Pre-Calculus 11

Solving Quadratic Functions by Factoring

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What Does "Solving" Mean?

Definition

- Finding a value for "x" (Variable) so that both sides of an equation will be equal
- **Example:** Solve for "x"

$$\begin{aligned} 2x + 1 &= 5 \\ 2x &= 4 \\ x &= 2 \end{aligned}$$

- **Check:**

$$\begin{aligned} 2(2) + 1 &= 5 \\ 4 + 1 &= 5 \\ 5 &= 5 \quad \checkmark \end{aligned}$$

Solving Trinomials by Factoring

Key Concept

- When you have the product of two brackets equal to zero, you can solve this equation easily
- **Example:** Solve $(x+3)(2x-5)=0$

$$x+3=0 \quad \text{or} \quad 2x-5=0$$
$$x=-3 \quad \text{or} \quad x=2.5$$

- **Important Rule:** Zero times anything is always equal to zero

Practice: Solving Trinomials

Solve Each of the Following

1. $(x+4)(x-3)=0$
2. $(2x+5)(3x-2)=0$
3. $(x-6)(x+2)=0$

Solutions: Solving Trinomials

Detailed Solutions

1. $(x+4)(x-3)=0$

$$x+4=0 \quad \text{or} \quad x-3=0$$
$$x=-4 \quad \text{or} \quad x=3$$

2. $(2x+5)(3x-2)=0$

$$2x+5=0 \quad \text{or} \quad 3x-2=0$$
$$x=-\frac{5}{2} \quad \text{or} \quad x=\frac{2}{3}$$

3. $(x-6)(x+2)=0$

$$x-6=0 \quad \text{or} \quad x+2=0$$
$$x=6 \quad \text{or} \quad x=-2$$

Steps for Solving Quadratic Equations

Key Steps

1. Move all terms to one side and make it equal to zero
2. Factor the equation into two binomials
3. Make each binomial equal to zero
4. Solve for "x" from each bracket

Example

Solve: $x^2 + 10x = 9$

$$
\begin{aligned}
x^2 + 10x - 9 &= 0 \\
(x+3)(x-3) &= 0 \\
x + 3 = 0 \quad &\text{or} \quad x - 3 = 0 \\
x = -3 \quad &\text{or} \quad x = 3
\end{aligned}
$$

Practice: Advanced Factoring

Solve Each of the Following

1. $x^2 + 7x + 12 = 0$
2. $2x^2 - 5x - 3 = 0$
3. $3x^2 + 10x + 7 = 0$

Solutions: Advanced Factoring

Detailed Solutions

1. $x^2 + 7x + 12 = 0$

$$(x+4)(x+3) = 0$$
$$x+4 = 0 \quad \text{or} \quad x+3 = 0$$
$$x = -4 \quad \text{or} \quad x = -3$$

2. $2x^2 - 5x - 3 = 0$

$$(2x+1)(x-3) = 0$$
$$2x+1 = 0 \quad \text{or} \quad x-3 = 0$$
$$x = -\frac{1}{2} \quad \text{or} \quad x = 3$$

3. $3x^2 + 10x + 7 = 0$

$$(3x+7)(x+1) = 0$$

Word Problems

Solve Each of the Following

1. The product of two consecutive numbers is 132. Find the numbers.
2. The length of a rectangle is 5 meters more than twice its width. If the area is 75 square meters, find the dimensions.
3. A 32m tall tree is broken during a storm. The distance from the base to the tip is 16m. At what height did the tree break?

Word Problem 1

Problem

The product of two consecutive numbers is 132. Find the numbers.

Word Problem 1 - Solution

Detailed Solution

The product of two consecutive numbers is 132. Find the numbers.
Solution:

1. Let x be the first number
2. Then $x + 1$ is the second number
3. $x(x + 1) = 132$
4. $x^2 + x - 132 = 0$
5. $(x + 12)(x - 11) = 0$
6. $x = -12$ or $x = 11$

Answer: The numbers are 11 and 12, or -12 and -11

Word Problem 2

Problem

The length of a rectangle is 5 meters more than twice its width. If the area is 75 square meters, find the dimensions.

Word Problem 2 - Solution

Detailed Solution

The length of a rectangle is 5 meters more than twice its width. If the area is 75 square meters, find the dimensions.

Solution:

1. Let x be the width
2. Then $2x + 5$ is the length
3. $x(2x + 5) = 75$
4. $2x^2 + 5x - 75 = 0$
5. $(2x + 15)(x - 5) = 0$
6. $x = 5$ meters (width)
7. Length $= 2(5) + 5 = 15$ meters

Answer: The dimensions are 5 meters by 15 meters

Word Problem 3

Problem

A 32m tall tree is broken during a storm. The distance from the base to the tip is 16m. At what height did the tree break?

Word Problem 3 - Solution

Detailed Solution

A 32m tall tree is broken during a storm. The distance from the base to the tip is 16m. At what height did the tree break?

Solution:

1. Let x be the height where the tree broke
2. Then $32 - x$ is the length of the fallen part
3. Using Pythagorean theorem:

$$\begin{aligned} x^2 + 16^2 &= (32 - x)^2 \\ x^2 + 256 &= 1024 - 64x + x^2 \\ 64x &= 768 \\ x &= 12 \end{aligned}$$

Answer: The tree broke at 12 meters

Practice Problems

Solve Each of the Following

1. $2x^2 + 5x = 3$
2. $3x^2 - 8x = -4$
3. $4x^2 + 12x = -9$
4. $x^2 - 6x = 16$
5. $5x^2 + 7x = -2$
6. $2x^2 - 9x = 5$

Practice Problems - Solutions Part 1

Detailed Solutions

1. $2x^2 + 5x = 3$

$$
\begin{aligned}
2x^2 + 5x - 3 &= 0 \\
(2x - 1)(x + 3) &= 0 \\
x = \frac{1}{2} \quad &\text{or} \quad x = -3
\end{aligned}
$$

2. $3x^2 - 8x = -4$

$$
\begin{aligned}
3x^2 - 8x + 4 &= 0 \\
(3x - 2)(x - 2) &= 0 \\
x = \frac{2}{3} \quad &\text{or} \quad x = 2
\end{aligned}
$$

Practice Problems - Solutions Part 2

Detailed Solutions

3. $4x^2 + 12x = -9$

$$\begin{aligned} 4x^2 + 12x + 9 &= 0 \\ (2x+3)^2 &= 0 \\ x &= -\frac{3}{2} \end{aligned}$$

4. $x^2 - 6x = 16$

$$\begin{aligned} x^2 - 6x - 16 &= 0 \\ (x-8)(x+2) &= 0 \\ x = 8 \quad &\text{or} \quad x = -2 \end{aligned}$$

Practice Problems - Solutions Part 3

Detailed Solutions

5. $5x^2 + 7x = -2$

$$5x^2 + 7x + 2 = 0$$
$$(5x + 2)(x + 1) = 0$$
$$x = -\frac{2}{5} \quad \text{or} \quad x = -1$$

6. $2x^2 - 9x = 5$

$$2x^2 - 9x - 5 = 0$$
$$(2x + 1)(x - 5) = 0$$
$$x = -\frac{1}{2} \quad \text{or} \quad x = 5$$

Summary

Key Concepts

- Understanding what it means to solve an equation
- Solving trinomials by factoring
- Steps for solving quadratic equations
- Word problems and applications
- Practice with various types of problems

Challenge Problems

Solve Each of the Following

1. The difference of two numbers is 6. The sum of their squares is 90. Find the numbers.
2. A rectangular garden is 10 meters long and 7 meters wide. A path of uniform width is to be built around the garden. If the area of the path is 54 square meters, find the width of the path.
3. A rectangular field has an area of 120 square meters. Its length is 8 meters more than its width. Find the dimensions of the field.

Challenge Problem 1 - Solution Part 1

Detailed Solution (Part 1)

The difference of two numbers is 6. The sum of their squares is 90. Find the numbers.

Solution:

1. Let the two numbers be x and y.
2. From the first condition: $x - y = 6 \Rightarrow y = x - 6$
3. From the second condition: $x^2 + y^2 = 90$
4. Substitute $y = x - 6$ into the second equation:

$$\begin{aligned} x^2 + (x-6)^2 &= 90 \\ x^2 + (x^2 - 12x + 36) &= 90 \\ 2x^2 - 12x + 36 - 90 &= 0 \\ 2x^2 - 12x - 54 &= 0 \end{aligned}$$

Challenge Problem 1 - Solution Part 2

Detailed Solution (Part 2)

The difference of two numbers is 6. The sum of their squares is 90. Find the numbers.
Solution (Cont.):

5. Simplify and factor the quadratic equation:

$$x^2 - 6x - 27 = 0$$
$$(x - 9)(x + 3) = 0$$

6. Solve for x:

$$x - 9 = 0 \quad \text{or} \quad x + 3 = 0$$
$$x = 9 \quad \text{or} \quad x = -3$$

7. Find the corresponding values for y:
   - If $x = 9$, then $y = 9 - 6 = 3$.
   - If $x = -3$, then $y = -3 - 6 = -9$.

Challenge Problem 2

Problem

A rectangular garden is 10 meters long and 7 meters wide. A path of uniform width is to be built around the garden. If the area of the path is 54 square meters, find the width of the path.

Challenge Problem 2 - Solution Part 1

Detailed Solution (Part 1)

A rectangular garden is 10 meters long and 7 meters wide. A path of uniform width is to be built around the garden. If the area of the path is 54 square meters, find the width of the path.

Solution:

1. Let w be the uniform width of the path.
2. Original garden dimensions: Length $= 10$ m, Width $= 7$ m.
3. Original garden area $= 10 \times 7 = 70$ square meters.
4. When a path of width w is built around the garden, the new dimensions of the garden including the path will be:
   - New Length $= 10 + 2w$ (add w to each side)
   - New Width $= 7 + 2w$ (add w to each side)

Challenge Problem 2 - Solution Part 2

Detailed Solution (Part 2)

A rectangular garden is 10 meters long and 7 meters wide. A path of uniform width is to be built around the garden. If the area of the path is 54 square meters, find the width of the path.

Solution (Cont.):

5. The total area (garden + path) is $(10 + 2w)(7 + 2w)$ square meters.
6. The area of the path is the total area minus the original garden area.
7. Area of path $= (10 + 2w)(7 + 2w) - 70$
8. We are given that the area of the path is 54 square meters.

$$\begin{aligned} (10 + 2w)(7 + 2w) - 70 &= 54 \\ 70 + 20w + 14w + 4w^2 - 70 &= 54 \\ 4w^2 + 34w &= 54 \end{aligned}$$

Challenge Problem 2 - Solution Part 3

Detailed Solution (Part 3)

A rectangular garden is 10 meters long and 7 meters wide. A path of uniform width is to be built around the garden. If the area of the path is 54 square meters, find the width of the path.

Solution (Cont.):

9. Rearrange the equation into standard quadratic form ($Aw^2 + Bw + C = 0$):

$$4w^2 + 34w - 54 = 0$$

10. Divide the entire equation by 2 to simplify (optional, but good practice):

$$2w^2 + 17w - 27 = 0$$

Challenge Problem 2 - Solution Part 4

Detailed Solution (Part 4)

A rectangular garden is 10 meters long and 7 meters wide. A path of uniform width is to be built around the garden. If the area of the path is 54 square meters, find the width of the path.

Solution (Cont.):

11. Use the quadratic formula to solve for w:

$$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$w = \frac{-17 \pm \sqrt{17^2 - 4(2)(-27)}}{2(2)}$$

$$w = \frac{-17 \pm \sqrt{289 + 216}}{4}$$

$$w = \frac{-17 \pm \sqrt{505}}{4}$$

Challenge Problem 2 - Solution Part 5

Detailed Solution (Part 5)

A rectangular garden is 10 meters long and 7 meters wide. A path of uniform width is to be built around the garden. If the area of the path is 54 square meters, find the width of the path.

Solution (Cont.):

12. Calculate the two possible values for w:

$$w_1 = \frac{-17 + \sqrt{505}}{4} \approx \frac{-17 + 22.47}{4} \approx \frac{5.47}{4} \approx 1.3675$$

$$w_2 = \frac{-17 - \sqrt{505}}{4} \approx \frac{-17 - 22.47}{4} \approx \frac{-39.47}{4} \approx -9.8675$$

13. Since the width cannot be negative, we discard $w_2 \approx -9.8675$.

Answer: The width of the path is approximately 1.37 meters.

Challenge Problem 3

Problem

A rectangular field has an area of 120 square meters. Its length is 8 meters more than its width. Find the dimensions of the field.

Challenge Problem 3 - Solution Part 1

Detailed Solution (Part 1)

A rectangular field has an area of 120 square meters. Its length is 8 meters more than its width. Find the dimensions of the field.

Solution:

1. Let the width of the field be x meters.
2. The length of the field is $x + 8$ meters.
3. The area of the field is given by Length $\times$ Width, so $x(x + 8) = 120$.
4. Expand and rearrange the equation into standard quadratic form:

$$x^2 + 8x = 120$$
$$x^2 + 8x - 120 = 0$$

5. Identify the coefficients for the quadratic formula: $a = 1$, $b = 8$, $c = -120$.

Challenge Problem 3 - Solution Part 2

Detailed Solution (Part 2)

A rectangular field has an area of 120 square meters. Its length is 8 meters more than its width. Find the dimensions of the field.

Solution (Cont.):

6. Use the quadratic formula to solve for x:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-8 \pm \sqrt{8^2 - 4(1)(-120)}}{2(1)}$$
$$x = \frac{-8 \pm \sqrt{64 + 480}}{2}$$
$$x = \frac{-8 \pm \sqrt{544}}{2}$$

7. Simplify the square root: $\sqrt{544} = \sqrt{16 \times 34} = 4\sqrt{34}$.