Implicit Differentiation

A Powerful Technique for Complex Functions

Differential Calculus

Outline

What is Implicit Differentiation?

- A technique to find derivatives when functions are not explicitly solved for y
- Used when you have an equation relating x and y but can't solve for y easily
- Also useful even when you have an explicit formula but the equation is simpler
- The key idea: differentiate both sides of the equation with respect to x

When to Use Implicit Differentiation

Two Main Cases

1. **No explicit formula:** When you can't solve for y in terms of x
2. **Complicated explicit formula:** When the equation is simpler than the explicit form

Examples:

- $x^2 + y^2 = 25$ (circle)
- $x^3 + y^3 = 6xy$ (folium of Descartes)
- $y = y^3 + xy + x^3$ (cubic equation)

The Basic Method

Step-by-Step Process

1. Start with an equation relating x and y
2. Differentiate both sides with respect to x
3. Remember that y is a function of x, so use the chain rule
4. Solve for $\frac{dy}{dx}$ (or y')

Key Rule: When differentiating terms with y, remember to multiply by $\frac{dy}{dx}$

Example 1: Circle

Find $\frac{dy}{dx}$ **for the circle** $x^2 + y^2 = 25$

Solution to Example 1

Solution:

$$
\begin{aligned}
x^2 + y^2 &= 25 \\
\frac{d}{dx}(x^2 + y^2) &= \frac{d}{dx}(25) \\
2x + 2y \cdot \frac{dy}{dx} &= 0 \\
2y \cdot \frac{dy}{dx} &= -2x \\
\frac{dy}{dx} &= -\frac{x}{y}
\end{aligned}
$$

Example 2: Ellipse

Find $\frac{dy}{dx}$ **for the ellipse** $3x^2 + 5y^2 = 7$

Solution to Example 2

Solution:

$$\begin{aligned} 3x^2 + 5y^2 &= 7 \\ \frac{d}{dx}(3x^2 + 5y^2) &= \frac{d}{dx}(7) \\ 6x + 10y \cdot \frac{dy}{dx} &= 0 \\ 10y \cdot \frac{dy}{dx} &= -6x \\ \frac{dy}{dx} &= -\frac{3x}{5y} \end{aligned}$$

Example 3: Cubic Equation

Find $\frac{dy}{dx}$ **for** $y = y^3 + xy + x^3$

Solution to Example 3

Solution:

$$
\begin{aligned}
y &= y^3 + xy + x^3 \\
\frac{dy}{dx} &= 3y^2 \cdot \frac{dy}{dx} + x \cdot \frac{dy}{dx} + y + 3x^2 \\
\frac{dy}{dx} - 3y^2 \cdot \frac{dy}{dx} - x \cdot \frac{dy}{dx} &= y + 3x^2 \\
\frac{dy}{dx}(1 - 3y^2 - x) &= y + 3x^2 \\
\frac{dy}{dx} &= \frac{y + 3x^2}{1 - 3y^2 - x}
\end{aligned}
$$

Finding Tangent Lines

Method

1. Find the point (x_0, y_0) on the curve
2. Use implicit differentiation to find $\frac{dy}{dx}$
3. Evaluate $\frac{dy}{dx}$ at (x_0, y_0) to get the slope
4. Use point-slope form: $y = y_0 + m(x - x_0)$

Example: Tangent to Circle

Find the tangent line to $x^2 + y^2 = 25$ **at** $(3, 4)$

Solution: Tangent to Circle

Solution:

$$\text{From earlier: } \frac{dy}{dx} = -\frac{x}{y}$$

$$\text{At } (3,4): \frac{dy}{dx} = -\frac{3}{4}$$

$$\begin{aligned}\text{Tangent line: } y &= 4 - \frac{3}{4}(x-3) \\ y &= 4 - \frac{3}{4}x + \frac{9}{4} \\ y &= -\frac{3}{4}x + \frac{25}{4}\end{aligned}$$

Example: Astroid

Find $\frac{dy}{dx}$ **for the astroid** $x^{2/3} + y^{2/3} = 1$

Solution: Astroid

Solution:

$$
\begin{aligned}
x^{2/3} + y^{2/3} &= 1 \\
\frac{d}{dx}(x^{2/3} + y^{2/3}) &= \frac{d}{dx}(1) \\
\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3} \cdot \frac{dy}{dx} &= 0 \\
\frac{2}{3}y^{-1/3} \cdot \frac{dy}{dx} &= -\frac{2}{3}x^{-1/3} \\
\frac{dy}{dx} &= -\frac{x^{-1/3}}{y^{-1/3}} \\
\frac{dy}{dx} &= -\left(\frac{y}{x}\right)^{1/3}
\end{aligned}
$$

Example: Folium of Descartes

Find $\frac{dy}{dx}$ **for** $x^3 + y^3 = 6xy$

Solution: Folium of Descartes

Solution:

$$
\begin{aligned}
x^3 + y^3 &= 6xy \\
\frac{d}{dx}(x^3 + y^3) &= \frac{d}{dx}(6xy) \\
3x^2 + 3y^2 \cdot \frac{dy}{dx} &= 6y + 6x \cdot \frac{dy}{dx} \\
3y^2 \cdot \frac{dy}{dx} - 6x \cdot \frac{dy}{dx} &= 6y - 3x^2 \\
\frac{dy}{dx}(3y^2 - 6x) &= 6y - 3x^2 \\
\frac{dy}{dx} &= \frac{6y - 3x^2}{3y^2 - 6x} \\
\frac{dy}{dx} &= \frac{2y - x^2}{y^2 - 2x}
\end{aligned}
$$

Practice: 1 and 2

Practice 1:

$$\text{Find } \frac{dy}{dx} \text{ for } x^2 + y^2 = 16$$

Practice 2:

$$\text{Find } \frac{dy}{dx} \text{ for } 4x^2 + 9y^2 = 36$$

Practice: 3 and 4

Practice 3:

$$\text{Find } \frac{dy}{dx} \text{ for } x^2 - y^2 = 9$$

Practice 4:

$$\text{Find } \frac{dy}{dx} \text{ for } xy = 4$$

Practice: 5 and 6

Practice 5:

$$\text{Find } \frac{dy}{dx} \text{ for } x^2 + xy + y^2 = 3$$

Practice 6:

$$\text{Find } \frac{dy}{dx} \text{ for } x^3 + y^3 = 8$$

Practice: 7 and 8

Practice 7:

Find the tangent line to $x^2 + y^2 = 25$ at $(4, 3)$

Practice 8:

Find the tangent line to $x^2 - y^2 = 7$ at $(4, 3)$

Practice: 9 and 10

Practice 9:

$$\text{Find } \frac{dy}{dx} \text{ for } x^2 + y^2 = 2xy$$

Practice 10:

$$\text{Find } \frac{dy}{dx} \text{ for } x^3 + y^3 = 3xy$$

Practice: 11 and 12

Practice 11:

$$\text{Find } \frac{dy}{dx} \text{ for } x^2 + y^2 = x^2 y^2$$

Practice 12:

$$\text{Find } \frac{dy}{dx} \text{ for } x^4 + y^4 = 16$$

Practice: 13 and 14

Practice 13:

$$\text{Find } \frac{dy}{dx} \text{ for } x^2 + y^2 = e^{xy}$$

Practice 14:

$$\text{Find } \frac{dy}{dx} \text{ for } \sin(xy) = x + y$$

Practice: 15 and 16

Practice 15:

$$\text{Find } \frac{dy}{dx} \text{ for } x^2 + y^2 = \ln(xy)$$

Practice 16:

$$\text{Find } \frac{dy}{dx} \text{ for } x^3 + y^3 = 6xy^2$$

Practice: 17 and 18

Practice 17:

$$\text{Find } \frac{dy}{dx} \text{ for } x^{2/3} + y^{2/3} = 4$$

Practice 18:

$$\text{Find } \frac{dy}{dx} \text{ for } x^3 + y^3 = 3x^2y$$

Practice: 19 and 20

Practice 19:

$$\text{Find } \frac{dy}{dx} \text{ for } x^2 + y^2 = \sin(xy)$$

Practice 20:

$$\text{Find } \frac{dy}{dx} \text{ for } e^{x^2} + e^{y^2} = e^{xy}$$

Practice: 21 and 22

Practice 21:

$$\text{Find } \frac{dy}{dx} \text{ for } x^4 + y^4 = x^2y^2$$

Practice 22:

$$\text{Find } \frac{dy}{dx} \text{ for } \ln(x^2 + y^2) = 2xy$$

Practice: 23 and 24

Practice 23:

$$\text{Find } \frac{dy}{dx} \text{ for } x^3 + y^3 = 3xy + 1$$

Practice 24:

$$\text{Find } \frac{dy}{dx} \text{ for } x^2 + y^2 = \cos(xy)$$

Practice: 25

Practice 25:

$$\text{Find } \frac{dy}{dx} \text{ for } x^5 + y^5 = 5x^2y^3$$

Solution to Practice 1

Practice 1:
Find $\frac{dy}{dx}$ for $x^2 + y^2 = 16$
Solution:

$$\begin{aligned} x^2 + y^2 &= 16 \\ 2x + 2y \cdot \frac{dy}{dx} &= 0 \\ 2y \cdot \frac{dy}{dx} &= -2x \\ \frac{dy}{dx} &= -\frac{x}{y} \end{aligned}$$

Solution to Practice 2

Practice 2:
Find $\frac{dy}{dx}$ for $4x^2 + 9y^2 = 36$
Solution:

$$\begin{aligned} 4x^2 + 9y^2 &= 36 \\ 8x + 18y \cdot \frac{dy}{dx} &= 0 \\ 18y \cdot \frac{dy}{dx} &= -8x \\ \frac{dy}{dx} &= -\frac{4x}{9y} \end{aligned}$$

Solution to Practice 3

Practice 3:
Find $\frac{dy}{dx}$ for $x^2 - y^2 = 9$
Solution:

$$\begin{aligned} x^2 - y^2 &= 9 \\ 2x - 2y \cdot \frac{dy}{dx} &= 0 \\ -2y \cdot \frac{dy}{dx} &= -2x \\ \frac{dy}{dx} &= \frac{x}{y} \end{aligned}$$

Solution to Practice 4

Practice 4:
Find $\frac{dy}{dx}$ for $xy = 4$
Solution:

$$\begin{aligned} xy &= 4 \\ x \cdot \frac{dy}{dx} + y \cdot 1 &= 0 \\ x \cdot \frac{dy}{dx} &= -y \\ \frac{dy}{dx} &= -\frac{y}{x} \end{aligned}$$

Solution to Practice 5

Practice 5:
Find $\frac{dy}{dx}$ for $x^2 + xy + y^2 = 3$
Solution:

$$\begin{aligned}
x^2 + xy + y^2 &= 3 \\
2x + x \cdot \frac{dy}{dx} + y + 2y \cdot \frac{dy}{dx} &= 0 \\
x \cdot \frac{dy}{dx} + 2y \cdot \frac{dy}{dx} &= -2x - y \\
\frac{dy}{dx}(x + 2y) &= -2x - y \\
\frac{dy}{dx} &= -\frac{2x + y}{x + 2y}
\end{aligned}$$

Solution to Practice 6

Practice 6:
Find $\frac{dy}{dx}$ for $x^3 + y^3 = 8$
Solution:

$$\begin{aligned} x^3 + y^3 &= 8 \\ 3x^2 + 3y^2 \cdot \frac{dy}{dx} &= 0 \\ 3y^2 \cdot \frac{dy}{dx} &= -3x^2 \\ \frac{dy}{dx} &= -\frac{x^2}{y^2} \end{aligned}$$

Solution to Practice 7

Practice 7:
Find the tangent line to $x^2 + y^2 = 25$ at $(4, 3)$
Solution:

$$\frac{dy}{dx} = -\frac{x}{y}$$

$$\text{At } (4,3): \frac{dy}{dx} = -\frac{4}{3}$$

$$\text{Tangent line: } y = 3 - \frac{4}{3}(x - 4)$$

$$y = 3 - \frac{4}{3}x + \frac{16}{3}$$

$$y = -\frac{4}{3}x + \frac{25}{3}$$

Solution to Practice 8

Practice 8:

Find the tangent line to $x^2 - y^2 = 7$ at $(4, 3)$

Solution:

$$\frac{dy}{dx} = \frac{x}{y}$$

$$\text{At } (4,3): \frac{dy}{dx} = \frac{4}{3}$$

$$\text{Tangent line: } y = 3 + \frac{4}{3}(x - 4)$$

$$y = 3 + \frac{4}{3}x - \frac{16}{3}$$

$$y = \frac{4}{3}x - \frac{7}{3}$$

Solution to Practice 9

Practice 9:
Find $\frac{dy}{dx}$ for $x^2 + y^2 = 2xy$
Solution:

$$
\begin{aligned}
x^2 + y^2 &= 2xy \\
2x + 2y \cdot \frac{dy}{dx} &= 2x \cdot \frac{dy}{dx} + 2y \\
2y \cdot \frac{dy}{dx} - 2x \cdot \frac{dy}{dx} &= 2y - 2x \\
\frac{dy}{dx}(2y - 2x) &= 2y - 2x \\
\frac{dy}{dx} &= 1
\end{aligned}
$$

Solution to Practice 10

Practice 10:
Find $\frac{dy}{dx}$ for $x^3 + y^3 = 3xy$
Solution:

$$
\begin{aligned}
x^3 + y^3 &= 3xy \\
3x^2 + 3y^2 \cdot \frac{dy}{dx} &= 3x \cdot \frac{dy}{dx} + 3y \\
3y^2 \cdot \frac{dy}{dx} - 3x \cdot \frac{dy}{dx} &= 3y - 3x^2 \\
\frac{dy}{dx}(3y^2 - 3x) &= 3y - 3x^2 \\
\frac{dy}{dx} &= \frac{y - x^2}{y^2 - x}
\end{aligned}
$$

Solution to Practice 11

Practice 11:
Find $\frac{dy}{dx}$ for $x^2 + y^2 = x^2y^2$
Solution:

$$x^2 + y^2 = x^2y^2$$

$$2x + 2y \cdot \frac{dy}{dx} = 2x \cdot y^2 + x^2 \cdot 2y \cdot \frac{dy}{dx}$$

$$2y \cdot \frac{dy}{dx} - 2x^2y \cdot \frac{dy}{dx} = 2xy^2 - 2x$$

$$\frac{dy}{dx}(2y - 2x^2y) = 2xy^2 - 2x$$

$$\frac{dy}{dx} = \frac{xy^2 - x}{y - x^2y}$$

Solution to Practice 12

Practice 12:
Find $\frac{dy}{dx}$ for $x^4 + y^4 = 16$
Solution:

$$\begin{aligned} x^4 + y^4 &= 16 \\ 4x^3 + 4y^3 \cdot \frac{dy}{dx} &= 0 \\ 4y^3 \cdot \frac{dy}{dx} &= -4x^3 \\ \frac{dy}{dx} &= -\frac{x^3}{y^3} \end{aligned}$$

Solution to Practice 13

Practice 13:
Find $\frac{dy}{dx}$ for $x^2 + y^2 = e^{xy}$
Solution:

$$\begin{aligned}
x^2 + y^2 &= e^{xy} \\
2x + 2y \cdot \frac{dy}{dx} &= e^{xy} \cdot \left(x \cdot \frac{dy}{dx} + y\right) \\
2x + 2y \cdot \frac{dy}{dx} &= xe^{xy} \cdot \frac{dy}{dx} + ye^{xy} \\
2y \cdot \frac{dy}{dx} - xe^{xy} \cdot \frac{dy}{dx} &= ye^{xy} - 2x \\
\frac{dy}{dx}(2y - xe^{xy}) &= ye^{xy} - 2x \\
\frac{dy}{dx} &= \frac{ye^{xy} - 2x}{2y - xe^{xy}}
\end{aligned}$$

Solution to Practice 14

Practice 14:
Find $\frac{dy}{dx}$ for $\sin(xy) = x + y$
Solution:

$$\begin{aligned}
\sin(xy) &= x + y \\
\cos(xy) \cdot \left(x \cdot \frac{dy}{dx} + y\right) &= 1 + \frac{dy}{dx} \\
x\cos(xy) \cdot \frac{dy}{dx} + y\cos(xy) &= 1 + \frac{dy}{dx} \\
x\cos(xy) \cdot \frac{dy}{dx} - \frac{dy}{dx} &= 1 - y\cos(xy) \\
\frac{dy}{dx}(x\cos(xy) - 1) &= 1 - y\cos(xy) \\
\frac{dy}{dx} &= \frac{1 - y\cos(xy)}{x\cos(xy) - 1}
\end{aligned}$$

Solution to Practice 15

Practice 15:
Find $\frac{dy}{dx}$ for $x^2 + y^2 = \ln(xy)$
Solution:

$$x^2 + y^2 = \ln(xy)$$

$$2x + 2y \cdot \frac{dy}{dx} = \frac{1}{xy} \cdot (x \cdot \frac{dy}{dx} + y)$$

$$2x + 2y \cdot \frac{dy}{dx} = \frac{1}{y} \cdot \frac{dy}{dx} + \frac{1}{x}$$

$$2y \cdot \frac{dy}{dx} - \frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{x} - 2x$$

$$\frac{dy}{dx}(2y - \frac{1}{y}) = \frac{1}{x} - 2x$$

$$\frac{dy}{dx} = \frac{\frac{1}{x} - 2x}{2y - \frac{1}{y}}$$

Solution to Practice 16

Practice 16:
Find $\frac{dy}{dx}$ for $x^3 + y^3 = 6xy^2$
Solution:

$$\begin{aligned} x^3 + y^3 &= 6xy^2 \\ 3x^2 + 3y^2 \cdot \frac{dy}{dx} &= 6x \cdot 2y \cdot \frac{dy}{dx} + 6y^2 \\ 3y^2 \cdot \frac{dy}{dx} - 12xy \cdot \frac{dy}{dx} &= 6y^2 - 3x^2 \\ \frac{dy}{dx}(3y^2 - 12xy) &= 6y^2 - 3x^2 \\ \frac{dy}{dx} &= \frac{6y^2 - 3x^2}{3y^2 - 12xy} \end{aligned}$$

Solution to Practice 17

Practice 17:
Find $\frac{dy}{dx}$ for $x^{2/3} + y^{2/3} = 4$
Solution:

$$\begin{aligned}
x^{2/3} + y^{2/3} &= 4 \\
\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3} \cdot \frac{dy}{dx} &= 0 \\
\frac{2}{3}y^{-1/3} \cdot \frac{dy}{dx} &= -\frac{2}{3}x^{-1/3} \\
\frac{dy}{dx} &= -\frac{x^{-1/3}}{y^{-1/3}} \\
\frac{dy}{dx} &= -\left(\frac{y}{x}\right)^{1/3}
\end{aligned}$$

Solution to Practice 18

Practice 18:
Find $\frac{dy}{dx}$ for $x^3 + y^3 = 3x^2y$
Solution:

$$\begin{aligned}
x^3 + y^3 &= 3x^2y \\
3x^2 + 3y^2 \cdot \frac{dy}{dx} &= 6xy + 3x^2 \cdot \frac{dy}{dx} \\
3y^2 \cdot \frac{dy}{dx} - 3x^2 \cdot \frac{dy}{dx} &= 6xy - 3x^2 \\
\frac{dy}{dx}(3y^2 - 3x^2) &= 6xy - 3x^2 \\
\frac{dy}{dx} &= \frac{6xy - 3x^2}{3y^2 - 3x^2}
\end{aligned}$$

Solution to Practice 19

Practice 19:
Find $\frac{dy}{dx}$ for $x^2 + y^2 = \sin(xy)$
Solution:

$$\begin{aligned}
x^2 + y^2 &= \sin(xy) \\
2x + 2y \cdot \frac{dy}{dx} &= \cos(xy) \cdot (x \cdot \frac{dy}{dx} + y) \\
2x + 2y \cdot \frac{dy}{dx} &= x\cos(xy) \cdot \frac{dy}{dx} + y\cos(xy) \\
2y \cdot \frac{dy}{dx} - x\cos(xy) \cdot \frac{dy}{dx} &= y\cos(xy) - 2x \\
\frac{dy}{dx}(2y - x\cos(xy)) &= y\cos(xy) - 2x \\
\frac{dy}{dx} &= \frac{y\cos(xy) - 2x}{2y - x\cos(xy)}
\end{aligned}$$

Solution to Practice 20

Practice 20:
Find $\frac{dy}{dx}$ for $e^{x^2} + e^{y^2} = e^{xy}$
Solution:

$$\begin{aligned}
e^{x^2} + e^{y^2} &= e^{xy} \\
e^{x^2} \cdot 2x + e^{y^2} \cdot 2y \cdot \frac{dy}{dx} &= e^{xy} \cdot (x \cdot \frac{dy}{dx} + y) \\
2xe^{x^2} + 2ye^{y^2} \cdot \frac{dy}{dx} &= xe^{xy} \cdot \frac{dy}{dx} + ye^{xy} \\
2ye^{y^2} \cdot \frac{dy}{dx} - xe^{xy} \cdot \frac{dy}{dx} &= ye^{xy} - 2xe^{x^2} \\
\frac{dy}{dx}(2ye^{y^2} - xe^{xy}) &= ye^{xy} - 2xe^{x^2} \\
\frac{dy}{dx} &= \frac{ye^{xy} - 2xe^{x^2}}{2ye^{y^2} - xe^{xy}}
\end{aligned}$$

Practice 21:
Find $\frac{dy}{dx}$ for $x^4 + y^4 = x^2y^2$
Solution:

$$\begin{aligned}
x^4 + y^4 &= x^2y^2 \\
4x^3 + 4y^3 \cdot \frac{dy}{dx} &= 2x \cdot y^2 + x^2 \cdot 2y \cdot \frac{dy}{dx} \\
4x^3 + 4y^3 \cdot \frac{dy}{dx} &= 2xy^2 + 2x^2y \cdot \frac{dy}{dx} \\
4y^3 \cdot \frac{dy}{dx} - 2x^2y \cdot \frac{dy}{dx} &= 2xy^2 - 4x^3 \\
\frac{dy}{dx}(4y^3 - 2x^2y) &= 2xy^2 - 4x^3 \\
\frac{dy}{dx} &= \frac{2xy^2 - 4x^3}{4y^3 - 2x^2y}
\end{aligned}$$

Solution to Practice 22

Practice 22:
Find $\frac{dy}{dx}$ for $\ln(x^2+y^2)=2xy$
Solution:

$$\begin{aligned}
\ln(x^2+y^2) &= 2xy\\
\frac{1}{x^2+y^2}\cdot(2x+2y\cdot\frac{dy}{dx}) &= 2x\cdot\frac{dy}{dx}+2y\\
\frac{2x}{x^2+y^2}+\frac{2y}{x^2+y^2}\cdot\frac{dy}{dx} &= 2x\cdot\frac{dy}{dx}+2y\\
\frac{2y}{x^2+y^2}\cdot\frac{dy}{dx}-2x\cdot\frac{dy}{dx} &= 2y-\frac{2x}{x^2+y^2}\\
\frac{dy}{dx}(\frac{2y}{x^2+y^2}-2x) &= 2y-\frac{2x}{x^2+y^2}\\
\frac{dy}{dx} &= \frac{2y-\frac{2x}{x^2+y^2}}{\frac{2y}{x^2+y^2}-2x}
\end{aligned}$$

Practice 23:
Find $\frac{dy}{dx}$ for $x^3 + y^3 = 3xy + 1$
Solution:

$$\begin{aligned} x^3 + y^3 &= 3xy + 1 \\ 3x^2 + 3y^2 \cdot \frac{dy}{dx} &= 3x \cdot \frac{dy}{dx} + 3y \\ 3y^2 \cdot \frac{dy}{dx} - 3x \cdot \frac{dy}{dx} &= 3y - 3x^2 \\ \frac{dy}{dx}(3y^2 - 3x) &= 3y - 3x^2 \\ \frac{dy}{dx} &= \frac{y - x^2}{y^2 - x} \end{aligned}$$

Solution to Practice 24

Practice 24:
Find $\frac{dy}{dx}$ for $x^2 + y^2 = \cos(xy)$
Solution:

$$\begin{aligned}
x^2 + y^2 &= \cos(xy) \\
2x + 2y \cdot \frac{dy}{dx} &= -\sin(xy) \cdot (x \cdot \frac{dy}{dx} + y) \\
2x + 2y \cdot \frac{dy}{dx} &= -x\sin(xy) \cdot \frac{dy}{dx} - y\sin(xy) \\
2y \cdot \frac{dy}{dx} + x\sin(xy) \cdot \frac{dy}{dx} &= -y\sin(xy) - 2x \\
\frac{dy}{dx}(2y + x\sin(xy)) &= -y\sin(xy) - 2x \\
\frac{dy}{dx} &= \frac{-y\sin(xy) - 2x}{2y + x\sin(xy)}
\end{aligned}$$

Solution to Practice 25

Practice 25:
Find $\frac{dy}{dx}$ for $x^5 + y^5 = 5x^2y^3$
Solution:

$$
\begin{aligned}
x^5 + y^5 &= 5x^2y^3 \\
5x^4 + 5y^4 \cdot \frac{dy}{dx} &= 10x \cdot y^3 + 5x^2 \cdot 3y^2 \cdot \frac{dy}{dx} \\
5x^4 + 5y^4 \cdot \frac{dy}{dx} &= 10xy^3 + 15x^2y^2 \cdot \frac{dy}{dx} \\
5y^4 \cdot \frac{dy}{dx} - 15x^2y^2 \cdot \frac{dy}{dx} &= 10xy^3 - 5x^4 \\
\frac{dy}{dx}(5y^4 - 15x^2y^2) &= 10xy^3 - 5x^4 \\
\frac{dy}{dx} &= \frac{10xy^3 - 5x^4}{5y^4 - 15x^2y^2}
\end{aligned}
$$

Key Points - Implicit Differentiation

- **When to use:** When functions are not explicitly solved for y
- **Method:** Differentiate both sides with respect to x
- **Key rule:** Remember that y is a function of x, so use chain rule
- **Goal:** Solve for $\frac{dy}{dx}$

Common Applications

- **Conic sections:** Circles, ellipses, hyperbolas
- **Curves:** Astroids, foliums, and other complex curves
- **Tangent lines:** Finding slopes and equations
- **Related rates:** When variables are related by equations

Implicit differentiation is a powerful tool for finding derivatives when explicit formulas are difficult or impossible to obtain.

Thank You!

Questions?

Implicit differentiation opens up a whole new world of functions to differentiate!