PHYS 158 Study Notes Electricity and Magnetism

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Contents

1. Circuits

1.1. Basic Components

Power Supplies (DC/AC) Direct Current and Alternating Current.

Resistors (R) Resists current, consumes power. Light bulbs, lamps are also resistors.

Capacitors (C) Stores electric charge and energy. Not to be confused with batteries: Batteries release energy in a slow manner; capacitors can discharge energy in a short burst.

Inductors/Stabilizers (L) Generates induced current, opposing passing current.

1.2. Current, Voltage, Capacitance, and Resistance

Charge (Q) The amount of electric charge, measured in Coulombs (C).

Current (I) The flow of electric charge, $I = \frac{\mathrm{d}Q}{\mathrm{d}t}$, measured in Amperes (A). It is generated by a voltage difference.

Voltage (V) The potential difference between two points, measured in Volts (V).

Capacitance (C) The ability to store electric charge, measured in Farads (F).

Resistance (R) The opposition to the flow of electric current, measured in Ohms (Ω).

$$V = IR \tag{1}$$

Resistance of a resistor depends on its material, length L, and cross-sectional area A.

$$R = \rho \frac{L}{A} \tag{2}$$

where ρ is the resistivity of the material.

For multiple resistors,

$$R_{\text{series}} = R_1 + R_2 + \dots \tag{3}$$

$$\frac{1}{R_{\text{parallel}}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots \tag{4}$$

1.3. Voltage Drops

Electromotive Force (EMF) The voltage difference between the positive and negative terminals of a DC power supply (battery) is V or ε, specified on the battery.

Resistance The voltage drop across a resistor is

$$\Delta V_R = IR \tag{5}$$

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Capacitance The voltage drop across a capacitor is

$$\Delta V_C = \frac{Q}{C} \tag{6}$$

where C is the capacitance.

Inductance The voltage drop across an inductor is

$$\Delta V_L = -L\frac{\mathrm{d}I}{\mathrm{d}t} \tag{7}$$

where L is the inductance.

1.4. Reading Resistors

There will be maximum 5 color bands on a resistor. From the left to the right:

- 1-3: The first three digits of the resistance value.
- 4: The number of zeros following the first three digits (multiplier).
- 5: The tolerance of the resistance value.

Sometimes, there will be 4 bands only, where there are only 2 digits represented, followed by the number of zeros.

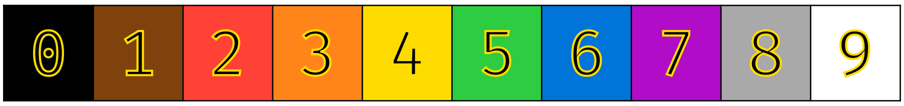

Figure 1: Resistor digit colors

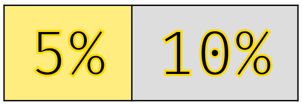

Figure 2: Resistor tolerance colors

1.5. Kirchhoff's Laws

When traveling in the direction of the current, the voltage change through a resistor is negative, $\Delta V = -IR$, and positive through a battery, $\Delta V = +\varepsilon$. When traveling against the direction of the current, invert the signs.

Kirchhoff's Current Law (KCL) The sum of currents entering a node is equal to the sum of currents leaving the node.

$$\Sigma I_{\text{in}} = \Sigma I_{\text{out}} \tag{8}$$

Kirchhoff's Voltage Law (KVL) The sum of voltage drops in a closed loop is equal to the sum of voltage rises.

$$\Sigma V_{\text{drop}} = \Sigma V_{\text{rise}} \tag{9}$$

A 'closed loop' is a path that starts and ends at the same point, no matter the direction of current.

These two laws are crucial in analyzing circuits (solving problems), especially when appliances are not clearly connected in series or parallel.

1.6. Short, Open Circuits and Proportionality

Short Circuit A circuit with no resistance, causing a large current to flow. The voltage drop across a short circuit is zero.

Open Circuit A circuit with infinite resistance, causing no current to flow.

In case of a parallel circuit, the voltage across each component is the same, while the current is inversely proportional to the resistance. While in a series circuit, the current across each component is the same, while the voltage is inversely proportional to the resistance.

For instance, 3 A of current flows through a 2 Ω and a 1 Ω resistors, the current through each will be 1 A and 2 A.

Combining this knowledge with Kirchhoff's laws, we can solve even more complex circuits.

1.7. Real Batteries

Internal Resistance (r) The resistance within a battery, causing a voltage drop.

$$\begin{aligned} V_{\text{battery}} &= \varepsilon - Ir \\ I &= \frac{\varepsilon}{r + R}. \end{aligned} \tag{10}$$

Hence, the terminal voltage of a battery is

$$\begin{aligned} V_{\text{terminal}} &= \varepsilon - \frac{\varepsilon}{r + R} r \\ &= \varepsilon \frac{R}{r + R}. \end{aligned} \tag{11}$$

1.8. Power

Power The rate at which energy is consumed or produced, measured in Watts (W).

$$P = IV = I^2 R = \frac{V^2}{R} \tag{12}$$

1.9. Grounding

Ground A reference point in a circuit, usually at zero voltage. It is used to measure the voltage of other points in the circuit.

Grounding Connecting a circuit to the ground or other big conductors to send away excess energy, usually to prevent electric shock. It is also used to stabilize the voltage of a circuit.

Addition of a ground symbol in a circuit diagram does not affect the circuit itself, our calculations stay the same. However, our **zero reference point changes**, and we must measure the voltage of other points in the circuit with respect to the ground!

1.10. Capacitor

Any collection of conductors that can store electric charge & energy.

1.11. Capacitance (C)

The ability to store electric charge, measured in Farads (F).

Capacitance of a capacitor depends on its material, area A, and distance d between plates.

$$C = \varepsilon \frac{A}{d} \tag{13}$$

where ε is the electric permittivity of the material.

$$Q = CV \tag{14}$$

$$I = \frac{\mathrm{d}Q}{\mathrm{d}t} = C \frac{\mathrm{d}V}{\mathrm{d}t} \tag{15}$$

For multiple capacitors,

$$C_{\text{series}} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \ldots} \tag{16}$$

$$C_{\text{parallel}} = C_1 + C_2 + \ldots \tag{17}$$

1.11.1. Capacitors in Parallel

As they are in parallel, the voltage drop across them should be the same, regardless of the capacitance.

$$\begin{aligned} V &= V_1 = V_2 = \ldots \\ Q &= C_1 V_1 + C_2 V_2 + \ldots \\ &= C_1 V + C_2 V + \ldots \\ &= (C_1 + C_2 + \ldots) V \\ \frac{Q}{V} &= C_1 + C_2 + \ldots \end{aligned} \tag{18}$$

which leads to Equation 17.

Imagine all the parallel capacitors as one big capacitor with the sum of capacitances. Now the formula makes sense.

1.11.2. Capacitors in Series

When connected in series, the capacitors can be viewed as one beginning plate with positive charges and one ending plate with negative charges, plus all the plates in between, with charges but adding up to zero.

In this case, charges across the capacitors are the same.

$$
\begin{aligned}
Q &= Q_1 = Q_2 = \dots \\
V &= V_1 + V_2 + \dots \\
&= \frac{Q}{C_1} + \frac{Q}{C_2} + \dots \\
&= Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \dots\right) \\
\frac{V}{Q} &= \frac{1}{C_1} + \frac{1}{C_2} + \dots
\end{aligned} \tag{19}
$$

which leads to Equation 16.

1.11.3. Work and Energy in Capacitors

Batteries charge capacitors bit by bit, by $\mathrm{d}q$. Thus, we can say the work done is

$$
\begin{aligned}
\mathrm{d}W &= \Delta V \, \mathrm{d}q \\
&= \frac{q}{C} \, \mathrm{d}q
\end{aligned} \tag{20}
$$

$$
U = \frac{1}{C} \int_0^Q q \, \mathrm{d}q = \frac{Q^2}{2C} \tag{21}
$$