

Chapter 5.3: Adding and Subtracting Rational Expressions

Rational Expressions - Lesson 3

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June 16, 2025

Adding and Subtracting Rational Expressions

Key Concepts

- To add or subtract two rational expressions, they **MUST** have a common denominator (LCD).
- LCD is **ONLY** needed for adding subtracting, **NOT** for multiplying/dividing.
- To get a common denominator, multiply each fraction by the missing terms.

Example: Find LCD and Add/Subtract

Example 1

Add:

$$\frac{7}{2x} - \frac{5}{3y}$$

Find LCD: $6xy$

$$\frac{7}{2x} \times \frac{3y}{3y} = \frac{21y}{6xy}$$

$$\frac{5}{3y} \times \frac{2x}{2x} = \frac{10x}{6xy}$$

Combine:

$$\frac{21y - 10x}{6xy}$$

Practice: Add or Subtract by finding the LCD

Practice Problems

Q1. $\frac{4}{3x} - \frac{6}{x}$

Q2. $\frac{5}{2m} + \frac{8}{3n}$

Q3. $\frac{5}{6m} - \frac{3}{4m}$

Q4. $\frac{y^2}{3x^2} + \frac{5x}{y}$

Q5. $\frac{3}{2ab} - \frac{2}{3bc} + \frac{4}{abc}$

Finding LCD with Binomials

Key Concepts

- If the denominator has a binomial, multiply in the missing binomial for each fraction.
- You cannot just add or subtract the missing term; you must multiply.

Example: Binomial Denominators

Example 2

Add:

$$\frac{4}{x-3} + \frac{2}{x}$$

LCD: $(x-3)x$

$$\frac{4}{x-3} \times \frac{x}{x} = \frac{4x}{x(x-3)}$$

$$\frac{2}{x} \times \frac{x-3}{x-3} = \frac{2(x-3)}{x(x-3)}$$

Combine:

$$\frac{4x + 2(x-3)}{x(x-3)} = \frac{4x + 2x - 6}{x(x-3)} = \frac{6x - 6}{x(x-3)}$$

Practice: Find the LCD and then Simplify

Practice Problems

Q6. $\frac{5}{x} + \frac{3}{x-2}$

Q7. $\frac{4}{x+1} - \frac{2}{x}$

Q8. $\frac{2}{x-1} + \frac{8}{x+4}$

Q9. $\frac{5x}{x-3} - \frac{2x}{x}$

Factor the Denominators

Key Concepts

- If the denominator is a trinomial or a difference of squares, always factor them first!
- After factoring, look for any missing terms or binomials to get the LCD.

Example: Factor Denominators

Example 3

Add:

$$\frac{2}{x^2 - 16} + \frac{8}{x + 4}$$

Factor: $x^2 - 16 = (x + 4)(x - 4)$ LCD: $(x + 4)(x - 4)$

$$\frac{2}{(x + 4)(x - 4)} + \frac{8(x - 4)}{(x + 4)(x - 4)} = \frac{2 + 8(x - 4)}{(x + 4)(x - 4)} = \frac{8x - 30}{(x + 4)(x - 4)}$$

Practice: Factor, Find LCD, and Simplify

Practice Problems

Q10. $\frac{2}{x^2-9} + \frac{5}{x+3}$

Q11. $\frac{4}{x^2-4} - \frac{2}{x-2}$

Q12. $\frac{3}{x^2-1} + \frac{2}{x-1}$

Q13. $\frac{5x}{x^2-4x+3} - \frac{2x}{x-1}$

Simplifying R.E. with Fractions

Key Concepts

- When simplifying rational expressions with fractions, multiply ALL terms by the LCD to cancel out denominators.
- Distribute and expand as needed.

Example: Simplifying with Fractions

Example 4

Simplify:

$$\frac{4}{2x} - \frac{3}{6x} + 1$$

LCD: $6x$

$$\frac{4}{2x} \times 3 = \frac{12}{6x}$$

$$\frac{3}{6x} \times 1 = \frac{3}{6x}$$

Combine:

$$\frac{12 - 3}{6x} + 1 = \frac{9}{6x} + 1$$

Practice: Simplify Each of the Following

Practice Problems

Q14. $\frac{4}{2x} - \frac{3}{6x} + 1$

Q15. $\frac{2}{2x} + \frac{8}{10x} + 1$

Q16. $\frac{7}{3x} - \frac{4}{2x} + 2$

Challenge Problem Q17

Challenge Q17

Simplify and state all NPV's:

$$\frac{2}{x^2 - 5x + 6} + \frac{3}{x^2 - 4}$$

Challenge Q17: Solution (Part 1)

Step-by-Step Solution (Part 1)

Step 1: Factor denominators

$$x^2 - 5x + 6 = (x - 2)(x - 3) \quad x^2 - 4 = (x - 2)(x + 2)$$

Step 2: LCD is $(x - 2)(x - 3)(x + 2)$ Step 3: Rewrite each fraction with LCD

$$\frac{2}{(x - 2)(x - 3)} \times \frac{x + 2}{x + 2} = \frac{2(x + 2)}{(x - 2)(x - 3)(x + 2)}$$

$$\frac{3}{(x - 2)(x + 2)} \times \frac{x - 3}{x - 3} = \frac{3(x - 3)}{(x - 2)(x - 3)(x + 2)}$$

Challenge Q17: Solution (Part 2)

Step-by-Step Solution (Part 2)

Step 4: Combine numerators

$$\frac{2(x+2) + 3(x-3)}{(x-2)(x-3)(x+2)} = \frac{2x+4+3x-9}{(x-2)(x-3)(x+2)} = \frac{5x-5}{(x-2)(x-3)(x+2)}$$

Step 5: Factor numerator if possible

$$5x - 5 = 5(x - 1)$$

Final Answer:

$$\frac{5(x-1)}{(x-2)(x-3)(x+2)}$$

NPV: $x \neq 2, 3, -2$

Challenge Problem Q18

Challenge Q18

Simplify and state all NPV's:

$$\frac{3x}{x^2 - 4x + 3} - \frac{2}{x^2 - 1} + \frac{5}{x^2 - 9}$$

Challenge Q18: Solution (Part 1)

Step-by-Step Solution (Part 1)

Step 1: Factor denominators

$$x^2 - 4x + 3 = (x - 3)(x - 1) \quad x^2 - 1 = (x - 1)(x + 1) \quad x^2 - 9 = (x - 3)(x + 3)$$

Step 2: LCD is $(x - 3)(x - 1)(x + 1)(x + 3)$ Step 3: Rewrite each fraction with LCD

$$\frac{3x}{(x - 3)(x - 1)} \times \frac{(x + 1)(x + 3)}{(x + 1)(x + 3)} = \frac{3x(x + 1)(x + 3)}{(x - 3)(x - 1)(x + 1)(x + 3)}$$

$$\frac{2}{(x - 1)(x + 1)} \times \frac{(x - 3)(x + 3)}{(x - 3)(x + 3)} = \frac{2(x - 3)(x + 3)}{(x - 3)(x - 1)(x + 1)(x + 3)}$$

$$\frac{5}{(x - 3)(x + 3)} \times \frac{(x - 1)(x + 1)}{(x - 1)(x + 1)} = \frac{5(x - 1)(x + 1)}{(x - 3)(x - 1)(x + 1)(x + 3)}$$

Challenge Q18: Solution (Part 2)

Step-by-Step Solution (Part 2)

Step 4: Combine numerators

$$\frac{3x(x+1)(x+3) - 2(x-3)(x+3) + 5(x-1)(x+1)}{(x-3)(x-1)(x+1)(x+3)}$$

Step 5: Expand and simplify numerator

$$3x(x+1)(x+3) = 3x(x^2 + 4x + 3) = 3x^3 + 12x^2 + 9x$$

$$-2(x-3)(x+3) = -2(x^2 - 9) = -2x^2 + 18$$

$$5(x-1)(x+1) = 5(x^2 - 1) = 5x^2 - 5$$

$$\text{Sum: } 3x^3 + 12x^2 + 9x - 2x^2 + 18 + 5x^2 - 5 = 3x^3 + (12x^2 - 2x^2 + 5x^2) + 9x + (18 - 5) = 3x^3 + 15x^2 + 9x + 13$$

Final Answer:

$$\frac{3x^3 + 15x^2 + 9x + 13}{(x-3)(x-1)(x+1)(x+3)}$$

NPV: $x \neq 3, 1, -1, -3$