

# Pre-Calculus 11

## 6.5 Solving Equations with Two or More Absolute Values

Created by Yi-Chen Lin

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## What We Will Learn

- 1 Understand how to solve equations with two or more absolute values
- 2 Learn to set up and solve all possible cases
- 3 Check solutions for extraneous roots
- 4 Use graphical methods to verify solutions

## Key Questions:

- How do we set up cases for multiple absolute values?
- Why do we need to check for extraneous solutions?
- How can graphs help us solve these equations?

# Solving Equations with Two Absolute Values

## Key Idea

When an equation contains two absolute values, consider all possible sign combinations (cases) for each absolute value expression.

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## General Steps:

- 1 Isolate absolute value expressions if possible
- 2 Set up all possible cases (both positive/negative)
- 3 Solve each case for  $x$
- 4 Check each solution in the original equation (extraneous roots)

## Example: $|x - 2| + |x + 6| = 11$

Since there are two absolute values, consider all four cases:

- Case 1:  $x - 2 \geq 0, x + 6 \geq 0$  ( $x \geq -6$ )
- Case 2:  $x - 2 \geq 0, x + 6 < 0$  ( $2 \leq x < -6$ ) (impossible)
- Case 3:  $x - 2 < 0, x + 6 \geq 0$  ( $-6 \leq x < 2$ )
- Case 4:  $x - 2 < 0, x + 6 < 0$  ( $x < -6$ )

**Set up equations for each valid case:**

- Case 1:  $(x - 2) + (x + 6) = 11$
- Case 3:  $-(x - 2) + (x + 6) = 11$
- Case 4:  $-(x - 2) - (x + 6) = 11$

## Example Solution: $|x - 2| + |x + 6| = 11$ (Part 1)

**Case 1:**  $x \geq -6$

$$(x - 2) + (x + 6) = 11$$

$$2x + 4 = 11$$

$$2x = 7$$

$$x = 3.5$$

Check:  $|3.5 - 2| + |3.5 + 6| = 1.5 + 9.5 = 11$  (Valid) **Case 3:**  $-6 \leq x < 2$

$$-(x - 2) + (x + 6) = 11$$

$$-x + 2 + x + 6 = 11$$

$$8 = 11$$

No solution in this case.

## Example Solution: $|x - 2| + |x + 6| = 11$ (Part 2)

**Case 4:**  $x < -6$

$$-(x - 2) - (x + 6) = 11$$

$$-x + 2 - x - 6 = 11$$

$$-2x - 4 = 11$$

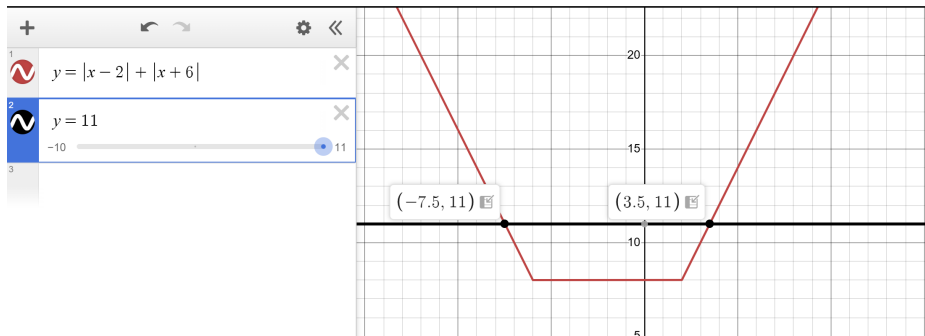
$$-2x = 15$$

$$x = -7.5$$

Check:  $|-7.5 - 2| + |-7.5 + 6| = |-9.5| + |-1.5| = 9.5 + 1.5 = 11$  (Valid)

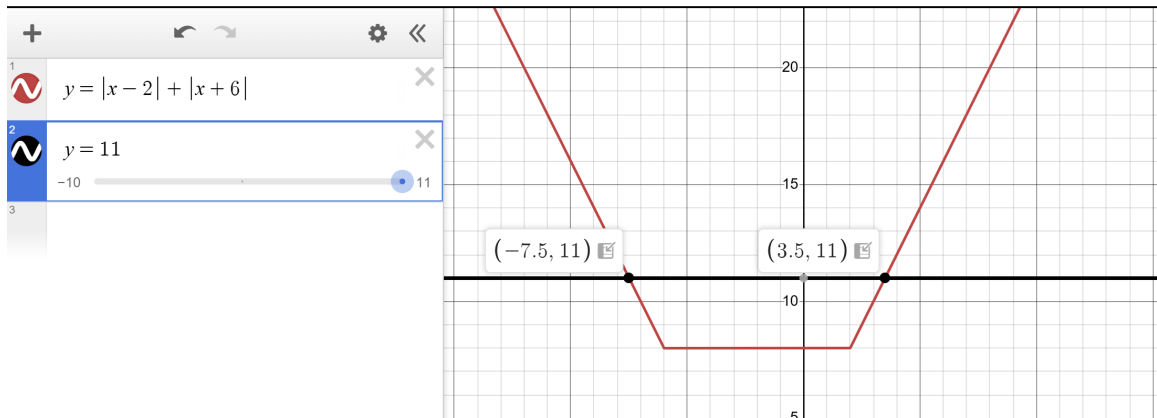
# Graphical Solution

**Graph:**  $y_1 = |x - 2| + |x + 6|$ ,  $y_2 = 11$



**Intersections:**  $x = -7.5$ ,  $x = 3.5$

# Graphical Solution (Enlarged)





Practice:  $|x - 1| + |x + 5| = 8$

**Try:** Solve  $|x - 1| + |x + 5| = 8$  by considering all cases.  
**(Solutions on next slide)**

## Solution: $|x - 1| + |x + 5| = 8$ (Part 1)

**Case 1:**  $x \geq -5$

$$(x - 1) + (x + 5) = 8$$

$$2x + 4 = 8$$

$$2x = 4$$

$$x = 2$$

Check:  $|2 - 1| + |2 + 5| = 1 + 7 = 8$  (Valid) **Case 3:**  $-5 \leq x < 1$

$$-(x - 1) + (x + 5) = 8$$

$$-x + 1 + x + 5 = 8$$

$$6 = 8$$

No solution in this case.

## Solution: $|x - 1| + |x + 5| = 8$ (Part 2)

**Case 4:**  $x < -5$

$$-(x - 1) - (x + 5) = 8$$

$$-x + 1 - x - 5 = 8$$

$$-2x - 4 = 8$$

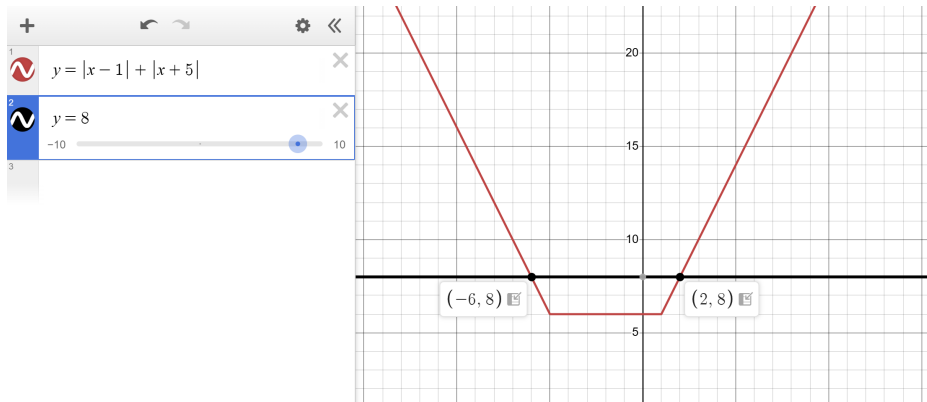
$$-2x = 12$$

$$x = -6$$

Check:  $|-6 - 1| + |-6 + 5| = 7 + 1 = 8$  (Valid)

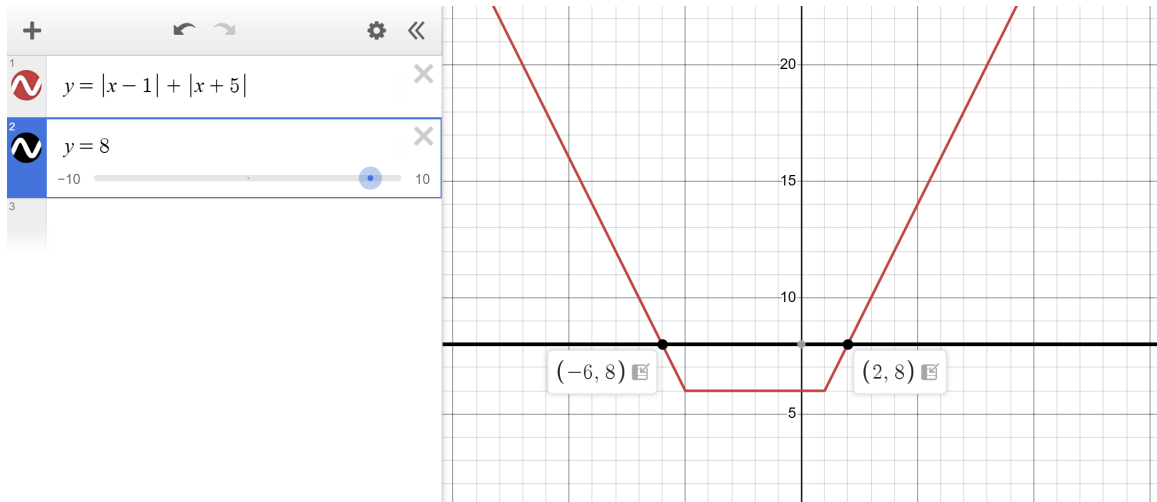
# Graphical Solution

**Graph:**  $y_1 = |x - 1| + |x + 5|$ ,  $y_2 = 8$



**Intersections:**  $x = -6$ ,  $x = 2$

# Graphical Solution (Enlarged)



Practice:  $|x - 3| + |x + 4| = 9$

**Try:** Solve  $|x - 3| + |x + 4| = 9$  by considering all cases.  
**(Solutions on next slide)**

## Solution: $|x - 3| + |x + 4| = 9$ (Part 1)

**Case 1:**  $x \geq -4$

$$(x - 3) + (x + 4) = 9$$

$$2x + 1 = 9$$

$$2x = 8$$

$$x = 4$$

Check:  $|4 - 3| + |4 + 4| = 1 + 8 = 9$  (Valid) **Case 3:**  $-4 \leq x < 3$

$$-(x - 3) + (x + 4) = 9$$

$$-x + 3 + x + 4 = 9$$

$$7 = 9$$

No solution in this case.

## Solution: $|x - 3| + |x + 4| = 9$ (Part 2)

**Case 4:**  $x < -4$

$$-(x - 3) - (x + 4) = 9$$

$$-x + 3 - x - 4 = 9$$

$$-2x - 1 = 9$$

$$-2x = 10$$

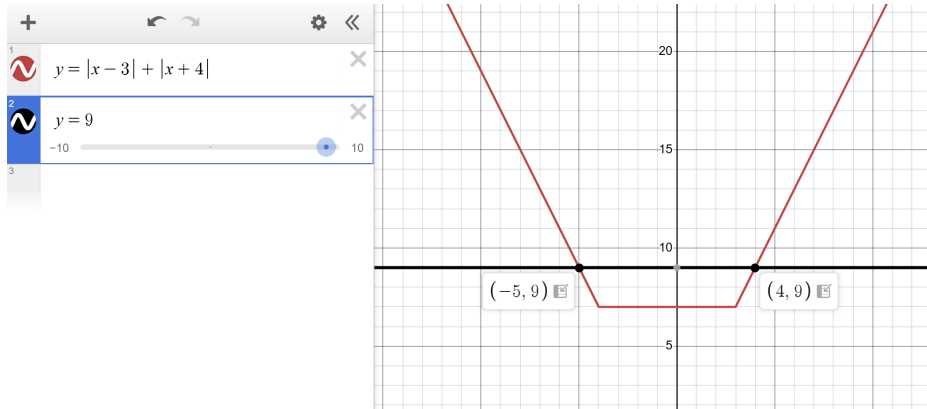
$$x = -5$$

Check:  $|-5 - 3| + |-5 + 4| = 8 + 1 = 9$  (Valid)



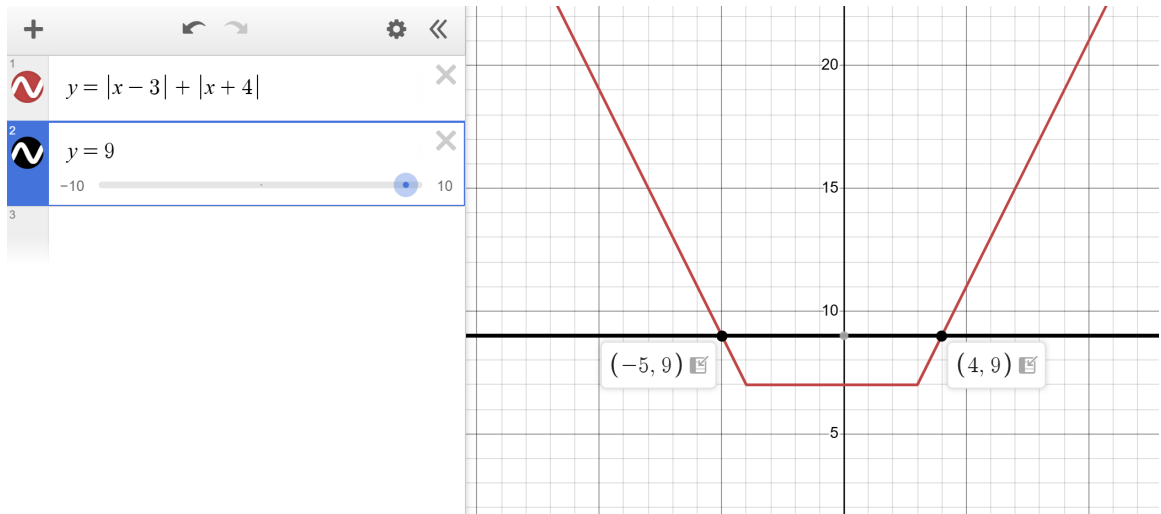
# Graphical Solution

**Graph:**  $y_1 = |x - 3| + |x + 4|$ ,  $y_2 = 9$



**Intersections:**  $x = -5$ ,  $x = 4$

# Graphical Solution (Enlarged)



## Key Points

- 1 For equations with two or more absolute values, always consider all possible sign cases
- 2 Solve each case and check for extraneous solutions
- 3 Use graphs to visualize and confirm solutions
- 4 Solutions must satisfy the original equation and the domain of each case

## Practice: $|x - 7| + |x + 2| = 13$

Solve  $|x - 7| + |x + 2| = 13$  by considering all cases.  
**(Solution on next slide)**

## Solution: $|x - 7| + |x + 2| = 13$ (Part 1)

**Case 1:**  $x \geq 7$

$$(x - 7) + (x + 2) = 13$$

$$2x - 5 = 13$$

$$2x = 18$$

$$x = 9$$

Check:  $|9 - 7| + |9 + 2| = 2 + 11 = 13$  (Valid) **Case 3:**  $-2 \leq x < 7$

$$-(x - 7) + (x + 2) = 13$$

$$-x + 7 + x + 2 = 13$$

$$9 = 13$$

No solution in this case.

## Solution: $|x - 7| + |x + 2| = 13$ (Part 2)

**Case 4:**  $x < -2$

$$-(x - 7) - (x + 2) = 13$$

$$-x + 7 - x - 2 = 13$$

$$-2x + 5 = 13$$

$$-2x = 8$$

$$x = -4$$

Check:  $|-4 - 7| + |-4 + 2| = 11 + 2 = 13$  (Valid)

## Practice: $|x - 6| + |x + 1| = 10$

Solve  $|x - 6| + |x + 1| = 10$  by considering all cases.  
**(Solution on next slide)**

## Solution: $|x - 6| + |x + 1| = 10$ (Part 1)

**Case 1:**  $x \geq 6$

$$(x - 6) + (x + 1) = 10$$

$$2x - 5 = 10$$

$$2x = 15$$

$$x = 7.5$$

Check:  $|7.5 - 6| + |7.5 + 1| = 1.5 + 8.5 = 10$  (Valid) **Case 3:**  $-1 \leq x < 6$

$$-(x - 6) + (x + 1) = 10$$

$$-x + 6 + x + 1 = 10$$

$$7 = 10$$

No solution in this case.



## Solution: $|x - 6| + |x + 1| = 10$ (Part 2)

**Case 4:**  $x < -1$

$$\begin{aligned} -(x - 6) - (x + 1) &= 10 \\ -x + 6 - x - 1 &= 10 \\ -2x + 5 &= 10 \\ -2x &= 5 \\ x &= -2.5 \end{aligned}$$

Check:  $|-2.5 - 6| + |-2.5 + 1| = 8.5 + 1.5 = 10$  (Valid)

## Practice: $|x - 8| + |x + 3| = 15$

Solve  $|x - 8| + |x + 3| = 15$  by considering all cases.  
**(Solution on next slide)**

## Solution: $|x - 8| + |x + 3| = 15$ (Part 1)

**Case 1:**  $x \geq 8$

$$(x - 8) + (x + 3) = 15$$

$$2x - 5 = 15$$

$$2x = 20$$

$$x = 10$$

Check:  $|10 - 8| + |10 + 3| = 2 + 13 = 15$  (Valid) **Case 3:**  $-3 \leq x < 8$

$$-(x - 8) + (x + 3) = 15$$

$$-x + 8 + x + 3 = 15$$

$$11 = 15$$

No solution in this case.

## Solution: $|x - 8| + |x + 3| = 15$ (Part 2)

**Case 4:**  $x < -3$

$$-(x - 8) - (x + 3) = 15$$

$$-x + 8 - x - 3 = 15$$

$$-2x + 5 = 15$$

$$-2x = 10$$

$$x = -5$$

Check:  $|-5 - 8| + |-5 + 3| = 13 + 2 = 15$  (Valid)