

Pre-Calculus 11

Chapter 8.1: Graphing Linear Inequalities with Two Variables

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What is a Linear Inequality in Two Variables?

Definition

A linear inequality in two variables is an inequality that can be written in the form $ax+by < c$, $ax+by \leq c$, $ax+by > c$, or $ax+by \geq c$.

- The solution is a region (half-plane) on the coordinate plane.
- The boundary is a straight line, which may be solid (included) or dotted (not included).

Examples:

- $y > 2x - 1$
- $3x + 2y \leq 6$

How to Graph a Linear Inequality

Step-by-Step Method

- 1 **Graph the boundary line:**
 - Use a **dotted line** for $<$ or $>$ (not included)
 - Use a **solid line** for \leq or \geq (included)
- 2 **Pick a test point** not on the line (often $(0, 0)$)
- 3 Substitute the test point into the inequality:
 - If it makes the inequality true, shade that side
 - If false, shade the opposite side
- 4 **The shaded region is the solution set.**

Example: Graph $3x + 2y > 6$

Step 1: Rearrange

$$3x + 2y > 6$$

$$2y > -3x + 6$$

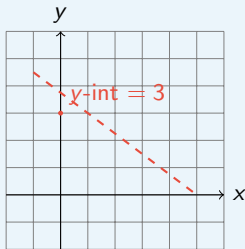
$$y > -\frac{3}{2}x + 3$$

Slope: $-\frac{3}{2}$, y-intercept: 3

Example: Graph $3x + 2y > 6$

Step 2: Graph the Line

Dotted line (since $>$, not \geq)



Example: Graph $3x + 2y > 6$

Step 3: Test Point

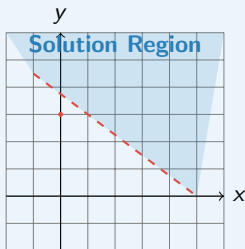
Use $(0, 0)$ as a test point:

$$3(0) + 2(0) > 6 \quad 0 > 6 \quad \text{False}$$

So, shade the side **not** containing $(0, 0)$.

Example: Graph $3x + 2y > 6$

Step 4: Shading Solution Region



Practice: Graphing Linear Inequality

Practice

Graph the solution set for $y \leq \frac{1}{2}x - 2$.

- Identify the boundary line and whether it is solid or dotted.
- Pick a test point and determine which side to shade.

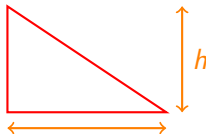
Graphing Systems of Linear Inequalities

Systems of Linear Inequalities

- If you have two or more equations in a system of inequalities:
- Graph each inequality separately
- Shade the common area of all the inequalities involved
- The solution will be the common area
- If asked to find the area of the common shaded zone, use area formulas:

Area of a Triangle:

$$A = \frac{b \times h}{2}$$



Example: Graph the System and Find the Area

System:

$$x > 2$$

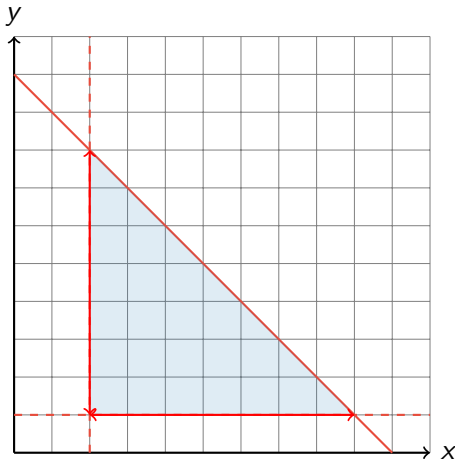
$$y > 1$$

$$x + y \leq 10$$

BASE: $= 9 - 2 = 7$

HEIGHT: $= 8 - 1 = 7$

A $= \frac{7 \times 7}{2} = 24.5 \text{ units}^2$

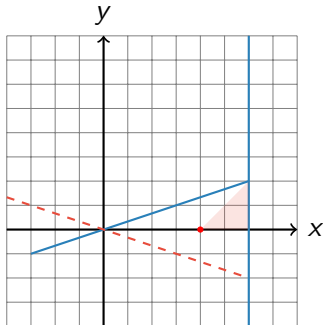


Example: Write the System of Inequalities for the Shaded Region

Pick a point in the shaded region (e.g., $(4, 0)$).
Use the point to determine the inequality sign.

Dotted line: $3y > -x$

Solid line: $x \leq 6$, $3y \leq x$



Problems Involving Linear Inequalities

Translating Words to Inequalities

- Two numbers add *up to* 100: $x + y \leq 100$
- The sum is *at least* 99: $x + y \geq 99$
- The sum is *more than* 50: $x + y > 50$
- Jack's age is less than 35: $x < 35$
- Some variables cannot be negative: $t > 0$ (time, age, quantity)

Application Example: Work Hours

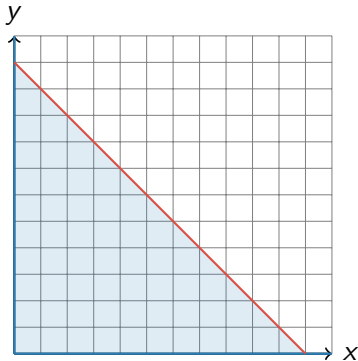
John has 2 jobs: a stock broker for BMO and a financial planner for London Life. He works up to 55 hours a week. Let x be the hours at BMO, y at London Life.

$$x + y \leq 55$$

$$x \geq 0$$

$$y \geq 0$$

Graph each equation separately. The solution is the common area.



Summary: Graphing Linear Inequalities

Key Points

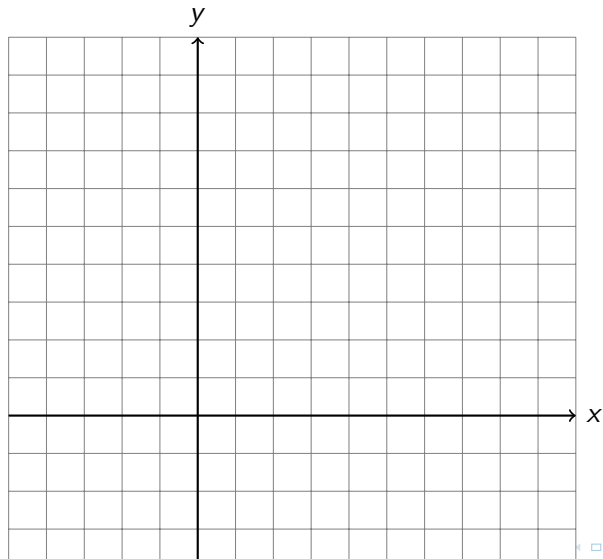
- Graph the boundary line (solid for \leq, \geq ; dotted for $<, >$)
- Pick a test point (often $(0, 0)$)
- Shade the region where the test point makes the inequality true
- The shaded area is the solution set

Practice Problem 1

Practice 1

Graph the solution set for $2x - y < 4$.

Practice 1: Blank Grid



Practice Problem 2

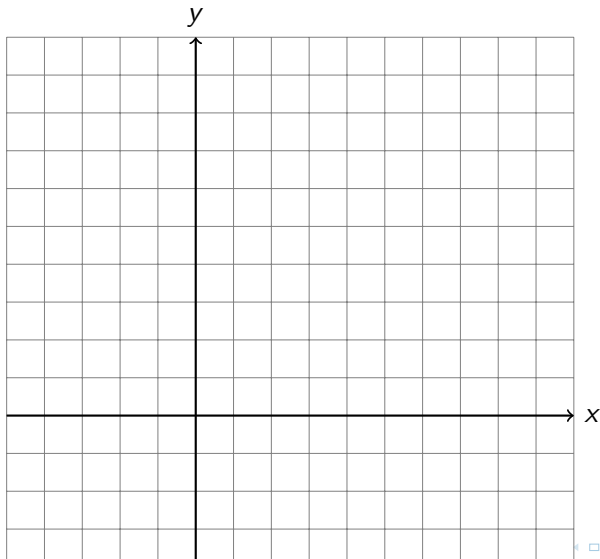
Practice 2

Graph the solution set for the system:

$$x + y \geq 2$$

$$y < 3x - 1$$

Practice 2: Blank Grid



Practice Problem 3

Practice 3

A company can produce up to 100 units of product A and B combined. At least 30 units of A and at least 20 units of B must be produced.

Write and graph the system of inequalities for the feasible region.

Practice 3: Blank Grid

