Pre-Calculus 11

Prerequisite Skills Review - Lesson 1.2

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Combining Like-Terms

Definition

- You can only add or subtract two terms if they are like-terms.
- **Like-terms:** Two algebraic terms that have the same variables and the exponents of the corresponding variables are the same.
- If they are not like-terms, then you cannot add or subtract them.

Combining Like-Terms - Practice

Practice Problems

Indicate which of the following terms are like-terms. If they are like-terms, combine them:

1. $3x$ and x^3
2. x^2y^2 and $-5x^2y^3$
3. $10a^2 - 4a$
4. $5y + 12y^2 + 10y - 3y^2$
5. $4a^2 - 3ab + 6b^2 - 8a^2 + 15ab + 2b^2$

Combining Like-Terms - Solutions Part 1

Detailed Solutions

1. $3x$ and x^3 **Solution:** Not Like-terms because the exponents for "x" are different.
2. x^2y^2 and $-5x^2y^3$ **Solution:** Not Like-terms because they don't have the same variables (exponents for y are different).
3. $10a^2 - 4a$ **Solution:** Not Like-terms (cannot be combined).

Detailed Solutions

4. $5y + 12y^2 + 10y - 3y^2$ **Solution:**
   - Identify like-terms: $5y$ and $10y$ are like-terms, $12y^2$ and $-3y^2$ are also like-terms.
   - Combine: $(5y + 10y) + (12y^2 - 3y^2) = 15y + 9y^2$
5. $4a^2 - 3ab + 6b^2 - 8a^2 + 15ab + 2b^2$ **Solution:**
   - Identify like-terms: $4a^2$ and $-8a^2$; $-3ab$ and $15ab$; $6b^2$ and $2b^2$.
   - Combine: $(4a^2 - 8a^2) + (-3ab + 15ab) + (6b^2 + 2b^2) = -4a^2 + 12ab + 8b^2$

FOIL/Expansion - Multiplying Binomials

Multiplying Binomials

- When multiplying two binomials, you can visualize it as finding the area of a rectangle.
- **Example:** $(x + 4)(x + 5)$

	x	5
x	x^2	$5x$
4	$4x$	20

- The area is $x^2 + 5x + 4x + 20 = x^2 + 9x + 20$.

FOIL/Expansion - Another Method: FOIL

Another Method: FOIL

- **F**irst: Multiply the first terms of each binomial.
- **O**utside: Multiply the outer terms.
- **I**nside: Multiply the inner terms.
- **L**ast: Multiply the last terms of each binomial.
- **Example:** $(x+4)(x+5) = \underbrace{x \cdot x}_{F} + \underbrace{x \cdot 5}_{O} + \underbrace{4 \cdot x}_{I} + \underbrace{4 \cdot 5}_{L} = x^2 + 5x + 4x + 20 = x^2 + 9x + 20.$

Expand Binomials - Practice

Practice Problems

Expand the Binomials:

1. $(x-5)(x-2)$
2. $(2x-3)(5x-8)$
3. $(5x+4)(7x-6)-(8x-2)(x+3)$

Expand Binomials - Solutions Part 1

Detailed Solutions

1. $(x-5)(x-2)$ **Solution:**

$$\begin{aligned}(x-5)(x-2) &= x(x) + x(-2) + (-5)(x) + (-5)(-2)\\ &= x^2 - 2x - 5x + 10\\ &= x^2 - 7x + 10\end{aligned}$$

2. $(2x-3)(5x-8)$ **Solution:**

$$\begin{aligned}(2x-3)(5x-8) &= 2x(5x) + 2x(-8) + (-3)(5x) + (-3)(-8)\\ &= 10x^2 - 16x - 15x + 24\\ &= 10x^2 - 31x + 24\end{aligned}$$

Detailed Solutions

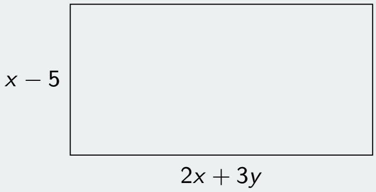
3. $(5x+4)(7x-6)-(8x-2)(x+3)$ **Solution:**

$$
\begin{aligned}
&(5x+4)(7x-6)-(8x-2)(x+3)\\
&=(35x^2-30x+28x-24)-(8x^2+24x-2x-6)\\
&=(35x^2-2x-24)-(8x^2+22x-6)\\
&=35x^2-2x-24-8x^2-22x+6\\
&=(35x^2-8x^2)+(-2x-22x)+(-24+6)\\
&=27x^2-24x-18
\end{aligned}
$$

Area and Perimeter - Practice

Practice Problem

Given the dimensions of the solid, find an algebraic expression for the area and PERIMETER!!:



- Area $=$?
- Perimeter $=$?

Area and Perimeter - Solutions

Detailed Solutions

- **Area:**

$$\begin{aligned}\text{Area} &= (x-5)(2x+3y)\\ &= x(2x)+x(3y)+(-5)(2x)+(-5)(3y)\\ &= 2x^2+3xy-10x-15y\end{aligned}$$

- **Perimeter:**

$$\begin{aligned}\text{Perimeter} &= 2(x-5)+2(2x+3y)\\ &= 2x-10+4x+6y\\ &= (2x+4x)+6y-10\\ &= 6x+6y-10\end{aligned}$$

Basic Factoring: GCF

Introduction

- Factoring is the opposite of expanding.
- When factoring, you are dividing out the greatest common factor (GCF) between several terms.

Basic Factoring: GCF - Practice 1

Practice Problems

First indicate the GCF and then Factor out the GCF for each of the following:

1. $20x^3 + 8xy$
2. $21x^3y^2 + 35xy + 42y^4$

Basic Factoring: GCF - Solutions 1

Detailed Solutions

1. $20x^3 + 8xy$ **Solution:**
   - Both terms are multiples of 4.
   - Both terms have one x variable.
   - GCF $= 4x$
   - Factored: $4x(5x^2 + 2y)$
2. $21x^3y^2 + 35xy + 42y^4$ **Solution:**
   - All three terms are multiples of 7.
   - All three terms have a "y" variable.
   - GCF $= 7y$
   - Factored: $7y(3x^3y + 5x + 6y^3)$

Practice Problems

Find the Greatest common factor and then factor out the GCF for each of the following:

1. $50x^4y^3 + 30x^6y^4$
2. $90a^4b^7 + 25ab^6 + 65a^2b^5$
3. $30(y - x) + 40x(y - x)$
4. $28(x + y)^4 - 63(x + y)^3$

Basic Factoring: GCF - Solutions 2 Part 1

Detailed Solutions

1. $50x^4y^3 + 30x^6y^4$ **Solution:**
   - GCF $= 10x^4y^3$
   - Factored: $10x^4y^3(5 + 3x^2y)$
2. $90a^4b^7 + 25ab^6 + 65a^2b^5$ **Solution:**
   - GCF $= 5ab^5$
   - Factored: $5ab^5(18a^3b^2 + 5b + 13a)$

Basic Factoring: GCF - Solutions 2 Part 2

Detailed Solutions

3. $30(y-x)+40x(y-x)$ **Solution:**
   - $\text{GCF} = (y-x)$
   - Factored: $(y-x)(30+40x) = 10(y-x)(3+4x)$
4. $28(x+y)^4 - 63(x+y)^3$ **Solution:**
   - $\text{GCF} = 7(x+y)^3$
   - Factored: $7(x+y)^3(4(x+y)-9) = 7(x+y)^3(4x+4y-9)$

Factoring Difference of Squares: Conjugates

Conjugates

Two binomials are conjugates if they have the same terms but a different sign in between them:

- Example: $x + 5 \to x - 5$
- Example: $7x - 10 \to 7x + 10$
- Example: $9a - 20 \to 9a + 20$

Difference of Squares: Multiplication Property

What happens when you multiply a binomial with its conjugate?

1. $(x+5)(x-5)$ **Solution:**

$$\begin{aligned}(x+5)(x-5) &= x^2 - 5x + 5x - 25 \\ &= x^2 - 25\end{aligned}$$

2. $(7x-10)(7x+10)$ **Solution:**

$$\begin{aligned}(7x-10)(7x+10) &= 49x^2 + 70x - 70x - 100 \\ &= 49x^2 - 100\end{aligned}$$

- The middle two terms will always cancel each other out.
- The first and last terms are always perfect squares.
- The middle sign is always a subtraction.

Indicate the Missing Terms

1. $(x-7)(x+7) = x^2 - \_\_\_\_\_\_$
2. $(15a-9b)(15a+9b) = 225a^2 - \_\_\_\_\_\_$
3. $144x^2y^2 - 169 = (\_\_\_\_\_\_ - \_\_\_\_\_\_)(\_\_\_\_\_\_ + \_\_\_\_\_\_)$
4. $49p^2q^2 - 25q^2 = (\_\_\_\_\_\_ - \_\_\_\_\_\_)(\_\_\_\_\_\_ + \_\_\_\_\_\_)$
5. $9y^4 - 100 = (\_\_\_\_\_\_ - \_\_\_\_\_\_)(\_\_\_\_\_\_ + \_\_\_\_\_\_)$

Difference of Squares: Missing Terms - Solutions

Detailed Solutions

1. $(x-7)(x+7) = x^2 - \underline{49}$
2. $(15a-9b)(15a+9b) = 225a^2 - \underline{81b^2}$
3. $144x^2y^2 - 169 = (\underline{12xy} - \underline{13})(\underline{12xy} + \underline{13})$
4. $49p^2q^2 - 25q^2 = (\underline{7pq} - \underline{5q})(\underline{7pq} + \underline{5q})$
5. $9y^4 - 100 = (\underline{3y^2} - \underline{10})(\underline{3y^2} + \underline{10})$

Rule

- When you have two perfect squares subtracting each other, you can factor it to a product of two conjugate binomials: $a^2 - b^2 = (a-b)(a+b)$
- If the two perfect squares have a common factor, you can factor out the common factor first and then separate it as a product of conjugate binomials.

Factor Completely - Practice

Practice Problems

FACTOR COMPLETELY:

1. $144x^2y^2 - 49$
2. $27a^2b^2 - 75$
3. $49p^2q^5 - 81q$
4. $16z^4 - 1$

Factor Completely - Solutions Part 1

Detailed Solutions

1. $144x^2y^2 - 49$ **Solution:**

$$\begin{aligned} 144x^2y^2 - 49 &= (12xy)^2 - (7)^2 \\ &= (12xy - 7)(12xy + 7) \end{aligned}$$

2. $27a^2b^2 - 75$ **Solution:**

$$\begin{aligned} 27a^2b^2 - 75 &= 3(9a^2b^2 - 25) \\ &= 3((3ab)^2 - (5)^2) \\ &= 3(3ab - 5)(3ab + 5) \end{aligned}$$

Detailed Solutions

3. $49p^2q^5 - 81q$ **Solution:**

$$\begin{aligned} 49p^2q^5 - 81q &= q(49p^2q^4 - 81) \\ &= q((7pq^2)^2 - (9)^2) \\ &= q(7pq^2 - 9)(7pq^2 + 9) \end{aligned}$$

4. $16z^4 - 1$ **Solution:**

$$\begin{aligned} 16z^4 - 1 &= (4z^2)^2 - (1)^2 \\ &= (4z^2 - 1)(4z^2 + 1) \\ &= ((2z)^2 - (1)^2)(4z^2 + 1) \\ &= (2z - 1)(2z + 1)(4z^2 + 1) \end{aligned}$$

Summary

Key Concepts

- Combining Like-Terms
- Expanding Polynomials (FOIL/Box Method)
- Basic Factoring (Greatest Common Factor - GCF)
- Factoring Difference of Squares