Chapter 5.5: Applications of Rational Functions

Rational Expressions - Lesson 5

Created by Yi-Chen Lin

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Speed, Time, Distance (STD) Applications

Key Concepts

- Rational functions are used in problems involving speed, time, and distance.
- $D = S \times T$, $T = \frac{D}{S}$, $S = \frac{D}{T}$
- Applications include travel, fuel efficiency, wind/drag, and more.

Example Q1: Basic STD (Part 1)

Example Q1 (Part 1)

A cyclist travels 60 km at a speed of x km/h. Write an expression for the time required.

Example Q1: Basic STD (Part 2)

Example Q1 (Part 2)

Solution:

$$T = \frac{D}{S} = \frac{60}{x} \text{ hours}$$

Applications: Rate of Work Problems

Key Concepts

- Rate of work: If one person can do a job in a hours, their rate is $\frac{1}{a}$ jobs/hour.
- For n people: $\frac{x}{a} + \frac{x}{b} + \ldots = 1$ (where x is the time working together)

Example Q2: Rate of Work (Part 1)

Example Q2 (Part 1)

Sam can paint a wall in 4 hours, Tim in 6 hours, and Sara in 5 hours. How long will it take them to paint it together?

Example Q2: Rate of Work (Part 2)

Example Q2 (Part 2)

Let x be the time to finish together:

$$\frac{x}{4} + \frac{x}{6} + \frac{x}{5} = 1$$

Step 1: Find LCD: 60

$$\frac{15x}{60} + \frac{10x}{60} + \frac{12x}{60} = 1$$

$$(15x + 10x + 12x) = 6037x = 60x = \frac{60}{37} \approx 1.62 \text{ hours}$$

Practice Q3: Rate of Work

Practice Q3

Mike can mow a lawn in 2 hours, his brother Tim in 4 hours. How long if they work together?

Solution Q3 (Part 1)

Let x be the time together:

$$\frac{x}{2} + \frac{x}{4} = 1$$

Step 1: Find LCD: 4

$$2x + x = 43x = 4$$

Solution Q3 (Part 2)

Solution Q3 (Part 2)

Step 2: Solve:

$$x = \frac{4}{3} \text{ hours}$$

Applications: Travelling with Different Speeds

Key Concepts

- When two vehicles travel the same distance at different speeds, time difference can be modeled with rational functions.
- Use $T = \frac{D}{S}$ for each.

Example Q4: Travelling with Different Speeds (Part 1)

Example Q4 (Part 1)

A car travels 400 km at $8x$ km/h. Tom runs at x km/h and takes 4.5 hours longer. Find x.

Example Q4: Travelling with Different Speeds (Part 2)

Example Q4 (Part 2)

Step 1: Write time for each:

$$T_{car} = \frac{400}{8x}, \quad T_{Tom} = \frac{400}{x}$$

Step 2: Set up equation:

$$\frac{400}{x} - \frac{400}{8x} = 4.5$$

Step 3: Simplify:

$$\frac{400(1-\frac{1}{8})}{x} = 4.5\frac{400 \times \frac{7}{8}}{x} = 4.5\frac{350}{x} = 4.5x = \frac{350}{4.5} \approx 77.78 \text{ km/h}$$

Practice Q5: Travelling

Practice Q5

Jack and Steven leave Portland for Bellingham (300 km). Jack takes a plane, Steven drives. Jack is 10 times faster and arrives 4.5 hours earlier. How fast is each travelling?

Solution Q5 (Part 1)

Let x be Steven's speed. Jack's speed is $10x$.

$$T_{Jack} = \frac{300}{10x}, \quad T_{Steven} = \frac{300}{x}$$

Time difference:

$$\frac{300}{x} - \frac{300}{10x} = 4.5$$

Step 1: Simplify:

$$\frac{300(1 - \frac{1}{10})}{x} = 4.5 \frac{300 \times \frac{9}{10}}{x} = 4.5 \frac{270}{x} = 4.5$$

Solution Q5 (Part 2)

Solution Q5 (Part 2)

Step 2: Solve:

$$x = \frac{270}{4.5} = 60 \text{ km/h}$$

Jack's speed:

$$10x = 600 \text{ km/h}$$

Applications: Plane Questions with Wind Speed

Key Concepts

- For round trips with wind, use $T = \frac{D}{S}$ for each leg.
- Let x be plane speed, w wind speed. Outbound: $x + w$, return: $x - w$.
- Total time: $T = \frac{D}{x+w} + \frac{D}{x-w}$

Example Q6: Plane with Wind (Part 1)

Example Q6 (Part 1)

A plane flies 3400 km to Toronto with a tailwind, returns with a headwind. Wind speed is w km/h. Write an equation for total round trip time if plane speed is x km/h.

Example Q6: Plane with Wind (Part 2)

Example Q6 (Part 2)

Total time:

$$T = \frac{3400}{x + w} + \frac{3400}{x - w}$$

Practice Q7: Plane with Wind

Practice Q7

The distance from Vancouver to Hong Kong is 10300 km. Wind speed is 60 km/h. If a round trip takes 25 hours, how fast must the plane fly?

Solution Q7 (Part 1)

Let x be the plane's speed.

$$25 = \frac{10300}{x+60} + \frac{10300}{x-60}$$

Step 1: Multiply both sides by $(x+60)(x-60)$:

$$25(x+60)(x-60) = 10300(x-60) + 10300(x+60)$$

Solution Q7 (Part 2)

Solution Q7 (Part 2)

Step 2: Expand and solve:

$$25(x^2 - 3600) = 10300x - 618000 + 10300x + 61800025x^2 - 90000 = 20600x25x^2 - 20600x - 90000 = 0$$

Step 3: Quadratic formula:

$$x = \frac{20600 \pm \sqrt{20600^2 + 4 \times 25 \times 90000}}{2 \times 25}$$

Challenge Q8: River Current

Challenge Q8

A boat travels 120 km downstream and returns upstream. The current is y km/h. The boat's speed in still water is b km/h. The round trip takes 10 hours. If $y = 4$, find b.

Solution Q8 (Part 1)

Step 1: Write time for each leg:

$$T_{down} = \frac{120}{b+4}, \quad T_{up} = \frac{120}{b-4}$$

Step 2: Set up equation:

$$\frac{120}{b+4} + \frac{120}{b-4} = 10$$

Solution Q8 (Part 2)

Step 3: Multiply both sides by $(b+4)(b-4)$:

$$120(b-4)+120(b+4)=10(b+4)(b-4)$$

Step 4: Expand and solve:

$$120b-480+120b+480=10(b^2-16)240b=10b^2-16010b^2-240b-160=0$$

Step 5: Quadratic formula:

$$b=\frac{240\pm\sqrt{240^2+4\times10\times160}}{2\times10}$$