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Asynchronous H_∞ filtering of continuous-time Markov jump systems

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Summary

The paper investigates the asynchronous H_∞ filtering design problem for continuous-time linear systems with Markov jump. The hidden Markov jump principle is applied to represent the asynchronous situation between the target system and the designed filter. Via a Lyapunov technique, two sufficient conditions are developed to guarantee that the filtering error system is stochastically stable with a prescribed H_∞ noise attenuation level. Furthermore, three filtering design approaches are developed in the form of linear matrix inequalities. Finally, one example is provided to show the effectiveness and feasibility of the developed methods.

KEYWORDS

asynchronous H_∞ filtering, Lyapunov technique, Markov jump

1 | INTRODUCTION

The state estimation or filtering problem is a significant issue in signal processing and control areas.[1-3] To tackle it, many effective techniques have been proposed such as the H_∞ filtering approach and the Kalman filtering method. Compared with the Kalman filtering method, H_∞ estimation shows great advantages in managing bounded disturbances with unknown statistic information. Recently, it has been widely applied in various areas, for example, networked control systems[4] and two-dimensional uncertain fuzzy systems.[5]

In fact, practical systems often undergo abrupt environmental noises, uncertain devices' failures, changes in operational points, and so on. These unexpected factors often result in sudden changes of system construction or parameters. The Markov jump principle has been regarded as an efficient and appropriate technique to model such complex circumstances, and many works on Markov jump systems (MJSs) have been reported.[6-12] The optimal mean square filter design issue has been addressed in the work of Vergés and Fragoso[13] for Markov jump linear systems (MJLSs). The optimal sampled-data control via state feedback has been studied in the work of Geromel and Gabriel[14] for MJLSs. As for positive MJLSs, researchers have analyzed the stochastic stability in the work of Bolzern et al.[15]

For MJSs, mode-dependent filters or controllers receive much more popularity than mode-independent ones because they make full use of jump knowledge and bring less conservatism. Mode-dependence means synchronization, which implies that we must ensure the accessibility of system modes. However, reasons including time delays and uncertainties make this condition hard to satisfy and result in the inaccessibility of jump modes. This implies that there are plenty of nonsynchronous phenomena between the original system and the devised controller or filter.[16,17] Thus, the investigation on asynchronization becomes more important. Dong et al[18] have investigated the asynchronous dissipative control issue for continuous-time fuzzy MJSs via applying the hidden Markov model (HMM). Using HMM, an asynchronous and periodic filter has been designed in the work of Shen et al[19] for periodic MJSs. Stadtmann and Costa[20] have also adopted HMM to detect jump modes when designing an H_∞ controller for MJLSs.

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This paper analyzes an asynchronous H_∞ filtering design issue for continuous-time MJLSs. A nonsynchronous filter model is constructed via HMM, whose modes are acquired from observation of the target system jump. Applying Lyapunov function and convex optimization techniques, we devise an asynchronous H_∞ filter to ensure that the filtering error system is stochastically stable, and the designed filter can also meet the requirement of a given H_∞ noise attenuation level. At last, one example is given to show the validity and correctness of the derived approaches. Comparing with some published papers, two primary contributions are given as follows. (1) As for continuous-time MJLSs, two sufficient conditions are proposed to guarantee the stochastic stability of the filtering error systems with a given H_∞ performance level. (2) Three sufficient conditions for solving filter gains have been developed. The first one is to select an appropriate Lyapunov function matrix P with special structure, which has been adopted by many reported works such as those of Chang et al.[21,22] The second is to use some slack matrices for eliminating the coupling between system matrices and Lyapunov function matrix and removing some constraints on Lyapunov function matrix. Then, based on the developed second method, we further manage to remove some unnecessary matrices, decrease the number of unknown variables, and obtain the third algorithm. The comparisons among three different design approaches are given in the simulation section.

Notation. Define $(\Omega, \mathcal{F}, \mathcal{P})$ as a probability space carrying its natural filtration $\{\mathcal{F}_t,\ t \in R^+\}$, where $\Omega, \mathcal{F}, \mathcal{P}$ are the sample space, the σ-algebra of subsets of the sample space, and the probability measure on $\mathcal{F}$, respectively. $E\{\cdot\}$ is the mathematical expectation. The superscript "T" stands for the matrix transpose. $X > 0$ implies that matrix X is real symmetric and positive definite. The symbol $*$ means a symmetric term in a matrix. $Her(X)$ denotes X^T+X. l_2 represents the space of square Lebesgue integrable function. For $w(t) \in l_2$, its 2-norm is defined as $\|w(t)\|_2 \triangleq \sqrt{\int_0^\infty |w(t)|^2 dt}$, where $|\cdot|$ is the Euclidean norm of a vector.

2 | PRELIMINARY ANALYSIS

Consider the following MJLSs on the probability space $(\Omega, \mathcal{F}, \mathcal{P})$:

$$\begin{cases} \dot{x}(t) = A_{\delta(t)}x(t) + B_{\delta(t)}w(t), \\ y(t) = C_{\delta(t)}x(t) + D_{\delta(t)}w(t), \\ z(t) = E_{\delta(t)}x(t) + F_{\delta(t)}w(t), \end{cases} \tag{1}$$

where variables $x(t) \in R^n$ is the state, $y(t) \in R^p$ is the measured output, $z(t) \in R^q$ is the signal to be estimated, and $w(t) \in R^g$ is the disturbance belonging to $l_2[0, +\infty)$. The parameter $\delta(t)$ ($\delta(t) \in \mathcal{R} = \{1, 2, \dots, r\}$) is applied to describe a homogenous Markov jump process with right continuous trajectories, and it satisfies the stationary transition rate matrix $\Phi = [\theta_{ij}]$ with

$$\Pr\{\delta(t+\Delta t) = j | \delta(t) = i\} = \begin{cases} \theta_{ij}\Delta t + o(\Delta t), & j \neq i, \\ 1 + \theta_{ii}\Delta t + o(\Delta t), & j = i, \end{cases} \tag{2}$$

where Δt ($\Delta t > 0$) satisfies $\lim_{\Delta t \to 0} \frac{o(\Delta t)}{\Delta t} = 0$. The variable θ_{ij} is the transition rate for mode i at time t to mode j at time $t + \Delta t$ with $\theta_{ij} \geq 0$, $i \neq j$ and $\theta_{ii} = -\sum_{j=1, j\neq i}^{r} \theta_{ij}$.

In this work, we are dedicated to designing a nonsynchronous filter as follows:

$$\begin{cases} \dot{\hat{x}}(t) = \hat{A}_{\rho(t)}\hat{x}(t) + \hat{B}_{\rho(t)}y(t), \\ \hat{z}(t) = \hat{E}_{\rho(t)}\hat{x}(t) + \hat{F}_{\rho(t)}y(t), \end{cases} \tag{3}$$

where $\hat{x}(t) \in R^n$ is the filter state and $\hat{z}(t) \in R^q$ is the filter output. Because information of system modes is not directly accessible to the designed filter, we adopt a stochastic variable $\rho(t)$ to detect jump mode $\delta(t)$. Though this differs from Markov process $(\delta(t), t)$, it relies on $\delta(t)$ and satisfies the conditional transition probability matrix $\Psi = [\pi_{im}]$ ($\pi_{im} \geq 0$) with

$$\Pr\{\rho(t) = m | \delta(t) = i\} = \pi_{im}, \quad \sum_{m=1}^{s} \pi_{im} = 1, \tag{4}$$

where $\rho(t)$ takes values in $\mathcal{S} = \{1, 2, \dots, s\}$. Thus, based on the work of Rabiner,[23] it is clearly observed that the set $(\delta(t), \rho(t), \Phi, \Psi)$ builds an HMM.

Remark 1. Hidden Markov model (HMM) is adopted to deal with the inaccessibility of jump modes in original systems (1). Based on HMM, filter modes in this paper are obtained by observing $\delta(t)$ via a stochastic variable $\rho(t)$. Because there is a great probability of the case $\delta(t) \neq \rho(t)$, we regard the designed filter (3) as an asynchronous one, which runs asynchronously with original systems (1). It is clear to find that the designed asynchronous filtering model (3) covers two special cases: When $\mathcal{S} = \{1\}$, a mode-independent filter is devised, and when $\Pr\{\rho(t) = i|\delta(t) = i\} = \pi_{ii} = 1$ and $\mathcal{S} = \mathcal{R}$, we can construct a mode-dependent (or synchronous) filter. Hidden Markov model (HMM) is also adopted to investigate the cluster observation problem, where only part of modes is accessible and thus clusters (disjoint sets) of observations are shaped. Many works have been published, such as H_2/H_∞ control,[24-26] mean-square stability analysis,[27] and H_∞ filtering design.[28]

Considering (1) and (3) with $\delta(t) = i$ and $\rho(t) = m$, we have the following filtering error system:

$$\begin{cases} \dot{\xi}(t) = \bar{A}_{im}\xi(t) + \bar{B}_{im}w(t), \\ e(t) = \bar{E}_{im}\xi(t) + \bar{F}_{im}w(t), \end{cases} \tag{5}$$

where

$$\bar{A}_{im} = \begin{bmatrix} A_i & 0 \\ \hat{B}_m C_i & \hat{A}_m \end{bmatrix}, \quad \bar{B}_{im} = \begin{bmatrix} B_i \\ \hat{B}_m D_i \end{bmatrix}, \quad \xi(t) = \begin{bmatrix} x^T(t) & \hat{x}^T(t) \end{bmatrix}^T,$$
$$\bar{E}_{im} = \begin{bmatrix} E_i - \hat{F}_m C_i - \hat{E}_m \end{bmatrix}, \quad \bar{F}_{im} = F_i - \hat{F}_m D_i, \quad e(t) = z(t) - \hat{z}(t).$$

We introduce three lemmas that are useful for investigating.

Lemma 1 (See the work of Ding et al[29]). *Let $x \in R^n$, $H \in R^{m\times n}$ with rank$(H) = r < n$ and $P = P^T \in R^{n\times n}$. The following statements are equivalent.*

1. $x^T P x < 0$ *for all* $x \neq 0$ *and* $Hx = 0$.
2. $\exists \mathcal{Y} \in R^{n\times m}$ *such that* $P + \mathcal{Y}H + H^T\mathcal{Y}^T < 0$.

Lemma 2 (See the work of Boyd et al[30]). *Given G, U, V, there exists X satisfying*

$$G + UXV^T + VX^TU^T > 0$$

if and only if

$$\hat{U}^T G \hat{U} > 0, \quad \hat{V}^T G \hat{V} > 0$$

hold, where $\hat{U}$ and $\hat{V}$ are orthogonal complements of U and V, respectively.

Lemma 3. *Let A, B, R, Q, W be known real matrices. Then, there exists $P = P^T > 0$ satisfying*

$$\begin{bmatrix} PA + A^TP + Q & PB + R \\ * & W \end{bmatrix} < 0 \tag{6}$$

if there exist $P = P^T > 0$, H, G satisfying

$$\begin{bmatrix} -G - G^T & P - H^T + GA & GB \\ * & HA + A^TH^T + Q & HB + R \\ * & * & W \end{bmatrix} < 0. \tag{7}$$

Proof. Suppose that (7) holds for some matrices H, G, $P = P^T > 0$. Define

$$\mathcal{A} = \begin{bmatrix} A & B \\ I & 0 \\ 0 & I \end{bmatrix}.$$

Premultiply and postmultiply (7) by $\mathcal{A}^T$ and $\mathcal{A}$. We obtain the inequality (6). □

Remark 2. In the control theory, "P" is commonly regarded as Lyapunov function matrix. It is clearly obvious from Lemma 3 that coupling between "P" and "A" (or "P" and "B") is removed by using slack matrices H and G. This method is similar to the technique that is to devise a controller or filter for discrete-time systems by introducing some other slack matrices, which can be seen in the works of Dong et al[4] and Wu et al.[31]

In this paper, our purpose is to construct a nonsynchronous H_∞ filter, satisfying the following.

1. System (5) is stochastically stable when $w(t) \equiv 0$ if $E\{\|\xi(t)\|_2\} < \infty$ or $E\{|\xi(t)|^2\} \to 0$ as $t \to \infty$.[26,32]
2. System (5) guarantees the H_∞ noise attenuation level γ, namely, under zero initial condition, for any nonzero $w(t) \in l_2[0, \infty)$, $e(t)$ satisfies $E\left\{\|e(t)\|_2^2\right\} < \gamma^2 \|w(t)\|_2^2$.[32]

3 | MAIN RESULTS

We begin to study the stochastic stability with a prescribed H_∞ performance of system (5). Then, we will set up to develop filtering design approaches for the nonsynchronous H_∞ filtering issue.

Theorem 1. *If there exist $P_i > 0$, R_{im} for any $i \in \mathcal{R}$ and $m \in \mathcal{S}$ satisfying*

$$\sum_{m=1}^{s} \pi_{im} R_{im} < \gamma^2 I, \tag{8}$$

$$\begin{bmatrix} \Gamma_{im}^1 & P_i \bar{B}_{im} & \bar{E}_{im}^T \\ * & -R_{im} & \bar{F}_{im}^T \\ * & * & -I \end{bmatrix} < 0, \tag{9}$$

system (5) is stochastically stable with a desired H_∞ performance γ, where

$$\Gamma_{im}^1 = Her(P_i \bar{A}_{im}) + \sum_{j=1}^{r} \theta_{ij} P_j.$$

Proof. Using Schur complement to (9), we have

$$\begin{bmatrix} \Gamma_{im}^1 & P_i \bar{B}_{im} \\ * & -R_{im} \end{bmatrix} + \begin{bmatrix} \bar{E}_{im}^T \\ \bar{F}_{im}^T \end{bmatrix} \begin{bmatrix} \bar{E}_{im}^T \\ \bar{F}_{im}^T \end{bmatrix}^T < 0. \tag{10}$$

Due to $\sum_{m=1}^{s} \pi_{im} = 1$ and $\pi_{im} \geq 0$, we have

$$\sum_{m=1}^{s} \pi_{im} \left(\begin{bmatrix} \Gamma_{im}^1 & P_i \bar{B}_{im} \\ * & -R_{im} \end{bmatrix} + \begin{bmatrix} \bar{E}_{im}^T \\ \bar{F}_{im}^T \end{bmatrix} \begin{bmatrix} \bar{E}_{im}^T \\ \bar{F}_{im}^T \end{bmatrix}^T \right) < 0. \tag{11}$$

Further from (8), it follows that

$$\Lambda_i = \sum_{m=1}^{s} \pi_{im} \left(\begin{bmatrix} \Gamma_{im}^1 & P_i \bar{B}_{im} \\ * & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} \bar{E}_{im}^T \\ \bar{F}_{im}^T \end{bmatrix} \begin{bmatrix} \bar{E}_{im}^T \\ \bar{F}_{im}^T \end{bmatrix}^T \right) < 0, \tag{12}$$

and $\sum_{m=1}^{s} \pi_{im} \Gamma_{im}^1 < 0$.

Construct the following Lyapunov function:

$$V(t)=\xi^T(t)P(\delta(t))\xi(t). \tag{13}$$

Along system (5) trajectory with $w(t)\equiv 0$, we achieve

$$\begin{aligned}
\mathcal{L}V(t) &= \lim_{\Delta t\to 0}\frac{1}{\Delta t}E\{V(t+\Delta t)-V(t)\}\\
&= \lim_{\Delta t\to 0}\frac{1}{\Delta t}\left(\xi^T(t+\Delta t)\left(\sum_{j=1}^{r}(\theta_{ij}\Delta t+o(\Delta t))P_j\right)\xi(t+\Delta t)\right.\\
&\quad\left.+(\xi^T(t+\Delta t)-\xi^T(t))P_i\xi(t+\Delta t)+\xi^T(t)P_i(\xi(t+\Delta t)-\xi(t))\right)\\
&= \xi^T(t)\left(\sum_{j=1}^{r}\theta_{ij}P_j\right)\xi(t)+2\dot{\xi}^T(t)P(\delta(t))\xi(t)\\
&= \xi^T(t)\left(\sum_{m=1}^{s}\pi_{im}\Gamma^1_{im}\right)\xi(t)<0,
\end{aligned} \tag{14}$$

where $\mathcal{L}$ is the weak infinitesimal generator of the random process $\{\xi(t),\delta(t)\}$. It implies $\lim_{t\to\infty}E\{|\xi(t)|^2\}=0$, further reflecting that system (5) is stochastically stable.

Next, supposing that $\xi(0)=0$, we will prove that system (5) can satisfy the H_∞ performance with a given scalar γ for $w(t)\neq 0$. By computing, we have

$$E\{\mathcal{L}V(t)+e^T(t)e(t)-\gamma^2w^T(t)w(t)\}=\begin{bmatrix}\xi(t)\\ w(t)\end{bmatrix}^T\Lambda_i\begin{bmatrix}\xi(t)\\ w(t)\end{bmatrix}. \tag{15}$$

According to (12), we obtain

$$E\{\mathcal{L}V(t)+e^T(t)e(t)-\gamma^2w^T(t)w(t)\}<0. \tag{16}$$

Via integral computation with $\xi(0)=0$, we achieve

$$\begin{aligned}
E\left\{\int_0^\infty e^T(t)e(t)\mathrm{d}t\right\} &< -E\{V(\infty)\}+\gamma^2\int_0^\infty w^T(t)w(t)\mathrm{d}t\\
&<\gamma^2\int_0^\infty w^T(t)w(t)\mathrm{d}t,
\end{aligned} \tag{17}$$

which ensures that $E\left\{\|e(t)\|_2^2\right\}<\gamma^2\|w(t)\|_2^2$ holds. Accordingly, it is clearly found that system (5) is stochastically stable with H_∞ performance. The proof is complete. □

Then, on the basis of Theorem 1, we will set up to develop the asynchronous filtering approach through selecting suitable Lyapunov function matrices.

Theorem 2. *If there exist* $P_i=\begin{bmatrix}P_{1i} & P_2\\ P_2 & P_2\end{bmatrix}>0$, R_{im}, $\mathcal{A}_m$, $\mathcal{B}_m$, $\mathcal{E}_m$, *and* $\mathcal{F}_m$ *for any* $i\in\mathcal{R}$ *and* $m\in\mathcal{S}$ *satisfying*

$$\sum_{m=1}^{s}\pi_{im}R_{im}<\gamma^2 I, \tag{18}$$

$$\begin{bmatrix}\Sigma_{1im} & \Sigma_{2im} & \Sigma_{3im}\\ * & -R_{im} & \Sigma_{4im}\\ * & * & -I\end{bmatrix}<0, \tag{19}$$

system (5) is stochastically stable with a desired H_∞ performance γ. Furthermore, filter gains can be calculated from

$$\left[\begin{array}{c|c}\hat{A}_m & \hat{B}_m \\ \hline \hat{E}_m & \hat{F}_m\end{array}\right] = \left[\begin{array}{c|c}P_2^{-1}\mathcal{A}_m & P_2^{-1}\mathcal{B}_m \\ \hline \mathcal{E}_m & \mathcal{F}_m\end{array}\right], \tag{20}$$

where

$$\begin{aligned}
\Sigma_{1im} &= \begin{bmatrix}\Sigma_{1im}^{11} & \mathcal{A}_m + (P_2A_i + \mathcal{B}_mC_i)^T \\ * & Her(\mathcal{A}_m)\end{bmatrix}, \\
\Sigma_{2im} &= \begin{bmatrix}P_{1i}B_i + \mathcal{B}_mD_i \\ P_2B_i + \mathcal{B}_mD_i\end{bmatrix}, \quad \Sigma_{3im} = \begin{bmatrix}E_i^T - C_i^T\mathcal{F}_m^T \\ -\mathcal{E}_m^T\end{bmatrix}, \\
\Sigma_{1im}^{11} &= \sum_{j=1}^{r}\theta_{ij}P_{1i} + Her(P_{1i}A_i + \mathcal{B}_mC_i), \\
\Sigma_{4im} &= F_i^T - D_i^T\mathcal{F}_m^T.
\end{aligned}$$

Proof. Define

$$P_i = \begin{bmatrix}P_{1i} & P_2 \\ P_2 & P_2\end{bmatrix}, \quad \left[\begin{array}{c|c}\mathcal{A}_m & \mathcal{B}_m \\ \hline \mathcal{E}_m & \mathcal{F}_m\end{array}\right] = \left[\begin{array}{c|c}P_2\hat{A}_m & P_2\hat{B}_m \\ \hline \hat{E}_m & \hat{F}_m\end{array}\right]. \tag{21}$$

Moreover, associating (5), it is easy to find that (19) is equivalent to (9). If there is an solution to linear matrix inequalities (LMIs) (18) and (19), the filter gains will be computed from (20). The proof is complete. □

Remark 3. Theorem 2 provides one LMI-based approach to design an asynchronous filter gains. However, there is a constraint on the structure of Lyapunov function matrix $P = \begin{bmatrix}P_1 & P_2 \\ P_2^T & P_3\end{bmatrix}$, namely, $P_2 = P_3$, which will bring some conservatism. Most published works have adopted this approach such as those of Chang et al.[21,22] Some have used the matrix congruence transformation and changed P into $\hat{P}$ with $\hat{P}_2 = \hat{P}_3$.[33-35] In the following, an alternative method is utilized to overcome this constraint by utilizing some slack matrices.

Theorem 3. *If there exist $P_i > 0$, R_{im}, G_m, H_m for any $i \in \mathcal{R}$ and $m \in \mathcal{S}$ satisfying*

$$\sum_{m=1}^{s}\pi_{im}R_{im} < \gamma^2 I, \tag{22}$$

$$\begin{bmatrix}-Her(G_m) & \Upsilon_{1im} & G_m\bar{B}_{im} & 0 \\ * & \Upsilon_{2im} & H_m\bar{B}_{im} & \bar{E}_{im}^T \\ * & * & -R_{im} & \bar{F}_{im}^T \\ * & * & * & -I\end{bmatrix} < 0, \tag{23}$$

system (5) is stochastically stable with a desired H_∞ performance γ, where

$$\begin{aligned}
\Upsilon_{1im} &= P_i - H_m^T + G_m\bar{A}_{im}, \\
\Upsilon_{2im} &= \sum_{j=1}^{r}\theta_{ij}P_j + Her(H_m\bar{A}_{im}).
\end{aligned}$$

Proof. From (22)-(23), it is easily observed that

$$\phi_i^1 + Her\left(\sum_{m=1}^{s}\pi_{im}\chi_m^1\begin{bmatrix}-I & \bar{A}_{im}\end{bmatrix}\right) < 0, \tag{24}$$

and

$$\phi_i^2 + Her\left(\sum_{m=1}^{s} \pi_{im}\chi_m^2 \begin{bmatrix} -I & \bar{A}_{im} & \bar{B}_{im} & 0 \\ 0 & \bar{E}_{im} & \bar{F}_{im} & -I \end{bmatrix}\right) < 0, \tag{25}$$

where

$$\phi_i^1 = \begin{bmatrix} 0 & P_i \\ P_i & \sum_{j=1}^{r} \theta_{ij}P_j \end{bmatrix}, \quad \chi_m^1 = \begin{bmatrix} G_m \\ H_m \end{bmatrix},$$
$$\phi_i^2 = \begin{bmatrix} 0 & P_i & 0 & 0 \\ P_i & \sum_{j=1}^{r} \theta_{ij}P_j & 0 & 0 \\ 0 & 0 & -\gamma^2 I & 0 \\ 0 & 0 & 0 & I \end{bmatrix}, \quad \chi_m^2 = \begin{bmatrix} G_m & 0 \\ H_m & 0 \\ 0 & 0 \\ 0 & I \end{bmatrix}.$$

Using the same Lyapunov function in (13), we have

(1) when $w(t) \equiv 0$,

$$E\{\mathcal{L}V(t)\} = \begin{bmatrix} \dot{\xi}(t) \\ \xi(t) \end{bmatrix}^T \phi_i^1 \begin{bmatrix} \dot{\xi}(t) \\ \xi(t) \end{bmatrix}, \tag{26}$$

and (2) when $w(t) \neq 0$,

$$E\{\mathcal{L}V(t) + e^T(t)e(t) - \gamma^2 w^T(t)w(t)\} = \begin{bmatrix} \dot{\xi}(t) \\ \xi(t) \\ w(t) \\ e(t) \end{bmatrix}^T \phi_i^2 \begin{bmatrix} \dot{\xi}(t) \\ \xi(t) \\ w(t) \\ e(t) \end{bmatrix}. \tag{27}$$

When $w(t) \equiv 0$ with (5), it infers that

$$\begin{bmatrix} -I & \bar{A}_{im} \end{bmatrix} \begin{bmatrix} \dot{\xi}(t) \\ \xi(t) \end{bmatrix} = 0. \tag{28}$$

Considering $\sum_{m=1}^{s} \pi_{im} = 1$ ($\pi_{im} \geq 0$), if there exist matrices G_m, H_m such that (24) holds, it will follow from Lemma 1 and (26) that $E\{\mathcal{L}V(t)\} < 0$ holds. It further implies that system (5) is stochastically stable.

For any nonzero $w(t)$, from (5), we have

$$\begin{bmatrix} -I & \bar{A}_{im} & \bar{B}_{im} & 0 \\ 0 & \bar{E}_{im} & \bar{F}_{im} & -I \end{bmatrix} \begin{bmatrix} \dot{\xi}(t) \\ \xi(t) \\ w(t) \\ e(t) \end{bmatrix} = 0. \tag{29}$$

If there exist matrices G_m, H_m such that (25) holds, by Lemma 1 again and from (27), we will have that $E\{\mathcal{L}V(t) + e^T(t)e(t) - \gamma^2 w^T(t)w(t)\} < 0$. The proof is complete. □

Remark 4. Adopting Schur complement to (9) and (23), respectively, we have

$$\begin{bmatrix} \Gamma_{im}^1 + \bar{E}_{im}^T\bar{E}_{im} & P_i\bar{B}_{im} + \bar{E}_{im}^T\bar{F}_{im} \\ * & -R_{im} + \bar{F}_{im}^T\bar{F}_{im} \end{bmatrix} < 0, \tag{30}$$

and

$$\begin{bmatrix} -Her(G_m) & \Upsilon_{1im} & G_m\bar{B}_{im} \\ * & \Upsilon_{2im} + \bar{E}_{im}^T\bar{E}_{im} & H_m\bar{B}_{im} + \bar{E}_{im}^T\bar{F}_{im} \\ * & * & -R_{im} + \bar{F}_{im}^T\bar{F}_{im} \end{bmatrix} < 0. \tag{31}$$

Based on Lemma 3, it is obvious that Theorem 1 can be achieved through Theorem 3.

The filtering design algorithm will be given as follows.

Theorem 4. *If there exist* $P_i = \begin{bmatrix} P_{1i} & P_{2i} \\ P_{2i}^T & P_{3i} \end{bmatrix} > 0$, R_{im}, G_{km}, H_{km} $(k \in \{1,2\})$, Y_m, $\mathcal{A}_m$, $\mathcal{B}_m$, $\mathcal{E}_m$, *and* $\mathcal{F}_m$ *for any* $i \in \mathcal{R}$ *and* $m \in \mathcal{S}$ *satisfying*

$$\sum_{m=1}^{s} \pi_{im} R_{im} < \gamma^2 I, \tag{32}$$

$$\Xi_{im} = \begin{bmatrix} \Xi_{im}^{11} & \Xi_{im}^{12} & \Xi_{im}^{13} & \Xi_{im}^{14} & \Xi_{im}^{15} & 0 \\ * & \Xi_{im}^{22} & \Xi_{im}^{23} & \Xi_{im}^{24} & \Xi_{im}^{25} & 0 \\ * & * & \Xi_{im}^{33} & \Xi_{im}^{34} & \Xi_{im}^{35} & \Xi_{im}^{36} \\ * & * & * & \Xi_{im}^{44} & \Xi_{im}^{45} & -\mathcal{E}_m^T \\ * & * & * & * & -R_{im} & \Xi_{im}^{56} \\ * & * & * & * & * & -I \end{bmatrix} < 0, \tag{33}$$

system (5) is stochastically stable with a desired H_∞ *performance* γ*. Furthermore, filter gains can be calculated from*

$$\left[\begin{array}{c|c} \hat{A}_m & \hat{B}_m \\ \hline \hat{E}_m & \hat{F}_m \end{array}\right] = \left[\begin{array}{c|c} Y_m^{-1}\mathcal{A}_m & Y_m^{-1}\mathcal{B}_m \\ \hline \mathcal{E}_m & \mathcal{F}_m \end{array}\right], \tag{34}$$

where

$$\Xi_{im}^{11} = -Her(G_{1m}), \quad \Xi_{im}^{12} = -Y_m - G_{2m}^T,$$

$$\Xi_{im}^{22} = -Her(Y_m), \quad \Xi_{im}^{44} = \sum_{j=1}^{r} \theta_{ij} P_{3j} + Her(\mathcal{A}_m),$$

$$\Xi_{im}^{13} = P_{1i} - H_{1m}^T + G_{1m}A_i + \mathcal{B}_m C_i,$$

$$\Xi_{im}^{23} = P_{2i}^T - Y_m^T + G_{2m}A_i + \mathcal{B}_m C_i,$$

$$\Xi_{im}^{14} = P_{2i} - H_{2m}^T + \mathcal{A}_m, \quad \Xi_{im}^{24} = P_{3i} - Y_m^T + \mathcal{A}_m,$$

$$\Xi_{im}^{33} = \sum_{j=1}^{r} \theta_{ij} P_{1j} + Her(H_{1m}A_i + \mathcal{B}_m C_i),$$

$$\Xi_{im}^{34} = \sum_{j=1}^{r} \theta_{ij} P_{2j} + \mathcal{A}_m + (H_{2m}A_i + \mathcal{B}_m C_i)^T,$$

$$\Xi_{im}^{15} = G_{1m}B_i + \mathcal{B}_m D_i, \quad \Xi_{im}^{25} = G_{2m}B_i + \mathcal{B}_m D_i,$$

$$\Xi_{im}^{35} = H_{1m}B_i + \mathcal{B}_m D_i, \quad \Xi_{im}^{45} = H_{2m}B_i + \mathcal{B}_m D_i,$$

$$\Xi_{im}^{36} = E_i^T - C_i^T \mathcal{F}_m^T, \quad \Xi_{im}^{56} = F_i^T - D_i^T \mathcal{F}_m^T.$$

Proof. Define

$$\begin{aligned} P_i &= \begin{bmatrix} P_{1i} & P_{2i} \\ P_{2i}^T & P_{3i} \end{bmatrix}, \quad \left[\begin{array}{c|c} \mathcal{A}_m & \mathcal{B}_m \\ \hline \mathcal{E}_m & \mathcal{F}_m \end{array}\right] = \left[\begin{array}{c|c} Y_m\hat{A}_m & Y_m\hat{B}_m \\ \hline \hat{E}_m & \hat{F}_m \end{array}\right], \\ G_m &= \begin{bmatrix} G_{1m} & Y_m \\ G_{2m} & Y_m \end{bmatrix}, \quad H_m = \begin{bmatrix} H_{1m} & Y_m \\ H_{2m} & Y_m \end{bmatrix}. \end{aligned} \tag{35}$$

With system matrices in (5), we can deduce that (23) is equivalent to (33). If there is an solution to LMIs (32) and (33), the filter gains will be computed from (34). The proof is complete. □

Next, we will adopt Lemma 2 to eliminate some variables in LMIs of Theorem 4, which can be seen as another filter design approach.

Theorem 5. *If there exist* $P_i = \begin{bmatrix} P_{1i} & P_{2i} \\ P_{2i}^T & P_{3i} \end{bmatrix} > 0$, R_{im}, Y_m, $\mathcal{A}_m$, $\mathcal{B}_m$, $\mathcal{E}_m$, *and* $\mathcal{F}_m$ *for any* $i \in \mathcal{R}$ *and* $m \in \mathcal{S}$ *satisfying*

$$\sum_{m=1}^{s} \pi_{im} R_{im} < \gamma^2 I, \tag{36}$$

$$\Lambda_1^T \Omega_{im} \Lambda_1 < 0, \tag{37}$$

$$\Lambda_{2i}^T \Omega_{im} \Lambda_{2i} < 0, \tag{38}$$

system (5) is stochastically stable with a desired H_∞ *performance* γ*. Furthermore, filter gains can be calculated from*

$$\left[\begin{array}{c|c} \hat{A}_m & \hat{B}_m \\ \hline \hat{E}_m & \hat{F}_m \end{array}\right] = \left[\begin{array}{c|c} Y_m^{-1}\mathcal{A}_m & Y_m^{-1}\mathcal{B}_m \\ \hline \mathcal{E}_m & \mathcal{F}_m \end{array}\right], \tag{39}$$

where

$$\Omega_{im} = \begin{bmatrix} 0 & -Y_m & \Omega_{im}^{13} & \Omega_{im}^{14} & \mathcal{B}_m D_i & 0 \\ * & \Xi_{im}^{22} & \Omega_{im}^{23} & \Xi_{im}^{24} & \mathcal{B}_m D_i & 0 \\ * & * & \Omega_{im}^{33} & \Omega_{im}^{34} & \mathcal{B}_m D_i & \Xi_{im}^{36} \\ * & * & * & \Xi_{im}^{44} & \mathcal{B}_m D_i & -\mathcal{E}_m^T \\ * & * & * & * & -R_{im} & \Xi_{im}^{56} \\ * & * & * & * & * & -I \end{bmatrix},$$

$$\Omega_{im}^{13} = P_{1i} + \mathcal{B}_m C_i, \quad \Omega_{im}^{23} = P_{2i}^T - Y_m^T + \mathcal{B}_m C_i,$$

$$\Omega_{im}^{33} = \sum_{j=1}^{r} \theta_{ij} P_{1j} + Her(\mathcal{B}_m C_i), \quad \Omega_{im}^{14} = P_{2i} + \mathcal{A}_m,$$

$$\Omega_{im}^{34} = \sum_{j=1}^{r} \theta_{ij} P_{2j} + \mathcal{A}_m + (\mathcal{B}_m C_i)^T,$$

$$\Lambda_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ I & 0 \\ 0 & I \end{bmatrix}, \quad \Lambda_{2i} = \begin{bmatrix} 0 & A_i & 0 & B_i & 0 \\ I & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix}.$$

Proof. Define

$$\Xi_{im} = \Omega_{im} + Her\left(N \mathcal{H}_m M_i^T\right) < 0, \tag{40}$$

where

$$N = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \mathcal{H}_m = \begin{bmatrix} G_{1m} \\ G_{2m} \\ H_{1m} \\ H_{2m} \end{bmatrix}, \quad M_i = \begin{bmatrix} -I \\ 0 \\ A_i^T \\ 0 \\ B_i^T \\ 0 \end{bmatrix}.$$

Through calculation, we can find that Λ_1 and Λ_{2i} are orthogonal complements of N and M_i, respectively. According to Lemma 2, it is clear that if (40) holds for some $\mathcal{H}_m$, so do inequalities (37)-(38). The proof is complete. □

Remark 5. Unnecessary matrices G_{km}, H_{km} ($k \in \{1,2\}$) are removed, and the number of variables to solve is reduced but the quantity of LMIs is increased by sr. Through analysis, the asynchronous H_∞ filtering problem is formulated as a convex issue, as shown in Theorems 2, 4, and 5. The optimal disturbance attenuation γ^* can be computed via minimizing γ^2 subject to LMIs in the corresponding theorem via LMI Toolbox. Based on HMM, the work of Rodrigues et al[28] has dealt with the H_∞ filter design problem for MJLSs in view of the cluster observation and the developed approach is nonconvex if ϵ_i is not given a priori. The solution to filter gains is nonconvex in the work of Rodrigues et al[36] as well when studying the detector-based H_∞ filter design issue for MJLSs. By contrast, our proposed methods are convex, which is more easier to compute. Lyapunov matrix function P_i in this paper is only related to the jump mode $\delta(t)$, which may result in conservatism. Thus, it is worth investigating how to construct Lyapunov matrix function by using both $\delta(t)$ and $\rho(t)$ to lower conservatism in the future.

4 | ILLUSTRATIVE EXAMPLE

This section presents one example to illustrate the effectiveness and feasibility of the developed approaches. The mechanical system is plotted in Figure 1, containing two springs and two masses.[37,38] The stiffness of the springs is denoted by k_1, k_2. The positions of masses m_1, m_2 are represented by x_1, x_2.

In order to study the performance of MJLSs, we assume that masses of the system change over time by obeying the stochastic Markov process. System matrices are

$$
\begin{aligned}
&A_i = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k_1+k_2}{m_{1i}} & \frac{k_2}{m_{1i}} & -\frac{c}{m_{1i}} & 0 \\ \frac{k_2}{m_{2i}} & -\frac{k_2}{m_{2i}} & 0 & -\frac{c}{m_{2i}} \end{bmatrix}, \quad B_i = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{m_{1i}} \\ 0 \end{bmatrix}, \\
&C_i = [1 \quad 0 \quad 0 \quad 0], \quad E_i = [0 \quad 1 \quad 0 \quad 0], \quad D_i = F_i = 1, \\
&k_1 = k_2 = 1, \quad c = 2, \quad m_{11} = 1, \quad m_{12} = 2, \\
&m_{21} = 0.5, \quad m_{22} = 1, \quad i \in \{1,2\}.
\end{aligned}
$$

Here, c is the viscous friction coefficient and state is $x(t) = [x_1 \quad x_2 \quad \dot{x}_1 \quad \dot{x}_2]^T$. The transition rate matrix of the mass is supposed to be

$$
\Phi = \begin{bmatrix} -5 & 5 \\ 4 & -4 \end{bmatrix}.
$$

Moreover, we assume that the mode of the designed filer is subject to

$$
\Psi = \begin{bmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{bmatrix}.
$$

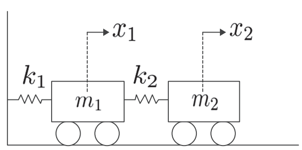

FIGURE 1 Mass-spring system

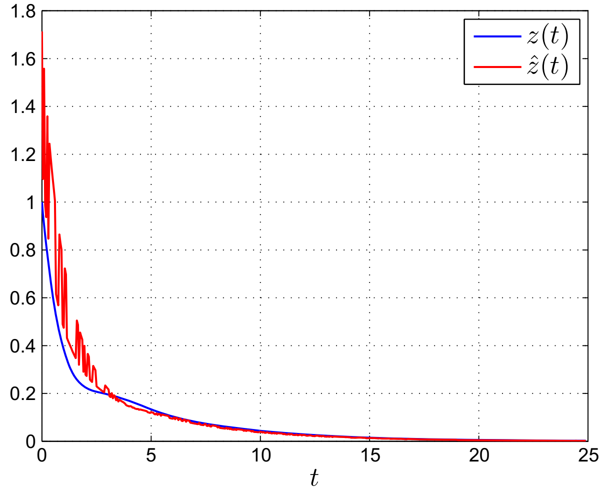

FIGURE 2 Original signal $z(t)$ and estimation signal $\hat{z}(t)$ [Colour figure can be viewed at wileyonlinelibrary.com]

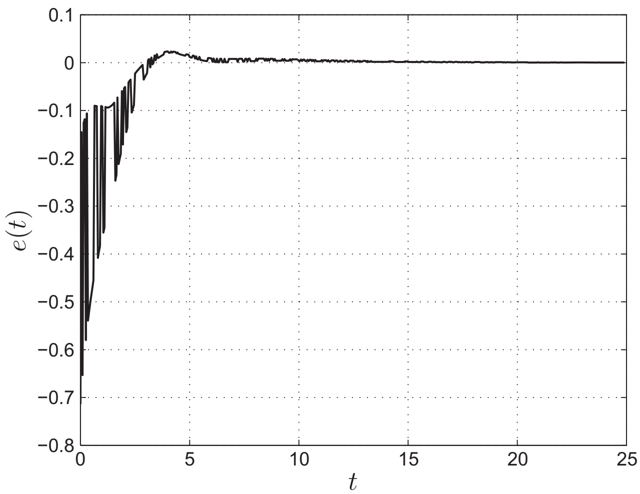

FIGURE 3 Estimation error $e(t)$

Assuming the noise attenuation level $\gamma = 0.9000$, then solving LMIs (36)-(38) in Theorem 5, we obtain the filter parameters as follows:

$$\hat{A}_1 = \begin{bmatrix} -0.2675 & 0.1437 & 0.9100 & 0.1017 \\ 0.4376 & -0.5195 & -0.1823 & 0.2278 \\ -1.6369 & 0.2022 & -1.5798 & -0.5100 \\ 0.4723 & -0.3601 & 0.5053 & -1.3168 \end{bmatrix},$$

$$\hat{A}_2 = \begin{bmatrix} -0.2452 & 0.1228 & 0.9504 & 0.0607 \\ 0.4419 & -0.5248 & -0.2191 & 0.2342 \\ -1.5547 & 0.2617 & -1.5458 & -0.4492 \\ 0.4915 & -0.3653 & 0.5510 & -1.3504 \end{bmatrix},$$

$$\hat{B}_1 = [-0.0518 \quad -0.0360 \quad -0.7540 \quad 0.0608]^T,$$

$$\hat{B}_2 = [-0.0649 \quad -0.0211 \quad -0.6242 \quad 0.0568]^T,$$

$$\hat{E}_1 = [0.7193 \quad -0.5293 \quad 0.1461 \quad 0.4867],$$

$$\hat{E}_2 = [1.0668 \quad -0.4452 \quad -0.1007 \quad 0.6407],$$

$$\hat{F}_1 = 1.1578, \quad \hat{F}_2 = 1.7139.$$

In simulation, states of the system and the designed filter are supposed to be zero, and the external noise $w(t)$ is taken as e^{-t}. It is clearly observed from Figure 2 that $\hat{z}(t)$ can exactly approximate the signal $z(t)$. Signal $e(t)$ tends to zero with time, plotted in Figure 3. Figures 2 and 3 demonstrate the effectiveness of our developed method.

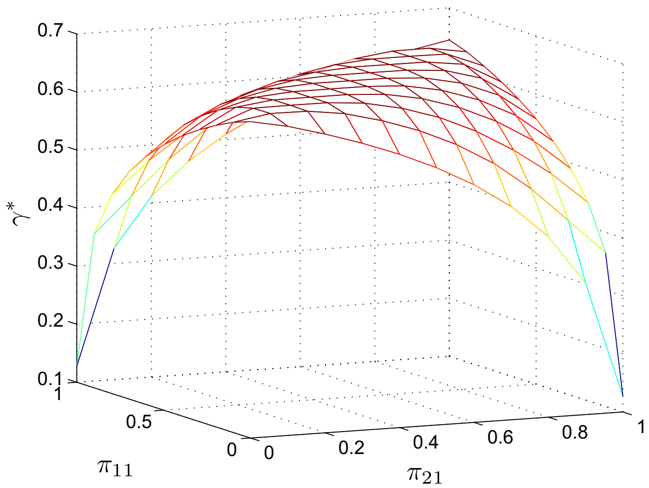

FIGURE 4 Optimal H_∞ filtering performance γ^* on the whole space Ψ [Colour figure can be viewed at wileyonlinelibrary.com]

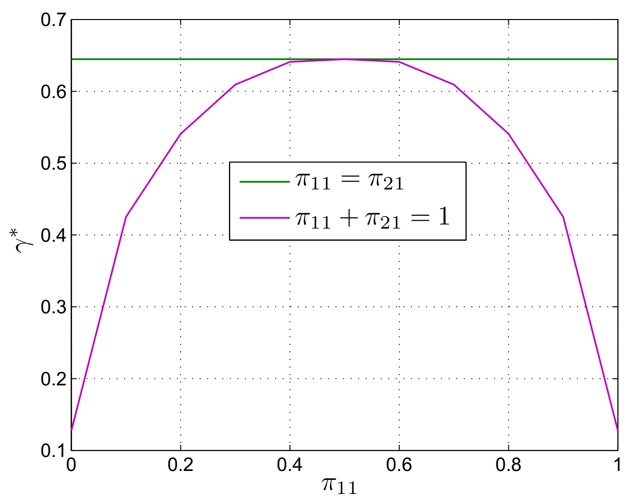

FIGURE 5 Optimal H_∞ filtering performance γ^* of the two special cases [Colour figure can be viewed at wileyonlinelibrary.com]

TABLE 1 Conditional transition probability matrix Ψ

Case I	Case II	Case III	Case IV	Case V
$\begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{bmatrix}$	$\begin{bmatrix} 0.9 & 0.1 \\ 0.3 & 0.7 \end{bmatrix}$	$\begin{bmatrix} 0.9 & 0.1 \\ 0.5 & 0.5 \end{bmatrix}$	$\begin{bmatrix} 0.9 & 0.1 \\ 0.7 & 0.3 \end{bmatrix}$	$\begin{bmatrix} 0.9 & 0.1 \\ 0.9 & 0.1 \end{bmatrix}$

Figure 4 shows that the optimal H_∞ filtering performance γ^* changes with the conditional probability matrix $\Psi = \begin{bmatrix} \pi_{11} & 1-\pi_{11} \\ \pi_{21} & 1-\pi_{21} \end{bmatrix}$, which ranges from the lowest value 0.1275 to the highest value 0.6448. Two special cases are plotted in Figure 5: When $\pi_{11} = \pi_{21}$, γ^* remains the peak value, implying that r^* is independent of filter jumps in this case, and the curve of r^* is symmetrical in the case of $\pi_{11} + \pi_{21} = 1$.

In the following, we will focus on comparing differences and advantages of three developed techniques, namely, Theorems 2, 4, and 5.

From Table 2, we can clearly observe that γ^* obtained from these three techniques all increase with asynchronous level. In Table 1, Ψ is taken as $\begin{bmatrix} 0.9 & 0.1 \\ 1-\pi_{22} & \pi_{22} \end{bmatrix}$ and the decrease of π_{22} means that the designed filter is less likely to operate synchronously with the original plant. H_∞ performance index γ^* achieved from Theorem 5 is the smallest one among three approaches. On the other hand, it is obvious from Tables 2 and 3 that although Theorems 2 and 4 share the same number of LMIs, and the number of unknown variables of Theorem 4 is over three times as many as that of Theorem 2, γ^* computed by Theorem 4 is much less than that by Theorem 2. This implies that Theorem 4 can bring much better performance than Theorem 2. There is an opposite phenomenon in Theorems 4 and 5. The obtained γ^* from Theorem 4 is much bigger than that from Theorem 5 though Theorem 4 has more variables to be decided and fewer LMIs than Theorem 5.

Ψ	Case I	Case II	Case III	Case IV	Case V
Theorem 2	0.4769	0.5675	0.6219	0.6527	0.6616
Theorem 4	0.4272	0.5359	0.6042	0.6465	0.6576
Theorem 5	0.4252	0.5321	0.5972	0.6355	0.6448

TABLE 2 Optimal performance γ^* with different Ψ

	Variables	LMIs
Theorem 2	85	6
Theorem 4	287	6
Theorem 5	159	10

TABLE 3 Quantities of variables and linear matrix inequalities (LMIs) for three methods

5 | CONCLUSION

The issue of asynchronous H_∞ filtering design has been analyzed for continuous-time MJLSs in this paper. By utilizing HMM, an asynchronous filter model has been constructed. Then, via a Lyapunov function technique, we have developed two sufficient conditions to ensure the stochastic stability of filtering error systems with H_∞ performance. Three filtering design methods have been given, and the filtering issue has been formulated into a convex optimization issue. Finally, simulation results have been provided to verify the feasibility of developed approaches. The future work is to investigate how to extend the developed filtering design techniques to complex nonlinear systems with uncertainties and Markov jump.

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