# Pre-Calculus 11
## 6.1 Basics with Absolute Values and Solving Equations with ABS

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# What is an Absolute Value?

> **Definition**
>
> The Absolute value notation is defined as the distance of any value from zero.
> - The Absolute value of a positive number stays positive
> - The ABS of a negative number becomes positive
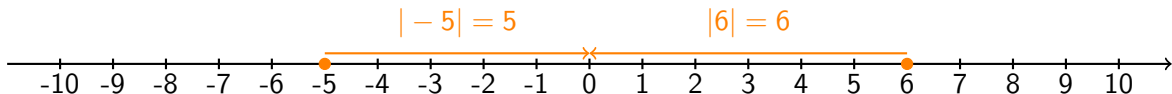
**Examples:**
- $|-19| = 19$
- $|20| = 20$
- $|-81| = 81$
- $|-32| = 32$

# Absolute Value on a Number Line

> **Visualizing Absolute Value**
>
> The absolute value notation is defined as the distance of any value from zero.

$$|-5| = 5 \qquad\qquad |6| = 6$$



- The absolute value of a positive number stays positive, but the ABS of a negative number becomes positive.

# Definition of an Absolute Value

## Two Definitions

1. $|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$

2. $|x| = \sqrt{x^2}$

# Evaluating ABS Expressions

## Practice

Evaluate:

- $|25 - 17| = 8$
- $|42 - 12| = 30$
- $|12 - 317| = 305$
- $|432 - 45| = 387$
- $|318 - 24| = 294$

# Comparing Absolute Values

## Which is bigger?

Compare:

- $|132|$ vs $|178|$
- $|13|$ vs $|48|$
- $|23|$ vs $|-23|$
- $|13|$ vs $|-13|$

# Ordering Numbers

## Example 2

Order from least to greatest:

- $|-7.5|, |-8.2|, |-3.4|, |-14.6|, |-4.3|, |-6.7|, |-12.9|$
- $|-5.6|, |-2.1|, |-13.4|, |-24.5|, |-12.3|, |-36.8|, |-25.9|$

# Solving Equations with ABS Value

## Key Points

- For $|x| = 10$, $x$ can be 10 or $-10$
- For $|x - 5| = 3$, $x - 5$ can be 3 or $-3$
- Always check for extraneous roots

## Solve for $x$

Solve for $x$, indicate if there are any extraneous roots:

1. $|x + 5| = 12$
2. $|2x - 4| = 9$
3. $|3x - 7| = 4$
4. $|x^2 - 9| = 9$
5. $|2x^2 - 12| = 4$

# Multiple Choice Questions

## Practice Questions

1. If $|x + 3| = 12$, what are the possible value(s) of $x$?

   - $x = 9$
   - $x = -9$
   - $x = 15$
   - $x = -15$
   - $x = -15$ or $x = 9$
   - $x = -9$ or $x = 15$

2. If $|x + 5|^2 = 289$, what is the value of $x$?

   - $x = 12$
   - $x = -12$
   - $x = 22$
   - $x = -22$
   - $x = -22$ or $x = 12$
   - $x = -12$ or $x = 22$

# Challenge Practice

## Challenge Problems

Q1. Solve for $x$: $|2x - 7| = |x + 4|$

Q2. Solve for $x$: $|x^2 - 5x + 6| = 2$

# Challenge Q1 Solution

## Solution to Q1

$|2x - 7| = |x + 4|$
**Case 1:** $2x - 7 = x + 4$

$$2x - 7 = x + 4 \quad 2x - x = 4 + 7 \quad x = 11$$

**Case 2:** $2x - 7 = -(x + 4)$

$$2x - 7 = -x - 4 \quad 2x + x = -4 + 7 \quad 3x = 3 \quad x = 1$$

**Check for extraneous roots:**

- $x = 11$: $|2 \times 11 - 7| = |22 - 7| = |15| = 15$, $|11 + 4| = |15| = 15$ (valid)
- $x = 1$: $|2 \times 1 - 7| = |2 - 7| = |-5| = 5$, $|1 + 4| = |5| = 5$ (valid)

**Final Answer:** $x = 1$ or $x = 11$

# Challenge Q2 Solution

## Solution to Q2

$|x^2 - 5x + 6| = 2$
Let $y = x^2 - 5x + 6$
**Case 1:** $x^2 - 5x + 6 = 2$

$$x^2 - 5x + 6 = 2x^2 - 5x + 4 = 0(x - 4)(x - 1) = 0x = 4, \ 1$$

**Case 2:** $x^2 - 5x + 6 = -2$

$$x^2 - 5x + 6 = -2x^2 - 5x + 8 = 0$$

Use quadratic formula:

$$x = \frac{5 \pm \sqrt{25 - 32}}{2} = \frac{5 \pm \sqrt{-7}}{2}$$

No real solutions for this case.
**Final Answer:** $x = 1$ or $x = 4$