A Survey of Multiobjective Evolutionary Algorithms Based on Decomposition

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Abstract—Decomposition is a well-known strategy in traditional multiobjective optimization. However, the decomposition strategy was not widely employed in evolutionary multiobjective optimization until Zhang and Li proposed multiobjective evolutionary algorithm based on decomposition (MOEA/D) in 2007. MOEA/D proposed by Zhang and Li decomposes a multiobjective optimization problem into a number of scalar optimization subproblems and optimizes them in a collaborative manner using an evolutionary algorithm (EA). Each subproblem is optimized by utilizing the information mainly from its several neighboring subproblems. Since the proposition of MOEA/D in 2007, decomposition-based MOEAs have attracted significant attention from the researchers. Investigations have been undertaken in several directions, including development of novel weight vector generation methods, use of new decomposition approaches, efficient allocation of computational resources, modifications in the reproduction operation, mating selection and replacement mechanism, hybridizing decomposition- and dominance-based approaches, etc. Furthermore, several attempts have been made at extending the decomposition-based framework to constrained multiobjective optimization, many-objective optimization, and incorporate the preference of decision makers. Additionally, there have been many attempts at application of decomposition-based MOEAs to solve complex real-world optimization problems. This paper presents a comprehensive survey of the decomposition-based MOEAs proposed in the last decade.

Index Terms—Decomposition approach, evolutionary algorithm (EA), many-objective optimization, multiobjective optimization.

I. INTRODUCTION

A MULTIOBJECTIVE optimization problem (MOP) can be represented as follows:

$$\text{minimize } F(x) = (f_1(x), \ldots, f_m(x))^T$$
$$\text{subject to} \quad x \in \Omega \qquad (1)$$

where Ω is the search space and x is the decision variable vector. $F : \Omega \rightarrow \Re^m$, where m is the number of objective functions, and $\Re^m$ is the objective space.

In most practical optimization problems, the objectives defined in (1) are mutually conflicting. A single solution, thus, is not available which can minimize all the objectives. The presence of conflicting objectives in an MOP gives rise to a set of tradeoff optimal solutions known as Pareto-optimal (P-O) solutions [1, p. 21]. Since, it is practically not possible and desirable to obtain the entire set of P-O solutions, an approximation set to the Pareto front (PF, the set of all P-O solutions) is rather obtained. Classical multiobjective optimization methods are based on transforming the MOP to a single-objective optimization problem by constructing aggregation functions and obtaining one particular P-O solution at a time. The widely used methods for constructing aggregation functions are the weighted sum (WS) approach and the Tchebycheff (TCH) approach [2, p. 97]. However, the disadvantage of classical multiobjective optimization approach is that to obtain the approximation set, the method has to be applied multiple times in the hope of finding a different P-O solution in each simulation run.

Since, an evolutionary algorithm (EA) because of its population based nature, can obtain multiple P-O solutions in a single simulation run, multiobjective EAs (MOEAs) have been very popular in solving MOPs [3], [4]. The three goals of an MOEA are: 1) to find a set of solutions as close as possible to the PF (known as convergence); 2) to find a well distributed set of solutions (known as diversity); and 3) to cover the entire PF (known as coverage). To achieve these goals, several MOEAs have been proposed in the literature which can be broadly categorized under three categories.

1) *Domination-Based Framework:* In this framework, an MOP is optimized by simultaneously optimizing all the objectives. The assignment of fitness to solutions is based on Pareto-dominance principle which plays a key role in the convergence of domination-based MOEAs. Further, an explicit diversity preservation scheme is necessary in order to maintain a diverse set of solutions. Some of the remarkable MOEAs based on the dominance-based framework are NSGA-II [5], SPEA2 [6], etc.

2) *Indicator-Based Framework:* In this framework, a performance indicator such as the hypervolume (HV) indicator is used to measure the fitness of a solution by assessing its contribution to the combined convergence

and diversity measure of an MOEA. Some of the well known MOEAs based on the indicator-based framework are IBEA [7], SIBEA [8], etc.

3) *Decomposition-Based Framework:* In this framework, scalarizing functions such as the weighted TCH are used to convert the MOP into single-objective optimization subproblems and the subproblems are solved in a single run using an EA. The decomposition-based MOEAs utilize aggregated fitness value of solutions in the selection. Some of the famous representative MOEAs based on decomposition are MOEA/D [9], MOGLS [10], C-MOGA [11], etc.

Although, the dominance- and the indicator-based framework are quite popular, they have certain limitations. One of the main drawbacks of the dominance-based framework is that it not suitable for many-objective optimization. This is because in presence of many-objectives, almost all the solutions in the population become nondominated with one another, thereby reducing the selection pressure and hampering the evolutionary process [12], [13]. In the indicator-based framework, the HV indicator is generally adopted due to its established theoretical properties. The limitation of the HV indicator-based MOEAs is the computational cost related to evaluation of the HV indicator, which grows exponentially with the increase in number of objectives. Although, it is worthwhile noting that there have been some attempts at reducing the computational costs of HV indicator and making the HV indicator-based MOEAs more effective for many-objective optimization [14], [15]. Further, some studies have suggested replacing the HV indicator with other indicators which are much less computationally expensive and have good theoretical properties as well (e.g., $R2$ [16] and $\triangle_p$ [17]). However, the framework which has attracted the most attention of researchers in the evolutionary multiobjective optimization community in the last decade is the decomposition-based framework.

Although the idea of decomposition for solving MOPs has been implemented to a certain extent in several metaheuristics [10], [11], [18]–[23], it became popular with the introduction of MOEA based on decomposition by Zhang and Li [9]. In the original framework proposed by Zhang and Li [9], termed as MOEA/D, an MOP is decomposed into several scalar optimization subproblems, which are formulated by decomposition approach such as the TCH using uniformly distributed weight vectors. In MOEA/D, all the subproblems are solved simultaneously by employing an EA and evolving a population of solutions. The characteristic features of MOEA/D framework are that a neighborhood relation is defined among the subproblems based on the distance between their weight vectors and local mating as well as local replacement is implemented in a steady-state manner.

Since the proposition of the original MOEA/D framework, many studies have been conducted in the literature to: 1) overcome the limitations in the design components of the original MOEA/D; 2) improve the performance of MOEA/D; 3) present novel decomposition-based MOEAs; and 4) adapt decomposition-based MOEAs for different type of problems. For example, studies on decomposition-based MOEAs have

been carried out to incorporate novel weight vector generation methods [24]–[26], include improved decomposition methods [27], [28], integrate efficient computational resource allocation strategies [24], [29]–[31], modify the reproduction operators [32]–[34], enhance the mating selection [35], and the replacement procedure [36], [37]. Furthermore, decomposition-based MOEAs have been proposed for many-objective optimization problems (MaOPs) [38], [39], constrained MOPs [40], [41], and incorporating preference of decision makers (DMs) [42]–[44]. Moreover, decomposition-based MOEAs have been extended to solve several real-world MOPs as well. This paper presents a comprehensive survey of work conducted on decomposition-based MOEAs, particularly after Zhang and Li [9] proposed MOEA/D framework in 2007.

The remainder of this paper is systematically divided into different sections. Section II presents the basics of the original MOEA/D framework. Section III presents an overview of different lines of research along decomposition-based MOEAs. In Section IV, the studies on weight vector generation methods in the decomposition-based framework are reviewed. Section V presents a review of the decomposition methods investigated in the decomposition-based framework. Section VI reviews the strategies proposed for efficient computational resource allocation in MOEAs based on decomposition. Section VII reviews the studies presented on modifying the reproduction operators in the MOEA/D framework. In Section VIII, the studies conducted on investigations into mating selection and replacement mechanism in the MOEA/D framework are discussed. Section IX discusses the studies conducted on extending the decomposition-based framework to many-objective optimization. Section X reviews the studies presented for extending the decomposition-based MOEAs to constrained MOPs. In Section XI, the preference incorporation strategies in the decomposition-based framework are reviewed. Section XII summarizes the application of decomposition-based MOEAs to real-world MOPs. Finally, Section XIII presents the conclusions and the directions for future research.

II. ORIGINAL MOEA/D FRAMEWORK

In the original MOEA/D framework [9], a set of uniformly spread weight vectors $\lambda_1, \lambda_2, \ldots, \lambda_N$ are generated, where $\lambda_i = (\lambda_i^1, \ldots, \lambda_i^m)^T$, subject to the conditions $\lambda_i^j \geq 0$ and $\sum_{j=1}^m \lambda_i^j = 1$ for all $i = 1, \ldots, N$ and for all $j = 1, \ldots, m$. The MOP (1) is decomposed into N scalar optimization subproblems by decomposition methods (discussed below) using uniformly distributed weight vectors.

A. Decomposition Methods

The original study on MOEA/D investigated three decomposition methods namely—the WS, the weighted TCH, and the penalty-based boundary intersection (PBI), as briefly described below.

1) Weighted Sum Approach: In this approach, the ith subproblem is defined in the form

$$\text{minimize } g^{ws}(x|\lambda_i) = \sum_{j=1}^m \lambda_i^j f_j(x). \qquad (2)$$

This approach works well for convex PFs (for minimization problems). However, it cannot approximate the entire PF for nonconvex PFs [9].

2) Tchebycheff Approach: In this approach, the ith subproblem is defined in the form

$$\text{minimize } g^{te}(x\,|\,\lambda_i, z^*) = \max_{1\le j\le m}\left\{\lambda_i^j\left|f_j(x) - z_j^*\right|\right\} \qquad (3)$$

where $z^* = (z_1^*, \ldots, z_m^*)^T$ is the ideal reference point with

$$z_j^* < \min\{f_j(x)|x \in \Omega\} \quad \text{for } j = 1, 2, \ldots, m. \qquad (4)$$

3) Penalty-Based Boundary Intersection Approach: In this approach, the ith subproblem is defined in the form

$$\text{minimize} \quad g^{\text{pbi}}(x|\lambda_i, z^*) = d_1 + \theta d_2$$
$$\text{where} \quad d_1 = \frac{\left\|(F(x) - z^*)^T\lambda_i\right\|}{\|\lambda_i\|}$$
$$d_2 = \left\|F(x) - \left(z^* - d_1\frac{\lambda_i}{\|\lambda_i\|}\right)\right\|. \qquad (5)$$

In (5), z^* is the reference point as defined in the TCH approach and θ is a penalty parameter. A drawback of the PBI decomposition approach is that the penalty parameter θ is to be properly tuned.

B. MOEA/D Framework

In the following discussion on MOEA/D framework, it is assumed that the TCH approach is adopted. According to the TCH approach, the solution to the target MOP is equivalent to optimizing N scalar optimization subproblems, where the objective function of the jth subproblem is given by

$$g^{te}(x|\lambda_i, z^*) = \max_{1\le j\le m}\left\{\lambda_i^j\left|f_j(x) - z_j^*\right|\right\}. \qquad (6)$$

MOEA/D minimizes all these N objective functions simultaneously in a single run.

In MOEA/D, a neighborhood of weight vector λ_i is defined as the T closest weight vectors in $\lambda_1, \lambda_2, \ldots, \lambda_N$. Thus, the neighborhood of the ith subproblem consists of all the subproblems with the weight vectors from the neighborhood of λ_i. A population of N solutions is randomly generated and each solution is randomly allocated to a particular subproblem. In the reproduction operation corresponding to a subproblem i, two solutions are randomly selected from the neighborhood of the subproblem i and genetic operators are applied to generate an offspring. Thereafter, in a steady-state manner, the offspring is used to update the neighborhood of the subproblem i. Thus, the population is composed of the best solution found so far for each subproblem.

The original MOEA/D requires the following input.
Input:
1) MOP given by (1);
2) *N:* The number of subproblems considered, i.e., the population size;
3) $\lambda_1, \lambda_2, \ldots, \lambda_N$: A set of N uniformly distributed weight vectors;
4) *T:* The neighborhood size (NS).

At each generation, MOEA/D maintains the following.
1) A population of N solutions $x_1, \ldots, x_N$, where x_i is the current solution to the ith subproblem, and $F(x_1), \ldots, F(x_N)$.
2) Reference point $z = (z_1, \ldots, z_m)$, where z_j is the best value found so far for objective f_j $\forall j = 1, \ldots, m$.

The steps of the original MOEA/D are as follows.

Step 1 (Initialization):
  a) *Step 1.1:* Compute the Euclidean distances between any two weight vectors and then determine the T closest weight vector to each λ_i. For each $i = 1, \ldots, N$, set $B(i) = \{i_1, \ldots, i_T\}$, where λ_j, $\forall j \in B(i)$ are T closest vectors to λ_i.
  b) *Step 1.2:* Randomly generate the initial population.
  c) *Step 1.3:* Evaluate $F(x_i)$ $\forall i = 1, \ldots, N$.
  d) *Step 1.4:* Initialize z: the initial reference point $(z_1, \ldots, z_m)$ according to the condition: $z_j = \min_{1\le i\le N} f_j(x_i)$ $\forall j = 1, \ldots, m$.

Step 2 (Update):
For $i = 1, \ldots, N$, do
  a) *Step 2.1 (Reproduction):* Randomly select two indices k and l from $B(i)$, and generate a child solution x_{child} from parents x_k and x_l by applying genetic operators.
  b) *Step 2.2 (Repair):* Repair x_{child} by applying problem-specific repair/improvement heuristic.
  c) *Step 2.3 (Function Evaluation):* Evaluate $F(x_{\text{child}})$.
  d) *Step 2.4 (Update of z):* For each $j = 1, \ldots, m$, if $z_j > f_j(x_{\text{child}})$ then set $z_j = f_j(x_{\text{child}})$.
  e) *Step 2.5 (Replacement/Update of Solutions):* For each index $j \in B(i)$, if $g^{te}(x_{\text{child}}|\lambda_j, z) \le g^{te}(x_j|\lambda_j, z)$ then set $x_j = x_{\text{child}}$ and $F(x_j) = F(x_{\text{child}})$.

Step 3 (Stopping Criteria):
If termination criterion is satisfied, then obtain approximation to P-O solutions: $\{x_1, \ldots, x_N\}$ and PF: $\{F(x_1), \ldots, F(x_N)\}$ else go to step 2.

C. Origin of the MOEA/D Framework

The idea of decomposition for solving MOPs has been adopted by several metaheuristics (see [10], [11], [18]–[23]) before Zhang and Li [9] proposed MOEA/D. However, the original MOEA/D framework [9] has its origins, particularly in the cellular multiobjective genetic algorithm (C-MOGA) presented by Murata and Gen [11]. To be specific, MOEA/D is similar to C-MOGA in aspects such as decomposition of MOP into several single-objective optimization subproblems, weight vector generation method, and the idea of neighborhood for mating selection. However, there are two differences between MOEA/D and C-MOGA. First, in C-MOGA, a newly generated offspring corresponding to a cell (i.e., index) is compared with only the current solution of the cell. However, in MOEA/D, it is compared with its neighbors as well. Thus, in MOEA/D, along with the mating neighborhood structure, there is a replacement neighborhood structure as well. Ishibuchi *et al.* [45] demonstrated that employing local replacement neighborhood structure plays a very crucial role in the performance of MOEA/D. Second, Murata and Gen [11]
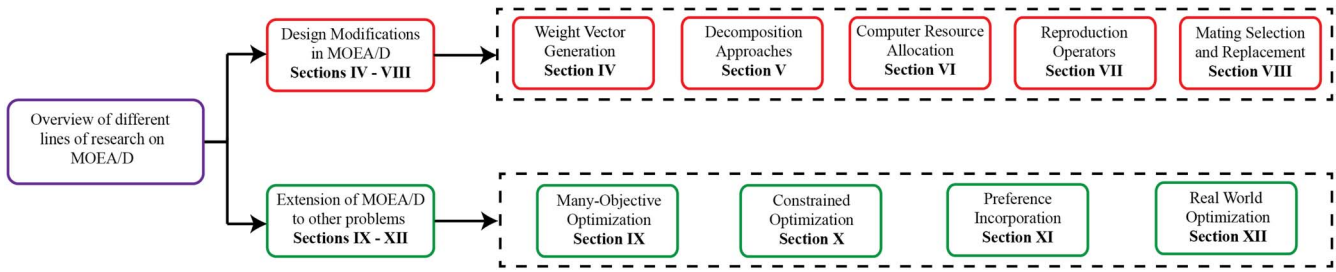
Fig. 1. Overview of different lines of research on MOEAs based on decomposition.

employed only the WS scalarizing function in C-MOGA whereas Zhang and Li [9] investigated the WS, the TCH, and the PBI scalarizing functions in MOEA/D.

III. OVERVIEW OF DIFFERENT LINES OF RESEARCH ALONG DECOMPOSITION-BASED MOEAs

The main design components of the MOEA/D framework are: 1) weight vector generation method; 2) decomposition method; 3) computational resource allocation strategy; 4) mating selection mechanism; 5) reproduction operators; and 6) replacement procedure. However, there are several limitations in the above design components embedded in the original MOEA/D framework [9]. Furthermore, Zhang and Li [9] tested the original MOEA/D primarily on unconstrained MOPs with 2–3 objectives and multiobjective knapsack problems (MOKPs) with 2–4 objectives. Thus, different lines of research along decomposition-based MOEAs have mainly surfaced to: 1) address the drawbacks in the design components of the original MOEA/D; 2) enhance the performance of MOEA/D; 3) present new decomposition-based MOEAs; and 4) extend decomposition-based MOEAs to other category of problems such as many-objective optimization, constrained optimization, preference incorporation, and real-world optimization. This section discusses the limitations existing in the original MOEA/D framework and presents an overview of the different lines of research along decomposition-based MOEAs. For the convenience of readers, the different lines of research along decomposition-based MOEAs are illustrated in Fig. 1.

1) *Weight Vector Generation:* In the original MOEA/D version, the weight vectors are generated using the simplex lattice design method [46]. The main drawbacks of this method are that: a) the population size increases nonlinearly with the number of objectives and the population size cannot be set at will [47] and b) the distribution of the resulting weight vectors is not very uniform for three or more objectives [47]. To overcome these limitations, several studies have been conducted on weight vector generation methods in the decomposition-based MOEAs, as discussed in Section IV.

2) *Decomposition Approach:* The original MOEA/D study [9] investigated three decomposition approaches namely—the WS, the TCH, and the PBI. However, these traditional decomposition approaches have certain shortcomings such as performance on MaOPs [48], handling problems with disparately scaled objectives [9],

and association of same solution with several different subproblems [28]. Furthermore, the choice of decomposition method is critical to the performance of MOEA/D on a particular problem [9] and it is not an easy task to select an appropriate method for different problems [27]. To overcome these shortcomings, several studies have been carried out on decomposition methods, as discussed in Section V.

3) *Computational Resource Allocation:* In the original MOEA/D [9], all the subproblems are treated equally and receive uniform computational effort at each generation. However, some regions of the PF can be more difficult to approximate than the others [29]. Moreover, the original MOEA/D [9] adopts a constant set of uniformly distributed weight vectors for every problem irrespective of the characteristics of the PF. However, uniformly distributed weight vectors cannot produce uniformly distributed P-O solutions when the PF is complex [24] or irregular [49]. Section VI presents a review of the studies which have investigated employing efficient computational resource allocation strategies in the decomposition-based MOEAs.

4) *Reproduction Operation:* In the original MOEA/D [9], simulated binary crossover (SBX) and polynomial mutation operators, i.e., genetic operators are incorporated as the variation operators. To improve the performance of MOEA/D, several studies have attempted at replacing the genetic operators with other reproduction operators such as DE [32], adaptive operator selection (AOS) method [33], etc., as discussed in Section VII.

5) *Mating Selection and Replacement Mechanism:* The neighborhood structure as well as the NS play an important role in MOEA/D [9]. In the original MOEA/D version, the neighborhood of each subproblem is determined in the weight vector space before the execution of the algorithm and thereafter, the neighborhood, and the NS remain constant for each subproblem throughout the evolutionary process. However, these assumptions may be misleading to the algorithm [26], [50]. Moreover, in the original MOEA/D, a single good offspring solution can replace several inferior neighboring solutions which can result in deterioration of the population diversity [28]. Thus, there are several shortcomings in the mating selection and the replacement mechanism of the original MOEA/D. Section VIII presents a review of the studies which aimed at enhancing the

mating selection or/and replacement mechanism in the decomposition-based framework.

6) *Many-Objective Optimization:* The original MOEA/D [9] and most of its subsequent variants are suitable for MOPs. However, MaOPs, which refer to the class of problems with four or more number of objectives, present a number of challenges to MOEAs developed for solving MOPs [12]. Thus, studies have been conducted to extend the decomposition-based MOEAs to solve MaOPs, as reviewed in Section IX.

7) *Constrained Optimization:* The original MOEA/D [9] and most of its subsequent variants have been tested extensively on unconstrained MOPs. However, most of the real world problems are constrained in nature. Thus, some studies have extended the decomposition-based MOEAs to solve constrained optimization problems, as discussed in Section X.

8) *Preference Incorporation:* Traditional MOEAs generally focus on obtaining the entire PF so that the DM can have a complete idea of all the available solutions and choose the desired solution. However, the search can be effectively focused within a small area if the DM's preference can be incorporated within the evolutionary multiobjective optimization framework [51]–[54]. Therefore, several studies have attempted to integrate the preference of DM in decomposition-based MOEAs, as discussed in Section XI.

9) *Application to Real World Optimization Problems:* The real world optimization problems are generally complex and present significant challenges to the MOEAs, which are often validated on numerical benchmark problems only. Several studies have proposed MOEAs based on decomposition to solve real world optimization problems, as summarized in Section XII.

IV. Studies on Weight Vector Generation Methods

The original version of MOEA/D [9] and many of its subsequent variants employ Das and Dennis's [46] systematic approach, known as the simplex-lattice design method, to generate evenly distributed weight vectors. However, in the simplex-lattice design method [9], the population size dramatically grows as the number of objectives increase and the setting of population size is not flexible [47]. Furthermore, in the simplex-lattice design method, the distribution of the weight vectors is not very uniform for problem with three or more objectives [47]. Another weight vector generation method widely employed in many MOEA/D variants (such as MOEA/D-DRA [29]) is based on uniform random sampling paradigm [19]. The advantage of the uniform random sampling method over simplex-lattice design is that the setting of population size is flexible. This section presents a review of the studies which have been undertaken to develop novel weight vector generation methods.

Qi *et al.* [24] proposed a novel weight vector initialization method, named WS transformation, based on the geometric relationship between the weight vectors and the corresponding optimal solutions under the TCH approach. The authors illustrated that when the PF shape is close to the hyperplane $\sum_{i=1}^{m} f_i = 1$, decomposition-based MOEAs should adopt uniformly distributed solution mapping vectors that result from WS transformation applied to uniformly distributed weight vectors. The experimental study on three-objective DTLZ1–DTLZ4 [55] problems (which have simple PF shape) demonstrated that the WS transformation technique helps MOEA/D obtain much better uniformly distributed P-O solutions

Tan *et al.* [47] proposed a new version of MOEA/D with uniform design, termed UMOEA/D, for solving MaOPs. In UMOEA/D, uniform design for experiment with mixtures (UDEMs) is employed to generate the weight vectors. With UDEM, the weight vectors yield minimum discrepancy in their distribution and thus the weight vectors are more uniformly distributed than the simplex lattice design employed in MOEA/D. Furthermore, with UDEM, the population size is decoupled with the number of objectives m. The experimental study on DTLZ1–DTLZ4 [55] test problems and two newly proposed test problems (F1 and F2) having complicated PS shapes, with 3–5 objectives, illustrated that UMOEA/D is superior to MOEA/D [9] and NSGA-II [5], particularly for higher dimensional problems and complicated PS shapes. In addition, UMOEA/D is also found to significantly outperform state-of-the-art MOEAs on MOKP with 2–4 objectives.

Ma *et al.* [56] proposed MOEA/D with uniform decomposition measurement, termed MOEA/D-UDM, to solve MaOPs. MOEA/D-UDM is based on novel weight vector initialization using uniform decomposition measurements. In MOEA/D-UDM, the UDEM method employed in [47] is combined with the simplex-lattice design method to generate alternative weight vectors. Thereafter, the required number of weight vectors are selected from the set of alternative weight vectors. It is noted that MOEA/D-UDM adopts the reverse TCH (rTCH) [36] decomposition approach to construct scalar optimization subproblems. The experimental study demonstrated that MOEA/D-UDM significantly outperforms MOEA/D [9] and UMOEA/D [47] on DTLZ1–DTLZ4 [55] and WFG4–WFG9 [57] test problems with 3–6 objectives.

Giagkiozis *et al.* [25] presented a novel method for weight vector generation, known as the generalized decomposition (gD). The main feature of the gD method is that under the assumption that a reference PF exists, the optimal set of weight vectors can be derived corresponding to a particular scalarizing function. In this study, the authors presented the gD method with respect to the TCH scalarizing function, due to its guarantee of producing P-O solution for every weight vector [2, p. 34]. The experimental study first compared the gD method with weight vector generation method based on simplex-lattice design [9] and uniform random sampling [19], on test problems with objectives ranging from 2 to 11, all with a PF of linear geometry. The experimental results indicated that with respect to generating evenly distributed solutions on the PF, the gD method remarkably outperforms the other two methods on problems with three or more objectives. Thus, the experimental results disproved the general notion that an

evenly [9] or uniformly [19] distributed set of weight vectors can result in a well-distributed set of P-O solutions for MOP with three or more objectives. A limitation of the gD method is that it requires *a priori* information about the PF geometry. However, the authors argued that in presence of *a priori* information about the PF geometry, the gD method can perform remarkably well. Nevertheless, if such information is unavailable, the gD method can still perform very well using linear PF as the reference PF.

Giagkiozis *et al.* [26] extended their work on the gD method presented in [25] and proposed a gD framework using which DM can obtain solutions on the entire PF or in specific regions of interest with the desired distribution. Giagkiozis *et al.* [26] further highlighted that if *a priori* information about the PF geometry is unavailable, the performance of the gD method can still be better than the weight vector generation method based on simplex lattice design [9] and uniform random sampling [19]. To demonstrate this, the authors presented an experimental study comparing the gD method with the other two methods on test problems with objectives ranging from 2 to 11, and with a number of PF geometries. In the experimental study, corresponding to the gD method, a reference PF with affine geometry is assumed. The experimental results confirmed that the gD method is remarkably better than the other two methods for a wide range of PF geometries, and different number of objective functions. Based on the concept of the gD and an estimation of distribution algorithm (EDA), known as cross entropy (CE) [58], the authors proposed an algorithm termed as many-objective CE based on gD (MACE-gD). In the experimental study, MACE-gD is compared against original MOEA/D [9], regularity-model-based EDA (RM-MEDA) [59], and MACE with evenly spaced weight vectors [9], on WFG2–WFG9 [57] test problems with 2–11 objectives. The experimental results demonstrated that MACE-gD outperforms both MOEA/D and RM-MEDA on many of the test instances while on the rest of the instances, the performance of MACE-gD is quite competitive.

Li *et al.* [39] presented a two-layer weight vector generation method for MaOPs. In this method, a set of N_1 and N_2 weight vectors are separately generated for boundary and inside layers, respectively, using the simplex-lattice design method (such that $N_1 + N_2 = N$). Thereafter, using co-ordinate transformation, the co-ordinates of weight vectors in the inside layer are shrunk. The set of weight vectors for the boundary and the inside layer are then combined to represent the final weight vector set. The two-layer weight vector generation method overcomes the limitation of the simplex-lattice design method in generating relatively small number of evenly spread weight vectors for MaOPs. It is noted that the two-layer weight vector generation method has been employed in decomposition-based MOEAs such as [39], [40], and [60] for MaOPs.

A. Summary

In summary, the survey of studies on weight vector generation methods reveal the following.
1) The distribution of the obtained P-O solutions is highly dependent on the weight vector generation method employed in the decomposition-based framework.
2) The assumption that an evenly [9] or uniformly [19] distributed weight vectors can result in uniformly distributed P-O solutions has been comprehensively refuted, particularly for MaOPs [25], [26].
3) The weight vector generation methods based on UDEM [47], uniform decomposition measurement [56], gD [25], [26], and two-layer generation method [39] decouple the population size with the number of objectives and make the population size setting flexible.
4) The gD method [25], [26] is particularly interesting as it has been validated to be significantly better than the weight vector generation method based on the simplex lattice design [9] and uniform random sampling [19], on problems with three or more objectives.
5) The two-layer weight vector generation method [39] extends the simplex-lattice design method for many-objective optimization and overcomes the limitation of the latter in generating relatively small number of evenly spread weight vectors for MaOPs.

A summary of studies on weight vector generation methods for decomposition-based MOEAs is presented in Table S1 in the supplementary document.

V. STUDIES ON DECOMPOSITION APPROACHES

The original MOEA/D framework [9] investigated three decomposition methods, namely, the WS, the TCH, and the PBI. The major shortcomings with the traditional decomposition approaches is their applicability to MaOPs [48] and that the improvement region corresponding to these methods may be too large in some problems, resulting in low population diversity [28]. However, the most important challenge is to determine an appropriate decomposition method for a particular problem. In this section, the studies which conducted investigation on decomposition approaches are reviewed.

A. Studies on Improved Decomposition Approaches

Sato [61] argued that the conventional decomposition approaches encounter difficulty in approximating widely spread PF in some problems like MOKPs. To overcome this problem and to design a decomposition method effective for many-objective optimization, Sato [61] extended conventional PBI approach and proposed inverted PBI (IPBI) decomposition method. In the conventional decomposition methods such as the TCH and the PBI, solutions are evolved toward the reference point z by minimizing the scalarizing function value. On the contrary, in the IPBI approach, solutions are evolved from the nadir point n by maximizing the scalarizing function value. The experimental study on MOKPs and WFG4 problem [57], with 2–8 objectives, illustrated that the IPBI approach can better approximate widely spread PF in comparison to other scalarizing approaches. The limitation of the IPBI approach, just like the PBI approach, is that it involves the parameter θ which needs to be appropriately tuned.

One of the main drawbacks of the PBI decomposition approach is that there is no unique setting of the penalty parameter θ that works well on different types of problems with different number of objectives [62]. Yang *et al.* [63] suggested two new penalty schemes, namely adaptive penalty scheme (APS) and subproblem-based penalty scheme (SPS), to set the value of the parameter θ. In the APS, θ is linearly increased with the number of generations from $\theta_{\min}$ (=1) to $\theta_{\max}$ (=10). A small value of θ is adopted initially so as to emphasize convergence and drive the search toward PF as fast as possible. The value of θ is gradually increased so as to emphasize diversity toward the later search stage. In the SPS scheme, each subproblem is assigned a different (but fixed) penalty value. In particular, the extreme subproblems (which are expected to represent the extreme portions of the PF) are assigned a larger, i.e., stricter penalty value in comparison to the intermediate subproblems. The experimental study on MOPs F1–F6 [63] demonstrated that both penalty schemes help to improve the performance of MOEA/D-PBI, particularly in terms of better coverage of the PF. Furthermore, the SPS scheme is found to perform better than the APS scheme.

Wang *et al.* [28] defined the *improvement region* of a current solution x_i for subproblem i as the region in the objective space, in which if a new solution y is produced then the current solution x_i can get replaced. The study demonstrated that the improvement regions corresponding to the conventional decomposition approaches (WS, TCH, and PBI) may be too large for some problems. Thus, a single new good solution can lead to replacement of several old solutions, and result in deterioration of the population diversity. To overcome this limitation, Wang *et al.* [28] suggested to impose constraints on the subproblems to reduce the volumes of the improvement regions. In the formulation presented for the constrained optimization subproblem, θ^i is a control parameter for defining the improvement region corresponding to subproblem i. To handle the constraint, the replacement step in MOEA/D is modified such that when a child solution y is compared with a randomly picked solution x from its neighborhood, rules similar to the constraint binary tournament selection [5] are applied. Apart from MOEA/D-CD (i.e., MOEA/D with constrained decomposition approach) which requires appropriately tuning of the θ value, the study presented an adaptive strategy for adjusting the θ^i value. The experimental study demonstrated that MOEA/D-CD and MOEA/D-ACD (i.e., MOEA/D with adaptive constrained decomposition approach) perform significantly better than MOEA/D-DE [32] on several test problems from different test suites.

Cheng *et al.* [41] introduced a new scalarization approach, termed angle penalized distance (APD). In the APD scalarization approach, the convergence criterion is measured by the distance between the candidate solution and the ideal point, as in the PBI approach. However, the diversity criterion is measured by the acute angle between the candidate solution and the reference vector. In the PBI approach, the penalty term θ is a fixed parameter and there is no unique setting of parameter θ that works well on different types of problems with different number of objectives [62]. In contrast, the penalty function in the APD is adaptive to the search process

as well as the number of objectives. In particular, the penalty function is designed to emphasize convergence at the early stage of the search process and diversity at the later stage of the search process. In the APD, α is a user-defined parameter that controls the rate of change of the penalty function. The experimental study demonstrated that the APD performs robustly well on a variety of problems with different number of objectives without changing the setting for parameter α.

B. Studies on Combination/Adaptation of Scalarizing Functions

Jiang and Yang [35] proposed a two-phase (TP) strategy for MOEA/D to tackle problems having convex PFs with sharp peak and low tail. In the first phase, $M_r\%$ of the entire computing resources are dedicated and MOEA/D with the TCH approach is implemented. At the end of the first phase, the uniformity of solutions is evaluated using a crowding-based method to determine if the problem is probably convex. If the problem is convex, in the second phase, the reverse TCH approach is adopted which is more suitable for convex optimization problems. In the reverse TCH approach, solutions are evolved from the nadir point n by maximizing the scalarizing function value. The experimental study on two- and three-objective problems with convex PFs having sharp peak and low tail confirmed the efficacy of the TP strategy.

Ishibuchi *et al.* [27] proposed an approach based on combining different scalarizing functions within the MOEA/D framework so as to overcome the problem of choosing an appropriate scalarizing function for a particular problem. The authors suggested two methods of implementation: 1) a multigrid scheme where each scalarizing function has its own complete grid of weight vectors and 2) a single-grid scheme in which there is a single grid of weight vectors and different scalarizing functions are alternately assigned for each weight vector. The experimental study demonstrated that MOEA/D based on simultaneous use of the WS and the TCH scalarizing functions outperforms MOEA/D using only the WS or the TCH approach on MOKP with two-, four-, and six-objective.

Zhang and Li [9] demonstrated that MOEA/D-WS performs remarkably better than MOEA/D-TCH on MOKPs with 2–4 objectives. However, it is well known that MOEA/D-WS is not suitable for MOPs with nonconvex PFs. Motivated by these observations, Ishibuchi *et al.* [64] proposed an adaptive scalarizing function based approach for MOEA/D which automatically employs the WS and the TCH scalarizing function approach for subproblems along nonconvex and convex regions of PF, respectively. The experimental study on modified MOKPs with nonconvex PFs demonstrated the effectiveness of the idea of adapting scalarizing functions in MOEA/D framework.

Saborido *et al.* [65] presented an algorithm, named global weighting achievement scalarizing function (ASF) genetic algorithm (global WASF-GA), to approximate the whole PF. Global WASF-GA is an extension of WASF-GA proposed by Ruiz *et al.* [66], which is a preference based MOEA. In global WASF-GA, the weight vectors employed are generated by taking inverse of a representative set of weight vectors

(for example, defined using the simplex-lattice design, etc.) such that the resulting weight vectors define uniform search directions. The distinctive feature of global WASF-GA is the use of ASF as the fitness function, wherein two reference points are considered simultaneously (the nadir point and an utopian point). In particular, at each generation, the parent offpsring population are combined and sorted into different fronts, as in NSGA-II [5]. However, unlike NSGA-II, the classification of the combined population into different fronts is performed according to the values that each solution takes on the ASF, for each one of the weight vectors. Thereafter, the population for the next generation is created using N solutions (where N is the population size) in the best fronts. The experimental study on two-, three-, and five-objective test problems from a variety of test suites validated that global WASF-GA outperforms both MOEA/D [9] and NSGA-II [5], particularly on three- and five-objective optimization problems. In this paper, the authors claimed that it is the simultaneous use of ASF with respect to both nadir point and utopian point that enables global WASF-GA to efficiently approximate the entire PF.

Giagkiozis and Fleming [13] presented a theoretical study on the working principles of Pareto-dominance based methods and the L_p weighted decomposition methods. This paper demonstrated that the Pareto-dominance based methods and the TCH decomposition method are equivalent in the respect that both have equal probability of finding superior solutions. Furthermore, the study illustrated that if the PF geometry is known *a priori*, then the optimal p value for the L_p weighted decomposition method can be determined. It is worthwhile noting that the claim of Giagkiozis and Fleming [13] regarding the optimal p value for the L_p weighted decomposition method determination has been later verified by Wang *et al.* [67], [68].

Wang *et al.* [67] investigated the property of a family of widely used scalarizing functions—the L_p weighted approaches. It is noted that in the L_p weighted approaches, $p = 1$ and $\lim p \to \infty$ represent the WS and the TCH approach, respectively. This study illustrated that there is a tradeoff dependent on the p value between the search ability of the L_p weighted approach and its robustness on PF geometry. Hence, the study recommended that an appropriate p value should be selected corresponding to each subproblem on the basis of PF geometry. Based on these observations, the authors proposed MOEA/D using Pareto-adaptive scalarizing (PaS) functions, termed MOEA/D-par. In MOEA/D-par, the p value is initialized as one for every subproblem at the beginning. Thereafter, at every iteration, the PF geometry is estimated based on PF approximation using a family of reference curves [69] corresponding to every subproblem, and the p values associated with the subproblems are adapted accordingly. The experimental study on two-, four-, and seven-objective instances of WFG41 (concave PF), WFG42 (convex PF), and WFG43 (hyperplane) exhibited that MOEA/D-par outperforms MOEA/D-TCH on all the test instances in terms of convergence.

Wang *et al.* [68] extended their work on PaS methods conducted in [67] and presented an enhanced online method, named PaS approximation, to approximate the optimal p value in the L_p weighted approaches. The advantage of *PaS* over the method presented earlier by Wang *et al.* [67] is that *PaS* method does not require PF estimation. Thus, the *PaS* method is simple and computationally efficient. The resulting algorithm based on *PaS* method is termed as MOEA/D-PaS and is investigated on several difficult test problems with two-, four-, and seven-objectives, and different PF geometries. The experimental results illustrated that MOEA/D-PaS outperforms MOEA/D-SS [27] and MOEA/D-AS [64].

C. Studies on Alternate Ways of Decomposition

Recently, several works have adopted alternate ways of decomposition, in which the reference vectors are used to partition the objective space into multiple small subspaces. The motivation behind such an approach is to preserve and emphasize good solutions in each of the subspaces, thereby maintaining a balance between convergence and diversity.

Liu *et al.* [70] proposed an algorithm, termed MOEA/D-M2M, which does not require any aggregation methods as in MOEA/D. MOEA/D-M2M requires the user to choose K unit vectors, depending upon which the objective space is divided into K subregions. Thus, MOEA/D-M2M decomposes an MOP into a set of K simple multiobjective optimization subproblems. At each generation, MOEA/D-M2M maintains K subpopulations with S solutions corresponding to each subproblem in order to maintain population diversity. Another characteristic feature of MOEA/D-M2M is that at each generation it merges the parent population and the offspring population and allocates solutions to the appropriate subproblems. It is noted that, if required, the nondominated sorting principle of NSGA-II is used to ensure that each subpopulation gets exactly S solutions. The experimental study demonstrated that MOEA/D-M2M remarkably outperforms MOEA/D-DE [32] and NSGA-II [5] on MOP1-MOP7 test instances.

Like MOEA/D-M2M [70], NSGA-III [40], MOEA/DD [39], reference vector guided EA (RVEA) [41], and MOEA/D-AM2M [71] also employ reference vectors to divide the objective space into a number of small subspaces. However, in comparison to MOEA/D-M2M [70], these MOEAs incorporate more sophisticated update procedures and are thoroughly investigated on MaOPs as well. Hence, these MOEAs are discussed in detail in Section IX which covers many-objective optimization.

D. Summary

In summary, the survey of studies presented in this section reveal the following.
1) In the L_p weighted approach, there is a tradeoff dependent on the p value between the search ability of the approach and its robustness on PF geometry [67], [68].
2) The adaptation of scalarizing functions, in particular the recently proposed PaS approximation has overcome the challenge of choosing an appropriate scalarizing function for a particular problem.

3) The constrained decomposition approach proposed by Wang *et al*. [28] and the MOEA/D variants based on it (i.e., MOEA/D-CD and MOEA/D-ACD) seem to be highly promising.

4) The studies MOEA/D-M2M [70], NSGA-III [40], MOEA/DD [39], and RVEA [41] provide new direction in which the objective space is partitioned into small subspaces using reference vectors, and good solutions are emphasized in each of the subspaces to maintain a balance between convergence and diversity.

A summary of studies on decomposition approaches is presented in Table S2 in the supplementary document.

VI. STUDIES ON COMPUTATIONAL RESOURCE ALLOCATION

In the original MOEA/D [9], a fixed set of weight vectors are utilized irrespective of the characteristics of the PF. Moreover, all the subproblems are allocated equal computational effort at each generation. However, depending upon the complexity of the problem, the algorithm may find some regions of the PF more difficult to approximate than the others. Furthermore, it is intuitive that consideration of fixed set of weight vectors may not be ideal in some cases such as problems with disconnected PFs. Thus, uniform treatment of all the subproblems and considering a fixed set of weight vectors may lead to wastage of computational resources, and in turn result in inferior performance of the algorithm. In this section, the studies which proposed efficient computational resource allocation strategies in the MOEA/D framework are reviewed under two categories. In the first category, computational resource allocation strategies considering fixed weight vectors are reviewed. In the second category, computational resource allocation strategies considering weight vector adaptation technique are reviewed.

A. Computational Resource Allocation With Fixed Weight Vectors

Zhang *et al*. [29] proposed a version of MOEA/D, named MOEA/D-DRA, based on dynamic allocation of computational resources to different subproblems. In MOEA/D-DRA, a utility function is used to represent the relative improvement of the scalarizing function for each subproblem, and is computed in interval of every 50 generations for each subproblem. Further, a ten-tournament selection step is introduced to select a set I of subproblems. In the tournament selection, the subproblem with the highest utility function value from ten randomly selected subproblems enter the set I for exploration of the search space. Thus, the computational resources are dynamically allocated to those subproblems for which the utility function is higher. It is worthwhile noting that MOEA/D-DRA is the winner of the unconstrained MOEA competition in the CEC 2009 [72].

Zhou and Zhang [30] extended the work on MOEA/D-DRA [29] and presented MOEA/D with generalized resource allocation (GRA) strategy, termed MOEA/D-GRA. In MOEA/D-GRA, each subproblem is associated with a probability of improvement (PoI) vector. At each generation,

computational resources are assigned to some subproblems selected according to the PoI vector. The authors introduced both offline and online resource allocation strategy in MOEA/D, termed as OFRA and ONRA, respectively. The experimental study on some selected test problems first demonstrated that the ONRA strategy is superior to OFRA and NORA (no resource allocation strategy). The authors named MOEA/D-DE with ONRA strategy as MOEA/D-GRA. The experimental study on F1–F9 [32] and UF1–UF10 [72] test instances demonstrated that MOEA/D-GRA significantly outperforms both MOEA/D-DE [32] and MOEA/D-DRA [29] on most of the test instances.

Cai *et al*. [73] proposed an external archive guided MOEA/D, termed EAG-MOEA/D. EAG-MOEA/D works with an internal (working) population which is evolved using MOEA/D, and an external archive which is updated using nondominated sorting and crowding distance principle of NSGA-II [5]. EAG-MOEA/D records the number of successful solutions each subproblem contributes to the external archive over L previous generations and the subproblems are probabilistically allocated computational resources depending upon their respective contribution. Thus, the external archive guides MOEA/D in allocating computation resources depending upon the historical convergence and diversity information. The experimental study exhibited that EAG-MOEA/D significantly outperforms NSGA-II [5], MOEA/D [9], and MOEA/D-DRA [29] on multiobjective software next release problem (MNRP) and multiobjective traveling salesman problem (MTSP).

Li *et al*. [49] proposed an MOEA/D variant with random weights, named MOEA/D-RW, in which both fixed and random weight vectors are used. More specifically, MOEA/D-RW is a modified MOEA/D-DE [32] variant in which an external population based on adaptive ϵ-dominance strategy [49] is maintained to store nondominated solutions found during the search. If there is no improvement in solution to a subproblem i for G number of generations, then the parent solutions for generation of offspring solution y corresponding to subproblem i are randomly selected from the external archive. The offspring solution y thus generated is used to update the subproblem corresponding to which y results in maximum improvement. It is worthwhile noting that when subproblems get stuck for certain number of generations, MOEA/D-RW does not explicitly adapt weight vectors or introduce new subproblems with random weights, rather MOEA/D-RW generates new solution in random search direction with the help of nondominated solutions in external archive. The experimental study on several test problems with irregular PFs illustrated that MOEA/D-RW outperforms MOEA/D-DE [32] and NSGA-II [5] on all the problems.

Li *et al*. [74] proposed a bi-criterion evolution framework, termed BCE, which involves collaborative working of a Pareto criterion (PC) population and a non-PC (NPC) population. In BCE framework, the two individual populations evolve based on their own criterion but communicate and exchange information with each other in a generational manner. In particular, each new individual produced in any of the populations is considered in both sides of BCE to

check if it could be preserved in their own population. The population in the PC evolution not only maintains a representative set of nondominated individuals but also explores some promising areas which are under developed in the NPC population. This study selected two representative non-Pareto-based algorithms, IBEA [7] and MOEA/D [32], and presented three algorithms based on the BCE framework, BCE-IBEA, BCE-MOEA/D+TCH, and BCE-MOEA/D-PBI. The experimental study on a wide variety of test problems demonstrated that the three BCE algorithms outperform their corresponding non-Pareto-based algorithms. The BCE framework is categorized under this section because in this framework, the limited computational resources are efficiently utilized by the collaboration of the PC and the NPC population which results in effective approximation of the PF.

Zhang *et al.* [75] proposed MOEA/D with Gaussian stochastic process model, named MOEA/D-EGO, for solving expensive MOPs. In MOEA/D-EGO, at each iteration, a Gaussian stochastic process model is built for each subproblem based on the data obtained from the previous search. MOEA/D-EGO simultaneously optimizes the expected improvement metric of the subproblems and at each iteration evaluates the objective function of only few solutions. Further, MOEA/D-EGO employs a fuzzy clustering-based modeling method to improve the prediction quality without significantly increasing the computational burden. The experimental study on some benchmark problems (such as ZDT, etc.) demonstrated that when limited number of function evaluations are allowed, the performance of MOEA/D-EGO is comparable to that of ParEGO [76] and SMS-EGO [77].

Martínez and Coello [78] proposed a radial basis function (RBF) networks assisted MOEA/D, named MOEA/D-RBF, for solving expensive MOPs. The characteristic feature of MOEA/D-RBF is that in order to improve the function prediction, the Gaussian, the multiquadratic, and the inverse multiquadratic kernels are used in a cooperative manner. The experimental study on ZDT [79] test problems demonstrated that MOEA/D-RBF outperforms the original MOEA/D and MOEA/D-EGO [75] when limited number of function evaluations are allowed. The experimental study further validated MOEA/D-RBF on an airfoil design problem which is an expensive real-world MOP.

Lin *et al.* [80] presented an MOEA/D variant with classification based on support vector machine (SVM), termed MOEA/D-SVM. In MOEA/D-SVM, a classification model is built on the search space, wherein for the training set, the solutions in the current population are regarded as the promising solutions and the recently discarded solutions for each subproblem as unpromising ones. MOEA/D-SVM classifies all new generated solutions and performs the function evaluation of all the promising solutions, while unpromising solutions are evaluated with a small probability for exploration purpose. Thus, MOEA/D-SVM allocates computational resources mainly to the promising solutions. The experimental study demonstrated that the proposed classification approach can significantly improve the performance of MOEA/D.

B. Computational Resource Allocation With Weight Vector Adaptation

Inspired by the Pareto-adaptive ϵ-dominance method, Jiang *et al.* [69] proposed a mechanism known as the Pareto-adaptive weight vectors ($pa\lambda$) and incorporated it in MOEA/D framework. In the resulting algorithm, termed as $pa\lambda$-MOEA/D, the weight vectors automatically adapt according to the geometrical properties of the PF. The experimental results on problems from ZDT [79] and DTLZ test suite [55] demonstrated that $pa\lambda$-MOEA/D is superior to NSGA-II [5], and MOEA/D [9].

Qi *et al.* [24] argued that uniformly distributed weight vectors in MOEA/D cannot ensure uniform distribution of the P-O solutions when the PF is complex (i.e., discontinuous PF or PF with sharp peak or low tail). Thus, Qi *et al.* [24] proposed an adaptive weight vector adjustment (AWA) strategy for MOPs with complex PFs and integrated the strategy within MOEA/D-DRA [29]. The resulting algorithm, termed as MOEA/D-AWA, is based on a two-stage strategy in which at first, a set of predetermined weight vectors are employed until the algorithm converges to a certain extent. Thereafter, the weight vectors are adjusted such that some subproblems are removed from the crowded parts of the PF, and some new subproblems are added into actual sparse regions instead of the pseudo-sparse or discontinuous regions of the PF. MOEA/D-AWA utilizes an external population to store the visited nondominated solutions and to guide the algorithm in removal and addition of subproblems. The experimental study exhibited that MOEA/D-AWA outperforms several state-of- the-art MOEAs on MOPs with discontinuous PFs (two-objective ZDT3 and three-objective DTLZ6), and two newly constructed MOPs with sharp peak and low tail (two-objective F1 and three-objective F2). Furthermore, the experimental study on two many-objective optimization test problems, DTLZ5(3, 6) [81] and its variation DTLZ4(3, 6), illustrated that MOEA/D-AWA is capable of tackling MaOPs as well.

Jiang *et al.* [82] presented an MOEA/D variant with a fast HV archive, termed FV-MOEA/D. In FV-MOEA/D, a fast HV archive is introduced to store nondominated solutions, wherein candidate solutions are inserted or deleted such that the HV of the archive is maximized. The main idea of FV-MOEA/D is to periodically adapt weight vectors based on the solutions in the external archive. The experimental study exhaustively demonstrated the efficiency of FV-MOEA/D in tackling MOPs with different PF shapes.

Giagkiozis *et al.* [83] argued that adapting the weight vectors in decomposition-based MOEAs may introduce a new difficulty to the algorithm. This is because when the weight vectors are fixed, the subproblems to be solved also remain fixed. However, when the weight vectors are adapted, the associated subproblems also change. Thus, the authors recommended that the adaptive weight vector strategies must be carefully investigated before implementation.

Jain and Deb [84] presented a many-objective EA with weight vector adaptation, termed adaptive NSGA-III. It is noted that adaptive NSGA-III is reviewed in Section IX.

C. Summary

In summary, the survey of studies presented in this section reveal the following.

1) Depending upon the hardness of the problem, different subproblems may require different computational budget in order to be efficiently solved [30].
2) Strategies based on dynamic computational resource allocation to different subproblems can significantly improve the performance of the decomposition-based MOEAs [29], [30].
3) Apart from considering improvement of a subproblem as the utility function, alternate utility functions based on considering population distribution in the objective space or/and decision space can also be considered [30].
4) Weight vector adaptation is essential in the decomposition-based framework when the target MOP has a complex or irregular PF (e.g., discontinuous PF, PF with sharp peak, and low tail) [24].
5) Weight vector adaptation strategies which involve online estimation of PF shape can be computationally expensive [69].
6) Use of dual populations or an external archive to store nondominated solutions and guide the internal working population of MOEA/D as in MOEA/D-AWA [24], EAG-MOEA/D [73], BCE framework [74], FV-MOEA/D [82] is highly promising.

A summary of studies on computational resource allocation strategies introduced for decomposition-based MOEAs is presented in Table S3 in the supplementary document.

VII. STUDIES ON MODIFICATIONS IN THE REPRODUCTION OPERATORS

In the original MOEA/D [9], SBX and polynomial mutation operators, i.e., genetic operators are incorporated as the variation operators. However, it is well known that there is no single EA which outperforms all other EAs across different problems. Thus, several studies have aimed at modifying the reproduction operators in order to improve the performance of MOEAs based on decomposition. In this section, a review of such studies is presented.

A. Reproduction Operation Based on DE

Li and Zhang [32] proposed an enhanced version of MOEA/D using differential evolution (DE) algorithm, termed MOEA/D-DE, to handle complicated Pareto set (PS) shapes. The authors introduced nine test instances (F1–F9) with complicated PS shapes for investigating the efficiency of MOEA/D-DE. The experimental study demonstrated that MOEA/D-DE significantly outperforms NSGA-II-DE on all the test instances. Zapotecas-Martinez *et al.* [85] proposed integrating geometric DE (gDE) [86], the discrete generalization of DE, into the MOEA/D framework for solving combinatorial MOPs. Huang and Li [87] investigated the influence of using different DE schemes in MOEA/D-DE [32].

B. Reproduction Operation Based on ACO

A combination of ant colony optimization (ACO) algorithm and MOEA/D, termed MOEA/D-ACO, has been proposed by Ke *et al.* [34] for solving combinatorial optimization problems. The experimental study comprehensively investigated the efficiency of MOEA/D-ACO on MOKP and MTSP. The experimental results demonstrated that MOEA/D-ACO outperforms MOEA/D-GA [9] with conventional genetic operators and a local search operator [19] on several test instances of MOKP, and bi-criterion ant algorithm [88] on several test instances of bi-objective TSP.

C. Reproduction Operation Based on Adaptive Operator Selection

AOS [33] is a method that dynamically determines the rate of application of different operators considering the performance history of the operators in the optimization process. AOS comprises of two parts, namely—credit assignment and operator selection. The credit assignment task is related to rewarding an operator based on its recent performance while the operator selection task determines the operator to be applied next based on the reward information accumulated during the optimization process. The popular credit assignment methods set the reward amount as the average of the fitness improvements [89] or employ rank-based schemes [90] such as area under the curve, sum of ranks, and fitness rate rank (FRR). The operator selection is generally based on probabilistic methods such as probability matching (PM) [91] and adaptive pursuit (AP) [92], or multiarmed bandit (MAB) methods. It is noted that the most popular MAB algorithms are based on the upper confidence bound (UCB) [93] algorithm.

Venske *et al.* [94] incorporated within MOEA/D the mechanism of self-adaptation of DE mutation strategies proposed in AdapSS [95] and presented an algorithm named, adaptive DE for MOPs (ADEMO/D). In ADEMO/D, the candidate strategy pool consists of *DE/rand/1*, *DE/rand/2*, and *DE/nonlinear* [97]. The authors investigated combining two operator selection strategies namely, PM and AP, with four credits assignment techniques based on relative fitness improvements, and observed that the ADEMO/D version based on combination of PM and extreme absolute reward for AOS works best on UF [72] test instances. The experimental study further demonstrated the superiority of the best version of ADEMO/D against several state-of-the-art MOEAs.

Li *et al.* [33] proposed an AOS method, termed as FRR-based MAB (FRRMAB), which utilizes FRR-based credit assignment scheme and MAB based operator selection scheme. In FRRMAB, a sliding window strategy is used to capture the dynamics of the search process and provide reward to operators based on current situation of the search. The authors incorporated the proposed FRRMAB method within MOEA/D-DRA and named the resulting algorithm as MOEA/D-FRRMAB. In MOEA/D-FRRMAB, the operator pool consists of four different DE variants namely *DE/rand/1*, *DE/rand/2*, *DE/current-to-rand/1*, and *DE/current-to-rand/2*. The experimental study

demonstrated that MOEA/D-FRRMAB significantly out-performs MOEA/D-DE [32], MOEA/D-DRA [29], and ENS-MOEA/D [50] on UF [72] test instances.

Gonçalves *et al*. [97] proposed two MOEA/D variants based on AOS selection method, namely MOEA/D-UCB-tuned and MOEA/D-UCB-V. The proposed algorithms utilize the same pool of DE operators and FRR credit assignment scheme as MOEA/D-FRRMAB [33]. However, the MAB method employed in MOEA/D-FRRMAB is UCB1 algorithm [93], while in MOEA/D-UCB-tuned and MOEA/D-UCB-V is UCB-tuned and UCB-V algorithm, respectively. The experimental study on UF [72] test instances demonstrated that MOEA/D-UCB-tuned is the most consistent algorithm when compared with MOEA/D-UCB-V and MOEA/D-FRRMAB. Further, the experimental comparison against several state-of-the-art MOEAs demonstrated the superiority of MOEA/D-UCB-tuned on most of the UF test instances.

Shim *et al*. [98] presented a multimethod multiobjective approach based on adaptive synergistic combination of GA, DE, and EDA. The multimethod approach is similar to AMALGAM [99] but differs in incorporating progressively control paradigm with respect to the number of offspring an individual algorithm contributes at each generation and using evolutionary gradient search (EGS) as the local search algorithm. The authors presented two algorithms, namely mNSEA and mMOEA/D, by integrating the proposed adaptive memetic algorithm within NSGA-II and MOEA/D, respectively. The experimental study on test instances from several test suites demonstrated that across all the test problems, mMOEA/D and mNSEA are the best algorithms in comparison to several state-of-the-art MOEAs.

D. Reproduction Operation Based on Other Strategies

In [100], Li *et al*. dealt with the bias feature in MOPs. The authors argued that to deal with the bias feature, an algorithm should be able to maintain good balance between exploration and exploitation. Thus, the authors proposed an MOEA/D variant, named MOEA/D-CMA, in which both CMA-ES and DE are used as the variation operators. In order to reduce the computational complexity, MOEA/D-CMA clusters the single-objective subproblems into several groups, and employs CMA-ES to only one subproblem from each group at every generation. The rest of the subproblems are optimized by DE. The authors proposed a set of nine new multiobjective test instances with bias, named BT1–BT9, and demonstrated the efficacy of MOEA/D-CMA in dealing with the bias feature in MOPs.

E. Summary

In summary, the survey of studies presented in this section display the following.
1) Several reproduction operators have been investigated within the decomposition-based framework.
2) Extensive work is being conducted on incorporating AOS in the MOEA/D framework [33], [94], [97], etc., indicating that MOEA/D framework is highly compatible with integrating AOS.

3) The MOEA/D variants based on AOS such as MOEA/D-FRRMAB [33], MOEA/D-UCB-tuned [97], etc. show remarkable performance.
4) Utilization of different reproduction operators within the MOEA/D framework such that one promotes convergence while other promotes diversity (e.g., MOEA/D-NL&DE [101] and MOEA/D-CMA [100]) is a promising idea for maintaining balance between convergence and diversity.

Due to space constraints, review of reproduction operation strategies based on PSO [102], SA [103], hyper-heuristics [104], [105], and probability models [106], [107], etc. is presented in the supplementary document. A summary of studies on modifications in the reproduction operation is presented in Table S4 in the supplementary document.

VIII. STUDIES ON MATING SELECTION AND REPLACEMENT MECHANISM

The neighborhood structure as well as the NS play an important role in MOEA/D [9]. This is because the selection of parents for mating and the selection of solutions to be replaced is dependent on the neighborhood structure and the NS. In the original MOEA/D version, the neighborhood relationship is defined in the weight vector space and the neighborhood structure as well as the NS remain fixed throughout the evolutionary process. However, these assumptions may be misleading to the algorithm [26], [50]. Moreover, in the original MOEA/D, a single good offspring solution can replace several inferior neighboring solutions which can result in deterioration of the population diversity [28]. Thus, there are certain shortcomings in the mating selection and the replacement mechanism of the original MOEA/D. In this section, the studies which aimed at modifying the mating selection or/and the replacement principle in the decomposition-based framework are reviewed.

A. Studies on Improved Mating Mechanism

Besides the TP strategy (discussed in Section V), Jiang and Yang [35] presented a niche-guided scheme for the setting of mating selection range. In this scheme, each individual's niche count is computed over its T neighboring subproblems. If the niche count of an individual is over a certain threshold, it means that the individual is similar to its T neighboring subproblems and thus the mating parents corresponding to the individual are selected from outside its neighborhood. The experimental study exhibited that the proposed niche-guided mating selection strategy is particularly beneficial on problems having disconnected PFs. The resulting algorithm based on TP strategy and niche guided scheme (termed MOEA/D-TPN) is found to be superior to state-of-the-art MOEAs on two- and three-objective problems with complex PFs.

B. Studies on Improved Replacement Mechanism

In the original version of MOEA/D [9], the number of solutions that are considered for selection and replacement corresponding to a subproblem i are the same as the NS.

In the study on MOEA/D-DE, Li and Zhang [32] argued that in order to maintain population diversity, the replacement NS should be smaller than the selection NS and set it to 2 in the experimental study. However, the authors did not present an experimental study to demonstrate that the replacement NS setting of 2 is appropriate for all the problems under investigation.

Wang et al. [108] argued that the new solution x_i^{new} of subproblem i may not be the most suitable solution for its neighboring subproblems $B(i)$. Thus, the authors proposed a global replacement (GR) scheme for MOEA/D and named the resulting algorithm as MOEA/D-GR. In this study, two different neighborhoods, i.e., mating neighborhood (of size T_m) and replacement neighborhood (of size T_r), are considered for each subproblem i, and the effect of T_r on the performance of MOEA/D is investigated. In the GR scheme, corresponding to a newly generated solution x_i^{new}, the most appropriate subproblem j is determined. Thereafter, T_r closest subproblems to subproblem j are selected to form the replacement neighborhood, i.e., $B_r(j)$. Finally, the solutions of subproblems belonging to $B_r(j)$ are updated by the newly generated solution x_i^{new}. The experimental study on incorporating GR scheme in MOEA/D indicated that the appropriate replacement NS, i.e., T_r is generally distinct for different problems.

Wang et al. [109] extended the GR scheme proposed in [108] and developed an adaptive GR scheme. The authors argued that a small T_r is good for exploration at the beginning of the search process while a large T_r is good for exploitation toward the end of the search process. The study investigated three different adaptive schemes for adjusting T_r, based on linear, exponential, and sigmoid functions, and found the sigmoid function based adaptive scheme to be the best. Furthermore, based on the adaptive replacement strategy, both a steady-state algorithm (named MOEA/D-AGR) and a generational algorithm (named gMOEA/D-AGR) are presented. The proposed algorithms are extensively compared against MOEA/D-GR [108] and several state-of-the-art MOEAs on different test suites. The experimental results demonstrated that the algorithms based on GR scheme consistently outperform the competitor algorithms on all the test suites. Among the algorithms based on GR scheme, MOEA/D-AGR is found to perform the best across all the test problems.

Li et al. [36] suggested incorporating a stable matching model (STM) to co-ordinate the selection of promising solutions for subproblems in MOEA/D. In the resulting algorithm, termed MOEA/D-STM, each subproblem ranks all the solutions in the solution pool (i.e., the parent and the offspring solutions) using its aggregation function values, and expresses its preference for the solutions with better aggregation function values, thus encouraging convergence. On the contrary, each solution ranks all the subproblems according to its distance to the direction vectors of the subproblems, and expresses its preference for the subproblems with smaller distance, thus promoting diversity. The STM model links each subproblem to one single solution such that a balance between convergence and diversity is maintained. The experimental study demonstrated that MOEA/D-STM is significantly superior to several state-of-the-art MOEAs on UF [72] test instances.

Li et al. [37] extended MOEA/D-STM [36] and presented MOEA/D-IR, based on incorporating interrelationship based selection in MOEA/D. Like MOEA/D-STM, MOEA/D-IR is also based on defining mutual preferences between subproblems and solutions. However, as compared to MOEA/D-STM, a modification relating the preference definition of a subproblem to a solution is introduced in MOEA/D-IR. The experimental study demonstrated that MOEA/D-IR is significantly superior to several state-of-the-art MOEA variants including MOEA/D-STM [36] on several test-suites.

Gee et al. [110] presented an online diversity metric to enhance the diversity of the solution set obtained by an MOEA. The authors introduced a diversity measurement quantity, called as maximum relative diversity loss (MRDL), to estimate the diversity loss of a solution to the whole population. In order to validate the proposed diversity metric, the authors incorporated the method in MOEA/D framework. In particular, a new selection operator is introduced in MOEA/D, wherein each offspring solution is checked for its MRDL before the replacement step is executed. If the MRDL corresponding to an offspring solution is higher than the predefined threshold γ, the parent solution is selected instead of offspring in order to preserve the population diversity.

C. Studies on Improved Mating Selection and Replacement Mechanism

Besides the introduction of DE operators in MOEA/D, Li and Zhang [32] refined the MOEA/D framework by introducing two extra measures. The first measure allows parent solutions to be selected during reproduction with a low probability from the whole population (i.e., outside the neighborhood). The second measure puts an upper bound (n_r) on the maximal number of solutions that can be replaced by a child solution during the update of neighboring solutions. The introduction of these extra measures help to maintain the population diversity.

Ishibuchi et al. [45] considered MOEA/D as a cellular algorithm where each cell has its own scalarizing fitness function with a different weight vector. In standard cellular EAs, an offspring solution is only compared with the current solution at the cell. In this study, the authors investigated the effect of local replacement on the search ability of cellular MOEA/D. In particular, the authors examined the impact of adopting different neighborhood structures in MOEA/D for selection and replacement. The experimental study on MOKP with two-, four-, and six-objectives demonstrated that the local replacement neighborhood plays a key role in the performance of MOEA/D. Thus, the authors recommended that the performance of standard cellular EAs can be improved by incorporating local replacement structure.

To overcome the problem of choosing a suitable NS for different problems, Zhao et al. [50] proposed an algorithm known as ENS-MOEA/D. In ENS-MOEA/D, different values of NSs are used in the form of an ensemble and the selection probabilities of NSs are dynamically adjusted based on their historical performances of generating promising solutions. The experimental study demonstrated the superiority

of ENS-MOEA/D against MOEA/D-DRA with fixed NSs on UF [72] test instances.

Giagkiozis *et al.* [26] presented an algorithm named MACE-gD, as discussed in Section IV. An important feature of MACE-gD is that the neighborhood structure of a subproblem is controlled by parameter ρ. The parameter ρ indicates the percentage of top solutions in the current population with respect to a subproblem, which are used in building the probability model for CE method. Thus, the neighborhood relationship is dynamically updated with respect to the objective space. Furthermore, in the replacement step in MACE-gD, a new solution to subproblem i is only compared with the current solution to subproblem i.

Inspired from the regularity property of MOPs [59], Zhang *et al.* [111] proposed an MOEA/D variant, termed SMOEA/D, based on self-organizing reproduction mechanism (SRM). In SRM, a self-organizing map is applied at every generation to determine the population distribution structure (in the decision space) and construct a mating pool for every solution. Another characteristic feature of SRM is the use of ensemble of NSs and adaptive adjustment of the probability of selecting different NS based on their performance in generating solutions over certain number of previous generations. The replacement strategy in SMOEA/D is based on a greedy strategy according to which for every new generated solution y, the two subproblems for which solution y can show maximum improvement in terms of aggregated fitness value are updated. The experimental study on different test suites involving test instances with complicated PS shapes and PF shapes, revealed that SMOEA/D is superior to MOEA/D-DE [32] and RM-MEDA [59] on each test suite. The limitation of SMOEA/D is its higher computational complexity and that it introduces four additional parameters.

D. Summary

In summary, the survey of studies presented in this section display the following.

1) Extensive research has been conducted to improve the mating selection and the replacement mechanism in the MOEA/D framework.
2) The neighborhood relationship in the weight vector space, as defined in the original MOEA/D framework, can be deceptive to the algorithm [26].
3) The neighborhood relationship should be rather defined in the objective space and should be adaptive such that solutions which participate in mating procedure are close in the objective space [26].
4) The niche-guided mating selection strategy in MOEA/D-TPN [35] is particularly useful when the target MOP involves disconnected PF.
5) MOEA/D based on adaptive GR scheme (MOEA/D-AGR) [109] and SMOEA/D [111] based on SRM seem to be highly promising.

A summary of studies on mating selection and replacement mechanism in MOEA/D framework is presented in Table S5 in the supplementary document.

IX. STUDIES ON MANY-OBJECTIVE OPTIMIZATION

MaOPs refer to the class of MOPs with four or more number of objectives. Since the last decade, a considerable amount of effort has been spent by researchers in developing efficient algorithms for solving MaOPs [12], [112]–[121]. In this section, the studies which proposed decomposition-based MOEAs for solving MaOPs are reviewed.

Deb and Jain [40] presented a reference-point-based many-objective EA, termed NSGA-III. In particular, the basic framework of NSGA-III is similar to NSGA-II [5] with significant modifications in the replacement step. Like weight vector generation in MOEA/D, NSGA-III also utilizes a set of reference points that spread over the objective space. NSGA-III uses a generational replacement scheme in which the combined parent offspring population is classified into different nondominated levels. In NSGA-III, solutions in all the levels starting from level 1 except the last acceptable level are included in the next population (like in NSGA-II [5]). To select the solutions from the last acceptable level, a niche-preservation operator is used, in which solutions associated with less crowded reference point have a higher chance to be selected. It is worthwhile noting that NSGA-III does not decompose the MOP explicitly into single-objective subproblems like MOEA/D. However, the reference points implicitly decompose the objective space in such a way that diversity of the solutions close to every possible reference point and thus along the entire P-O surface is attempted to be maintained. The experimental study compared NSGA-III with MOEA/D-TCH, MOEA/D-PBI, and MOEA/D-DE [32] on several MaOPs ranging from 3–15 objectives. The experimental results demonstrated that: 1) MOEA/D-TCH and MOEA/D-DE perform quite poorly on most of the problems; 2) NSGA-III performs better on some test problems while MOEA/D-PBI performs better on some other problems; and 3) NSGA-III significantly outperforms both MOEA/D-PBI and MOEA/D-TCH variants on scaled test problems.

Jain and Deb [84] argued that in many constrained or even unconstrained problems, there will be some reference points with no P-O solution associated with them. On the other hand, there will be some reference points which have more than one P-O solution associated with them. Hence, NSGA-III may not be able to generate uniformly distributed P-O solutions over then entire PF for such problems. To overcome this difficulty, Jain and Deb [84] modified NSGA-III presented in [40], and proposed an adaptive NSGA-III, named A-NSGA-III. A characteristic feature of A-NSGA-III is the addition of new reference points around a crowded reference point which has more than one population member associated with it. Another characteristic feature of A-NSGA-III is the deletion of reference points which have no population member associated with them. It is noted that A-NSGA-III always preserves the original reference points. The experimental study on inverted DTLZ1, constrained DTLZ2 problem, and two real world problems, demonstrated that A-NSGA-III works extremely well, and outperforms NSGA-III [40] in generating uniformly distributed P-O solutions.

Li *et al.* [39] proposed a unified paradigm, termed MOEA/DD, based on combination of dominance- and

decomposition-based approaches for many-objective optimization. In MOEA/DD, the decomposition method employed is PBI, primarily because of its promising performance reported in [40] for many-objective optimization. Each weight vector in MOEA/DD along with defining a subproblem, also specifies a unique subregion in the objective space. Thus, each solution is always associated with a unique subregion and the problem of diversity management is addressed by local density estimation of the subregions. The characteristic feature of MOEA/DD is its update procedure which is carried out in a steady-state hierarchical manner, depending on Pareto dominance, local density estimation, and scalarizing functions, sequentially. The experimental study demonstrated that MOEA/DD significantly outperforms state-of-the-art MOEAs including NSGA-III [40] on several MaOPs.

Cheng et al. [41] presented an RVEA, for many-objective optimization. In RVEA, a set of predefined uniformly distributed reference vectors are used to partition the objective space into a number of small subspaces. RVEA inherits an elitism strategy similar to NSGA-II in which the parent population and the offspring population are combined at every generation to undergo an elitist selection. However, the characteristic feature of RVEA lies in reference vector guided selection. RVEA partitions the combined parent offspring population into N subpopulations by associating each solution with its closest reference vector, where N is the number of reference vectors. Thereafter, one elitist solution is selected from each subpopulation to enter the next generation. As discussed in Section V, Cheng et al. [41] introduced a new scalarization approach, APD. The elitist solution selected from each subpopulation is the one with the lowest APD. The experimental study demonstrated that RVEA performs robustly on benchmark problems from different test suites and with objectives ranging from 3 to 10.

Asafuddoula et al. [38] proposed an improved decomposition-based EA, termed I-DBEA, for many-objective optimization. In I-DBEA, every solution is associated with two distance measures—d_1 and d_2, as in the PBI approach. However, in I-DBEA, the penalty parameter θ of the PBI approach is eliminated. Instead, in the replacement step, d_2 is given simple precedence over d_1, in order to emphasize diversity. Furthermore, the concept of neighborhood for mating and replacement is completely eliminated in I-DBEA. In I-DBEA, the entire population is considered as a neighborhood and a first-encounter replacement strategy is incorporated. In particular, a child solution y is compared in a steady-state manner with all solutions in the population in a random manner until it makes a successful replacement or has been compared with all solutions. Moreover, to emphasize convergence, a child solution y is allowed to enter the replacement step only if it is nondominated with respect to all solutions in the population. The experimental study thoroughly validated the efficiency of I-DBEA in solving MaOPs.

Yuan et al. [60] presented an algorithm, termed MOEA/D-DU, for many-objective optimization. The characteristic feature of MOEA/D-DU is its update procedure, in which the perpendicular distance from the solution to the weight vector in the objective space is explicitly exploited to maintain a better balance between convergence and diversity. In particular, upon generation of a new solution y, its perpendicular distances to each weight vector λ_i, i.e., $d_{i,2}(y)$, $i = 1, 2, \ldots, N$, are evaluated, respectively. Thereafter, from all these N distances, K minimum distances and the corresponding indices of the K weight vectors are selected, and the K minimum distances are arranged in the increasing order. Next, the solution y is compared one by one with the solutions at the K indices with respect to the rTCH [36] scalarizing function value, until y replaces one of the solutions. It is noted that K is a parameter in MOEA/D-DU, where $K \ll N$. The experimental study on several test problems from DTLZ and WFG test suites demonstrated the superiority of MOEA/D-DU with respect to several state-of-the-art MOEAs.

Yuan et al. [60] also presented an enhanced version of ensemble fitness ranking (EFR) [122] algorithm with ranking restriction scheme, named as EFR-RR. In EFR-RR, at every generation t, each solution in the combined parent offspring population (U_t) is evaluated on K ($K \ll N$) different fitness functions whose corresponding weight vectors are close to it in the objective space in terms of perpendicular distance. It is noted that each function corresponds to the rTCH [36] scalarizing function of a subproblem. Thus, each solution in (U_t) is associated with K different fitness values. For each fitness function, all the K closest solutions are sorted in the nondecreasing order with respect to the fitness values, and each solution is assigned a rank. Hence, each solution is allotted K ranking positions. In EFR-RR, the ensemble ranking scheme, known as maximum ranking, is adopted which assigns the best rank $R_g(x)$ out of the K ranking positions to each solution x in U_t. Next, the merged population U_t is sorted into different fronts based on the R_g value, and the new population is created as in NSGA-II. However, unlike NSGA-II, EFR-RR randomly selects solutions in the last accepted front. The experimental study on several test problems from DTLZ and WFG test suites demonstrated the superiority of EFR-RR in comparison to several state-of-the-art MOEAs, including MOEA/D-DU [60].

Cai et al. [123] presented an algorithm on decomposition-based sorting (DBS) and angle-based-selection (ABS), named MOEA/D-SAS, for evolutionary multiobjective and many-objective optimization. In MOEA/D-SAS, at every generation t, each subproblem j chooses its L closest solutions from the combined parent offspring population (U_t) based on the acute angle between the corresponding weight vector and the solutions. For each subproblem, the chosen L solutions are sorted in an ascending order of the aggregation function values. Thereafter, the solutions are sorted into L fronts, $Q^1, \ldots, Q^k, \ldots, Q^L$, where Q^k contains the solutions with the kth best aggregation function values, for every subproblem j. Based on the sorted population, the new population is created as in NSGA-II. In the last accepted front, ABS is implemented which selects solutions with the largest angle to the already created new population. The experimental study first demonstrated the efficiency of MOEA/D-SAS on MOPs (UF1–UF10 and three-objective DTLZ1–DTLZ7). Thereafter, the authors demonstrated the superiority of MOEA/D-SAS in comparison

to NSGA-III on MaOPs from DTLZ test suite with up to 15 objectives.

Yuan *et al.* [118] presented a new dominance-relation based EA, named θ-DEA, for many-objective optimization. In θ-DEA, a clustering operator is used to split the combined parent offspring population into N clusters corresponding to the subproblems. A solution x is assigned to the cluster C_j if its perpendicular distance with the reference vector λ_j in the normalized objective space is minimum. In this paper, the authors proposed a new dominance relation, termed θ-dominance, which employs PBI fitness function in the normalized objective space and is defined with respect to a cluster. In θ-DEA, the combined parent offspring population is partitioned into different θ-nondomination levels based on the θ-dominance relation, and the new population is constructed as in NSGA-II. However, unlike NSGA-II, θ-DEA randomly selects solutions in the last accepted front. The experimental study on MaOPs from DTLZ and WFG test suites with up to 15 objectives, revealed the robustness of θ-DEA in comparison to several many-objective optimizers, including NSGA-III and MOEA/D-PBI.

Liu *et al.* [71] extended MOEA/D-M2M [70] and proposed a many-objective algorithm with adaptive region decomposition, termed MOEA/D-AM2M. The characteristic feature of MOEA/D-AM2M is that the K direction vectors are adaptively adjusted every certain generations using a method termed as max–min method. Depending upon the K direction vectors, the objective space is divided into K subregions. At each generation, MOEA/D-M2M maintains K subpopulations with S_k solutions corresponding to subregion K. Like MOEA/D-M2M [70], MOEA/D-AM2M also merges the parent population and the offspring population at each generation, and allocates solutions to the appropriate subregions. Another characteristic feature of MOEA/D-AM2M is the periodic adaptation of S_k weight vectors in each subregion K using the max–min method. It is noted that the weight vectors in a subregion are used to select S_k solutions when the number of solutions in subregion K is more than S_k. In this paper, the authors presented five degenerated MaOPs with disconnected PFs, and demonstrated the efficacy of MOEA/D-AM2M in comparison to MOEA/D-DE [32] and MOEA/D-M2M [70].

Ishibuchi *et al.* [124] conducted an exhaustive performance analysis of decomposition-based many-objective algorithms to demonstrate their sensitivity to PF shapes. The authors first presented performance comparison results of many-objective algorithms such as NSGA-III, MOEA/DD, θ-DEA, MOEA/D-PBI, etc., on DTLZ1–DTLZ4 and WFG4–WFG9 problems with up to ten objectives. The experimental study showed that θ-DEA, MOEA/DD, and NSGA-III perform very well on most of the test problems. To demonstrate the sensitivity of decomposition-based many-objective algorithms to PF shapes, the authors slightly changed the DTLZ and WFG problem formulations by multiplying (-1) to each objective of DTLZ and WFG problems. The experimental study on DTLZ^{-1} and WFG^{-1} problems demonstrated that in comparison to the original test problems, totally different performance comparison results are obtained among the algorithms on their minus versions. In particular, the performance of

θ-DEA, MOEA/DD, and NSGA-III significantly deteriorated on DTLZ^{-1} and WFG^{-1} problems. The authors highlighted this to the reason that DTLZ1–DTLZ4 and WFG4–WFG9 have triangular shape PFs, and thus their is a similarity between the shape of distribution of the weight vectors and the shape of the PFs of these test problems. On the other hand, DTLZ1–DTLZ4^{-1} and WFG4–WFG9^{-1} have rotated triangular shape PFs. Based on these observations, the study recommended use of a wide variety of test problems with various PF shapes. The authors also suggested use of adaptation mechanism for the weight vectors and scalarizing function in the decomposition-based framework.

Sato *et al.* [125] argued that for MaOPs, the solutions in the population have to be sparsely distributed in the objective space and the variable space to approximate a high-dimensional PF. Thus, each solution faces difficulty in playing the role of variable information resource to generate offspring. To overcome this limitation, the authors proposed an enhanced MOEA/D framework, wherein each weight vector is assigned supplemental weight vectors, and supplemental solutions are maintained as additional variable information resource to enhance the solution search. The experimental study on MOKPs with 4–6 objectives demonstrated that the introduction of the supplemental weight vectors and solutions significantly improves the performance of MOEA/D.

It is noted that several other algorithms discussed in the previous sections successfully tackled MaOPs to a certain extent. For example, MACE-gD [26] tackled WFG2–WFG9 problems with 2–11 objectives, IPBI [61] tackled MOKPs and WFG4 problem up to 8 objectives, global WASF-GA [65] tackled several five-objective test problems, and MOEA/D-PaS [68] tackled four- and seven-objective test problems from several test suites.

A. Summary

In summary, the survey of studies presented in this section display the following.
1) On MaOPs, MOEA/D-TCH, and MOEA/D-DE do not perform well while the performance of MOEA/D-PBI is quite well on a range of test problems [40].
2) The studies NSGA-III [40], MOEA/DD [39], and RVEA [41] provide new direction in which the high-dimensional objective space in MaOPs can be partitioned into small subspaces using reference vectors, and sophisticated update procedures can be employed to preserve diversity in all subspaces.
3) Efficiently combining dominance- and decomposition-based approaches can result in high performance many-objective optimizers (e.g., NSGA-III [40], MOEA/DD [39], and I-DBEA [38]).
4) The perpendicular distance from the solution to the weight vector in the objective space, i.e., the distance measure—d_2 of the PBI approach can be exploited to efficiently preserve diversity in MaOPs (e.g., I-DBEA [38] and MOEA/D-DU [60]).
5) The genetic variation operators, i.e., SBX crossover operator and polynomial mutation operators, are

employed in most of the many-objective optimizers (e.g., NSGA-III [40], MOEA/DD [39], MOEA/D-DU [60], and RVEA [41]).

A summary of studies on MOEAs based on decomposition for many-objective optimization is presented in Table S6 in the supplementary document.

X. Studies on Constrained Optimization

The original MOEA/D [9] and many of its subsequent variants have been tested extensively on unconstrained MOPs. However, most of the real world problems are constrained in nature. In this section, the studies which extended decomposition-based MOEAs to tackle constrained optimization problems are reviewed.

Jain and Deb [84] extended NSGA-III [40] to tackle constrained MaOPs. In constrained NSGA-III, the constraint binary tournament selection operator of NSGA-II [5] is utilized while selecting parents for mating. Further, in the elitist selection operator, the constraint-domination principle of NSGA-II [5] is adopted to classify the combined parent offspring population into nondomination levels. Thereafter, if the number of feasible solutions (N_f) in the combined population is less than the population size (N), then all the feasible solutions are selected and the remaining population slots are filled with infeasible solutions having smaller constrain violation (CV) values. On the other hand, if $N_f > N$, then all the infeasible solutions are ignored and the unconstrained NSGA-III selection procedure is followed with feasible solution set. In this study, the authors proposed three different types of scalable constrained test MaOPs. The experimental study demonstrated the efficacy of constrained NSGA-III on test problems with large number of objectives.

In [84], apart from proposing constrained NSGA-III, Jain and Deb [84] also proposed a constraint-MOEA/D, named C-MOEA/D. The constraint handling mechanism in C-MOEA/D is based on the modification of the replacement step in which a child solution y is compared with a randomly picked solution x from its neighborhood using similar rules as in constraint binary tournament selection [5]. Thus, the scalarizing function is used for comparing y and x only when both solutions are feasible. The experimental study demonstrated that C-MOEA/D performs well on several constrained many-objective test problems. However, the authors adopted the parameter values for C-MOEA/D as suggested in the original MOEA/D study [9] and recommended that a parametric study is required to determine if the performance of C-MOEA/D can be further improved on the challenging constrained many-objective test problems.

Li *et al*. [39] extended MOEA/DD (discussed in Section IX) and proposed C-MOEA/DD to solve constrained MaOPs. In C-MOEA/DD, the constraint binary tournament selection of NSGA-II [5] is adopted to select solutions during the mating procedure. Further, the steady-state update procedure of MOEA/DD is modified such that feasible solutions are always emphasized over infeasible solutions. Additionally, the survival of infeasible solutions is determined using the CV values as well as the niching mechanism. In particular, the infeasible solution with the largest CV and which is not associated with an isolated subregion is removed from the population. The experimental study demonstrated that C-MOEA/DD is significantly superior to C-NSGA-III [84] and C-MOEA/D [84] on constrained DTLZ1–DTLZ4 [55] test problems with large number of objectives.

Cheng *et al*. [41] extended RVEA (discussed in Section IX) and proposed C-RVEA to solve constrained MaOPs. In C-RVEA, the elitist selection strategy of RVEA is updated to handle constraints. In particular, during selection of one elitist solution from a subpopulation to survive into the next generation, the solution with the lowest CV is preferred if all solutions in the subpopulation are infeasible. Otherwise, the feasible solution with minimum APD value is selected. The experimental study demonstrated that C-RVEA and C-MOEA/DD show comparable performance on constrained DTLZ1–DTLZ3 [55] test problems with large number of objectives.

A. Summary

In summary, the survey of studies presented in this section display the following.

1) Constrained NSGA-III [84], C-MOEA/DD [39], and C-RVEA [41] have been found to be considerably successful on constrained MaOPs.
2) A parametric study is required to determine if the performance of C-MOEA/D [84] can be further improved on constrained MaOPs.

A summary of studies on extending decomposition-based MOEAs to constrained optimization is presented in Table S7 in the supplementary document.

XI. Studies on Preference Incorporation

Traditional MOEAs generally focus on obtaining the entire PF so that the DM can have a complete idea of all the available solutions and choose the desired solution. However, the DM may not be always interested in finding the entire PF and may rather wish to obtain specific regions of the PF [51]–[54]. Preference articulation is particularly meaningful in many-objective optimization as it is difficult to obtain a good approximation of a high-dimensional PF using limited population size [41]. In this section, the studies which incorporated DM preference in decomposition-based MOEAs are reviewed.

Deb and Jain [40] investigated the potential of NSGA-III to find a preferred part of PF. To do so, instead of using reference points that spread over the entire objective space, a few representative reference points are chosen to represent the preferred region of user. The experimental study on three- and ten-objective DTLZ1 and DTLZ2 problems exhibited that NSGA-III is very effective in finding preferred part of PF. This is because one of the main characteristic feature of NSGA-III is to emphasize solutions that are associated with each of the reference points.

Cheng *et al*. [41] demonstrated the capability of RVEA to handle DM preferences. The authors presented a modified reference vector generation method to uniformly generate

reference vectors in a user specified subspace of the objective space. In particular, to specify a preferred subspace, the DM needs to identify and provide a central vector v_c and a radius r, where v_c is a unit vector. Thereafter, the reference vectors inside the specified subspace are uniformly generated. The experimental study on three-objective DTLZ1 and DTLZ2 problems illustrated that RVEA can efficiently incorporate DM preferences and return P-O solutions in the preferred region.

Li *et al.* [36] extended MOEA/D-STM to incorporate the preference of DM and presented an algorithm, termed r-MOEA/D-STM. In r-MOEA/D-STM, the DM provides his/her preference information in terms of reference point in the objective space. At first, a large number of weight vectors are generated using the simplex-lattice design method. Thereafter, a small number of weight vectors closest to the provided reference point are selected to define N subproblems for r-MOEA/D-STM. The experimental study on ZDT [79] and three-objective DTLZ [55] test instances demonstrated that r-MOEA/D-STM can efficiently obtain well-distributed solutions close to the provided reference point.

Mohammadi *et al.* [42] proposed an algorithm, termed R-MEAD, based on integration of reference point approach [126] and MOEA/D [9]. In R-MEAD, a set of reference points in the objective space is provided *a priori* by the DM as a representative of his/her preferred regions. R-MEAD is at first executed (with population size denoted by *size*1) for small number of iterations in order to obtain the base weight vectors. The base weight vectors are the corresponding weight vectors of the closest point to each reference point in the objective space. Thereafter, a new set of (*size*2 number of) weight vectors (where *size*2 is much smaller than *size*1) are generated around the base weight vectors in a region whose spread is determined by the parameter *radius*. Until the termination criteria is met, the weight vectors associated with each reference point are dynamically updated in a round-robin manner such that the algorithm converges close to the preferred regions. It is noted that contrary to the original MOEA/D, R-MEAD does not adopt a neighborhood structure for mating and replacement. Furthermore, R-MEAD works with a relatively much smaller population size as only a limited set of solutions are to be obtained near the reference point. The experimental study on problems from the ZDT test suite [79] and few three-objective problems from the DTLZ test suite demonstrated that R-MEAD can efficiently obtain solutions close to the region specified by the DM.

Mohammadi *et al.* [127] improved the scalability of R-MEAD [42] and proposed an algorithm, termed R-MEAD2, for solving MaOPs. Like MOEA/D [9], the drawback of R-MEAD [42] is that the simplex lattice design method is used to generate weight vectors, wherein the population size dramatically grows as the number of objectives increase. To overcome this limitation, in R-MEAD2, a uniform random number generator method is used to generate weight vectors. The experimental study on DTLZ1–DTLZ6 [55] test problems with 4–10 objectives demonstrated that both R-MEAD2-TCH and R-MEAD2-PBI significantly outperform R-NSGA-II [128] on most of the test problems.

Pilát and Neruda [44] proposed an extension of MOEA/D with co-evolution of weights, named cwMOEA/D, to incorporate user preferences. In cwMOEA/D, the user needs to specify his/her preference in the form of a function, which evaluates the solutions at every iteration and returns a negative number for preferred solutions and a positive number otherwise. The co-evolutionary step in cwMOEA/D evolves and dynamically adjusts the weights through Gaussian mutation and selection such that the algorithm explores solutions more preferred by the user. The experimental results on ZDT1 [79], WFG2, and WFG4 [57] benchmark functions demonstrated that cwMOEA/D is very effective in obtaining solutions according to the user-specified preference.

Ruiz *et al.* [66] presented an algorithm, termed weighting ASF genetic algorithm (WASF-GA), to incorporate the preference information of DM. In particular, WASF-GA is the precursor algorithm of global WASF-GA [65], which has been discussed in detail in Section V. Hence, here we only specify the difference between WASF-GA and global WASF-GA. In WASF-GA, the ASF utilizes the reference point as the user-specified reference point (which reflects the preference information of the DM). The experimental study on two- and three-objective test problems from a variety of test suites, thoroughly exhibited the efficacy of WASF-GA to approximate the region of interest of the PF defined by the reference point.

Interactive preference-based MOEAs are based on interactively incorporating the DM's preference to obtain the desired P-O solutions. An interactive version of MOEA/D, named iMOEA/D, is proposed by Gong *et al.* [43]. In iMOEA/D, during periodical interaction, a set of current solutions is offered to the DM, and the search is then guided by renewing the preferred weight region toward the neighborhood of the solution which is most preferred by the DM. In the experimental study, various utility functions are used to simulate the preference of DM and investigate the performance of iMOEA/D on few test problems from ZDT suite [79] and DTLZ suite [55]. The experimental study demonstrated that iMOEA/D can effectively converge to the preferred regions specified by the utility function.

A. Summary

In summary, the survey of studies presented in this section display the following.

1) Several studies have attempted to incorporate DM preferences in the decomposition-based framework using *a priori* reference point information approach.
2) However, only the study on WASF-GA [66] has conducted thorough investigation of the presented preference-based MOEA on wide variety of test problems. The rest of the studies have used only few test problems in the experimental study.
3) Only the studies on NSGA-III [40] and RMEAD-2 [127] have investigated the applicability of preference incorporation in decomposition-based framework on MaOPs. The rest of the studies have used only MOPs in the experimental study.

A summary of studies on preference incorporation strategies in the decomposition-based framework is presented in Table S8 in the supplementary document.

XII. Studies on Real-World Optimization Problems

In the literature, several studies have developed decomposition-based MOEAs for real-world optimization problems. However, due to space constraints, the discussion of such works is not included here. A summary of the studies on decomposition-based MOEAs for real-world optimization problems is presented in Table S9 in the supplementary document.

XIII. Conclusion

This paper presented a comprehensive survey of research works on decomposition-based MOEAs. The survey shows that since the proposition of the original MOEA/D framework by Zhang and Li [9], many decomposition-based MOEAs have been proposed in the literature. The main motivation behind the research works carried out has been either: 1) to overcome the several limitations that existed primarily in the design components of the original MOEA/D framework or 2) to improve the performance of MOEA/D or 3) to present novel decomposition-based MOEAs or 4) to extend decomposition-based MOEAs to other type of problems.

A summary of important conclusions that can be made with respect to the work conducted on decomposition-based MOEAs are as follows.

1) The distribution of the obtained P-O solutions is highly dependent on the weight vector generation method employed in the decomposition-based framework. The assumption that an evenly [9] or uniformly [19] distributed weight vectors can result in uniformly distributed P-O solutions has been comprehensively refuted, particularly for MaOPs [25], [26].

2) The two-layer weight vector generation method [39] extends the simplex-lattice design method for many-objective optimization and overcomes the limitation of the latter in generating relatively small number of evenly spread weight vectors for MaOPs.

3) The adaptation of scalarizing functions, particularly the recently proposed PaS [68] approximation to approximate the optimal p value in the L_p weighted approach, has overcome the challenge of choosing an appropriate scalarizing function for a particular problem.

4) The studies which proposed algorithms such as NSGA-III [40], MOEA/DD [39], RVEA [41], MOEA/D-AM2M [71] provide new direction in which the high-dimensional objective space in MaOPs can be partitioned into small subspaces using reference vectors, and sophisticated update procedures can be employed to preserve diversity in all subspaces.

5) Depending upon the hardness of the problem, different subproblems may require different computational budget in order to be efficiently solved [30]. Thus, strategies based on dynamic computational resource allocation

to different subproblems in the decomposition framework can significantly improve the performance of the algorithm [29], [30].

6) Weight vector adaptation is essential in the decomposition-based framework when the target MOP has a complex or irregular PF (e.g., discontinuous PF, PF with sharp peak, and low tail) [24].

7) Weight vector adaptation is also essential in the decomposition-based framework when the PF of the problem is such that there are some reference points which have no P-O solution associated with them [84], [124].

8) Extensive work is being conducted on incorporating AOS in the MOEA/D framework [33], [94], [97], etc. These MOEA/D variants based on AOS show remarkable improvement in performance on MOPs.

9) The neighborhood relationship in the weight vector space, as defined in the original MOEA/D framework, can be deceptive to the algorithm [26].

10) The neighborhood relationship should be rather defined in the objective space and should be adaptive such that solutions which participate in mating procedure are close in the objective space [26].

11) Efficiently combining decomposition- and indicator-based approaches (e.g., FV-MOEA/D [82]) or decomposition- and dominance-based approaches, (e.g., NSGA-III [40], MOEA/DD [39], I-DBEA [38], and BCE-MOEA/D [74]) can result in high performance algorithms for multiobjective and many-objective optimization.

The survey shows that researchers have presented several successful attempts to extend the decomposition-based framework to constrained MOPs, MaOPs, and incorporate DM preferences. Moreover, the survey shows that a number of attempts have been made to apply decomposition-based MOEAs to complex real world optimization problems. This clearly suggests that the decomposition-based framework is highly flexible. Thus, the decomposition-based framework will continue to attract researchers from the evolutionary multiobjective optimization community in the future.

A. Future Directions

The survey indicates that there are several directions to be pursued in the future, as summarized below.

1) Currently, only the study [63] has investigated into methods for avoiding tuning of the parameter θ in the PBI approach. However, this study tested the efficacy of the proposed APS and SBS penalty schemes only on few selected MOPs. Thus, thorough investigation of APS and SPS on other MOPs, MaOPs, and introduction of new methods to adaptively control parameter θ can be interesting future directions.

2) Combination or adaptation of scalarizing functions can be further studied. Further, efficacy of the existing MOEAs based on such concepts such as global WASF-GA [65] and MOEA/D-PaS [68] can be analyzed on different MaOPs with large number of objectives.

3) Genetic operators have been used in most of the successful many-objective optimizers (e.g., NSGA-III [40], MOEA/DD [39], MOEA/D-DU [60], and RVEA [41]). However, on MOPs, AOS has been found to be really beneficial [33], [97]. Thus, investigating AOS on MaOPs is an interesting future direction [33].

4) The role of neighborhood structure for mating and replacement in the decomposition-based MOEAs can be further studied for MaOPs. This is because I-DBEA [38] has been found to be considerably successful on MaOPs without involving the concept of neighborhood.

5) Most of the decomposition-based MOEAs proposed in the literature have been tested on MOPs. Thus, investigative studies can be undertaken to analyze if the enhanced MOEA/D variants such as MOEA/D-ACD [28], MOEA/D-GRA [30], MOEA/D-IR [37], MOEA/D-AGR [109], etc. scale well on MaOPs.

6) Further, new decomposition-based MOEAs can be developed to efficiently tackle MaOPs. To develop such many-objective optimizers, use of reference vectors to decompose the objective space into small multiple subspaces [39]–[41] and combining decomposition-based approach with dominance- or indicator-based approach can be highly important component.

7) Only a few decomposition-based MOEAs such as MOEA/D-AWA [24], adaptive NSGA-III [84], FV-MOEA/D [82] have been proposed with weight vector adaptation. Thus, more studies should be undertaken to develop weight vector adaptation strategies for decomposition-based framework.

8) Most of the decomposition-based many-objective optimizers are generally tested on DTLZ [55] and WFG [57] test problems. However, the study by Ishibuchi *et al.* [124] recommended that the performance of the algorithms should be tested on a wide variety of test problems with various PF shapes.

9) Only a limited number of theoretical studies have been undertaken on analyzing the decomposition-based MOEAs [13], [129]. Thus, theoretical aspects of decomposition-based MOEAs can be further studied.

10) Most of the decomposition-based MOEAs have been tested primarily on unconstrained MOPs and only few studies have investigated incorporating constraint handling strategies. Thus, constraint handling strategies can be proposed for enhanced decomposition-based MOEAs such as MOEA/D/ACD [28], MOEA/D-GRA [30], MOEA/D/IR [37], MOEA/D-AGR [109], etc., to extend them to constrained MOPs.

11) The survey shows that only the study on WASF-GA [66] has conducted thorough investigation of the presented preference-based MOEA on wide variety of test problems. Further, only the studies on NSGA-III [40] and RMEAD-2 [127] have investigated the applicability of preference incorporation in decomposition-based framework on MaOPs. Thus, further investigation of preference incorporation in decomposition-based framework can be an important future direction.

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