Pre-Calculus 11

Lesson 3: Graphing Quadratic Functions

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Key Concepts

- **Linear Functions**
  - Straight Lines
  - General Form: $y = mx + b$
  - Highest degree for "x" is one
  - m: Slope
  - b: Y-intercept
- **Quadratic Functions**
  - Curved, Shape of a "Parabola"
  - Highest Degree for "x" is two
  - General Form: $y = ax^2 + bx + c$, where a, b, c are real numbers
  - Example: $y = x^2 + x - 10$ or $y = -x^2 - 4x + 10$

Understanding the Parabola Shape

- If we make a Table of Values (TOV), plot the coordinates, and connect the dots, the resulting shape is a Parabola.

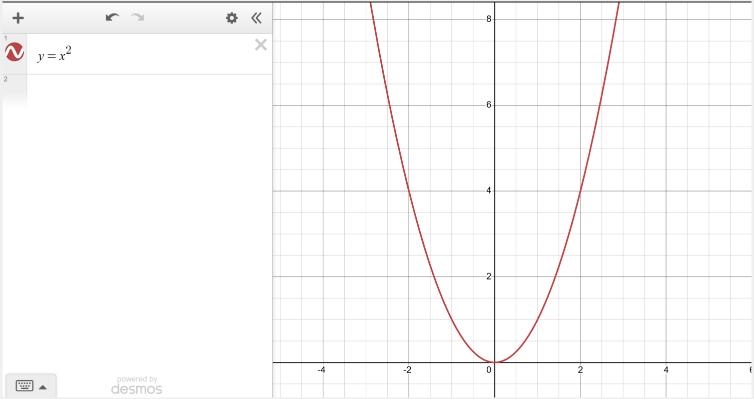
Example TOV for $y = x^2$

x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

(Graph will be on the next page)

Graph of $y = x^2$

Visual Representation



II) COMPONENTS OF A PARABOLA

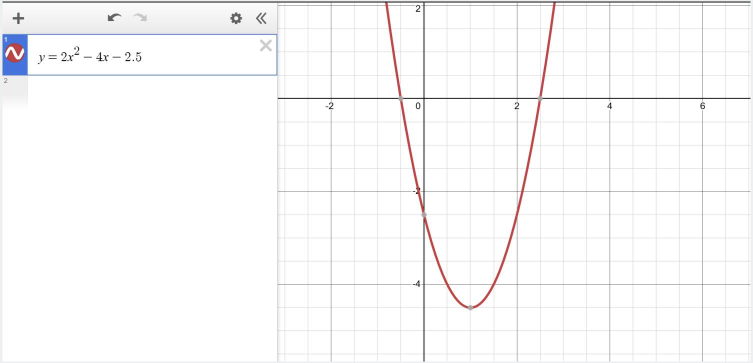
Key Components

- **Vertex:** The coordinates at either the top (maximum) or bottom (minimum) of the parabola. Always in the middle.
- **Axis of Symmetry (AOS):** A vertical line that cuts the graph in the middle. Must be an equation (e.g., $x = k$).
- **X-intercepts:** Intersection points between the parabola and the x-axis (where $y = 0$). Can have zero, one, or two x-intercepts. Represented as $(x_1, 0)$ and $(x_2, 0)$.
- **Y-intercept:** Intersection point between the parabola and the y-axis (where $x = 0$). Always one y-intercept. Represented as $(0, y_0)$.

(Insert diagram of a parabola with labeled components: Vertex, Axis of Symmetry, X-intercepts, Y-intercept)

Ex: Given each parabola, indicate the components

Example 1 - Graph



Parabola 1: (Opening Up)

Ex: Given each parabola, indicate the components - Example 1 Details

Example 1 - Components

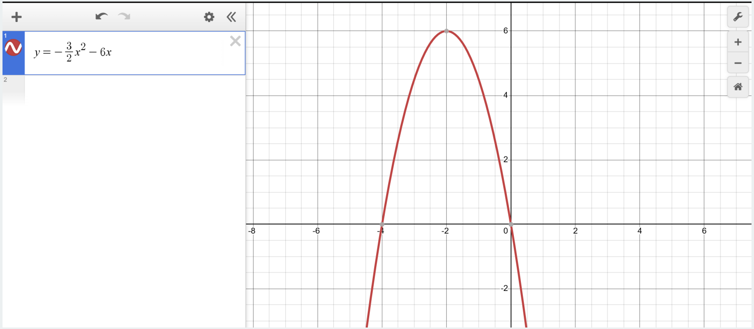
Parabola 1: (Opening Up)

- **Vertex:** (1, -3)
- **Axis of Symmetry:** $x = 1$
- **X-intercepts:** (-0.5, 0) and (2.5, 0)
- **Y-intercept:** (0, -2.5)

Ex: Given each parabola, indicate the components

Example 2 - Graph

Parabola 2: (Opening Down)



Ex: Given each parabola, indicate the components - Example 2 Details

Example 2 - Components

Parabola 2: (Opening Down)

- **Vertex:** (-2, 6)
- **Axis of Symmetry:** $x = -2$
- **X-intercepts:** (-4, 0) and (0, 0)
- **Y-intercept:** (0, 0)

GRAPHING PARABOLAS WITH XAVIER'S METHOD

Xavier's Method Steps

- First find the vertex using **X.A.V.**
  - **X:** x-intercepts by factoring (set $y = 0$)
  - **A:** Axis of Symmetry (average of x-intercepts: $x = \frac{x_1+x_2}{2}$)
  - **V:** Vertex (substitute AOS x-value into the original equation to find y-coordinate)
- Use the constant "a" from $y = ax^2 + bx + c$ to determine which way the graph opens:
  - If $a > 0$ (positive), graph Opens Up (U-shape)
  - If $a < 0$ (negative), graph Opens Down (inverted U-shape)
- Plot a couple of extra points for a better graph (e.g., y-intercept and its symmetric point).
- **Note:** For Quadratic Functions that do not have x-intercepts, we will learn to graph them in the next section.

EX: FIND THE X INTERCEPTS, AOS, VERTEX, AND GRAPH $y = x^2 - 4x - 12$

Step-by-Step Solution

Given: $y = x^2 - 4x - 12$

1. **Find X-intercepts (by factoring):** Set $y = 0$:

$$x^2 - 4x - 12 = 0$$
$$(x - 6)(x + 2) = 0$$

   So, $x - 6 = 0 \Rightarrow x = 6$ (X-intercept: $(6, 0)$) And, $x + 2 = 0 \Rightarrow x = -2$ (X-intercept: $(-2, 0)$)

2. **Find Axis of Symmetry (AOS) (Equation):** Average of x-intercepts:

$$x = \frac{6 + (-2)}{2}$$
$$x = \frac{4}{2}$$
$$x = 2$$

EX: FIND THE X INTERCEPTS, AOS, VERTEX, AND GRAPH $y = x^2 - 4x - 12$

Step-by-Step Solution (Cont.)

Given: $y = x^2 - 4x - 12$

3. **Find Vertex (Coordinates):** Substitute AOS $x = 2$ into the original equation:

$$y = (2)^2 - 4(2) - 12$$
$$y = 4 - 8 - 12$$
$$y = -16$$

   Vertex: $(2, -16)$

4. **Determine opening direction:** Since $a = 1$ (positive), the parabola opens UP.

EX: FIND THE X INTERCEPTS, AOS, VERTEX, AND GRAPH $y = x^2 - 4x - 12$

Graph - y

Points to plot:

- X-intercepts: $(-2, 0)$ and $(6, 0)$
- Vertex: $(2, -16)$
- Y-intercept: Set $x = 0$, $y = (0)^2 - 4(0) - 12 = -12$. So $(0, -12)$.
- Additional points (symmetric to y-intercept): Since AOS is $x = 2$, point symmetric to $(0, -12)$ is at $x = 4$ (distance of 2 from AOS). For $x = 4$, $y = (4)^2 - 4(4) - 12 = 16 - 16 - 12 = -12$. So $(4, -12)$.

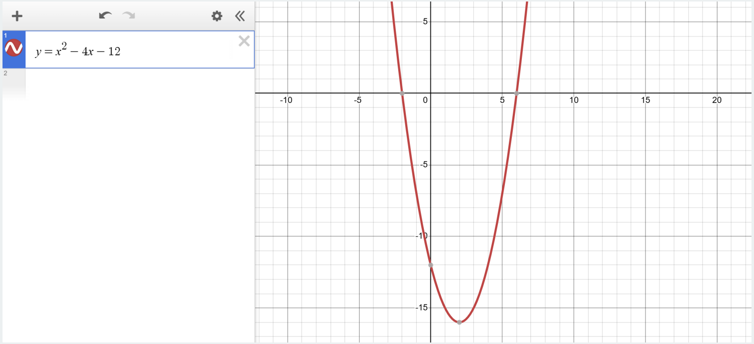
(Graph will be on the next page)

Domain and Range:

- **Domain (D):** $x \in \mathbb{R}$ (All real numbers)
- **Range (R):** $y \geq -16$ (Since it opens up, y-values are greater than or equal to the vertex's y-coordinate)

Graph for $y = x^2 - 4x - 12$

Visual Representation



PRACTICE: FIND THE X INTERCEPTS, AOS, VERTEX, AND GRAPH $y = 3x^2 + 5x - 2$

Problem

Find the X-intercepts, Axis of Symmetry, Vertex and Graph the following: $y = 3x^2 + 5x - 2$

PRACTICE: Solution $y = 3x^2 + 5x - 2$ (Part 1)

Detailed Solution

Given: $y = 3x^2 + 5x - 2$

1. **Find X-intercepts (by factoring):** Set $y = 0$:

$$3x^2 + 5x - 2 = 0$$
$$(3x - 1)(x + 2) = 0$$

So, $3x - 1 = 0 \Rightarrow 3x = 1 \Rightarrow x = \frac{1}{3}$ (X-intercept: $(\frac{1}{3}, 0)$) And, $x + 2 = 0 \Rightarrow x = -2$ (X-intercept: $(-2, 0)$)

Detailed Solution (Part 2)

Given: $y = 3x^2 + 5x - 2$

2. **Find Axis of Symmetry (AOS) (Equation):** Average of x-intercepts:

$$x = \frac{\frac{1}{3} + (-2)}{2}$$

$$x = \frac{\frac{1}{3} - \frac{6}{3}}{2}$$

$$x = \frac{-\frac{5}{3}}{2}$$

$$x = -\frac{5}{6}$$

AOS: $x = -\frac{5}{6}$

PRACTICE: Solution $y = 3x^2 + 5x - 2$ (Part 3)

Detailed Solution (Cont.)

Given: $y = 3x^2 + 5x - 2$

3. **Find Vertex (Coordinates):** Substitute AOS $x = -\frac{5}{6}$ into the original equation:

$$y = 3(-\frac{5}{6})^2 + 5(-\frac{5}{6}) - 2$$
$$y = 3(\frac{25}{36}) - \frac{25}{6} - 2$$
$$y = \frac{25}{12} - \frac{50}{12} - \frac{24}{12}$$
$$y = \frac{25 - 50 - 24}{12}$$
$$y = -\frac{49}{12}$$

Vertex: $(-\frac{5}{6}, -\frac{49}{12})$

4. **Determine opening direction:** Since $a = 3$ (positive), the parabola opens UP.

PRACTICE: Graph $y = 3x^2 + 5x - 2$

Graph - y

Points to plot:

- X-intercepts: $(-2, 0)$ and $(\frac{1}{3}, 0)$
- Vertex: $(-\frac{5}{6}, -\frac{49}{12})$
- Y-intercept: Set $x = 0$, $y = 3(0)^2 + 5(0) - 2 = -2$. So $(0, -2)$.
- Additional points (symmetric to y-intercept): Since AOS is $x = -\frac{5}{6}$, point symmetric to $(0, -2)$ is at $x = -\frac{5}{6} + (0 - (-\frac{5}{6})) = -\frac{5}{6} + \frac{5}{6} = -\frac{10}{6} = -\frac{5}{3}$. For $x = -\frac{5}{3}$,
$y = 3(-\frac{5}{3})^2 + 5(-\frac{5}{3}) - 2 = 3(\frac{25}{9}) - \frac{25}{3} - 2 = \frac{25}{3} - \frac{25}{3} - 2 = -2$. So $(-\frac{5}{3}, -2)$.

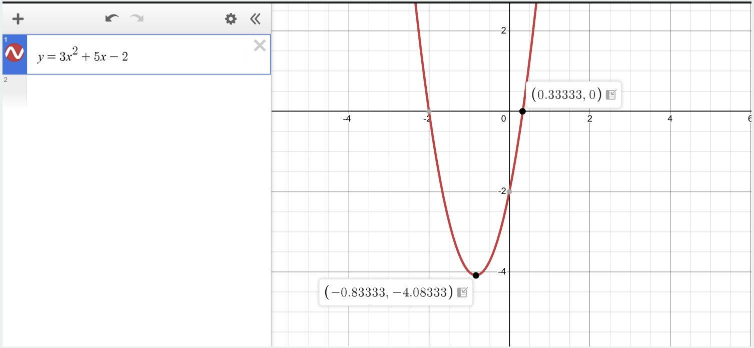
(Graph will be on the next page)

Domain and Range:

- **Domain (D):** $x \in \mathbb{R}$ (All real numbers)
- **Range (R):** $y \geq -\frac{49}{12}$ (Since it opens up, y-values are greater than or equal to the vertex's y-coordinate)

Graph for $y = 3x^2 + 5x - 2$

Visual Representation



REVIEW: DOMAIN AND RANGE

Definitions

- **Domain:** The collection of all possible X-values (input) that a function can have. Look at the graph horizontally.
- **Range:** The collection of all possible Y-values (output) that a function can have. Look at the graph vertically.

Example for finding Domain: *(Insert image of a graph that starts at x=-2 and continues to the right, e.g., a ray)*

- This graph starts at $x = -2$ and continues to the right.
- **Domain:** $x \geq -2$

Example for finding Range: *(Insert image of a graph with a lowest point at y=0, and then goes up, e.g., an upward opening parabola fragment or a V-shape)*

- The lowest point of this graph is at $y = 0$, and then it goes up.
- **Range:** $y \geq 0$

DOMAIN AND RANGE OF PARABOLAS:

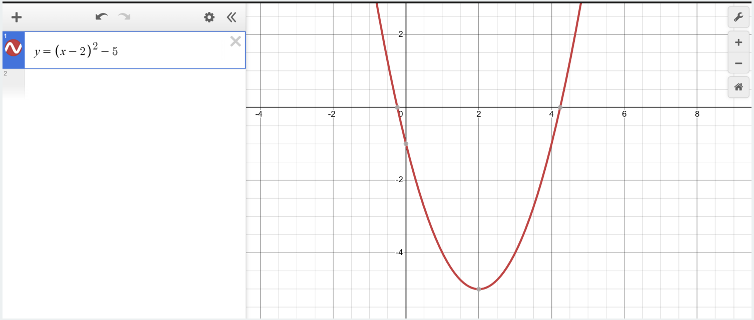
Key Rules for Parabolas

- Since a parabola that opens up or down extends infinitely to the left and right, it can take on any x-value.
- So, the **Domain** of a parabola is always: **All Real Numbers** ($x \in \mathbb{R}$).
- The **Range** of a parabola depends on which way the graph opens and the y-coordinate of its vertex:
    - If it opens **up** ($a > 0$), range will be: $y \geq$ lowest y-value (vertex y-coordinate)
    - If it opens **down** ($a < 0$), range will be: $y \leq$ highest y-value (vertex y-coordinate)

(Insert diagram of an upward opening parabola with Domain: $x \in \mathbb{R}$ and Range: $y \geq -3$ labeled)

drex1: Parabola 1 Graph

Visual Representation



Ex: Indicate the domain and range for Parabola 1

Problem

Given the parabola, indicate its domain and range. **Parabola 1: (Opening Up, Vertex at (2, -5))** *(Refer to previous page for graph)*

Ex: Indicate the domain and range for Parabola 1 - Solution

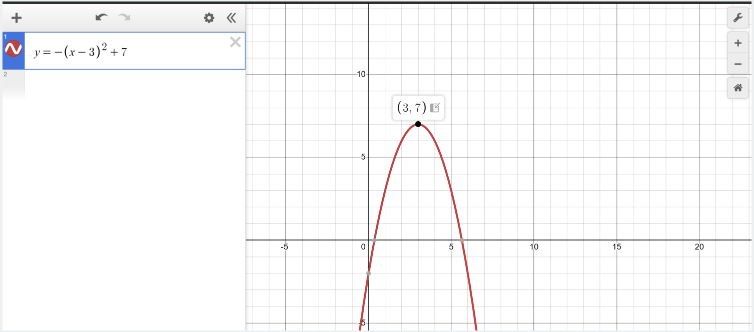
Detailed Solution

For Parabola 1:

- **Domain:** $x \in \mathbb{R}$
- **Range:** $y \geq -5$

drex2: Parabola 2 Graph

Visual Representation



Ex: Indicate the domain and range for Parabola 2

Problem

Given the parabola, indicate its domain and range. **Parabola 2: (Opening Down, Vertex at (3, 7))** *(Refer to previous page for graph)*

Ex: Indicate the domain and range for Parabola 2 - Solution

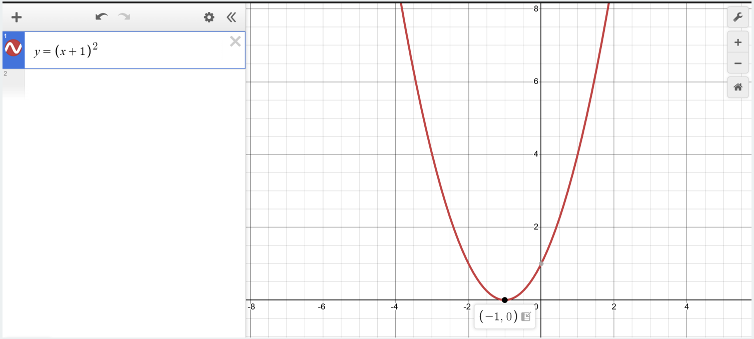
Detailed Solution

For Parabola 2:

- **Domain:** $x \in \mathbb{R}$
- **Range:** $y \leq 7$

drex3: Parabola 3 Graph

Visual Representation



Ex: Indicate the domain and range for Parabola 3

Problem

Given the parabola, indicate its domain and range. **Parabola 3: (Opening Up, Vertex at (-1, 0))** *(Refer to previous page for graph)*

Ex: Indicate the domain and range for Parabola 3 - Solution

Detailed Solution

For Parabola 3:

- **Domain:** $x \in \mathbb{R}$
- **Range:** $y \geq 0$

Ex: Match each equation with the description

Match the following

Equations:

a) $y = x^2 - 9$

b) $y = -2x^2 + 4x + 6$

c) $y = x^2 + 4x + 3$

Descriptions:

i) Range is $y \geq -1$, Axis of symmetry is $x = -2$

ii) X-intercepts at 3 and -3, Axis of symmetry is the Y-axis

iii) Range is $y \leq 8$, Axis of symmetry is $x = 1$

iv) Graph opens up and vertex at (4,-5)

Ex: Match each equation with the description - Solution

Detailed Solutions

a) $y = x^2 - 9$

- Vertex: $(0, -9)$
- AOS: $x = 0$
- X-intercepts: $x^2 - 9 = 0 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3$. So $(3, 0), (-3, 0)$.
- Range: $y \geq -9$ (opens up)
- Matches **ii) X-intercepts at 3 and -3, Axis of symmetry is the Y-axis**

b) $y = -2x^2 + 4x + 6$

- Opens down $(a = -2)$
- AOS: $x = \frac{-4}{2(-2)} = \frac{-4}{-4} = 1$. So $x = 1$.
- Vertex y-coord: $y = -2(1)^2 + 4(1) + 6 = -2 + 4 + 6 = 8$. Vertex: $(1, 8)$.
- Range: $y \leq 8$
- Matches **iii) Range is $y \leq 8$, Axis of symmetry is $x = 1$**

Detailed Solutions (Cont.)

c) $y = x^2 + 4x + 3$

- Opens up ($a = 1$)
- AOS: $x = \frac{-4}{2(1)} = \frac{-4}{2} = -2$. So $x = -2$.
- Vertex y-coord: $y = (-2)^2 + 4(-2) + 3 = 4 - 8 + 3 = -1$. Vertex: $(-2, -1)$.
- Range: $y \geq -1$
- Matches **i) Range is $y \geq -1$, Axis of symmetry is $x = -2$**

Description **iv) Graph opens up and vertex at (4,-5)** does not match any equation provided.

THINGS TO REMEMBER:

Key Reminders

- The **vertex** is always in the middle between the two X-intercepts (if they exist).
- The **Axis of Symmetry** must always be an equation: $x = k$, where "k" is the x-coordinate of the vertex.
- The **domain** of a parabola that opens up or down will always be: $x \in \mathbb{R}$ (All Real Numbers).
- To find the **Y-coordinate of the vertex**, plug the x-value of the AOS into the original equation and solve for "y".
- The vertex, x-intercepts, and y-intercept should be provided as a **pair of coordinates** (a,b).
- The y-coordinate of the Vertex will be used for the **range** of the function.

WHAT DO YOU DO IF YOU CAN'T FACTOR THE TRINOMIAL?

Next Steps

- **Answer:** Use the Quadratic Formula (next lesson)!
- **Quadratic Formula:**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$