Pre-Calculus 11

Solving for Angles in All Four Quadrants

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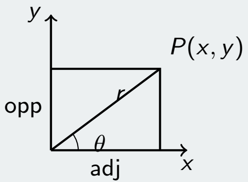
Review: SOH-CAH-TOA

Key Trig Ratios

$$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$$
$$\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$
$$\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}$$

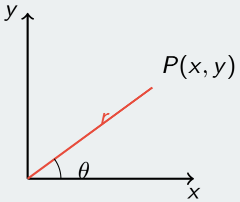


- Pythagorean Theorem: $a^2 + b^2 = c^2$
- Circle Equation: $x^2 + y^2 = r^2$

Trig Ratio for Any Angle

Coordinates on the Circle

- For angle θ in standard position, any point $P(x, y)$ on the circle of radius r:
- $x = r\cos\theta,\ y = r\sin\theta$
- On the unit circle $(r = 1)$: $P(\cos\theta, \sin\theta)$



Example: Coordinates for Given Angles

Question

Find the coordinates of point $P(x, y)$ for the following angles in standard position:

- $\theta = 215^\circ$
- $\theta = 150^\circ$

Example: Coordinates for Given Angles

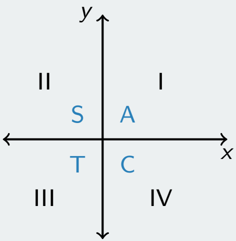
Solution

- $\theta = 215^\circ$: $x = \cos 215^\circ \approx -0.819$, $y = \sin 215^\circ \approx -0.573$
- $\theta = 150^\circ$: $x = \cos 150^\circ \approx -0.866$, $y = \sin 150^\circ \approx 0.5$

Sine, Cosine, Tangent in Different Quadrants

CAST Rule

- Sine, cosine, and tangent can be positive or negative depending on the quadrant.
- Use the CAST rule to remember which is positive:
- Quadrant I: All positive
- Quadrant II: Sine only
- Quadrant III: Tangent only
- Quadrant IV: Cosine only



Example: Which Quadrant?

Question

Determine which quadrants the angle θ can be in for the following trig functions:

- $\sin\theta = -2/3$
- $\cos\theta = 2/5$
- $\tan\theta = -3/7$

Example: Which Quadrant?

Solution

- $\sin\theta = -2/3$ (negative): θ in Q3 Q4
- $\cos\theta = 2/5$ (positive): θ in Q1 Q4
- $\tan\theta = -3/7$ (negative): θ in Q2 Q4

Steps for Solving for Angles

Step-by-Step

1. Identify the trig function (sine, cosine, tangent)
2. Check if the ratio is positive or negative
3. Use CAST to determine possible quadrants
4. Use inverse trig to find the reference angle
5. Find the second angle in the other quadrant
6. State both solutions

Example: Solve for the Angle

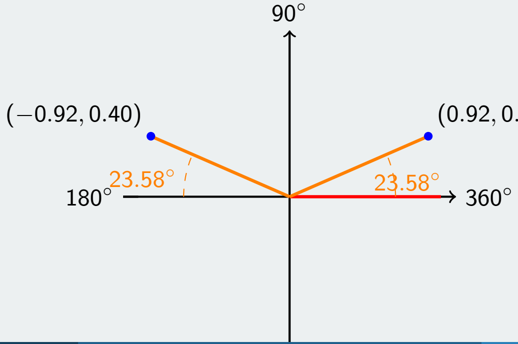
Question

Solve for θ given $\sin\theta = 2/5$.

Example: Solve for the Angle

Solution

$\sin\theta = 2/5$ (positive)
θ in Q1 and Q2
$\theta_1 = \sin^{-1}(2/5) \approx 23.58^\circ$
$\theta_2 = 180^\circ - 23.58^\circ = 156.42^\circ$



Example: Solve for the Angle (Cos Negative)

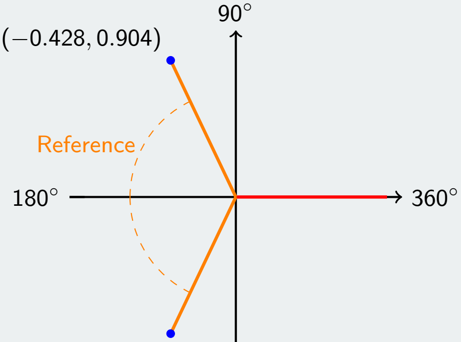
Question

Solve for θ given $\cos\theta = -3/7$.

Example: Solve for the Angle (Cos Negative)

Solution

$\cos\theta = -3/7$ (negative)
θ in Q2 and Q3
$\theta_1 = \cos^{-1}(-3/7) \approx 115.54^\circ$
$\theta_2 = 360^\circ - 115.54^\circ = 244.46^\circ$



Example: $\tan\theta = 4/9$

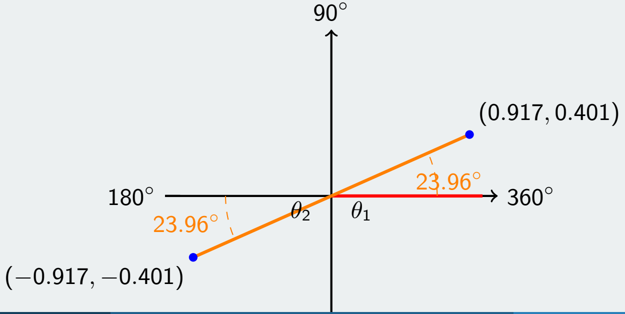
Question

Given $\tan\theta = 4/9$, find all possible coordinates for point $P(x, y)$ in the unit circle.

Example: $\tan\theta = 4/9$

Solution

$\tan\theta = 4/9$ (positive)
θ in Q1 and Q3
$x = 0.917,\ y = 0.401$ (Q1), $x = -0.917,\ y = -0.401$ (Q3)
Reference angle: $\theta_1 = \tan^{-1}(4/9) \approx 23.96^\circ$
$\theta_2 = 180^\circ + 23.96^\circ = 203.96^\circ$



Practice: Find the Angles

Practice

Given $0^\circ \leq \theta \leq 360^\circ$, find the angles:

- $\tan\theta = -2/5$
- $\sin\theta = -5/3$
- $\cos\theta = -5/3$

Q: One or No Solution?

Concept Check

Which of the following have one solution? Which have none?

- $\sin\theta = 0.856$
- $\sin\theta = 1$
- $\cos\theta = 1$
- $\cos\theta = 2.345$
- $\tan\theta = -9.856$
- $\tan\theta = 0$
- $\sin\theta = -1$

Steps

1. Isolate the trig function
2. Determine possible quadrants using CAST
3. Find the reference angle
4. State all solutions

Example: Algebraic Trig Equation

Question

Solve for θ given $8\sin\theta + 7 = 12$.

Example: Algebraic Trig Equation

Solution

$8\sin\theta + 7 = 12$
$8\sin\theta = 5$
$\sin\theta = 5/8$
θ in Q1 and Q2
$\theta_1 = \sin^{-1}(5/8) \approx 38.68^\circ$
$\theta_2 = 180^\circ - 38.68^\circ = 141.32^\circ$

Practice: Solve for θ

Practice

Given $0^\circ \leq \theta \leq 360^\circ$, solve:

- $5\cos\theta + 13 = 11$
- $3\cos\theta = 8\sin\theta$
- $4\tan^2\theta + 7 = 10$
- $9\sin^2\theta - 4 = 0$

Sine, Cosine, Tangent of $0°$, $90°$, $180°$, $270°$, $360°$

Special Angles

- $\sin\theta$ is the y-coordinate, $\cos\theta$ is the x-coordinate
- $\cos 0^\circ = 1, \ \sin 0^\circ = 0$
- $\cos 90^\circ = 0, \ \sin 90^\circ = 1$
- $\cos 180^\circ = -1, \ \sin 180^\circ = 0$
- $\cos 270^\circ = 0, \ \sin 270^\circ = -1$
- $\cos 360^\circ = 1, \ \sin 360^\circ = 0$

What is $\tan\theta$ at Special Angles?

Special Tangent Values

- $\tan\theta = \frac{\sin\theta}{\cos\theta}$
- $\tan 0^\circ = 0$
- $\tan 90^\circ$ is undefined
- $\tan 180^\circ = 0$
- $\tan 270^\circ$ is undefined

Finding Coordinates of P on Terminal Arm

Method 1: Using Central Angle

- $P(x, y) = (r\cos\theta, r\sin\theta)$
- On the unit circle: $P(\cos\theta, \sin\theta)$
- There are usually two angles (reference angles)

Method 2: Using Pythagorean Theorem

- Draw a right triangle in the correct quadrant
- Use the given ratio to find missing side
- Use $a^2 + b^2 = c^2$ to solve for x or y

Example: $\cos\theta = 3/5$

Question

Given $\cos\theta = 3/5$, find all possible coordinates for point $P(x, y)$ in the unit circle.

Example: $\cos\theta = 3/5$

Solution

$\cos\theta = 3/5$ (positive)
θ in Q1 and Q4
$x = 0.6$, $y = 0.8$ (Q1), $y = -0.8$ (Q4)
Angles: $\theta_1 = \cos^{-1}(3/5) \approx 53.13^\circ$, $\theta_2 = 360^\circ - 53.13^\circ = 306.87^\circ$

Practice: $\sin\theta = -2/5$

Practice

Given $\sin\theta = -2/5$, show the angle in standard position and the coordinates of P on the unit circle.

Practice: $\sin\theta = -2/5$

Solution

$\sin\theta = -2/5$ (negative)
θ in Q3 and Q4
$x = 0.447$, $y = -0.894$ (Q4), $y = -0.894$ (Q3)

Example: $\sin\theta = -6/11$ (Exact Value)

Question

Given $\sin\theta = -6/11$, find the exact value of the coordinates of point $P(x, y)$ in the unit circle.

Example: $\sin\theta = -6/11$ (Exact Value)

Solution

$\sin\theta = -6/11$ (negative)
θ in Q3
Draw triangle: $\sin\theta = \frac{\text{opp}}{\text{hyp}} = -6/11$
$x = -5,\ y = -6,\ r = 11$

Example: $\tan\theta = 3/5$

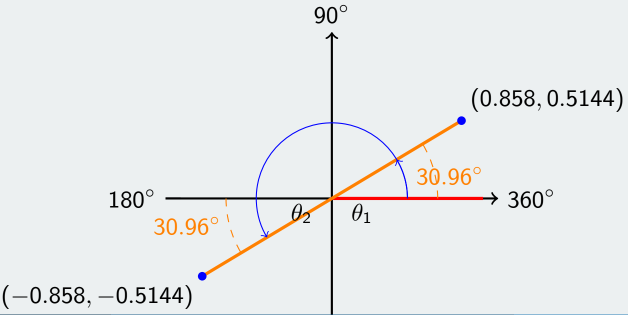
Question

Given $\tan\theta = 3/5$, find all possible coordinates for point $P(x, y)$ in the unit circle.

Example: $\tan\theta = 3/5$

Solution

$\tan\theta = 3/5$ (positive)
θ in Q1 and Q3
$x = 0.858$, $y = 0.514$ (Q1), $x = -0.858$, $y = -0.514$ (Q3)
Reference angle: $\theta_1 = \tan^{-1}(3/5) \approx 30.96^\circ$
$\theta_2 = 180^\circ + 30.96^\circ = 210.96^\circ$



Practice: Find Coordinates

Practice

Given the trig value, show the angle in standard position and the coordinates of P on the unit circle:

- $\sin\theta = -2/5$
- $\sin\theta = -6/11$
- $\tan\theta = 3/5$

Practice: Solve for θ

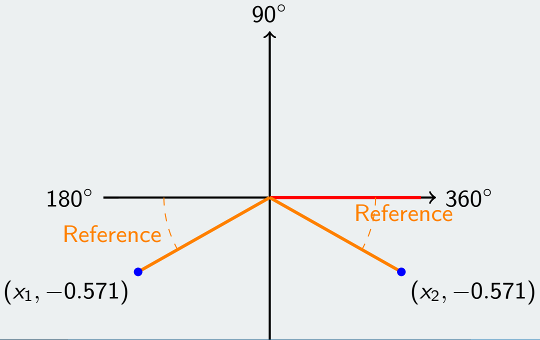
Practice

Solve for θ given $\sin\theta = -4/7$.

Practice: Solve for θ

Solution

$\sin\theta = -4/7$ (negative)
θ in Q3 and Q4
$\theta_1 = 360^\circ - \sin^{-1}(4/7) \approx 209.46^\circ$
$\theta_2 = 360^\circ + \sin^{-1}(4/7) \approx 330.54^\circ$



Practice: Solve for θ

Practice

Solve for θ given $\cos\theta = 5/13$.

Practice: Solve for θ

Solution

$\cos\theta = 5/13$ (positive)
θ in Q1 and Q4
$\theta_1 = \cos^{-1}(5/13) \approx 67.38^\circ$
$\theta_2 = 360^\circ - 67.38^\circ = 292.62^\circ$