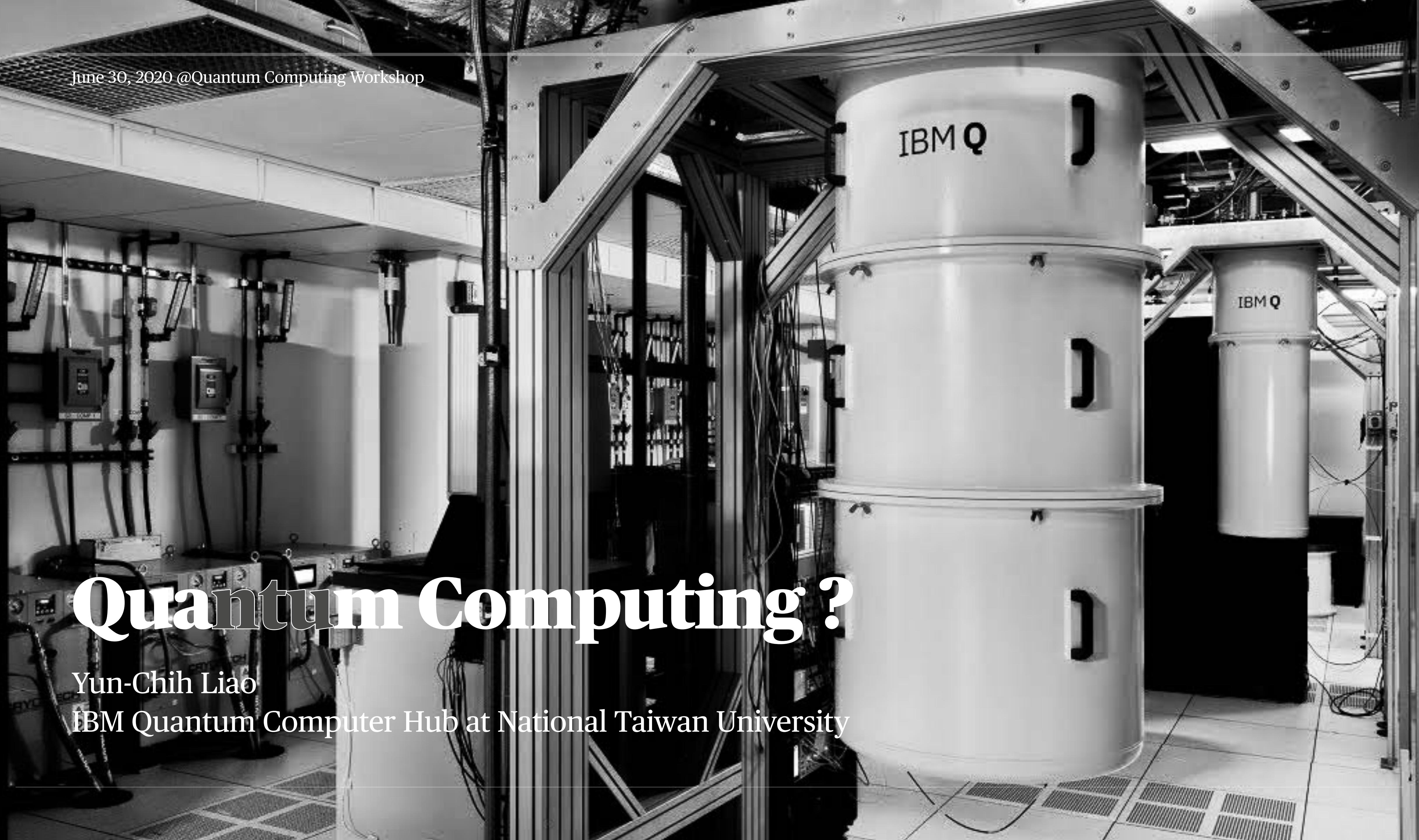


June 30, 2020 @Quantum Computing Workshop

Quantum Computing?

Yun-Chih Liao

IBM Quantum Computer Hub at National Taiwan University



Outline

History

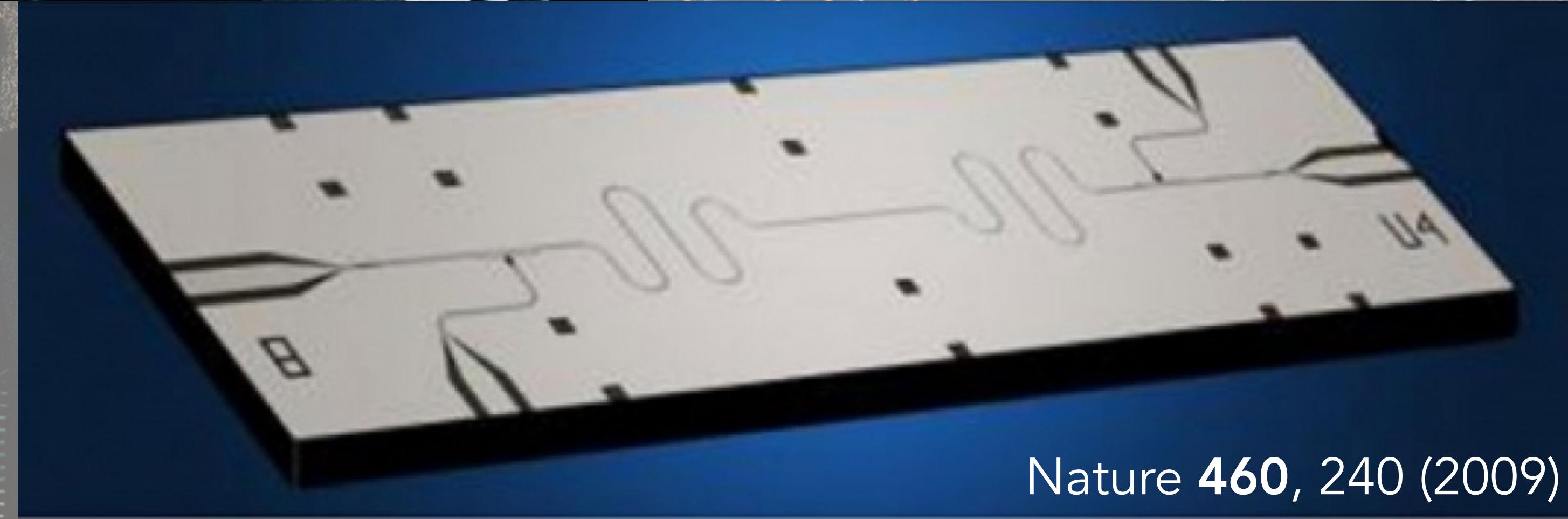
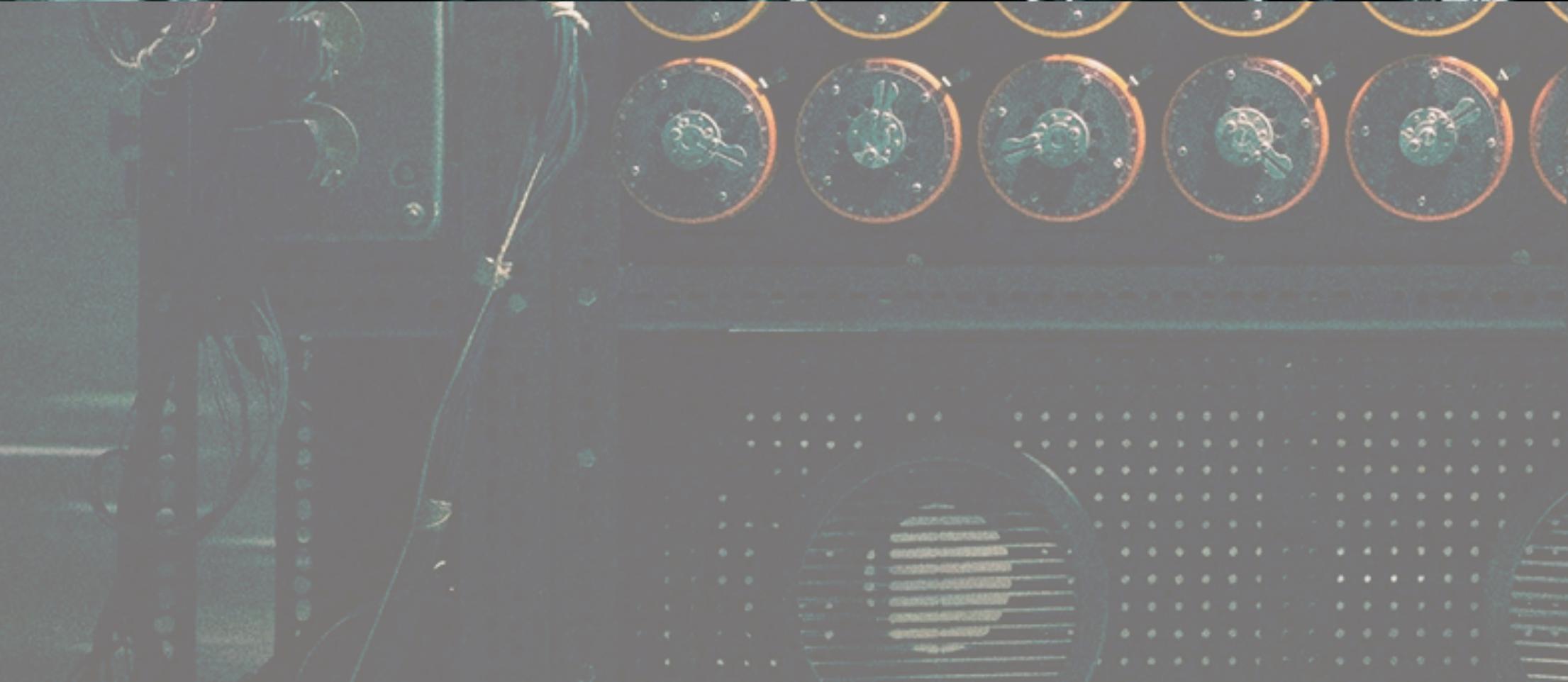
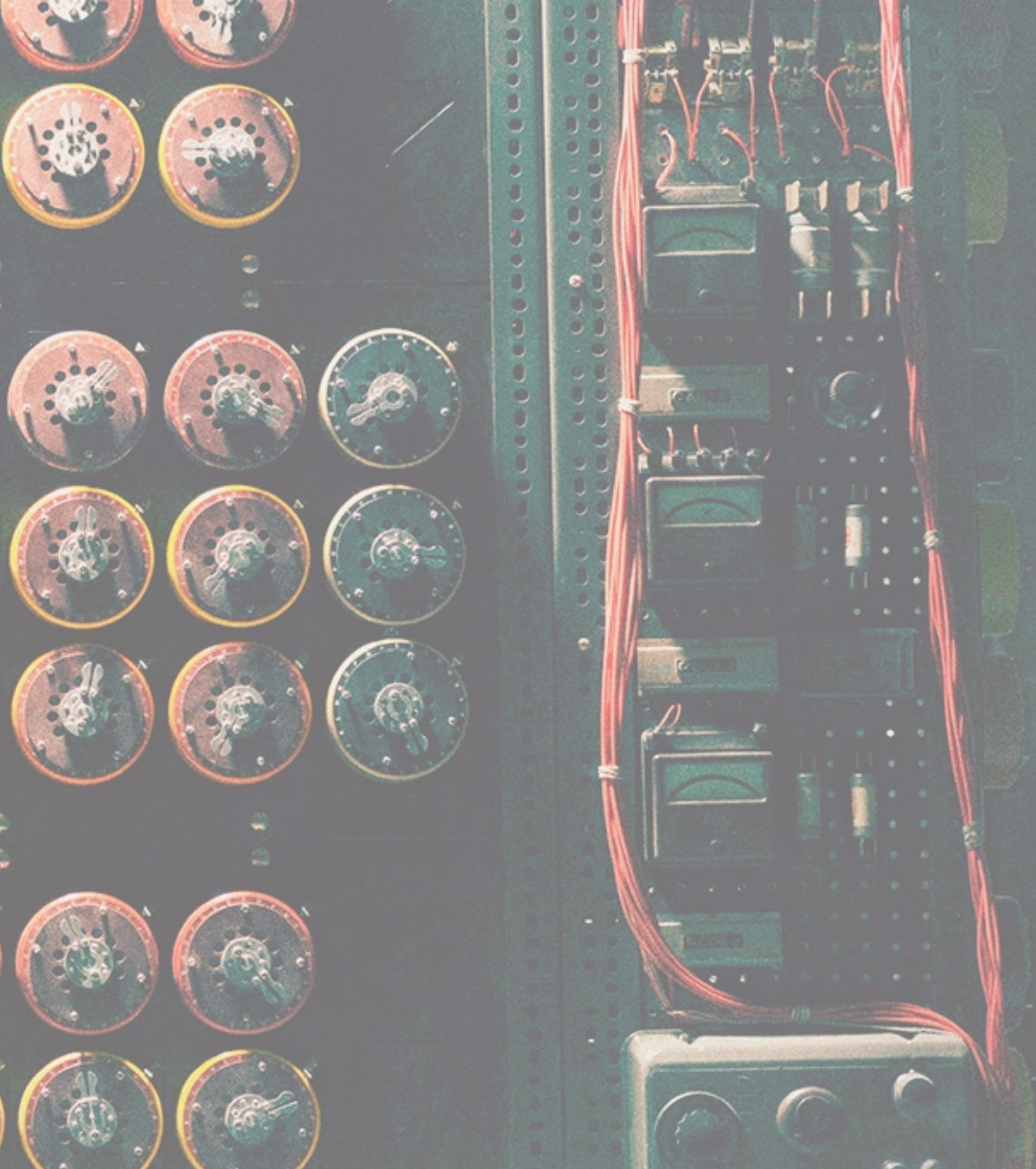
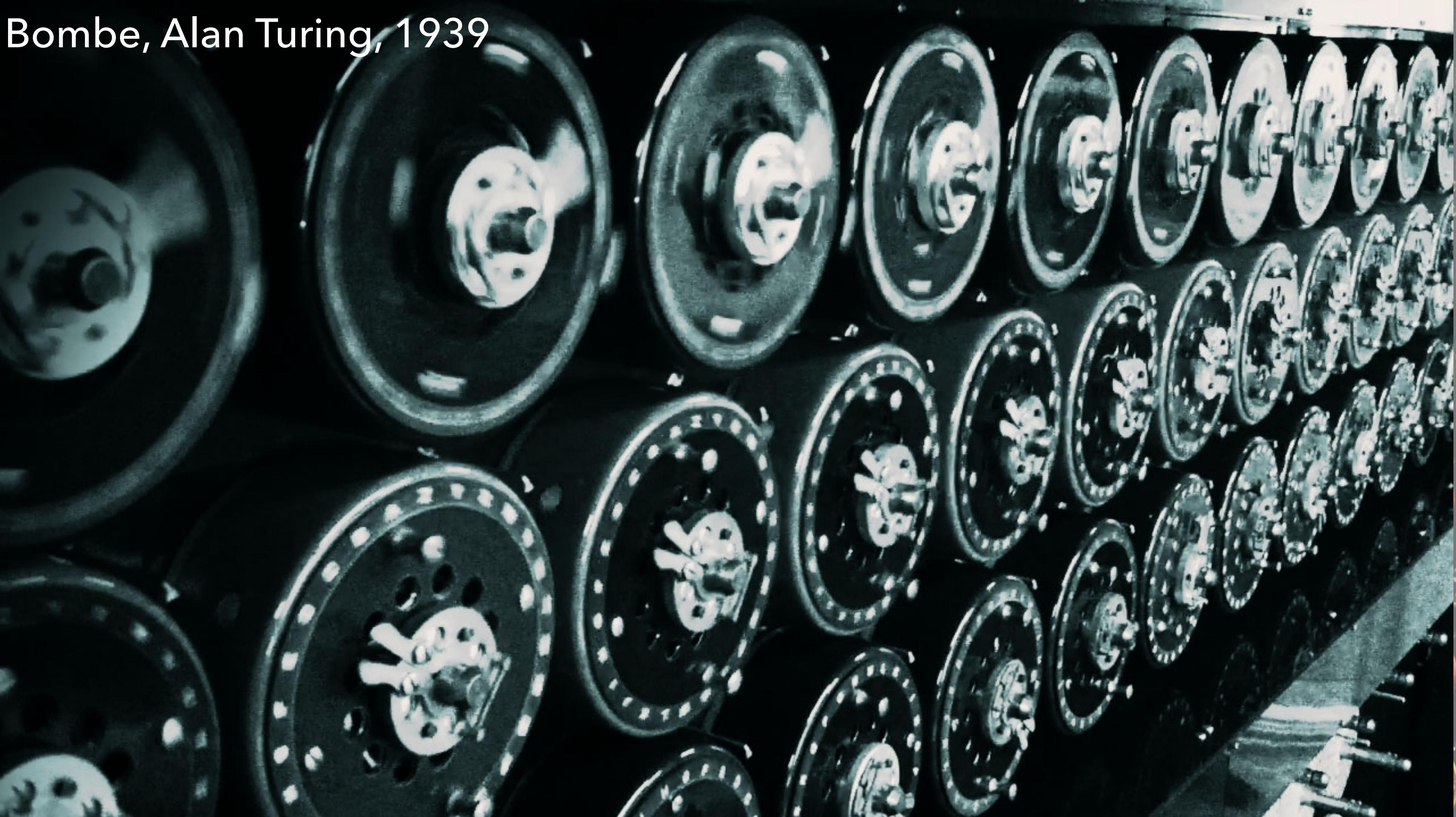
Background

Quantum Language

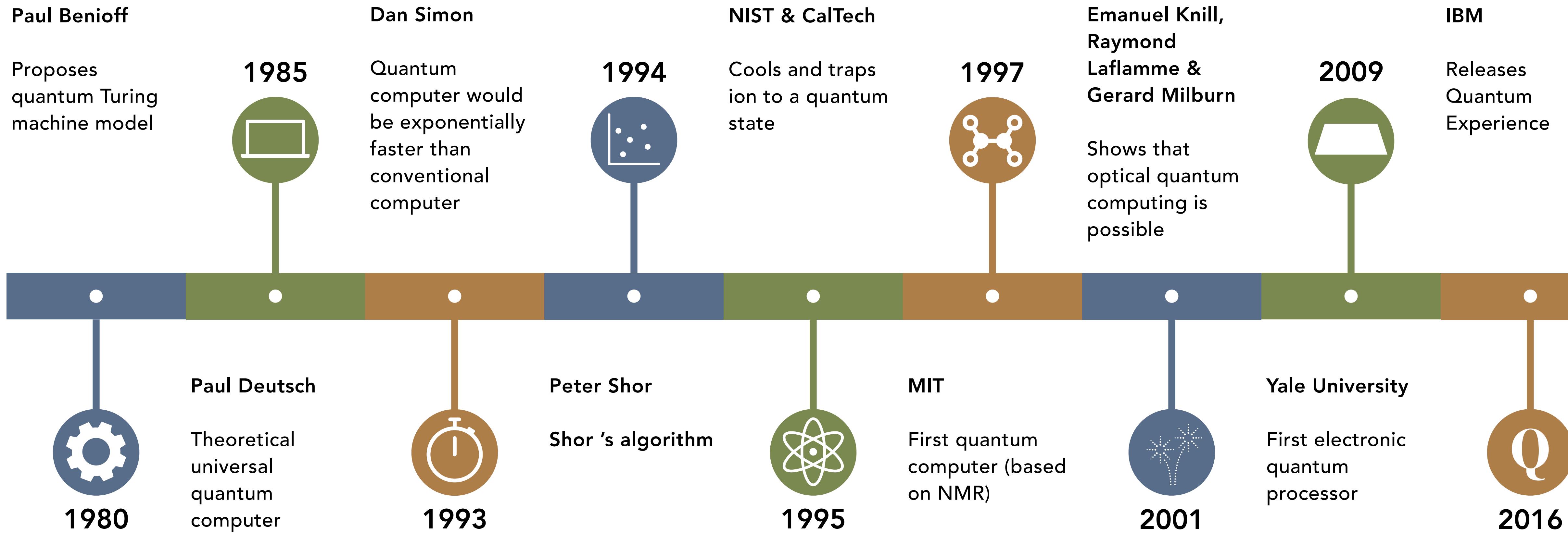
Quantum Computers



Bombe, Alan Turing, 1939

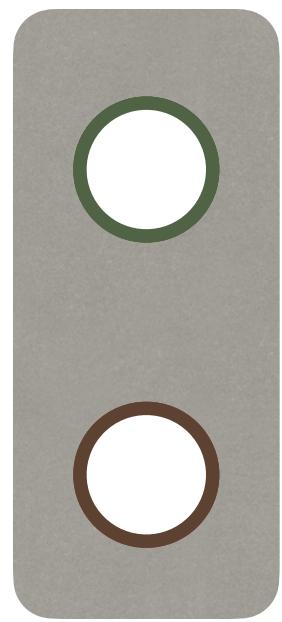


History

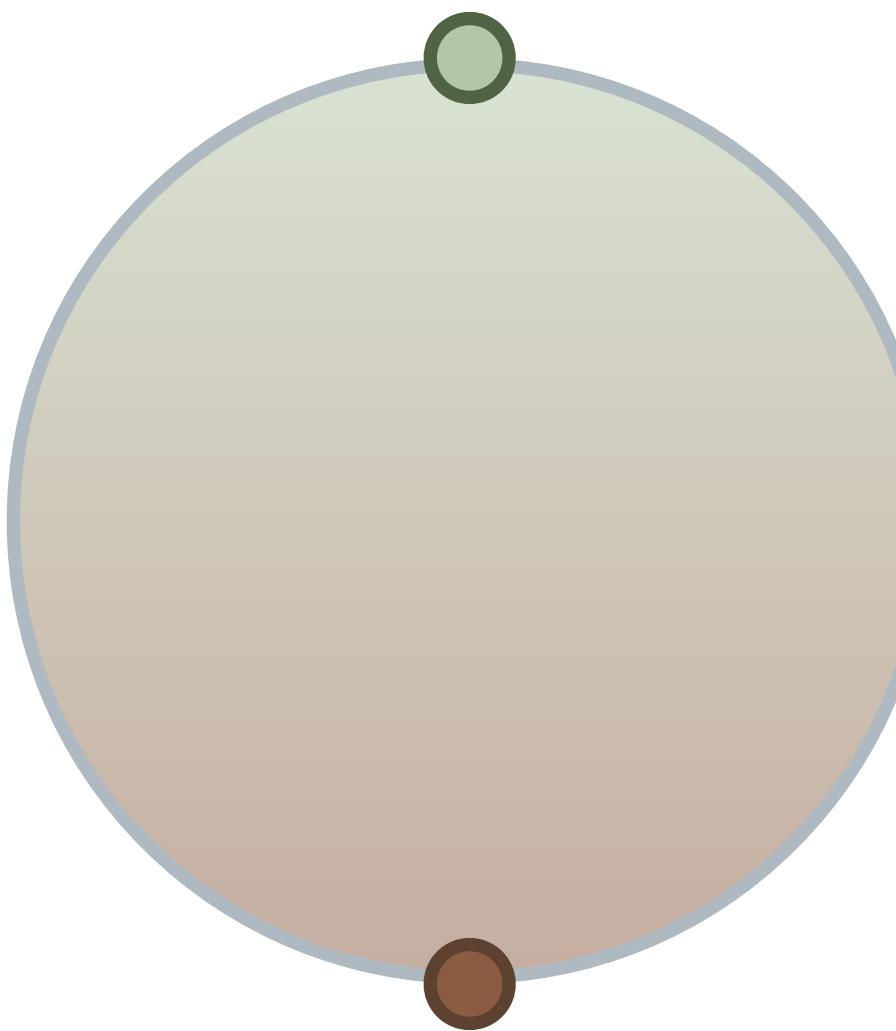


Classical v.S. Quantum

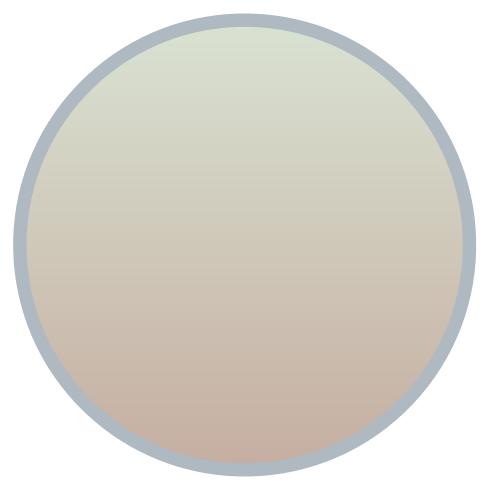
- Bit



- Qubit

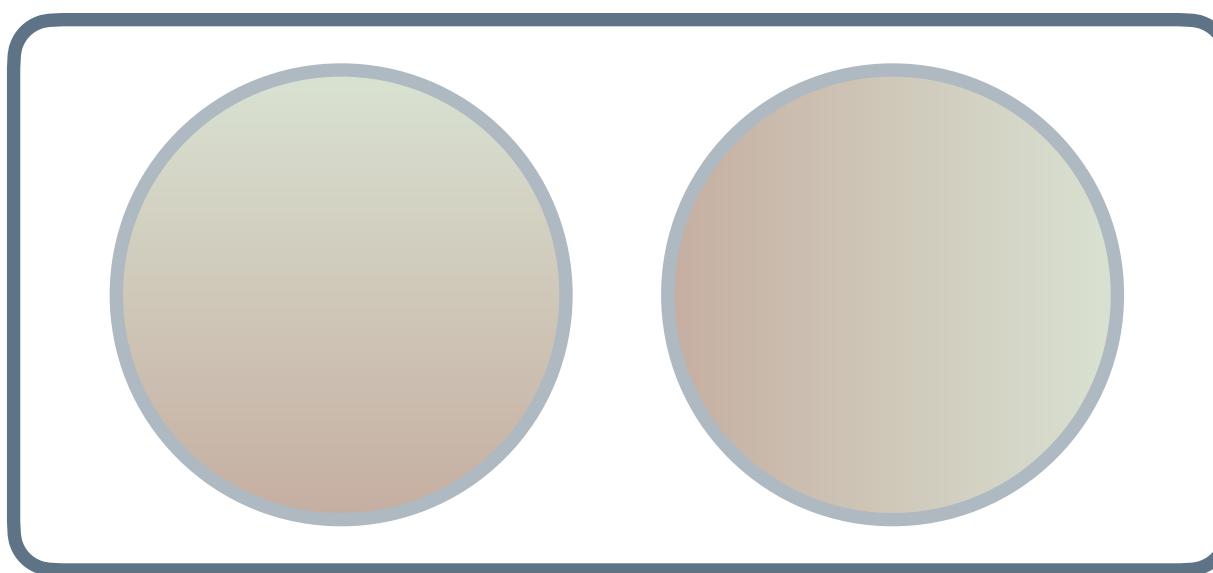


States



$$|\psi\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

Basis: $\{|0\rangle, |1\rangle\}$

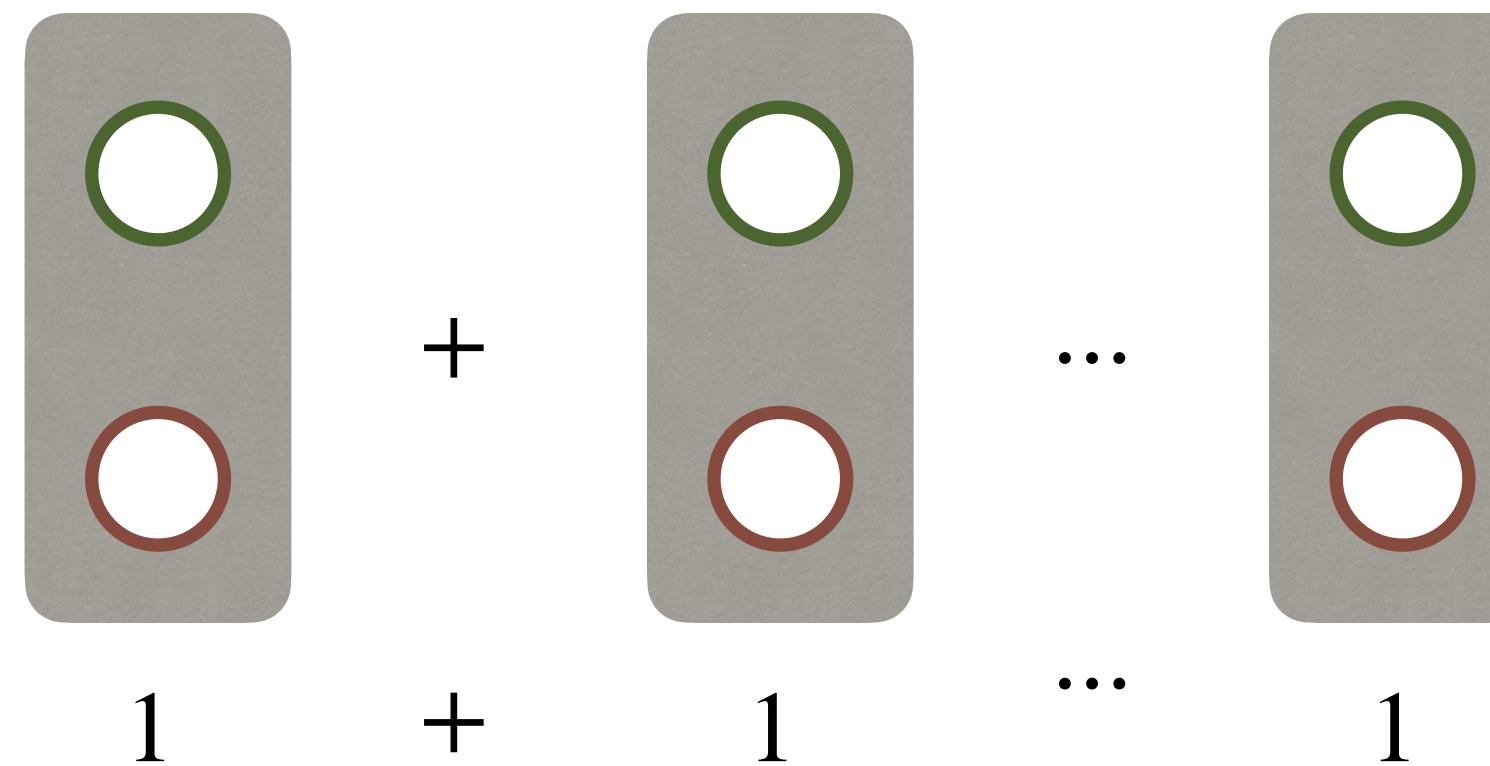


$$|\psi_1\rangle \otimes |\psi_2\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \otimes \begin{bmatrix} \gamma \\ \delta \end{bmatrix} = |\psi_1\psi_2\rangle$$

Basis: $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$

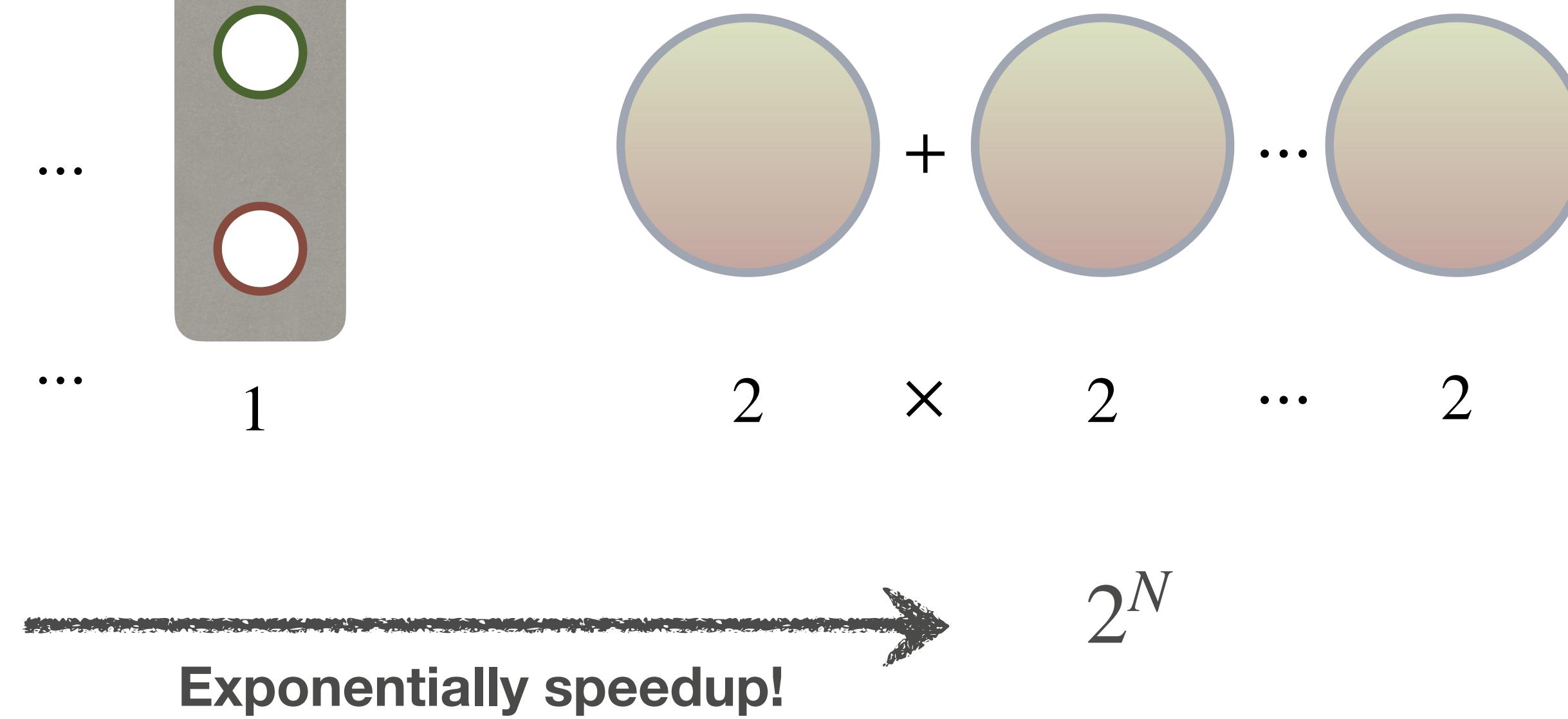
Speedup computational time

- Conventional



N

- Quantum

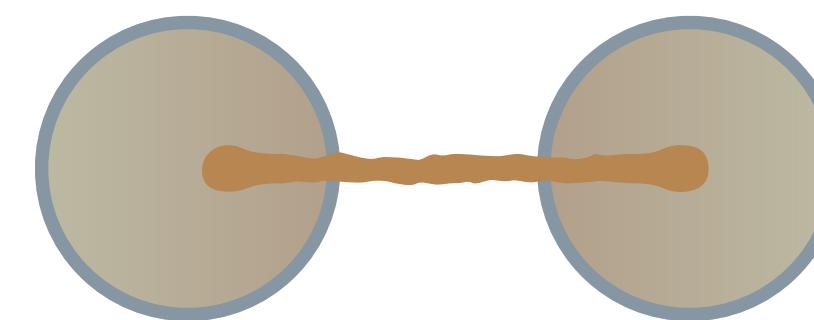
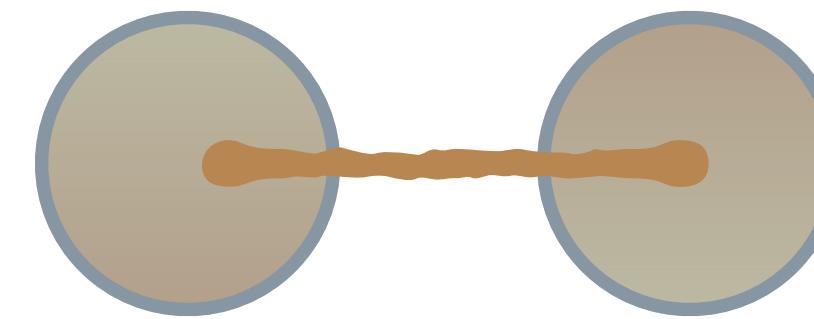
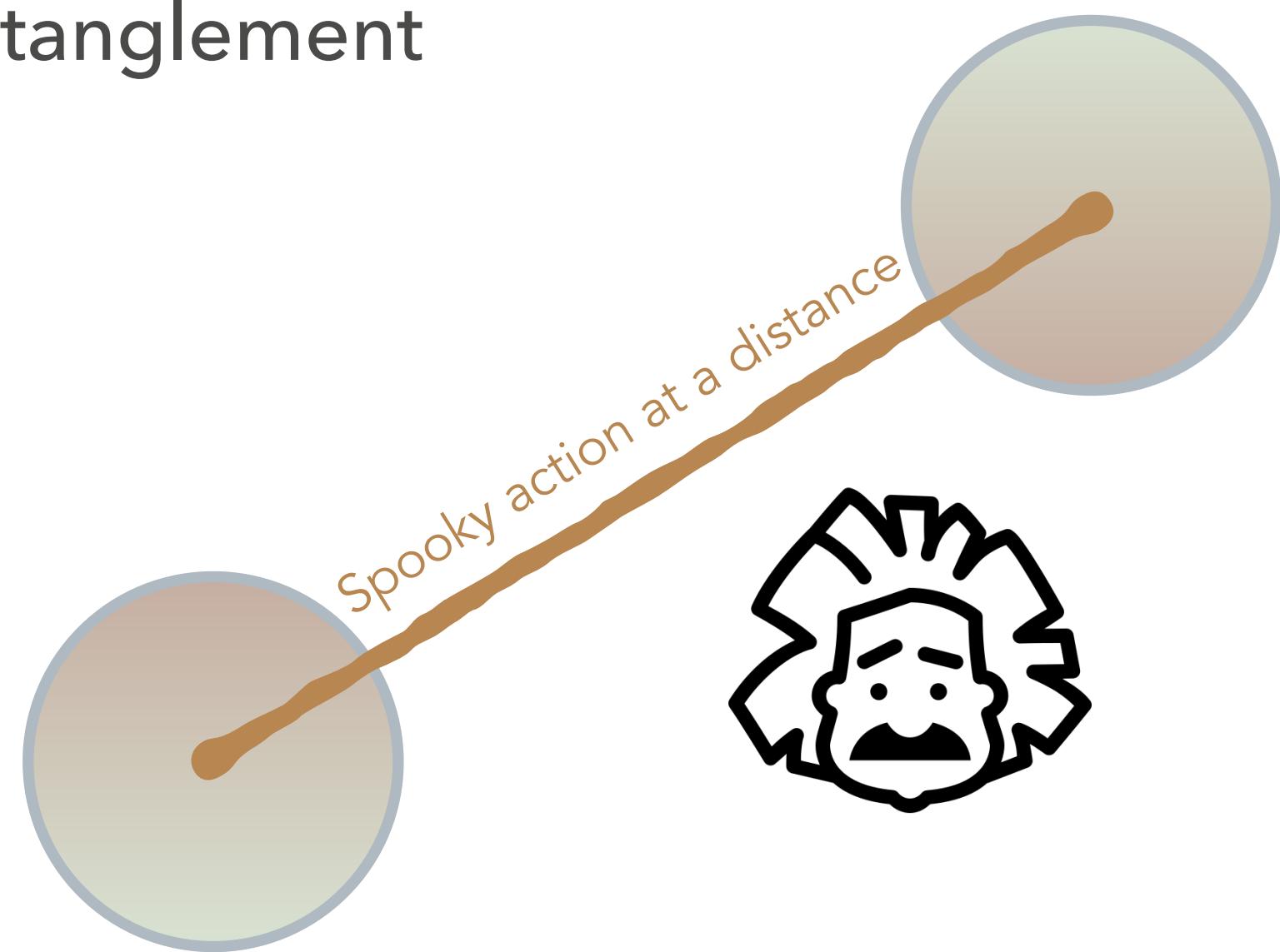


2^N

Exponentially speedup!

Why so powerful ?

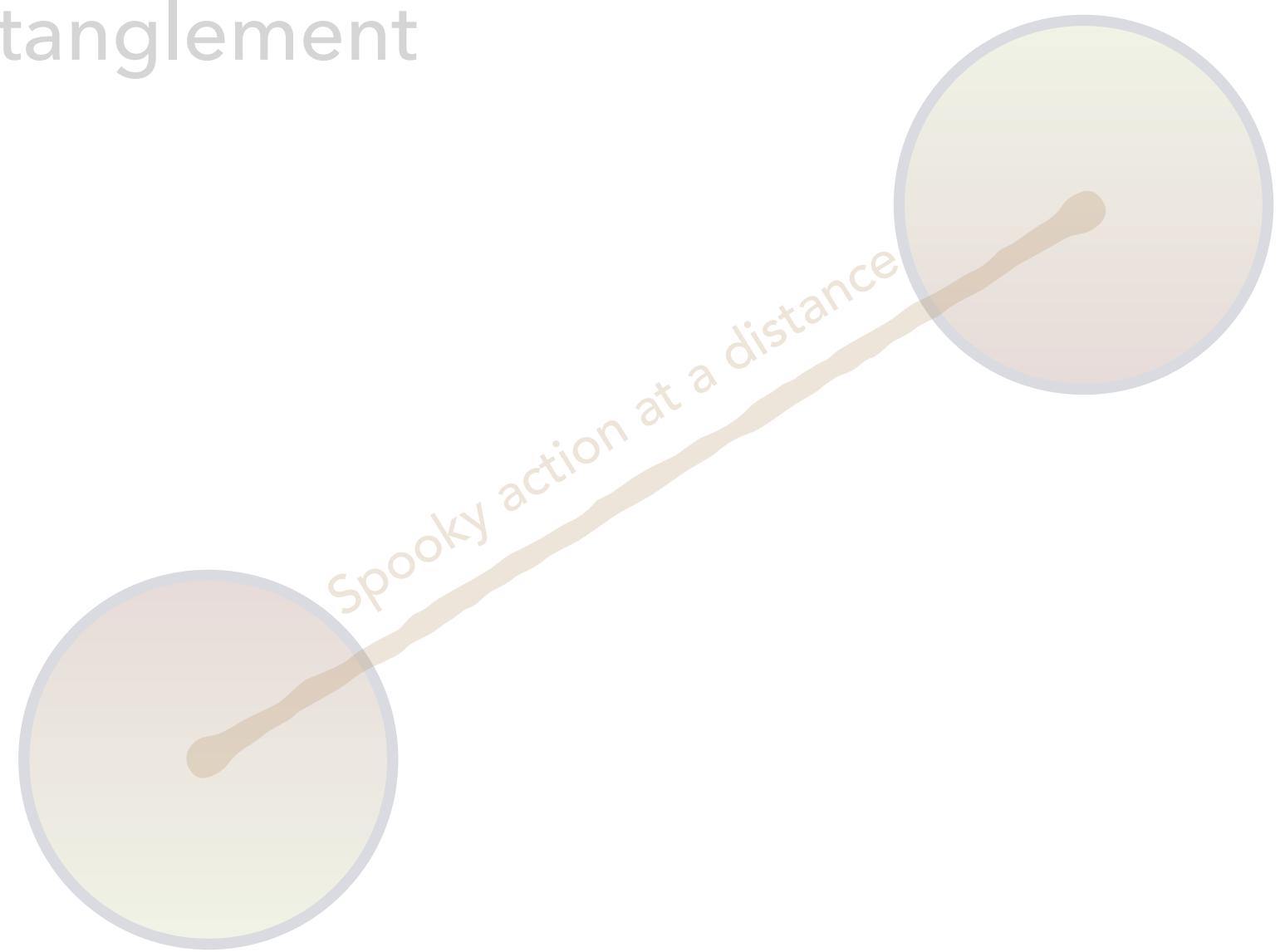
- Entanglement



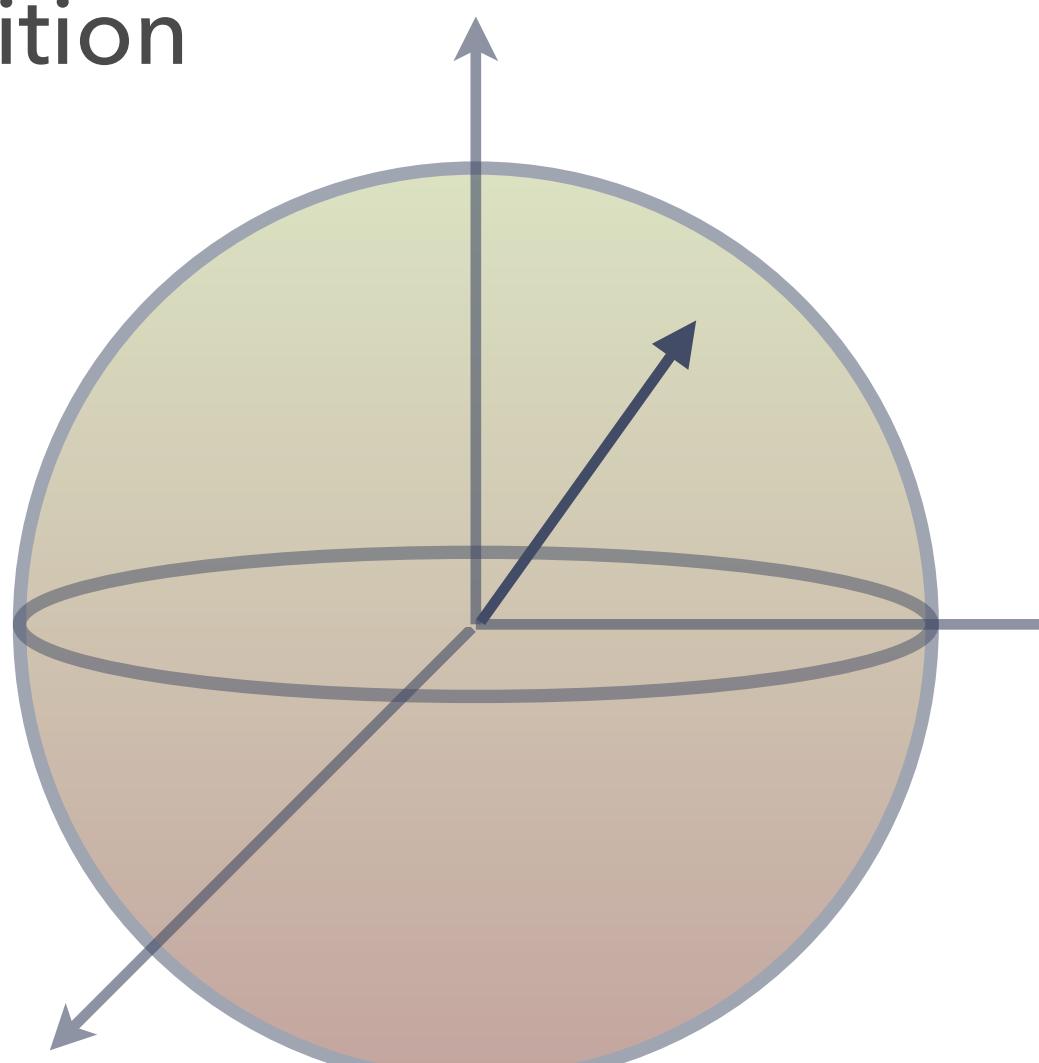
speedup quantum algorithms

Why so powerful ?

- Entanglement

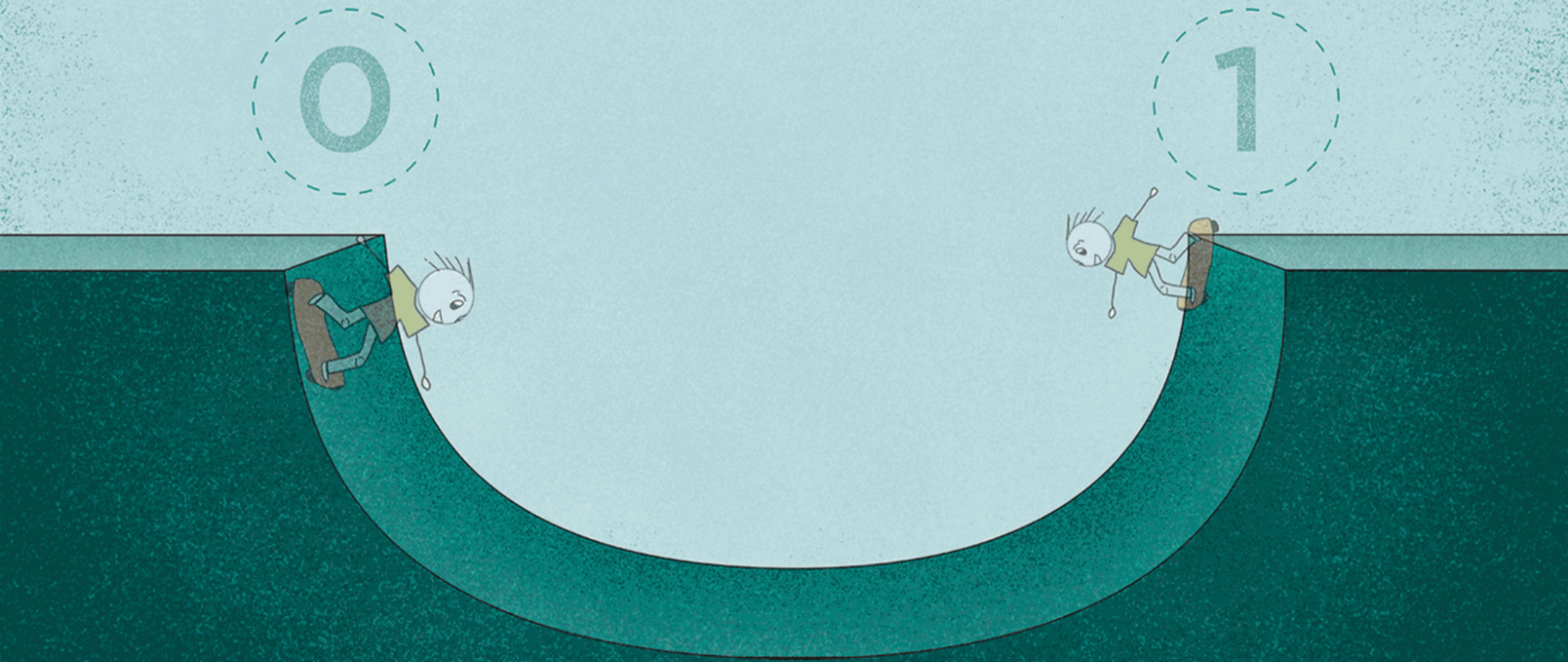


- Superposition



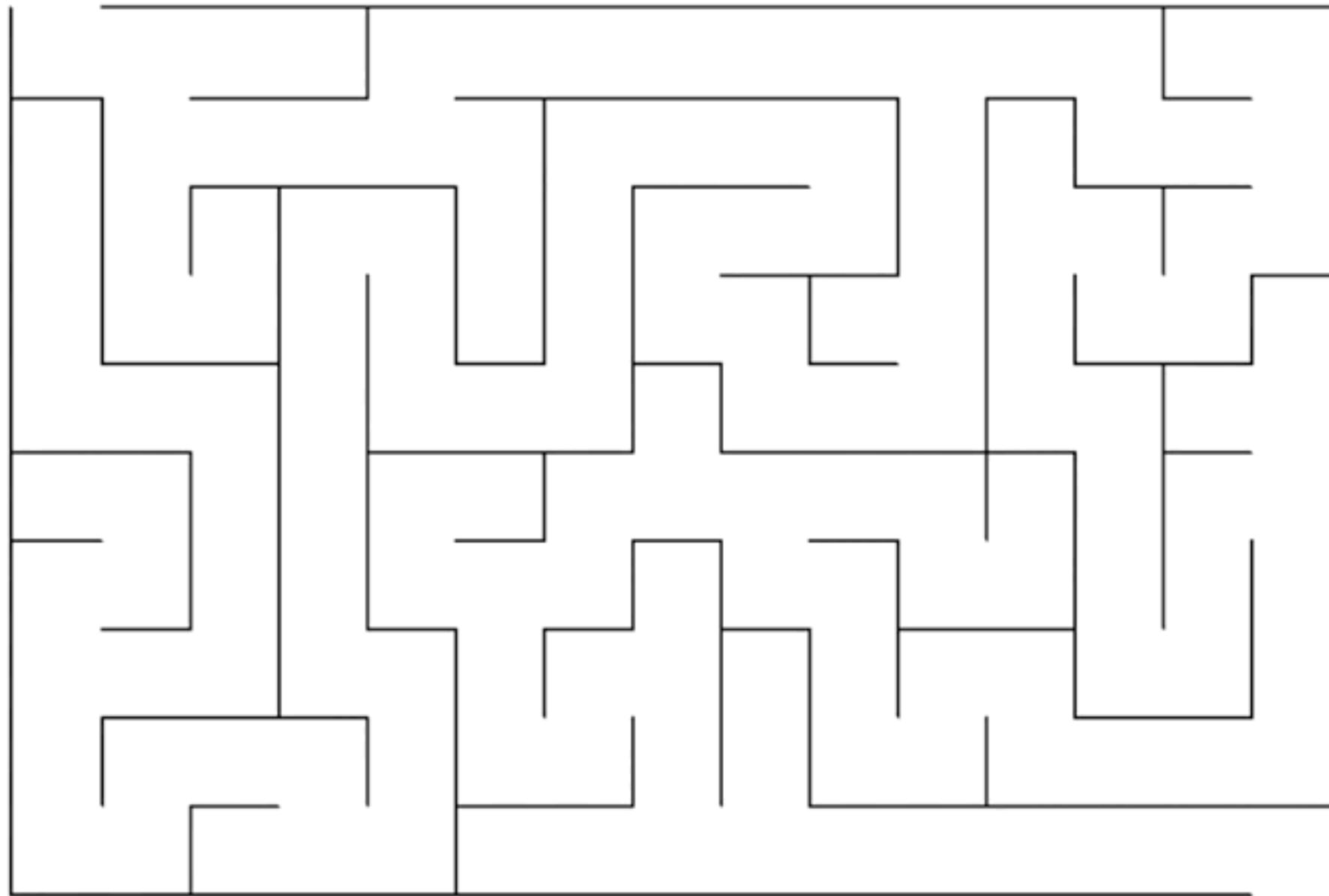
$$\alpha |0\rangle + \beta |1\rangle$$

SUPERPOSITION

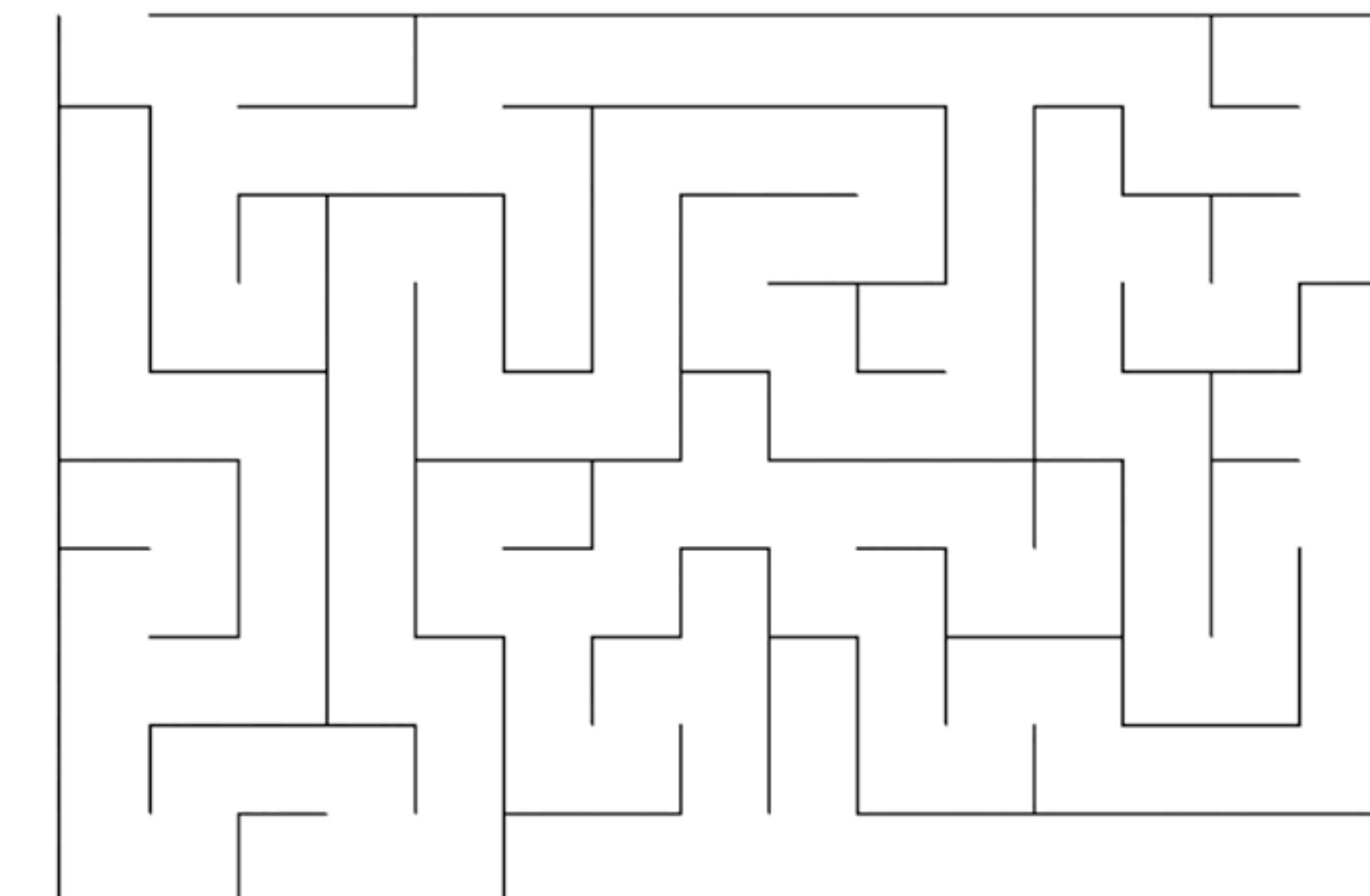


When we are in a complex maze.....

- Conventional Computer



- Quantum Computer



Quantum Language

Notations

- **States**

- $|\psi\rangle$: ket-vector $|\psi\rangle = \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}$
- $\langle\psi|$: bra-vector $\langle\psi| = [\alpha^* \quad \beta^* \quad \gamma^*]$
- $|\psi_1\rangle |\psi_2\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$: tensor product

- **Operators**

- $U^\dagger = (U^T)^* = (U^*)^T$: Hermitian conjugate
- $\langle\psi| U |\psi\rangle = \langle\psi| U^\dagger |\psi\rangle \equiv \langle U \rangle$: expectation value

$$= [\alpha^* \quad \beta^* \quad \gamma^*] \begin{bmatrix} a & b & c \\ d & f & g \\ h & j & k \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}$$

$$|\psi_1\rangle = \begin{bmatrix} 1 & -1 \\ 0 & i \\ -1+i & 1-i \end{bmatrix} \quad |\psi_2\rangle = \begin{bmatrix} i \\ -1 \end{bmatrix}$$

$$|\psi_1\rangle \otimes |\psi_2\rangle = \begin{bmatrix} 1 \cdot \begin{bmatrix} i \\ -1 \end{bmatrix} & -1 \cdot \begin{bmatrix} i \\ -1 \end{bmatrix} \\ 0 \cdot \begin{bmatrix} i \\ -1 \end{bmatrix} & i \cdot \begin{bmatrix} i \\ -1 \end{bmatrix} \\ (-1+i) \cdot \begin{bmatrix} i \\ -1 \end{bmatrix} & (1-i) \cdot \begin{bmatrix} i \\ -1 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} i & -i \\ -1 & 1 \\ 0 & -1 \\ 0 & -i \\ -1-i & 1+i \\ 1-i & -1+i \end{bmatrix}$$

$$U = \begin{bmatrix} a & b & c \\ d & f & g \\ h & j & k \end{bmatrix}$$

Bloch sphere

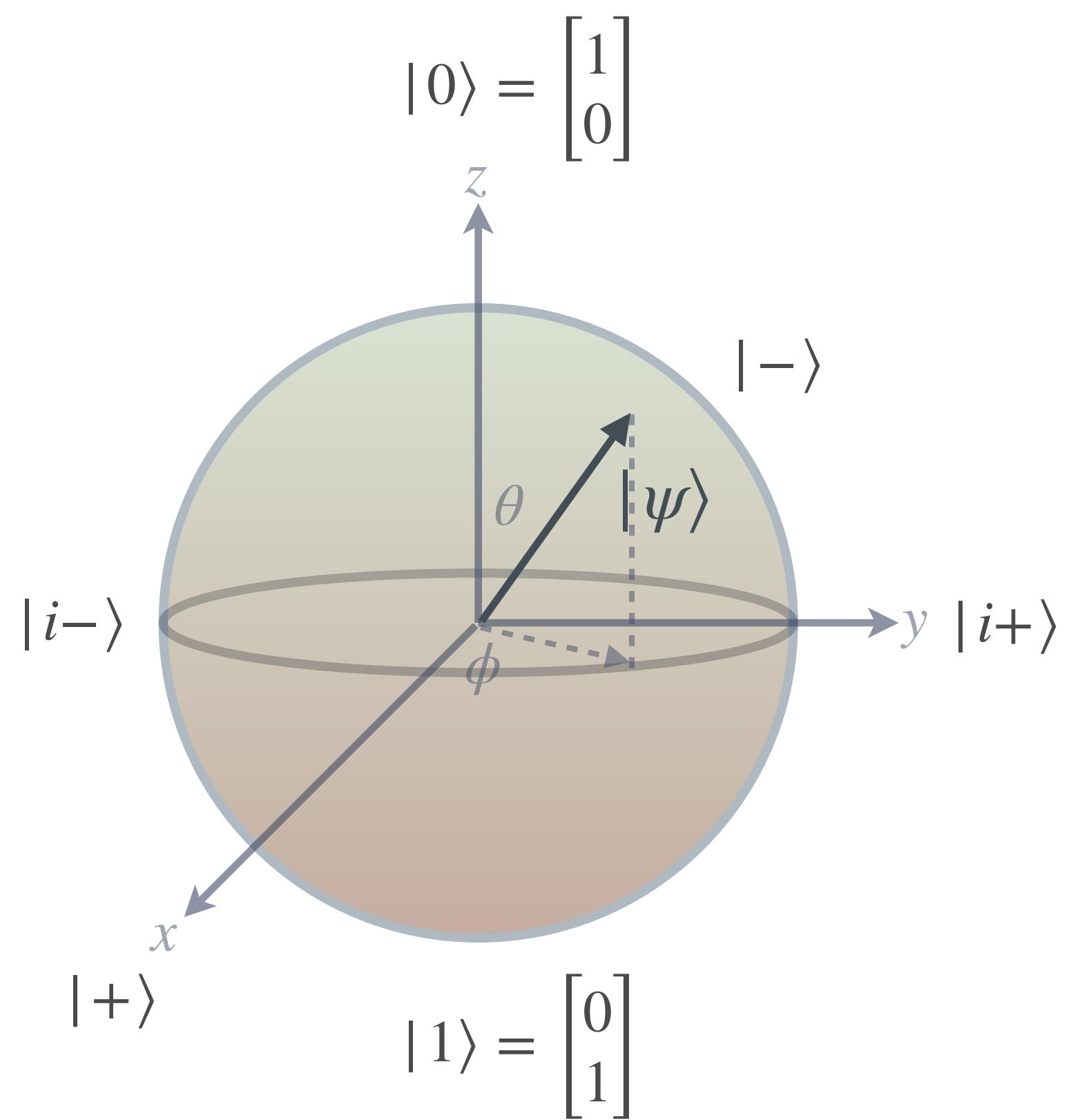
- Geometrical representation of the state

$$|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$$

$$= e^{ir} \left(\cos \frac{\theta}{2} + e^{i\phi} \sin \frac{\theta}{2} \right)$$

$$|\pm\rangle = \frac{1}{\sqrt{2}} (|0\rangle \pm |1\rangle)$$

$$|i\pm\rangle = \frac{1}{\sqrt{2}} (|0\rangle \pm i|1\rangle)$$



Operations - Quantum gates

- Single-qubit gate

- Hadamard 

- Phase 

- $\pi/8$ 

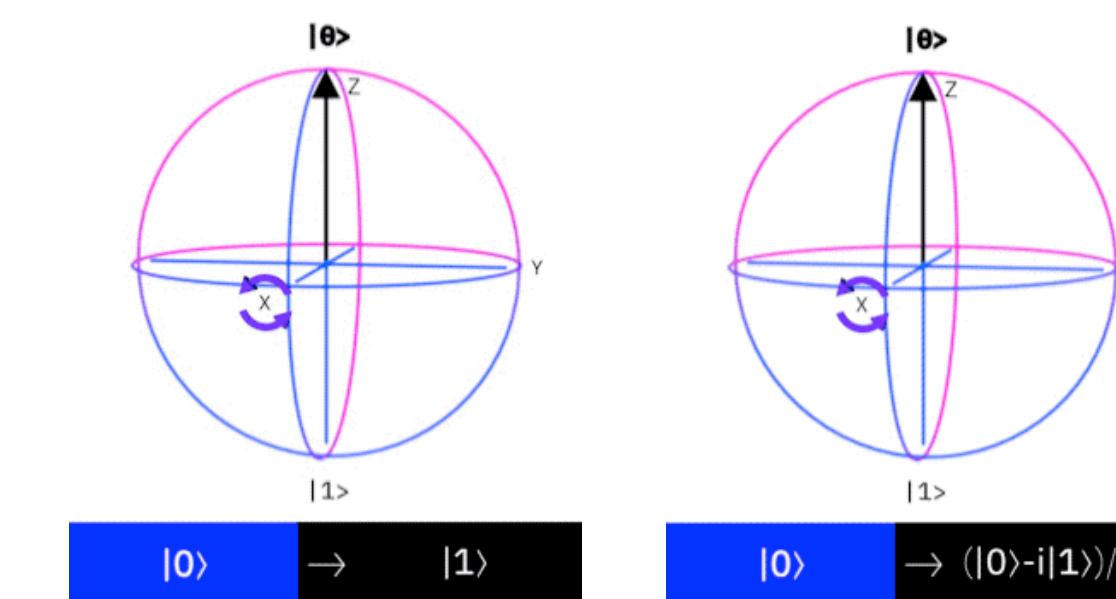
- Rotation      

- Multi-qubit gate

- CNOT 

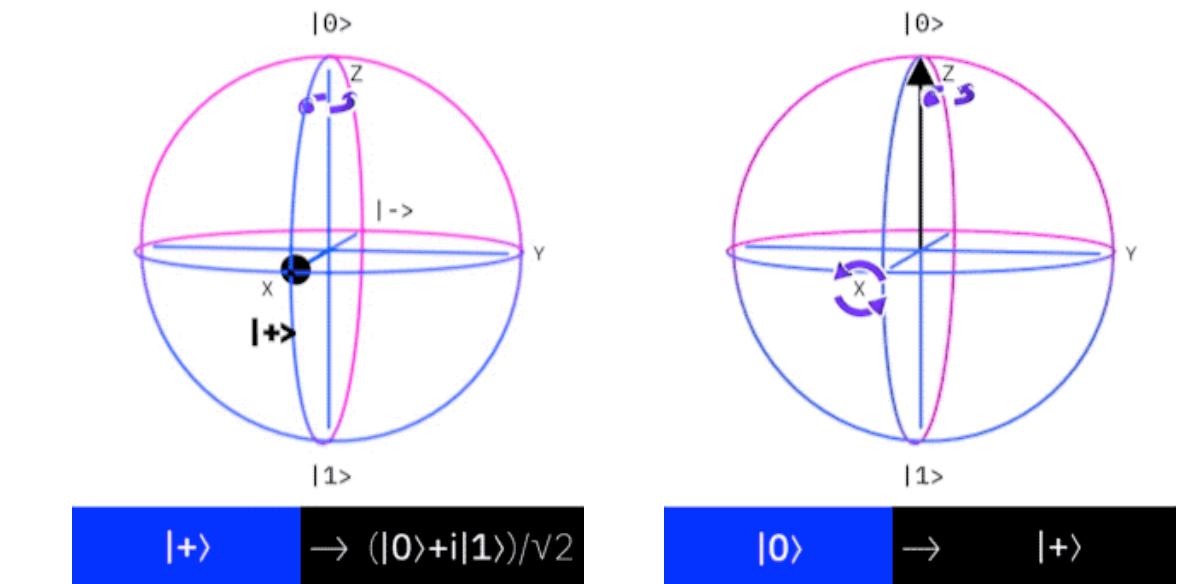
- Controlled-rotation  

- SWAP 



$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha & \beta \\ 0 & 0 & \gamma & \delta \end{bmatrix}$$

controlled-rotation



Input		Output	
Control	Target	Control	Target
$ 0\rangle$	$ 0\rangle$	$ 0\rangle$	$ 0\rangle$
$ +\rangle$	$ +\rangle$	$(0\rangle + i 1\rangle)/\sqrt{2}$	
$ 0\rangle$	$ 1\rangle$	$ 0\rangle$	$ 1\rangle$
$ 0\rangle$	$ 0\rangle_0$	$ 1\rangle$	$ 0\rangle$
$ 0\rangle$	$ 1\rangle_0$	$ 0\rangle$	$ 1\rangle$
$ 1\rangle$	$ 0\rangle_0$	$ 1\rangle$	$ 1\rangle$
$ 0\rangle$	$ 1\rangle_1$	$ 1\rangle$	$ 0\rangle$

SWAP
Resources/Circuit Composer/Block glossary

Operate only one of the qubits ?

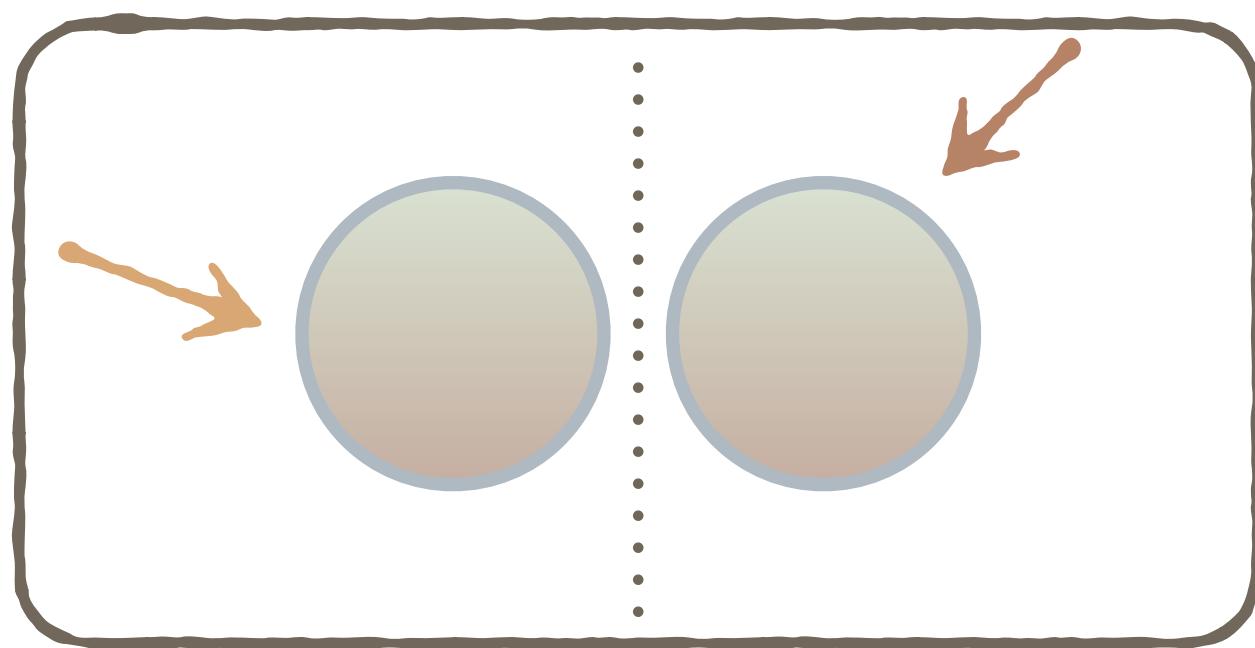
- Do “nothing to Q1 + NOT gate on Q2” ?

- nothing = Identity gate ID

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- operate independently:

$$I^{(1)}X^{(2)} = X^{(2)} = (I \otimes I)(I \otimes X) = (I \otimes X)$$



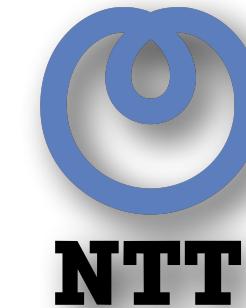
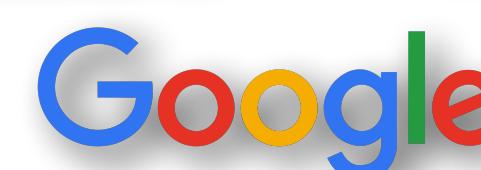
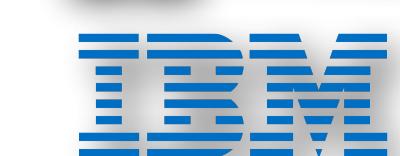
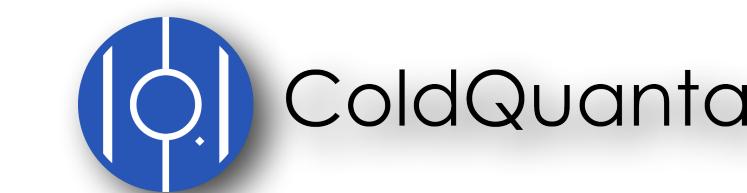
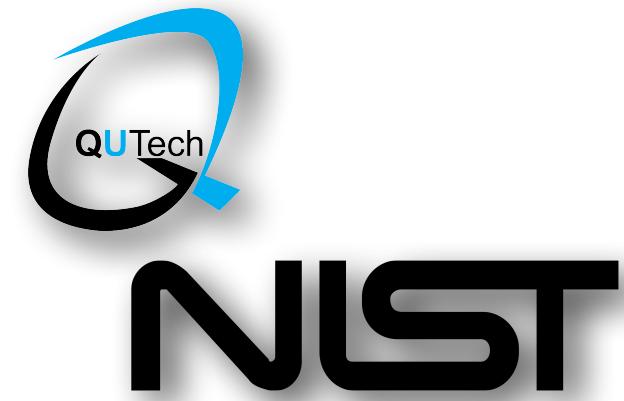
Quantum Computers

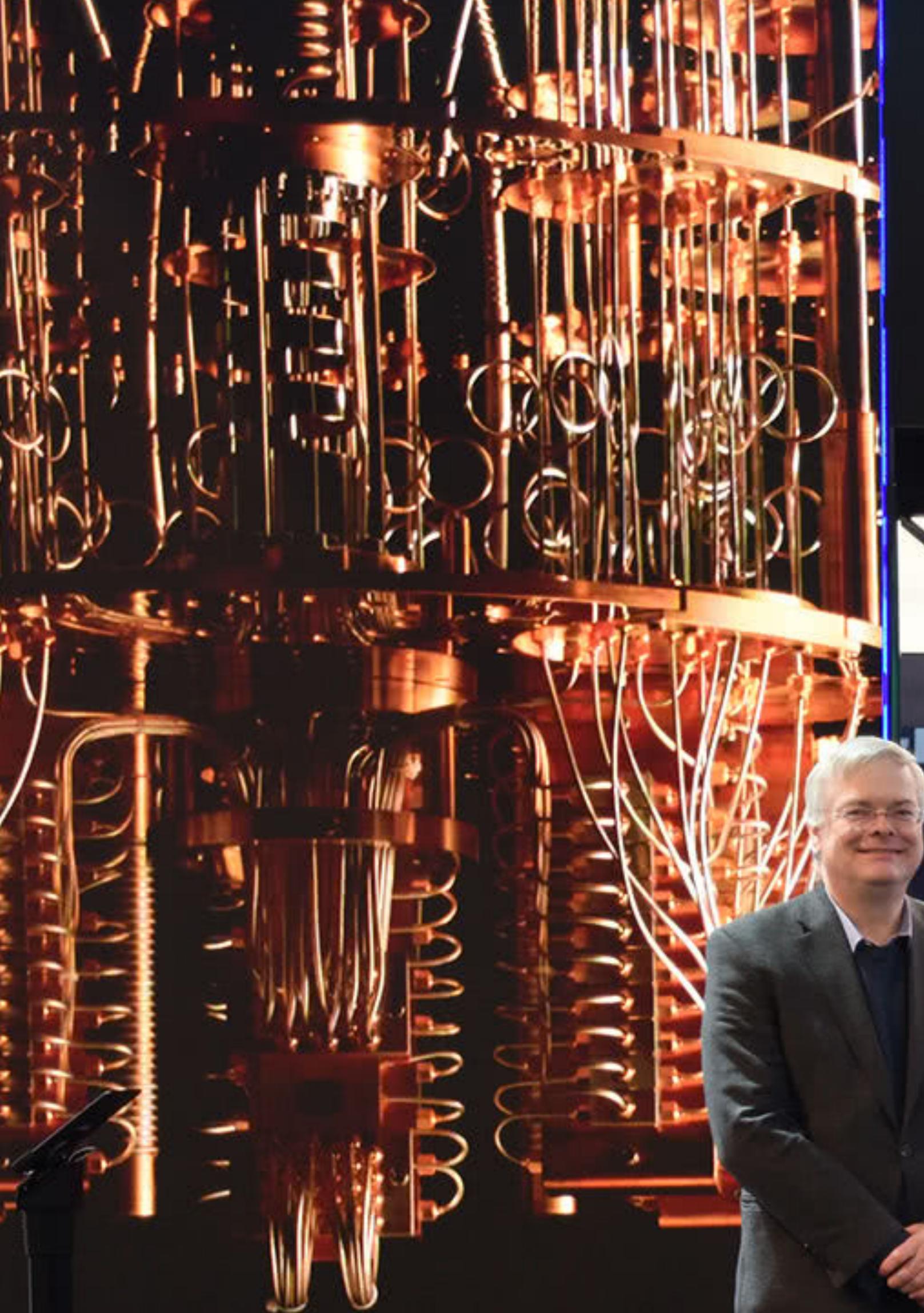
Requirements for physical realization

- Scalable physical system
- Ability to initialize the state
- Long decoherence time > operation time
- A universal set of quantum gates
- Capability to qubit-specific measurement

Implements

- Photons, electrons, nucleus
- NMR
- Quantum dots
- Nature atoms
- Diamonds
- Semiconductors
- Superconductors
- and so on.....



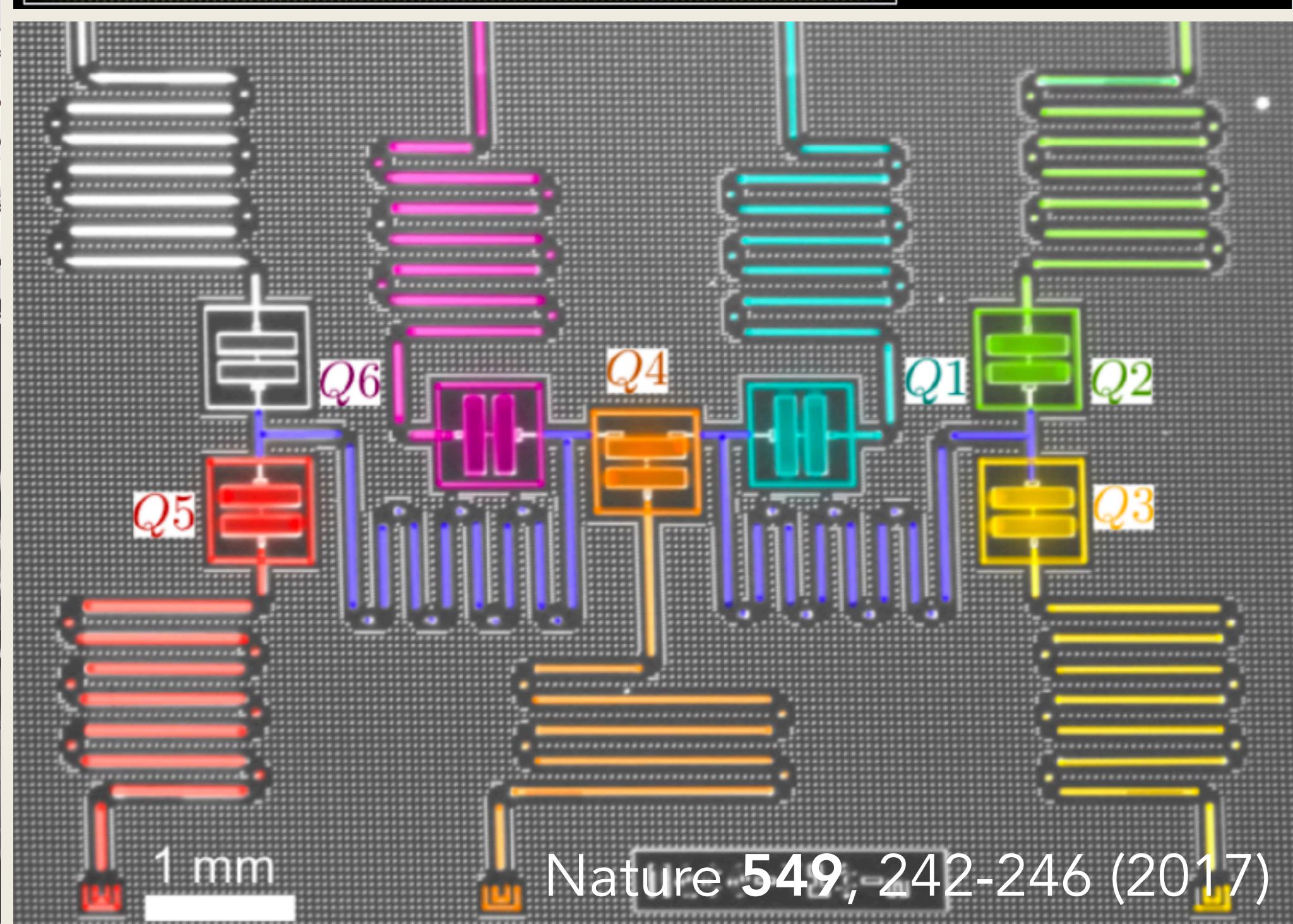
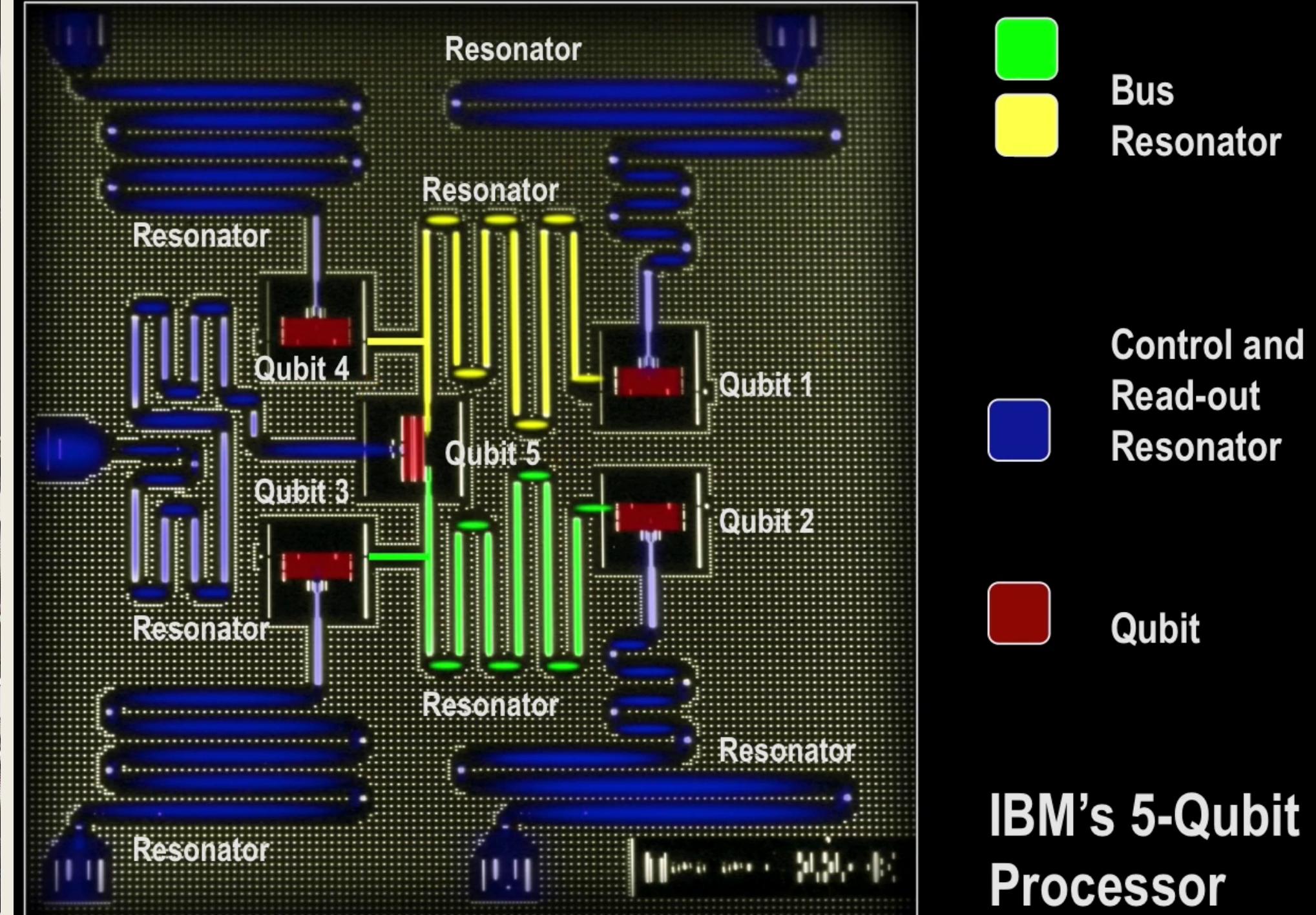
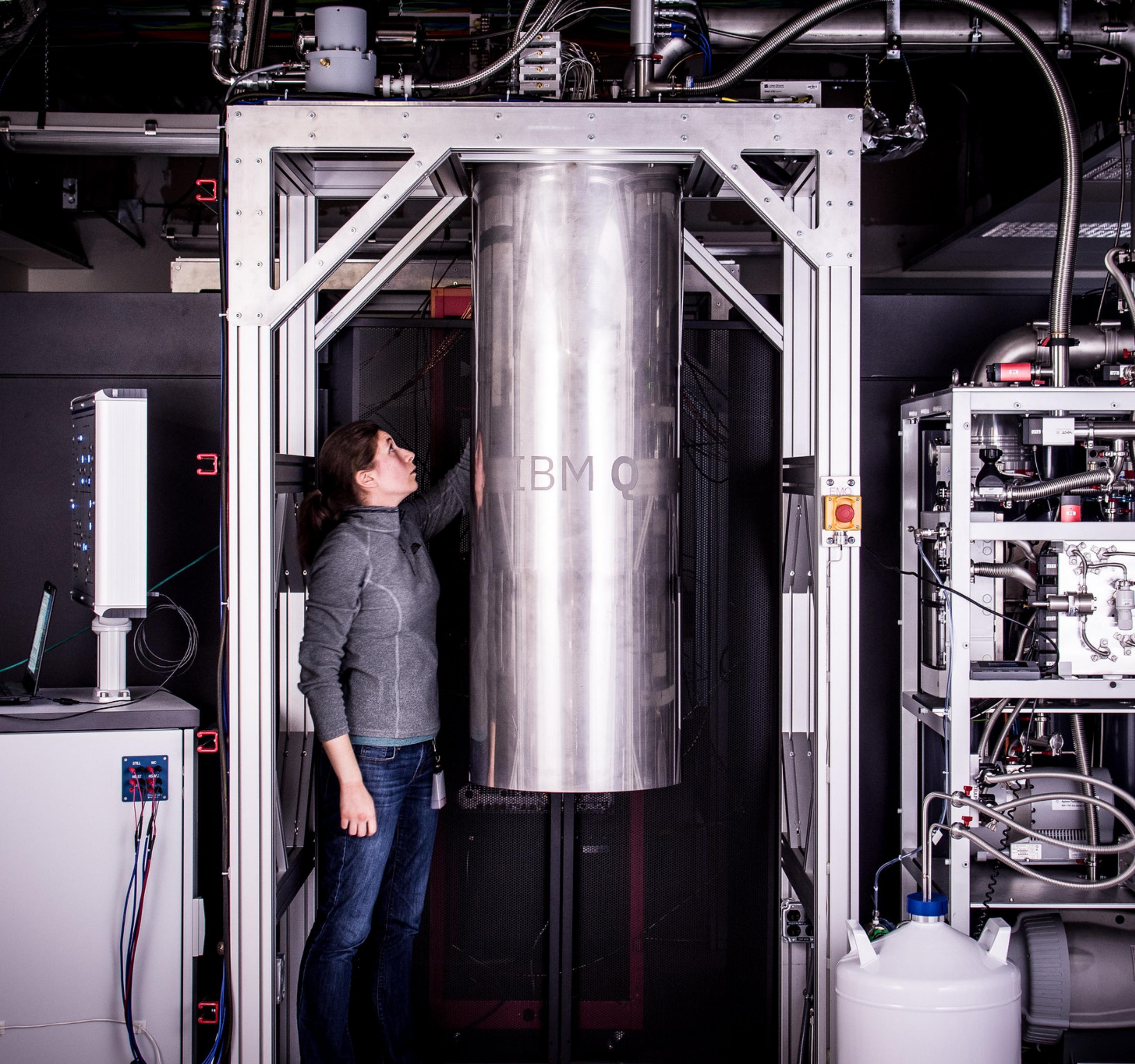


Quantum Computing
—
Intersecting science,
systems and design.

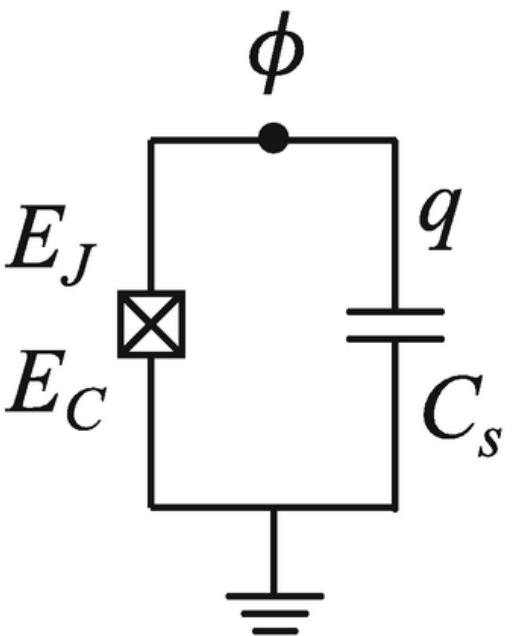
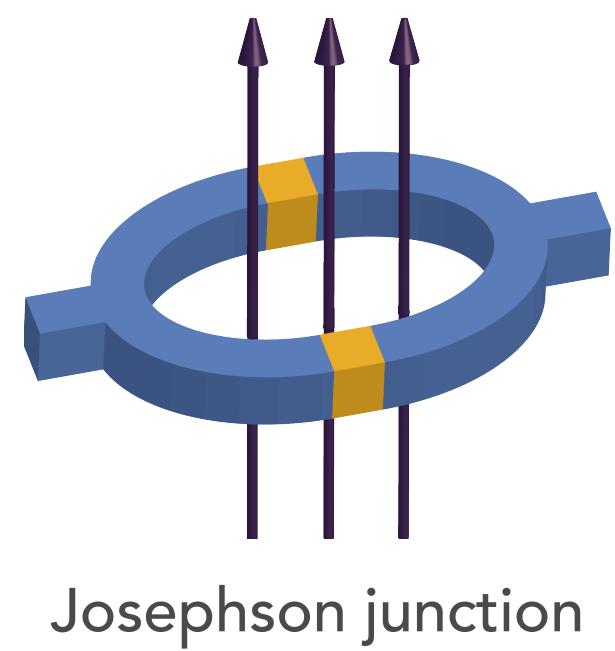


IBM Q
System One

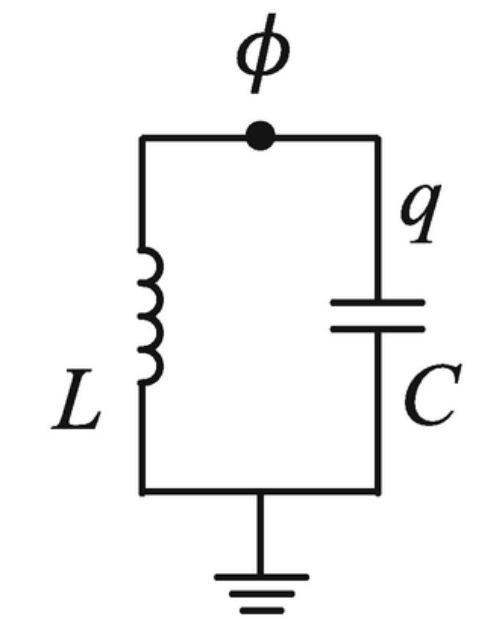




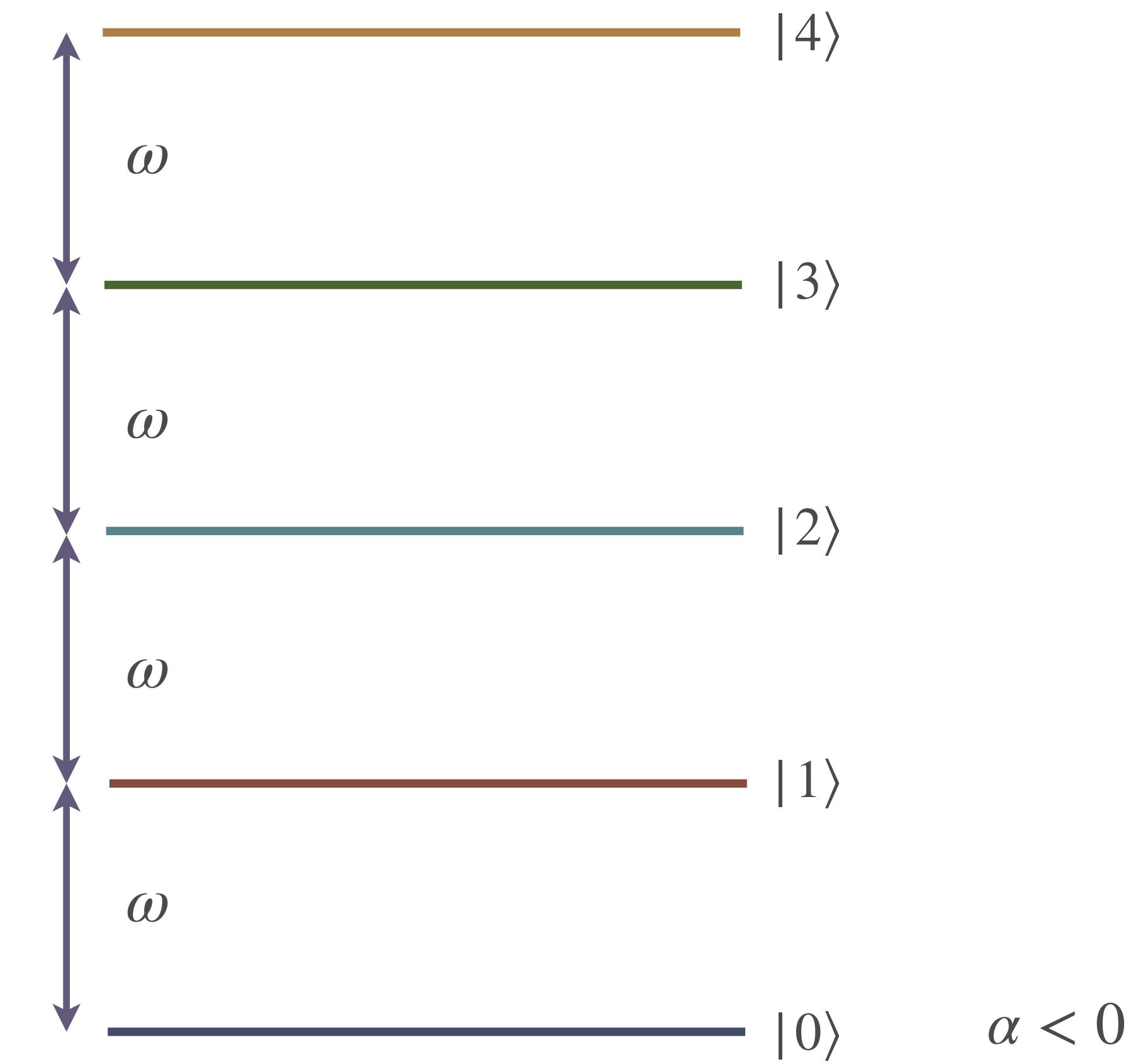
Superconducting circuit



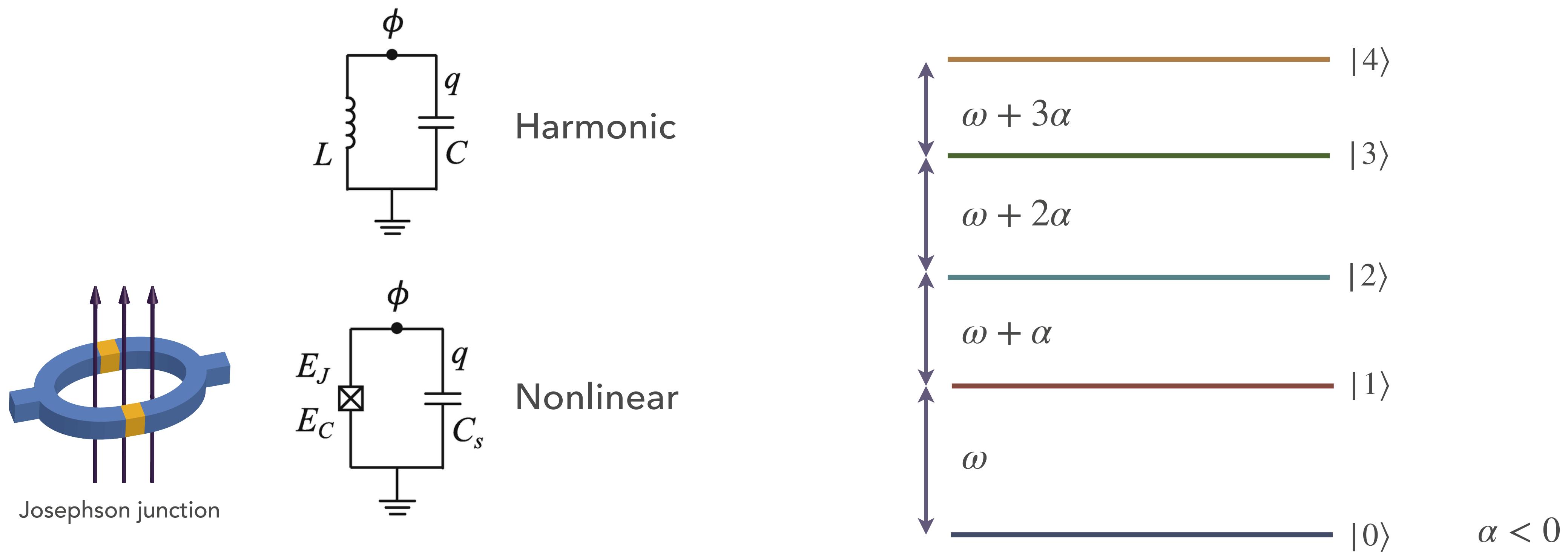
Nonlinear

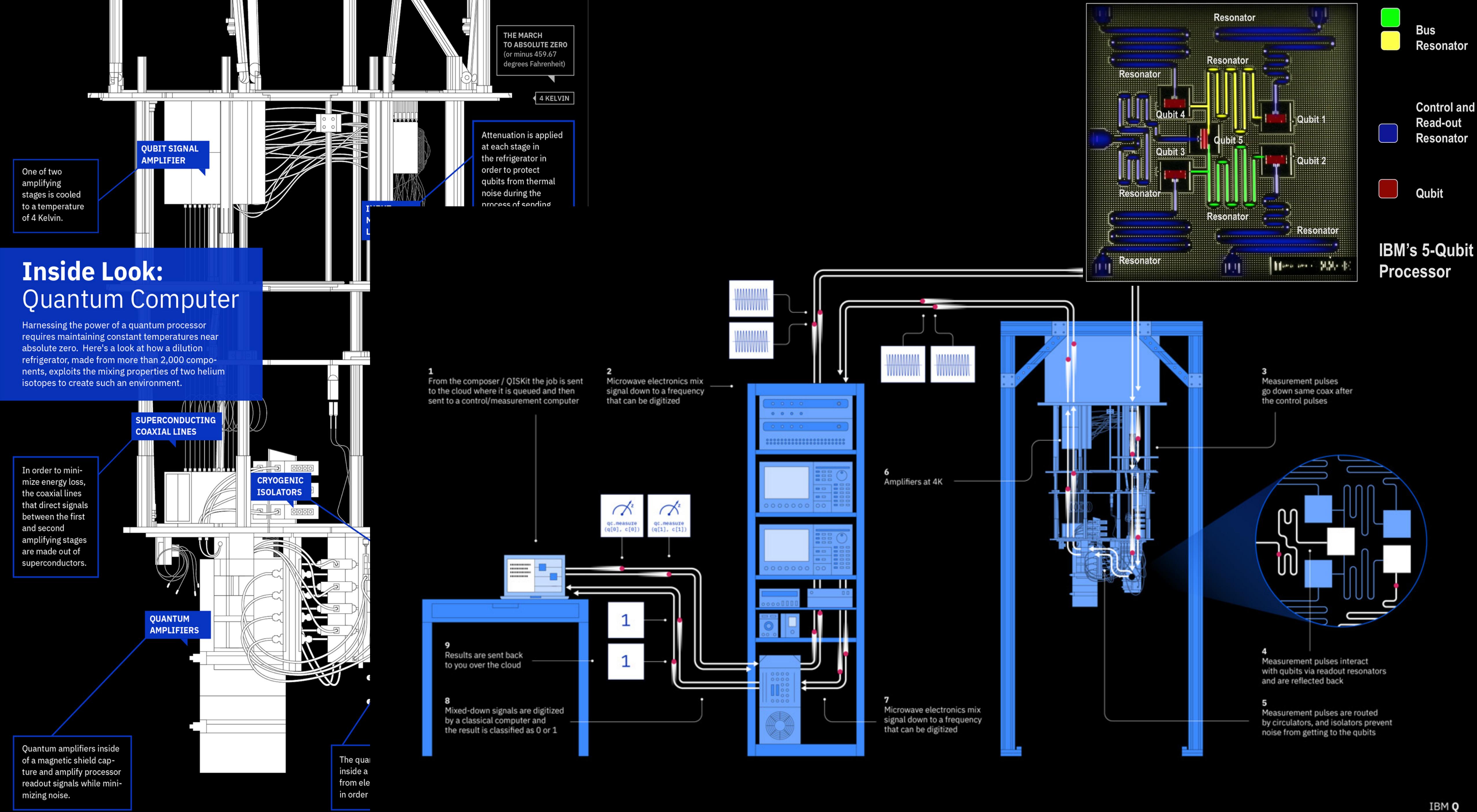


Harmonic



Superconducting circuit



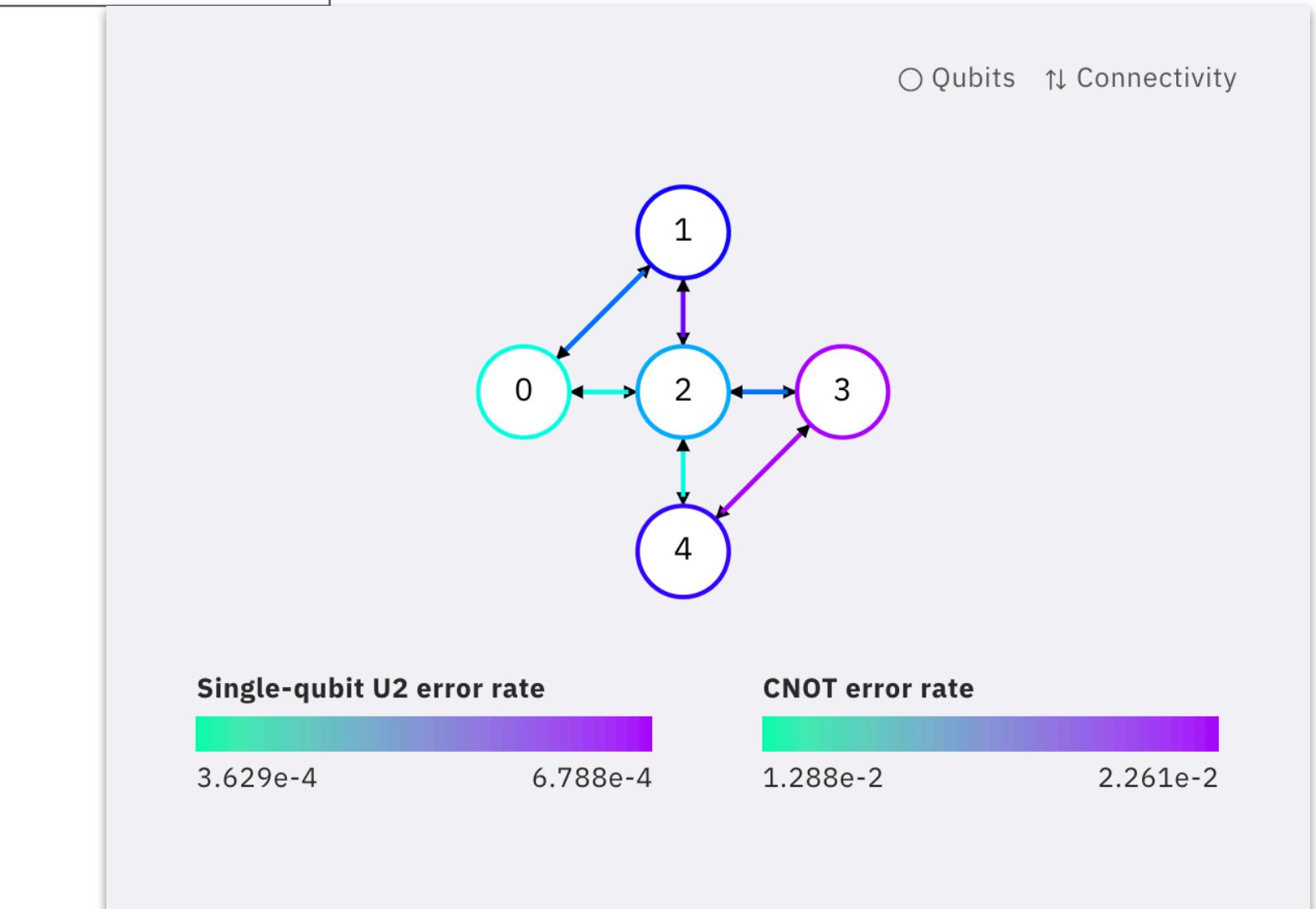


Nutrition Fact of Qubit

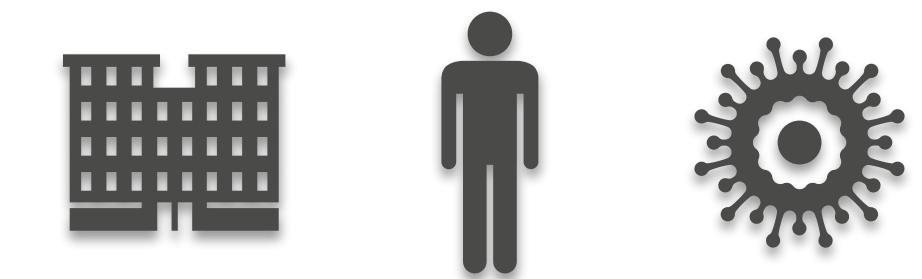
#	T1 (μs)	T2 (μs)	Frequency (GHz)	Readout error	Single-qubit U2 error rate
0	73.38	95.40	5.286	8.500E-03	3.629E-04
1	72.95	77.21	5.238	3.050E-02	5.608E-04
2	60.03	76.74	5.030	2.100E-02	4.458E-04
3	40.19	39.87	5.296	3.750E-02	6.788E-04
4	60.25	56.50	5.084	1.500E-02	5.997E-04

#	CNOT error rate
0	cx0_1: 1.615E-02, cx0_2: 1.380E-02
1	cx1_0: 1.615E-02, cx1_2: 2.076E-02
2	cx2_0: 1.380E-02, cx2_1: 2.076E-02, cx2_3: 1.628E-02, cx2_4: 1.288E-02
3	cx3_2: 1.628E-02, cx3_4: 2.261E-02
4	cx4_2: 1.288E-02, cx4_3: 2.261E-02

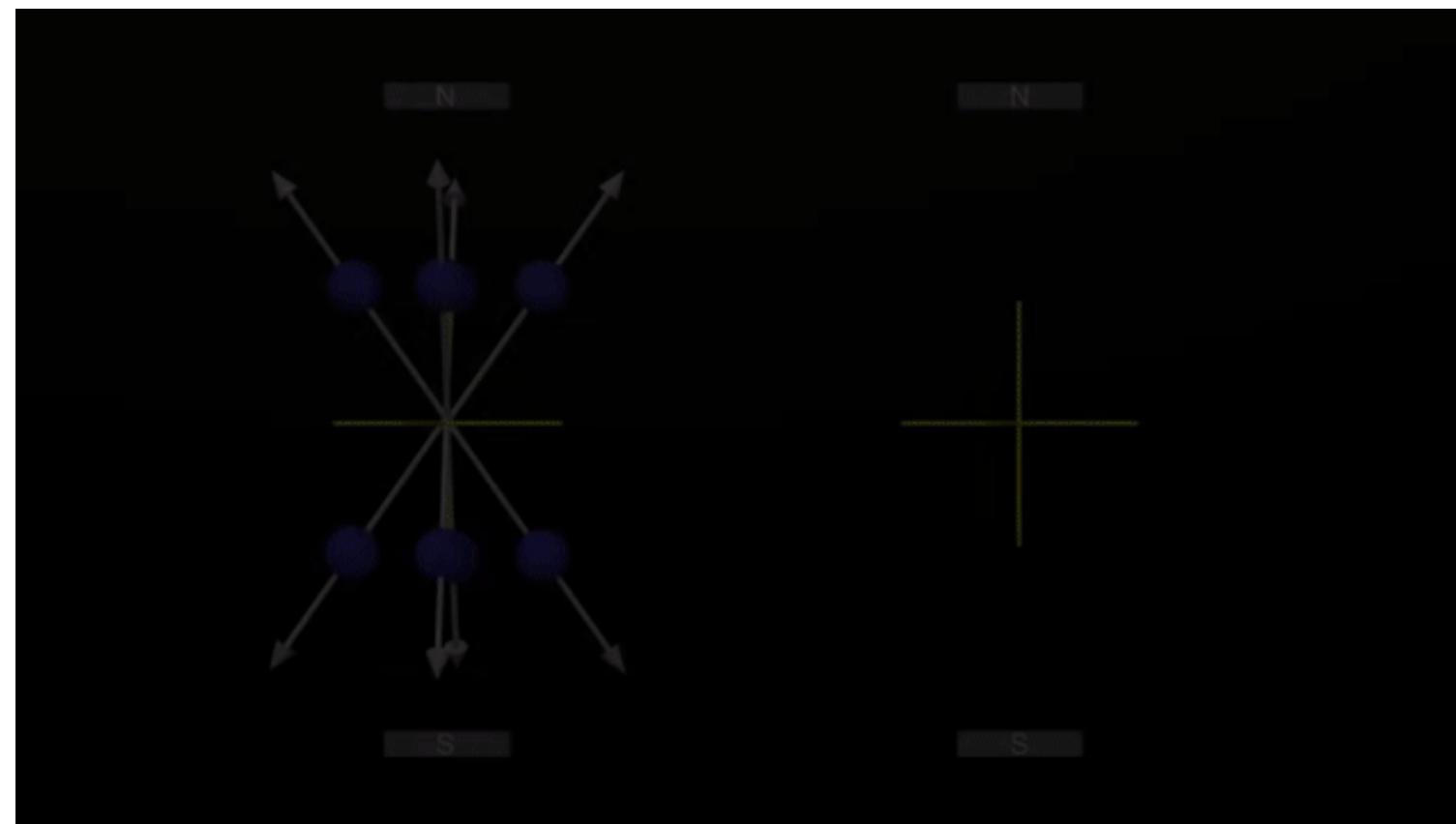
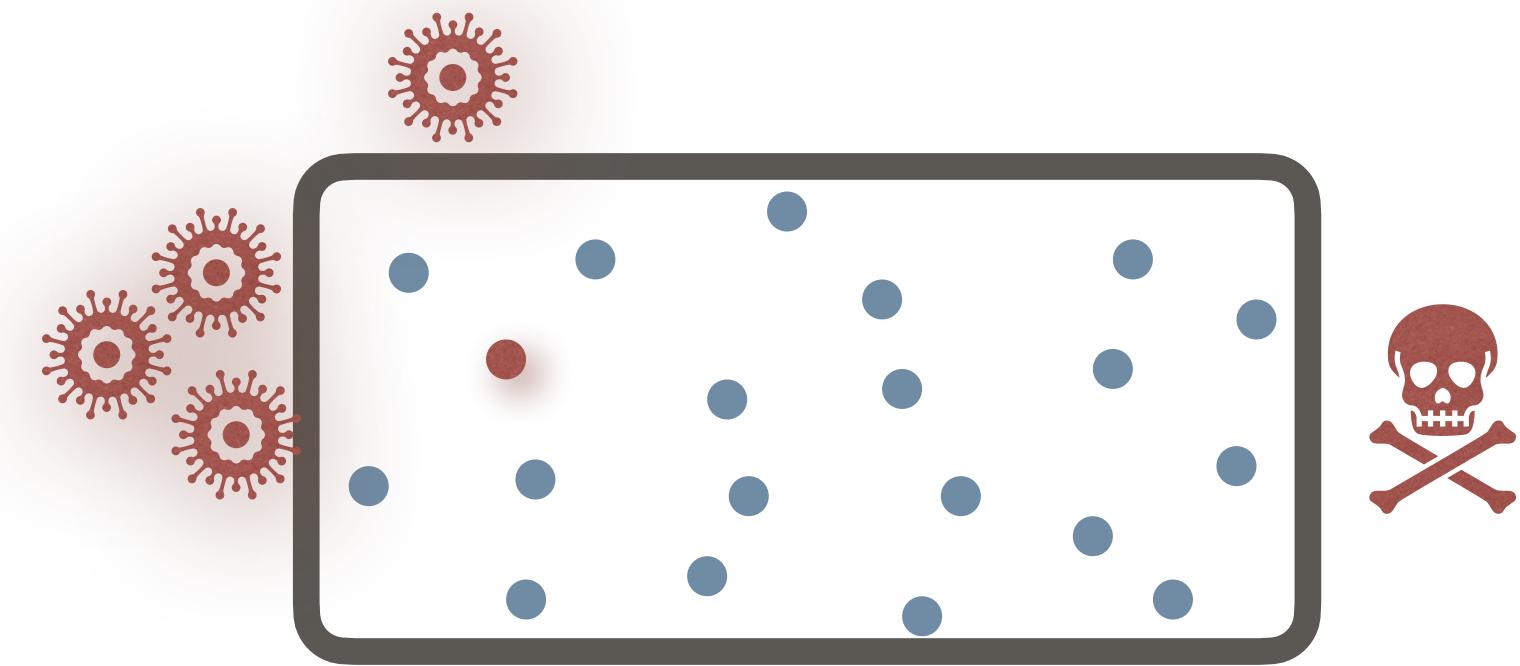
March 24, 2020 21:23:06 GMT +0800
 Dashboard/Your Backends/[device_name]/Download Calibrations



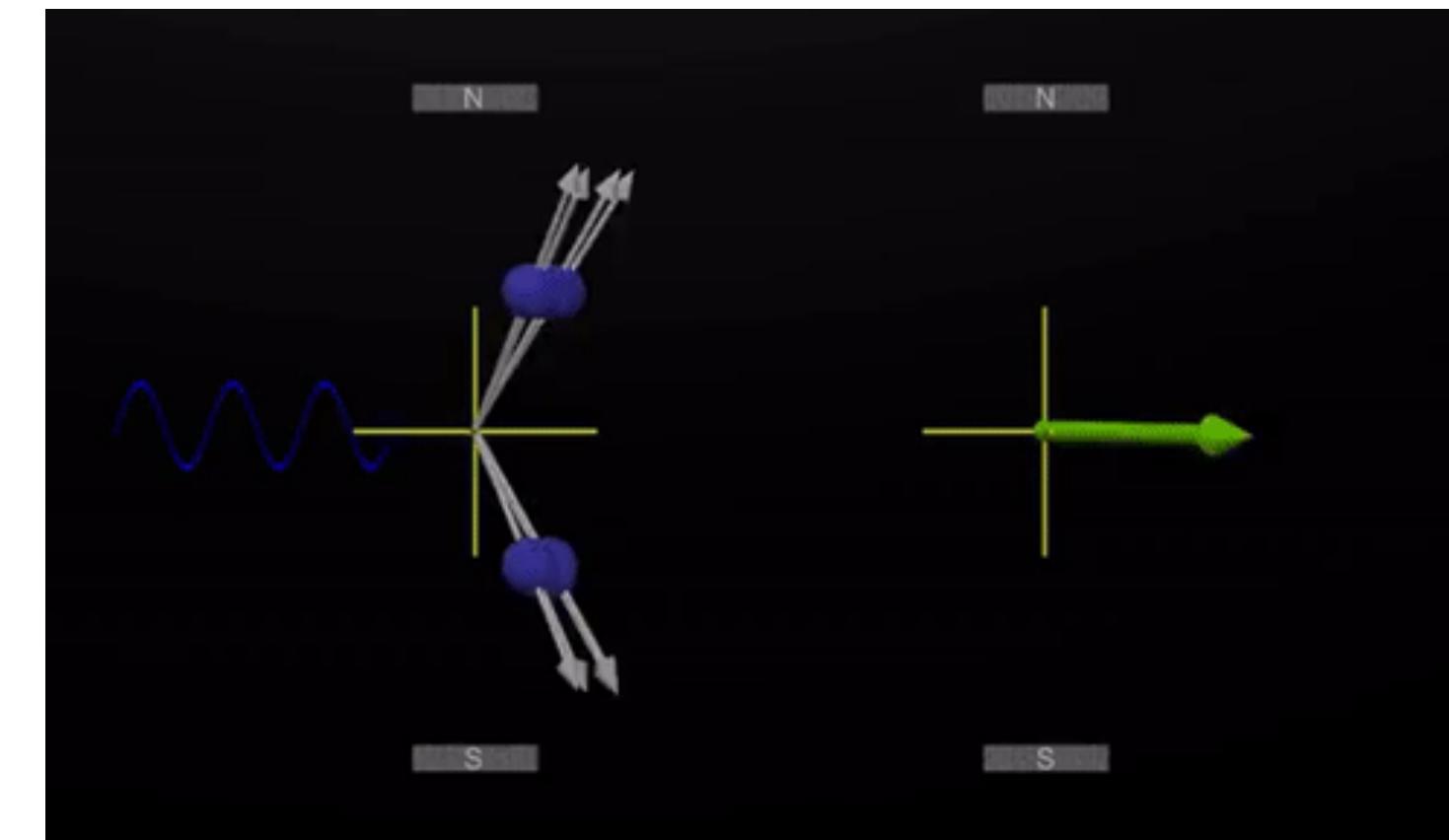
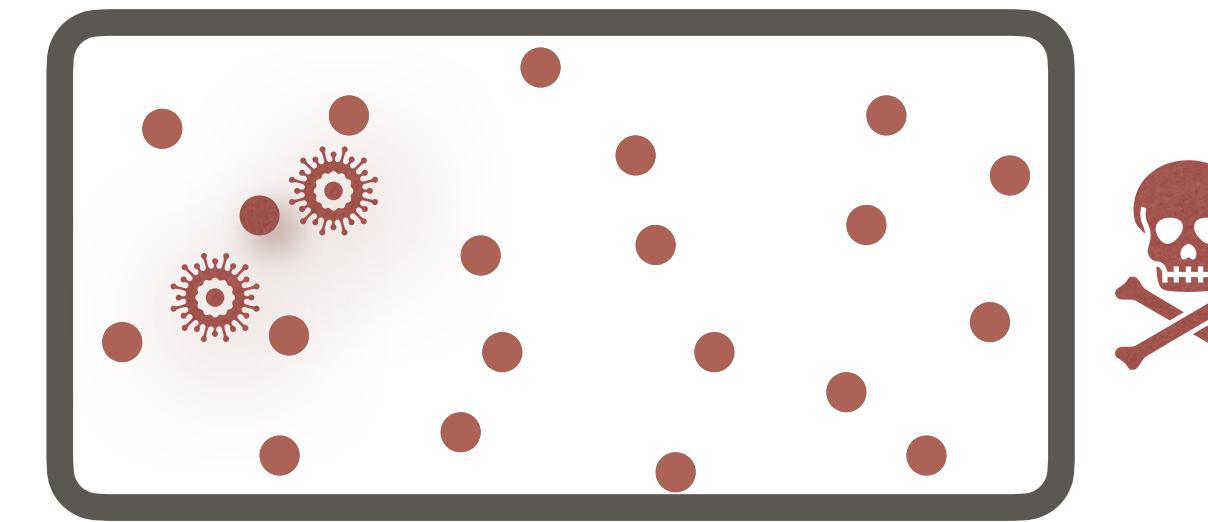
Relaxation



- qubit-environment


 T_1

- qubit-qubit


 T_2

$$\begin{bmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{bmatrix}$$



IBM

IBM Quantum Experience

IBM Q
System One

Example - Entangled States

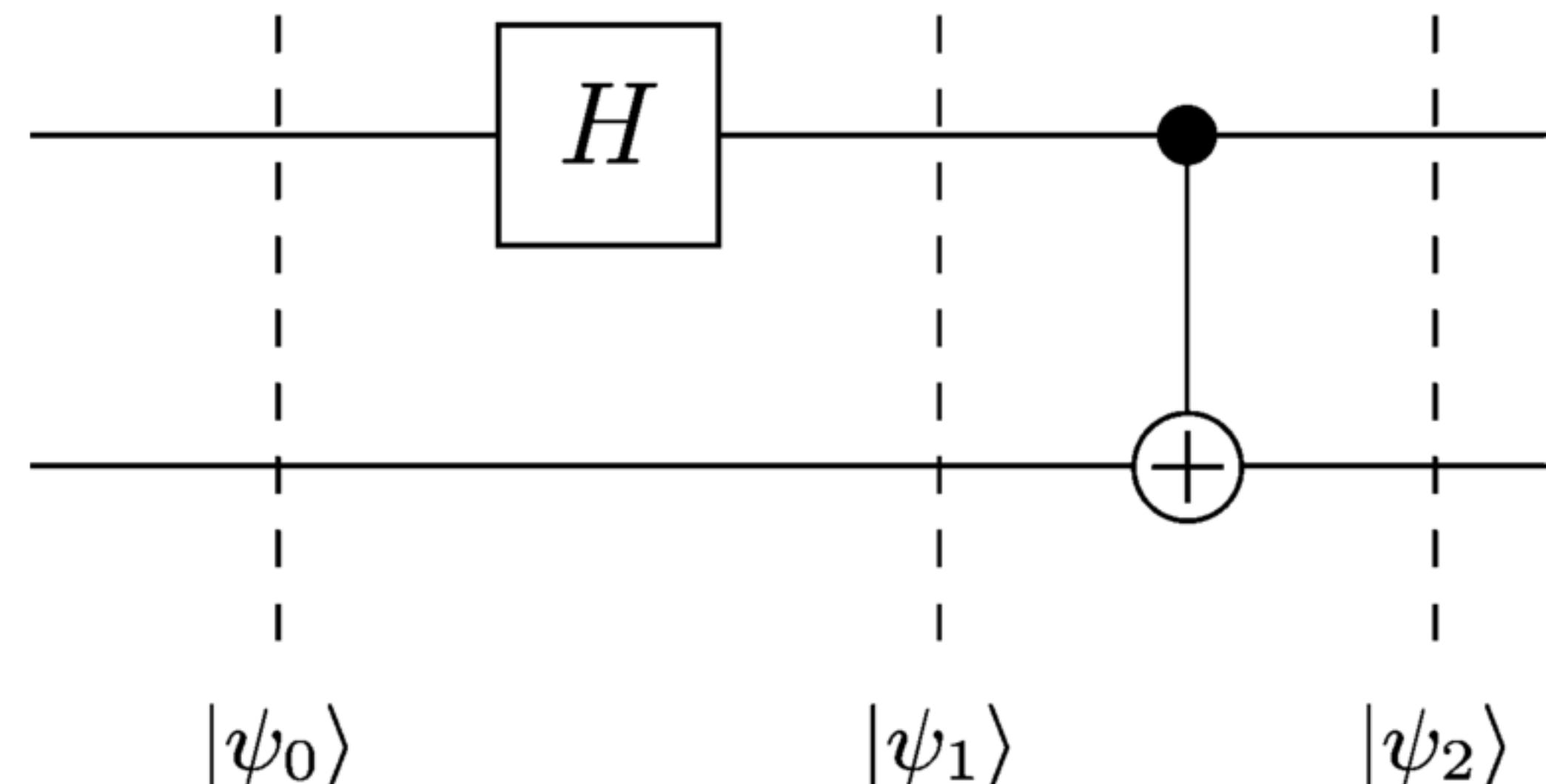
- Bell states

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle)$$

$$|\Phi^-\rangle = \frac{1}{\sqrt{2}} (|00\rangle - |11\rangle)$$

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}} (|01\rangle + |10\rangle)$$

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}} (|01\rangle - |10\rangle)$$



$$|\psi_2\rangle = CNOT \cdot (H \otimes I) |\psi_0\rangle$$

Measurement - projection

- Projection operator

$$\sum_m \mathbb{P}_m = \sum_m |m\rangle \underline{\underline{\langle m|}}$$

measurement basis

- Probability of outcome m

$$p(m) = \langle \psi | \mathbb{P}_m \underline{\underline{|\psi\rangle}}$$

system state

- Post-measurement state

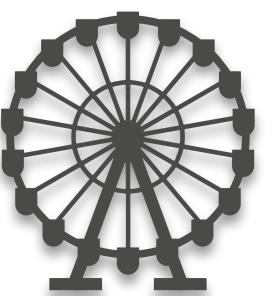
$$|\psi'\rangle = \frac{\mathbb{P}_m}{\sqrt{p(m)}} |\psi\rangle$$

- Default measurement basis

$$\begin{matrix} \text{z} \\ \text{x} \end{matrix}$$

$\{|0\rangle, |1\rangle\}$

How to change the measurement basis,
e.g., to Bell states ?



Questions ?

Alan Turing

