Pre-Calculus 11

3.7 Cosine Law /

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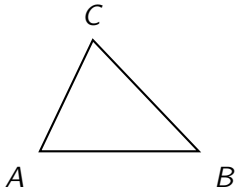
I) What is the Cosine Law? /

Cosine Law

- The Cosine Law is for solving triangles that are **not right triangles** ().
- Use Cosine Law when you have:
  - 2 sides and the angle in between (SAS)
  - All 3 sides and need to find an angle (SSS)

Cosine Law Formulas:

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$



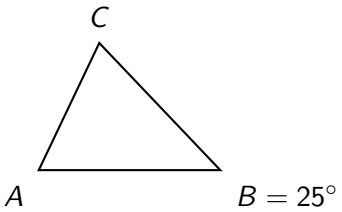
To find an angle:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Example 1: Find the Missing Side / 1

Question

In $\triangle ABC$, $B = 25°$, $c = 15$, $a = 7$. Find b.



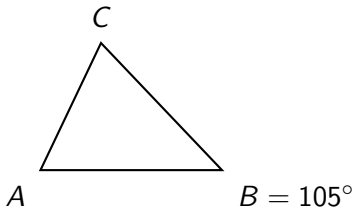
Example 1: Solution /

$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$b^2 = 7^2 + 15^2 - 2 \times 7 \times 15\cos 25^\circ$$
$$b^2 = 49 + 225 - 210 \times 0.9063$$
$$b^2 = 274 - 190.32 = 83.68$$
$$b = \sqrt{83.68} \approx 9.15$$

Example 2: Find the Missing Side / 2

Question

In $\triangle ABC$, $B = 105°$, $a = 17$, $c = 14$. Find b.



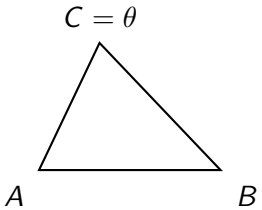
Example 2: Solution /

$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$b^2 = 17^2 + 14^2 - 2 \times 17 \times 14\cos 105^\circ$$
$$b^2 = 289 + 196 - 476 \times (-0.2588)$$
$$b^2 = 485 + 123.20 = 608.20$$
$$b = \sqrt{608.20} \approx 24.66$$

II) Finding Angles with Cosine Law /

Question

In $\triangle ABC$, $a = 9$, $b = 8$, $c = 14$. Find θ (angle opposite c).



II) Solution /

$$\cos\theta = \frac{a^2 + b^2 - c^2}{2ab}$$

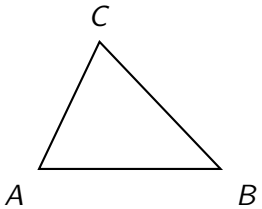
$$\cos\theta = \frac{9^2 + 8^2 - 14^2}{2 \times 9 \times 8} = \frac{81 + 64 - 196}{144} = \frac{-51}{144} \approx -0.3542$$

$$\theta = \cos^{-1}(-0.3542) \approx 110.74^\circ$$

Practice: Find the Missing Angle /

Practice

In $\triangle ABC$, $a = 12$, $b = 15$, $c = 8$. Find C.



Practice: Solution /

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos C = \frac{12^2 + 15^2 - 8^2}{2 \times 12 \times 15}$$

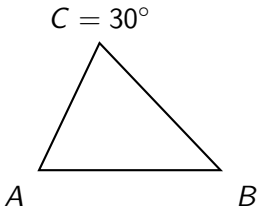
$$\cos C = \frac{144 + 225 - 64}{360} = \frac{305}{360} \approx 0.8472$$

$$C = \cos^{-1}(0.8472) \approx 32.06^\circ$$

Example: Find the Missing Side /

Question

In $\triangle ABC$, $a = 8$, $b = 5$, $C = 30^\circ$. Find c.



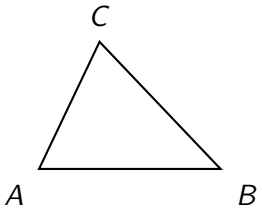
Example: Solution /

$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$c^2 = 8^2 + 5^2 - 2 \times 8 \times 5\cos 30^\circ$$
$$c^2 = 64 + 25 - 80 \times 0.8660$$
$$c^2 = 89 - 69.28 = 19.72$$
$$c = \sqrt{19.72} \approx 4.44$$

Practice: Find the Area /

Practice

In $\triangle ABC$, $a = 12$ cm, $b = 27$ cm, $c = 35$ cm. Find the area of the triangle.



Practice: Solution /

Use Heron's formula or $Area = \frac{1}{2}ab\sin C$ after finding an angle.

$$s = \frac{12 + 27 + 35}{2} = 37$$

$$Area = \sqrt{37(37-12)(37-27)(37-35)} = \sqrt{37 \times 25 \times 10 \times 2} \approx 136.01$$

(units^2)

Proof for the Cosine Law /

Proof Sketch

Let h be the height from C to AB

$b^2 = h^2 + (x)^2$

$c^2 = h^2 + (a - x)^2$

Expand:

$c^2 = h^2 + a^2 - 2ax + x^2$

Substitute $b^2 = h^2 + x^2$:

$c^2 = b^2 + a^2 - 2ax$

But $x = b \cos A$

$c^2 = a^2 + b^2 - 2ab \cos A$