ISA Transactions 63 (2016) 204–210

Contents lists available at ScienceDirect

ISA Transactions

journal homepage: www.elsevier.com/locate/isatrans

H_∞ filtering of Markov jump linear systems with general transition probabilities and output quantization

Mouquan Shen [a,b], Ju H. Park [c,*]

[a] College of Electrical Engineering and Control Science, Nanjing Technology University, Nanjing, 211816, China
[b] Key Laboratory of Measurement and Control of Complex Systems of Engineering, Ministry of Education, Southeast University, Nanjing 210096, China
[c] Department of Electrical Engineering, Yeungnam University, Kyonsan 38541, Korea

ARTICLE INFO

Article history:
Received 12 January 2016
Received in revised form
18 March 2016
Accepted 8 April 2016
Available online 26 April 2016
This paper was recommended for publication by Dr. Jeff Pieper

Keywords:
Markov jump system
H_∞ filtering
Output quantization

ABSTRACT

This paper addresses the H_∞ filtering of continuous Markov jump linear systems with general transition probabilities and output quantization. S-procedure is employed to handle the adverse influence of the quantization and a new approach is developed to conquer the nonlinearity induced by uncertain and unknown transition probabilities. Then, sufficient conditions are presented to ensure the filtering error system to be stochastically stable with the prescribed performance requirement. Without specified structure imposed on introduced slack variables, a flexible filter design method is established in terms of linear matrix inequalities. The effectiveness of the proposed method is validated by a numerical example.

1. Introduction

Markov jump linear systems (MJLSs) belong to the category of stochastic systems and are able to present practical plants of the multi-mode nature [1]. Since this kind of system has widespread applications in the fields of robotics, mathematical finance, flight systems, etc, a plenty of well documented studies on MJLSs have been reported in [1–14] and references therein. It is to note that transition probabilities (TPs) in most of referred results are presumed to be known. However, this hypothesis is not realistic in engineering situation. For example, in networked control systems (NCSs), it is common to model networked induced time-delays or packet dropouts as Markov chain [15]. Since the induced delays or the packet dropouts are obscure and random in different running periods, it leads that TPs are hard or costly to obtain. In line with this situation, [16] presents a robust controller design method for MJLSs with uncertain TPs formulated by norm-bounded type. Sufficient conditions are given in [17] to calculate the possible perturbation bounds of uncertain TPs. Unfortunately, the structure or nominal terms of the uncertain TPs are required to be set in advance. To patch up this deficiency, [18] proposes a different approach where TPs are allowed to be known and unknown. Based on this approach, the free-connection weighting matrix technique is adopted by [19] to get less conservative conditions for stability analysis. Finite-time boundedness filtering of discrete-time MJLSs subject to partly unknown TPs is discussed in [20]. Nevertheless, uncertain with known bounds case is treated as completely unknown [18–20]. To shorten this gap, new analysis and synthesis conditions are provided in [21]. Once uncertain TPs has variation [16], no effective measurement has been supplied in the existing results.

As mentioned above, in NCSs, Markov chain is utilized to model sensor-to-controller and controller-to-actuator delays and the resulting closed-loop system is transformed to MJLSs [15,22,23]. On the other hand, sensor signals in the network environment are transmitted in digital frame, namely, system outputs are always quantized before they are sent out. It is known that the quantization may deteriorate system performance or even make systems unstable [24–27]. Consequently, the study on the MJLSs with quantized consideration is an important and valuable problem [24,27]. Specifically, the problem of filter design for uncertain stochastic systems with logarithmic quantized output is studied in [24]. Ref. [27] concerns the quantized dynamic output controller design for a class of semi-Markovian jump systems with repeated scalar nonlinearities. However, TPs in these results are required to be known beforehand.

On another research front line, H_∞ filtering has been recognized as a powerful way to estimate system states once the energy of the external noise is bounded [13]. Regarding to the H_∞ filtering of MJLSs, asynchronous filtering of discrete-time stochastic Markov

* Corresponding author.
E-mail addresses: mouquanshen@gmail.com (M. Shen), jessie@ynu.ac.kr (J.H. Park).

http://dx.doi.org/10.1016/j.isatra.2016.04.007
0019-0578/

jump systems with sensor nonlinearity is concerned in [28]. Ref. [29] investigates the resilient asynchronous H_∞ filtering for Markov jump neural networks with unideal measurements and multiplicative noises. Adopting the input-output technique, the delay-dependent H_∞ filter design method for a class of continuous-time MJLSs with time-varying delay is demonstrated in [30]. Mode-dependent H_∞ filtering treatments for discrete-time MJLSs with partly known TPs is developed in [18,20]. Although accessible methods have been proposed in these results in terms of LMIs, specified structures are imposed on the introduced slack variables related to filter parameters.

Inspired by the above observations, the H_∞ filtering of Markov jump linear systems with general TPs and output quantization is delivered in this paper. The filter input is quantized by a logarithmic quantizer which depends on system mode and TPs are allowed to be known, uncertain with variations and completely unknown. To get a better system performance, the nonlinearity induced by quantization is tackled by S-procedure. By making full use of the transition probability matrix property, new techniques are employed to deal with the nonlinearity induced by uncertain and unknown TPs. Based on these effective strategies and Finsler lemma, sufficient conditions are established to guarantee the filtering error system to be stochastically stable with the prescribed H_∞ performance. Contrast to the existing results where the structures of the induced slack variables are specified, a flexible method to H_∞ filter design is obtained in the framework of linear matrix inequalities (LMIs). Moreover, the established result includes the existing one as a special case. A numerical example is given to show the effectiveness of the proposed method. Therefore, the highlights of this paper are summarized as below

- The general transition probabilities cover known, uncertain with variations and unknown.
- Some new measurements are developed to deal with the nonlinearities induced by quantization and unknown transition probabilities.
- Without specified the structures of introduced slack variables, a flexible filter design method is established in terms of LMIs.

The organization of this paper is as follows. The system model and the quantized filtering problem are stated in Section 2. A flexible approach to the desired filter design is established in Section 3. In Section 4, the validity of the proposed approach is illustrated by a numerical example. Lastly, Section 5 concludes this paper.

Notation: Throughout this paper, M^T represents the transpose of matrix M. The notation $X \le Y$ ($X < Y$) where X and Y are symmetric matrices, means that $X - Y$ is negative semi-definite (negative definite) respectively. $\mathcal{E}$ is a mathematical expectation operator. I and 0 represent identity matrix and zero matrix, respectively. $\mathcal{L}_2$ denotes the space of square integrable vector functions of a given dimension over $[0, \infty)$, with norm $\mathcal{E}\{|x|_2^2\} = \mathcal{E}\left\{\int_0^\infty x(t)^T x(t)\, dt\right\} < \infty$. $\star$ denotes the entries of matrices implied by symmetry. Finally, the symbol $He(X)$ is used to represent $(X + X^T)$.

2. Preliminaries

Consider the following physical plant represented by continuous MJLS

$$\begin{cases} \dot{x}(t) = A(\eta(t))x(t) + B(\eta(t))w(t) \\ z(t) = C_1(\eta(t))x(t) + D_1(\eta(t))w(t) \\ y(t) = C_2(\eta(t))x(t) + D_2(\eta(t))w(t) \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the system state; $w(t) \in R^q$ is the noise signal that is assumed to be the arbitrary signal in $\mathcal{L}_2[0\ \infty)$; $z(t) \in R^m$ is the signal to be estimated; $y(t) \in R^p$ is the measurement output. The random form process $\{\eta(t)\}$ is a continuous-time discrete-state Markov process taking values in a finite set $\mathcal{I} = \{1, 2, \dots, s\}$. The transition probability $\Pi = [\pi_{ij}]_{i,j \in \mathcal{I}}$ satisfies

$$\mathbb{P}\{\eta(t+\Delta t) = i \,|\, \eta(t) = j\} = \begin{cases} \pi_{ij}\Delta t + o(\Delta t) & \text{if } j \ne i \\ 1 + \pi_{ii}\Delta t + o(\Delta t) & \text{if } j = i \end{cases}$$

where $\Delta t > 0$, $\lim_{\Delta t \to 0}(o(\Delta t)/\Delta t) = 0$, $\mathbb{P}\{\cdot\}$ is the probability and $\pi_{ij} \ge 0$ for $i \ne j$, $\pi_{ii} = -\sum_{l=1, l \ne k}^{s} \pi_{lj}$.

In this paper, the TPs are known, uncertain and unknown. To see the considered TPs clearly, the TPs for system (1) with four operation modes may be

$$\begin{bmatrix} \pi_{11} + \Delta_{11} & ? & \pi_{13} & ? \\ ? & ? & \pi_{23} & \pi_{24} \\ \pi_{31} & \pi_{32} + \Delta_{32} & \pi_{33} + \Delta_{33} & \pi_{34} + \Delta_{34} \\ \pi_{41} & \pi_{42} + \Delta_{42} & ? & \pi_{44} + \Delta_{44} \end{bmatrix} \tag{2}$$

where "?", π_{ij} and Δ_{ij} mean that the corresponding elements are unknown, known and uncertain. Moreover, Δ_{ij} satisfies $\Delta_{ij} \in [-\delta_{ij}\ \delta_{ij}]$ where δ_{ij} is known. Subsequently, for brief presentation, let $\overline{\pi}_{ij} = \pi_{ij} + \Delta_{ij}$ and $\hat{\pi}_{ij}$ include all possible cases (known, uncertain and unknown) of TPs in ith row. Since the boundary information of uncertain TPs is accessible, the following descriptions are employed to distinguish the availability of TPs:

$$\mathcal{I}_k^i = \{j : \pi_{ij} \textit{ is known or uncertain}\},$$

$$\mathcal{I}_{uk}^i = \{j : \pi_{ij} \textit{ is unknown}\}$$

When the system transits to the ith mode, namely, $\eta(t) = i$, the corresponding system matrices are denoted as A_i, B_i, C_{1i}, C_{2i}, D_{1i}, D_{2i}.

To estimate $z(t)$, the following mode-dependent filter with quantization is adopted

$$\begin{cases} \dot{x}_f(t) = A_{fi}x_f(t) + B_{fi}q_i(y(t)) \\ z_{qf}(t) = C_{fi}x_f(t) + D_{fi}q_i(y(t)) \end{cases} \tag{3}$$

where A_{fi}, B_{fi}, C_{fi}, and D_{fi} are filter parameters to be designed. $q_i(y(t))$ is the quantized measurement output of $y(t)$ by a mode-dependent logarithmic quantizer. It is denoted as

$$q_i(y(t)) = \left[q_i^1(y(t)), q_i^2(y(t)), \dots, q_i^p(y(t))\right], \quad i \in \mathcal{I}$$

and satisfies

$$q_i^c(y_c(t)) = -q_i^c(-y_c(t)), \quad c = 1, \dots, p$$

For each $i \in \mathcal{I}$, the quantized level set of $q_i^c(\bullet)$ is presented by

$$\mathcal{U}_c = \left\{ \pm U_d^{(i,c)}, U_d^{(i,c)} = \{\rho_i^c\}^d U_0^{(i,c)}, d = \pm 1, \cdots \right\} \cup \left\{ \pm U_0^{(i,c)} \right\} \cup \{0\}, \rho \in (0,1), \eta_0^{(i,c)} > 0$$

where $\rho_i{}^c$ is the quantizer density of the subquantizer $q_i^c(\bullet)$. At mode i, each of the quantization level corresponds to a segment such that the quantizer maps the whole segment to this quantization level. The associated quantizer $q_i^c(\bullet)$ for mode i is defined as

$$q_i^c(y_c(t)) = \begin{cases} U_d^{(i,c)}, & y_c(t) \in \left(\dfrac{U_d^{(i,c)}}{1+\sigma_i^c}, \dfrac{U_d^{(i,c)}}{1-\sigma_i^c} \right) \\ 0, & y_c(t) = 0 \\ -q_i^c(-y_c(t)), & y_c(t) < 0 \end{cases} \tag{4}$$

where $\sigma_i^c = \frac{1-\rho_i^c}{1+\rho_i^c}$.

To see the filter structure clearly, a diagram is given in Fig. 1 to show this point.

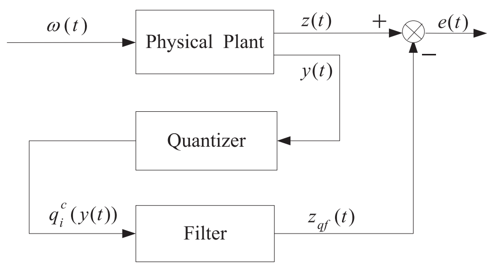

Fig. 1. The quantized filter structure.

Similar to [24], indicating $\Theta_i = \text{diag}\{\sigma_i^1, \ldots, \sigma_i^p\}$, $q_i(y(t))$ is decomposed as

$$q_i(y(t)) = K_i y(t) + q_i^h(y(t)) \tag{5}$$

and

$$(q_i^h(y(t)))^T (q_i^h(y(t)) - 2\Theta_i y(t)) \le 0 \tag{6}$$

where $K_i = I_p - \Theta_i$.

Combining (1), (3) and (5), one has the filtering error system

$$\begin{cases} \dot{\xi}(t) = \overline{A}_i \xi(t) + \overline{B}_{1i} w(t) + \overline{B}_{2i} q_i^h(y(t)) \\ e_q(t) = \overline{C}_i \xi(t) + \overline{D}_i w(t) + \overline{D}_{2i} q_i^h(y(t)) \end{cases} \tag{7}$$

where $\xi(t) = \left[x^T(t)\ x_f^T(t) \right]^T$, $e_q(t) = z(t) - z_{qf}(t)$ is the estimation error, and

$$\overline{A}_i = \begin{bmatrix} A_i & 0 \\ B_{fi} K_i C_{2i} & A_{fi} \end{bmatrix}, \quad \overline{B}_{1i} = \begin{bmatrix} B_i \\ B_{fi} K_i D_{2i} \end{bmatrix}, \quad \overline{B}_{2i} = \begin{bmatrix} 0 \\ B_{fi} \end{bmatrix},$$

$$\overline{C}_i = \begin{bmatrix} C_{1i} - D_{fi} K_i C_{2i} \\ -C_{fi} \end{bmatrix}^T, \quad \overline{D}_{1i} = [D_{1i} - D_{fi} K_i D_{2i}], \quad \overline{D}_{2i} = -D_{fi}.$$

The objective of this study is to consider (1) with quantized measurement output subject to general TPs (2), design a quantized H_∞ filter (3) such that the resulted filtering error system (7) is stochastically stable with a prescribed H_∞ performance level.

To realize this aim, some definitions and technique lemmas are introduced firstly.

Definition 1 (*Shi and Li [1]*). The system (7) with $w(t) = 0$ is said to be stochastically stable if

$$\mathcal{E}\left\{ \int_0^\infty \xi^T(t)\xi(t)\, dt \,|\, \xi(0), r(0) \right\} < \infty$$

for every initial condition $\xi(0)$ and $r(0)$.

Definition 2 (*Shi and Li [1]*). Given a scalar $\gamma > 0$, the filter error system (7) is said to be stochastically stable with disturbance attenuation level γ if, under zero initial conditions,

$$\mathcal{E}\left\{ \int_0^\infty e_q(t)^T e_q(t)\, dt \right\} < \gamma^2 \mathcal{E}\left\{ \int_0^\infty w(t)^T w(t)\, dt \right\} \tag{8}$$

holds for all non-zero $w(t) \in \mathcal{L}_2$.

Lemma 1 (*Chang and Yang [31]*). *$\mathcal{T} + \mathcal{P}\mathcal{A} + (\mathcal{P}\mathcal{A})^T < 0$ can be obtained from the following inequality:*

$$\begin{bmatrix} \mathcal{T} + He(\mathcal{M}\mathcal{A}) & \star \\ \mathcal{P} - \mathcal{M}^T + \mathcal{G}\mathcal{A} & He(-\mathcal{G}) \end{bmatrix} < 0 \tag{9}$$

Lemma 2 (*(Finsler Lemma) Boyd et al. [32]*). *The following statement holds:*

$$\varpi^T(t) \Gamma \varpi(t) + \digamma(\varpi(t)) < 0, \quad \forall \mathcal{B}\varpi(t) = 0, \quad \varpi(t) \ne 0 \tag{10}$$

where Γ is a symmetric matrix, $\mathcal{B} \in \mathcal{R}^{m \times n}$ and $\digamma(\varpi(t))$ is a scalar function, if there exists $\mathcal{X} \in \mathcal{R}^{n \times m}$ such that:

$$\varpi^T(t)\left(\Gamma + \mathcal{X}\mathcal{B} + (\mathcal{X}\mathcal{B})^T\right)\varpi(t) + \digamma(\varpi(t)) < 0, \quad \varpi(t) \ne 0 \tag{11}$$

3. Main results

In this section, to make the filtering error system (7) be stochastically stable with a prescribed H_∞ performance index, sufficient conditions are proposed in Theorem 1 firstly. Then, based on the attained conditions, a flexible approach to the desired filter design is established subsequently.

Theorem 1. *Consider the system (1) and let γ be a given positive scalar. Then, there exists a filter (3) such that the filtering error system (7) is stochastically stable with the prescribed H_∞ performance index γ if there exist matrices $P_i > 0$, $T_{ij} > 0$, G_i, and F_i such that the following LMIs hold:*

$$\Pi_i = \begin{bmatrix} \Pi_i(1,1) & \Pi_i(1,2) \\ * & \Pi_i(2,2) \end{bmatrix} < 0 \tag{12}$$

$$P_l \le P_i (i \in \mathcal{I}_{uk}^i, l \in \mathcal{I}_{uk}^i) \tag{13}$$

where $j_a \in \mathcal{I}_k^i, j_a \ne i$, $l \in \mathcal{I}_{uk}^i$

$$\Pi_i(1,1) = \begin{bmatrix} He(-G_i) & G_i\overline{B}_{2i} & 0 & (4,1) & G_i\overline{B}_{1i} \\ * & -\tau_i I & \overline{D}_{2i} & (4,2) & \tau_i\Theta_i D_{1i} \\ * & * & -I & \overline{C}_i^T & \overline{D}_{1i}^T \\ * & * & * & (4,4) & F_i\overline{B}_{1i} \\ * & * & * & * & -\gamma^2 I \end{bmatrix}$$

$$(4,1) = G_i\overline{A}_i - F_i^T + P_i, \quad (4,2) = (F_i\overline{B}_2 i)^T + \tau_i\Theta_i C_{1i}[I\ 0]$$

$$(4,4) = He\left(F_i\overline{A}_i\right) + \begin{cases} \sum_{j \in \mathcal{I}_k^i} \frac{\delta_{ij}^2}{4} T_{ij} + \sum_{j \in \mathcal{I}_k^i} \pi_{ij}(P_j - P_l)(i \in \mathcal{I}_k^i) \\ \sum_{j \in \mathcal{I}_k^i} \frac{\delta_{ij}^2}{4} T_{ij} + \sum_{j \in \mathcal{I}_k^i} \pi_{ij}(P_j - P_i)(i \in \mathcal{I}_{uk}^i) \end{cases}$$

$$\Pi_i(1,2) = \begin{cases} \begin{bmatrix} 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ P_{j_1} - P_l - P_i & \cdots & P_{j_m} - P_l - P_i \\ 0 & \cdots & 0 \end{bmatrix} (i \in \mathcal{I}_k^i) \\ \begin{bmatrix} 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ P_{j_1} - P_i & \cdots & P_{j_m} - P_i \\ 0 & \cdots & 0 \end{bmatrix} (i \in \mathcal{I}_{uk}^i) \end{cases}$$

$$\Pi_i(2,2) = \begin{bmatrix} -T_{ij_1} & \cdots & 0 \\ * & \ddots & \vdots \\ * & * & -T_{ij_m} \end{bmatrix}$$

Proof. Choose the following Lyapunov function

$$V(\xi(t), i) = \xi^T(t) P_i \xi(t)$$

where $P_i > 0$. Calculating its differential yields

$$\mathcal{E}\{\dot{V}(\xi(t), i)\} = \dot{\xi}^T(t) P_i \xi(t) + \xi^T(t) P_i \dot{\xi}(t) + \sum_{j=1}^{s} \hat{\pi}_{ij} P_j. \tag{14}$$

Associated with the differential of the required H_∞ performance

(8), defining $J=\mathcal{E}\{\dot{V}(\xi(t),i)+e_q^T(t)e_q(t)-\gamma^2 w(t)w(t)\}$ gives

$$J=\varsigma^T(t)\Phi_i\varsigma(t)$$

where

$$\varsigma(t)=\left[\dot{\xi}^T(t)\ \ (q_i^h(y(t)))^T\ \ e_q^T(t)\ \ \xi^T(t)\ \ w^T(t)\right]^T$$

and

$$\Phi_i=\begin{bmatrix} 0 & 0 & 0 & P_i & 0 \\ * & 0 & 0 & 0 & 0 \\ * & * & 0 & I & 0 \\ * & * & * & \sum_{j=1}^{s}\hat{\pi}_{ij}P_j & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix}$$

To achieve the aim that the filtering error system is stochastically stable with the required H_∞ performance, our main task is to guarantee $J<0$.

On the other hand, (6) is rewritten as

$$\varsigma^T(t)\mathcal{L}_{1i}\varsigma(t)\le 0 \tag{15}$$

where

$$\mathcal{L}_{1i}=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ * & I & 0 & -\Theta_i\overline{C}_i[I\ 0] & -\Theta_i\overline{D}_{1i} \\ * & * & 0 & 0 & 0 \\ * & * & * & 0 & 0 \\ * & * & * & * & 0 \end{bmatrix}$$

Employing S-procedure to (15), for any $\tau_i\ge 0$, $J<0$ can beensured from the following inequality:

$$\varsigma^T(t)\Phi_i^1\varsigma^T(t)<0 \tag{16}$$

where

$$\Phi_i^1=\begin{bmatrix} 0 & 0 & 0 & P_i & 0 \\ * & -\tau_i I & 0 & \tau_i\Theta_i\overline{C}_i[I\ 0] & \tau_i\Theta_i\overline{D}_{1i} \\ * & * & I & 0 & 0 \\ * & * & * & \sum_{j=1}^{s}\pi_{ij}P_j & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix}$$

Additionally, the filtering error system (7) is revisited as

$$\mathcal{L}_{2i}\varsigma(t)=0 \tag{17}$$

where

$$\mathcal{L}_{2i}=\begin{bmatrix} -I & \overline{B}_{2i} & 0 & \overline{A}_i & \overline{B}_{1i} \\ 0 & \overline{D}_{2i} & -I & \overline{C}_i & \overline{D}_{1i} \end{bmatrix}.$$

Utilizing Finsler Lemma to (16) and (17), $\Phi_i^1<0$ is satisfied once the following inequality holds:

$$\Phi_i^2=\begin{bmatrix} He(-G_i) & G_i\overline{B}_{2i} & 0 & \Phi_i^2(4,1) & G_i\overline{B}_{1i} \\ * & -\tau_i I & \overline{D}_{2i} & \Phi_i^2(4,2) & \tau_i\Theta_i D_{1i} \\ * & * & -I & \overline{C}_i^T & \overline{D}_{1i}^T \\ * & * & * & \Phi_i^2(4,4) & \overline{D}_{1i}^T \\ * & * & * & * & -\gamma^2 I \end{bmatrix}<0$$

where $\Phi_i(4,1)=G_i\overline{A}_i-F_i^T+P_i$, $\Phi_i(4,2)=(F_i\overline{B}_2 i)^T+\tau_i\Theta_i C_{1i}[I\ 0]$, $\Phi_i(4,4)=He\left(F_i\overline{A}_i\right)+\sum_{j=1}^{s}\hat{\pi}_{ij}P_j$.

Since there exists interconnection between $\hat{\pi}_{ij}$ and P_j, the stability analysis conditions are shown in nonlinearity form. To overcome this difficulty, two cases are considered individually.

Case I: $\hat{\pi}_{ii}$ is known, namely, $\hat{\pi}_{ii}\in\mathcal{I}_k^i$.

Applying $\left\{\sum_{j\in\mathcal{I}_{uk}^i}\hat{\pi}_{ij}\right\}\Big/\left\{-\sum_{j\in\mathcal{I}_k^i}\hat{\pi}_{ij}\right\}=1$, Φ_i^2 is transformed as $\Phi_i^2=\left\{\sum_{l\in\mathcal{I}_{uk}^i}\hat{\pi}_{il}\right\}\Big/\left\{-\sum_{i\in\mathcal{I}_k^i}\hat{\pi}_{ij}\right\}\Phi_i^3$ with

$$\Phi_i^3=\begin{bmatrix} He(-G_i) & G_i\overline{B}_{2i} & 0 & \Phi_i^2(4,1) & G_i\overline{B}_{1i} \\ * & -\tau_i I & \overline{D}_{2i} & \Phi_i^2(4,2) & \tau_i\Theta_i D_{1i} \\ * & * & -I & \overline{C}_i^T & \overline{D}_{1i}^T \\ * & * & * & \Phi_i^3(4,4) & F_i\overline{B}_{1i} \\ * & * & * & * & -\gamma^2 I \end{bmatrix}$$

and $\Phi_i^3(4,4)=He(F_i\overline{A}_i)+\sum_{j\in\mathcal{I}_k^i}\hat{\pi}_{ij}(P_j-P_l)$.

Thus, $\Phi_i^2<0$ holds if $\Phi_i^3<0$.

Before the further proceeding, Δ_{ij} is transformed as follows:

$$\sum_{j\in\mathcal{I}_k^i}\Delta_{ij}(P_j-P_l)=\sum_{j\in\mathcal{I}_k^i,j\ne i}\Delta_{ij}(P_j-P_l)+\Delta_{ii}(P_i-P_l). \tag{18}$$

Applying $\sum_{j\in\mathcal{I}}\Delta_{ij}=0$, one gets $\Delta_{ii}\le-\sum_{j\in\mathcal{I}_k^i,j\ne i}\Delta_{ij}$. Taking this fact to (18) yields

$$\sum_{j\in\mathcal{I}_k^i}\Delta_{ij}(P_j-P_l)\le\sum_{j\in\mathcal{I}_k^i,j\ne i}\Delta_{ij}(P_j-P_l-P_i)+\Delta_{ii}(-P_l)=\sum_{j\in\mathcal{I}_k^i}\Delta_{ij}(P_j-P_l-P_i). \tag{19}$$

Consequently, adopting the well-known fact $X^TY+Y^TX\le X^TX+Y^TY$ to (19), $\Phi_i^3<0$ is satisfied if the following inequality holds:

$$\Phi_i^4=\begin{bmatrix} He(-G_i) & G_i\overline{B}_{2i} & 0 & \Phi_i^2(4,1) & G_i\overline{B}_{1i} \\ * & -\tau_i I & \overline{D}_{2i} & \Phi_i^2(4,2) & \tau_i\Theta_i D_{1i} \\ * & * & -I & \overline{C}_i^T & \overline{D}_{1i}^T \\ * & * & * & \Phi_i^4(4,4) & \overline{D}_{1i}^T \\ * & * & * & * & -\gamma^2 I \end{bmatrix}<0$$

where

$$\Phi_i^4(4,4)=He\left(F_i\overline{A}_i\right)+\sum_{j\in\mathcal{I}_k^i}\pi_{ij}(P_j-P_l)$$
$$+\sum_{j\in\mathcal{I}_k^i}\left\{\frac{\delta_{ij}^2}{4}T_{ij}+(P_j-P_l-P_i)T_{ij}^{-1}(P_j-P_l-P_i)\right\}$$

which is just (12) for $i\in\mathcal{I}_k^i$, with the help of Schur complement.

Case II: $i\in\mathcal{I}_{uk}^i$. In this case, use $\hat{\pi}_{ii}=-\sum_{j\in\mathcal{I}_k^i,j\ne i}\hat{\pi}_{ij}-\sum_{l\in\mathcal{I}_{uk}^i,l\ne i}\hat{\pi}_{il}$ and adopt the similar measurements as Case 1, (12) and (13) can be established.□

Remark 1. To handle the difficulty incurred by uncertain TPs, the transition probability property is fully utilized with the scale technique.

Remark 2. Thanks to Finsler lemma, two sets of slack variables are introduced to separate the Lyapunov variables from system matrices. Meanwhile, with the help of S-procedure, the influence of the quantization is made full consideration.

Remark 3. If TPs are known, by resorting to a matrix operation, the proposed conditions are reduced to those of [24].

With the above obtained stability analysis conditions, a new filter design method is proposed in the following Theorem.

Theorem 2. *Consider MJSs (1) and the filter (3). The filtering error system (7) with general TPs is stochastically stable with a guaranteed H_∞ performance γ, if, for given scalars b_{1i}, b_{2i}, b_{3i} and b_{4i}, there exist approximate dimension matrices G_{i11}, G_{i12}, G_{i21}, G_{i22}, F_{i11}, F_{i12}, F_{i21}, F_{i22}, R_i, a_{fi}, b_{fi}, c_{fi}, d_{fi} and positive-definite symmetric matrices P_i, τ_i and β_i satisfying the following inequalities:*

$$\Sigma_i = \begin{bmatrix} \Sigma_{i1} & \Pi_i(1,2) \\ * & \Pi_i(2,2) \end{bmatrix} < 0, \tag{20}$$

$$P_l \le P_i (i, l \in \mathcal{I}_{uk}^i). \tag{21}$$

where

$$\Sigma_{i1} = \begin{bmatrix} \Sigma_{i1}^{11} & \Sigma_{i1}^{12} & \Sigma_{i1}^{13} & 0 & \Sigma_{i1}^{15} & \Sigma_{i1}^{16} & \Sigma_{i1}^{17} & \Sigma_{i1}^{18} \\ * & \Sigma_{i1}^{22} & \Sigma_{i1}^{23} & 0 & \Sigma_{i1}^{25} & \Sigma_{i1}^{26} & \Sigma_{i1}^{27} & \Sigma_{i1}^{28} \\ * & * & -\tau_i I & -d_{fi} & \Sigma_{i1}^{35} & \Sigma_{i1}^{36} & \Sigma_{i1}^{37} & \Sigma_{i1}^{38} \\ * & * & * & -I & \Sigma_{i1}^{45} & \Sigma_{i1}^{46} & \Sigma_{i1}^{47} & 0 \\ * & * & * & * & \Sigma_{i1}^{55} & \Sigma_{i1}^{56} & \Sigma_{i1}^{57} & \Sigma_{i1}^{58} \\ * & * & * & * & * & \Sigma_{i1}^{66} & \Sigma_{i1}^{67} & \Sigma_{i1}^{68} \\ * & * & * & * & * & * & -\gamma^2 I & \Sigma_{i1}^{78} \\ * & * & * & * & * & * & * & \Sigma_{i1}^{88} \end{bmatrix}$$

$\Sigma_{i1}^{11} = He(-G_{i11})$, $\Sigma_{i1}^{12} = -G_{i12} - G_{i21}^T$, $\Sigma_{i1}^{13} = b_{1i} b_{fi}$, $\Sigma_{i1}^{15} = G_{i11} A_i + b_{1i} b_{fi} K_i C_{1i} + P_{i11} - F_{i11}^T$, $\Sigma_{i1}^{16} = b_{1i} a_{fi} + P_{i12} - F_{i21}^T$, $\Sigma_{i1}^{17} = G_{i11} B_i + b_{1i} b_{fi} K_i D_{1i}$, $\Sigma_{i1}^{18} = \beta_i (G_{i12} - b_{1i} R_i)$, $\Sigma_{i1}^{22} = He(-G_{i22})$, $\Sigma_{i1}^{23} = b_{2i} b_{fi}$, $\Sigma_{i1}^{25} = G_{i21} A_i + b_{2i} b_{fi} K_i C_{1i} + P_{i12}^T - F_{i12}^T$, $\Sigma_{i1}^{26} = b_{2i} a_{fi} + P_{i22} - F_{i22}^T$, $\Sigma_{i1}^{27} = G_{i21} B_i + b_{2i} b_{fi} K_i D_{1i}$, $\Sigma_{i1}^{28} = \beta_i (G_{i22} - b_{2i} R_i)$, $\Sigma_{i1}^{35} = (b_{3i} b_{fi})^T + \tau_i \Theta_i C_{1i}$, $\Sigma_{i1}^{36} = (b_{4i} b_{fi})^T$, $\Sigma_{i1}^{37} = \tau_i \Theta_i D_{1i}$, $\Sigma_{i1}^{38} = b_{fi}^T$, $\Sigma_{i1}^{45} = C_{2i} - d_{fi} K_i C_{1i}$, $\Sigma_{i1}^{46} = -c_{fi}$, $\Sigma_{i1}^{47} = D_{2i} - d_{fi} K_i D_{1i}$, $\Sigma_{i1}^{55} = He(F_{i11} A_i + b_{3i} b_{fi} K_i C_{1i}) + \Lambda_i^{12}$, $\Sigma_{i1}^{56} = (F_{i21} A_i + b_{4i} b_{fi} K_i C_{1i})^T + b_{3i} a_{fi} + \Lambda_i^{12}$, $\Sigma_{i1}^{57} = F_{i11} B_i + b_{3i} b_{fi} K_i D_{1i}$, $\Sigma_{i1}^{58} = \beta_i (F_{i12} - b_{3i} R_i) + (b_{fi} K_i C_{1i})^T$, $\Sigma_{i1}^{66} = He(b_{4i} a_{fi}) + \Lambda_i^{22}$, $\Sigma_{i1}^{67} = F_{i21} B_i + b_{4i} b_{fi} K_i D_{1i}$, $\Sigma_{i1}^{68} = \beta_i (F_{i22} - b_{4i} R_i) + a_{fi}^T$, $\Sigma_{i1}^{78} = (b_{fi} K_i D_{1i})^T$, $\Sigma_{i1}^{88} = He(-\beta_i R_i)$.

Moreover, the filter gains matrices are solved by

$$A_{fi} = R_i^{-1} a_{fi}, \quad B_{fi} = R_i^{-1} b_{fi}, \quad C_{fi} = c_{fi}, \quad D_{fi} = d_{fi}.$$

Proof. Based on Theorem 1, choose the structure of G_i and F_i in (12) as following:

$$G_i = \begin{bmatrix} G_{i11} & G_{i12} \\ G_{i21} & G_{i22} \end{bmatrix}, \quad F_i = \begin{bmatrix} F_{i11} & F_{i12} \\ F_{i21} & F_{i22} \end{bmatrix} \tag{22}$$

Then, one gets

$$\Gamma_i = \begin{bmatrix} \Gamma_{i1} & \Pi_i(1,2) \\ * & \Pi_i(2,2) \end{bmatrix} < 0 \tag{23}$$

where

$$\Gamma_{i1} = \begin{bmatrix} \Gamma_{i1}^{11} & \Gamma_{i1}^{12} & \Gamma_{i1}^{13} & 0 & \Gamma_{i1}^{15} & \Gamma_{i1}^{16} & \Gamma_{i1}^{17} \\ * & \Gamma_{i1}^{22} & \Gamma_{i1}^{23} & 0 & \Gamma_{i1}^{25} & \Gamma_{i1}^{26} & \Gamma_{i1}^{27} \\ * & * & -\tau_i I & -d_{fi} & \Gamma_{i1}^{35} & \Gamma_{i1}^{36} & \Gamma_{i1}^{37} \\ * & * & * & -I & \Gamma_{i1}^{45} & \Gamma_{i1}^{46} & \Gamma_{i1}^{47} \\ * & * & * & * & \Gamma_{i1}^{55} & \Gamma_{i1}^{56} & \Gamma_{i1}^{57} \\ * & * & * & * & * & \Gamma_{i1}^{66} & \Gamma_{i1}^{67} \\ * & * & * & * & * & * & -\gamma^2 I \end{bmatrix}$$

with $\Gamma_{i1}^{11} = He(-G_{i11})$, $\Gamma_{i1}^{12} = -G_{i12} - G_{i21}^T$, $\Gamma_{i1}^{13} = G_{i12} B_{fi}$, $\Gamma_{i1}^{15} = G_{i11} A_i + G_{i12} B_{fi} K_i C_{1i} + P_{i11} - F_{i11}^T$, $\Gamma_{i1}^{16} = G_{i12} A_{fi} + P_{i12} - F_{i21}^T$, $\Gamma_{i1}^{17} = G_{i11} B_i + G_{i12} B_{fi} K_i D_{1i}$, $\Gamma_{i1}^{22} = He(-G_{i22})$, $\Gamma_{i1}^{23} = G_{i22} B_{fi}$, $\Gamma_{i1}^{25} = G_{i21} A_i + G_{i22} B_{fi} K_i C_{1i} + P_{i12}^T - F_{i11}^T$, $\Gamma_{i1}^{26} = G_{i22} A_{fi} + P_{i22} - F_{i22}^T$, $\Gamma_{i1}^{27} = G_{i21} B_i + G_{i22} B_{fi} K_i D_{1i}$, $\Gamma_{i1}^{35} = (F_{i12} B_{fi})^T + \tau_i \Theta_i C_{1i}$, $\Gamma_{i1}^{36} = (F_{i22} B_{fi})^T$, $\Gamma_{i1}^{37} = \tau_i \Theta_i D_{1i}$, $\Gamma_{i1}^{45} = C_{2i} - D_{fi} K_i C_{1i}$, $\Gamma_{i1}^{46} = -D_{fi}$, $\Gamma_{i1}^{47} = D_{2i} - D_{fi} K_i D_{1i}$, $\Gamma_{i1}^{55} = He(F_{i11} A_i + F_{i12} B_{fi} K_i C_{1i}) + \Lambda_i^{11}$, $\Gamma_{i1}^{56} = (F_{i21} A_i + G_{i22} B_{fi} K_i C_{1i})^T + F_{i22} A_{fi} + \Lambda_i^{12}$, $\Gamma_{i1}^{57} = F_{i11} B_i + F_{i12} B_{fi} K_i D_{1i}$, $\Gamma_{i1}^{66} = He(F_{i22} A_{fi}) + \Lambda_i^{22}$, $\Gamma_{i1}^{67} = F_{i21} B_i + F_{i22} B_{fi} K_i D_{1i}$.

To separate the filter parameters A_{fi} and B_{fi}, (23) is rewritten as

$$\Gamma_{i1} = \Gamma_{i2} + He(\mathcal{L}_{3i} \mathcal{L}_{4i}) \tag{24}$$

where

$$\Gamma_{i2} = \begin{bmatrix} \Gamma_{i2}^{11} & \Gamma_{i2}^{12} & \Gamma_{i2}^{13} & 0 & \Gamma_{i2}^{15} & \Gamma_{i2}^{16} & \Gamma_{i2}^{17} \\ * & \Gamma_{i2}^{22} & \Gamma_{i2}^{23} & 0 & \Gamma_{i2}^{25} & \Gamma_{i2}^{26} & \Gamma_{i2}^{27} \\ * & * & -\tau_i I & -d_f & \Gamma_{i2}^{35} & \Gamma_{i2}^{36} & \Gamma_{i2}^{37} \\ * & * & * & -I & \Gamma_{i2}^{45} & \Gamma_{i2}^{46} & \Gamma_{i2}^{47} \\ * & * & * & * & \Gamma_{i2}^{55} & \Gamma_{i2}^{56} & \Gamma_{i2}^{57} \\ * & * & * & * & * & \Gamma_{i2}^{66} & \Gamma_{i2}^{67} \\ * & * & * & * & * & * & -\gamma^2 I \end{bmatrix}$$

$\mathcal{L}_{3i} = \begin{bmatrix} \mathcal{L}_{3i}^1 & \mathcal{L}_{3i}^2 & 0 & 0 & \mathcal{L}_{3i}^3 & \mathcal{L}_{3i}^4 & 0 \end{bmatrix}^T$, $\mathcal{L}_{4i} = \begin{bmatrix} 0 & 0 & B_{fi} & 0 & B_{fi} K_i C_{1i} & A_{fi} & B_{fi} K_i D_{1i} \end{bmatrix}$, $\Gamma_{i2}^{11} = He(-G_{i11})$, $\Gamma_{i2}^{12} = -G_{i12} - G_{i21}^T$, $\Gamma_{i2}^{13} = b_{1i} R_i B_{fi}$, $\Gamma_{i2}^{15} = G_{i11} A_i + b_{1i} R_i B_{fi} K_i C_{1i} + P_{i11} - F_{i11}^T$, $\Gamma_{i2}^{16} = b_{1i} R_i A_{fi} + P_{i12} - F_{i21}^T$, $\Gamma_{i2}^{17} = G_{i11} B_i + b_{1i} R_i B_{fi} K_i D_{1i}$, $\Gamma_{i2}^{22} = He(-G_{i22})$, $\Gamma_{i2}^{23} = b_{2i} R_i B_{fi}$, $\Gamma_{i2}^{25} = G_{i21} A_i + b_{2i} R_i B_{fi} K_i C_{1i} + P_{i12}^T - F_{i12}^T$, $\Gamma_{i2}^{26} = b_{2i} R_i A_{fi} + P_{i22} - F_{i22}^T$, $\Gamma_{i2}^{27} = G_{i21} B_i + b_{2i} R_i B_{fi} K_i D_{1i}$, $\Sigma_{i1}^{35} = (b_{3i} R_i B_{fi})^T + \tau_i \Theta_i C_{1i}$, $\Gamma_{i2}^{36} = (b_{4i} R_i B_{fi})^T$, $\Gamma_{i2}^{37} = \tau_i \Theta_i D_{1i}$, $\Gamma_{i2}^{45} = C_{2i} - D_{fi} K_i C_{1i}$, $\Gamma_{i2}^{46} = -C_{fi}$, $\Gamma_{i2}^{47} = D_{2i} - D_{fi} K_i D_{1i}$, $\Gamma_{i2}^{55} = He(F_{i11} A_i + b_{3i} R_i B_{fi} K_i C_{1i}) + \Lambda_i^{11}$, $\Gamma_{i2}^{56} = (F_{i21} A_i + b_{4i} R_i B_{fi} K_i C_{1i})^T + b_{3i} R_i A_{fi} + \Lambda_i^{12}$, $\Gamma_{i2}^{57} = F_{i11} B_i + b_{3i} R_i B_{fi} K_i D_{1i}$, $\Gamma_{i2}^{66} = He(b_{4i} R_i A_{fi}) + \Lambda_i^{22}$, $\Gamma_{i2}^{67} = F_{i21} B_i + b_{4i} R_i A B_{fi} K_i D_{1i}$, $\mathcal{L}_{3i}^1 = (G_{i12} - b_{1i} R_i)^T$, $\mathcal{L}_{3i}^2 = (G_{i22} - b_{1i} R_i)^T$, $\mathcal{L}_{3i}^3 = (F_{i12} - b_{1i} R_i)^T$, $\mathcal{L}_{3i}^4 = (F_{i22} - b_{1i} R_i)^T$.

Employing Lemma 1 and setting $a_{fi} = R_i A_{fi}$, $b_{fi} = R_i B_{fi}$, $c_{fi} = C_{fi}$ and $d_{fi} = D_{fi}$ to (24), one gets (20). Moreover, (21) is just (13). □

Remark 4. Since the filter parameters are unified to R_i, no specified structure is imposed on the slack variables G_i and F_i. If choose $G_{i12} = G_{i22} = F_{i12} = F_{i22}$ and $R_i = 0$, then the proposed method is reduced to [30].

Remark 5. It is to note that there are four sets scalars to be tuned which result the obtained conditions in nonconvex. For simplicity, they are equal to 1. Once these scalars are fixed, conditions are in terms of LMIs and H_∞ performance index γ can be optimized.

Remark 6. Since the accessability of Markov mode to the designed filter may not be ensured, a mode-independent filter is deemed to be a favourite solution. In this scenario, the corresponding result can be attained by choosing $R_i = R$ in Theorem 2.

4. Numerical example

Consider MJLS (1) with four operation modes and the data:

$$A_1 = \begin{bmatrix} -0.15 & -0.2 \\ -0.3 & -0.5 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -0.2 & -0.2 \\ -0.3 & -0.4 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} -0.5 & 0.3 \\ 0.4 & -0.5 \end{bmatrix}, \quad A_4 = \begin{bmatrix} -0.3 & 0.2 \\ -0.1 & -0.2 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} -0.2 \\ 0.4 \end{bmatrix}, \quad B_2 = \begin{bmatrix} -0.4 \\ -0.1 \end{bmatrix}, \quad B_3 = \begin{bmatrix} 0.8 \\ 0.5 \end{bmatrix},$$

$$B_4 = \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix}, \quad C_{11}^T = \begin{bmatrix} -0.1 \\ 0.7 \end{bmatrix}, \quad C_{12}^T = \begin{bmatrix} 0.5 \\ -0.2 \end{bmatrix},$$

$$C_{13}^T = \begin{bmatrix} 0.1 \\ 0.6 \end{bmatrix}, \quad C_{14}^T = \begin{bmatrix} 0.8 \\ 0.2 \end{bmatrix}, \quad C_{21}^T = \begin{bmatrix} 0.4 \\ -0.2 \end{bmatrix},$$

$$C_{22}^T = \begin{bmatrix} -0.1 \\ 0.4 \end{bmatrix}, \quad C_{23}^T = \begin{bmatrix} 0.3 \\ -0.3 \end{bmatrix}, \quad C_{24}^T = \begin{bmatrix} -0.2 \\ 0.2 \end{bmatrix},$$

$D_{11} = 0.2, \quad D_{12} = -0.2, \quad D_{13} = -0.2, \quad D_{14} = 0.2,$
$D_{21} = -0.2, \quad D_{22} = 0.6, \quad D_{23} = 0.3, \quad D_{24} = 0.4.$

Table 1
H_∞ performance indices for different methods.

Method	[30]	Theorem 2
γ	1.2366	1.0985

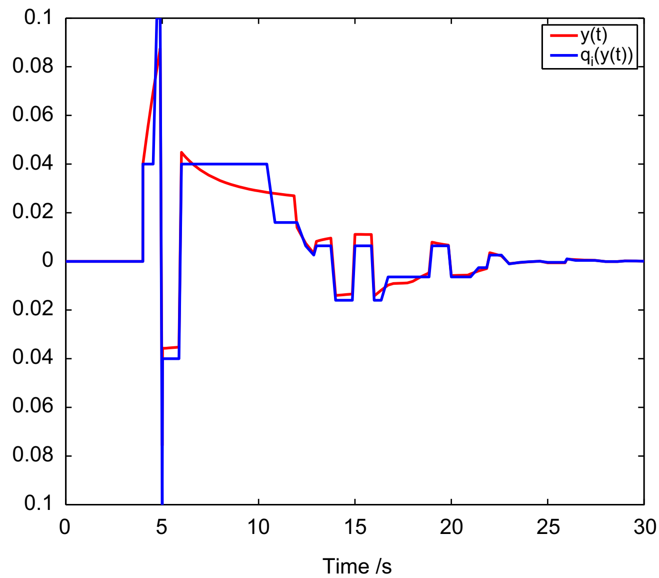

Fig. 2. Curves of $y(t)$ and $q_i(y(t))$.

The general transition probability matrix with $\Delta_{21} \in [-0.002\ 0.002]$ is given below:

$$\begin{bmatrix} -1.5 & 0.2+\Delta_{21} & ? & ? \\ ? & ? & 0.5 & 0.3 \\ 0.7 & ? & -1.3 & ? \\ 0.2 & ? & ? & ? \end{bmatrix},$$

where "?" means that the corresponding elements are unknown.

Our purpose is to design a mode-dependent full-order H_∞ filter in the form of (3) such that the resulting filtering error system is stochastically stable and has a guaranteed H_∞ performance level.

For the logarithmic quantizer (4), the quantizer densities are chosen as $\rho_1=\rho_2=\rho_3=\rho_4=0.4$ and the initial quantizer points are chosen as $U_0^1=U_0^2=U_0^3=U_0^4=0.1$. The H_∞ performance indices for methods proposed in [30] and Theorem 2 (where $b_{1i}=b_{2i}=b_{3i}=b_{4i}=1$) are given in the following Table 1.

According to this table, it is seen that the H_∞ performance index obtained with the proposed approach is better than the existing method. The reason is that no extra constraint is imposed on the introduced slack variables in Theorem 2, while special structures are imposed on them in [30].

Accompanying with the obtained H_∞ performance index $\gamma=1.0985$ solved by Theorem 2, the corresponding filter parameters are given below

$$A_{f1}=\begin{bmatrix} -4.6372 & -7.5682 \\ -6.6424 & -11.3406 \end{bmatrix}, \quad A_{f2}=\begin{bmatrix} -6.8542 & -7.0106 \\ -6.6155 & -9.8295 \end{bmatrix},$$

$$A_{f3}=\begin{bmatrix} -54.2647 & 66.6409 \\ 66.7405 & -84.5150 \end{bmatrix}, A_{f4}=\begin{bmatrix} -12.8384 & 2.1223 \\ 0.5186 & -0.9518 \end{bmatrix},$$

$$B_{f1}=\begin{bmatrix} -0.6962 \\ -1.5679 \end{bmatrix}, B_{f2}=\begin{bmatrix} -2.5870 \\ 0.6569 \end{bmatrix}, \quad B_{f3}=\begin{bmatrix} 21.6850 \\ -29.7489 \end{bmatrix},$$

$$B_{f4}=\begin{bmatrix} -8.9636 \\ 0.3282 \end{bmatrix}, \quad C_{f1}=\begin{bmatrix} -0.3621 \\ 0.0143 \end{bmatrix}^T, \quad C_{f2}=\begin{bmatrix} -0.0930 \\ -0.2608 \end{bmatrix}^T,$$

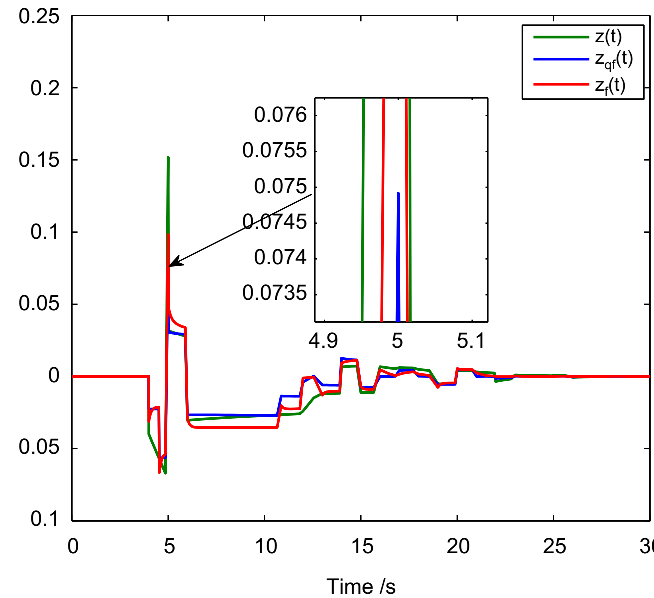

Fig. 3. Curves of $z(t)$, $z_{qf}(t)$ and $z_f(t)$.

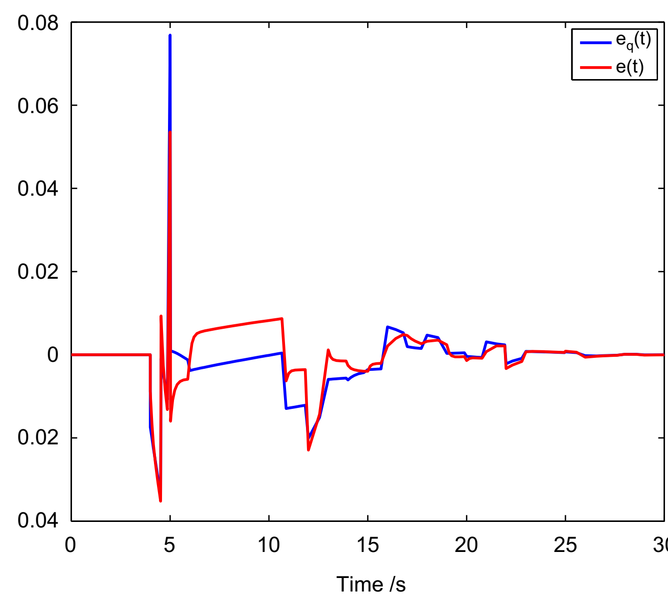

Fig. 4. Curves of $e_q(t)$ and $e(t)$.

$$C_{f3}=\begin{bmatrix} -0.2768 \\ -0.2344 \end{bmatrix}^T, C_{f4}=\begin{bmatrix} -0.1249 \\ -0.2900 \end{bmatrix}^T, \quad D_{f1}=-0.3058,$$

$$D_{f2}=-0.4923, D_{f3}=-0.5597, \quad D_{f4}=-0.1883.$$

Under zero initial condition, simulation curves of $y(t)$ and $q_i(y(t))$ are drawn in Fig. 2, the estimated outputs $z_{qf}(t)$ (with quantization consideration) and $z_f(t)$ (without quantization consideration) also compared with $z(t)$ are shown in Fig. 3 and the filtering error response curves $e_{qf}(t)$ and $e(t)$ are given in Fig. 4. These figures are listed at the bottom of the paper.

From Fig. 2, it is found that the nonlinearity induced by quantization could deteriorates system performance. Fig. 3 shows that the estimated output $z_{qf}(t)$ with quantization consideration is better than $z_f(t)$, which is further verified in Fig. 4. Therefore, to maintain the desired system performance, it is necessary to take into account the adverse influence of quantization when systems operate in network environment.

5. Conclusions

The quantized H_∞ filtering of continuous MJLSs is considered in this paper. A mode-dependent logarithmic quantizer is adopted to quantize the system output before it sends to filter and TPs are allowed to be known, uncertain with variations and unknown. Some effective strategies are developed to deal with the nonlinearities caused by both quantization and uncertain/unknown TPs. Sufficient conditions are established to ensure the filtering error system to be stochastically stable with the prescribed H_∞ performance requirement. A new filter design method is presented in terms of LMIs without imposing constraint on introduced slack variables. The validity of the proposed method is demonstrated by a numerical example.

Acknowledgments

The authors would like to thank the editor, the associate editor and the reviewers for their valuable comments and suggestions which help to significantly improve the quality and presentation of this paper. This work was supported in part by the Basic Science Research Program through the National Research Foundation of Korea funded by the Ministry of Education under Grant 2013R1A1A2A10005201, in part by the National Natural Science Foundation of China under Grant 61403189, in part by the Natural Science Foundation of Jiangsu Province of China under Grant BK20130949, in part by the Doctoral Foundation of Ministry of Education of China under Grant 20133221120012, in part by the Natural Science Foundation of Jiangsu Provincial Universities of China under Grant 13KJB120004, in part by the Jiangsu Postdoctoral Science Foundation under Grant 1401015B, in part by the China Postdoctoral Science Foundation under Grant 2015M570397, in part by the peak of six talents in Jiangsu Province under Grant 2015XXRJ-011, in part by the MCCSE2015A03.

References

[1] Shi P, Li F. A survey on Markovian jump systems: modeling and design. Int J Control Autom Syst 2015;13:1–16.
[2] Wu H, Cai K. Mode-independent robust stabilization for uncertain Markovian jump nonlinear systems via fuzzy control. IEEE Trans Syst Man Cybern Part B (Cybern) 2005;36:509–19.
[3] Wang Y, Wang Q, Zhou P, Duan D. Robust guaranteed cost control for singular Markovian jump systems with time-varying delay. ISA Trans 2012;51:559–65.
[4] Shen H, Su L, Park JH. Further results on stochastic admissibility for singular Markov jump systems using a dissipative constrained condition. ISA Trans 2015;59:65–71.
[5] Li H, Chen B, Zhou Q, Qian W. Robust stability for uncertain delayed fuzzy hopfield neural networks with Markovian jumping parameters. IEEE Trans Syst Man Cybern Part B (Cybern) 2009;41:94–102.
[6] Zhu Q, Cao J. Exponential stability of stochastic neural networks with both Markovian jump parameters and mixed time delays. IEEE Trans Syst Man Cybern Part B (Cybern) 2011;41:341–53.
[7] Wu Z, Shi P, Su H, Chu J. Passivity analysis for discrete-time stochastic Markovian jump neural networks with mixed time-delays. IEEE Trans Neural Netw 2011;22:1566–75.
[8] Li H, Gao H, Shi P, Zhao X. Fault-tolerant control of Markovian jump stochastic systems via the augmented sliding mode observer approach. Automatica 2014;50:1825–34.
[9] Li F, Wu L, Shi P, Lim C-C. State estimation and sliding mode control for semi-Markovian jump systems with mismatched uncertainties. Automatica 2015;51:385–93.
[10] Wu Z, Shi P, Su H, Chu J. Stochastic synchronization of Markovian jump neural networks with time-varying delay using sampled data. IEEE Trans Cybern 2013;43:1796–806.
[11] Yin Y, Shi P, Liu F, Teo KL, Lim C-C. Robust filtering for nonlinear nonhomogeneous Markov jump systems by fuzzy approximation approach. IEEE Trans Cybern 2014;45:1706–16.
[12] Li L, Zhang Q, Zhu B. Fuzzy stochastic optimal guaranteed cost control of bio-economic singular Markovian jump systems. IEEE Trans Cybern 2015;45:2512–21.
[13] Liu M, Shi P, Zhang L, Zhao X. Fault-tolerant control for nonlinear Markovian jump systems via proportional and derivative sliding mode observer technique. IEEE Trans Circuits Syst I: Regul Pap 2011;58:2755–64.
[14] Wang Y, Shi P, Wang Q, Duan D. Exponential filtering for singular Markovian jump systems with mixed mode-dependent time-varying delay. IEEE Trans Circuits Syst I: Regul Pap 2013;60:2440–52.
[15] Zhang L, Shi Y, Chen T, Huang B. A new method for stabilization of networked control systems with random delays. IEEE Trans Autom Control 2005;50:1177–81.
[16] Xiong J, Lam J, Gao H, Ho DC. On robust stabilization of Markovian jump systems with uncertain switching probabilities. Automatica 2005;41:897–903.
[17] Karan M, Shi P, Kaya C. Transition probability bounds for the stochastic stability robustness of continuous and discrete time Markovian jump linear systems. Automatica 2006;42:2159–68.
[18] Zhang L, Boukas EK, Lam J. Analysis and synthesis of Markov jump linear systems with time-varying delays and partially known transition probabilities. IEEE Trans Autom Control 2009;53:2458–64.
[19] Zhang Y, He Y, Wu M, Zhang J. Stabilization for Markovian jump systems with partial information on transition probability based on free-connection weighting matrices. Automatica 2011;47:79–84.
[20] Zhong Q, Bai J, Wen B, Li S, Zhong F. Finite-time boundedness filtering for discrete-time Markovian jump system subject to partly unknown transition probabilities. ISA Trans 2014;53:1107–18.
[21] Shen M, Yang G. New analysis and synthesis conditions for continuous Markov jump linear systems with partly known transition probabilities. IET Control Theory Appl 2012;6:2318–25.
[22] Li J, Yuan J, Lu. J. Observer-based H_∞ control for networked nonlinear systems with random packet losses. ISA Trans 2010;49:39–46.
[23] Zhang L, Shi Y, Chen T, Huang B. A new method for stabilization of networked control systems with random delays. IEEE Trans Autom Control 2005;50:1177–81.
[24] Liu M, Ho DWC, Niu Y. Robust filtering design for stochastic system with mode-dependent output quantization. IEEE Trans Signal Process 2010;58:6410–6.
[25] Yu J, Nan L, Tang X, Wang P. Model predictive control of non-linear systems over networks with data quantization and packet loss. ISA Trans 2015;59:1–9.
[26] Lu R, Xu Y, Xue A, Zheng J. Networked control with state reset and quantized measurements: observer-based case. IEEE Trans Ind Electron 2013;60:5206–13.
[27] Li F, Shi P, Wu L, Basin MV, Lim C. Quantized control design for cognitive radio networks modeled as nonlinear semi-Markovian jump systems. IEEE Trans Ind Electron 2015;62:2330–40.
[28] Wu Z, Shi P, Su H, Chu J. Asynchronous l_2-l_∞ filtering for discrete-time stochastic Markov jump systems with randomly occurred sensor nonlinearities. Automatica 2014;50:180–6.
[29] Zhang L, Zhu Y, Shi P, Zhao Y. Resilient asynchronous H_∞ filtering for Markov jump neural networks with unideal measurements and multiplicative noises. IEEE Trans Cybern 2015;45:2840–52.
[30] Wei Y, Qiu J, Karimi HR, Wang M. A new design of H_∞ filtering for continuous-time Markovian jump systems with time-varying delay and partially accessible mode information. Signal Process 2013;93:2392–407.
[31] Chang X, Yang G. Nonfragile H_∞ filtering of continuous-time fuzzy systems. IEEE Trans Signal Process 2011;59:1528–38.
[32] Boyd S, Ghaoui LEL, Feron E, Balakrishnan V. Linear Matrix Inequalities in System and Control Theory. Philadelphia, PA: SIAM; 1994.