Quantum Random Walks

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Disney·PIXAR

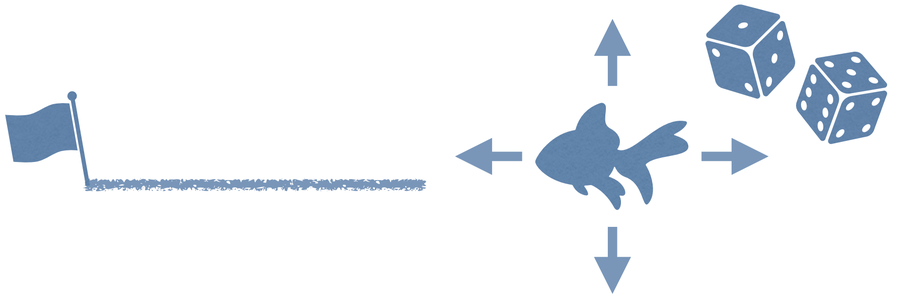
Outline

- Introduction
- Theory & Simulations
  - Standard
  - Lackadaisical
- Walks on Real Quantum Computers
- Summary

Introduction

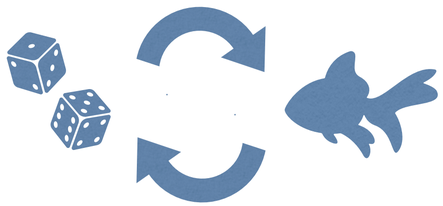
Motivation

- Dory is finding home……
- Classical methodology
  - Poor memory ⟶ random walks
  - Simplify case ⟶ 1-dimension path

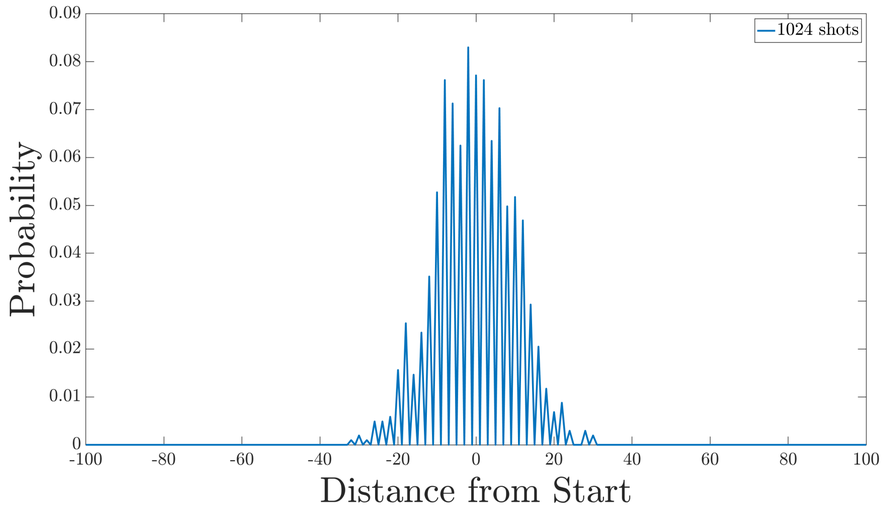


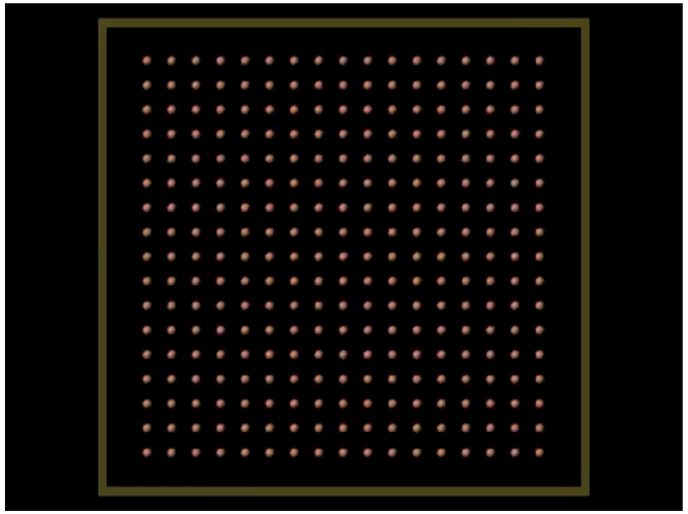
Classical random walks

- Scheme



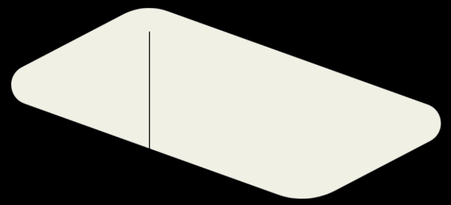
- Simulations





I am sorry about Dory……

WHO can help?



Quantum.

What can we do with quantum ?

- Markov process (memoryless)
- Make decisions randomly
- Superposition of choices

Theory & Simulation

System setup

- States: coin + position

$$|c\rangle \otimes |k\rangle$$

- Dimension
  - coin: 2 for 1-D path (standard)
  - position: dependes on #nodes

- Operators
  - Coin → entangles two choices $|\uparrow\rangle$, $|\downarrow\rangle$
    1. Hadamard coin
    2. Grover coin
    3. General coin
  - Shift → moves according to Coin

$$\hat{S} = |n+1\rangle\langle n| \otimes |\uparrow\rangle\langle\uparrow| + |n-1\rangle\langle n| \otimes |\downarrow\rangle\langle\downarrow|$$

Standard quantum walks

- Move step by step

coin: Hadamard coin

shift: $|\uparrow\rangle \Longrightarrow |n+1\rangle$, $|\downarrow\rangle \Longrightarrow |n-1\rangle$

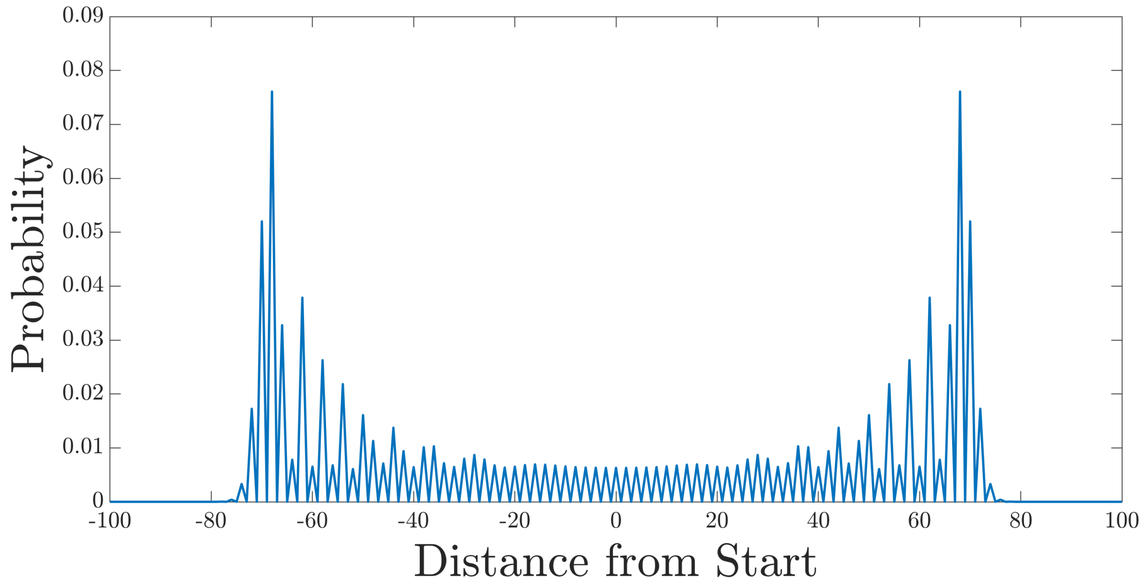
$$\frac{1}{\sqrt{2}}\left(|\uparrow\rangle\otimes|1\rangle - |\downarrow\rangle\otimes|-1\rangle\right)$$

$$\frac{1}{2}\left[|\uparrow\rangle\otimes|2\rangle - \left(|\uparrow\rangle - |\downarrow\rangle\right)\otimes|0\rangle + |\downarrow\rangle\otimes|-2\rangle\right]$$

$$\frac{1}{2\sqrt{2}}\left(|\uparrow\rangle\otimes|3\rangle + |\downarrow\rangle\otimes|1\rangle + |\uparrow\rangle\otimes|-1\rangle - 2|\downarrow\rangle\otimes|-1\rangle - |\downarrow\rangle\otimes|-3\rangle\right)$$

	-3	-2	-1	0	1	2	3
0				1			
1			1/2		1/2		
2		1/4		1/2		1/4	
3	1/8		5/8		1/8		1/8

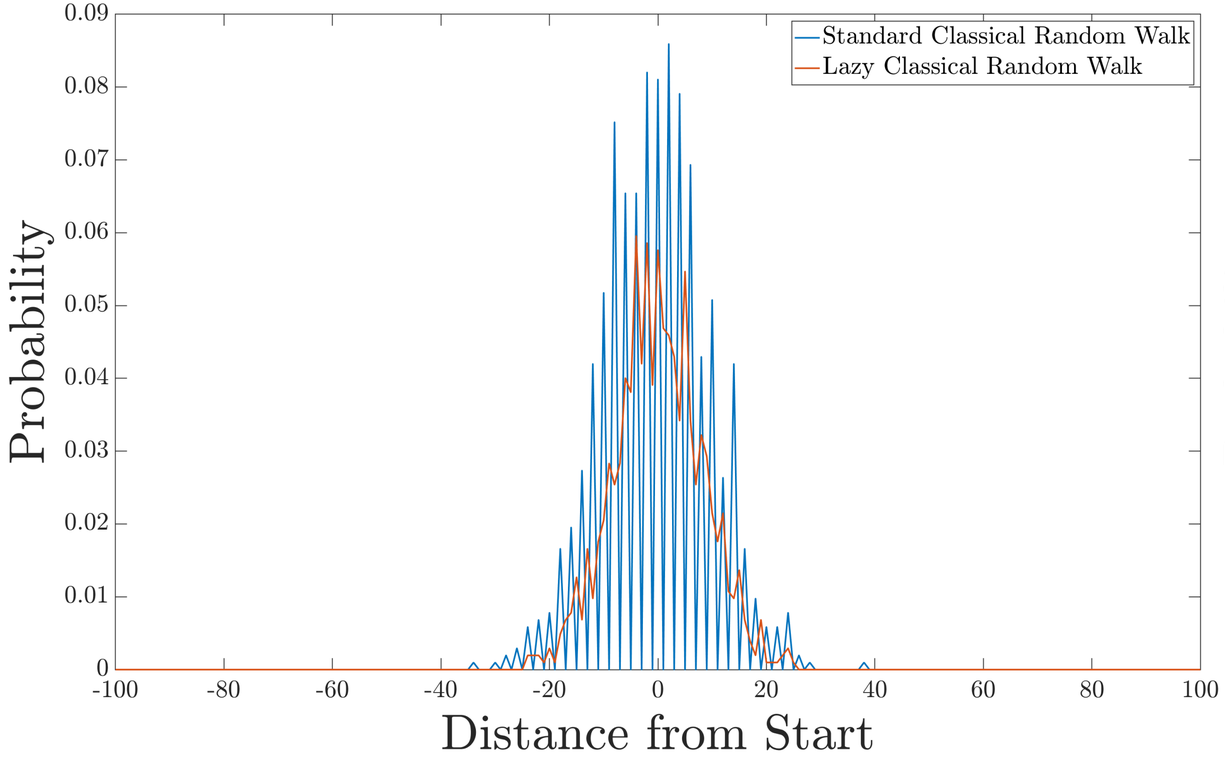
- Hadamard coin $\hat{C} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

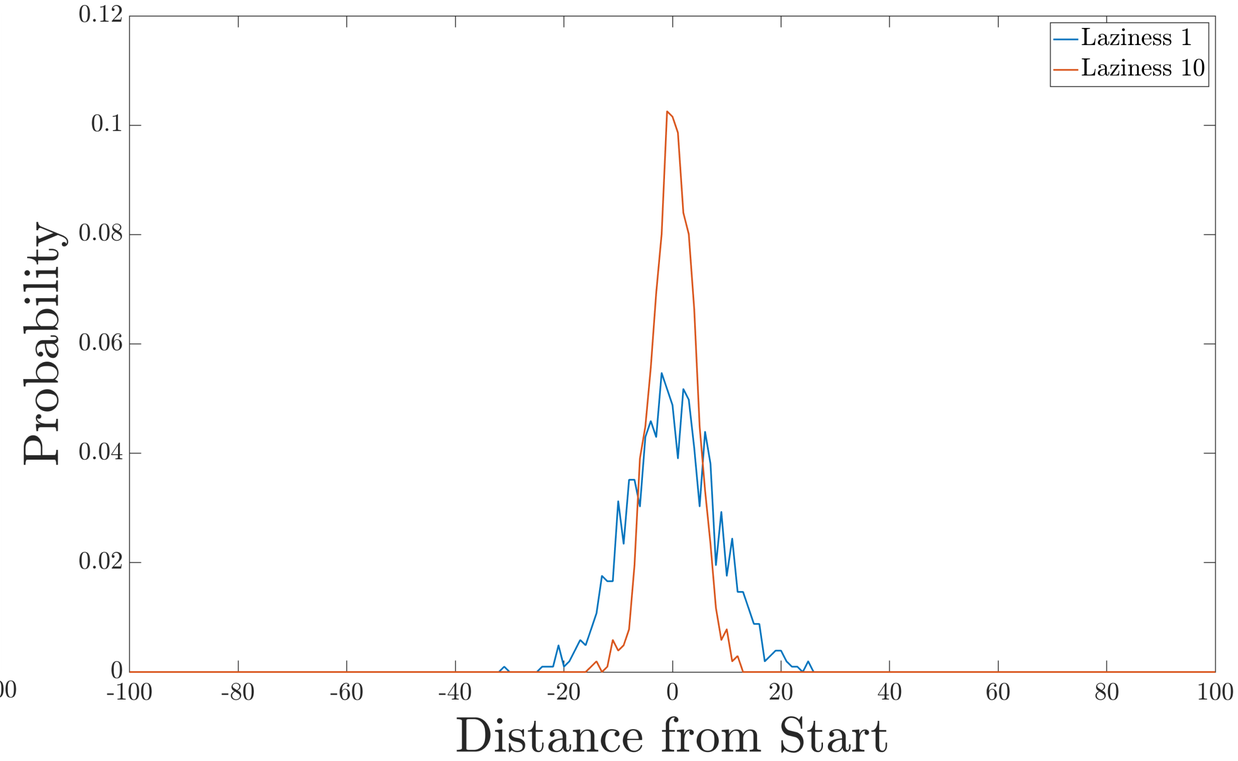


general coin $\hat{C} = \begin{pmatrix} \sqrt{\rho} & \sqrt{1-\rho}\,e^{i\theta} \\ \sqrt{1-\rho}\,e^{i\phi} & -\sqrt{\rho}\,e^{i(\theta+\phi)} \end{pmatrix}$


Rest during traveling……

Lazy Dory


Standard Classical Random Walk
Lazy Classical Random Walk
Probability
Distance from Start
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
-100
-80
-60
-40
-20
0
20
40
60
80
100

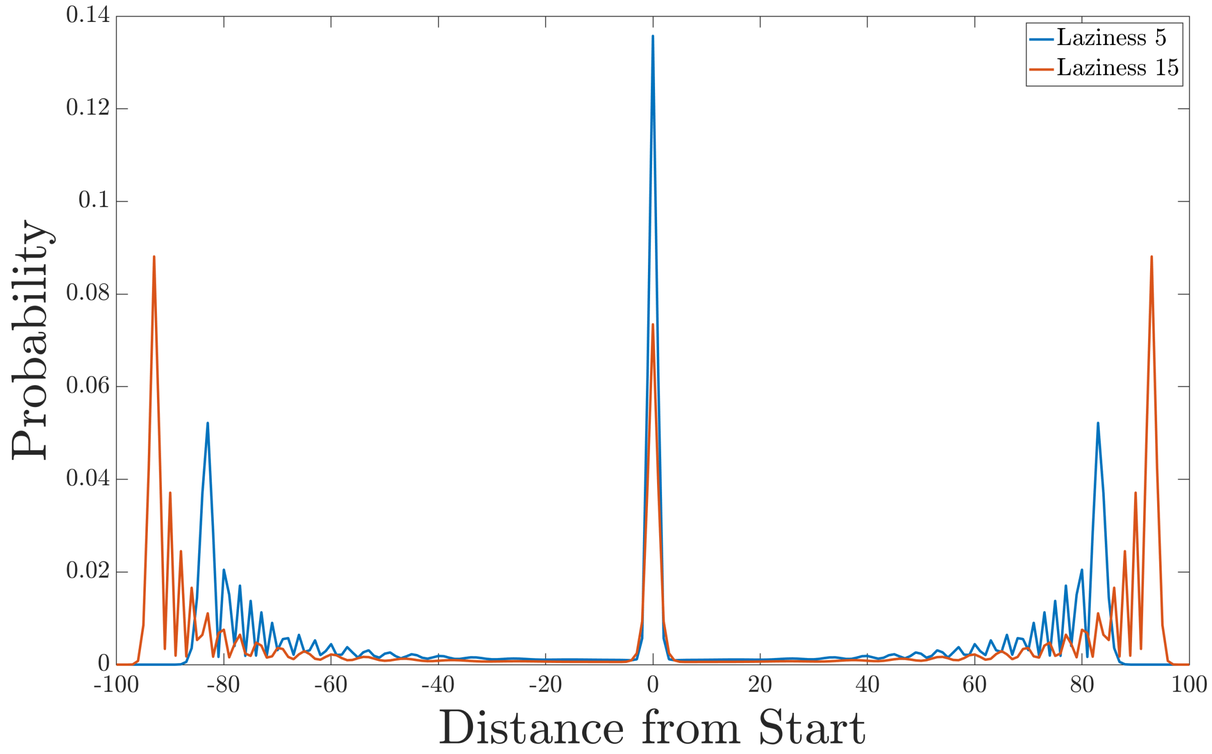

Laziness 1
Laziness 10
Probability
Distance from Start
0.12
0.1
0.08
0.06
0.04
0.02
0
-100
-80
-60
-40
-20
0
20
40
60
80
100

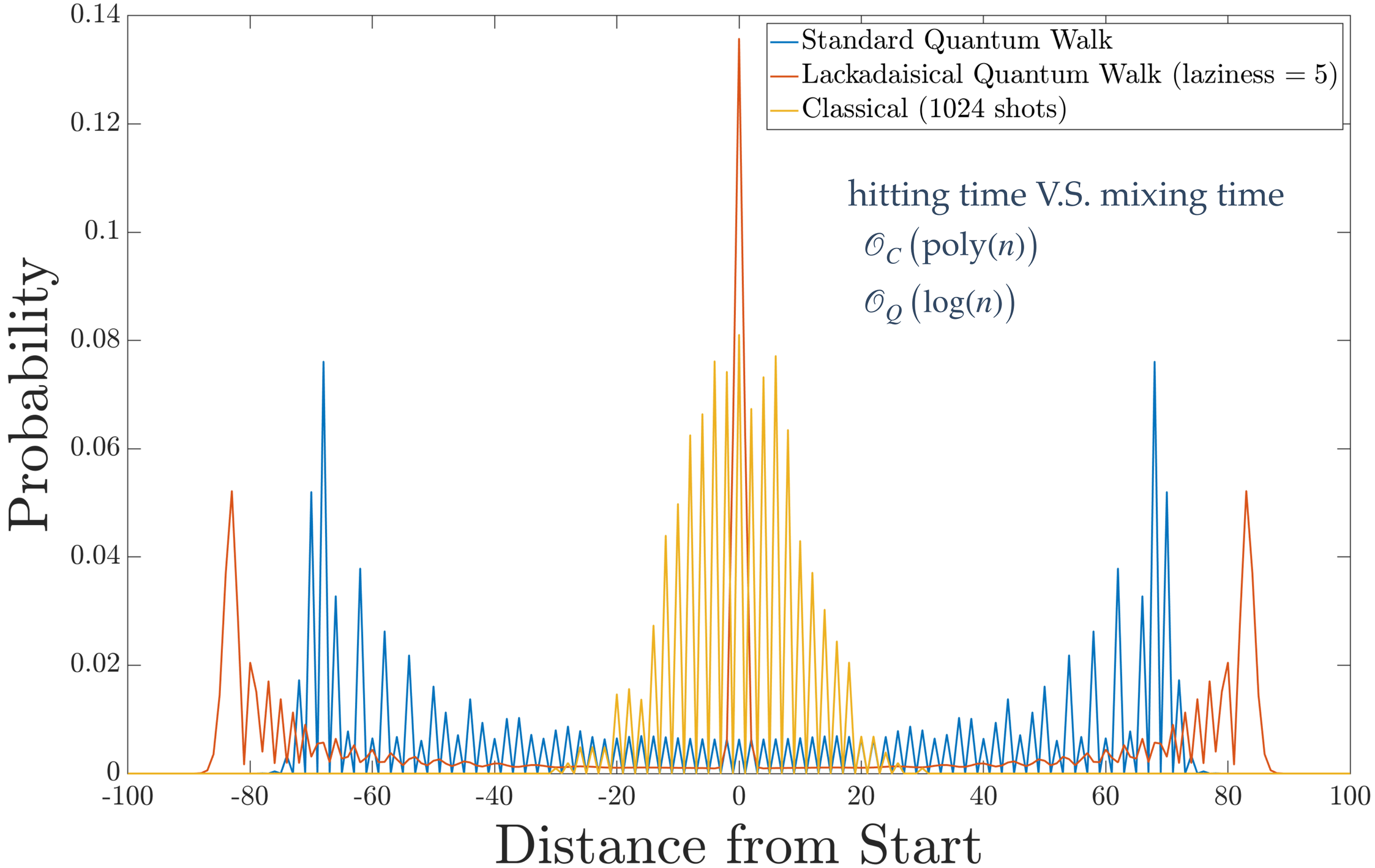
Lazy Dory is quantum……

- Lackadaisical quantum walks
  - Laziness τ

$$\hat{S} = |n-1\rangle\langle n| \otimes |1\rangle\langle 1| + |n+1\rangle\langle n| \otimes |2\rangle\langle 2| + \sum_{j=3}^{\tau+2} |n\rangle\langle n| \otimes |j\rangle\langle j|$$

$$\hat{C} = \frac{1}{\tau+2}\begin{pmatrix} -\tau & 2 & \cdots & 2 \\ 2 & -\tau & \cdots & 2 \\ \vdots & \cdots & \vdots & \vdots \\ 2 & 2 & \cdots & -\tau \end{pmatrix}$$




Standard Quantum Walk
Lackadaisical Quantum Walk (laziness = 5)
Classical (1024 shots)
hitting time V.S. mixing time
$\mathcal{O}_C\left(\text{poly}(n)\right)$
$\mathcal{O}_Q\left(\log(n)\right)$
Probability
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
-100
-80
-60
-40
-20
0
20
40
60
80
100
Distance from Start

Walks on Real Quantum Computers

Quantum Programming

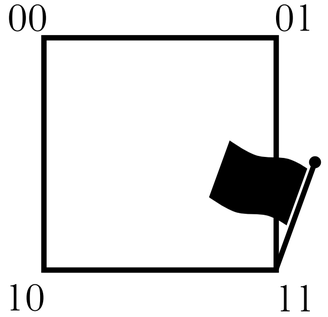
Prerequisite

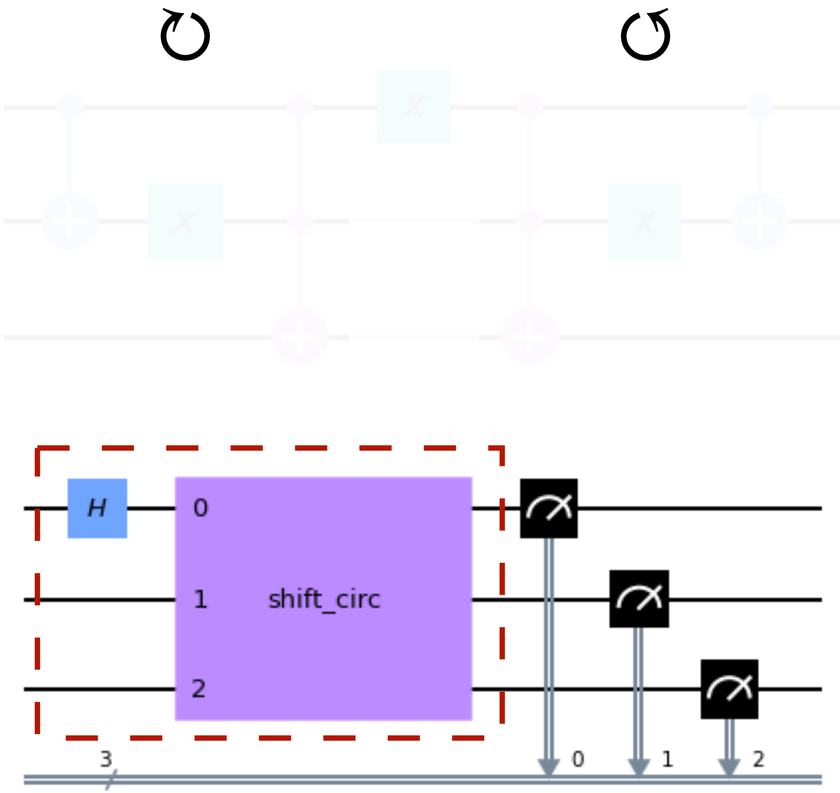
- States: coin + position

$$|c\rangle \otimes |k\rangle$$

- Operations
  - Coin: $\hat{C}\,|k,q\rangle = \left(\hat{\mathbb{I}} \otimes \hat{H}\right)|k,q\rangle$
  - Shift: $\hat{S}\,|k,q\rangle = |k+(-1)^q, q \oplus 1\rangle$

$$\Big\} \hat{U} = \left(\hat{S}\hat{C}\right)^T$$





Become a real programmer

- Arbitrary coin ?
- More steps, nodes ?

- Single-qubit gate
  - XYZ rotation X Y Z

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

  - General unitary gate U1 U2 U3

flip bit bit & phase phase

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle = \begin{pmatrix} \cos\frac{\theta}{2} \\ e^{i\phi}\sin\frac{\theta}{2} \end{pmatrix}$$

- Position-dependent coin

$$|\psi\rangle = U|\psi_0\rangle$$

$$U = \begin{pmatrix} \cos n\theta & \sin n\theta \\ \sin n\theta & -\cos n\theta \end{pmatrix}$$

$$U = \begin{pmatrix} \cos\frac{\theta}{2} & -e^{i\lambda}\sin\frac{\theta}{2} \\ e^{i\lambda}\sin\frac{\theta}{2} & e^{i(\lambda+\phi)}\cos\frac{\theta}{2} \end{pmatrix} \Longrightarrow U(\theta, \phi, \lambda)$$

Become a real programmer

- Arbitrary coin ?
- More steps, nodes ?

- Textbook: https://qiskit.org/textbook/preface.html
- Document: https://qiskit.org/documentation/index.html

Next step……

- *N*-node cycle ~ lone line path
  - Shift operator
- Two- or three- dimensional lattice
  - #Coin operators
- Type of random walks

- 140.112.102.245:9529

Summary

Quantum random walks

- **Reach the edges**
  quickly touch the edges of space
  speeds up the searching processes

- **Potential applications**
  biology, algorithms, universal computation, analysis, etc.

Questions ?