Pre-Calculus 11

6.2 Absolute Value Functions

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Graphing the Absolute Value of a Function

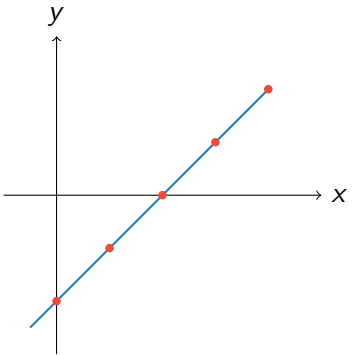
Key Idea

When graphing $y = |f(x)|$, any point with a negative y-coordinate is reflected above the x-axis. Points with positive y-coordinates remain unchanged.

Example: $y = x - 2$ and $y = |x - 2|$

Example: Linear Function and Its Absolute Value

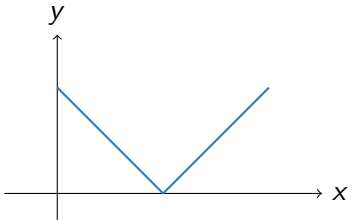
Original: $y = x - 2$



Piecewise:

$$y = \begin{cases} x - 2, & x \geq 2 \\ -(x - 2), & x < 2 \end{cases}$$

Absolute Value: $y = |x - 2|$



Steps to Find Piecewise Function for |Parabola|

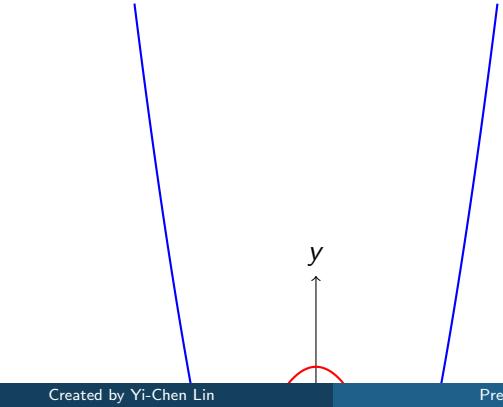
How to Write the Piecewise Function

1. **Find the x-intercepts** of the parabola by setting $y = 0$ and solving for x. Use these intercepts to split the domain into (Left, Middle, Right).
2. **Write the equation inside the absolute value** for each domain interval, **without** the absolute value sign.
3. **Graph the parabola** and indicate which parts are below the x-axis (these will be flipped up).
4. For the part(s) that were flipped, **place a negative sign in front of the entire equation** (with brackets) for those intervals.

Example: Parabola and Its Absolute Value

Reflecting Below the X-axis

To graph $y = |f(x)|$ for a parabola, reflect any part below the x-axis upward.

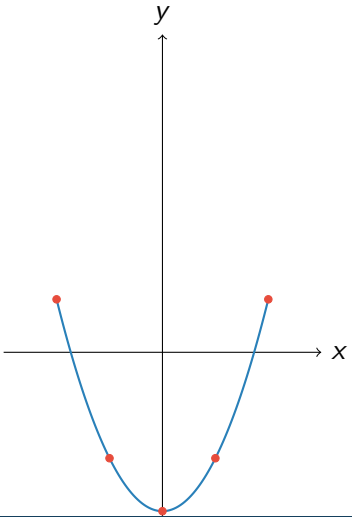


Explanation

Blue dashed: Original part below x-axis
Blue solid: Reflected upward
Red: Original part above x-axis (unchanged)

Example: Quadratic Function and Its Absolute Value

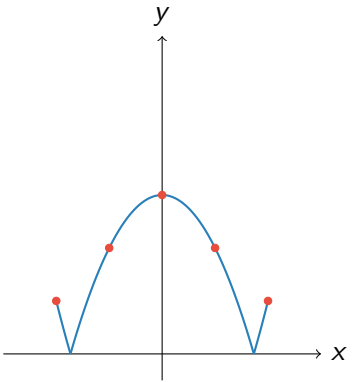
Original: $y = x^2 - 3$



Piecewise:

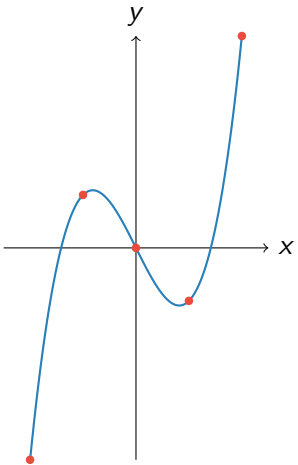
$$y = \begin{cases} x^2 - 3, & |x| \geq \sqrt{3} \\ -(x^2 - 3), & |x| < \sqrt{3} \end{cases}$$

Absolute Value: $y = |x^2 - 3|$



Example: Cubic Function and Its Absolute Value

Original: $y = x^3 - 2x$

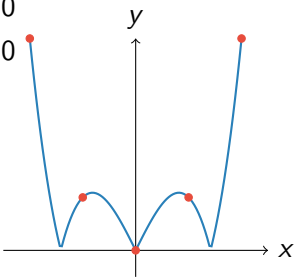


Piecewise:

$$y = \begin{cases} x^3 - 2x, & x^3 - 2x \geq 0 \\ -(x^3 - 2x), & x^3 - 2x < 0 \end{cases}$$

Absolute Value:

$y = |x^3 - 2x|$



Practice: Graph and Analyze

Practice

Graph $y = |x^2 - 4|$ and:

1. Determine the x- and y-intercepts
2. State the domain and range
3. Write the piecewise function

Multiple Choice: Piecewise Function

Which is the correct piecewise function for y

1. $y = x^2 - 4$ for $x \leq -2$ or $x \geq 2$; $y = -(x^2 - 4)$ for $-2 < x < 2$
2. $y = x^2 - 4$ for $x < 0$; $y = -(x^2 - 4)$ for $x \geq 0$
3. $y = x^2 - 4$ for $x > 0$; $y = -(x^2 - 4)$ for $x \leq 0$

Practice: Graph the Absolute Value

Practice 1

Graph $y = |x + 1|$.

Practice: Graph the Absolute Value

Practice 2

Graph $y = |-2x + 3|$.

Practice: Graph the Absolute Value

Practice 3

Graph $y = |x^2 - 2|$.

Practice: Graph the Absolute Value

Practice 4

Graph $y = |-x^2 + 4|$.

Practice: Graph the Absolute Value

Practice 5

Graph $y = |x^2 - 4x + 3|$.