Non-stochastic one-dimensional lackadaisical quantum random walks

Yun-Chih Liao[1], Hsi-Sheng Goan[3,4], Hao-Chung Cheng[2,1]

[1]Graduate Institute of Communication Eigeneering, National Taiwan University, Taipei 106319, Taiwan
[2]Department of Electrical Engineering, National Taiwan University, Taipei 106319, Taiwan
[3]Department of Physics and Center for Theoretical Sciences, National Taiwan University, Taipei 106319, Taiwan
[4]Center for Quantum Science and Engineering, National Taiwan University, Taipei 106319, Taiwan

We investigate the evolution of discrete-time one-dimensional lackadaisical quantum random walk (LQW) with step- and position- dependent coins. The coins are characterized by the phases, which depend on the step of the walk or the position of the quantum walker, respectively. For different phases, such a coin leads to diverse probability distributions of the walks, and spreads faster. We explore the entropy dynamics associated with the position space and the walker's internal degrees of freedom space, the so-called coin space. We demonstrate how the probability distributions vary with the coins. This enables us to properly control the LQW, which might lead to future technological applications.

Quantum random walks

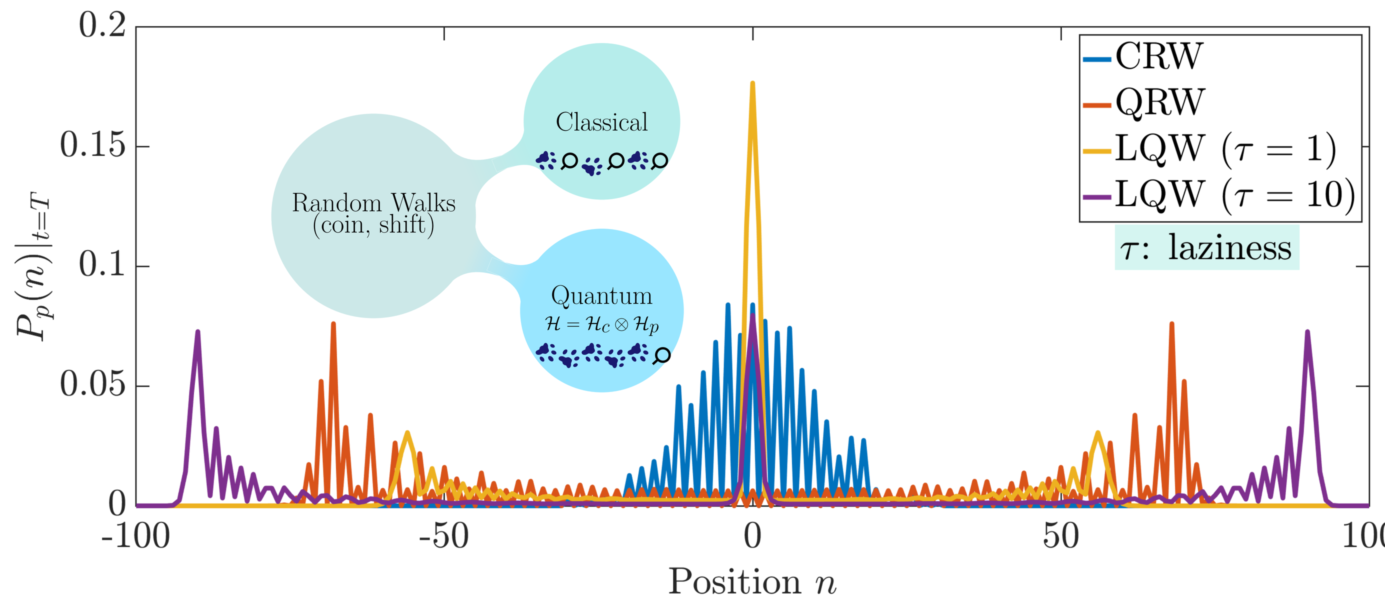

Figure 1:Probability distribution of different types of one-dimensional random walks: classical random walk (CRW), standard quantum random walk (QRW), and lackadaisical quantum walks with different laziness values τ.

Quantum random walk breaks the localization limitation and thus spreads wider comparing to its classical analogue [1], so is beneficial to database searching problems [6]. Considering the 1D LQW [5] with 3 options: moving *right*, going *left*, and *staying* at its current location, the shift operator reads

$$\mathcal{S} = \sum_n |R\rangle\langle R| \otimes |n+1\rangle\langle n| + |L\rangle\langle L| \otimes |n-1\rangle\langle n| \; \boxed{+ |S\rangle\langle S| \otimes |n\rangle\langle n|} \,. \quad (1)$$

The coin operator discribes the preference of moves. In the case that the walker's internal degrees of freedom is 3, a common choice is Grover's diffusion operator [4, 5]

$$\mathcal{C} = \mathcal{C}_{\mathrm{G}} = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} \otimes \hat{\mathbb{I}}_p. \quad (2)$$

The time evolution of each step is $\mathcal{U} = \mathcal{S}\mathcal{C}$. Hence the system after evolving totally T steps can be evaluated with the propagator

$$\hat{U} = \mathcal{U}^T = (\mathcal{S}\mathcal{C})^T \,. \quad (3)$$

Novelty: path-dependent coin $\mathcal{C}_{\mathrm{DC}}$

Quantum walks can be utilized for simulating quantum systems or programming and engineering quantum algorithms, for which high controllability is required [2, 3]. To enhance it, we introduce two path-dependent coins,

$$\mathcal{C}_{\mathrm{DCs}}(x) = e^{i\mathcal{C}_{\mathrm{G}} \cdot x} \quad (4)$$

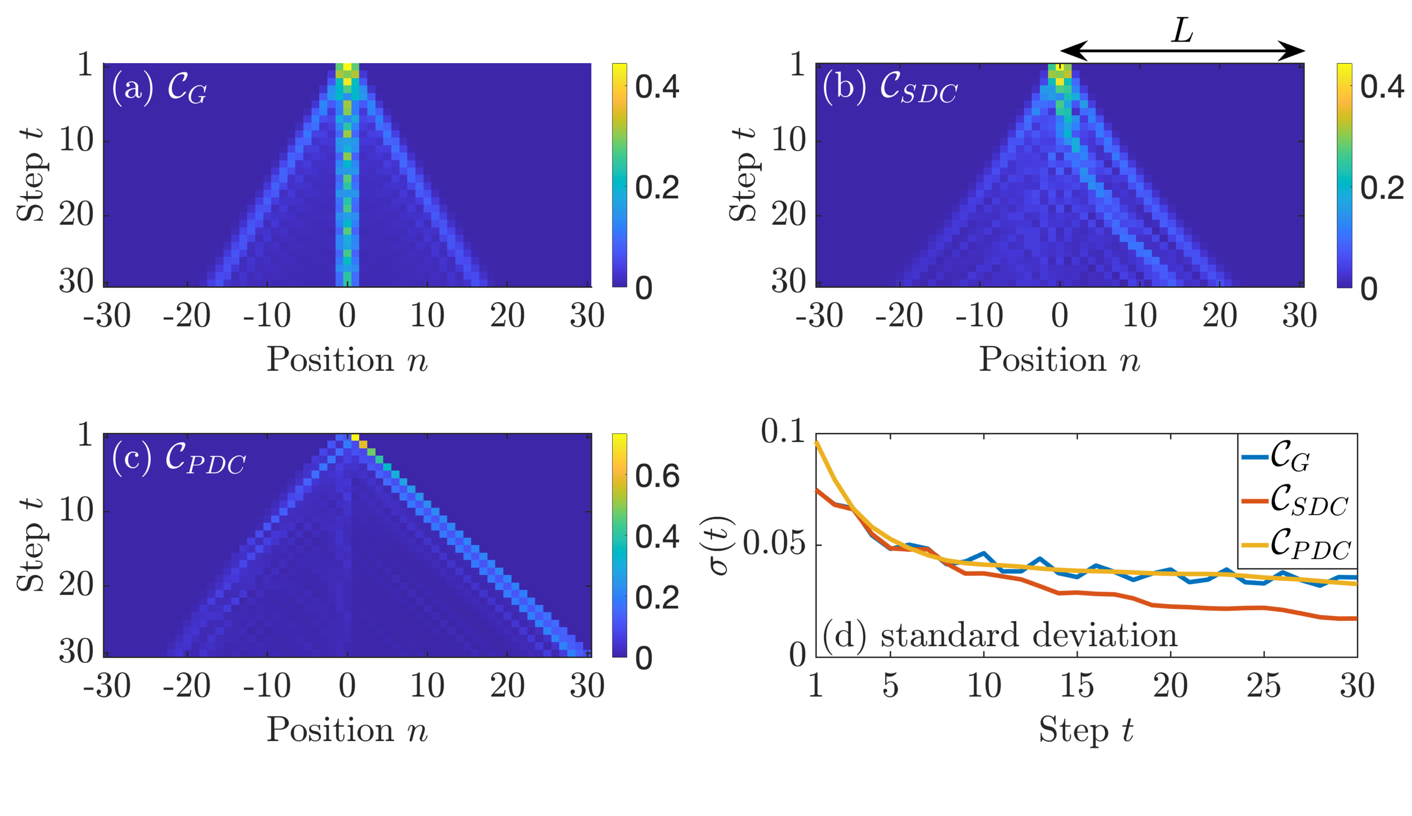

Figure 2:Probability dynamics of (a) full-stochastic process and the walks with (b)(c) path-dependent coins for $T = 30$ steps. (d) shows the standard deviations of position distributions. The evolution begins from $|\Psi(0)\rangle = 1/\sqrt{2}\,(|L\rangle + i\,|R\rangle)_c \otimes |0\rangle_p$ that initially leads to symmetric distribution.

where $x = t/T$ for the step-dependent coin (SDC) with the t-th step, and $x = n/L$ for the position-dependent coin (PDC) with position n and the half-length of the 1D line L.

Entropy dynamics

- Shannon entropy: measures the amount of uncertainty that is present in the state of a physical system. That of the probability describes the level of disorder.
  - To achieve a more coherent and wider position distribution, the Shannon entropy S_p is suggested to be as large as possible.
- von Neumann entropy: characterizes the entropy of entanglement, i.e., how "mixed" is the density state of the system.
  - The process of quantum walks is driven by $\mathcal{C}$, and hence H_p, H_c of the same case are supposed to be identical.
- Kullback-Leibler divergence: describes how one probability distribution diverges from the other one.
  - The $P_p^{\mathcal{C}_{\mathrm{SDC}}}$ distributes more similar to $P_p^{\mathcal{C}_{\mathrm{G}}}$ rather than to $P_p^{\mathcal{C}_{\mathrm{PDC}}}$, for which the middle peak still remains and thus more uniformly.

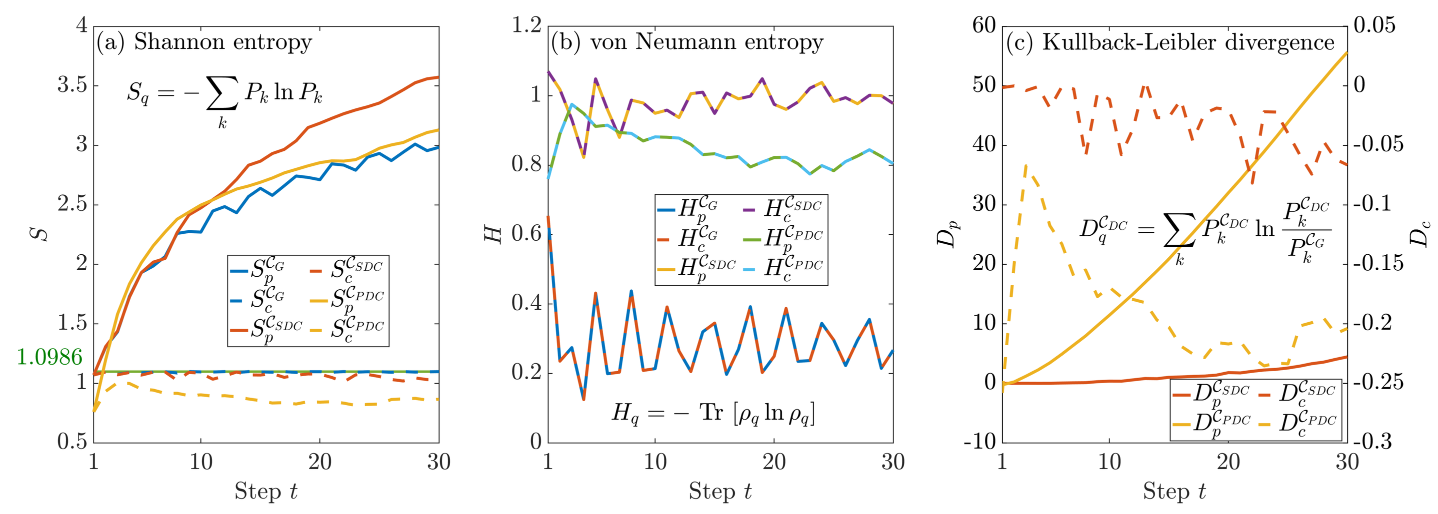

Figure 3:(a) Shannon entropies S_q of coin (dashed) and position (solid) probabilities for the three coins. In the expression of S_q, q can be either c or p associated to coin or position subspace, respectively. The summation is taken over all corresponding probabilities P_k. Due to the fact that $\mathcal{C}$ determines the walker's movements, S_c gets closer to its asymptotic value (green) for the faster distributed case. (b) von Neumann entropies of the reduced density states for the three processes. The simulation result provides an evidence that $\mathcal{C}$ rules the patterns of quantum walks. (c) The relative entropy of P_q and P_q'. The value vanishes when the P_q's of the two walks are identical.

Main contributions

Two models $\mathcal{C}_{\mathrm{SDC}}$ and $\mathcal{C}_{\mathrm{PDC}}$ walk further than $\mathcal{C}_{\mathrm{G}}$ without expanding $\mathcal{H}_c$:

- $\mathcal{C}_{\mathrm{SDC}}$: probability distribution spreads more uniformly than $\mathcal{C}_{\mathrm{G}}$, which may be beneficial to creating coherent resources.
- $\mathcal{C}_{\mathrm{PDC}}$: distributes much faster than both $\mathcal{C}_{\mathrm{G}}$ and $\mathcal{C}_{\mathrm{SDC}}$ cases. The behavior might accelerate the database searching processes by reaching to its edges more quickly.

References

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