This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TAC.2019.2915827, IEEE Transactions on Automatic Control

Model Reduction of Markovian Jump Systems with Uncertain Probabilities

Ying Shen, Zheng-Guang Wu, Peng Shi, Choon Ki Ahn

Abstract—**This paper studies the problem of model reduction for nonhomogeneous Markovian jump systems. The transition probability matrix of the nonhomogeneous Markovian chain has the characteristic of a polytopic structure. An asynchronous reduced-order model is considered, and the asynchronization is modeled by a hidden Markov model with a partially unknown conditional probability matrix. Under this framework, a new sufficient condition is proposed to ensure the augmented system is stochastically mean-square stable with a specified level of H_∞ performance. Finally, a numerical example is provided to show the effectiveness and advantages of the theoretic results obtained.**

Index Terms—**model reduction, nonhomogeneous Markovian chain, asynchronization, hidden Markov model, partially unknown conditional probabilities**

I. Introduction

Markovian jump systems (MJSs) have been given great concern in academic community for a long time (see [1], [2] and the references therein), which is primarily because MJS is powerful and effective in modeling sudden changes of systems. For example, MJS has been recognized to be efficient to model communication-constrained systems [3]. In MJS, the system suffering sudden changes is represented by a family of subsystems or modes, where a Markovian process is used to describe the stochastic changes or transitions among these modes. The Markovian process is said to be homogeneous if the corresponding transition probability matrix (TPM, in discrete-time system) or transition rate matrix (TRM, in continuous-time system) is time-independent, otherwise, it is nonhomogeneous [4]. In [5], two cases that the transition probabilities vary arbitrarily or periodically have been considered for stability analysis of MJSs. Besides, as for the nonhomogeneous Markovian process, the most common ones include those with piecewise homogeneous TPMs (see, e.g., [6], [7]) or polytope-structured TPMs (see, e.g., [8]–[11]). In fact, the polytope-structured TPM can be used to describe the uncertainty in TPM. Also, we would like to mention another two uncertain cases, in which the TPMs or TRMs have partially unknown entries [12], [13] or have entries bounded in an interval [14], [15]. These two uncertain cases have been studied simultaneously in [16], [17].

Large-scale systems are ubiquitous in both engineering and nature (e.g., large chemical process, social networks), and hence high-order models are required to describe such systems precisely. In control science and engineering, high-order models will bring about high complexity of analysis and synthesis, sometimes even unsolvability of the problem. As a consequence, model reduction becomes a focal issue in control domain, the primary goal of which is to find a mathematical model with lower state dimension and preserving input-output precision as much as possible in terms of certain measures. With respect to model reduction, a variety of methods have been put forward, for example, balanced truncation [18] as well as Hankel-norm approximation [19]. Thanks to the development of linear matrix inequality (LMI) solvers, various results concerning model reduction have been derived in terms of LMIs, e.g., [20]–[26], and in many of them, the H_∞ performance has been concerned. Model reduction has also been investigated for switched systems or MJSs [20]–[23]. Note that the work in [23] has developed two model reduction approaches for continuous-time MJSs, i.e., a convex linearization approach and an iterative approach, where the TRM contains both polytope-like uncertainty and unknown entries.

In the existing research concerning MJS, it is usually assumed that the controller/filter/reduced-order model is synchronous with the original plant [20]–[23], [27], [28], that is, these units are able to access the modes of the original plant without any error or delay. As a matter of fact, it is rather tough to attain full accessibility. In some works (e.g., [29], [30]), mode-independent ones are applied, which, indeed, can circumvent the aforementioned dilemma, whereas some conservatism has been introduced since the mode information has not been fully utilized. In recent years, the asynchronization phenomenon has been increasingly considered. We note that there are mainly three methods to characterize the asynchronization phenomenon. In [31], it is the time delay that gives rise to asynchronization. The asynchronization in [32] has been characterized by a piecewise homogeneous Markovian chain. In [33]–[35], a hidden-Markov-model-based description has been employed, where a conditional probability matrix (CPM) reflects the asynchronous relation between the original plant and the controller. In fact, before the works [33]–[35], some important results based on the hidden Markov model have been published, see for example, [2], [36], [37]. Also, readers

This work was partially supported by the Science Fund for Creative Research Groups (61621002), the National Nature Science Foundation of China (61773131, U1509217), and the Australian Research Council (DP170102644). (Corresponding author: Zheng-Guang Wu)

Y. Shen and Z.-G. Wu are with the State Key Laboratory of Industrial Control Technology, Institute of Cyber-Systems and Control, Zhejiang University, Yuquan Campus, Hangzhou Zhejiang, 310027, China (e-mail: yingshen@zju.edu.cn; nashwzhg@zju.edu.cn).

P. Shi is with the School of Electrical and Electronic Engineering, University of Adelaide, Adelaide, SA 5005, Australia (e-mail: peng.shi@adelaide.edu.au).

C. K. Ahn is with the School of Electrical Engineering, Korea University, Seoul 136-701, Korea (e-mail: hironaka@korea.ac.kr).

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TAC.2019.2915827, IEEE Transactions on Automatic Control

can refer to some newly published works [38], [39] and the references therein. Compared with [33]–[35], the framework applied in [2], [36]–[39] is slightly different, where the controller's mode is a detected value of the plant's mode. Besides, it is worth mentioning the work [40] which has elaborated on the hidden Markov model and its application to biological problems. Despite all this, to authors' best knowledge, hidden Markov model with partially unknown CPM has not been fully investigated yet. In addition to the above methods, using filtering technique to estimate the plant's mode is an alternative method to deal with the discrepancy between the plant's mode and controller's mode [41]. Unlike the methods in [31]–[39] which try to adapt the overall system to the asynchronization, the filtering method will actively narrow the gap between the plant's and controller's mode states. It would be interesting and also challenging to show the connection between these two methods, which will be our future work.

In view of the above-mentioned facts, this paper is concerned with the problem of asynchronous model reduction for nonhomogeneous MJSs. This work distinguishes itself from existing works on model reduction by the following two points: 1) The Markovian chain of the original system is nonhomogeneous, whose TPM is polytope-structured; and 2) Hidden-Markov-model-based asynchronization between the original system and the reduced-order model is considered, where some entries of the CPM are unknown. These two aspects together contribute to the uniqueness and novelty of this work. Although TPMs in a polytopic style have been considered in [8], [9], [11], they were considered for filtering or control problems in synchronous situations. On the other hand, the second aspect above has not been considered in existing works on model reduction. For example, it is assumed in [20]–[23] that the modes of the original system and the reduced-order model are synchronous. Moreover, the conditional probabilities or detection probabilities in [33]–[39] are completely accessible. To our knowledge, there are no results reported on partially accessible CPMs when considering the problem of model reduction. Under such a framework, an asynchronous reduced-order model is established and a new sufficient condition is obtained such that the augmented system is stochastically mean-square stable with a certain H_∞ performance. Finally, the theoretical results are verified by a numerical example.

II. Preliminaries

In the current paper, we are interested in the following system:

$$\begin{cases} x(k+1) = A(\alpha_k)x(k) + B(\alpha_k)v(k), \\ y(k) = C(\alpha_k)x(k) + D(\alpha_k)v(k), \end{cases} \tag{1}$$

which is defined on a probability space $(\Omega, \mathscr{F}, \{\mathscr{F}_k\}, \mathscr{P})$ with a filtration $\mathscr{F}_k \subset \mathscr{F}$, $k = 0, 1, 2, \cdots$. In system (1), $x(k) \in \mathbb{R}^{n_x}$, $y(k) \in \mathbb{R}^{n_y}$, and $v(k) \in \mathbb{R}^{n_v}$ denote system state, output, and stochastic input, respectively; and $v(k)$ belongs to $l_2(\Omega, \mathscr{F}, \{\mathscr{F}_k\}, \mathscr{P})$, i.e., satisfying $\sum_{k=0}^{\infty} \mathbb{E}\{||v(k)||^2\} < \infty$, $\mathbb{E}\{\cdot\}$ denotes expectation operation. $A(\alpha_k)$, $B(\alpha_k)$, $C(\alpha_k)$, and $D(\alpha_k)$ are known system matrices which depend on the Markovian parameter α_k.

We intend to approximate system (1) by establishing a reduced-order model formulated as follows:

$$\begin{cases} \bar{x}(k+1) = \bar{A}(\beta_k)\bar{x}(k) + \bar{B}(\beta_k)v(k), \\ \bar{y}(k) = \bar{C}(\beta_k)\bar{x}(k) + \bar{D}(\beta_k)v(k), \end{cases} \tag{2}$$

where $\bar{x}(k)$ is the reduced-order state vector with dimension $n_{\bar{x}} < n_x$. $\bar{y}(k)$, with the same dimension as $y(k)$, represents the output of the reduced-order model. $\bar{A}(\beta_k)$, $\bar{B}(\beta_k)$, $\bar{C}(\beta_k)$, and $\bar{D}(\beta_k)$ are undetermined system matrices with compatible dimensions, and they rest with the parameter β_k.

The parameters α_k and β_k play an important role in this work. Next, we will elaborate on the characteristics of α_k and β_k. Before proceeding, we define $\mathcal{H}^{n\times m}$ as a set of matrices $H = [h_{is}] \in \mathbb{R}^{n\times m}$ satisfying following restrictions:

$$h_{is} \geq 0, \ \sum_{s=1}^{m} h_{is} = 1 \tag{3}$$

for $\forall i \in \mathscr{N} = \{1, 2, \cdots, n\}$ and $s \in \mathscr{M} = \{1, 2, \cdots, m\}$. Specially, $\mathcal{H}^L$, a special case of $\mathcal{H}^{n\times m}$, denotes the set of row vectors $H = [h_l] \in \mathbb{R}^L$ with similar restrictions as (3), i.e., $h_l \geq 0$, $\sum_{l=1}^{L} h_l = 1$. The Markovian parameter α_k varies in the set $\mathscr{N}$ with TPM $\Phi(k)$ defined as

$$\Phi(k) = \Phi(\rho(k)) = \sum_{l=1}^{L} \rho_l(k)\Phi^{(l)}, \tag{4}$$

where $\rho(k) = \begin{bmatrix} \rho_1(k) & \rho_2(k) & \cdots & \rho_L(k) \end{bmatrix} \in \mathcal{H}^L$, $\Phi^{(l)} = [\phi_{ij}^{(l)}] \in \mathcal{H}^{n\times n}$, $l = 1, 2, \cdots, L$, are known and real matrices. Then, it is not difficult to verify that $\Phi(k)$ also belongs to the set $\mathcal{H}^{n\times n}$. The entry $\phi_{ij}(k)$ of $\Phi(k)$ signifies the transition probability and can be calculated as

$$\phi_{ij}(k) = \Pr\{\alpha_{k+1} = j | \alpha_k = i\} = \sum_{l=1}^{L} \rho_l(k)\phi_{ij}^{(l)}, \tag{5}$$

where $i, j \in \mathscr{N}$. Another parameter β_k takes values in the finite set $\mathscr{M}$, and its transitions depend on the value of α_k. Denote $\tilde{x}(k) = \begin{bmatrix} x^{\mathrm{T}}(k) & \bar{x}^{\mathrm{T}}(k) \end{bmatrix}^{\mathrm{T}}$. Let $\tilde{\mathscr{F}}_0$ be the σ-field produced by $\{\tilde{x}(0), v(0), \alpha_0\}$ and $\tilde{\mathscr{F}}_k$ be the σ-field produced by $\{\tilde{x}(0), v(0), \alpha_0, \beta_0, \cdots, \tilde{x}(k), v(k), \alpha_k\}$. Note that $\tilde{\mathscr{F}}_k$ does not include β_k. The relationship of α_k and β_k is assumed to satisfy the following assumption:

Assumption 1: [39] The joint process $(\alpha, \beta) \triangleq \{(\alpha_k, \beta_k, \mathscr{F}_k);\ k = 0, 1, 2, \cdots\}$ is Markovian, and for all $i \in \mathscr{N}, s \in \mathscr{M}, k = 0, 1, 2, \cdots$

$$\Pr\{\beta_k = s | \tilde{\mathscr{F}}_k\} = \Pr\{\beta_k = s | \alpha_k\} = \theta_{\alpha_k s} \triangleq \theta_{is}, \tag{6}$$

holds, where θ_{is} is called conditional probability.

Define $\Theta = [\theta_{is}]$ as a CPM. It is obvious that $\Theta \in \mathcal{H}^{n\times m}$. We assume that the entries of Θ are partially accessible, i.e., Θ may have following form:

$$\Theta = \begin{bmatrix} ? & \theta_{12} & ? & \theta_{14} \\ \theta_{21} & \theta_{22} & ? & ? \\ \theta_{31} & ? & \theta_{33} & ? \end{bmatrix}, \tag{7}$$

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TAC.2019.2915827, IEEE Transactions on Automatic Control

where "?" denotes that corresponding conditional probability is unavailable. For $i \in \mathscr{N}$, we denote $\mathscr{M} = \mathscr{M}_{\mathcal{K}}^i \cup \mathscr{M}_{\mathcal{U}}^i$, where

$$\begin{cases} \mathscr{M}_{\mathcal{K}}^i = \{s : \theta_{is} \ is \ known\}, \\ \mathscr{M}_{\mathcal{U}}^i = \{s : \theta_{is} \ is \ unknown\}. \end{cases} \tag{8}$$

Remark 1: The nonhomogeneous Markovian chain $\{\alpha_k, k \geq 0\}$ with time-varying TPM in a polytopic structure is considered in this paper. It has been stated in [8] that this polytope-structured model covers the piecewise homogeneous case with arbitrary variations. Accordingly, results proposed in Section III can be applied to the case with piecewise homogeneous Markovian chain.

Remark 2: Actually, Assumption 1 defines a hidden Markov model, also called hidden Markov process [40]. This model will play a significant role in derivations of main results in Section III. In our framework, the mode of the original system is seen as a hidden state, and the mode of the reduced-order model only depends on the present mode of the original system. Although this assumption will introduce some conservatism, it seems reasonable and has some practical significance. Its applications include linguistics, speech processing, bioinformatics and so on.

In the following, all the matrices related to the ith mode of system (1), i.e. $\alpha_k = i$, will use subscript i instead of α_k, e.g., $A(\alpha_k)$ is abbreviated as A_i. Similarly, we make simplification for the reduced-order model (2) by using subscript s instead of β_k.

By denoting $\tilde{x}(k) = \begin{bmatrix} x^{\mathrm{T}}(k) & \bar{x}^{\mathrm{T}}(k) \end{bmatrix}^{\mathrm{T}}$, $\tilde{y}(k) = y(k) - \bar{y}(k)$, we integrate (1) and (2), and then have the following augmented system:

$$\begin{cases} \tilde{x}(k+1) = \tilde{A}_{is}\tilde{x}(k) + \tilde{B}_{is}v(k), \\ \quad\ \ \tilde{y}(k) = \tilde{C}_{is}\tilde{x}(k) + \tilde{D}_{is}v(k), \end{cases} \tag{9}$$

where

$$\tilde{A}_{is} = \begin{bmatrix} A_i & 0 \\ 0 & \bar{A}_s \end{bmatrix}, \ \tilde{B}_{is} = \begin{bmatrix} B_i \\ \bar{B}_s \end{bmatrix},$$
$$\tilde{C}_{is} = \begin{bmatrix} C_i & -\bar{C}_s \end{bmatrix}, \ \tilde{D}_{is} = D_i - \bar{D}_s.$$

Next, we introduce some definitions for further development of this paper.

Definition 1: System (9) is said to be stochastically mean-square stable if

$$\mathbb{E}\Big\{ \sum_{k=0}^{\infty} \| \tilde{x}(k) \|^2 \, \big| \tilde{x}(0), \alpha_0, \beta_0 \Big\} < \infty \tag{10}$$

holds under $v(k) \equiv 0$ for any initial condition $(\tilde{x}(0), \alpha_0, \beta_0)$.

Definition 2: Under the premise that system (9) is stochastically mean-square stable, for a given scalar $\gamma > 0$, (9) is said to have H_∞ performance γ if

$$\sum_{k=0}^{\infty} \mathbb{E}\{\|\tilde{y}(k)\|^2\} < \gamma^2 \sum_{k=0}^{\infty} \|v(k)\|^2 \tag{11}$$

holds under zero initial state $\tilde{x}(0) = 0$ and any nonzero $v(k) \in l_2[0, +\infty)$.

Given all the above, we formulate the problem to be addressed in this paper as follows: Given a stochastically mean-square stable system (1), establish a reduced-order model in the form of (2) to approximate system (1) such that the augmented system (9) is stochastically mean-square stable with H_∞ performance γ.

Remark 3: As specified above, this work will investigate model reduction based on H_∞ theory under the framework of hidden Markov model. Note that H_∞ performance and hidden-Markov-model-based framework are also concerned in [34], [35], [39]. What is different is that [34], [35], [39] studied H_∞ control. The issues of control and model reduction are quite different. The main objective of H_∞ control is to stabilize a system with a certain H_∞ performance. For H_∞ based model reduction, the aim is to approximate a high-order system by finding a lower-order model, and the augmented system is required to be stable and have a prescribed H_∞ performance. It is worth noting that the asynchronization framework in this work is slightly different from that in [39]. In [39], the mode $\hat{\theta}_k$ of the controller is a detected value of the plant's mode θ_k. However, in our work, the asynchronization is induced by unreliable transmission. Besides, in this work, the TPM is polytopic and the CPM is partially accessible, which is not considered in [34], [35], [39].

III. Main Results

To start with, we present a new sufficient condition ensuring the stochastic mean-square stability and H_∞ performance of system (9) provided $\bar{A}_s$, $\bar{B}_s$, $\bar{C}_s$, and $\bar{D}_s$ in the reduced-order model (2) are known.

Theorem 1: The augmented system (9) is stochastically mean-square stable with an H_∞ performance γ ($\gamma > 0$), if there exist matrices $P_i^{(l)} > 0$, $R_{is} > 0$, such that the following conditions hold: for $\forall i \in \mathscr{N}$, $s \in \mathscr{M}_{\mathcal{U}}^i$, $l = 1, 2, \cdots, L$,

$$\mathcal{R}_i^{\mathcal{K}} + (1 - \theta_i^{\mathcal{K}})R_{is} - P_i^{(l)} < 0, \tag{12}$$

and for $\forall i \in \mathscr{N}$, $s \in \mathscr{M}$, $l, t = 1, 2, \cdots, L$,

$$\begin{bmatrix} -(\bar{P}_i^{(l,t)})^{-1} & 0 & \tilde{A}_{is} & \tilde{B}_{is} \\ * & -I & \tilde{C}_{is} & \tilde{D}_{is} \\ * & * & -R_{is} & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0, \tag{13}$$

where

$$\mathcal{R}_i^{\mathcal{K}} = \sum_{s \in \mathscr{M}_{\mathcal{K}}^i} \theta_{is}R_{is}, \ \theta_i^{\mathcal{K}} = \sum_{s \in \mathscr{M}_{\mathcal{K}}^i} \theta_{is}, \ \bar{P}_i^{(l,t)} = \sum_{j=1}^{n} \phi_{ij}^{(l)} P_j^{(t)}.$$

Proof: Firstly, we will have some useful derivations from (12) and (13). The following inequality

$$\sum_{s=1}^{m} \theta_{is}R_{is} - P_i^{(l)} < 0 \tag{14}$$

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TAC.2019.2915827, IEEE Transactions on Automatic Control

is ensured by (12) with $\theta_i^{\mathcal{K}} < 1$ due to that

$$\sum_{s=1}^{m} \theta_{is} R_{is} - P_i^{(l)} = \mathcal{R}_i^{\mathcal{K}} + (1-\theta_i^{\mathcal{K}}) \sum_{s\in\mathscr{M}_{\mathcal{U}}^i} \frac{\theta_{is}}{1-\theta_i^{\mathcal{K}}} R_{is} - P_i^{(l)} = \sum_{s\in\mathscr{M}_{\mathcal{U}}^i} \frac{\theta_{is}}{1-\theta_i^{\mathcal{K}}} \Big\{ \mathcal{R}_i^{\mathcal{K}} + (1-\theta_i^{\mathcal{K}}) R_{is} - P_i^{(l)} \Big\}. \tag{15}$$

And it is obvious that (12) and (14) are equivalent when $\theta_i^{\mathcal{K}} = 1$. Then, multiplying (14) by $\rho_l(k)$ and adding all inequalities, we have

$$\sum_{s=1}^{m} \theta_{is} R_{is} < \sum_{l=1}^{L} \rho_l(k) P_i^{(l)}. \tag{16}$$

Using Schur Complement to (13), we obtain (17) and (18):

$$-R_{is} + \tilde{A}_{is}^{\mathrm{T}} \bar{P}_i^{(l,t)} \tilde{A}_{is} < 0, \tag{17}$$

$$\begin{bmatrix} -R_{is} & 0 \\ 0 & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} \tilde{A}_{is}^{\mathrm{T}} \\ \tilde{B}_{is}^{\mathrm{T}} \end{bmatrix} \bar{P}_i^{(l,t)} \begin{bmatrix} \tilde{A}_{is} & \tilde{B}_{is} \end{bmatrix} + \begin{bmatrix} \tilde{C}_{is}^{\mathrm{T}} \\ \tilde{D}_{is}^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} \tilde{C}_{is} & \tilde{D}_{is} \end{bmatrix} < 0, \tag{18}$$

It follows from (17) and (18) that

$$\tilde{A}_{is}^{\mathrm{T}} \sum_{l=1}^{L} \sum_{t=1}^{L} \rho_l(k) \omega_t(k) \bar{P}_i^{(l,t)} \tilde{A}_{is} < R_{is}, \tag{19}$$

and

$$\begin{bmatrix} \tilde{A}_{is}^{\mathrm{T}} \\ \tilde{B}_{is}^{\mathrm{T}} \end{bmatrix} \sum_{l=1}^{L} \sum_{t=1}^{L} \rho_l(k) \omega_t(k) \bar{P}_i^{(l,t)} \begin{bmatrix} \tilde{A}_{is} & \tilde{B}_{is} \end{bmatrix} + \begin{bmatrix} \tilde{C}_{is}^{\mathrm{T}} \\ \tilde{D}_{is}^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} \tilde{C}_{is} & \tilde{D}_{is} \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & \gamma^2 I \end{bmatrix} < \begin{bmatrix} R_{is} & 0 \\ 0 & 0 \end{bmatrix} \tag{20}$$

holds, where $\omega_t(k) \geq 0$ will be specified later.

Next, we will check whether the stochastic mean-square stability and H_∞ performance of system (9) are ensured by (16), (19), and (20). We introduce following Lyapunov function:

$$V(k) = \tilde{x}^{\mathrm{T}}(k) \mathcal{P}_i(\rho(k)) \tilde{x}(k), \tag{21}$$

where $\mathcal{P}_i(\rho(k)) = \sum_{l=1}^{L} \rho_l(k) P_i^{(l)}$. In $V(k+1)$, we define

$$\mathcal{P}_j(\rho(k+1)) = \sum_{l=1}^{L} \rho_l(k+1) P_j^{(l)} = \sum_{t=1}^{L} \omega_t(k) P_j^{(t)}, \tag{22}$$

i.e., $\omega_t(k) \triangleq \rho_l(k+1)$. Then, the expectation of $V(k+1)$ can be calculated as

$$\begin{aligned} &\mathbb{E}\{V(k+1)|\tilde{x}(k), \alpha_k = i\} \\ =&\mathbb{E}\{\tilde{x}^{\mathrm{T}}(k+1)\mathcal{P}_j(\rho(k+1))\tilde{x}(k+1)|\tilde{x}(k), \alpha_k = i\} \\ =&\mathbb{E}\Big\{ \begin{bmatrix} \tilde{x}(k) \\ v(k) \end{bmatrix}^{\mathrm{T}} \Big(\sum_{s=1}^{m} \theta_{is} \begin{bmatrix} \tilde{A}_{is}^{\mathrm{T}} \\ \tilde{B}_{is}^{\mathrm{T}} \end{bmatrix} \sum_{j=1}^{n} \phi_{ij}(k) \mathcal{P}_j(\rho(k+1)) \\ &\cdot \begin{bmatrix} \tilde{A}_{is} & \tilde{B}_{is} \end{bmatrix} \Big) \begin{bmatrix} \tilde{x}(k) \\ v(k) \end{bmatrix} \Big\}. \end{aligned} \tag{23}$$

When $v(k) = 0$, it follows from (19) that

$$\begin{aligned} &\mathbb{E}\{V(k+1)|\tilde{x}(k), \alpha_k = i\} \\ =&\mathbb{E}\Big\{ \tilde{x}^{\mathrm{T}}(k) \Big(\sum_{s=1}^{m} \theta_{is} \tilde{A}_{is}^{\mathrm{T}} \sum_{j=1}^{n} \phi_{ij}(k) \mathcal{P}_j(\rho(k+1)) \tilde{A}_{is} \Big) \tilde{x}(k) \Big\} \\ <&\mathbb{E}\Big\{ \tilde{x}^{\mathrm{T}}(k) \sum_{s=1}^{m} \theta_{is} R_{is} \tilde{x}(k) \Big\}. \end{aligned} \tag{24}$$

Then, we obtain following inequality:

$$\begin{aligned} &\mathbb{E}\{\Delta V(k)\} \\ =&\mathbb{E}\{V(k+1) - V(k)|\tilde{x}(k), \alpha_k = i\} \\ <&\mathbb{E}\Big\{ \tilde{x}^{\mathrm{T}}(k) \Big(\sum_{s=1}^{m} \theta_{is} R_{is} - \mathcal{P}_i(\rho(k)) \Big) \tilde{x}(k) \Big\} \\ \leq& -\mu \mathbb{E}\{\|\tilde{x}(k)\|^2\}, \end{aligned} \tag{25}$$

where $\mu = \min_{i\in\mathscr{N},k} \big\{ \lambda_{min} \big(\mathcal{P}_i(\rho(k)) - \sum_{s=1}^{m} \theta_{is} R_{is} \big) \big\}$, and (16) implies $\mu > 0$. Thus, we can arrive at the conclusion

$$\mathbb{E}\Big\{ \sum_{k=0}^{\infty} \|\tilde{x}(k)\|^2 \Big\} < -\frac{1}{\mu} \mathbb{E}\Big\{ \sum_{k=0}^{\infty} \Delta V(k) \Big\} \leq \frac{1}{\mu} \mathbb{E}\{V(0)\} < \infty, \tag{26}$$

which implies that system (9) is stochastically mean-square stable according to Definition 1.

In the following, we will continue to investigate H_∞ performance of system (9) with zero initial state and nonzero $v(k)$. We examine the following performance index:

$$\begin{aligned} J =& \sum_{k=0}^{\infty} \mathbb{E}\{\tilde{y}^{\mathrm{T}}(k)\tilde{y}(k) - \gamma^2 v^{\mathrm{T}}(k) v(k)\} \\ \leq& \sum_{k=0}^{\infty} \mathbb{E}\{\tilde{y}^{\mathrm{T}}(k)\tilde{y}(k) - \gamma^2 v^{\mathrm{T}}(k) v(k) + \Delta V(k)\} \\ =& \sum_{k=0}^{\infty} \mathbb{E}\Bigg\{ \begin{bmatrix} \tilde{x}(k) \\ v(k) \end{bmatrix}^{\mathrm{T}} \sum_{s=1}^{m} \theta_{is} \Bigg(\begin{bmatrix} \tilde{A}_{is}^{\mathrm{T}} \\ \tilde{B}_{is}^{\mathrm{T}} \end{bmatrix} \sum_{j=1}^{n} \phi_{ij}(k) \mathcal{P}_j(\rho(k+1)) \\ &\cdot \begin{bmatrix} \tilde{A}_{is} & \tilde{B}_{is} \end{bmatrix} + \begin{bmatrix} \tilde{C}_{is}^{\mathrm{T}} \\ \tilde{D}_{is}^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} \tilde{C}_{is} & \tilde{D}_{is} \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & \gamma^2 I \end{bmatrix} \Bigg) \begin{bmatrix} \tilde{x}(k) \\ v(k) \end{bmatrix} \\ &- \tilde{x}^{\mathrm{T}}(k) \mathcal{P}_i(\rho(k)) \tilde{x}(k) \Bigg\} \\ <& \mathbb{E}\Big\{ \tilde{x}^{\mathrm{T}}(k) \Big(\sum_{s=1}^{m} \theta_{is} R_{is} - \mathcal{P}_i(\rho(k)) \Big) \tilde{x}(k) \Big\} \\ <& 0, \end{aligned} \tag{27}$$

where the last two "<" hold as a result of (20) and (16), respectively. Then, (11) is proved by (27). This completes the proof. □

Remark 4: Note that the TPM of Markov chain α_k in system (1) is in a polytopic form (4). Then, a $\rho(k)$-dependent Lypapunov function is used in the proof of Theorem 1. We use the same technique as that in [8] to deal with $\{\rho(k), k \geq 0\}$. Except for this point, this work is quite different from [8]:

1) Different issues are considered in [8] and the present work, i.e., stability and stabilization, and model reduction are investigated, respectively.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TAC.2019.2915827, IEEE Transactions on Automatic Control

2) Besides the stochastic mean square stability, H_∞ performance is further analyzed in this work, which is not considered in [8].
3) Most importantly, compared with [8], this work takes into consideration the mode asynchronization between the plant and the reduced-order model. Moreover, this work is based on a unified framework, i.e., hidden Markov model, which also includes synchronous and mode-independent cases.

Next, the matrices $\bar{A}_s$, $\bar{B}_s$, $\bar{C}_s$, and $\bar{D}_s$ in the reduced-order model (2) will be designed in Theorem 2. Before proceeding, we firstly introduce a following slack matrix:

$$Q_{is} = \begin{bmatrix} X_{is} & UZ_s \\ Y_{is} & Z_s \end{bmatrix}, \tag{28}$$

where $X_{is} \in \mathbb{R}^{n_x \times n_x}$, $Z_s \in \mathbb{R}^{n_{\bar{x}} \times n_{\bar{x}}}$, and $U = \begin{bmatrix} I_{n_{\bar{x}}} & 0 \end{bmatrix}^{\mathrm{T}}$.

Theorem 2: The augmented system (9) is stochastically mean-square stable with an H_∞ performance γ ($\gamma > 0$), if there exist matrices $\check{A}_s$, $\check{B}_s$, $\check{C}_s$, $\check{D}_s$, Q_{is}, $P_i^{(l)} > 0$, and $R_{is} > 0$, such that condition (12) and the following condition hold for $\forall i \in \mathscr{N}$, $s \in \mathscr{M}$, $l, t = 1, 2, \cdots, L$,

$$\begin{bmatrix} \bar{P}_i^{(l,t)} - Q_{is} - Q_{is}^{\mathrm{T}} & 0 & \mathscr{A}_{is} & \mathscr{B}_{is} \\ * & -I & \mathscr{C}_{is} & \mathscr{D}_{is} \\ * & * & -R_{is} & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0, \tag{29}$$

where

$$\mathscr{A}_{is} = \begin{bmatrix} X_{is}A_i & U\check{A}_s \\ Y_{is}A_i & \check{A}_s \end{bmatrix}, \ \mathscr{B}_{is} = \begin{bmatrix} X_{is}B_i + U\check{B}_s \\ Y_{is}B_i + \check{B}_s \end{bmatrix},$$
$$\mathscr{C}_{is} = \begin{bmatrix} C_i & -\check{C}_s \end{bmatrix}, \ \mathscr{D}_{is} = D_i - \check{D}_s,$$

Moreover, if the LMIs (12) and (29) have feasible solutions, then the parameters in the reduced-order model (2) can be formulated as

$$\bar{A}_s = Z_s^{-1}\check{A}_s, \ \bar{B}_s = Z_s^{-1}\check{B}_s, \ \bar{C}_s = \check{C}_s, \ \bar{D}_s = \check{D}_s. \tag{30}$$

Proof: We know that

$$\bar{P}_i^{(l,t)} - Q_{is} - Q_{is}^{\mathrm{T}} \geq -Q_{is}(\bar{P}_i^{(l,t)})^{-1}Q_{is}^{\mathrm{T}} \tag{31}$$

holds. Then, (29) together with (31) yields that

$$\begin{bmatrix} -Q_{is}(\bar{P}_i^{(l,t)})^{-1}Q_{is}^{\mathrm{T}} & 0 & \mathscr{A}_{is} & \mathscr{B}_{is} \\ * & -I & \mathscr{C}_{is} & \mathscr{D}_{is} \\ * & * & -R_{is} & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0. \tag{32}$$

We can infer from (29) that Q_{is} and Z_s are nonsingular. Using the following notations,

$$\check{A}_s = Z_s\bar{A}_s, \ \check{B}_s = Z_s\bar{B}_s, \ \check{C}_s = \bar{C}_s, \ \check{D}_s = \bar{D}_s, \tag{33}$$

we rewrite (32) as

$$\begin{bmatrix} -Q_{is}(\bar{P}_i^{(l,t)})^{-1}Q_{is}^{\mathrm{T}} & 0 & Q_{is}\tilde{A}_{is} & Q_{is}\tilde{B}_{is} \\ * & -I & \tilde{C}_{is} & \tilde{D}_{is} \\ * & * & -R_{is} & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0. \tag{34}$$

Pre- and post-multiplying (34) by $diag\{Q_{is}^{-1}, I, I, I\}$ and its transpose leads to (13). Hence, conditions (12) and (29) are sufficient to ensure the system (9) is stochastically mean-square stable with a prescribed H_∞ performance. Furthermore, once we obtain $\check{A}_s$, $\check{B}_s$, $\check{C}_s$, and $\check{D}_s$ by solving (12) and (29), $\bar{A}_s$, $\bar{B}_s$, $\bar{C}_s$, and $\bar{D}_s$ can be determined according to (30). This completes the proof. □

Remark 5: Note that the number of decision variables N_{dv} and the number of LMIs N_{lmi} in Theorem 2 are respectively:

$$\begin{aligned} N_{dv} &= (2n_{\bar{x}}^2 + n_{\bar{x}}n_v + n_y n_{\bar{x}} + n_y n_v) \cdot m \\ &\quad + (n_x^2 + n_x n_{\bar{x}} + \bar{n}^2) \cdot n \cdot m + \bar{n}^2 \cdot n \cdot L, \\ N_{lmi} &= L \cdot \sum_{i=1}^{n} |\mathscr{M}_{\mathcal{U}}^i| + n \cdot m \cdot L^2, \end{aligned} \tag{35}$$

where $\bar{n} = n_x + n_{\bar{x}}$, $|\mathscr{M}_{\mathcal{U}}^i|$ denotes the number of elements in the set $\mathscr{M}_{\mathcal{U}}^i$. We can observe from (35) that the computational burden is aggravated due to asynchronization (m), polytope-structured TPM (L) as well as partially unknown CPM ($|\mathscr{M}_{\mathcal{U}}^i|$).

IV. Numerical Example

In this section, a numerical example is simulated to show the efficiency of the proposed model reduction method. This numerical example is composed of four modes:

$$A_1 = \begin{bmatrix} 0.0550 & -0.2750 & 0.4400 & 0.3850 \\ 0.5500 & 0.3300 & 0.3850 & 0.5500 \\ 0.1100 & 0.1650 & 0.2750 & 0.4400 \\ 0.0550 & 0.2200 & 0.1650 & 0.1100 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0.8100 & -0.0650 & 0.2400 & 0.5120 \\ -0.2155 & 0.0550 & 0.2200 & 0.3213 \\ 0.0550 & 0.4650 & 0.5750 & 0.2400 \\ 0.1650 & 0.0550 & 0.0550 & -0.1100 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 0.1280 & -0.0550 & 0.1100 & -0.3850 \\ -0.5500 & 0.3210 & 0.6850 & 0.7500 \\ 0.4100 & 0.6650 & 0.1100 & 0.2200 \\ 0.3550 & 0.1200 & 0.2010 & -0.0550 \end{bmatrix},$$

$$A_4 = \begin{bmatrix} 0.1550 & -0.0650 & 0.4900 & 0.2860 \\ 0.2500 & 0.0950 & 0.4200 & -0.5500 \\ 0.4550 & 0.3850 & 0.4250 & 0.1650 \\ 0.1580 & 0.4450 & 0.1750 & -0.2550 \end{bmatrix},$$

$$\begin{aligned} B_1 &= \begin{bmatrix} 0.19 & -0.31 & -0.15 & 0.16 \end{bmatrix}^{\mathrm{T}}, \\ B_2 &= \begin{bmatrix} 0.25 & -0.33 & 0.22 & -0.26 \end{bmatrix}^{\mathrm{T}}, \\ B_3 &= \begin{bmatrix} 0.24 & -0.36 & 0.46 & -0.18 \end{bmatrix}^{\mathrm{T}}, \\ B_4 &= \begin{bmatrix} 0.25 & -0.33 & 0.22 & -0.26 \end{bmatrix}^{\mathrm{T}}, \\ C_1 &= \begin{bmatrix} 1.50 & 0.71 & 0.18 & -0.70 \end{bmatrix}, \ D_1 = 1.9, \\ C_2 &= \begin{bmatrix} 1.40 & 0.66 & 0.14 & -0.89 \end{bmatrix}, \ D_2 = 1.4 \\ C_3 &= \begin{bmatrix} 1.00 & 0.95 & 0.10 & -1.39 \end{bmatrix}, \ D_3 = 1.5 \\ C_4 &= \begin{bmatrix} 1.35 & 0.85 & 0.15 & -0.90 \end{bmatrix}, \ D_4 = 1.0 \end{aligned}$$

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TAC.2019.2915827, IEEE Transactions on Automatic Control

The TPM $\Phi(k)$ varies in a polytope with following vertices:

$$\Phi^1 = \begin{bmatrix} 0.25 & 0.15 & 0.3 & 0.3 \\ 0.5 & 0.35 & 0 & 0.15 \\ 0.05 & 0.4 & 0.5 & 0.05 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\Phi^2 = \begin{bmatrix} 0.17 & 0.33 & 0.4 & 0.1 \\ 0.45 & 0.55 & 0 & 0 \\ 0.1 & 0.1 & 0.3 & 0.5 \\ 0.2 & 0.1 & 0.7 & 0 \end{bmatrix}.$$

In the following, four cases with different CPMs will be considered:

Case 1:

$$\Theta = \begin{bmatrix} 0.2 & 0.1 & 0.3 & 0.4 \\ 0.1 & 0.3 & 0.5 & 0.1 \\ 0.6 & 0.1 & 0.2 & 0.1 \\ 0.25 & 0.4 & 0.3 & 0.05 \end{bmatrix},$$

Case 2:

$$\Theta = \begin{bmatrix} 0.2 & ? & ? & 0.4 \\ 0.1 & 0.3 & ? & ? \\ 0.6 & 0.1 & 0.2 & 0.1 \\ 0.25 & 0.4 & 0.3 & 0.05 \end{bmatrix},$$

Case 3:

$$\Theta = \begin{bmatrix} 0.2 & ? & ? & 0.4 \\ 0.1 & 0.3 & ? & ? \\ ? & 0.1 & ? & ? \\ ? & 0.4 & 0.3 & ? \end{bmatrix},$$

Case 4:

$$\Theta = \begin{bmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{bmatrix}.$$

Note that the CPM Θ is completely known and completely unknown in Case 1 and Case 4, respectively, and Θ in Case 3 has more unknown entries than that in Case 2.

A four-mode reduced-order model will be designed by virtue of the design method presented in Theorem 2. Note that the parameter γ is pre-given in Theorem 2. In fact, γ can be optimized by replacing γ^2 with $\check{\gamma}$ in Theorem 2, provided it is not pre-given. Denote the optimized γ as γ^*. Then, these four cases are simulated and corresponding γ^* are displayed in Table I. It can be concluded from Table I that a better H_∞ performance will be achieved if more information concerning CPM Θ is available. Besides, the parameters of the reduced-order model in Case 4 are listed below:

$$\left[\begin{array}{c|c} \bar{A}_s & \bar{B}_s \\ \hline \bar{C}_s & \bar{D}_s \end{array}\right] = \left[\begin{array}{cc|c} 0.8275 & 0.3457 & -0.1341 \\ 0.0558 & 0.1845 & -0.3884 \\ \hline -1.1023 & -0.9192 & 1.4178 \end{array}\right]$$

for $s = 1, 2, 3, 4$. Case 4 indicates that a mode-independent model will be valid if Θ is completely unknown.

Furthermore, the designed reduced-order model in Case 3 is given as follows:

$$\left[\begin{array}{c|c} \bar{A}_1 & \bar{B}_1 \\ \hline \bar{C}_1 & \bar{D}_1 \end{array}\right] = \left[\begin{array}{cc|c} 0.7432 & 0.1903 & -0.2306 \\ 0.0968 & 0.5443 & -0.0344 \\ \hline -1.0359 & -0.7424 & 1.4942 \end{array}\right],$$

$$\left[\begin{array}{c|c} \bar{A}_2 & \bar{B}_2 \\ \hline \bar{C}_2 & \bar{D}_2 \end{array}\right] = \left[\begin{array}{cc|c} 0.9262 & 0.1682 & -0.2647 \\ 0.2529 & 0.6531 & 0.0503 \\ \hline -1.3084 & -0.7728 & 1.4364 \end{array}\right],$$

$$\left[\begin{array}{c|c} \bar{A}_3 & \bar{B}_3 \\ \hline \bar{C}_3 & \bar{D}_3 \end{array}\right] = \left[\begin{array}{cc|c} 0.7106 & 0.1643 & -0.3689 \\ 0.1386 & 0.5627 & 0.0974 \\ \hline -1.0931 & -0.8423 & 1.1298 \end{array}\right],$$

$$\left[\begin{array}{c|c} \bar{A}_4 & \bar{B}_4 \\ \hline \bar{C}_4 & \bar{D}_4 \end{array}\right] = \left[\begin{array}{cc|c} 0.7471 & 0.1961 & -0.2211 \\ 0.0905 & 0.5358 & -0.0474 \\ \hline -1.0352 & -0.7384 & 1.5074 \end{array}\right].$$

The initial condition is given as: $\bar{x}(0) = \begin{bmatrix} 0 & 0 \end{bmatrix}^{\mathrm{T}}$, $x(0) = \begin{bmatrix} 0.3 & -0.5 & 0.1 & 0.6 \end{bmatrix}^{\mathrm{T}}$, $\alpha_0 = 2$. The system input is as follows:

$$v(k) = \begin{cases} 0.9^k \tilde{v}(k), & 0 \le k \le 20 \\ -0.5, & 50 \le k \le 55 \\ 0, & else \end{cases}$$

where $\tilde{v}(k)$ is a random signal between -1 and 1. Note that $v(k)$ includes a step-like signal, which is always considered as a tough situation for control systems. Then, the evolving curves of $y(k)$, $\bar{y}(k)$, and $\tilde{y}(k)$ are produced, which are displayed in Fig.1. We can observe that the error $\tilde{y}(k)$ is small and converges rapidly to zero even with a step-like disturbance, which means that the reduced-order model approximates the original system very well. In addition, we plot the mode transitions in Fig.2 to show the asynchronization.

TABLE I
MINIMAL H_∞ PERFORMANCE FOR DIFFERENT Θ

	Case 1	Case 2	Case 3	Case 4
γ^*	2.1987	2.3208	2.5575	2.7330

V. CONCLUSION

The issue of model reduction has been investigated for nonhomogeneous MJSs. The concerned MJS is characterized by a nonhomogeneous Markovian chain with a polytope-structured TPM. The designed reduced-order model is asynchronous with the original system. The mode transition follows a hidden Markov model, in which the CPM is assumed to be partially accessible. By Lyapunov method, a new sufficient condition has been obtained such that the augmented system is stochastically mean-square stable and has a certain level of H_∞ performance. Moreover, the reduced-order model has been parameterized by LMI approach. Finally, a numerical example is presented to illustrate the effectiveness of the proposed design method in this work.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TAC.2019.2915827, IEEE Transactions on Automatic Control

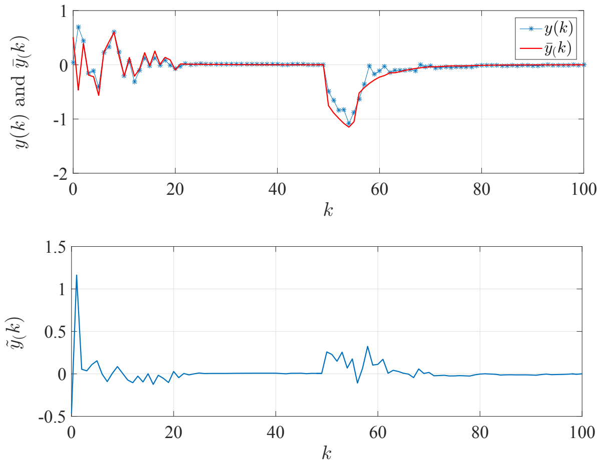

Fig. 1. The evolving curves of $y(k)$, $\bar{y}(k)$ and $\tilde{y}(k)$

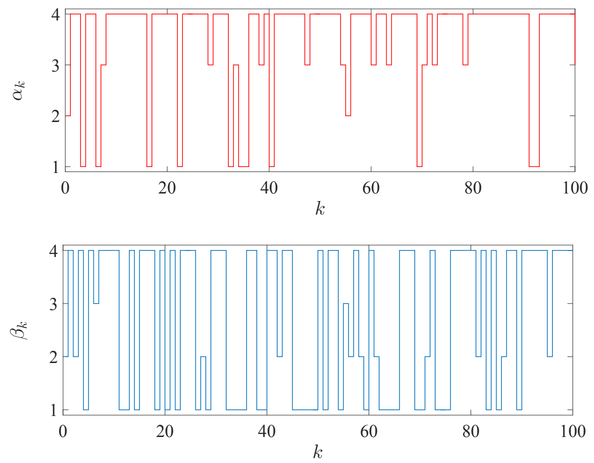

Fig. 2. Mode transitions of the original system and reduced-order model

REFERENCES

[1] O. L. V. Costa, M. D. Fragoso and M. G. Todorov, *Continuous-time Markov jump linear systems*, Springer, 2013.

[2] O. L. V. Costa, M. D. Fragoso and R. P. Marques, *Discrete-time Markov jump linear systems*, Springer, 2005.

[3] S. Aberkane and V. Dragan, "H_∞ filtering of periodic Markovian jump systems: Application to filtering with communication constraints," *Automatica*, vol. 48, no. 12, pp. 3151–3156, 2012.

[4] L. Zhang, T. Yang, P. Shi and Y. Zhu, *Analysis and design of Markov jump systems with complex transition probabilities*, Springer, 2016.

[5] S. Aberkane, "Bounded real lemma for nonhomogeneous Markovian jump linear systems," *IEEE Transactions on Automatic Control*, vol. 58, no. 3, pp. 797–801, 2013.

[6] L. Zhang, "H_∞ estimation for discrete-time piecewise homogeneous Markov jump linear systems," *Automatica*, vol. 45, no. 11, pp. 2570–2576, 2009.

[7] Z.-G. Wu, P. Shi, H. Su and J. Chu, "Passivity analysis for discrete-time stochastic Markovian jump neural networks with mixed time delays," *IEEE Transactions on Neural Networks*, vol. 22, no. 10, pp. 1566-1575, 2011.

[8] S. Aberkane, "Stochastic stabilization of a class of nonhomogeneous Markovian jump linear systems," *Systems & Control Letters*, vol. 60, no. 3, pp. 156-160, 2011.

[9] Y. Yin, P. Shi, F. Liu, K. Teo and C. Lim, "Robust filtering for nonlinear nonhomogeneous Markov jump systems by fuzzy approximation approach," *IEEE Transactions on Cybernetics*, vol. 45, no. 9, pp. 1706–1716, 2015.

[10] Y. Z. Lun, A. D'Innocenzo, and M. D. Di Benedetto, "Robust stability of time-inhomogeneous Markov jump linear systems," *IFAC-PapersOnLine*, vol. 50, no. 1, pp. 3418–3423, 2017.

[11] D. Zhang, J. Cheng, J. H. Park, and J. Cao, "Robust H_∞ control for nonhomogeneous Markovian jump systems subject to quantized feedback and probabilistic measurements," *Journal of the Franklin Institute*, vol. 355, pp. 6992–7010, 2018.

[12] L. Zhang, E.-K. Boukas, and J. Lam, "Analysis and synthesis of Markov jump linear systems with time-varying delays and partially known transition probabilities," *IEEE Transactions on Automatic Control*, vol. 53, no. 10, pp. 2458–2464, 2008.

[13] Y. Li, H. R. Karimi, D. Zhao, Y. Xu, and P. Zhao, "H_∞ fault detection filter design for discrete-time nonlinear Markovian jump systems with missing measurements," *European Journal of Control*, 2018, Doi: 10.1016/j.ejcon.2018.09.017.

[14] M. Karan, P. Shi, and C. Y. Kaya, "Transition probability bounds for the stochastic stability robustness of continuous- and discrete-time Markovian jump linear systems," *Automatica*, vol. 42, pp. 2159–2168, 2006.

[15] S. Chitraganti, S. Aberkane, and C. Aubrun, "Mean square stability of non-homogeneous Markov jump linear systems using interval analysis," *In 2013 European Control Conference (ECC)*, Zurich, Switzerland, pp. 3724–3729, 2013.

[16] X. Li, H. R. Karimi, Y. Wang, D. Lu, and S. Guo, "Robust fault estimation and fault-tolerant control for Markovian jump systems with general uncertain transition rates," *Journal of the Franklin Institute*, vol. 355, pp. 3508–3540, 2018.

[17] Y. Jiang, B. Kao, H. R. Karimi, and C. Gao, "Stability and stabilization for singular switching semi-Markovian jump systems with generally uncertain transition Rates," *IEEE Transactions on Automatic Control*, vol. 63, no. 11, pp. 3919–3926, 2018.

[18] B. Moore, "Principal component analysis in linear systems: controllability, observability, and model reduction," *IEEE Transactions on Automatic Control*, vol. 26, no. 1, pp. 17–32, 1981.

[19] S. Kung and D. Lin, "Optimal Hankel-norm model reductions: Multivariable systems," *IEEE Transactions on Automatic Control*, vol. 26, no. 4, pp. 832–852, 1981.

[20] L. Zhang and P. Shi, "l_2-l_∞ model reduction for switched LPV systems with average dwell time," *IEEE Transactions on Automatic Control*, vol. 53, no. 10, pp. 2443–2448, 2008.

[21] X. Li, J. Lam, H. Gao and P. Li, "Improved results on H_∞ model reduction for Markovian jump systems with partly known transition probabilities," *Systems & Control Letters*, vol. 70, pp. 109–117, 2014.

[22] L. Zhang, B. Huang and J. Lam, "H_∞ model reduction of Markovian jump linear systems," *Systems & Control Letters*, vol. 50, no. 2, pp. 103–118, 2003.

[23] Y. Wei, J. Qiu, H. R. Karimi, and M. Wang, "H_∞ model reduction for continuous-time Markovian jump systems with incomplete statistics of mode information," *International Journal of Systems Sciences*, vol. 45, no. 7, pp. 1496–1507, 2014.

[24] S. Xu and J. Lam, "H_∞ model reduction for discrete-time singular systems," *Systems & Control Letters*, vol. 48, no. 2, pp. 121–133, 2003.

[25] J. Shen and J. Lam, "Improved results on H_∞ model reduction for continuous-time linear systems over finite frequency ranges," *Automatica*, vol. 53, pp. 79–84, 2015.

[26] P. Li, J. Lam, Z. Wang and P. Data, "Positivity-preserving H_∞ model reduction for positive systems," *Automatica*, vol. 47, no. 7, pp. 1504–1511, 2011.

[27] D. P. de Farias, J. C. Geromel, J. B. R. do Val and O. L. V. Costa, "Output feedback control of Markov jump linear systems in continuous-time," *IEEE Transactions on Automatic Control*, vol. 45, no. 5, pp. 944–949, 2000.

[28] T. Hou and H. Ma, "Exponential stability for discrete-time infinite Markov jump systems," *IEEE Transactions on Automatic Control*, vol. 61, no. 12, pp. 4241–4246, 2016.

[29] R. C. L. F. Oliveira, A. N. Vargas, J. B. R. Do Val and P. L. D. Peres, "Mode-independent H_2 Control of a DC motor modeled as a Markov jump linear system," *IEEE Transactions on Control Systems Technology*, vol. 22, no. 5, pp. 1915–1919, 2014.

[30] X. Zhong, H. He, H. Zhang and Z. Wang, "Optimal control for unknown discrete-time nonlinear Markov jump systems using adaptive dynamic programming," *IEEE Transactions on Neural Networks & Learning Systems*, vol. 25, no. 12, pp. 2141–2155, 2014.

[31] L. Zhang and H. Gao, "Asynchronously switched control of switched linear systems with average dwell time," *Automatica*, vol. 46, no. 5, pp. 953–958, 2010.
[32] Z.-G. Wu, P. Shi, H. Su and J. Chu, "Asynchronous l_2-l_∞ filtering for discrete-time stochastic Markov jump systems with randomly occurred sensor nonlinearities," *Automatica*, vol. 50, no. 1, pp. 180–186, 2014.
[33] Z.-G. Wu, P. Shi, Z. Shu, H. Su and R. Lu, "Passivity-based asynchronous control for Markov jump systems," *IEEE Transactions on Automatic Control*, vol. 62, no. 4, pp. 2020–2025, 2017.
[34] Z.-G. Wu, Y. Shen, P. Shi, Z. Shu, and H. Su, "H_∞ control for 2D Markov jump systems in Roesser model," *IEEE Transactions on Automatic Control*, vol. 64, no. 1, pp. 427–432, 2019.
[35] Y. Shen, Z.-G. Wu, P. Shi, Z. Shu, and H. R. Karimi, "H_∞ control of Markov jump time-delay systems under asynchronous controller and quantizer," *Automatica*, vol. 99, pp. 352–360, 2019.
[36] M. D. Fragoso and O. L. V. Costa, "Mean square stabilizability of continuous-time linear systems with partial information on the Markovian jumping parameters," *Stochastic Analysis and Applications*, vol. 22, no. 1, pp. 99–111, 2004.
[37] O. L. V. Costa, M. D. Fragoso, and M. G. Todorov, "A detector-based approach for the H_2 control of Markov jump linear systems with partial information," *IEEE Transactions on Automatic Control*, vol. 65, no. 5, pp. 1219–1234, 2015.
[38] A. M. Oliveira, and O. L. V. Costa, "Mixed H_2/H_∞ control of hidden Markov jump systems," *International Journal of Robust and Nonliear Control*, vol. 28, pp. 1261–1280, 2017.
[39] M. G. Todorov, M. D. Fragoso, and O. L. V. Costa, "Detector-based H_∞ results for discrete-time Markov jump linear systems with partial observations," *Automatica*, vol. 91, pp. 159–172, 2018.
[40] M. Vidyasagar, *Hidden Markov processes: theory and applications to biology*, Princeton: Princeton University Press, 2014.
[41] F. V. Verges and M. D. Fragoso, "Optimal linear mean square filter for the operation mode of continuous-time Markovian jump linear systems," *In 56th IEEE Conference on Decision and Control*, Melbourne, Australia, pp. 5876–5881, 2017.