Lesson 7: Word Problems with Quadratic Functions

Yi-Chen Lin

June 10, 2025

Table of Contents

1. Recap
2. Applications
3. Example 1
4. Example 2
5. Example 3
6. Example 4
7. Practice Problems

I) Recap on What We Learned So Far

Key Concepts

- The vertex is for finding the Max/Minimum of something
- The x-intercepts is when the height is zero
- The y-intercept is when $t=0$ or $x=0$, height at the beginning
- The axis of symmetry is the x-value or the time when an object is at the vertex (highest point) or lowest point

I) General Form and APQ Form

Forms

General Form:

$$y = ax^2 + bx + c$$

$$x = -\frac{b}{2a} \text{ (Axis of Symmetry)}$$

APQ Form:

$$y = a(x-p)^2 + q$$

Vertex: (p, q) A.O.S. $x = p$

II) Applications of Quadratic Functions

Applications

- Finance and Business:
  - Maximize Revenue and Profit
- Construction and Infrastructure
  - Maximize the area of a park, building, room
- Coding and Programming
  - Maximize efficiency: minimize resources

II) Common Terms in Word Problems

Common Terms

- Sum $\to$ Add: $x + y = 10$
- Difference $\to$ Subtract: $x - y = 8$
- Product $\to$ Multiply: $M = x \cdot y$
- Sum of squares $\to$ $Min = x^2 + y^2$
- Perimeter $\to$ $P = 2L + 2W$

III) Example 1: Product of Numbers

Problem

The sum of two numbers is 95. Their product is 2100. Find the numbers.

III) Example 1: Solution - Part 1

Solution - Part 1

1. Write expressions:
   - 1st value: x
   - 2nd value: $95 - x$
2. Product equation:

$$x(95 - x) = 2100$$

Solution - Part 2

$$
\begin{aligned}
95x - x^2 &= 2100 \\
-x^2 + 95x - 2100 &= 0 \\
x^2 - 95x + 2100 &= 0 \\
(x - 47.5)^2 &= 156.25 \\
x - 47.5 &= \pm 12.5 \\
x &= 47.5 \pm 12.5 \\
x &= 60 \text{ or } 35
\end{aligned}
$$

The two numbers are 35 and 60.

IV) Example 2: Sum of Squares

Problem

The difference of two numbers is 12. The sum of their squares is 74. Find the numbers.

IV) Example 2: Solution - Part 1

Solution - Part 1

1. Write expressions:
   - 1st value: x
   - 2nd value: $x + 12$
2. Sum of squares equation:

$$x^2 + (x + 12)^2 = 74$$

IV) Example 2: Solution - Part 2

Solution - Part 2

$$\begin{aligned}
x^2 + x^2 + 24x + 144 &= 74 \\
2x^2 + 24x + 70 &= 0 \\
x^2 + 12x + 35 &= 0 \\
(x+6)^2 &= 1 \\
x + 6 &= \pm 1 \\
x &= -6 \pm 1
\end{aligned}$$

The two sets of numbers are:

$$x = -5, y = 7$$

$$x = -7, y = 5$$

V) Example 3: Projectile Motion

Problem

The height, "H" metres, of a baseball "T" seconds after being hit is given by:

$$H = 35T - 5T^2$$

Find:

1. Height after 3 seconds
2. Time at height of 25m
3. Maximum height and when it occurs
4. When the ball hits the ground

V) Example 3: Solution - Part 1

Solution - Part 1

1. Height after 3 seconds:

$$H = 35(3) - 5(3)^2 = 105 - 45 = 60 \text{ metres}$$

2. Time at height of 25m:

$$25 = 35T - 5T^2$$
$$5T^2 - 35T + 25 = 0$$
$$T = 0.82 \text{ sec or } 6.18 \text{ sec}$$

V) Example 3: Solution - Part 2

Solution - Part 2

1. Maximum height:

$$T = -\frac{b}{2a} = -\frac{35}{2(-5)} = 3.5 \text{ seconds}$$

$$H = 35(3.5) - 5(3.5)^2 = 122.5 - 61.25 = 61.25 \text{ metres}$$

2. Time to hit ground:

$$0 = 35T - 5T^2$$

$$T = 0 \text{ or } 7 \text{ seconds}$$

VI) Example 4: Maximum Area

Problem

A farmer wants to build a rectangular barn using 120 meters of fencing for his cows and chickens. He needs to separate the two groups and make the largest possible area. Determine the dimensions.

VI) Example 4: Solution - Part 1

Solution - Part 1

1. Perimeter equation: $2L + 3W = 120$
2. Area equation: $A = L \times W$
3. Isolate L: $L = -1.5W + 60$
4. Substitute into area equation: $A = W(-1.5W + 60)$

VI) Example 4: Solution - Part 2

Solution - Part 2

1. Complete the square:
$$A = -1.5(W - 20)^2 + 600$$
2. Maximum area: $600m^2$
3. Dimensions:
   - Width: $20m$
   - Length: $30m$

VII) Practice Problem 1: Rectangle Area

Problem

A rectangle has a perimeter of 80 meters. Find the dimensions that will give the maximum area.

VII) Practice Problem 1: Solution

Solution

1. Perimeter equation: $2L + 2W = 80$
2. Area equation: $A = L \times W$
3. Isolate L: $L = 40 - W$
4. Substitute: $A = W(40 - W)$
5. Complete the square:
$$A = -(W - 20)^2 + 400$$
6. Maximum area: $400m^2$
7. Dimensions: $20m \times 20m$ (square)

VII) Practice Problem 2: Number Product

Problem

Find two numbers whose sum is 50 and whose product is a maximum.

VII) Practice Problem 2: Solution

Solution

1. Let first number be x
2. Second number is $50 - x$
3. Product equation: $P = x(50 - x)$
4. Complete the square:
$$P = -(x - 25)^2 + 625$$
5. Maximum product: 625
6. Numbers: 25 and 25

Problem

A ball is thrown upward from a height of 2 meters with an initial velocity of 40 m/s. The height is given by:

$$H = -5T^2 + 40T + 2$$

Find:

1. Maximum height
2. Time to reach maximum height
3. When the ball hits the ground

VII) Practice Problem 3: Solution

Solution

1. Time to maximum height:

$$T = -\frac{b}{2a} = -\frac{40}{2(-5)} = 4 \text{ seconds}$$

2. Maximum height:

$$H = -5(4)^2 + 40(4) + 2 = 82 \text{ meters}$$

3. Time to hit ground:

$$0 = -5T^2 + 40T + 2$$

$$T = 8.05 \text{ seconds}$$