Mode-Independent $\mathcal{H}_2$-Control of a DC Motor Modeled as a Markov Jump Linear System

Ricardo C. L. F. Oliveira, Alessandro N. Vargas, João B. R. do Val, and Pedro L. D. Peres

***Abstract*— This brief presents a control strategy for Markov jump linear systems (MJLS) with no access to the Markov state (or mode). The controller is assumed to be in the linear state-feedback format and the aim of the control problem is to design a static mode-independent gain that minimizes a bound to the corresponding $\mathcal{H}_2$-cost. This approach has a practical appeal since it is often difficult to measure or to estimate the actual operating mode. The result of the proposed method is compared with that of a previous design, and its usefulness is illustrated by an application that considers the velocity control of a DC motor device subject to abrupt failures that is modeled as an MJLS.**

***Index Terms*— Control systems, Markov jump linear systems (MJLS).**

I. Introduction

LET us consider the discrete-time Markov jump linear system (MJLS), defined on a filtered probability space $(\Omega, \mathcal{F}, \mathcal{F}_k, P)$, as follows:

$$\mathcal{S} : \begin{cases} x_{k+1} = A_{\theta_k} x_k + B_{\theta_k} u_k + E_{\theta_k} w_k \\ y_k = C_{\theta_k} x_k + D_{\theta_k} u_k, \quad k \geq 0, \quad \theta_0 \sim \pi(0), \; x_0 \in \mathcal{R}^r \end{cases}$$

where the sequences $\{x_k\}$ on $\mathcal{R}^r$, $\{y_k\}$ on $\mathcal{R}^q$, $\{u_k\}$ on $\mathcal{R}^s$, and $\{w_k\}$ on $\mathcal{R}^m$, represent the system state, output, control, and noise input, respectively. The Markov chain state is represented by $\{\theta_k\}$ and the underlying stochastic matrix is denoted by $\mathbb{P} = [p_{ij}]$, in that $i, j \in \mathcal{I} := \{1, \ldots, \sigma\}$, $\sigma > 1$. The initial distribution is denoted by $\pi(0)$. For each $k \geq 0$, the state process θ_k takes values in the set $\mathcal{I}$, in such a way that $\theta_k = i$ points out to a set of matrices $(A_i, B_i, C_i, D_i, E_i)$ that is assumed to be given for each $i \in \mathcal{I}$.

Many of the real systems that can be described within the MJLS setting face adversity when the design of a useful control is the primary concern. Actually, it is often difficult to determine the exact mode of the Markov chain because of the physical limitations on sensors or nonexistence of realistic measurement instruments for that task. However, most of the results in the literature do not take this practical difficulty into account because they assume that the controllers have full access to the mode at each instant of time [1]–[7]. It is also worth mentioning recent papers dealing with other aspects of MJLS, such as 2-D dynamics, singularity, and presence of time delays [8]–[11].

Thus, from this standpoint, it is reasonable to consider the design of controllers that do not require access to the Markov mode at all. This subject represents the main line of investigation in this brief.

The discussion above stresses the need of simple control synthesis in real-world implementations, and we choose the control law to be in the linear static state-feedback format with no mode observation

$$u_k = G x_k \quad \forall k \geq 0 \tag{1}$$

where G is a fixed matrix of dimensions $s \times r$ to be determined.

With the aid of simple calculations, the $\mathcal{H}_2$-control problem can be cast as that of finding G, the solution of the following optimization problem (Proposition II-A):

$$\beta^* = \inf_G \sum_{i=1}^{\sigma} \operatorname{tr} \left\{ \pi_i(0) E_i' \left(\sum_{j=1}^{\sigma} p_{ij} P_j \right) E_i \right\} \tag{2}$$

$$\text{s.t. } (A_i + B_i G)' \left(\sum_{j=1}^{\sigma} p_{ij} P_j \right) (A_i + B_i G) + C_i' C_i - P_i = 0, \quad i = 1, \ldots, \sigma \tag{3}$$

where $P_i = P_i' \geq 0$, $i = 1, \ldots, \sigma$, are matrices with dimensions $r \times r$. The problem of minimizing the $\mathcal{H}_2$-cost as in (2) and (3) is, to the best of our knowledge, open. Our approach provides a step toward in finding the solution for this problem because, although it does not calculate the optimal solution, our approach is capable of generating less conservative results when compared with the previous method in [12]. This sets the main theoretical novelty of this brief.

There are two main contributions of this brief. First, a novel sufficient linear matrix inequality (LMI) relaxation is proposed for the computation of a mode-independent state-feedback stabilizing gain with a guaranteed $\mathcal{H}_2$-cost, say $\beta > 0$, such that $\beta \geq \beta^*$.

The method is inspired by the two-step design procedure developed in the context of deterministic systems for output feedback control in [13]–[15], which in [16] and [17] have been extended to incorporate polynomially parameter-dependent matrices. In this brief, the method is adapted to cope with MJLS control design as follows: first determine a mode-dependent stabilizing gain; then use this gain as an input parameter for an LMI-based procedure (called second stage) that, if feasible, provides a mode-independent stabilizing gain associated with an $\mathcal{H}_2$ guaranteed cost.

Manuscript received October 1, 2013; accepted November 20, 2013. Manuscript received in final form November 29, 2013. Date of publication December 20, 2013; date of current version July 24, 2014. This work was supported in part by the Brazilian Agencies CAPES, FAPESP, and CNPq, in part by the Spanish agency Fundación Carolina – Programa Movilidad de Profesores e Investigadores Brasil-España.C.2010, and in part by the Basque Center for Applied Mathematics–BCAM Visiting Fellow Program. Recommended by Associate Editor H. Gao.

R. C. L. F. Oliveira, J. B. R. do Val, and P. L. D. Peres are with the School of Electrical and Computer Engineering, University of Campinas - UNICAMP, 13083-852, Campinas, SP, Brazil (e-mail: ricfow@dt.fee.unicamp.br; jbosco@dt.fee.unicamp.br; peres@dt.fee.unicamp.br).

A. N. Vargas is with the Universidade Tecnológica Federal do Paraná - UTFPR, 86300-000, Cornélio Procópio, PR, Brazil, and also with the Basque Center for Applied Mathematics, BCAM, Bilbao E-48009, Spain (e-mail: avargas@utfpr.edu.br).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TCST.2013.2293627

As the second contribution, the theoretical design procedure proposed in this brief is validated in practice, by controlling the angular velocity of a DC motor apparatus in real-time. During the experiments, the DC motor was subject to failures that evolved according to a homogeneous Markov chain.

The rest of this brief is organized as follows. Section II quotes the basic notions of mean square stability and evaluation of the $\mathcal{H}_2$-norm of MJLS. Section III presents the main result based on the novel LMI method. Finally, Section IV illustrates the result via a workbench experiment involving a DC motor subject to failures. The method is applied to design a mode-independent control rule to control the shaft speed.

II. Notation, Definitions, and Basic Result

The r-th dimensional Euclidean space is represented by $\mathcal{R}^r$ and $\|\cdot\|$ stands for the corresponding norm; $e_s \in \mathcal{R}^r$ represents the standard basis pointing in the direction of the s-th coordinate. The linear space made up by all $r \times s$ $(r \times r)$ real matrices is denoted by $\mathcal{M}^{r,s}$ $(\mathcal{M}^r)$. Let $\mathcal{I} := \{1, \ldots, \sigma\}$ be an index set, and let $\mathbb{M}^{r,s}$ denote the linear space formed by σ matrices belonging to $\mathcal{M}^{r,s}$, i.e., $\mathbb{M}^{r,s} = \{U = (U_1, \ldots, U_\sigma) : U_i \in \mathcal{M}^{r,s}, i \in \mathcal{I}\}$. Take, in particular, $\mathbb{M}^r \equiv \mathbb{M}^{r,r}$.

We employ the ordering $U > V$ $(U \geq V)$ for elements of $\mathbb{M}^r$, meaning that $U_i - V_i$ is positive definite (semi-definite) for all $i \in \mathcal{I}$, and similarly for other mathematical relations. If $V \in \mathcal{M}^{r,s}$ and $U \in \mathbb{M}^{s,r}$, then the multiplication VU results in $(VU_1, \ldots, VU_\sigma) \in \mathbb{M}^{r,r}$. The trace operator is denoted by $\operatorname{tr}\{\cdot\}$. The identity matrix is denoted by I.

The Markov chain $\{\theta_k\}$ is driven by the probability distribution $\pi_i(k) := \Pr(\theta_k = i)$, for all $k \geq 0$ and each $i \in \mathcal{I}$. Notice that $\pi(0) = \{\pi_1(0), \ldots, \pi_\sigma(0)\}$.

Let ℓ_2 denote the Hilbert space formed by the sequence $y = \{y_k\}$, a second-order real-valued stochastic process that is $\{\mathcal{F}_k\}$-adapted and satisfies

$$\|y\|^2 := \sum_{k=0}^{\infty} \mathrm{E}[\|y_k\|^2] < \infty.$$

Assumption 1: The noise input $\{w_k\}$ in $\mathcal{S}$ belongs to the class ℓ_2.

A. Preliminary Results for the System With No Control

Next we recall the definition of mean square stability.

Definition 1 [2], [18], [19]: MS-stability. We say the system $\mathcal{S}$ with $u_k \equiv 0$ is mean square stable (MS-stable) if

$$\mathrm{E}[\|x_k\|^2] \to 0 \quad \text{as} \quad k \to \infty \tag{4}$$

for each $x_0 \in \mathbb{R}^r$ and each $\theta_0 \in \mathcal{I}$.

Proposition 1 ([2, Th. 3.9], [18, Th. 2]): The following assertions are equivalent.

1) The system $\mathcal{S}$ with $u_k \equiv 0$ is MS-stable.
2) For some $V \in \mathbb{M}^{r,r}$, $V = V' > 0$, there holds

$$A_i' \left(\sum_{j=1}^{\sigma} p_{ij} V_j \right) A_i - V_i < 0, \quad i = 1, \ldots, \sigma. \tag{5}$$

Next we present the definition of the $\mathcal{H}_2$-norm for MJLS.

Definition 2 [2, Ch. 4.4], [12]: The $\mathcal{H}_2$-norm associated with the system $\mathcal{S}$ is represented by the value

$$\|\mathcal{S}\|^2 = \sum_{s=1}^{m} \sum_{i=1}^{\sigma} \pi_i(0) \|y^{s,i}\|^2$$

where $y^{s,i}$ denotes the output $y = \{y_k\}$ due to the specific input $w_0 = e_s \in \mathcal{R}^m$, $w_k = 0, \forall k \geq 1$, and initial condition $x_0 = 0$ and $\theta_0 = i$.

The next result presents an expression for the evaluation of the $\mathcal{H}_2$-norm.

Proposition 2 [12, Appendix A]: If $\mathcal{S}$ with $u_k \equiv 0$ is MS-stable and there exists $P \in \mathbb{M}^{r,r}$, $P = P' \geq 0$, the unique solution of the equation

$$A_i' \left(\sum_{j=1}^{\sigma} p_{ij} P_j \right) A_i + C_i' C_i - P_i = 0, \quad i = 1, \ldots, \sigma \tag{6}$$

then $\|\mathcal{S}\|^2 = \sum_{i=1}^{\sigma} \sum_{j=1}^{\sigma} \pi_i(0) p_{ij} \operatorname{tr}\{E_i' P_j E_i\}$.

Remark 1: If the $\|\mathcal{S}\|^2$-norm is finite, then the system $\mathcal{S}$ is MS-stable. Indeed, this assertion comes from the fact that, with $\|\mathcal{S}\|^2$ finite, (6) has a solution and implies that (5) holds true.

B. Control Structure

The LMI method introduced in the sequel requires an input data, in the sense that the method should be initialized with a gain that depends on the Markov mode. To compute a gain that does not depend on the mode, i.e., some gain $G_{\text{out}} \in \mathcal{M}^{s,r}$, a mode-dependent gain $G_{\text{ini}} \in \mathbb{M}^{s,r}$ is used in the design conditions. The single condition imposed on $G_{\text{ini}} \in \mathbb{M}^{s,r}$ is that it is MS-stabilizing, a basic concept as defined next.

Definition 3 ([20, p. 1283]): A gain $G \in \mathbb{M}^{s,r}$ is called MS-stabilizing if the resulting closed-loop system in $\mathcal{S}$ (i.e., replacing A by $A + BG$) is MS-stable.

Remark 2: Recall that there exists a solution to the coupled Riccati equations if and only if the corresponding gain is MS-stabilizing [20, Prop. 2]. This stabilizing property of the Riccati gain will be used in the design control project of Section IV.

III. Main Result

This section is devoted to presenting and proving the main result based on an LMI strategy. An advantage of our LMI method is that it generates less conservative results than the existing ones (see Table II for a pragmatic comparison). Next we present the main result of this brief.

Theorem 1: Let $K = \{K_1, \ldots, K_\sigma\} \in \mathbb{M}^{s,r}$ be a given MS-stabilizing gain. If there exist a set of matrices $P \in \mathbb{M}^{r,r}$, $P = P' > 0$, $W \in \mathbb{M}^{m,m}$, $F \in \mathbb{M}^{r,r}$, $H \in \mathbb{M}^{q,q}$, matrices $R \in \mathcal{M}^{s,s}$, $L \in \mathcal{M}^{s,r}$, and a scalar β such that the following LMIs hold:

$$\beta > \sum_{i=1}^{\sigma} \pi_i(0) \operatorname{tr}(W_i) \tag{7}$$

$$W_i - E_i' \left(\sum_{j=1}^{\sigma} p_{ij} P_j \right) E_i > 0, \ i = 1, \ldots, \sigma \tag{8}$$

$$\begin{bmatrix} -P_i & (A_i'+K_i'B_i')F_i & (C_i'+K_i'D_i')H_i & L'-K_i'R' \\ \star & \left(\sum_{j=1}^{\sigma} p_{ij}P_j\right) - F_i - F_i' & 0 & F_i'B_i \\ \star & \star & I - H_i - H_i' & H_i'D_i \\ \star & \star & \star & -R-R' \end{bmatrix} < 0, \quad i = 1,\ldots,\sigma \tag{9}$$

then $G = R^{-1}L$ is a mode-independent MS-stabilizing gain and $\sqrt{\beta}$ is an upper bound (guaranteed cost) for the $\mathcal{H}_2$-norm of the system $\mathcal{S}$.

Proof: By multiplying (9) on the left by T and on the right by T', with

$$T = \begin{bmatrix} I & 0 & 0 & S' \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \end{bmatrix}, \quad S = R^{-1}L - K$$

we obtain

$$\begin{bmatrix} -P_i & A_{cl_i}'F_i & C_{cl_i}'H_i \\ \star & \left(\sum_{j=1}^{\sigma} p_{ij}P_j\right) - F_i - F_i' & 0 \\ \star & \star & I - H_i - H_i' \end{bmatrix} < 0 \tag{10}$$

with $A_{cl_i} \equiv A_i + B_iR^{-1}L$ and $C_{cl_i} \equiv C_i + D_iR^{-1}L$. Now, multiply (10) on the left by $\begin{bmatrix} I & A_{cl_i}' & C_{cl_i}' \end{bmatrix}$ and on the right by its transpose, and as a result we get

$$A_{cl_i}'\left(\sum_{j=1}^{\sigma} p_{ij}P_j\right)A_{cl_i} - P_i + C_{cl_i}'C_{cl_i} < 0 \quad \forall i \in \mathcal{I}. \tag{11}$$

The inequalities in (7), (8), and (11) ensure that the gain matrix $G = R^{-1}L$ is MS-stabilizing (Remark 1) and that $\sqrt{\beta}$ is a guaranteed cost for the $\mathcal{H}_2$-norm of the system $\mathcal{S}$. ■

Remark 3: Notice that Theorem 1 is a two-step procedure, i.e., it requires as input data a previously calculated MS-stabilizable gain (with full mode observation), and in the second step the set of LMIs in (7)–(9), if feasible, generates a corresponding MS-stabilizing gain (with no mode observation). This is the main novelty of our design method. As illustrated in the next section, the numerical evaluation confirms that the result can be less conservative than the one available in the literature [12]. To the best of our knowledge, the result in [12] for the design of mode-independent controllers presents the least conservative result described so far in the literature in the context of this brief.

IV. Practical Application: Control of Velocity in a DC Motor Device

This section describes a practical experiment of controlling the angular velocity of a DC motor device subject to abrupt failures. These failures change the equipment behavior from the normal mode of operation to the failure modes, and vice versa, in which a computer is responsible to make these triggers to occur. To control the velocity of the DC motor in this scenario, we implement the control strategy suggested by Theorem 1 and the control design method from [12] for sake of comparison. As we will see in the sequel, the control strategy from Theorem 1 produces a better response for the DC motor device under abrupt failures.

The laboratory testbed used in this section is composed of the DC motor Module 2208 (Datapool Eletronica Ltda, Brazil) linked with a National Instruments USB-6008 data card and a computer running an instance of MATLAB. This laboratory setup was used previously by some of the authors in a time-varying feedback experiment, see [21].

TABLE I
Discrete-Time MJLS That Models the DC Motor Device

Parameters	$i=1$	$i=2$	$i=3$
$a_{11}^{(i)}$	-0.4799	-1.6026	0.6346
$a_{12}^{(i)}$	5.1546	9.1632	0.9178
$a_{21}^{(i)}$	-3.8162	-0.5918	-0.5056
$a_{22}^{(i)}$	14.4723	3.0317	2.4811
$a_{31}^{(i)}$	0.1399	0.0740	0.3865
$a_{33}^{(i)}$	-0.9255	-0.4338	0.0982
$b_1^{(i)}$	5.8705	10.2851	0.7874
$b_2^{(i)}$	15.5010	2.2282	1.5302
$\gamma^{(i)}$	0.1176	-0.1328	0.1632

We design a controller based on Theorem 1, considering the regulator problem, and then implement it in the DC motor testbed with a nonzero constant plus a sinusoid reference. The idea of the experiment is to adjust the DC motor to track this sinusoid reference with null steady-state error. To accomplish this goal, we implemented the proportional-integrative (PI) scheme as described in [22, Sec. 1.8.2, p. 56] to cope with the steady-state error and it is included in the synthesis by augmenting the system state. Let us set the system state x_k as follows: take $x_k \equiv [v_k \ i_k \ x_{3,k}]'$, where v_k and i_k represent the angular velocity of the rotor and the electrical current consumed by the motor, and the third element denotes the integrator term from the PI scheme.

The failures occur on the power delivered to the shaft, which are triggered by a computer. They are based on three distinct operation modes, i.e., the normal ($\theta_k = 1$), low ($\theta_k = 2$), and medium ($\theta_k = 3$) power modes. In fact, the DC motor Module 2208 has a round potentiometer, coupled with an electronic driver. We used this configuration to simulate abrupt changes in the power delivery to the shaft, respecting the physical limits of the equipment. We arrange the circuitry to command the potentiometer by a computer, and choose the jumps corresponding to the three levels: 1) 0% of rotary (normal mode); 2) +20% of rotary for improving the power (low mode); and 3) −40% of rotary for decreasing the power (medium mode).

These elements allow us to represent the DC motor device with abrupt failures as the MJLS

$$\begin{aligned} x_{k+1} &= A_{\theta_k}x_k + B_{\theta_k}u_k + E_{\theta_k}w_k + \Gamma_{\theta_k}r_k \\ y_k &= C_{\theta_k}x_k + D_{\theta_k}u_k, \quad x_0 \in \mathcal{R}^3 \\ A_i &= \begin{bmatrix} a_{11}^{(i)} & a_{12}^{(i)} & 0 \\ a_{21}^{(i)} & a_{22}^{(i)} & 0 \\ a_{31}^{(i)} & 0 & a_{33}^{(i)} \end{bmatrix} B_i = \begin{bmatrix} b_1^{(i)} \\ b_2^{(i)} \\ 0 \end{bmatrix} \end{aligned} \tag{12}$$

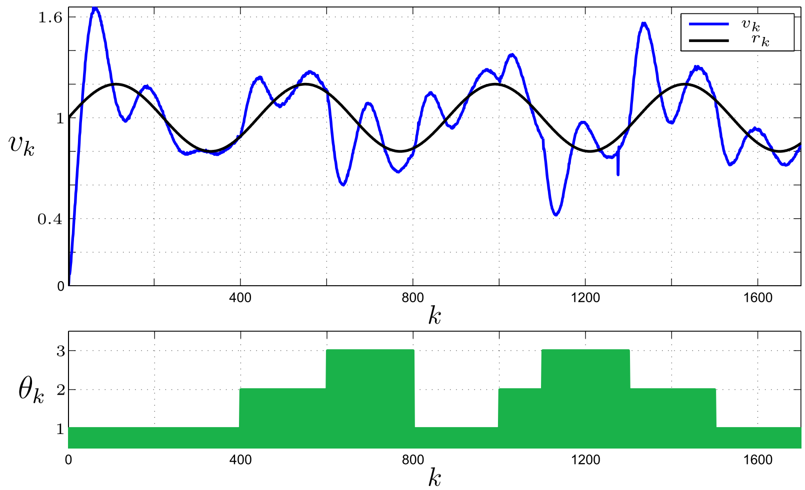


Fig. 1. Top: the curve in blue represents the velocity (rad/s) of the DC motor measured in practice and the black one represents the reference. Bottom: the states of the Markov chain evaluated in the experiment.

$$\Gamma_i = \begin{bmatrix} 0 \\ 0 \\ \gamma^{(i)} \end{bmatrix}, E_i \equiv 0.1I, \quad i = 1, 2, 3.$$

The parameters are given in Table I (for details concerning the modeling, see [21, Sec. 4]). To design the mode-independent controller, let us define

$$C_i \equiv \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1.5 & 0 & 0 & 0 \\ 0 & 0 & 2.5 & 0 & 0 \end{bmatrix}', D_i \equiv \begin{bmatrix} 0 & 0 & 0 & 0 & 0.5 \end{bmatrix}'$$

where the values of the matrices were adopted as a trade-off between the cost due to the system state and that due to the control effort.

The sequences $\{u_k\}$ on $\mathcal{R}$ and $\{w_k\}$ on $\mathcal{R}^2$ obey the definitions of the previous sections, whereas $\{r_k\}$ on $\mathcal{R}$ represents a reference tracking input. The Markov chain has the initial distribution $\pi_0 = [1\ 0\ 0]'$ and the transition probability matrix $\mathbb{P}$ is defined as

$$\mathbb{P} = \begin{bmatrix} 0.95 & 0.05 & 0 \\ 0.36 & 0.6 & 0.04 \\ 0.1 & 0.1 & 0.8 \end{bmatrix}.$$

The controller applied to the DC motor device is in the static state-feedback format

$$u_k = Gx_k \quad \forall k \geq 0 \tag{13}$$

where the value of the gain $G \in \mathcal{M}^{1,3}$ is determined according to Theorem 1 (taking (12) with $r_k \equiv 0$). Indeed, we use the Riccati gains as the first step, as described in Remark 2. In the second step, we solve the LMIs in Theorem 1 with the Riccati gains to obtain the $\mathcal{H}_2$-cost and the corresponding admissible gain G, which stabilizes the system and provides the $\mathcal{H}_2$-cost in Table II. Note that the $\mathcal{H}_2$-cost provided by the mode-independent gain obtained with Theorem 1 represents a good improvement when compared with the one associated to the mode-independent gain obtained with the method in [12]. In fact, in [12], the slack variables in the LMI conditions must be fixed to allow the determination of a mode-independent control gain. In our approach, the LMI conditions at the second step have more slack variables when compared to the LMIs in [12]

TABLE II
$\mathcal{H}_2$-COSTS PROVIDED BY THE MODE-INDEPENDENT CONTROLLERS FROM THEOREM 1 AND [12, P. 347, $G_i = G$]

Method	$\mathcal{H}_2$-cost	Gain
[12, p. 347]	3.8119	$G = [0.1889\ \ -0.8817\ \ -3.4384 \times 10^{-3}]$
Theorem 3.1	0.7963	$G = [0.2192\ \ -0.9492\ \ -2.3497 \times 10^{-3}]$

and, moreover, allow part of them to be mode-dependent, potentially providing less conservative design conditions. Note also that the optimal $\mathcal{H}_2$ performance provided by the mode-dependent controller in [12] is equal to 0.7265, while our mode-independent controller provides 0.7963.

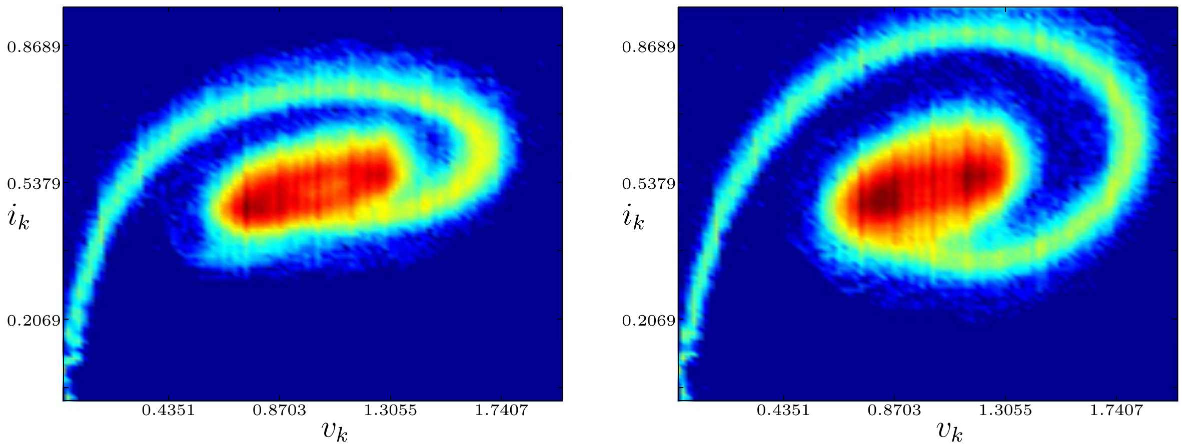
In practice, the failures tend to deviate the velocity of the DC motor device from its nominal path. To illustrate how the designed controller deals with these fluctuations, we set the apparatus to run with the sinusoid reference signal

$$r_k = 1 + 0.2\sin(k/70) \quad \forall k \geq 0.$$

Although the failures produce a significant deviation of the system from its desired path in the moment of their occurrence, we observed in practice that the control rule (13) tends to restore the equilibrium of the system (Fig. 1).

The experiment using this sinusoid signal was repeated for 700 distinct realizations of the Markov chain, running the mode-independent controllers proposed by Theorem 1 and by the method in [12]. The corresponding path density in Fig. 2 suggests that the velocity v_k and electrical current i_k have finite values for mean and standard deviation for all $k \geq 0$ for both methods. This fact indicates that the two controllers for the DC motor device are uniformly second moment stable [23].

In addition, Fig. 2 shows that the controller from Theorem 1 produces an outcome with less statistical dispersion when compared with the one from [12]. In fact, the better behavior provided by the controller obtained from Theorem 1 is related to the $\mathcal{H}_2$-bounds, as shown in Table II. In practice, the controller of Theorem 1 induces a fast transient to the system,



(a) (b)

Fig. 2. Phase portraits representing the electrical current i_k (A) and velocity v_k (rad/s) measured from the DC motor device, corresponding to 700 distinct realizations. The shading of the colors represents the statistical dispersion of the variables produced by the power failures. Their statistical means follow a spiral path, and reach an ellipsoidal limit cycle (colored in red). A comparison between the two responses indicates that the result of Theorem 1 generates an improved and less dispersive phase portrait. (a) Response due to the mode-independent controller of Theorem 1. (b) Response due to the mode-independent controller of [12, p. 347, $G_i = G$].

with less oscillations around the tracking signal, when failures occur.

The experiments described in this section emphasize the applicability of the derived approach to design controllers for stochastic systems subject to real-time failures.

V. Conclusion

This note presented an LMI formulation to design mode-independent controllers for MJLS. For the corresponding control problem associated with the $\mathcal{H}_2$-cost, our LMI approach has been shown to improve a previous result from the literature [12, mode-independent case] since it is able to produce lower $\mathcal{H}_2$-costs, as illustrated by the application of Section IV. Our approach proves to be useful in the control of real-time processes, as verified by the experiments shown in this brief.

References

[1] V. Dragan, T. Morozan, and A. Stoica, "H^2 optimal control for linear stochastic systems," *Automatica*, vol. 40, no. 7, pp. 31103–1113, Jul. 2004.

[2] O. L. V. Costa, M. D. Fragoso, and R. P. Marques, *Discrete-Time Markovian Jump Linear Systems*. New York, NY, USA: Springer-Verlag, 2005.

[3] E. F. Costa, A. N. Vargas, and J. B. R. do Val, "Quadratic costs and second moments of jump linear systems with general Markov chain," *Math. Control, Signals, Syst.*, vol. 23, nos. 1–3, pp. 141–157, Dec. 2011.

[4] A. N. Vargas, W. Furloni, and J. B. R. do Val, "Second moment constraints and the control problem of Markov jump linear systems," *Numer. Linear Algebra Appl.*, vol. 20, no. 2, pp. 357–368, Mar. 2013.

[5] R. C. L. F. Oliveira, A. N. Vargas, J. B. R. do Val, and P. L. D. Peres, "Robust stability, $\mathcal{H}_2$ analysis and stabilisation of discrete-time Markov jump linear systems with uncertain probability matrix," *Int. J. Control*, vol. 82, no. 3, pp. 470–481, Jun. 2009.

[6] J. B. R. do Val and E. F. Costa, "Stabilizability and positiveness of solutions of the jump linear quadratic problem and the coupled algebraic Riccati equation," *IEEE Trans. Autom. Control*, vol. 50, no. 5, pp. 691–695, May 2005.

[7] O. L. V. Costa and W. L. de Paulo, "Indefinite quadratic with linear costs optimal control of Markov jump with multiplicative noise systems," *Automatica*, vol. 43, no. 4, pp. 587–597, Apr. 2007.

[8] L. Wu, P. Shi, H. Gao, and C. Wang, "H_∞ filtering for 2D Markovian jump systems," *Automatica*, vol. 44, no. 7, pp. 1849–1858, Jul. 2008.

[9] L. Wu, X. Yao, and W. X. Zheng, "Generalized H_2 fault detection for Markovian jumping two-dimensional systems," *Automatica*, vol. 48, no. 8, pp. 1741–1750, Aug. 2012.

[10] L. Wu, X. Su, and P. Shi, "Sliding mode control with bounded $\mathcal{L}_2$ gain performance of Markovian jump singular time-delay systems," *Automatica*, vol. 48, no. 8, pp. 1929–1933, Aug. 2012.

[11] L. Wu, P. Shi, and H. Gao, "State estimation and sliding-mode control of Markovian jump singular systems," *IEEE Trans. Autom. Control*, vol. 55, no. 5, pp. 1213–1219, May 2010.

[12] J. B. R. do Val, J. C. Geromel, and A. P. Gonçalves, "The H_2 control for jump linear systems: Cluster observations of the Markov state," *Automatica*, vol. 38, no. 2, pp. 343–349, Feb. 2002.

[13] D. Peaucelle and D. Arzelier, "An efficient numerical solution for $\mathcal{H}_2$ static output feedback synthesis," in *Proc. Eur. Control Conf.*, Porto, Portugal, Sep. 2001, pp. 3800–3805.

[14] D. Arzelier, D. Peaucelle, and S. Salhi, "Robust static output feedback stabilization for polytopic uncertain systems: Improving the guaranteed performance bound," in *Proc. 4th IFAC Symp. Robust Control Des.*, Milan, Italy, Jun. 2003, pp. 425–430.

[15] D. Mehdi, E. K. Boukas, and O. Bachelier, "Static output feedback design for uncertain linear discrete time systems," *IMA J. Math. Control Inf.*, vol. 21, no. 1, pp. 1–13, Mar. 2004.

[16] H. R. Moreira, R. C. L. F. Oliveira, and P. L. D. Peres, "Robust $\mathcal{H}_2$ static output feedback design starting from a parameter-dependent state feedback controller for time-invariant discrete-time polytopic systems," *Opt. Control Appl. Methods*, vol. 32, no. 1, pp. 1–13, Jan. 2011.

[17] C. M. Agulhari, R. C. L. F. Oliveira, and P. L. D. Peres, "LMI relaxations for reduced-order robust $\mathcal{H}_\infty$ control of continuous-time uncertain linear systems," *IEEE Trans. Autom. Control*, vol. 57, no. 6, pp. 1532–1537, Jun. 2012.

[18] O. L. V. Costa and M. D. Fragoso, "Stability results for discrete-time linear systems with Markovian jumping parameters," *J. Math. Anal. Appl.*, vol. 179, no. 1, pp. 154–178, Oct. 1993.

[19] J.-W. Lee and G. E. Dullerud, "A stability and contractiveness analysis of discrete-time Markovian jump linear systems," *Automatica*, vol. 43, no. 1, pp. 168–173, Jan. 2007.

[20] E. F. Costa and J. B. R. do Val, "Weak detectability and the linear-quadratic control problem of discrete-time Markov jump linear systems," *Int. J. Control*, vol. 75, nos. 16–17, pp. 1282–1292, 2002.

[21] A. N. Vargas, E. F. Costa, and J. B. R. do Val, "On the control of Markov jump linear systems with no mode observation: Application to a DC motor device," *Int. J. Robust Nonlinear Control*, vol. 23, no. 10, pp. 1136–115, Jul. 2013.

[22] E. Ostertag, *Mono- and Multivariable Control and Estimation: Linear, Quadratic and LMI Methods*. New York, NY, USA: Springer-Verlag, 2011.

[23] A. N. Vargas and J. B. R. do Val, "Average cost and stability of time-varying linear systems," *IEEE Trans. Autom. Control*, vol. 55, no. 3, pp. 714–720, Mar. 2010.