Chapter 4.3: Solving Radical Functions

Pre-Calculus 11 - Lesson 3

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Overview: Solving Radical Functions

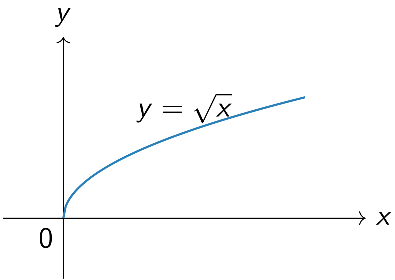
Key Concepts

- A root (radical) function is like a sideways parabola (e.g., $y = \sqrt{x}$)
- The domain starts where the radicand is zero
- To solve radical equations: isolate, square both sides, solve, and check for extraneous roots

I) Understanding Root Functions

Key Points

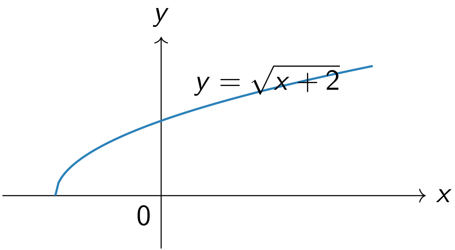
- $y = \sqrt{x}$ is the top half of a sideways parabola
- $x = y^2$ is a horizontal parabola
- The graph of $y = \sqrt{x}$ starts at $x = 0$ and only exists for $x \geq 0$

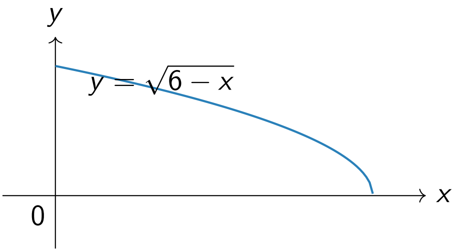


II) Where Does the Radical Function Begin?

Domain of Radical Functions

- The function $y = \sqrt{x-a}$ begins at $x = a$
- The function $y = \sqrt{2x+4}$ begins at $x = -2$
- The function $y = \sqrt{6-x}$ begins at $x = 6$





III) Steps for Solving Radical Equations

Solving Steps

1. Isolate the radical
2. Square both sides to eliminate the radical
3. Solve for x
4. Check for extraneous roots by plugging back into the original equation

Example: Solving a Radical Equation

Example

Solve $\sqrt{2x - 1} = 4$.

$$\begin{aligned} \sqrt{2x-1} &= 4 \\ 2x - 1 &= 16 \\ 2x &= 17 \\ x &= 8.5 \end{aligned}$$

Check: $\sqrt{2 \times 8.5 - 1} = \sqrt{17 - 1} = \sqrt{16} = 4$ (valid)

Practice: Solve the Radical Equations

Practice Problems

1. $\sqrt{x+4} = 6$
1. $\sqrt{2x-3} = 5$
1. $\sqrt{3x+1} = x-2$
1. $\sqrt{x-1} + 2 = 7$

Practice: Solve the Radical Equations - Solution 1a

Solution 1a

Solving:

$$\sqrt{x+4} = 6$$
$$(\sqrt{x+4})^2 = 6^2$$
$$x + 4 = 36$$
$$x = 32$$

Checking:

$$\sqrt{32+4} = \sqrt{36} = 6$$

True!

Practice: Solve the Radical Equations - Solution 1b

Solution 1b

Solving:

$$\sqrt{2x-3} = 5$$
$$(\sqrt{2x-3})^2 = 5^2$$
$$2x - 3 = 25$$
$$2x = 28$$
$$x = 14$$

Checking:

$$\sqrt{2(14)-3} = \sqrt{28-3} = \sqrt{25} = 5$$

True!

Practice: Solve the Radical Equations - Solution 1c (Solving)

Solution 1c – Solving

$\sqrt{3x+1} = x - 2$

$$
\begin{aligned}
&\sqrt{3x+1} = x - 2 \\
&(\sqrt{3x+1})^2 = (x-2)^2 \\
&3x + 1 = x^2 - 4x + 4 \\
&0 = x^2 - 7x + 3
\end{aligned}
$$

Using quadratic formula:

$$
\begin{aligned}
&x = \frac{7 \pm \sqrt{49-12}}{2} \\
&x = \frac{7 \pm \sqrt{37}}{2} \\
&x \approx 6.54 \text{ or } x \approx 0.46
\end{aligned}
$$

Practice: Solve the Radical Equations - Solution 1c (Checking)

Solution 1c – Checking

Checking:

- For $x = 6.54$: $\sqrt{3(6.54)+1} \approx 4.54$ and $6.54 - 2 = 4.54$ ✓
- For $x = 0.46$: $\sqrt{3(0.46)+1} \approx 1.54$ and $0.46 - 2 = -1.54$ ×

Therefore, only $x = 6.54$ is valid.

Practice: Solve the Radical Equations - Solution 1d

Solution 1d

Solving:

$$\sqrt{x-1}+2=7$$
$$\sqrt{x-1}=5$$
$$(\sqrt{x-1})^2=5^2$$
$$x-1=25$$
$$x=26$$

Checking:

$$\sqrt{26-1}+2=\sqrt{25}+2=5+2=7$$

True!

Practice: Extraneous Roots or No Solution?

Practice Problems

1. $\sqrt{x+3} = -2$
1. $\sqrt{2x-5} = x-3$
1. $\sqrt{x-2} = 2-x$
1. $\sqrt{4-x} = x+1$

Practice: Extraneous Roots or No Solution? - Solution 2a

Solution 2a

$\sqrt{x+3} = -2$

- No solution: square root cannot be negative

Practice: Extraneous Roots or No Solution? - Solution 2b (Solving)

Solution 2b – Solving

$\sqrt{2x-5} = x - 3$

$$
\begin{aligned}
&\sqrt{2x-5} = x - 3 \\
&(\sqrt{2x-5})^2 = (x-3)^2 \\
&2x - 5 = x^2 - 6x + 9 \\
&0 = x^2 - 8x + 14
\end{aligned}
$$

Using quadratic formula:

$$
\begin{aligned}
x &= \frac{8 \pm \sqrt{64-56}}{2} \\
x &= \frac{8 \pm \sqrt{8}}{2} \\
x &= 4 \pm \sqrt{2}
\end{aligned}
$$

Practice: Extraneous Roots or No Solution? - Solution 2b (Checking)

Solution 2b – Checking

Checking:

- For $x = 5.41$: $\sqrt{2(5.41) - 5} \approx 2.41$ and $5.41 - 3 = 2.41$ ✓
- For $x = 2.59$: $\sqrt{2(2.59) - 5} \approx 0.41$ and $2.59 - 3 = -0.41$ $\times$

Therefore, only $x = 5.41$ is valid.

Practice: Extraneous Roots or No Solution? - Solution 2c (Solving)

Solution 2c – Solving

$\sqrt{x-2} = 2 - x$

$$
\begin{aligned}
\sqrt{x-2} &= 2 - x \\
(\sqrt{x-2})^2 &= (2-x)^2 \\
x - 2 &= 4 - 4x + x^2 \\
0 &= x^2 - 5x + 6
\end{aligned}
$$

Using quadratic formula:

$$
\begin{aligned}
x &= \frac{5 \pm \sqrt{25-24}}{2} \\
x &= \frac{5 \pm 1}{2} \\
x &= 3 \text{ or } x = 2
\end{aligned}
$$

Practice: Extraneous Roots or No Solution? - Solution 2c (Checking)

Solution 2c – Checking

Checking:

- For $x = 3$: $\sqrt{3-2} = 1$ and $2 - 3 = -1$ $\times$
- For $x = 2$: $\sqrt{2-2} = 0$ and $2 - 2 = 0$ $\checkmark$

Therefore, only $x = 2$ is valid.

Practice: Extraneous Roots or No Solution? - Solution 2d (Solving)

Solution 2d – Solving

$\sqrt{4-x} = x+1$

$$\sqrt{4-x} = x+1$$
$$(\sqrt{4-x})^2 = (x+1)^2$$
$$4-x = x^2+2x+1$$
$$0 = x^2+3x-3$$

Using quadratic formula:

$$x = \frac{-3 \pm \sqrt{9+12}}{2}$$
$$x = \frac{-3 \pm \sqrt{21}}{2}$$
$$x \approx 0.79 \text{ or } x \approx -3.79$$

Practice: Extraneous Roots or No Solution? - Solution 2d (Checking)

Solution 2d – Checking

Checking:

- For $x = 0.79$: $\sqrt{4 - 0.79} \approx 1.79$ and $0.79 + 1 = 1.79$ ✓
- For $x = -3.79$: $\sqrt{4 - (-3.79)} \approx 2.79$ and $-3.79 + 1 = -2.79$ ×

Therefore, only $x = 0.79$ is valid.

Practice: Expand/FOIL with Radicals

Practice Problems

1. $(x+\sqrt{3})(x-\sqrt{3})$
1. $(x+2\sqrt{2})(x-2\sqrt{2})$
1. $(2+\sqrt{5})(2-\sqrt{5})$
1. $(3+\sqrt{7})(3-\sqrt{7})$

Practice: Expand/FOIL with Radicals - Solutions (1/2)

Solutions

$(x+\sqrt{3})(x-\sqrt{3})$

$$\begin{aligned} &= x^2 - x\sqrt{3} + x\sqrt{3} - (\sqrt{3})^2 \\ &= x^2 - 3 \end{aligned}$$

$(2+\sqrt{5})(2-\sqrt{5})$

$$\begin{aligned} &= 4 - 2\sqrt{5} + 2\sqrt{5} - (\sqrt{5})^2 \\ &= 4 - 5 \\ &= -1 \end{aligned}$$

Practice: Expand/FOIL with Radicals - Solutions (2/2)

Solutions

$(x + 2\sqrt{2})(x - 2\sqrt{2})$

$$
\begin{aligned}
&= x^2 - 2x\sqrt{2} + 2x\sqrt{2} - (2\sqrt{2})^2 \\
&= x^2 - 8
\end{aligned}
$$

$(3 + \sqrt{7})(3 - \sqrt{7})$

$$
\begin{aligned}
&= 9 - 3\sqrt{7} + 3\sqrt{7} - (\sqrt{7})^2 \\
&= 9 - 7 \\
&= 2
\end{aligned}
$$

Practice: Solve and Extraneous Roots

Step-by-Step Example

i) $-4+\sqrt{3x+4}=2$

$$-4+\sqrt{3x+4}=2$$
$$\sqrt{3x+4}=6$$
$$3x+4=36$$
$$3x=32$$
$$x=10.66$$

ii) $\sqrt{3x+1}=2x-6$

$$(\sqrt{3x+1})^2=(2x-6)^2$$
$$3x+1=(2x-6)(2x-6) \quad \text{(FOIL)}$$
$$3x+1=4x^2-24x+36$$
$$0=4x^2-27x+35$$
$$4x^2-27x+35=0$$

The 'Extraneous Root' is a solution that does not satisfy the original equation. Always check your answers!

Graphical Meaning of Extraneous Roots

Key Points:

- The extraneous root is at the intersection on the bottom side of the parabola.
- It does not satisfy the original radical equation.
- We only want the intersection on the top (actual root).

We don't want the "Extraneous Root"!