PHYS 170 Study Notes

Mechanics

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Contents

1. General Principles

1.1. The Four Horseman of Mechanics

- Length
- Mass
- Time
- Force

So you basically take three of them and solve the 1 left.

1.2. US Customary Units

Length	Mass	Time	Force
meter m	kilogram kg	second s	force $\mathrm{kg\,m\,s^{-2}}$
foot ft	slug $\mathrm{lb\,s^{2}\,ft^{-1}}$	second s	pound lb

Table 1: SI and US Customary (FPS) Units for Mechanics

1.3. Gravity

$$F = G\frac{m_1 m_2}{r^2} \tag{1.1}$$

$$F = ma \tag{1.2}$$

In this course, we will use

$$g = 9.81\ \mathrm{m\,s^{-2}} \tag{2}$$

which happens to be true for Vancouver.

1.4. Vector Notation

In this course, vectors are upright bold, and vector magnitudes are italicized bold, while unit vectors are italics with an hat over.

$$\mathbf{A} \text{ has a magnitude of } \boldsymbol{A} \text{ in direction } \hat{\boldsymbol{A}}. \tag{3}$$

1.5. Angle Unit

In this course, angles are in degrees.

2. Force Vectors

Force, having both magnitude and direction, is a vector. Intuitively, we can apply all kinds of vector operations to forces, as you would learn in MATH 152.

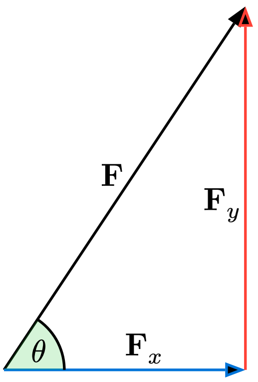
2.1. Addition

Use "tip to tail" for triangular method of addition: draw the vectors head to tail, and the resultant vector is the vector from the tail of the first vector to the head of the last vector.

2.2. Force Components

$$\mathbf{F} = \boldsymbol{x}\hat{\boldsymbol{\imath}} + \boldsymbol{y}\hat{\boldsymbol{\jmath}} \tag{4}$$

where x, y are magnitudes of the force in the $\hat{\boldsymbol{\imath}}, \hat{\boldsymbol{\jmath}}$ directions.



Force $\mathbf{F}$ can be represented as a combination of $\mathbf{F}_x$ and $\mathbf{F}_y$

$$\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y \tag{5}$$

or as a polar coordinate of angle $\theta = \arctan\left(\frac{\boldsymbol{F}_y}{\boldsymbol{F}_x}\right)$ and magnitude $\boldsymbol{F}$

$$\mathbf{F} = \boldsymbol{F}(\cos(\theta) + \sin(\theta)). \tag{6}$$

To generalize it, we can write it as

$$\mathbf{F} = \boldsymbol{F}_x\hat{\boldsymbol{\imath}} + \boldsymbol{F}_y\hat{\boldsymbol{\jmath}} \tag{7.1}$$

$$= \boldsymbol{F}\left(\cos(\theta)\hat{\boldsymbol{\imath}} + \sin(\theta)\hat{\boldsymbol{\jmath}}\right) \tag{7.2}$$

where $\hat{\boldsymbol{\imath}}, \hat{\boldsymbol{\jmath}}$ are unit vectors in the x, y directions. This is the Cartesian form of a vector.

For a force with 2 dimensions, we call it a coplanar force.

Sometimes, non-linear equations arise from problems involving forces. Gladly use math solvers for those.

2.3. Unit Vector

To disregard magnitude and only focus on direction, we use unit vector, which we divide a vector by its magnitude, $\hat{\boldsymbol{u}} = \frac{\mathbf{A}}{\boldsymbol{A}}$.

2.4. 3D Forces

Forces in 3D are $\mathbf{F} = \boldsymbol{F}_x\hat{\boldsymbol{\imath}} + \boldsymbol{F}_y\hat{\boldsymbol{\jmath}} + \boldsymbol{F}_z\hat{\boldsymbol{k}}$, with their magnitudes being $\boldsymbol{F} = \sqrt{\boldsymbol{F}_x^2 + \boldsymbol{F}_y^2 + \boldsymbol{F}_z^2}$.

To determine orientation of the axis, we use the right-hand rule: make a thumb up using your right hand, the side of the curling fingers is x, the arm is y, and the thumb is z.

2.4.1. Direction of Cartesian Vector

The direction of a Cartesian vector is the angles between the vector and the **positive** axis. α, β, γ each corresponds to the angle from the positive x, y, z axis.

$$\cos(\alpha) = \frac{\boldsymbol{F}_x}{\boldsymbol{F}} \tag{8.1}$$

$$\cos(\beta) = \frac{\boldsymbol{F}_y}{\boldsymbol{F}} \tag{8.2}$$

$$\cos(\gamma) = \frac{\boldsymbol{F}_z}{\boldsymbol{F}} \tag{8.3}$$

Therefore,

$$\hat{\boldsymbol{u}} = \cos(\alpha)\hat{\boldsymbol{i}} + \cos(\beta)\hat{\boldsymbol{j}} + \cos(\gamma)\hat{\boldsymbol{k}} \tag{9}$$

and

$$\mathbf{F} = \boldsymbol{F}\hat{\boldsymbol{u}} \tag{10.1}$$

$$= \boldsymbol{F}\left(\cos(\alpha)\hat{\boldsymbol{i}} + \cos(\beta)\hat{\boldsymbol{j}} + \cos(\gamma)\hat{\boldsymbol{k}}\right) \tag{10.2}$$

The directions satisfy $-180° < \alpha, \beta, \gamma < 180°$ and have identity

$$\cos^2(\alpha) + \cos^2(\beta) + \cos^2(\gamma) = 1. \tag{11}$$

2.4.2. Determining 3D Force Components

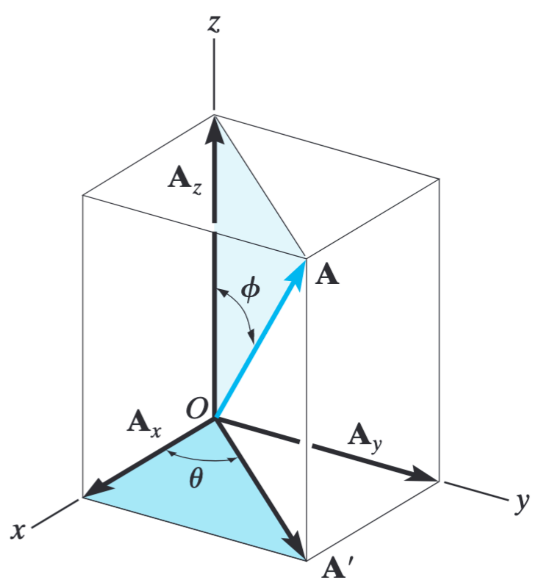


Figure 1: A Cartesian Vector

With magnitude $\boldsymbol{F}$ and angles from the positive z-axis φ and from the positive x-axis θ, we can determine the force components by first solving for $\boldsymbol{F}_z$, then $\boldsymbol{F}_{xy}$ followed by $\boldsymbol{F}_x$ and $\boldsymbol{F}_y$.

$$\boldsymbol{F}_z = \boldsymbol{F}\cos(\varphi) \tag{12.1}$$

$$\boldsymbol{F}_{xy} = \boldsymbol{F}\sin(\varphi) \tag{12.2}$$

$$\boldsymbol{F}_x = \boldsymbol{F}_{xy}\cos(\theta) \tag{12.3}$$

$$\boldsymbol{F}_y = \boldsymbol{F}_{xy}\sin(\theta) \tag{12.4}$$

Or instead, given 2 (β, γ) of the 3 Cartesian angles, we can determine the force by

$$\cos(\alpha) = \sqrt{1 - \cos^2(\beta) - \cos^2(\gamma)} \tag{13.1}$$

$$\mathbf{F} = \boldsymbol{F}\Big(\cos(\alpha)\hat{\boldsymbol{\imath}} + \cos(\beta)\hat{\boldsymbol{\jmath}} + \cos(\gamma)\hat{\boldsymbol{k}}\Big). \tag{13.2}$$

2.5. Position Vectors

Position vectors are vectors that describe the position of a point in space relative to a reference point.

As obvious, we need 3 coordinates to locate a point in 3D space. Point $P(x, y, z)$ has position vector $\mathbf{r} = x\hat{\boldsymbol{\imath}} + y\hat{\boldsymbol{\jmath}} + z\hat{\boldsymbol{k}}$ relative to the origin.

Note that the position vector does not always come from the origin, it can be relative to arbitrary points. Given $A(x_A, y_A, z_A)$ and $B(x_B, y_B, z_B)$, the position vector of B relative to A is

$$\mathbf{r} = (x_B - x_A)\hat{\boldsymbol{\imath}} + (y_B - y_A)\hat{\boldsymbol{\jmath}} + (z_B - z_A)\hat{\boldsymbol{k}}. \tag{14}$$

Connecting to unit vectors, $\mathbf{u} = \frac{\mathbf{F}}{\boldsymbol{F}}$,

$$\mathbf{F} = \boldsymbol{F}\mathbf{u} = \boldsymbol{F}\frac{\mathbf{r}}{\boldsymbol{r}}. \tag{15}$$

To simplify calculation, let $X = \frac{\boldsymbol{F}}{\boldsymbol{r}}$,

$$\mathbf{F} = X\mathbf{r} \tag{16.1}$$

$$\boldsymbol{F} = X\boldsymbol{r}. \tag{16.2}$$

2.6. Vector Operations

Mostly taught in MATH 152, but here again anyways.

2.6.1. Dot Product & Angle Between Vectors

$$\mathbf{A} \cdot \mathbf{B} = \boldsymbol{AB}\cos(\theta) \tag{17.1}$$

$$= A_x B_x + A_y B_y + A_z B_z \tag{17.2}$$

2.6.2. Parallel & Perpendicular Components

Two vectors are parallel if their cross product is a zero vector, and perpendicular if their dot product is zero.

Given a vector $\mathbf{A}$, its parallel component is

$$\mathbf{A}_{\parallel} = (\mathbf{A} \cdot \hat{\boldsymbol{u}})\hat{\boldsymbol{u}} \tag{18}$$

and its perpendicular component is

$$\mathbf{A}_{\perp} = \mathbf{A} - \mathbf{A}_{\parallel}. \tag{19}$$

2.6.3. Projection

The projection of $\mathbf{A}$ onto $\mathbf{B}$ is

$$\mathbf{A}_{\text{proj on B}} = (\mathbf{A} \cdot \hat{\boldsymbol{u}}_{\boldsymbol{B}})\hat{\boldsymbol{u}}_{\boldsymbol{B}}. \tag{20}$$

Note the similarity to the parallel component formula.

3. Equilibrium of a Particle

4. Force System Resultants

5. Equilibrium of a Rigid Body

6. Friction

7. Kinematics of a Particle

8. Kinetics of a Particle: Force and Acceleration

9. Kinetics of a Particle: Work and Energy

10. Kinetics of a Particle: Impulse and Momentum