範圍： 3.5（虛像法不考）～4全

12/19

Solution of Electromagnetics (1) Final

1. 如圖一所示有一接地的無限長的空心金屬管其軸心位於 z 軸且內徑及外徑分別為 5cm 和 8cm，在軸心有一電荷線其線電荷密度為ρ_l=1nC/m，請問在空間中任一點的電壓值為何? 10%

Apply Gauss's Law, 0<r<5cm

$$\oint_S \bar{E}\cdot d\bar{s}=\frac{\int_l \rho_l dl}{\varepsilon_0}\Rightarrow 2\pi rLE_r=\frac{1\times10^{-9}}{\varepsilon_0}\Rightarrow E_r=\frac{1\times10^{-9}}{2\pi r\varepsilon_0}=\frac{18}{r}(V/m),$$

$$V=-\int_{0.05}^{r}\frac{18}{r}dr=18\ln(0.05/r)$$

5cm≤r≤8cm

V=0

8cm<r, E are the same

V=-18ln(r/0.08) (V)

2. 有一球狀電荷半徑為 9cm 且其內部之電場為 $\bar{E}=\bar{a}_R\frac{1}{R}$ 而外部的電場 $\bar{E}=\bar{a}_R\frac{0.09}{R^2}$，請問此球包含的靜電能有多少? 10%

Solution:

$$W_e=\frac{1}{2}\int\varepsilon_0|E_R|^2dv$$

$$=\cancel{\frac{1}{2}}\varepsilon_0\left(\int_0^{0.09}\int_0^{2\pi}\int_0^{\pi}(\frac{1}{R})^2R^2\sin\theta d\theta d\phi dR+\int_{0.09}^{\infty}\int_0^{2\pi}\int_0^{\pi}(\frac{0.09}{R^2})^2R^2\sin\theta d\theta d\phi dR\right)$$

$$=\cancel{4}\pi\varepsilon_0\left(\int_0^{0.09}dR+\int_{0.09}^{\infty}R^{-2}dR\right)=\cancel{4}\pi\varepsilon_0\left(0.09+0.09^2R^{-1}\Big|_{\infty}^{0.09}\right)=4\pi\varepsilon_0\left(0.09+\frac{0.09^2}{0.09}\right)=\cancel{2}\times10^{-11}(J)$$

2 2 1

3. 如圖二所示為四分之一空心圓之介質並內徑為 1mm 而其外徑為 1.649mm 且厚度(T)為 2mm，請問(a)在 A 平面與 B 平面加上均匀之電壓且兩面之間電壓差為 1V 時，在介質內的電壓分佈函數為何? (b)若是在 C 平面與 D 平面加上均匀之電壓且兩面之間電壓差也為 10V 時其電壓分佈函數為何? 30%

(Sol) Solve Laplace Eq. $\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial V}{\partial r})+\frac{1}{r^2}\frac{\partial^2 V}{\partial\phi^2}+\frac{\partial^2 V}{\partial z^2}=0$

(a) 在 z 與 r 方向為均匀電壓⇒∂V/∂z=∂V/∂r=0

$\frac{\partial^2 V}{\partial\phi^2}$=0⇒V(ϕ)=Aϕ+B；V(ϕ=0)=0⇒B=0, V(ϕ=π/2)=1⇒1=Aπ/2；A=2/π

V(ϕ)=2ϕ/π

(b) 在ϕ與 z 方向為均匀電壓⇒∂V/∂z=∂V/∂ϕ=0

$\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial V}{\partial r})$=0⇒$\frac{\partial V(r)}{\partial r}=\frac{A}{r}$,V(r)=Alnr+B；

V(r=1.649cm)=0⇒ Aln(1.649mm)+B, V(r=1mm)=1⇒1= Aln(1.0mm)+B；

1=-Aln(1.649)⇒A=-2, B=2ln(1.649mm)

V(ϕ)=-2lnr+2ln(1.649mm)=2ln(1.649mm/r) = -1.2815

A ln(1) + B = 0

A ln(1.649) + B = 1

A ln(1.649) = 1

A 0.5 = 1 A = 2

考試時間: 112年12月19日下午2:10~4:00舉行，考試時間110分鐘。

考試範圍: Chap3.11.1~3.11.4及Chap.4.

地點：第一醫學大樓M204教室.

考試規定: Close book，但可帶一張A4紙上面可有(1)教科書封底之公式表及(2)各座標向量轉換及標示轉換公式. (3)Eq. (3-28), (3-31), (3-36), (3-38)~(3-40),(3-57), (3-59),(3-63), (3-72), (3-75), (3-97), (3-101) , (3-106), (3-126), (3-130),(4-9),(4-10),(4-11),(4-27),(4-30), (4-34), (4-35), (4-38).

4. 圖三所示兩片平行金屬板中間有兩層不同物質，若在上下兩平面加上 V_0 電壓，(a) 請由 Laplace 方程式找出此結構中的電壓分佈函數；(b)請用 V_0 及其兩導體之導電度來表示此結構上下兩平面間的等效電阻及電流密度。 30%

(Sol) Solve Laplace Eq. $\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2}=0$,

在 x 與 z 方向為均勻電壓$\Rightarrow \partial V/\partial z=\partial V/\partial z=0$

$\frac{\partial^2 V}{\partial y^2}=0\Rightarrow$在下層 $V_1(y)=Ay+B$；在上層 $V_2(y)=Cy+D$；$V_1(y=0)=B=0$, $V_2(d)=Cd+D=V_0$

$D_{1y}=D_{2y}$，$-\varepsilon_1\frac{\partial V_1}{\partial y}=-\varepsilon_2\frac{\partial V_2}{\partial y}\Rightarrow\varepsilon_1 A=\varepsilon_2 C$；$V_2(\frac{d}{2})=V_1(\frac{d}{2})\Rightarrow A\frac{d}{2}=C\frac{d}{2}+V_0-Cd=V_0-C\frac{d}{2}$

$\Rightarrow A=\frac{2V_0}{d}-C\Rightarrow A(\frac{\varepsilon_1+\varepsilon_2}{\varepsilon_2})=\frac{2V_0}{d}$，$A=\frac{2V_0\varepsilon_2}{d(\varepsilon_2+\varepsilon_1)}$, $C=\frac{2V_0\varepsilon_1}{d(\varepsilon_2+\varepsilon_1)}$，$d=\frac{V_0(\varepsilon_2-\varepsilon_1)}{(\varepsilon_2+\varepsilon_1)}$

在下層 $V_1(y)=\frac{2V_0\varepsilon_2}{d(\varepsilon_2+\varepsilon_1)}y$；在上層 $V_2(y)=\frac{2V_0\varepsilon_1}{d(\varepsilon_2+\varepsilon_1)}y+\frac{V_0(\varepsilon_2-\varepsilon_1)}{(\varepsilon_2+\varepsilon_1)}$

假設 I 在兩層導體間，$\vec{J}=-\vec{a}_y\frac{I}{W\times l}$

$\vec{E}_2=\vec{J}_2/\sigma_2$ 且 $\vec{E}_1=\vec{J}_2/\sigma_1\Rightarrow V=(\int_0^{d/2}\frac{I}{W\times l\times\sigma_1}dy+\int_{d/2}^{d}\frac{I}{W\times l\times\sigma_2}dy)$

$R=V/I=\frac{d(\sigma_1+\sigma_2)}{2Wl\sigma_1\sigma_2}$, $I=V_0/R\Rightarrow\vec{J}=-\vec{a}_y\frac{2V_0\sigma_1\sigma_2}{d(\sigma_1+\sigma_2)}$

5. 假設在空間中有兩個介質在 x-y 平面相接，在 $z\geq 0$ 的電流密度為 $\vec{J}=\vec{a}_x 20+\vec{a}_y 5-\vec{a}_z 10(A/m^2)$ 且其導電度為 σ_1=20mS，若在 $z<0$ 的區域導電度為 σ_2=4mS，請問在 $z<0$ 的區域其電流密度為何?及在各區域的電場強度為何? 10%?

(Sol)$J_{2n}=J_{1n}\Rightarrow J_{2z}=J_{1z}=-10$, $\frac{J_{2t}}{\sigma_2}=\frac{J_{1t}}{\sigma_1}\Rightarrow J_{2x}=\sigma_2 J_{1x}/\sigma_1=4$, $J_{2y}=\sigma_2 J_{1y}/\sigma_1=1$

$z<0$ 區域 $\vec{J}=\vec{a}_x 4+\vec{a}_y 1-\vec{a}_z 10(A/m^2)$。

$\vec{E}=\vec{J}/\sigma$, $z\geq 0$ 區域 $\vec{E}=\vec{a}_x 1+\vec{a}_y .25-\vec{a}_z 0.5(KV/m)$；

$z<0$ 區域 $\vec{E}=\vec{a}_x 1+\vec{a}_y 0.25-\vec{a}_z 0.25(KV/m)$

6. 有一 n 型半導體其電子濃度為 $8.8\times10^3\ C/m^3$，電洞濃度為 $1.76\times10^2\ C/m^3$，若此半導體的相對介電常數為 12，且電子及電洞遷移速率分別為 $0.12\ m^2/V\text{-}s$ 與 $0.03\ m^2/V\text{-}s$ 時，請問當有少數載子其濃度為 ρ_0 進入此半導體時，需要多久時間少數載子濃度會衰減至 $\rho_0 e^{-10}$? 15%

(Sol) $\sigma=-\rho_e\mu_h+\rho_h\mu_h=-(-8.8\times10^3)\times0.12+1.76\times10^2\times0.03=1.061\times10^3$

$\tau=12\varepsilon_0/\sigma=\frac{1}{3\pi}\times10^{-9}/1.061\times10^3=1\times10^{-13}$, $t=10\tau=1ps$

7. 空心圓柱內含兩層導體如圖四所示，此導體的內徑為 1cm 外徑為 4.485cm，其內層導體的厚度為 1.72cm，內外導體之導電度(conductivity)分別為 10mS 與 2mS 請計算此圓柱內外兩層導體所構成的單位長度等效電阻。 15%

(Sol) 假設 I 在兩層導體間，$\vec{J}=\vec{a}_r I/2\pi r$ 在內外兩層的電場強度分別為 $\vec{E}_1=\vec{J}_1/\sigma_1$ 與 $\vec{E}_2=\vec{J}_2/\sigma_2$

兩導體間由 I 形成的電壓為 $V=-(\int_{1cm}^{2.72cm}\frac{I}{2\pi\sigma_1 r}dr+\int_{2.72cm}^{4.485cm}\frac{I}{2\pi\sigma_2 r}dr)=-\frac{I}{2\pi}(\frac{\ln 2.72}{1\times10^{-2}}+\frac{\ln 1.649}{2\times10^{-3}})$

兩導體間的電壓差$=\frac{I}{2\pi}(100+250)\Rightarrow R=V/I=350/2\pi\approx55.7(\Omega)$

電磁學(I)總成績				
B0827137	67		B1027222	75
B0927124	54		B1027223	65
B0927130	61		B1027224	60
B0927153	53		B1027225	36
B0927207	68		B1027226	90
B0927229	49		B1027228	60
B1027202	67		B1027231	66
B1027203	34		B1027232	37
B1027204	76		B1027233	60
B1027206	77		B1027234	91
B1027208	63		B1027235	80
B1027209	93		B1027236	86
B1027210	54		B1027237	79
B1027211	67		B1027238	31
B1027212	63		B1027239	69
B1027213	89		B1027241	70
B1027214	64		B1027242	61
B1027215	50		B1027248	61
B1027216	68		B1027249	33
B1027219	60		B1027250	36
B1027220	60		B1027251	65
B1027221	66		B1027254	63

科目 電磁學(一)

長庚大學期中、期末考試答案用紙

112 學年度 第 1 學期 末考 電子工程學系 姓名 林永濬 學號 B1027234

1. ① $r>8cm$

$2\pi r L|\vec{E_1}| = \frac{\rho_\ell L}{\varepsilon_0}$

$\Rightarrow |\vec{E_1}| = \frac{\rho_\ell}{2\pi\varepsilon_0 r}$

$V_1 = -\int_\infty^r \frac{\rho_\ell}{2\pi\varepsilon_0 r}\cdot dr$

$= \frac{\rho_\ell}{2\pi\varepsilon_0}(0 - \ln r)$

$= -\frac{\rho_\ell}{2\pi\varepsilon_0}\ln r$

$= -18\ln r$ (V)

② $8cm > r > 5cm$

∵ 接地

$\Rightarrow V_2 = 0$

③ $5cm > r$

$|\vec{E_3}| = \frac{\rho_\ell}{2\pi\varepsilon_0 r}$

$V_3 = V_2 - \int_{5cm}^r \frac{\rho_\ell}{2\pi\varepsilon_0 r}\cdot dr$

$= 0 - 18\cdot \ln r\Big|_{5cm}^r$

$= -18(\ln r - \ln 5cm)$

$= -18\ln r - 53.92$ (V)

+7

2.

$W_{e內} = \frac{1}{2}\varepsilon_0\int_0^{2\pi}\int_0^{\pi}\int_0^{9cm}\varepsilon|\vec{E}|^2 R^2\sin\theta\, dR\, d\theta\, d\phi$

$= \frac{\varepsilon}{2}\int_0^{2\pi}\int_0^{\pi}\sin\theta\cdot 9\times10^{-2}\, d\theta\, d\phi$

$= \frac{9\times10^{-2}\varepsilon}{2}(-\cos\theta)\Big|_0^{\pi}\times 2\pi$

$= \frac{9\times10^{-2}\varepsilon}{2}[+1-(-1)]\times 2\pi$

$= 2\pi\times 9\times10^{-2}\varepsilon$

$W_{e外} = \frac{1}{2}\varepsilon_0\int_0^{2\pi}\int_0^{\pi}\int_{9cm}^{\infty}\varepsilon\frac{0.09^2}{R^4}R^2\sin\theta\, dR\, d\theta\, d\phi$

$= \frac{1}{2}\varepsilon\times0.09\times\frac{-1}{R}\Big|_{9cm}^{\infty}\times(-\cos\theta)\Big|_0^{\pi}\times 2\pi$

$= \frac{1}{2}\varepsilon\times0.09\times\left[0-\frac{-1}{0.09}\right]\times[1+1]\times2\pi$

$= 2\pi\varepsilon$

$\therefore W_{total} = W_{e內} + W_{e外}$

$= 2\pi 9\times10^{-2}\varepsilon_0 + 2\pi\varepsilon_0$

$\doteq 6.06\times10^{-11}$ J

+9

3. (a) 使用圓柱座標

$\frac{\partial V}{\partial r} = \frac{\partial^2 V}{\partial z^2} = 0$

$\therefore \nabla^2 V = 0$

$\Rightarrow \frac{1}{r^2}\frac{\partial^2 V}{\partial\phi^2} = 0$

$\Rightarrow \frac{\partial^2 V}{\partial\phi^2} = 0$

$\Rightarrow \frac{\partial V}{\partial\phi} = C_1$

$\Rightarrow V = C_1\phi + C_2$

$V_A = 1 = C_1\frac{\pi}{2} + C_2 \Rightarrow C_1 = \frac{2}{\pi}$

$V_B = 0 = C_1 0 + C_2 \Rightarrow C_2 = 0$

$\therefore V = \frac{2}{\pi}\phi$ (V)

~~$\vec{E} = -\nabla V = -\vec{a_\phi}\frac{1}{r}\frac{\partial V}{\partial\phi} = -\vec{a_\phi}\frac{1}{r}\frac{2}{\pi}$~~

~~$\vec{J} = \sigma\vec{E} = -\vec{a_\phi}\frac{1}{r}\frac{2}{\pi}\sigma$~~

~~$I = \int_s \vec{J}\cdot d\vec{s} = \int_0^{4cm}\int_{1mm}^{1.649}\frac{1}{r}\frac{2}{\pi}\sigma\cdot dr\, dz$~~

~~$= \frac{2}{\pi}\sigma$~~

(b) 使用圓柱座標

$\frac{\partial^2 V}{\partial\phi^2} = \frac{\partial^2 V}{\partial z^2} = 0$

$\nabla^2 V = 0 \Rightarrow \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) = 0$

$\Rightarrow r\frac{\partial V}{\partial r} = C_1$

$\Rightarrow \frac{\partial V}{\partial r} = \frac{1}{r}C_1$

$\Rightarrow V = \ln r C_1 + C_2$

$V_C = 1 = \ln(1mm)C_1 + C_2$ — ①

$V_D = 0 = \ln(1.649mm)C_1 + C_2$ — ②

由② $\Rightarrow C_2 = -\ln(1.649mm)C_1$

代入①

$\Rightarrow 1 = \ln(1mm)C_1 - \ln(1.649mm)C_1$

$\Rightarrow C_1 = \frac{1}{\ln(1mm)-\ln(1.649mm)}$

$= -1.999 \doteq -2$

$C_2 = -12.81$

$\therefore V = -1.999\ln r - 12.81$ (V)

+30

(請翻面繼續作答)

長庚大學期中、期末考試答案用紙

______ 學年度 第 ______ 學期 ______ 考 ______ 系 姓名 ______ 學號 ______

4.(a) $\frac{d^2}{dx^2}V = \frac{d^2V}{dz^2} = 0$

$\nabla^2 V = 0 \Rightarrow \frac{d^2}{dy^2}V = 0$

$\Rightarrow V_1 = C_1 y + C_2,\ \frac{d}{2} \le y \le d$

$V_2 = C_3 y + C_4,\ 0 \le \frac{d}{2} \le d$

$\vec{E_1} = -\nabla V_1 = -\vec{a_y} C_1$

$\vec{E_2} = -\nabla V_2 = -\vec{a_y} C_3$

$V_0 = C_1 d + C_2$ —— ①，由① $\Rightarrow C_2 = V_0 - C_1 d$ 代入③

$0 = C_4$ —— ②

$C_1 \frac{d}{2} + C_2 = C_3 \frac{d}{2}$ —— ③

$\cancel{J_{1n} = J_{2n}}$

$\cancel{\Rightarrow \sigma_1 E_{1n} = \sigma_2 E_{2n}}$

$\cancel{\Rightarrow E_{1n} = \frac{\sigma_2}{\sigma_1} E_{2n}}$

$D_{1n} = D_{2n}$

$\Rightarrow \varepsilon_1 |\vec{E_1}| = \varepsilon_2 |\vec{E_2}|$

$\Rightarrow \varepsilon_1 C_1 = \varepsilon_2 C_3$

$\Rightarrow C_3 = \frac{\varepsilon_1}{\varepsilon_2} C_1$ 代入③

$\therefore C_1 \frac{d}{2} + V_0 - C_1 d - \frac{\varepsilon_1}{\varepsilon_2} C_1 \cdot \frac{d}{2} = 0$

$\Rightarrow C_1 \left(\frac{d}{2} - d - \frac{\varepsilon_1}{\varepsilon_2}\frac{d}{2}\right) = -V_0$

$\Rightarrow C_1 = \frac{-V_0}{\frac{d}{2} - d - \frac{\varepsilon_1}{\varepsilon_2}\frac{d}{2}}$

$\Rightarrow C_2 = V_0 - C_1 d = V_0 + \frac{V_0}{\frac{1}{2} - 1 - \frac{\varepsilon_1}{\varepsilon_2}\cdot\frac{1}{2}}$

$C_3 = \frac{\varepsilon_1}{\varepsilon_2} C_1 = \frac{-\varepsilon_1 V_0}{\varepsilon_2\left(\frac{d}{2} - d - \frac{\varepsilon_1}{\varepsilon_2}\frac{d}{2}\right)}$

-1.75

$\therefore \frac{d}{2} \le y \le d,\ V = \frac{-V_0}{\frac{d}{2} - d - \frac{\varepsilon_1}{\varepsilon_2}\frac{d}{2}} y + V_0 + \frac{V_0}{-\frac{1}{2} - \frac{1}{2}\frac{\varepsilon_1}{\varepsilon_2}}$ #

$0 \le y \le \frac{d}{2},\ V = \frac{-\varepsilon_1 V_0}{\frac{d}{2}\varepsilon_2 - \varepsilon_2 d - \varepsilon_1 \frac{d}{2}} y$ #

(b) $\vec{E_1} = -\vec{a_y} C_1 = \vec{a_y} \frac{V_0}{\frac{d}{2} - d \frac{\varepsilon_1}{\varepsilon_2}\frac{d}{2}}$

$\vec{E_2} = -\vec{a_y} C_3 = \vec{a_y} \frac{\varepsilon_1 V_0}{\varepsilon_2\left(\frac{d}{2} - d - \frac{\varepsilon_1}{\varepsilon_2}\frac{d}{2}\right)}$

$\vec{J_1} = \sigma_1 \vec{E_1} = \vec{a_y} \frac{V_0 \sigma_1}{\frac{d}{2} - d - \frac{\varepsilon_1}{\varepsilon_2}\frac{d}{2}}$ #

$\vec{J_2} = \sigma_2 \vec{E_2} = \vec{a_y} \frac{\varepsilon_1 V_0 \sigma_2}{\varepsilon_2\left(\frac{d}{2} - d - \frac{\varepsilon_1}{\varepsilon_2}\frac{d}{2}\right)}$ #

$I_1 = |\vec{J_1}| \cdot w \cdot l = \frac{V_0 \sigma_1 w l}{\frac{d}{2} - d - \frac{\varepsilon_1}{\varepsilon_2}\frac{d}{2}}$

$I_2 = |\vec{J_2}| \cdot wl = \frac{\varepsilon_1 \sigma_2 V_0 wl}{\varepsilon_2\left(\frac{d}{2} - d - \frac{\varepsilon_1}{\varepsilon_2}\frac{d}{2}\right)}$

$R = \frac{V_0}{I_1 + I_2} = \frac{V_0}{\frac{V_0 \sigma_1 wl}{\frac{d}{2} - d - \frac{\varepsilon_1}{\varepsilon_2}\frac{d}{2}} + \frac{\varepsilon_1 \sigma_2 V_0 wl}{\varepsilon_2\left(\frac{d}{2} - d - \frac{\varepsilon_1}{\varepsilon_2}\frac{d}{2}\right)}}$ #

$I_1 = I_2$

長庚大學期中、期末考試答案用紙

科目＿＿＿＿

＿＿學年度 第＿＿學期＿＿考＿＿＿＿系 姓名 林永濬 學號 B1021234

5. $J_{1n} = J_{2n}$

$\Rightarrow J_{1z} = J_{2z} = -10$

$\frac{J_{1t}}{\sigma_1} = \frac{J_{2t}}{\sigma_2} \Rightarrow J_{2t} = \frac{\sigma_2}{\sigma_1} J_{1t}$

$= \frac{4m}{20m} J_{1t}$

$= \frac{1}{5} J_{1t}$

$\therefore J_{2x} = \frac{1}{5} J_{1x} = \frac{1}{5} \times 20 = 4$

$J_{2y} = \frac{1}{5} J_{1y} = \frac{1}{5} \times 5 = 1$

(a)

$\therefore$ $z<0$ 時，$\vec{J} = \vec{a_x} 4 + \vec{a_y} - \vec{a_z} 10 \ A/m^2$

+5

(b)

$|\vec{J_1}| = \sigma_1 |\vec{E_1}| \Rightarrow |\vec{E_1}| = \frac{|\vec{J_1}|}{\sigma_1} = \frac{5\sqrt{21}}{20M} = 1.146 \times 10^3 \ V/m$

$|\vec{E_2}| = \frac{|\vec{J_2}|}{\sigma_2} = \frac{3\sqrt{13}}{4M} = 2.704 \times 10^3 \ V/m$

6.

$\sigma = -\rho_e \mu_e + \rho_h \mu_h = 8.8 \times 10^3 \times 0.12 + 1.76 \times 10^2 \times 0.03 = 1061.28$

$\rho_0 e^{-\frac{\sigma}{\varepsilon} t} = \rho_0 e^{-10}$

$\Rightarrow -\frac{\sigma}{\varepsilon} t = -10$

$\Rightarrow \frac{1061.28}{12\varepsilon_0} t = 10$

$\Rightarrow t = 9.998 \times 10^{-13} \ s$

+14

7. 使用圓柱座標

↓背面

(請翻面繼續作答)

長庚大學期中、期末考試答案用紙 $V=2\pi$ 科目

學年度 第 學期 考 系 姓名 學號

7. 〈法1〉使用圓柱座標

$\frac{\partial^2 V}{\partial \phi^2} = \frac{\partial^2 V}{\partial z^2} = 0$

$\therefore \nabla^2 V = 0$

$\Rightarrow \frac{1}{r}\frac{d}{dr}\left(r\frac{dV}{dr}\right) = 0$

$\Rightarrow r\frac{dV}{dr} = C_1$

$\Rightarrow V_1 = \ln r\, C_1 + C_2$, $1cm < r < 2.72cm$ —— ④

$V_2 = \ln r\, C_3 + C_4$, $2.72cm < r < 4.485cm$ —— ⑤

~~$V_1 = \ln r\, C_1$~~

〈法2〉$J = \frac{I}{2\pi r}$

$E = \frac{I}{2\pi r\sigma}$

$V_1 = -\int \frac{I}{2\pi r\sigma}$

V_2

$R = \frac{V_1+V_2}{I}$

令內徑上電位為V_0且與外徑的電位差V_0

$\therefore V_0 = \ln(1\times10^{-2})C_1 + C_2 \Rightarrow C_2 = V_0 - \ln(1cm)C_1$ —— ①

$0 = \ln(4.485\times10^{-2})C_3 + C_4 \Rightarrow C_4 = -\ln(4.485cm)C_3$ —— ②

$\ln(2.72cm)C_1 + C_2 = \ln(2.72cm)C_3 + C_4$

$\vec{E_1} = -\nabla V_1 = -\vec{a_r}\frac{1}{r^2}C_1$

$\vec{E_2} = -\nabla V_2 = -\vec{a_r}\frac{1}{r^2}C_3$

~~$D_1 = D_2 \Rightarrow \varepsilon_1 C_1 = \varepsilon_2 C_3$~~

$\neq$

$J_{1n} = J_{2n}$

$\Rightarrow \sigma_1 C_1 = \sigma_2 C_3$

$\Rightarrow 5C_1 = C_3$ —— ③

+8

$\therefore \ln(2.72cm)C_1 + V_0 - \ln(1cm)C_1 = [\ln(2.72cm)]5C_1 - \ln(4.485\times10^{-2})5C_1$

$\Rightarrow C_1 = \frac{V_0}{[\ln(2.72\times10^{-2})]\times5 - [\ln(4.485\times10^{-2})]5 - \ln(2.72\times10^{-2}) + \ln(1\times10^{-2})} = \frac{V_0}{-0.2856}$

$C_2 = V_0 - \ln(1\times10^{-2})C_1$

由①、②、③、④、⑤可知電壓函數，再由$\vec{E} = -\nabla V$，$\vec{J} = \sigma\vec{E}$，$I = JS$，$R = \frac{V_0}{I} \Rightarrow$ 等效電阻

(請翻面繼續作答)