Pre-Calculus 11

Lesson 3.3: Special Triangles and Angles in Standard Position

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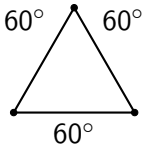
June 15, 2025

Review: Equilateral, Isosceles, and Right Triangles

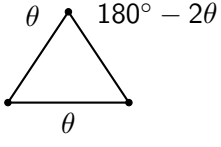
Triangle Types

- **Equilateral Triangle**: All 3 sides and all angles are equal (60° each)
- **Isosceles Triangle**: Two sides are equal, two angles are equal
- **Isosceles Right Triangle**: Both isosceles and right triangle (45°, 45°, 90°)

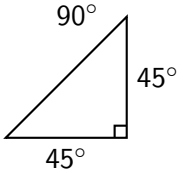
Equilateral



Isosceles



Isosceles Right



What are Special Triangles?

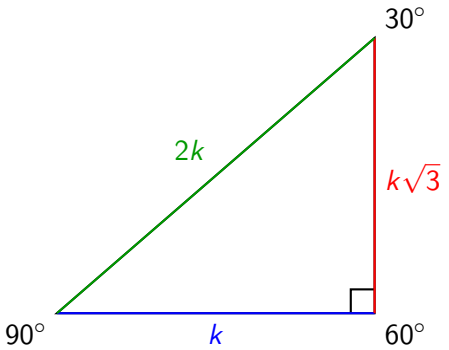
Special Triangles

- **Two types:**
  - $30°$-$60°$-$90°$ triangle (from equilateral)
  - $45°$-$45°$-$90°$ triangle (isosceles right)
- The side ratios of these triangles are used to find exact values of sine, cosine, and tangent for $30°$, $45°$, $60°$

30-60-90 Triangle: Diagram and Ratios

30-60-90 Triangle

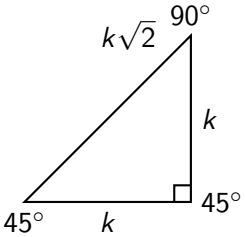
- Start with an equilateral triangle (all sides k)
- Cut in half to get a 30°-60°-90° triangle
- Side ratios: $1 : \sqrt{3} : 2$



45-45-90 Triangle: Diagram and Ratios

45-45-90 Triangle

- Isosceles right triangle: two sides equal, one 90° angle
- Side ratios: $1 : 1 : \sqrt{2}$



Exact Trig Values Using Special Triangles

Exact Values

- Use the side ratios to find exact values:

$$\sin 30^\circ = \frac{1}{2} \qquad \cos 30^\circ = \frac{\sqrt{3}}{2} \qquad \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\sin 45^\circ = \frac{1}{\sqrt{2}} \qquad \cos 45^\circ = \frac{1}{\sqrt{2}} \qquad \tan 45^\circ = 1$$

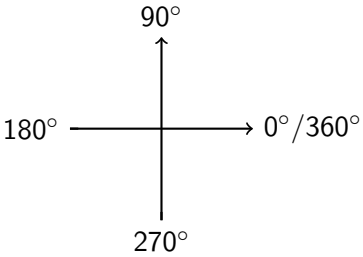
$$\sin 60^\circ = \frac{\sqrt{3}}{2} \qquad \cos 60^\circ = \frac{1}{2} \qquad \tan 60^\circ = \sqrt{3}$$

Using Special Triangles for Reference Angles

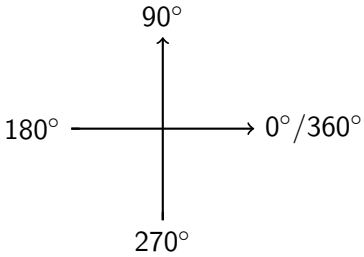
Reference Angles

- If an angle has a reference angle of $30°$, $45°$, or $60°$, use special triangles to find exact trig values
- Example: $\sin 330°$, $\cos 225°$, $\tan 120°$

Example: $\sin 330°$



Example: $\cos 225°$



Practice: Special Triangles

Practice

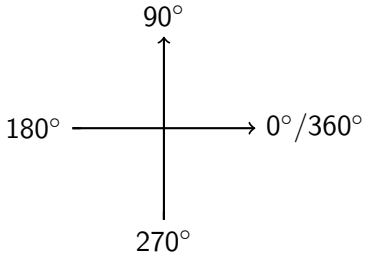
Use special triangles to find the exact value of each:

- $\sin 120^\circ$
- $\cos 300^\circ$
- $\tan 135^\circ$
- $\sin 225^\circ$

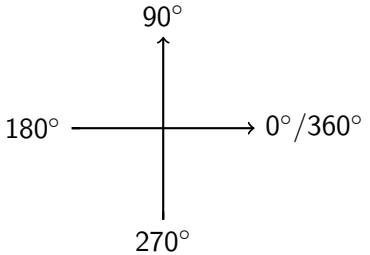
Practice: Draw and Label

Practice

Draw and label the special triangle for 60° and 45° in standard position. Indicate the side ratios and angles.



60° Triangle



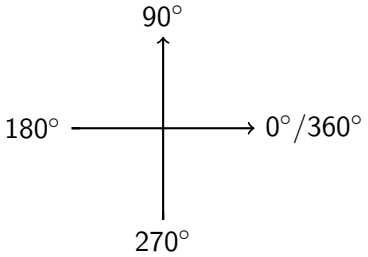
45° Triangle

Practice: Special Triangle for 30°

Practice

Draw and label the special triangle for 30° in standard position. Indicate the side ratios and angles.

Blank Axis:

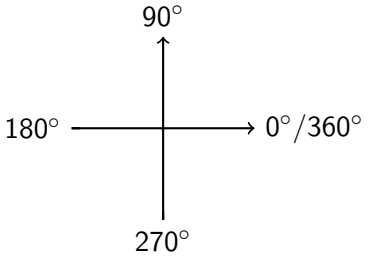


Practice: Special Triangle for 120°

Practice

Draw and label the special triangle for 120° in standard position. Indicate the side ratios and angles.

Blank Axis:

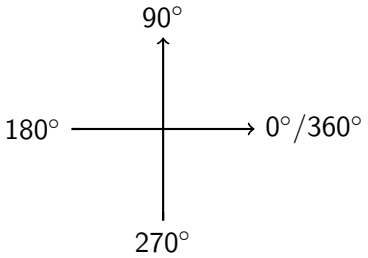


Practice: Exact Value of $\sin 150^\circ$

Practice

Find the exact value of $\sin 150^\circ$ using a special triangle. Draw and label the triangle.

Blank Axis:

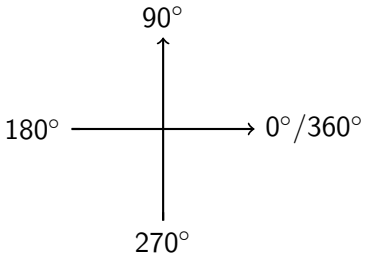


Practice: Exact Value of $\cos 315^\circ$

Practice

Find the exact value of $\cos 315^\circ$ using a special triangle. Draw and label the triangle.

Blank Axis:

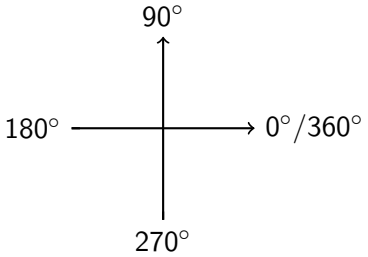


Practice: Special Triangle for 45°

Practice

Draw and label the special triangle for 45° in standard position. Indicate the side ratios and angles.

Blank Axis:

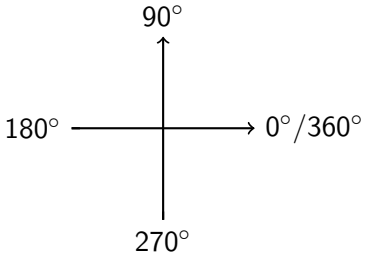


Practice: Special Triangle for 135°

Practice

Draw and label the special triangle for 135° in standard position. Indicate the side ratios and angles.

Blank Axis:

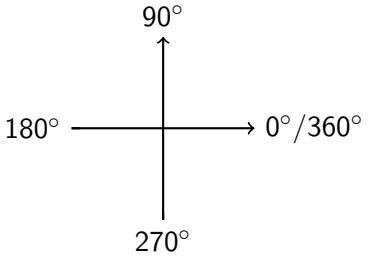


Practice: Special Triangle for 210°

Practice

Draw and label the special triangle for 210° in standard position. Indicate the side ratios and angles.

Blank Axis:

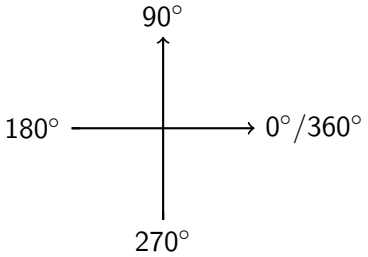


Practice: Special Triangle for 225°

Practice

Draw and label the special triangle for 225° in standard position. Indicate the side ratios and angles.

Blank Axis:

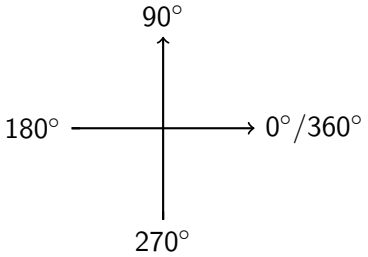


Practice: Special Triangle for 240°

Practice

Draw and label the special triangle for 240° in standard position. Indicate the side ratios and angles.

Blank Axis:

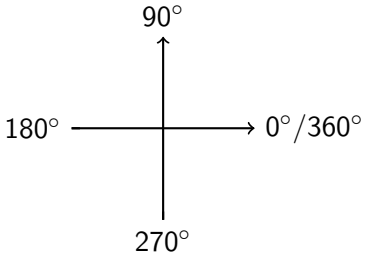


Practice: Special Triangle for 300°

Practice

Draw and label the special triangle for 300° in standard position. Indicate the side ratios and angles.

Blank Axis:



Practice: Special Triangle for 330°

Practice

Draw and label the special triangle for 330° in standard position. Indicate the side ratios and angles.

Blank Axis: