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具有丢包的未知转移概率 Markov 跳变系统鲁棒 H_∞ 滤波

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摘 要: 研究一类具有数据包丢失的部分未知转移概率离散时间马尔可夫跳变系统(MJSs)鲁棒 H_∞ 滤波问题。假定系统丢包发生在传感器至滤波器之间的通信信道且丢包概率服从伯努利分布,基于 Delta 算子离散化方法构造具有不确定参数的离散时间马尔可夫跳变系统及模态相关的全阶滤波器。引入松弛矩阵变量解决系统矩阵与 Lyapunov 函数中正定矩阵之间的耦合问题。利用 Lyapunov 函数、Schur 补引理及线性矩阵不等式方法获得系统随机稳定且满足 H_∞ 性能的充分条件。已知系统丢包率,分别求得 Delta 算子系统及移位算子系统最优 H_∞ 性能指标。当丢包概率取值越低时系统的鲁棒性能越好,并且在相同丢包概率下,Delta 算子系统最优 H_∞ 性能总是优于移位算子系统最优 H_∞ 性能。数值仿真结果表明所提方法不仅有效可行,还具有一定的优越性。

关键词: 马尔可夫跳变系统; 不确定参数; 数据包丢失; Delta 算子

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0 引言

网络控制系统以网络作为信息传输的媒介,将传感器获得的数据经由网络发送至控制设备,控制设备对收到的信息进行处理后交由执行器执行[1]。网络控制系统改善了传统控制系统布线复杂、系统不稳定等诸多问题[2]。然而,随着网络控制系统的结构变得越来越复杂,节点的数量持续增加,信号在网络中传输易受到噪声干扰从而引起数据丢失和系统的不稳定[3-4]。因此,对网络控制系统进行滤波估计以还原真实传输信号就显得尤为重要。

卡尔曼滤波[5]可以处理非平稳信号和随机信号,倘若噪声信号的统计信息未知,那么卡尔曼滤波将失去最佳观测性能。而 H_∞ 滤波不需要确切已知噪声信号的种类和统计特性[6],其将滤波问题与系统的 H_∞ 范数相结合,确保系统在受到噪声干扰时,估计误差的能量增益小于预选定的性能指标。相对于卡尔曼滤波,H_∞ 滤波具有更好的鲁棒性[7]。

马尔可夫跳变系统(Markov jump systems, MJSs)通过一组马尔科夫链来描述系统在多个不同模态之间的跳变过程[8-9]。MJSs 中跳变的模态可以很好地表示网络控制系统中不确定的系统状态。Niu 等[10]在转移概率完全已知的情况下,研究具有丢包及不确定参数的离散时间 MJSs 鲁棒 H_∞ 滤波问题。然而,在实际的 MJSs 中难以准确获得系统模态转移概率。不仅如此,对于鲁棒性能较差的闭环系统,当丢包概率超过某一固定值时会造成系统的不稳定,因此研究具有丢包的 MJSs 鲁棒滤波及稳定性控制已成为当前主要任务。

作为对普通移位算子的改进,Delta 算子在高速采样下能避免出现极限环振荡等不稳定状态。然而,目前采用 Delta 算子描述具有通信丢包和不确定参数的马尔可夫跳变系统 H_∞ 滤波的研究尚不多见。

基于以上讨论,本文主要研究具有通信丢包和不确定参数的转移概率部分未知的 Delta 算子马尔可夫跳变系统 H_∞ 滤波问题。

1 问题描述

Delta 算子定义:

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$$\delta \boldsymbol{x}(t)=\begin{cases}\dfrac{\mathrm{d}\boldsymbol{x}(t)}{\mathrm{d}t}, & h=0;\\ \dfrac{\boldsymbol{x}(t+h)-\boldsymbol{x}(t)}{h}, & h\neq 0。\end{cases} \tag{1}$$

式中:h为采样周期。

考虑具有通信丢包及不确定参数的离散MJSs,建立基于Delta算子的状态空间模型:

$$\begin{cases}\delta \boldsymbol{x}(k)=[\boldsymbol{E}(r_k)+\Delta\boldsymbol{E}(k,r_k)]\boldsymbol{x}(k)+\\ \qquad [\boldsymbol{F}(r_k)+\Delta\boldsymbol{F}(k,r_k)]\boldsymbol{\omega}(k);\\ \boldsymbol{y}(k)=\alpha(k)\boldsymbol{H}(r_k)\boldsymbol{x}(k)+\boldsymbol{L}(r_k)\boldsymbol{\omega}(k);\\ \boldsymbol{z}(k)=\boldsymbol{G}(r_k)\boldsymbol{x}(k)。\end{cases} \tag{2}$$

式中:$\boldsymbol{x}(k)\in\mathbf{R}^n$为系统状态矢量;$\boldsymbol{y}(k)\in\mathbf{R}^m$为测量输出;$\boldsymbol{\omega}(k)\in \boldsymbol{l}_2[0,+\infty)$为外部干扰信号;$\boldsymbol{z}(k)\in\mathbf{R}^p$为被估计的目的信号。$\{r_k,k\geqslant 0\}$为离散时间MJSs模态,在有限集合$S=\{1,2,\cdots,N\}$中取值,当$r_k=i,r_{k+1}=j$时,对应的马尔可夫跳变系统转移概率$Pr(r_{k+1}=j|r_k=i)=u_{ij},u_{ij}\geqslant 0$。

为简化起见,将与模态相关的系统矩阵参数$\boldsymbol{E}_i:=\boldsymbol{E}(r_k=i)$;$\boldsymbol{F}_i:=\boldsymbol{F}(r_k=i)$;$\boldsymbol{H}_i:=\boldsymbol{H}(r_k=i)$;$\boldsymbol{L}_i:=\boldsymbol{L}(r_k=i)$;$\boldsymbol{G}_i:=\boldsymbol{G}(r_k=i)$。$\Delta\boldsymbol{E}(k,r_k)$和$\Delta\boldsymbol{F}(k,r_k)$为未知的实矩阵,满足范数有界不确定性:

$$\begin{cases}[\Delta\boldsymbol{E}(k,r_k)\quad \Delta\boldsymbol{F}(k,r_k)]=\\ \qquad \boldsymbol{M}(r_k)\boldsymbol{X}(k,r_k)[\boldsymbol{N}_a(r_k)\quad \boldsymbol{N}_b(r_k)];\\ \boldsymbol{X}(k,r_k)\boldsymbol{X}(k,r_k)^{\mathrm{T}}\leqslant \boldsymbol{I},\quad \forall k>0。\end{cases} \tag{3}$$

式中:$\boldsymbol{M}(r_k)$、$\boldsymbol{N}_a(r_k)$和$\boldsymbol{N}_b(r_k)$为已知具有适当维度的实矩阵;$\boldsymbol{X}(k,r_k)$为满足范数有界条件的未知矩阵。

式(2)中随机变量$\alpha(k)$服从Bernoulli分布,取值为0或1,即:

$$\begin{cases}\mathrm{Prob}\{\alpha(k)=1\}=E\{\alpha(k)\}=\beta;\\ \mathrm{Prob}\{\alpha(k)=0\}=1-E\{\alpha(k)\}=1-\beta。\end{cases} \tag{4}$$

且$\beta\in[0\quad 1]$是一个已知正常量,E为数学期望,满足:

$$\begin{cases}E\{\alpha(k)-\beta\}=0;\\ E\{[\alpha(k)-\beta]^2\}=\beta(1-\beta)。\end{cases} \tag{5}$$

显然,$\alpha(k)=0$表示信道传输中发生丢包。

考虑以下形式的滤波器模型:

$$\begin{cases}\delta \boldsymbol{x}_F(k)=\boldsymbol{C}_F(r_k)\boldsymbol{x}_F(k)+\\ \qquad \boldsymbol{D}_F(r_k)[\boldsymbol{y}(k)-\beta\boldsymbol{H}(r_k)\boldsymbol{x}_F(k)];\\ \boldsymbol{z}_F(k)=\boldsymbol{B}_F(r_k)\boldsymbol{x}_F(k)。\end{cases} \tag{6}$$

式中:$\boldsymbol{x}_F(k)\in\mathbf{R}^n$为滤波器的状态矢量;$\boldsymbol{y}_k\in\mathbf{R}^m$为滤波器的输入;$\boldsymbol{z}_F(k)$为信号$\boldsymbol{z}(k)$的估计;$\boldsymbol{C}_{Fi}$、$\boldsymbol{D}_{Fi}$、$\boldsymbol{B}_{Fi}$为待确定的滤波器参数。联立式(2)、(6)可得以下滤波误差系统:

$$\begin{cases}\delta\bar{\boldsymbol{x}}(k)=\bar{\boldsymbol{E}}_i\bar{\boldsymbol{x}}(k)+[\alpha(k)-\beta]\bar{\boldsymbol{E}}_{1i}\bar{\boldsymbol{x}}(k)+\bar{\boldsymbol{F}}_i\boldsymbol{\omega}(k);\\ \boldsymbol{e}(k)=\bar{\boldsymbol{L}}_i\bar{\boldsymbol{x}}(k)。\end{cases} \tag{7}$$

式中:$\bar{\boldsymbol{E}}_i=\tilde{\boldsymbol{E}}_i+\Delta\tilde{\boldsymbol{E}}_{ik}$;$\bar{\boldsymbol{F}}_i=\tilde{\boldsymbol{F}}_i+\Delta\tilde{\boldsymbol{F}}_{ik}$;

$$\bar{\boldsymbol{E}}_{1i}=\begin{bmatrix}\boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{D}_{Fi}\boldsymbol{H}_i & \boldsymbol{0}\end{bmatrix};\bar{\boldsymbol{L}}_i=[\boldsymbol{G}_i\ -\boldsymbol{B}_{Fi}];$$

$$\tilde{\boldsymbol{E}}_i=\begin{bmatrix}\boldsymbol{E}_i & \boldsymbol{0}\\ \beta\boldsymbol{D}_{Fi}\boldsymbol{H}_i & \boldsymbol{C}_{Fi}-\beta\boldsymbol{D}_{Fi}\boldsymbol{H}_i\end{bmatrix};$$

$$\bar{\boldsymbol{x}}(k)=[\boldsymbol{x}^{\mathrm{T}}(k),\boldsymbol{x}_F^{\mathrm{T}}(k)]^{\mathrm{T}};\boldsymbol{e}(k)=\boldsymbol{z}(k)-\boldsymbol{z}_F(k);$$

$$\Delta\tilde{\boldsymbol{E}}_{ik}=\tilde{\boldsymbol{M}}_i\boldsymbol{X}_{ik}\tilde{\boldsymbol{N}}_{ai};\tilde{\boldsymbol{F}}_i=\begin{bmatrix}\boldsymbol{F}_i\\ \boldsymbol{D}_{Fi}\boldsymbol{L}_i\end{bmatrix};\tilde{\boldsymbol{M}}_i=\begin{bmatrix}\boldsymbol{M}_i\\ \boldsymbol{0}\end{bmatrix};$$

$$\Delta\tilde{\boldsymbol{F}}_{ik}=\tilde{\boldsymbol{M}}_i\boldsymbol{X}_{ik}\tilde{\boldsymbol{N}}_{bi};\tilde{\boldsymbol{N}}_{ai}=[N_{ai}\quad 0];\tilde{\boldsymbol{N}}_{bi}=\boldsymbol{N}_{bi}。$$

定义1[11] 若对于每个初始条件$x_0\in\mathbf{R}^n$和$r_k\in\boldsymbol{S}$,在$\boldsymbol{\omega}(k)\equiv 0$的情况下,有以下不等式成立,则系统(见式(7))具有鲁棒随机稳定性:

$$E\left\{\sum_{k=0}^{\infty}\|\tilde{\boldsymbol{x}}(k)\|^2\tilde{\boldsymbol{x}}_0,r_0\right\}<\infty。 \tag{8}$$

定义2[12] 给定标量$\gamma>0$,如果在零初始条件下对于所有非零$\boldsymbol{\omega}(k)\in\boldsymbol{l}_2[0,+\infty)$,有以下不等式成立,则系统(见式(7))鲁棒随机稳定,且满足给定的H_∞性能。

$$E\left\{\sum_{k=0}^{\infty}\|\boldsymbol{e}(k)\|^2<\gamma^2\sum_{k=0}^{\infty}\|\boldsymbol{\omega}(k)\|^2\right\}。 \tag{9}$$

2 主要结果

定理1 考虑离散马尔可夫跳变系统,给定常数$\gamma>0$,如果存在矩阵$\boldsymbol{P}_i>0$,使得以下不等式成立,则滤波误差系统鲁棒随机稳定,并满足给定的H_∞性能。

$$\begin{bmatrix}-h\bar{\boldsymbol{P}}_k^i & \boldsymbol{0} & \boldsymbol{0} & \bar{\boldsymbol{P}}_k^i(h\bar{\boldsymbol{E}}_i+\boldsymbol{I}) & h\bar{\boldsymbol{P}}_k^i\bar{\boldsymbol{F}}_i\\ * & -h\bar{\boldsymbol{P}}_k^i & \boldsymbol{0} & h\theta\bar{\boldsymbol{P}}_k^i\bar{\boldsymbol{E}}_{1i} & \boldsymbol{0}\\ * & * & -u_k^i & u_k^i\bar{\boldsymbol{L}}_i & \boldsymbol{0}\\ * & * & * & -\dfrac{1}{h}u_k^i\boldsymbol{P}_i & \boldsymbol{0}\\ * & * & * & * & -u_k^i\gamma^2\boldsymbol{I}\end{bmatrix}<0; \tag{10}$$

$$\sum_{j\in S_{uk}^i}u_{ij}\begin{bmatrix}-hP_j & 0 & 0 & P_j(h\bar{E}_i+I) & hP_j\bar{F}_i\\ * & -hP_j & 0 & h\theta P_j\bar{E}_{1i} & 0\\ * & * & -I & \bar{L}_i & 0\\ * & * & * & -\frac{1}{h}P_i & 0\\ * & * & * & * & -\gamma^2 I\end{bmatrix}<0。\tag{11}$$

式中: $\theta=[(1-\beta)\beta]^{\frac{1}{2}}$; $u_k^i=\sum_{j\in S_k^i}u_{ij}$; $P_i=\sum_{j=1}^{N}u_{ij}P_j$。

证明　对于滤波误差系统,构造 Lyapunov 函数:

$$v(\bar{x}(k),r_k)=\bar{x}^{\mathrm{T}}(k)P_i\bar{x}(k);\tag{12}$$

$$E\{\delta v(\bar{x}(k),r_k)\}=\frac{1}{h}\left\{E\left[\bar{x}_{k+1}^{\mathrm{T}}\sum_{j\in S_k^i}u_{ij}P_j\bar{x}_{k+1}\right]-\bar{x}_k^{\mathrm{T}}\sum_{j\in S_k^i}u_{ij}P_i\bar{x}_k\right\}+\frac{1}{h}\left\{E\left[\bar{x}_{k+1}^{\mathrm{T}}\sum_{j\in S_{uk}^i}u_{ij}P_j\bar{x}_{k+1}\right]-\bar{x}_k^{\mathrm{T}}\sum_{j\in S_{uk}^i}u_{ij}P_i\bar{x}_k\right\}。\tag{13}$$

式中:

$$\frac{1}{h}\left\{E\left[\bar{x}_{k+1}^{\mathrm{T}}\sum_{j\in S_k^i}u_{ij}P_j\bar{x}_{k+1}\right]-\bar{x}_k^{\mathrm{T}}\sum_{j\in S_k^i}u_{ij}P_i\bar{x}_k\right\}=\frac{1}{h}\left\{E\left[\bar{x}_{k+1}^{\mathrm{T}}\bar{P}_k^i\bar{x}_{k+1}\right]-\bar{x}_k^{\mathrm{T}}u_k^iP_i\bar{x}_k\right\}。$$

当 $\omega(k)=0$ 时,由式(10)可得

$$\begin{bmatrix}-h\bar{P}_k^i & 0 & \bar{P}_k^i(h\bar{E}_i+I)\\ * & -h\bar{P}_k^i & h\theta\bar{P}_k^i\bar{E}_{1i}\\ * & * & -\frac{1}{h}u_k^iP_i\end{bmatrix}<0。\tag{14}$$

由 Schur 补引理可知,式(14)等价于:

$$\frac{1}{h}(h\bar{E}_i^{\mathrm{T}}+I)\bar{P}_k^i(h\bar{E}_i+I)-\frac{1}{h}u_k^iP_i+h\theta^2\bar{E}_{1i}^{\mathrm{T}}\bar{P}_k^i\bar{E}_{1i}<0。\tag{15}$$

同理可得

$$\frac{1}{h}\left\{E\left[\bar{x}_{k+1}^{\mathrm{T}}\sum_{j\in S_{uk}^i}u_{ij}P_j\bar{x}_{k+1}\right]-\bar{x}_k^{\mathrm{T}}\sum_{j\in S_{uk}^i}u_{ij}P_i\bar{x}_k\right\}=\sum_{j\in S_{uk}^i}u_{ij}\frac{1}{h}E\{[\bar{x}_{k+1}^{\mathrm{T}}P_j\bar{x}_{k+1}]-\bar{x}_k^{\mathrm{T}}P_i\bar{x}_k\}。$$

当 $\omega(k)=0$ 时,由不等式(11)及 Schur 补引理可得

$$\frac{1}{h}(h\bar{E}_i^{\mathrm{T}}+I)P_j(h\bar{E}_i+I)-\frac{1}{h}P_i+h\theta^2\bar{E}_{1i}^{\mathrm{T}}P_j\bar{E}_{1i}<0。\tag{16}$$

因此,将式(15)、(16)代入式(13)得

$$E[\Delta v]\leqslant-(a_1+a_2)\bar{x}_k^{\mathrm{T}}\bar{x}_k=-(a_1+a_2)\|\bar{x}_k\|^2。\tag{17}$$

式中: a_1、a_2 分别为式(15)、(16)的最小特征值下界。

定义 $a=a_1+a_2$,则由式(17)可得,对于任意 $\tau\geqslant1$, $E\{\sum_{k=0}^{\tau}\|\bar{x}_k\|^2\}\leqslant\frac{1}{a}\{E[v(\bar{x}_0,0)]-E[v(\bar{x}_{\tau+1},\tau+1)]\}\leqslant\infty$。因此,由定义 1 可知,该系统是随机稳定的。

为证明系统的 H_∞ 性能,有以下等式:

$$J=E\{\sum_{k=0}^{\infty}[e^{\mathrm{T}}(k)e(k)-\gamma^2\omega^{\mathrm{T}}(k)\omega(k)]\};\tag{18}$$

$$J\leqslant E\{\sum_{k=0}^{\infty}[e^{\mathrm{T}}(k)e(k)-\gamma^2\omega^{\mathrm{T}}(k)\omega(k)+\Delta v]\}=\sum_{k=0}^{\infty}\eta_k^{\mathrm{T}}\phi_i\eta_k。\tag{19}$$

式中: $\eta_k=[\bar{x}_k^{\mathrm{T}}\quad\omega_k^{\mathrm{T}}]^{\mathrm{T}}$; $\phi_i=\begin{bmatrix}\phi_{11} & \phi_{12}\\ * & \phi_{22}\end{bmatrix}$;

$$\phi_{11}=\frac{1}{h}(h\bar{E}_i^{\mathrm{T}}+I)\tilde{P}_i(h\bar{E}_i+I)+h\theta^2\bar{E}_{1i}^{\mathrm{T}}\tilde{P}_i\bar{E}_{1i}+\bar{L}_i^{\mathrm{T}}\bar{L}_i-\frac{1}{h}\tilde{P}_i;$$

$$\phi_{12}=(h\bar{E}_i^{\mathrm{T}}+I)\tilde{P}_i\bar{F}_i;\ \phi_{22}=h\bar{F}_i^{\mathrm{T}}\tilde{P}_i\bar{F}_i-\gamma^2I;$$

$$\tilde{P}_i\overset{\mathrm{def}}{=}\sum_{j\in S_k^i}u_{ij}P_j+\sum_{j\in S_{uk}^i}u_{ij}P_j=\bar{P}_k^i+\sum_{j\in S_{uk}^i}u_{ij}P_j。$$

由 Schur 补引理可知,不等式(10)与式(11)成立,可得 $\phi_i<0$。

定理 2　对于给定 $\gamma>0$,若存在矩阵 $P_i>0$,且 Q_i 使得以下不等式成立,则滤波误差系统是随机稳定的,并满足给定的 H_∞ 性能。

$$\begin{bmatrix}\Omega_{11} & 0 & 0 & \Omega_{14} & \Omega_{15}\\ * & \Omega_{11} & 0 & \Omega_{24} & 0\\ * & * & -I & \bar{L}_i & 0\\ * & * & * & -\frac{1}{h}P_i & 0\\ * & * & * & * & -\gamma^2I\end{bmatrix}<0。\tag{20}$$

式中: $\Omega_{11}=h(\Xi_j-Q_i-Q_i^{\mathrm{T}})$; $\Omega_{14}=Q_i(h\bar{E}_i+I)$; $\Omega_{15}=hQ_i\bar{F}_i$; $\Omega_{24}=h\theta Q_i\bar{E}_{1i}$; $\Xi_j\overset{\mathrm{def}}{=}\frac{1}{u_k^i}\bar{P}_k^i,\ \forall j\in S_k^i$; $\Xi_j\overset{\mathrm{def}}{=}P_j,\ \forall j\in S_{uk}^i$。

证明　不等式(10)等价于:

$$\begin{bmatrix} \boldsymbol{\pi}_{11} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{\pi}_{14} & \boldsymbol{\pi}_{15} \\ * & \boldsymbol{\pi}_{11} & \boldsymbol{0} & \boldsymbol{\pi}_{24} & \boldsymbol{0} \\ * & * & -\boldsymbol{I} & \bar{\boldsymbol{L}}_i & \boldsymbol{0} \\ * & * & * & -\frac{1}{h}\boldsymbol{P}_i & \boldsymbol{0} \\ * & * & * & * & -\gamma^2\boldsymbol{I} \end{bmatrix} < 0。 \quad (21)$$

式中：$\boldsymbol{\pi}_{11} = -h\frac{1}{u_k^i}\bar{\boldsymbol{P}}_k^i$；$\boldsymbol{\pi}_{14} = \frac{1}{u_k^i}\bar{\boldsymbol{P}}_k^i(h\bar{\boldsymbol{E}}_i + \boldsymbol{I})$；$\boldsymbol{\pi}_{15} = h\frac{1}{u_k^i}\bar{\boldsymbol{P}}_k^i\bar{\boldsymbol{F}}_i$；$\boldsymbol{\pi}_{24} = h\theta\frac{1}{u_k^i}\bar{\boldsymbol{P}}_k^i\bar{\boldsymbol{E}}_{1i}$。

对于任一矩阵 $\boldsymbol{Q}_i$，$\forall i \in \boldsymbol{S}$，有以下不等式成立：$\boldsymbol{\Xi}_j - \boldsymbol{Q}_i - \boldsymbol{Q}_i^{\mathrm{T}} \geqslant -\boldsymbol{Q}_i\boldsymbol{\Xi}_j^{-1}\boldsymbol{Q}_i^{\mathrm{T}}$。所以由式(21)可得：

$$\begin{bmatrix} -h\boldsymbol{Q}_i\boldsymbol{\Xi}_j^{-1}\boldsymbol{Q}_i^{\mathrm{T}} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{\Omega}_{14} & \boldsymbol{\Omega}_{15} \\ * & -h\boldsymbol{Q}_i\boldsymbol{\Xi}_j^{-1}\boldsymbol{Q}_i^{\mathrm{T}} & \boldsymbol{0} & \boldsymbol{\Omega}_{24} & \boldsymbol{0} \\ * & * & -\boldsymbol{I} & \bar{\boldsymbol{L}}_i & \boldsymbol{0} \\ * & * & * & -\frac{1}{h}\boldsymbol{P}_i & \boldsymbol{0} \\ * & * & * & * & -\gamma^2\boldsymbol{I} \end{bmatrix} < 0。 \quad (22)$$

对式(22)左右两边同乘以对角阵 $\mathrm{diag}\{\boldsymbol{\Xi}_j\boldsymbol{Q}_i^{-1}, \boldsymbol{\Xi}_j\boldsymbol{Q}_i^{-1}, \boldsymbol{I}, \boldsymbol{I}, \boldsymbol{I}\}$ 可得不等式(23)。

定理3 给定常数 $\gamma > 0$，对于式(3)中参数的整个不确定域，若存在矩阵 $\boldsymbol{P}_i > 0$，$\boldsymbol{Q}_i$ 和正标量 ε_i 使得以下不等式成立，则滤波误差系统鲁棒随机稳定且满足 H_∞ 性能。

$$\begin{bmatrix} \boldsymbol{\Omega}_{11} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{\varphi}_{14} & \boldsymbol{\varphi}_{15} & \boldsymbol{\varphi}_{16} \\ * & \boldsymbol{\Omega}_{11} & \boldsymbol{0} & \boldsymbol{\varphi}_{24} & \boldsymbol{0} & \boldsymbol{0} \\ * & * & -\boldsymbol{I} & \bar{\boldsymbol{L}}_i & \boldsymbol{0} & \boldsymbol{0} \\ * & * & * & -\frac{1}{h}\boldsymbol{P}_i + \boldsymbol{\varphi}_{44} & \boldsymbol{\varphi}_{45} & \boldsymbol{0} \\ * & * & * & * & -\gamma^2\boldsymbol{I} + \boldsymbol{\varphi}_{55} & \boldsymbol{0} \\ * & * & * & * & * & -\varepsilon_i\boldsymbol{I} \end{bmatrix} < 0。 \quad (23)$$

式中：

$$\boldsymbol{\Omega}_{11} = h(\boldsymbol{\Xi}_j - \boldsymbol{Q}_i - \boldsymbol{Q}_i^{\mathrm{T}});$$

$$\boldsymbol{\varphi}_{14} = \boldsymbol{Q}_i(h\tilde{\boldsymbol{E}}_i + \boldsymbol{I});$$

$$\boldsymbol{\varphi}_{15} = h\boldsymbol{Q}_i\tilde{\boldsymbol{F}}_i;\ \boldsymbol{\varphi}_{16} = \sqrt{h}\boldsymbol{Q}_i\tilde{\boldsymbol{M}}_i;$$

$$\boldsymbol{\varphi}_{24} = h\theta\boldsymbol{Q}_i\bar{\boldsymbol{E}}_{1i};\ \boldsymbol{\varphi}_{44} = h\varepsilon_i\tilde{\boldsymbol{N}}_{ai}^{\mathrm{T}}\tilde{\boldsymbol{N}}_{ai};$$

$$\boldsymbol{\varphi}_{45} = h\varepsilon_i\tilde{\boldsymbol{N}}_{ai}^{\mathrm{T}}\tilde{\boldsymbol{N}}_{bi};\ \boldsymbol{\varphi}_{55} = h\varepsilon_i\tilde{\boldsymbol{N}}_{bi}^{\mathrm{T}}\tilde{\boldsymbol{N}}_{bi}。$$

证明 式(19)可写成如下形式：

$$\boldsymbol{R}_i + \boldsymbol{\Gamma}_{1i}\boldsymbol{X}_{ik}\boldsymbol{\Gamma}_{2i} + \boldsymbol{\Gamma}_{2i}^{\mathrm{T}}\boldsymbol{X}_{ik}^{\mathrm{T}}\boldsymbol{\Gamma}_{1i}^{\mathrm{T}} < 0。 \quad (24)$$

式中：

$$\boldsymbol{\Gamma}_{1i} = [\sqrt{h}\tilde{\boldsymbol{M}}_i^{\mathrm{T}}\boldsymbol{Q}_i^{\mathrm{T}} \quad \boldsymbol{0} \quad \boldsymbol{0} \quad \boldsymbol{0} \quad \boldsymbol{0}]^{\mathrm{T}};$$

$$\boldsymbol{\Gamma}_{2i} = [\boldsymbol{0} \quad \boldsymbol{0} \quad \boldsymbol{0} \quad \sqrt{h}\tilde{\boldsymbol{N}}_{ai} \quad \sqrt{h}\tilde{\boldsymbol{N}}_{bi}];$$

$$\boldsymbol{R}_i = \begin{bmatrix} \boldsymbol{\Omega}_{11} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{\varphi}_{14} & \boldsymbol{\varphi}_{15} \\ * & \boldsymbol{\Omega}_{11} & \boldsymbol{0} & \boldsymbol{\varphi}_{24} & \boldsymbol{0} \\ * & * & -\boldsymbol{I} & \bar{\boldsymbol{L}}_i & \boldsymbol{0} \\ * & * & * & -\frac{1}{h}\boldsymbol{P}_i & \boldsymbol{0} \\ * & * & * & * & -\gamma^2\boldsymbol{I} \end{bmatrix} < 0。$$

当存在正标量 ε_i 使得不等式(23)成立，由Schur补引理可得不等式(20)成立。

定理4 对于具有通信丢包和不确定参数的离散马尔可夫跳变系统(见式(2))，若存在矩阵 $\boldsymbol{P}_{1i}$、$\boldsymbol{P}_{2i}$、$\boldsymbol{P}_{3i}$、$\boldsymbol{C}_{fi}$、$\boldsymbol{D}_{fi}$、$\boldsymbol{B}_{fi}$、$\boldsymbol{U}_i$、$\boldsymbol{V}_i$、$\boldsymbol{W}_i$ 及正标量 ε_i 满足以下不等式，则滤波误差系统随机稳定且满足给定的 H_∞ 性能指标 γ。

$$\begin{bmatrix} \boldsymbol{\Lambda}_{11} & \boldsymbol{\Lambda}_{12} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{\Lambda}_{16} & \boldsymbol{\Lambda}_{17} & \boldsymbol{\Lambda}_{18} & \boldsymbol{\Lambda}_{19} \\ * & \boldsymbol{\Lambda}_{22} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{\Lambda}_{26} & \boldsymbol{\Lambda}_{27} & \boldsymbol{\Lambda}_{28} & \boldsymbol{\Lambda}_{29} \\ * & * & \boldsymbol{\Lambda}_{33} & \boldsymbol{\Lambda}_{34} & \boldsymbol{0} & \boldsymbol{\Lambda}_{36} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ * & * & * & \boldsymbol{\Lambda}_{44} & \boldsymbol{0} & \boldsymbol{\Lambda}_{46} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ * & * & * & * & -\boldsymbol{I} & \boldsymbol{G}_i & -\boldsymbol{B}_{fi} & \boldsymbol{0} & \boldsymbol{0} \\ * & * & * & * & * & \boldsymbol{\Lambda}_{66} & \boldsymbol{\Lambda}_{67} & \boldsymbol{\Lambda}_{68} & \boldsymbol{0} \\ * & * & * & * & * & * & \boldsymbol{\Lambda}_{77} & \boldsymbol{0} & \boldsymbol{0} \\ * & * & * & * & * & * & * & \boldsymbol{\Lambda}_{88} & \boldsymbol{0} \\ * & * & * & * & * & * & * & * & \boldsymbol{\Lambda}_{99} \end{bmatrix} < 0。 \quad (25)$$

式中：

$$\boldsymbol{\Lambda}_{11} = \boldsymbol{\Lambda}_{33} = h(\boldsymbol{\Xi}_{1j} - \boldsymbol{U}_i - \boldsymbol{U}_i^{\mathrm{T}});$$

$$\boldsymbol{\Lambda}_{12} = \boldsymbol{\Lambda}_{34} = h(\boldsymbol{\Xi}_{2j} - \boldsymbol{W}_i - \boldsymbol{V}_i^{\mathrm{T}});$$

$$\boldsymbol{\Lambda}_{16} = \boldsymbol{U}_i(h\boldsymbol{E}_i + \boldsymbol{I}) + h\beta\boldsymbol{D}_{fi}\boldsymbol{H}_i;$$

$$\boldsymbol{\Lambda}_{17} = \boldsymbol{\Lambda}_{27} = h\boldsymbol{C}_{fi} - h\beta\boldsymbol{D}_{fi}\boldsymbol{H}_i + \boldsymbol{W}_i;$$

$$\boldsymbol{\Lambda}_{18} = h\boldsymbol{U}_i\boldsymbol{F}_i + h\boldsymbol{D}_{fi}\boldsymbol{L}_i;\ \boldsymbol{\Lambda}_{19} = \sqrt{h}\boldsymbol{U}_i\boldsymbol{M}_i;$$

$$\boldsymbol{\Lambda}_{22} = \boldsymbol{\Lambda}_{44} = h(\boldsymbol{\Xi}_{3j} - \boldsymbol{W}_i - \boldsymbol{W}_i^{\mathrm{T}});$$

$$\boldsymbol{\Lambda}_{26} = \boldsymbol{V}_i(h\boldsymbol{E}_i + \boldsymbol{I}) + h\beta\boldsymbol{D}_{fi}\boldsymbol{H}_i;$$

$$\boldsymbol{\Lambda}_{28} = h\boldsymbol{V}_i\boldsymbol{F}_i + h\boldsymbol{D}_{fi}\boldsymbol{L}_i;\ \boldsymbol{\Lambda}_{29} = \sqrt{h}\boldsymbol{V}_i\boldsymbol{M}_i;\ \boldsymbol{\Lambda}_{36} = \boldsymbol{\Lambda}_{46} = h\theta\boldsymbol{D}_{fi}\boldsymbol{H}_i;$$

$$\boldsymbol{\Lambda}_{66} = -\frac{1}{h}\boldsymbol{P}_{1i} + h\varepsilon_i\boldsymbol{N}_{ai}^{\mathrm{T}}\boldsymbol{N}_{ai};\ \boldsymbol{\Lambda}_{67} = -\frac{1}{h}\boldsymbol{P}_{2i};\ \boldsymbol{\Lambda}_{68} = h\varepsilon_i\boldsymbol{N}_{ai}^{\mathrm{T}}\boldsymbol{N}_{bi};$$

$$\boldsymbol{\Lambda}_{77} = -\frac{1}{h}\boldsymbol{P}_{3i};\ \boldsymbol{\Lambda}_{88} = -\gamma^2\boldsymbol{I} + h\varepsilon_i\boldsymbol{N}_{bi}^{\mathrm{T}}\boldsymbol{N}_{bi};\ \boldsymbol{\Lambda}_{99} = -\varepsilon_i\boldsymbol{I};$$

$$\begin{cases} \boldsymbol{\Xi}_{ij} \overset{\mathrm{def}}{=\!=} \frac{1}{u_k^i}\sum_{j \in S_k^i} u_{ij}\boldsymbol{P}_{ij}, & u_k^i \neq 0; \\ \boldsymbol{\Xi}_{ij} \overset{\mathrm{def}}{=\!=} \boldsymbol{P}_{ij}, & \forall j \in u_{uk}^i。 \end{cases}$$

滤波器的参数为

$$\boldsymbol{C}_{Fi} = \boldsymbol{W}_i^{-1}\boldsymbol{C}_{fi};\ \boldsymbol{D}_{Fi} = \boldsymbol{W}_i^{-1}\boldsymbol{D}_{fi};\ \boldsymbol{B}_{Fi} = \boldsymbol{B}_{fi}。$$

证明　矩阵 $\boldsymbol{P}_i$ 和 $\boldsymbol{Q}_i$ 具有以下形式:

$$\boldsymbol{P}_i=\begin{bmatrix}\boldsymbol{P}_{1i} & \boldsymbol{P}_{2i}\\ * & \boldsymbol{P}_{3i}\end{bmatrix}>0;\quad \boldsymbol{Q}_i=\begin{bmatrix}\boldsymbol{U}_i & \boldsymbol{W}_i\\ \boldsymbol{V}_i & \boldsymbol{W}_i\end{bmatrix}。$$

其中,$\boldsymbol{W}_i$ 为可逆矩阵。将滤波器参数代入不等式(25)可得不等式(23),证毕。

3　数值仿真

考虑具有 4 个模态的离散 MJSs(见式(2)),代入以下初始数据:

$$\boldsymbol{E}_1=\begin{bmatrix}-0.3000 & -0.5000\\ 0.8100 & -0.9000\end{bmatrix};$$

$$\boldsymbol{E}_2=\begin{bmatrix}-0.3000 & -0.3000\\ 0.8100 & -1.0800\end{bmatrix};$$

$$\boldsymbol{E}_3=\begin{bmatrix}-0.3000 & -0.8100\\ 0.8100 & -0.9720\end{bmatrix};$$

$$\boldsymbol{E}_4=\begin{bmatrix}-0.6000 & -0.6000\\ 0.8100 & -1.8900\end{bmatrix};$$

$$\boldsymbol{F}_1=\begin{bmatrix}-0.5000\\ 1.0000\end{bmatrix};\ \boldsymbol{F}_2=\begin{bmatrix}-0.3000\\ 0.8000\end{bmatrix};$$

$$\boldsymbol{F}_3=\begin{bmatrix}-0.5000\\ 1.1000\end{bmatrix};\ \boldsymbol{F}_4=\begin{bmatrix}-0.4000\\ 0.9000\end{bmatrix};$$

$\boldsymbol{H}_1=[0.4000\quad 0.5000]$; $\boldsymbol{H}_2=[0.3000\quad 0.2000]$;
$\boldsymbol{H}_3=[0.1000\quad 0.2000]$; $\boldsymbol{H}_4=[0.2000\quad 0.4000]$;
$L_1=0.8000$; $L_2=0.9000$; $L_3=0.8100$; $L_4=0.8500$;
$\boldsymbol{G}_1=[0.1000\quad -0.2000]$; $\boldsymbol{G}_2=[0.1000\quad -0.2000]$;
$\boldsymbol{G}_3=[0.1000\quad -0.2000]$; $\boldsymbol{G}_4=[0.1000\quad -0.2000]$;

$$\boldsymbol{M}_1=\boldsymbol{M}_2=\boldsymbol{M}_3=\boldsymbol{M}_4=[0.3000\quad 0.3000];$$

$$\boldsymbol{N}_{a1}=\boldsymbol{N}_{a2}=\boldsymbol{N}_{a3}=\boldsymbol{N}_{a4}=[0.2000\quad 0.1000];$$

$$N_{b1}=N_{b2}=N_{b3}=N_{b4}=0.1。$$

系统的转移概率矩阵如下所示:

$$\boldsymbol{\Pi}=\begin{bmatrix}0.1 & 0.4 & 0.2 & 0.3\\ 0.3 & p & p & 0.2\\ p & 0.1 & p & 0.3\\ 0.3 & 0.2 & p & p\end{bmatrix}。\qquad (26)$$

式中:p 表示存在但不可预知的系统转移概率。

表 1 为不同传输概率下系统最优 H_∞ 性能指标。其中 γ_1、γ_2 分别为移位算子系统、Delta 算子系统的最优 H_∞ 性能指标。由表 1 可知:β 取值越高,系统对于干扰的抑制度 γ 越小,即系统的性能越好。在相同传输率下,Delta 算子系统最优 H_∞ 性能总是优于移位算子系统最优 H_∞ 性能。当 $\beta=0.6$ 时,选择初始条件 $\boldsymbol{x}_0=[0.4\quad 0.6\quad 0\quad 0]^{\mathrm{T}}$,能量有限噪声信号 $\omega(k)=0.5\exp(-0.1k)$,则滤波系统误差响应如图 1 所示。图 1 表示在 MJSs 中 4 个模态的滤波误差响应分别收敛。

表 1　不同传输概率下系统最优 H_∞ 性能指标

Table 1　Optimal H_∞ performance index under different transmission probabilities

β	γ_1	γ_2	β	γ_1	γ_2
0.1	1.0361	0.0137	0.6	0.8582	0.0100
0.2	1.0865	0.0132	0.7	0.7820	0.0089
0.3	1.0526	0.0124	0.8	0.6969	0.0083
0.4	0.9950	0.0120	0.9	0.5959	0.0063
0.5	0.9289	0.0109	1.0	0.5054	0.0048

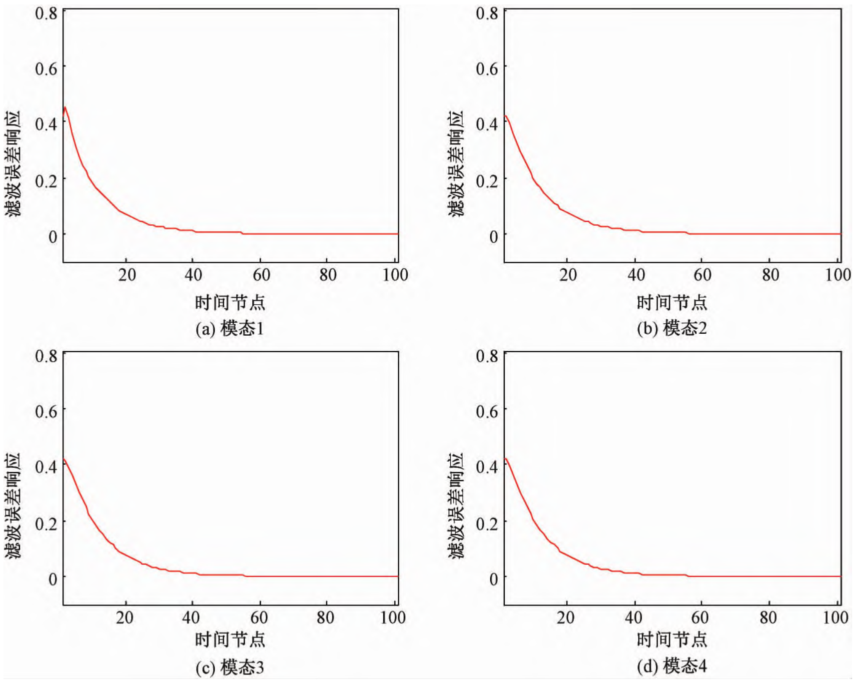

图 1　4 个模态的滤波误差响应

Figure 1　Four-mode filtering error responses

图 2 表明,在按照表 1 所示的部分未知转移概率跳变滤波误差系统的滤波误差响应可以在给定时间内收敛。图 3 中曲线分别表示 MJSs 的输出响应 $z(k)$ 及模态相关滤波器的输出响应 $z_F(k)$。图 4 表示传输概率取值为 0.1~1.0 时,Delta 算子系统最优 H_∞ 性能指标 γ_2 取值分布。

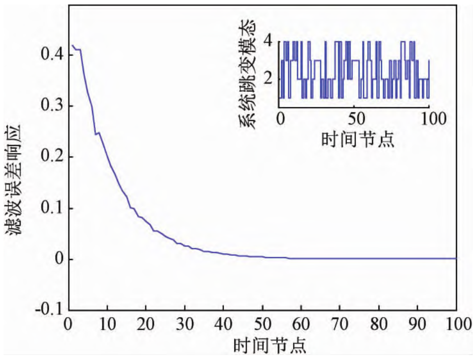

图 2 跳变系统滤波误差响应(β=0.6)

Figure 2 Filtering error response of jumping system(β=0.6)

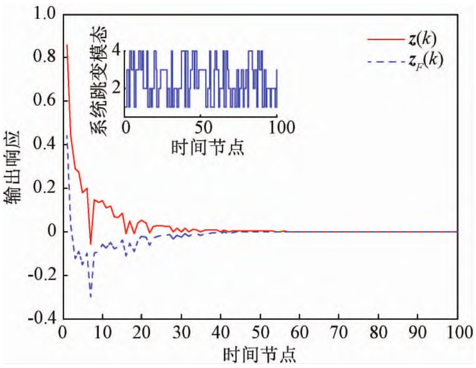

图 3 跳变系统及滤波器输出响应(β=0.6)

Figure 3 Jump system and filter output responses(β=0.6)

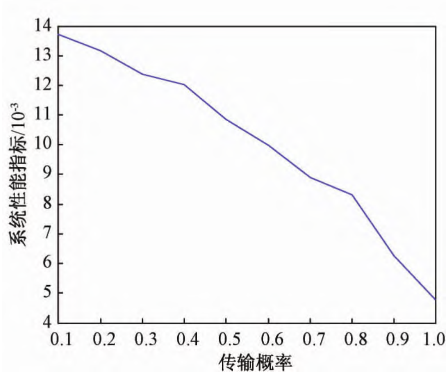

图 4 Delta 算子系统最优 H_∞ 性能指标

Figure 4 Optimal H_∞ performance index of Delta operator system

4 结论

本文对具有通信丢包和不确定参数的离散 MJSs H_∞ 滤波器进行分析与设计。离散 MJSs 的模态转移概率部分未知且丢包概率服从伯努利分布,在相同丢包概率下 Delta 算子系统最优 H_∞ 性能总是优于移位算子系统最优 H_∞ 性能。转移概率部分未知的 Delta 算子 MJSs 随机稳定且满足给定的 H_∞ 性能。

参考文献:

[1] 刘艳红,杨东伟.基于观测器的网络控制系统均方指数稳定控制器设计[J].郑州大学学报(工学版),2018,39(3):10-14.

[2] PAN Y N,YANG G H.Novel event-triggered filter design for nonlinear networked control systems[J].Journal of the Franklin institute,2018,355(3):1259-1277.

[3] 刘义才,刘斌,张永,等.具有双边随机时延和丢包的网络控制系统稳定性分析[J].控制与决策,2017,32(9):1565-1573.

[4] 李艳辉,李玉龙.具有随机时延和丢包的网络化离散系统的动态输出反馈 H_∞ 控制[J].化工自动化及仪表,2017,44(1):6-11,15.

[5] SHI L,XIE L H,MURRAY R M.Kalman filtering over a packet-delaying network: a probabilistic approach[J].Automatica,2009,45(9):2134-2140.

[6] WANG G P,XU H L,ZHANG G Z,et al.H_∞ filtering for spatially interconnected time-delay systems with interconnected chains in finite frequency domains[J]. Asian journal of control,2020,22(1):511-520.

[7] CHANG X H,XIONG J,PARK J H.Resilient H_∞ filtering for discrete-time systems[J].Signal processing,2016,127:71-79.

[8] YIN Y Y,SHI P,LIU F,et al.Filtering for discrete-time nonhomogeneous Markov jump systems with uncertainties[J].Information sciences,2014,259:118-127.

[9] ZHU J, WU X H, LI C X, et al. State and mode feedback control strategy for discrete-time Markovian jump linear systems with time-varying controllable mode transition probability matrix[J]. International journal of robust and nonlinear control,2020,30(8):3501-3519.

[10] NIU Y J,DONG W,JI Y D.Robust H_∞ filtering for discrete-time Markov jump linear system with missing measurements[EB/OL].(2020-12-03) [2015-05-28].https://doi.org/10.1155/2015/671491.

[11] 张端金,刘雪,范鑫.基于 Delta 算子时变时延网络控制系统的 H_∞ 滤波[J].郑州大学学报(工学版),2016,37(2):10-14.

[12] ZHANG L X,BOUKAS E K.Stability and stabilization of Markovian jump linear systems with partly unknown transition probabilities[J].Automatica,2009,45(2):463-468.

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CUDA-based Parallel Swarm Intelligence Method for Solving Weighted MTSP

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Abstract: To solve the low running speed of the multi-traveling salesman problem (MTSP) using the heuristic method, a CUDA-based hybrid particle swarm clustering-ant colony algorithm (GPSO-AC) was proposed by integrating their parallel characteristics with programming techniques optimally. GPSO-AC used GPU′s instruction architecture with multiple stream processors (SM) and single instruction multithreading (SIMT) to parallel the search process of numerous independent individuals, so as to accelerate the execution speed of the hybrid iterative method. GPSO-AC was tested on 6 datasets compared with other methods, such as PSO-AC, TPHA and *K*-means-AC. Then the influence of cost equilibrium constraint on the convergence performance of the optimal solution of weighted MTSP problem was discussed. Furthermore, the cost standard deviations obtained from GPSO-AC on chn31 with different traveling salesmen, were 1 165. 26, 54. 97 and 6. 74 in the three cases respectively. The experimental results showed that the proposed algorithm was much faster than other CPU based algorithms and the advantage becomed more obvious with the expansion of the model size, and the convergence precision of the algorithm was better than the similar algorithms for solving MTSP problems.

Keywords: multiple traveling salesman problem(MTSP) ; CUDA parallel algorithm; cost-balanced; particle swarm clustering; ant colony algorithm

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Robust H_∞ Filtering for Markov Jump Systems with Unknown Transition Probabilities and Packet Dropouts

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Abstract: The robust H_∞ filtering problem of Markov jump systems with partly unknown transition probabilities and packet dropouts was investigated. Assuming that the probability of packet dropouts would obey Bernoulli distribution, a discrete-time Markov jump system with uncertain parameters and mode-dependent full-order filter were constructed based on the Delta operator. The slack matrix variables were introduced to solve the cross coupling between the system matrices and the Lyapunov positive matrices. The Lyapunov function, Schur complement and linear matrix inequalities were used to obtain sufficient conditions for the system to be stochastically stable and satisfy H_∞ performance. The optimal H_∞ performance index of the Delta operator system and the shifting operator system were obtained respectively with the known probability of packet dropouts. When the probability of the packet dropouts were lower, the robust performance as well as the optimal H_∞ performance of Delta operator system were better than the shift operator system. Numerical simulation proved that the method proposed in this paper not only was effective and feasible, but also had certain advantages.

Keywords: Markov jumping systems; uncertain parameters; packet dropouts; Delta operator