, the system becomes a discrete-time switched linear system under arbitrary switching. Also, the stability conditions contained in Lemma 1 are reduced to $A_i^{\mathrm{T}}P_jA_i - P_i < 0, \forall i \times j \in \mathcal{I} \times \mathcal{I}$, which is the criterion obtained in Daafouz, Riedinger, and Iung (2002) by a switched Lyapunov function approach to guarantee the system is globally uniformly asymptotically stable in discrete-time context. In addition, note that it is hard to use Lemma 1 to design the desired filter due to the cross coupling of matrix product terms among different system operation modes, as shown in (5) and (6). To overcome this difficulty, the technique using slack matrix developed in Zhang et al. (2006) can be adopted here to obtain the following improved BRL for system (4).

Lemma 2. *Consider system (4) with partly unknown transition probabilities (2) and let $\gamma > 0$ be a given constant. If there exist matrix $P_i > 0$, and $R_i, \forall i \in \mathcal{I}$ such that*

$$\begin{bmatrix} \Upsilon_j - R_i - R_i^{\mathrm{T}} & 0 & R_i\tilde{A}_i & R_i\tilde{B}_i \\ * & -I & \tilde{C}_i & \tilde{D}_i \\ * & * & -P_i & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0 \tag{13}$$

where if $\pi^i_{\mathcal{K}} = 0$, $\Upsilon_j \triangleq P_j, j \in \mathcal{I}^i_{\mathcal{UK}}$, otherwise,

$$\begin{cases} \Upsilon_j \triangleq \dfrac{1}{\pi^i_{\mathcal{K}}}\mathcal{P}^i_{\mathcal{K}}, \\ \Upsilon_j \triangleq P_j, \forall j \in \mathcal{I}^i_{\mathcal{UK}} \end{cases} \tag{14}$$

and $\mathcal{P}^i_{\mathcal{K}}$ is denoted in Lemma 1, then the filtering error system (4) is stochastically stable (or stable for any switching sequence if $\mathcal{I}^i_{\mathcal{K}} = \emptyset$, for all $i \in \mathcal{I}$) with an H_∞ performance index γ.

Proof. First of all, by Lemma 1, we conclude that system (4) is stochastically stable with an H_∞ performance index γ if the inequalities (5) and (6) hold. Notice that (5) can be rewritten as:

$$\begin{bmatrix} -\dfrac{1}{\pi^i_{\mathcal{K}}}\mathcal{P}^i_{\mathcal{K}} & 0 & \dfrac{1}{\pi^i_{\mathcal{K}}}\mathcal{P}^i_{\mathcal{K}}A_i & \dfrac{1}{\pi^i_{\mathcal{K}}}\mathcal{P}^i_{\mathcal{K}}B_i \\ * & -I & C_i & D_i \\ * & * & -P_i & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0. \tag{15}$$

From the other side, for an arbitrary matrix $R_i, \forall i \in \mathcal{I}$, we have the following facts:

$$\left(\frac{1}{\pi^i_{\mathcal{K}}}\mathcal{P}^i_{\mathcal{K}} - R_i\right)^{\mathrm{T}}\left(\frac{1}{\pi^i_{\mathcal{K}}}\mathcal{P}^i_{\mathcal{K}}\right)^{-1}\left(\frac{1}{\pi^i_{\mathcal{K}}}\mathcal{P}^i_{\mathcal{K}} - R_i\right) \geq 0,$$

$$(P_j - R_i)^{\mathrm{T}}P_j^{-1}(P_j - R_i) \geq 0,$$

then by using (14), one has $\Upsilon_j - R_i - R_i^{\mathrm{T}} \geq -R_i^{\mathrm{T}}\Upsilon_j^{-1}R_i$. Furthermore, from (13), we can obtain that

$$\begin{bmatrix} -R_i^{\mathrm{T}}\Upsilon_j^{-1}R_i & 0 & R_i\tilde{A}_i & R_i\tilde{B}_i \\ * & -I & \tilde{C}_i & \tilde{D}_i \\ * & * & -P_i & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0.$$

Performing now a congruence transformation using $diag\{R_i^{-1}\Upsilon_j, I, I, I\}$ yields (15) and (6) for $j \in \mathcal{I}^i_{\mathcal{K}}$ and $j \in \mathcal{I}^i_{\mathcal{UK}}$, respectively (note that R_i is invertible if it satisfies (13)). This completes the proof. □

Remark 4. Note that in Lemmas 1 and 2, the stochastic stability for the underlying system is actually guaranteed by the two aspects, i.e., efficiently utilizing the partly known transition probabilities (see (9)) together with the requirements that $V_j(\tilde{x}_{k+1}, k+1) - V_i(\tilde{x}_k, k) < 0, \forall j \in \mathcal{I}^i_{\mathcal{UK}}$ on the latent Lyapunov function $V_i(\tilde{x}_k, k) = \tilde{x}_k^{\mathrm{T}}P_i\tilde{x}_k, \forall i \in \mathcal{I}$ (see (10), where if $j \neq i$, the time k will be the mode switching times).

3.2. H_∞ filter design

The following Theorem presents sufficient conditions for the existence of an admissible mode-dependent H_∞ filter with the form (3).

Theorem 1. *Consider system (1) with partly unknown transition probabilities (2) and let $\gamma > 0$ be a given constant. If there exist matrices $P_{1i} > 0$, and $P_{3i} > 0, \forall i \in \mathcal{I}$, and matrices $P_{2i}, X_i, Y_i, Z_i, A_{fi}, B_{fi}, C_{fi}, D_{fi}, \forall i \in \mathcal{I}$, such that*

$$\begin{bmatrix} \Upsilon_{1j} - X_i - X_i^{\mathrm{T}} & \Upsilon_{2j} - Y_i - Z_i^{\mathrm{T}} & 0 & X_iA_i + B_{fi}C_i & A_{fi} & X_iB_i + B_{fi}D_i \\ * & \Upsilon_{3j} - Y_i - Y_i^{\mathrm{T}} & 0 & Z_iA_i + B_{fi}C_i & A_{fi} & Z_iB_i + B_{fi}D_i \\ * & * & -I & H_i - D_{fi}C_i & -C_{fi} & L_i - D_{fi}D_i \\ * & * & * & -P_{1i} & -P_{2i} & 0 \\ * & * & * & * & -P_{3i} & 0 \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix} < 0 \tag{16}$$

where if $\pi^i_{\mathcal{K}} = 0$, $\Upsilon_{1j} \triangleq P_{1j}, \Upsilon_{2j} \triangleq P_{2j}, \Upsilon_{3j} \triangleq P_{3j}, j \in \mathcal{I}^i_{\mathcal{UK}}$, otherwise,

$$\begin{cases} \Upsilon_{1j} \triangleq \dfrac{1}{\pi^i_{\mathcal{K}}}\mathcal{P}^{1i}_{\mathcal{K}} \triangleq \dfrac{1}{\pi^i_{\mathcal{K}}}\sum_{j\in\mathcal{I}^i_{\mathcal{K}}}\pi_{ij}P_{1j} \\ \Upsilon_{2j} \triangleq \dfrac{1}{\pi^i_{\mathcal{K}}}\mathcal{P}^{2i}_{\mathcal{K}} = \dfrac{1}{\pi^i_{\mathcal{K}}}\sum_{j\in\mathcal{I}^i_{\mathcal{K}}}\pi_{ij}P_{2j} \\ \Upsilon_{3j} \triangleq \dfrac{1}{\pi^i_{\mathcal{K}}}\mathcal{P}^{3i}_{\mathcal{K}} = \dfrac{1}{\pi^i_{\mathcal{K}}}\sum_{j\in\mathcal{I}^i_{\mathcal{K}}}\pi_{ij}P_{3j} \end{cases}, \tag{17}$$

$$\begin{cases} \Upsilon_{1j} \triangleq P_{1j} \\ \Upsilon_{2j} \triangleq P_{2j} \\ \Upsilon_{3j} \triangleq P_{3j} \end{cases}, \quad \forall j \in \mathcal{I}^i_{\mathcal{UK}} \tag{18}$$

then, there exists a mode-dependent full-order filter such that the resulting filtering error system (4) is stochastically stable (or stable for any switching sequence if $\mathcal{I}^i_{\mathcal{K}} = \emptyset$, for all $i \in \mathcal{I}$) with an H_∞ performance. Moreover, if the LMIs (16) have a feasible solution, the gains of an admissible filter in the form (3) are given by

$$A_{Fi} = Y_i^{-1}A_{fi}, \qquad B_{Fi} = Y_i^{-1}B_{fi}, \qquad C_{Fi} = C_{fi}, \qquad D_{Fi} = D_{fi} \tag{19}$$

Proof. Consider filtering error system (4) and assume that the matrices P_i, R_i in Lemma 2 have the following forms:

$$P_i \triangleq \begin{bmatrix} P_{1i} & P_{2i} \\ * & P_{3i} \end{bmatrix}, \qquad R_i \triangleq \begin{bmatrix} X_i & Y_i \\ Z_i & Y_i \end{bmatrix}$$

then we have

$$\mathcal{P}^i_{\mathcal{K}} \triangleq \sum_{j \in \mathcal{I}^i_{\mathcal{K}}} \pi_{ij} P_j = \sum_{j \in \mathcal{I}^i_{\mathcal{K}}} \pi_{ij} \begin{bmatrix} P_{1j} & P_{2j} \\ * & P_{3j} \end{bmatrix} \triangleq \begin{bmatrix} \mathcal{P}^{1i}_{\mathcal{K}} & \mathcal{P}^{2i}_{\mathcal{K}} \\ * & \mathcal{P}^{3i}_{\mathcal{K}} \end{bmatrix}.$$

Further define matrix variables $A_{fi} = Y_i A_{Fi}, B_{fi} = Y_i B_{Fi}, C_{fi} = C_{Fi}, D_{fi} = D_{Fi}$ and $\boldsymbol{\Upsilon}_j \triangleq \begin{bmatrix} \boldsymbol{\Upsilon}_{1j} & \boldsymbol{\Upsilon}_{2j} \\ * & \boldsymbol{\Upsilon}_{3j} \end{bmatrix}$, where $\boldsymbol{\Upsilon}_{1j}, \boldsymbol{\Upsilon}_{2j}$ and $\boldsymbol{\Upsilon}_{3j}$ are denoted in (17) and (18) for $j \in \mathcal{I}^i_{\mathcal{K}}$ and $j \in \mathcal{I}^i_{\mathcal{UK}}$, respectively, one can readily obtain (16) replacing $\tilde{A}_i, \tilde{B}_i, \tilde{C}_i, \tilde{D}_i, \boldsymbol{\Upsilon}_j, P_i$ and R_i into (13), namely, if (16) holds, the filtering error system (4) will be stochastically stable with an H_∞ performance under the Markovian Chain with partly unknown transition probabilities (2). Meanwhile, if a solution of (16) exists, the parameters of admissible filter are given by (19). This completes the proof. □

Remark 5. As the two extreme cases, i.e., when all the transition probabilities are known and unknown, the underlying systems are the traditional MJLS and the switched linear systems under arbitrary switching, respectively. Correspondingly, the filtering results can be found in some existing references, see Zhang et al. (2008) and Zhang et al. (2006) for discrete-time switched systems based on general filter type and (de Souza & Fragoso, 1997; Wang et al., 2004a) for discrete-time MJLS based on observer type. Also, without uncertainties of system matrices, the results in Zhang et al. (2006) will be just the extreme case of Theorem 1 when all transition probabilities are unknown. In view of these, the proposed systems and the obtained filter design are more general in this sequel. In addition, note that the slack matrix R_i is constructed with a special structure as shown above, which allows us to obtain a solution of filtering problem within strict LMI framework for the proposed systems.

Remark 6. By setting $\delta = \gamma^2$ and minimizing δ subject to (16), we can obtain the optimal H_∞ noise attenuation performance index γ ($\gamma = \sqrt{\delta}$) and the corresponding filter gains as well. Also, it can be deduced from (16) that, given different degree of unknown elements in the transition probabilities matrix, the optimal γ achieved for system (4) and the corresponding filter gains solved for system (2) should be different, which we will illustrate via a numerical example in next section.

4. Numerical example

Consider the MJLS (1) with four operation modes and the following data:

$$A_1 = \begin{bmatrix} 0 & -0.4050 \\ 0.8100 & 0.8100 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 0 & -0.2673 \\ 0.8100 & 1.1340 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 0 & -0.8100 \\ 0.8100 & 0.9720 \end{bmatrix}, \qquad A_4 = \begin{bmatrix} 0 & -0.1863 \\ 0.8100 & 0.8910 \end{bmatrix},$$

$$B_1 = B_2 = B_3 = B_4 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0 \end{bmatrix},$$

$$C_1 = C_2 = C_3 = C_4 = [1 \quad 0],$$
$$D_1 = D_2 = D_3 = D_4 = H_1 = H_2 = H_3 = H_4 = [0 \quad 1],$$
$$L_1 = L_2 = L_3 = L_4 = [0 \quad 0].$$

The four cases for the transition probabilities matrix will be considered in this example as shown in Table 1.

Table 1
Different transition probabilities matrices.

Completely known

	1	2	3	4
1	0.3	0.2	0.1	0.4
2	0.3	0.2	0.3	0.2
3	0.1	0.1	0.5	0.3
4	0.2	0.2	0.1	0.5

Partly unknown (case I)

	1	2	3	4
1	0.3	0.2	0.1	0.4
2	?	?	0.3	0.2
3	0.1	0.1	0.5	0.3
4	0.2	?	?	?

Partly unknown (case II)

	1	2	3	4
1	0.3	0.2	0.1	0.4
2	?	?	0.3	0.2
3	?	0.1	?	0.3
4	0.2	?	?	?

Completely unknown

	1	2	3	4
1	?	?	?	?
2	?	?	?	?
3	?	?	?	?
4	?	?	?	?

Table 2
Minimum γ^* for different transition probabilities cases.

Transition probabilities	Completely known	Partly unknown (Case I)	Partly unknown (Case II)	Completely unknown
γ^*	1.8556	3.8215	4.2793	4.4624

Our purpose here is to design a mode-dependent full-order H_∞ filter in the form of (3) such that the resulting filtering error system is stochastically stable and has a guaranteed H_∞ performance. By solving (16), the optimal H_∞ performance indices are obtained for the four different transition probabilities cases. The corresponding computation results are listed in Table 2.

From Table 2, it is easily seen that the more transition probabilities knowledge the system has, the smaller performance index the system can achieve. Therefore, by means of our ideas and approaches, a tradeoff can be easily built in practice between the complexity to obtain transition probabilities and the system performance benefits.

The desired filter corresponding to the optimal H_∞ performance index can be also solved using (16), for brevity, the gains are omitted here. Applying the obtained filters and giving two possible time sequences of the mode jumps, we obtain the error response of the resulting filtering error systems in Figs. 1 and 2 for given initial condition $x = [-1.2\ 0.6\ 0\ 0]^{\mathrm{T}}$ and noise signal

$$w(k) = \begin{bmatrix} 0.7 \exp(-0.1k) \sin(0.001\pi k) \\ 0.5 \exp(-0.1k) \sin(0.01\pi k) \end{bmatrix}.$$

It is clearly observed from the simulation curves that for the above energy-bounded disturbance $w(k)$, the filtering error system is stable against different partly unknown transition probabilities, which implies that our designed filter is feasible and effective.

5. Conclusions

The H_∞ filtering problem for the discrete-time MJLS with partly unknown transition probabilities is investigated in this paper. The systems under consideration are more general than the MJLS with completely known or completely unknown transition probabilities as two special cases. The LMI-based BRL for the underlying filtering error system is derived and its improved version is further

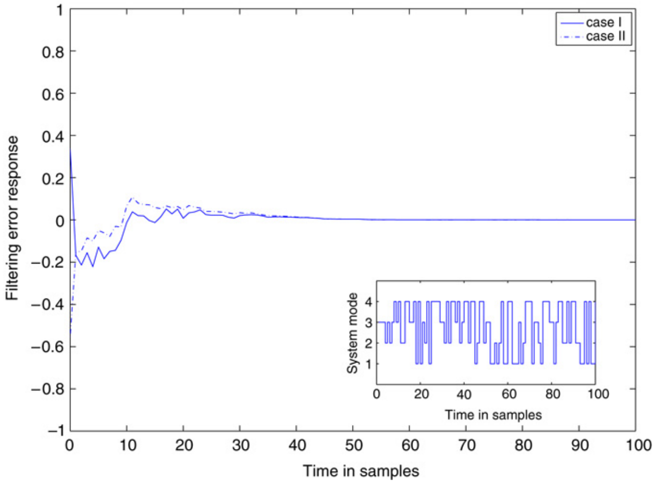

Fig. 1. Filtering error response for mode evolution r_k^1.

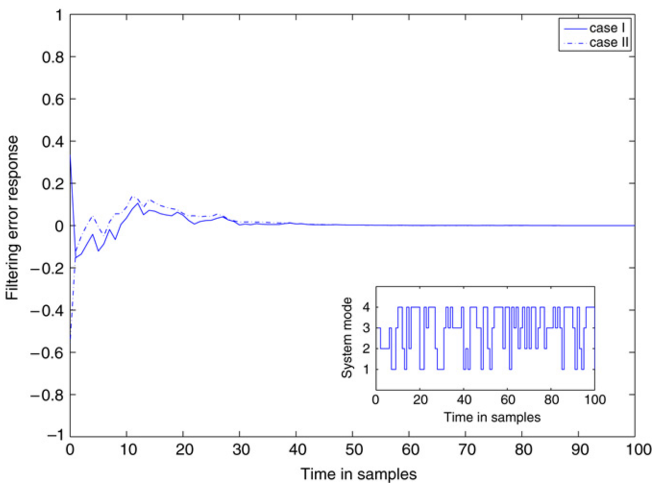

Fig. 2. Filtering error response for mode evolution r_k^2.

given by means of additional slack matrix variables to eliminate the cross coupling between the Lyapunov positive matrices and system matrices. Despite the partly unknown elements in the transition probabilities matrix, the mode-dependent full-order filter is designed and the existence conditions of the desired filter are obtained such that the resulting filtering error system is stochastically stable and has a guaranteed H_∞ performance index. A numerical example is given to illustrate the effectiveness and potential of the developed theoretical results.

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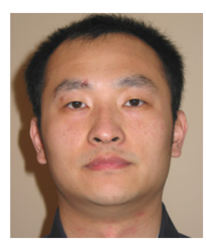

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