Chapter 4.4: Chapter 4 Review

Pre-Calculus 11 - Review

Created by Yi-Chen Lin

June 16, 2025

Overview: Chapter 4 Review

Key Concepts

- Simplifying radicals and fractional exponents
- Multiplying, dividing, and rationalizing radicals
- Solving radical equations and checking for extraneous roots
- FOIL and expanding expressions with radicals

Summary of Key Concepts

Summary

- $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$
- $\frac{a\sqrt{b}}{c\sqrt{d}} = \frac{a}{c} \times \sqrt{\frac{b}{d}}$
- Rationalize denominators by multiplying by the radical or conjugate
- To solve $\sqrt{ax+b} = c$, isolate, square both sides, solve, and check
- Always check for extraneous roots in radical equations
- FOIL: $(a+\sqrt{b})(a-\sqrt{b}) = a^2 - b$

Review Practice Problems

Practice Problems

1. Simplify: $\sqrt{50} + 2\sqrt{8} - \sqrt{18}$
2. Multiply: $3\sqrt{2} \times 4\sqrt{3}$
3. Divide: $\frac{6\sqrt{27}}{2\sqrt{3}}$
4. Rationalize: $\frac{5}{\sqrt{2}}$

1. Solve: $\sqrt{2x+3} = 5$
2. Solve: $\sqrt{x-1} + 2 = 7$
3. Expand: $(x + \sqrt{5})(x - \sqrt{5})$
4. Identify extraneous roots: $\sqrt{x+4} = x - 2$

Review Practice Solutions (1/2)

Solutions

1. $\sqrt{50} + 2\sqrt{8} - \sqrt{18} = 5\sqrt{2} + 4\sqrt{2} - 3\sqrt{2} = 6\sqrt{2}$
2. $3\sqrt{2} \times 4\sqrt{3} = 12\sqrt{6}$
3. $\frac{6\sqrt{27}}{2\sqrt{3}} = \frac{6\times 3\sqrt{3}}{2\sqrt{3}} = \frac{18\sqrt{3}}{2\sqrt{3}} = 9$
4. $\frac{5}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$

Solutions

1. Solve $\sqrt{2x+3} = 5$

$$\begin{aligned} \sqrt{2x+3} &= 5 \\ 2x+3 &= 25 \\ 2x &= 22 \\ x &= 11 \end{aligned}$$

2. Solve $\sqrt{x-1} + 2 = 7$

$$\begin{aligned} \sqrt{x-1} &= 5 \\ x-1 &= 25 \\ x &= 26 \end{aligned}$$

3. Expand $(x+\sqrt{5})(x-\sqrt{5}) = x^2 - 5$

Solution: $\sqrt{x+4}$

Solving:

$$\sqrt{x+4} = x - 2$$
$$x + 4 = (x-2)^2$$
$$x + 4 = x^2 - 4x + 4$$
$$0 = x^2 - 5x$$
$$x(x-5) = 0$$
$$x = 0 \text{ or } x = 5$$

Checking:
For $x = 0$:

$$\sqrt{0+4} = 2$$
$$0 - 2 = -2$$

$2 \neq -2$ (not valid)
For $x = 5$:

$$\sqrt{5+4} = 3$$
$$5 - 2 = 3$$

$3 = 3$ (valid)
Conclusion: Only $x = 5$ is a valid solution.

More Practice Problems

1. Simplify: $2\sqrt{18} - 3\sqrt{8} + \sqrt{32}$
2. Multiply: $5\sqrt{7} \times 2\sqrt{14}$
3. Divide: $\frac{8\sqrt{45}}{4\sqrt{5}}$
4. Rationalize: $\frac{3}{2\sqrt{3}}$
5. Solve: $\sqrt{3x-2} = 4$
6. Solve: $\sqrt{5x+1} + 1 = 6$
7. Expand: $(2+\sqrt{3})(2-\sqrt{3})$
8. Identify extraneous roots: $\sqrt{2x+5} = x - 1$
9. Word Problem: The area of a square is 50 cm^2. What is the length of one side in simplest radical form?
10. Conceptual: Explain why $\sqrt{x^2} = |x|$ for all real x.

More Review Practice Solutions (1/5)

Solutions

1. $2\sqrt{18} - 3\sqrt{8} + \sqrt{32}$

$$\begin{aligned} &= 2 \times 3\sqrt{2} - 3 \times 2\sqrt{2} + 4\sqrt{2} \\ &= 6\sqrt{2} - 6\sqrt{2} + 4\sqrt{2} \\ &= 4\sqrt{2} \end{aligned}$$

2. $5\sqrt{7} \times 2\sqrt{14}$

$$\begin{aligned} &= 10\sqrt{98} \\ &= 10 \times 7\sqrt{2} \\ &= 70\sqrt{2} \end{aligned}$$

More Review Practice Solutions (2/5)

Solutions

3. $\frac{8\sqrt{45}}{4\sqrt{5}}$

$$= \frac{8 \times 3\sqrt{5}}{4\sqrt{5}}$$
$$= \frac{24\sqrt{5}}{4\sqrt{5}}$$
$$= 6$$

4. $\frac{3}{2\sqrt{3}}$

$$= \frac{3}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$
$$= \frac{3\sqrt{3}}{2 \times 3}$$
$$= \frac{\sqrt{3}}{}$$

More Review Practice Solutions (3/5)

Solutions

5. Solve $\sqrt{3x-2}=4$

$$\sqrt{3x-2}=4$$
$$3x-2=16$$
$$3x=18$$
$$x=6$$

6. Solve $\sqrt{5x+1}+1=6$

$$\sqrt{5x+1}=5$$
$$5x+1=25$$
$$5x=24$$
$$x=\frac{24}{5}$$

More Review Practice Solutions (4/5)

Solutions

7. Expand $(2+\sqrt{3})(2-\sqrt{3})$

$$\begin{aligned} &= 4 - 2\sqrt{3} + 2\sqrt{3} - 3 \\ &= 4 - 3 \\ &= 1 \end{aligned}$$

More Review Practice Solutions (Checking Extraneous Roots)

Solution: $\sqrt{2x+5}$

Solving:

$$\sqrt{2x+5} = x - 1$$
$$2x + 5 = (x-1)^2$$
$$2x + 5 = x^2 - 2x + 1$$
$$0 = x^2 - 4x - 4$$
$$x^2 - 4x - 4 = 0$$

Quadratic formula:

$$x = \frac{4 \pm \sqrt{16+16}}{2}$$
$$x = \frac{4 \pm \sqrt{32}}{2}$$
$$x = \frac{4 \pm 4\sqrt{2}}{2}$$

Checking:

For $x = 2 + 2\sqrt{2}$:

$$x - 1 = (2 + 2\sqrt{2}) - 1 = 1 + 2\sqrt{2}$$
$$(1 + 2\sqrt{2})^2 = 1 + 4\sqrt{2} + 8 = 9 + 4\sqrt{2}$$
$$\text{So } \sqrt{9 + 4\sqrt{2}} = 1 + 2\sqrt{2} \text{ (valid)}$$

For $x = 2 - 2\sqrt{2}$:

$$x - 1 = (2 - 2\sqrt{2}) - 1 = 1 - 2\sqrt{2}$$
$$(1 - 2\sqrt{2})^2 = 1 - 4\sqrt{2} + 8 = 9 - 4\sqrt{2}$$
$$\text{So } \sqrt{9 - 4\sqrt{2}} = 1 - 2\sqrt{2} \text{ (valid)}$$

Conclusion: Both $x = 2 + 2\sqrt{2}$ and $x = 2 - 2\sqrt{2}$ are valid solutions (no extraneous roots).

More Review Practice Solutions (5/5)

Solutions

9. Word Problem: The area of a square is 50 cm^2. What is the length of one side in simplest radical form?

$$s^2 = 50$$
$$s = \sqrt{50} = 5\sqrt{2}\text{ cm}$$

10. Conceptual: Explain why $\sqrt{x^2} = |x|$ for all real x.

The square root of x^2 is always non-negative, so $\sqrt{x^2}$ gives the absolute value of x.