

Pre-Calculus 11

Chapter 2 Review: Factoring and Quadratic Functions

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Chapter 2 Review

Chapter Overview

- 2.1 Factoring Trinomials
- 2.2 Solving Quadratic Equations by Factoring
- 2.3 Graphing Quadratic Functions by Factoring
- 2.4 The Quadratic Formula
- 2.5 Graphing Quadratic Functions in APQ form
- 2.6 Completing the Square
- 2.7 Word Problems on Max/Min
- 2.8 Quadratic Inequalities

Section 1: Factoring Review

Practice Problems

Factor each of the following expressions:

- 1 $6x^2 + 17x + 5$ (Basic trinomial)
- 2 $4x^2 - 12x + 9$ (Perfect square)
- 3 $9x^2 - 16$ (Difference of squares)
- 4 $8x^3 - 27$ (Difference of cubes)
- 5 $x^4 - 16$ (Multiple factoring steps)
- 6 $12x^2y - 18xy^2$ (Common factor)

Section 1: Solutions (Part 1)

Detailed Solutions

① $6x^2 + 17x + 5$

$$= (3x + 1)(2x + 5)$$

② $4x^2 - 12x + 9$

$$= (2x - 3)^2$$

Section 1: Solutions (Part 2)

Detailed Solutions (Continued)

③ $9x^2 - 16$

$$= (3x + 4)(3x - 4)$$

④ $8x^3 - 27$

$$= (2x - 3)(4x^2 + 6x + 9)$$

Section 1: Solutions (Part 3)

Detailed Solutions (Continued)

5 $x^4 - 16$

$$= (x^2 + 4)(x^2 - 4)$$

$$= (x^2 + 4)(x + 2)(x - 2)$$

6 $12x^2y - 18xy^2$

$$= 6xy(2x - 3y)$$

Section 2: Solving Quadratic Equations

Practice Problems

Solve each quadratic equation using the most appropriate method:

- 1 $x^2 + 5x + 6 = 0$ (by factoring)
- 2 $2x^2 - 5x + 3 = 0$ (using quadratic formula)
- 3 $x^2 + 6x + 2 = 0$ (by completing the square)
- 4 $3x^2 - 12 = 0$ (by square root method)
- 5 $x^2 - 4x + 4 = 0$ (perfect square)
- 6 $2x^2 + 7x - 4 = 0$ (mixed methods)

Section 2: Solutions (Part 1)

Detailed Solutions

① $x^2 + 5x + 6 = 0$

$$(x + 2)(x + 3) = 0$$

$$x = -2 \text{ or } x = -3$$

② $2x^2 - 5x + 3 = 0$

$$x = \frac{5 \pm \sqrt{25 - 24}}{4}$$

$$= \frac{5 \pm 1}{4}$$

$$x = \frac{3}{2} \text{ or } x = 1$$

Section 2: Solutions (Part 2)

Detailed Solutions (Continued)

③ $x^2 + 6x + 2 = 0$

$$x^2 + 6x = -2$$

$$x^2 + 6x + 9 = 7$$

$$(x + 3)^2 = 7$$

$$x = -3 \pm \sqrt{7}$$

④ $3x^2 - 12 = 0$

$$3x^2 = 12$$

$$x^2 = 4$$

$$x = \pm 2$$

Section 2: Solutions (Part 3)

Detailed Solutions (Continued)

5 $x^2 - 4x + 4 = 0$

$$(x - 2)^2 = 0$$

$x = 2$ (double root)

6 $2x^2 + 7x - 4 = 0$

$$x = \frac{-7 \pm \sqrt{49 + 32}}{4}$$

$$= \frac{-7 \pm 9}{4}$$

$$x = \frac{1}{2} \text{ or } x = -4$$

Section 3: Graphing and Applications

Practice Problems

- 1 Find the vertex and axis of symmetry of $y = x^2 - 4x + 3$
- 2 A ball is thrown upward from a height of 6 feet with an initial velocity of 40 feet per second. The height h of the ball after t seconds is given by $h = -16t^2 + 40t + 6$. Find the maximum height of the ball.
- 3 Solve the inequality: $x^2 - 4x - 5 > 0$
- 4 Find the domain and range of $y = -2x^2 + 8x - 5$
- 5 A rectangular garden has a perimeter of 100 feet. Find the dimensions that maximize the area.
- 6 Solve the system of inequalities:

$$y > x^2 - 4x + 3$$

$$y < -x^2 + 6x - 5$$

Section 3: Solutions (Part 1)

Detailed Solutions

① $y = x^2 - 4x + 3$

Vertex form: $y = (x - 2)^2 - 1$

Vertex: $(2, -1)$

Axis of symmetry: $x = 2$

② Maximum height problem:

$$h = -16t^2 + 40t + 6$$

$$t = \frac{-40}{2(-16)} = 1.25 \text{ seconds}$$

$$\begin{aligned} h &= -16(1.25)^2 + 40(1.25) + 6 \\ &= 31 \text{ feet} \end{aligned}$$

Section 3: Solutions (Part 2)

Detailed Solutions (Continued)

3 $x^2 - 4x - 5 > 0$

$$(x - 5)(x + 1) > 0$$

$$x < -1 \text{ or } x > 5$$

4 Domain and Range:

Domain: $(-\infty, \infty)$

Range: $(-\infty, 3]$ (since vertex is at $(2, 3)$)

Section 3: Solutions (Part 3)

Detailed Solutions (Continued)

5 Garden problem:

Let $x =$ width

Then $50 - x =$ length

$$A = x(50 - x) = -x^2 + 50x$$

Maximum at $x = 25$ feet

Dimensions: 25 ft by 25 ft

6 System of inequalities:

Solution region is between the parabolas
where $1 < x < 4$

Final Review Questions

Comprehensive Review

- 1 Factor completely: $12x^2 - 27$
- 2 Solve: $3x^2 + 7x - 6 = 0$
- 3 Find the vertex and graph: $y = -2x^2 + 8x - 5$
- 4 Solve the inequality: $2x^2 - 5x - 3 \leq 0$
- 5 A projectile is launched from ground level. Its height h in meters after t seconds is given by $h = -4.9t^2 + 20t$. Find the maximum height and time to reach it.
- 6 Solve the equation: $\sqrt{x^2 - 4x + 4} = x - 2$

Final Review Solutions (Part 1)

Detailed Solutions

① $12x^2 - 27$

$$\begin{aligned} &= 3(4x^2 - 9) \\ &= 3(2x + 3)(2x - 3) \end{aligned}$$

② $3x^2 + 7x - 6 = 0$

$$\begin{aligned} x &= \frac{-7 \pm \sqrt{49 + 72}}{6} \\ &= \frac{-7 \pm 11}{6} \\ x &= \frac{2}{3} \text{ or } x = -3 \end{aligned}$$

Final Review Solutions (Part 2)

Detailed Solutions (Continued)

③ $y = -2x^2 + 8x - 5$

Vertex: $(2, 3)$

Axis of symmetry: $x = 2$

④ $2x^2 - 5x - 3 \leq 0$

$$(2x + 1)(x - 3) \leq 0$$

$$-\frac{1}{2} \leq x \leq 3$$

Final Review Solutions (Part 3)

Detailed Solutions (Continued)

5 Projectile problem:

$$t = \frac{-20}{2(-4.9)} \approx 2.04 \text{ seconds}$$

$$h = -4.9(2.04)^2 + 20(2.04) \\ \approx 20.4 \text{ meters}$$

6 $\sqrt{x^2 - 4x + 4} = x - 2$

$$\sqrt{(x - 2)^2} = x - 2$$

$$|x - 2| = x - 2$$

$$x \geq 2$$