

# Pre-Calculus 11

## Factoring Trinomials

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# Factoring Trinomials

## Introduction

- In this section, you will be factoring trinomials where the coefficient of  $x^2$  is not equal to one.
- **Examples:**
  - $5x^2 + 17x + 6$  (Can't factor out common factors like the previous section.)
  - $7x^2 - 18x + 8$
- There are 3 different methods for factoring Trinomials:
  - **B.U.M. Method**
    - Easiest, straight-forward, Long
  - **Criss-Cross Method**
    - Fast, Quick with Numbers, Hard
  - **Grouping Method**
    - Textbook, standard method

# BUM Method - Example

## Example: Factor using BUM Method

Factor the following Trinomial using the BUM Method:

$$7x^2 - 18x + 8$$

# BUM Method - Example Solution

## Detailed Solution

Factor  $7x^2 - 18x + 8$  using the BUM Method:

Bring the First term to the Last term and Multiply them

$$= x^2 - 18x + 56$$

Factor, two numbers that multiply to 56 and adds to -18

$$= (x - 14)(x - 4)$$

Bring the First term back in front of each x

$$= (7x - 14)(7x - 4)$$

Factor/Bum out any common factors in each binomial:

$$= \left( \frac{7x - 14}{7} \right) \left( \frac{7x - 4}{1} \right)$$

$$= (x - 2)(7x - 4)$$

## Practice: Factor using BUM Method

Factor each of the following Trinomials using the BUM Method:

①  $15x^2 + 22x + 8$

②  $14x^2 - 27x + 9$

## Detailed Solutions

①  $15x^2 + 22x + 8$     **Solution:**

$$\begin{aligned} &15x^2 + 22x + 8 \\ &= x^2 + 22x + 120 \\ &= (x + 12)(x + 10) \\ &= (15x + 12)(15x + 10) \\ &= \left(\frac{15x + 12}{3}\right) \left(\frac{15x + 10}{5}\right) \\ &= (5x + 4)(3x + 2) \end{aligned}$$

## Detailed Solutions

②  $14x^2 - 27x + 9$     **Solution:**

$$\begin{aligned} &14x^2 - 27x + 9 \\ &= x^2 - 27x + 126 \\ &= (x - 21)(x - 6) \\ &= (14x - 21)(14x - 6) \\ &= \left(\frac{14x - 21}{7}\right) \left(\frac{14x - 6}{2}\right) \\ &= (2x - 3)(7x - 3) \end{aligned}$$

# Grouping Method - Example

## Example: Factor using Grouping Method

Factor the following Trinomial using the Grouping Method:

$$9x^2 + 15x + 4$$



# Grouping Method - Example Solution (Part 1)

## Detailed Solution

Factor  $9x^2 + 15x + 4$  using the Grouping Method:

Multiply the First & Last Numbers

Find 2 numbers that MULTIPLY to 36 and ADDS to 15

$3 \times 12 \rightarrow$  Adds to 15

$$= 9x^2 + 3x + 12x + 4$$

Split the  $15x$  into the two factors

# Grouping Method - Example Solution (Part 2)

## Detailed Solution (Cont.)

Factor  $9x^2 + 15x + 4$  using the Grouping Method (Cont.):

$$= (9x^2 + 3x) + (12x + 4)$$

Group the First 2 and Last 2 terms

$$= 3x(3x + 1) + 4(3x + 1)$$

Factor out any common factors from each bracket

$$= (3x + 1)(3x + 4)$$

The Binomial is a GCF. Factor it out

# Grouping Method - Practice

## Practice: Factor using Grouping Method

Factor each of the following using the Grouping Method:

①  $12x^2 + 13x - 14$

②  $9x^2 + 21x - 8$

# Grouping Method - Solutions Part 1

## Detailed Solutions

①  $12x^2 + 13x - 14$     **Solution:**

$$12x^2 + 13x - 14$$

Multiply  $12 \times -14 = -168$

Find two numbers that multiply to -168 and add to 13: 21, -8

$$= 12x^2 + 21x - 8x - 14$$

$$= (12x^2 + 21x) + (-8x - 14)$$

$$= 3x(4x + 7) - 2(4x + 7)$$

$$= (4x + 7)(3x - 2)$$

# Grouping Method - Solutions Part 2

## Detailed Solutions

2  $9x^2 + 21x - 8$  **Solution:**

$$9x^2 + 21x - 8$$

Multiply  $9 \times -8 = -72$

Find two numbers that multiply to -72 and add to 21: 24, -3

$$= 9x^2 + 24x - 3x - 8$$

$$= (9x^2 + 24x) + (-3x - 8)$$

$$= 3x(3x + 8) - 1(3x + 8)$$

$$= (3x + 8)(3x - 1)$$

# Criss-Cross Method - Example

## Example: Factor using Criss-Cross Method

Factor the following using the Criss-Cross Method:

$$24x^2 + 2x - 15$$

# Criss-Cross Method - Example Solution (Part 1)

## Detailed Solution

Factor  $24x^2 + 2x - 15$  using the Criss-Cross Method:

Pick 2 numbers that multiply to the FIRST term ( $24x^2$ ):

$4x$  and  $6x$

Pick 2 numbers that multiply to the LAST term ( $-15$ ):

$-3$  and  $5$

Multiply sides ways or Criss-Cross:

$$4x \rightarrow -3 = -12x$$

$$6x \rightarrow 5 = 30x$$

$18x$  (Sum must equal the middle term) c

# Criss-Cross Method - Example Solution (Part 2)

## Detailed Solution (Cont.)

The sum  $18x$  does not equal the middle term  $2x$ . Let's try different factors:

Pick 2 numbers that multiply to the FIRST term ( $24x^2$ ):

$4x$  and  $6x$

Pick 2 numbers that multiply to the LAST term ( $-15$ ):

$5$  and  $-3$

Multiply sides ways or Criss-Cross:

$$4x \rightarrow 5 = 20x$$

$$6x \rightarrow -3 = -18x$$

$2x$  (Sum must equal the middle term)

$$(4x - 3)(6x + 5)$$

Numbers on the left go in front of each bracket



# Criss-Cross Method - Practice

## Practice: Factor using Criss-Cross Method

Factor the following using the Criss-Cross Method:

①  $8x^2 - 26x + 15$

②  $6x^2 - 17x + 5$

# Criss-Cross Method - Solutions Part 1 (Cont.)

## Detailed Solutions

①  $8x^2 - 26x + 15$     **Solution:**

$$2x \rightarrow -5 = -10x$$

$$4x \rightarrow -3 = -12x$$

$$-22x \quad (\text{Incorrect sum})$$

Therefore, the factors are  $(2x - 5)(4x - 3)$ .

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# Criss-Cross Method - Solutions Part 2 (Cont.)

## Detailed Solutions

①  $8x^2 - 26x + 15$     **Solution:**

$$2x \rightarrow -3 = -6x$$

$$4x \rightarrow -5 = -20x$$

$$-26x \quad (\text{Correct sum})$$

Therefore, the factors are  $(2x - 5)(4x - 3)$ .

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# Mixed Practice: Factor Trinomials

Factor Each of the Following Trinomials

①  $20x^3 - 80x^2 + 35x$

②  $6x^4 - 17x^2y - 10y^2$

# Mixed Practice: Solutions Part 1

## Detailed Solutions

①  $20x^3 - 80x^2 + 35x$     **Solution:**

$$20x^3 - 80x^2 + 35x$$

$$= 5x(4x^2 - 16x + 7)$$

Factor  $4x^2 - 16x + 7$  (BUM Method:  $4 \times 7 = 28$ ; add to -16: -14, -2)

$$= 5x(4x - 14)(4x - 2)$$

$$= 5x \left( \frac{4x - 14}{2} \right) \left( \frac{4x - 2}{2} \right)$$

$$= 5x(2x - 7)(2x - 1)$$

# Mixed Practice: Solutions Part 2

## Detailed Solutions

②  $6x^4 - 17x^2y - 10y^2$     **Solution:**

$$6x^4 - 17x^2y - 10y^2$$

Treat  $x^2$  as  $A$  and  $y$  as  $B$  :  $6A^2 - 17AB - 10B^2$

BUM Method:  $6 \times -10 = -60$ ; add to  $-17$ :  $-20, 3$

$$= (6A - 20B)(6A + 3B)$$

$$= \left( \frac{6A - 20B}{2} \right) \left( \frac{6A + 3B}{3} \right)$$

$$= (3A - 10B)(2A + B)$$

Substitute back  $A = x^2$  and  $B = y$

$$= (3x^2 - 10y)(2x^2 + y)$$

# Mixed Practice: Factor Each of the Following

## Factor Each of the Following

①  $14x^2 - 23x + 3$

②  $18x^2 + 27x + 10$

③  $30x^2 + x - 1$

④  $12x^2 + 29xy + 14y^2$

⑤  $5x^2 + 11xy + 6y^2$

⑥  $4x^4 - 25x^2y^2 + 36y^4$

# Mixed Practice: Solutions Part 3

## Detailed Solutions

①  $14x^2 - 23x + 3$     **Solution:**  $(7x - 1)(2x - 3)$

②  $18x^2 + 27x + 10$     **Solution:**  $(3x + 2)(6x + 5)$

③  $30x^2 + x - 1$     **Solution:**  $(5x + 1)(6x - 1)$



# Mixed Practice: Solutions Part 4

## Detailed Solutions

④  $12x^2 + 29xy + 14y^2$     **Solution:**  $(3x + 2y)(4x + 7y)$

⑤  $5x^2 + 11xy + 6y^2$     **Solution:**  $(x + y)(5x + 6y)$

⑥  $4x^4 - 25x^2y^2 + 36y^4$     **Solution:**  $(x - 2y)(x + 2y)(2x - 3y)(2x + 3y)$

# Application Problem: Area and Perimeter

## Area and Perimeter Application

Ex: Given that the area of a rectangle is  $12x^2 + 23x + 10$ , then which of the following expressions is the perimeter?

- ①  $14x + 12$
- ②  $14x + 14$
- ③  $7x + 7$
- ④  $28x + 28$

# Application Problem: Area and Perimeter - Solution

## Detailed Solution

Ex: Given that the area of a rectangle is  $12x^2 + 23x + 10$ , then which of the following expressions is the perimeter?

**Solution:**

Factor the area:  $12x^2 + 23x + 10$

Multiply  $12 \times 10 = 120$

Find two numbers that multiply to 120 and add to 23: 8, 15

$$= (12x + 8)(12x + 15)$$

$$= \left( \frac{12x + 8}{4} \right) \left( \frac{12x + 15}{3} \right)$$

$$= (3x + 2)(4x + 5)$$

So, Length =  $4x + 5$  and Width =  $3x + 2$ .

$$\text{Perimeter} = 2(\text{Length} + \text{Width})$$

# Application Problem: Integral Values of $k$

## Integral Values of $k$

Ex: For which integral values of  $k$  can  $6x^2 + kx + 1$  be factored?

- ① 5, 7 *only*
- ②  $\pm 5, \pm 7$  *only*
- ③  $-5, -7$  *only*
- ④ all integers between  $-7$  and  $5$ , inclusive

# Application Problem: Integral Values of $k$ - Solution

## Detailed Solution

Ex: For which integral values of  $k$  can  $6x^2 + kx + 1$  be factored?

**Solution:** We need to find combinations of factors for  $6x^2$  and 1 that produce integral values for  $k$ .

- Factors of  $6x^2$ :  $(x, 6x), (2x, 3x)$
- Factors of 1:  $(1, 1), (-1, -1)$

Possible cross products for  $k$  (middle term):

- $(x + 1)(6x + 1) \rightarrow 1x + 6x = 7x \rightarrow k = 7$
- $(x - 1)(6x - 1) \rightarrow -1x - 6x = -7x \rightarrow k = -7$
- $(2x + 1)(3x + 1) \rightarrow 2x + 3x = 5x \rightarrow k = 5$
- $(2x - 1)(3x - 1) \rightarrow -2x - 3x = -5x \rightarrow k = -5$

The possible integral values for  $k$  are  $\pm 5, \pm 7$ . The correct option is **b)**  $\pm 5, \pm 7$  only.

## Key Concepts

- Understanding Trinomial Factoring when  $a \neq 1$
- B.U.M. Method for factoring trinomials
- Grouping Method for factoring trinomials
- Criss-Cross Method for factoring trinomials
- Mixed practice involving various factoring scenarios