Pre-Calculus 11

The Ambiguous Case of the Sine Law

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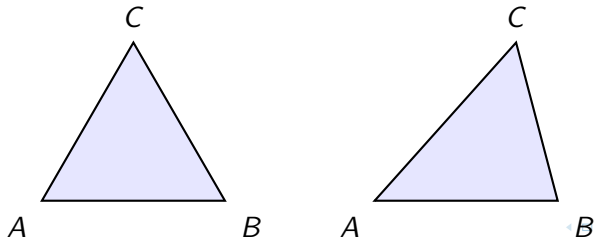
What is the Ambiguous Case?

What is the Ambiguous Case?

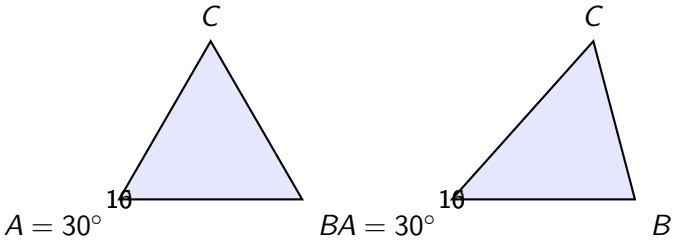
- The ambiguous case occurs when using the Sine Law to solve a triangle and two different triangles can be constructed with the given information.
- This happens when you are given two sides and a non-included angle (SSA case), and the side opposite the given angle is shorter than the other given side.
- You must consider both possible triangles.



Example: Two Possible Triangles

Question

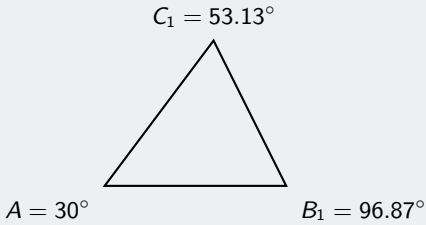
In $\triangle ABC$, $A = 30°$, $a = 10$, $c = 16$. Find all possible values for B and b.



Example: Two Possible Triangles

Solution (Triangle 1)

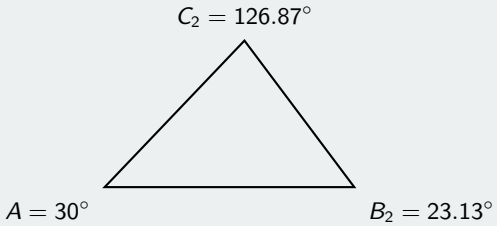
$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{10}{\sin 30^\circ} = \frac{16}{\sin C}$$
$$\sin C = \frac{16 \cdot \sin 30^\circ}{10} = 0.8$$
$$C_1 = \sin^{-1}(0.8) \approx 53.13^\circ$$
$$B_1 = 180^\circ - 30^\circ - 53.13^\circ = 96.87^\circ$$
$$\frac{a}{\sin A} = \frac{b_1}{\sin B_1}$$
$$b_1 = \frac{10 \cdot \sin 96.87^\circ}{\sin 30^\circ} \approx 19.93$$



Example: Two Possible Triangles

Solution (Triangle 2)

$$C_2 = 180^\circ - 53.13^\circ = 126.87^\circ$$
$$B_2 = 180^\circ - 30^\circ - 126.87^\circ = 23.13^\circ$$
$$b_2 = \frac{10 \cdot \sin 23.13^\circ}{\sin 30^\circ} \approx 7.86$$



Practice: Ambiguous Case

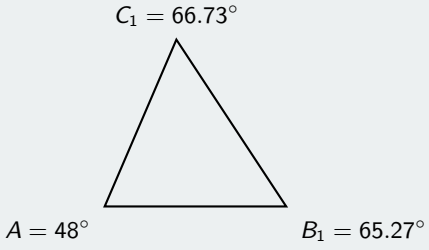
Practice

In $\triangle ABC$, $A = 48^\circ$, $a = 9$, $c = 11$. Find all possible values for B and C.

Practice: Ambiguous Case

Solution (Triangle 1)

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{9}{\sin 48^\circ} = \frac{11}{\sin C}$$
$$\sin C = \frac{11 \cdot \sin 48^\circ}{9} \approx 0.914$$
$$C_1 = \sin^{-1}(0.914) \approx 66.73^\circ$$
$$B_1 = 180^\circ - 48^\circ - 66.73^\circ = 65.27^\circ$$

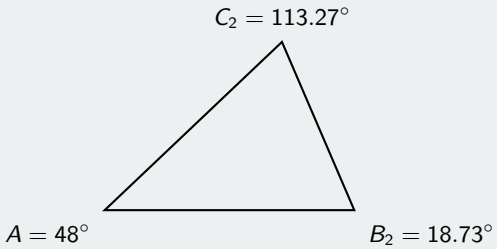


Practice: Ambiguous Case

Solution (Triangle 2)

$C_2 = 180^\circ - 66.73^\circ = 113.27^\circ$

$B_2 = 180^\circ - 48^\circ - 113.27^\circ = 18.73^\circ$



When is the Ambiguous Case Possible?

Summary

- The ambiguous case (SSA) is possible when:
  - The side opposite the given angle is shorter than the other given side, but longer than the height from the given angle.
  - $a < c$ and $a > c \sin A$
- If the side opposite the angle is longer than the other given side, only one triangle is possible.
- If the side opposite the angle is shorter than the height, no triangle is possible.