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Brief paper

State-feedback control of Markov jump linear systems with hidden-Markov mode observation[☆]

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ABSTRACT

In this paper, we study state-feedback control of Markov jump linear systems with partial information about the mode signal responsible for switching between dynamic modes. We assume that the controller can only access random samples of the mode signal according to a hidden-Markov observation process. Our formulation provides a novel framework to analyze and design feedback control laws for various Markov jump linear systems previously studied in the literature, such as the cases of (*i*) clustered observations, (*ii*) detector-based observations, and (*iii*) periodic observations. We present a procedure to transform the closed-loop system with hidden-Markov observations into a standard Markov jump linear system while preserving stability, H_2 norm, and H_∞ norm. Furthermore, based on this transformation, we propose a set of Linear Matrix Inequalities (LMI) to design feedback control laws for stabilization, H_2 suboptimal control, and H_∞ suboptimal control of discrete-time Markov jump linear systems under hidden-Markov observations of the mode signals. We conclude by illustrating our results with some numerical examples.

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1. Introduction

Markov jump linear systems (Costa, Fragoso, & Marques, 2005) are an important class of switched systems in which the *mode signal*, responsible for controlling the switch among dynamic modes, is modeled by a time-homogeneous Markov chain. This class of systems has been widely used in multiple areas, such as robotics (Siqueira & Terra, 2004), economics (Blair & Sworder, 1975), and networked control (Hespanha, Naghshtabrizi, & Xu, 2007). Solutions to standard controller synthesis problems for Markov jump linear systems, such as stabilization, quadratic optimal control, H_2 optimal control, and H_∞ optimal control (see, e.g., Costa et al., 2005), can be found in the literature. These works are based on the assumption that the controller has full knowledge about the mode signal at any time instant. However, this assumption is not realistic in many practical scenarios.

To overcome this issue, several papers have investigated the effect of limited and/or uncertain knowledge about the mode signal. For example, do Val, Geromel, and Gonçalves (2002) studied H_2 suboptimal control of discrete-time Markov jump linear systems when the state space of the mode signal is partitioned into subsets, called *clusters*, and the controller only knows in which cluster the mode signal is at a given time. Similar studies in the context of H_∞ suboptimal control can be found in Fioravanti, Gonçalves, and Geromel (2014) and Gonçalves, Fioravanti, and Geromel (2012). Vargas, Costa, and do Val (2013) investigated quadratic optimal control problems in the extreme case of having a single mode cluster (i.e., when the mode signal cannot be observed). Many of the above works can be analyzed in a framework recently proposed by Costa, Fragoso, and Todorov (2015) in the context of H_2 suboptimal control, as long as the mode signal can be observed at any time instant. In a complementary line of work, we find some papers assuming that the mode signal can only be observed at particular sampling times, instead of at any time instant. In this direction, Cetinkaya and Hayakawa (2015) designed almost-surely stabilizing state-feedback gains when the sampling times follow a renewal process. Similarly, Cetinkaya and Hayakawa (2014) derived stabilizing state-feedback gains using Lyapunov-like functions under periodic observations.

In this paper, we propose a novel framework to analyze and design state-feedback control laws for discrete-time Markov jump

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linear systems when the observations of the mode signal by the controller are both clustered and randomized over time. Specifically, we assume that the random samples of the mode signal are obtained when, and only when, the mode signal takes particular values. The proposed observation process, called *hidden-Markov* – due to its similitude with hidden-Markov chains (Ephraim & Merhav, 2002) – recovers many relevant cases previously studied in the literature, such as those in Cetinkaya and Hayakawa (2014, 2015), Costa et al. (2015), do Val et al. (2002) and Gonçalves et al. (2012). It is important to remark that, since the observation process is hidden-Markovian, existing methods for analysis and control of Markov jump linear systems, such as those in Costa et al. (2005), do Val et al. (2002) and Gonçalves et al. (2012), do not apply to this case.

One of the main purposes of this paper is to show that, despite the generality of hidden-Markov observation processes, the resulting closed-loop system can be equivalently transformed into a (standard) Markov jump linear system while preserving important closed-loop properties, including mean-square stability, H_2 norm, and H_∞ norm. Furthermore, based on this transformation, we propose a set of Linear Matrix Inequalities (LMI) to design feedback control laws for stabilization, H_2 suboptimal control, and H_∞ suboptimal control of discrete-time Markov jump linear systems under hidden-Markov observations of the mode signal.

The paper is organized as follows. In Section 2, we formulate the state-feedback control problem for Markov jump linear systems with hidden-Markov observations of the mode signal. We show in Section 3 that the resulting closed-loop system can be transformed into a standard Markov jump linear system by embedding the (possibly non-Markovian) stochastic processes responsible for the random observation process into a standard Markov chain. Based on this transformation, in Section 4, we present an LMI formulation to design state-feedback gains for stabilization, H_2 suboptimal control, and H_∞ suboptimal control. We conclude by illustrating the obtained results by numerical simulations in Section 5.

The notation used in this paper is standard. Let $\mathbb{Z}$ and $\mathbb{N}$ denote the set of integers and nonnegative integers, respectively. The number of the elements of a finite set X is denoted by $|X|$. Let $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the spaces of real n-vectors and $n \times m$ matrices, respectively. By $\|\cdot\|$, we denote the Euclidean norm on $\mathbb{R}^n$. $\Pr(\cdot)$ is used to denote the probability of an event. The probability of an event conditional on another event $\mathcal{A}$ is denoted by $\Pr(\cdot \mid \mathcal{A})$. Expectations are denoted by $E[\cdot]$. The identity matrix with dimension d is denoted by I_d. Let A be a real matrix. Define $\mathrm{He}(A) = A + A^\top$. When A is positive definite, we write $A > 0$. The symbol $\star$ is used to denote the symmetric blocks of partitioned symmetric matrices. Finally, indicator functions are denoted by $\mathbb{1}(\cdot)$.

2. Problem formulation

In this section, we formulate the problems under study. Let $r = \{r(k)\}_{k=0}^\infty$ be a time-homogeneous Markov chain taking values in a finite set Θ with transition probabilities $\Pr(r(k+1) = \theta' | r(k) = \theta) = p_{\theta\theta'}$ for $\theta, \theta' \in \Theta$. Let n, m, q, and ℓ be positive integers and, for each $\theta \in \Theta$, let $A_\theta \in \mathbb{R}^{n\times n}$, $B_\theta \in \mathbb{R}^{n\times m}$, $C_\theta \in \mathbb{R}^{\ell\times n}$, $D_\theta \in \mathbb{R}^{\ell\times m}$, and $E_\theta \in \mathbb{R}^{n\times q}$. Consider the Markov jump linear system Σ given as

$$\begin{aligned} x(k+1) &= A_{r(k)}x(k) + B_{r(k)}u(k) + E_{r(k)}w(k), \\ z(k) &= C_{r(k)}x(k) + D_{r(k)}u(k). \end{aligned} \tag{1}$$

We call x and r the *state* and the *mode* of Σ, respectively. The signal w represents an exogenous disturbance, u is the control input, and z is the controlled output. The initial conditions are denoted by $x(0) = x_0$ and $r(0) = r_0$. We will assume that x_0 and r_0 are either deterministic constants or random variables, depending on the particular control problem considered.

2.1. Hidden-Markov mode observation

In this paper, we consider the situation where the controller cannot measure the mode signal at every time instant. To study this case, we model the times at which the controller can observe the mode by a stochastic process $t = \{t_i\}_{i=0}^\infty$ taking values in $\mathbb{N} \cup \{\infty\}$. We call t the *observation process* and each t_i an *observation time*. For each i, we assume either $t_i < t_{i+1}$ or $t_i = t_{i+1} = \infty$. It is understood that, if $t_i < t_{i+1} = \infty$, then no observation will be performed after time t_i. In this paper, we focus on the following class of observation processes:

Definition 1. We say that an observation process t is *hidden-Markovian* if there exists a subset $\Theta_o \subset \Theta$ such that $t_0 = \min\{k \geq 0 : r(k) \in \Theta_o\}$ and, for every $i \geq 0$, $t_{i+1} = \min\{k > t_i : r(k) \in \Theta_o\}$, where the minimum of the empty set is understood to be ∞.

We can interpret Θ_o as the set of modes that are observable from the controller. In the extreme case of $\Theta_o = \Theta$, the mode is observed at every time instant. Although Definition 1 requires the observation process to be correlated with the dynamics of the plant, in practice, it is possible to use an observation process independent of the plant, as illustrated in the following example:

Example 2. Consider a Markov jump linear system Σ_P given by

$$\begin{aligned} x(k+1) &= A_{P,r_P(k)}x(k) + B_{P,r_P(k)}u(k) + E_{P,r_P(k)}w(k), \\ z(k) &= C_{P,r_P(k)}x(k) + D_{P,r_P(k)}u(k), \end{aligned} \tag{2}$$

for a Markov chain r_P having a finite state space Θ_P and appropriately defined coefficient matrices $A_{P,\theta_P}, \dots$, and E_{P,θ_P} for $\theta_P \in \Theta_P$. Let us consider the situation where the observation process t is defined independently of the given system. Specifically, assume the existence of another Markov chain r_K, defined over a finite set Θ_K and independent of r_P, and a subset $\Theta_{K,o} \subset \Theta_K$ such that $t_0 = \min\{k \geq 0 : r_K(k) \in \Theta_{K,o}\}$ and, for every $i \geq 0$, $t_{i+1} = \min\{k > t_i : r_K(k) \in \Theta_{K,o}\}$. We can show that this observation process, which we call an *independent hidden-Markov observation process*, can be regarded as specifying a hidden-Markov observation process in the sense of Definition 1, as we see below. For all $\theta_P \in \Theta_P$ and $\theta_K \in \Theta_K$, define $A_{(\theta_P,\theta_K)} = A_{P,\theta_P}$, $B_{(\theta_P,\theta_K)} = B_{P,\theta_P}$, $\dots$, and $E_{(\theta_P,\theta_K)} = E_{P,\theta_P}$. Let us also introduce the extended Markov chain $r = (r_P, r_K)$ taking values in $\Theta = \Theta_P \times \Theta_K$. We can then see that Σ_P is equivalent to the Markov jump linear system Σ (given in (1)). Also, we can confirm that the above observation process can be realized as the observation process in the sense of Definition 1 if we set $\Theta_o = \Theta_P \times \Theta_{K,o} \subset \Theta$. As we see in Section 5, the flexibility of being able to design observation processes that are independent of the plant to be controlled allows us to recover interesting cases in the literature.

Apart from the random uncertainties in the observation times described above, we also consider the situation in which the controller may only observe partial information about the mode signal. Specifically, we assume that the set Θ_o of observable modes is divided into nonempty subsets $\mathcal{C}_1, \dots, \mathcal{C}_N$ called clusters (do Val et al., 2002), and that the controller can only observe to which cluster the mode signal belongs at each observation time. In the particular situation where Θ_o equals the entire space Θ, the proposed observation process reduces to the case studied in do Val et al. (2002) and Gonçalves et al. (2012). Throughout the paper, we let $\pi : \Theta_o \to \{1, \dots, N\}$ be defined by

$$\pi(\theta) = k, \ \text{if } \theta \in \mathcal{C}_k, \tag{3}$$

which denotes the mapping of a mode into the integer index of the cluster the mode belongs to.

The combination of a hidden-Markov observation process and clustered observations enables us to realize the detector-based observations studied by Costa et al. (2015):

Table 1
Relationship to other observation processes.

Observation process	Our case
Perfect observation	$\Theta_o = \Theta$ and $N = \lvert\Theta_o\rvert$
Clustered observation	$\Theta_o = \Theta$ and $N < \lvert\Theta_o\rvert$
No observation	$\Theta_o = \emptyset$
Detector-based observation	Example 3
Periodic observation	Example 17

Example 3. Consider the Markov jump linear system Σ_P in (2). We also consider the situation where the controller receives information about the mode signal r_P through a "detector" (Costa et al., 2015), which emits the random process r_K taking values in a finite set Θ_K in such a way that $\Pr(r_K(k) = \theta_K \mid r_P(k) = \theta_P) = \alpha_{\theta_P\theta_K}$ for all $k \geq 0$, $\theta_P \in \Theta_P$, and $\theta_K \in \Theta_K$. In this example, we allow the detector to emit no signal, which occurs with probability $\beta_{\theta_P} = 1 - \sum_{\theta_K\in\Theta_K}\alpha_{\theta_P\theta_K} \geq 0$ given $r_P(k) = \theta_P$. In order to realize this observation process as a hidden-Markov observation, we extend the set Θ_K as $\bar\Theta_K = \Theta_K \cup \{\psi\}$ for an arbitrarily chosen $\psi \notin \Theta_K$. We also define $\alpha_{\theta_P\psi} = \beta_{\theta_P}$, and consider the Markov chain r taking values in $\Theta_P \times \bar\Theta_K$ and having the transition probabilities $\Pr(r(k+1) = (\theta_P', \bar\theta_K') \mid r(k) = (\theta_P, \bar\theta_K)) = p_{\theta_P\theta_P'}\alpha_{\theta_P'\bar\theta_K'}$ for all $k \geq 0$, $\theta_P, \theta_P' \in \Theta_P$, and $\bar\theta_K, \bar\theta_K' \in \bar\Theta_K$. If we define the matrices $A, B, \ldots$, and E in the same way as in Example 2, then we can see that the Markov jump linear system Σ (given in (1)) has the same dynamics as system Σ_P. In order to realize the given detector-based observation process as a hidden-Markov observation, we assume that the set of the modes is clustered as $\Theta_P \times \bar\Theta_K = \bigcup_{\bar\theta_K\in\bar\Theta_K}\mathcal{C}_{\bar\theta_K}$ for the clusters $\mathcal{C}_{\bar\theta_K} = \Theta_P \times \{\bar\theta_K\}$. We can then see that $\Pr(\pi(r(k)) = \bar\theta_K) = \alpha_{\theta_P\bar\theta_K}$ provided $A_{r(k)} = A_{\theta_P}$, recovering the detector-based observation. In order to realize the situation where the detector does not emit an output, we finally let the observable set of modes to be $\Theta_o = \Theta_P \times \Theta_K$.

We summarize in Table 1 how other observation processes studied in the literature can be recovered from our framework. It is remarked that the first three classes of observation processes in the table are special cases of detector-based observation.

2.2. State-feedback controller

In this subsection, we describe the behavior of the state-feedback controller studied in this paper. In order to specify the behavior of the controller between two consecutive observation times, we introduce the following stochastic processes: Given an observation process $t = \{t_i\}_{i=0}^\infty$, we define the stochastic process $\tau = \{\tau(k)\}_{k=0}^\infty$ where $\tau(k) = \max\{t_i : t_i \leq k, i \geq 0\}$ if $k \geq t_0$, and $\tau(k) = \tau_0$ otherwise, where τ_0 is an arbitrary integer satisfying

$$\begin{cases}\tau_0 = 0, & \text{if } t_0 = 0,\\ \tau_0 < 0, & \text{otherwise.}\end{cases} \tag{4}$$

For each time k, the above defined $\tau(k)$ represents the most recent time the controller observes a mode cluster. We, in particular, have $\tau(t_i) = t_i$ for every $i \geq 0$, provided $t_i < \infty$. Notice that, for $k < t_0$, we augment the process τ with a negative integer τ_0. This is because, before time $k = t_0$, no observation has been performed by the controller yet. This augmentation is not needed if $t_0 = 0$, in which case we set $\tau_0 = 0$ as in (4).

We also define the stochastic process $\hat r = \{\hat r(k)\}_{k=0}^\infty$ taking values in Θ_o and defined as $\hat r(k) = r(\tau(k))$ if $k \geq t_0$, and $\hat r(k) = \hat r_0$ otherwise for an arbitrarily chosen $\hat r_0 \in \Theta_o$. Then, for each k, the random variable $\pi(\hat r(k))$ represents the most-updated information about the mode signal at time k. Notice that, by the same reason indicated above, $\hat r$ is augmented by an arbitrary $\hat r_0$ before the first observation time $k = t_0$.

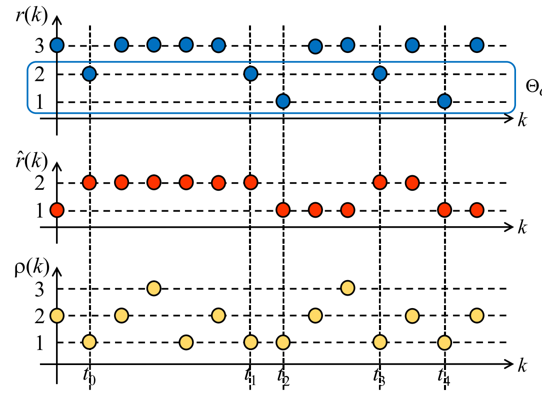

Fig. 1. An observation of the mode signal r whose state space is $\Theta = \{1, 2, 3\}$. The observation times t_i are determined by the observable set $\Theta_o = \{1, 2\}$. Until the first observation time $t_0 = 1$, the most recent observation $\hat r$ is temporarily set to $\hat r_0 = 1$. The internal clock ρ is incremented whenever an observation fails, and reset to 1 whenever the clock exceeds the constant $T = 3$.

We now present the state-feedback control scheme studied in this paper. We assume that the controller has access to the following pieces of information at each time $k \geq 0$: (*i*) the state variable $x(k)$, (*ii*) the most recent clustered observation $\pi(\hat r(k))$ of the mode r, and (*iii*) the time elapsed since the last observation, i.e., $k-\tau(k)$. It is further assumed that, due to limitations of memory space in the controller, the elapsed time is reset to 0 whenever it reaches a given threshold $T > 0$. Therefore, the controller stores the residue $\rho(k) = \lfloor k - \tau(k)\rfloor_T$ of the time $k - \tau(k)$ modulo T, instead of the elapsed time itself (see Fig. 1 for an illustration of the stochastic processes described so far). Specifically, the state-feedback controller under consideration takes the form

$$u(k) = K_{\pi(\hat r(k)),\rho(k)}x(k), \tag{5}$$

where $K_{i,\delta} \in \mathbb{R}^{m\times n}$ for each $i = 1, \ldots, N$ and $\delta = 0, \ldots, T-1$. Notice that the first subindex $\pi(\hat r(k))$ of K in (5) allows the gain to be reset whenever the controller observes the mode. We also remark that we can naturally expect an improvement in the performance of the closed-loop system as the "memory capacity" T of the controller increases, as numerically observed later in Section 5. Before proceeding to the next subsection, we summarize the relevant stochastic processes presented so far in Table 2.

Remark 4. Since $\hat r(k) = r(k - d(k))$ for $d(k) = k - \tau(k) \geq 0$, we can regard the state-feedback control (5) as a feedback control law subject to time-varying delays in its mode measurements. Although many papers in the literature address the feedback control of Markov jump linear systems subject to time-varying delays (see, e.g., Wang, Shi, Lim, and Xue (2015); Zhang, Shi, Chen, and Huang (2005)), most of the results assume uniform boundedness on the length of the delays. This restriction prevents us from relying on the existing approaches in the literature, since our problem setting allows the delay $d(k)$ to grow without a bound (as we will illustrate in Example 16).

2.3. Closed-loop properties

We now introduce several properties of the closed-loop system that will be used to evaluate the state-feedback control law (5). Let Σ_K denote the closed-loop system composed of the system Σ and the feedback control law (5). We introduce the notation

$$s(k) = (r(k), \hat r(k), \rho(k)), \tag{6}$$

which collects the discrete-valued stochastic processes relevant to the closed-loop system Σ_K. Also, define Ξ as the set of triples

Table 2
Roles of the stochastic processes.

Stochastic process	Role
$r(k) \in \Theta$	Mode signal of the Markov jump linear system Σ
$\hat{r}(k) \in \Theta_o$	Most recent observation of the mode signal (before clustering)
$\tau(k) \in \mathbb{Z}$	Most recent observation time
$\rho(k) \in \{0, \dots, T-1\}$	Counter in the controller to measure the time elapsed since the last observation time

$(\theta, \hat{\theta}, \delta) \in \Theta \times \Theta_o \times \{0, \dots, T-1\}$ such that, if $\theta \in \Theta_o$, then $\theta = \hat{\theta}$ and $\delta = 0$. The set Ξ contains all possible values that can be taken by the stochastic process $s = \{s(k)\}_{k\geq 0}$. We denote the initial condition of s as $s(0) = s_0$. We sometimes denote the trajectories x and z of Σ_K by $x(\cdot; x_0, s_0, w)$ and $z(\cdot; x_0, s_0, w)$, respectively, whenever we need to clarify the initial conditions as well as the disturbance w. We finally remark that, by the conditions in (4), s_0 is determined by r_0, $\hat{r}_0$, and ρ_0 as

$$s_0 = \begin{cases} (r_0, r_0, 0), & \text{if } r_0 \in \Theta_o, \\ (r_0, \hat{r}_0, \rho_0), & \text{otherwise.} \end{cases} \tag{7}$$

The first property under consideration is the mean square stability:

Definition 5. We say that Σ_K is *mean square stable* if there exist $C > 0$ and $\lambda \in [0, 1)$ such that $E[\|x(k)\|^2] \leq C\lambda^k \|x_0\|^2$ for all x_0, s_0, and $k \geq 0$, provided that $w \equiv 0$.

In order to define the second property under consideration, we first need to introduce the space of square summable stochastic processes, as follows. Let $\mathfrak{F}_k$ be the σ-algebra generated by the random variables $\{r(k), \dots, r(0)\}$. Define $\ell^2(\mathbb{R}^n)$ (ℓ^2 for short) as the space of $\mathbb{R}^n$-valued stochastic processes $f = \{f(k)\}_{k\geq 0}$ such that $f(k)$ is an $\mathbb{R}^n$-valued and $\mathfrak{F}_k$-measurable random variable for each $k \geq 0$ and, moreover, $\sum_{k=0}^{\infty} E[\|f(k)\|^2]$ is finite. For $f \in \ell^2$, define its norm $\|f\|_2$ by $\|f\|_2^2 = \sum_{k=0}^{\infty} E[\|f(k)\|^2]$. In what follows, we extend the definition of the H_2 norm of a Markov jump linear system introduced in do Val et al. (2002), as follows:

Definition 6. Assume that s_0 follows a probability distribution μ_s. Define the H_2 *norm* of Σ_K by $\|\Sigma_K\|_2 = (\sum_{j=1}^{q}\sum_{\xi\in\Xi}\mu_s(\xi)\| z(\cdot; 0, \xi, \phi_j)\|_2^2)^{1/2}$, where the sequence ϕ_j is defined by $\phi_j = \{e_j, 0, 0, \dots\}$ with e_j being the jth standard unit vector in $\mathbb{R}^q$.

Our third and last property is the H_∞ performance. In our context, we use the following definition, which is an extension of the one for standard Markov jump linear systems (Seiler & Sengupta, 2003):

Definition 7. Assume that Σ_K is mean square stable and $x_0 = 0$. Define the H_∞ *norm* of Σ_K by $\|\Sigma_K\|_\infty = \sup_{s_0\in\Xi}\sup_{w\in\ell^2(\mathbb{R}^q)\setminus\{0\}} \|z\|_2/\|w\|_2$.

We remark that Σ_K is no longer a standard Markov jump linear system due to the nature of the processes $\hat{r}$ and ρ. Therefore, we cannot use existing techniques in the literature Cetinkaya and Hayakawa (2014, 2015), Costa et al. (2005), do Val et al. (2002) and Gonçalves et al. (2012) to synthesize a control law, or, even to analyze the closed-loop system.

3. Equivalent reduction to a Markov jump linear system

In this section, we reduce the closed-loop system Σ_K to a standard Markov jump linear system by embedding the stochastic processes appearing in the closed-loop system (which are not necessarily Markovian) into an extended Markov chain with a larger state space. We furthermore show that the obtained Markov jump linear system shares the same closed-loop properties, namely, mean square stability, H_2 norm, and H_∞ norm, as the original closed-loop system.

The following proposition plays a key role in our reduction:

Proposition 8. *The stochastic process s defined by (6) is a time-homogeneous Markov chain. Moreover, its transition probability $q_{\xi\xi'} = \Pr(s(k+1) = \xi' \mid s(k) = \xi)$ for $\xi = (\theta, \hat{\theta}, \delta)$ and $\xi' = (\theta', \hat{\theta}', \delta')$ in Ξ is given by*

$$q_{\xi\xi'} = \begin{cases} \mathbb{1}(\theta' = \hat{\theta}',\ \delta' = 0)p_{\theta\theta'}, & \text{if } \theta' \in \Theta_o, \\ \mathbb{1}(\hat{\theta}' = \hat{\theta},\ \delta' {=_T} \delta + 1)p_{\theta\theta'}, & \text{otherwise,} \end{cases} \tag{8}$$

where $=_T$ denotes the congruence modulo T.

Proof. Let $k_0 \in \mathbb{N}$, $k \geq k_0$, and $\xi_i = (\theta_i, \hat{\theta}_i, \delta_i) \in \Xi$ ($i = k_0, \dots, k+1$) be arbitrary. For each i, define the events $\mathcal{A}_i$ and $\mathcal{B}_i$ as $\mathcal{A}_i = \{\omega : s(i) = \xi_i, \dots, s(k_0) = \xi_{k_0}\}$ and $\mathcal{B}_i = \{\omega : s(i) = \xi_i\}$. In order to show (8), let us evaluate the probability $\Pr(\mathcal{A}_{k+1})$. We first consider the case $\theta_{k+1} \in \Theta_o$. If either $\theta_{k+1} = \hat{\theta}_{k+1}$ or $\delta_{k+1} = 0$ does not hold, then the set $\mathcal{A}_{k+1}$ is trivially a null set by the definitions of the stochastic processes $\hat{r}$ and τ. Let us assume that both $\theta_{k+1} = \hat{\theta}_{k+1}$ and $\delta_{k+1} = 0$ hold. Then, since $\theta_{k+1} \in \Theta_o$, the events $\mathcal{B}_{k+1}$ and $\{\omega : r(k+1) = \theta_{k+1}\}$ are equivalent by the definition of the stochastic processes $\hat{r}$ and τ. Hence, we can evaluate the probability $\Pr(\mathcal{A}_{k+1})$ as

$$\begin{aligned} \Pr(\mathcal{A}_{k+1}) &= \Pr(\mathcal{A}_k \cap \{\omega : r(k+1) = \theta_{k+1}\}) \\ &= \Pr(\mathcal{A}_k)\Pr(r(k+1) = \theta_{k+1} \mid \mathcal{A}_k) \\ &= \Pr(\mathcal{A}_k)\Pr(r(k+1) = \theta_{k+1} \mid r(k) = \theta_k) \\ &= \Pr(\mathcal{A}_k)p_{\theta_k\theta_{k+1}} \end{aligned} \tag{9}$$

where, in the third last equation, we have used the fact that r is a Markov chain and the values of the stochastic processes $\hat{r}$ and ρ are determined by the current or past values of r. Summarizing, we can conclude that $\Pr(s(k+1) = \xi_{k+1} \mid \mathcal{A}_k) = \mathbb{1}(\theta_{k+1} = \hat{\theta}_{k+1}, \delta_{k+1} = 0)p_{\theta_k\theta_{k+1}}$ for the case of $\theta_{k+1} \in \Theta_o$, proving the first part of (8) since k_0 was arbitrary.

Second, let us consider the case $\theta_{k+1} \notin \Theta_o$. If either $\hat{\theta}_{k+1} = \hat{\theta}_k$ or $\delta_{k+1} {=_T} \delta_k + 1$ does not hold, then the event $\mathcal{A}_{k+1}$ is of probability zero. In the sequel, we assume that both $\hat{\theta}_{k+1} = \hat{\theta}_k$ and $\delta_{k+1} {=_T} \delta_k + 1$ hold. Under this assumption, since $\theta_{k+1} \notin \Theta_o$, we see that the events $\mathcal{B}_{k+1}$ and $\{\omega : r(k+1) = \theta_{k+1}\}$ are equivalent and, therefore, the identity (9) holds for the same reason as in the first part of the proof of this proposition. This implies that, if $\theta_{k+1} \notin \Theta_o$, then $\Pr(s(k+1) = \xi_{k+1} \mid \mathcal{A}_k) = \mathbb{1}(\hat{\theta}_{k+1} = \hat{\theta}_k, \delta_{k+1} {=_T} \delta_k + 1)p_{\theta_k\theta_{k+1}}$, completing the proof of the second part of (8). □

Remark 9. We remark that the transition probabilities $q_{\xi\xi'}$ are well-defined because, for every $\xi = (\theta, \hat{\theta}, \delta) \in \Xi$,

$$\begin{aligned} \sum_{\xi'\in\Xi} q_{\xi\xi'} &= \sum_{\theta'\in\Theta_o}\sum_{\hat{\theta}'\in\Theta_o}\sum_{\delta'=0}^{T-1} \mathbb{1}(\theta' = \hat{\theta}', \delta' = 0)p_{\theta\theta'} \\ &\quad + \sum_{\theta'\notin\Theta_o}\sum_{\hat{\theta}'\in\Theta_o}\sum_{\delta'=0}^{T-1} \mathbb{1}(\hat{\theta}' = \hat{\theta}, \delta' {=_T} \delta + 1)p_{\theta\theta'} \\ &= \sum_{\theta'\in\Theta_o} p_{\theta\theta'} + \sum_{\theta'\notin\Theta_o} p_{\theta\theta'} = 1. \end{aligned}$$

Proposition 8 indicates that the closed-loop system Σ_K can be represented as a Markov jump linear system with its mode being the extended Markov chain s. This indication leads us to the following definitions. Let us introduce the Markov jump linear system

$$\bar{\Sigma}_K : \begin{cases} \bar{x}(k+1) = A_{K,\bar{s}(k)}\bar{x}(k) + E_{K,\bar{s}(k)}\bar{w}(k), \\ \bar{z}(k) = C_{K,\bar{s}(k)}\bar{x}(k), \end{cases}$$

where $\bar{s}$ is a time-homogeneous Markov chain taking values in Ξ and having the following transition probabilities $\Pr(\bar{s}(k+1) = \xi' \mid \bar{s}(k) = \xi) = q_{\xi\xi'}$. The matrices $A_{K,\xi}$, $C_{K,\xi}$, and $E_{K,\xi}$ are defined by $A_{K,\xi} = A_\theta + B_\theta K_{\pi(\hat{\theta}),\delta}$, $C_{K,\xi} = C_\theta + D_\theta K_{\pi(\hat{\theta}),\delta}$ and $E_{K,\xi} = E_\theta$ for each $\xi = (\theta, \hat{\theta}, \delta) \in \Xi$. We sometimes denote $\bar{x}$ and $\bar{z}$ by $\bar{x}(\cdot; \bar{x}_0, \bar{s}_0, \bar{w})$ and $\bar{z}(\cdot; \bar{x}_0, \bar{s}_0, \bar{w})$ whenever we need to clarify initial conditions and disturbances $\bar{w}$. Then, the next theorem shows the stochastic equivalence of the two systems Σ_K and $\bar{\Sigma}_K$:

Theorem 10. *The following statements hold true:*

(1) *Assume that $x_0 = \bar{x}_0$, w and $\bar{w}$ have the same probability distribution, and s_0 and $\bar{s}_0$ have the same probability distribution. Then, the stochastic processes $x(\cdot; x_0, s_0, w)$ and $\bar{x}(\cdot; \bar{x}_0, \bar{s}_0, \bar{w})$ have the same probability distribution. Also, under the same assumptions, the stochastic processes $z(\cdot; x_0, s_0, w)$ and $\bar{z}(\cdot; \bar{x}_0, \bar{s}_0, \bar{w})$ have the same probability distribution.*
(2) *Σ_K is mean square stable if and only if $\bar{\Sigma}_K$ is mean square stable.*
(3) *If Σ_K and $\bar{\Sigma}_K$ are mean square stable, then the following statements are true: (a) If s_0 and $\bar{s}_0$ follow the same distribution, then $\|\Sigma_K\|_2 = \|\bar{\Sigma}_K\|_2$; and (b) $\|\Sigma_K\|_\infty = \|\bar{\Sigma}_K\|_\infty$.*

Proof. Let us prove the first statement. By the assumption, the Markov chains s and $\bar{s}$ have the same initial distribution. These chains also have the same transition probabilities from Proposition 8 and the definition of $\bar{s}$. Therefore, s and $\bar{s}$ have the same probability distribution. Also, notice that, by the definition of the matrices $A_{K,\xi}$, $C_{K,\xi}$, and $E_{K,\xi}$, the closed-loop system Σ_K admits the representation $x(k+1) = A_{K,s(k)}x(k) + E_{s(k)}w(k)$ and $z(k) = C_{K,s(k)}x(k)$. Therefore, Σ_K has the same dynamics as $\bar{\Sigma}_K$. In conclusion, the claim holds true under the assumptions stated in the theorem.

Then we prove the second statement on mean square stability. Assume that $\bar{\Sigma}_K$ is mean square stable. In order to show that Σ_K is also mean square stable, let us take arbitrary $x_0 \in \mathbb{R}^n$ and $s_0 \in \Xi$. Then, by the first statement of the theorem and the mean square stability of $\bar{\Sigma}_K$, we can show $E[\|x(k; x_0, s_0, 0)\|^2] = E[\|\bar{x}(k; x_0, s_0, 0)\|^2] \leq C\lambda^k\|x_0\|^2$ for some $C > 0$ and $\lambda \in [0, 1)$, which implies the mean square stability of Σ_K. We can prove the other direction in the same way.

Let us finally prove the last statement. Assume that Σ_K and $\bar{\Sigma}_K$ are mean square stable. By the first claim of the theorem and the definition of the H_2 norm, it is immediate to see that $\|\Sigma_K\|_2 = \|\bar{\Sigma}_K\|_2$, provided that s_0 and $\bar{s}_0$ follow the same distribution. In order to prove $\|\Sigma_K\|_\infty = \|\bar{\Sigma}_K\|_\infty$, we let $\bar{\mathfrak{F}}_k$ denote the σ-algebra generated by the random variables $\{\bar{s}(k), \dots, \bar{s}(0)\}$. Define $\bar{\ell}^2$ as the space of stochastic processes $\bar{f} = \{\bar{f}(k)\}_{k=0}^\infty$ such that $\sum_{k=0}^\infty E[\|\bar{f}(k)\|^2]$ is finite and, for each $k \geq 0$, $\bar{f}(k)$ is an $\mathbb{R}^n$-valued and $\bar{\mathfrak{F}}_k$-measurable random variable. Define the norm of $\bar{f} \in \bar{\ell}^2$ by $\|\bar{f}\|_2^2 = \sum_{k=0}^\infty E[\|\bar{f}(k)\|^2]$. Then, the H_∞ norm of $\bar{\Sigma}_K$ is given by $\|\bar{\Sigma}_K\|_\infty = \sup_{\bar{s}_0\in\Xi}\sup_{\bar{w}\in\bar{\ell}^2\setminus\{0\}}(\|\bar{z}(\cdot; 0, \bar{s}_0, \bar{w})\|_2/\|\bar{w}\|_2)$. To show that this norm equals $\|\Sigma_K\|_\infty$, it is sufficient to show

$$\sup_{w\in\ell^2\setminus\{0\}} \frac{\|z(\cdot; 0, s_0, w)\|_2}{\|w\|_2} = \sup_{\bar{w}\in\bar{\ell}^2\setminus\{0\}} \frac{\|\bar{z}(\cdot; 0, \bar{s}_0, \bar{w})\|_2}{\|\bar{w}\|_2},$$

provided $s_0 = \bar{s}_0$. By the first claim of the theorem, the only difference between both sides of this equality is the spaces ℓ^2 and $\bar{\ell}^2$. In other words, to complete the proof, it is sufficient to show that $\mathfrak{F}_k$, the σ-algebra generated by the random variables $\{r(k), \dots, r(0)\}$, coincides with the one generated by $\{\bar{s}(k), \dots, \bar{s}(0)\}$. This is obvious because $\hat{r}(k)$ and $\rho(k)$, contained in $\bar{s}(k)$, are images of $(r(k), r(k-1), \dots, r(0))$ under measurable functions. This completes the proof of the theorem. □

Theorem 10 shows that the closed-loop system Σ_K, whose collective dynamics is not necessarily trivial, can be in fact regarded as a Markov jump linear system, while the important closed-loop characteristics are preserved. We will show in the next section that this fact allows us to design state-feedback controllers for stabilization, H_2, and H_∞ performance for Markov jump linear systems under a hidden-Markov observation in a straightforward manner.

4. Design of feedback gains via linear matrix inequalities

As an application of the equivalent representation of Σ_K as a Markov jump linear system, we now show that previous results available in the literature, such as do Val et al. (2002) and Gonçalves et al. (2012), can be readily utilized to design feedback control laws for stabilization, H_2 suboptimal control, and H_∞ suboptimal control of Markov jump linear systems under hidden-Markov observations of the mode signals. The next proposition provides an LMI formulation to solve the stabilization problem:

Proposition 11. *Assume that the matrices $R_\xi \in \mathbb{R}^{n\times n}$, $G_{i,\delta} \in \mathbb{R}^{n\times n}$, and $F_{i,\delta} \in \mathbb{R}^{m\times n}$ ($\xi \in \Xi$, $i = 1, \dots, N$, and $\delta = 0, \dots, T-1$) satisfy the following LMI*

$$\begin{bmatrix} R_\xi & A_\theta G_{\pi(\hat{\theta}),\delta} + B_\theta F_{\pi(\hat{\theta}),\delta} \\ \star & \mathrm{He}(G_{\pi(\hat{\theta}),\delta}) - \mathcal{D}_\xi(R) \end{bmatrix} > 0, \tag{10}$$

for all $\xi = (\theta, \hat{\theta}, \delta) \in \Xi$, where $\mathcal{D}_\xi(R) = \sum_{\xi'\in\Xi} q_{\xi'\xi}R_{\xi'}$. For each $i = 1, \dots, N$ and $\delta = 0, \dots, T-1$, define

$$K_{i,\delta} = F_{i,\delta}G_{i,\delta}^{-1}. \tag{11}$$

Then, Σ_K is mean square stable.

Proof. Since $R_\xi \in \mathbb{R}^{n\times n}$, $G_{i,\delta} \in \mathbb{R}^{n\times n}$, and $F_{i,\delta} \in \mathbb{R}^{m\times n}$ satisfy (10), in the same way as in the argument proposed in do Val et al. (2002), we can show that $\bar{\Sigma}_K$ is mean square stable. Therefore, by Theorem 10, the closed-loop system Σ_K is mean square stable. □

Secondly, we consider H_2 suboptimal control. In this problem, we assume that the distribution of r_0, denoted by μ_r, is given. Thus, the parameters to be designed are feedback gains K and the distribution ν of the pair $(\hat{r}_0, \rho_0)$. We remark that, given the distribution μ_r, we can find from (7) the probability distribution μ_s of s_0 as

$$\mu_s(\xi) = \begin{cases} \mu_r(\theta), & \text{if } \theta \in \Theta_o, \\ \mu_r(\theta)\nu(\hat{\theta}, \delta), & \text{otherwise}, \end{cases} \tag{12}$$

for every $\xi = (\theta, \hat{\theta}, \delta) \in \Xi$. The next proposition provides an LMI formulation to solve the H_2 suboptimal control problem. We omit its proof because it is almost identical as the one in do Val et al. (2002):

Proposition 12. *Let $\zeta > 0$ be arbitrary. Assume that $W_\xi \in \mathbb{R}^{\ell\times\ell}$, $R_\xi \in \mathbb{R}^{n\times n}$, $F_{i,\delta} \in \mathbb{R}^{m\times n}$, $G_{i,\delta} \in \mathbb{R}^{n\times n}$, and $\nu(\hat{\theta}, \delta) \geq 0$ ($\xi \in \Xi$, $\hat{\theta} \in \Theta_o$, $i = 1, \dots, N$, and $\delta = 0, \dots, T-1$) satisfy the following LMI's*

$$\begin{bmatrix} R_\xi - \mu_s(\xi)E_\theta E_\theta^\top & A_\theta G_{\pi(\hat{\theta}),\delta} + B_\theta F_{\pi(\hat{\theta}),\delta} \\ \star & \mathrm{He}(G_{\pi(\hat{\theta}),\delta}) - \bar{\mathcal{D}}_\xi(R) \end{bmatrix} > 0,$$

$$\begin{bmatrix} W_\xi & C_\theta G_{\pi(\hat\theta),\delta} + D_\theta F_{\pi(\hat\theta),\delta} \\ \star & \operatorname{He}(G_{\pi(\hat\theta),\delta}) - \bar{\mathcal{D}}_\xi(R) \end{bmatrix} > 0,$$

$$\sum_{\xi\in\Xi} \operatorname{tr}(W_\xi) < \zeta, \quad \sum_{\hat\theta\in\Theta_0}\sum_{\delta=0}^{T-1} \nu(\hat\theta,\delta) = 1, \tag{13}$$

for every $\xi = (\theta, \hat\theta, \delta) \in \Xi$. Define $K_{i,\delta}$ by (11) for all $i = 1, \dots, N$ and $\delta = 0, \dots, T-1$. Then, Σ_K is mean square stable and satisfies $\|\Sigma_K\|_2^2 < \zeta$.

Remark 13. The (in)equalities in Proposition 12 are indeed linear with respect to the design variables. The linearity with respect to the matrix variables W_ξ, R_ξ, $F_{i,\delta}$, and $G_{i,\delta}$ is obvious. The linearity with respect to ν follows from (12). We also remark that the last equation in (13) makes ν a probability measure.

Finally, the next proposition provides an LMI formulation to solve the H_∞ suboptimal control problem. We again omit its proof because it is almost identical as the one in Gonçalves et al. (2012):

Proposition 14. *Let $\zeta > 0$ be arbitrary. Assume that $H_\xi \in \mathbb{R}^{n\times n}$, $X_\xi \in \mathbb{R}^{n\times n}$, $Z_{\xi,\xi'} \in \mathbb{R}^{n\times n}$, $G_{i,\delta} \in \mathbb{R}^{n\times n}$, and $F_{i,\delta} \in \mathbb{R}^{m\times n}$ ($\xi, \xi' \in \Xi$, $1 \le i \le N$, and $0 \le \delta \le T-1$) satisfy the following LMI's*

$$\begin{bmatrix} \operatorname{He}(G_{\pi(\hat\theta),\delta}) - X_\xi & \star & \star & \star \\ O & \zeta I_q & \star & \star \\ A_\theta G_{\pi(\hat\theta),\delta} + B_\theta F_{\pi(\hat\theta),\delta} & E_\theta & \operatorname{He}(H_\xi) - \mathcal{F}_\xi(Z) & \star \\ C_\theta G_{\pi(\hat\theta),\delta} + D_\theta F_{\pi(\hat\theta),\delta} & O & O & I_\ell \end{bmatrix} > 0,$$

$$\begin{bmatrix} Z_{\xi,\xi'} & \star \\ H_\xi & X_{\xi'} \end{bmatrix} > 0,$$

for all $\xi = (\theta, \hat\theta, \delta)$ and ξ' in Ξ, where $\mathcal{F}_\xi(Z) = \sum_{\xi'\in\Xi} q_{\xi\xi'} Z_{\xi,\xi'}$. Define $K_{i,\delta}$ by (11) for all $i = 1, \dots, N$ and $\delta = 0, \dots, T-1$. Then, Σ_K is mean square stable and satisfies $\|\Sigma_K\|_\infty^2 < \zeta$.

Remark 15. In the case of clustered observations considered in do Val et al. (2002), one can check that the LMI's in Propositions 11 and 12 (for stabilization and H_2 control, respectively) are equivalent to the LMI's presented in do Val et al. (2002). Likewise, we can also see that the linear matrix inequalities given in Proposition 14 equivalently reduce to the LMI's presented in Gonçalves et al. (2012) for H_∞ control under clustered observations.

5. Numerical examples

In this section, we present two numerical examples to illustrate the results obtained in the previous section. Our first example studies the control of a Markov jump linear system with a mode observation through a Gilbert–Elliot channel (Gilbert, 1960):

Example 16. Let us consider the H_2 control of the Markov jump linear system studied in Costa et al. (2015, Example 1). The Markov jump linear system has the two modes labeled as 1 and 2, and its mode signal is identical and independently distributed with probabilities $\Pr(r_P(k) = 1) = 0.6942$ and $\Pr(r_P(k) = 2) = 0.3058$. We refer the readers to Costa et al. (2015) for the parameters of the system. We assume that the controller observes the mode through a Gilbert–Elliot channel (Gilbert, 1960) that behaves independently of the Markov jump linear system to be controlled. We, for simplicity, shall focus on the following simplified case where the channel has two possible states: the good (G) and bad (B) states (see, e.g., Gonçalves, Fioravanti, & Geromel, 2010, for a general description of the channel). Specifically, we assume that, when the channel is at the G state, it transmits the mode signal to the controller; in contrast, when it is at state B, it does not transmit. Let $p, q \in [0, 1]$ be the transition probabilities from G to B and B to G, respectively.

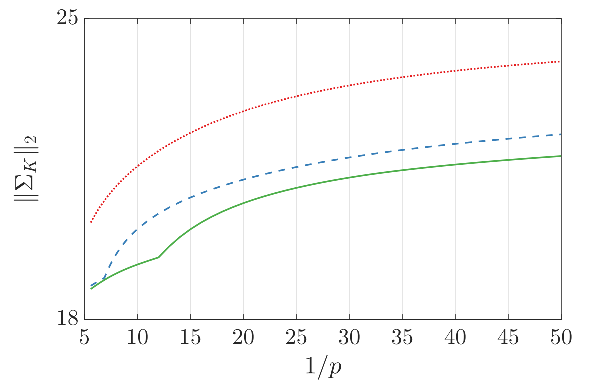

Fig. 2. The H_2 norm of the closed-loop system Σ_K versus the expected duration $1/p$. Dotted: $T = 1$, dashed: $T = 3$, solid: $T = 5$.

In order to design an H_2 controller for this system, we formulate this channel as the independent hidden-Markov observation process given in Example 2, by using a Markov chain r_K having the state space $\Theta_K = \{1, 2\}$ and the parameter $\Theta_{K,o} = \{1\}$. Since the observation is not clustered, we set the mapping π in (3) to be the identity mapping. For simplicity in our presentation, we let $p = q > 0$. Notice that, whatever value p takes, the limiting distribution of the Markov chain r_K is the uniform distribution on the set $\{1, 2\}$; in other words, the asymptotic frequency of the controller observing the mode signal r_P is $1/2$. In addition, the expected duration of the chain r_K staying at either Good or Bad state depends on p, and is equal to $1/p$. We assume that the initial distribution of r_K is the uniform distribution.

We use the procedure described in Example 2 to reduce the given Markov jump linear system and the independent hidden-Markov observation process into a single Markov jump linear system with a hidden-Markov observation process (given in Definition 1). We then use Proposition 12 to design the stabilizing feedback gains and the initial distribution ν in order to achieve a small H_2 norm for the closed-loop system Σ_K by finding parameters W_ξ, R_ξ, $F_{i,\delta}$, $G_{i,\delta}$, and $\nu(\hat\theta, \delta)$ that minimize ζ while satisfying the LMI's (13). Fig. 2 shows the H_2 norms of the optimized closed-loop system for several values of T. As expected, the larger the memory capacity T of the controller, the smaller the attained H_2 norm. We can also see that the H_2 norm of the closed-loop system increases as the expected duration $1/p$ increases, even though the stationary distribution of r_K does not depend on p. We remark that this feature, which is a consequence of the Markovian property in the observation process, cannot be captured by the framework in Costa et al. (2015), where mode observations at different time instants are assumed to be independent events with identical probabilities.

In our second example, we consider the control of a linear time-invariant system with a failure-prone controller under periodic monitoring (Gertsbakh, 2000):

Example 17. Consider a plant modeled as the linear time-invariant system $x(k+1) = A_0 x(k) + B_0 u(k) + E_0 w(k)$, $z(k) = x(k)$ having the coefficient matrices

$$A_0 = \begin{bmatrix} -0.1 & -0.4 \\ 0 & -0.6 \end{bmatrix}, \; B_0 = \begin{bmatrix} 1.5 & 0 \\ 0 & -0.4 \end{bmatrix}, \; E_0 = \begin{bmatrix} 0.3 \\ -0.5 \end{bmatrix}.$$

We consider the situation where the actuator can experience a component-wise failure, in which a component of the actuator fails

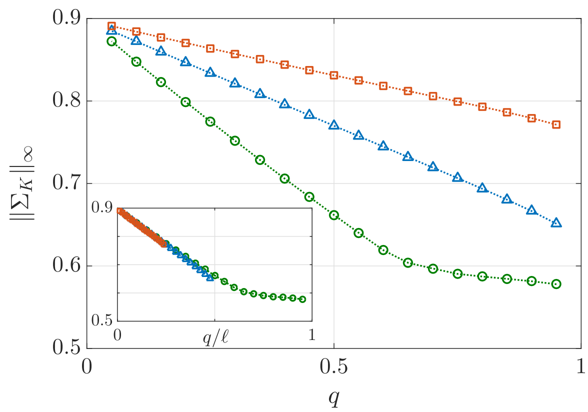

Fig. 3. H_∞ norms of the closed-loop system versus q (main figure) and q/ℓ (inset figure). Circles: $\ell = 1$, triangles: $\ell = 2$, squares: $\ell = 4$.

to transmit control signals to the plant. We model this situation by the state-feedback control

$$u(k) = \begin{bmatrix} r_1(k) & 0 \\ 0 & r_2(k) \end{bmatrix} K(k)x(k),$$

where $K(k)$ is the state-feedback gain to be designed and r_1, r_2 are $\{0, 1\}$-valued independent Markov chains representing the failures of the first and the second components of the actuator. We specifically let $r_i(k) = 0$ if the ith component is experiencing a failure at time k and $r_i(k) = 1$ otherwise. Denoting $r_P = (r_1, r_2)$, we can represent the system to be controlled as the Markov jump linear system (2) having the coefficient matrices $A_{P,(i,j)} = A_0$,

$$B_{P,(i,j)} = \begin{bmatrix} 1.5i & 0 \\ 0 & -0.4j \end{bmatrix},$$

$C_{P,(i,j)} = I$, $D_{P,(i,j)} = O$, and $E_{P,(i,j)} = E_0$ for all $i, j \in \{0, 1\}$.

We here focus on the situation where monitoring of the failures is costly, and therefore the observation of the mode signal r_P is performed in an imperfect manner. We specifically assume that the observation of the mode is performed periodically (Gertsbakh, 2000; Nakagawa, 1986) with a period ℓ and, also, that each observation fails with probability $1 - q \in [0, 1]$, in which case the controller does not receive a mode signal. We can realize this observation as an independent hidden-Markov observation process using a Markov chain r_K having the state space $\Theta_K = \{1, \dots, \ell+1\}$, the transition probability matrix

$$\left[\begin{array}{c:c} & e^\top \\ & I_{\ell-1} \\ \hdashline q \quad 1-q & \end{array}\right],$$

where e denotes the first element in the canonical basis of $\mathbb{R}^{\ell-1}$, and the parameter $\Theta_{K,o} = \{1\}$. We remark that, when $q = 0$, this observation process recovers the case of periodic mode-observations studied in Cetinkaya and Hayakawa (2014). We further consider a more realistic scenario where the observer of the mode signals, due to limitations on its performance, cannot distinguish different failures occurring on distinct components. We realize this situation by a clustered observation having the clusters $\mathcal{C}_1 = \{(1, 1)\}$ (representing the case of no failures) and $\mathcal{C}_2 = \{(1, 0), (0, 1), (0, 0)\}$ (representing the existence of a failure).

In this numerical simulation, we consider the H_∞ control of the above described system. Using the procedure given in Example 2, we equivalently transform the above problem into the problem of controlling a Markov jump linear system under a hidden-Markov observation. Then, using Proposition 14, we design suboptimal feedback gains for the H_∞ control of the system for various values of ℓ and q, while fixing the probabilities of failure and recovery of the components in the actuator to be $p_f = 0.1$ and $p_r = 0.5$. Fig. 3 shows the H_∞ norms of the resulting closed-loop systems versus q and the ratio q/ℓ. While we observe that a longer monitoring period ℓ deteriorates the closed-loop performance, we can also see from the inset figure that the performance is almost fully determined by the "effective" monitoring rate q/ℓ of the mode signal.

Remark 18. Using Proposition 12, we can derive alternative linear matrix inequalities for designing state-feedback gains for the H_2 control of Markov jump linear systems subject to detector-based mode observations (Costa et al., 2015). However, our extensive numerical simulations suggest that the closed-loop system designed by our framework tends to have a larger H_2 norm compared with the one designed by the method given in Costa et al. (2015). This observation suggests a possible trade-off between applicability (i.e., the admissible class of observation processes) and conservativeness: Although the methods in Costa et al. (2015) can perform well in the case of detector-based mode observations, our framework can be applied to a broader class of problems at the expense of potential underperformance. We have found a similar tendency from an extensive numerical comparison with the results in Cetinkaya and Hayakawa (2014) for the stabilization of Markov jump linear systems subject to periodic mode observations. We leave the problem of exploring this potential conservativeness of our framework as an open problem.

6. Conclusion

In this paper, we have proposed a general framework to analyze and design state-feedback control laws for Markov jump linear systems with hidden-Markov and clustered observations of the mode signals. This observation model recovers many relevant cases previously studied in the literature of Markov jump linear systems, such as the cases with clustered observations, detector-based observations, and periodic observations of the mode signal. We have first shown that the resulting closed-loop system can be equivalently transformed to a standard Markov jump linear system. Based on this fact, we have then presented an LMI framework to design feedback gains for stabilization, H_2 suboptimal control, and H_∞ suboptimal control. Finally, we have illustrated the effectiveness of this framework with numerical examples.

A possible direction for future research is the analysis and control of the particular class of Markov jump linear systems having identical and independently distributed mode signals. For this specific class of Markov jump linear systems, it is of theoretical interest to investigate whether we can reduce conservativeness of the LMI-based state-feedback design, as achieved in Costa et al. (2015). Other direction of interest is the simplification of the systems of LMI's introduced in Section 4. In this direction, since the transition probability matrix of the extended Markov chain s is rather sparse, we plan to exploit the sparsity of the associated LMI's to reduce the computational complexity and the conservativeness of our designs.

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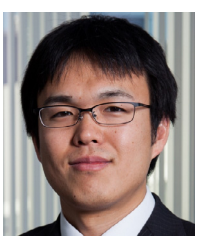

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