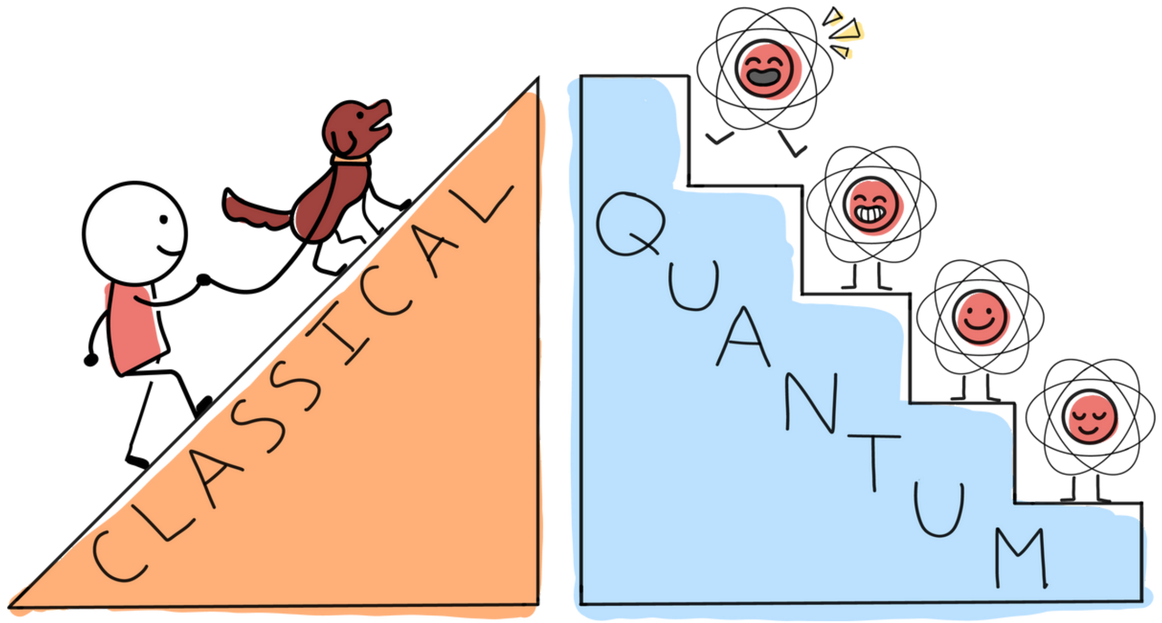
Throwback - DAY01

Quantum Technology Summer School

Sep. 16, 2021

Quantum vs. classical

- Representation of system state
  - Classical: traffic lights 🚦
  - Quantum: Bloch sphere 🌏
- Two things that classical don’t know
  - Superposition
  - Entanglement



Postulates of quantum mechanics

- State vector Can be **superposition**!

$$|\Psi\rangle = c_1 |\psi_1\rangle + c_2 |\psi_2\rangle$$

- Observable Observable is **operator**

$$\mathscr{A} : |\psi\rangle \mapsto |\psi'\rangle = \mathscr{A} |\psi\rangle$$

- Measurement Outcome is **one of the eigenvalues** to the operator(observable)

$$\mathscr{A} \to \{a\} : \mathscr{A} |\psi_a\rangle = a |\psi_a\rangle$$

- Born rule Tell us the **probability** of getting some measurement outcome

$$P_a = |\langle \psi_a | \psi \rangle|^2$$

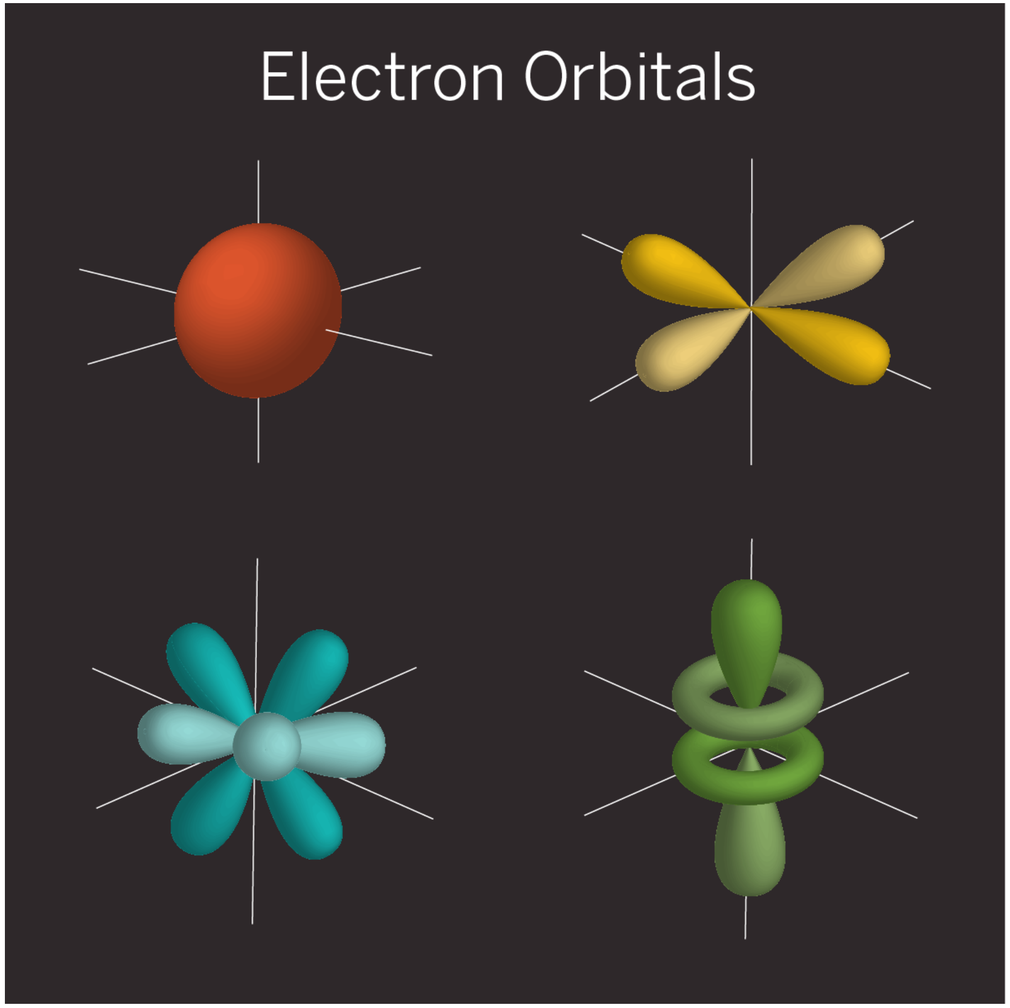
- Wavefunction collapse

Measurement kills the superposition

$$\mathscr{A} : a \implies |\psi_a\rangle$$

- Time evolution How the system evolves

$$|\psi(t)\rangle = \mathscr{U}(t, t_0) |\psi(t_0)\rangle$$



Probability wave

Measurement

Notations

- State: vector
  - ket-vector (column vector)

$$|\psi\rangle = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_n \end{pmatrix}$$

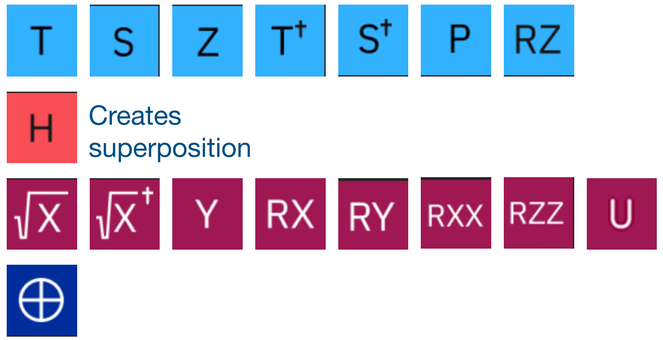
- Observable: operator
  - block matrix

$$\mathscr{A} = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & A_{22} & \cdots & A_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ A_{n1} & A_{n2} & \cdots & A_{nn} \end{pmatrix}$$

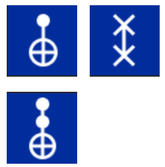
Quantum gates

Operations Glossary

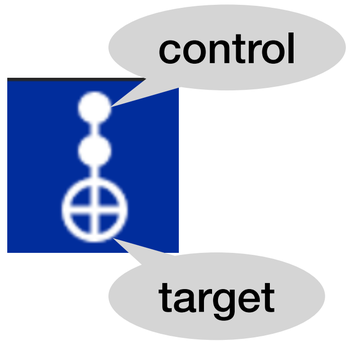
- Single-qubit gates



- Multi-qubit gates



Phase gates, Hadamard gate
Quantum gates, Classical gates

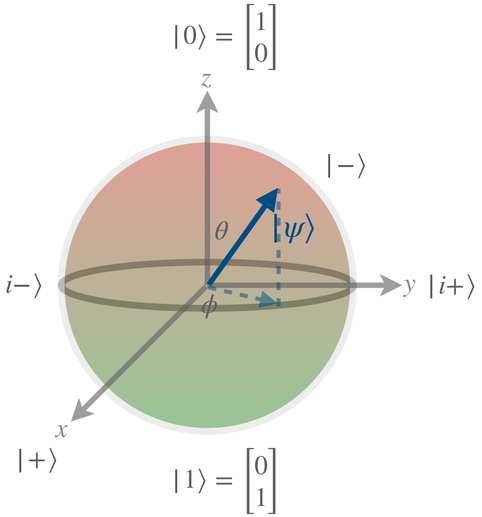


- Quantum gates are unitary: **Reversible!**

$$\mathcal{U}^{-1} = \mathcal{U}^{\dagger} \quad \text{or} \quad \mathcal{U}\mathcal{U}^{\dagger} = \mathbb{I}$$

- Gates operations are **rotations** Matrices

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$$
$$= e^{ir}\left(\cos\frac{\theta}{2} + e^{i\phi}\sin\frac{\theta}{2}\right)$$



Ready?