

Chapter 5.4: Solving Rational Equations

Rational Expressions - Lesson 4

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Solving Rational Equations

Key Concepts

- Rational equations have an equal sign and require finding the value of x that makes both sides equal.
- To solve, find the LCD and multiply all terms by the LCD to clear denominators (only when solving, not simplifying).
- Always check for NPV (Non-Permissible Values). If a solution is an NPV, it is called an "Extraneous Root" and must be excluded.

Example Q1: Using LCD to Clear Denominators (Part 1)

Example Q1 (Part 1)

Solve for x :

$$\frac{2}{x} + \frac{3}{x+1} = 4$$

Step 1: Find LCD: $x(x+1)$

$$\frac{2}{x} \times (x+1) + \frac{3}{x+1} \times x = 4(x)(x+1)$$

Step 2: Multiply both sides by LCD:

$$2(x+1) + 3x = 4x(x+1)$$

Step 3: Expand:

$$2x + 2 + 3x = 4x^2 + 4x5x + 2 = 4x^2 + 4x$$

Example Q1: Using LCD to Clear Denominators (Part 2)

Example Q1 (Part 2)

Step 4: Rearrange and solve:

$$0 = 4x^2 - x - 2$$

Step 5: Solve the quadratic:

$$4x^2 - x - 2 = 0$$

Step 6: Check for NPV: $x \neq 0, -1$

Example Q2: Cross-Multiplication (Part 1)

Example Q2 (Part 1)

Solve for x :

$$\frac{4}{x+1} = \frac{3}{x}$$

Step 1: Cross-multiply:

$$4x = 3(x+1)$$

Step 2: Expand:

$$4x = 3x + 3$$

Example Q2: Cross-Multiplication (Part 2)

Example Q2 (Part 2)

Step 3: Solve:

$$4x - 3x = 3x = 3$$

Step 4: NPV: $x \neq 0, -1$ **Final Answer:** $x = 3$ (valid)

Example Q3: Using LCM (Part 1)

Example Q3 (Part 1)

Solve for x :

$$\frac{1}{6} + \frac{1}{8} = \frac{5}{x}$$

Step 1: Find LCM of 6, 8, and x (assume x is a positive integer): LCM = $24x$ **Step 2: Multiply both sides by LCM:**

$$\frac{1}{6} \times 24x + \frac{1}{8} \times 24x = \frac{5}{x} \times 24x$$

Step 3: Simplify:

$$4x + 3x = 5 \times 24 \quad x = 120$$

Example Q3: Using LCM (Part 2)

Example Q3 (Part 2)

Step 4: Solve:

$$x = \frac{120}{7}$$

Step 5: NPV: $x \neq 0$ **Final Answer:** $x = \frac{120}{7}$ (valid)

Practice: Solve for x

Practice Problems

Q4. $\frac{3}{x} + \frac{4}{x+2} = 1$

Q5. $\frac{2}{x} + \frac{1}{x-1} = 3$

Q6. $\frac{5}{x-2} - \frac{2}{x+3} = 1$

Q7. $\frac{1}{6} + \frac{1}{8} = \frac{5}{x}$

Q8. $\frac{2}{x+1} = \frac{3}{x-2}$

Solution Q4 (Part 1)

Solution Q4 (Part 1)

Solve for x :

$$\frac{3}{x} + \frac{4}{x+2} = 1$$

Step 1: Find LCD: $x(x+2)$ **Step 2: Multiply both sides by LCD:**

$$\frac{3}{x} \times (x+2) + \frac{4}{x+2} \times x = 1 \times x(x+2)$$

Step 3: Expand:

$$3(x+2) + 4x = x^2 + 2x$$

Solution Q4 (Part 2)

Solution Q4 (Part 2)

Step 4: Simplify and solve:

$$3x + 6 + 4x = x^2 + 2x7x + 6 = x^2 + 2x0 = x^2 - 5x - 6$$

Step 5: Factor and solve:

$$x^2 - 5x - 6 = 0(x - 6)(x + 1) = 0x = 6, -1$$

Step 6: NPV: $x \neq 0, -2$ **Final Answer:** $x = 6$ (valid), $x = -1$ (valid)

Solution Q5 (Part 1)

Solution Q5 (Part 1)

Solve for x :

$$\frac{2}{x} + \frac{1}{x-1} = 3$$

Step 1: Find LCD: $x(x-1)$ **Step 2: Multiply both sides by LCD:**

$$\frac{2}{x} \times (x-1) + \frac{1}{x-1} \times x = 3x(x-1)$$

Step 3: Expand:

$$2(x-1) + x = 3x^2 - 3x$$

Solution Q5 (Part 2)

Solution Q5 (Part 2)

Step 4: Simplify and solve:

$$2x - 2 + x = 3x^2 - 3x3x - 2 = 3x^2 - 3x0 = 3x^2 - 6x + 2$$

Step 5: Quadratic formula:

$$x = \frac{6 \pm \sqrt{36 - 24}}{6} = \frac{6 \pm 2\sqrt{3}}{6} = 1 \pm \frac{\sqrt{3}}{3}$$

Step 6: NPV: $x \neq 0, 1$ Final Answer: $x = 1 + \frac{\sqrt{3}}{3}$, $x = 1 - \frac{\sqrt{3}}{3}$ (both valid)

Solution Q6 (Part 1)

Solution Q6 (Part 1)

Solve for x :

$$\frac{5}{x-2} - \frac{2}{x+3} = 1$$

Step 1: Find LCD: $(x-2)(x+3)$ **Step 2: Multiply both sides by LCD:**

$$\frac{5}{x-2} \times (x+3) - \frac{2}{x+3} \times (x-2) = (x-2)(x+3)$$

Step 3: Expand:

$$5(x+3) - 2(x-2) = (x-2)(x+3)$$

Solution Q6 (Part 2)

Solution Q6 (Part 2)

Step 4: Simplify and solve:

$$5x + 15 - 2x + 4 = x^2 + 3x - 2x - 63x + 19 = x^2 + x - 60 = x^2 - 2x - 25$$

Step 5: Factor or quadratic formula:

$$x = \frac{2 \pm \sqrt{4 + 100}}{2} = \frac{2 \pm \sqrt{104}}{2} = \frac{2 \pm 2\sqrt{26}}{2} = 1 \pm \sqrt{26}$$

Step 6: NPV: $x \neq 2, -3$ **Final Answer:** $x = 1 + \sqrt{26}, x = 1 - \sqrt{26}$ (both valid)

Solution Q7 (Part 1)

Solution Q7 (Part 1)

Solve for x :

$$\frac{1}{6} + \frac{1}{8} = \frac{5}{x}$$

Step 1: Find LCM: $24x$ **Step 2: Multiply both sides by LCM:**

$$4x + 3x = 5 \times 24$$

Step 3: Simplify:

$$7x = 120$$

Solution Q7 (Part 2)

Solution Q7 (Part 2)

Step 4: Solve:

$$x = \frac{120}{7}$$

Step 5: NPV: $x \neq 0$ **Final Answer:** $x = \frac{120}{7}$ (valid)

Solution Q8 (Part 1)

Solution Q8 (Part 1)

Solve for x :

$$\frac{2}{x+1} = \frac{3}{x-2}$$

Step 1: Cross-multiply:

$$2(x-2) = 3(x+1)$$

Step 2: Expand:

$$2x - 4 = 3x + 3$$

Solution Q8 (Part 2)

Solution Q8 (Part 2)

Step 3: Solve:

$$2x - 4 = 3x + 3 - 4 - 3 = 3x - 2x - 7 = xx = -7$$

Step 4: NPV: $x \neq -1, 2$ **Final Answer:** $x = -7$ (valid)

Example Q9: Extraneous Solution (Part 1)

Example Q9 (Part 1)

Solve for x :

$$\frac{3}{x+2} = \frac{5}{x+2}$$

Step 1: LCD is $x+2$

$$3 = 5$$

Example Q9: Extraneous Solution (Part 2)

Example Q9 (Part 2)

No solution (all x cancel, both sides not equal).

Solution Q10 (Part 1)

Solution Q10 (Part 1)

Solve for x :

$$\frac{2}{x-1} = \frac{2}{x-1}$$

Step 1: Both sides are identical. Step 2: Any $x \neq 1$ is a solution.

Solution Q10 (Part 2)

Solution Q10 (Part 2)

Final Answer: Infinite solutions for $x \neq 1$ (NPV: $x \neq 1$)

Solution Q11 (Part 1)

Solution Q11 (Part 1)

Solve for x :

$$\frac{4}{x+2} = \frac{4}{x+2}$$

Step 1: Both sides are identical. Step 2: Any $x \neq -2$ is a solution.

Solution Q11 (Part 2)

Solution Q11 (Part 2)

Final Answer: Infinite solutions for $x \neq -2$ (NPV: $x \neq -2$)

Solution Q12 (Part 1)

Solution Q12 (Part 1)

Solve for x :

$$\frac{3}{x} = \frac{5}{x}$$

Step 1: Subtract $\frac{5}{x}$ from both sides:

$$\frac{3}{x} - \frac{5}{x} = 0$$

Step 2: Combine:

$$\frac{-2}{x} = 0$$

Solution Q12 (Part 2)

Solution Q12 (Part 2)

Step 3: $-2 = 0$ is a contradiction. **Final Answer:** No solution (NPV: $x \neq 0$)

Extraneous, No, and Infinite Solutions

Special Cases

- If a solution is an NPV, it is extraneous and not valid.
- If all x cancel and both sides are not equal, there are no solutions.
- If all x cancel and both sides are equal, there are infinite solutions.

Practice: Special Cases

Practice Problems

Q10. $\frac{2}{x-1} = \frac{2}{x-1}$

Q11. $\frac{4}{x+2} = \frac{4}{x+2}$

Q12. $\frac{3}{x} = \frac{5}{x}$

Solving Rational Equations by Graphs

Graphical Solution

- Let y_1 be the left side and y_2 be the right side of the equation.
- The solution is the x -value where $y_1 = y_2$ (the intersection point).
- Always check for NPV.

Challenge Problem Q13

Challenge Q13

Solve for x and state all NPV's:

$$\frac{2}{x^2 - 4} + \frac{3}{x + 2} = 1$$

Challenge Q13: Solution (Part 1)

Step-by-Step Solution (Part 1)

Step 1: Factor denominators

$$x^2 - 4 = (x + 2)(x - 2)$$

Step 2: LCD is $(x + 2)(x - 2)$ Step 3: Rewrite each fraction with LCD

$$\frac{2}{(x + 2)(x - 2)} + \frac{3(x - 2)}{(x + 2)(x - 2)} = 1$$

Challenge Q13: Solution (Part 2)

Step-by-Step Solution (Part 2)

Step 4: Combine numerators

$$\begin{aligned}\frac{2 + 3(x - 2)}{(x + 2)(x - 2)} &= 1 \\ 2 + 3x - 6 &= (x + 2)(x - 2) \\ 3x - 4 &= x^2 - 4 \\ 0 &= x^2 - 3x \\ x(x - 3) &= 0\end{aligned}$$

Step 5: Solutions: $x = 0, x = 3$ **Step 6: NPV:** $x \neq 2, -2$ **Final Answer:** $x = 0, x = 3$ (both are valid)

Challenge Problem Q14

Challenge Q14

Solve for x and state all NPV's:

$$\frac{1}{x-1} + \frac{2}{x+2} = \frac{3x+1}{x^2+x-2}$$

Challenge Q14: Solution (Part 1)

Step-by-Step Solution (Part 1)

Step 1: Factor denominators

$$x^2 + x - 2 = (x - 1)(x + 2)$$

Step 2: LCD is $(x - 1)(x + 2)$ Step 3: Rewrite each fraction with LCD

$$\frac{1}{x - 1} \times \frac{x + 2}{x + 2} = \frac{x + 2}{(x - 1)(x + 2)}$$

$$\frac{2}{x + 2} \times \frac{x - 1}{x - 1} = \frac{2(x - 1)}{(x - 1)(x + 2)}$$

Challenge Q14: Solution (Part 2)

Step-by-Step Solution (Part 2)

Step 4: Combine numerators

$$\frac{x + 2 + 2(x - 1)}{(x - 1)(x + 2)} = \frac{x + 2 + 2x - 2}{(x - 1)(x + 2)} = \frac{3x}{(x - 1)(x + 2)}$$

Step 5: Set equal to right side and solve:

$$\frac{3x}{(x - 1)(x + 2)} = \frac{3x + 1}{(x - 1)(x + 2)}$$

$$3x = 3x + 1$$

No solution (contradiction). **Step 6: NPV:** $x \neq 1, -2$