Explore IBM Q Devices

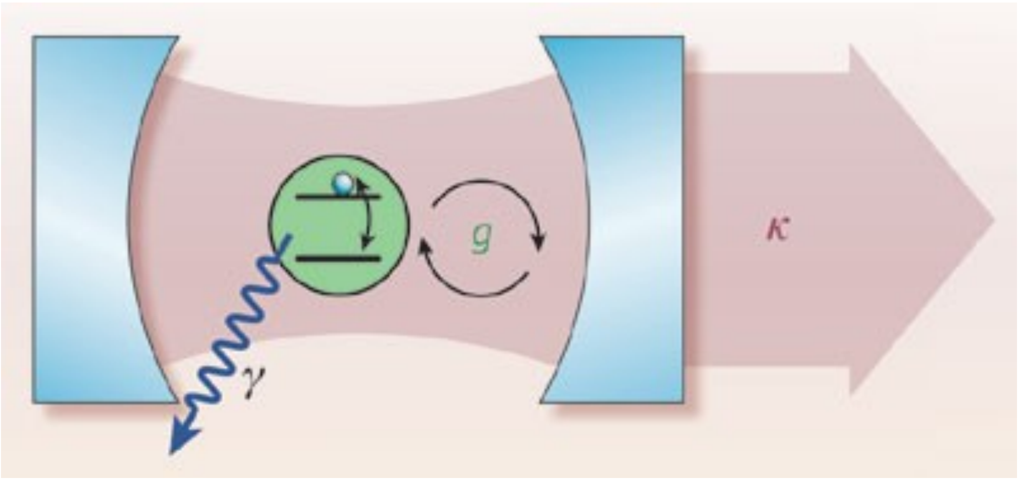
Insight of hardwares & Quantum random walks

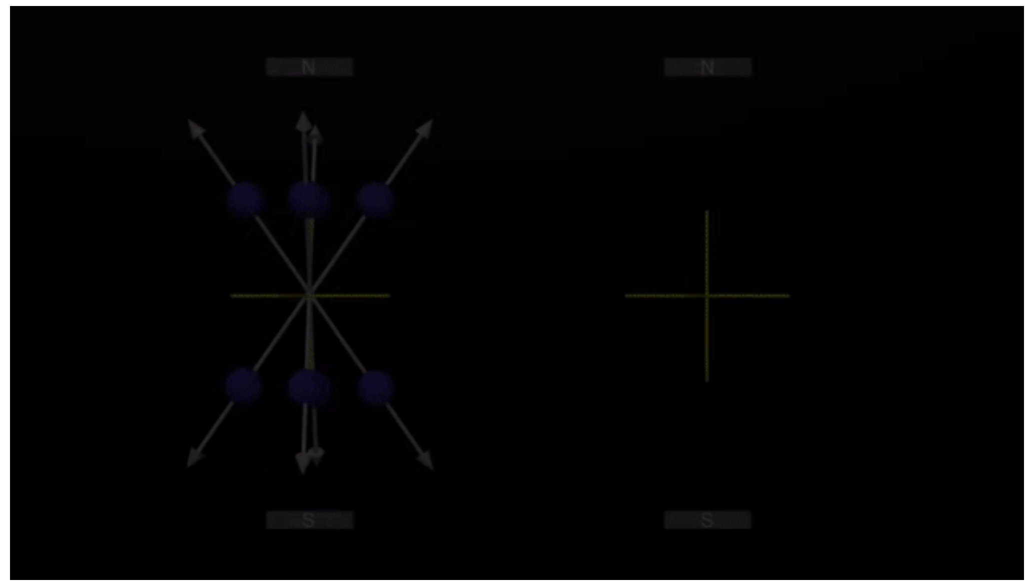
Sep 16, 2021

Quantum Technology Summer School

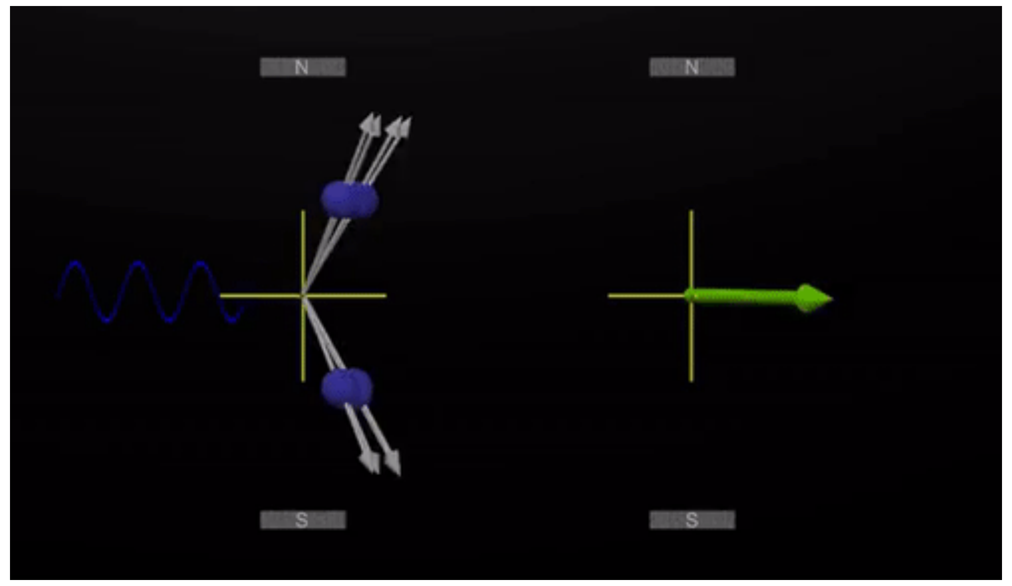
Errors

- Dissipation
  - Spontaneous emission
  - Cavity leakage
  - Imperfect two-level
- <u>Lifetime</u>
  - Population relaxation: T_1
  - Coherence: T_2
- Driving errors
- Measurements errors





T_1



T_2

Superposition of states and decoherence

GHZ state

Exercise

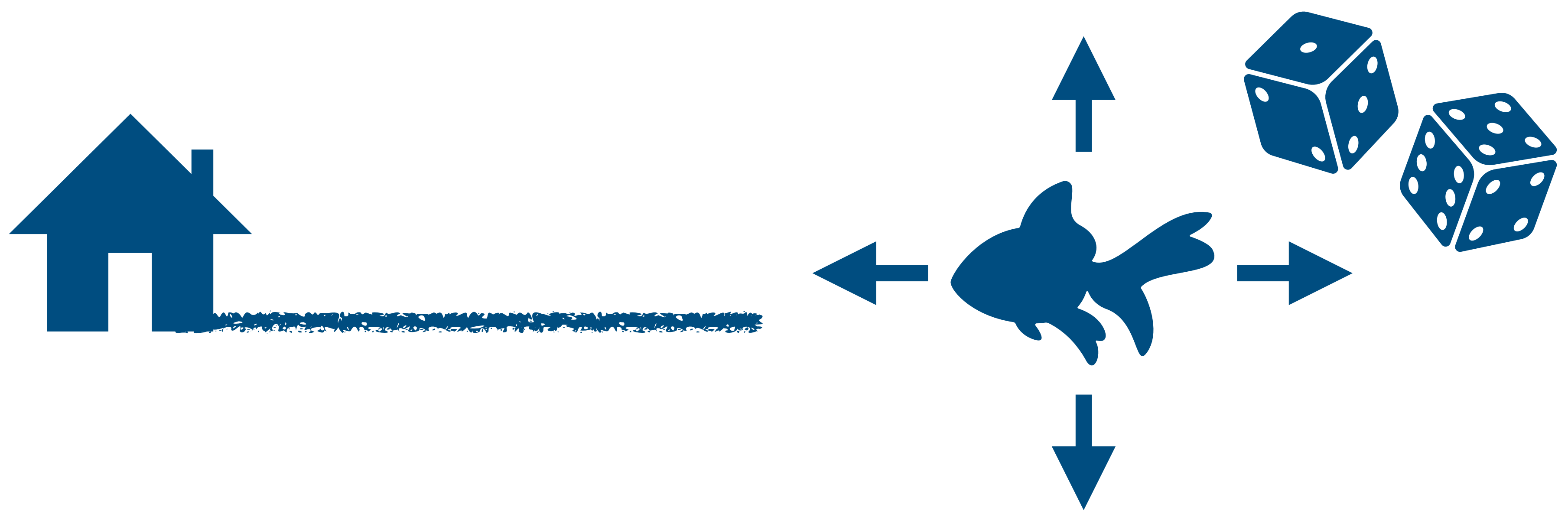
Quantum random walks

Demonstrations


Dory is finding home…

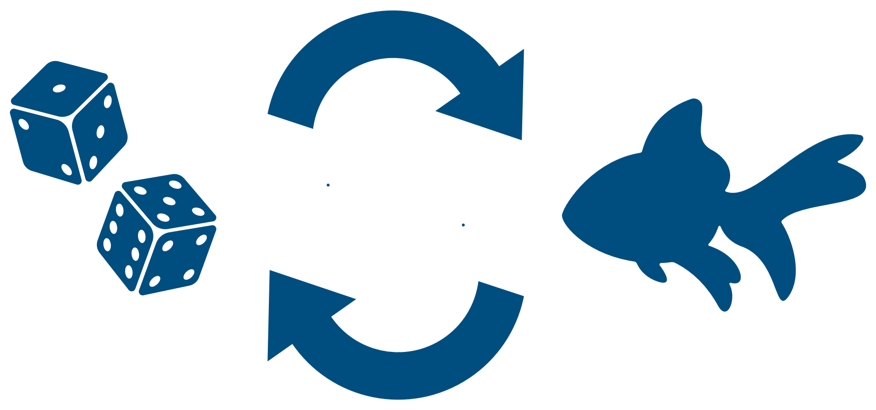
Scheme

- Task: Dory wants to go home
- Method
  - Poor memory $\longrightarrow$ random walks
  - Simplify situation $\longrightarrow$ 1D path

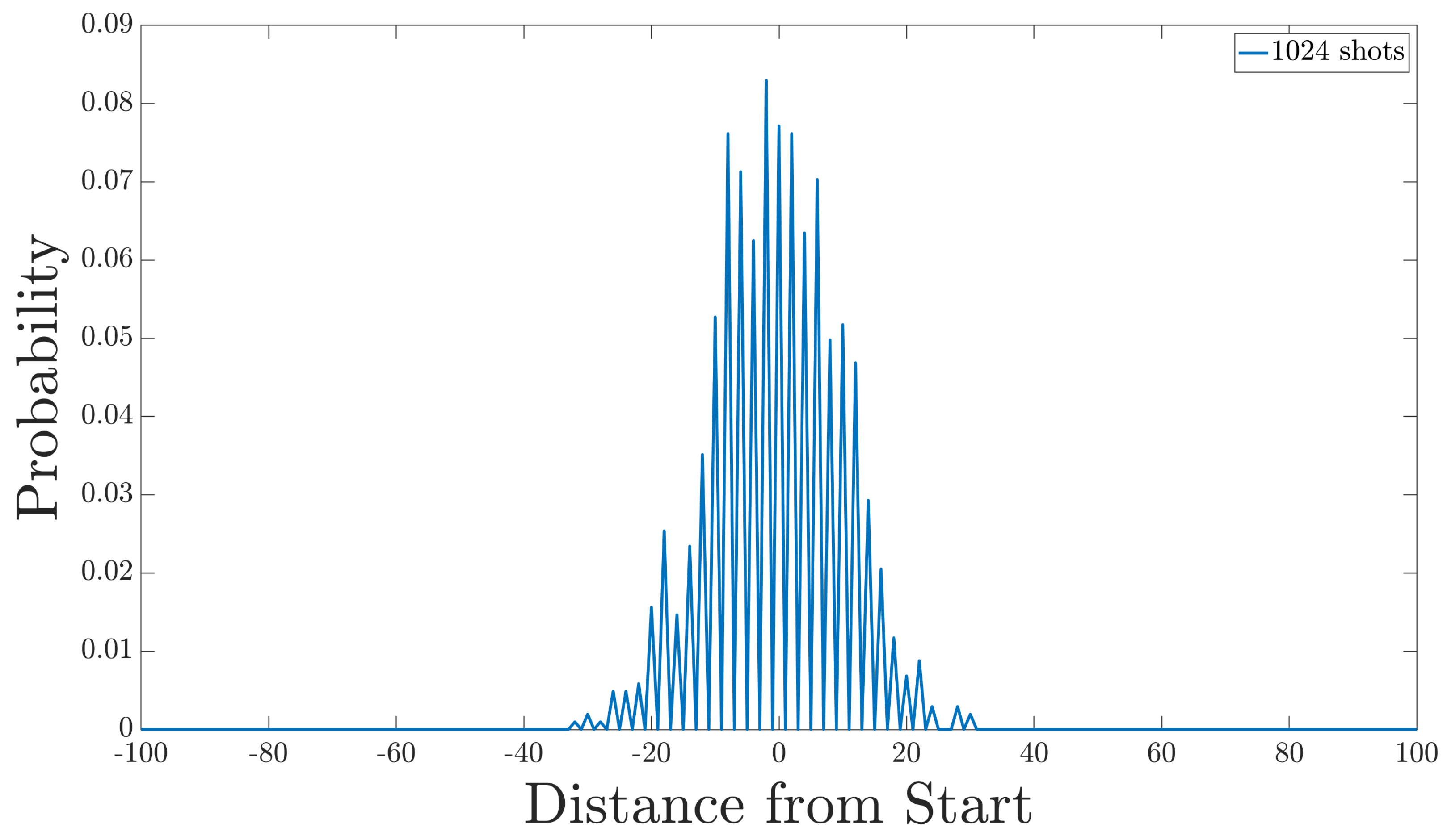


Classically……

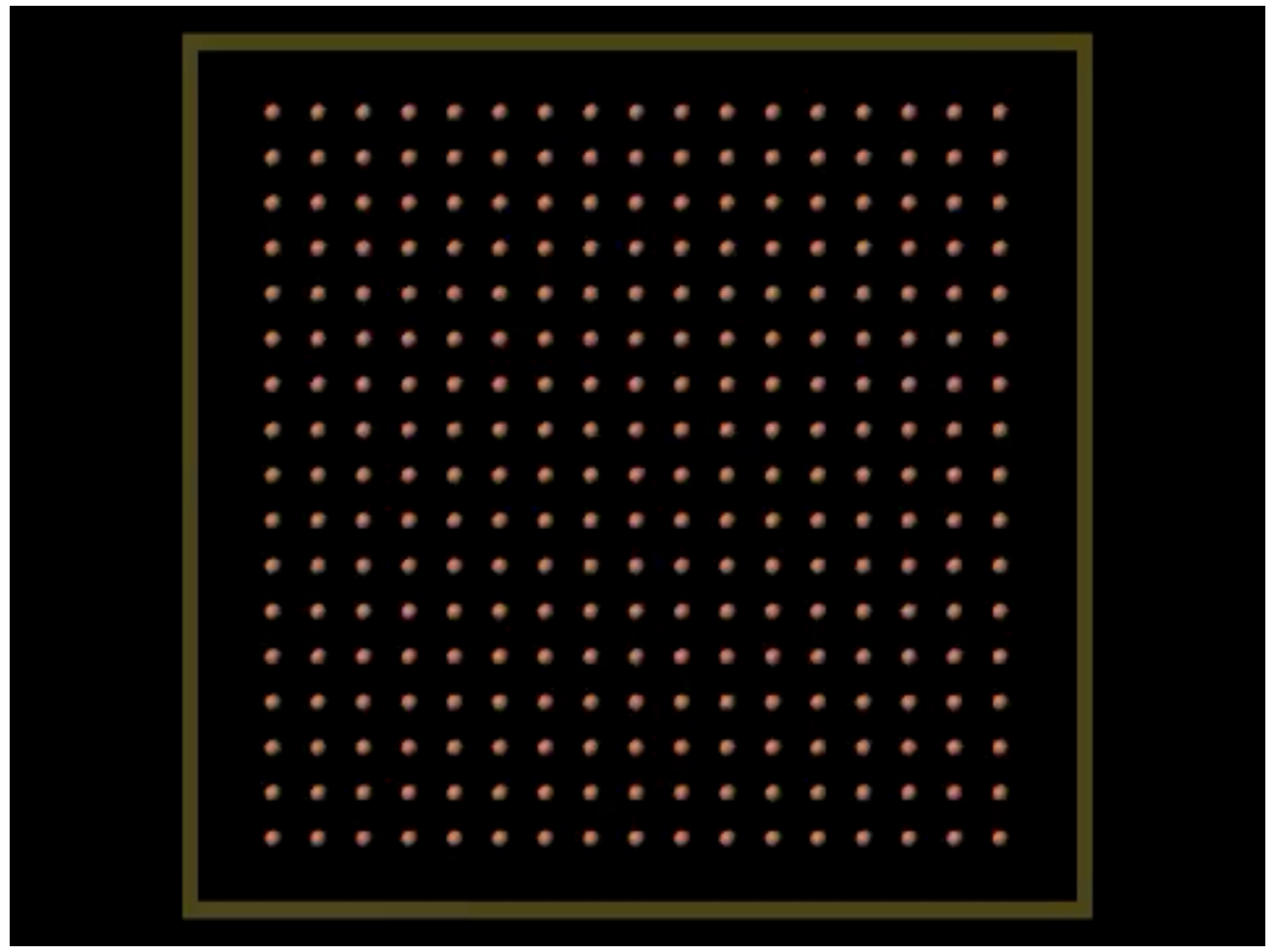
- Scheme



- Simulation



I am sorry about Dory :(



Brownian motions

WHO can help?

Quantum.

What can we do with quantum?

- Memoryless → Markov process
- Make decision randomly → probability → quantum!
- Superposition of choices

$$\frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle), \quad \frac{1}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle),$$

$$\frac{1}{\sqrt{2}}(|\uparrow\rangle + i|\downarrow\rangle), \quad \frac{1}{\sqrt{2}}(|\uparrow\rangle - i|\downarrow\rangle), \cdots$$

Basic elements

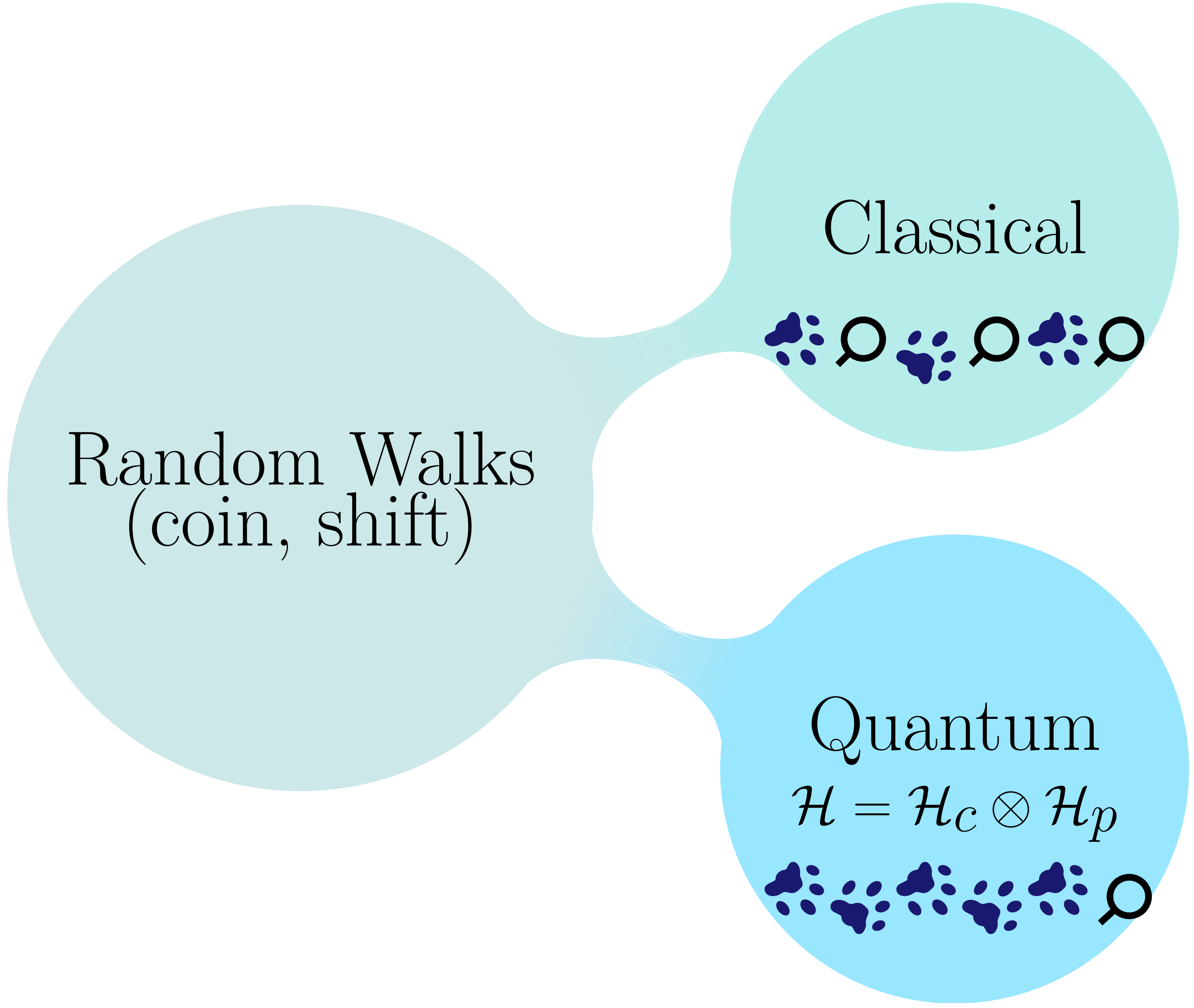
Case: 1D walk

- Coin $\mathscr{C}$
  - 2 directions: forward, backward
  - Dimension: 2x2
  - Common choice: Hadamard coin (or any 2-by-2 block matrix)

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

- Shift $\mathscr{S}$
  - 2 options: forward, backward
  - Conditional action related to the coin state

$$\mathscr{S} := \sum_n \left(|\uparrow\rangle \otimes |n+1\rangle\langle n| + |\downarrow\rangle \otimes |n-1\rangle\langle n| \right)$$



Reminder: some math

- Complete, orthonormal basis $\{\phi\}$

$\langle \phi | \phi \rangle = 1$ and $\langle \phi_j | \phi_k \rangle = \delta_{jk}$

$$|\psi\rangle = \sum_n c_n |\phi_n\rangle$$

$$c_j = \langle \phi_j | \psi \rangle = \sum_n c_n \langle \phi_n | \phi_j \rangle$$

$$\implies |\psi\rangle = \sum_n \langle \phi_n | \psi \rangle \cdot |\phi_n\rangle$$

- Example: Identity matrix

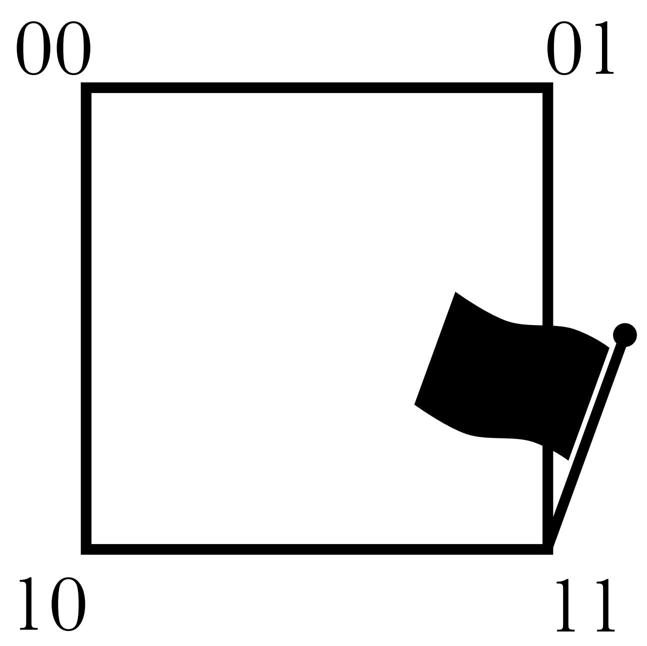
$$I = \begin{pmatrix} 1 & & \\ & \ddots & \\ & & 1 \end{pmatrix} = \sum_n |n\rangle\langle n|$$

- Example

$$\begin{aligned} \big(|n+1\rangle\langle n| \big) \cdot |n\rangle &= |n+1\rangle \cdot \big(\langle n | n \rangle \big) \\ &= |n+1\rangle \end{aligned}$$

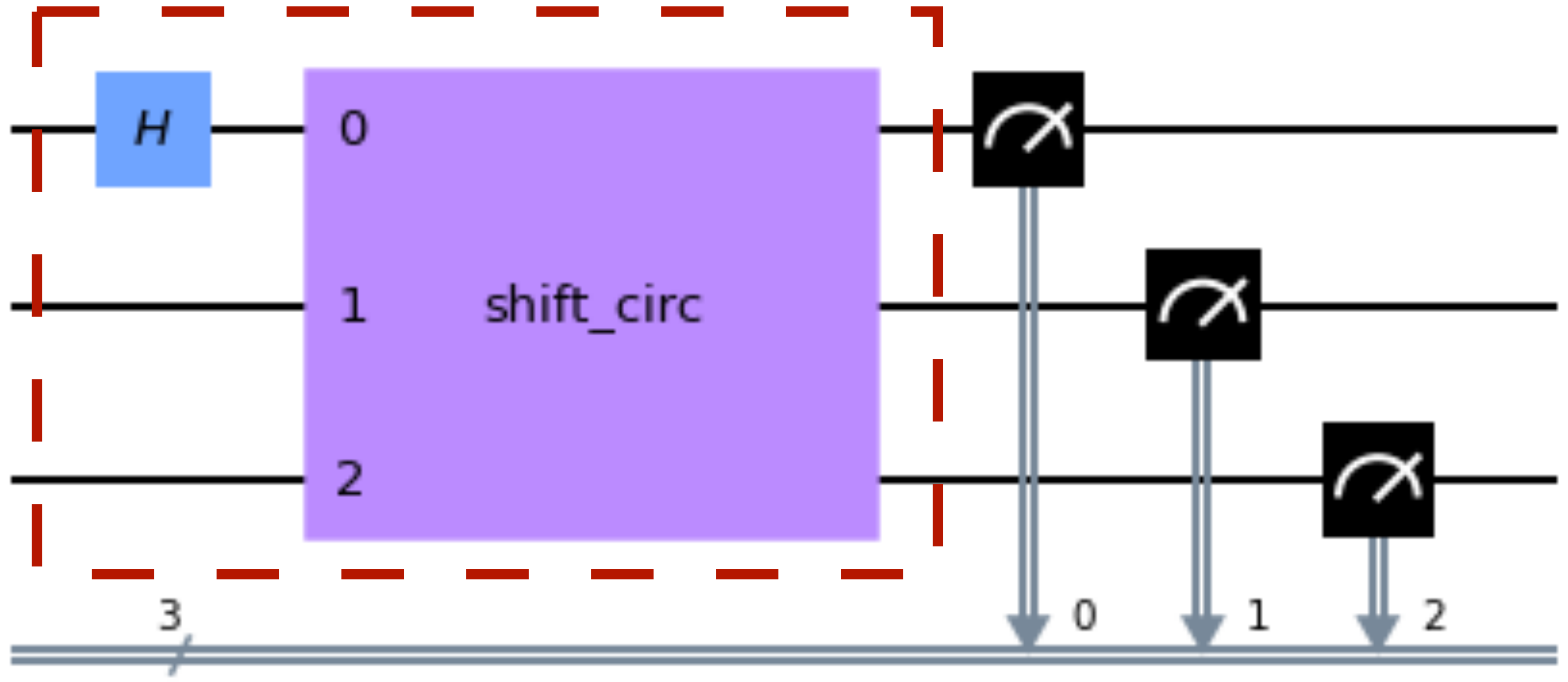
Shift operator

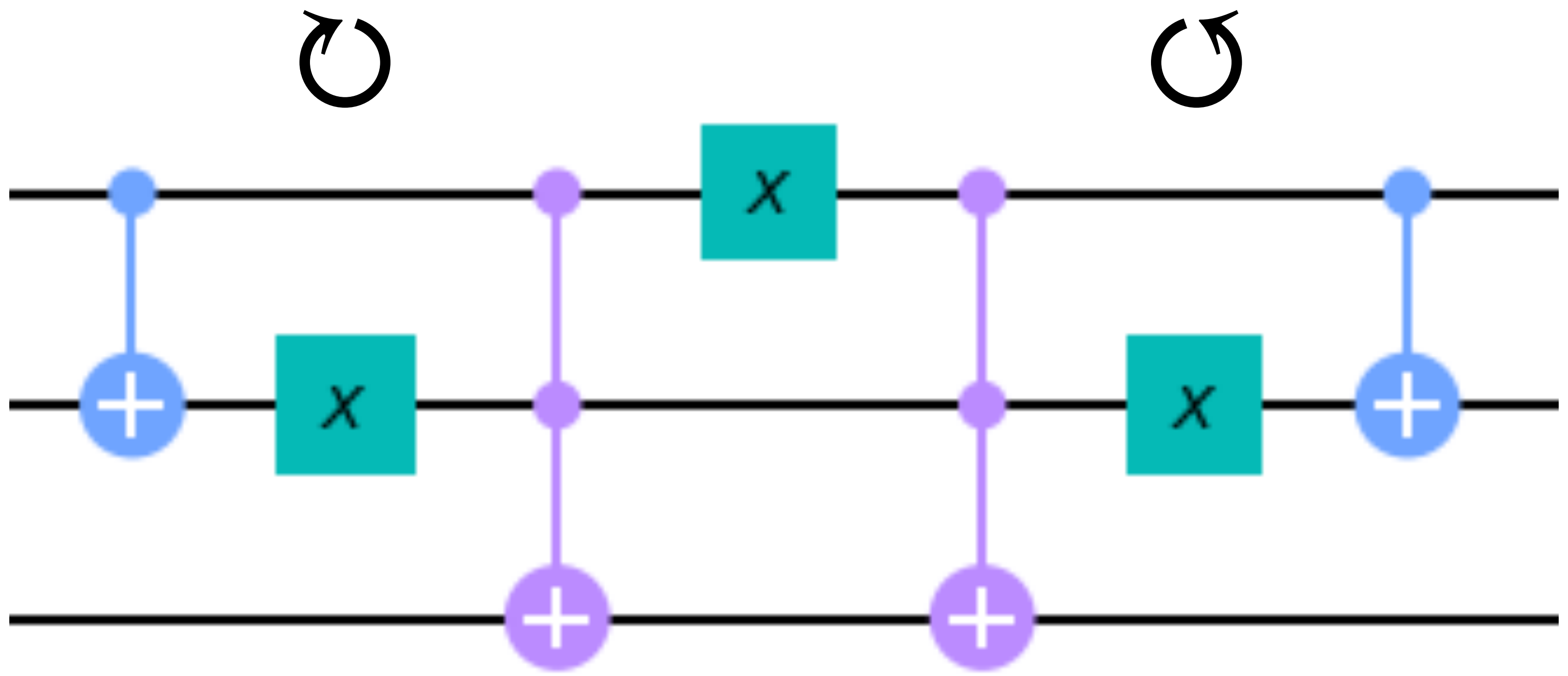
Case: 4-vertice circle



- Vertex labels: 00, 01, 10, 11 (binary!)
- Oracle: the order of $\mathscr{C}$ and $\mathscr{S}$ cannot switch!!

$$\mathscr{U} = \left(\mathscr{S}\mathscr{C}\right)^T$$





- *Question.* How to expand the operator to larger position space?

System state

- Subspaces: $\mathscr{H} = \mathscr{H}_c \otimes \mathscr{H}_p$
  - Coin state $|\psi\rangle_c$
  - Position state $|\psi\rangle_p$
  - Total state $|\Psi\rangle = |\psi\rangle_c \otimes |\psi\rangle_p$
- *Example*: at 0, superposition coin

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\big(|\uparrow\rangle + i|\downarrow\rangle\big) \otimes |0\rangle$$

- Recall operations

$$\begin{cases} H|\uparrow\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle) \\ H|\downarrow\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle) \end{cases}, |\uparrow\rangle = \begin{pmatrix}1\\0\end{pmatrix}, |\downarrow\rangle = \begin{pmatrix}0\\1\end{pmatrix}$$

- Related phase between $|\uparrow\rangle, |\downarrow\rangle$
  - Interference (real / imaginary axis)

State evolution

Starts from origin

- Classical

$50\,\%\,(1) + 50\,\%\,(-1)$

$25\,\%\,(2) + 50\,\%\,(0) + 25\,\%\,(-2)$

$12.5\,\%\,(3) + 37.5\,\%\,(1) + 37.5\,\%\,(-1) + 12.5\,\%\,(-3)$

	-3	-2	-1	0	1	2	3
0				1			
1			1/2		1/2		
2		1/4		1/2		1/4	
3	1/8		3/8		3/8		1/8

- Quantum - Hadamard coin

coin: $\mathscr{C} = H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

shift: $|\uparrow\rangle \implies |n+1\rangle$
$|\downarrow\rangle \implies |n-1\rangle$

$$\frac{1}{\sqrt{2}}\left(|\uparrow\rangle \otimes |1\rangle - |\downarrow\rangle \otimes |-1\rangle\right)$$

$$\frac{1}{2}\left[|\uparrow\rangle \otimes |2\rangle - \left(|\uparrow\rangle - |\downarrow\rangle\right) \otimes |0\rangle + |\downarrow\rangle \otimes |-2\rangle\right]$$

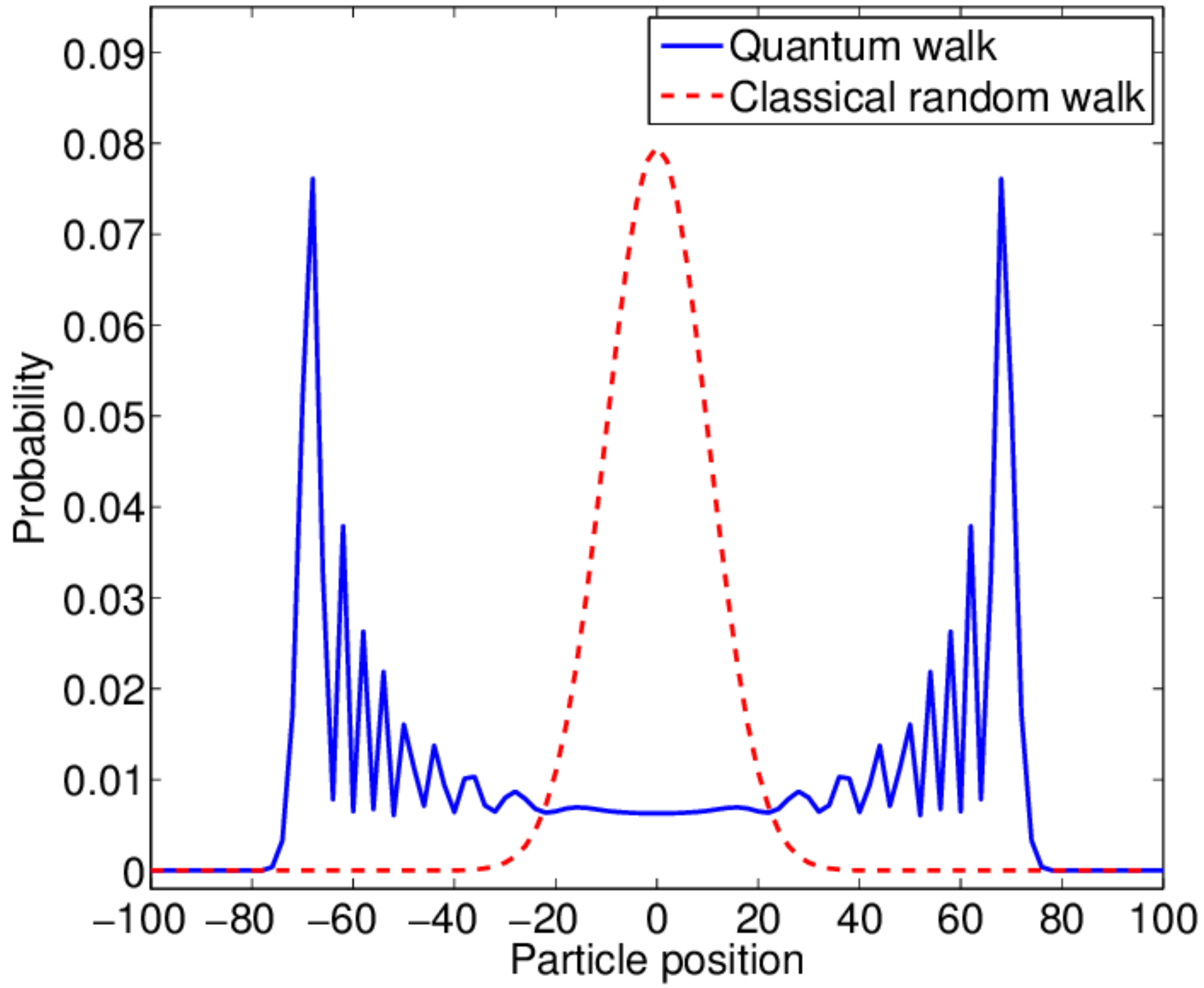
$$\frac{1}{2\sqrt{2}}\left(|\uparrow\rangle \otimes |3\rangle + |\downarrow\rangle \otimes |1\rangle + |\uparrow\rangle \otimes |-1\rangle - 2\,|\downarrow\rangle \otimes |-1\rangle - |\downarrow\rangle \otimes |-3\rangle\right)$$

	-3	-2	-1	0	1	2	3
0				1			
1			1/2		1/2		
2		1/4		1/2		1/4	
3	1/8		5/8		1/8		1/8

Peep at final position

Simulation results

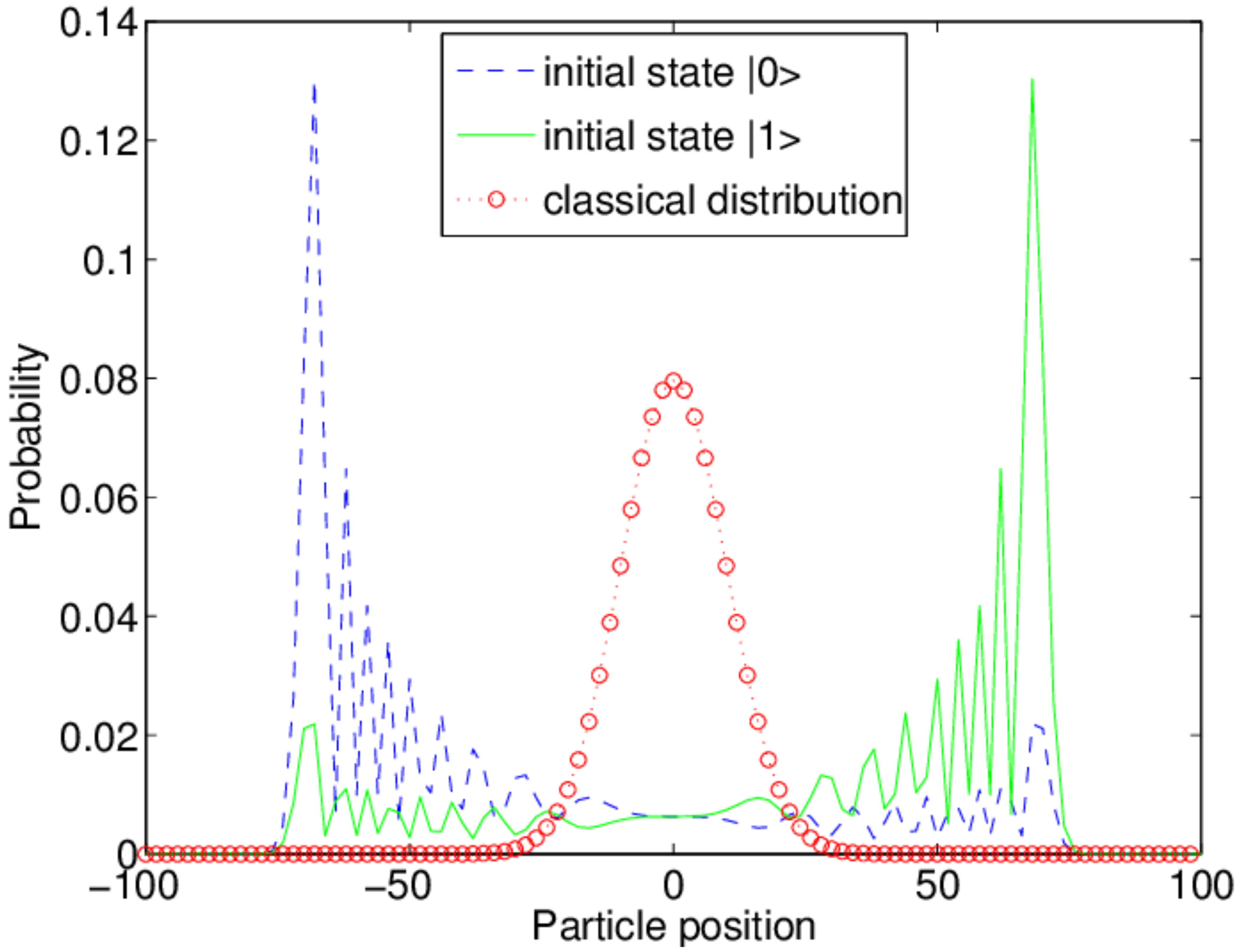
- What cause the difference?
  - path - wavefunction
  - Interference among paths
- Will the quantum walk reduces to the classical analogue? How?
  - measurement timing V.S. wavefunction collapse



Probability distribution of quantum and classical random walks.

Exercise: initial coin V.S. probability distribution

- Classical
  - $\sigma^2 \sim T$
- Quantum
  - $\sigma^2 \sim T^2$
  - quadratically speedup



Different input coin state leads to diverge position distribution.

Project: Randomly walk in Quantum and Classical

Can be 1D, 2D, or even 3D

- Total qubit number contains…
    - Vertex number - dimension of the position space
    - Coin - 2^d, d: dimension of the spatial space

- Discussion may include but not limited to…
    - Compare with the classical case. Plot the probability histogram of both cases.
        - Who travels faster? How to quantify the speed? Mean distance V.S. root-mean-square distance. Don’t forget to include the concept of crossing probabilities with respect to each final position.
        - Analyze the outcome of the probability distribution, what cause the asymmetric distribution if your coin is symmetric?
    - What if the related phase between two coin base state is not chosen properly? Discuss the mechanism behind that phase. Can we come out with the same probability distribution with different coin?
    - The two operators $\mathscr{C}$ and $\mathscr{S}$ are not commutable (can be explained bu the process of QRW). What if we switch the order of them? Please view the outcome probability and discuss about the meaning behind that.

Project reminder

- Default input: all zeros
    - NOT gate - 1
    - Hadamard gate - superposition
- Output format: $|q_n q_{n-1} \cdots q_1 q_0\rangle$
- Run experiment
    - Platform: simulator / real devices

- How many shots provides a reliable outcome?

- Multi-controlled multi-target gate

```
from qiskit.circuit.library import MCMT

MCMT(gate, num_ctrl_qubits, num_target_qubits, label=None)
```

Prerequisite with Quantum Lab

- Load IBMid

```
# import packages
from qiskit import IBMQ
from qiskit.tools.jupyter import *

# load IBM account if run via LAB
IBMQ.load_account()

# internal account
provider = IBMQ.get_provider(hub='ibm-q', group='open',
project='main')
```

- Some useful packages

```
# execution
from qiskit import Aer, BasicAer, execute
from qiskit.tools.monitor import job_monitor
# quantum gates, circuits
from qiskit import QuantumCircuit, QuantumRegister,
ClassicalRegister
from qiskit.providers.aer import QasmSimulator

# math calculation
import numpy as np

# plotting
import matplotlib.pyplot as plt
%matplotlib inline

# view qiskit version
import qiskit.tools.jupyter
from qiskit.visualization import plot_histogram, plot_state_city
%qiskit_version_table
```

- Backend setting

```
backend = provider.get_backend('ibmq_armonk')
simulator = Aer.get_backend('qasm_simulator')
```