

MOEA/D

MOEA/D - Restart Strategy

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What is MOP?

Multiobjective Optimization Problem have m multiple objective functions that must be optimized simultaneously.

Maximize¹ $F(x) = (f_1(x), f_2(x), \dots, f_m(x))$,
subject to x in Ω .

- ▶ $F(x)$ objective functions;
- ▶ f_i is the i -th objective to be maximized;
- ▶ x is the decision vector;
- ▶ Ω is the decision space.

¹ All definitions are for maximization. Following inequalities should be reversed if the goal is to minimize.

Why is MOP interesting?

1. Many real-world scientific and engineering are MOP.
 - ▶ Water quality control, Groundwater pollution re-mediation, Design of marine vehicles, ... Coello et al. [2007].
 - ▶ Petrol extraction.
2. Hard problems: to balance the interests of the multi-objective as a whole is hard.

What is Pareto Set?

Objectives may be conflicting - The goal is to find good trade-off.

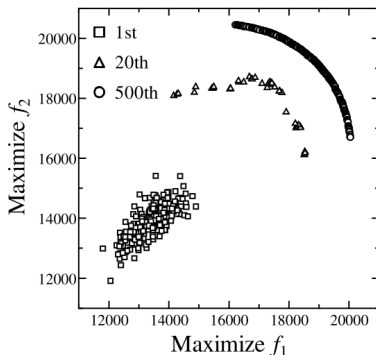
- Set of solutions.

Set of *optimum solutions* - Pareto set.

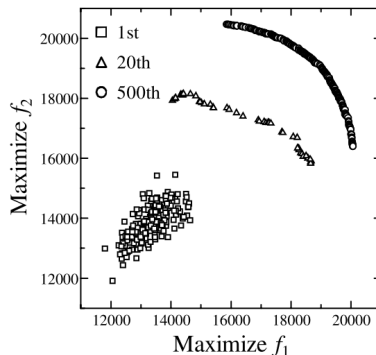
- Non-dominated solutions: no single solution provides a better trade-off in all objectives.

Pareto Set

What is Pareto Set?



(a) The weighted sum.



(b) The weighted Tchebycheff.

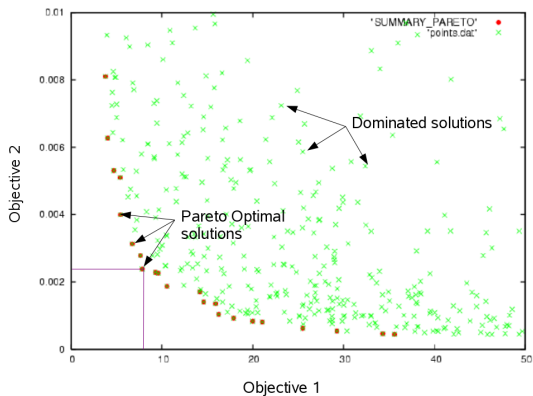
From Ishibuchi et al. [2009].

Non-dominated solutions

- Let $u = (u_1, \dots, u_m)$ and $v = (v_1, \dots, v_m)$ vectors in Ω (the decision space).
 - ▶ $\forall i : u$ dominates v if $f_i(u) \leq f_i(v)$ and $\exists j : f_j(u) < f_j(v)$.
 - ▶ u dominates v , v is dominated by u , u is better than v .
- A point x^* in Ω is called *Pareto Optimal* if no other point dominates x^* .

Pareto Set

Pareto Front



From:

<http://www.cenaero.be/Page.asp?docid=27103&langue=EN>

Pareto Front

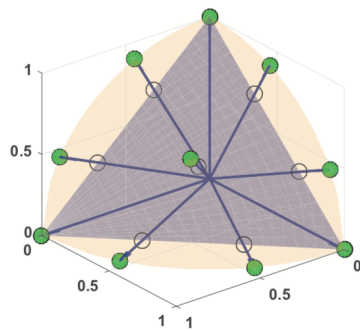
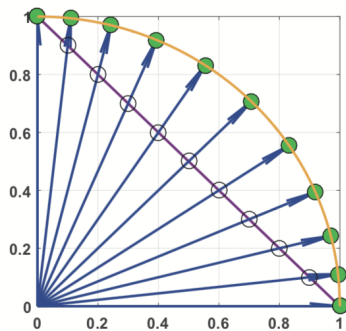
1. The set of all Pareto Optimal is called the *Pareto Set*.
 - ▶ $P^* = \{x \in \Omega : \nexists y \in \Omega \text{ and } F(y) \leq F(x)\}$
2. **Pareto Front** is the image of the Pareto Set in the objective space.
 - ▶ $PF = \{F(x) = (f_1(x), \dots, f_m(x)) : x \in P^*\}$

Decompostion

MOEA/D represents a class of population-based meta-heuristics for solving Multi Objective Problems (MOPs).

- ▶ It is based on decomposition - one kind of scalarizing function
- ▶ One multi-objective problem becomes various single-objective sub-problems.
- ▶ A decomposition strategy generates weight vectors that defines the sub-problems.

Decomposition - 2 and 3 objective functions



From: Chugh [2017].

Why use decomposition?

- ▶ It may be good at generating an even distribution of solutions in MOPs;
- ▶ It reduces the computation complexity when compared to other algorithms (NSGA-II) (at each generation), Zhang et al. [2009];
- ▶ Fitness assignment and diversity maintenance become easier to handle.

Decomposition + Aggregation Function

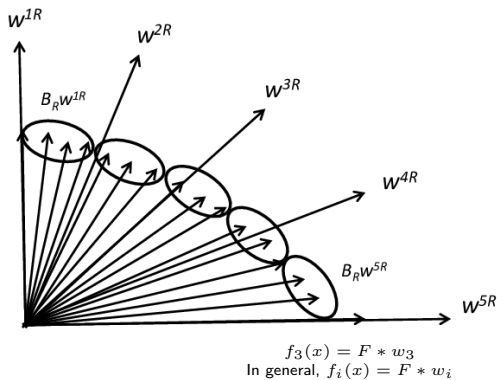


Figure from: Chugh [2017].

Components of the MOEA/D

- ▶ Decomposition strategy: decomposes w/ weight vectors;
- ▶ Aggregation function: weight vector \Rightarrow single-objective sub-problems;
- ▶ Neighbourhood assignment strategy: Relationship between sub-problems;
- ▶ Variation Stack: New candidates solutions;
- ▶ Update Strategy: Maintain/discard candidate solutions;
- ▶ Constraint handling method: Constraint violation;
- ▶ Termination Criteria: when to stop the search.

Modifications Already integrated

1. Cellular GA- proposed in the context of MOEA/D by Ishibuchi et al. [2009].
2. Latin Hypercube Sample - an alternative approach in initializing populations.
3. On-line Resource Allocation - proposed in the context of MOEA/D by Zhou and Zhang [2016].
4. Bet-and-Run: A kind of restart strategy - in the context of single-objective problems (SOP) by Friedrich et al. [2017]

Cellular GA and MOEA/D Ishibuchi et al. [2009]

- ▶ Why? MOEA/D can be viewed as a Cellular GA (cGA).
 - ▶ A cell can be seen as a specific “Neighbourhood assignment strategy”, where each cell has its own weight vector.
- ▶ cGA is well explored in the context of SOP.

MOEA/D as cGA

- ▶ The main characteristic feature of MOEA/D as a cGA is the use of **local replacement** in addition to local selection.
 - ▶ Generated offspring for a cell is compared with not only the current solution of the cell but also its neighbours for possible replacement.
- ▶ Local replacement neighbourhood has greater effect on the performance than local selection neighbourhood.
 - ▶ Increasing its size tends to be better.

What is Latin Hypercube Sample

- ▶ Latin Hypercube Sample (LHS) was developed to generate a distribution of collections of parameter values from a multidimensional distribution, for more information see Stein [1987].

How it affects MOEA/D

- ▶ As defined in McKay et al. [1979], it could be a good method to use for selecting values of input variables.
- ▶ Therefore we expect that by using it, the initial population (ours input variable) would be better distributed along the search space.

What is Online Resource Allocation

- ▶ On-line Resource Allocation (ONRA) is an adaptation strategy that aim to adjust the behaviour of an algorithm in an on-line manner to suit the problem in question.

How it affects MOEA/D - Zhou and Zhang [2016].

- ▶ Some sub-problems can be more difficult to approximate than others. To better explore them, different computational resources are allocated to different sub-problems.
- ▶ The resources re-allocated is *the number of functions evaluations*.
 - ▶ From an equal amount to every sub-problem to an amount related to the difficulty of the sub-problem.

What is Restart Strategy

- ▶ Restart Strategy is a strategy used to avoid heavy-tailed running time distributions Gomes et al. [2000].
- ▶ If a execution of an algorithm does not conclude within a pre-determined limit or if the solution quality is unsatisfactory, we restart the algorithm Lissovoi et al. [2017].

Bet-and-Run framework

- ▶ It is defined in Fischetti and Monaci [2014]. as a number of short runs with randomized initial conditions, bet on the most promising run, and bring it to completion.
- ▶ To the best of our knowledge, only applied with EA in the context of SOP.

How it affects MOEA/D - Lissovoi et al. [2017].

- ▶ Initialisation can have a small beneficial effect even on very easy functions.
- ▶ Countermeasure when problems with promising and deceptive regions are encountered.
- ▶ Additional speed-up heuristic.

Unary Indicators

- ▶ Measure Pareto Sets independently.
 - ▶ Power is restricted.
 - ▶ Cannot tell in general if a set is better than another.
 - ▶ Focus on problem dependent and specifics.
 - ▶ Assumptions and knowledge should be specified.
1. Hyper-volume.
 2. Error ratio.
 3. Distance from reference set.

Binary Indicators

- ▶ Theoretically have no limitations.
 - ▶ Analysis and presentation of results more difficult.
1. R1, R2, R3 indicators.
 2. ϵ -Indicator.
 3. Binary Hyper-volume.

Considerations

- ▶ Is complete - If, and only if $HV(A) > HV(B) \implies A$ is not worse than B .
- ▶ Is weakly compatible - $HV(A) > HV(B) \implies B$ dominates A .
- ▶ Assumptions - All points of a Pareto Set under consideration dominate the reference point.
 - ▶ Ishibuchi ? proposed a method to specify the reference point from a viewpoint of fair performance comparison.

Considerations

- ▶ A large population size is **always** more beneficial than a small one.
- ▶ Measures both the convergence toward the Pareto Front and the diversity of non-dominated solutions.
- ▶ A monotonic increase of the hyper-volume over time cannot always be ensured.
 - ▶ For MOEA/D that is always true.

Considerations

- ▶ It compares 2 Pareto Sets.
 - ▶ It indicates which set is better and how much better
- ▶ If A is better than $B \implies I_{\epsilon}(B, A) > 0$.
- ▶ If $I_{\epsilon}(A, B) \leq 0$ and $I_{\epsilon}(B, A) > 0 \implies A$ is better than B .

Experimental Design

1. Simple experiments - Check my understand and get insights.
2. DTLZ1, DTLZ2, DTLZ6 and DTLZ7 MOP benchmark functions - Available from the MOEADr package.
3. Every variation will be discussed based on the pilot data showed in the next slide by a box-plot figure.
4. Number of evaluations: $5 * 10^4$.
5. Based on the common variation: MOEA/D (variations 1 and 2 from MOEADr) and MOEA/D-DE.

MOP

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MOEA/D

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Modifications - MOEA/D

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Evaluation Metrics

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Pilot Experiments

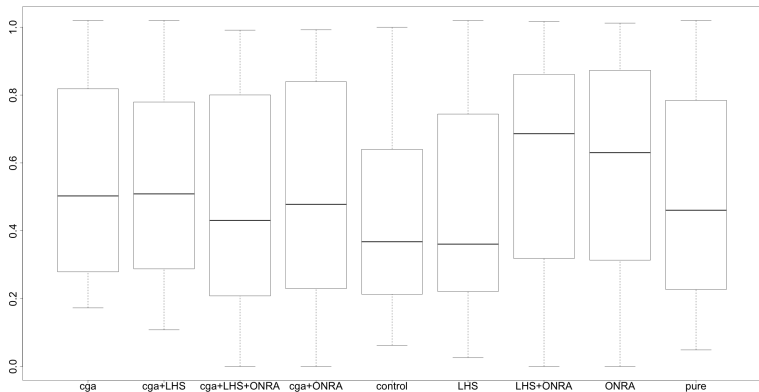
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References

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Preliminary Results

Boxplot - HV



cGA

1. MOEA/D as cGA has a high sensitivity on the parameters of local replacement and local selection.
2. Decreasing the size of competition neighbourhood increases the non-dominated solutions, but degrades the search ability of the MOEA/D. **As already observed in Ishibuchi et al. [2009] .**

LHS

1. In most cases improves the results by a little - All but with naive restart strategy.
 2. It is cheap in terms of computational cost - it is only used once at each execution.
- This improve seems not to be significant.

ONRA

1. Computational costly -> more interactions than without it and we need to calculate the resources allocation every interaction.
2. It was beneficial in a few cases, while in others the overall quality decreased.

Bet-and-Run Strategy

1. Overall, this strategy combined with the MOEA/D lead to better final quality results.
2. Its performances become better when combined with other variations, specially with cGA+LHS.

Future works

1. cGA - On the fly parameter adaptation.
2. ONRA
 - ▶ Review my implementation and try the other methods proposed in Zhou and Zhang [2016]. Only the one considered to be the “best” was implemented.
3. Bet-and-Run strategy
 - ▶ Use this strategy to add some adaptive technique.
 - ▶ Use more instances based on the best one, instead of only one.
 - ▶ dynamic bet-and-run.
 - ▶ Hierarchical bet-and-run - here it has only 2 phases.

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