Pre-Calculus 11

Factoring Trinomials

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Factoring Trinomials

Introduction

- In this section, you will be factoring trinomials where the coefficient of x^2 is not equal to one.
- **Examples:**
  - $5x^2 + 17x + 6$ (Can't factor out common factors like the previous section.)
  - $7x^2 - 18x + 8$
- There are 3 different methods for factoring Trinomials:
  - **B.U.M. Method**
    - Easiest, straight-forward, Long
  - **Criss-Cross Method**
    - Fast, Quick with Numbers, Hard
  - **Grouping Method**
    - Textbook, standard method

BUM Method - Example

Example: Factor using BUM Method

Factor the following Trinomial using the BUM Method:
$7x^2 - 18x + 8$

BUM Method - Example Solution

Detailed Solution

Factor $7x^2 - 18x + 8$ using the BUM Method:

Bring the First term to the Last term and Multiply them

$= x^2 - 18x + 56$

Factor, two numbers that multiply to 56 and adds to -18

$= (x - 14)(x - 4)$

Bring the First term back in front of each x

$= (7x - 14)(7x - 4)$

Factor/Bum out any common factors in each binomial:

$= \left(\frac{7x - 14}{7}\right)\left(\frac{7x - 4}{1}\right)$

$= (x - 2)(7x - 4)$

BUM Method - Practice

Practice: Factor using BUM Method

Factor each of the following Trinomials using the BUM Method:

1. $15x^2 + 22x + 8$
2. $14x^2 - 27x + 9$

BUM Method - Solutions Part 1

Detailed Solutions

1. $15x^2 + 22x + 8$ **Solution:**

$$
\begin{aligned}
& 15x^2 + 22x + 8 \\
&= x^2 + 22x + 120 \\
&= (x + 12)(x + 10) \\
&= (15x + 12)(15x + 10) \\
&= \left(\frac{15x + 12}{3}\right)\left(\frac{15x + 10}{5}\right) \\
&= (5x + 4)(3x + 2)
\end{aligned}
$$

BUM Method - Solutions Part 2

Detailed Solutions

2. $14x^2 - 27x + 9$ **Solution:**

$$
\begin{aligned}
& 14x^2 - 27x + 9 \\
&= x^2 - 27x + 126 \\
&= (x - 21)(x - 6) \\
&= (14x - 21)(14x - 6) \\
&= \left(\frac{14x - 21}{7}\right)\left(\frac{14x - 6}{2}\right) \\
&= (2x - 3)(7x - 3)
\end{aligned}
$$

Grouping Method - Example

Example: Factor using Grouping Method

Factor the following Trinomial using the Grouping Method:
$9x^2 + 15x + 4$

Grouping Method - Example Solution (Part 1)

Detailed Solution

Factor $9x^2 + 15x + 4$ using the Grouping Method:

Multiply the First & Last Numbers

Find 2 numbers that MULTIPLY to 36 and ADDS to 15

$3 \times 12 \rightarrow$ Adds to 15

$= 9x^2 + 3x + 12x + 4$

Split the 15x into the two factors

Grouping Method - Example Solution (Part 2)

Detailed Solution (Cont.)

Factor $9x^2 + 15x + 4$ using the Grouping Method (Cont.):

$= (9x^2 + 3x) + (12x + 4)$

Group the First 2 and Last 2 terms

$= 3x(3x + 1) + 4(3x + 1)$

Factor out any common factors from each bracket

$= (3x + 1)(3x + 4)$

The Binomial is a GCF. Factor it out

Grouping Method - Practice

Practice: Factor using Grouping Method

Factor each of the following using the Grouping Method:

1. $12x^2 + 13x - 14$
2. $9x^2 + 21x - 8$

Grouping Method - Solutions Part 1

Detailed Solutions

1. $12x^2 + 13x - 14$ **Solution:**

$12x^2 + 13x - 14$

Multiply $12 \times -14 = -168$

Find two numbers that multiply to -168 and add to 13: $21, -8$

$= 12x^2 + 21x - 8x - 14$

$= (12x^2 + 21x) + (-8x - 14)$

$= 3x(4x + 7) - 2(4x + 7)$

$= (4x + 7)(3x - 2)$

Grouping Method - Solutions Part 2

Detailed Solutions

2. $9x^2 + 21x - 8$ **Solution:**

$9x^2 + 21x - 8$

Multiply $9 \times -8 = -72$

Find two numbers that multiply to -72 and add to 21: $24, -3$

$= 9x^2 + 24x - 3x - 8$

$= (9x^2 + 24x) + (-3x - 8)$

$= 3x(3x + 8) - 1(3x + 8)$

$= (3x + 8)(3x - 1)$

Criss-Cross Method - Example

Example: Factor using Criss-Cross Method

Factor the following using the Criss-Cross Method:
$24x^2 + 2x - 15$

Criss-Cross Method - Example Solution (Part 1)

Detailed Solution

Factor $24x^2 + 2x - 15$ using the Criss-Cross Method:

Pick 2 numbers that multiply to the FIRST term ($24x^2$):

$4x$ and $6x$

Pick 2 numbers that multiply to the LAST term (-15):

-3 and 5

Multiply sides ways or Criss-Cross:

$4x \rightarrow -3 = -12x$

$6x \rightarrow 5 = 30x$

$18x$ (Sum must equal the middle term) c

Criss-Cross Method - Example Solution (Part 2)

Detailed Solution (Cont.)

The sum $18x$ does not equal the middle term $2x$. Letś try different factors:

Pick 2 numbers that multiply to the FIRST term ($24x^2$):

$4x$ and $6x$

Pick 2 numbers that multiply to the LAST term (-15):

5 and -3

Multiply sides ways or Criss-Cross:

$4x \rightarrow 5 = 20x$

$6x \rightarrow -3 = -18x$

$2x$ (Sum must equal the middle term)

$$(4x + 5)(6x - 3)$$

Numbers on the left go in front of each bracket

Criss-Cross Method - Practice

Practice: Factor using Criss-Cross Method

Factor the following using the Criss-Cross Method:

1. $8x^2 - 26x + 15$
2. $6x^2 - 17x + 5$

Criss-Cross Method - Solutions Part 1 (Cont.)

Detailed Solutions

1. $8x^2 - 26x + 15$ **Solution:**

$$
\begin{array}{lcll}
2x & \rightarrow & -5 & = -10x \\
4x & \rightarrow & -3 & = -12x \\
 & & \text{-22x} & \text{(Incorrect sum)}
\end{array}
$$

Therefore, the factors are $(2x - 5)(4x - 3)$.

c

Criss-Cross Method - Solutions Part 2 (Cont.)

Detailed Solutions

1. $8x^2 - 26x + 15$ **Solution:**

$$\begin{array}{lll} 2x & \rightarrow & -3 \quad = -6x \\ 4x & \rightarrow & -5 \quad = -20x \\ & & \text{-26x (Correct sum)} \end{array}$$

Therefore, the factors are $(2x - 5)(4x - 3)$.

c

Mixed Practice: Factor Trinomials

Factor Each of the Following Trinomials

1. $20x^3 - 80x^2 + 35x$
2. $6x^4 - 17x^2y - 10y^2$

Mixed Practice: Solutions Part 1

Detailed Solutions

1. $20x^3 - 80x^2 + 35x$ **Solution:**

$$20x^3 - 80x^2 + 35x$$
$$= 5x(4x^2 - 16x + 7)$$

Factor $4x^2 - 16x + 7$ (BUM Method: $4 \times 7 = 28$; add to -16: -14, -2)

$$= 5x(4x - 14)(4x - 2)$$
$$= 5x\left(\frac{4x - 14}{2}\right)\left(\frac{4x - 2}{2}\right)$$
$$= 5x(2x - 7)(2x - 1)$$

Mixed Practice: Solutions Part 2

Detailed Solutions

2. $6x^4 - 17x^2y - 10y^2$ **Solution:**

$6x^4 - 17x^2y - 10y^2$

Treat x^2 as A and y as B : $6A^2 - 17AB - 10B^2$

BUM Method: $6 \times -10 = -60$; add to -17: -20, 3

$= (6A - 20B)(6A + 3B)$

$= \left(\frac{6A - 20B}{2}\right)\left(\frac{6A + 3B}{3}\right)$

$= (3A - 10B)(2A + B)$

Substitute back $A = x^2$ and $B = y$

$= (3x^2 - 10y)(2x^2 + y)$

Mixed Practice: Factor Each of the Following

Factor Each of the Following

1. $14x^2 - 23x + 3$
2. $18x^2 + 27x + 10$
3. $30x^2 + x - 1$
4. $12x^2 + 29xy + 14y^2$
5. $5x^2 + 11xy + 6y^2$
6. $4x^4 - 25x^2y^2 + 36y^4$

Mixed Practice: Solutions Part 3

Detailed Solutions

1. $14x^2 - 23x + 3$ **Solution:** $(7x - 1)(2x - 3)$
2. $18x^2 + 27x + 10$ **Solution:** $(3x + 2)(6x + 5)$
3. $30x^2 + x - 1$ **Solution:** $(5x + 1)(6x - 1)$

Mixed Practice: Solutions Part 4

Detailed Solutions

4. $12x^2 + 29xy + 14y^2$ **Solution:** $(3x + 2y)(4x + 7y)$
5. $5x^2 + 11xy + 6y^2$ **Solution:** $(x + y)(5x + 6y)$
6. $4x^4 - 25x^2y^2 + 36y^4$ **Solution:** $(x - 2y)(x + 2y)(2x - 3y)(2x + 3y)$

Application Problem: Area and Perimeter

Area and Perimeter Application

Ex: Given that the area of a rectangle is $12x^2 + 23x + 10$, then which of the following expressions is the perimeter?

1. $14x + 12$
2. $14x + 14$
3. $7x + 7$
4. $28x + 28$

Application Problem: Area and Perimeter - Solution

Detailed Solution

Ex: Given that the area of a rectangle is $12x^2 + 23x + 10$, then which of the following expressions is the perimeter?

Solution:

Factor the area: $12x^2 + 23x + 10$

Multiply $12 \times 10 = 120$

Find two numbers that multiply to 120 and add to 23: $8, 15$

$$= (12x + 8)(12x + 15)$$

$$= \left(\frac{12x + 8}{4}\right)\left(\frac{12x + 15}{3}\right)$$

$$= (3x + 2)(4x + 5)$$

So, Length $= 4x + 5$ and Width $= 3x + 2$.

$$\text{Perimeter} = 2(\text{Length} + \text{Width})$$

Application Problem: Integral Values of k

Integral Values of k

Ex: For which integral values of k can $6x^2 + kx + 1$ be factored?

1. $5, 7 only$
2. $\pm 5, \pm 7 only$
3. $-5, -7 only$
4. all integers between -7 and 5, inclusive

Application Problem: Integral Values of k - Solution

Detailed Solution

Ex: For which integral values of k can $6x^2 + kx + 1$ be factored?

Solution: We need to find combinations of factors for $6x^2$ and 1 that produce integral values for k.

- Factors of $6x^2$: $(x, 6x), (2x, 3x)$
- Factors of 1: $(1, 1), (-1, -1)$

Possible cross products for k (middle term):

- $(x + 1)(6x + 1) \rightarrow 1x + 6x = 7x \rightarrow k = 7$
- $(x - 1)(6x - 1) \rightarrow -1x - 6x = -7x \rightarrow k = -7$
- $(2x + 1)(3x + 1) \rightarrow 2x + 3x = 5x \rightarrow k = 5$
- $(2x - 1)(3x - 1) \rightarrow -2x - 3x = -5x \rightarrow k = -5$

The possible integral values for k are $\pm 5, \pm 7$. The correct option is **b)** $\pm 5, \pm 7 only$.

Summary

Key Concepts

- Understanding Trinomial Factoring when $a \neq 1$
- B.U.M. Method for factoring trinomials
- Grouping Method for factoring trinomials
- Criss-Cross Method for factoring trinomials
- Mixed practice involving various factoring scenarios