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Robust Fault Detection H_∞ Filter for Markovian Jump Linear Systems with Partial Information on the Jump Parameter

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Abstract: The present work focus on the Robust Fault Detection (RFD) problem in the Markovian Jump Linear System framework for the discrete-time domain, in which the Markov parameter $\theta(k)$ is considered not accessible. The assumption that the Markov Chain is not accessible brings a challenge where the filter designed for the RFD should not be dependent on the Markov Chain parameter. In order to represent this kind of situation, the implementation of a Hidden Markov Chain to model the system mode $\theta(k)$ and the estimated mode $\hat{\theta}(k)$ is used. The main result presented in this work is the design of a H_∞ MJLS Robust Fault Detection filter that depends only on the estimated mode $\hat{\theta}(k)$ obtained through LMI formulation. In order to illustrate the feasibility of the proposed solution a numerical example is also included.

Keywords: MJLS, Hidden Markov Chain, Robust Fault Detection, H_∞ Filtering

1. INTRODUCTION

In the last decades the frameworks that consider abrupt changes in dynamic systems have been heavily studied, due to the wide range of applicability of this kind of theory, like the fields in engineering such as chemical, nuclear, aerospace, and automotive applications, for instance, see the works Venkatasubramanian et al. [2003], Kim and Bartlett [1996], Favre [1994], Isermann et al. [2002], respectively. A particular field of study that tackles this type of situation is the Robust Fault Detection and Isolation (RFDI for short). This approach has the main purpose of sensing and rearranging the system in order to minimize the possible losses and/or chances of accidents, see for instance Hwang et al. [2010]. The RFDI working scheme can be explained as follows: generating a residual signal obtained from the sensor in the plant (commonly through a filter), pre-setting a threshold (usually after observing the plant behavior in the nominal work mode, Patton et al. [2013]) and whenever the residual signal surpasses the threshold value it is considered that a failure occurred. Keeping in mind these informations we are able to envision important characteristic that the RFDI must have for the process involved in the creation of the residual signal: first, the approach must be robust since the noise presence may notably influence the RFDI scheme performance; second, the communication between the equipment that composes the RFDI must be reliable because the communication loss also can largely impact in the RFDI performance.

To mention a few works that study the MJLS framework associated with the FDI, we can point out Zhong et al. [2005] where the design of a H_∞ residual filter in the discrete-time MJLS is studied and also Wang and Yin [2017] that considered the synthesis of H_∞ residual filters for continuous-time MJLS. In the first one the Markov Chain modes are assumed to be accessible and in the second one it is considered the assumption that the modes of operation of the filter are unmatched in relation to the system being observed. Consider a generic MJLS as

$$\mathcal{G}:\begin{cases} x(k+1) = A_{\theta_k}x(k) + J_{\theta_k}w(k) \\ y(k) = C_{\theta_k}x(k) + E_{\theta_k}w(k) \\ x(0) = x_0, \quad \theta(0) = \theta_0 \end{cases} \tag{1}$$

The system (1) is subject to abrupt changes, which are modeled as a Markov chain denoted by the variable $\theta(k)$. Hereafter, the variable $\theta(k)$ is assumed to be not accessible.

The primary motivation to consider that the Markov Chain mode not accessible is the possibility to model the communication made through a network where the network mode is not accessible to the components responsible to the robust fault detection. It is possible to consider the variable $\theta(k)$ as the network mode and the variable $\hat{\theta}(k)$ as the estimated network mode.

In this paper, we consider that the Markov Chain mode is not accessible. In order to tackle this assumption, the Hidden Markov Chain model is used, where a detector

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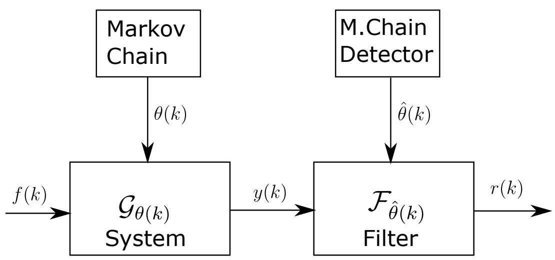

Figure 1. Simplified block diagram.

provides an estimation of the Markov Chain mode denoted by $\hat{\theta}(k)$. Given that the only variable accessible is the mode estimation provided by the detector, the RFD filter designed to work under this assumption may only depend on the parameter provided by the detector $\hat{\theta}(k)$. In this paper we study a particular case of Robust Fault Detection problem where we assume that the Markov Chain mode is not accessible. Keeping that information in mind, and observing the Fig. 1, we have three distinct variables:

- The mode information of the system (1), denoted by the index $\theta(k)$, that is unavailable.
- the estimated system mode $\hat{\theta}(k)$ provided by a detector.
- $f(k)$ that represents the fault to be detected by a robust fault detection filter.

The main goal of this work is to provide a robust fault detection filter that can work under the assumption that it is not possible to access the system (1) information's mode. Inspired by the works de Oliveira and Costa [2017b] Todorov et al. [2015], a new LMI formulation to design a RFD filter is provided. The RFD filter obtained via the LMI formulation proposed in this work depends only on the detector parameter $\hat{\theta}(k)$. The novelty of this paper is, as far as the author are aware of, the presentation of a RFD filter that depends only on detector parameter $\hat{\theta}(k)$.

This paper is organized as follows: Section 2 presents the notation, Section 3 presents the necessary theoretical background, Section 4 describes the RFD problem, Section 5 presents the detector-dependent case, Section 6 illustrates the obtained results with a numerical example, and Section 7 concludes the paper with some final comments. In the Appendix A the proof for the Theorem 1 is presented.

2. NOTATION

The notation used in this work is standard, the operator ($'$) denotes the transpose of a matrix or vector, the symbol ($\bullet$) denotes a symmetric block of a symmetric matrix. The representation of a Markov chain states set is $\mathbb{K} = \{1, 2, \ldots, N\}$. The convex combinations of the matrix X_j and the weight ρ_{ij} is given by $\varepsilon_i(X) = \sum_{j=1}^{N} \rho_{ij} X_j$ for $i \in \mathbb{K}$. The symbol $\varepsilon()$ represents mathematical expectation. Considering the stochastic signal $z(k)$, its norm is defined by $\|z\|_2^2 = \sum_{k=0}^{\infty} \varepsilon\{z(k)'z(k)\}$. The set of signals $z(k) \in \mathbb{R}^p$ defined for all $k \in \mathbb{N}$, such that $\|z\|_2 < \infty$ is indicated by $\mathcal{L}^2$.

3. MEAN SQUARE STABILITY AND H_∞ NORM

We define in this section the concept of mean square stability and H_∞ norm. For that we consider the following general discrete-time Markovian Jump Linear System (MJLS)

$$\mathcal{G} : \begin{cases} x(k+1) = A_{\theta(k)} x(k) + J_{\theta(k)} w(k) \\ z(k) = C_{z\theta(k)} x(k) + D_{z\theta(k)} w(k) \end{cases} \tag{2}$$

where $x(k) \in \mathbb{R}^n$ is the state, $y(k) \in \mathbb{R}^q$ is the measured output, $z(k) \in \mathbb{R}^p$ is the estimated output, $w(k) \in \mathbb{R}^m$ is the exogenous input. We also consider that $w \in \mathcal{L}_2$. We define the transition probability matrix by $\Psi = [\rho_{ij}]$ where $\rho_{ij} = Pr[\theta_{k+1} = j | \theta_k = i]$ and $\sum_{j=1}^{N} \rho_{ij} = 1$ for all $i \in \mathbb{K}$.

3.1 Mean Square Stability

The definition of Mean Square Stability presented in Costa and Marques [1998] is:

Definition: System (2) is Mean Square Stable (MSS) if for any initial condition $x(0) = x_0 \in \mathbb{R}^n$, initial distribution $\theta(0) = \theta_0 \in \mathbb{K}$ and $w \equiv 0$ we have that

$$\lim_{k\to\infty} \varepsilon\{x(k)'x(k) | x_0, \theta_0\} = 0. \tag{3}$$

3.2 H_∞ Norm

Assuming that system $\mathcal{G}$ as in (2) is MSS, the H_∞ norm of $\mathcal{G}$ is given by (see, Fragoso and Costa [2005], Costa and Marques [1998])

$$\|\mathcal{G}\|_\infty^2 = \sup_{0 \neq w \in \mathcal{L}_2, \theta_0 \in \mathbb{K}} \frac{\|z\|_2^2}{\|w\|_2^2} \tag{4}$$

where w represents the inputs and z represents the outputs. Notice that when $\mathbb{K} = \{1\}$, that is, there is only one state for the Markov chain, (4) corresponds to the H_∞ norm for the deterministic case.

3.3 Hidden Markov Chain

In Todorov et al. [2015, 2018], it was studied the H_∞ control problem for MJLS considering that the Markov Chain state $\theta(k)$ is not accessible. The way to tackle this problem is to consider a Hidden Markov Chain, where a detector observes the Markov Chain and provides an estimated Markov Chain state, denoted by $\hat{\theta}(k)$. The set $\mathbb{M}_i$ represents all the possible values of $\hat{\theta}(k)$, whenever $\theta(k) = i$, where $\mathbb{M}_i \subseteq \mathbb{M}$ and $\mathbb{M} = \{1, \ldots, M\}$ is the set of all possible outputs from the detector.

This combination of $\theta(k)$ and $\hat{\theta}(k)$ is inspired from the Hidden Markov Chain model, see, for instance, Ross [2014]. Considering that $\theta(k)$ corresponds to a Markov chain state that is not accessible, then $\hat{\theta}(k)$ is the observable sequence of emissions depending on the state $\theta(k)$. Observe, as usual for an HMC, that there is a transition probability matrix Ψ related to $\theta(k)$, and the detection probability matrix $\Omega = [a_{il}]$, where $a_{ij} = Pr[\hat{\theta}_k = j | \theta_k = i]$ and $\sum_{j=1}^{M} a_{ij} = 1$. This detector determines the possible values of $\hat{\theta}(k)$ given the current value of the state of the Markov Chain $\theta(k)$.

4. PROBLEM FORMULATION

The MJLS we consider in this work is represented by

$$\mathcal{G}_a : \begin{cases} x(k+1) = A_{\theta_k}x(k) + B_{\theta_k}u(k) + B_{d\theta_k}d(k) + B_{f\theta_k}f(k) \\ y(k) = C_{\theta_k}x(k) + D_{d\theta_k}d(k) + D_{f\theta_k}f(k) \\ x(0) = x_0, \quad \theta(0) = \theta_0 \end{cases} \tag{5}$$

where $x(k) \in \mathbb{R}^n$ is the state, $y(k) \in \mathbb{R}^q$ is the measured output, $u(k) \in \mathbb{R}^m$ is the known input, $d(k) \in \mathbb{R}^p$ is the exogenous input and $f(k) \in \mathbb{R}^t$ is the fault vector which is considered as an unknown function of time. We also consider that $f(k), d(k) \in \mathcal{L}^2$. Usually the Fault Detection system is divided into two distinct stages, a residual generator and a residual evaluation.

4.1 Residual Generator

The filter responsible to generate the residual signal $r(k)$ is a Markovian observer defined as

$$\mathcal{F} : \begin{cases} \eta(k+1) = A_{\eta\hat{\theta}_k}\eta(k) + M_{\eta\hat{\theta}_k}u(k) + B_{\eta\hat{\theta}_k}y(k) \\ r(k) = C_{\eta\hat{\theta}_k}\eta(k) + D_{\eta\hat{\theta}_k}y(k) \\ \eta(0) = \eta_0 \end{cases} \tag{6}$$

where $\eta(k) \in \mathbb{R}^n$ represents the filter states and $r(k) \in \mathbb{R}^l$ is the filter residue. We point out that this filter structure exclusively depends on the detector mode $\hat{\theta}_k$.

The major goal in this paper is to design the matrices $A_{\eta l}$, $B_{\eta l}$, $C_{\eta l}$, $D_{\eta l}$, $M_{\eta l}$ so that the residual generator (6) is mean square stable when $x(0) = 0$, $u(0) = 0$, $d(0) = 0$ and $f(0) = 0$ and minimizes the value of γ in

$$\sup_{w \neq 0,\ w \in \mathcal{L}^2,\ \theta_0 \in \mathbb{N}} \frac{\|r - \hat{f}\|_2}{\|w\|_2} < \gamma \tag{7}$$

where $w(k) = [u'(k) \quad d'(k) \quad f'(k)]$, $\hat{f}(k) = f(k)$, or more generally, $\hat{f}(z) = W_f(z)f(z)W_f(z)$, where W_f is a given by weighting matrix. We call the problem with $\hat{f}(k) = f(k)$ Robust Fault Identification, while when using $\hat{f}(k) = W_f f(k)$ by Robust Fault Detection filter problem. It is possible to observe that the RFI problem is a special case of the RFDF problem where $W_f(k) = I$, see Zhong et al. [2005].

Similarly to the continuous-time case presented in Chen and Patton [2000] and the discrete-time case in Zhong et al. [2005], a weighting matrix $W_f(k)$ is used with the intention to increase the chance of detecting the failure by limiting the frequency interval, in which the fault should be identified, where $\hat{f}(k) = W_f(k)f(k)$. A minimal realization is

$$\mathcal{W}_f : \begin{cases} x_f(k+1) = A_{wf}x_f(k) + B_{wf}f(k) \\ \hat{f}(k) = C_{wf}x_f(k) + D_{wf}f(k) \\ x_f(0) = 0 \end{cases} \tag{8}$$

where $x_f(k) \in \mathbb{R}^t$ is the filter state, and $f(k)$ is the same signal as in (5). The block diagram presented below represents the equivalent system:

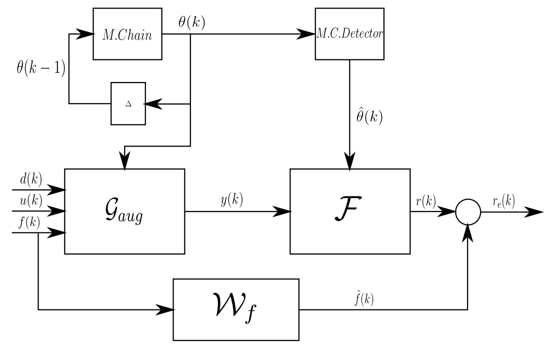

Figure 2. Block diagram.

Considering $r_e(k) = r(k) - \hat{f}(k)$ the equivalent system can be written in the augmented form as

$$\mathcal{G}_{aug} : \begin{cases} \bar{x}(k+1) = \tilde{A}_{\theta_k\hat{\theta}_k}\bar{x}(k) + \tilde{B}_{\theta_k\hat{\theta}_k}\bar{w}(k) \\ r_e(k) = \tilde{C}_{\theta_k\hat{\theta}_k}\bar{x}(k) + \tilde{D}_{\theta_k\hat{\theta}_k}\bar{w}(k) \end{cases} \tag{9}$$

where the augmented state is $\bar{x}(k) = [x'(k)\ \eta'(k)\ x'_f(k)]'$ and $\bar{w}(k) = [u'(k)\ d'(k)\ \hat{f}'(k)]'$ and

$$\left[\begin{array}{c|c} \tilde{A}_{\theta_k\hat{\theta}_k} & \tilde{B}_{\theta_k\hat{\theta}_k} \\ \hline \tilde{C}_{\theta_k\hat{\theta}_k} & \tilde{D}_{\theta_k\hat{\theta}_k} \end{array}\right] = \left[\begin{array}{ccc|ccc} A_{\theta_k} & 0 & 0 & B_{\theta_k} & B_{d\theta_k} & B_{f\theta_k} \\ B_{\eta\hat{\theta}_k}C_{\theta_k} & A_{\eta\hat{\theta}_k} & 0 & M_{\eta\hat{\theta}_k} & B_{\eta\hat{\theta}_k}D_{d\theta_k} & B_{\eta\hat{\theta}_k}D_{f\theta_k} \\ 0 & 0 & A_{wf} & 0 & 0 & B_{wf} \\ \hline D_{\eta\hat{\theta}_k}C_{\theta_k} & C_{\eta\hat{\theta}_k} & -C_{wf} & 0 & D_{\eta\hat{\theta}_k}D_{d\theta_k} & D_{\eta\hat{\theta}_k}D_{f\theta_k} - D_{wf} \end{array}\right] \tag{10}$$

Summing up, the Robust Fault Detection filter problem corresponds to an optimization problem to obtain the matrices that compose the observer (6) in such a way that the system (9) is MSS and γ is as small as possible in the feasibility of

$$\sup_{\|w\|_2 \neq 0,\ w \in \mathcal{L}_2,\ \theta_0 \in \mathbb{N}} \frac{\|r_e\|_2}{\|w\|_2} < \gamma, \quad \gamma > 0. \tag{11}$$

4.2 Residual Evaluation

The evaluation stage is composed by two main parts, the threshold and the signal. In order to make the evaluation more effective, an evaluation function that depends on the residual signal is defined by $J(\bar{r}(k))$, as well as a threshold that is defined in terms of the evaluation function. These definitions are characterized as described in Zhong et al. [2005]. We consider L as the evaluation time step, and with that, we are able to separate the evaluation process into two distinct cases, the first one is defined by $k - L \geq 0$ and the second one, $k - L < 0$. Therefore, we define the auxiliary vectors for each case as

$$\begin{cases} \text{for } k - L \geq 0,\ \bar{r}(k) = [r(k)'\ r(k-1)'\ \ldots\ r(k-L)'] \\ \text{for } k - L < 0,\ \bar{r}(k) = [r(k)'\ r(k-1)'\ \ldots\ r(0)'] \end{cases} \tag{12}$$

and, the evaluation functions for each case are set as

$$\begin{cases} \text{for } k - L \geq 0,\ J(\bar{r}(k)) = \left\{ \displaystyle\sum_{\sigma=k}^{\sigma=k-L} \bar{r}(\sigma)'\bar{r}(\sigma) \right\}^{\frac{1}{2}}, \\ \text{for } k - L < 0,\ J(\bar{r}(k)) = \left\{ \displaystyle\sum_{\sigma=k}^{0} \bar{r}(\sigma)'\bar{r}(\sigma) \right\}^{\frac{1}{2}}. \end{cases} \tag{13}$$

The threshold is defined as

$$J_{th}(k) = \sup_{d \in \mathcal{L}^2,\ f=0} \varepsilon(J(\bar{r}_{f=0}(k))), \tag{14}$$

where $\bar{r}(k)_{f=0}$ represents the residual signal when the system is operating on the nominal state, meaning that no fault occurs.

The occurrence of faults can be detected by analyzing the value of $J(\bar{r}(k))$ as follow:

$$\begin{cases} J(\bar{r}(k)) < J_{th}(k), \text{ means that the system is in the nominal mode} \\ J(\bar{r}(k)) \geq J_{th}(k) \text{ means that a fault occurred at the instant } k. \end{cases} \tag{15}$$

5. MAIN RESULTS

This section introduces the main contribution of this paper, a Robust Fault Detection filter in the MJLS framework considering the partial access to the Markov Chain state that represents the dynamic of the system abrupt transitions. Theorem 1 allows us to design a RFDF that depends only on the estimated state $\hat{\theta}(k)$.

Theorem 1. There exists a filter in form of (6) such that $\|\mathcal{G}_{aug}\|_\infty < \gamma$ if there exist symmetric matrices Z_i, X_i, $\tilde{M}_{il}$, $\tilde{N}_{il}$, $\tilde{S}_{il}$, W, and the matrices Δ_l, O_l, F_l, G_l with compatible dimensions that satisfy the LMI constraint (16), (17)

$$\begin{bmatrix} Z_i & \bullet & \bullet & \bullet & \bullet & \bullet \\ Z_i & X_i & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & W_i & \bullet & \bullet & \bullet \\ 0 & 0 & 0 & \gamma^2 I & \bullet & \bullet \\ 0 & 0 & 0 & 0 & \gamma^2 I & \bullet \\ 0 & 0 & 0 & 0 & 0 & \gamma^2 I \end{bmatrix} > \sum_{l \in \mathcal{M}_l} a_{il} \begin{bmatrix} M_{il}^{11} & \bullet & \bullet & \bullet & \bullet & \bullet \\ M_{il}^{21} & M_{il}^{22} & \bullet & \bullet & \bullet & \bullet \\ M_{il}^{31} & M_{il}^{32} & M_{il}^{33} & \bullet & \bullet & \bullet \\ N_{il}^{11} & N_{il}^{12} & N_{il}^{13} & S_{il}^{11} & \bullet & \bullet \\ N_{il}^{21} & N_{il}^{22} & N_{il}^{23} & S_{il}^{21} & S_{il}^{22} & \bullet \\ N_{il}^{31} & N_{il}^{32} & N_{il}^{33} & S_{il}^{31} & S_{il}^{32} & S_{il}^{33} \end{bmatrix} \tag{16}$$

$$\begin{bmatrix} M_{il}^{11} & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ M_{il}^{21} & M_{il}^{22} & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ M_{il}^{31} & M_{il}^{32} & M_{il}^{33} & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ N_{il}^{11} & N_{il}^{12} & N_{il}^{13} & S_{il}^{11} & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ N_{il}^{21} & N_{il}^{22} & N_{il}^{23} & S_{il}^{21} & S_{il}^{22} & \bullet & \bullet & \bullet & \bullet & \bullet \\ N_{il}^{31} & N_{il}^{32} & N_{il}^{33} & S_{il}^{31} & S_{il}^{32} & S_{il}^{33} & \bullet & \bullet & \bullet & \bullet \\ \varepsilon_i(Z)A_i & \varepsilon_i(Z)A_i & 0 & \varepsilon_i(Z)B_i & \varepsilon_i(Z)B_{di} & \varepsilon_i(Z)B_{fi} & \varepsilon_i(Z) & \bullet & \bullet & \bullet \\ R_lA_i + \Delta_lC_i + O_l & R_lA_i + \Delta_lC_i & 0 & R_lB_i + H_l & R_lB_{di} + \Delta_lD_{di} & R_lB_{fi} + \Delta_lD_{fi} & 0 & R_l + R_l' + \varepsilon_i(Z) - \varepsilon_i(X) & \bullet & \bullet \\ 0 & 0 & \varepsilon_i(W)A_{wf} & 0 & 0 & \varepsilon_i(W)B_{wf} & 0 & 0 & \varepsilon_i(W) & \bullet \\ G_lC_i + F_l & G_lC_i & -C_{wf} & 0 & G_lD_{di} & G_lD_{di} - D_{wf} & 0 & 0 & 0 & I \end{bmatrix} > 0 \tag{17}$$

If a feasible solution is found, then a suitable RFD filter is given by $A_{\eta l} = R_l^{-1}O_l$, $B_{\eta l} = R_l^{-1}\Delta_l$, $M_{\eta l} = R_l^{-1}H_l$, $C_{\eta l} = F_l$, $D_{\eta l} = G_l$.

Proof: The proof is presented in the Appendix A.

6. NUMERICAL EXAMPLE

In this section we present a numerical example in order to illustrate that the approach presented in this work is a viable alternative to the Robust Fault Detection filter without the knowledge of Markov Chain problem. The numerical example is partially extracted from Zhong et al. [2005], with the system matrices presented as below:

$$A_1 = \begin{bmatrix} 0.1 & 0 & 1 & 0 \\ 0 & 0.1 & 0 & 0.5 \\ 0 & 0 & 0.2 & 0 \\ 0 & 0 & 0 & 0.2 \end{bmatrix},\ A_2 = \begin{bmatrix} 0.3 & 0 & -1 & 0 \\ -0.1 & 0.2 & 0 & -0.5 \\ 0 & 0 & -0.2 & 0 \\ 0 & 0 & 0 & -0.5 \end{bmatrix},$$

$$B_d = \begin{bmatrix} 0.8 \\ -2.4 \\ 1.6 \\ 0.8 \end{bmatrix},\quad B_f = \begin{bmatrix} 1 \\ 1 \\ 2 \\ -2 \end{bmatrix},\ C_i = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix},$$

$$D_d = \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix} \quad D_f = \begin{bmatrix} 2 \\ -1 \end{bmatrix},\ \Psi = \begin{bmatrix} 0.9 & 0.1 \\ 0.7 & 0.3 \end{bmatrix},$$

$$A_{wf} = 0.5,\quad B_{wf} = 0.25,\ C_{wf} = 1,\ D_{wf} = 0.5.$$

6.1 Simulations

In order to fully analyze the proposed solution introduced in this work three tests were performed: the first one is the analysis of the norm behavior when we vary the detector matrix; the second one is a simulation with three distinct configuration for the detector matrix; the third and last simulation is a Monte Carlo simulation with 2000 rounds.

H_∞ Norm Behavior This first evaluation denotes the norm variation when the probabilities in the transition matrix Ψ vary, considering also the variation behavior for three distinct values of the detector matrix Ω, represented below:

$$\Omega_1 = \begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{bmatrix},\ \Omega_2 = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix},\ \Omega_3 = \begin{bmatrix} 0.1 & 0.9 \\ 0.9 & 0.1 \end{bmatrix} \tag{18}$$

In (18) it is shown three distinct values of Ω, where Ω_1 and Ω_3 are direct opposed to each other and Ω_2 is the worst case scenario, regarding the detector performance. The matrix Ψ structure is given by,

$$\Psi = \begin{bmatrix} \rho_1 & 1 - \rho_1 \\ 1 - \rho_2 & \rho_2 \end{bmatrix}. \tag{19}$$

The results obtained in this subsection were achieved using Theorems 1. Considering $\rho_1, \rho_2 \in [0\ 1]$ it was obtained the surface presented in Fig. 3. In Fig.3 it is possible to

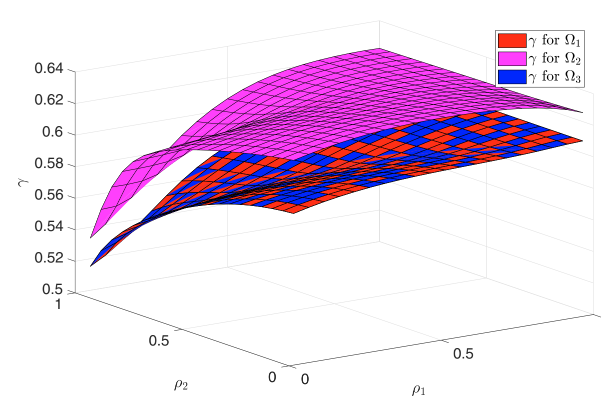

Figure 3. H_∞ upperbound γ.

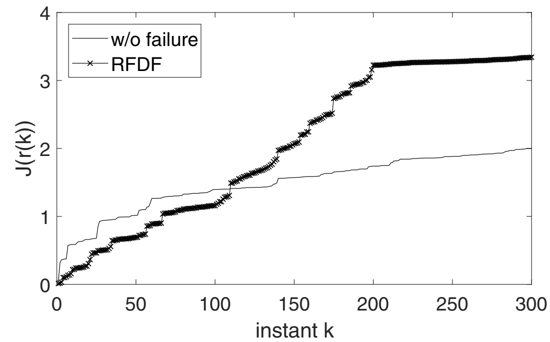

Figure 4. J for all cases and an example without failure occurrence.

draw the following conclusions: the norm value for the case using Ω_2 is higher, this phenomenon occurs due the higher entropy, the second information is the norm value for the case using Ω_1 and Ω_3 were practically equal, both results are similar to the results obtained in de Oliveira and Costa [2017b,a]. Summarizing, Theorem 1 provides results that are in line with the literature, regarding the norm behavior.

Single Sample Simulation In this subsection we present the results for a simulation with the example extracted from Zhong et al. [2005], which was described at the beginning of this section, the noise signal used is the white noise with mean equal to 0 and variance equal to 0.5^2 and multiplied by an exponential. The value of $\Omega = [0.8\ 0.2;\ 0.7\ 0.3]$, where the cyan curve represents RFDF designed using the Theorem 1 and the black curve represents the a situation where the fault does not occur. The number of instants k necessary to the RFDF identify the fault occurrence was $k = 107$.

The matrices that compose the filter obtained using Theorem 1 in the form presented in (6) are presented in (20)

$$A_{\eta 1} = \begin{bmatrix} 0.08 & 0.80 & 0.23 & 0.66 \\ 0.59 & 1.15 & 0.73 & 1.45 \\ 0.28 & 1.08 & 0.27 & 0.95 \\ -0.44 & -1.79 & -0.40 & -1.53 \end{bmatrix},\ A_{\eta 2} = \begin{bmatrix} 0.46 & 0.71 & -0.05 & 0.62 \\ 0.20 & 0.85 & -0.53 & 0.26 \\ 0.45 & 0.86 & -0.20 & 0.60 \\ -0.63 & -1.51 & 0.45 & -0.92 \end{bmatrix},$$

$$B_{\eta 1} = \begin{bmatrix} -1.26 & -0.73 \\ -1.12 & -0.34 \\ -1.09 & -0.37 \\ 1.82 & 0.48 \end{bmatrix},\quad B_{\eta 2} = \begin{bmatrix} -1.31 & -0.97 \\ -0.89 & -0.01 \\ -1.04 & -0.48 \\ 1.79 & 0.63 \end{bmatrix},\ M_{\eta 1} = M_{\eta 2} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

$$C_{\eta 1} = \begin{bmatrix} -0.15 \\ -0.37 \\ -0.13 \\ -0.38 \end{bmatrix}',\quad C_{\eta 2} = \begin{bmatrix} -0.01 \\ -0.06 \\ -0.02 \\ -0.06 \end{bmatrix}',\ D_{\eta 1} = \begin{bmatrix} 0.32 \\ 0.11 \end{bmatrix}',\quad D_{\eta 2} = \begin{bmatrix} 0.11 \\ 0.03 \end{bmatrix}',$$

$$\gamma = 0.5664.$$

Monte Carlo Simulation The results obtained with a Monte Carlo simulation with 2000 rounds are shown below. The example is the same as previously mentioned in this section.

In Fig.5 the results for the Monte Carlo simulation are presented, where the cyan curve represents RFDF designed using the Theorem 1 and the black curve represents the a situation where the fault does not occur. The values of the average number of instants necessary to detect a failure that starts at $k = 100$ using the designed RFDF was $k = 103.3930$.

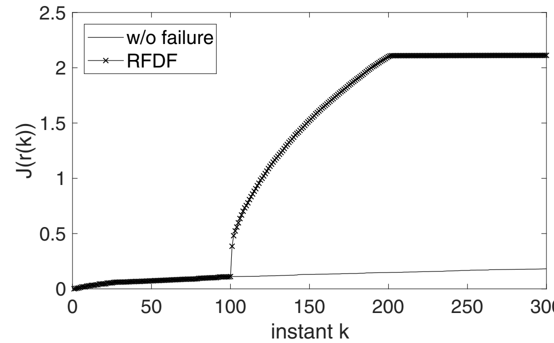

Figure 5. J for all cases and an example without failure occurrence.

7. CONCLUSION

In this paper, the Robust Fault Detection Problem associated with Markovian Jump Linear System in the discrete-time domain with partial mode access is studied. The proposed solution consists in designing a residual generator via LMI and formulated as an H_∞ MJLS filter with partial access to the Markov Chain mode. A numerical example is presented with the intention of illustrate that our approach can provide a viable solution to the RFDF problem. The next step along this line of research is to formulate the H_2 RFDF that also considers the partial access to the Markov Chain modes in order to evaluate its efficiency when compared to the H_∞ approach. Another possible work would be the formulation of a Mixed H_∞ and H_2 RFDF.

Appendix A. PROOF OF THEOREM 1

Consider the structure for the matrices

$$\tilde{P}_i = \begin{bmatrix} X_i & \bullet & \bullet \\ U_i' & \hat{X}_i & \bullet \\ 0 & 0 & P_i^{33} \end{bmatrix}, \tilde{P}_i^{-1} = \begin{bmatrix} Y_i & \bullet & \bullet \\ V_i' & \hat{Y}_i & \bullet \\ 0 & 0 & P_i^{33\ -1} \end{bmatrix} \quad \text{(A.1)}$$

$$\varepsilon_i(\tilde{P})^{-1} = \begin{bmatrix} \hat{T}_{1i} & \bullet & \bullet \\ \hat{T}_{2i}' & \hat{T}_{3i} & \bullet \\ 0 & 0 & \varepsilon_i(P_i^{33})^{-1} \end{bmatrix} \quad \text{(A.2)}$$

and the linearization matrices

$$\tau_i = \begin{bmatrix} I & I & 0 \\ V_i' Y_i^{-1} & 0 & 0 \\ 0 & 0 & I \end{bmatrix},\ \iota = \begin{bmatrix} \hat{T}_{1i}^{-1} & \varepsilon_i(X) & 0 \\ 0 & \varepsilon_i(U)' & 0 \\ 0 & 0 & \varepsilon_i(P_i^{33}) \end{bmatrix} \text{(A.3)}$$

that leads to

$$\tau_i' \tilde{P}_i \tau_i = \begin{bmatrix} Y_i^{-1} & Y_i^{-1} & 0 \\ Y_i^{-1} & X_i & 0 \\ 0 & 0 & P_i^{33} \end{bmatrix}, \quad \text{(A.4)}$$

$$\iota_i' \varepsilon_i(\tilde{P})^{-1} \iota_i = \begin{bmatrix} \varepsilon_i(Z) & \bullet & \bullet \\ \varepsilon_i(Z) & \varepsilon_i(X) & \bullet \\ 0 & 0 & \varepsilon_i(P_i^{33}) \end{bmatrix} \quad \text{(A.5)}$$

Considering the constraint (17), and $U_i = Z_i - X_i$, $\hat{X}_i = -U_i$, $V_i' Y_i^{-1}$ and from (16) we can say that $\varepsilon_i(X) - \varepsilon_i(Z)$ is invertible since $X_i > Z_i$. This observation also allows us to write $R_l(\varepsilon_i(X) - \varepsilon_i(Z))^{-1} R_l' > R_l + R_l' + \varepsilon_i(Z) - \varepsilon_i(X)$, (see de Oliveira et al. [1999]), in such a way that the term $Her(R_l) + \varepsilon_i(Z) - \varepsilon_i(X)$ can be change by $R_l(\varepsilon_i(X) - \varepsilon_i(Z))^{-1}R_l'$ in the constraint (17). Defining the matrix Q_{il} as,

$$Q_{il} = \begin{bmatrix} I_n & I_n & 0 \\ 0 & (R_l^{-1})'(\varepsilon_i(X) - \varepsilon_i(Z)) & 0 \\ 0 & 0 & I \end{bmatrix}. \quad \text{(A.6)}$$

a similar structure is presented in Gonçalves et al. [2011] and first presented in Gonçalves et al. [2010]. Applying the congruence transformation $diag(I, Q_{il}, I, I)$ in (17), with term $R_l(\varepsilon_i(X) - \varepsilon_i(Z))^{-1}R_l'$, we obtain the constraints where $O_l = R_l A_{\eta l}$, $\Delta_l = R_l B_{\eta l}$, $H_l = R_l M_l$, $F_l = C_{\eta l}$, $G_l = D_{\eta l}$. As presented in de Oliveira and Costa [2017b] and the references therein, $\hat{T}_{1i}^{-1} = \varepsilon_i(X) - \varepsilon_i(U)\varepsilon_i(\hat{X})^{-1}\varepsilon_i(U)$, and we also have that $\varepsilon_i(U) = -\varepsilon_i(\hat{X})$. Therefore, $\hat{T}_{1i}^{-1} = \varepsilon_i(Z) = \varepsilon_i(X) + \varepsilon_i(U)$, and so the constraint (16) and (17) can be also described as

$$\begin{bmatrix} \tau_i'\tilde{P}\tau_i & 0 \\ 0 & \gamma^2 I \end{bmatrix} > \sum_{l \in \mathbb{M}_i} \begin{bmatrix} \tau_i'\tilde{M}_{il}\tau_i & \bullet \\ \tilde{N}_{il}\tau_i & \tilde{S}_{il} \end{bmatrix} \quad \text{(A.7)}$$

$$\begin{bmatrix} \tau_i'\tilde{M}_{il}\tau_i & \bullet & \bullet & \bullet \\ \tilde{N}_{il}\tau_i & \tilde{S}_{il} & \bullet & \bullet \\ \iota_i'\tilde{A}_{il}\tau_i & \iota_i'\tilde{J}_{il} & \iota_i'\varepsilon(\tilde{P})^{-1}\iota_i & \bullet \\ \tilde{C}_{il}\tau_i & \tilde{E}_{il} & 0 & I \end{bmatrix} > 0 \quad \text{(A.8)}$$

Using the congruence transformations $diag(\tau_i^{-1}, I)$ in (A.7) and $diag(\tau_i^{-1}, I, \iota_i^{-1}, I)$ in (A.8), concludes the proof.

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