# MATH 101 Study Notes Integral Calculus with Applications

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# 1. General Principles

In this course,

•  $\log is \ln$ 

# 1.1. Sigma Notation

$$\sum_{i=1}^{n} i \tag{1}$$

means "the sum of i from 1 to n, where i is an integer".

## 2. Definite Integral

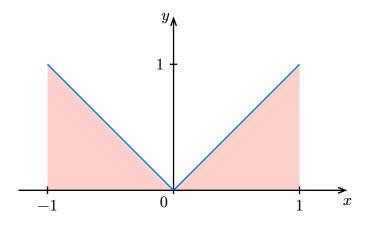
$$\int_{a}^{b} f(x) \, \mathrm{d}x \tag{2}$$

means "the integral of f(x) from a to b".

Take f(x) = |x|, its integral from -1 to 1 is:

$$\int_{-1}^{1} |x| \, \mathrm{d}x \tag{3}$$

looks like:



### 2.1. Estimation

Right Riemann Sum (RRS) is an estimation of the area under the curve using rectangles with the right endpoint as the height.

For example,

$$\int_0^8 \sqrt{x} \, \mathrm{d}x \,. \tag{4}$$

Using RRS with 4 rectangles, we have:

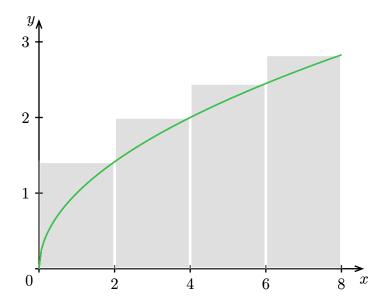
- 1. each rectangle has width  $\frac{8-0}{4} = 2$ ,
- 2. the right endpoints are 2, 4, 6, 8,
- 3. the heights are  $\sqrt{2}$ ,  $\sqrt{4}$ ,  $\sqrt{6}$ ,  $\sqrt{8}$ .

That gives us the estimation:

$$\sum_{i=1}^{4} 2\sqrt{2i} = 2\sqrt{2} + 2\sqrt{4} + 2\sqrt{6} + 2\sqrt{8}$$

$$\approx 2.83 + 4 + 5.29 + 5.66$$

$$= 17.78.$$
(5)



Similarly, Left Riemann Sum (LRS), Midpoint Riemann Sum (MRS), and Trapezoidal Riemann Sum (TRS) exist.

The generalized formula with n rectangles/trapeziums from a to b are:

$$\begin{aligned} & \text{RRS}(a,b,n) = \sum_{i=1}^{n} f(x_i) \Delta x \\ & \text{LRS}(a,b,n) = \sum_{i=1}^{n} f(x_{i-1}) \Delta x \\ & \text{MRS}(a,b,n) = \sum_{i=1}^{n} f\left(\frac{x_{i-1} + x_i}{2}\right) \Delta x \\ & \text{TRS}(a,b,n) = \sum_{i=1}^{n} (f(x_{i-1}) + f(x_i)) \Delta \frac{x}{2}, \end{aligned} \tag{6}$$

where  $\Delta x = \frac{b-a}{n}$  and  $x_i = a + i\Delta x$ .

For an increasing function (f'(x) > 0), RRS is an overestimation, and LRS is an underestimation. For a function concave up (f''(x) > 0), TRS is an overestimation.

### 2.2. Signed Area

If the 'area under the curve' is below the x-axis, it can be called 'negative'. Hence, the integral of a function can be interpreted as the signed area of a curve, which can be positive or negative.

Say, if we have an odd function ( $\pi$  rotation symmetry about the origin), then its signed area/integral over a symmetric interval is 0.

### 2.3. Precise Calculation

Using Riemann Sums, we can see that the more rectangles we use, the closer the estimation is to the actual value.

So, let's bust up n to infinity:

$$\begin{split} \lim_{n \to \infty} \mathrm{RRS}(a,b,n) &= \lim_{n \to \infty} \sum_{i=1}^n f(x_i) \Delta x \\ &= \text{the actual signed area} \\ &= \text{wait isn't this the definition of the integral?} \\ &= \int_a^b f(x) \, \mathrm{d}x \,. \end{split} \tag{7}$$