Pre-Calculus 11

Sine Law

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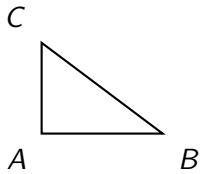
Sine Law

The Sine Law

- The Sine Law is used for solving triangles that are not right triangles.
- Name each side with the letter opposite its angle: a opposite A, b opposite B, c opposite C.
- **Sine Law:**

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

- You can use the Sine Law when you are given one angle and its opposite side.



Example: Solving for a Side

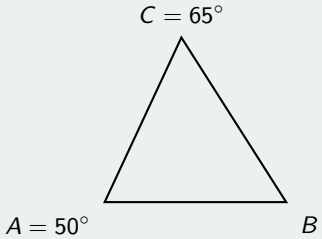
Question

In $\triangle ABC$, $a = 10$ cm, $A = 50^\circ$, $C = 65^\circ$. Solve for side c.

Example: Solving for a Side

Solution

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{10}{\sin 50^\circ} = \frac{c}{\sin 65^\circ}$$
$$c = \frac{10 \cdot \sin 65^\circ}{\sin 50^\circ} \approx 12.36 \text{ cm}$$



Practice: Find the Value of x

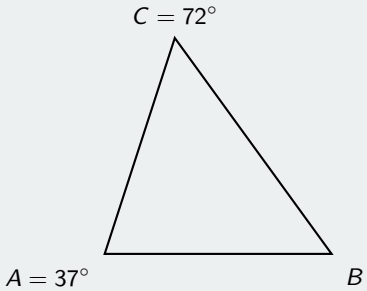
Practice

In $\triangle ABC$, $a = 15$ m, $A = 37^\circ$, $C = 72^\circ$, $c = x$. Find x.

Practice: Find the Value of x

Solution

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{15}{\sin 37^\circ} = \frac{x}{\sin 72^\circ}$$
$$x = \frac{15 \cdot \sin 72^\circ}{\sin 37^\circ} \approx 24.18 \text{ m}$$



Finding Missing Angles

Finding Angles with Sine Law

- To find an angle, use the inverse sine function.
- If the angle is obtuse, the answer is in Quadrant II: $\theta = 180^\circ - \sin^{-1}(\text{ratio})$.

Example: Solving for an Angle

Question

In $\triangle ABC$, $a = 11$ in, $A = 40^\circ$, $c = 8$ in. Solve for angle C.

Example: Solving for an Angle

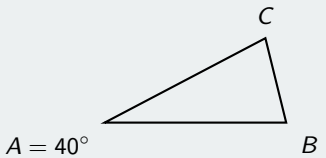
Solution

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{11}{\sin 40^\circ} = \frac{8}{\sin C}$$

$$\sin C = \frac{8 \cdot \sin 40^\circ}{11} \approx 0.4646$$

$$C = \sin^{-1}(0.4646) \approx 27.7^\circ$$



Practice: Solve for the Missing Angle

Practice

In $\triangle ABC$, $a = 7.5$, $A = 50^\circ$, $c = 15$. Solve for angle C.

Practice: Solve for the Missing Angle

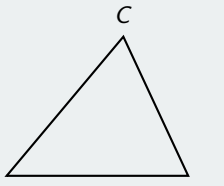
Solution

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{7.5}{\sin 50^\circ} = \frac{15}{\sin C}$$

$$\sin C = \frac{15 \cdot \sin 50^\circ}{7.5} \approx 1.148$$

No solution: $\sin C > 1$



Triangles with Obtuse Angles

Obtuse Angles

- An obtuse angle is greater than 90°.
- In the xy-plane, the angle will be in Quadrant II.
- If $\sin B = x$ and B is obtuse, $B = 180^\circ - \sin^{-1}(x)$.

Example: Obtuse Angle

Question

In $\triangle ABC$, $a = 13$ m, $b = 18.5$ m, $A = 40^\circ$. Solve for angle B.

Example: Obtuse Angle

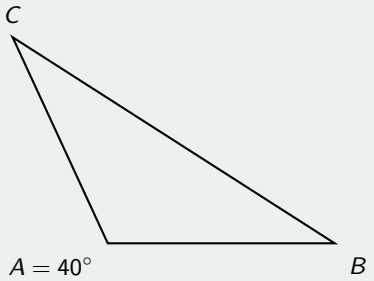
Solution

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{13}{\sin 40^\circ} = \frac{18.5}{\sin B}$$

$$\sin B = \frac{18.5 \cdot \sin 40^\circ}{13} \approx 0.914$$

$$B = 180^\circ - \sin^{-1}(0.914) \approx 114.8^\circ$$



Practice: Find the Obtuse Angle

Practice

In $\triangle ABC$, $a = 12$ m, $b = 19$ m, $A = 35^\circ$. Solve for angle B (obtuse).

Practice: Find the Obtuse Angle

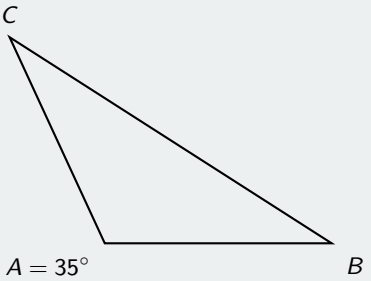
Solution

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{12}{\sin 35^\circ} = \frac{19}{\sin B}$$

$$\sin B = \frac{19 \cdot \sin 35^\circ}{12} \approx 0.908$$

$$B = 180^\circ - \sin^{-1}(0.908) \approx 114.8^\circ$$



Proof of the Sine Law

Proof

- Draw an altitude from one vertex and use right triangle trigonometry to show:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$