Chapter 4.1: Radicals

Pre-Calculus 11

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Review: Basic Radicals

Evaluate each of the following without a calculator

1. $\sqrt{4}$
2. $\sqrt{25}$
3. $\sqrt{121}$
4. $\sqrt{64}$

1. $\sqrt{49}$
2. $\sqrt[3]{8}$
3. $\sqrt[3]{27}$
4. $\sqrt[3]{64}$

1. $\sqrt[4]{16}$
2. $\sqrt{2} \times \sqrt{25}$
3. $\sqrt{242}$
4. $\sqrt{192}$

1. $\sqrt{98}$
2. $\sqrt[3]{24}$
3. $\sqrt[3]{54}$
4. $\sqrt[3]{320}$
5. $\sqrt[4]{48}$

Review: Basic Radicals - Answers

Solutions

1. $\sqrt{4} = 2$
2. $\sqrt{25} = 5$
3. $\sqrt{121} = 11$
4. $\sqrt{64} = 8$

1. $\sqrt{49} = 7$
2. $\sqrt[3]{8} = 2$
3. $\sqrt[3]{27} = 3$
4. $\sqrt[3]{64} = 4$

1. $\sqrt[4]{16} = 2$
2. $\sqrt{2} \times \sqrt{25} = 5\sqrt{2}$
3. $\sqrt{242} = 11\sqrt{2}$
4. $\sqrt{192} = 8\sqrt{3}$

1. $\sqrt{98} = 7\sqrt{2}$
2. $\sqrt[3]{24} = 2\sqrt[3]{3}$
3. $\sqrt[3]{54} = 3\sqrt[3]{2}$
4. $\sqrt[3]{320} = 4\sqrt[3]{5}$
5. $\sqrt[4]{48} = 2\sqrt[4]{3}$

Fractional Exponents and Nth Roots

Key Concepts

- The square root of a number is equivalent to having an exponent of $\frac{1}{2}$
- The cube root of a number is equivalent to having an exponent of $\frac{1}{3}$
- The fourth root of a number is equivalent to having an exponent of $\frac{1}{4}$
- The nth root of a number is equivalent to having an exponent of $\frac{1}{n}$

$$\sqrt{x} = x^{\frac{1}{2}}$$

$$\sqrt[3]{x} = x^{\frac{1}{3}}$$

$$\sqrt[4]{x} = x^{\frac{1}{4}}$$

$$\sqrt[n]{x} = x^{\frac{1}{n}}$$

Simplifying Radicals

Practice Problems

1. $\sqrt{5} \times \sqrt{5}$
2. $\sqrt[3]{8} \times \sqrt[3]{8} \times \sqrt[3]{8}$

1. $\sqrt{150}$
2. $\sqrt[3]{108}$

Simplifying Radicals - Answers

Solutions

1. $\sqrt{5} \times \sqrt{5} = 5$
2. $\sqrt[3]{8} \times \sqrt[3]{8} \times \sqrt[3]{8} = 8$

1. $\sqrt{150} = \sqrt{25 \times 6} = 5\sqrt{6}$
2. $\sqrt[3]{108} = \sqrt[3]{27 \times 4} = 3\sqrt[3]{4}$

Adding & Subtracting Radicals

Practice Problems

1. $3\sqrt{2} + 5\sqrt{2} - 7\sqrt{2} + 11\sqrt{2}$
2. $7\sqrt{3} + 9\sqrt{3} - 12\sqrt{3} - 15\sqrt{3}$

1. $3\sqrt{5} - \sqrt{20} + \sqrt{45} + 12\sqrt{5} - \sqrt{180}$
2. $3\sqrt{12} - 2\sqrt{45} + \sqrt{75} - 6\sqrt{80} + \sqrt{243}$

Adding & Subtracting Radicals - Answers

Solutions

1. $3\sqrt{2} + 5\sqrt{2} - 7\sqrt{2} + 11\sqrt{2} = 12\sqrt{2}$
2. $7\sqrt{3} + 9\sqrt{3} - 12\sqrt{3} - 15\sqrt{3} = -11\sqrt{3}$

1. $3\sqrt{5} - \sqrt{20} + \sqrt{45} + 12\sqrt{5} - \sqrt{180} = 10\sqrt{5}$
2. $3\sqrt{12} - 2\sqrt{45} + \sqrt{75} - 6\sqrt{80} + \sqrt{243} = 20\sqrt{3} - 30\sqrt{5}$

Different Types of Roots - Practice

Practice Problems

1. $\sqrt{72}$
2. $\sqrt[3]{81}$
3. $\sqrt[4]{48}$
4. $\sqrt[5]{96}$

1. $\sqrt{200}$
2. $\sqrt[3]{128}$
3. $\sqrt[4]{162}$
4. $\sqrt[5]{243}$

Different Types of Roots - Answers

Solutions

1. $\sqrt{72} = 6\sqrt{2}$
2. $\sqrt[3]{81} = 3\sqrt[3]{3}$
3. $\sqrt[4]{48} = 2\sqrt[4]{3}$
4. $\sqrt[5]{96} = 2\sqrt[5]{3}$

1. $\sqrt{200} = 10\sqrt{2}$
2. $\sqrt[3]{128} = 4\sqrt[3]{2}$
3. $\sqrt[4]{162} = 3\sqrt[4]{2}$
4. $\sqrt[5]{243} = 3$

Variable Radicals - Practice

Practice Problems

1. $\sqrt{x^2}$
2. $\sqrt[3]{x^3}$
3. $\sqrt{x^4}$
4. $\sqrt[3]{x^6}$

1. $\sqrt{x^2y^4}$
2. $\sqrt[3]{x^3y^6}$
3. $\sqrt{x^4y^2}$
4. $\sqrt[3]{x^6y^9}$

Variable Radicals - Answers

Solutions

1. $\sqrt{x^2} = |x|$
2. $\sqrt[3]{x^3} = x$
3. $\sqrt{x^4} = x^2$
4. $\sqrt[3]{x^6} = x^2$

1. $\sqrt{x^2y^4} = |x|y^2$
2. $\sqrt[3]{x^3y^6} = xy^2$
3. $\sqrt{x^4y^2} = x^2|y|$
4. $\sqrt[3]{x^6y^9} = x^2y^3$

Mixed to Entire Radicals - Practice

Practice Problems

1. $2\sqrt{3}$
2. $3\sqrt[3]{2}$
3. $4\sqrt{5}$
4. $2\sqrt[3]{3}$

1. $3\sqrt{2}$
2. $2\sqrt[3]{4}$
3. $5\sqrt{3}$
4. $3\sqrt[3]{2}$

Mixed to Entire Radicals - Answers

Solutions

1. $2\sqrt{3} = \sqrt{12}$
2. $3\sqrt[3]{2} = \sqrt[3]{54}$
3. $4\sqrt{5} = \sqrt{80}$
4. $2\sqrt[3]{3} = \sqrt[3]{24}$

1. $3\sqrt{2} = \sqrt{18}$
2. $2\sqrt[3]{4} = \sqrt[3]{32}$
3. $5\sqrt{3} = \sqrt{75}$
4. $3\sqrt[3]{2} = \sqrt[3]{54}$

Entire to Mixed Radicals - Practice

Practice Problems

1. $\sqrt{12}$
2. $\sqrt[3]{54}$
3. $\sqrt{80}$
4. $\sqrt[3]{24}$

1. $\sqrt{18}$
2. $\sqrt[3]{32}$
3. $\sqrt{75}$
4. $\sqrt[3]{54}$

Entire to Mixed Radicals - Answers

Solutions

1. $\sqrt{12} = 2\sqrt{3}$
2. $\sqrt[3]{54} = 3\sqrt[3]{2}$
3. $\sqrt{80} = 4\sqrt{5}$
4. $\sqrt[3]{24} = 2\sqrt[3]{3}$

1. $\sqrt{18} = 3\sqrt{2}$
2. $\sqrt[3]{32} = 2\sqrt[3]{4}$
3. $\sqrt{75} = 5\sqrt{3}$
4. $\sqrt[3]{54} = 3\sqrt[3]{2}$

Simplify Each Entire Radical to a Mixed Radical

Practice Problems

1. $\sqrt[3]{a^5 b^7}$
2. $\sqrt[4]{x^9 y^5}$
3. $\sqrt[5]{m^8 n^{12}}$
4. $\sqrt[3]{-b^6}$

Simplify Each Entire Radical to a Mixed Radical - Solutions (1/2)

Solutions

1. $\sqrt[3]{a^5b^7}$

$$= \sqrt[3]{a^3 \cdot a^2 \cdot b^6 \cdot b}$$
$$= ab^2 \times \sqrt[3]{a^2 b}$$

2. $\sqrt[4]{x^9y^5}$

$$= \sqrt[4]{x^8 \cdot x \cdot y^4 \cdot y}$$
$$= x^2y \times \sqrt[4]{xy}$$

Simplify Each Entire Radical to a Mixed Radical - Solutions (2/2)

Solutions

3. $\sqrt[5]{m^8 n^{12}}$

$$= \sqrt[5]{m^5 \cdot m^3 \cdot n^{10} \cdot n^2}$$
$$= mn^2 \times \sqrt[5]{m^3 n^2}$$

4. $\sqrt[3]{-b^6}$

$$= \sqrt[3]{(-1)b^6}$$
$$= -1 \cdot b^2$$
$$= -b^2$$

Practice: Simplify each radical to a mixed radical

Practice Problems

1. $\sqrt{a^6 b^3 c^5}$
2. $\sqrt{18 a^2 b^7 c^3}$
3. $\sqrt[3]{-a^4 b^5 c^2}$
4. $\sqrt[3]{-54 a^7 b^6}$
5. $\sqrt[4]{a^5 b^8 c^3}$

Practice: Simplify each radical to a mixed radical - Solutions

Solutions

1. $\sqrt{a^6b^3c^5} = a^3bc^2\sqrt{bc}$
2. $\sqrt{18a^2b^7c^3} = 3ab^3c\sqrt{2bc}$
3. $\sqrt[3]{-a^4b^5c^2} = -ab\sqrt[3]{ab^2c^2}$
4. $\sqrt[3]{-54a^7b^6} = -3a^2b^2\sqrt[3]{2ab^2}$
5. $\sqrt[4]{a^5b^8c^3} = ab^2c\sqrt[4]{ac^3}$

Practice: Match each radical with its corresponding Mixed radical

Match Problems

1. $\sqrt{72}$
2. $\sqrt{98}$
3. $-\sqrt{128}$
4. $\sqrt{242}$
5. $\sqrt[3]{-216}$

1. $6\sqrt{2}$
2. $7\sqrt{2}$
3. $-8\sqrt{2}$
4. $11\sqrt{2}$
5. $-6\sqrt{2}$
6. $2\sqrt{6}$
7. $-2\sqrt[3]{27}$
8. -6
9. $-2\sqrt[3]{27}$
10. $-2\sqrt{2}$

Practice: Match each radical with its corresponding Mixed radical - Answers

Solutions

1. $\sqrt{72} = 6\sqrt{2}$
2. $\sqrt{98} = 7\sqrt{2}$
3. $-\sqrt{128} = -8\sqrt{2}$
4. $\sqrt{242} = 11\sqrt{2}$
5. $\sqrt[3]{-216} = -6$

Simplify the Following (Challenging)

Practice Problems

1. $3\sqrt{72} + 5\sqrt{32}$
2. $2\sqrt[3]{54} - 4\sqrt[3]{128} + 7\sqrt[3]{250}$
3. $4\sqrt{98} - 2\sqrt{50} + 6\sqrt{18} - 3\sqrt{200}$

Simplify the Following (Challenging) - Solutions (1/2)

Full Solutions

1. $3\sqrt{72} + 5\sqrt{32}$

$$
\begin{aligned}
&= 3\sqrt{36 \times 2} + 5\sqrt{16 \times 2} \\
&= 3 \times 6\sqrt{2} + 5 \times 4\sqrt{2} \\
&= 18\sqrt{2} + 20\sqrt{2} \\
&= 38\sqrt{2}
\end{aligned}
$$

2. $2\sqrt[3]{54} - 4\sqrt[3]{128} + 7\sqrt[3]{250}$

$$
\begin{aligned}
&= 2\sqrt[3]{27 \times 2} - 4\sqrt[3]{64 \times 2} + 7\sqrt[3]{125 \times 2} \\
&= 2 \times 3\sqrt[3]{2} - 4 \times 4\sqrt[3]{2} + 7 \times 5\sqrt[3]{2} \\
&= 6\sqrt[3]{2} - 16\sqrt[3]{2} + 35\sqrt[3]{2} \\
&= 25\sqrt[3]{2}
\end{aligned}
$$

Simplify the Following (Challenging) - Solutions (2/2)

Full Solutions

3. $4\sqrt{98} - 2\sqrt{50} + 6\sqrt{18} - 3\sqrt{200}$

$$
\begin{aligned}
&= 4\sqrt{49 \times 2} - 2\sqrt{25 \times 2} + 6\sqrt{9 \times 2} - 3\sqrt{100 \times 2} \\
&= 4 \times 7\sqrt{2} - 2 \times 5\sqrt{2} + 6 \times 3\sqrt{2} - 3 \times 10\sqrt{2} \\
&= 28\sqrt{2} - 10\sqrt{2} + 18\sqrt{2} - 30\sqrt{2} \\
&= 6\sqrt{2}
\end{aligned}
$$