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$\mathcal{H}_\infty$ Filtering for Markovian Jump Linear Systems*

Carlos E. de Souza

Department of Electrical and Computer Engineering
The University of Newcastle
NSW 2308, Australia
email: eeced@cc.newcastle.edu.au

Marcelo D. Fragoso

National Laboratory for Scientific Computing
Rua Lauro Müller 455
22290-160 Rio de Janeiro, RJ, Brazil
email: frag@server01.lncc.br

ABSTRACT

The problem of $\mathcal{H}_\infty$ filtering for continuous-time linear systems with Markovian jumps is investigated. It is assumed that the jumping parameter is available. We propose a methodology for designing Markovian jump linear filters which ensure a prescribed bound on the $\mathcal{L}_2$-induced gain from the noise signals to the estimation error. The main result is tailored via linear matrix inequalities.

1. INTRODUCTION

It is a well known fact that in a great variety of stochastic modelling problems, it is very difficult to know *precisely* the *statistics* of the additive noise actuating in the system. This is a particularly important issue when we are dealing with what is known in the specialized literature as the *filtering problem.* One way to deal with this issue is to use a nowadays very popular measure of performance, the $\mathcal{H}_\infty$-norm, which has been introduced in the robust control setting (see, e.g., [6], and the references therein). In this context, the filtering problem is known in the literature as the $\mathcal{H}_\infty$ *filtering problem*, and its success can be confirmed, in part, by the amount of available literature on this subject; see, for example, [1], [2], [10], [14]-[16], [20], [21] and the references therein. Roughly speaking, in the $\mathcal{H}_\infty$ filtering approach the noise sources one considers are arbitrary signals with bounded energy, or bounded average power, and the estimator is designed to guarantee that the $\mathcal{L}_2$-induced gain from the noise signals to the estimation error be less than a certain prescribed level.

The subject matter of this paper is to study the problem of $\mathcal{H}_\infty$ filtering for a class of linear continuous-time systems whose structures are subject to abrupt parameters changes (jumps), modelled here via a continuous-time finite-state Markov chain (it is also known in the literature as the class of Markovian jump linear systems). These changes may be a consequence of random component failures or repairs, abrupt environmental disturbances, changes in the operating point of a nonlinear plant, etc. This can be found, for instance, in control of solar thermal central receivers, robotic manipulator systems, aircraft control systems, large flexible structures for space stations (such as antenna, solar arrays), etc. Several authors have analyzed different aspects of such a class and some successful applications have, in part, spurred a considerable interest on it (see, e.g., [3]-[5], [8], [12], [13], [17]-[19], [22] and the references therein).

*This work was carried out while the first author was visiting the National Laboratory for Scientific Computing, LNCC/CNPq, Rio de Janeiro, Brazil.

In particular with regard to the filtering problem, minimum variance filtering schemes for discrete-time systems have been studied in, for instance, [3], [4], [18] and [22]. To the best of the authors's knowledge, to date the problem of $\mathcal{H}_\infty$ filtering for this class of systems has not yet been addressed.

The problem we consider in this paper is the design of a Markovian jump linear filter for the above class of Markovian jump linear systems, which provides a mean square stable error dynamics and a prescribed bound on the $\mathcal{L}_2$-induced gain from the noise signals to the estimation error. A linear matrix inequality (LMI) approach is proposed for solving this $\mathcal{H}_\infty$ filtering problem.

Notation. Throughout the paper the superscript 'T' stands for matrix transposition, $\Re^n$ denotes the n dimensional Euclidean space, $\Re^{n\times m}$ is the set of all $n \times m$ real matrices, $\|\cdot\|$ denotes the induced matrix 2-norm, and $\mathcal{L}_2$ stands for the space of square integrable vector functions over $[0,\infty)$. For a real matrix P, $P>0$ (respectively, $P<0$), means that P is symmetric and positive definite (respectively, negative definite).

2. PROBLEM FORMULATION

Fix a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$ and consider the following class of stochastic systems:

$$\dot{x}(t) = A(\theta_t)x(t) + B(\theta_t)w(t); \quad x(0) = x_0, \quad \theta_0 = i_0 \qquad (1)$$

$$y(t) = C(\theta_t)x(t) + D(\theta_t)w(t) \qquad (2)$$

$$z(t) = L(\theta_t)x(t) \qquad (3)$$

where $x(t) \in \Re^n$ is the state, $x_0 \in \Re^n$ is an unknown initial state, $w(t) \in \Re^q$ is the noise signal which is assumed to be an arbitrary signal in $\mathcal{L}_2$, $y(t) \in \Re^m$ is the measurement, and $z(t) \in \Re^p$ is the signal to be estimated. $\{\theta_t\}$ is a homogeneous Markov process with right continuous trajectories and taking values on the finite set $\phi = \{1, 2, \ldots, N\}$ with stationary transition probabilities:

$$\mathrm{Prob}\{\theta_{t+h} = j \,|\, \theta_t = i\} = \begin{cases} \lambda_{ij}h + o(h), & i \neq j \\ 1 + \lambda_{ii}h + o(h), & i = j \end{cases}$$

where $h > 0$ and $\lambda_{ij} \geq 0$ is the transition rate from the state i to j, $i \neq j$, and

$$\lambda_{ii} = -\sum_{\substack{j=1 \\ j\neq i}}^{N} \lambda_{ij}. \qquad (4)$$

The set ϕ comprises the various operation modes of system (1)-(3) and for each possible value of $\theta_t = i$, $i \in \phi$, we will denote the matrices associated with the "i-th mode" by:

$$A_i \triangleq A(\theta_t = i), \quad B_i \triangleq B(\theta_t = i), \quad C_i \triangleq C(\theta_t = i),$$
$$D_i \triangleq D(\theta_t = i), \quad L_i \triangleq L(\theta_t = i)$$

where A_i, B_i, C_i, D_i and L_i are constant matrices for any $i \in \phi$.

It is assumed that the jumping process, $\{\theta_t\}$, is accessible, i.e. the operation mode of system (1)-(3) is known for every $t \geq 0$.

We are concerned with obtaining an estimate, $\hat{z}(t)$, of $z(t)$, via a causal Markovian jump linear filter using the measurement y and which provides a uniformly small estimation error, $z - \hat{z}$, for all $w \in \mathcal{L}_2$.

In order to put the $\mathcal{H}_\infty$ filtering problem for system (1)-(3) in a stochastic setting, we introduce the space $\mathcal{L}_2[\Omega, \mathcal{F}, \mathcal{P}]$ of $\mathcal{F}$-measurable processes, $z(t) - \hat{z}(t)$, for which

$$\|z - \hat{z}\|_2 \triangleq \left\{ E\left[\int_0^\infty [z(t) - \hat{z}(t)]^T [z(t) - \hat{z}(t)] dt \right] \right\}^{\frac{1}{2}} < \infty$$

where $E[\cdot]$ stands for the mathematical expectation. For the sake of notation simplification, we shall use indistinctly $\|\cdot\|_2$ to denote the norm either in $\mathcal{L}_2[\Omega, \mathcal{F}, \mathcal{P}]$ or in $\mathcal{L}_2$, defined by

$$\|w\|_2 \triangleq \left[\int_0^\infty w^T(t) w(t) dt \right]^{\frac{1}{2}}, \quad \text{for } w \in \mathcal{L}_2.$$

Before formulating the $\mathcal{H}_\infty$ filtering problem, we recall the notion of internal mean square stability.

Definition 2.1 System (1) is said to be *internally mean square stable*, if the solution of

$$\dot{x}(t) = A(\theta_t) x(t)$$

is such that $E(\|x(t)\|^2) \to 0$, as $t \to \infty$ for arbitrary initial condition (x_0, θ_0). □

The filtering problem we address in this paper is as follows:

Given an a priori estimate, $\hat{x}_0$, of the initial state, x_0, design a Markovian jump linear filter that provides an estimate, $\hat{z}(t)$, of $z(t)$ based on $\{y(\tau),\ 0 \leq \tau \leq t\}$ and $\{\theta_\tau,\ 0 \leq \tau \leq t\}$ such that the estimation error system is internally mean square stable and for all $w \in \mathcal{L}_2$ and $x_0 \in \Re^n$:

$$\|z - \hat{z}\|_2 \leq \gamma \left[\|w\|_2^2 + (x_0 - \hat{x}_0)^T R (x_0 - \hat{x}_0) \right]^{\frac{1}{2}} \tag{5}$$

where $R > 0$ is a given weighting matrix for the initial state estimation error, and $\gamma \geq 0$ is a given scalar which specifies the level of "noise" attenuation in the estimation error.

The weighting matrix R is a measure of the degree of confidence in the estimate $\hat{x}_0$ relative to the uncertainty in w. A 'large' value of R indicates that $\hat{x}_0$ is very close to x_0.

In the case of filtering problems where the effect of the initial state is ignored, without loss of generality, x_0 and $\hat{x}_0$ can be set to zero and (5) is replaced by

$$\|z - \hat{z}\|_2 \leq \gamma \|w\|_2. \tag{6}$$

Attention will be focused on the design of an n-th order filter. Since the matrices in system (1)-(3) are known at time t (as θ_t is available) and it is required that in the absence of w,

$$E[\|x(t) - \hat{x}(t)\|] \to 0 \quad \text{as} \quad t \to \infty$$

where $\hat{x}(t)$ is the state of the filter, irrespective of the internal mean square stability of (1), without loss of generality, the following structure for the Markovian jump linear filter will be adopted:

$$\dot{\hat{x}}(t) = A(\theta_t)\hat{x}(t) + K(\theta_t)[y(t) - C(\theta_t)\hat{x}(t)], \quad \hat{x}(0) = \hat{x}_0 \tag{7}$$
$$\hat{z}(t) = L(\theta_t)\hat{x}(t) \tag{8}$$

where the filter gain matrix $K(\theta_t)$ is to be determined.

Remark 2.1 We observe that no "non-singularity assumption", namely

$$D(\theta_t) D^T(\theta_t) > 0, \quad \forall \theta_t \in \phi, \quad \text{and} \quad \forall t \geq 0$$

is imposed to the filtering problem treated in this paper. This is in contrast with the $\mathcal{H}_\infty$ filtering approaches in [14], [15] and [21] for linear systems without jumps. □

3. THE $\mathcal{H}_\infty$ MARKOVIAN JUMP FILTER

Before presenting our filter design, the motivation for the approach we adopted will be discussed. First, recall that since θ_t is accessible, the system (1)-(3) could be seen as a linear time-varying system defined by matrices $A(t), B(t), C(t), D(t)$ and $L(t)$ which can be computed online based on the state of the process $\{\theta_t\}$ at time t. Indeed

$$A(t) = A_i, \quad B(t) = B_i, \quad C(t) = C_i, \quad D(t) = D_i, \quad L(t) = L_i$$

when $\theta(t) = i$. This implies that well known results on $\mathcal{H}_\infty$ filtering for linear time-varying systems, such as those in [14], could be used to solve the $\mathcal{H}_\infty$ filtering problem for system (1)-(3). Subject to the assumption that

$$S(t) \triangleq D(t) D^T(t) > 0, \quad \forall t \geq 0,$$

it follows from the results in [14] that this $\mathcal{H}_\infty$ filtering problem is solvable via a causal linear filter if and only if there exists a bounded symmetric positive definite solution $P(t)$ over $[0, \infty)$ to the Riccati differential equation:

$$\begin{aligned}\dot{P} = &(A - BD^T S^{-1} C)P + P(A - BD^T S^{-1} C)^T + P(\gamma^{-2} L^T L \\ &- C^T S^{-1} C)P + B(I - D^T S^{-1} D)B^T; \quad P(0) = R^{-1}\end{aligned} \tag{9}$$

such that the time-varying system

$$\dot{\eta}(t) = \left[A - (PC^T + BD^T)S^{-1}C + \gamma^{-2}PL^T L\right]\eta(t) \tag{10}$$

is exponentially stable. Note that, for simplicity of notation, we omitted the time-dependence in the matrices of (9) and (10).

Further, a suitable filter is of the form:

$$\dot{\hat{x}}(t) = A(t)\hat{x}(t) + K(t)[y(t) - C(t)\hat{x}(t)]; \quad \hat{x}(0) = \hat{x}_0 \tag{11}$$

$$\hat{z}(t) = L(t)\hat{x}(t) \tag{12}$$

where

$$K(t) = [P(t)C(t) + B(t)D^T(t)]S^{-1}(t). \tag{13}$$

Note that in view of the above, the filter gain, $K(t)$, of (13) will not be constant while system (1)-(3) remains in a certain operation mode, i.e. the jumping process remains in a certain state, $j \in \phi$. Thus, the filter of (11)-(12) is not a Markovian jump linear system, which is highly undesirable, as the underlying system (1)-(3) is a Markovian jump linear system. Another undesirable feature of the above filter is that since the matrices of system (1)-(3) are not available *a priori*, the existence of a bounded solution to the Riccati differential equation (9) with the required stability property, i.e. the existence of a filter, cannot be ascertained off-line. Also note that the computation of the filter gain, $K(t)$, if it exists, can only be carried out on-line and would involve the solution of a quadratic matrix differential equation with time-varying matrix coefficients, which is numerically unattractive. The filter design proposed in this paper will not exhibit the above undesirable features.

The approach developed here to solve the $\mathcal{H}_\infty$ filtering problem for system (1)-(3) has the following features:

(a) The design leads to a Markovian jump linear filter;

(b) The existence of a filter can be ascertained off-line;

(c) The filter gain, $K(\theta_t)$, can assume N possible values, $K_i = K(\theta_t)$ when $\theta_t = i$, and the matrices $K_i, \forall\, i \in \phi$, can be calculated off-line.

The following theorem presents a solution to the $\mathcal{H}_\infty$ filtering problem for the Markovian jumping linear system (1)-(3).

Theorem 3.1 Consider the system (1)-(3) and let $\gamma \geq 0$ be a given scalar. Let $\hat{x}_0$ be an a-priori estimate of the initial state and $R > 0$ a given initial state error weighting matrix. Then there exists a Markovian jump filter of the form of (7)-(8) such that the estimation error system is internally mean square stable and

$$\|z - \hat{z}\|_2 \leq \gamma\left[\|w\|_2^2 + (x_0 - \hat{x}_0)^T R (x_0 - \hat{x}_0)\right]^{\frac{1}{2}}$$

for all $w \in \mathcal{L}_2$ and $x_0 \in \Re^n$, if for all $i \in \phi$ there exist matrices $X_i > 0$ and Y_i satisfying the following LMIs:

$$\begin{bmatrix} M_i & X_iB_i - Y_iD_i \\ B_i^TX_i - D_i^TY_i^T & -\gamma^2 I \end{bmatrix} < 0, \quad \forall\, i \in \phi \tag{14}$$

$$X_{i_0} - \gamma^2 R \leq 0 \tag{15}$$

where

$$M_i = A_i^TX_i + X_iA_i - C_i^TY_i^T - Y_iC_i + \sum_{j=1}^{N} \lambda_{ij}X_j + L_i^TL_i$$

and i_0 is the state assumed by $\{\theta_t\}$ at $t = 0$. Moreover, a suitable filter is given by

$$\dot{\hat{x}}(t) = A_i\hat{x}(t) + K_i\left[y(t) - C_i\hat{x}(t)\right]; \quad \hat{x}(0) = \hat{x}_0 \tag{16}$$

$$\hat{z}(t) = L_i\hat{x}(t) \tag{17}$$

for $\theta_t = i$, $i \in \phi$, where

$$K_i = K(\theta_t = i) = X_i^{-1}Y_i. \tag{18}$$

Proof. First note that by defining $\tilde{x} \triangleq x - \hat{x}$ and considering (1)-(3) and (16)-(17), it follows that a state-space model for the estimation error, $z - \hat{z}$, is given by

$$\dot{\tilde{x}}(t) = \tilde{A}(\theta_t)\tilde{x}(t) + \tilde{B}(\theta_t)w(t); \quad \tilde{x}(0) = x_0 - \hat{x}_0 \tag{19}$$

$$z(t) - \hat{z}(t) = L(\theta_t)\tilde{x}(t) \tag{20}$$

where

$$\tilde{A}(\theta_t) = A(\theta_t) - K(\theta_t)C(\theta_t), \quad \tilde{B}(\theta_t) = B(\theta_t) - K(\theta_t)D(\theta_t). \tag{21}$$

In view of (14) and taking into account (18), we have that for all $i \in \phi$

$$(A_i - K_iC_i)^T X_i + X_i(A_i - K_iC_i) + \sum_{j=1}^{N} \lambda_{ij}X_j < 0.$$

By Theorem 3.1 and Proposition 3.5 in [7], this implies that the error system of (19) is internally mean square stable.

Now for $T > 0$, define the following cost function:

$$J(T) \triangleq E\left\{\int_0^T \left[(z - \hat{z})^T(z - \hat{z}) - \gamma^2 w^T w\right] dt\right\}. \tag{22}$$

Note that by Proposition A.1 in [5], $\{\tilde{x}(t),\ \theta_t\}$ is a Markov process with infinitesimal operator given by

$$\tilde{V}\cdot g(\tilde{x}(t), \theta_t) = g_t(\tilde{x}(t), \theta_t) + \left[\tilde{A}(\theta_t)\tilde{x} + \tilde{B}(\theta_t)w(t)\right]^T \cdot g_{\tilde{x}}(\tilde{x}(t), \theta_t) + \sum_{j=1}^{N} \lambda_{\theta_t j}\, g(\tilde{x}(t), j) \tag{23}$$

where $g(\tilde{x}(t), \theta_t)$ is a real continuous, bounded, functional with partial derivatives:

$$g_t \triangleq \frac{\partial g}{\partial t} \quad \text{and} \quad g_{\tilde{x}} \triangleq \left[\frac{\partial g}{\partial \tilde{x}_1}, \frac{\partial g}{\partial \tilde{x}_2}, \cdots, \frac{\partial g}{\partial \tilde{x}_n}\right]^T$$

where $\tilde{x}_j$ denotes the j-th component of $\tilde{x}$.

Adding and subtracting $\tilde{V}\cdot[\tilde{x}^T(t)X(\theta_t)\tilde{x}(t)]$ to (22), where $X(\theta_t) = X^T(\theta_t) > 0$, $\forall\, \theta_t \in \phi$, and considering (20), (21) and (23), we have:

$$J(T) = E\left\{\int_0^T \{\tilde{x}^T(t)\left[(A(\theta_t) - K(\theta_t)C(\theta_t))^T X(\theta_t)\right.\right.$$
$$+ X(\theta_t)(A(\theta_t) - K(\theta_t)C(\theta_t)) + L^T(\theta_t)L(\theta_t)$$
$$+ \sum_{j=1}^{N} \lambda_{\theta_t j} X_j]\tilde{x}(t) + \tilde{x}^T(t)X(\theta_t)[B(\theta_t) - K(\theta_t)D(\theta_t)]w(t)$$
$$+ w^T(t)[B(\theta_t) - K(\theta_t)D(\theta_t)]^T X(\theta_t)\tilde{x}(t) - \gamma^2 w^T(t)w(t)$$
$$\left. - \tilde{\mathcal{V}}\cdot[\tilde{x}^T(t)X(\theta_t)\tilde{x}(t)]\} \, dt\right\}. \tag{24}$$

It was shown in [9] that, since the system of (19) is internally mean square stable and $w \in \mathcal{L}_2$, it follows that

$$\lim_{T\to\infty} E\left[\tilde{x}^T(T)X(\theta_T)\tilde{x}(T)\right] = 0. \tag{25}$$

This implies that $J(T)$ of (22) is well defined as $T \to \infty$.

Using Dynkin's formula (see, e.g., [11]) together with (25) and defining

$$Y(\theta_t) = X(\theta_t)K(\theta_t), \quad \forall\, \theta_t \in \phi \tag{26}$$

it results from (24) that

$$||z - \hat{z}||_2^2 - \gamma^2\left[||w||_2^2 + \tilde{x}^T(0)R\tilde{x}(0)\right] =$$
$$E\left\{\int_0^\infty [\tilde{x}^T \;\; w^T]\,\Psi(\theta_t)\begin{bmatrix}\tilde{x}\\ w\end{bmatrix} dt\right\} + \tilde{x}^T(0)\left(X_{i_0} - \gamma^2 R\right)\tilde{x}(0) \tag{27}$$

where

$$\Psi(\theta_t) = \begin{bmatrix} M(\theta_t) & N(\theta_t) \\ N^T(\theta_t) & -\gamma^2 I \end{bmatrix}$$

and

$$M(\theta_t) = A^T(\theta_t)X(\theta_t) + X(\theta_t)A(\theta_t) - C^T(\theta_t)Y^T(\theta_t)$$
$$- Y(\theta_t)C(\theta_t) + \sum_{j=1}^{N} \lambda_{\theta_t j} X_j + L^T(\theta_t)L(\theta_t).$$

$$N(\theta_t) = X(\theta_t)B(\theta_t) - Y(\theta_t)D(\theta_t)$$

Finally, considering the inequalities (14) and (15), the desired result follows from (26) and (27). $\nabla\nabla\nabla$

Remark 3.1 Theorem 3.1 provides a method for designing $\mathcal{H}_\infty$ Markovian jump linear filters for linear systems subject to Markovian jumping parameters. The proposed design is given in terms of linear matrix inequalities, which has the advantage that it can be solved numerically very efficiently using recently developed algorithms for solving LMIs.

We observe that the problem of designing an optimal $\mathcal{H}_\infty$ Markovian jump linear filter, i.e. for the smallest possible $\gamma \geq 0$, can be easily solved via the following linear programming problem in γ, X_i and Y_i, $\forall\, i \in \phi$:

$$\begin{array}{ll} \text{minimize} & \gamma \\ \text{subject to} & \gamma \geq 0, \;\; X_i > 0, \;\; (14) \text{ and } (15). \end{array}$$ □

When the effect of the initial state is ignored, without loss of generality, x_0 and $\hat{x}_0$ can be set to zero. Thus, the inequality of (15) will no longer be required as this case corresponds to choosing a sufficient large R (in the sense that its smallest eigenvalue approaches infinity). In such situation, Theorem 3.1 specializes as follows:

Corollary 3.1 Consider the system (1)-(3) with $x(0) = 0$ and let $\gamma \geq 0$ be a given scalar. Then there exists a Markovian jump filter of the form of (7)-(8) such that the estimation error system is internally mean square stable and

$$\|z - \hat{z}\|_2 \leq \gamma\|w\|_2$$

for all $w \in \mathcal{L}_2$, if for all $i \in \phi$ there exist matrices $X_i > 0$ and Y_i such that the LMI (14) is satisfied. Moreover, a suitable filter is given by (16)-(18) with $\hat{x}_0 = 0$. □

In the case of one mode operation, i.e. there no jumps in system (1)-(3), we have $N = 1$, $\phi = \{1\}$ and $\lambda_{11} = 0$. Denoting the matrices of system (1)-(3) by A, B, C, D and L, Theorem 3.1 reduces to the following result:

Corollary 3.2 Consider the system (1)-(3) with no jumps and let $R > 0$ be a given initial state weighting matrix and $\gamma > 0$ a given scalar. Then there exists a causal linear filter such that the estimation error dynamics is asymptotically stable and

$$\|z - \hat{z}\|_2 \leq \gamma\left(\|w\|_2 + x_0^T R x_0\right)^{\frac{1}{2}}$$

for all $w \in \mathcal{L}_2$ and $x_0 \in \Re^n$, if there exist matrices $X > 0$ and Y satisfying the following LMIs:

$$\begin{bmatrix} A^T X + XA - C^T Y^T - YC & XB - YD \\ B^T X - D^T Y^T & -\gamma^2 I \end{bmatrix} < 0 \tag{28}$$

$$X - \gamma^2 R \leq 0 \tag{29}$$

Moreover, a suitable filter is given by

$$\dot{\hat{x}}(t) = A\hat{x}(t) + K\left[y(t) - C\hat{x}(t)\right]; \qquad \hat{x}(0) = 0$$
$$\hat{z}(t) = L\hat{x}(t)$$

where

$$K = X^{-1}Y.$$

Remark 3.2 Corollary 3.2 provide a methodology for designing $\mathcal{H}_\infty$ filters for linear systems in terms of linear matrix inequalities. This is in contrast with the approaches developed in [14], [15] and [21] which are based on algebraic Riccati equations and are also restricted to "non-singular" $\mathcal{H}_\infty$ filtering problems. □

4. CONCLUSION

In this paper we have addressed the problem of $\mathcal{H}_\infty$ filtering for a class of continuous-time Markovian jump linear systems. We have derived a methodology for designing $\mathcal{H}_\infty$ Markovian jump linear filters. The filter design is given in terms of linear matrix inequalities.

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