Support

Hinges that prevents translation in the same direction does NOT produce couple moment, but force in that direction.

Friction

 $F_s = \mu_s$ (impending motion), $F_k = \mu_k N$. Use moment equilibrium to determine point of normal force.

Tangential Coord.

$$\rho = \frac{\left[1 + \left(\frac{\mathrm{d}x}{\mathrm{d}y}\right)^2\right]^{\frac{3}{2}}}{\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}}$$

$$\vec{v} = v\vec{u}_t$$

$$\dot{\theta} = \frac{v}{\rho}$$

$$\vec{a} = \dot{v}\vec{u}_t + v\vec{u}_t$$

$$= \dot{v}\vec{u}_t + \frac{v^2}{\rho}\vec{u}_n$$

Cylindrical
Coord.
Polar but
with an
additional

axis, z.
$$\vec{u}_z = \vec{u}_\theta \times \vec{u}_r$$

Linear Momentum and Impulse

$$\vec{L} = m\vec{v}$$

$$\vec{I} = \int_{t_1}^{t_2} \vec{F}(t) dt$$

$$m\boldsymbol{v}_1 + \sum \int_{t_1}^{t_2} \boldsymbol{F} \, \mathrm{d}t = m\boldsymbol{v}_2$$

Can be conserved.

<u>Vector Components</u>

$$\begin{split} \vec{v}_{\parallel} &= (\vec{v} \cdot \hat{u}) \hat{u} \quad \vec{v}_{\perp} = \vec{v} - \vec{v}_{\parallel} \\ \vec{v}_{1} \cdot \vec{v}_{2} &= v_{1} v_{2} \cos(\theta) \quad \vec{v}_{1} \times \vec{v}_{2} = v_{1} v_{2} \sin(\theta) \vec{n} \text{ (right-hand)} \end{split}$$

Tangential-Polar

Let ψ be the angle between \vec{r} and \vec{u}_t , η be the angle between the tangential and the polar axis.

$$\tan(\psi) = \frac{r\theta}{\dot{r}} = \frac{r}{\frac{dr}{d\theta}}$$
$$\eta = 90^{\circ} - \psi$$
$$\vec{u}_n = \vec{u}_r \cos(\eta)$$

Polar Coord.

$$\begin{split} \vec{v} &= \dot{\vec{r}} \\ &= v_r \vec{u}_r + v_\theta \vec{u}_\theta \\ &= \dot{r} \vec{u}_r + r \dot{\theta} \vec{u}_\theta \\ \vec{a} &= \left(\ddot{r} - r \dot{\theta}^2 \right) \vec{u}_r + \left(r \ddot{\theta} + 2 \dot{r} \dot{\theta} \right) \end{split}$$

Energetics

$$U_{\text{const}} = F \cos(\theta)(b - a)$$

$$U_{\text{var}} = \int_{a}^{b} F \cos(\theta) \, ds$$

$$U_{\text{spring}} = \frac{1}{2} k \left(s_{b}^{2} - s_{a}^{2} \right)$$

 $T = \frac{1}{2}mv^{2}$ $V_{g} = Wh = mgh$ $V_{s} = +\frac{1}{2}ks^{2}$

Can be conserved.

$$T_1 + V_1 = T_2 + V_2$$

<u>SI-Imperial</u>

Constants:

$$g = 9.81 \,\mathrm{m \, s^{-2}} = 32.2 \,\mathrm{m \, s^{-2}}$$

Units:

 $mass: kg, s^2 ft^{-1}$

force: $kg m s^{-2}$,

Moment

$$\overrightarrow{M} - \overrightarrow{r} \times \overrightarrow{F}$$

$$M = rF\sin(\theta) = Fd$$

Vector moments of the same point add up like forces.

Couple moments can move to anywhere. To move a force, add a pair of force on the target point, then make a couple moment.

Reduction to a Wrench

- 1. Get a resultant force;
- 2. get a resultant moment to a point;
- 3. split the moment into parallel and perpendicular components to the force:
- 4. remove the perpendicular components by moving the force (M = Fd);
- 5. move the parallel moment to the force.