APR. 27, 2020
Quantum Computing Workshop @NTU PHYS.

MAY 01, 2020
Seminar @NTHU EE.

STEP INTO QUANTUM

YUN-CHIH LIAO
IBM Quantum Computer Hub at NTU

Outline

- History
- Basic Tools
- From Classical to Quantum
- Quantum Language
  - Your First Quantum Circuit
- Quantum Computers

History

1980

Paul Benioff

Proposes quantum Turing machine model

1985

Paul Deutsch

Theoretical universal quantum computer

1993

Dan Simon

Quantum computer would be exponentially faster than conventional computer

1994

Peter Shor

Shor 's algorithm

1995

NIST & CalTech

Cools and traps ion to a quantum state

1997

MIT

First quantum computer (based on NMR)

2001

Emanuel Knill, Raymond Laflamme & Gerard Milburn

Shows that optical quantum computing is possible

2009

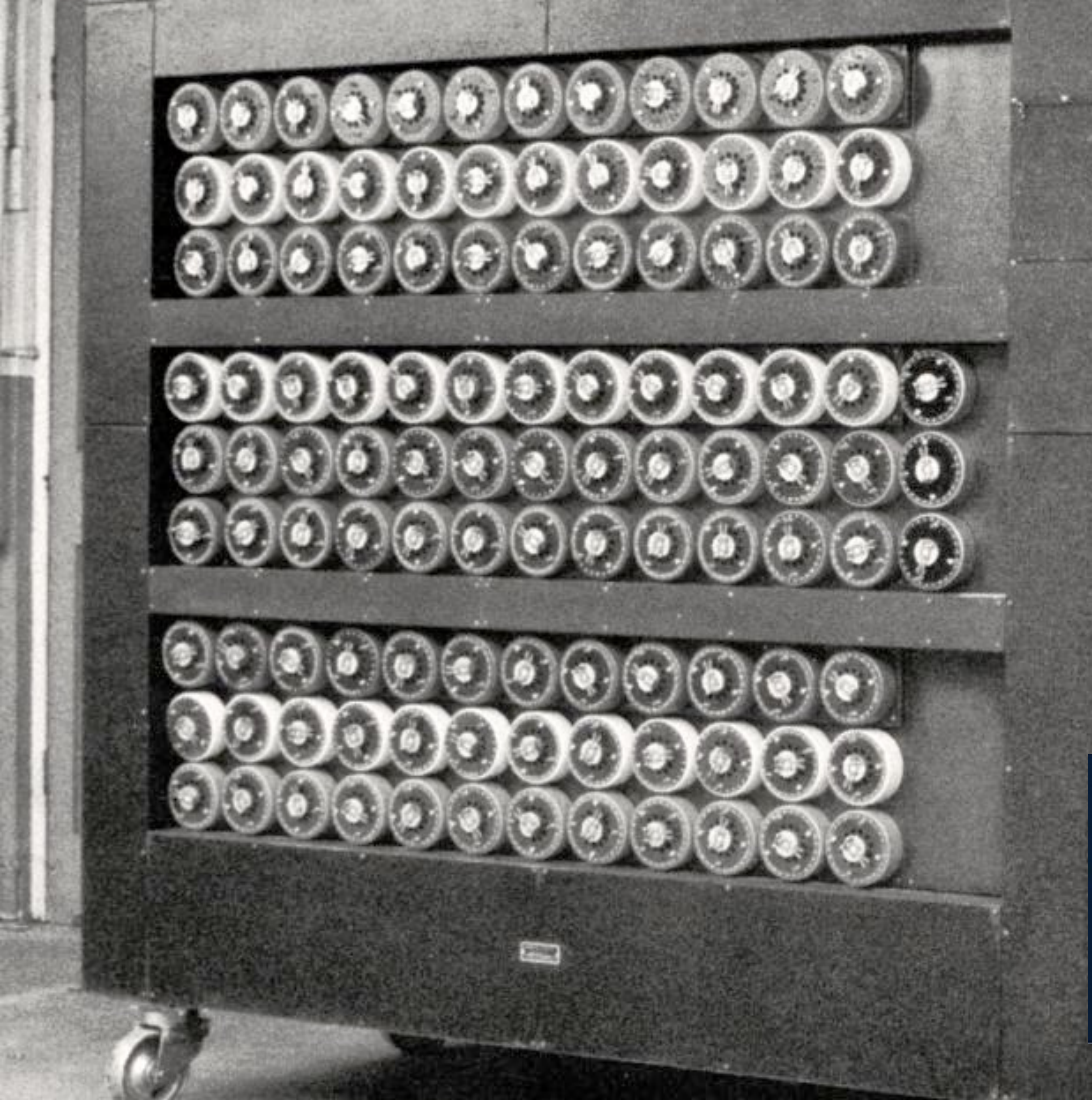
Yale University

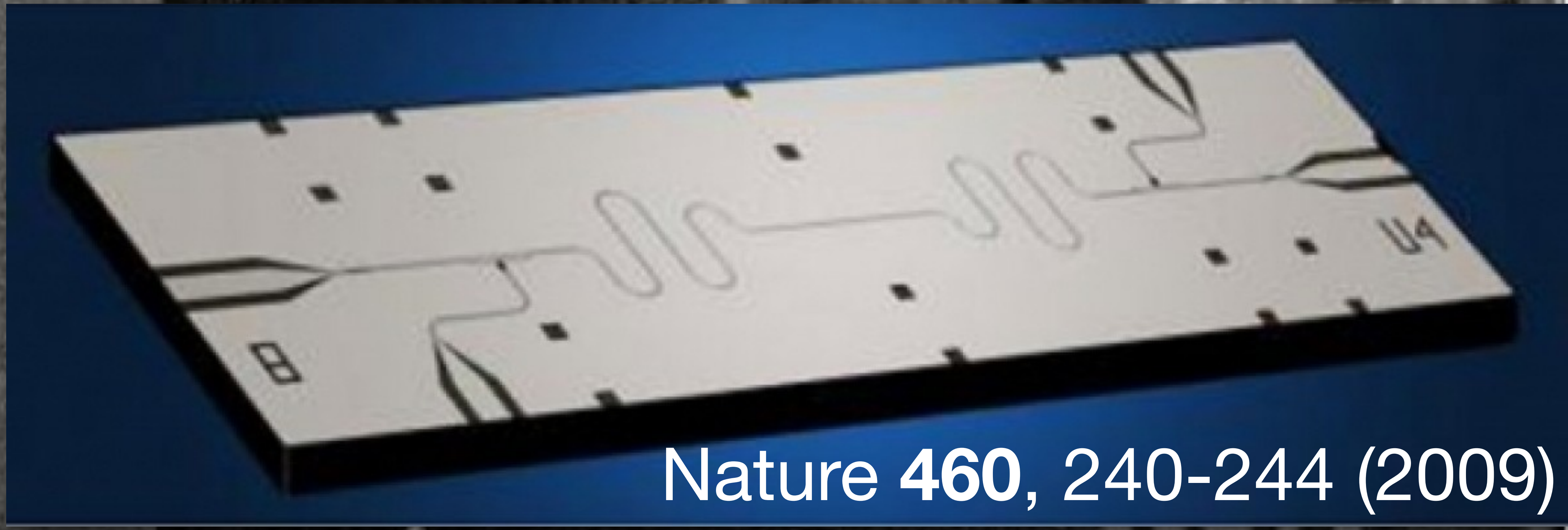
First electronic quantum processor

2016

IBM

Releases Quantum Experience





Nature **460**, 240-244 (2009)

Produced by Alan Turing at 1939

Basic Tools

It’s all about matrices!

Notations & Operations

- $|\psi\rangle$: ket-vector $\quad |\psi\rangle = \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}$
- $\langle\psi|$: bra-vector $\quad \langle\psi| = \begin{bmatrix} \alpha* & \beta* & \gamma* \end{bmatrix}$
- $|\psi_1\rangle\,|\psi_2\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$: tensor product
- $U^\dagger = \left(U^T\right)^* = (U*)^T$: Hermitian conjugate
- $\langle\psi|\,U\,|\psi\rangle = \langle\psi|\,U^\dagger\,|\psi\rangle \equiv \langle U\rangle$: expectation value

$$= \begin{bmatrix} \alpha* & \beta* & \gamma* \end{bmatrix} \begin{bmatrix} a & b & c \\ d & f & g \\ h & j & k \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}$$

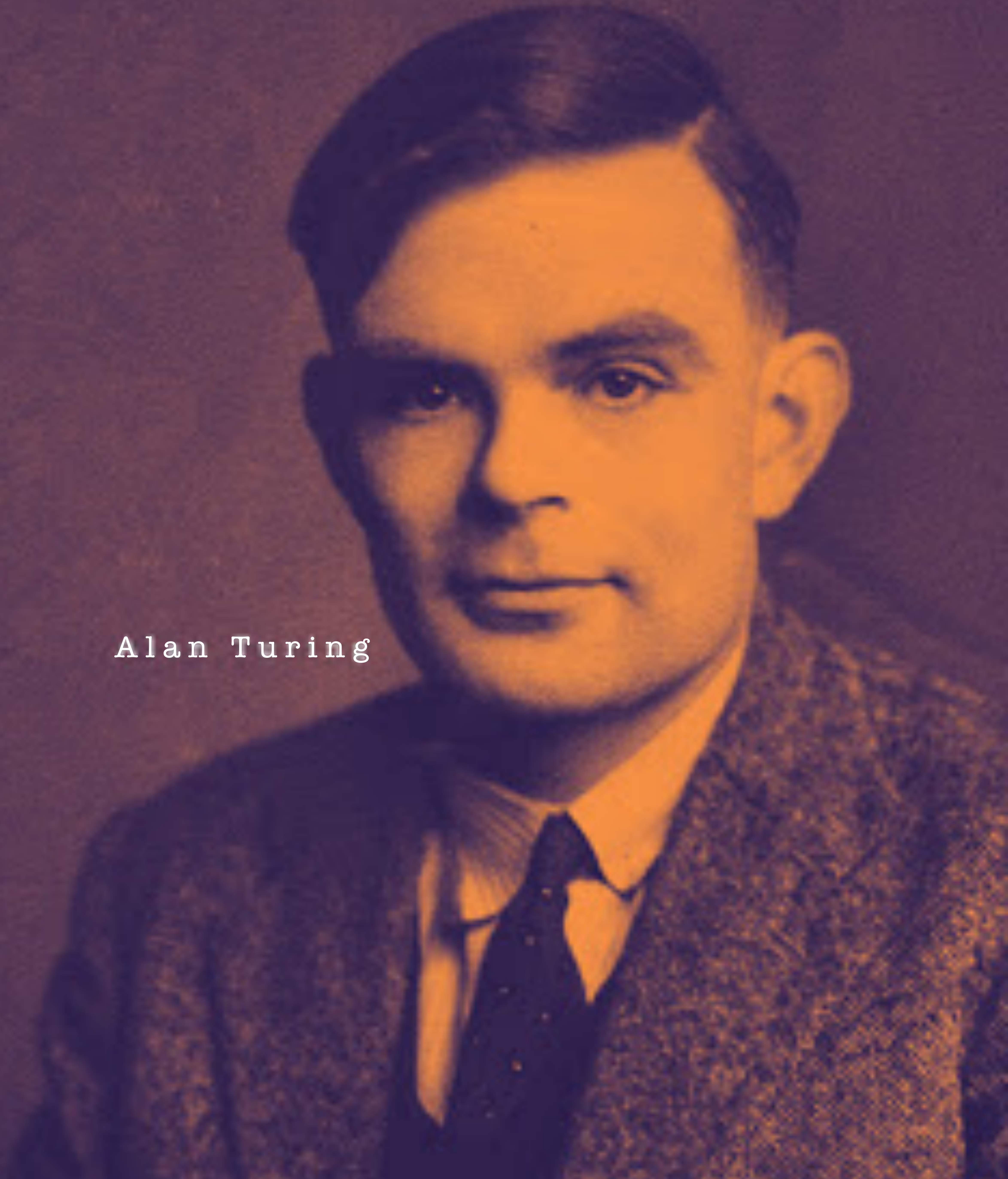
$$|\psi_1\rangle = \begin{bmatrix} 1 & -1 \\ 0 & i \\ -1+i & 1-i \end{bmatrix} \qquad |\psi_2\rangle = \begin{bmatrix} i \\ -1 \end{bmatrix}$$

$$|\psi_1\rangle\,|\psi_2\rangle = \begin{bmatrix} 1 \cdot \begin{bmatrix} i \\ -1 \end{bmatrix} & -1 \cdot \begin{bmatrix} i \\ -1 \end{bmatrix} \\ 0 \cdot \begin{bmatrix} i \\ -1 \end{bmatrix} & i \cdot \begin{bmatrix} i \\ -1 \end{bmatrix} \\ (-1+i) \cdot \begin{bmatrix} i \\ -1 \end{bmatrix} & (1-i) \cdot \begin{bmatrix} i \\ -1 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} i & -i \\ -1 & 1 \\ 0 & -1 \\ 0 & -i \\ -1-i & 1+i \\ 1-i & -1+i \end{bmatrix}$$

$$U = \begin{bmatrix} a & b & c \\ d & f & g \\ h & j & k \end{bmatrix}$$

A man provided with paper, pencil, and rubber, and subject to strict discipline, is in effect a universal Turing Machine.

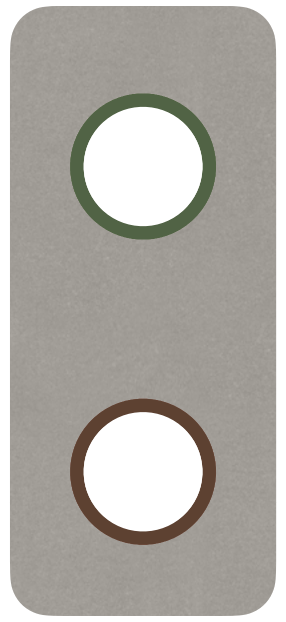
Alan Turing



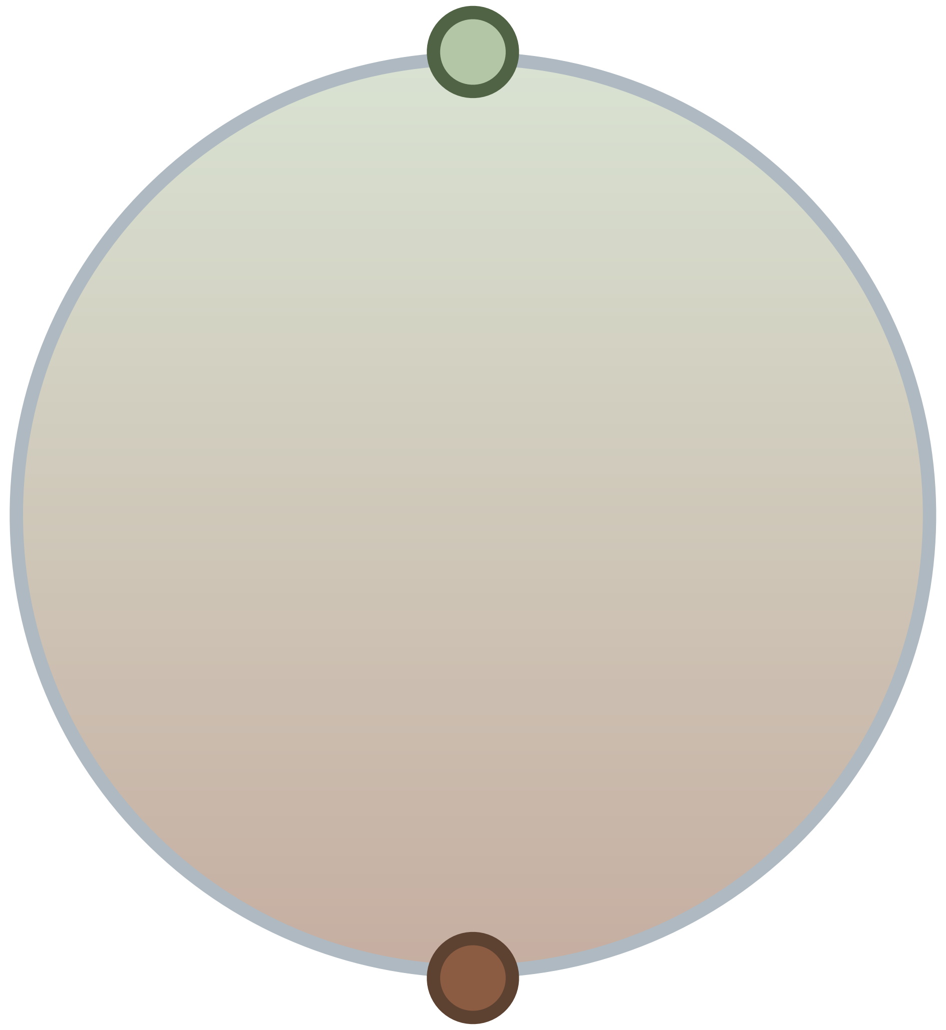
From Classical to Quantum

Classical V.S. Quantum

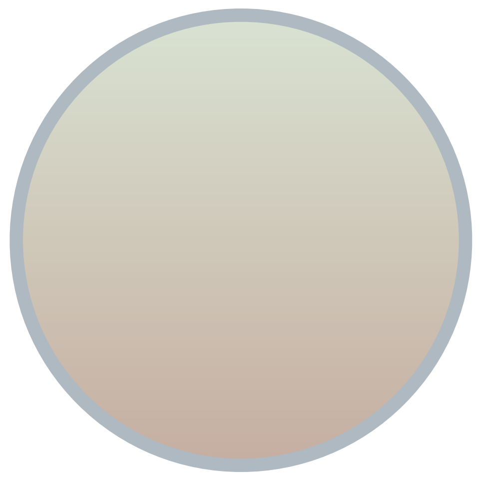
- Bit

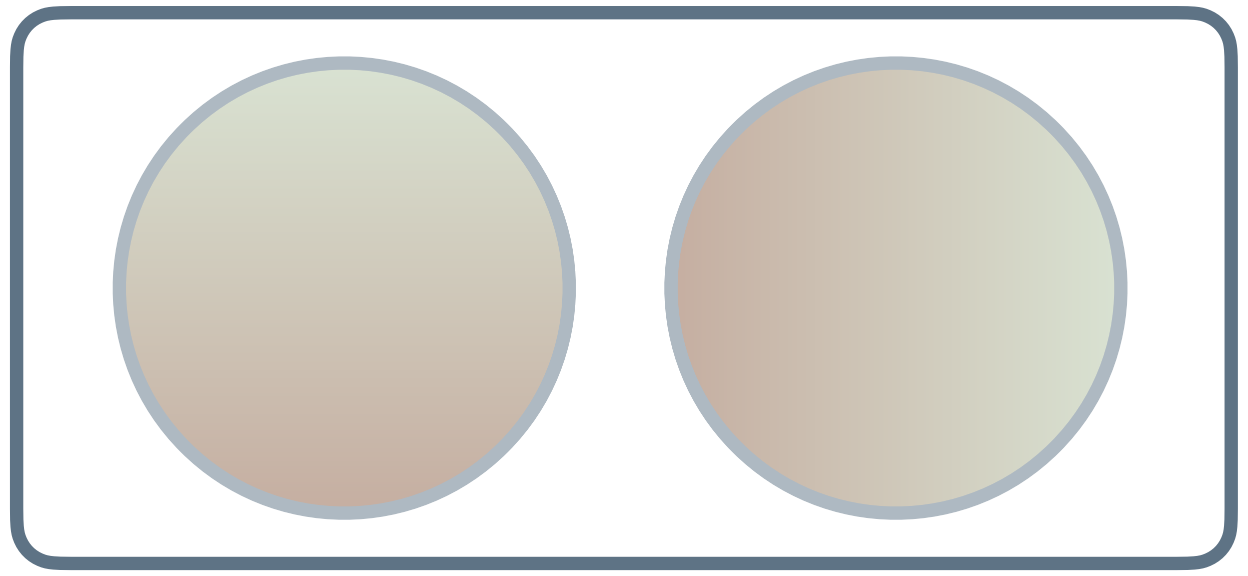


- Qubit



States





$$|\psi\rangle = \begin{bmatrix}\alpha \\ \beta\end{bmatrix}$$

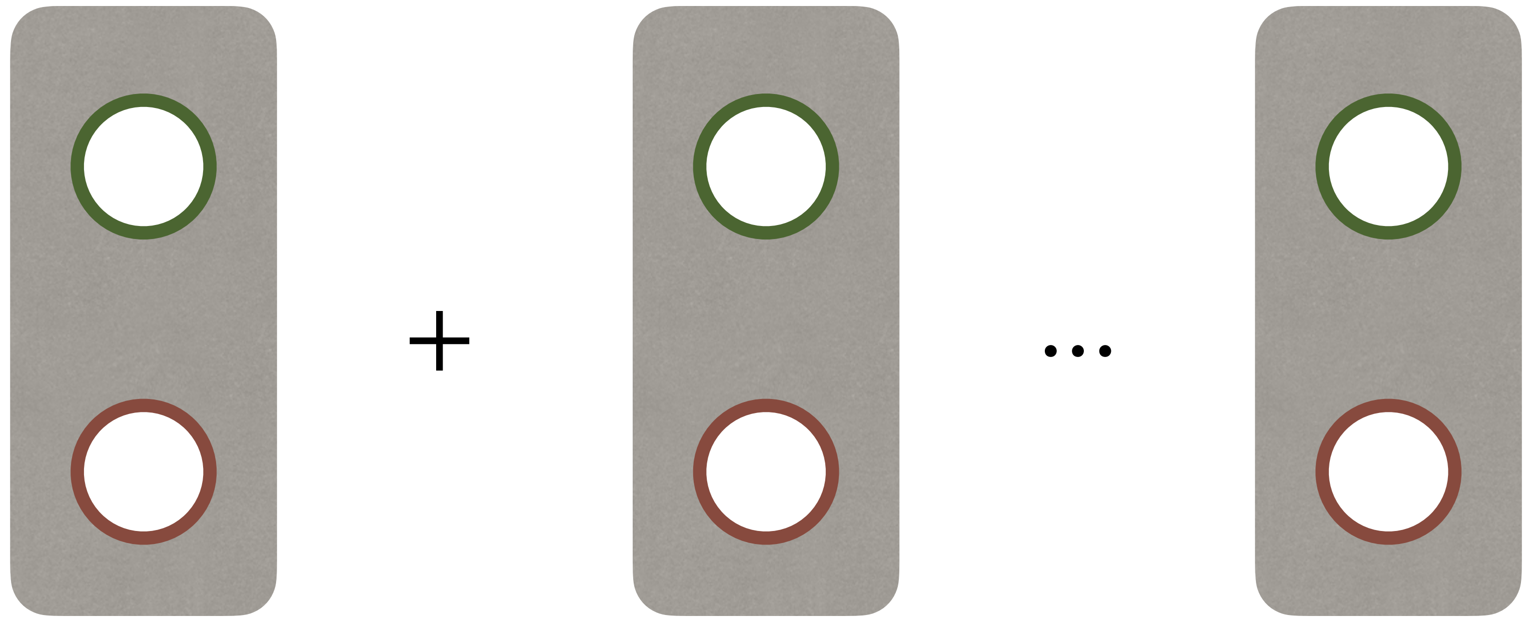
Basis: $\{\,|0\rangle, |1\rangle\,\}$

$$|\psi_1\rangle \otimes |\psi_2\rangle = \begin{bmatrix}\alpha \\ \beta\end{bmatrix} \otimes \begin{bmatrix}\gamma \\ \delta\end{bmatrix} = |\psi_1\psi_2\rangle$$

Basis: $\{\,|00\rangle, |01\rangle, |10\rangle, |11\rangle\,\}$

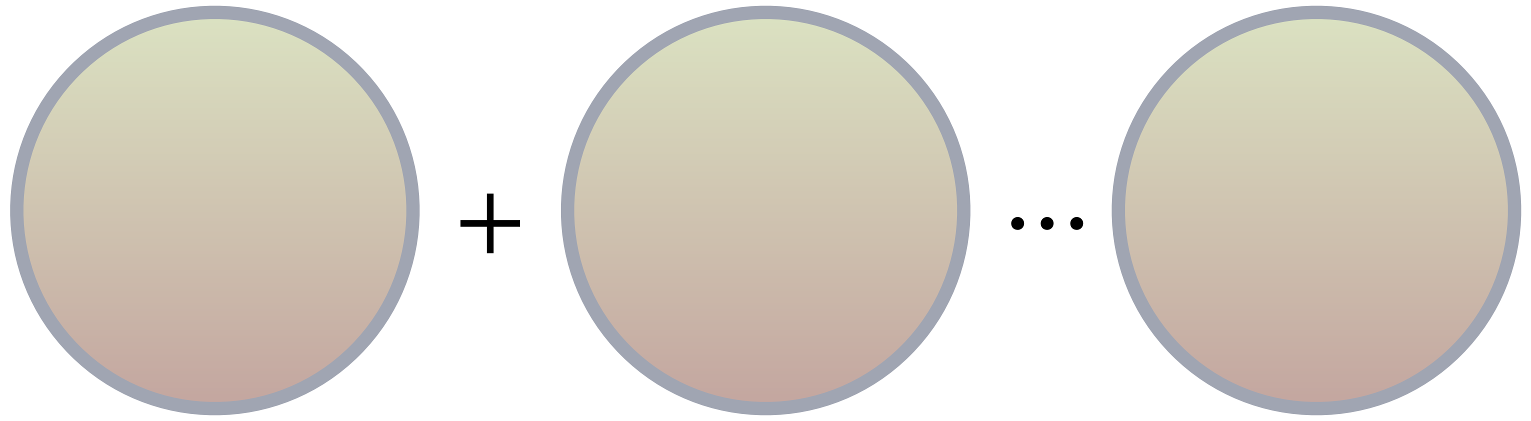
Speedup computational time

- Conventional



$1 + 1 \cdots 1$

- Quantum



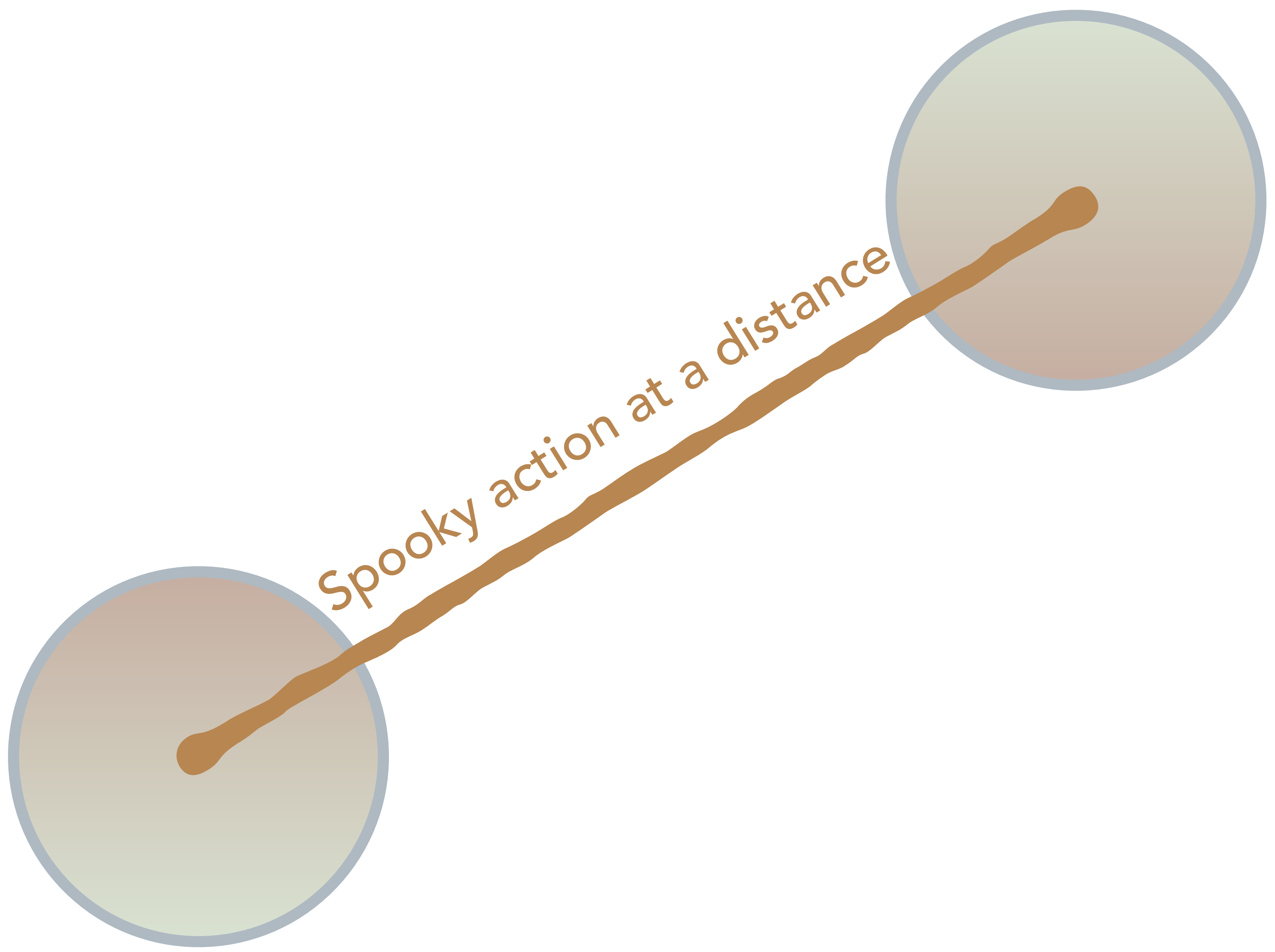
$2 \times 2 \cdots 2$

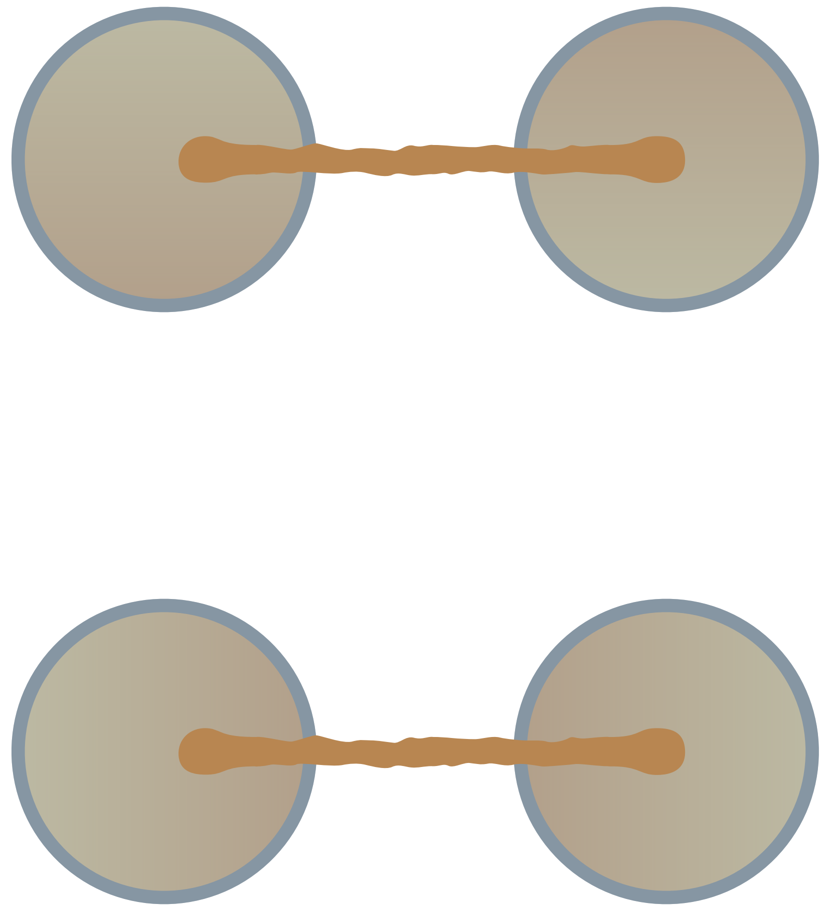
$N \longrightarrow 2^N$

Exponentially speedup!

Why so powerful ?

- Entanglement



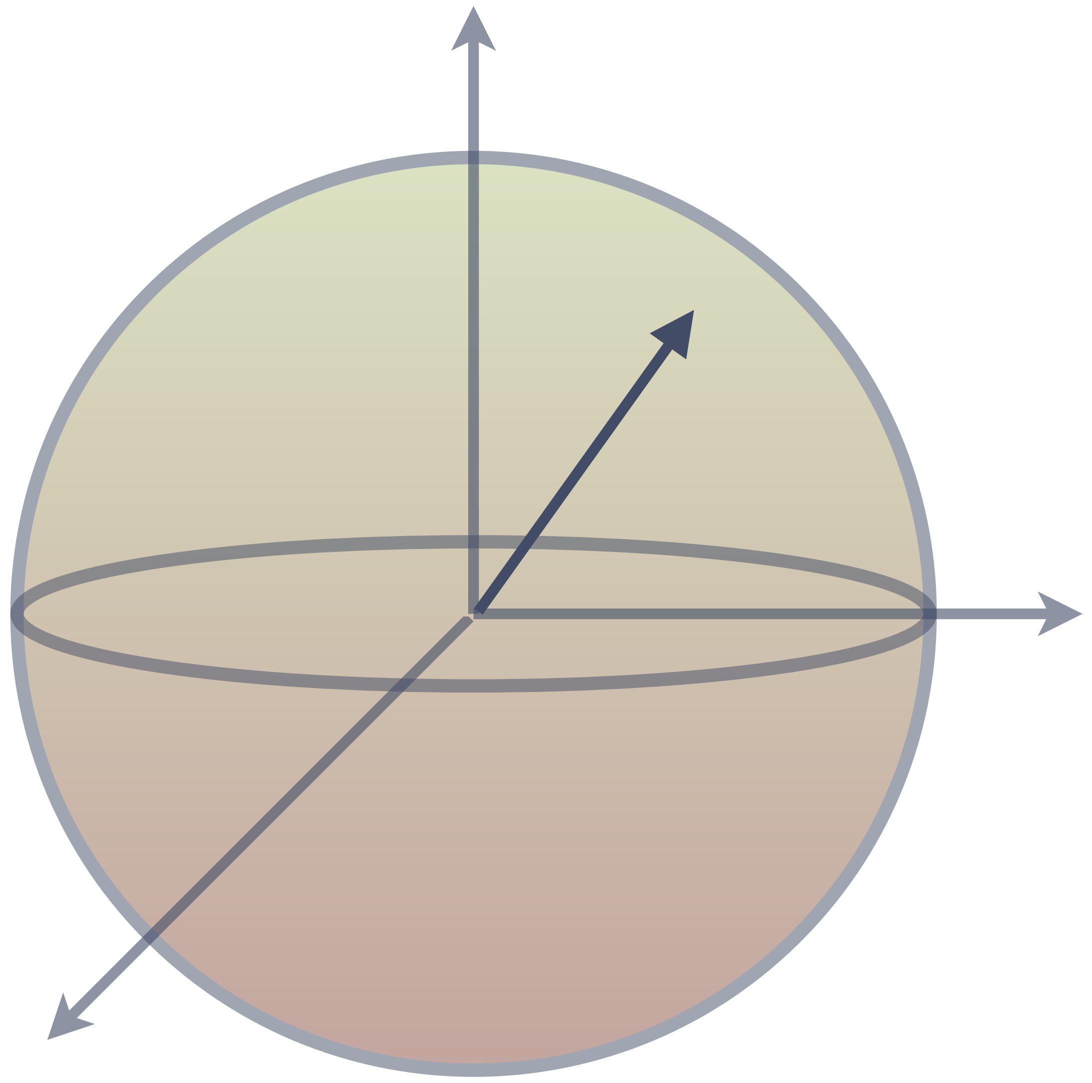


speedup quantum algorithms

Why so powerful ?

- Entanglement

- Superposition

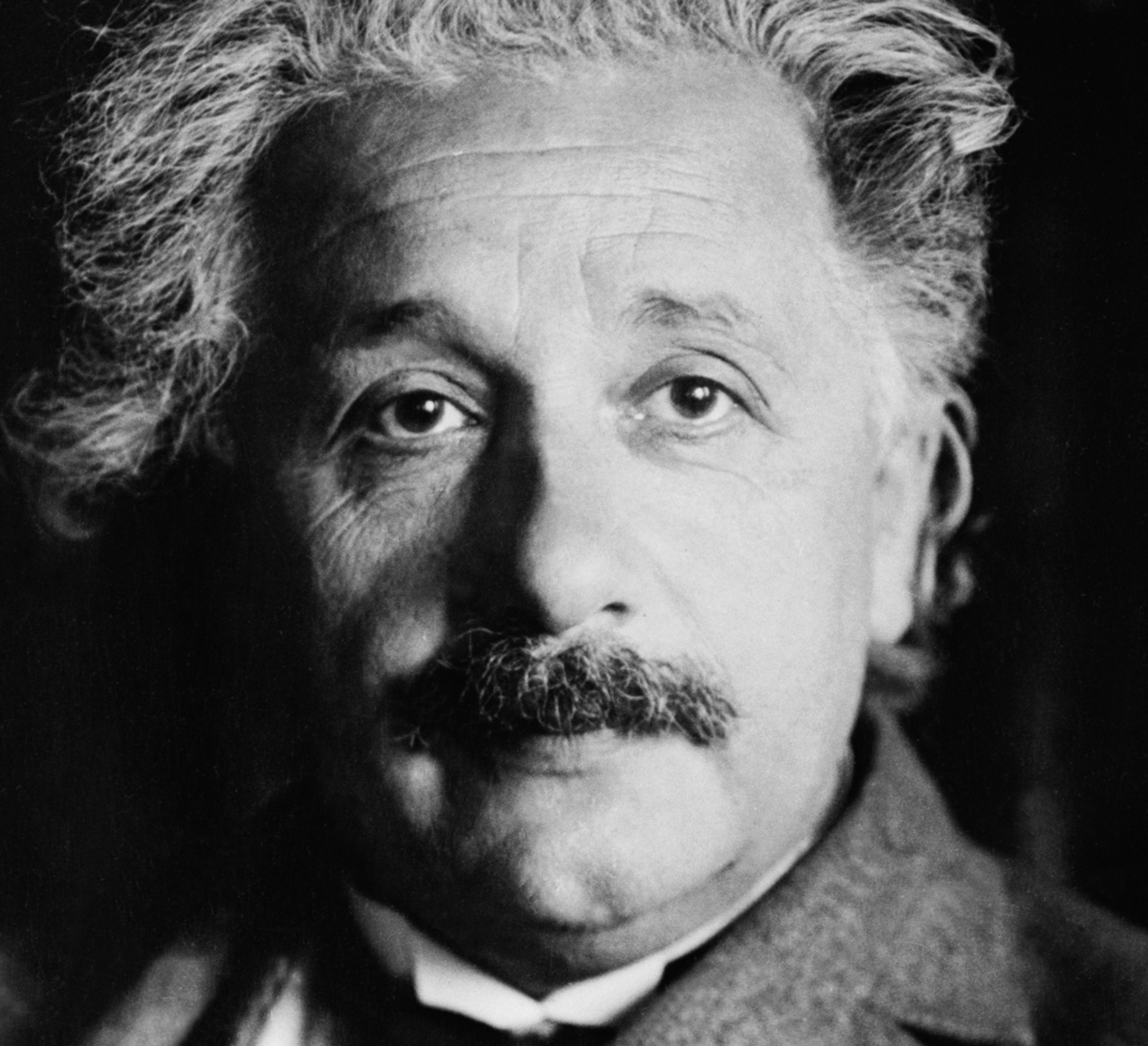


$$\alpha\,|0\rangle + \beta\,|1\rangle$$


SUPERPOSITION
0
1
Natasha Hanacek/NIST

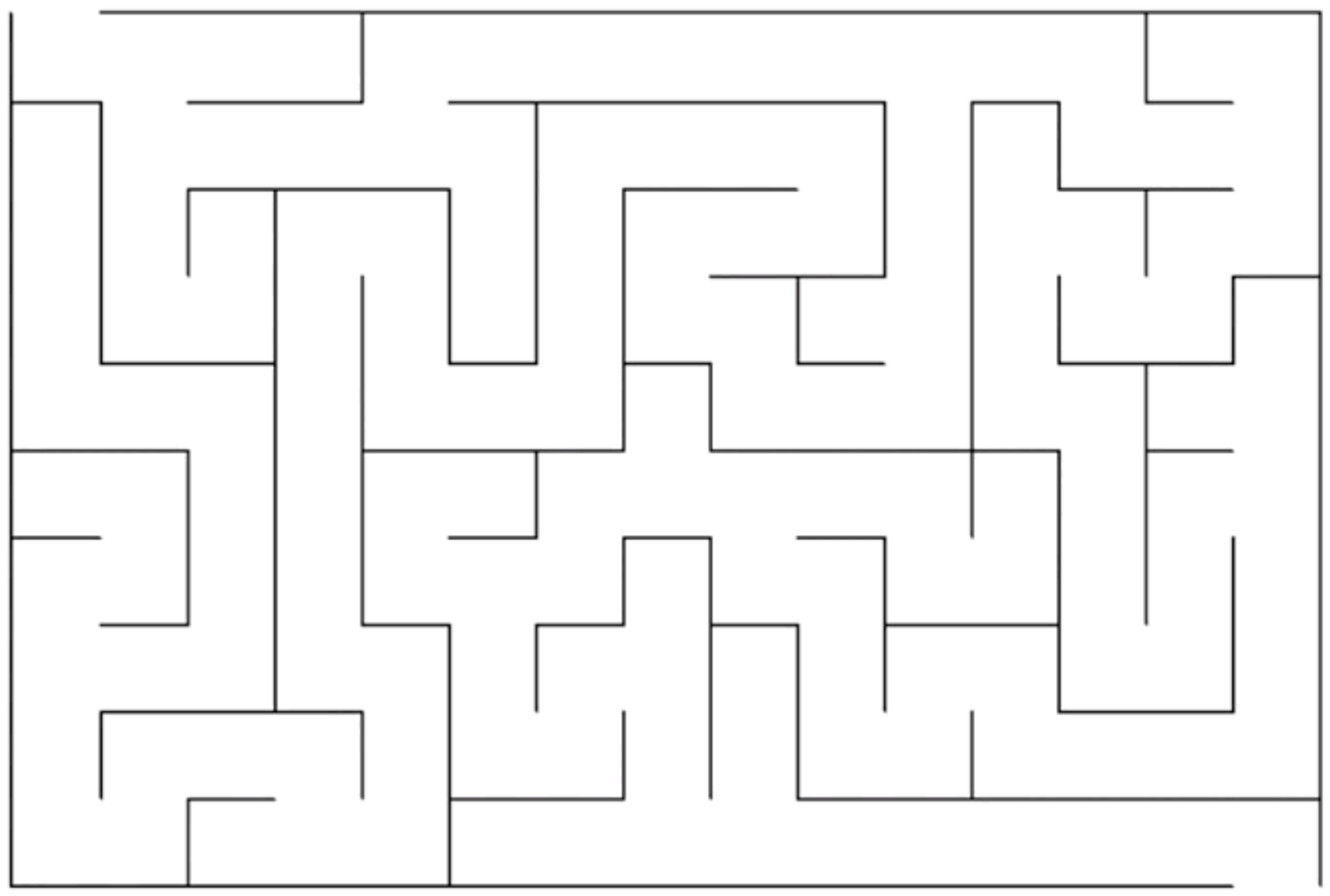
Do you believe that the moon isn’t there when nobody looks ?

—Albert Einstein

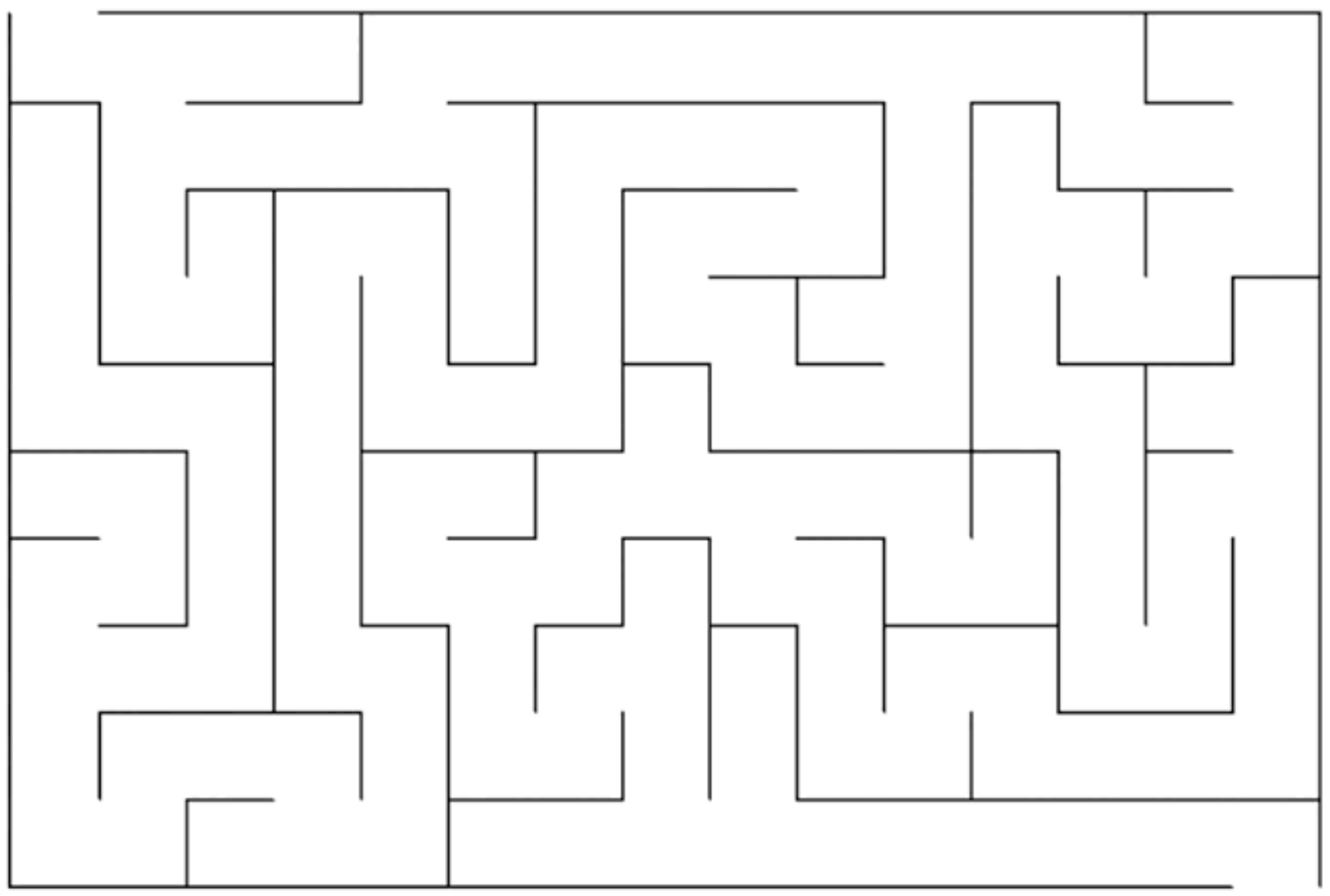


When we are in a complex maze……

- Conventional Computer



- Quantum Computer



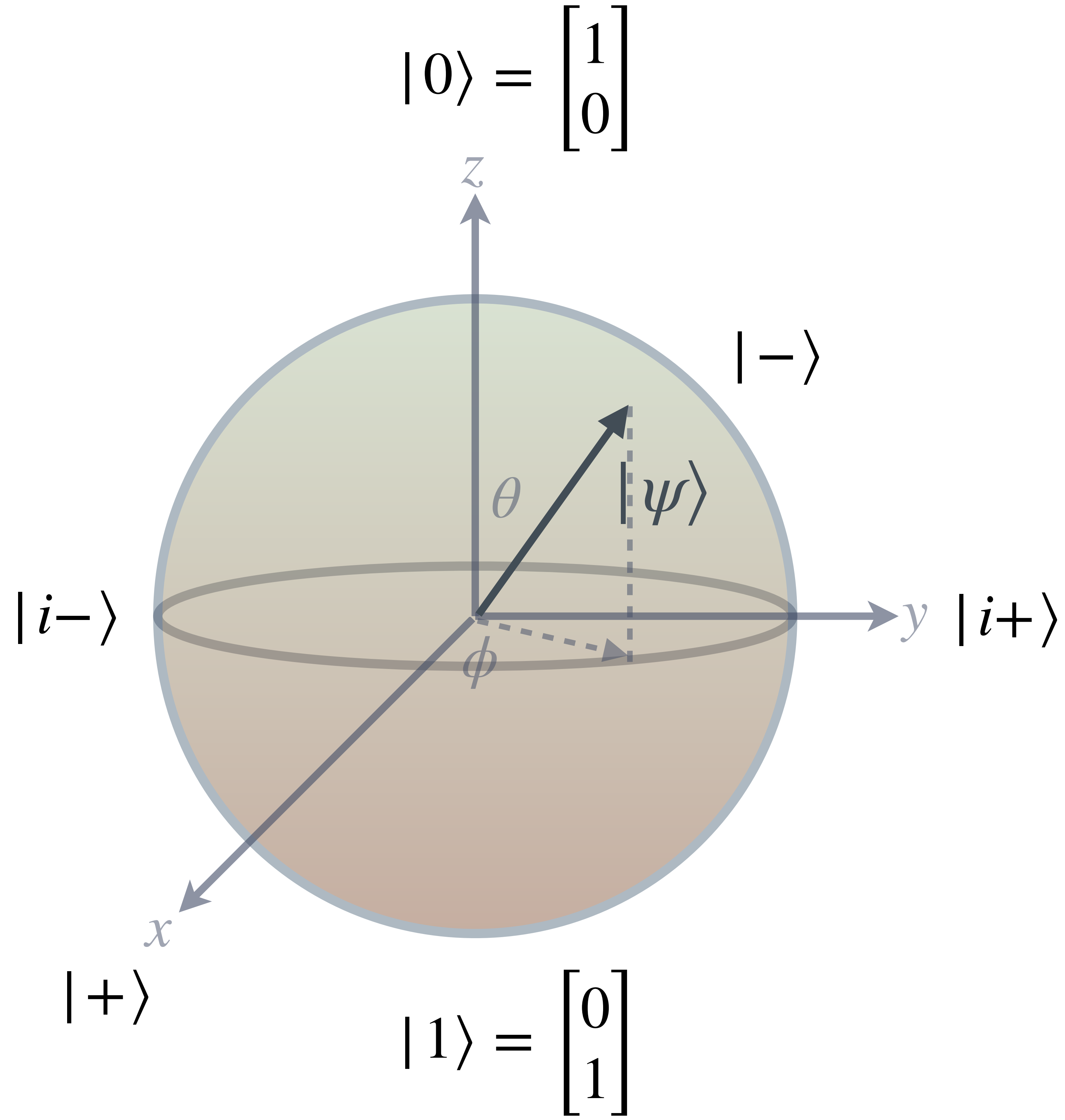
Quantum Language

Bloch sphere

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$$
$$= e^{ir}\left(\cos\frac{\theta}{2} + e^{i\phi}\sin\frac{\theta}{2}\right)$$

$$|\pm\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle \pm |1\rangle\right)$$

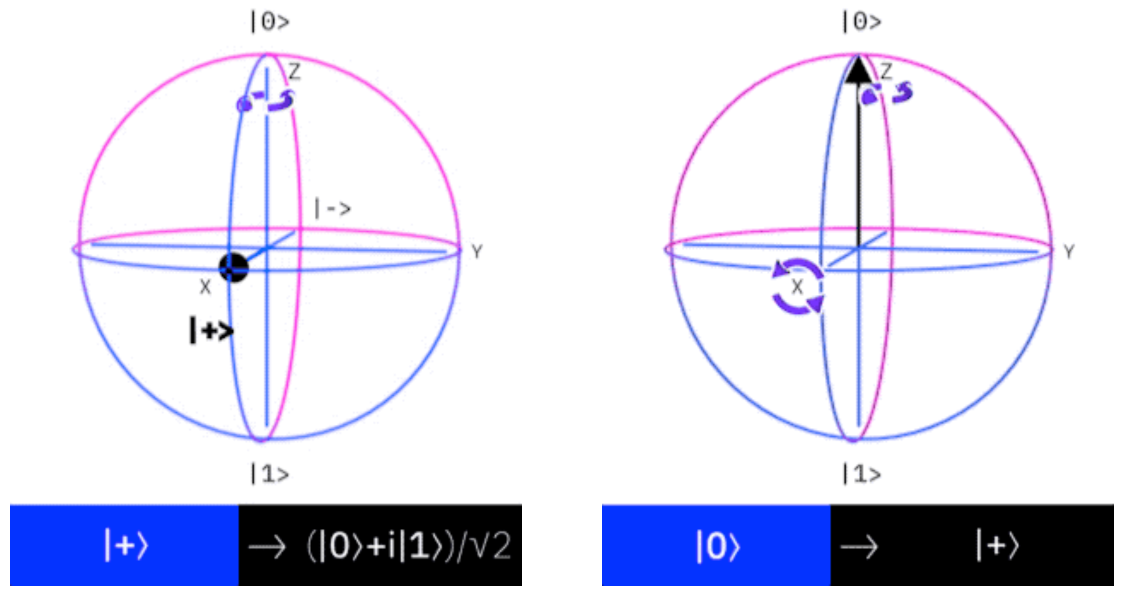
$$|i\pm\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle \pm i\,|1\rangle\right)$$

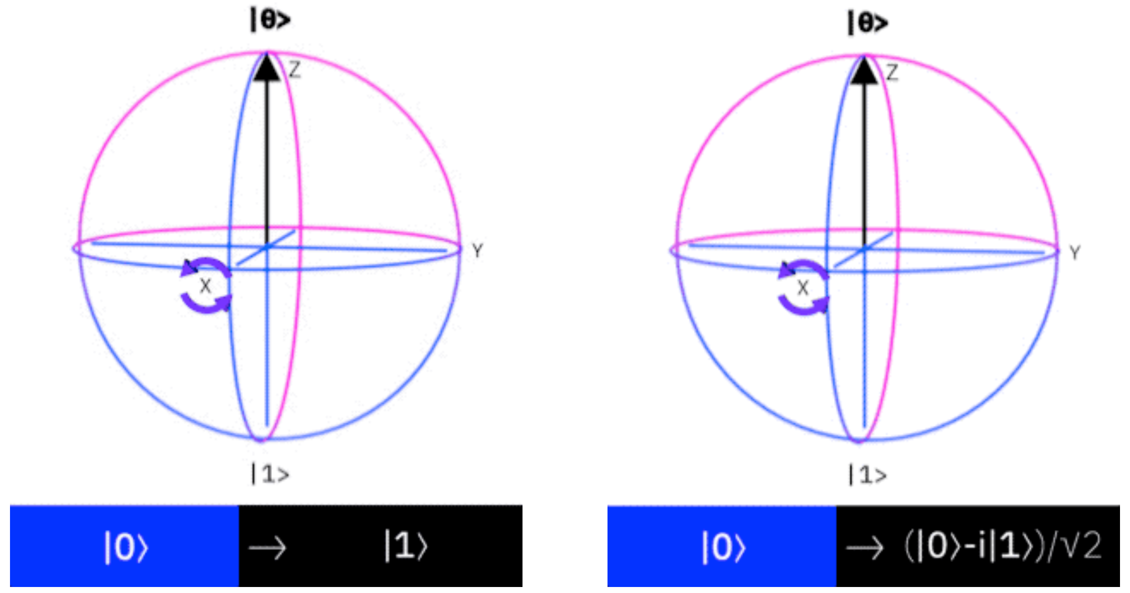


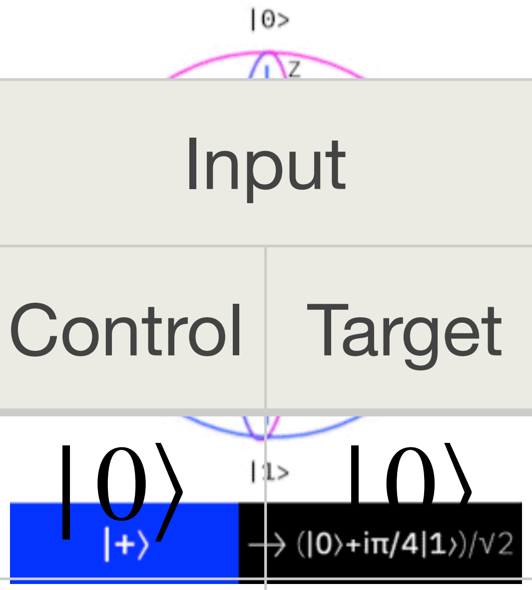
Operations - Quantum gates

unitary: $UU^{\dagger} =$ ▯ reversible!

- Single-qubit gate
  - Hadamard H
  - Phase S
  - $\pi/8$ T
  - Rotation X Y Z Rx Ry Rz
- Multi-qubit gate
  - CNOT
  - Controlled-rotation cH cRz
  - SWAP







Input		Output	
Control	Target	Control	Target
$\vert 0\rangle$	$\vert 0\rangle$	$\vert 0\rangle$	$\vert 0\rangle$
$\vert 0\rangle$	$\vert 1\rangle$	$\vert 0\rangle$	$\vert 1\rangle$
$\vert 1\rangle$	$\vert 0\rangle$	$\vert 1\rangle$	$\vert 1\rangle$
$\vert 1\rangle$	$\vert 1\rangle$	$\vert 1\rangle$	$\vert 0\rangle$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha & \beta \\ 0 & 0 & \gamma & \delta \end{bmatrix}$$

controlled-rotation

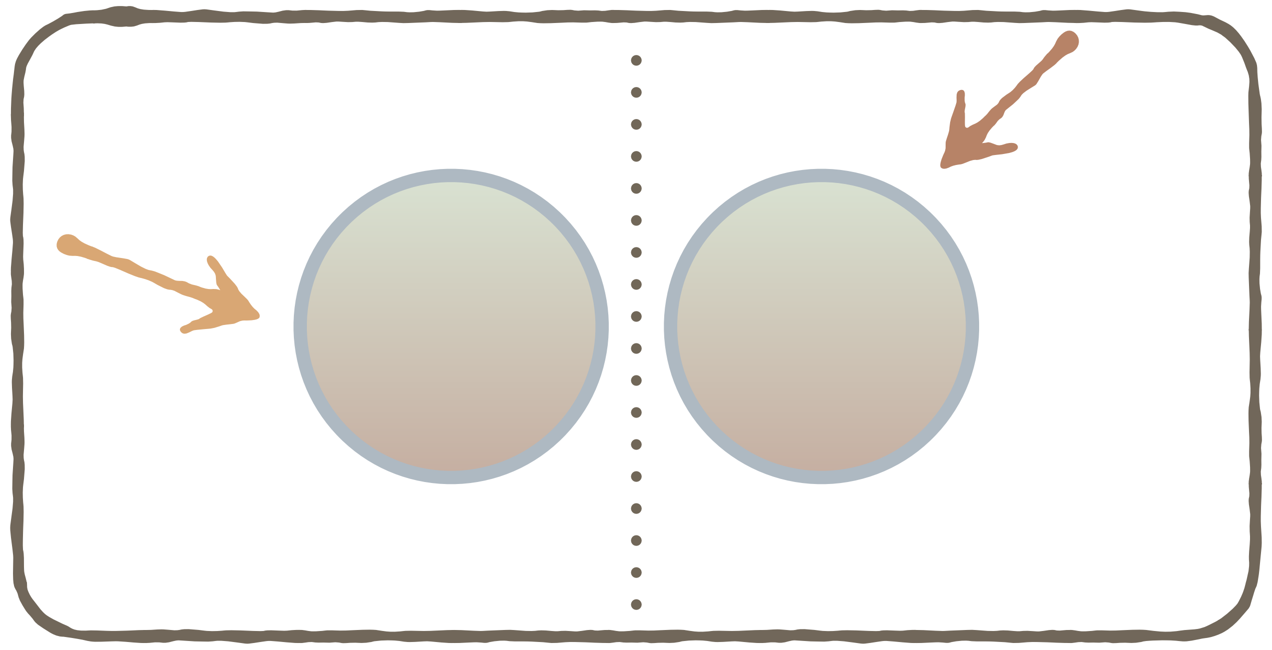
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

SWAP

Resources/Circuit Composer/Block glossary

Operate only one of the qubits ?

- Do "nothing to Q1 + NOT gate on Q2" ?
- nothing = Identity gate ID



$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- operate independently: $I^{(1)}X^{(2)} = X^{(2)} = (I \otimes I)\,(I \otimes X) = (I \otimes X)$

"NOT gate on Q1 +
Hadamard gate on Q2" ?

Quantum Computers

Requirements for physical realization

- Scalable physical system
- Ability to initialize the state
- Long decoherence time $>$ operation time
- A universal set of quantum gates
- Capability to qubit-specific measurement

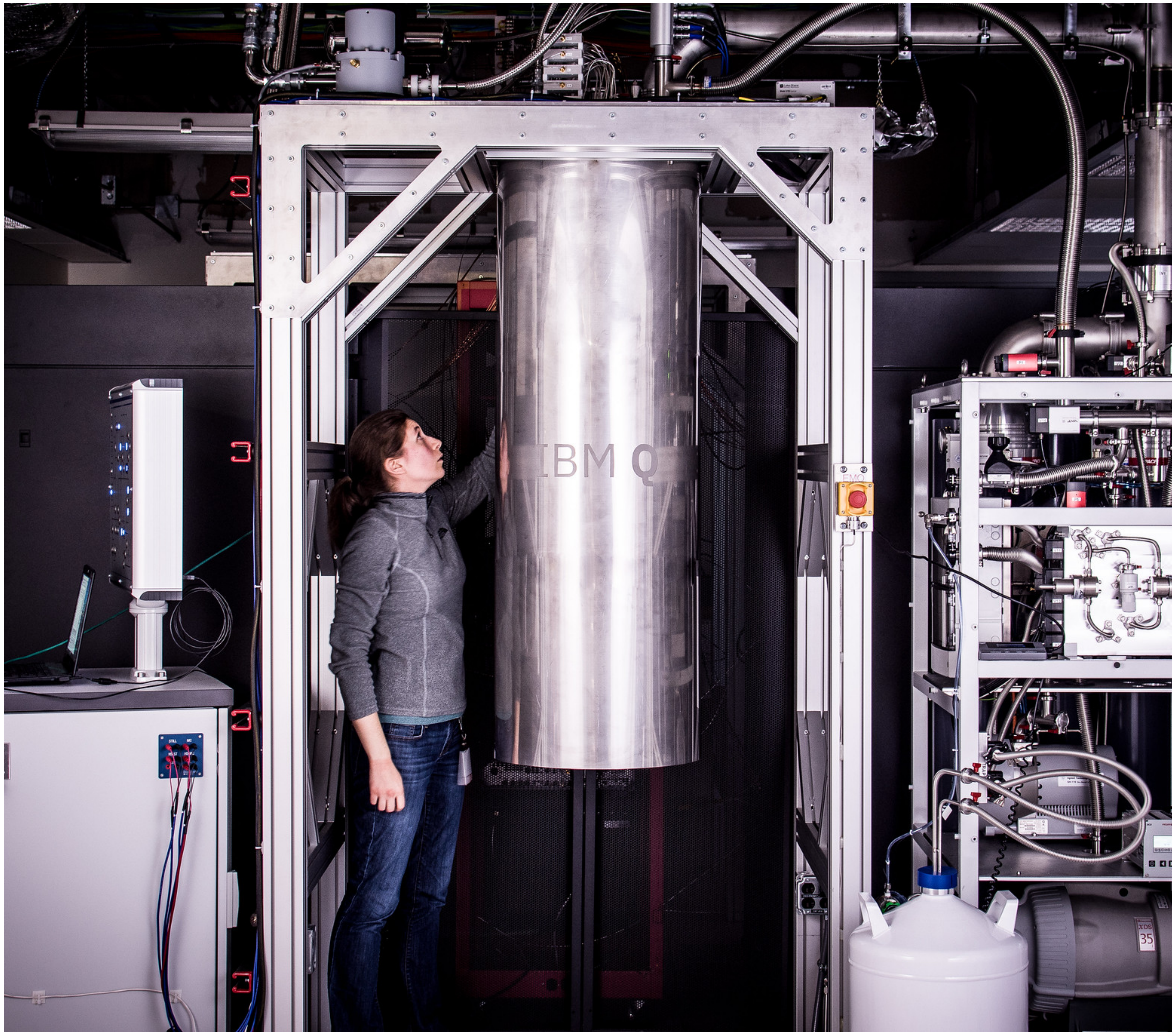
Fortschr. Phys. **48**, 771-783 (2000)

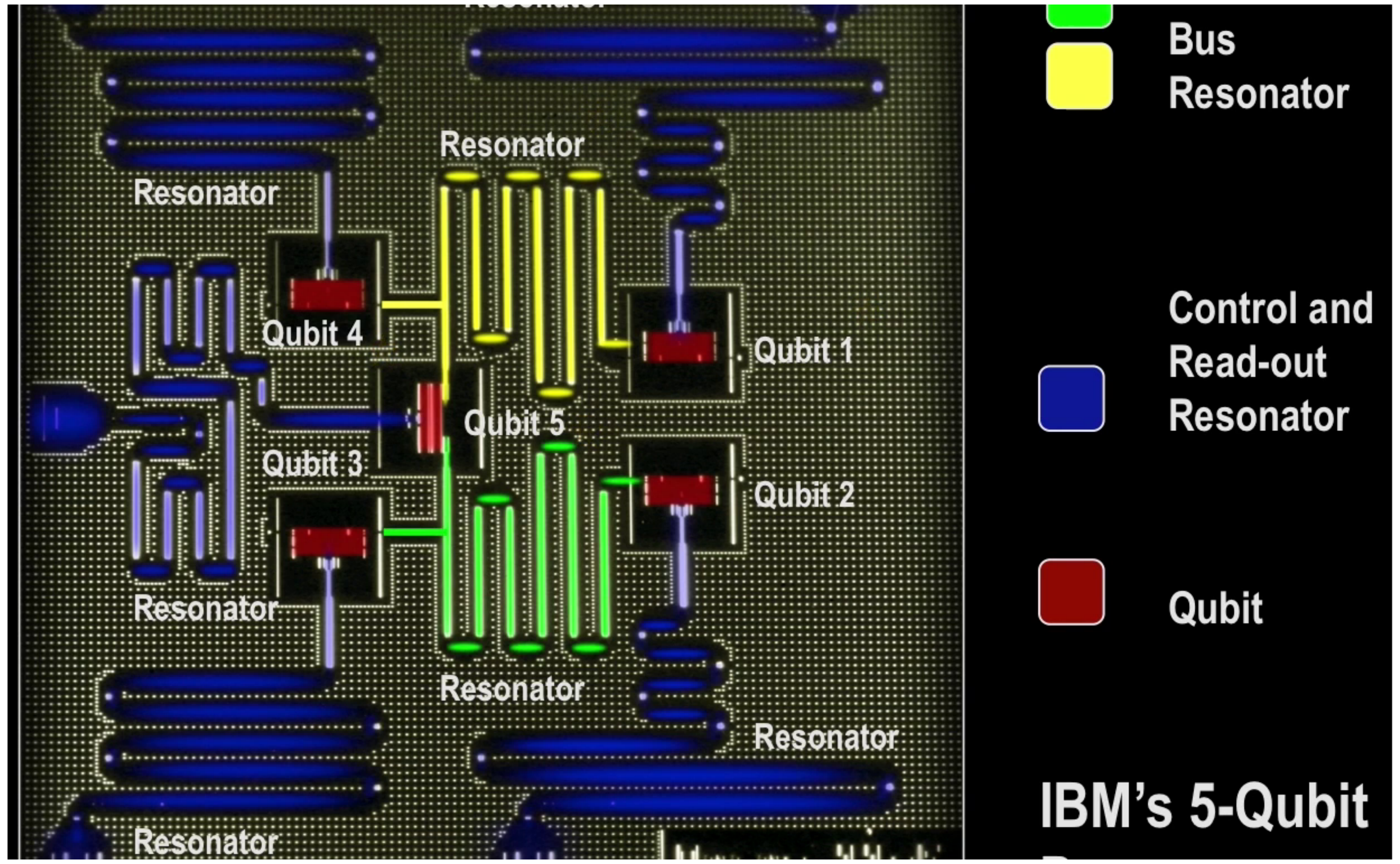
Implement

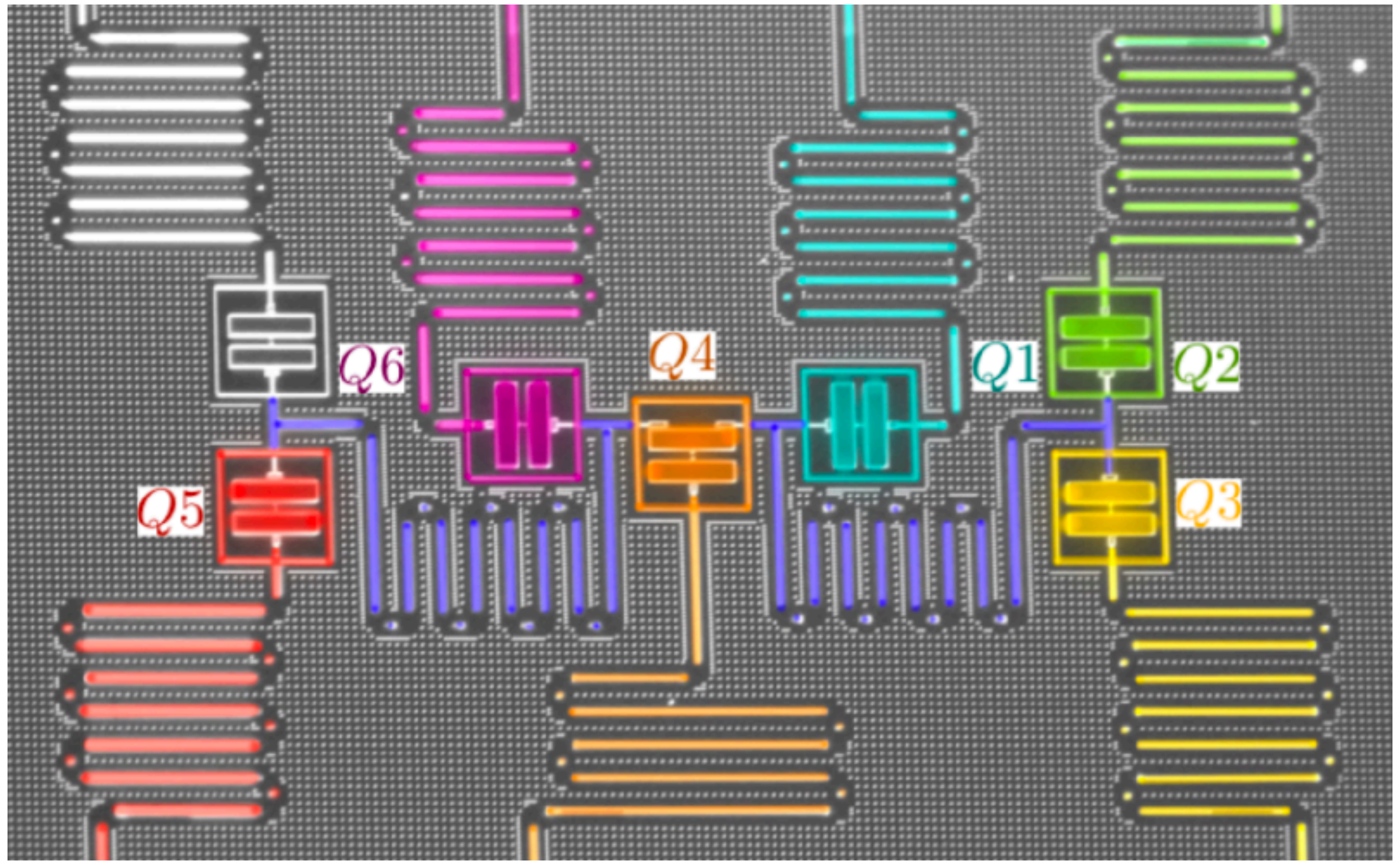
- Photons, Electrons, Nucleus
- NMR
- Quantum dots
- Nature atoms
- Diamond
- Semiconductors
- Superconductors

and so on……


Quantum Computing
Intersecting science, systems and design.
IBM Q
System One
IBM



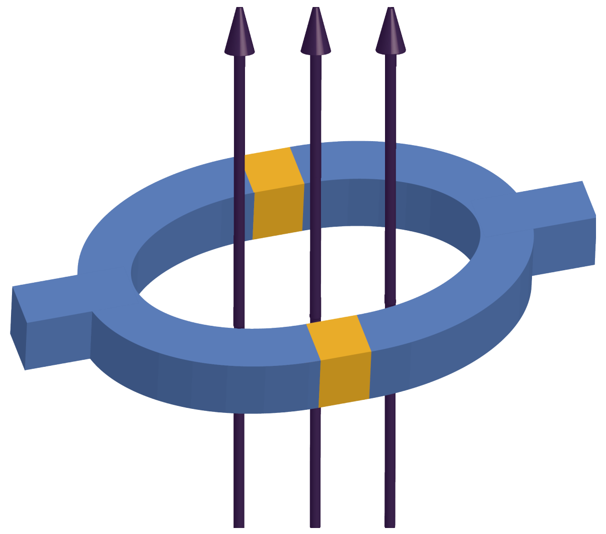


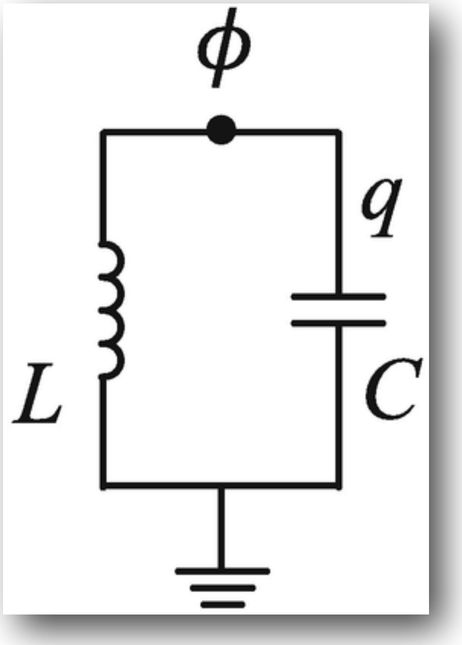


Nature **549**, 242-246 (2017)

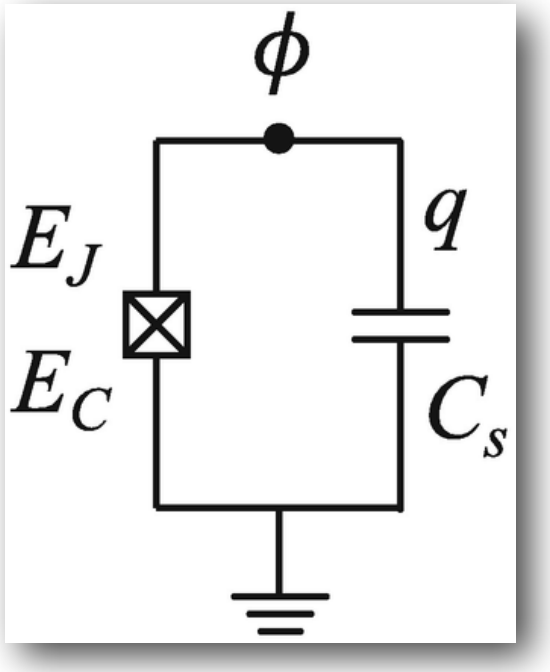
Superconducting circuit

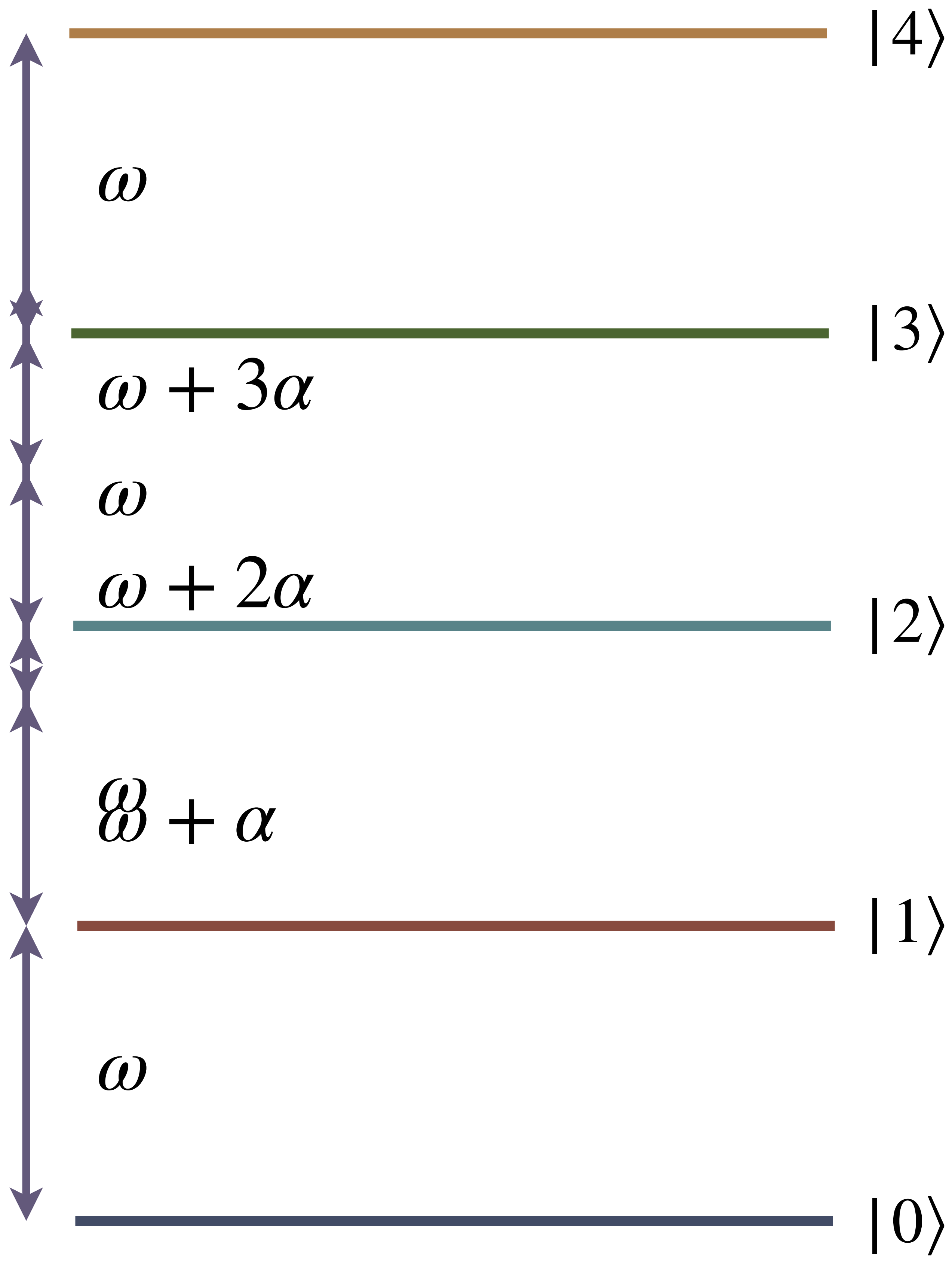
Harmonic → Nonlinear



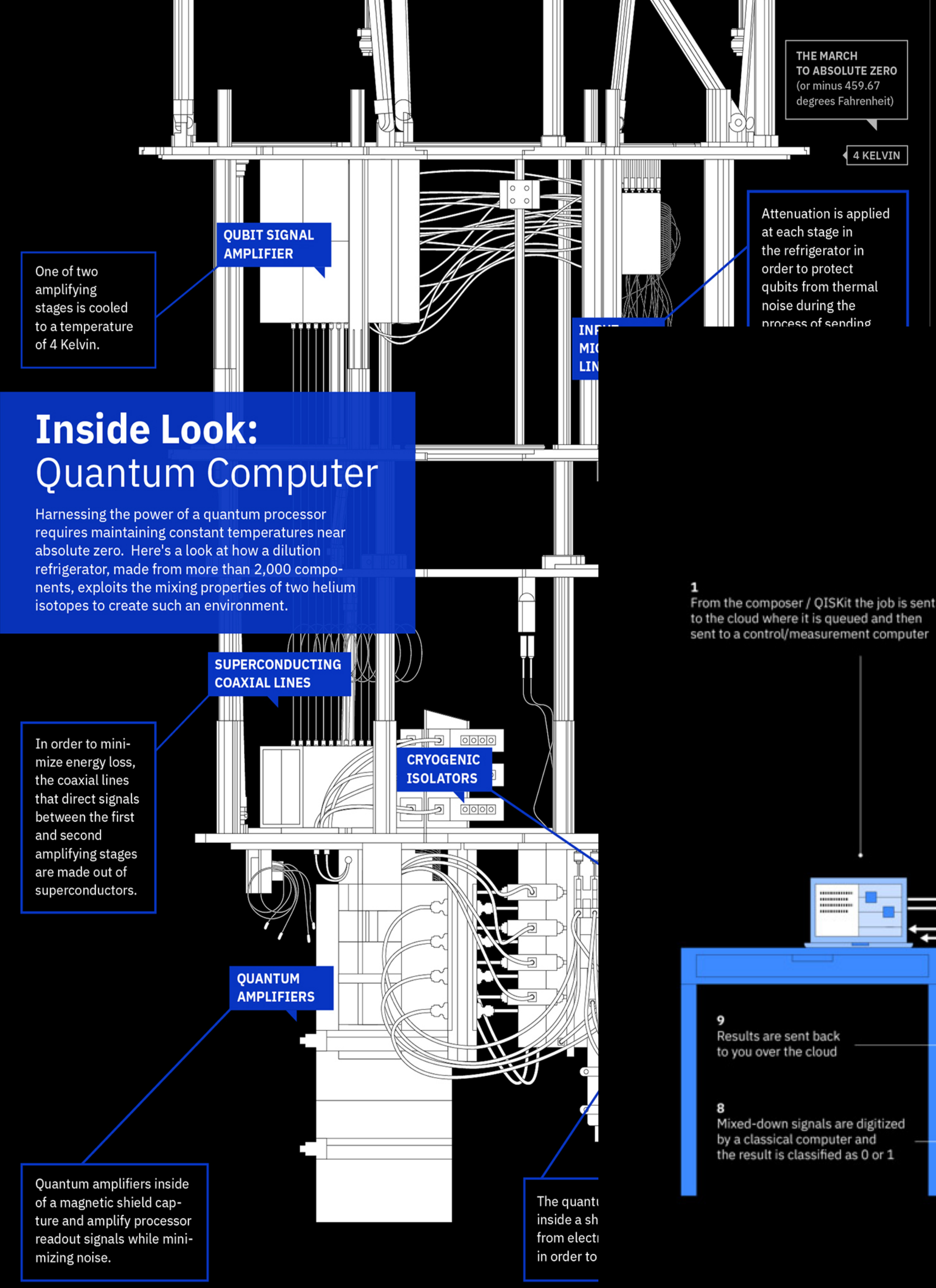


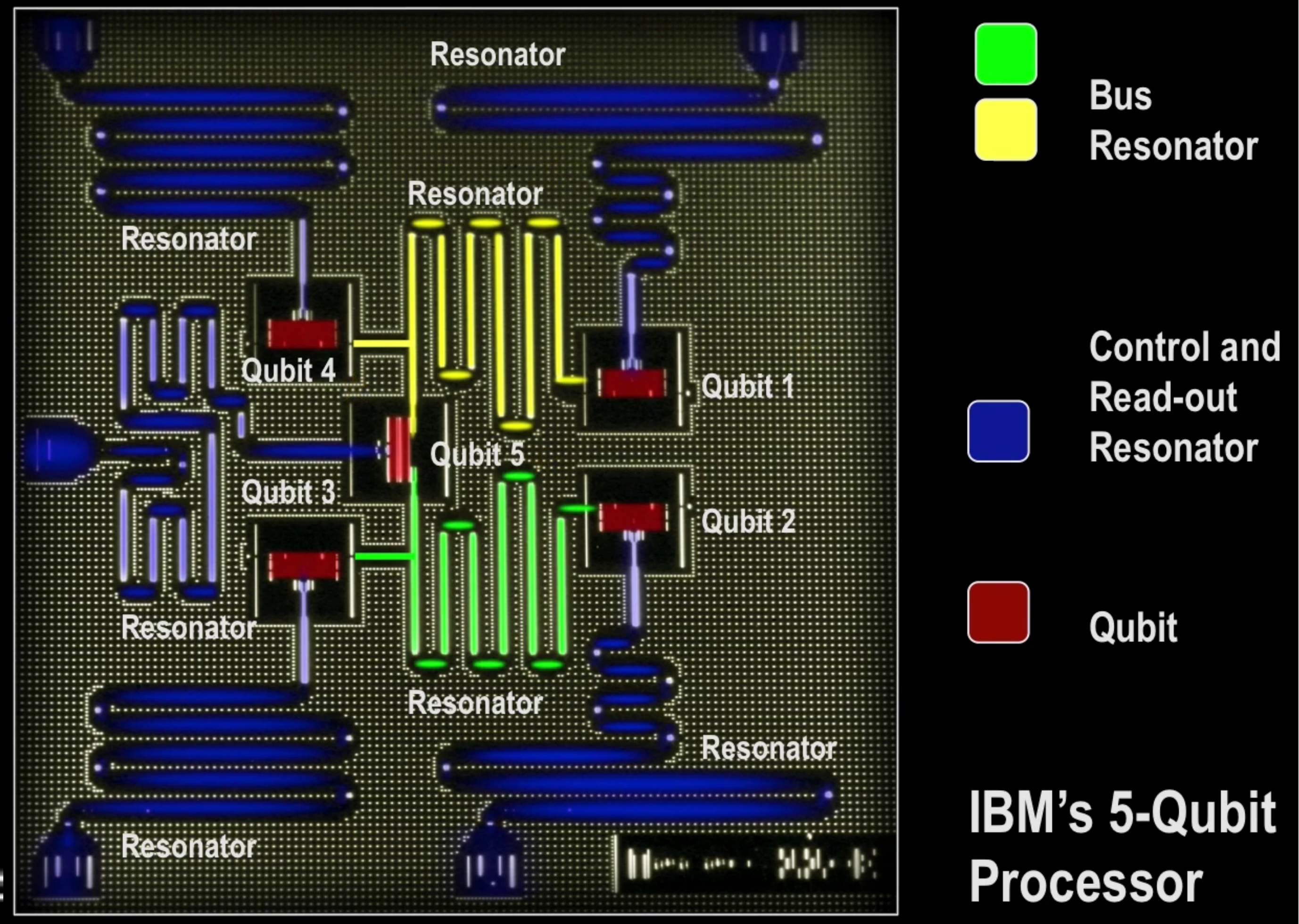
Josephson junction

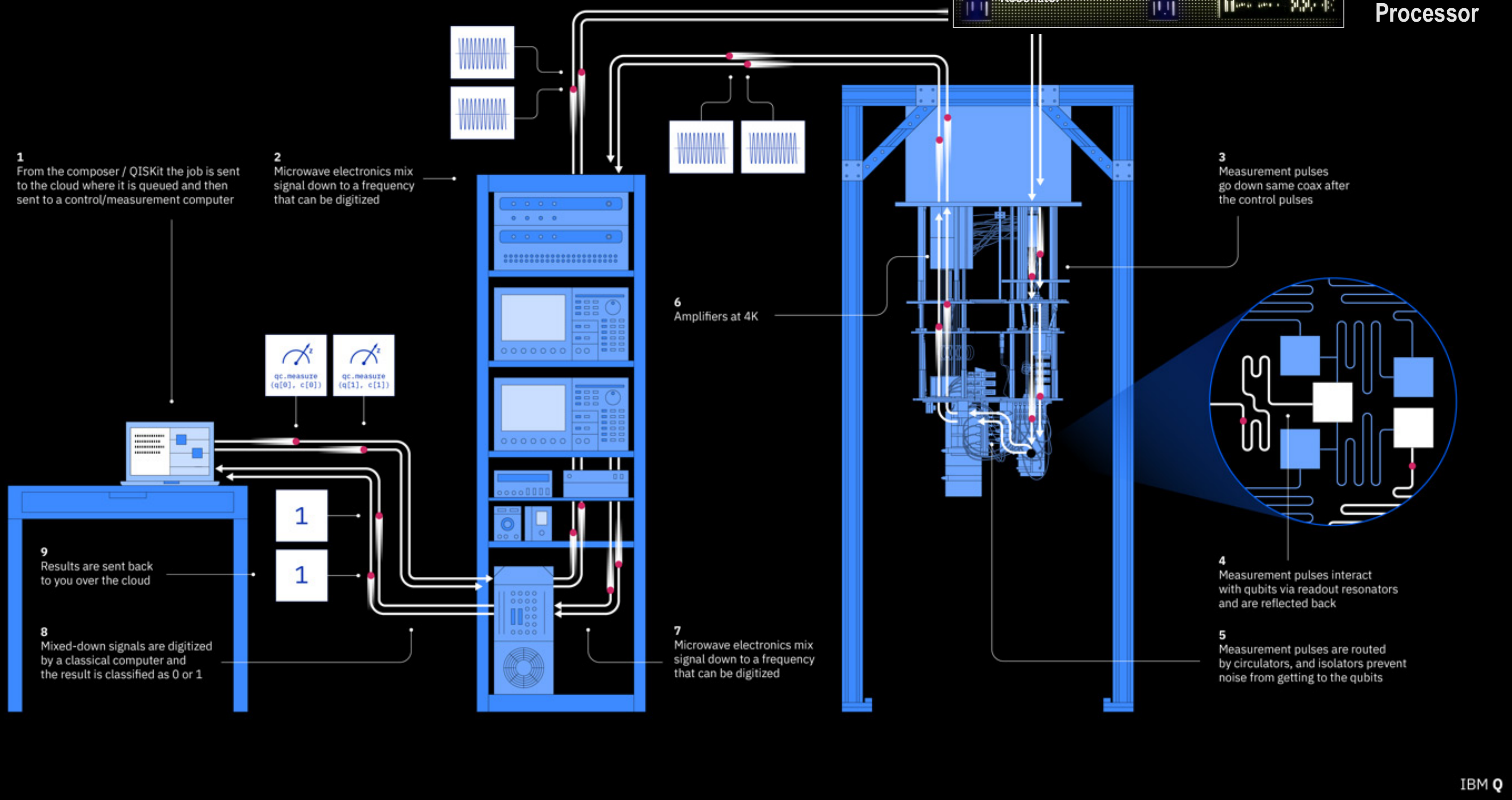




$\alpha < 0$


THE MARCH
TO ABSOLUTE ZERO
(or minus 459.67
degrees Fahrenheit)
4 KELVIN
QUBIT SIGNAL
AMPLIFIER
One of two amplifying stages is cooled to a temperature of 4 Kelvin.
Attenuation is applied at each stage in the refrigerator in order to protect qubits from thermal noise during the
Inside Look:
Quantum Computer
Harnessing the power of a quantum processor requires maintaining constant temperatures near absolute zero. Here's a look at how a dilution refrigerator, made from more than 2,000 components, exploits the mixing properties of two helium isotopes to create such an environment.
SUPERCONDUCTING
COAXIAL LINES
In order to minimize energy loss, the coaxial lines that direct signals between the first and second amplifying stages are made out of superconductors.
CRYOGENIC
ISOLATORS
QUANTUM
AMPLIFIERS
Quantum amplifiers inside of a magnetic shield capture and amplify processor readout signals while minimizing noise.


Resonator
Resonator
Resonator
Qubit 4
Qubit 1
Qubit 5
Qubit 3
Qubit 2
Resonator
Resonator
Resonator
Resonator
Bus
Resonator
Control and
Read-out
Resonator
Qubit
IBM's 5-Qubit
Processor


1
From the composer / QISKit the job is sent to the cloud where it is queued and then sent to a control/measurement computer
2
Microwave electronics mix signal down to a frequency that can be digitized
3
Measurement pulses go down same coax after the control pulses
qc.measure (q[0], c[0])
qc.measure (q[1], c[1])
6
Amplifiers at 4K
1
1
9
Results are sent back to you over the cloud
8
Mixed-down signals are digitized by a classical computer and the result is classified as 0 or 1
7
Microwave electronics mix signal down to a frequency that can be digitized
4
Measurement pulses interact with qubits via readout resonators and are reflected back
5
Measurement pulses are routed by circulators, and isolators prevent noise from getting to the qubits
IBM Q

Your First Quantum Circuit

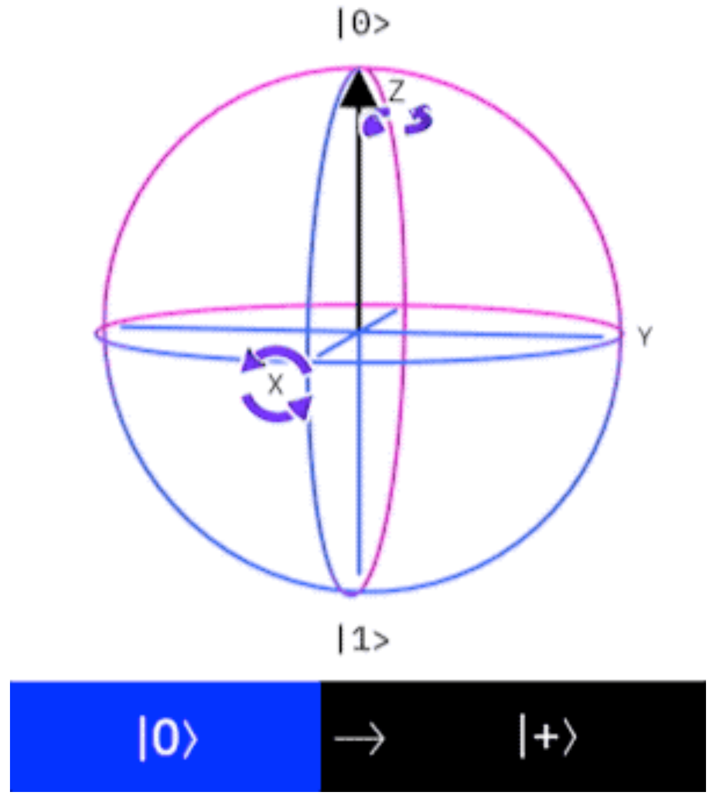
IBM Quantum Experience

Demonstration - Superposition state

- Hadamard gate

$$-\boxed{H}- \longmapsto \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$$

$$H\,|0\rangle = |+\rangle$$
$$H\,|1\rangle = |-\rangle$$



- Gate operation

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\cdot\begin{bmatrix}1\\ 0\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\ 1\end{bmatrix}$$

$$H\,|0\rangle = \frac{1}{\sqrt{2}}\big(|0\rangle + |1\rangle\big) = |+\rangle$$

$$H\,|1\rangle = |-\rangle$$

What is the equivalent for two H gates ?

Demonstration - Entangled state

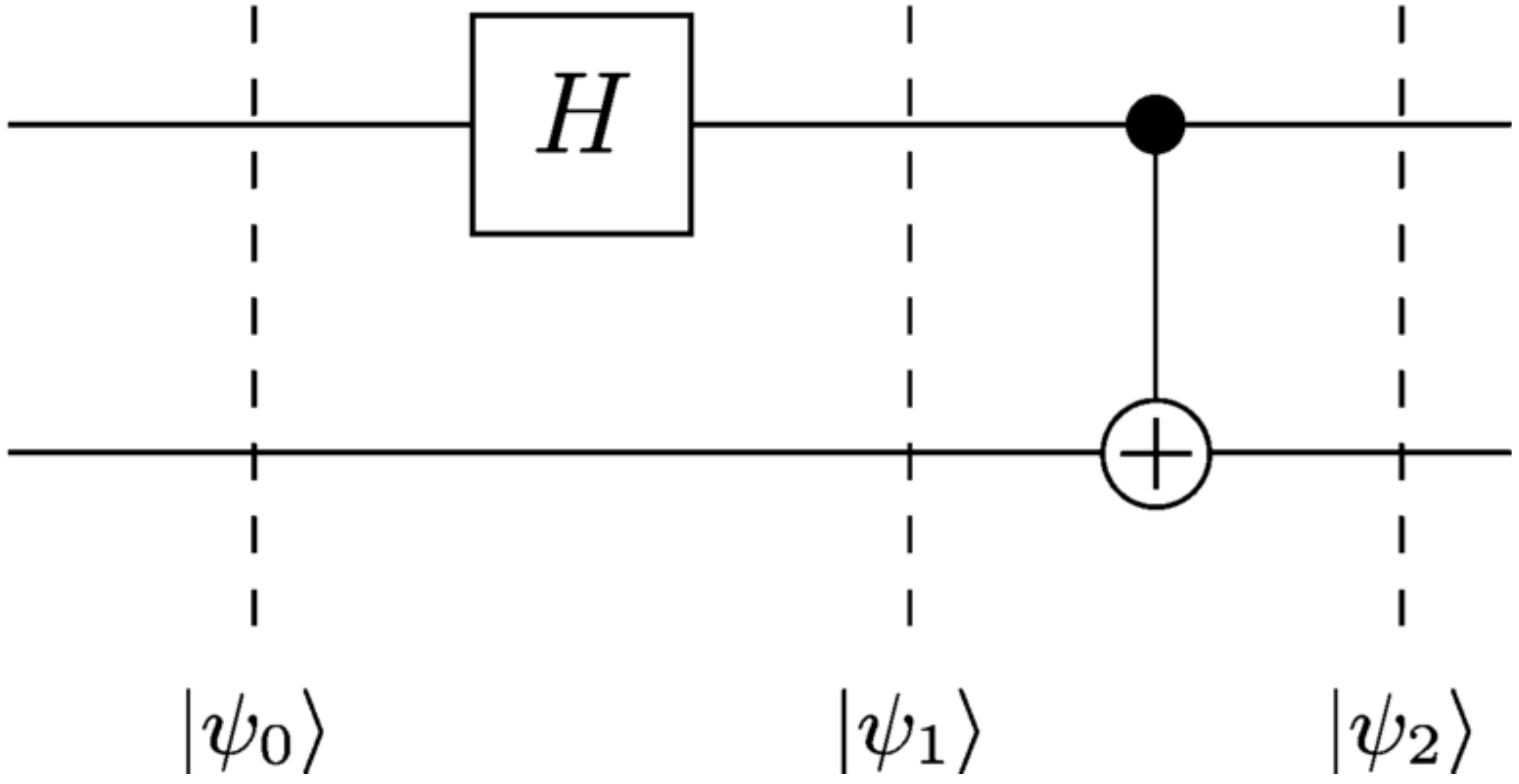
- Bell states

$$|\Phi^{+}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

$$|\Phi^{-}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle - |11\rangle\right)$$

$$|\Psi^{+}\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle + |10\rangle\right)$$

$$|\Psi^{-}\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right)$$



- Gate operation

$$|\psi_2\rangle = CNOT \cdot (H \otimes I)\,|\psi_0\rangle$$

What are the initials ?

Measurement

Projective measurement

- Projection operator

$$\sum_m \mathbb{P}_m = \sum_m |m\rangle \langle m|$$

measurement basis

- Probability of outcome m

$$p(m) = \langle \psi | \mathbb{P}_m | \psi \rangle$$

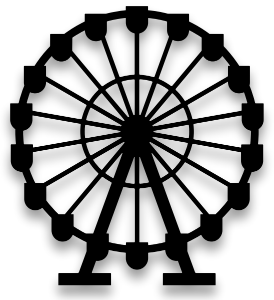
system state

- Post-measurement state

$$|\psi'\rangle = \frac{\mathbb{P}_m}{\sqrt{p(m)}} |\psi\rangle$$

- Default measurement basis 

$$\{ |0\rangle, |1\rangle \}$$

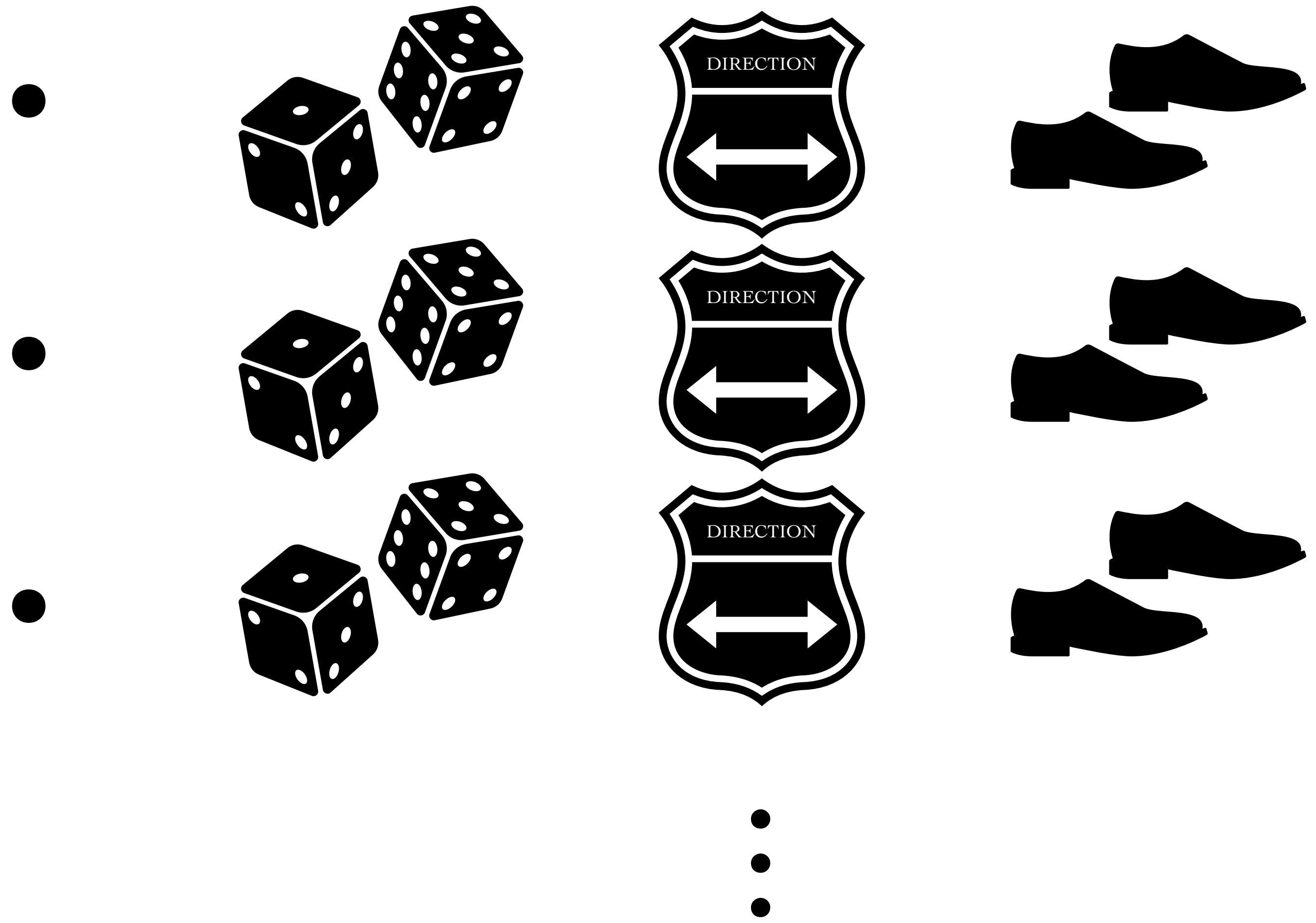
How to change the measurement basis, e.g., to Bell states ? 

Demonstration - Quantum random walks

Story of a drunk man

Can he reach home?

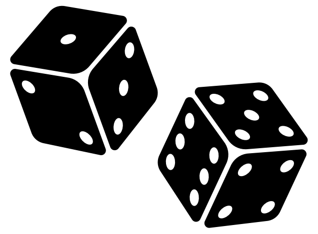
- Scheme



Demonstration - Quantum random walks

Prediction: for 1-D case

- Classical



$50\,\%\,(1) + 50\,\%\,(-1)$

$25\,\%\,(2) + 50\,\%\,(0) + 25\,\%\,(-2)$

$12.5\,\%\,(3) + 37.5\,\%\,(1) + 37.5\,\%\,(-1) + 12.5\,\%\,(-3)$

	-3	-2	-1	0	1	2	3
0				1			
1			1/2		1/2		
2		1/4		1/2		1/4	
3	1/8		3/8		3/8		1/8

- Quantum

coin: H

shift: $|\uparrow\rangle \implies |k+1\rangle$
$|\downarrow\rangle \implies |k-1\rangle$

$$\frac{1}{\sqrt{2}}\left(|\uparrow\rangle \otimes |1\rangle - |\downarrow\rangle \otimes |-1\rangle\right)$$

$$\frac{1}{2}\left[|\uparrow\rangle \otimes |2\rangle - \left(|\uparrow\rangle - |\downarrow\rangle\right) \otimes |0\rangle + |\downarrow\rangle \otimes |-2\rangle\right]$$

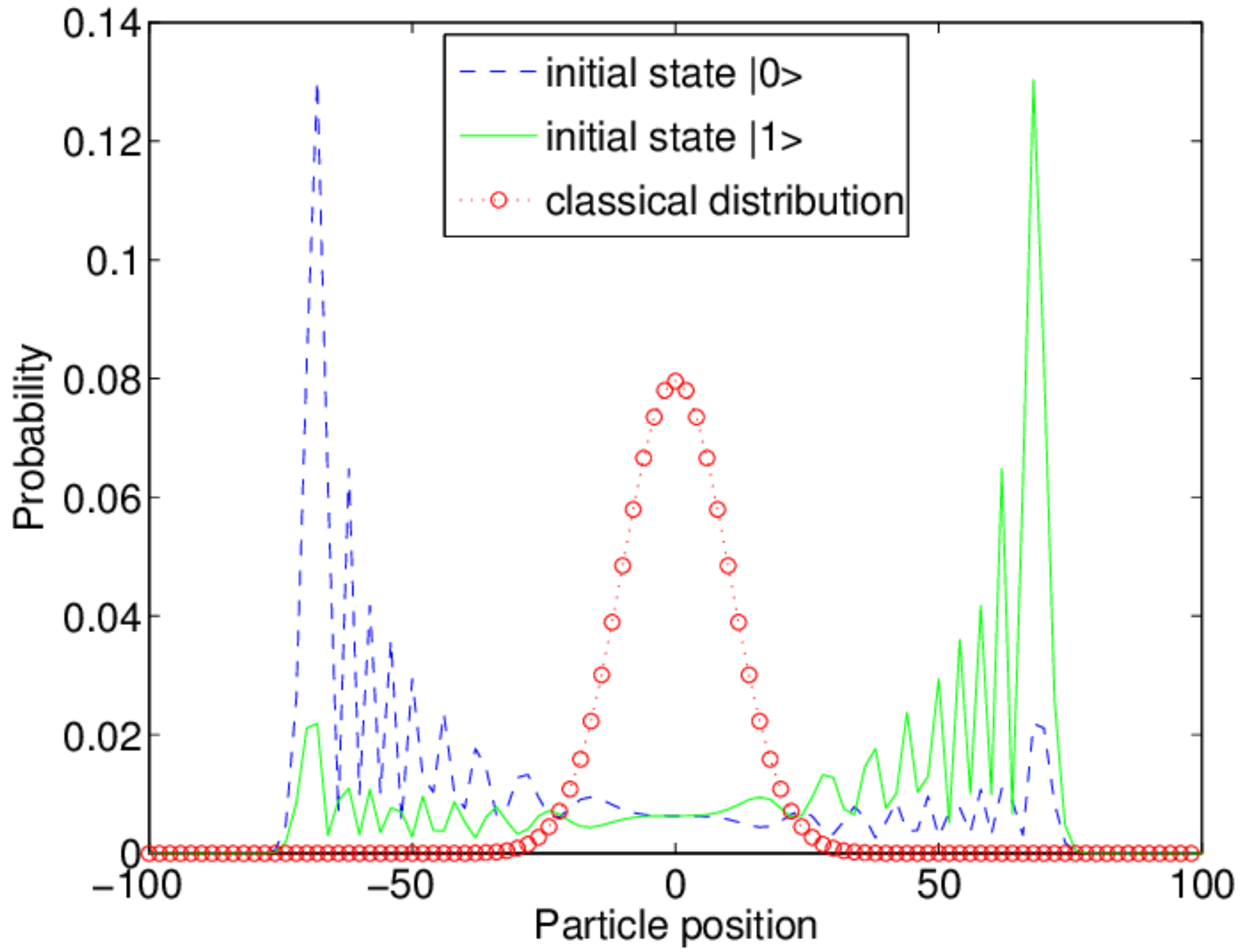
$$\frac{1}{2\sqrt{2}}\left(|\uparrow\rangle \otimes |3\rangle + |\downarrow\rangle \otimes |1\rangle + |\uparrow\rangle \otimes |-1\rangle - 2\,|\downarrow\rangle \otimes |-1\rangle - |\downarrow\rangle \otimes |-3\rangle\right)$$

	-3	-2	-1	0	1	2	3
0				1			
1			1/2		1/2		
2		1/4		1/2		1/4	
3	1/8		5/8		1/8		1/8

Demonstration - Quantum random walks

Probability distribution

- Classical
  - $\sigma^2 \sim T$
- Quantum
  - $\sigma^2 \sim T^2$
  - quadratically faster!



Chandrashekar, C. M.. (2010). Discrete Time Quantum Walk - Dynamics and Applications.

Demonstration - Quantum random walks

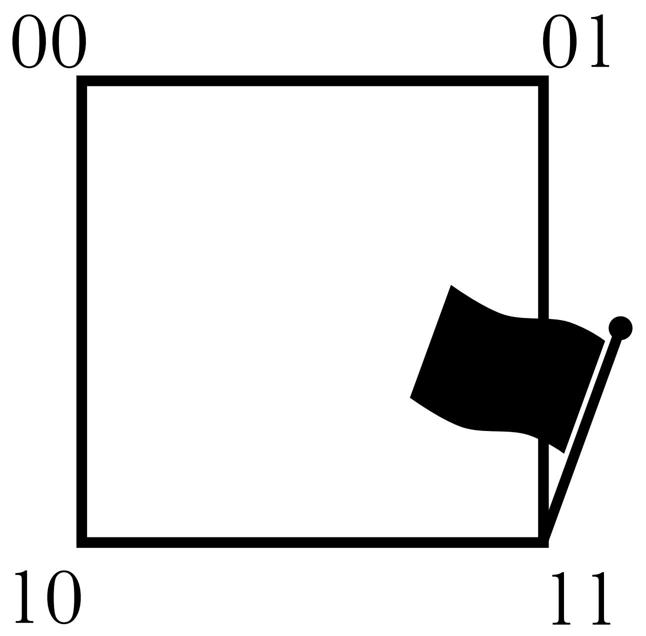
Quantum view: operations, circuits

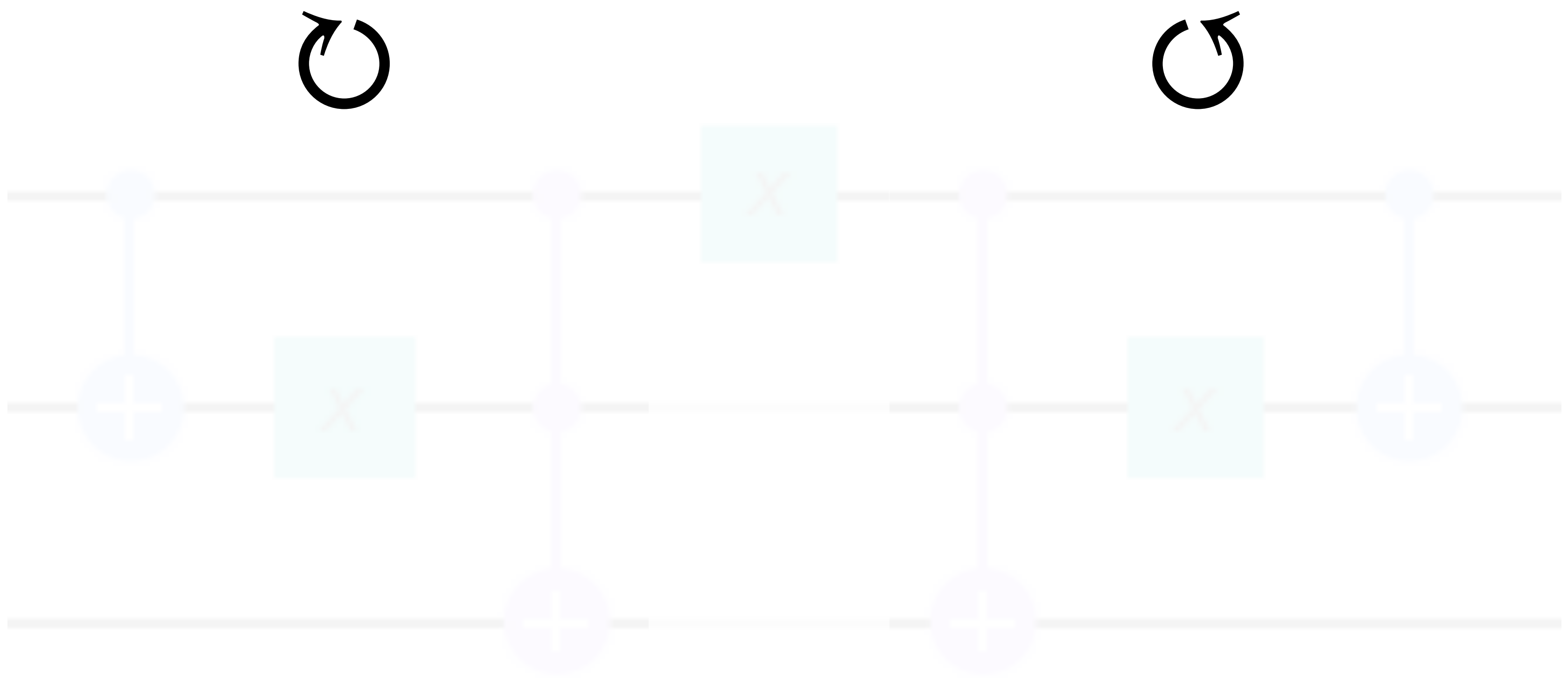
- States: node $\otimes$ coin

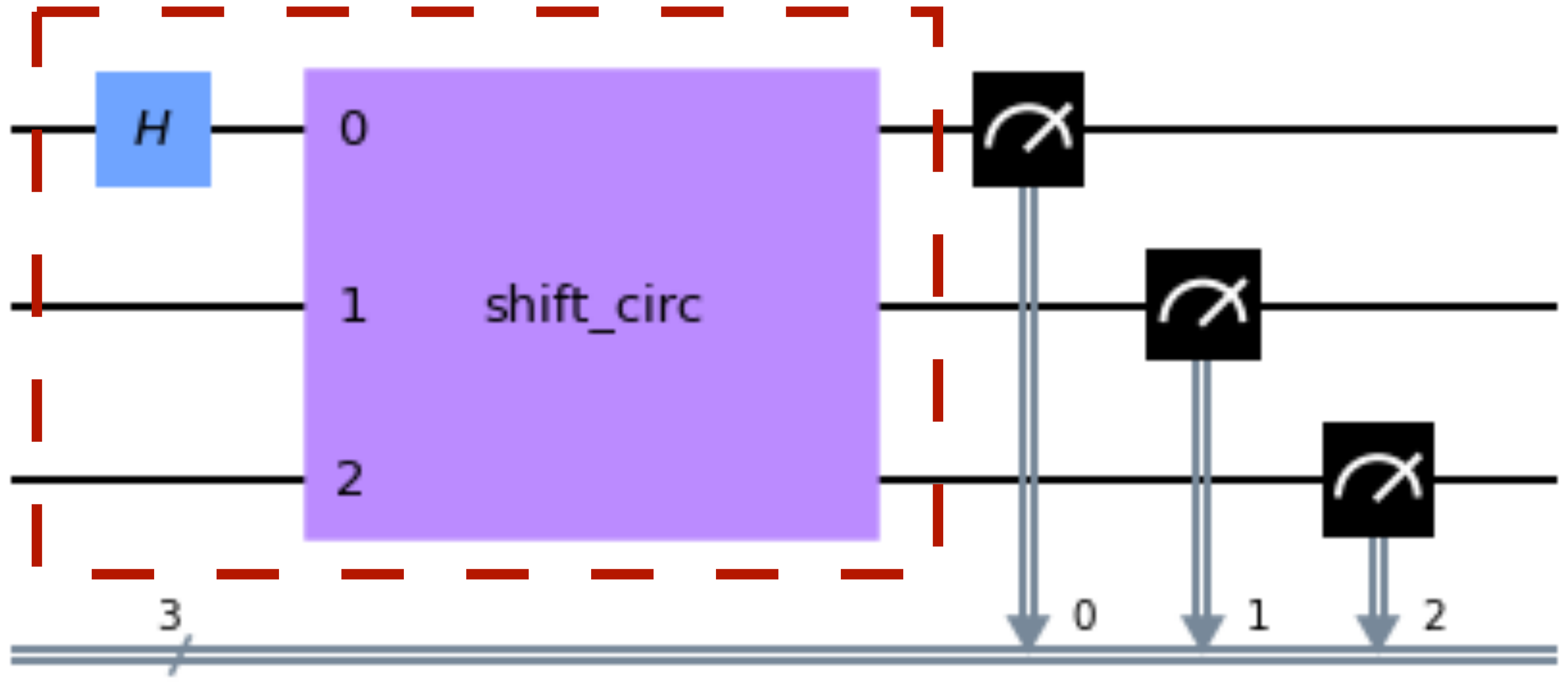
$$|k\rangle \otimes |q\rangle$$

- Operations
  - Coin: $C\,|k,q\rangle = (\mathbb{1} \otimes H)\,|k,q\rangle$
  - Shift: $S\,|k,q\rangle = |k + (-1)^q, q \oplus 1\rangle$

$$\left.\begin{array}{l} C \\ S \end{array}\right\} U = (SC)^T$$



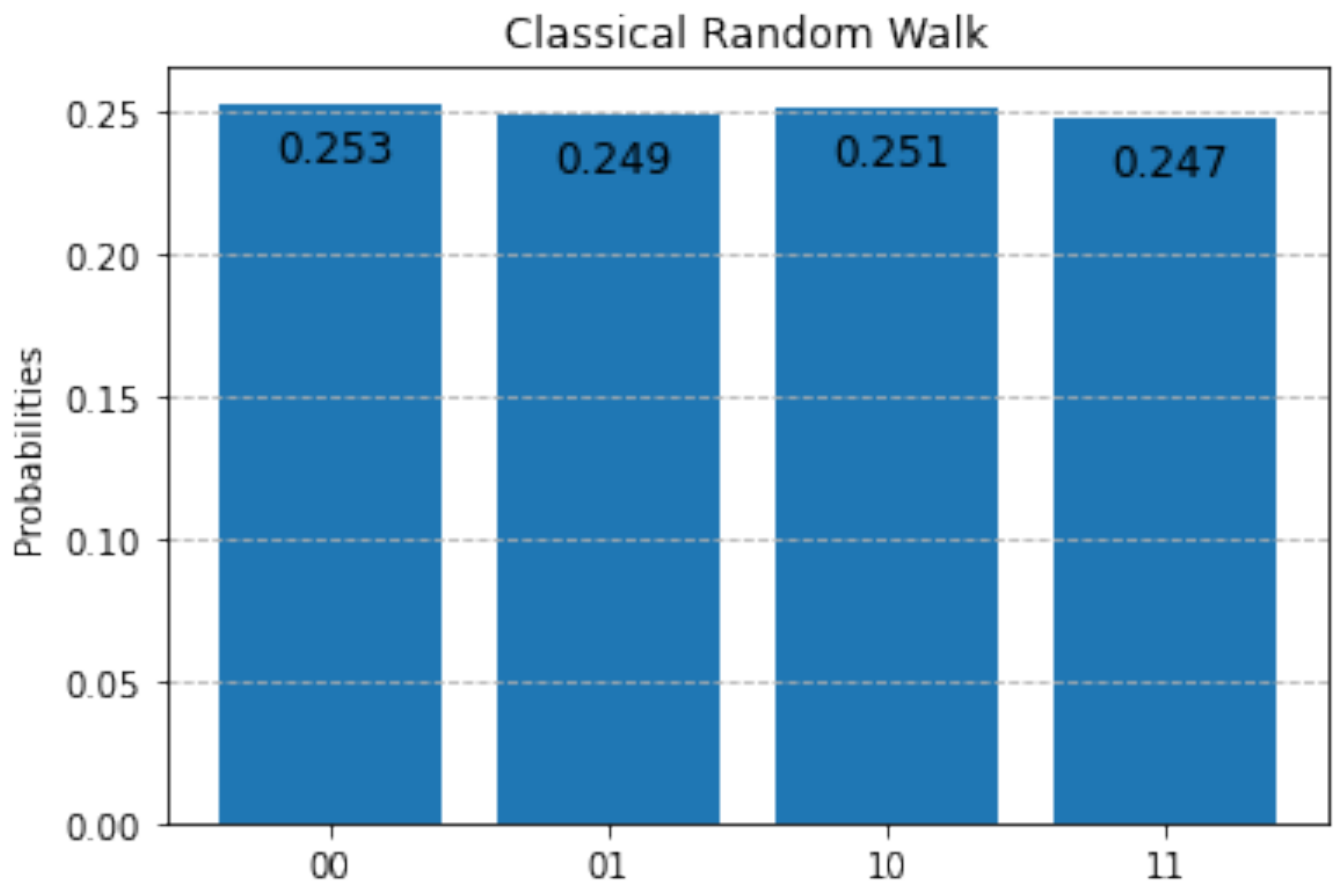




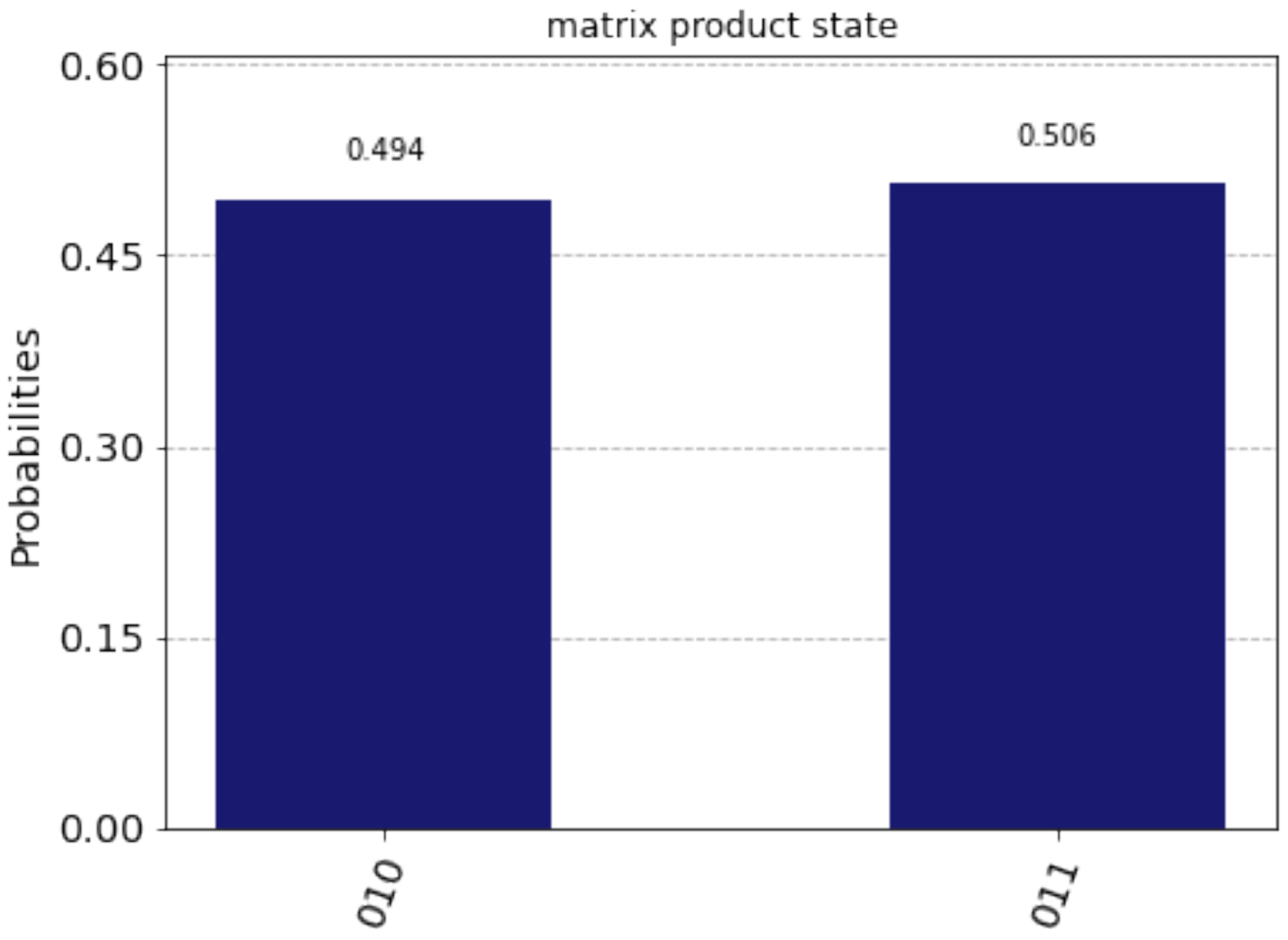
Demonstration - Quantum random walks

Simulation results: 3-step / 8192 shots

- Classical



- Quantum

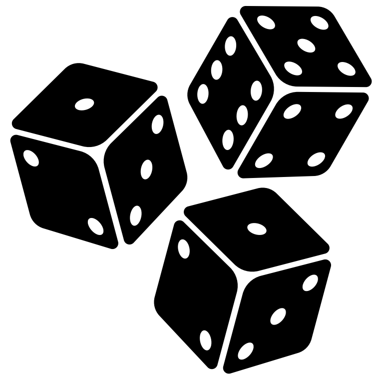


Build your quantum circuit

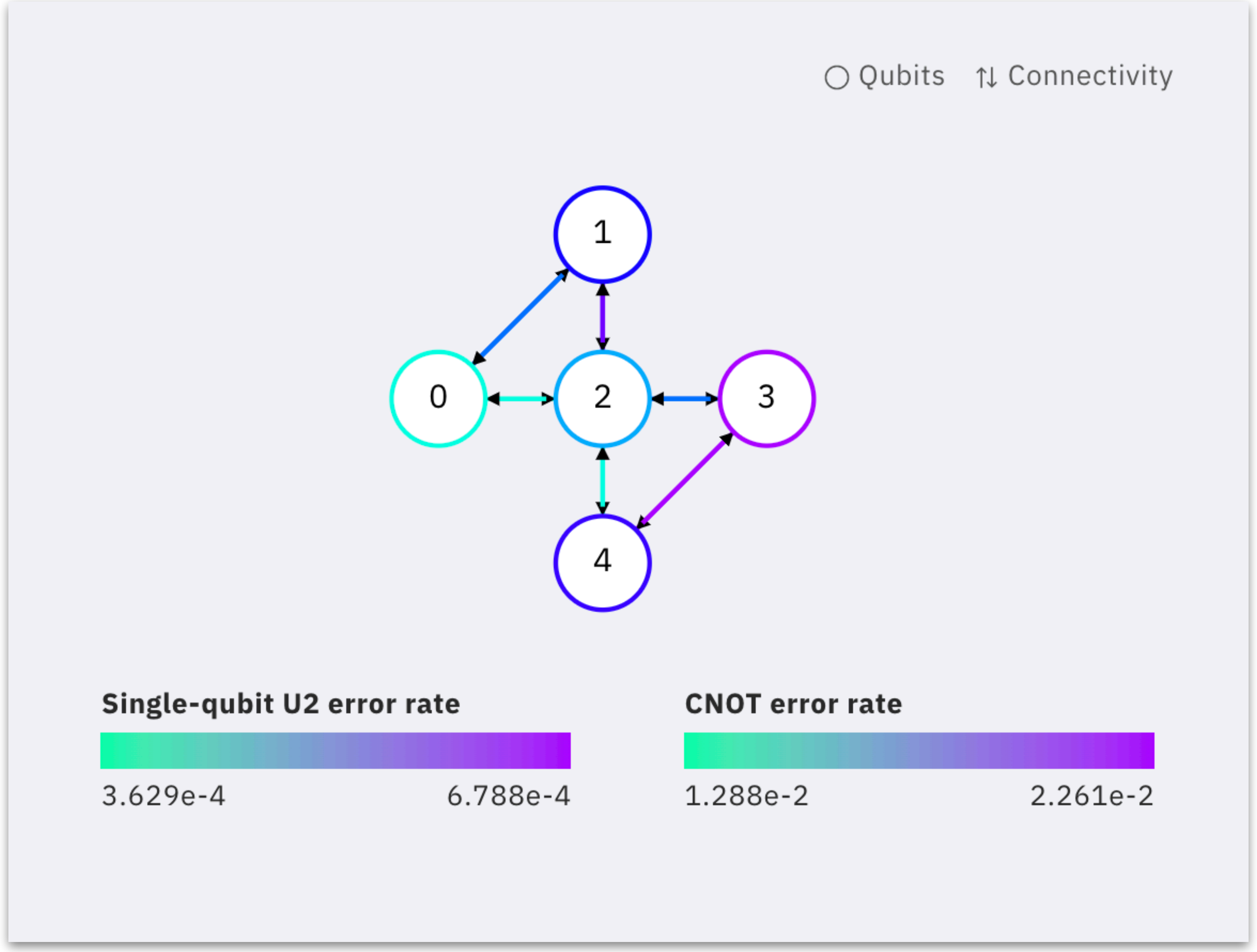
- Create IBMid
  - https://quantum-computing.ibm.com/
- Compose circuit
  - Operation / Measurement boxes
  - Qiskit Notebooks
- Run experiment
  - Platform: simulator / real devices
  - View / Compare results

- Default input: all zeros
  - Initial V.S. result
- Output state: $|q_n q_{n-1} \cdots q_1 q_0\rangle$

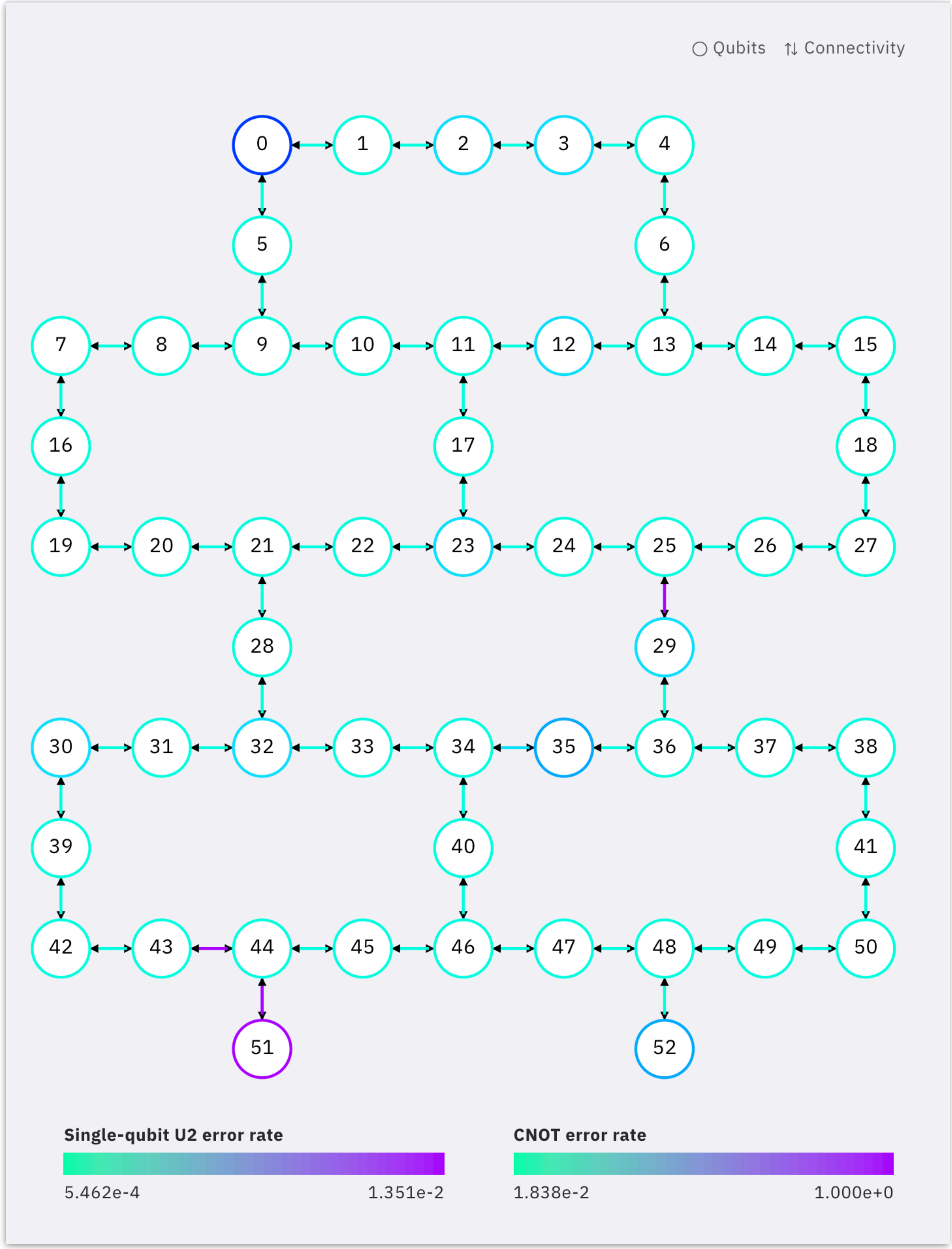
How many shots for an experiment provide a reliable outcome ?



Calibration



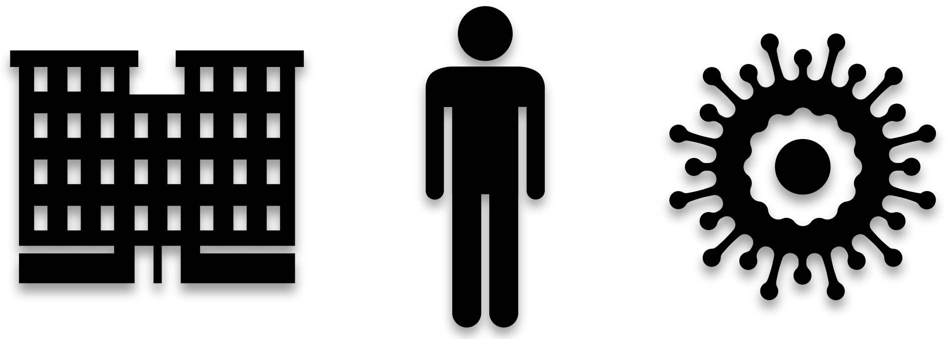
ibmq_5_yorktown - ibmqx2



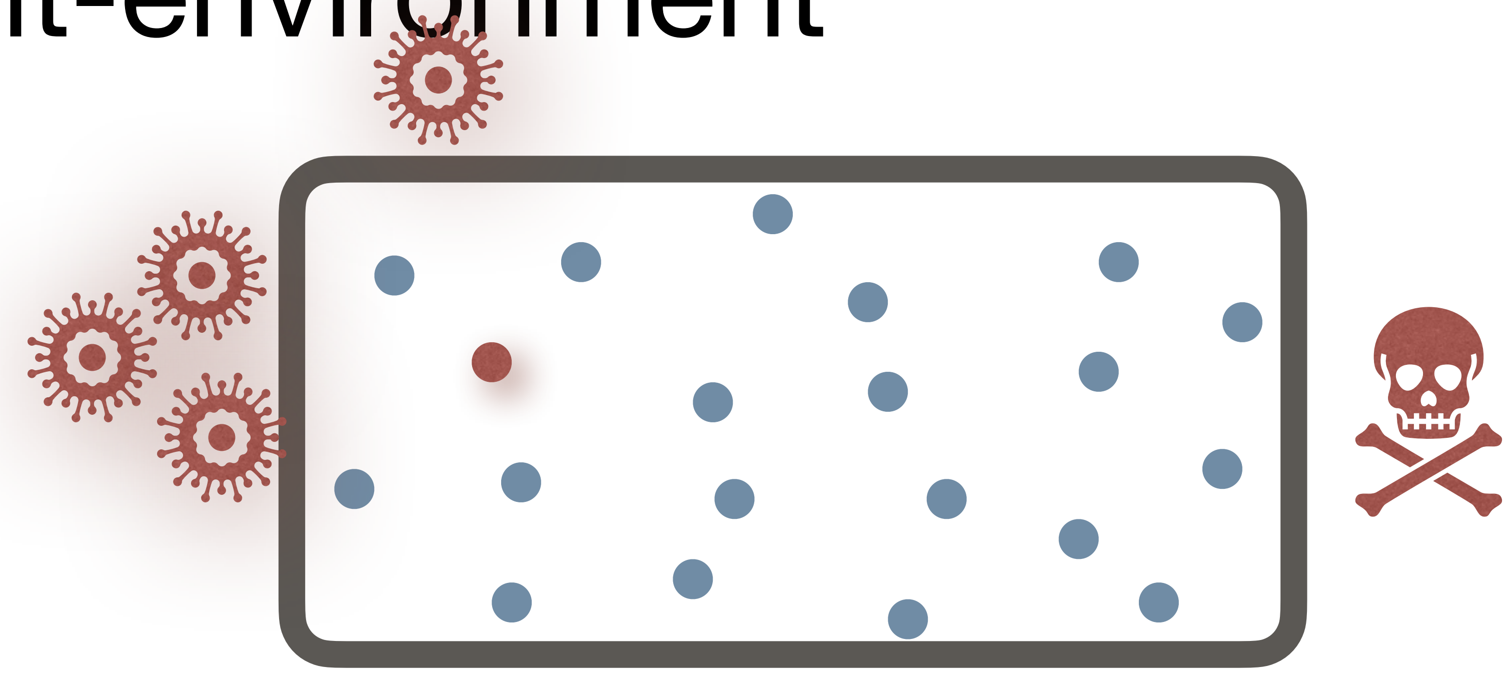
ibmq_rochester

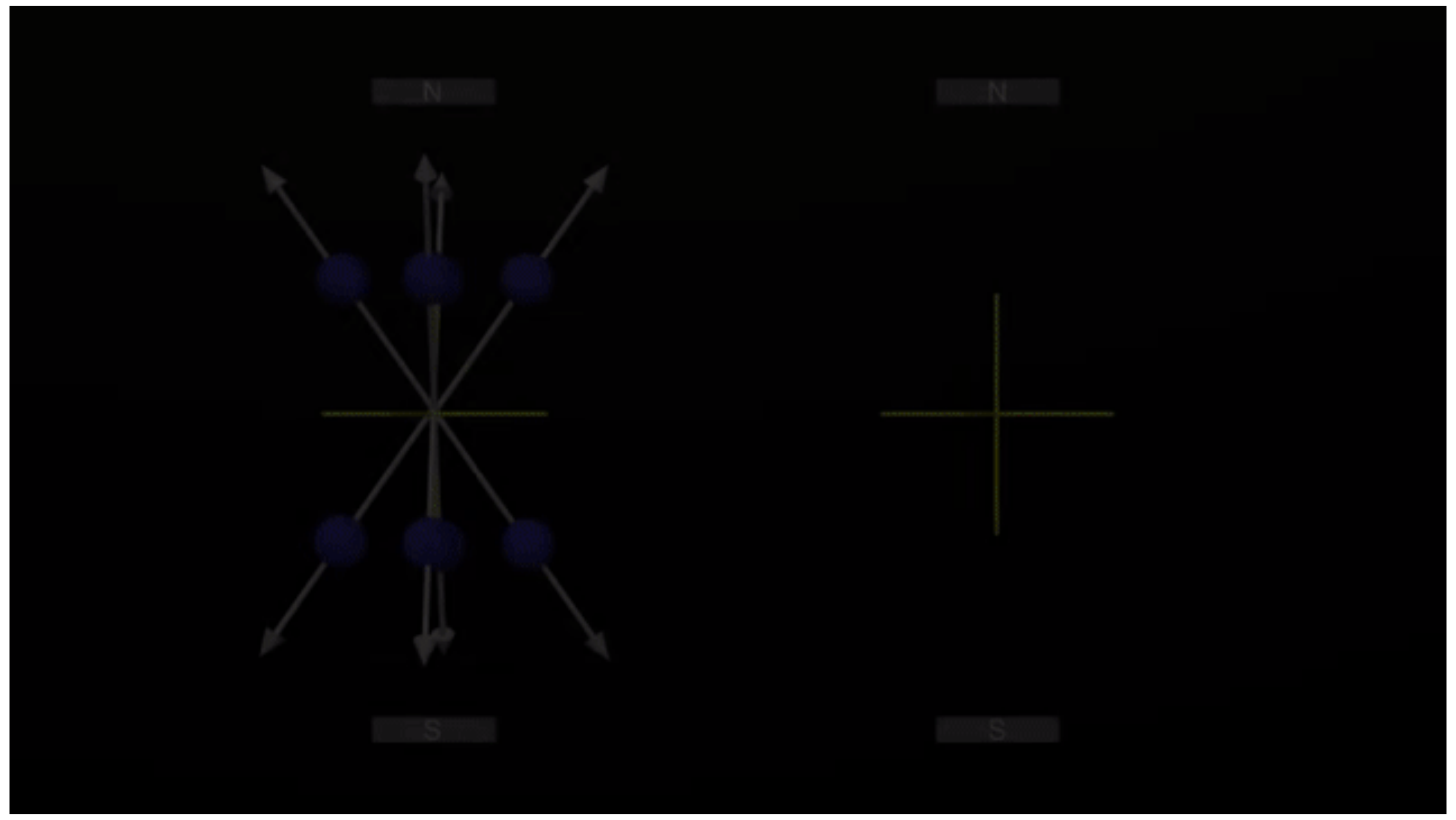
Relaxation

COVID-19 Model



- qubit-environment

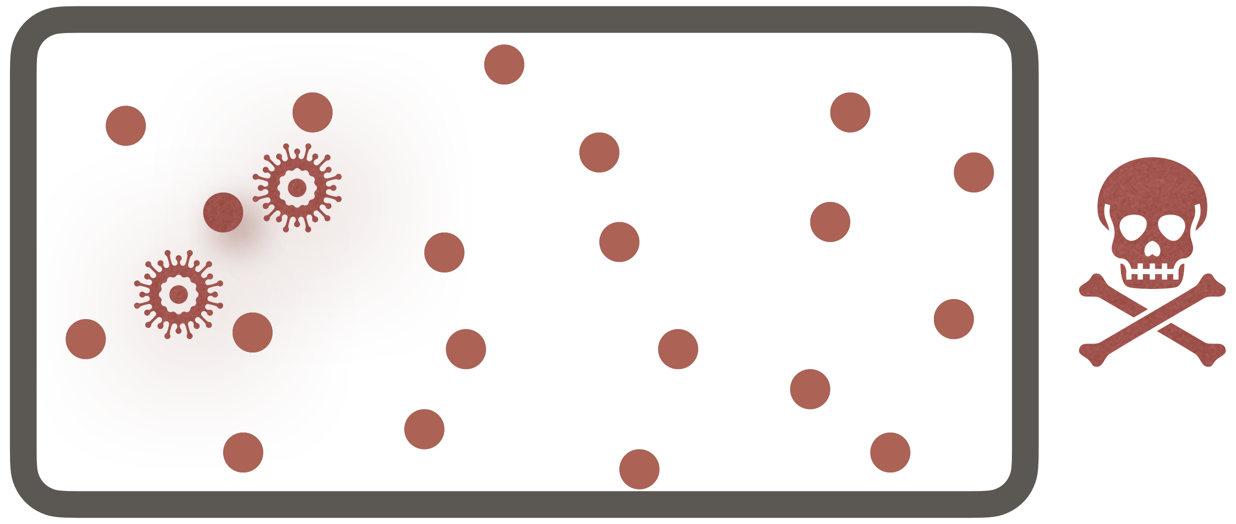


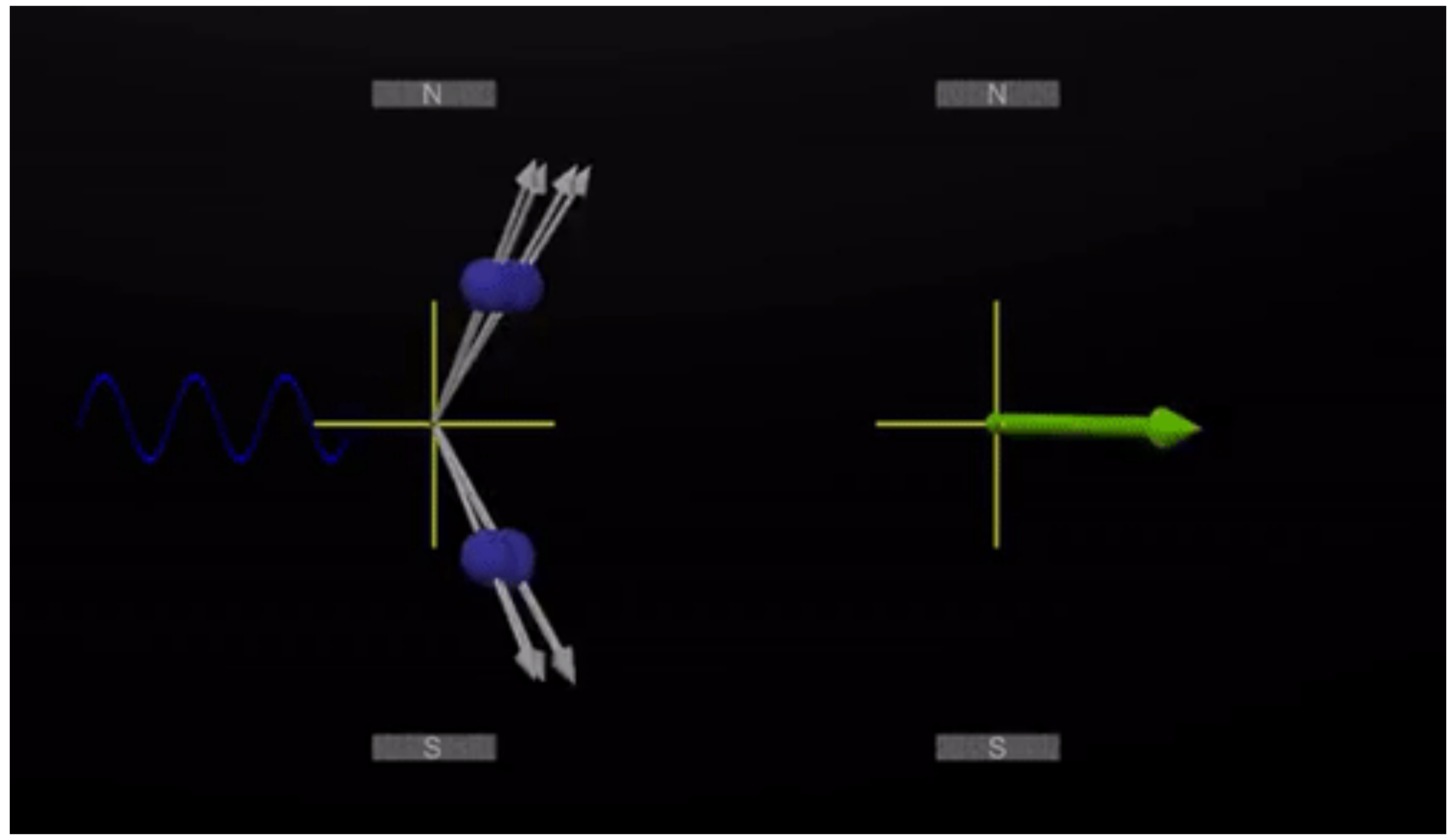


T_1

$$\begin{bmatrix}\rho_{11} & \rho_{12}\\ \rho_{21} & \rho_{22}\end{bmatrix}$$

- qubit-qubit





T_2

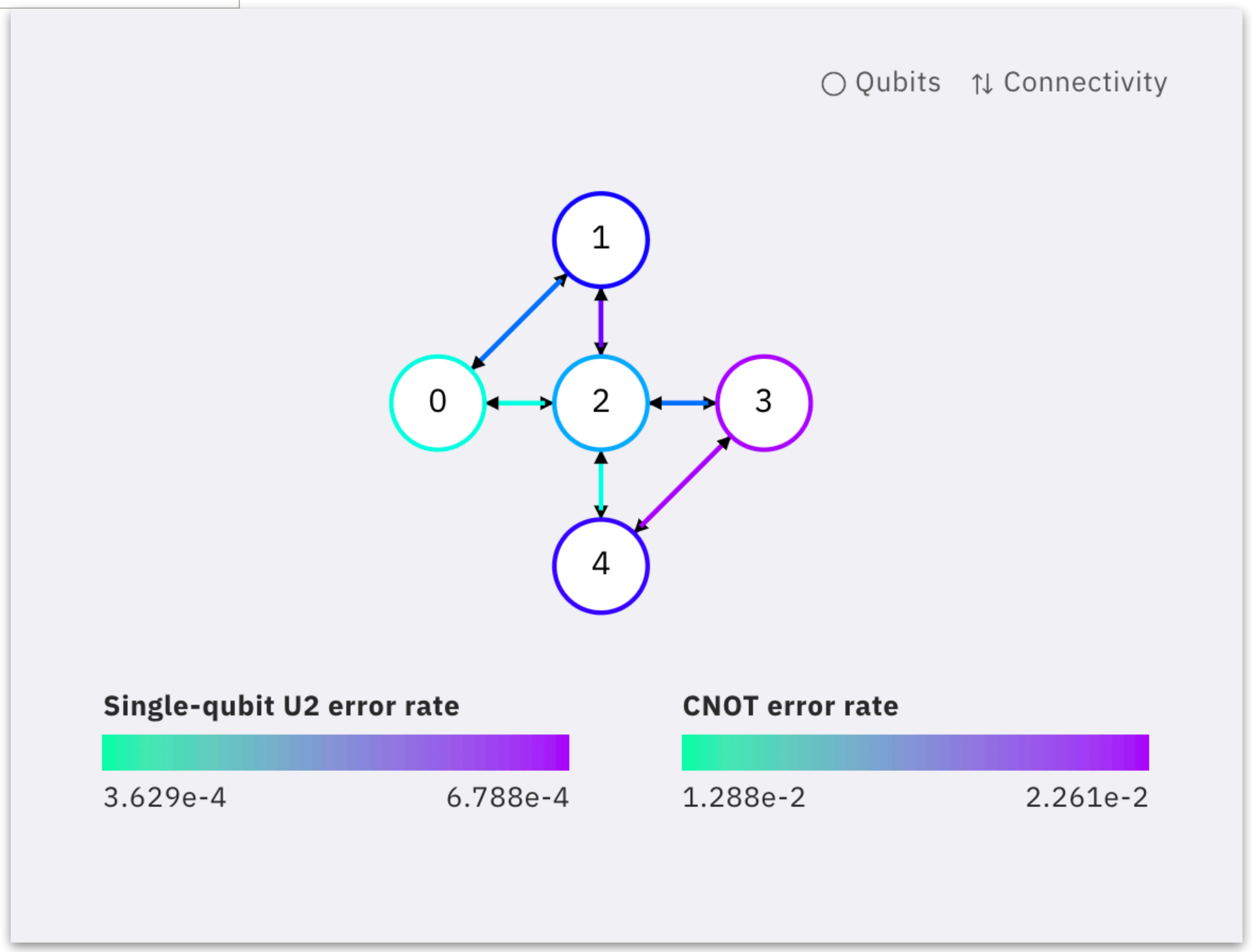
Nutrition Fact of Qubit

#	T1 (μs)	T2 (μs)	Frequency (GHz)	Readout error	Single-qubit U2 error rate
0	73.38	95.40	5.286	8.500E-03	3.629E-04
1	72.95	77.21	5.238	3.050E-02	5.608E-04
2	60.03	76.74	5.030	2.100E-02	4.458E-04
3	40.19	39.87	5.296	3.750E-02	6.788E-04
4	60.25	56.50	5.084	1.500E-02	5.997E-04

#	CNOT error rate
0	cx0_1: 1.615E-02, cx0_2: 1.380E-02
1	cx1_0: 1.615E-02, cx1_2: 2.076E-02
2	cx2_0: 1.380E-02, cx2_1: 2.076E-02, cx2_3: 1.628E-02, cx2_4: 1.288E-02
3	cx3_2: 1.628E-02, cx3_4: 2.261E-02
4	cx4_2: 1.288E-02, cx4_3: 2.261E-02

March 24, 2020 21:23:06 GMT +0800

Dashboard/Your Backends/[device_name]/Download Calibrations



ibmq_5_yorktown - ibmqx2

Questions ?

Supplement

Superposition of states and decoherence

The postulates of quantum mechanics

- Postulate 1

The state of an isolated physical system is represented by a state vector

$$|\psi(t)\rangle$$

in a Hilbert space. The state vector is unit, by which the system is completely described.

- Postulate 3

If the physical system is in a state $|\psi\rangle$, measurement of the variable Ω will yield one of the eigenvalues ω with probability

$$p(m) \propto \left|\langle\omega|\psi\rangle\right|^2.$$

The post-measurement state will be $|\omega\rangle$.

Principles of Quantum Mechanics 2 Ed., Plenum Press, 1994 , ISBN 0306447908

Why do we need quantum computers?

Some applications

- Shor's algorithm
- Grover's algorithm
- HHL algorithm
- Quantum simulation
- Adiabatic optimization

Duffing oscillator model
Second quantization

- Drift Hamiltonian

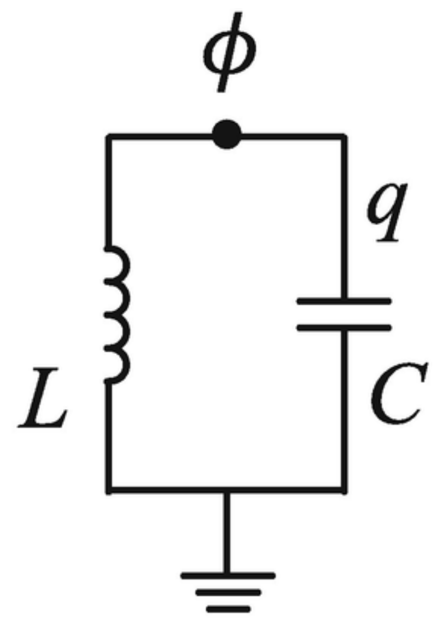
$$H = \frac{Q^2}{2C} + \frac{\Phi^2}{2L} = \hbar\omega\left(a^\dagger a + \frac{1}{2}\right)$$

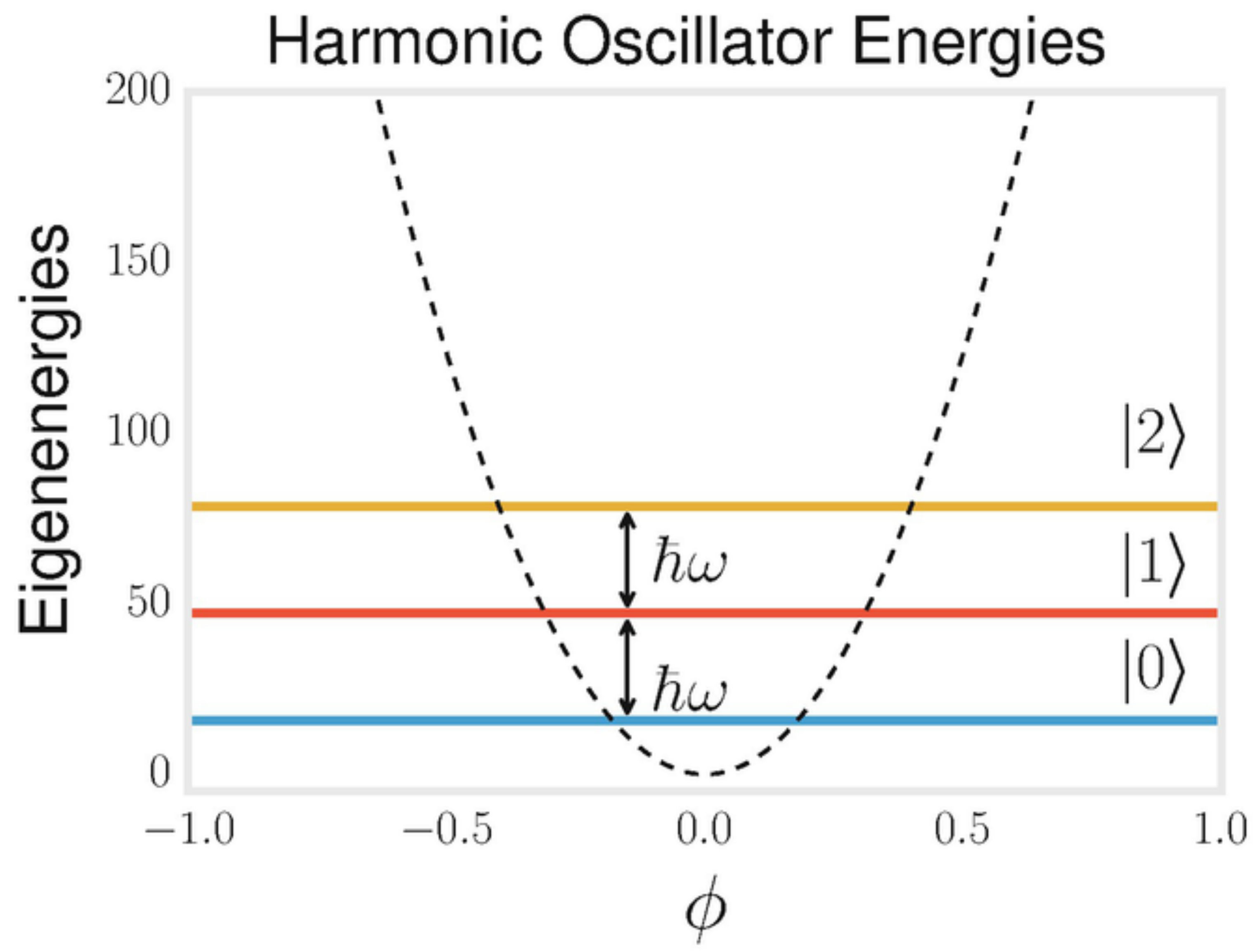
$$\uparrow$$

$$a = \frac{1}{\sqrt{2\hbar Z}}\left(\Phi + iZQ\right) \qquad a^\dagger = \frac{1}{\sqrt{2\hbar Z}}\left(\Phi - iZQ\right)$$

Impedance $Z = \sqrt{\frac{L}{C}}$

Frequency $\omega = \frac{1}{\sqrt{LC}}$





An Introduction to Superconducting Qubits and Circuit Quantum Electrodynamics, Springer International Publishing, Cham, 2018, ISBM 9783319927268

System Hamiltonian
LAB frame

- Individual

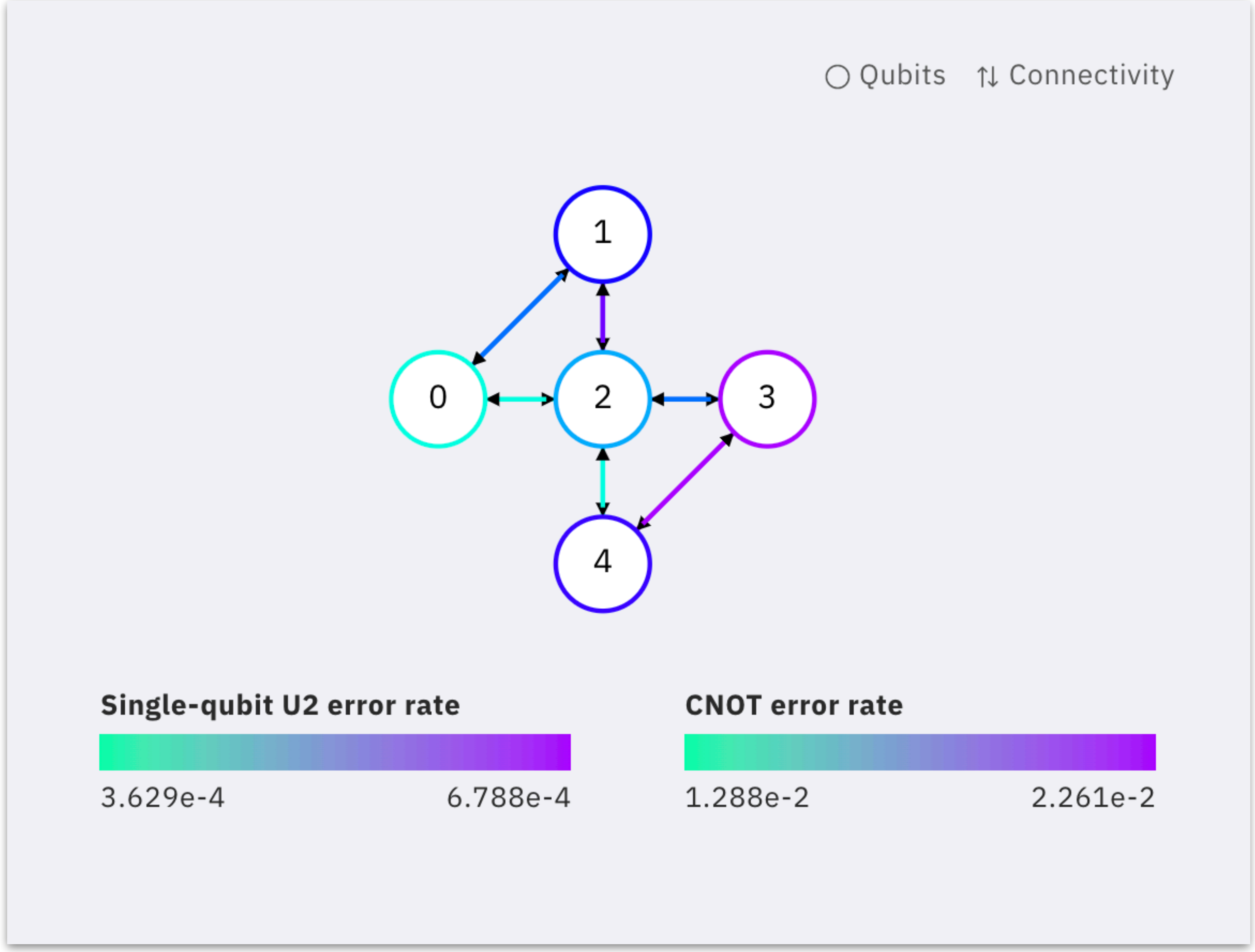
$$H_0/\hbar = \sum_{n=0}^{4} \omega_q^{(n)} a_n^\dagger a_n + \frac{\alpha_n}{2}\left(a_n^\dagger a_n - \mathbb{I}\right) a_n^\dagger a_n, \quad (\alpha_n < 0)$$

- Coupling

$$H_J/\hbar = \sum_{n,m} J_{nm}\left(a_n^\dagger + a_n\right)\left(a_m^\dagger + a_m\right)$$

- Pumping

$$H_d/\hbar = \sum_{n=0}^{4} \Omega_d^{(n)} D^{(n)}(t)\left(a_n^\dagger + a_n\right)$$



ibmq_5_yorktown - ibmqx2

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- YouTube: Qiskit URL https://www.youtube.com/Qiskit