

# Contribution Title<sup>\*</sup>

First Author<sup>1</sup>[0000–1111–2222–3333], Second Author<sup>2,3</sup>[1111–2222–3333–4444], and  
Third Author<sup>3</sup>[2222–3333–4444–5555]

<sup>1</sup> Princeton University, Princeton NJ 08544, USA

<sup>2</sup> Springer Heidelberg, Tiergartenstr. 17, 69121 Heidelberg, Germany  
`lncs@springer.com`

<http://www.springer.com/gp/computer-science/lncs>

<sup>3</sup> ABC Institute, Rupert-Karls-University Heidelberg, Heidelberg, Germany  
`{abc,lncs}@uni-heidelberg.de`

**Abstract.** bet-and-run and moea/d

## 1 Introduction

Multiobjective Optimization Problem have  $m$  multiple objective functions that must be optimized simultaneously.

Maximize<sup>1</sup>  $F(x) = (f_1(x), f_2(x), \dots, f_m(x))$ ,  
subject to  $x$  in  $\Omega$ .

-  $F(x)$  objective functions; -  $f_i$  is the  $i$ -th objective to be maximized; -  $x$  is the decision vector; -  $\Omega$  is the decision space.

<sup>1</sup> All definitions are for maximization. Following inequalities should be reversed if the goal is to minimize.

Many real-world scientific and engineering are MOP. Water quality control, Ground-water pollution re-mediation, Design of marine vehicles [3]. Petrol extraction. Hard problems: to balance the interests of the multi-objective as a whole is hard.

Objectives may be conflicting - The goal is to find good trade-off.

- Set of solutions.

Set of \*optimum solutions\* - Pareto set.

- Non-dominated solutions: no single solution provides a better trade-off in all objectives.

Let  $u = (u_1, \dots, u_m)$  and  $v = (v_1, \dots, v_m)$  vectors in  $\Omega$  (the decision space). -  $\forall i : u$  dominates  $v$  if  $f_i(u) \leq f_i(v)$  and  $\exists j : f_j(u) < f_j(v)$ . -  $u$  dominates  $v$ ,  $v$  is dominated by  $u$ ,  $u$  is better than  $v$ .

A point  $x^*$  in  $\Omega$  is called \*Pareto Optimal\* if no other point dominates  $x^*$ .

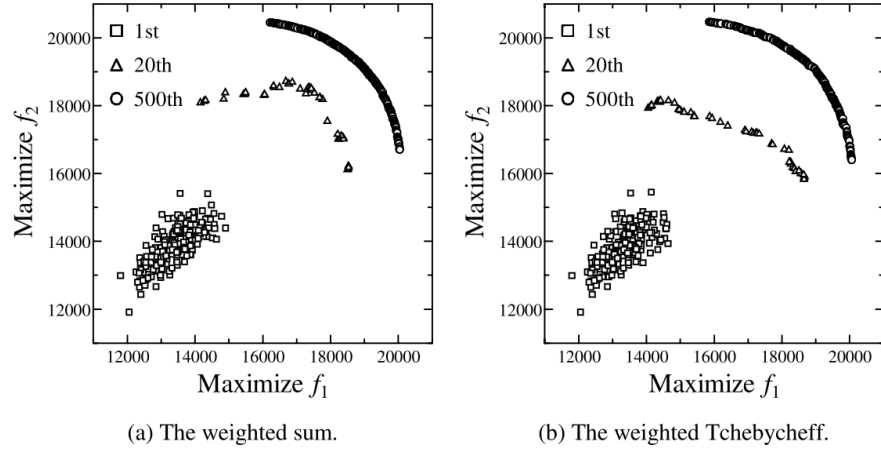
The set of all Pareto Optimal is called the Pareto Set.

$P^* = \{x \in \Omega : \nexists y \in \Omega \text{ and } F(y) \leq F(x)\}$

Pareto Front is the image of the Pareto Set in the objective space.  $PF = F(x) = (f_1(x), \dots, f_m(x)) : x \in P^*$

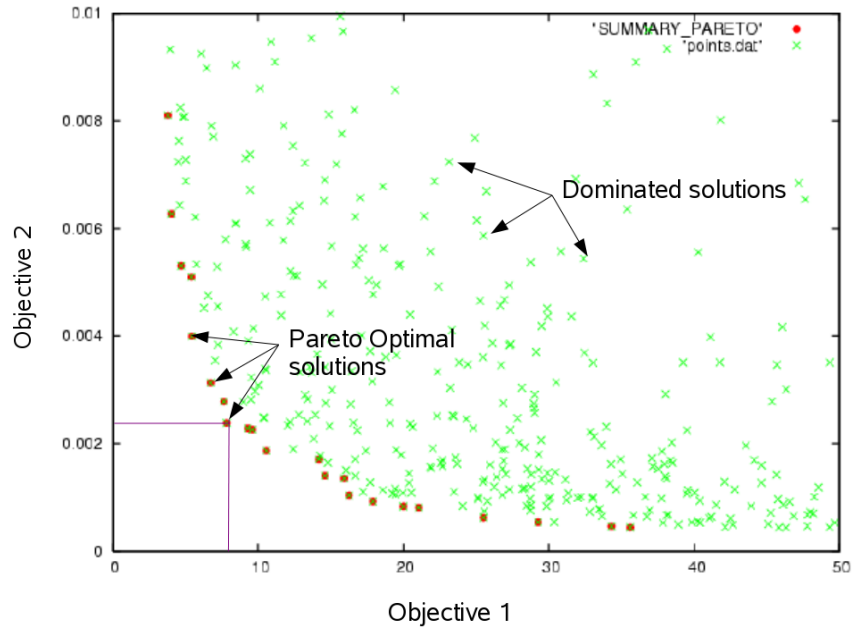
---

<sup>\*</sup> Supported by organization x.



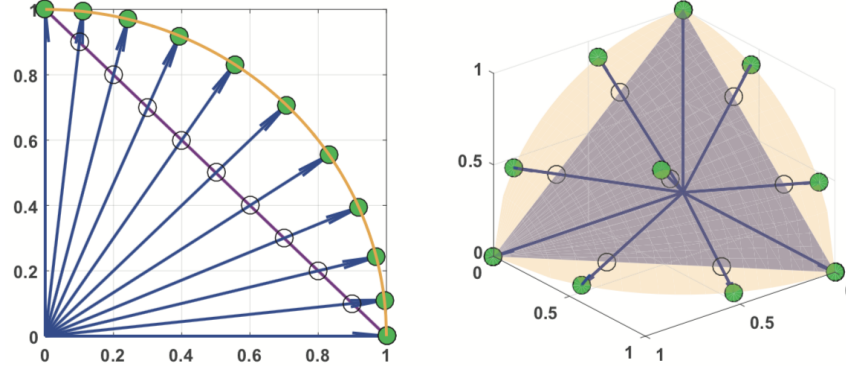
**Fig. 1.** Pareto Set - [6]

“r fig.width=3, fig.height=15,echo=FALSE



## 2 MOEA/D

MOEA/D represents a class of population-based meta-heuristics for solving Multi Objective Problems (MOPs).



**Fig. 2.** Decomposition - 2 and 3 objectives, [2]

It is based on decomposition - one kind of scalarizing function. One multi-objective problem becomes various single-objective sub-problems. All sub-problems are solved in parallel. A decomposition strategy generates weight vectors that define the sub-problems.

Why use decomposition?

It may be good at generating an even distribution of solutions in MOPs. It reduces the computation complexity when compared to other algorithms (NSGA-II) [8]. An optimal solution of a set of scalar optimization problems can be a Pareto optimal solution, under mild conditions. All solutions can be compared based on their objective function values. It is simple to find a solution to multi single-objective problems than for a multi-objective problem. Fitness assignment and diversity maintenance become easier to handle.

$$f_3(x) = F * w_3$$

$$\text{In general, } f_i(x) = F * w_i$$

Components of the MOEA/D

- Decomposition strategy: decomposes w/ weight vectors;
- Aggregation function: weight vector =  $\lambda$  single-objective sub-problems;
- Neighbourhood assignment strategy: Relationship between sub-problems;
- Variation Stack: New candidate solutions;
- Update Strategy: Maintain/discard candidate solutions;
- Constraint handling method: Constraint violation;
- Termination Criteria: when to stop the search.

Variations Already Integrated

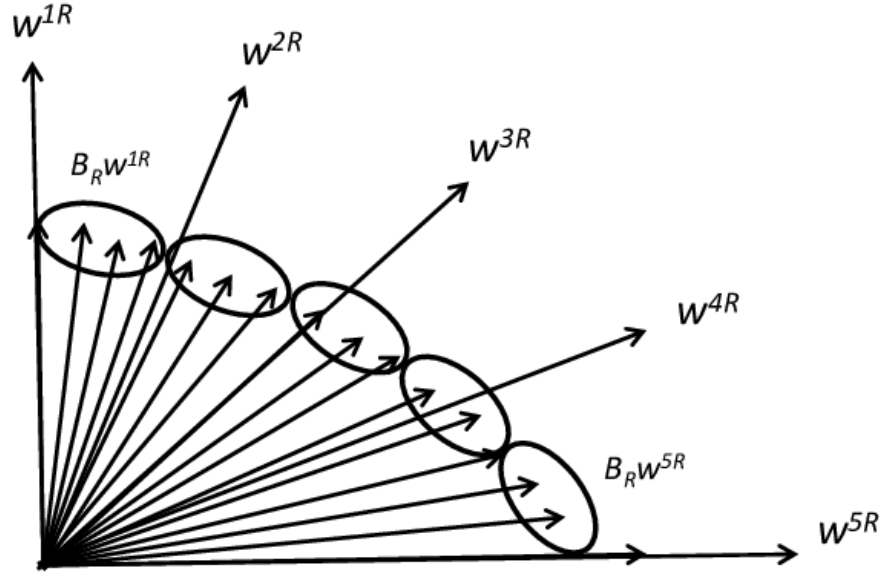
On-line Resource Allocation - proposed in the context of MOEA/D by [9]. Bet-and-Run: A kind of restart strategy - in the context of single-objective problems (SOP) by [4].

What is Online Resource Allocation

- On-line Resource Allocation (ONRA) is an adaptation strategy that aims to adjust the behaviour of an algorithm in an on-line manner to suit the problem in question.

How it affects MOEA/D [9].

- Some sub-problems can be more difficult to approximate than others. To better explore them, different computational resources are allocated to different sub-problems.



**Fig. 3.** Decomposition and Aggregation Function - [2].

- The resources re-allocated is \*the number of functions evaluations\*. - From an equal amount to every sub-problem to an amount related to the difficulty of the sub-problem.

Restart Strategy

- Restart Strategy is a strategy used to avoid heavy-tailed running time distributions [5].

- If a execution of an algorithm does not conclude within a pre-determined limit or if the solution quality is unsatisfactory, we restart the algorithm @lissovoi2017theoretical.

Bet-and-Run framework

- It is defined in @fischetti2014exploiting. as a number of short runs with randomized initial conditions, bet on the most promising run, and bring it to completion.

- To the best of our knowledge, only applied with EA in the context of SOP.

How it affects MOEA/D - [7].

- Initialisation can have a small beneficial effect even on very easy functions.

- Countermeasure when problems with promising and deceptive regions are encountered.

- Additional speed-up heuristic.

### 3 Evaluation Metrics

Evaluation Metrics

Unary Indicators

- Measure Pareto Sets independently. - Power is restricted. - Cannot tell in general if a set is better than another. - Focus on problem dependent and specifics. - Assumptions

and knowledge should be specified. 1. Hyper-volume. 2. Error ratio. 3. Distance from reference set.

Binary Indicators

- Theoretically have no limitations. - Analysis and presentation of results more difficult.

1. R1, R2, R3 indicators. 2.  $\epsilon$ -Indicator. 3. Binary Hyper-volume.

Hypervolume Considerations

- Is complete - If, and only if  $HV(A) > HV(B) \implies A$  is not worse than  $B$ . - Is weakly compatible -  $HV(A) > HV(B) \implies B$  dominates  $A$ . - Assumptions - All points of a Pareto Set under consideration dominate the reference point. - @ishibuchi2018specify proposed a method to specify the reference point from a viewpoint of fair performance comparison.

Considerations - A large population size is **\*\*always\*\*** more beneficial than a small one. - Measures both the convergence toward the Pareto Front and the diversity of non-dominated solutions. - A monotonic increase of the hyper-volume over time cannot always be ensured. - For MOEA/D that is always true.

$\epsilon$ -Indicator Considerations - It compares 2 Pareto Sets. - It indicates which set is better and how much better - If  $A$  is better than  $B \implies I_\epsilon(B, A) > 0$ . - If  $I_\epsilon(A, B) \leq 0$  and  $I_\epsilon(B, A) > 0 \implies A$  is better than  $B$ .

The benchmark used are the DTLZ and the ZDT group of functions.

DTLZ are easy [1].

## 4 Experiment Design

4. maxiter 300. 5. Based on the common variation: MOEA/D (variations 1 and 2 from MOEADr) and MOEA/D-DE.

Overview of the experiments

DTLZ[1-7] MOP benchmark- Available from the MOEADr package. both 3 and 2 objectives. ZDT[1-6] MOP benchmark- Available from the MOEADr package, only for 2 objectives.

Number of evaluations: 50000 or 100000 for 4 objectives Compared with their HV - normalized between 0 and 1 (based on the fair comparison paper). 30 repetitions. box-plots Kruskal-Wallis (data non-normal data, used in the literature)

Configurations and Parameters

Control - Based on the common variation: MOEA/D (variation1) and MOEA/D-DE as in preset\_moead Control and ONRA - parameters: dt = 20 Ben-and-run Ben-and-run and ONRA - parameters: dt = 20

Dt - interval that control the resources allocation. From the proposal paper, there is no much sensibility. Decomposition method used - SLD, with H being 199 for 2D and 19 for 3D number of dimensions - 60 All other parameters are defined by preset\_moead

Bet-and-run

Phase 1 of the bet-and-run strategy is using the epsilon indicator. 40 instances. It needs two Pareto sets. The first is the Pareto set of a bet instance while the other is the Pareto set from the control algorithm executed with 1% of the number of interactions. Phase 2 uses the 60% rest of max interactions.

## 5 Results

Analysis are done with

## 6 Conclusion

## References

1. Bezerra, L.C., López-Ibáñez, M., Stützle, T.: Comparing decomposition-based and automatically component-wise designed multi-objective evolutionary algorithms. In: International Conference on Evolutionary Multi-Criterion Optimization. pp. 396–410. Springer (2015)
2. Chugh, T.: Handling expensive multiobjective optimization problems with evolutionary algorithms. *Jyväskylä studies in computing* 263. (2017)
3. Coello, C.A.C., Lamont, G.B., Van Veldhuizen, D.A., et al.: Evolutionary algorithms for solving multi-objective problems, vol. 5. Springer (2007)
4. Friedrich, T., Kötzling, T., Wagner, M.: A generic bet-and-run strategy for speeding up stochastic local search. In: AAAI. pp. 801–807 (2017)
5. Gomes, C.P., Selman, B., Crato, N., Kautz, H.: Heavy-tailed phenomena in satisfiability and constraint satisfaction problems. *Journal of automated reasoning* **24**(1-2), 67–100 (2000)
6. Ishibuchi, H., Sakane, Y., Tsukamoto, N., Nojima, Y.: Adaptation of scalarizing functions in moea/d: An adaptive scalarizing function-based multiobjective evolutionary algorithm. In: International Conference on Evolutionary Multi-Criterion Optimization. pp. 438–452. Springer (2009)
7. Lissovoi, A., Sudholt, D., Wagner, M., Zarges, C.: Theoretical results on bet-and-run as an initialisation strategy. In: Proceedings of the Genetic and Evolutionary Computation Conference. pp. 857–864. ACM (2017)
8. Zhang, Q., Liu, W., Li, H.: The performance of a new version of moea/d on cec09 unconstrained mop test instances. In: Evolutionary Computation, 2009. CEC’09. IEEE Congress on. pp. 203–208. IEEE (2009)
9. Zhou, A., Zhang, Q.: Are all the subproblems equally important? resource allocation in decomposition-based multiobjective evolutionary algorithms. *IEEE Transactions on Evolutionary Computation* **20**(1), 52–64 (2016)