Pre-Calculus 11
Chapter 2: Factoring and Quadratic Functions

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Chapter Overview

This chapter covers various methods of factoring and solving quadratic functions, including:

- Factoring trinomials
- Solving quadratic equations
- Graphing quadratic functions
- Applications of quadratic functions

1 2.1 Factoring Trinomials

Key Concepts

Methods of Factoring

1. **B.U.M. Method**
   - Bring the First term to the Last term and Multiply
   - Factor the resulting trinomial
   - Bring the First term back in front of each x
   - Factor out any common factors
2. **Criss-Cross Method**
   - Find factors of first and last terms
   - Cross multiply and check if sum equals middle term
   - Write the factors in the correct positions
3. **Grouping Method**
   - Multiply first and last coefficients
   - Find two numbers that multiply to this product and add to middle coefficient
   - Split the middle term
   - Factor by grouping

Special Cases

Special Factoring Patterns

- **Perfect Square Trinomials**

$$a^2 + 2ab + b^2 = (a + b)^2$$
$$a^2 - 2ab + b^2 = (a - b)^2$$

- **Difference of Squares**

$$a^2 - b^2 = (a + b)(a - b)$$

- **Difference of Cubes**

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

2 2.2 Solving Quadratic Equations by Factoring

Key Concepts

Steps to Solve by Factoring

1. Write equation in standard form: $ax^2 + bx + c = 0$
2. Factor the quadratic expression
3. Set each factor equal to zero
4. Solve for x

Zero Product Property

Important Property

If $ab = 0$, then either $a = 0$ or $b = 0$ (or both)

3 2.3 Graphing Quadratic Functions by Factoring

Key Concepts

Graphing Steps

1. Find x-intercepts by factoring
2. Find y-intercept by setting x = 0
3. Find vertex using symmetry
4. Plot points and draw parabola

Important Features

Graph Features

- **Vertex**: The highest or lowest point
- **Axis of Symmetry**: Vertical line through vertex
- **x-intercepts**: Points where y = 0
- **y-intercept**: Point where x = 0

4 2.4 The Quadratic Formula

Key Concepts

Quadratic Formula

For $ax^2 + bx + c = 0$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Discriminant

Discriminant Analysis

- $b^2 - 4ac > 0$: Two real solutions
- $b^2 - 4ac = 0$: One real solution
- $b^2 - 4ac < 0$: No real solutions

5 2.5 Graphing Quadratic Functions in APQ Form

Key Concepts

Vertex Form

$$y = a(x - h)^2 + k$$

where (h, k) is the vertex

Graphing Steps

Steps to Graph

1. Identify vertex (h, k)
2. Determine direction of opening
3. Find y-intercept
4. Plot additional points if needed

6 2.6 Completing the Square

Key Concepts

Steps to Complete the Square

1. Write in form $x^2 + bx + c$
2. Add and subtract $(\frac{b}{2})^2$
3. Factor perfect square trinomial
4. Simplify

Example

Example

$$\begin{aligned} x^2 + 6x + 2 &= 0 \\ x^2 + 6x &= -2 \\ x^2 + 6x + 9 &= 7 \\ (x+3)^2 &= 7 \\ x &= -3 \pm \sqrt{7} \end{aligned}$$

7 2.7 Word Problems on Max/Min

Key Concepts

Problem-Solving Steps

1. Identify variables
2. Write equation
3. Find vertex (maximum/minimum)
4. Interpret solution

Common Applications

Applications

- Projectile motion
- Area optimization
- Revenue maximization
- Cost minimization

8 2.8 Quadratic Inequalities

Key Concepts

Solving Steps

1. Write in standard form
2. Find critical points
3. Test intervals
4. Write solution

Solution Methods

Methods

- **Graphical**: Plot and shade
- **Algebraic**: Test points
- **Sign Analysis**: Use number line

Chapter Summary

Key Takeaways

- Multiple methods for factoring trinomials
- Various ways to solve quadratic equations
- Different forms of quadratic functions
- Applications in real-world problems
- Methods for solving inequalities