Variational Quantum Eigensolver

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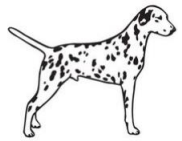
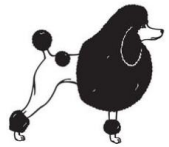
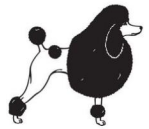
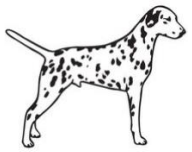
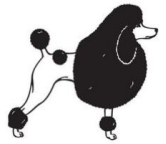
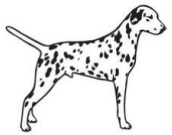
Outline

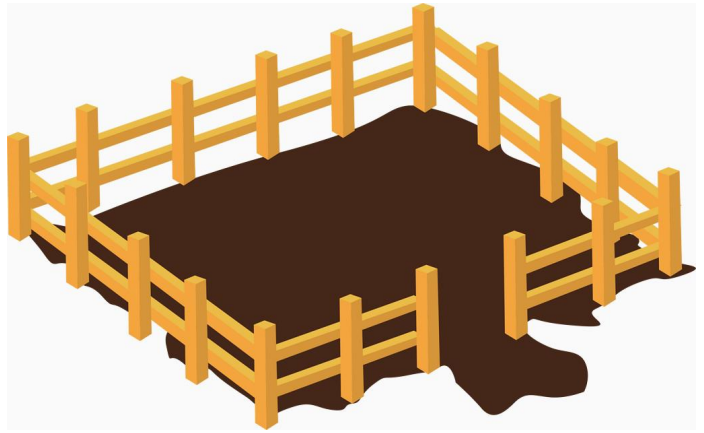
1. 介紹問題(Max-cut)
2. Encode
3. VQE/QAOA
4. Example

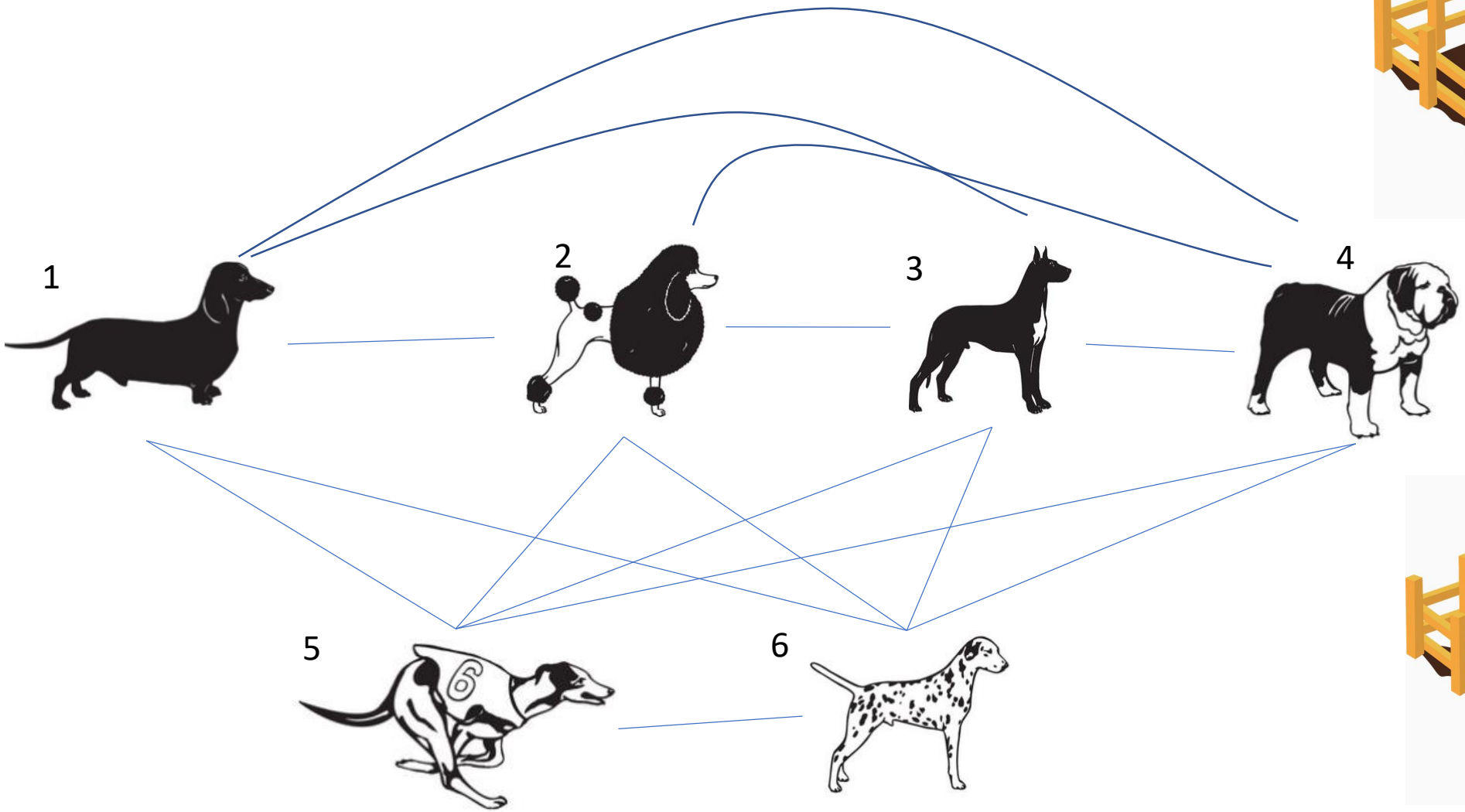
Problem

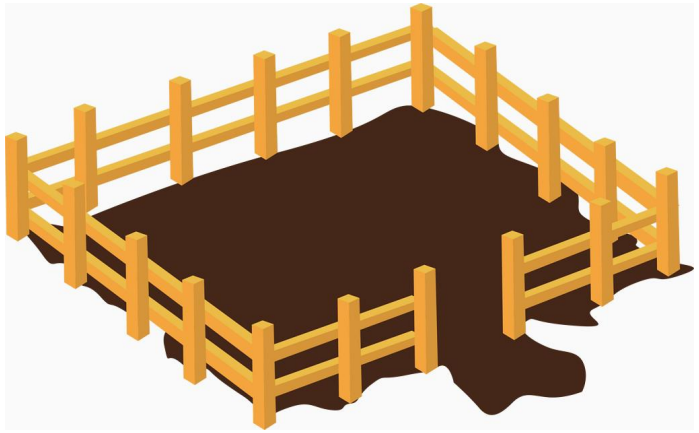
最大切割問題(Max-cut)

		仇恨值			仇恨值
		0.6			0.76
		0.8			0.7
		0.2			0.85
		0.9			

最大切割問題(Max-cut)

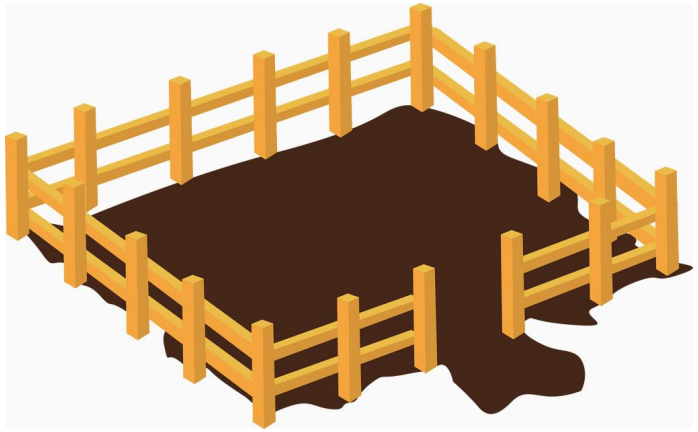






Encode

0



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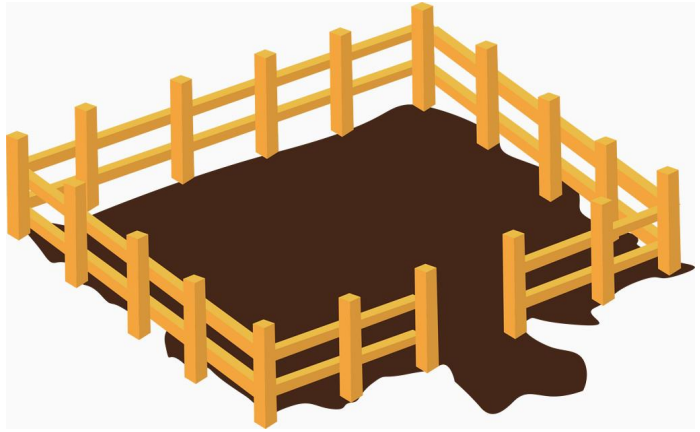
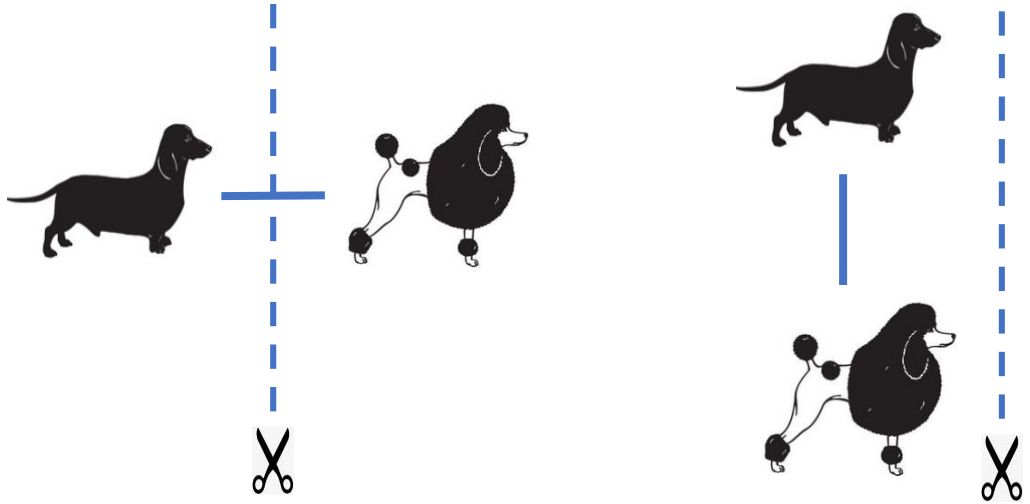
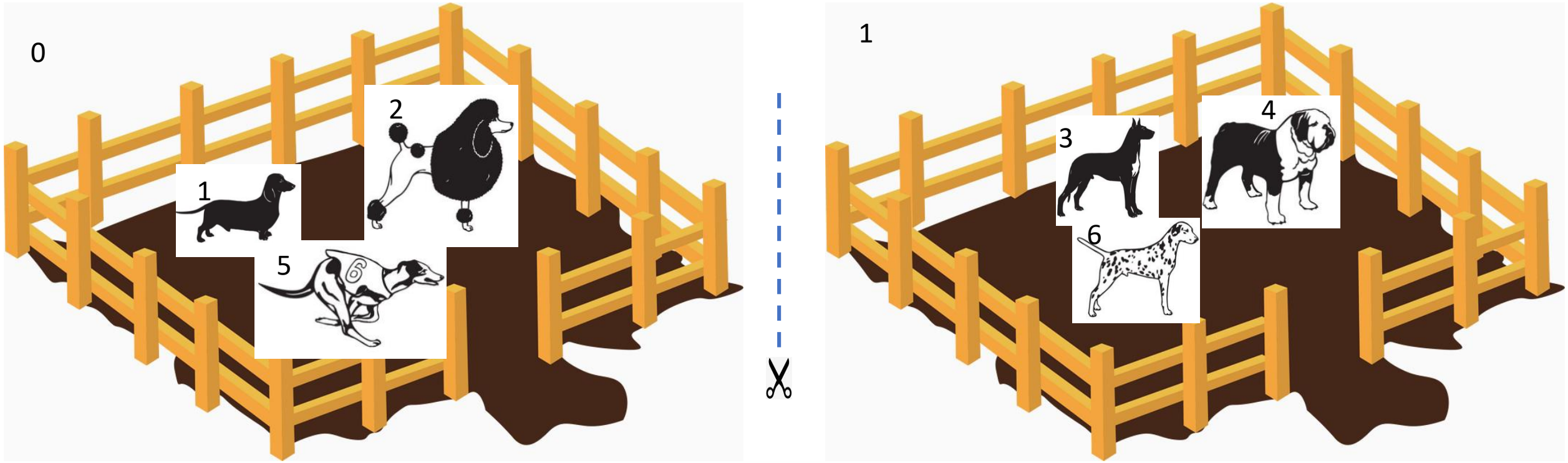




$$\frac{1}{2}(I - \sigma_z^1 \otimes \sigma_z^0) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{matrix}$$

Target function:

$$H = \sum_{i,j} w_{i,j} \sigma_z^i \sigma_z^j$$



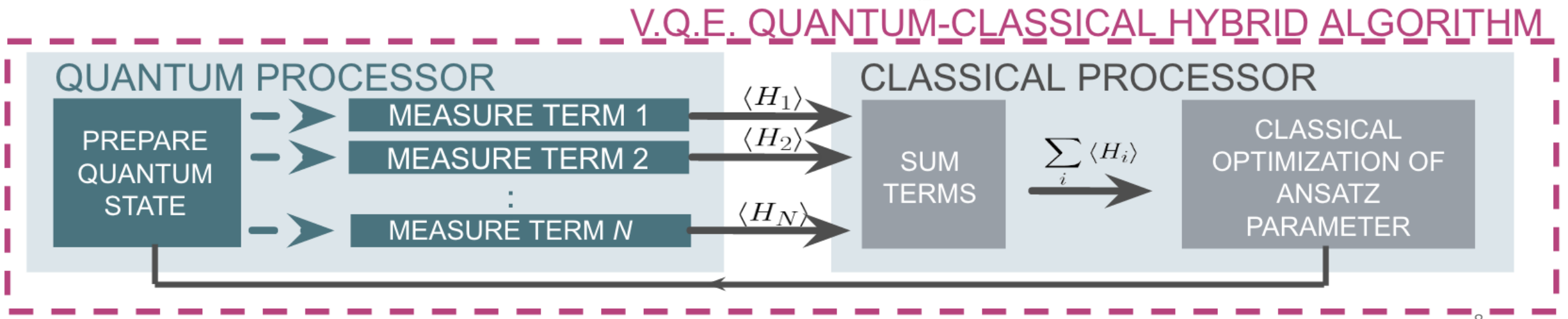
$$|\psi\rangle = |q_1, q_2, q_3, q_4, q_5, q_6\rangle$$
$$= |0, 0, 1, 1, 0, 1\rangle$$

$\min_\theta \langle\psi(\theta)|H|\psi(\theta)\rangle$ → $|\psi(\theta)\rangle$ is your solution!!

VQE:1234

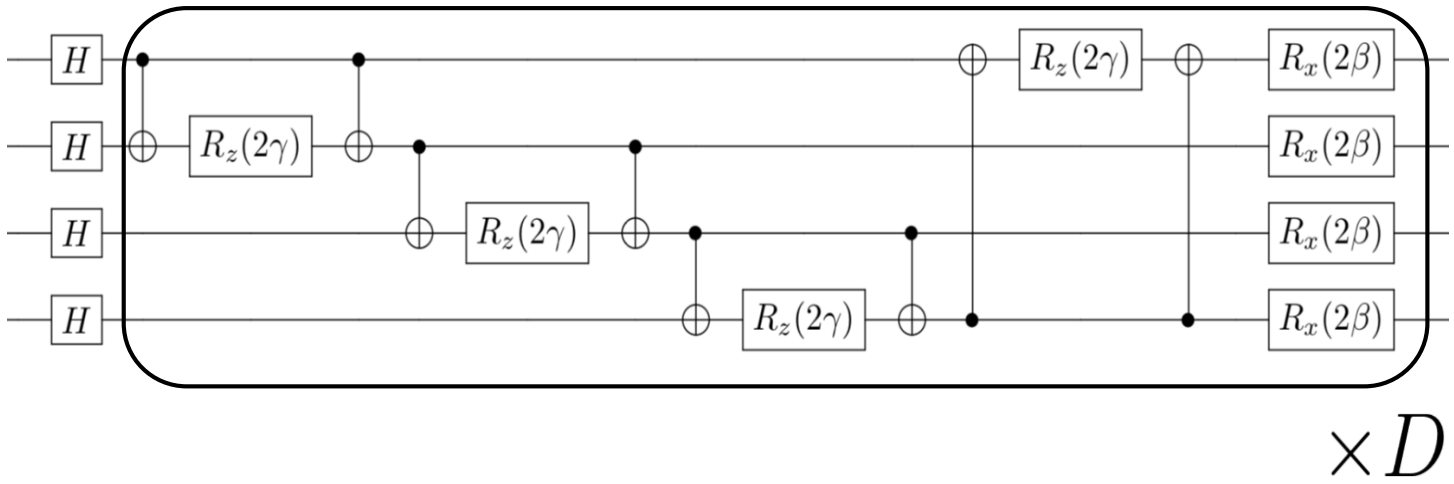
- One core concept: Variational principle
- Two subroutine: Quantum and Classical
- Three steps: Ansatz preparation, measure expectation and optimization
- And…….. For Success!

$$H = \sum_{i,j} w_{i,j} \sigma_z^i \sigma_z^j \qquad \min_\theta \langle \psi(\theta) | H | \psi(\theta) \rangle$$

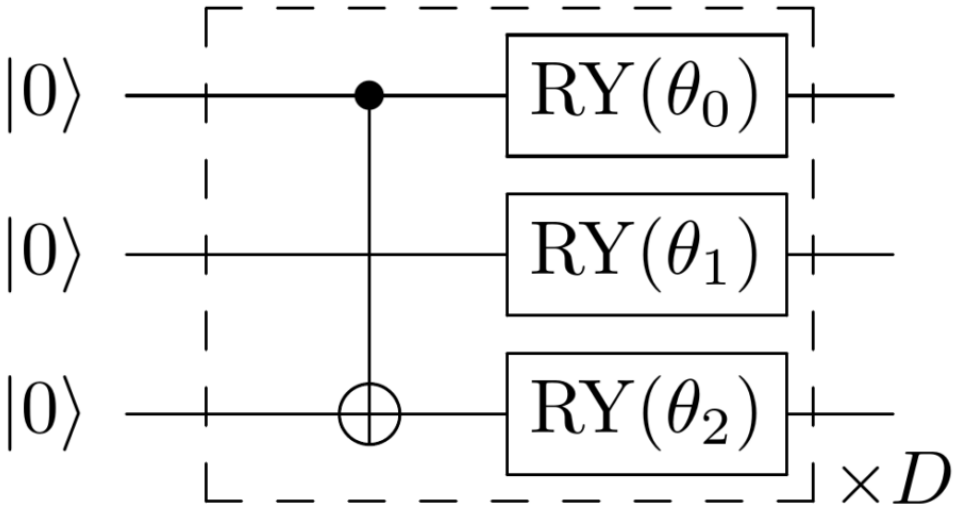


Prepare quantum state: QAOA/Ry ansatzs

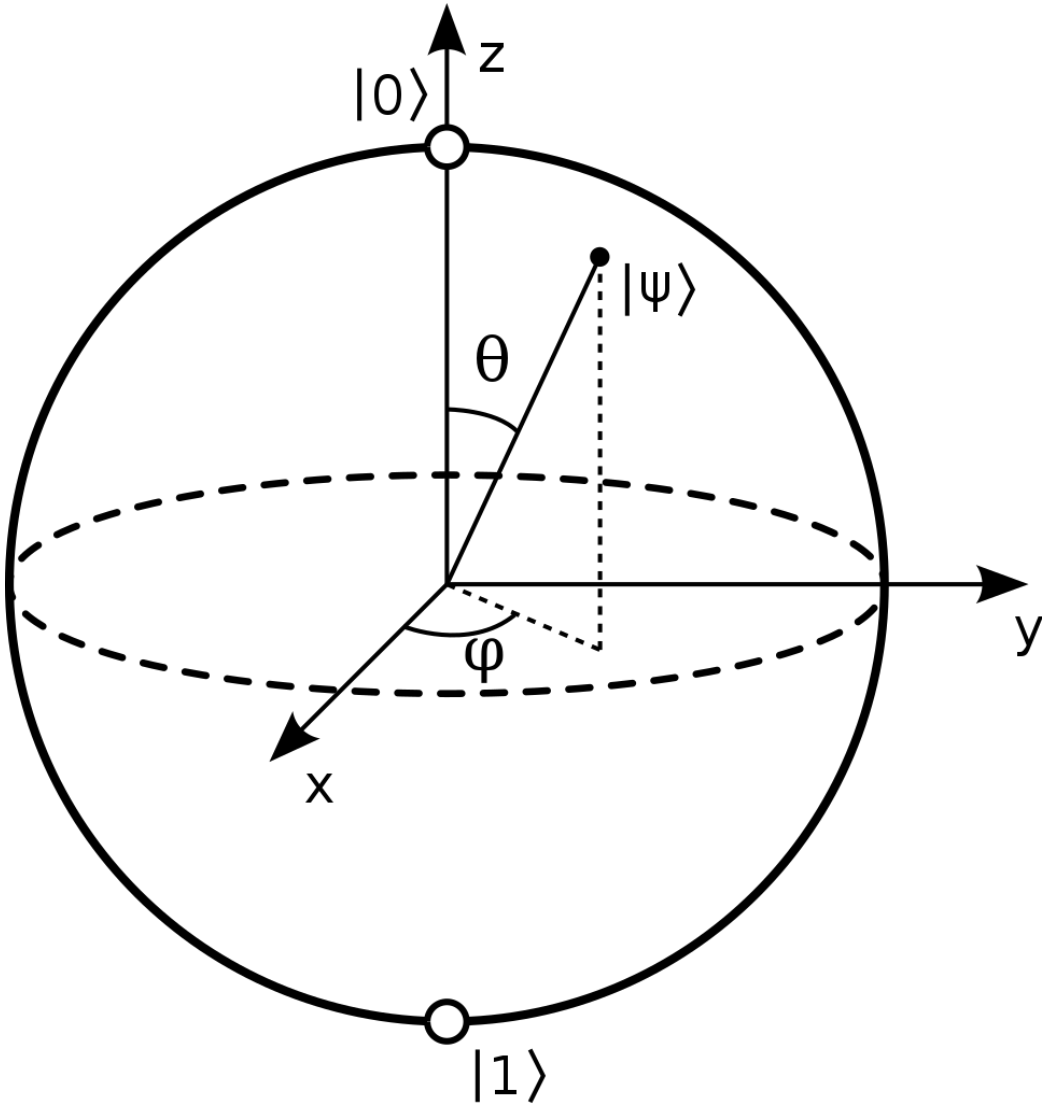
QAOA:



Ry:



Quantum measurement



$$\langle\psi|Z|\psi\rangle = \mathbb{P}_0 - \mathbb{P}_1$$

$$\langle\psi|X|\psi\rangle = \langle\psi'|Z|\psi'\rangle$$

$$|\psi'\rangle = R_y(-\pi/2)|\psi\rangle$$

$$\langle\psi|Y|\psi\rangle = \langle\psi'|Z|\psi'\rangle$$

$$R_x(\pi/2)|\psi\rangle = |\psi'\rangle$$

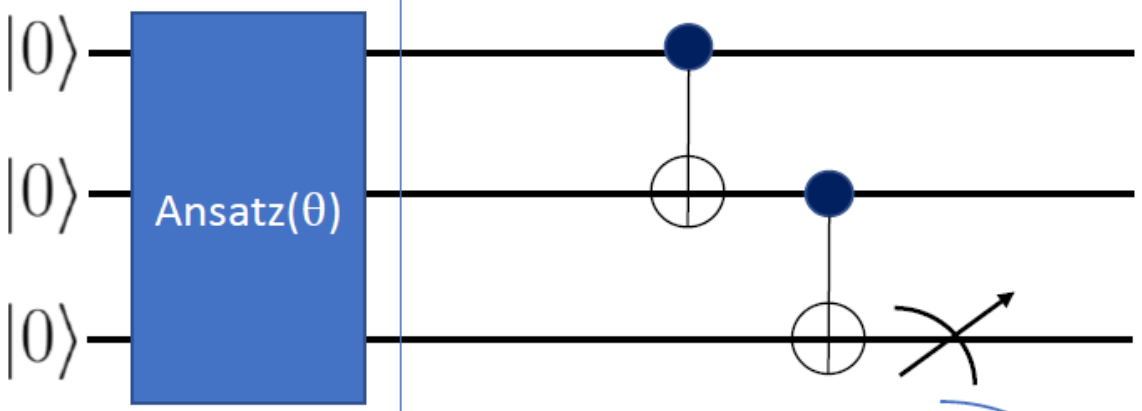
Quantum measurement

$$|\psi\rangle = a|000\rangle + b|001\rangle + c|010\rangle + d|011\rangle + f|100\rangle + g|101\rangle + h|110\rangle + h|111\rangle$$

$$\langle\psi|ZZZ|\psi\rangle$$

$$= \underbrace{|a|^2 + |d|^2 + |g|^2 + |h|^2}_{\mathbb{P}_0} - \underbrace{|b|^2 - |c|^2 - |f|^2 - |h|^2}_{\mathbb{P}_1}$$

Generate a state by ansatz.



sum of the bit string is even

Eigenvalue=1

$a|000\rangle$

$d|011\rangle$

$g|101\rangle$

$h|110\rangle$

sum of the bit string is odd

Eigenvalue=-1

$b|001\rangle$

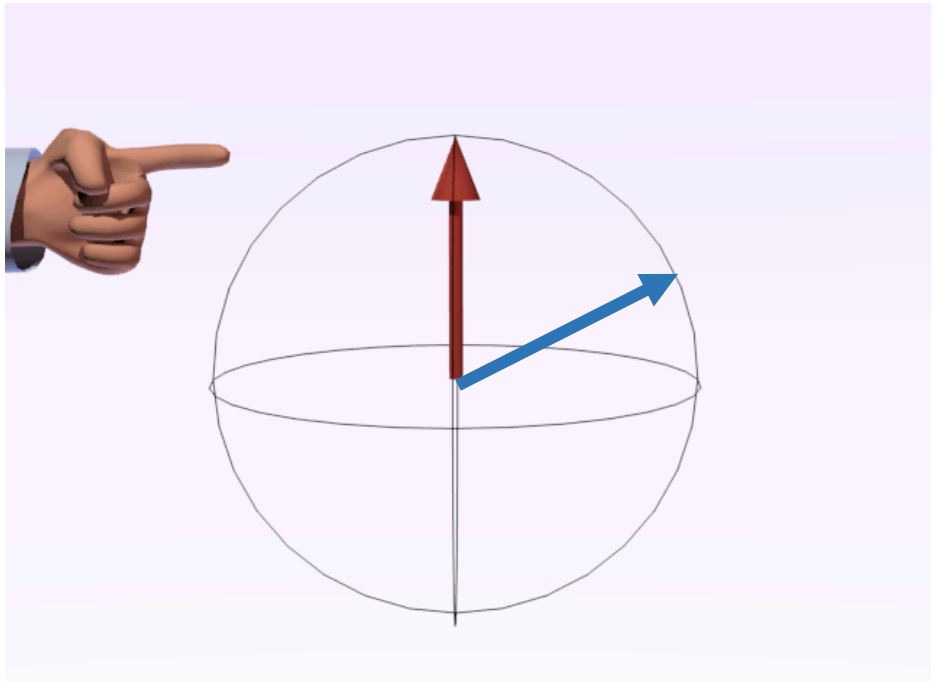
$c|010\rangle$

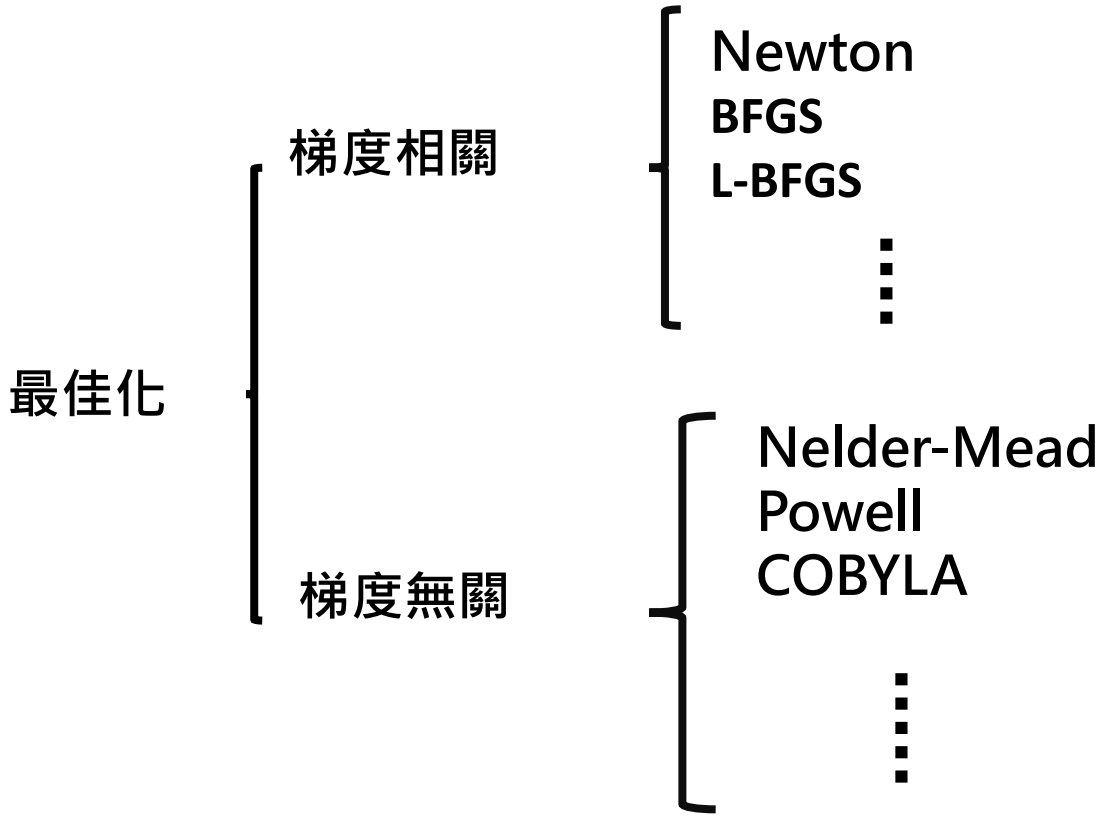
$f|100\rangle$

$h|111\rangle$

$$\langle ZZZ\rangle = \mathbb{P}_0 - \mathbb{P}_1$$

Optimization





Application

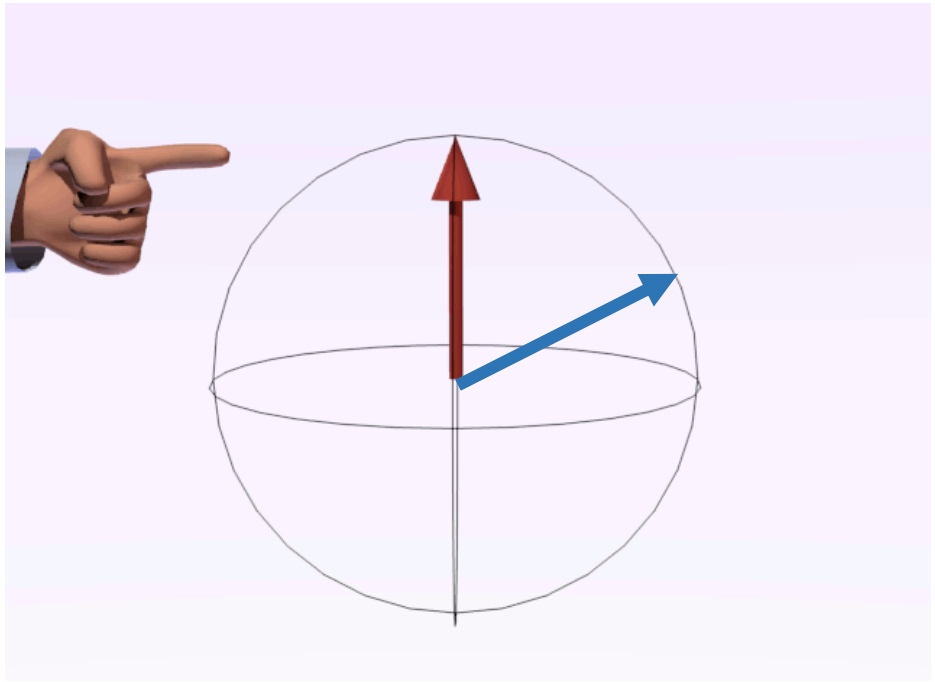
- quantum chemistry
- portfolio optimization
- support vector machine
- Recurrent Neural Network(QNN)
- traveling salesman problem

QUBO

Ising model

$$H = \sum_{i,j} w_{i,j} \sigma_z^i \sigma_z^j$$

Example:

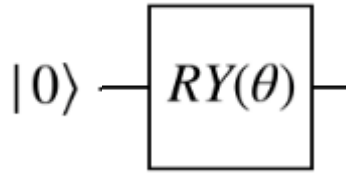


Problem:

$$M = \begin{bmatrix} -0.2524859 & 0.18121 \\ 0.18121 & -1.8318639 \end{bmatrix}$$
$$= -1.0421749I + 0.789689Z + 0.181210X$$

Trail wavefunction:

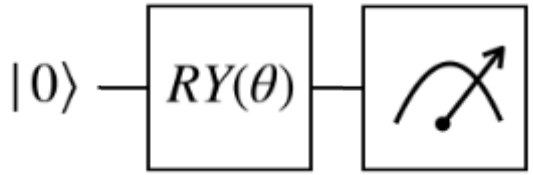
$$|\psi(\theta)\rangle = R_y(\theta)|0\rangle = \cos(\theta/2)|0\rangle + \sin(\theta/2)|1\rangle = \begin{bmatrix} \cos(\frac{\theta}{2}) \\ \sin(\frac{\theta}{2}) \end{bmatrix}$$



Measurement & optimization

$$M_0 = \min_\theta \langle 0|R_y(\theta)^\dagger M R_y(\theta)|0\rangle = \min_\theta M(\theta)$$

<Z>



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