Chapter 4.2: Multiplying, Dividing, and Rationalizing Radicals

Pre-Calculus 11 - Lesson 2

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Overview: Multiplying, Dividing, and Rationalizing Radicals

Key Concepts

- Multiplying radicals: outside $\times$ outside, inside $\times$ inside; same radicals multiply to whole numbers
- Dividing radicals: divide outside and inside separately, never leave a radical in the denominator
- Rationalizing radicals: multiply numerator and denominator by the radical (monomial) or conjugate (binomial) to eliminate radicals in the denominator

I) Multiplying Radicals

Rules

- $\sqrt{a} \times \sqrt{a} = a$
- $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$
- $(a\sqrt{b}) \times (c\sqrt{d}) = ac\sqrt{bd}$
- Numbers outside stay outside, numbers inside stay inside

Multiplying Radicals - Practice

Practice Problems

- $2\sqrt{3} \times 6\sqrt{2}$
- $\sqrt{5} \times \sqrt{20}$
- $3\sqrt{7} \times 8\sqrt{5}$
- $\sqrt[3]{4} \times \sqrt[3]{6}$
- $2\sqrt{10} \times 7\sqrt{90}$
- $\sqrt{8x} \times \sqrt{2x^3}$

Multiplying Radicals - Solutions

Solutions

- $2\sqrt{3} \times 6\sqrt{2} = 12\sqrt{6}$
- $\sqrt{5} \times \sqrt{20} = \sqrt{100} = 10$
- $3\sqrt{7} \times 8\sqrt{5} = 24\sqrt{35}$
- $\sqrt[3]{4} \times \sqrt[3]{6} = \sqrt[3]{24}$
- $2\sqrt{10} \times 7\sqrt{90} = 14\sqrt{900} = 14 \times 30 = 420$
- $\sqrt{8x} \times \sqrt{2x^3} = \sqrt{16x^4} = 4x^2$

II) Dividing Radicals

Rules

- $\frac{a\sqrt{b}}{c\sqrt{d}} = \frac{a}{c} \times \sqrt{\frac{b}{d}}$
- Simplify outside and inside separately
- Never leave a radical in the denominator

Dividing Radicals - Practice

Practice Problems

- $\frac{6\sqrt{50}}{3\sqrt{2}}$
- $\frac{8\sqrt{18}}{4\sqrt{3}}$
- $\frac{5\sqrt{12x}}{10\sqrt{3x}}$
- $\frac{\sqrt[3]{54}}{\sqrt[3]{2}}$
- $\frac{7\sqrt{45}}{14\sqrt{5}}$
- $\frac{\sqrt{32y^3}}{4\sqrt{2y}}$

Dividing Radicals - Solutions

Solutions

- $\frac{6\sqrt{50}}{3\sqrt{2}} = 2\sqrt{25} = 10$
- $\frac{8\sqrt{18}}{4\sqrt{3}} = 2\sqrt{6}$
- $\frac{5\sqrt{12x}}{10\sqrt{3x}} = \frac{1}{2}\sqrt{4} = 1$
- $\frac{\sqrt[3]{54}}{\sqrt[3]{2}} = \sqrt[3]{27} = 3$
- $\frac{7\sqrt{45}}{14\sqrt{5}} = \frac{1}{2}\sqrt{9} = \frac{3}{2}$
- $\frac{\sqrt{32y^3}}{4\sqrt{2y}} = \frac{1}{4}\sqrt{16y^2} = \frac{1}{4} \times 4y = y$

III) Rationalizing Radicals

Rules

- If denominator is a monomial: multiply top and bottom by the radical in the denominator
- If denominator is a binomial: multiply top and bottom by the conjugate of the denominator
- This eliminates the radical in the denominator

Rationalizing Radicals (Monomial) - Practice

Practice Problems

a) $\frac{1}{\sqrt{3}}$

b) $\frac{5}{2\sqrt{5}}$

c) $\frac{7}{\sqrt{2x}}$

d) $\frac{3}{\sqrt[3]{4}}$

e) $\frac{2x}{\sqrt{8x}}$

f) $\frac{4}{2\sqrt{y}}$

Rationalizing Radicals (Monomial) - Solutions

Solutions

a) $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

b) $\frac{5}{2\sqrt{5}} = \frac{5\sqrt{5}}{2\times 5} = \frac{\sqrt{5}}{2}$

c) $\frac{7}{\sqrt{2x}} = \frac{7\sqrt{2x}}{2x}$

d) $\frac{3}{\sqrt[3]{4}} = \frac{3\sqrt[3]{16}}{4}$

e) $\frac{2x}{\sqrt{8x}} = \frac{2x\sqrt{8x}}{8x} = \frac{\sqrt{8x}}{4}$

f) $\frac{4}{2\sqrt{y}} = \frac{2}{\sqrt{y}} = \frac{2\sqrt{y}}{y}$

Rationalizing Radicals (Binomial) - Practice

Practice Problems

- $\frac{1}{1+\sqrt{2}}$
- $\frac{2}{1-\sqrt{7}}$
- $\frac{3}{2-\sqrt{5}}$
- $\frac{4}{2+\sqrt{y}}$
- $\frac{5}{\sqrt{3}-1}$
- $\frac{6}{\sqrt{5}+\sqrt{2}}$

Rationalizing Radicals (Binomial) - Solutions

Solutions

- $\frac{1}{1+\sqrt{2}} = \frac{1\times(1-\sqrt{2})}{(1+\sqrt{2})(1-\sqrt{2})} = \frac{1-\sqrt{2}}{1-2} = \frac{1-\sqrt{2}}{-1} = \sqrt{2}-1$
- $\frac{3}{2-\sqrt{5}} = \frac{3(2+\sqrt{5})}{(2-\sqrt{5})(2+\sqrt{5})} = \frac{6+3\sqrt{5}}{4-5} = -6-3\sqrt{5}$
- $\frac{5}{\sqrt{3}-1} = \frac{5(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{5\sqrt{3}+5}{3-1} = \frac{5\sqrt{3}+5}{2}$
- $\frac{2}{1-\sqrt{7}} = \frac{2(1+\sqrt{7})}{(1-\sqrt{7})(1+\sqrt{7})} = \frac{2+2\sqrt{7}}{1-7} = -\frac{2+2\sqrt{7}}{6}$
- $\frac{4}{2+\sqrt{y}} = \frac{4(2-\sqrt{y})}{(2+\sqrt{y})(2-\sqrt{y})} = \frac{8-4\sqrt{y}}{4-y}$
- $\frac{6}{\sqrt{5}+\sqrt{2}} = \frac{6(\sqrt{5}-\sqrt{2})}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})} = \frac{6\sqrt{5}-6\sqrt{2}}{5-2} = 2\sqrt{5}-2\sqrt{2}$

Rationalize:

Practice Problems

a. $\dfrac{2+\sqrt[3]{5}}{\sqrt[3]{16}}$

b. $\dfrac{7}{\sqrt[3]{9}+\sqrt[3]{12}}$

Rationalize - Solutions

Solutions

- $\dfrac{2+\sqrt[3]{5}}{\sqrt[3]{16}} = \dfrac{2+\sqrt[3]{5}}{2} = 1 + \dfrac{1}{2}\sqrt[3]{5}$
- $\dfrac{7}{\sqrt[3]{9}+\sqrt[3]{12}}$ (Multiply numerator and denominator by $\sqrt[3]{9^2} - \sqrt[3]{9}\sqrt[3]{12} + \sqrt[3]{12^2}$ to rationalize, or leave as a challenge for students)

FOIL: Expand the Following

Practice Problems

- $\left(2+\sqrt{3}\right)\left(5-\sqrt{3}\right)$
- $\left(6+\sqrt{7}\right)\left(2-\sqrt{7}\right)$
- $\left(4+2\sqrt{2}\right)\left(4-2\sqrt{2}\right)$
- $\left(8-\sqrt{5}\right)\left(8+\sqrt{5}\right)$

FOIL: Expand the Following - Solutions

Solutions

- $\left(2+\sqrt{3}\right)\left(5-\sqrt{3}\right) = 2\times 5 - 2\times\sqrt{3} + 5\times\sqrt{3} - (\sqrt{3})^2 = 10 + 3\sqrt{3} - 3 = 7 + 3\sqrt{3}$
- $\left(4+2\sqrt{2}\right)\left(4-2\sqrt{2}\right) = 16 - 8\sqrt{2} + 8\sqrt{2} - 8 = 16 - 8 = 8$
- $\left(6+\sqrt{7}\right)\left(2-\sqrt{7}\right) = 12 - 6\sqrt{7} + 2\sqrt{7} - 7 = 12 - 7 - 4\sqrt{7} = 5 - 4\sqrt{7}$
- $\left(8-\sqrt{5}\right)\left(8+\sqrt{5}\right) = 64 + 8\sqrt{5} - 8\sqrt{5} - 5 = 64 - 5 = 59$