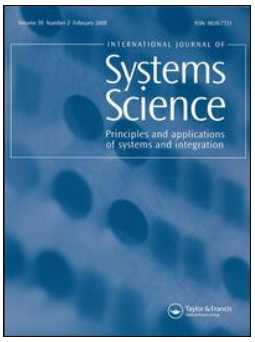

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Tracking control of discrete-time Markovian jump systems

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ABSTRACT

This paper is concerned with the model reference tracking control (MRTC) problem of discrete-time Markovian jump systems under external disturbance and input constraints. A state feedback controller is proposed to guarantee that the state vector of the system tracks precisely a given state vector of a reference model. The tracking problem is formulated as an optimisation problem and an approach to design the state feedback controller which stabilises the augmented system and rejects the external disturbance simultaneously. Sufficient conditions for stochastic stability and stabilisation of the augmented systems are derived via linear matrix inequalities. Then, under the assumption of completely unknown system modes, a common MRTC approach is developed to guarantee the stochastic stability, the disturbance rejection and the input constraints. Finally, a simulation example is provided to illustrate the effectiveness of the proposed method.

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Discrete-time; Markovian jump systems; tracking control; state feedback; linear matrix inequality (LMI)

1. Introduction

In the past four decades, the study of Markovian jump systems has been an attractive subject of research because their applications cover diverse fields including economics, biomedicine, fault-tolerant systems, and communication networks. It has been shown that as a special kind of hybrid systems, Markovian jump systems can better describe the systems subject to abrupt variation in their structures or parameters induced by external causes, for example, sudden environmental changes, changing subsystem interconnections and component failures (Luan et al., 2018). A number of results related to Markovian jump systems have been established to meet various requirements, such as H_∞ control (Boukas, 2009a), finite-time control (Cheng et al., 2015; Y. Zhang, Shi, & Nguang, 2014; Y. Zhang, Shi, Nguang, & Song, 2014; Y. Zhang et al., 2012, 2016, 2018), fault-tolerant control (H. Li et al., 2015; X. Li et al., 2018), state estimation (W. Liu, 2017), sliding mode control (Hu, Zhang, Yu, et al., 2019; Hu, Zhang, Kao, et al., 2019) and event-based dissipative analysis (Y. Zhang et al., 2020).

On the other hand, model reference tracking control (MRTC) has been a focus of the study, which can be applied to different types of systems, such as multivariable linear systems (De La Torre et al., 2016), uncertain linear systems (Xiao & Dong, 2019), anti-linear systems (Wu et al., 2014), descriptor linear systems (Duan & Zhang, 2007), linear parameter-varying systems (Abdullah, 2018) and chaotic systems (S. Y. Liu et al., 2017). It is notable that most results in the existing literature are limited to linear systems (Samigulina et al., 2015; Zhong & Lin, 2017). However, in some practical situations, Markovian jump systems can widely model the behaviour of practical systems. For this reason, the discrete-time Markovian jump systems have received more and more attention (Y. Li et al., 2018).

Recently, few results have been obtained for MRTC of Markovian jump systems. A MRTC problem of continuous-time Markovian jump systems with external finite energy disturbance was considered in Boukas (2009b). By using a class of linear matrix inequalities, a state-feedback controller is designed to guarantee the state vector of the system tracks precisely a given state vector of a reference model. Fan et al. (2015) investigated the design of fault-tolerant tracking controller for continuous-time Markovian jump systems with time-varying delay and actuator fault. The adaptive laws are constructed with a novel structure so that the fault estimation is fast enough and the high-frequency oscillations can be reduced

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effectively. Fu and Li (2016) dealt with the spacecraft trajectory tracking control problem with a stochastic thruster fault. Based on the theory of linear matrix inequalities and generalised Sylvester equations, a general complete parametric expression for the controller is presented to guarantee the stability and the input constraints.

In this paper, the MRTC problem of discrete-time Markovian jump systems is considered under external disturbance and input constraints. It is necessary to point out the differences between this paper and the existing relative papers (Boukas, 2009a, 2009b; Fan et al., 2015; Fu & Li, 2016; Hu, Zhang, Yu, et al., 2019; Hu, Zhang, Kao, et al., 2019). First, the literatures (Boukas, 2009a; Hu, Zhang, Yu, et al., 2019; Hu, Zhang, Kao, et al., 2019) study the sliding-mode control problem or H_∞ control problem for discrete-time Markovian jump systems, while this paper mainly considers the MRTC problem of discrete-time Markovian jump systems. Second, with regard to the MRTC problem of continuous-time systems (Boukas, 2009b; Fan et al., 2015; Fu & Li, 2016), the continuous-time systems usually have to be implemented digitally to design the controller in practice (Chen et al., 2017). Nevertheless, some MRTC methods of continuous-time systems cannot be transplanted into discrete-time systems directly, which brings some challenges in the MRTC design (Costa et al., 2005). To the best of our knowledge, the problem of MRTC for discrete-time Markovian jump systems under external disturbance and input constraints has not been discussed before, which is still an open problem, so it motivates us to research the problem in this paper. The main contributions of this paper are given as follows: (1) based on a set of linear matrix inequalities, a state feedback controller is designed to guarantee the state vector of the system tracks precisely a given state vector of a reference model and reject the external disturbance simultaneously; (2) under the assumption of completely unknown system modes, a common MRTC approach is developed to guarantee the stability, the disturbance rejection and the input constraints.

The structure of the rest of this paper is as follows: in Section 2, we give a formulation of the problem considered in this paper together with some relevant preliminaries. In Section 3, the main results are stated, including the design of a state-feedback controller and the method for solving the MRTC problem. In Section 4, a simulation example is presented to show the usefulness of the proposed method. Concluding remarks are eventually given in Section 5.

Notation 1.1: The standard notations are used throughout this paper. The superscript 'T' stands for matrix transposition. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. $\mathbb{R}^m \times \mathbb{R}^n$ is the set of all real $m \times n$ matrices. Given a probability space $(\Omega, \mathcal{F}, \Theta)$, Ω represents the sample space, $\mathcal{F}$ is the algebra of events and Θ is the probability measure defined on $\mathcal{F}$. $\Xi\{\cdot\}$ stands for the mathematical expectation. $\mathfrak{L}$ represents the infinitesimal operator. The notation † means Moore–Penrose pseudoinverse. The matrix $P > 0$ ($P \geq 0$) means P is real symmetric and positive definite (semi-positive definite). Given any two real symmetric matrices A and B, $A > B$ refers to the fact that $A-B$ is positive definite. The identity matrix of order m is denoted as I_m (or simply I if no confusion arises).

2. Problem statement

Fix the probability space $(\Omega, \mathcal{F}, \Theta)$ and consider a class of discrete-time Markovian jump systems with external disturbance as follows:

$$\begin{aligned} x_p(k+1) &= A(r(k))x_p(k) + B(r(k))u(k) \\ &\quad + G(r(k))w(k), \\ x_p(0) &= x_{p0}, \end{aligned} \tag{1}$$

where $x_p(k) \in \mathbb{R}^n$ is the state, $x_p(0) \in \mathbb{R}^n$ is the initial state, $u(k) \in \mathbb{R}^m$ is the control input and $w(k) \in \mathbb{R}^l$ is an external disturbance. Let $(r(k), k)$ be a Markovian homogeneous chain taking values in a finite set $S = \{1, 2, \ldots, N\}$ with the mode transition probability as follows:

$$\Pr\left\{r(k+1) = j \,|\, r(k) = i\right\} = \pi_{ij},$$

$\pi_{ij} \geq 0$, $\forall\ i, j \in S$, and $\sum_{j=1}^{N} \pi_{ij} = 1$. Consequently, the Markovian process transition probabilities matrix (TPM) can be described as

$$\Pi = \begin{pmatrix} \pi_{11} & \pi_{12} & \cdots & \pi_{1N} \\ \pi_{21} & \pi_{22} & \cdots & \pi_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \pi_{N1} & \pi_{N2} & \cdots & \pi_{NN} \end{pmatrix}.$$

In the above, for $r(k) = i$, $i \in S$, the system matrices of the ith mode are denoted by $A(i)$, $B(i)$, $G(i)$, where $A(i), B(i), G(i)$ are real and known matrices with appropriate dimensions.

In this paper, the reference model take up the following form:

$$x_m(k+1) = A_m x_m(k) + B_m v(k),$$
$$x_m(0) = x_{m0}, \tag{2}$$

where $x_m(k) \in \mathbb{R}^n$ is the state of the reference model, $x_m(0) \in \mathbb{R}^n$ is the initial state, $v(k) \in \mathbb{R}^m$ is the reference input , and A_m and B_m are known matrices with appropriate dimensions.

Remark 2.1: It can be seen that the reference model we want to track is independent of the system modes. In the rest of this article, we will assume the complete access to the state vectors and the system modes when it is necessary for feedback.

The objective is to design a state feedback controller that forces the system state $x_p(k)$ to track the state of the reference model $x_m(k)$ precisely. The following definition will be introduced and adopted throughout this paper.

Lemma 2.1 (Boyed et al., 1994): *For any symmetric matrix*

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^{\mathrm{T}} & S_{22} \end{bmatrix},$$

the following conditions are equivalent:

(i) $S < 0$,
(ii) $S_{11} < 0$ *and* $S_{22} - S_{12}^{\mathrm{T}} S_{11}^{-1} S_{12} < 0$,
(iii) $S_{22} < 0$ *and* $S_{11} - S_{12} S_{22}^{-1} S_{12}^{\mathrm{T}} < 0$.

3. Main results

In this section, the main results will be presented. A state feedback controller is designed to stabilise the augmented system, some sufficient conditions are presented to guarantee the state of system (1) tracks the state of reference model (2) precisely, the errors can be defined as follows:

$$e(k) = x_p(k) - x_m(k). \tag{3}$$

The augmented system is given by the following differential equations:

$$\Psi(k+1) = \widetilde{A}(r(k))\Psi(k) + \widetilde{B}(r(k))u(k) + \widetilde{G}(r(k))\widetilde{w}(k),$$
$$e(k) = \widetilde{C}(r(k))\Psi(k), \tag{4}$$

where

$$\Psi(k) = \begin{bmatrix} x_p(k) \\ x_m(k) \end{bmatrix}, \quad \widetilde{w}(k) = \begin{bmatrix} w(k) \\ v(k) \end{bmatrix},$$
$$\widetilde{A}(r(k)) = \begin{bmatrix} A(r(k)) & 0 \\ 0 & A_m \end{bmatrix}, \quad \widetilde{B}(r(k)) = \begin{bmatrix} B(r(k)) \\ 0 \end{bmatrix},$$
$$\widetilde{G}(r(k)) = \begin{bmatrix} G(r(k)) & 0 \\ 0 & B_m \end{bmatrix}, \quad \widetilde{C}(r(k)) = [I \quad -I].$$

For further development, we will give some important lemmas which are needed in the development of the main results.

Lemma 3.1 (L. Zhang & Boukas, 2009): *System* (4) *is said to be stochastically stable if, for* $u(k) = 0, \widetilde{w}(k) = 0$ *and every initial condition* $\Psi(0) \in \mathbb{R}^{2n}$ *and* $r(0) \in S$, *the following holds*

$$\Xi \left\{ \sum_{k=0}^{\infty} \|\Psi(k)\|^2 \mid \Psi(0), r(0) \right\} < \infty.$$

Lemma 3.2 (Costa et al., 2005): *System* (4) (*with* $u(k) = 0, \widetilde{w}(k) = 0$) *is stochastically stable if, and only if, there exists a set of positive-definite matrices* $P(i)$, $i \in S$, *satisfying*

$$\widetilde{A}^{\mathrm{T}}(i)\mathbb{P}^{(i)}\widetilde{A}(i) - P(i) < 0,$$

where $\mathbb{P}^{(i)} \triangleq \sum_{j \in S} \pi_{ij} P(j)$.

Lemma 3.3 (Costa et al., 2005): *Let* δ *be a given positive constant, system* (4) (*with* $u(k) = 0$) *is said to be stochastically stable with* δ*-disturbance attenuation if system* (4) (*with* $u(k) = 0$) *is stochastically stable, then the following holds*

$$\|e\|_2^2 = \Xi \left\{ \sum_{k=0}^{\infty} \mathrm{e}^{\mathrm{T}}(k)e(k) \right\} < \delta^2 \Xi \left\{ \sum_{k=0}^{\infty} \widetilde{w}^{\mathrm{T}}(k)\widetilde{w}(k) \right\}$$
$$= \delta^2 \|\widetilde{w}\|_2^2.$$

The following results give the conditions to guarantee that the unforced system is stochastically stable and the disturbance rejection of level δ.

Theorem 3.4: *Assume that there exists a positive scalar* δ *and a set of positive-definite matrices* $P(i)$ *satisfying* *the following LMI for each $i \in S$:*

$$\begin{bmatrix} \widetilde{C}^{\mathrm{T}}(i)\widetilde{C}(i) + \widetilde{A}^{\mathrm{T}}(i)\mathbb{P}^{(i)}\widetilde{A}(i) - P(i) & \widetilde{A}^{\mathrm{T}}(i)\mathbb{P}^{(i)}\widetilde{G}(i) \\ \widetilde{G}^{\mathrm{T}}(i)\mathbb{P}^{(i)}\widetilde{A}(i) & \widetilde{G}^{\mathrm{T}}(i)\mathbb{P}^{(i)}\widetilde{G}(i) - \delta^2 I \end{bmatrix} < 0, \tag{5}$$

then augmented system (4) *with $u(k) = 0$ is stochastically stable and guarantees the validity of the following inequality:*

$$\|e\|_2^2 \leq \delta^2 \|\widetilde{w}\|_2^2, \tag{6}$$

which means that the system with $u(k) = 0$ for each $i \in S$ is stochastically stable with δ-disturbance attenuation.

Proof: From LMI (5) and using Schur complement, we get the following inequality:

$$\widetilde{C}^{\mathrm{T}}(i)\widetilde{C}(i) + \widetilde{A}^{\mathrm{T}}(i)\mathbb{P}^{(i)}\widetilde{A}(i) - P(i) < 0.$$

Since $\widetilde{C}^{\mathrm{T}}(i)\widetilde{C}(i) > 0$, then we can get the following:

$$\widetilde{A}^{\mathrm{T}}(i)\mathbb{P}^{(i)}\widetilde{A}(i) - P(i) < 0.$$

Based on Lemma 3.2, it proves that system (4) with $u(k) = 0$ is stochastically stable.

Let us prove that relation (6) is satisfied. Define a performance function candidate as follows:

$$J_T = \sum_{k=0}^{T} \left[\mathrm{e}^{\mathrm{T}}(k)e(k) - \delta^2 \widetilde{w}^{\mathrm{T}}(k)\widetilde{w}(k) \right].$$

Note that for a Lyapunov candidate

$$V(\Psi(k), r(k)) = \Psi^{\mathrm{T}}(k)P(r(k))\Psi(k).$$

If at time k, $r(k) = i$, $i \in S$, and $V(\Psi(k), r(k)) = \Psi(k)^{\mathrm{T}}P(i)\Psi(k)$, the infinitesimal operator acting on $V(\cdot)$ and emanating from the point $(\Psi(k), i)$ at time k is given by

$$\begin{aligned} &\Xi\left[\mathfrak{L}V(\Psi(k), i)\right] \\ &\quad = \Xi\left[V\left(\Psi(k+1), r(k+1)\right) - V(\Psi(k), i)\right] \\ &\quad = \Psi^{\mathrm{T}}(k+1)\mathbb{P}^{(i)}\Psi(k+1) - \Psi^{\mathrm{T}}(k)P(i)\Psi(k), \end{aligned}$$

then we can get the following:

$$\sum_{k=0}^{T} \{\Xi\left[\mathfrak{L}V(\Psi(k), i)\right]\} \triangleq \Xi\left[V\left(\Psi(T+1), r(T+1)\right) - V(\Psi(0), r(0))\right],$$

and

$$\begin{aligned} &\mathrm{e}^{T}(k)e(k) - \delta^2 \widetilde{w}^{\mathrm{T}}(k)\widetilde{w}(k) \\ &\quad = \left[\widetilde{C}(r(k))\Psi(k)\right]^{\mathrm{T}} \left[\widetilde{C}(r(k))\Psi(k)\right] - \delta^2 \widetilde{w}^{\mathrm{T}}(k)\widetilde{w}(k) \\ &\quad = \Psi^{T}(k)\widetilde{C}^{\mathrm{T}}(i)\widetilde{C}(i)\Psi(k) - \delta^2 \widetilde{w}^{\mathrm{T}}(k)\widetilde{w}(k), \end{aligned}$$

then

$$\begin{aligned} &\mathrm{e}^{\mathrm{T}}(k)e(k) - \delta^2 \widetilde{w}^{\mathrm{T}}(k)\widetilde{w}(k) + \Xi\left[\mathfrak{L}V(\Psi(k), i)\right] \\ &\quad = \Psi^{\mathrm{T}}(k)\widetilde{C}^{\mathrm{T}}(i)\widetilde{C}(i)\Psi(k) - \delta^2 \widetilde{w}^{\mathrm{T}}(k)\widetilde{w}(k) \\ &\qquad + \Xi\left\{V\left[\Psi(k+1), r(k+1)\right] - V(\Psi(k), i)\right\} \\ &\quad = \Psi^{\mathrm{T}}(k)\widetilde{C}^{\mathrm{T}}(i)\widetilde{C}(i)\Psi(k) - \delta^2 \widetilde{w}^{\mathrm{T}}(k)\widetilde{w}(k) \\ &\qquad + \Psi^{\mathrm{T}}(k+1)\mathbb{P}^{(i)}\Psi(k+1) - \Psi^{\mathrm{T}}(k)P(i)\Psi(k) \\ &\quad = \Psi^{\mathrm{T}}(k)\widetilde{C}^{\mathrm{T}}(i)\widetilde{C}(i)\Psi(k) - \delta^2 \widetilde{w}^{\mathrm{T}}(k)\widetilde{w}(k) \\ &\qquad - \Psi^{\mathrm{T}}(k)P(i)\Psi(k) \\ &\qquad + \left(\widetilde{A}(i)\Psi(k) + \widetilde{G}(i)\widetilde{w}(k)\right)^{\mathrm{T}} \\ &\qquad \times \mathbb{P}^{(i)} \left(\widetilde{A}(i)\Psi(k) + \widetilde{G}(i)\widetilde{w}(k)\right) \\ &\quad = \Psi^{\mathrm{T}}(k)\left[\widetilde{C}^{\mathrm{T}}(i)\widetilde{C}(i) + \widetilde{A}^{\mathrm{T}}(i)\mathbb{P}^{(i)}\widetilde{A}(i) - P(i)\right]\Psi(k) \\ &\qquad + \Psi^{\mathrm{T}}(k)\widetilde{A}^{\mathrm{T}}(i)\mathbb{P}^{(i)}\widetilde{G}(i)\widetilde{w}(k) \\ &\qquad + \widetilde{w}^{\mathrm{T}}(k)\widetilde{G}^{\mathrm{T}}(i)\mathbb{P}^{(i)}\widetilde{A}(i)\Psi(k) \\ &\qquad + \widetilde{w}^{\mathrm{T}}(k)\left[\widetilde{G}^{\mathrm{T}}(i)\mathbb{P}^{(i)}\widetilde{G}(i) - \delta^2 I\right]\widetilde{w}(k), \end{aligned}$$

which implies the following equality:

$$\begin{aligned} &\mathrm{e}^{\mathrm{T}}(k)e(k) - \delta^2 \widetilde{w}^{\mathrm{T}}(k)\widetilde{w}(k) + \Xi\left[\mathfrak{L}V(\Psi(k), i)\right] \\ &\quad = \eta^{\mathrm{T}}(k)\Upsilon(i)\eta(k), \end{aligned}$$

with

$$\Upsilon(i) = \begin{bmatrix} \widetilde{C}^{\mathrm{T}}(i)\widetilde{C}(i) + \widetilde{A}^{\mathrm{T}}(i)\mathbb{P}^{(i)}\widetilde{A}(i) - P(i) & \widetilde{A}^{\mathrm{T}}(i)\mathbb{P}^{(i)}\widetilde{G}(i) \\ \widetilde{G}^{\mathrm{T}}(i)\mathbb{P}^{(i)}\widetilde{A}(i) & \widetilde{G}^{\mathrm{T}}(i)\mathbb{P}^{(i)}\widetilde{G}(i) - \delta^2 I \end{bmatrix},$$

$$\eta^{\mathrm{T}}(k) = \begin{bmatrix} \Psi^{\mathrm{T}}(k) & \widetilde{w}^{\mathrm{T}}(k) \end{bmatrix}.$$

Hence, from the previous definitions, the performance function also read

$$\begin{aligned} J_T = &\sum_{k=0}^{T} \left\{ \mathrm{e}^{\mathrm{T}}(k)e(k) - \delta^2 w^{\mathrm{T}}(k)w(k) \right. \\ &+ \Xi\left[\mathfrak{L}V(\Psi(k), i)\right]\} - \sum_{k=0}^{T} \{\Xi\left[\mathfrak{L}V(\Psi(k), i)\right]\} \end{aligned}$$

$$= \sum_{k=0}^{T} \left[\eta^{\mathrm{T}}(k) \Upsilon(i) \eta(k) \right] - \Xi \left[V\left(\Psi(T+1), r(T+1)\right) - V(\Psi(0), r(0)) \right].$$

Since $\Upsilon(i) < 0$ for each $i \in S$, and $\Xi\{V[\Psi(T+1), r(T+1)]\} \geq 0$, then the following inequality holds

$$J_T \leq V\left(\Psi(0), r(0)\right),$$

which implies $J_\infty \leq V(\Psi(0), r(0))$.

If the initial conditions are equal to zero, it implies that $J_\infty \leq 0$, then we can get the following:

$$\|e\|_2^2 \leq \delta^2 \|\tilde{w}\|_2^2,$$

which ends the proof of the theorem. ∎

In what follows, based on the results of Theorem 3.4, we will design a state feedback controller that stabilises augmented system (4) and rejects the external disturbance. The controller is given by the following expression:

$$u(k) = K(r(k))\Psi(k), \tag{7}$$

The closed-loop augmented system will be stable and guarantees the disturbance attenuation of level δ if there exists a set of symmetric and positive-definite matrices $\widetilde{P}(i)$ such that the following condition holds for each $i \in S$:

$$\begin{bmatrix} \widetilde{C}^{\mathrm{T}}(i)\widetilde{C}(i) + \widetilde{A}_{cl}^{\mathrm{T}}(i) \widetilde{\mathbb{P}}^{(i)} \widetilde{A}_{cl}(i) - \widetilde{P}(i) & \widetilde{A}_{cl}^{\mathrm{T}}(i)\widetilde{\mathbb{P}}^{(i)}\widetilde{G}(i) \\ \widetilde{G}^{\mathrm{T}}(i)\widetilde{\mathbb{P}}^{(i)}\widetilde{A}_{cl}(i) & \widetilde{G}^{\mathrm{T}}(i)\widetilde{\mathbb{P}}^{(i)}\widetilde{G}(i) - \delta^2 I \end{bmatrix} < 0,$$

where $\widetilde{A}_{cl}(i) = \widetilde{A}(i) + \widetilde{B}(i)K(i)$, $\widetilde{\mathbb{P}}^{(i)} = \sum_{j \in S} \pi_{ij}\widetilde{P}(j)$.

Let $\overline{P}$ and $W(i)$ be defined as follows:

$$\overline{P} = \mathrm{diag}\left[\widetilde{P}(1), \ldots, \widetilde{P}(N)\right],$$
$$W(i) = [\sqrt{\pi_{i1}}I, \ldots, \sqrt{\pi_{iN}}I],$$

then the following coupled matrix holds

$$\begin{bmatrix} \widetilde{A}_{cl}^{\mathrm{T}}(i)W(i) \overline{P}W^{\mathrm{T}}(i)\widetilde{A}_{cl}(i) + \widetilde{C}^{\mathrm{T}}(i)\widetilde{C}(i) - \widetilde{P}(i) & \widetilde{A}_{cl}^{\mathrm{T}}(i)W(i)\overline{P} W^{\mathrm{T}}(i)\widetilde{G}(i) \\ \widetilde{G}^{\mathrm{T}}(i)W(i) \overline{P}W^{\mathrm{T}}(i)\widetilde{A}_{cl}(i) & \widetilde{G}^{\mathrm{T}}(i)W(i)\overline{P}W^{\mathrm{T}}(i) \widetilde{G}(i) - \delta^2 I \end{bmatrix} < 0.$$

This matrix inequality can be rewritten as follows:

$$\begin{bmatrix} -\widetilde{P}(i) + \widetilde{C}^{\mathrm{T}}(i)\widetilde{C}(i) & 0 & \widetilde{A}_{cl}^{\mathrm{T}}(i)W(i) \\ 0 & -\delta^2 I & \widetilde{G}^{\mathrm{T}}(i)W(i) \\ W^{\mathrm{T}}(i)\widetilde{A}_{cl}(i) & W^{\mathrm{T}}(i)\widetilde{G}(i) & -\overline{P}^{-1} \end{bmatrix} < 0. \tag{8}$$

Let $X(i) = \widetilde{P}^{-1}(i)$ and define $\mathfrak{X}$ and $Y(i)$ as follows:

$$\mathfrak{X} = \mathrm{diag}\left[X(1), \ldots, X(N)\right],$$
$$Y(i) = K(i)X(i).$$

Pre- and post-multiplying inequality (8) by diag$[X(i), I, I]$ and using the Schur complement, we get the results of the following theorem.

Theorem 3.5: *Assume that there exist a positive scalar δ, symmetric and positive-definite matrices $X(1), \ldots, X(N)$ and $Y(1), \ldots, Y(N)$ satisfying the following LMI for each $i \in S$:*

$$\begin{bmatrix} -X(i) & 0 & \left[\widetilde{A}(i)X(i) + \widetilde{B}(i)Y(i)\right]^{\mathrm{T}} W(i) & X(i)\widetilde{C}^{\mathrm{T}}(i) \\ 0 & -\delta^2 I & \widetilde{G}^{\mathrm{T}}(i)W(i) & 0 \\ W^{\mathrm{T}}(i)\left[\widetilde{A}(i)X(i) + \widetilde{B}(i)Y(i)\right] & W^{\mathrm{T}}(i)\widetilde{G}(i) & -\mathfrak{X} & 0 \\ \widetilde{C}(i)X(i) & 0 & 0 & -I \end{bmatrix} < 0, \tag{9}$$

then augmented system (4) under controller (7) with $K(i) = Y(i)X^{-1}(i)$ is stochastically stable and guarantees the validity of the following inequality:

$$\|e\|_2^2 \leq \delta^2 \|\tilde{w}\|_2^2,$$

which means that the system is stochastically stable with δ-disturbance attenuation.

From the practical point of view, it is important to design a controller to stabilise the system and guarantee the minimum disturbance rejection simultaneously. This controller can be obtained by solving the following optimisation problem:

$$Q: \begin{cases} \min\limits_{\substack{\delta>0 \\ X(1),\ldots,X(N)>0 \\ Y(1),\ldots,Y(N)>0}} & \delta, \\ \text{s.t.}: & \text{LMI (9) holds.} \end{cases}$$

Remark 3.1: The feasibility of conditions stated in Theorem 3.5 can be turned into the above LMIs based feasibility problem with a parameter δ. LMI technique plays an important role in the derivation of a less conservative criterion. The criteria is extended into convex optimisation problems to obtain the optimal disturbance level.

Most results in the existing literature were obtained under the assumption that the state feedback controller is dependent on system modes. However, in some practical situations, the exact values of only partial system modes can be accessible, or it would be very expensive to obtain all system modes. Thus, we will relax this assumption and try to develop a common MRTC approach with completely unknown system modes. The state feedback controller has the following form:

$$u(k) = \mathcal{K}\Psi(k), \tag{10}$$

where $\mathcal{K}$ is a constant gain.

Before designing the controller, let us give some sufficient conditions when a constant matrix independent on system modes is used in the Lyapunov function. The corresponding results are summarised by the following corollary:

Corollary 3.6: *Assume that there exists a positive scalar δ and a positive-definite matrix P satisfying the following LMI for each $i \in S$:*

$$\begin{bmatrix} \begin{matrix}\widetilde{C}^{\mathrm{T}}(i)\widetilde{C}(i)\\ +\widetilde{A}^{\mathrm{T}}(i)P\widetilde{A}(i) - P\end{matrix} & \widetilde{A}^{\mathrm{T}}(i)P\widetilde{G}(i) \\ \widetilde{G}^{\mathrm{T}}(i)P\widetilde{A}(i) & \widetilde{G}^{\mathrm{T}}(i)P\widetilde{G}(i) - \delta^2 I \end{bmatrix} < 0, \tag{11}$$

then augmented system (4) *with $u(k) = 0$ is stochastically stable and guarantees the validity of the following inequality*:

$$\|e\|_2^2 \le \delta^2 \, \|\widetilde{w}\|_2^2,$$

which means that the system with $u(k) = 0$ for each $i \in S$ is stochastically stable with δ-disturbance attenuation.

Proof: Let us consider the following candidate Lyapunov function:

$$V(\Psi(k), i) = \Psi^{\mathrm{T}}(k)P\Psi(k),$$

where P is a symmetric and positive-definite matrix.

Following the same steps as for Theorem 3.4 and using the fact that $\sum_{j=1}^{N} \pi_{ij} = 1$, we can get the results of this corollary. ■

Let us use the results of this corollary and focus on the design of the state feedback controller with constant gain. For this purpose, notice that the closed-loop system under this controller will be stochastically stable if there exists a symmetric and positive-definite matrix $\widetilde{P}$ such that the following inequality holds for each $i \in S$:

$$\begin{bmatrix} -\widetilde{P} + \widetilde{C}^{\mathrm{T}}(i)\widetilde{C}(i) & 0 & \widetilde{A}_{cl}^{\mathrm{T}}(i) \\ 0 & -\delta^2 I & \widetilde{G}^{\mathrm{T}}(i) \\ \widetilde{A}_{cl}(i) & \widetilde{G}(i) & -\widetilde{P}^{-1} \end{bmatrix} < 0.$$

Let $X = \widetilde{P}^{-1}$ and define Y as follows:

$$Y = \mathcal{K}X. \tag{12}$$

Pre- and post-multiplying the previous inequality by $\mathrm{diag}[X, I, I]$ and using Schur complement, we get the required LMI design conditions for such a state feedback controller. The corollary can be summarised as follows:

Corollary 3.7: *Assume that there exists a positive scalars δ, symmetric and positive-definite matrices X and Y satisfying the following LMI for each $i \in S$:*

$$\begin{bmatrix} -X & 0 & [\widetilde{A}(i)X + \widetilde{B}(i)Y]^{\mathrm{T}} & X\widetilde{C}^{\mathrm{T}}(i) \\ 0 & -\delta^2 I & \widetilde{G}^{\mathrm{T}}(i) & 0 \\ \widetilde{A}(i)X + \widetilde{B}(i)Y & \widetilde{G}(i) & -X & 0 \\ \widetilde{C}(i)X & 0 & 0 & -I \end{bmatrix} < 0, \tag{13}$$

then augmented system (4) *under controller* (10) *with $\mathcal{K} = YX^{-1}$ is stochastically stable and guarantees the validity of the following inequality*:

$$\|e\|_2^2 \le \delta^2 \, \|\widetilde{w}\|_2^2,$$

which means that the system is stochastically stable with δ-disturbance attenuation.

The results of this corollary will allow us to determine the state feedback gain $\mathcal{K}$, but there is no guarantee that the control will not exceed its bounds which are imposed by physics and hard constraints that we should always follow. Consider the control input constraints in practice, give

$$\|u(k)\|_2 < u_{\max}.$$

where $u_{\max}$ represents the maximum control input.

Remark 3.2: In the rest of this article, we will focus on the mode-independent controller. The developed results can be extended easily to the mode-dependent controller in Theorem 3.5.

Define ellipsoid set ϖ by

$$\varpi = \left\{ \Psi \in \mathbb{R}^{2n} \mid \Psi^{\mathrm{T}} X^{-1} \Psi \leq 1 \right\}.$$

When the initial state is known, we can find an upper bound of the control input, $u(k) = \mathcal{K}\Psi(k)$. Take the matrices X and Y obtained by solving the LMI (13) and add it to the following condition:

$$\Psi_0^{\mathrm{T}} X^{-1} \Psi_0 \leq 1.$$

Using Schur complement, we can get the following:

$$\begin{bmatrix} 1 & \Psi_0^{\mathrm{T}} \\ \Psi_0 & X \end{bmatrix} \geq 0.$$

This guarantees that the control vector will always remain in the ellipsoid ϖ, then we can get the upper bound of the control input

$$\begin{aligned} \max_{k\geq 0} \|u(k)\|_2^2 &= \max_{k\geq 0} \|\mathcal{K}\Psi(k)\|_2^2 = \max_{k\geq 0} \left\| YX^{-1}\Psi(k) \right\|_2^2 \\ &\leq \max_{\Psi_0 \in \varpi} \left\| YX^{-1}\Psi(k) \right\|_2^2 \\ &\leq \left\| YX^{-1/2} \right\|_2^2 \left\| X^{-1/2}\Psi(k) \right\|_2^2 \\ &= \left(YX^{-1}Y^{\mathrm{T}} \right) \Psi^{\mathrm{T}}(k) X^{-1} \Psi(k) \\ &\leq YX^{-1}Y^{\mathrm{T}}. \end{aligned}$$

The constraint $\|u(k)\|_2 < u_{\max}$ will be enforced if there exists a symmetric and positive-definite matrix Z such that

$$\begin{bmatrix} Z & Y \\ Y^{\mathrm{T}} & X \end{bmatrix} \geq 0,$$

with $Z \leq u_{\max}^2 I$.

Using these developments, the following corollary is proposed to show the conditions which guarantee the stability, the disturbance rejection and the input constraints of augmented system (4).

Corollary 3.8: *Assume that there exists a positive scalars δ, symmetric and positive-definite matrices X, Y and Z satisfying the following set of LMI for each $i \in S$:*

$$\begin{bmatrix} -X & 0 & [\widetilde{A}(i)X + \widetilde{B}(i)Y]^{\mathrm{T}} & X\widetilde{C}^{\mathrm{T}}(i) \\ 0 & -\delta^2 I & \widetilde{G}^{\mathrm{T}}(i) & 0 \\ \widetilde{A}(i)X + \widetilde{B}(i)Y & \widetilde{G}(i) & -X & 0 \\ \widetilde{C}(i)X & 0 & 0 & -I \end{bmatrix} < 0,$$

$$\begin{bmatrix} Z & Y \\ Y^{\mathrm{T}} & X \end{bmatrix} \geq 0,$$

$$\begin{bmatrix} 1 & \Psi_0^{\mathrm{T}} \\ \Psi_0 & X \end{bmatrix} \geq 0, \tag{14}$$

with $Z \leq u_{\max}^2 I$, then augmented system (4) under controller (10) with $\mathcal{K} = YX^{-1}$ is stochastically stable and guarantees the validity of the following inequalities:

$$\|e\|_2^2 \leq \delta^2 \|\widetilde{w}\|_2^2,$$
$$\|u(k)\|_2 < u_{\max},$$

which means that the system is stochastically stable with δ-disturbance attenuation and guarantees the input constraints.

Similarly, the state feedback controller that guarantees the stability, the minimum disturbance rejection and the input constraints of augmented system (4) is obtained by solving the following optimisation problem:

$$Q1 : \begin{cases} \min\limits_{\substack{\delta>0 \\ X>0 \\ Y>0 \\ Z>0}} & \delta, \\ \text{s.t.}: & \text{LMI (14) holds}, \quad Z \leq u_{\max}^2 I. \end{cases}$$

Let $\delta > 0$, X and Y be the solution of the optimisation problem Q1. Then, controller (12) with $\mathcal{K} = YX^{-1}$ stabilises augmented system (4); moreover, the system satisfies the minimum disturbance rejection of level δ.

Remark 3.3: When the input constraints and completely unknown system modes are present, the tracking relation cannot be guaranteed, controller (10) is feasible and effective to guarantee the stability, the disturbance rejection and the input constraints. At the same time, the performance index δ can be minimised and the tracking relation is optimal.

4. Numerical example

In this section, we give an example to demonstrate the effectiveness of the proposed method. Consider a system with two operation modes, described by models

$\mathcal{G}_1$ and $\mathcal{G}_2$ given by

$$\mathcal{G}_1 : x_p(k+1) = \begin{bmatrix} 0.8 & 0.9 \\ 1.2 & 1 \end{bmatrix} x_p(k) + \begin{bmatrix} 0.8 \\ 0.6 \end{bmatrix} u(k) + \begin{bmatrix} 0.3 \\ 0.2 \end{bmatrix} w(k),$$

$$\mathcal{G}_2 : x_p(k+1) = \begin{bmatrix} 0.7 & 1.5 \\ 1.3 & 0.9 \end{bmatrix} x_p(k) + \begin{bmatrix} 0.5 \\ 0.4 \end{bmatrix} u(k) + \begin{bmatrix} 0.15 \\ 0.2 \end{bmatrix} w(k).$$

The two cases of the transition probabilities matrix are considered as follows:

$$\Pi_1 = \begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix}, \quad \Pi_2 = \begin{pmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{pmatrix}.$$

The reference model that we would like to track is described by the following matrices:

$$\mathcal{G}_m : x_m(k+1) = \begin{bmatrix} -0.2 & 0.1 \\ 0.23 & 0.6 \end{bmatrix} x_p(k) + \begin{bmatrix} 0.2 \\ 0.15 \end{bmatrix} v(k).$$

And then, by LMI's Toolbox of Matlab, the optimal values $\delta_1 = 0.49$, $\delta_2 = 0.5$ can be solved by inequality (9) in Theorem 3.5 with the following control gains:

Case 1:

$$K(1) = [-1.2898 \quad -1.2816 \quad -0.0125 \quad 0.4538],$$
$$K(2) = [-2.0811 \quad -2.7250 \quad 0.0337 \quad 0.7775].$$

Case 2:

$$K(1) = [-1.3053 \quad -1.2901 \quad -0.0108 \quad 0.4502],$$
$$K(2) = [-2.0647 \quad -2.7318 \quad 0.0330 \quad 0.7845].$$

For simulation purpose, in case 1, we consider that the external disturbance is given by the following expression:

$$w(k) = 0.8 \times \sin(kT), \quad \forall\, k \geq 0, T \text{ is sample time.}$$

Figure 1 is the modes' evolution $r(k)^1$, Figure 2 is the state response of the corresponding closed-loop system for given initial condition $\Psi_0^T = [3 \;\; 5 \;\; 1 \;\; 2]$ and Figure 3 shows the tracking errors of the corresponding closed-loop system under the modes' evolution $r(k)^1$.

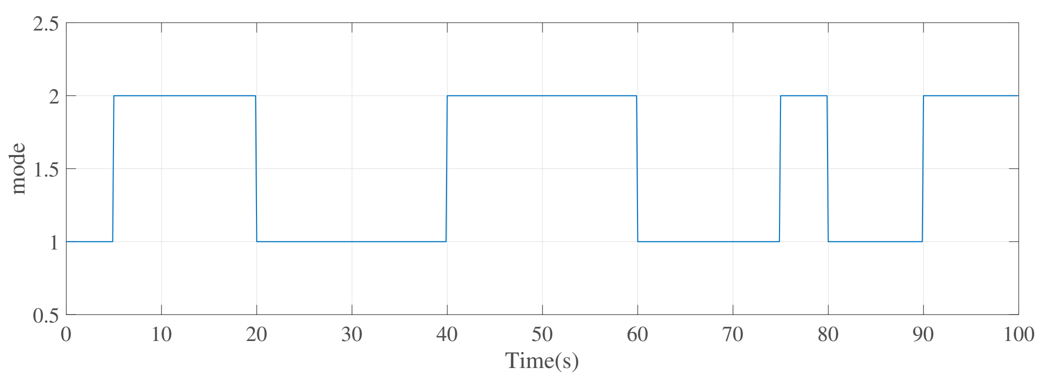

Figure 1. The Markov process under mode evolution $r(k)^1$.

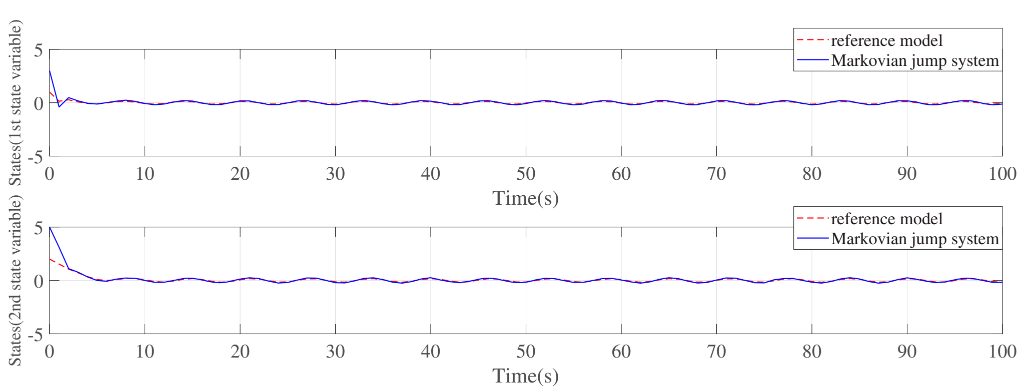

Figure 2. The state responses of the closed-loop system under mode evolution $r(k)^1$.

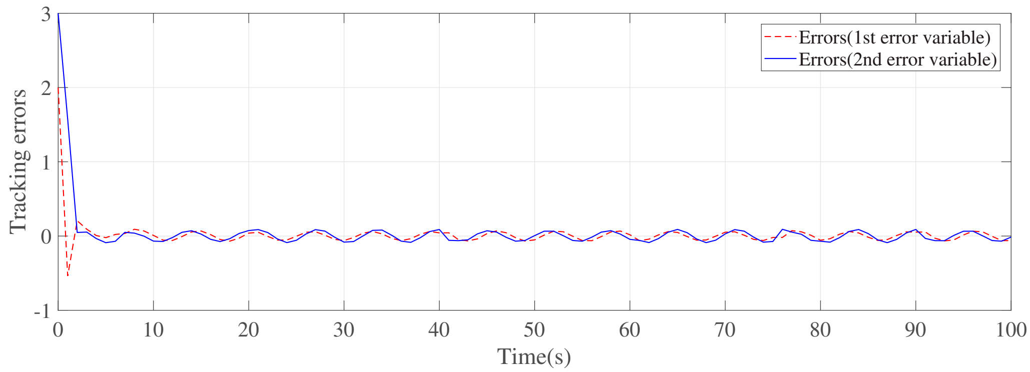

Figure 3. The tracking errors of the closed-loop system under mode evolution $r(k)^1$.

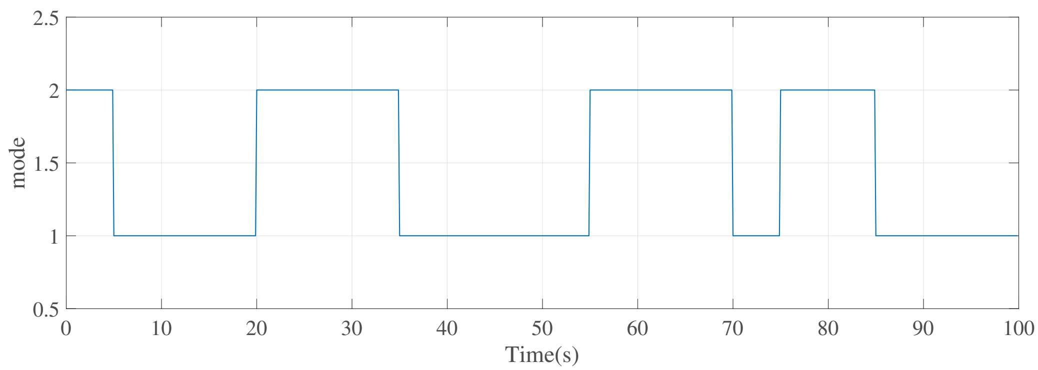

Figure 4. The Markov process under mode evolution $r(k)^2$.

In case 2, the external disturbance is given by the following expression:

$$w(k) = 2 \times \sin(0.8kT), \quad \forall\, k \geq 0, T \text{ is sample time}$$

the reference model input $v(k) = 5 \times \sin(0.8kT)$, Figures 4 is the modes' evolution $r(k)^2$, Figure 5 shows the state response of the corresponding closed-loop system for given initial condition $\Psi_0^T = [3 \;\; 5 \;\; 2 \;\; 3]$, Figure 6 shows the tracking errors of the corresponding closed-loop system under the modes' evolution $r(k)^2$.

From the curves in Figures 1–6, we can easily see that the designed controllers are feasible and effective to ensure that the closed-loop system is stable with the disturbance rejection of level 0.49 and 0.5, respectively.

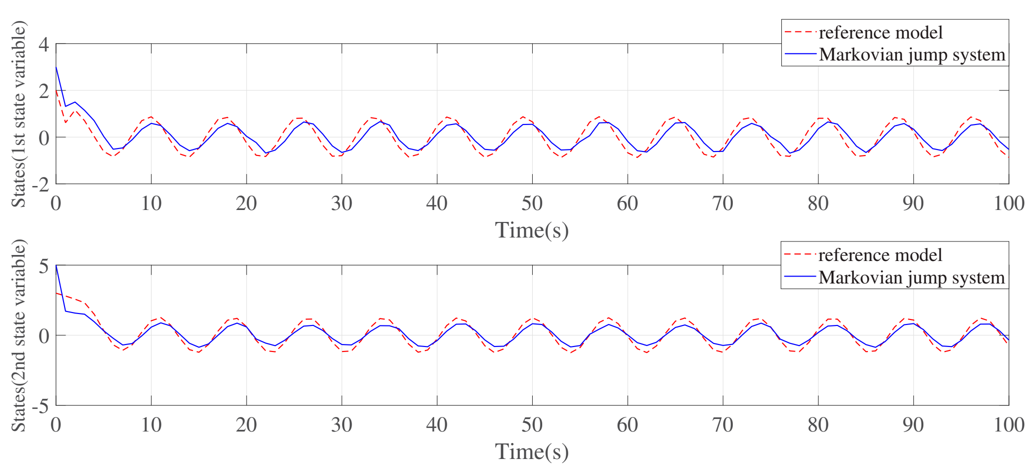

Figure 5. The state responses of the closed-loop system under mode evolution $r(k)^2$.

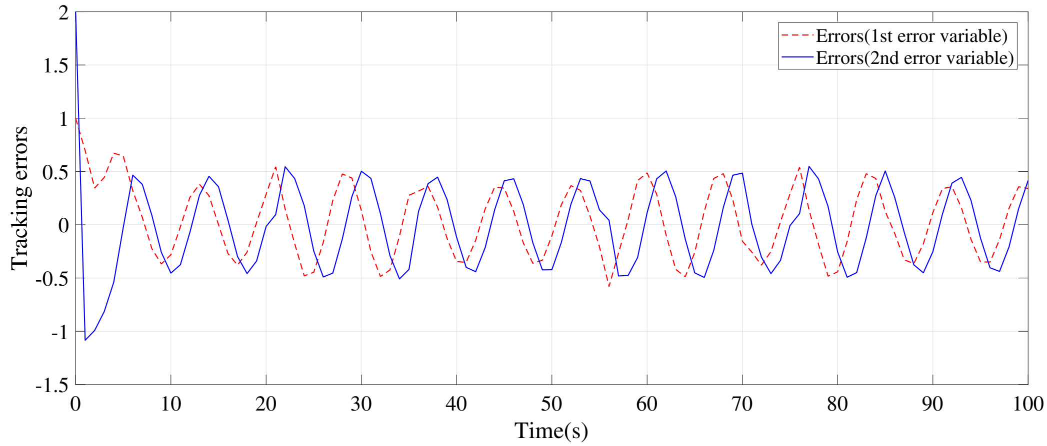

Figure 6. The tracking errors of the closed-loop system under mode evolution $r(k)^2$.

Remark 4.1: Increasing the reference input, the state responses of the corresponding closed-loop reference model become unstable, the designed controllers are feasible and effective, ensuring the resulting closed-loop systems can track the reference model. Compared with case 1, the tracking errors are larger in case 2 because the disturbances are larger. The relations between tracking errors and disturbances satisfy δ-disturbance attenuation both in case 1 and case 2.

5. Conclusion

The MRTC problem of discrete-time Markovian jump systems with the minimum disturbance rejection and input constraints is considered. Based on a set of LMI constraints, the state feedback controller is designed, and the augmented system is stochastically stable with δ-disturbance attenuation. The main results can also be used for the Markovian jump system in the general sense, i.e. the case of completely known system modes and the case of completely unknown system modes. A numerical example is given to demonstrate the applicability of the proposed method.

In the future, we will investigate the MRTC problem of Markovian jump nonlinear systems. On the other hand, the Markovian jump systems with a more general transition rate will be further considered.

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