練習1
第4章，第167頁

Fundamentals of Engineering Electromagnetics
ISBN：9780201566116
目錄

解答 已驗證

解答A 解答B

2年前提供

步驟1 步驟1 / 6

(a)

- Calculating the wire resistance,

$$R = \frac{V}{I}$$
$$= \frac{6}{\frac{1}{6}}$$
$$= 36\ \Omega$$

步驟2 步驟2 / 6

- Calculating the value of A,

$$A = \pi r^2$$
$$= \pi(0.5 \times 10^{-3})^2$$
$$= 2.5\pi \times 10^{-7}\ \text{m}^2$$

- Calculating the value of the wire conductivity,

$$\text{Conductivity} = \frac{l}{RA}$$
$$= \frac{1000}{36 \times 2.5\pi \times 10^{-7}}$$
$$= 3.5 \times 10^7$$

- Thus,

$$\boxed{\text{Conductivity} = 3.5 \times 10^7\ \text{S/m}}$$

步驟3 步驟3 / 6

(b)

- Calculating the field intensity,

$$E = \frac{V}{l}$$
$$= \frac{6}{1000}$$
$$= 6\ \text{mv/m}$$

- Thus,

$$\boxed{E = 6\ \text{mv/m}}$$

步驟4 步驟4 / 6

(c)

- Calculating the power dissipated,

$$P = VI$$
$$= 6 \times \frac{1}{6}$$
$$= 1\ \text{W}$$

- Thus,

$$\boxed{W = 1\ \text{W}}$$

步驟5 步驟5 / 6

(d)

- Calculating the value of v_d,

$$v_d = \mu.E$$
$$= 1.4 \times 10^{-3} \times 6 \times 10^{-3}$$
$$= 8.4 \times 10^{-6}\ \text{m/s}$$

- Thus,

$$\boxed{v_d = 8.4 \times 10^{-6}\ \text{m/s}}$$

結果 步驟6 / 6

$$(a)\ \text{Conductivity} = 3.5 \times 10^7\ \text{S/m}$$

$$(b)\ E = 6\ \text{mv/m}$$

$$(c)\ W = 1\ \text{W}$$

$$(d)\ v_d = 8.4 \times 10^{-6}\ \text{m/s}$$

2年前提供

步驟1

步驟1 / 5

In this problem we are given this data:

$$V = \ 6\,\mathrm{V}\, l = \ 1\,\mathrm{km}\, r = \ 0.5\,\mathrm{mm}\, I = \ \frac{1}{6}\,\mathrm{A}$$

We need to find these values:

a) the conductivity of the wire

We can find conductivity using the formula for resistance, and Ohm's Law:

$$R = \frac{l}{\sigma \cdot S} = \frac{V}{I} \rightarrow \sigma = \frac{l \cdot I}{V \cdot S}$$

We know the voltage, the current, and the length of a wire, and the radius, so we can find conductivity:

$$\begin{aligned} S &= r^2\pi \\ \sigma &= \frac{l \cdot I}{V \cdot S} = \frac{l \cdot I}{V \cdot r^2\pi} \\ &= \frac{10^3\,\mathrm{m} \cdot \frac{1}{6}\,\mathrm{A}}{6\,\mathrm{V} \cdot (0.5 \cdot 10^{-3}\,\mathrm{m})^2\pi} \\ &= 3.54 \cdot 10^7\,\mathrm{S/m} \end{aligned}$$

步驟2

步驟2 / 5

b) the electric field intensity in the wire

Electric field intensity inside the wire is constant, and it is equal to the potential difference across the wire and the length:

$$E = \frac{V}{l} = \frac{6\,\mathrm{V}}{10^3\,\mathrm{m}} = 6 \cdot 10^{-3}\,\mathrm{V/m}$$

步驟3

步驟3 / 5

c) the power dissipated in the wire

$$\begin{aligned} P &= V \cdot I = 6\,\mathrm{V} \cdot \frac{1}{6}\,\mathrm{A} \\ P &= 1\,\mathrm{W} \end{aligned}$$

步驟4

步驟4 / 5

d) the electron drift velocity, assuming electron mobility in the wire to be $\mu_e = 1.4 \cdot 10^{-3}$ m^2/Vs

To find the electron drift velocity,we can use equation (4-8) from the book:

$$\mathbf{u_e} = -\mu_\mathbf{e}\mathbf{E}$$

Using that equation, we can write:

$$\begin{aligned} u_e &= |\mu_e E| = 1.4 \cdot 10^{-3} \cdot 6 \cdot 10^{-3} \\ u_e &= 8.4 \cdot 10^{-6}\,\mathrm{m/s} \end{aligned}$$

結果

步驟5 / 5

$$\begin{aligned} &\text{a) } \sigma = 3.54 \cdot 10^7\,\mathrm{S/m} \\ &\text{b) } E = 6 \cdot 10^{-3}\,\mathrm{V/m} \\ &\text{c) } P = \ 1\,\mathrm{W} \\ &\text{d) } u_e = 8.4 \cdot 10^{-6}\,\mathrm{m/s} \end{aligned}$$

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解答 已驗證 2年前提供

步驟1 步驟1 / 7

(a)

- Considering the resistance of the wire R_1,

$$R_1 = \frac{1}{\sigma \pi a^2}$$

- Considering the resistance of the coated R_2,

$$R_2 = \frac{1}{0.1\sigma(\pi(a+b)^2 - \pi a^2)}$$

步驟2 步驟2 / 7

- For the resistance to be reduced to 50%,

$$R_1 = R_2$$

$$\frac{1}{\sigma \pi a^2} = \frac{1}{0.1\sigma(\pi(a+b)^2 - \pi a^2)}$$

$$10a^2 = b^2 + 2ab$$

$$b^2 + 2ab - 10a^2 = 0$$

  - Resolving b,

$$b = 2.32a$$

  - Thus,

$$\boxed{b = 2.32a}$$

步驟3 步驟3 / 7

(b)

- Calculating the value of J_1,

$$J_1 = \frac{I_1}{\pi a^2}$$

$$= \frac{\frac{I}{2}}{\pi a^2}$$

$$= \frac{I}{2\pi a^2}$$

- Thus,

$$\boxed{J_1 = \frac{I}{2\pi a^2}}$$

步驟4 步驟4 / 7

- Calculating the value of J_2,

$$J_2 = \frac{I_2}{(\pi(a+b)^2 - \pi a^2)}$$

$$= \frac{\frac{I}{2}}{(\pi(a+2.32a)^2 - \pi a^2)}$$

$$= \frac{I}{20\pi a^2}$$

- Thus,

$$\boxed{J_2 = \frac{I}{20\pi a^2}}$$

步驟5 步驟5 / 7

- Calculating the value of E_1,

$$E_1 = \frac{J_1}{\sigma}$$

$$= \frac{I}{2\pi a^2 \sigma}$$

- Thus,

$$\boxed{E_1 = \frac{I}{2\pi a^2 \sigma}}$$

步驟6 步驟6 / 7

- Calculating the value of E_2,

$$E_2 = \frac{J_2}{0.1\sigma}$$

$$= \frac{I}{2\pi a^2 \sigma}$$

- Thus,

$$\boxed{E_2 = \frac{I}{2\pi a^2 \sigma}}$$

結果 步驟7 / 7

$$(a)\ b = 2.32a$$

$$(b)\ J_1 = \frac{I}{2\pi a^2}$$

$$J_2 = \frac{I}{20\pi a^2}$$

$$E_1 = \frac{I}{2\pi a^2 \sigma}$$

$$E_2 = \frac{I}{2\pi a^2 \sigma}$$

練習3

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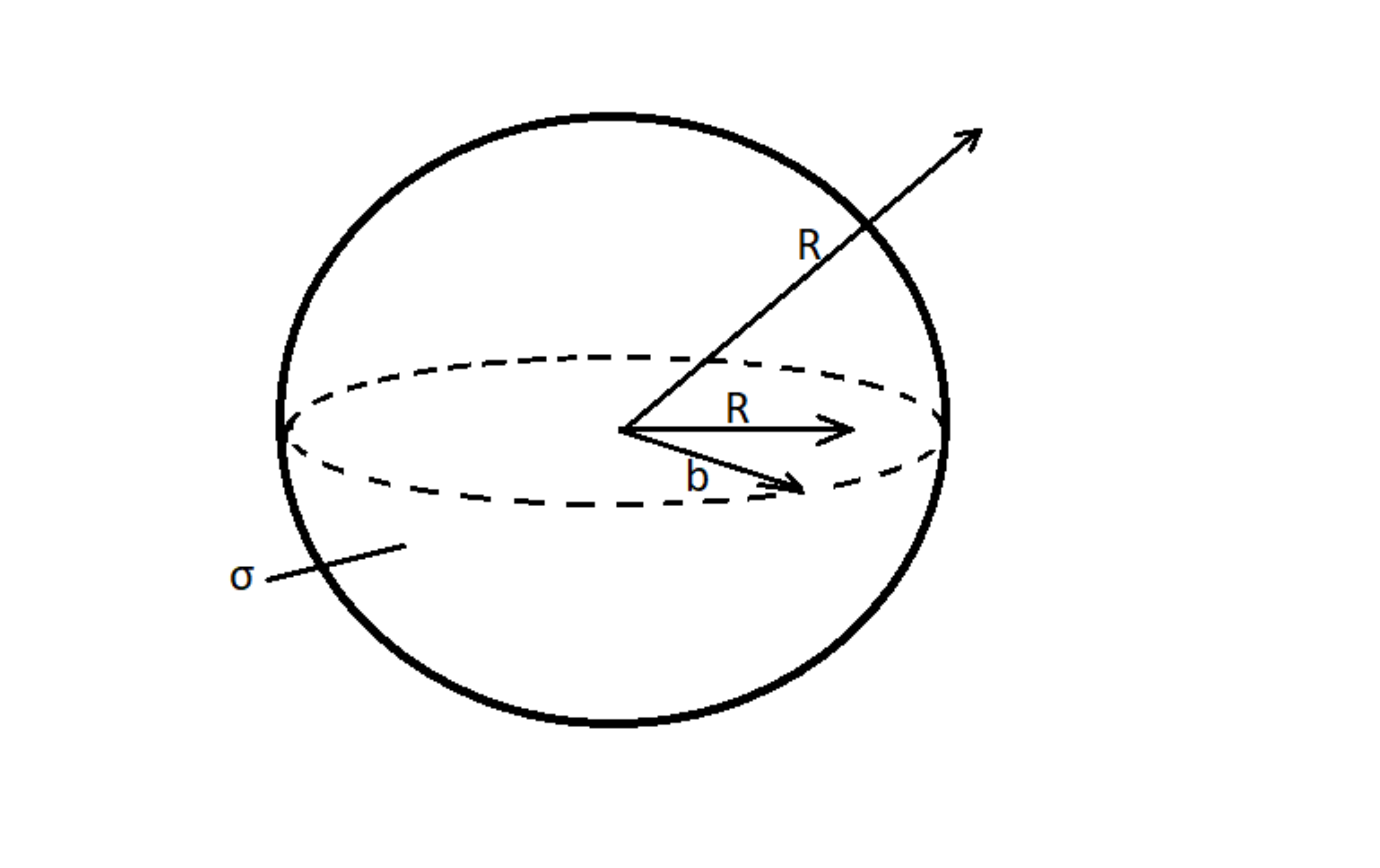
解答 已驗證 2年前提供

步驟1 步驟1 / 7

In this problem, we need to find an electric field intensity and the current density in the sphere.

The sphere has a radius b, and to find these values for inside and outside of the sphere, we will consider radius R for both cases.



步驟2 步驟2 / 7

The initial charge density in the sphere can be found like this:

$$\rho_0 = \frac{Q}{V} = \frac{Q_0}{\frac{4\pi}{3}\cdot b^3} = \frac{10^{-3}}{\frac{4\pi}{3}\cdot 0.1^3}$$
$$\rho_0 = 0.239\ \frac{\mathrm{C}}{\mathrm{m}^3}$$

Charge density at time t is decreasing exponentially, and is found using this equation:

$$\rho = \rho_0 \cdot e^{-\frac{\sigma}{\epsilon}\cdot t}$$

步驟3 步驟3 / 7

$a)$ - $R<b$ $$\begin{align*} \overrightarrow {E_{in} } =& \overrightarrow{ a_R } \cdot \dfrac{ Q }{ 4\pi \varepsilon R^2 } \\ =& \overrightarrow{ a_R } \cdot \dfrac{ \dfrac{4 \pi}{3} R^3 \rho }{ 4\pi \varepsilon R^2 } \\ =& \overrightarrow{ a_R } \cdot \dfrac{ R \cdot \rho_0 \cdot e^{ - \dfrac{ \sigma }{ \varepsilon } \cdot t } }{3\cdot 1.2 \, \varepsilon_0 } \\ =& \overrightarrow{ a_R } \cdot \dfrac{ R }{3 \cdot 1.2 \cdot 8.85 \cdot 10 ^{-12} } \cdot 0.239 \cdot e^{ - \dfrac{ 10 }{ 1.2 \, \varepsilon_0 } \cdot t } \\ \\ =& \overrightarrow{ a_R } \cdot7.5 \cdot 10 ^9 R \cdot e^{ -9.42 \cdot 10^{11} t } \ \ \ \mathrm{ V/m } \end{align*}$$

步驟4 步驟4 / 7

- $R > b$

$$\begin{aligned} \overrightarrow{E_{out}} &= \overrightarrow{a_R}\cdot\frac{Q_0}{4\pi\varepsilon_0 R^2} \\ &= \overrightarrow{a_R}\cdot\frac{10^{-3}}{4\pi\varepsilon_0 R^2} \\ &= \overrightarrow{a_R}\cdot\frac{9\cdot 10^6}{R^2}\quad \mathrm{V/m} \end{aligned}$$

步驟5 步驟5 / 7

$b)$ - $R<b$ $$\begin{align*} \overrightarrow{J_{in}} =& \sigma \, \overrightarrow{E_{in}} \\ =& 10 \cdot \, \overrightarrow{ a_R } \cdot7.5 \cdot 10 ^9 R \cdot e^{ -9.42 \cdot 10^{11} t } \\ =& \overrightarrow{ a_R } \cdot7.5 \cdot 10 ^{10} R \cdot e^{ -9.42 \cdot 10^{11} t } \ \ \ \mathrm{ A/m^2 } \\ \end{align*}$$

步驟6 步驟6 / 7

- $R > b$

$$\begin{aligned} \overrightarrow{J_{in}} &= \sigma\,\overrightarrow{E_{in}} \\ \overrightarrow{J_{in}} &= 0 \end{aligned}$$

結果 步驟7 / 7

$$a)$$
$$\overrightarrow{E_{in}} = \overrightarrow{a_R}\cdot 7.5\cdot 10^9 R\cdot e^{-9.42\cdot 10^{11}t}\quad \mathrm{V/m}$$
$$\overrightarrow{E_{out}} = \overrightarrow{a_R}\cdot\frac{9\cdot 10^6}{R^2}\quad \mathrm{V/m}$$
$$b)$$
$$\overrightarrow{J_{in}} = \overrightarrow{a_R}\cdot 7.5\cdot 10^{10} R\cdot e^{-9.42\cdot 10^{11}t}\quad \mathrm{V/m}$$
$$\overrightarrow{J_{out}} = 0$$

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步驟1 步驟1 / 7

In this problem, we are referring to problem **P.4-3** and using some values from there.

$a)$ To find the time it takes for charge density to diminish, we use this formula:

$$\rho = \rho_0 \cdot e^{-\left(\frac{\sigma}{\varepsilon}\right)\cdot t}$$

We need to find time t:

$$\begin{aligned}\frac{\rho}{\rho_0} &= e^{-\left(\frac{\sigma}{\varepsilon}\right)\cdot t}\\ \frac{0.01\,\rho_0}{\rho_0} &= e^{-\left(\frac{\sigma}{\varepsilon}\right)\cdot t}\\ \ln 0.01 &= -\left(\frac{\sigma}{\varepsilon}\right)\cdot t\\ t &= \frac{\ln 0.01}{-\frac{10}{1.2\,\varepsilon_0}} = 4.89\cdot 10^{-12}\ \mathrm{s}\\ \\ t &= 4.88\ \mathrm{ps}\end{aligned}$$

步驟2 步驟2 / 7

$b)$ We need to find out how electrostatic energy changes when charge density diminishes to 1%. Initial energy in the sphere is:

$$W_{in_0} = \frac{1}{2}\int_V \varepsilon E_{in}^2 dv$$

In problem **P.4-3** we found that initial energy is $E_{in} = \dfrac{R\rho_0}{3\varepsilon}$, and here we can also switch from dv to dR, because:

$$\begin{aligned}v &= \frac{4}{3}\pi R^3\\ \frac{dv}{dr} &= 4\pi R^2 \ \rightarrow\ dv = 4\pi R^2 dR\end{aligned}$$

步驟3 步驟3 / 7

Now we can write:

$$\begin{aligned}W_{in_0} &= \frac{1}{2}\int_V \varepsilon\left(\frac{R\rho_0}{3\varepsilon}\right)^2\cdot 4\pi R^2 dR\\ &= \frac{1}{2}\cdot\frac{\rho_0^2\cdot 4\pi}{9\,\varepsilon}\int R^4 dR\\ &= \frac{2\pi\cdot\rho_0^2\cdot R^5}{45\,\varepsilon}\end{aligned}$$

步驟4 步驟4 / 7

Energy after time $t = 4.89\cdot 10^{-12}$ is:

$$\begin{aligned}W_{in} &= \frac{1}{2}\int_V \varepsilon E_{in}^2 dv\\ &= \frac{1}{2}\int_V \varepsilon\left(\frac{R\cdot\rho_0\, e^{-\left(\frac{\sigma}{\varepsilon}\right)\cdot t}}{3\varepsilon}\right)^2\cdot 4\pi R^2 dR\\ &= \frac{1}{2}\int_V \varepsilon\left(\frac{R\rho_0}{3\varepsilon}\right)^2\cdot e^{-\left(\frac{\sigma}{\varepsilon}\right)\cdot t}\cdot 4\pi R^2 dR\\ &= W_{in_0}\cdot\left[e^{-\left(\frac{\sigma}{\varepsilon}\right)\cdot t}\right]^2\\ &= W_{in_0}\cdot\left[e^{-\left(\frac{10}{1.2\,\varepsilon_0}\right)\cdot 4.89\cdot 10^{-12}}\right]^2\\ &= W_{in_0}\cdot 10^{-4}\end{aligned}$$

步驟5 步驟5 / 7

Now we can find how the energy changed:

$$\begin{aligned}W_{in} &= W_{in_0}\cdot 10^{-4}\\ &= 0.01\%\ W_{in_0}\end{aligned}$$

Energy is dissipated, there is only 0.01% of the initial energy left. It turns into heat.

步驟6 步驟6 / 7

$c)$ To calculate the electrostatic energy stored outside the sphere, we use the general formula for energy:

$$W = \frac{1}{2}\int_V \varepsilon E^2\, dv$$

We use substitution from dv to dR and also we can express E through Q:

$$\begin{aligned}W_{out} &= \frac{1}{2}\int_b^\infty \varepsilon_0\cdot E_0^2\cdot 4\pi\, R^2 dR\\ &= \frac{1}{2}\int_b^\infty \varepsilon_0\cdot\left(\frac{Q_0}{4\pi\varepsilon_0 R^2}\right)^2\cdot 4\pi\, R^2 dR\\ &= \frac{1}{2}\int_b^\infty \varepsilon_0\cdot\frac{Q_0^2}{16\pi^2\varepsilon_0^2 R^4}\cdot 4\pi\, R^2 dR\\ &= \frac{Q_0^2}{8\pi\varepsilon_0}\int_b^\infty \frac{1}{R^2} dR\\ &= -\frac{Q_0^2}{8\pi\varepsilon_0 R}\end{aligned}$$

步驟7 步驟7 / 7

Instead of R we write b because that is the radius in this case, and we can also substitute other values:

$$\begin{aligned}W_{out} &= \frac{Q_0^2}{8\pi\varepsilon_0 b}\\ &= \frac{(10^{-3})^2}{8\pi\cdot 8.854\cdot 10^{-12}\cdot 0.1}\\ \\ &= 45\ \mathrm{kJ}\end{aligned}$$

We can notice that this value does not change and it is constant in time.

練習5

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ISBN：9780201566116
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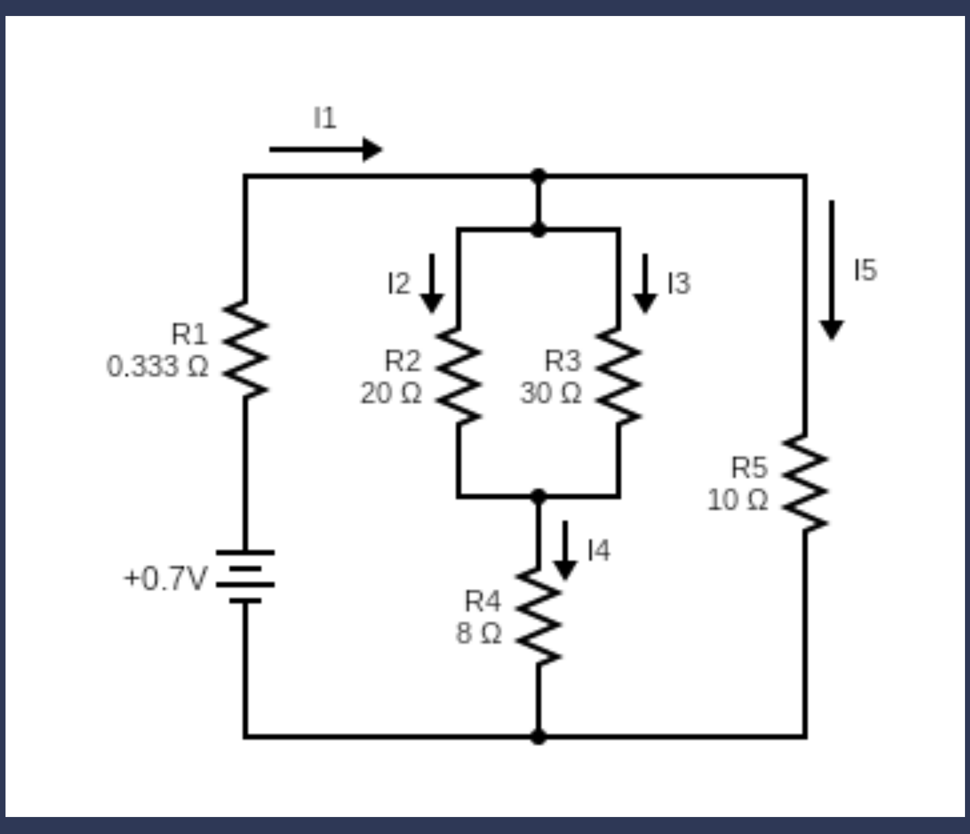
解答 已驗證 2年前提供

步驟1 步驟1 / 7

In this problem, we need to find current and heat dissipated in each of the five resistors in this circuit:



步驟2 步驟2 / 7

We can first calculate the total resistance looking from the source:

$$\begin{aligned} R_T =& R_1 + [(R_2 \mid\mid R_3) + R_4] \mid\mid R_5 \\ =& \frac{1}{3}\ \Omega + [(20\ \Omega \mid\mid 30\ \Omega) + 8\ \Omega] \mid\mid 10\ \Omega \\ =& \frac{1}{3}\ \Omega + (12\ \Omega + 8\ \Omega) \mid\mid 10\ \Omega \\ =& \frac{1}{3}\ \Omega + 20\ \Omega \mid\mid 10\ \Omega \\ =& \frac{1}{3}\ \Omega + \frac{20}{3}\ \Omega \\ =& 7\ \Omega \end{aligned}$$

步驟3 步驟3 / 7

Total current I_1 is:

$$I_1 = \frac{V_T}{R_T} = \frac{0.7\ \mathrm{V}}{7\ \Omega} = 0.1\ \mathrm{A} = 100\ \mathrm{mA}$$

I_5 can be found using current division. General formula when there are two currents:

$$\begin{aligned} I_X =& I_T \cdot \frac{R_T}{R_X} \\ I_5 =& I_1 \cdot \frac{[(R_2 \mid\mid R_3) + R_4] \mid\mid R_5}{R_5} \\ =& 100\ \mathrm{mA} \cdot \frac{\frac{20}{3}\ \Omega}{10\ \Omega} \\ =& 66.7\ \mathrm{mA} \end{aligned}$$

步驟4 步驟4 / 7

We can find I_4 using Kirchhoff's Current Law:

$$\begin{aligned} I_4 =& I_1 - I_5 \\ =& 100\ \mathrm{mA} - 66.7\ \mathrm{mA} \\ =& 33.3\ \mathrm{mA} \end{aligned}$$

步驟5 步驟5 / 7

Next, I_2 is:

$$\begin{aligned} I_2 =& I_4 \cdot \frac{R_2 \mid\mid R_3}{R_2} \\ =& 33.3\ \mathrm{mA} \cdot \frac{12\ \Omega}{20\ \Omega} \\ =& 20\ \mathrm{mA} \end{aligned}$$

$$\begin{aligned} I_3 =& I_4 - I_2 \\ =& 33.3\ \mathrm{mA} - 20\ \mathrm{mA} \\ =& 13.3\ \mathrm{mA} \end{aligned}$$

步驟6 步驟6 / 7

Now that we found all the currents, we can find the heat dissipated on each resistor using this formula:

$$P = I^2 R$$

$$P_1 = I_1^2 R_1 = (100\ \mathrm{mA})^2 \cdot \frac{1}{3}\ \Omega = 3.33\ \mathrm{mW}$$

$$P_2 = I_2^2 R_2 = (20\ \mathrm{mA})^2 \cdot 20\ \Omega = 8\ \mathrm{mW}$$

$$P_3 = I_3^2 R_3 = (13.3\ \mathrm{mA})^2 \cdot 30\ \Omega = 5.31\ \mathrm{mW}$$

$$P_4 = I_4^2 R_4 = (33.3\ \mathrm{mA})^2 \cdot 8\ \Omega = 8.87\ \mathrm{mW}$$

$$P_5 = I_5^2 R_5 = (66.7\ \mathrm{mA})^2 \cdot 10\ \Omega = 44.5\ \mathrm{mW}$$

結果 步驟7 / 7

$$\begin{aligned} I_1 &= 100\ \mathrm{mA},\ P_1 = 3.33\ \mathrm{mW} \\ I_2 &= 20\ \mathrm{mA},\quad P_2 = 8\ \mathrm{mW} \\ I_3 &= 13.3\ \mathrm{mA},\ P_3 = 5.31\ \mathrm{mW} \\ I_4 &= 33.3\ \mathrm{mA},\ P_4 = 8.87\ \mathrm{mW} \\ I_5 &= 66.7\ \mathrm{mA},\ P_5 = 44.5\ \mathrm{mW} \end{aligned}$$

練習6

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ISBN：9780201566116

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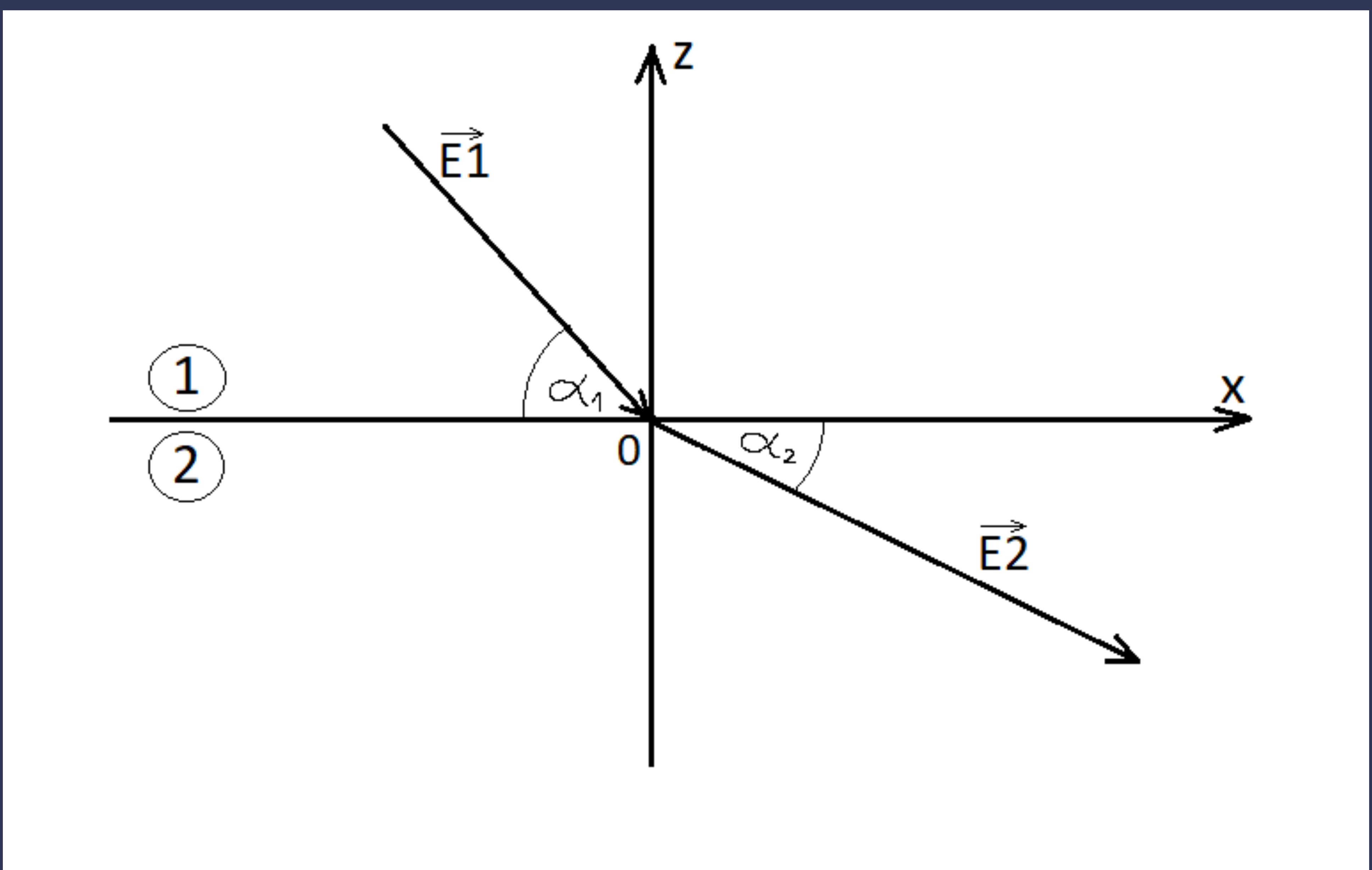
解答 已驗證 2年前提供

步驟1 步驟1 / 5

In this problem, there are two different media, separated by z-axis, and we will find the unknowns by using equations for boundary conditions.



步驟2 步驟2 / 5

$a)$ We know that $\vec{E}_1 = \vec{a_x}\,20 - \vec{a_z}\,50 \ \ (\mathrm{V/m})$, and now we need to find $\vec{E}_2$.

The tangential components of the electric field intensity vector do not change at the boundary, so we can write:

$$E_{1t} = E_{2t} = 20$$

We also know that at an interface between two different media, a divergenceless field has a continuous normal component:

$$J_{2n} = J_{1n} \rightarrow \sigma_2 E_{2n} = \sigma_1 E_{1n}$$
$$E_{2n} = \frac{\sigma_1}{\sigma_2} \cdot E_{1n} = \frac{15}{10} \cdot (-50) = 75$$
$$\vec{E}_2 = \vec{a_x}\,20 - \vec{a_z}\,75 \quad \frac{\mathrm{V}}{\mathrm{m}}$$

步驟3 步驟3 / 5

$b)$

$$\begin{aligned}\vec{J}_1 &= \sigma_1\,\vec{E}_1\\ &= 15 \cdot 10^{-3} \cdot (\vec{a_x}\,20 - \vec{a_z}\,50)\\ &= \vec{a_x}\,0.3 - \vec{a_z}\,0.75 \quad \frac{\mathrm{A}}{\mathrm{m}^2}\end{aligned}$$

$$\begin{aligned}\vec{J}_2 &= \sigma_2\,\vec{E}_2\\ &= 10 \cdot 10^{-3} \cdot (\vec{a_x}\,20 - \vec{a_z}\,75)\\ &= \vec{a_x}\,0.2 - \vec{a_z}\,0.75 \quad \frac{\mathrm{A}}{\mathrm{m}^2}\end{aligned}$$

步驟4 步驟4 / 5

$c)$

$$\alpha_1 = \tan^{-1}\left(\frac{50}{20}\right) = 68.2^\circ$$

$$\alpha_2 = \tan^{-1}\left(\frac{75}{20}\right) = 75.1^\circ$$

步驟5 步驟5 / 5

$d)$

$$\begin{aligned}D_{2n} - D_{1n} &= \rho_s\\ \varepsilon_2\,E_{2n} - \varepsilon_1\,E_{1n} &= \rho_s\\ \rho_s &= \varepsilon_0\,(-3 \cdot 75 + 2 \cdot 50)\\ &= -\,125\,\varepsilon_0\\ &= -\,1.105\,\frac{\mathrm{nC}}{\mathrm{m}}\end{aligned}$$

練習7
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ISBN：9780201566116
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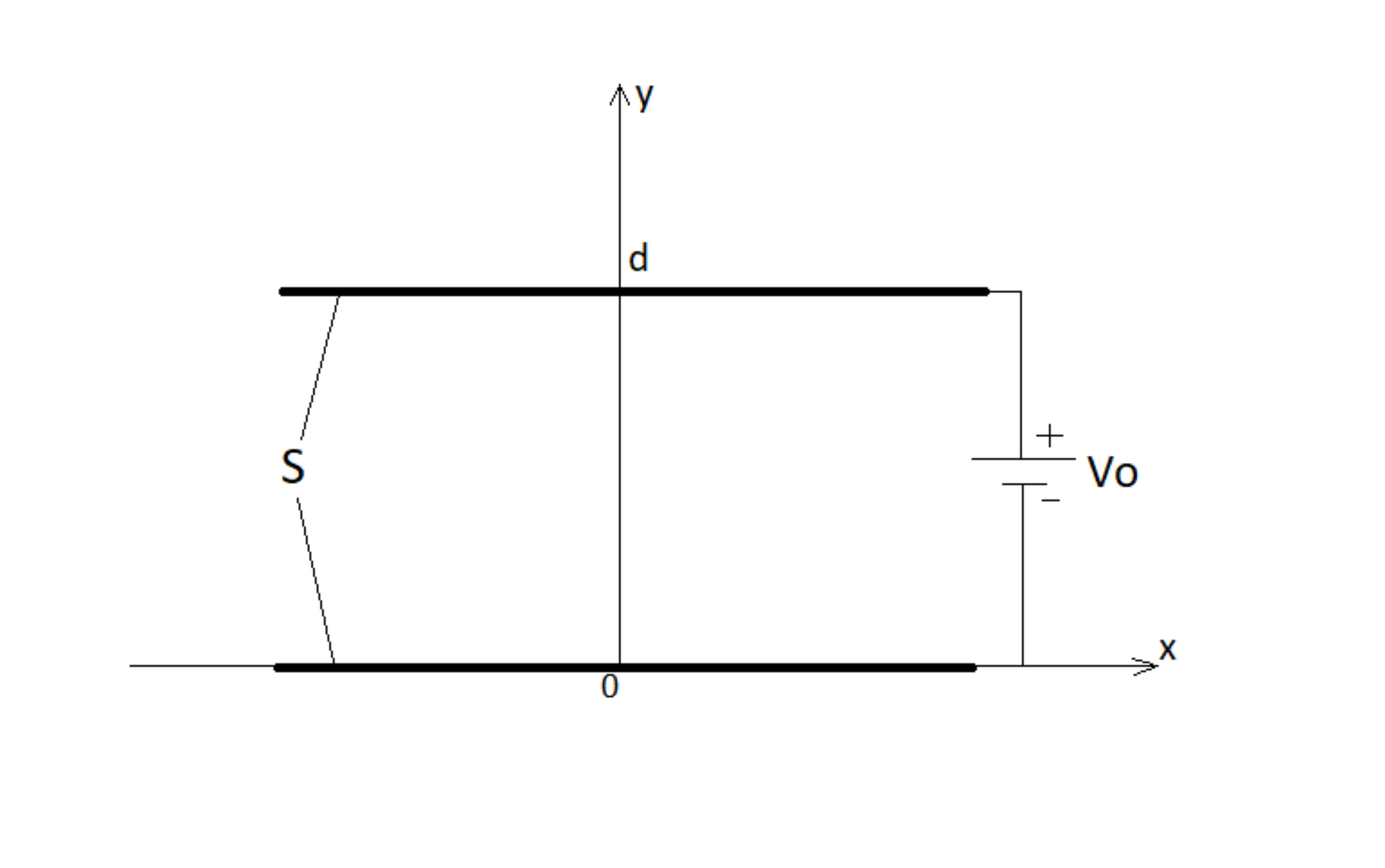
步驟1 步驟1 / 5

In this problem, we have two conducting plates, placed like this:



The area of the plates is S, the distance between them is d, and the space between is filled with an inhomogeneous medium, distributed linearly like this:

$$y = 0 \;\rightarrow\; \sigma = \sigma_1$$
$$y = d \;\rightarrow\; \sigma = \sigma_2$$

Which means: $\sigma(y) = \sigma_1 + (\sigma_2 - \sigma_1)\,\dfrac{y}{d}$

步驟2 步驟2 / 5

$a)$ We need to determine the total resistance between the plates, and we can use Ohm's law for that, but first, we need to find V.

$$V = -\int \vec{E}\,\vec{dl}$$
$$\vec{E} = \frac{\vec{J}}{\sigma} = \frac{-\vec{a_y}\,J_0}{\sigma(y)}$$
$$\begin{aligned} V &= -\int_0^d \vec{E}\,\vec{a_y}\,dy \\ &= -\int_0^d \frac{-\vec{a_y}\,J_0}{\sigma_1 + (\sigma_2-\sigma_1)\,\dfrac{y}{d}}\,\vec{a_y}\,dy \\ &= \int_0^d \frac{J_0\,d}{\sigma_1\,d + (\sigma_2-\sigma_1)\,y}\,dy \end{aligned}$$

步驟3 步驟3 / 5

To integrate we use this table integral:

$$\int \frac{1}{ax+b}\,dx = \frac{1}{a}\,\ln|ax+b|$$

$$\begin{aligned} V &= \int_0^d \frac{J_0\,d}{\sigma_1\,d + (\sigma_2-\sigma_1)\,y}\,dy \\ &= \frac{J_0\,d}{\sigma_2-\sigma_1}\,\ln[(\sigma_2-\sigma_1)\,y + \sigma_1\,d]\Big|_{y=0}^{y=d} \\ &= \frac{J_0\,d}{\sigma_2-\sigma_1}\,\ln(\sigma_2\,d) - \frac{J_0\,d}{\sigma_2-\sigma_1}\,\ln(\sigma_1\,d) \\ &= \frac{J_0\,d}{\sigma_2-\sigma_1}\,\ln\frac{\sigma_2}{\sigma_1} \end{aligned}$$

步驟4 步驟4 / 5

Now we can find the resistance:

$$R = \frac{V}{I} = \frac{V}{J_0\,S} = \frac{\dfrac{J_0\,d}{\sigma_2-\sigma_1}\,\ln\dfrac{\sigma_2}{\sigma_1}}{J_0\,S}$$

$$R = \frac{d}{(\sigma_2-\sigma_1)\,S}\,\ln\frac{\sigma_2}{\sigma_1}$$

步驟5 步驟5 / 5

$b)$ Here we need to find the surface charge densities on the plates. In part $a)$ we found:

$$V = \frac{J_0\,d}{\sigma_2-\sigma_1}\,\ln\frac{\sigma_2}{\sigma_1} \quad\rightarrow\quad J_0 = \frac{(\sigma_2-\sigma_1)\,V_0}{d\,\ln\dfrac{\sigma_2}{\sigma_1}}$$

On the upper plate (for $y = d$):

$$\begin{aligned} \rho_s &= \varepsilon_0\,E_y \\ &= \frac{\varepsilon_0\,J_0}{\sigma_2} \\ &= \frac{\varepsilon_0\cdot(\sigma_2-\sigma_1)\cdot V_0}{\sigma_2\cdot d\cdot\ln\dfrac{\sigma_2}{\sigma_1}} \end{aligned}$$

On the lower plate (for $y = 0$):

$$\begin{aligned} \rho_s &= \varepsilon_0\,E_y \\ &= \frac{\varepsilon_0\,J_0}{\sigma_1} \\ &= \frac{\varepsilon_0\cdot(\sigma_2-\sigma_1)\cdot V_0}{\sigma_1\cdot d\cdot\ln\dfrac{\sigma_2}{\sigma_1}} \end{aligned}$$

練習8

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ISBN：9780201566116

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解答 已驗證 2年前提供

步驟1

步驟1 / 4

$a)$ Because the normal component of current density $\vec{J}$ does not change, the current I also stays the same through different dielectrics. We can write Kirchhoff's voltage law for this circuit, and from there find J:

$$V_0 = (R_1 + R_2) \cdot I = \left(\frac{d_1}{\sigma_1 S} + \frac{d_2}{\sigma_2 S} \right) \cdot I$$

$$J = \frac{I}{S} = \frac{\dfrac{V_0}{\dfrac{d_1}{\sigma_1 S} + \dfrac{d_2}{\sigma_2 S}}}{S} = \frac{V_0}{\dfrac{d_1}{\sigma_1} + \dfrac{d_2}{\sigma_2}}$$

$$J = \frac{\sigma_1 \sigma_2 V_0}{\sigma_2 d_1 + \sigma_1 d_2}$$

步驟2

步驟2 / 4

$b)$ Now we need to find electric field intensities for both dielectrics. We can express two equations we know and find E from them:

$$V_0 = -\int \vec{E} d\vec{l} = E_1 d_1 + E_2 d_2$$

$$J_1 = J_2 \quad \rightarrow \quad \sigma_1 E_1 = \sigma_2 E_2$$

We will express E_2 from the second equation and include it in the first

$$E_2 = E_1 \frac{\sigma_1}{\sigma_2}$$

$$V_0 = E_1 d_1 + E_2 d_2$$

$$V_0 = E_1 d_1 + E_1 \frac{\sigma_1}{\sigma_2} d_2$$

$$V_0 = E_1 \left(d_1 + \frac{\sigma_1}{\sigma_2} d_2 \right)$$

$$E_1 = \frac{\sigma_2 V_0}{\sigma_2 d_1 + \sigma_1 d_2}$$

Now we can calculate E_2:

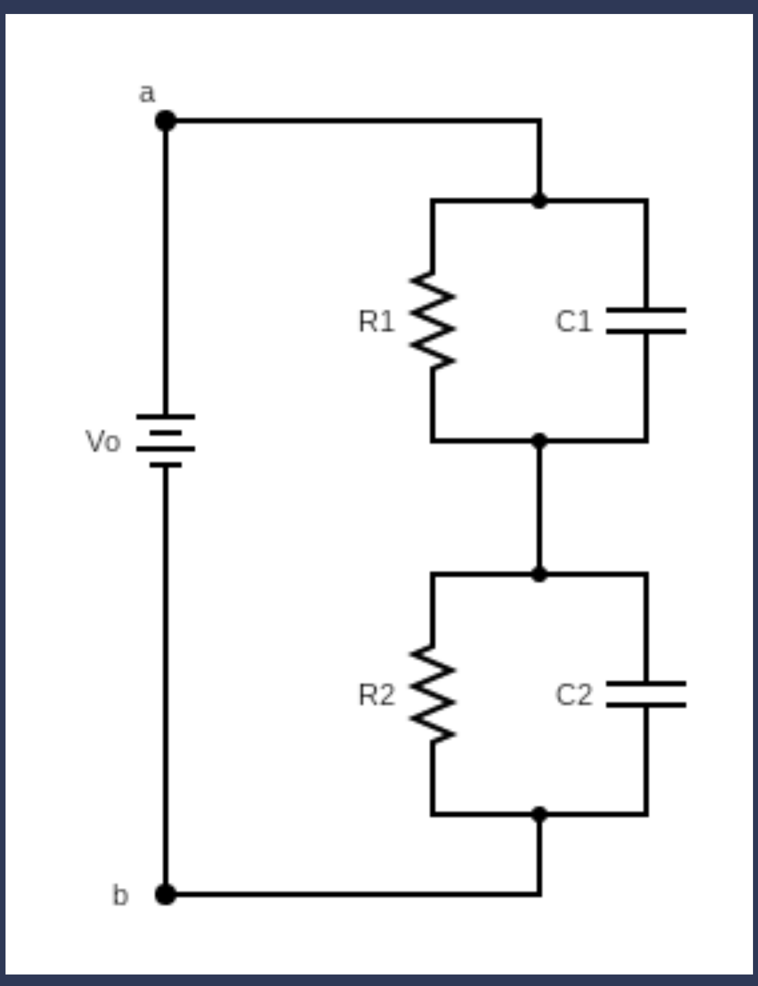
$$E_2 = E_1 \frac{\sigma_1}{\sigma_2} = \frac{\sigma_2 V_0}{\sigma_2 d_1 + \sigma_1 d_2} \cdot \frac{\sigma_1}{\sigma_2}$$

$$E_2 = \frac{\sigma_1 V_0}{\sigma_2 d_1 + \sigma_1 d_2}$$

步驟3

步驟3 / 4

The equivalent circuit is going to look like this:



步驟4

步驟4 / 4

Every dielectric is represented by parallel connection of resistor and capacitor, and they are found using these equations:

$$R = \frac{l}{\sigma S}$$

$$C = \frac{\epsilon}{R\sigma} = \frac{\epsilon}{\dfrac{l}{\sigma S}\sigma} = \frac{\epsilon S}{l}$$

For this circuit it is:

$$R_1 = \frac{d_1}{\sigma_1 S}$$

$$R_2 = \frac{d_2}{\sigma_2 S}$$

$$C_1 = \frac{\epsilon_1 S}{d_1}$$

$$C_2 = \frac{\epsilon_2 S}{d_2}$$

Fundamentals of Engineering Electromagnetics
ISBN：9780201566116
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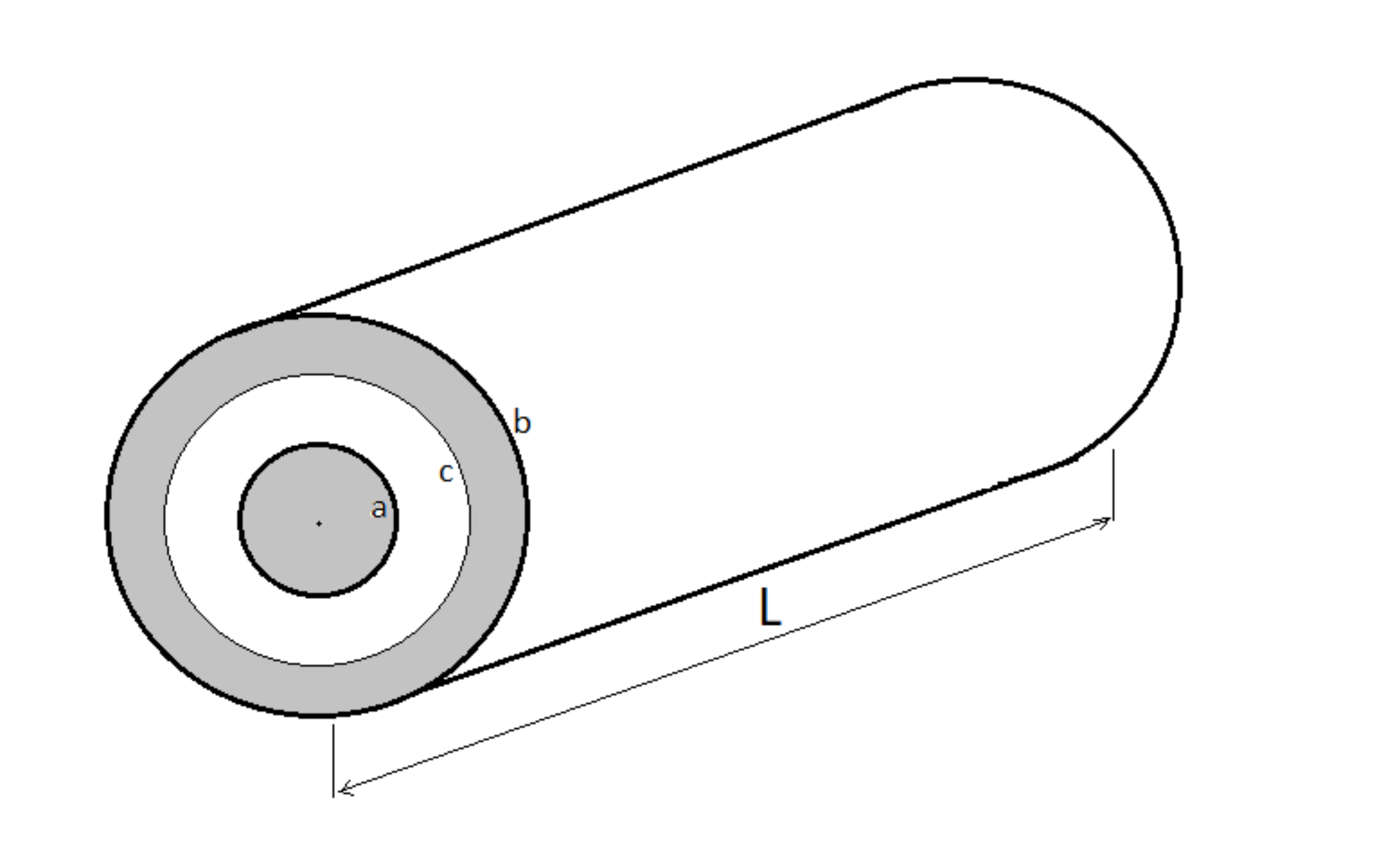
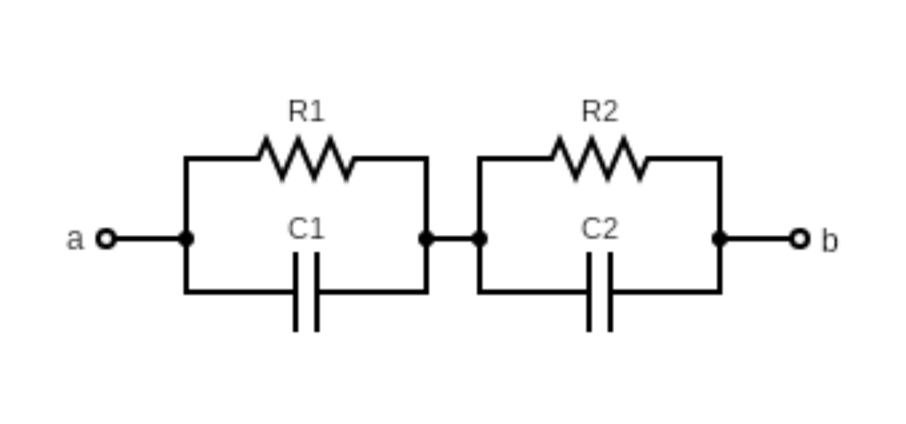
步驟1 步驟1 / 6

The cylindrical capacitor can be shown like this:



步驟2 步驟2 / 6

$a)$ Each conductor has resistance and capacitance, so the equivalent circuit between inner and outer conductors looks like this:



步驟3 步驟3 / 6

In Chapter 3 we learned that the capacitance of a cylindrical conductor is (3-90):

$$C = \frac{2\pi\varepsilon L}{\ln\left(\dfrac{b}{a}\right)}$$

We also know that $RC = \dfrac{\varepsilon}{\sigma}$, so R is:

$$R = \frac{\varepsilon}{\sigma}\cdot\frac{1}{C} = \frac{\varepsilon}{\sigma}\cdot\frac{\ln\left(\dfrac{b}{a}\right)}{2\pi\varepsilon L} = \frac{1}{2\pi\sigma L}\cdot\ln\left(\frac{b}{a}\right)$$

步驟4 步驟4 / 6

So for this circuit, it will be:

$$C_1 = \frac{2\pi\varepsilon_1 L}{\ln\left(\dfrac{c}{a}\right)}$$
$$R_1 = \frac{1}{2\pi\sigma_1 L}\cdot\ln\left(\frac{c}{a}\right)$$

$$C_2 = \frac{2\pi\varepsilon_2 L}{\ln\left(\dfrac{b}{c}\right)}$$
$$R_2 = \frac{1}{2\pi\sigma_2 L}\cdot\ln\left(\frac{b}{c}\right)$$

步驟5 步驟5 / 6

Current I and current density J are the same in both regions:

$$\begin{aligned} I &= \frac{V_0}{R} = V_0\,G \\ &= V_0\cdot\frac{1}{R_1+R_2} \\ &= V_0\cdot\frac{1}{\dfrac{1}{2\pi\sigma_1 L}\cdot\ln\left(\dfrac{c}{a}\right)+\dfrac{1}{2\pi\sigma_2 L}\cdot\ln\left(\dfrac{b}{c}\right)} \\ &= \frac{V_0\cdot 2\pi\,L\,\sigma_1\,\sigma_2}{\sigma_2\cdot\ln\left(\dfrac{c}{a}\right)+\sigma_1\cdot\ln\left(\dfrac{b}{c}\right)} \end{aligned}$$

步驟6 步驟6 / 6

$$\begin{aligned} J_1 = J_2 &= \frac{I}{2\pi r L} \\ &= \frac{V_0\cdot 2\pi L\,\sigma_1\,\sigma_2}{\sigma_2\cdot\ln\left(\dfrac{c}{a}\right)+\sigma_1\cdot\ln\left(\dfrac{b}{c}\right)}\cdot\frac{1}{2\pi r L} \\ &= \frac{V_0\cdot\sigma_1\,\sigma_2}{r\left[\sigma_2\cdot\ln\left(\dfrac{c}{a}\right)+\sigma_1\cdot\ln\left(\dfrac{b}{c}\right)\right]} \end{aligned}$$

練習10

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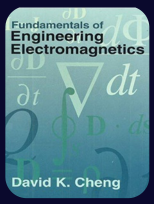

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ISBN：9780201566116

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步驟1

步驟1 / 3

In this problem, we are referring to Example 4-4 and Figure 4-4. To find the resistance of a quarter circular washer we use equation (4-16), for resistance of a straight homogeneous material of uniform cross-section:

$$R = \frac{l}{\sigma S}$$

Area S is found like this:

$$S = \frac{1}{4}(b^2\pi - a^2\pi) = \frac{\pi}{4}(b^2 - a^2)$$

步驟2

步驟2 / 3

This means that resistance between the top and bottom flat faces is:

$$R = \frac{h}{\sigma S} = \frac{h}{\sigma}\frac{4}{\pi(b^2 - a^2)}$$

$$R = \frac{4\,h}{\sigma\,\pi(b^2 - a^2)}$$

結果

步驟3 / 3

$$R = \frac{4\,h}{\sigma\,\pi(b^2 - a^2)}$$

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步驟1 步驟1 / 8

In this problem, we are referring to Example 4-4 and Figure 4-4. Here, we need to find the resistance of a quarter circular washer between the curved sides.

Laplaces equation:

$$\nabla^2 V = 0$$

The appropriate coordinate system for this problem is the cylindrical, and Laplace's equation in cylindrical coordinates is:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} = 0 \tag{1}$$

We are only using r in this calculation, so it reduces:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) = 0 \tag{2}$$

步驟2 步驟2 / 8

The general solution of (2) is:

$$V(r) = C_1 \ln(r) + C_2 \tag{3}$$

We can assume a potential difference of V_0 between the sides, say $V = 0$ at radius b and $V = V_0$ at radius a. Those are boundary conditions used to find C_1 and C_2.

$$\begin{aligned} V &= 0 \qquad \text{at} \qquad r = b, \\ V &= V_0 \qquad \text{at} \qquad r = a, \end{aligned}$$

步驟3 步驟3 / 8

Now we are solving equation (3) for those conditions:

$$\begin{aligned} V(r) &= C_1 \ln(r) + C_2 \\ 0 &= C_1 \ln(b) + C_2 \quad \rightarrow \quad C_2 = -C_1 \ln(b) \end{aligned}$$

$$\begin{aligned} V(r) &= C_1 \ln(r) + C_2 \\ V_0 &= C_1 \ln(a) - C_1 \ln(b) \\ V_0 &= C_1 \ln\left(\frac{a}{b}\right) \\ C_1 &= \frac{V_0}{\ln\left(\frac{a}{b}\right)} \end{aligned}$$

步驟4 步驟4 / 8

We will substitute C_1 in equation for C_2:

$$\begin{aligned} C_2 &= -C_1 \ln(b) \\ &= -\frac{V_0}{\ln\left(\frac{a}{b}\right)} \ln(b) \end{aligned}$$

Now that we found constants, we can solve equation (3):

$$\begin{aligned} V(r) &= C_1 \ln(r) + C_2 \\ &= \frac{V_0}{\ln\left(\frac{a}{b}\right)} \ln(r) - \frac{V_0}{\ln\left(\frac{a}{b}\right)} \ln(b) \\ &= \frac{V_0}{\ln\left(\frac{a}{b}\right)} \ln\left(\frac{r}{b}\right) \\ &= V_0 \frac{\ln\left(\frac{b}{r}\right)}{\ln\left(\frac{b}{a}\right)} \end{aligned}$$

步驟5 步驟5 / 8

We need to find current density, but before that the field intensity:

$$\begin{aligned} \vec{E}(r) &= -\vec{a_r}\frac{\partial V}{\partial r} \\ &= -\vec{a_r}\frac{\partial}{\partial r}\left[V_0 \frac{\ln\left(\frac{b}{r}\right)}{\ln\left(\frac{b}{a}\right)}\right] \\ &= -\vec{a_r}\frac{V_0}{\ln\left(\frac{b}{a}\right)} \cdot \frac{\partial}{\partial r} \ln\left(\frac{b}{r}\right) \\ &= -\vec{a_r}\frac{V_0}{\ln\left(\frac{b}{a}\right)} \cdot \frac{-1}{r} \\ &= \vec{a_r}\frac{V_0}{r\ln\left(\frac{b}{a}\right)} \end{aligned}$$

步驟6 步驟6 / 8

We will calculate the current here:

$$\begin{aligned} \vec{J}(r) &= \sigma \vec{E}(r) \\ &= \vec{a_r}\frac{V_0\,\sigma}{r\ln\left(\frac{b}{a}\right)} \end{aligned}$$

$$\begin{aligned} I &= \int_S \vec{J}\, d\vec{S} \\ &= \int_0^{2\pi} \vec{J}(\vec{a_r}\, h\, r\, d\phi) \\ &= \int_0^{2\pi} \vec{a_r}\frac{V_0\,\sigma}{r\ln\left(\frac{b}{a}\right)} (\vec{a_r}\, h\, r\, d\phi) \\ &= \int_0^{2\pi} \frac{\sigma\, h\, V_0}{\ln\left(\frac{b}{a}\right)}\, d\phi \\ &= \frac{\pi\,\sigma\, h\, V_0}{2\ln\left(\frac{b}{a}\right)} \end{aligned}$$

步驟7 步驟7 / 8

Now finally we can calculate the resistance:

$$\begin{aligned} R &= \frac{V_0}{I} \\ &= \frac{V_0}{\dfrac{\pi\,\sigma\, h\, V_0}{2\ln\left(\frac{b}{a}\right)}} \\ &= \frac{2\ln\left(\frac{b}{a}\right)}{\pi\,\sigma\, h} \end{aligned}$$

結果 步驟8 / 8

$$R = \frac{2}{\pi\,\sigma\, h} \ln\left(\frac{b}{a}\right)$$