Lesson 5: Graphing Quadratic Functions in APQ Form

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I) Quadratic Functions Written in Two Different Forms - Part 1

Key Forms

- **General Form:**

$$y = ax^2 + bx + c$$

  - Axis of Symmetry: $x = \frac{-b}{2a}$
  - Y-intercept: $(0, c)$
  - Not as easy finding the vertex

I) Quadratic Functions Written in Two Different Forms - Part 2

Key Forms (Cont.)

- **Vertex Form (APQ form):**

$$y = a(x - p)^2 + q$$

  - Finding the vertex is very easy!!
  - Graphing in APQ form is also easy
  - All you need to do is find the constants "a", "p", and "q"
  - Vertex: (p, q)
  - AOS: $x = p$

Example: Identifying Constants in APQ Form

Example

For the equation $y = 3(x-2)^2 + 7$:

- $a = 3$
- $p = 2$
- $q = 7$
- Vertex: $(2, 7)$
- AOS: $x = 2$

Example Graph: $y = 3(x-2)^2 + 7$

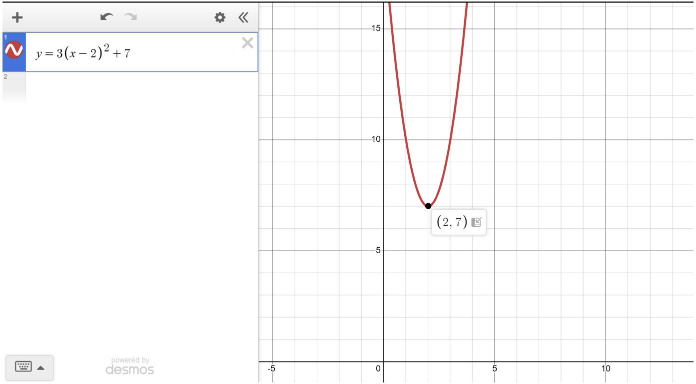


Figure: Example graph illustrating constants in APQ form for $y = 3(x-2)^2 + 7$.

II) Graphing Quadratic Functions in APQ Form

Key Points

- A Quadratic function in vertex form is much easier to graph
- Using constants "a", "p", & "q", we can find:
  - Vertex: (p, q)
  - Domain: $x \in \mathbb{R}$
  - Axis of Symmetry: $x = p$
  - Range: $y \geq q$ or $y \leq q$
  - Y-intercept: make $x = 0$, solve for y
  - X-intercept: make $y = 0$, solve for x

II.5) Understanding the Formula $y = a(x-p)^2 + q$ - Part 1

Formula Components

- **Basic Formula:** $y = a(x-p)^2 + q$
- **Each constant has a specific effect:**

II.5) Understanding the Formula $y = a(x-p)^2 + q$ - Part 2

Constant Effects (Cont.)

- a: Controls the shape and direction
  - If $a > 0$: Parabola opens up
  - If $a < 0$: Parabola opens down
  - Larger $|a|$: Narrower parabola
  - Smaller $|a|$: Wider parabola
- p: Controls horizontal position
  - Moves vertex left/right
  - Positive p: Shift right
  - Negative p: Shift left
- q: Controls vertical position
  - Moves vertex up/down
  - Positive q: Shift up
  - Negative q: Shift down

Graph Ideas and Key Points

Graphing Strategy

- **Step 1:** Identify the vertex (p, q)
- **Step 2:** Draw the axis of symmetry $x = p$
- **Step 3:** Use the value of a to determine:
  - Direction of opening
  - Width of parabola
  - Pattern of points
- **Step 4:** Plot points using the pattern:
  - For $a = 1$: 1, 3, 5, 7, ...
  - For $a = 2$: 2, 6, 10, 14, ...
  - For $a = 3$: 3, 9, 15, 21, ...
- **Step 5:** Connect points to form parabola

Example: Complete Analysis - Part 1

Example: y

- Equation: $y = 2(x-3)^2 - 5$
- **Constants:**
  - $a = 2$ (opens up, medium width)
  - $p = 3$ (shifts right 3 units)
  - $q = -5$ (shifts down 5 units)
- **Key Features:**
  - Vertex: $(3, -5)$
  - Axis of Symmetry: $x = 3$
  - Pattern: 2, 6, 10, 14, ...

Example: Complete Analysis - Part 2

Example: y

- **Graphing Steps:**
  1. Plot vertex at $(3, -5)$
  2. Draw AOS at $x = 3$
  3. Move right 1 unit, up 2 units
  4. Move right 1 more unit, up 6 more units
  5. Continue pattern
  6. Mirror points across AOS

Example Graph: Graphing $y = 2(x-3)^2 - 5$

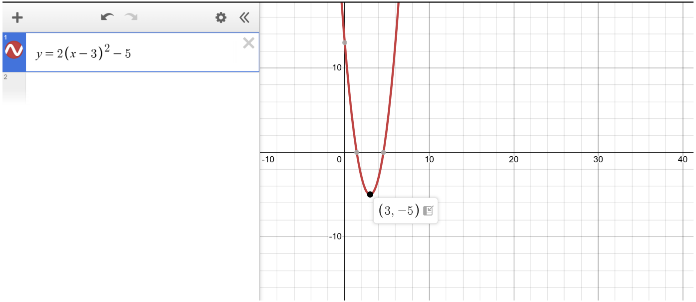


Figure: A visual representation of the graphing steps for $y = 2(x-3)^2 - 5$, showing the vertex, axis of symmetry, and parabolic shape.

III) Horizontal Translations

Key Concepts

- A parabola will shift left or right depending on the constant "p" that is placed inside the brackets with "x"
- Look inside the brackets to find what the value of "p" is
- When $p = 0$, the graph is centered on the Y-axis
- When $p > 0$, the graph shifts right
- When $p < 0$, the graph shifts left

IV) Vertical Translations (VT)

Key Concepts

- A Vertical shift (UP or Down) will occur if a constant is added to the equation outside of the brackets
- The value of "x" is squared first and then we add/subtract the constant
- When $q > 0$, the graph shifts up
- When $q < 0$, the graph shifts down

V) Summary for Constants "p" and "q"

Key Points

- The constant "p" affects the graph horizontally
  - When $p = 0$, the graph is centered on the Y-axis
  - When $p > 0$, the graph shifts right
  - When $p < 0$, the graph shifts left
- The constant "q" affects the graph vertically
  - When $q > 0$, the graph shifts up
  - When $q < 0$, the graph shifts down

VI) Constant "a" (Congruency Factor)

Key Points

- The constant "a" determines:
  - The width of the parabola (congruency)
  - Which way it opens
- If "a" is positive: Opens up
- If "a" is negative: Opens down
- If "a" is big: Skinny parabola
- If "a" is small: Wide parabola
- Congruency Factor Examples:
  - $a = 1$: 1, 3, 5, 7
  - $a = 2$: 2, 6, 10, 14
  - $a = 3$: 3, 9, 15, 21
  - $a = 0.5$: 0.5, 1.5, 2.5, 3.5
  - $a = 0.25$: 0.25, 0.75, 1.25, 1.75

Graph of $y = x^2$ $(a = 1)$

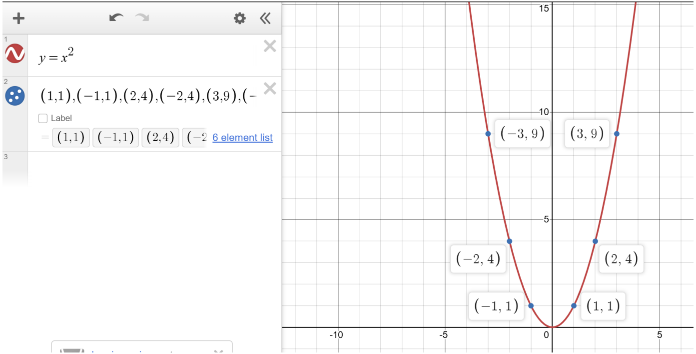


Figure: Parabola for $a = 1$ (e.g., $y = x^2$). Shows the standard width and upward opening.

Congruency Pattern

- For $a = 1$: 1, 3, 5, 7

Graph of $y = 2x^2$ $(a = 2)$

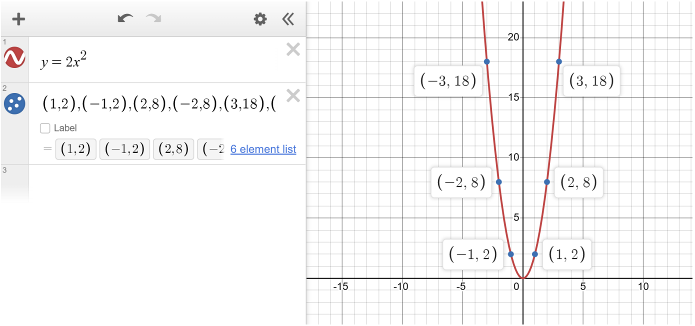


Figure: Parabola for $a = 2$ (e.g., $y = 2x^2$). Shows a narrower width compared to $a = 1$.

Congruency Pattern

- For $a = 2$: 2, 6, 10, 14

Graph of $y = \frac{1}{2}x^2$ $(a = 0.5)$

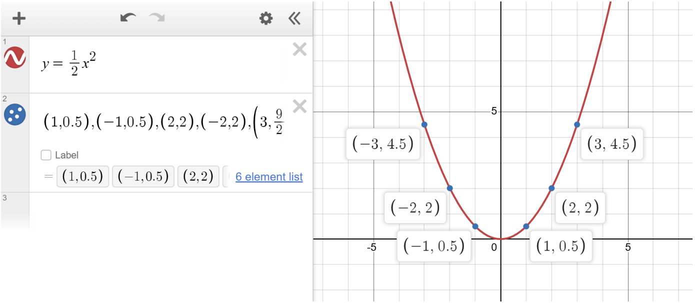


Figure: Parabola for $a = 0.5$ (e.g., $y = \frac{1}{2}x^2$). Shows a wider width compared to $a = 1$.

Congruency Pattern

- For $a = 0.5$: 0.5, 1.5, 2.5, 3.5

Finding the X-intercepts in APQ Form

Method

- Finding the X-intercepts in APQ form requires algebra
- At the X-intercepts, the y-coordinates are zero
- Steps:
  1. Make $y = 0$
  2. Isolate the squared term
  3. Take the square root of both sides
  4. Solve for x

Example: Finding X-intercepts

Example

For $y = 3(x-2)^2 - 14$:

$$\begin{aligned} 0 &= 3(x-2)^2 - 14 \\ 14 &= 3(x-2)^2 \\ \frac{14}{3} &= (x-2)^2 \\ \pm\sqrt{\frac{14}{3}} &= x - 2 \\ x &= 2 \pm \sqrt{\frac{14}{3}} \end{aligned}$$

Practice Problems

Problem Set

For each of the following equations, find:

1. Coordinates of the vertex
2. Equation of the Axis of Symmetry (AOS)
3. X and Y intercepts
4. Domain and Range

Practice Problem 1

Problem 1

$$y = (x-4)^2 + 3$$

Solution to Problem 1 - Analysis

Solution 1

For $y = (x-4)^2 + 3$, we have $a = 1$, $p = 4$, $q = 3$.

- **Vertex:** $(p, q) = (4, 3)$
- **AOS:** $x = p \implies x = 4$
- **Y-intercept:** Set $x = 0$

$$y = (0-4)^2 + 3$$
$$y = (-4)^2 + 3$$
$$y = 16 + 3 = 19$$

Y-intercept: $(0, 19)$

Solution 1 (Cont.)

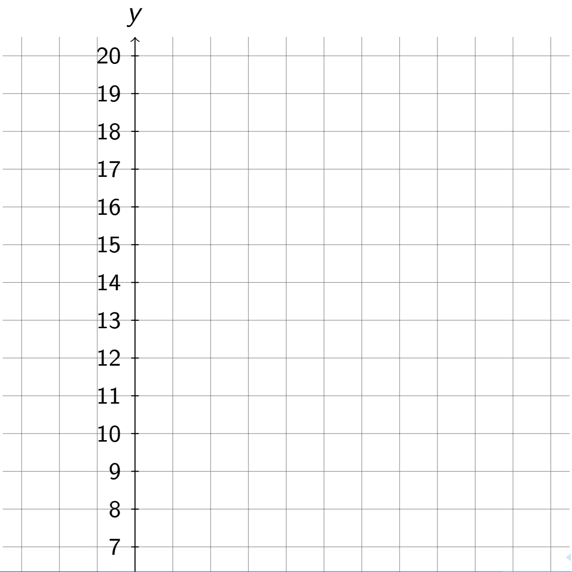
For $y = (x-4)^2 + 3$:

- **X-intercepts:** Set $y = 0$

$$\begin{aligned} 0 &= (x-4)^2 + 3 \\ -3 &= (x-4)^2 \end{aligned}$$

  Since a square cannot be negative, there are **No Real X-intercepts**.
- **Domain:** $x \in \mathbb{R}$
- **Range:** Since $a = 1 > 0$, the parabola opens up. $y \geq q \implies y \geq 3$

Solution to Problem 1 - Graphing Grid



Practice Problem 2

Problem 2

$$y = -2(x+1)^2 + 8$$

Solution to Problem 2 - Analysis

Solution 2

For $y = -2(x+1)^2 + 8$, we have $a = -2$, $p = -1$, $q = 8$.

- **Vertex:** $(p, q) = (-1, 8)$
- **AOS:** $x = p \implies x = -1$
- **Y-intercept:** Set $x = 0$

$$y = -2(0+1)^2 + 8$$
$$y = -2(1)^2 + 8$$
$$y = -2 + 8 = 6$$

Y-intercept: $(0, 6)$

Solution to Problem 2 - X-intercepts Domain/Range

Solution 2 (Cont.)

For $y = -2(x+1)^2 + 8$:

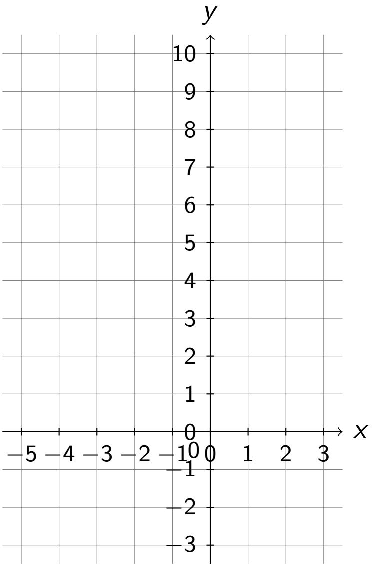
- **X-intercepts:** Set $y = 0$

$$\begin{aligned} 0 &= -2(x+1)^2 + 8 \\ -8 &= -2(x+1)^2 \\ 4 &= (x+1)^2 \\ \pm\sqrt{4} &= x+1 \\ \pm 2 &= x+1 \end{aligned}$$

  So, $x + 1 = 2 \implies x = 1$ and $x + 1 = -2 \implies x = -3$.
  X-intercepts: $(1, 0)$ and $(-3, 0)$

- **Domain:** $x \in \mathbb{R}$
- **Range:** Since $a = -2 < 0$, the parabola opens down. $y \leq q \implies y \leq 8$

Solution to Problem 2 - Graphing Grid



Practice Problem 3

Problem 3

$$y = \frac{1}{2}(x-3)^2 - 2$$

Solution to Problem 3 - Analysis

Solution 3

For $y = \frac{1}{2}(x-3)^2 - 2$, we have $a = \frac{1}{2}$, $p = 3$, $q = -2$.

- **Vertex:** $(p, q) = (3, -2)$
- **AOS:** $x = p \implies x = 3$
- **Y-intercept:** Set $x = 0$

$$y = \frac{1}{2}(0-3)^2 - 2$$
$$y = \frac{1}{2}(-3)^2 - 2$$
$$y = \frac{1}{2}(9) - 2$$
$$y = 4.5 - 2 = 2.5$$

Y-intercept: $(0, 2.5)$

Solution to Problem 3 - X-intercepts Domain/Range

Solution 3 (Cont.)

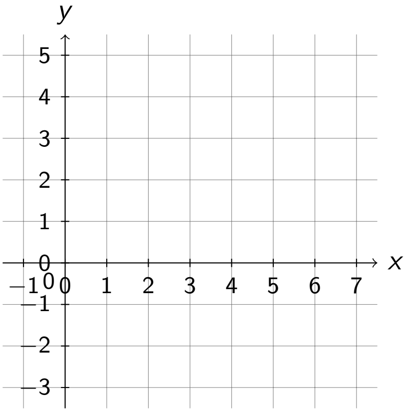
For $y = \frac{1}{2}(x-3)^2 - 2$:

- **X-intercepts:** Set $y = 0$

$$\begin{aligned} 0 &= \frac{1}{2}(x-3)^2 - 2 \\ 2 &= \frac{1}{2}(x-3)^2 \\ 4 &= (x-3)^2 \\ \pm\sqrt{4} &= x - 3 \\ \pm 2 &= x - 3 \end{aligned}$$

  So, $x - 3 = 2 \implies x = 5$ and $x - 3 = -2 \implies x = 1$.
  X-intercepts: $(5, 0)$ and $(1, 0)$
- **Domain:** $x \in \mathbb{R}$
- **Range:** Since $a = \frac{1}{2} > 0$, the parabola opens up. $y \geq q \implies y \geq -2$

Solution to Problem 3 - Graphing Grid



Practice Problem 4

Problem 4

$$y = -(x+5)^2 - 1$$

Solution 4

For $y = -(x+5)^2 - 1$, we have $a = -1$, $p = -5$, $q = -1$.

- **Vertex:** $(p, q) = (-5, -1)$
- **AOS:** $x = p \implies x = -5$
- **Y-intercept:** Set $x = 0$

$$y = -(0+5)^2 - 1$$
$$y = -(5)^2 - 1$$
$$y = -25 - 1 = -26$$

Y-intercept: $(0, -26)$

Solution to Problem 4 - X-intercepts Domain/Range

Solution 4 (Cont.)

For $y = -(x+5)^2 - 1$:

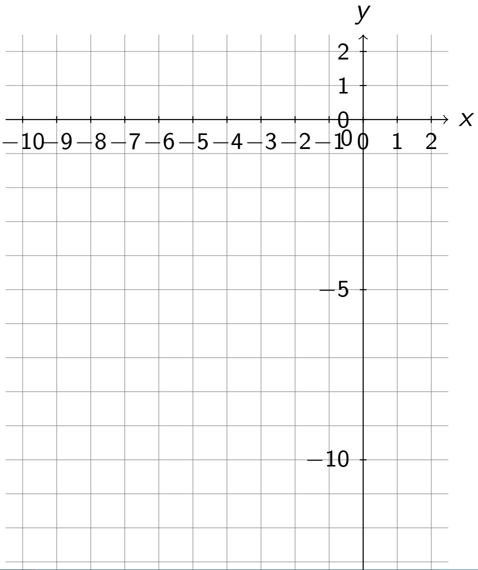
- **X-intercepts:** Set $y = 0$

$$\begin{aligned} 0 &= -(x+5)^2 - 1 \\ 1 &= -(x+5)^2 \\ -1 &= (x+5)^2 \end{aligned}$$

  Since a square cannot be negative, there are **No Real X-intercepts**.

- **Domain:** $x \in \mathbb{R}$
- **Range:** Since $a = -1 < 0$, the parabola opens down. $y \leq q \implies y \leq -1$

Solution to Problem 4 - Graphing Grid



Practice Problem 5

Problem 5

$$y = 3(x-1)^2$$

Solution to Problem 5 - Analysis

Solution 5

For $y = 3(x-1)^2$, we have $a = 3,\ p = 1,\ q = 0$.

- **Vertex:** $(p, q) = (1, 0)$
- **AOS:** $x = p \implies x = 1$
- **Y-intercept:** Set $x = 0$

$$y = 3(0-1)^2$$
$$y = 3(-1)^2$$
$$y = 3(1) = 3$$

Y-intercept: $(0, 3)$

Solution to Problem 5 - X-intercepts Domain/Range

Solution 5 (Cont.)

For $y = 3(x-1)^2$:

- **X-intercepts:** Set $y = 0$

$$\begin{aligned} 0 &= 3(x-1)^2 \\ 0 &= (x-1)^2 \\ 0 &= x-1 \\ x &= 1 \end{aligned}$$

  X-intercept: $(1, 0)$ (This is also the vertex, meaning the parabola touches the x-axis at one point).

- **Domain:** $x \in \mathbb{R}$
- **Range:** Since $a = 3 > 0$, the parabola opens up. $y \geq q \implies y \geq 0$

Solution to Problem 5 - Graphing Grid