Pre-Calculus 11

Chapter 8.2: Quadratic Inequalities with One Variable

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What is a Quadratic Inequality?

Definition

A quadratic inequality is an inequality that can be written in the form:

- $ax^2 + bx + c > 0$
- $ax^2 + bx + c \geq 0$
- $ax^2 + bx + c < 0$
- $ax^2 + bx + c \leq 0$

where a, b, and c are real numbers and $a \neq 0$

Examples:

- $x^2 > 16$
- $x^2 - 5x - 14 \leq 0$

How to Solve Square Inequalities

Step-by-Step Method

1. **Isolate the square term:**
   - Move all terms to one side
   - Square root both sides
2. **Consider both positive and negative roots**
3. **Draw a number line** and mark the roots
4. Test points in each domain
5. **Write the solution** using interval notation

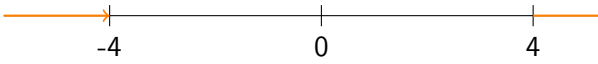
Example: Solve $x^2 > 16$

Solution

$$x^2 > 16$$
$$x < -4 \text{ or } x > 4$$

Test points:

- $(-5)^2 = 25 > 16$ [True]
- $(0)^2 = 0 < 16$ [False]
- $(5)^2 = 25 > 16$ [True]



Solving Quadratic Inequalities by Factoring

Method

1. **Factor** the quadratic expression
2. Find the **roots** [solutions]
3. Draw a **parabola** and mark the roots
4. Determine if points satisfy the inequality:
   - Above/Below/Equal to the X-axis
5. The domain that satisfies the inequality is the solution

Example: Solve $x^2 - 5x - 14 \leq 0$

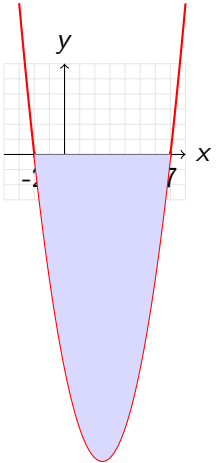
Solution

$$x^2 - 5x - 14 \leq 0$$
$$(x - 7)(x + 2) \leq 0$$

Roots: $x = -2$ and $x = 7$

Test (0): $(0 - 7)(0 + 2) = -14 \leq 0$ [True]

Solution: $-2 \leq x \leq 7$



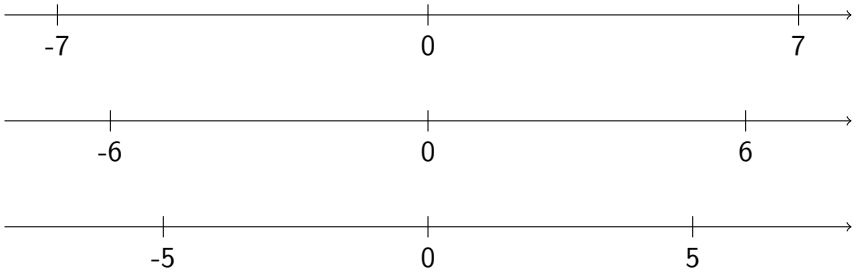
Practice Problem 1

Practice 1

Solve the following square inequalities:

1. $x^2 > 25$
2. $x^2 < 36$
3. $x^2 \geq 49$

Practice 1: Blank Number Lines



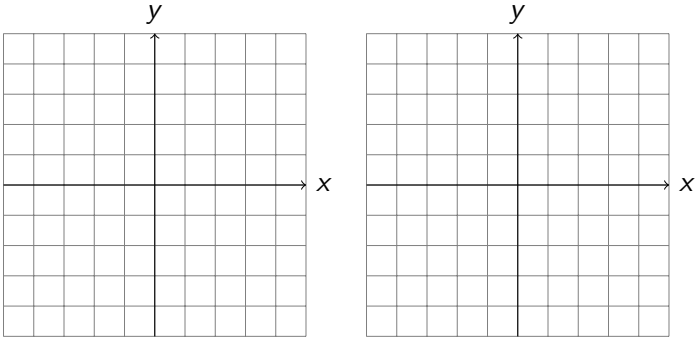
Practice Problem 2

Practice 2

Solve the following quadratic inequalities:

1. $x^2 + 3x - 10 > 0$
2. $2x^2 - 7x - 4 < 0$

Practice 2: Blank Coordinate Planes



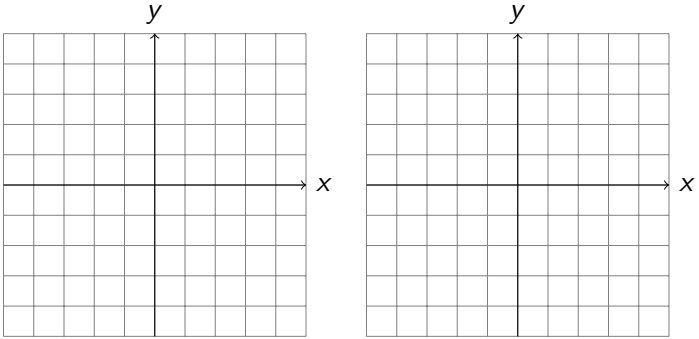
Practice Problem 3

Practice 3

Write a quadratic inequality with the following solutions:

1. $-3 \leq x \leq 2$
2. $x < -5$ or $x > 1$

Practice 3: Blank Coordinate Planes



Summary: Quadratic Inequalities

Key Points

- For square inequalities:
  - Take square root of both sides
  - Consider both positive and negative solutions
- For quadratic inequalities:
  - Factor first
  - Find roots
  - Test regions between and outside roots
- Remember to:
  - Always check your answer with test points
  - Pay attention to inequality signs
  - Consider the direction of the parabola