# Chapter 5.2: Multiplying and Dividing Rational Expressions

## Rational Expressions - Lesson 2

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# Multiplying and Dividing Rational Expressions

## Key Concepts

- When multiplying rational expressions, add exponents for like variables in the same position (top or bottom).
- If a variable appears in both numerator and denominator, subtract/cancel exponents.
- When dividing, change division to multiplication and flip (take the reciprocal of) the right-hand fraction only.
- After flipping, multiply as usual and simplify.

# Multiplying Example

## Example 1

Multiply:

$$\frac{x^3}{y^2} \times \frac{x^5}{y^3} = \frac{x^{3+5}}{y^{2+3}} = \frac{x^8}{y^5}$$

## Example 2

Multiply and simplify:

$$\frac{x^7y^5}{y^9} \times \frac{x^3}{x^4y^4} = \frac{x^{7+3}y^5}{x^4y^{9+4}} = \frac{x^{10}y^5}{x^4y^{13}} = \frac{x^6}{y^8}$$

# Dividing Example

## Example 3

Divide:

$$\frac{a^2b^3}{c^4} \div \frac{a^4b}{c^2} = \frac{a^2b^3}{c^4} \times \frac{c^2}{a^4b} = \frac{a^{2-4}b^{3-1}c^{2-4}}{1} = \frac{a^{-2}b^2c^{-2}}{1} = \frac{b^2}{a^2c^2}$$

# Practice: Multiply or Divide

## Practice Problems

Simplify by multiplying or dividing:

1. $\frac{6y^2}{5x} \times \frac{10x^3}{3y}$
2. $\frac{2x^6}{9} \div \frac{5x^{10}}{6}$
3. $\frac{15x^2y^3}{24y^2} \times \frac{6y^5}{5x}$
4. $\frac{12x^3}{16y^4} \div \frac{7y^2}{14x^2}$
5. $\frac{45x^3y^2}{30y^5} \times \frac{18y^4}{75x^2}$

# Practice Solutions

## Detailed Solutions

1. $\frac{6y^2}{5x} \times \frac{10x^3}{3y} = \frac{6 \times 10 \times y^2 \times x^3}{5 \times 3 \times x \times x \times y} = \frac{60x^3y^2}{15xy} = \frac{4x^2y}{1}$
2. $\frac{2x^6}{9} \div \frac{5x^{10}}{6} = \frac{2x^6}{9} \times \frac{6}{5x^{10}} = \frac{12x^6}{45x^{10}} = \frac{4}{15x^4}$
3. $\frac{15x^2y^3}{24y^2} \times \frac{6y^5}{5x} = \frac{15x^2y^3 \times 6y^5}{24y^2 \times 5x} = \frac{90x^2y^8}{120xy^2} = \frac{3xy^6}{4}$
4. $\frac{12x^3}{16y^4} \div \frac{7y^2}{14x^2} = \frac{12x^3}{16y^4} \times \frac{14x^2}{7y^2} = \frac{12x^3 \times 14x^2}{16y^4 \times 7y^2} = \frac{168x^5}{112y^6} = \frac{3x^5}{2y^6}$
5. $\frac{45x^3y^2}{30y^5} \times \frac{18y^4}{75x^2} = \frac{45x^3y^2 \times 18y^4}{30y^5 \times 75x^2} = \frac{810x^3y^6}{2250x^2y^5} = \frac{18xy}{50} = \frac{9xy}{25}$

# Factoring Rational Expressions

## Key Concepts

- When multiplying rational expressions with trinomials, factor every trinomial or difference of squares.
- Cancel out any common binomial in both the numerator and denominator.
- Always look for NPV's (Non-Permissible Values) from every bracket in the denominator.

# Factoring Example

## Example

Simplify and state the NPV:

$$\frac{x^2+5x+6}{x^2+3x+2} \times \frac{x^2+x-2}{x^2+2x-3}$$

Factor:

$$\frac{(x+2)(x+3)}{(x+1)(x+2)} \times \frac{(x+2)(x-1)}{(x+3)(x-1)}$$

Cancel common factors:

$$= \frac{1}{x+1}$$

NPV: $x \neq -2, -3, -1, 1$

# Practice: Simplify  Find NPV

## Practice

Simplify and find all NPV's:

1. $\frac{x^2-16}{x^2-9} \times \frac{x^2-4x+3}{x^2-1}$
2. $\frac{x^2-7x+12}{x^2-4} \div \frac{x^2-9}{x^2-1}$

# Practice Solutions: NPV

## Detailed Solutions

1. $\frac{x^2-16}{x^2-9} \times \frac{x^2-4x+3}{x^2-1}$

$$= \frac{(x-4)(x+4)}{(x-3)(x+3)} \times \frac{(x-3)(x-1)}{(x-1)(x+1)}$$
$$= \frac{(x+4)}{(x+3)(x+1)}$$

NPV: $x \neq 3, -3, 1, -1$

2. $\frac{x^2-7x+12}{x^2-4} \div \frac{x^2-9}{x^2-1}$

$$= \frac{(x-3)(x-4)}{(x-2)(x+2)} \times \frac{(x-1)(x+1)}{(x-3)(x+3)}$$
$$= \frac{(x-4)(x-1)(x+1)}{(x-2)(x+2)(x+3)}$$

NPV: $x \neq 2, -2, 3, -3, 1, -1$

# Extra Practice: Simplify  State NPV

## Extra Practice

Simplify each of the following and state the NPV's:

1. $\frac{x^2+3x-4}{x^2-9} \times \frac{x^2-4x+3}{x^2-1}$
2. $\frac{x^2+5x+6}{x^2-4} \div \frac{x^2-9}{x^2-1}$

# Challenge 1: Multiply and Divide

## Challenge 1

Simplify and state all NPV's:

$$\frac{x^2-9}{x^2-4} \times \frac{x^2-1}{x^2-5x+6} \div \frac{x^2-4x+3}{x^2-1}$$

# Challenge 1: Solution (Part 1)

## Step-by-Step Solution (Part 1)

**Step 1: Factor all numerators and denominators**

$$\frac{(x-3)(x+3)}{(x-2)(x+2)} \times \frac{(x-1)(x+1)}{(x-2)(x-3)} \div \frac{(x-1)(x-3)}{(x+1)}$$

**Step 2: Change division to multiplication and flip the third fraction**

$$= \frac{(x-3)(x+3)}{(x-2)(x+2)} \times \frac{(x-1)(x+1)}{(x-2)(x-3)} \times \frac{(x+1)}{(x-1)(x-3)}$$

**Step 3: Combine all numerators and denominators**

$$= \frac{(x-3)(x+3)(x-1)(x+1)(x+1)}{(x-2)(x+2)(x-2)(x-3)(x-1)(x-3)}$$

# Challenge 1: Solution (Part 2)

## Step-by-Step Solution (Part 2)

**Step 4: Cancel common factors**

$$= \frac{(x+3)(x+1)}{(x-2)(x+2)(x-3)}$$

**Step 5: State all NPV's (Non-Permissible Values)**

- $x \neq 2, -2, 3, -3, 1, -1$

# Challenge 2: Multiply and Divide

## Challenge 2

Simplify and state all NPV's:

$$\frac{a^2-4a+3}{a^2-1} \div \frac{a^2-9}{a^2-4} \times \frac{a^2-1}{a^2-2a}$$

# Challenge 2: Solution (Part 1)

## Step-by-Step Solution (Part 1)

**Step 1: Factor all numerators and denominators**

$$\frac{(a-1)(a-3)}{(a-1)(a+1)} \div \frac{(a-3)(a+3)}{(a-2)(a+2)} \times \frac{(a-1)(a+1)}{a(a-2)}$$

**Step 2: Change division to multiplication and flip the second fraction**

$$= \frac{(a-1)(a-3)}{(a-1)(a+1)} \times \frac{(a-2)(a+2)}{(a-3)(a+3)} \times \frac{(a-1)(a+1)}{a(a-2)}$$

**Step 3: Combine all numerators and denominators**

$$= \frac{(a-1)(a-3)(a-2)(a+2)(a-1)(a+1)}{(a-1)(a+1)(a-3)(a+3)a(a-2)}$$

# Challenge 2: Solution (Part 2)

## Step-by-Step Solution (Part 2)

**Step 4: Cancel common factors**

$$= \frac{(a+2)(a-1)}{(a+3)a}$$

**Step 5: State all NPV's (Non-Permissible Values)**

- $a \neq 1, -1, 3, -3, 2, -2, 0$

# Challenge 3: Multiply and Divide

## Challenge 3

Simplify and state all NPV's:

$$\frac{y^2-16}{y^2-9y+18} \times \frac{y^2-4y+3}{y^2-1} \div \frac{y^2-9}{y^2-4}$$

# Challenge 3: Solution (Part 1)

## Step-by-Step Solution (Part 1)

**Step 1: Factor all numerators and denominators**

$$\frac{(y-4)(y+4)}{(y-6)(y-3)} \times \frac{(y-1)(y-3)}{(y-1)(y+1)} \div \frac{(y-3)(y+3)}{(y-2)(y+2)}$$

**Step 2: Change division to multiplication and flip the third fraction**

$$= \frac{(y-4)(y+4)}{(y-6)(y-3)} \times \frac{(y-1)(y-3)}{(y-1)(y+1)} \times \frac{(y-2)(y+2)}{(y-3)(y+3)}$$

**Step 3: Combine all numerators and denominators**

$$= \frac{(y-4)(y+4)(y-1)(y-3)(y-2)(y+2)}{(y-6)(y-3)(y-1)(y+1)(y-3)(y+3)}$$

# Challenge 3: Solution (Part 2)

## Step-by-Step Solution (Part 2)

**Step 4: Cancel common factors**

$$= \frac{(y-4)(y+4)(y-2)}{(y-6)(y+1)(y+3)}$$

**Step 5: State all NPV's (Non-Permissible Values)**

- $y \neq 6, 3, 1, -1, 4, -4, 2, -2, -3$

# Challenge 4: Multiply and Divide

## Challenge 4

Simplify and state all NPV's:

$$\frac{m^2-25}{m^2-10m+25} \div \frac{m^2-6m+9}{m^2-1} \times \frac{m^2-1}{m^2-5m}$$

# Challenge 4: Solution (Part 1)

## Step-by-Step Solution (Part 1)

**Step 1: Factor all numerators and denominators**

$$\frac{(m-5)(m+5)}{(m-5)^2} \div \frac{(m-3)^2}{(m-1)(m+1)} \times \frac{(m-1)(m+1)}{m(m-5)}$$

**Step 2: Change division to multiplication and flip the second fraction**

$$= \frac{(m-5)(m+5)}{(m-5)^2} \times \frac{(m-1)(m+1)}{(m-3)^2} \times \frac{(m-1)(m+1)}{m(m-5)}$$

**Step 3: Combine all numerators and denominators**

$$= \frac{(m-5)(m+5)(m-1)(m+1)(m-1)(m+1)}{(m-5)^2(m-3)^2 m(m-5)}$$

# Challenge 4: Solution (Part 2)

## Step-by-Step Solution (Part 2)

**Step 4: Cancel common factors**

$$= \frac{(m+5)(m-1)(m+1)}{(m-5)(m-3)^2 m}$$

**Step 5: State all NPV's (Non-Permissible Values)**

- $m \neq 5, -5, 3, 1, -1, 0$