
Verifying the Impact of the Tournament Selection on Genetic Algorithms

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This is done by examining the results different values for the selection operator through simulations the 24 BBOB noise free benchmarks functions with 10, 20 and 40 dimensions and comparing their behavior. The results are XXX (here is where i should state that the tournsize = 2).

1 Introduction

Genetic Algorithm (GA) is a well-known meta-heuristic that is used to seek solutions given a search space. The GA commonly relies on operators such as the crossover operator, the mutation operator and the selection operator. Each of these operators have their own and specific role in the GA. Sawyerr et al. simplify the operators main principles, as follows: The selection operator selects solutions for mating based on the principle of “survival of the fittest”, while the crossover operator generates new solutions by combining the genetic materials of the selected parents and the mutation operator is an exploratory operator that is applied to the population to sustain diversity.

One of the most used selection operator is the Tournament operator. It selects individuals based on their fitness value and on a given parameter, named the tournament size. Although, it is widely used selection operator, it has not been proved, mathematically or by experimentation, what is the impact of the selection operator. As a rule of thumb, small values, such as 2 or 3, are widely used. We could trace back to 1991, and we found out that even from that time is has been a popular

choice. However, we must highlight that these values are chosen arbitrarily and has small scientific background. For instance, in 2009, Nicolau used a more pragmatic approach and set an empirical equation to his binary GA.

we would like to reconsider the idea that exists a trade-off between exploration versus exploitation and the tournament size by verifying, experimentally, the real impact of the tournament size of different domains.

Consequently, our goal in this paper, is to observe the performance of the tournament selection in a real-valued Genetic Algorithm, named the GA-BBOB. For that we applied the GA-BBOB to 24 noise-free BBOB benchmark functions with 10, 20 and 40 dimensions, for more information about these functions see ^[12]. To achieve that, we first explore the number of individuals (k) to be selected by the Tournament operator, with values from 2 to 25. These values are arbitrarily chosen. Then we analyze the results of the GA-BBOB to verify any relationship between all the benchmark functions. Then we propose an alternative, an auto-adaptative k-value “generator”. This alternative is done with the objective of choosing the ideal value of k and we understand that there is much more that could be done in this field of study.

From the works showed here, it is possible to understand that there is a preference to small values for the tournament size, as 2 or 3. We could trace back to 1991, and we found out that even from that time is has been a popular choice. However, we must highlight that these values are chosen arbitrarily and has small scientific background. For instance, in 2009, Nicolau used a more pragmatic approach and set an empirical equation to his binary GA.

These things are described here and there. we

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do this and that here and there too. results and other stuff are here and there.

2 Tournament Selection

In this work we want to analyse the impact of varying the tournament size. For that first, we need to understand why selection schemes are important and how they improve the quality of the Genetic Algorithms (GA).

The selection scheme is intended to improve the average quality of the GA by giving individuals of higher quality a higher probability of composing the next generation ⁶⁾. It is also an important factor to the convergence quality of the GA and the goal of any selection scheme is to favor the proliferation of good individuals in the population of the GA ¹³⁾.

Goldberg and Deb ¹⁰⁾ stated that there exists several selection schemes commonly used in GA, such as the rank selection, the roulette-wheel selection and the tournament selection. They also declare that the complexity of the tournament selection is $O(n)$ and say that tournaments are often held with tournament size $s = 2$, although larger tournament sizes may be analyzed.

2.1 The Tournament Selection Procedure

To be able to understand the impact of the Tournament Selection and to be capable to analyse this selection scheme we not only need to describe how it is conducted, but also we need to study applications that used the Tournament Selection. Here we describe how it operates, and we show the studies next.

Tournament selection is conducted as follows: choose some number, k , of individuals randomly from the population and copy the best individual from this group into an intermediate population, and repeat N times.

Often tournaments are held only between two individuals (binary tournament) but a generalization is possible to an arbitrary group size k called tournament size ^{6, 10)}. A tournament selection, as any selection mechanism in GA, simply favors the selection of better individuals of a population to influence the next generation.

Algorithm 1: Tournament Procedure

input : individuals, k , tournsize

- individuals: A list of individuals to select from.
- k : The number of individuals to select.
- tournsize: The number of individuals participating in each tournament.

output: list of selected individuals

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1 selected = list()
2 for  $i = 1$  to  $k$  do
3   candidates = choose_Random(individuals,
                                tournsize)
4   selected[i] = max(candidates, by: fitness
                     value)
5 end
6 return selected

```

2.2 Selection Schemes and BBOB functions

There are some examples of applications that applied these selection schemes with the BBOB functions. Next we show a few of these examples.

Holtschulte and Moses, in ¹⁵⁾ used rank selection in their GA implementation and applied it to the BBOB functions. They declared that rank selection should be replaced by an elitism selection scheme to improve the performance of their GA implementation by spreading of high quality solutions through the population.

One of these elitist selection scheme is the fitness-based roulette-wheel. However, it has been shown to lead to premature convergence ³⁾. Nicolau, in ¹⁹⁾ uses the tournament selection in his GA implementation also aiming to solve the BBOB functions. He declares that “for harder problem domains, such as the ones on the BBO-Benchmark suite, the tournament selection scheme is more appropriate.”.

Therefore, in this work we chose to analyse the tournament selection behavior in the BBOB functions. The main reasons for that are because the tournament selection is a widely used selection operator ^{9, 11, 1, 13, 23, 14, 8, 4, 16, 5, 19, 22, 2, 20)} and because, as showed previously, it is a more adequate selection scheme to by the BBOB benchmark functions. Likewise, it is simple to code, efficient in both parallel and non parallel architec-

tures, and its pressure is able to be adjusted for different domains ¹⁷⁾ .

2.3 Tournament Pressure

Muhlenbein et al., in ¹⁸⁾ , introduced the concept of selection intensity, which is to measure the pressure of selection schemes. The selection intensity is defined as the expected average fitness of the population after selection.

The selection pressure of tournament selection is expected to increase as the tournament size, k , becomes larger. In other words, the higher the selection pressure, the more the better individuals influences the next generation ¹⁷⁾ .

With higher tournament sizes, the selection pressure rises resulting in higher convergence rates, because the convergence rate of a GA is largely determined by the selection pressure. Accordingly, if the selection pressure is too low, the convergence rate will be slower if it is too high, there is an increased chance of the GA prematurely converging ¹⁷⁾ .

Then, we understand that it is important to explore the Tournament size, hence it is possible that its value would change respectively to the domain of the application that used Tournament Selection.

2.4 Tournament Size in the Literature

In 1991, Goldberg ⁹⁾ , analysed the tournament selection with size 2 in a real-coded GA. They recognized, empirically, that after some time only individuals with relatively high function values will be represented in the population. Also it compares the importance of mutation operators and the tournament selection operator and states that “the addition of some small amount of creeping mutation or other genetic operators should not materially affect these results”.

Goldberg et al. ¹¹⁾ , in 1993, implemented a binary GA using the tournament selection as a convenient way to control selection pressure. They wanted to study the theory aspects of the GA focusing in the mixing building block.

Agrawal ¹⁾ , in 1995, implemented real-coded GA with tournament selection with size 2. Their goal was to compare the results of the simulated binary crossover (SBX) with the binary single-point crossover and with the blend crossover. They con-

cluded that the SBX has search power similar to the single-point crossover and the SBX performs better in difficult test functions than the blend crossover.

In 1999, Tsutsui et al. ²³⁾ proposed a simplex crossover (SPX), a multi-parent recombination operator for real-coded genetic algorithms. For their experiments, they used a real-coded GA with tournament selection with size 2 and showed that “SPX works well on functions having multimodality and/or epistasis with a medium number of parents: 3-parent on a low dimensional function or 4 parents on high dimensional functions.”. In the same year, Harik et al. ¹⁴⁾ studied a simple GA and experimented with the tournament size. Their work shows that a given solution quality can be obtained faster by the tournament size equal to 4 or 8, instead of tournament size equal to 2 or 40. Also, Harik et al. ¹³⁾ implement a binary coded GA using tournament selection. They analyse the impact of the tournament size on their GA with a 100-bit one-max function. They inspected this implementation with tournament sizes of 2, 4, and 8. They observed that the proportion of the building blocks changed with the tournament size, being higher for smaller tournament sizes.

Later, in 2000, Deb ⁸⁾ , implemented a binary GA and also a real-coded GA, both with tournament size 2. He aimed to devise a penalty function approach that does not require any penalty parameter. Because all of their search spaces are defined in the real space, the real-coded GA were more suited in finding feasible solutions, but in all cases, that this approach “have a niche over classical methods to handle constraints”.

In 2001, Beyer et al. ⁴⁾ , analysed the results of using the simulated binary crossover (SBX), the fuzzy recombination operator and the blend crossover (BLX), all with no mutation and tournament size 2 in a real-coded GA. The results indicated that those crossovers exhibit similar performance and linear convergence order.

In 2007, Kaelo et al. ¹⁶⁾ compared the usage of the tournament size in a real-coded GA. They discuss the impact of using tournament with tournament size of 2 and tournament size of 3. In their

work, the results of comparing these two tournament sizes indicated that the tournament with size 3 is more greedy, reduces the number of functions evaluations greatly and also improves CPU usage, therefore, it impacts negatively in the success rate of their GA.

Bhunja et al. ⁵⁾, in 2009, created a tournament GA application to solve an economic production lot-size (EPL) model.

Also in 2009, Nicolau ¹⁹⁾ implemented a binary GA to solve the noise-free BBOB 2009 testbed. Trough a quick experimentation with a subset of benchmark functions which yielded the following equation:

$$k = P/500, \text{ where } P \text{ is the population size. } (1)$$

Sawyer et al. ²⁰⁾, in 2015, proposed the RC-GAu, hybrid real-coded genetic algorithm with “uniform random direction” search mechanism. They used tournament selection with size 3 to explore the suite of noiseless black-box optimization testbed and the results showed great performance in some functions as f_7 and f_{21} .

From the works showed here, it is possible to understand that there is a preference to small values for the tournament size. We traced back to 1991, and we found out that it has been a popular choice ever since. However, we must highlight that these values are chosen arbitrarily and that Nicolau used a more pragmatic approach and set an empirical equation to his binary GA.

3 Experiment Design

In this section we describe how the experiments were conducted.

3.1 GA-BBOB

In the work, we implemented the GA-BBOB a simple GA implementation. The GA-BBOB is a real-valued Genetic Algorithm to explore the search space of the 24 noise free N dimension BBOB benchmark functions ¹²⁾.

Genome Representation and Evolutionary Tools.

Each individual is represented as real valued array, where each element is one input to a N dimen-

sion noise free benchmark BBOB function. Therefore, each individual of the GA-BBOB has size of N real value elements. Each individual’s values is set to be into the interval $[-4, 4]$, once those function are limited by these values. It uses the Uniform crossover, elitism and Gaussian mutation are used as evolutionary operators. The relevant parameters were set as it is possible to see in the following Table 3.1 and Table 3.1.

Table 1 GA-BBOB Relevant Parameters

Number of Generations	800
Population Size	500
Crossover chance	0.9
Mutation Chance	0.1
Elitism size	1

Table 2 Gaussian Mutation parameters

Mean	0
Standard Deviation	1
Ind. Prob. Attribute to be Mutated	0.1

Fitness Function.

The fitness function considered are the N dimensions noise free benchmark BBOB function. For more information, please refer to the work of Hansen et al. ¹²⁾. Consequently, the GA-BBOB has 24 different fitness functions.

Implementation Details.

Chuang et al. ⁷⁾, proposed a Genetic Algorithm specific to run on the noise free BBOB benchmark functions. Some good practices presented in this work were incorporated to the GA-BBOB. These practice are the concept of “restart strategy” and the concept of “alleviate stagnation”.

The restart strategy concept is defined by Chuang et al. as: “For each restart, the initial population is uniformly and randomly sampled within the search space. Whenever the restart condition is met, the algorithm will be reinitialized without using any information about the last test run. This process is iterated until the stopping criteria are met, i.e., maximum number of function evaluations has been reached, or the function value is less than

the target precision.”

The restart condition used in the GA-BBOB is value of the standard deviation of the current population. If it is smaller than 10^{-12} , then the restart condition is triggered. Therefore, when it is contemplated the algorithm population is reinitialized randomly, as it is in the first initialization.

In the GA-BBOB, we consider information about the last run, by utilizing the Elitism strategy on the best individual prior to the restart. The maximum number of function evaluations chosen is $(4 * 10^5)$ and the target precision chosen is (10^{-8}) , values compatible with the ones in the previous work.

Genetic Algorithms and Benchmarks Functions.

IN 2009, Nicolau ¹⁹⁾ proposed the first Genetic Algorithm on the BBOB-2009 noiseless testbed. It was a simple binary Genetic Algorithm and its results showed good results on separable functions, but poor performance is achieved on the other functions.

In 2012, Chuang et al. ⁷⁾ presented the “DBRCGA”, a real coded Genetic Algorithm that uses relative fitness information to direct the crossover toward a direction that significantly improves the objective fitness. The DBRCGA was tested on the BBOB-2012 noiseless testbed. Their results showed that the DBRCGA performs with difficulty in getting a solution with the desired accuracy for high conditioning and multimodal functions within the specified maximum number of evaluations, the DBRCGA presents good performance in separable function and functions with low or moderate conditioning.

In 2013, Holtschulte et al. ¹⁵⁾ evaluated two Genetic Algorithm on the BBOB-2013 noiseless testbed, but none had results competitive with the best 2009 optimization algorithm. Their results highlight the importance of carefully chosen genetic operators.

Sawyer et al found similar results to the one achieved by Chuang et al. on their works in 2013 ²¹⁾ and in 2015 ²⁰⁾. In 2013, they proposed the “PRCGA”, or projection-based real-coded genetic algorithm. It incorporates exploratory search

mecanism based on vector projection. In 2015, they have studied a hybrid application of the Genetic Algorithm (GA) with the *uniform random direction* search, named “RCGAu”. Both the PRCGA and the RCGAu were tested on the BBOB-2013 noiseless testbed. They stated that both of them had difficulties to achieve solutions with the desired accuracy for high conditioning and multimodal functions, though they were able to solve the benchmark functions. Sawyer et al concluded that the RCGAu has excelled in solving the f_1, f_2, f_3, f_7 and f_{21} , tough for the other functions it achieved average results. They also state that real value GA do not efficiently solve highly conditioned problems and studies have currently been carried out to find out why ²⁰⁾.

3.2 Experiments

We made two experiments aiming to verify the impact of fluctuating the tournament size value. In the first experiment, we analysed the relation between the tournament size and the performance on the BBOB benchmark functions. The second experiment, we analyse the impact of fluctuating the values of the tournament size *during* the execution.

We used ANOVA to test whether any of the combinations shows a significant deviation in terms of tournament size values, represented as the quality of the result given functions and dimensions. In each of these tests, we set $\alpha = 0.05$.

Fixed tournament size

To analyze the impact of the tournament size into the GA-BBOB in terms of quality of result, the following experiments were performed. First we analyse different values for the tournament size applied to the BBOB benchmark functions with 10, 20 and 40 dimensions. The tournament sizes were select, arbitrarily, from 2 to 25. For each combination of BBOB benchmark functions, dimensions, and tournament size we performed 40 repetitions.

Fluctuating tournament size

To analyze the impact of fluctuating the tournament size *during* the execution of the GA-BBOB in terms of quality of result, the following experiments were performed. We start with the tour-

nament size equal to 2, as many of the work cited on section 2.4. Then when half of the evaluations are completed, the tournament size is changed to another value, chosen from 2 to 25. These values were applied to the BBOB benchmark functions with 10, 20 and 40 dimensions. For each combination of BBOB benchmark functions, dimensions, and fluctuating tournament size we performed 40 repetitions.

4 Results

To better understand the results, we performed an ANOVA analysis over all optimum values achieved from the GA-BBOB. These optimum values represent the values obtained from the 1728 different combinations paired by the 24 functions, the 3 dimensions and the 24 tournament sizes.

To get a finer intuition about these results, we show some visual examples separated in two groups of Figures. The first group, shows the mean value achieved by the GA-BBOB given a function. The second one, shows the convergence plot with the mean of the values found at each generation, the function target value with a given tournament size.

All Figures represent the mean of 40 repetitions given a certain dimension.

4.1 First Group

The Figure 1 and the Figure 2 exemplify that for the F1 function (with 10 dimensions) and F11 (with 20 dimensions), that changing the tournament size, does not lead to significant better final values found by the GA-BBOB.

4.2 Second Group

The Figure 3 exemplifies that for the F1 function (with 10 dimensions), the GA-BBOB is converging towards the optimum target value, represented by the red, vertical line. The same applies for the F11 function (with 20 dimensions), as is its possible to check on the Figure 4.

4.3 ANOVA

The ANOVA analysis showed that, as expected, number of dimensions, and e function were the variables that show some impact on the quality of the GA-BBOB outcome. Surprisingly enough, no significant variation was found when compar-

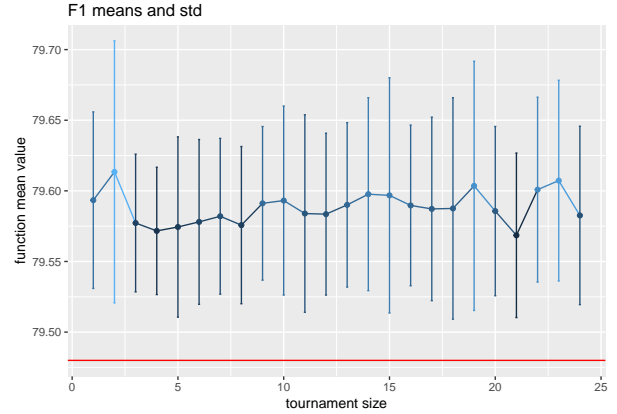


Fig. 1 Optimum values for the F1 function by the GA-BBOB. The mean of 40 repetitions is shown as bullets and the bars represent the standard deviation, with 10 dimensions.

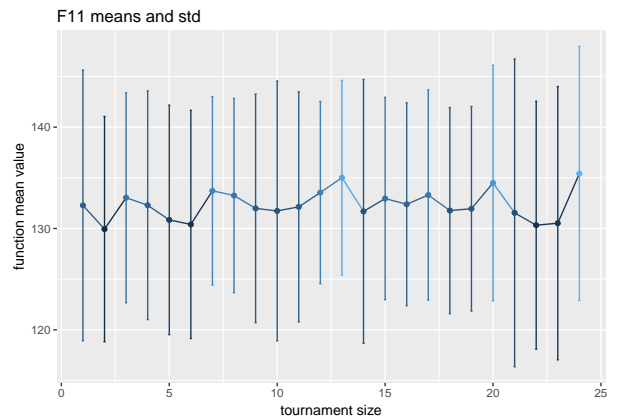


Fig. 2 Optimum values for the F11 function by the GA-BBOB. The mean of 40 repetitions is shown as bullets and the bars represent the standard deviation, with 20 dimensions.

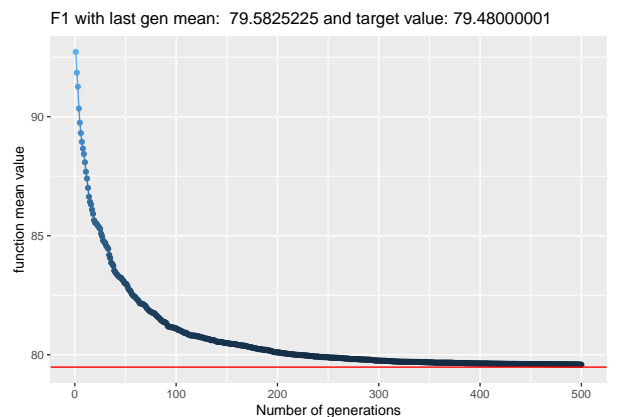


Fig. 3 GA-BBOB population convergence for function F1. The mean of 40 repetitions is shown, with 10 dimensions.

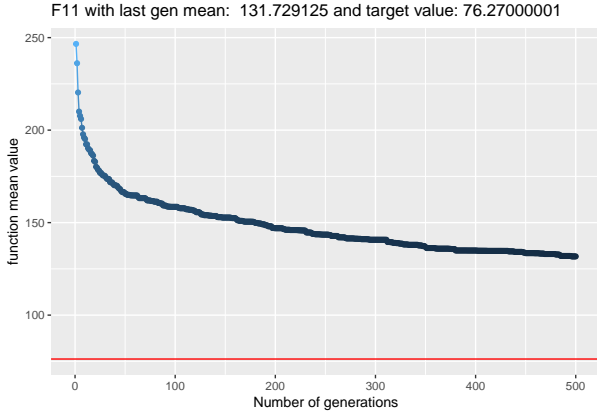


Fig. 4 GA-BBOB population convergence for function F11. The mean of 40 repetitions is shown, with 20 dimensions.

ing different tournament sizes. The data obtained from the ANOVA analysis can be seen (table/figure).

5 Conclusion

In this paper we proposed an experimental analysis of the impact of the tournament size on the BBOB benchmark functions. First, we verified that there is no mathematical or experimental study that backgrounds any possible choice of value for the tournament size. Then, we searched into the literature, and realized that, in most cases, the most common values for tournament size are 2 or 3, though these values were used as a popular principle.

Based on that, we projected our two experiments. In the first one, we analyzed a group of tournament size values, ranging from 2 to 24 and applied a simple GA, GA-BBOB, into the BBOB benchmark functions, given 10, 20 and 40 dimensions. The results showed that for any value of the tournament size did no impact was observed in improving the quality of the results for any functions in any dimension.

Further, we explored the fluctuation of the tournament size value through the execution of the GA-BBOB, by starting with the value 2 and then varied it, ranging from 2 to 24 and applied on the GA-BBOB, with the BBOB benchmark functions, given only 40 dimensions. This time also, no significant result was discovered.

We propose that more investigations need to be done, in order to truly understand the role of the tournament size value related to the quality of final results. For that, we suggest that another kind of fluctuation, a more elaborated one, should be explored. Also, we would like to understand if our results were mainly related with the GA-BBOB combination of parameters (crossover operator, mutation operator, selection operator, etc).

Hence most of the works already used tournament sizes equal to 2 or 3, and based on the results obtained allied with the complexity of the tournament selection being $O(n)$, we continue to advise that values of 2 or 3 should continue to be used as first choice.

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