Pre-Calculus 11

Chapter 3.8: Trigonometry Summary /

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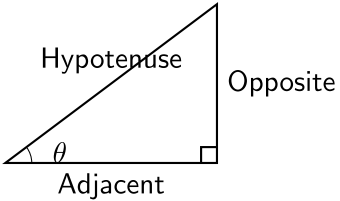
Chapter 3.8 Overview /

Topics Covered

- 3.1 Basic Trigonometric Functions
- 3.2 Angles in Standard Position
- 3.3 Special Triangles
- 3.4 Solving Angles in All Four Quadrants
- 3.5 Sine Law
- 3.6 Ambiguous Case of the Sine Law
- 3.7 Cosine Law

Key Points

- Sine, Cosine, Tangent: $\sin\theta$, $\cos\theta$, $\tan\theta$
- SOH-CAH-TOA: $\sin\theta = \frac{\text{Opp}}{\text{Hyp}}$, $\cos\theta = \frac{\text{Adj}}{\text{Hyp}}$, $\tan\theta = \frac{\text{Opp}}{\text{Adj}}$
- Pythagorean Theorem: $a^2 + b^2 = c^2$



3.1 Practice Problems

Practice

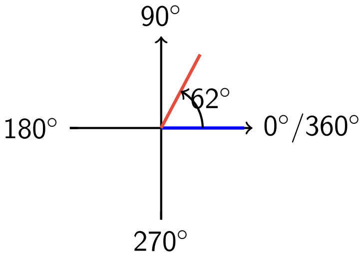
1. Find $\sin 30^\circ$, $\cos 60^\circ$, $\tan 45^\circ$.
2. In a right triangle, if $\theta = 37^\circ$ and hypotenuse $= 10$, find the length of the side opposite θ.
3. If $\sin\theta = 0.6$ and θ is acute, find $\cos\theta$.
4. Given a right triangle with legs 5 and 12, find all three basic trig ratios for the non-right angles.

Solutions

1. $\sin 30^\circ = 0.5, \ \cos 60^\circ = 0.5, \ \tan 45^\circ = 1$
2. $\text{Opposite} = 10 \times \sin 37^\circ \approx 6.02$
3. $\cos\theta = \sqrt{1 - (0.6)^2} = 0.8$
4. $\text{Hypotenuse} = \sqrt{5^2 + 12^2} = 13; \ \sin = 5/13, \ \cos = 12/13, \ \tan = 5/12$

Key Points

- Angles measured from the positive x-axis
- Quadrants I-IV
- Reference angle: always positive, between terminal arm and x-axis



Practice

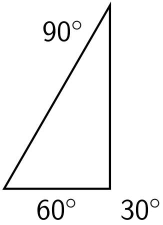
1. Draw 120° in standard position and label the reference angle.
2. What is the reference angle for 210°?
3. In which quadrant does -75° lie?
4. Find the reference angle for -135°.

Solutions

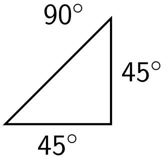
1. Reference angle is $180^\circ - 120^\circ = 60^\circ$
2. 210° is in QIII, reference angle $= 210^\circ - 180^\circ = 30^\circ$
3. -75° is in QIV
4. -135° is in QIII, reference angle $= 180^\circ - 135^\circ = 45^\circ$

Key Points

- 30°-60°-90° and 45°-45°-90° triangles
- Exact values for sin, cos, tan of special angles



30°-60°-90°



45°-45°-90°

Practice

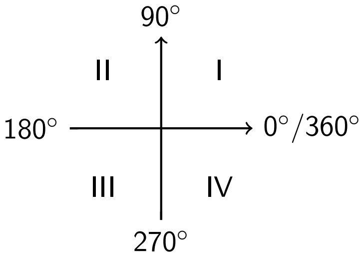
1. Find $\sin 60^\circ$ using a special triangle.
2. Find $\tan 30^\circ$ using a special triangle.
3. What are the side ratios for a 45°-45°-90° triangle?
4. Find $\cos 45^\circ$ using a special triangle.

Solutions

1. $\sin 60^\circ = \frac{\sqrt{3}}{2}$
2. $\tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
3. $1 : 1 : \sqrt{2}$
4. $\cos 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

Key Points

- ASTC rule: All Students Take Calculus (signs of trig functions in each quadrant)
- Reference angle method for finding all solutions



Practice

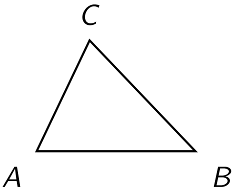
1. Find all angles $0^\circ < \theta < 360^\circ$ where $\sin\theta = 0.5$.
2. Find all angles $0^\circ < \theta < 360^\circ$ where $\tan\theta = -1$.
3. In which quadrants is $\cos\theta$ negative?
4. Find all solutions for $\sin\theta = -\frac{\sqrt{2}}{2}$ in $0^\circ < \theta < 360^\circ$.

Solutions

1. $\theta = 30^\circ, 150^\circ$
2. $\theta = 135^\circ, 315^\circ$
3. QII and QIII
4. $225^\circ, 315^\circ$

Key Points

- Sine Law: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
- Use for non-right triangles when you have an angle and its opposite side



Practice

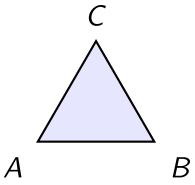
1. In $\triangle ABC$, $a = 7$, $A = 40^\circ$, $B = 60^\circ$. Find b.
2. In $\triangle ABC$, $a = 10$, $A = 30^\circ$, $c = 12$, $C = 80^\circ$. Find b.
3. In $\triangle ABC$, $a = 8$, $A = 50^\circ$, $b = 10$. Find B.
4. In $\triangle ABC$, $a = 9$, $A = 35^\circ$, $c = 11$. Find C.

Solutions

1. $\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow b = \frac{7 \sin 60^\circ}{\sin 40^\circ} \approx 9.25$
2. $\frac{a}{\sin A} = \frac{c}{\sin C} \Rightarrow b = \frac{a \sin B}{\sin A},\ B = 180^\circ - A - C = 70^\circ,\ b = \frac{10 \sin 70^\circ}{\sin 30^\circ} \approx 18.79$
3. $\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \sin B = \frac{b \sin A}{a} = \frac{10 \sin 50^\circ}{8} \approx 0.9589,\ B \approx 73.7^\circ$
4. $\frac{a}{\sin A} = \frac{c}{\sin C} \Rightarrow \sin C = \frac{11 \sin 35^\circ}{9} \approx 0.7005,\ C \approx 44.4^\circ$

Key Points

- SSA case: two sides and a non-included angle
- May yield 0, 1, or 2 possible triangles
- Check for ambiguous case when using Sine Law



Practice

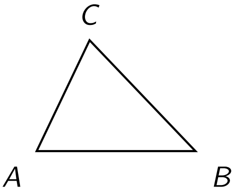
1. In $\triangle ABC$, $a = 8$, $b = 10$, $A = 30^\circ$. How many possible triangles?
2. In $\triangle ABC$, $a = 7$, $b = 12$, $A = 40^\circ$. Find all possible values for B.
3. In $\triangle ABC$, $a = 9$, $b = 8$, $A = 50^\circ$. Find all possible values for B.
4. In $\triangle ABC$, $a = 6$, $b = 5$, $A = 25^\circ$. How many possible triangles?

Solutions

1. $a < b$, $a > b \sin A$ so 2 triangles possible
2. $\sin B = \frac{12 \sin 40^\circ}{7} \approx 1.099$ (no triangle)
3. $\sin B = \frac{8 \sin 50^\circ}{9} \approx 0.682,\ B_1 = 43^\circ,\ B_2 = 137^\circ$
4. $\sin B = \frac{5 \sin 25^\circ}{6} \approx 0.352,\ B_1 = 20.6^\circ,\ B_2 = 159.4^\circ$

Key Points

- Cosine Law: $a^2 = b^2 + c^2 - 2bc\cos A$ (and cyclic)
- Use for non-right triangles with SAS or SSS



Practice

1. In $\triangle ABC$, $a = 7$, $b = 8$, $C = 60^\circ$. Find c.
2. In $\triangle ABC$, $a = 10$, $b = 12$, $c = 15$. Find A.
3. In $\triangle ABC$, $a = 9$, $b = 11$, $C = 45^\circ$. Find c.
4. In $\triangle ABC$, $a = 5$, $b = 7$, $c = 8$. Find B.

Solutions

1. $c^2 = 7^2 + 8^2 - 2 \times 7 \times 8 \cos 60^\circ = 49 + 64 - 112 \times 0.5 = 113 - 56 = 57,\ c = \sqrt{57} \approx 7.55$
2. $A = \cos^{-1}\left(\frac{10^2+12^2-15^2}{2\times10\times12}\right) = \cos^{-1}\left(\frac{100+144-225}{240}\right) = \cos^{-1}(-0.3375) \approx 110.7^\circ$
3. $c^2 = 9^2 + 11^2 - 2 \times 9 \times 11 \cos 45^\circ = 81 + 121 - 198 \times 0.7071 = 202 - 140 = 62,\ c = \sqrt{62} \approx 7.87$
4. $B = \cos^{-1}\left(\frac{5^2+8^2-7^2}{2\times5\times8}\right) = \cos^{-1}\left(\frac{25+64-49}{80}\right) = \cos^{-1}(0.5) = 60^\circ$

Comprehensive Review

1. Find $\sin 45^\circ$, $\cos 60^\circ$, $\tan 30^\circ$
2. Draw an angle of 120° in standard position and label the reference angle
3. Solve $\triangle ABC$ given $A = 40^\circ$, $a = 7$, $b = 10$
4. Use the Sine Law to find a missing side
5. Use the Cosine Law to find a missing angle
6. Explain the ambiguous case for Sine Law