第11週小考 RQ: 3-22 ~ 26 Exercise 13~17 Example 3-15~19

Q.3-22 Define *capacitance* and *capacitor*.

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目錄

解答 已驗證 2年前提供

步驟1 步驟1 / 2

Capacitance can be described from the following formula:

$$C = \frac{Q}{V_{12}}$$

This formula says us that capacitance is the ratio of the amount of electrical charge across the difference in electrical potential. The capacitance depends on the capacitor geometry and permittivity, that is why we have different formulas for different types of capacitors (parallel-plate, cylindrical capacitor, spherical capacitor).

步驟2 步驟2 / 2

A capacitor is a device that is constructed from two conductors separated by a dielectric medium, and it stores electrical energy in an electric field.

練習23

Q.3-23 Write the capacitance formula for a parallel-plate capacitor of area S whose plates are separated by a medium of dielectric constant ϵ_r and thickness d.

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解答 已驗證 2年前提供

The capacitance formula for a parallel-plate capacitor is:

$$C = \frac{\varepsilon_0 \varepsilon_r S}{d}$$

where:

1. ε_0 is a vacuum permittivity and it is a constant value $\left(\varepsilon_0 = 8.85 \cdot 10^{-12}\ \frac{\mathrm{F}}{\mathrm{m}}\right)$,
2. ε_r is a relative permittivity, and its value depends from material to material (dimensionless),
3. S is a surface of a plate $(\text{SI unit: } m^2)$, and
4. d is the distance between parallel plates $(\text{SI unit: } m)$.

練習24

Q.3-24 What is the definition of an *electron-volt*? How does it compare with a joule?

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解答 已驗證 2年前提供

An electron volt is the unit of work equal to the charge of electron times $1\ \mathrm{V}$.

$$\begin{aligned} 1eV &= 1.6 \cdot 10^{-19}\ \mathrm{C} \cdot 1\ \mathrm{V} \\ &= \boxed{1.6 \cdot 10^{-19}\ \mathrm{J}} \end{aligned}$$

Note: $\mathrm{C} \cdot \mathrm{V} = \mathrm{J}$

練習25

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Fundamentals of Engineering Electromagnetics

Q.3-25 Write the expression for electrostatic energy in terms of **E**.

目錄

解答 已驗證 2年前提供

步驟1 步驟1 / 2

The expression for electrostatic energy in terms of $\mathbf{E}$ is:

$$W_e = \frac{1}{2}\int_V \varepsilon E^2 dv$$

where:

1. ε is an absolute permittivity that denotes the electric polarization ability of a material. It can be expressed as $\varepsilon = \varepsilon_0 \varepsilon_r$, where ε_0 is a vacuum permittivity and ε_r is a relative permittivity. The SI Unit of ε is $\frac{\mathrm{F}}{\mathrm{m}}$.
2. E is the magnitude of an electric field where the electrostatic energy is stored. The SI Unit of E is $\frac{\mathrm{V}}{\mathrm{m}}$.
3. dv is a volume element that depends on a coordinate system.

結果 步驟2 / 2

$$W_e = \frac{1}{2}\int_V \varepsilon E^2 dv$$

練習26

第3章，第128頁

Q.3-26 Discuss the meaning and use of the *principle of virtual displacement*.

目錄

解答 已驗證 2年前提供

When one wants to calculate the forces between complex charged bodies, it is more reasonable to use the principle of virtual displacement than Coulomb's law. The reason for this is the easier calculation. According to the principle of virtual displacement, the system of complex charged bodies is put into an isolated system, i.e. no outside forces are affecting the system. An imaginary displacement of one of the bodies happens due to the electric forces in the system. The work needed to make this displacement is the product of the electric force acting on that body and the length of the (virtual) differential distance. Using this method, one can take advantage of the relation between the work and the force to calculate the electric force in the system.

EXERCISE 3.13 Determine the capacitance of the parallel-plate capacitor in Fig. 3-20 by starting with an assumed potential difference V_{12} between the upper and lower plates, then finding Q and taking the ratio Q/V_{12}.

解答 已驗證 2年前提供

步驟1

步驟1 / 6

In this exercise, we shall find the capacitance of a parallel plate capacitor assuming the potential difference between parallel plates. From figure 3-20, we can see that we have known area S and separation d.

步驟2

步驟2 / 6

We all know that the electric field of parallel plate capacitor is the potential over the distance, thus:

$$|\mathbf{E}| = \boxed{\frac{V_{12}}{d}}$$

or as a vector:

$$\mathbf{E} = \boxed{-\mathbf{a}_y \frac{V_{12}}{d}}$$

步驟3

步驟3 / 6

Now, we can apply the Gaus surface around the calculated enclosed charge:

$$\oint \mathbf{E} d\mathbf{S} = \frac{Q_{enc}}{\varepsilon}$$

步驟4

步驟4 / 6

We already know the surface S, so the integral becomes:

$$|\mathbf{E}|S = \frac{Q}{\varepsilon}$$
$$Q = \varepsilon|\mathbf{E}|S$$

步驟5

步驟5 / 6

By applying the previously calculated electric field, we have:

$$Q = \varepsilon \frac{V_{12}}{d} S$$
$$\frac{Q}{V_{12}} = \varepsilon \frac{S}{d}$$
$$C = \boxed{\varepsilon \frac{S}{d}}$$

結果

步驟6 / 6

$$C = \varepsilon \frac{S}{d}$$

練習14

EXERCISE 3.14 Assume the Earth to be a large conducting sphere (radius $= 6.37 \times 10^3$ km) surrounded by air. Find its capacitance referring to infinity.

ANS. 7.08×10^{-4} (F).

解答 已驗證 2年前提供

步驟1 步驟1 / 7

In this exercise, we shall calculate the capacitance of Earth if it is considered a large spherical conductor. First, we shall the general formula for the capacitance for the spherical capacitor. We shall start from Gauss law:

$$\oiint \mathbf{E} d\mathbf{S} = \frac{Q}{\varepsilon}$$

步驟2 步驟2 / 7

Since we have a spherical capacitor, the surface of the closed integral is:

$$S = 4\pi r^2$$

步驟3 步驟3 / 7

So, we have:

$$\begin{aligned} \oiint \mathbf{E} d\mathbf{S} &= \frac{Q}{\varepsilon} \\ ES &= \frac{Q}{\varepsilon} \\ E &= \frac{Q}{\varepsilon S} \\ &= \boxed{\frac{Q}{4\pi\varepsilon_0 r^2}} \end{aligned}$$

步驟4 步驟4 / 7

Now, we can compute the electrical potential:

$$\begin{aligned} V &= -\int_b^a \frac{Q}{4\pi\varepsilon_0 r^2} dr \\ &= -\frac{Q}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_b^a \\ &= \boxed{\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a}-\frac{1}{b}\right]} \end{aligned}$$

步驟5 步驟5 / 7

Now, we can calculate capacitance from the well-known relationship:

$$\begin{aligned} C &= \frac{Q}{V} \\ &= \frac{\cancel{Q}}{\frac{\cancel{Q}}{4\pi\varepsilon_0}\left[\frac{1}{a}-\frac{1}{b}\right]} \\ &= \boxed{\frac{4\pi\varepsilon_0}{\frac{1}{a}-\frac{1}{b}}} \end{aligned}$$

步驟6 步驟6 / 7

By applying the above formula and given parameters, the capacitance of the Earth, if $a = 6370$ km and $b = \infty$, is:

$$\begin{aligned} C &= \frac{4\pi\varepsilon}{\frac{1}{a}-\frac{1}{b}} \\ &= \frac{4\cancel{\pi} \cdot \frac{10^{-9}}{36\cancel{\pi}}}{\frac{1}{6370 \cdot 10^3}-\frac{1}{\infty}} \\ &= \frac{4\pi \cdot 10^{-9} \cdot 6370 \cdot 10^3}{36} \\ &= \boxed{707.78\ \mu\text{F}} \end{aligned}$$

結果 步驟7 / 7

$$C = 707.78\ \mu\text{F}$$

練習15

EXERCISE 3.15 Convert the kinetic energy of 2 (TeV) of the proton beam of a very powerful high-energy particle accelerator into joules.

ANS. 3.20×10^{-7} (J).

解答 已驗證 2年前提供

步驟1 步驟1 / 2

We shall convert $2\ \mathrm{TeV}$ into J. T stands a metric prefix for tera which is 10^{12}. eV stands for electron-volt which is $1.6 \cdot 10^{-19}\ \mathrm{J}$. Thus, we have:

$$\begin{aligned} 2\ \mathrm{TeV} &= 2 \cdot 10^{12} \cdot 1.6 \cdot 10^{-19} \\ &= \boxed{3.2 \cdot 10^{-7}\ \mathrm{J}} \end{aligned}$$

結果 步驟2 / 2

$$2\ \mathrm{TeV} = 3.2 \cdot 10^{-7}\ \mathrm{J}$$

練習16

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EXERCISE 3.16 Determine the amount of energy needed to arrange three point charges -1 (μC), 2 (μC), and 3 (μC) at the corners of an equilateral triangle of sides 10 (cm) in free space.

ANS. 0.09 (J).

解答 已驗證 2年前提供

步驟1 步驟1 / 3

The necessary work to keep three charges in equilibrium is given by the following relation:

$$W = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1Q_2}{R_{12}} + \frac{Q_1Q_3}{R_{13}} + \frac{Q_2Q_3}{R_{23}}\right]$$

步驟2 步驟2 / 3

Inserting the given parameters, we will get the work necessary to keep three charges in equilibrium:

$$\begin{aligned} W &= \frac{1}{4\pi \cdot \frac{10^{-9}}{36\pi}}\left[\frac{-10^{-12} \cdot 2 \cdot 10^{-12}}{0.1} + \frac{-10^{-12} \cdot 3 \cdot 10^{-12}}{0.1} + \frac{2 \cdot 10^{-12} \cdot 3 \cdot 10^{-12}}{0.1}\right] \\ &= 9 \cdot 10^{9} \cdot (-2 \cdot 10^{-11} - 3 \cdot 10^{-11} + 6 \cdot 10^{-11}) \\ &= 9 \cdot 10^{9} \cdot 10^{-11} \\ &= \boxed{0.09\ \mathrm{J}} \end{aligned}$$

結果 步驟3 / 3

$$W = 0.09\ \mathrm{J}$$

練習17

EXERCISE 3.17 Two capacitors having capacitances 20 (μF) and 40 (μF) are connected in series across a 60-(V) battery. Calculate the energy stored in each capacitor.

ANS. 16 (mJ), 8 (mJ).

解答 已驗證 2年前提供

步驟1 步驟1 / 5

Since we have the series connection, each capacitor will have its own voltage drop since $Q = const$. The voltage drop across the first capacitor $(20\ \mu C)$ can be calculated from the capacitor voltage divider:

$$\begin{aligned} V_1 &= V \cdot \frac{C_2}{C_1 + C_2} \\ &= 60 \cdot \frac{40}{20+40} \\ &= \boxed{40\ \mathrm{V}} \end{aligned}$$

步驟2 步驟2 / 5

The energy stored in the first capacitor is:

$$\begin{aligned} W_{e1} &= \frac{C_1 \cdot V_1^2}{2} \\ &= \frac{20 \cdot 10^{-6} \cdot (40)^2}{2} \\ &= \boxed{16\ \mathrm{mJ}} \end{aligned}$$

步驟3 步驟3 / 5

The voltage drop across the second capacitor $(40\ \mu C)$ can be calculated from the capacitor voltage divider:

$$\begin{aligned} V_2 &= V \cdot \frac{C_1}{C_1 + C_2} \\ &= 60 \cdot \frac{20}{20+40} \\ &= \boxed{20\ \mathrm{V}} \end{aligned}$$

步驟4 步驟4 / 5

The energy stored in the second capacitor is:

$$\begin{aligned} W_{e2} &= \frac{C_2 \cdot V_2^2}{2} \\ &= \frac{40 \cdot 10^{-6} \cdot (20)^2}{2} \\ &= \boxed{8\ \mathrm{mJ}} \end{aligned}$$

結果 步驟5 / 5

$$W_{e1} = 16\ \mathrm{mJ}$$
$$W_{e2} = 8\ \mathrm{mJ}$$