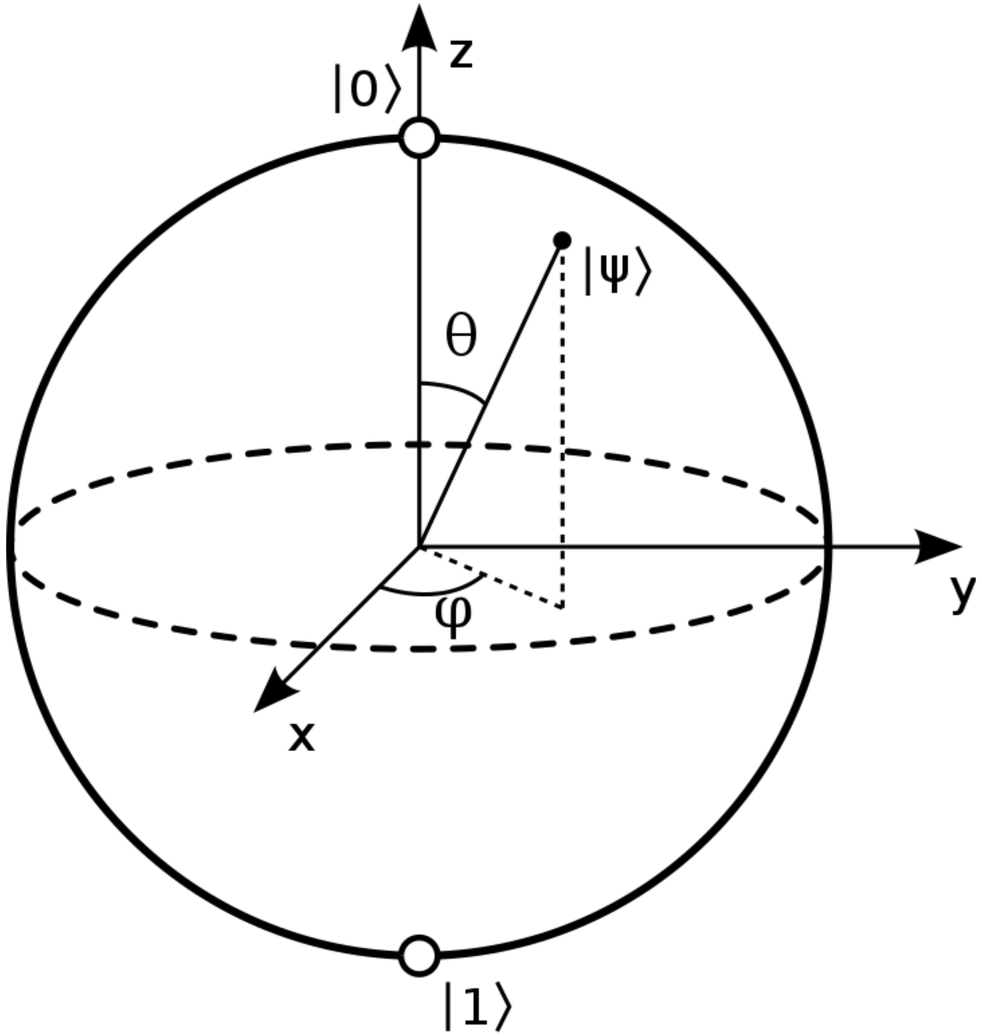
VQE & Rotation

Bloch Sphere



z-measurement

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\langle Z \rangle = \langle \psi | \, Z \, | \psi \rangle$$

$$= \begin{bmatrix} a^* & b^* \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = |a|^2 - |b|^2$$

$$= \mathbb{P}_0 - \mathbb{P}_1$$

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= #(measure: 0) - #(measure: 1)
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X-measurement?

$$\langle X \rangle = \langle \psi | \, X \, | \psi \rangle = \begin{bmatrix} a^* & b^* \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = a^* b - b^* a$$

Only $|a|^2$, $|b|^2$ are “measurable”!

(Some Math)

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$X \quad = \quad H \quad Z \quad H$$

X measurement

$$\langle\psi|\, X\, |\psi\rangle = \langle\psi|\, HZH\, |\psi\rangle$$
$$= (\langle\psi|\, H) Z (H\, |\psi\rangle) = \langle\psi'|\, Z\, |\psi'\rangle$$