Support

Hinges that prevents translation in the same direction does NOT produce couple moment, but force in that direction.

Friction

$F_s = \mu_s$ (impending motion), $F_k = \mu_k N$. Use moment equilibrium to determine point of normal force.

Tangential Coord.

$$\rho = \frac{\left[1 + \left(\frac{\mathrm{d}x}{\mathrm{d}y}\right)^2\right]^{\frac{3}{2}}}{\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}}$$

$$\vec{v} = v\vec{u}_t$$

$$\dot{\theta} = \frac{v}{\rho}$$

$$\begin{aligned}\vec{a} &= \dot{v}\vec{u}_t + v\dot{\vec{u}}_t \\ &= \dot{v}\vec{u}_t + \frac{v^2}{\rho}\vec{u}_n\end{aligned}$$

Cylindrical Coord.

Polar but with an additional axis, z.

$$\vec{u}_z = \vec{u}_\theta \times \vec{u}_r$$

Linear Momentum and Impulse

$$\vec{L} = m\vec{v}$$

$$\vec{I} = \int_{t_1}^{t_2} \vec{F}(t)\,\mathrm{d}t$$

$$m\boldsymbol{v}_1 + \sum \int_{t_1}^{t_2} \boldsymbol{F}\,\mathrm{d}t = m\boldsymbol{v}_2$$

Can be conserved.

Vector Components

$$\vec{v}_\parallel = (\vec{v}\cdot\hat{u})\hat{u} \quad \vec{v}_\perp = \vec{v} - \vec{v}_\parallel$$

$$\vec{v}_1 \cdot \vec{v}_2 = v_1 v_2 \cos(\theta) \quad \vec{v}_1 \times \vec{v}_2 = v_1 v_2 \sin(\theta)\vec{n} \text{ (right-hand)}$$

Tangential-Polar

Let ψ be the angle between $\vec{r}$ and $\vec{u}_t$, η be the angle between the tangential and the polar axis.

$$\tan(\psi) = \frac{r\dot{\theta}}{\dot{r}} = \frac{r}{\frac{\mathrm{d}r}{\mathrm{d}\theta}}$$

$$\eta = 90^\circ - \psi$$

$$\vec{u}_n = \vec{u}_r \cos(\eta)$$

Polar Coord.

$$\begin{aligned}\vec{v} &= \dot{\vec{r}} \\ &= v_r\vec{u}_r + v_\theta\vec{u}_\theta \\ &= \dot{r}\vec{u}_r + r\dot{\theta}\vec{u}_\theta\end{aligned}$$

$$\vec{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\vec{u}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\vec{u}_\theta$$

Energetics

$$U_{\text{const}} = F\cos(\theta)(b-a)$$

$$U_{\text{var}} = \int_a^b F\cos(\theta)\,\mathrm{d}s$$

$$U_{\text{spring}} = \frac{1}{2}k\left(s_b^2 - s_a^2\right)$$

$$T = \frac{1}{2}mv^2$$

$$V_g = Wh = mgh$$

$$V_s = +\frac{1}{2}ks^2$$

Can be conserved.

$$T_1 + V_1 = T_2 + V_2$$

SI-Imperial

Constants:

$$g = 9.81\,\mathrm{m\,s^{-2}} = 32.2\,\mathrm{m\,s^{-2}}$$

Units:

$$\text{mass} : \mathrm{kg}, \mathrm{s^2\,ft^{-1}}$$

$$\text{force} : \mathrm{kg\,m\,s^{-2}},$$

Moment

$$\overrightarrow{M} = \vec{r} \times \vec{F}$$

$$M = rF\sin(\theta) = Fd$$

Vector moments of the same point add up like forces.

Couple moments can move to anywhere. To move a force, add a pair of force on the target point, then make a couple moment.

Reduction to a Wrench

1. Get a resultant force;
2. get a resultant moment to a point;
3. split the moment into parallel and perpendicular components to the force;
4. remove the perpendicular components by moving the force ($M = Fd$);
5. move the parallel moment to the force.