Pre-Calculus 11

Chapter 6.6: Absolute Values and Reciprocal Functions Summary /

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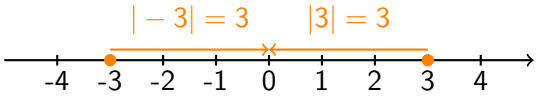
Chapter 6.6 Overview /

Topics Covered

- 6.1 Basics with Absolute Values
- 6.2 Absolute Value Functions
- 6.3 Solving Absolute Value Equations
- 6.4 Reciprocal Functions
- 6.5 Solving Equations with Two Absolute Values

Key Points

- $|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$
- $|x| = \sqrt{x^2}$
- Distance from zero on number line
- Always non-negative



Practice

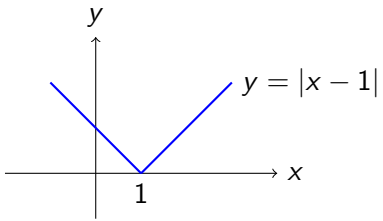
1. Evaluate $|25 - 17|$
2. Solve $|x + 5| = 12$
3. Solve $|2x - 4| = 9$
4. Order from least to greatest: $|-7.5|, |-8.2|, |-3.4|$

Solutions

1. $|25 - 17| = |8| = 8$
2. $x + 5 = 12$ or $x + 5 = -12$; $x = 7$ or $x = -17$
3. $2x - 4 = 9$ or $2x - 4 = -9$; $x = 6.5$ or $x = -2.5$
4. $|-3.4|, |-7.5|, |-8.2|$ (3.4, 7.5, 8.2)

Key Points

- $y = |f(x)|$ reflects negative parts above x-axis
- Piecewise definition: $y = \begin{cases} f(x), & f(x) \geq 0 \\ -f(x), & f(x) < 0 \end{cases}$
- Find x-intercepts to determine reflection points



6.2 Practice Problems /

Practice

1. Graph $y = |x^2 - 4|$
2. Write piecewise form for $y = |x + 2|$
3. Find x-intercepts of $y = |x^2 - 9|$
4. Graph $y = |x^3 - x|$

Solutions

1. V-shape with vertex at $(0, 4)$, x-intercepts at $x = \pm 2$
2. $y = \begin{cases} x + 2, & x \geq -2 \\ -(x + 2), & x < -2 \end{cases}$
3. $x = \pm 3$ (where $x^2 - 9 = 0$)
4. Reflects negative parts of $y = x^3 - x$ above x-axis

6.3 Solving Absolute Value Equations /

Key Points

- For $|x| = a$, set up two cases: $x = a$ and $x = -a$
- Always check for extraneous roots
- $|x| = -a$ has no solution if $a > 0$

Example: $|x - 3| = 7$

$x - 3 = 7$ or $x - 3 = -7$

$x = 10$ or $x = -4$

6.3 Practice Problems /

Practice

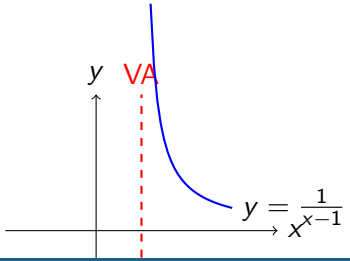
1. Solve $|x - 1| = x + 1$
2. Solve $|2x - 5| = |x + 4|$
3. Solve $|x^2 - 6x + 8| = 2$
4. Which has no solution: $|x + 3| = -5$ or $|x + 3| = 5$?

Solutions

1. Case 1: $x - 1 = x + 1$ (no solution); Case 2: $x - 1 = -(x + 1)$; $x = 0$
2. $x = 9$ or $x = \frac{1}{3}$
3. $x = 3 \pm \sqrt{3}$
4. $|x + 3| = -5$ has no solution (absolute value cannot be negative)

Key Points

- $y = \frac{1}{f(x)}$
- Vertical asymptotes where $f(x) = 0$
- Invariant points where $f(x) = \pm 1$
- Horizontal asymptote usually $y = 0$



6.4 Practice Problems /

Practice

1. Find vertical asymptotes of $y = \frac{1}{x^2-4}$
2. Find invariant points of $y = \frac{1}{x+2}$
3. Graph $y = \frac{1}{x^2}$
4. Domain of $y = \frac{1}{\sqrt{x-3}}$?

Solutions

1. $x = \pm 2$ (where $x^2 - 4 = 0$)
2. $x = -1$ and $x = -3$ (where $x + 2 = \pm 1$)
3. Hyperbola with VA at $x = 0$, HA at $y = 0$
4. $x > 3$ (to avoid division by zero and negative under square root)

Key Points

- Consider all possible sign combinations (4 cases)
- Use number line to determine valid intervals
- Always check solutions for extraneous roots

Example: $|x-2|+|x+6|=11$

Cases: $x \geq 2$, $-6 \leq x < 2$, $x < -6$

Solutions: $x = 3.5$, $x = -7.5$

Practice

1. Solve $|x-1|+|x+5|=8$
2. Solve $|x+3|-|x-2|=4$
3. Solve $|x-4|+|x+2|=10$
4. How many cases for $|x-a|+|x-b|=c$?

Solutions

1. $x = 2$ and $x = -6$
2. $x = 1.5$ (only valid solution)
3. $x = 6$ and $x = -4$
4. 3 cases: $x \geq \max(a, b)$, $\min(a, b) \leq x < \max(a, b)$, $x < \min(a, b)$

Final Review Questions /

Comprehensive Review

1. Evaluate $|-15|+|7|$
2. Solve $|2x-3|=7$
3. Graph $y=|x^2-1|$
4. Find asymptotes of $y=\frac{1}{x^2-9}$
5. Solve $|x-3|+|x+1|=6$
6. Explain the difference between $|f(x)|$ and $\frac{1}{f(x)}$

Final Review Solutions /

Solutions

1. $|-15| + |7| = 15 + 7 = 22$
2. $2x - 3 = 7$ or $2x - 3 = -7$; $x = 5$ or $x = -2$
3. V-shape with vertex at $(0, 1)$, x-intercepts at $x = \pm 1$
4. Vertical asymptotes: $x = \pm 3$; Horizontal asymptote: $y = 0$
5. $x = 4$ and $x = -2$
6. $|f(x)|$ reflects negative parts above x-axis; $\frac{1}{f(x)}$ creates reciprocal with asymptotes