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Brief paper

Design of H_∞ filter for Markov jumping linear systems with non-accessible mode information[☆]

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ABSTRACT

For Markov jumping linear systems (MJLS), it is often encountered that the jumping mode cannot be accessible for filtering. Therefore it is strongly desirable to design a filter which is independent of the jumping model. In this paper, a new deterministic filter design procedure is proposed, which shows less conservatism than existing results. The whole design procedure can be accomplished by solving a set of linear matrix inequalities (LMIs). Finally, numerical examples are given to illustrate the effectiveness of the proposed approach.

1. Introduction

Many dynamical systems have variable structures subject to random abrupt changes, which may result from abrupt phenomena such as random failures and repairs of the components, changes in the interconnections of subsystems, sudden environment changes, modification of the operating point of a linearized model of a nonlinear system, etc. Most of the general dynamic systems cannot deal with these random abrupt changes. It is known that hybrid systems, which involve both time-evolving and event-driven mechanisms, can be employed to model the above problems. In particular, a special class of hybrid systems may be in the form of a jumping system with many operation modes, and the system mode switching is governed by a Markov process. The parameter jumps among different modes can be seen as discrete events. This type of hybrid system is known as the so-called Markov jumping linear systems (MJLS). Recently, MJLS has been a subject of great practical importance since it is an appropriate class of systems to study the behavior of real systems (Boukas & Liu, 2002; Boukas, 2005; Wang, Qiao, & Burnham, 2002; Xiong, Lam, Gao, & Ho, 2005).

Filtering is a class of important approaches to estimate the state information when the plant is disturbed. Currently there are many approaches proposed for filter design, such as Kalman filtering (Mahmoud & Shi, 2003; Shi, Boukas, & Agarwal, 1999; Yang, Wang, & Hung, 2002), H_∞ filtering (Boukas & Liu, 2000; Gao & Wang, 2004; Yue & Han, 2004), L_2–L_∞ filtering (Gao & Wang, 2003), stationary filter (Fragoso & Rocha, 2004) etc. In the H_∞ filtering, the input is supposed to be an energy signal and the corresponding energy-to-energy gain can be minimized. Recently, many results on the H_∞ filtering for MJLS have been presented (Costa & Guerra, 2002; de Farias, Geromel, do Val, & Costa, 2000; Wang, Lam, & Liu, 2003). However, these results on the filtering for the MJLS require critical assumption on the accessibility of the jumping mode and belong to a class of mode-dependent stochastic filters. Practically, this assumption may sometimes be impossible to satisfy, such as those networked control systems without time stamp information (Kim, Park, & Ko, 2004; Xiong & Lam, 2007). In this case, the developed mode-dependent results are not applicable. An alternative approach to overcome this problem is to estimate the jumping mode by using the Monte Carlo approach (Andrieu, Davy, & Doucet, 2003; Doucet & Andrieu, 2001). However, these approaches cannot guarantee that the estimated jump mode is always correct and therefore make the strict performance analysis of the state filtering difficult. To tackle this problem, developing mode-independent deterministic filters is strongly desired. Currently there are only few results on deterministic filtering for MJLS. If the existing stochastic filtering approach is used to design the deterministic filter, (i.e., let the filters at different modes posses the same

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parameters), some bilinear matrix inequality (BMI) problems are required to be solved. This problem has been analyzed in El-Ghaoui and Ait-Rami (1996) for controller design. To avoid solving the BMI problem, (Boukas, 2005; El-Ghaoui & Ait-Rami, 1996; Kim et al., 2004) used the mode-independent Lyapunov function approach to design the deterministic state feedback controller. Though this approach is reasonable, it introduces very strong conservatism on the controller performance. In do Val and Fragoso (1994), the corresponding problem is cast into a set of interconnected Riccati equations, which are difficult to solve in practice. For filtering, if those similar approaches in Boukas (2005), El-Ghaoui and Ait-Rami (1996) and Kim et al. (2004) are used, such problems on solving BMI cannot be avoided. Very recently, Carlos E. de Souza and his colleagues made great progress in this field (de Souza, 2003; de Souza, Trofino, & Barbosa, 2004; de Souza, Trofino, & Barbosa, 2006). In de Souza (2003), they proposed a class of new approaches to design the mode-independent filter for discrete-time MJLS. In de Souza et al. (2004) and de Souza et al. (2006), they studied continuous-time MJLS, but the used approach is quite different from the discrete-time version (de Souza, 2003). In both cases, these approaches utilize mode-dependent Lyapunov function and show better performance than previous results.

In this paper, the deterministic filter design for continuous-time MJLS with non-accessible jumping mode information is revisited and a different design approach is developed. This approach is based on a rather straightforward matrix transformation technique, rather than Finsler's Lemma which was used in de Souza et al. (2004) and de Souza et al. (2006). The resulting approach is less conservative than the results in de Souza et al. (2004) and de Souza et al. (2006). In addition, another significant point is that the proposed approach can deal with MJLS with "terminal mode", which cannot be dealt with by de Souza et al. (2004, 2006).

The rest of the paper is organized as follows. In Section 2, the problem is stated and some preliminary results are developed to motivate the contribution of our paper. Section 3 presents the main results. Section 4 provides a numerical example to show the effectiveness of the established results.

Notation. Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n dimensional Euclidean space and the set of all $n \times m$ real matrices. The superscript "T" denotes matrix transposition and the notation $X > Y$ (respectively, $X < Y$) where X and Y are symmetric matrices, means that $X - Y$ is positive definite (respectively, negative definite). $\mathcal{L}_2$ is the space of integrable vector over $[0, \infty)$. $\|\cdot\|$ will refer to the Euclidean vector norm. For a symmetric block matrix, we use "*" as an ellipsis for the terms that are introduced by symmetry. diag{} stands for a block-diagonal matrix and $\mathbb{I}$ is for identity matrix with compatible dimensions. Finally, the symbol $\mathrm{He}\{X\}$ is used to represent $X + X^{\mathrm{T}}$.

2. Problem formulation

Given a probability space $(\Omega, \mathcal{F}, \mathcal{P})$, where Ω is the sample space, $\mathcal{F}$ is the algebra of events and $\mathcal{P}$ is the probability measure defined on $\mathcal{F}$. The investigated MJLS is described as:

$$\begin{cases} \dot{x}(t) = A(r(t))x(t) + B_w(r(t))w(t) \\ y(t) = C(r(t))x(t) + D_w(r(t))w(t) \\ z(t) = L(r(t))x(t), \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state variable, $w(t) \in \mathbb{R}^q$ is the disturbance input, which is a square integrable vector function over $[0, +\infty)$; $y(t) \in \mathbb{R}^p$ is the measured output; $z(t) \in \mathbb{R}^m$ is the signal to be evaluated. The mode jumping process $\{r(t)\}$ is a continuous-time, discrete-state homogeneous Markov process on the probability space, takes values in a finite state space $\mathbb{S} = \{1, 2, \ldots, s\}$, and has the mode transition probabilities

$$\Pr\{r(t+\Delta) = j | r(t) = i\} = \begin{cases} \pi_{ij}\Delta + o(\Delta), & i \neq j \\ 1 + \pi_{ii}\Delta + o(\Delta), & i = j, \end{cases}$$

where $\Delta > 0$, $\pi_{ij} \geq 0$ for $i \neq j$ and $\pi_{ii} = -\sum_{j=1, j\neq i}^{s} \pi_{ij}$ for each mode i. Note that if $\pi_{ii} = 0$ for some $i \in \mathbb{S}$, then the ith mode is called "terminal mode" (de Souza et al., 2004).

For each possible value of $r(t) = i \in \mathbb{S}$, we have $A(r(t)) = A_i$, $B_w(r(t)) = B_{wi}$, $C(r(t)) = C_i$, $D_w(r(t)) = D_{wi}$, $L(r(t)) = L_i$, where A_i, B_{wi}, C_i, D_{wi} and L_i are known constant matrices of appropriate dimensions.

Consider the following filter

$$\begin{cases} \dot{\hat{x}}(t) = \hat{A}\hat{x}(t) + \hat{B}y(t) \\ \hat{z}(t) = \hat{L}\hat{x}(t), \end{cases} \tag{2}$$

where $\hat{x}(t) \in \mathbb{R}^n$ is the filter state, $\hat{z}(t) \in \mathbb{R}^m$ is the estimation signal, $\hat{A}$, $\hat{B}$ and $\hat{L}$ are filter parameters to be determined. Note that (2) is a deterministic filter. It can be more useful when the jumping mode information $r(t)$ cannot be obtained.

To begin the study of the filtering problem, we define $\bar{x}(t) = [x^{\mathrm{T}}(t)\ \hat{x}^{\mathrm{T}}(t)]^{\mathrm{T}}$, $\bar{z}(t) = z(t) - \hat{z}(t)$, and

$$\bar{A}_i = \begin{bmatrix} A_i & 0 \\ \hat{B}C_i & \hat{A} \end{bmatrix}, \qquad \bar{B}_{wi} = \begin{bmatrix} B_{wi} \\ \hat{B}D_{wi} \end{bmatrix}, \tag{3}$$

$$\bar{L}_i = \begin{bmatrix} L_i & -\hat{L} \end{bmatrix}. \tag{4}$$

Then, from (1)–(4), we have the following augmented system:

$$\begin{cases} \dot{\bar{x}}(t) = \bar{A}(r(t))\bar{x}(t) + \bar{B}_w(r(t))w(t) \\ \bar{z}(t) = \bar{L}(r(t))\bar{x}(t), \end{cases} \tag{5}$$

where $\bar{A}(r(t)) = \bar{A}_i$, $\bar{B}_w(r(t)) = \bar{B}_{wi}$, $\bar{L}(r(t)) = \bar{L}_i$ for $r(t) = i$. Similarly to Boukas (2005), we introduce a definition on stochastic stability.

Definition 2.1. System (5) with $w(t) \equiv 0$ is said to be *stochastically stable*, if for any initial-finite state $\bar{x}_0 \in \mathbb{R}^{2n}$ and any initial mode $r_0 \in \mathbb{S}$, the following relation holds

$$\lim_{t\to+\infty} \mathbf{E}\left\{\|\bar{x}(t)\|^2 | \bar{x}_0, r_0\right\} = 0. \tag{6}$$

In the following of this paper, the augmented initial state will be set to $\bar{x}_0 = [x_0^{\mathrm{T}}, 0_{1\times n}]^{\mathrm{T}}$.

Consider the following relation (Boukas, 2005):

$$\mathbf{E}\left\{\int_0^{T_f} \bar{z}^{\mathrm{T}}(t)\bar{z}(t)\mathrm{d}t \middle| r_0\right\} < \gamma^2 \int_0^{T_f} w^{\mathrm{T}}(t)w(t)\mathrm{d}t, \tag{7}$$

we can introduce the following definitions:

Definition 2.2. A filter of the form (2) is called *deterministic* $\gamma - H_\infty$ *filter* for MJLS (1), if the corresponding augmented system (5) is stochastically stable and satisfies (7) under the zero-initial conditions $\bar{x}_0 = 0$, for all $T_f \in (0, +\infty)$ and any non-zero $w(t) \in \mathcal{L}_2[0, +\infty)$.

Remark 2.1. From Definition 2.1 we can see that if the augmented system (5) with $w(t) \equiv 0$ is stochastically stable, then the original system (1) with $w(t) \equiv 0$

$$\dot{x}(t) = A(r(t))x(t)$$

is also stochastically stable. Therefore, the stochastic stability of the original system is a necessary condition in this paper.

The objective of this paper is to design a deterministic $\gamma - H_\infty$ filter (2) for MJLS (1), where $\gamma > 0$ is a prescribed attenuation level. More specifically, we are interested in seeking the filter parameters $\hat{A}$, $\hat{B}$, $\hat{L}$ for deterministic filter (2).

3. Filter design

To design the filter, we will first concentrate our attention on the performance analysis of the augmented system (5). Here a well-known lemma is recalled:

Lemma 3.1 (*Li and Ugrinovskii (2007), Pan and Basar (1996) and de Souza et al. (2006)). Given a scalar $\gamma > 0$, the augmented MJLS (5) is stochastically stable and the H_∞ performance (7) can be satisfied, if and only if there exists a set of matrices $X_i \in \mathbb{R}^{2n\times 2n}$ satisfying $X_i > 0$ and*

$$\begin{bmatrix} \bar{A}_i^{\mathrm{T}} X_i + X_i\bar{A}_i + \sum_{j=1}^{s}\pi_{ij}X_j & * & * \\ \bar{B}_{wi}^{\mathrm{T}} X_i & -\gamma^2\mathbb{I} & * \\ \bar{L}_i & 0 & -\mathbb{I} \end{bmatrix} < 0. \tag{8}$$

Though this lemma provides a necessary and sufficient condition for the H_∞ performance of the augmented system (5), it is difficult to be used to design deterministic filter. To tackle this problem, we transform the above condition into another form:

Theorem 3.1. *Given a scalar $\gamma > 0$, the augmented MJLS (5) is stochastically stable and the H_∞ performance (7) can be satisfied, if and only if there exist a set of matrices $X_i \in \mathbb{R}^{2n\times 2n}$ satisfying $X_i > 0$, and two sets of matrices $G_i \in \mathbb{R}^{2n\times 2n}$, $Z_i \in \mathbb{R}^{2n\times 2n}$ such that*

$$\begin{bmatrix} G_i^{\mathrm{T}}\bar{A}_i + \bar{A}_i^{\mathrm{T}} G_i + \sum_{j=1}^{s}\pi_{ij}X_j & * & * & * \\ Z_i^{\mathrm{T}}\bar{A}_i + X_i - G_i & -Z_i - Z_i^{\mathrm{T}} & * & * \\ \bar{B}_{wi}^{\mathrm{T}} G_i & \bar{B}_{wi}^{\mathrm{T}} Z_i & -\gamma^2\mathbb{I} & * \\ \bar{L}_i & 0 & 0 & -\mathbb{I} \end{bmatrix} < 0. \tag{9}$$

Proof. The core of the proof is the equivalence of (8) and (9).

{(9) $\Rightarrow$ (8)}. Pre- and post-multiplying (9) with

$$\begin{bmatrix} \mathbb{I} & \bar{A}_i^{\mathrm{T}} & 0 & 0 \\ 0 & \bar{B}_{wi}^{\mathrm{T}} & \mathbb{I} & 0 \\ 0 & 0 & 0 & \mathbb{I} \end{bmatrix}$$

and its transpose, we can directly obtain (8).

{(8) $\Rightarrow$ (9)}. Since (8) holds, by continuity of LMI (Fridman, 2002), for arbitrary matrices Ω_i ($i = 1, 2, \ldots, s$) with compatible dimensions, there always exists a set of sufficiently small scalars $\epsilon_i > 0$ such that

$$\begin{bmatrix} \bar{A}_i^{\mathrm{T}} X_i + X_i\bar{A}_i + \sum_{j=1}^{s}\pi_{ij}X_j & * & * \\ \bar{B}_{wi}^{\mathrm{T}} X_i & -\gamma^2\mathbb{I} & * \\ \bar{L}_i & 0 & -\mathbb{I} \end{bmatrix} + \epsilon_i\Omega_i < 0. \tag{10}$$

If we select

$$\Omega_i = \frac{1}{2}\begin{bmatrix} \bar{A}_i^{\mathrm{T}} \\ \bar{B}_{wi}^{\mathrm{T}} \\ 0 \end{bmatrix}\begin{bmatrix} \bar{A}_i & \bar{B}_{wi} & 0 \end{bmatrix},$$

and

$$Z_i = \epsilon_i\mathbb{I}, \qquad G_i = X_i,$$

then (10) can be written as (9) by using Schur complements. This shows that (8) implies (9).

Therefore (8) and (9) are equivalent. By Lemma 3.1, the desired results are achieved. This completes the proof. □

Remark 3.1. In Cao and Lin (2004), a descriptor system approach has been proposed to obtain similar results, which is used to design robust controllers for uncertain linear systems. However, the approach proposed in this paper is different from Cao and Lin (2004) since the descriptor system approach is substituted by a more straightforward matrix transformation. The idea in Theorem 3.1 is a little similar with the results presented in de Oliveira, Bernussou, and Geromel (1999), which only treats discrete-time cases. To the best of the authors' knowledge, there has been no similar matrix transformation approach reported for continuous-time Markov jumping systems.

According to Theorem 3.1, the deterministic filter design problem reduces to determining $\hat{A}$, $\hat{B}$ and $\hat{L}$ from condition (9). Unfortunately, (9) cannot be effectively solved since it is highly nonlinear in the unknown variables $\hat{A}$, $\hat{B}$, $\hat{L}$, X_i, G_i and Z_i. In this section, an LMI formulation of (9) will be given.

Note that (9) implies $-G_i - G_i^{\mathrm{T}} < 0$ and therefore G_i is invertible. Further, let $Z_1 = Z_2 = \cdots = Z_s = G_1 = G_2 = \cdots = G_s = G$ and

$$Q = G^{-1}, \qquad P_i = Q^{\mathrm{T}} X_i Q.$$

Pre- and post-multiplying (9) with $\mathrm{diag}\{Q^{\mathrm{T}}, Q^{\mathrm{T}}, \mathbb{I}, \mathbb{I}\}$ and $\mathrm{diag}\{Q, Q, \mathbb{I}, \mathbb{I}\}$, respectively, give

$$\begin{bmatrix} \bar{A}_i Q + Q^{\mathrm{T}}\bar{A}_i^{\mathrm{T}} + \sum_{j=1}^{s}\pi_{ij}P_j & * & * & * \\ \bar{A}_i Q + P_i - Q^{\mathrm{T}} & -Q - Q^{\mathrm{T}} & * & * \\ \bar{B}_{wi}^{\mathrm{T}} & \bar{B}_{wi}^{\mathrm{T}} & -\gamma^2\mathbb{I} & * \\ \bar{L}_i Q & 0 & 0 & -\mathbb{I} \end{bmatrix} < 0. \tag{11}$$

Let

$$Q = \begin{bmatrix} R & M_1 \\ M_2^{\mathrm{T}} & U \end{bmatrix}, \qquad Q^{-1} = \begin{bmatrix} S & N_1 \\ N_2^{\mathrm{T}} & V \end{bmatrix}, \tag{12}$$

and define

$$\Pi_1 = \begin{bmatrix} S & \mathbb{I} \\ N_2^{\mathrm{T}} & 0 \end{bmatrix}, \qquad \Pi_2 = \begin{bmatrix} \mathbb{I} & R \\ 0 & M_2^{\mathrm{T}} \end{bmatrix}. \tag{13}$$

It is obvious that $Q\Pi_1 = \Pi_2$.

Pre- and post-multiplying (11) by $\mathrm{diag}\{\Pi_1^T, \Pi_1^T, \mathbb{I}, \mathbb{I}\}$ and $\mathrm{diag}\{\Pi_1, \Pi_1, \mathbb{I}, \mathbb{I}\}$ respectively, give

$$\begin{bmatrix} \begin{pmatrix} \Pi_1^{\mathrm{T}}\bar{A}_i\Pi_2 + \Pi_2^{\mathrm{T}}\bar{A}_i^{\mathrm{T}}\Pi_1 \\ +\sum_{j=1}^{s}\pi_{ij}\Pi_1^{\mathrm{T}}P_j\Pi_1 \end{pmatrix} & * & * & * \\ \begin{pmatrix} \Pi_1^{\mathrm{T}}\bar{A}_i\Pi_2 + \Pi_1^{\mathrm{T}}P_i\Pi_1 \\ -\Pi_2^{\mathrm{T}}\Pi_1 \end{pmatrix} & -\Pi_2^{\mathrm{T}}\Pi_1 - \Pi_1^{\mathrm{T}}\Pi_2 & * & * \\ \bar{B}_{wi}^{\mathrm{T}}\Pi_1 & \bar{B}_{wi}^{\mathrm{T}}\Pi_1 & -\gamma^2\mathbb{I} & * \\ \bar{L}_i\Pi_2 & 0 & 0 & -\mathbb{I} \end{bmatrix} < 0. \tag{14}$$

Note that (11) implies $-Q - Q^{\mathrm{T}} < 0$. By the decomposition (12), we have that $-Q - Q^{\mathrm{T}} < 0$ implies $-R - R^{\mathrm{T}} < 0$, and therefore R is also non-singular.

Define

$$Y = R^{-1}, \qquad \mathcal{M} = N_2\hat{A}M_2^{\mathrm{T}}Y, \qquad \mathcal{N} = Y^{\mathrm{T}}M_2N_2^{\mathrm{T}},$$
$$\mathcal{B} = N_2\hat{B}, \qquad \mathcal{K} = \hat{L}M_2^{\mathrm{T}}Y, \tag{15}$$

and

$$\begin{bmatrix} W_{11i} & W_{21i}^{\mathrm{T}} \\ W_{21i} & W_{22i} \end{bmatrix} = \begin{bmatrix} \mathbb{I} & 0 \\ 0 & Y^{\mathrm{T}} \end{bmatrix}\Pi_1^{\mathrm{T}}P_i\Pi_1\begin{bmatrix} \mathbb{I} & 0 \\ 0 & Y \end{bmatrix}. \tag{16}$$

Pre- and post-multiplying (14) by $\mathrm{diag}(\mathbb{I}, Y^T, \mathbb{I}, Y^T, \mathbb{I}, \mathbb{I})$ and $\mathrm{diag}(\mathbb{I}, Y, \mathbb{I}, Y, \mathbb{I}, \mathbb{I})$ yield

$$\begin{bmatrix} \Xi_{11i} & * & * & * & * & * \\ \Xi_{21i} & \Xi_{22i} & * & * & * & * \\ \Xi_{31i} & \Xi_{32i} & -\mathrm{He}(S) & * & * & * \\ \Xi_{41i} & \Xi_{42i} & -S-\mathcal{N}-Y^{\mathrm{T}} & -\mathrm{He}(Y) & * & * \\ B_{wi}^{\mathrm{T}}S + D_{wi}^{\mathrm{T}}\mathcal{B}^{\mathrm{T}} & B_{wi}^{\mathrm{T}}Y & B_{wi}^{\mathrm{T}}S + D_{wi}^{\mathrm{T}}\mathcal{B}^{\mathrm{T}} & B_{wi}^{\mathrm{T}}Y & -\gamma^2\mathbb{I} & * \\ L_i & L_i - \mathcal{K} & 0 & 0 & 0 & -\mathbb{I} \end{bmatrix} < 0, \tag{17}$$

where

$$\Xi_{11i} = \mathrm{He}(S^{\mathrm{T}}A_i + \mathcal{B}C_i) + \sum_{j=1}^{s} \pi_{ij}W_{11j},$$

$$\Xi_{21i} = Y^{\mathrm{T}}A_i + A_i^{\mathrm{T}}S + C_i^{\mathrm{T}}\mathcal{B}^{\mathrm{T}} + \mathcal{M}^{\mathrm{T}} + \sum_{j=1}^{s} \pi_{ij}W_{21j},$$

$$\Xi_{22i} = \mathrm{He}(A_i^{\mathrm{T}}Y) + \sum_{j=1}^{s} \pi_{ij}W_{22j},$$

$$\Xi_{31i} = S^{\mathrm{T}}A_i + \mathcal{B}C_i + W_{11i} - S,$$
$$\Xi_{32i} = S^{\mathrm{T}}A_i + \mathcal{B}C_i + \mathcal{M} + W_{21i}^{\mathrm{T}} - Y,$$
$$\Xi_{41i} = Y^{\mathrm{T}}A_i + W_{21i} - S - \mathcal{N},$$
$$\Xi_{42i} = Y^{\mathrm{T}}A_i + W_{22i} - Y.$$

Finally, the relation $P_i > 0$ is equivalent to

$$\begin{bmatrix} W_{11i} & W_{21i}^{\mathrm{T}} \\ W_{21i} & W_{22i} \end{bmatrix} > 0. \tag{18}$$

To summarize the above procedure, we can propose the following theorem.

Theorem 3.2. *Given the augmented MJLS (5) and a scalar $\gamma > 0$, if there exist matrices $Y, S, W_{11i}, W_{21i}, W_{22i}, \mathcal{B}, \mathcal{M}, \mathcal{N}$ and $\mathcal{K}$ satisfying (17) and (18) for $i = 1, 2, \ldots, s$, then (5) is stochastically stable and achieves the H_∞ performance (7).*

Since (17) and (18) are LMIs, it is easy to get $Y, S, W_{11i}, W_{21i}, W_{22i}, \mathcal{B}, \mathcal{M}, \mathcal{N}$ and $\mathcal{K}$. After obtaining these solutions, we can use the following procedure to obtain the filter parameters.

(1) Determine M_1 and N_2 using the relation

$$M_1N_2^{\mathrm{T}} = \mathbb{I} - Y^{-1}S. \tag{19}$$

This can be completed by standard SVD full-rank factorization.

The obtained M_1 and N_2 should be square invertible. If $\mathbb{I} - Y^{-1}S$ is not invertible, we can slightly perturb Y to $\tilde{Y} = (1+\varepsilon)Y$,where ε is a sufficiently small scalar which makes $\mathbb{I} - \tilde{Y}^{-1}S$ invertible and (17) still holds when Y is replaced by $\tilde{Y}$. This technique has been used in Chilali and Gahinet (1996) and Xu, Lam, and Zou (2003).

(2) Determine M_2 by using

$$M_2 = Y^{-T}\mathcal{N}N_2^{-T}$$

(3) Obtain $\hat{A}$, $\hat{B}$ and $\hat{L}$ as

$$\hat{A} = N_2^{-1}\mathcal{M}Y^{-1}M_2^{-T}, \qquad \hat{B} = N_2^{-1}\mathcal{B}, \qquad \hat{L} = \mathcal{K}Y^{-1}M_2^{-T}.$$

Remark 3.2. The deterministic filter design problem for Markov jumping linear systems was recently investigated by de Souza et al. (2004) and de Souza et al. (2006) by using Finsler's Lemma. In this paper, a different approach is proposed to tackle this problem, which is based on a more straightforward matrix transformation (see Theorem 3.1), and does not use Finsler's Lemma. It will be shown by numerical examples that the proposed approach is less conservative than that of de Souza et al. (2004, 2006). In addition, we note that in de Souza et al. (2004, 2006), an assumption $\pi_{ii} \neq 0$ for $i = 1, 2, \ldots, s$ is required. In this paper, the proposed deterministic filter design approach removes this assumption and therefore permits the existence of the "terminal mode" (for its definition, see de Souza et al. (2004, 2006)).

Remark 3.3. In Liu, Boukas, Sun, and Ho (2006), a descriptor system approach is proposed to design the mode-independent controller. However, the Markov property of the augmented system cannot be guaranteed. In this paper, a different approach, which is based on direct matrix transformation, is proposed to solve this problem.

Remark 3.4. Note that (17) are LMIs not only over the matrix variables, but also over the scalar γ^2. This implies that the scalar γ^2 can be included as one of the optimization variables to obtain the minimum noise-attenuation level bound. Then, the robust H_∞ filter with minimum guaranteed cost can be readily found by solving the following convex optimization problem:

$$\mathbf{OP}: \quad \min_{Y,S,W_{11i},W_{21i},W_{22i},\mathcal{B},\mathcal{M},\mathcal{N},\mathcal{K}} \delta$$

subject to $\delta > 0$ and the LMIs (17) and (18), where γ^2 is replaced by δ. The minimum guaranteed cost is given by $\gamma = \sqrt{\delta^*}$, where δ^* is the optimal value of δ.

4. Numerical example

Consider the following numerical example from de Souza et al. (2004, 2006) with small modifications.

$$A_1 = \begin{bmatrix} -3 & 1 & 0 \\ 0.3 & -2.5 & a \\ -0.1 & 0.3 & -3.8 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -2.5 & 0.5 & -0.1 \\ 0.1 & -3.5 & 0.3 \\ -0.1 & 1 & -2 \end{bmatrix},$$

$$B_{w1} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad B_{w2} = \begin{bmatrix} -0.6 \\ 0.5 \\ 0 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0.8 & 0.3 & 0 \end{bmatrix}, \quad C_2 = \begin{bmatrix} -0.5 & 0.2 & 0.3 \end{bmatrix},$$

$$D_{w1} = \begin{bmatrix} 0.2 \end{bmatrix}, \quad D_{w2} = \begin{bmatrix} 0.5 \end{bmatrix},$$

$$L_1 = \begin{bmatrix} 0.5 & -0.1 & 1 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 0 & 1 & 0.6 \end{bmatrix},$$

where the parameter a in matrix A_1 can take different values for extensive comparison purpose. The transition rate matrix Π is given by

$$\Pi = \begin{bmatrix} -0.5 & 0.5 \\ 0.3 & -0.3 \end{bmatrix}.$$

We run both the approach in de Souza et al. (2004, 2006) (we call this approach Souza's approach) and our approach. The obtained minimum attenuation levels versus different a are shown in Fig. 1, where the dashed line with "◇" represents Souza's results, and the solid line with "□" represents our results, which are obtained by solving **OP**. From this figure we can see that for different values of the parameter a, our approach can always obtain a smaller attenuation level than Souza's approach. Note that when $a > 14$, Souza's approach cannot give a feasible solution, while our approach works till $a = 25$.

Especially, when $a = 1$, this example reduces to the one in de Souza et al. (2004, 2006). The obtained minimum attenuation level is $\gamma_{\min} = 0.74038$ in de Souza et al. (2004, 2006), while our approach can obtain the minimal attenuation level $\gamma_{\min} = 0.3028$, with the filter parameters

$$\hat{A} = \begin{bmatrix} -3.0548 & -1.0834 & 4.0574 \\ -0.4024 & -1.0568 & -0.4447 \\ -0.0943 & 0.1215 & -1.7652 \end{bmatrix}, \quad \hat{B} = \begin{bmatrix} -0.5968 \\ 0.0902 \\ 0.0247 \end{bmatrix},$$

$$\hat{L} = \begin{bmatrix} -2.5527 & -2.0061 & 6.0951 \end{bmatrix}.$$

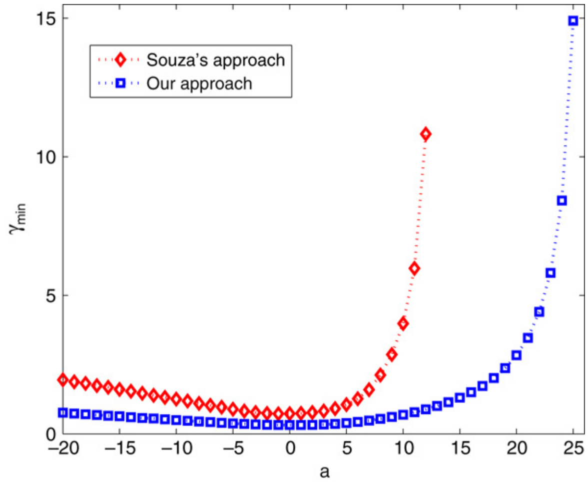

Fig. 1. H_∞ performance comparison between Souza's approach and our approach.

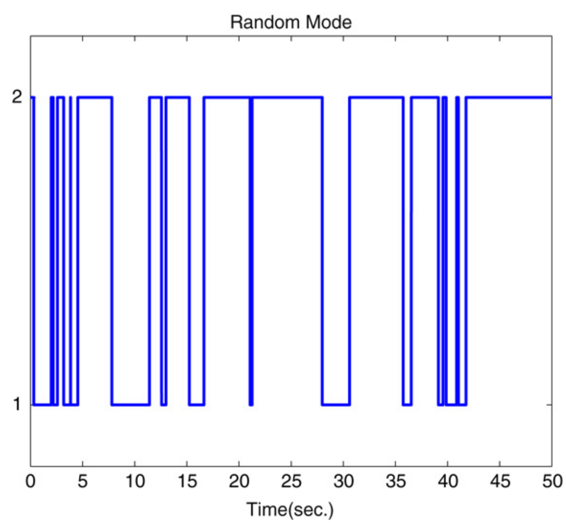

Fig. 2. Random jumping mode.

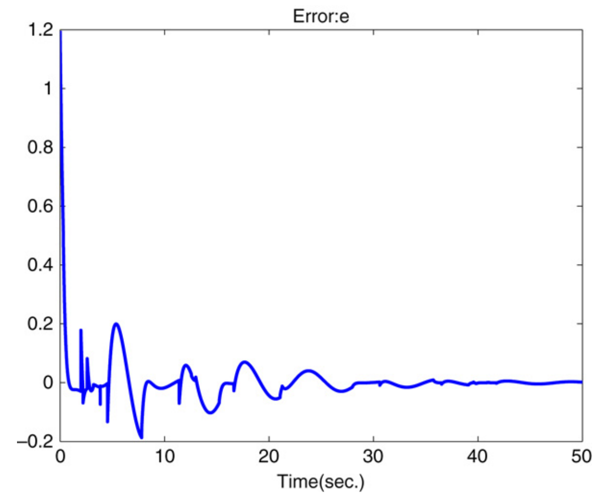

Fig. 3. Error signal $e(t) = z(t) - \hat{z}(t)$.

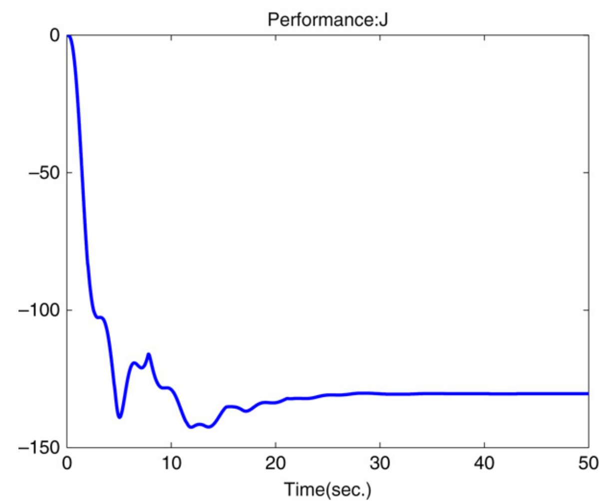

Fig. 4. Performance $J(T_f)$.

In this case, one of the possible realizations of the jumping mode is plotted in Fig. 2, where the initial mode is assumed to be $r_0 = 2$. We made some simulations of the behaviors of the filtering system under the disturbance $w(t) = \sin(t)\mathrm{e}^{-0.1t}$. The error signal $e(t) = z(t) - \hat{z}(t)$ under initial-state condition $x_0 = [1.7, 1.2, 1.0]^{\mathrm{T}}$ is plotted in Fig. 3. The functional cost $J(T_f) = \int_0^{T_f} \{z^{\mathrm{T}}(t)z(t) - \gamma_0^2 w(t)w(t)\}\,\mathrm{d}t$ under zero-initial-state condition, where $\gamma_0 = 0.3028$, is plotted in Fig. 4. Since the designed filter leads to negative function costs, it achieves the desired H_∞ performance in this simulation. We repeated this simulation many times under various mode realizations and observed similar phenomena.

In order to further show the difference between Souza's approach and ours, we change the transition rate matrix to

$$\Pi^{(1)} = \begin{bmatrix} 0 & 0 \\ 0.3 & -0.3 \end{bmatrix}, \qquad \Pi^{(2)} = \begin{bmatrix} -0.5 & 0.5 \\ 0 & 0 \end{bmatrix},$$

respectively. In these cases, Souza's approach will not work since there are zero elements in some diagonal positions. In fact, if the transition rate matrix is $\Pi^{(1)}$, there is no jump to or from the Markov jumping mode 1, i.e. if $r(t_0) = 1$ for some $t_0 > 0$, then $r(t)$ will be 1 for all $t > t_0$ and the whole jumping system reduces to a deterministic system. This mode is called "terminal mode" (de Souza et al., 2004, 2006), and however, these cases can still be dealt with using our approach. For example, for $\Pi^{(1)}$ or $\Pi^{(2)}$, we can solve **OP** to obtain the corresponding $\gamma_{\min} = 0.3421$ and $\gamma_{\min} = 0.3133$, with the filter parameters

$$\hat{A}^{(1)} = \begin{bmatrix} -3.0649 & 0.8565 & -2.1257 \\ 0.6338 & -2.7512 & 2.2281 \\ 0.2493 & -0.0564 & -1.5157 \end{bmatrix},$$

$$\hat{B}^{(1)} = \begin{bmatrix} -0.7271 \\ -0.1362 \\ -0.0600 \end{bmatrix},$$

$$\hat{L}^{(1)} = \begin{bmatrix} -2.0560 & 2.8923 & -5.3259 \end{bmatrix},$$

and

$$\hat{A}^{(2)} = \begin{bmatrix} -2.6387 & 0.3392 & -2.9309 \\ -0.6287 & -3.0526 & 4.3688 \\ -0.1539 & 0.0592 & -2.0116 \end{bmatrix},$$

$$\hat{B}^{(2)} = \begin{bmatrix} -0.8593 \\ 0.1533 \\ 0.0368 \end{bmatrix},$$

$$\hat{L}^{(2)} = \begin{bmatrix} -1.9218 & -2.9257 & 5.6807 \end{bmatrix},$$

respectively.

5. Conclusions

In this paper, a constructive approach to design deterministic filters for continuous-time MJLS with non-accessible jumping mode information is proposed. The design procedure can be cast into a set of LMIs, which can be solved effectively. The effectiveness of the procedure is demonstrated by some design examples.

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