Pre-Calculus 11

Lesson 2: Angles in Standard Position

Created by Yi-Chen Lin

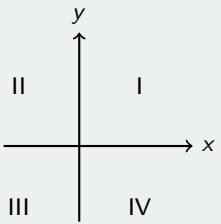
June 15, 2025

Naming Quadrants
X/Y Axis

Quadrants and Axes

- There are Four Quadrants in the XY plane
- On the X-axis: Right – Positive, Left - Negative
- On the Y-axis: Up – Positive, Bottom - Negative
- Center: Origin (0,0)

Quadrant	X Coordinates	Y Coordinates
1	+	+
2	-	+
3	-	-
4	+	-

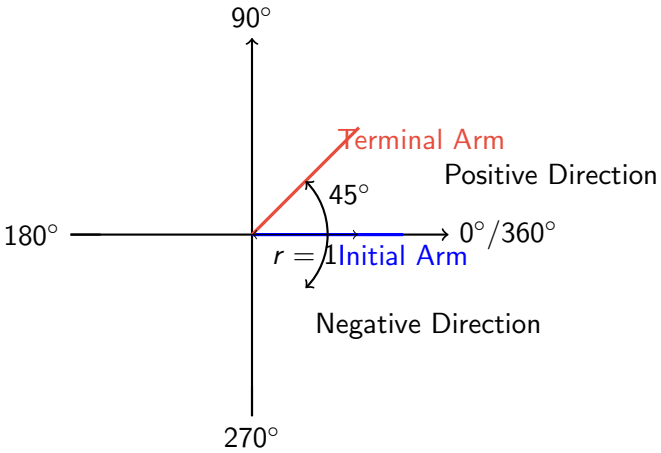


II) Angles in Standard Position

Angles in Standard Position

- Drawing an angle in standard position means that you start from the right side of the X-axis at 0 degrees
- When drawing the angle, indicate which direction it is rotating: Clockwise (Positive) and Counter Clockwise (Negative)
- The initial arm is the positive side of the X-axis
- The terminal arm is the line that rotates around the origin, with a radius of 1 (UNIT CIRCLE)

II) Angles in Standard Position (Diagram)

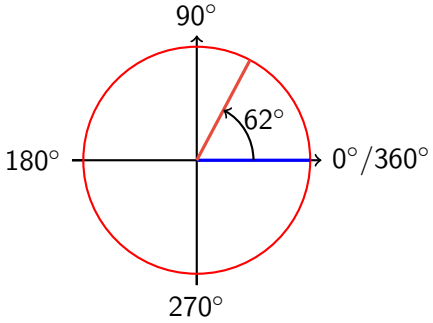


III) Drawing Angles in Standard Position

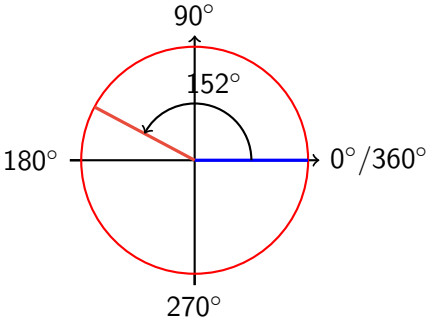
Drawing Angles in Standard Position

- When drawing angles in "standard position" make sure they begin at the Initial arm at 0°
- Draw the Terminal arm in the quadrant, approximate it
- Make sure you draw the curve indicating the direction and number of rotations

III) Drawing Angles in Standard Position: Examples

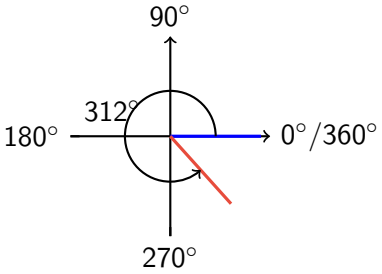


a) 62°

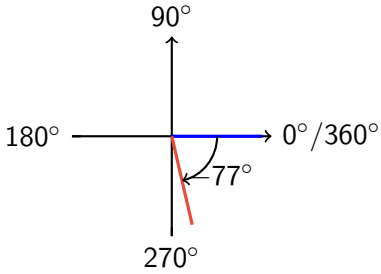


b) 152°

III) Drawing Angles in Standard Position: More Examples

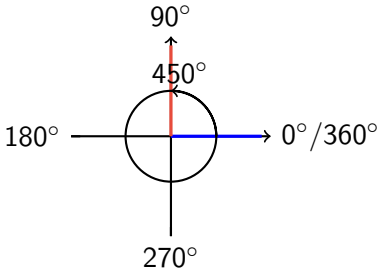


c) $312°$

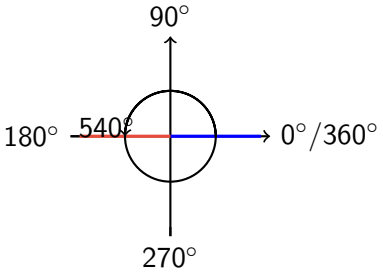


d) $-77°$

III) Drawing Angles in Standard Position: Even More Examples



e) 450°



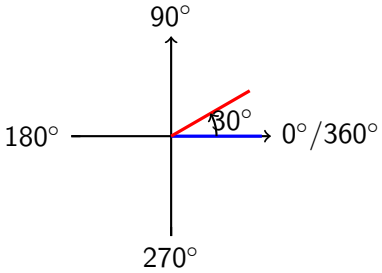
f) 540°

IV) Co-terminal Angles

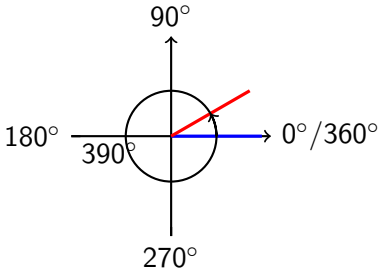
Co-terminal Angles

- Two angles are called "co-terminal" if they are located at the same position
- Co-terminal angles have a difference of 360° or multiples of 360° (Full circles)
- These angles are co-terminal with each other: 30°, 390°, 750°, -330°, -690°, etc.

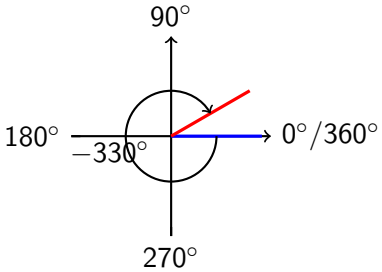
IV) Co-terminal Angle: 30°



IV) Co-terminal Angle: 390°



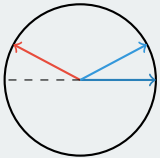
IV) Co-terminal Angle: -330°



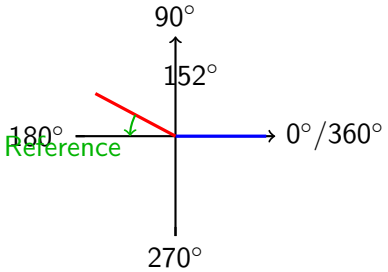
Reference Angles

Reference Angles

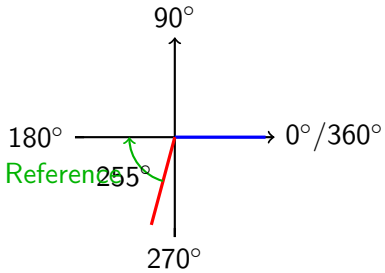
- Reference angle: angle between terminal arm and x-axis
- Must be in the same quadrant as the terminal arm
- Used for SINE, COSINE, TANGENT of angles $> 90^\circ$
- Ex: 152° reference: $180^\circ - 152^\circ = 28^\circ$
- Ex: 255° reference: $255^\circ - 180^\circ = 75^\circ$
- Ex: 420° reference: $420^\circ - 360^\circ = 60^\circ$
- Ex: -388° reference: $388^\circ - 360^\circ = 28^\circ$



V) Reference Angle Example: 152°



V) Reference Angle Example: 255°

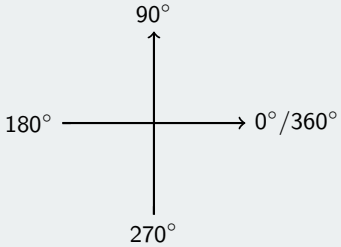


V) Reference Angles: Practice (a)

Practice: Find the Reference Angle

Draw 123° in standard position and find its reference angle.

Blank Axis:

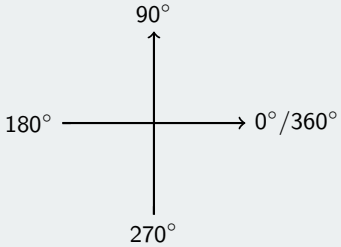


V) Reference Angles: Practice (b)

Practice: Find the Reference Angle

Draw -210° in standard position and find its reference angle.

Blank Axis:

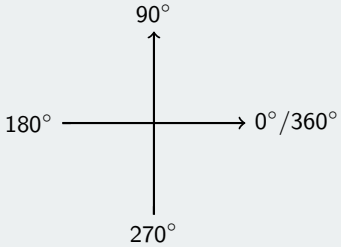


V) Reference Angles: Practice (c)

Practice: Find the Reference Angle

Draw 370° in standard position and find its reference angle.

Blank Axis:

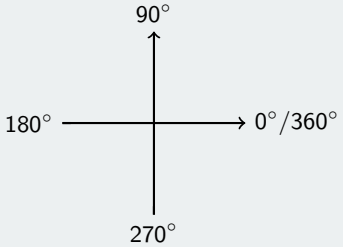


V) Reference Angles: Practice (d)

Practice: Find the Reference Angle

Draw -480° in standard position and find its reference angle.

Blank Axis:



V) Reference Angles: Practice (More)

Practice

Find the reference angle for each: 240°, -225°, 150°, 432°

V) Reference Angles: Practice (Same Reference)

Practice

Which of the following have the same reference angle: 195°, 285°, 165°, 345°, -15°, 105°, 85°, -735°

Application: Sine, Cosine, Tangent

Application

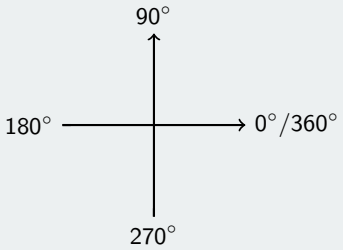
- If you want to SINE, COSINE, or TANGENT an angle greater than 90°, use the reference angle
- Example: Find $\sin 195^\circ$
- Step 1: Make a right triangle with the reference angle
- Step 2: Indicate what the "opp", "adj" and "hyp" sides are
- Note: UP/RIGHT (Positive), DOWN/LEFT (Negative), Hypotenuse is always positive

Application Practice: Standard Angle (a)

Practice: Draw the Angle

Draw 117° in standard position.

Blank Axis:

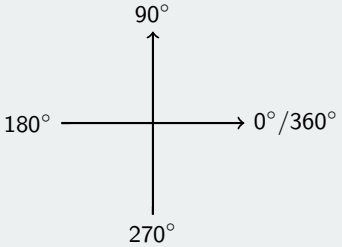


Application Practice: Standard Angle (b)

Practice: Draw the Angle

Draw -240° in standard position.

Blank Axis:

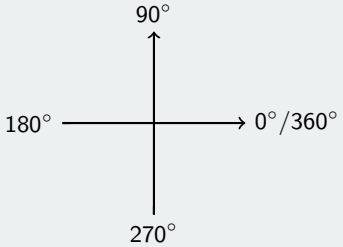


Application Practice: Standard Angle (c)

Practice: Draw the Angle

Draw 315° in standard position.

Blank Axis:

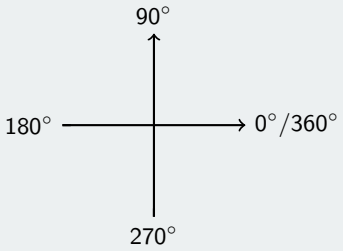


Application: Sine, Cosine, Tangent (a)

Application

Find $\sin 225^\circ$ by using its reference angle. Step 1: Make a right triangle with the reference angle.
Step 2: Indicate what the "opp", "adj" and "hyp" sides are.
Note: UP/RIGHT (Positive), DOWN/LEFT (Negative), Hypotenuse is always positive.

Blank Axis:

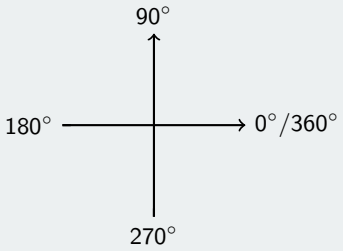


Application: Sine, Cosine, Tangent (b)

Application

Find $\cos 315^\circ$ by using its reference angle. Step 1: Make a right triangle with the reference angle.
Step 2: Indicate what the "opp", "adj" and "hyp" sides are.
Note: UP/RIGHT (Positive), DOWN/LEFT (Negative), Hypotenuse is always positive.

Blank Axis:

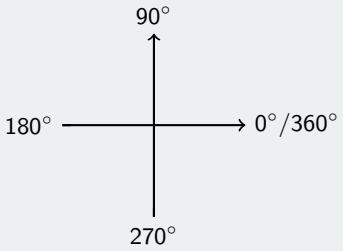


Application: Sine, Cosine, Tangent (c)

Application

Find $\tan 135^\circ$ by using its reference angle. Step 1: Make a right triangle with the reference angle.
Step 2: Indicate what the "opp", "adj" and "hyp" sides are.
Note: UP/RIGHT (Positive), DOWN/LEFT (Negative), Hypotenuse is always positive.

Blank Axis:

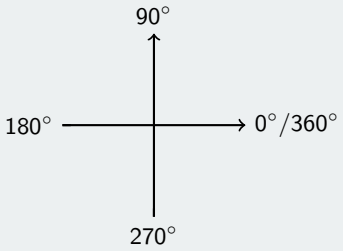


Application: Sine, Cosine, Tangent (d)

Application

Find $\sin 330^\circ$ by using its reference angle. Step 1: Make a right triangle with the reference angle.
Step 2: Indicate what the "opp", "adj" and "hyp" sides are.
Note: UP/RIGHT (Positive), DOWN/LEFT (Negative), Hypotenuse is always positive.

Blank Axis:

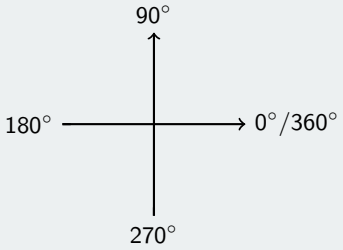


Application: Sine, Cosine, Tangent (e)

Application

Find $\cos 210^\circ$ by using its reference angle. Step 1: Make a right triangle with the reference angle.
Step 2: Indicate what the "opp", "adj" and "hyp" sides are.
Note: UP/RIGHT (Positive), DOWN/LEFT (Negative), Hypotenuse is always positive.

Blank Axis:



Practice: Co-terminal Angles

Practice

- Ex: A terminal arm is rotated 571° ccw around the origin in a unit circle
- What is the reference angle?
- What is the general formula for all coterminal angles?
- What is the base and height of the triangle created by the reference angle?
- Which one of the following is the general formula for all the coterminal angles with 415°?
  - $\varphi = 125^\circ \pm 360^\circ n$
  - $\varphi = 125^\circ \pm 180^\circ n$
  - $\varphi = 415^\circ \pm 360^\circ n$
  - $\varphi = 415^\circ \pm 180^\circ n$
  - $\varphi = 55^\circ \pm 360^\circ n$