CS350: Data Structures

Graphs

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Graph Data Structure

- **A graph is a data structure consisting of a set of vertices connected by edges**

$$\boxed{\begin{array}{l} G = (V, E) \\ \text{where } V \text{ is the set of vertices and } E \text{ is the set of edges} \end{array}}$$

- **The following can all be considered special cases of a graph data structure**
  - Linked lists
  - Trees
  - Skip lists

Graph Data Structure

- **Graphs have many uses:**
  - Representing the control-flow of a program
  - Underlying data structure for mapping/navigation systems
    - Each vertex represents an intersection
    - Each edge represents a road between intersections

- **A node and/or an edge in a graph may have some additional information associated with it**
  - For the mapping system:
    - Each vertex may have a GPS coordinate associated with it
    - Each edge may have a distance and/or a speed-limit associated with it
      - The value or values associated with an edge are sometimes referred to as the weight of the edge

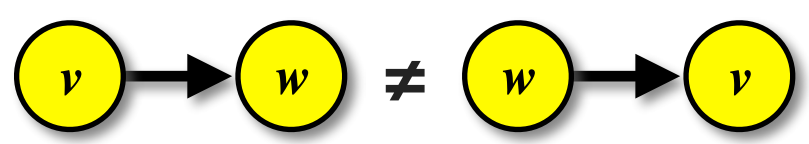
Graph Data Structure

- **A graph can be defined as follows:**

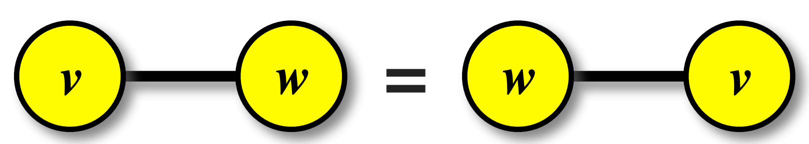
> $G = (V, E)$
> where V is the set of vertices and E is the set of edges

- **Each edge is a pair (v, w) where $v, w \in V$**

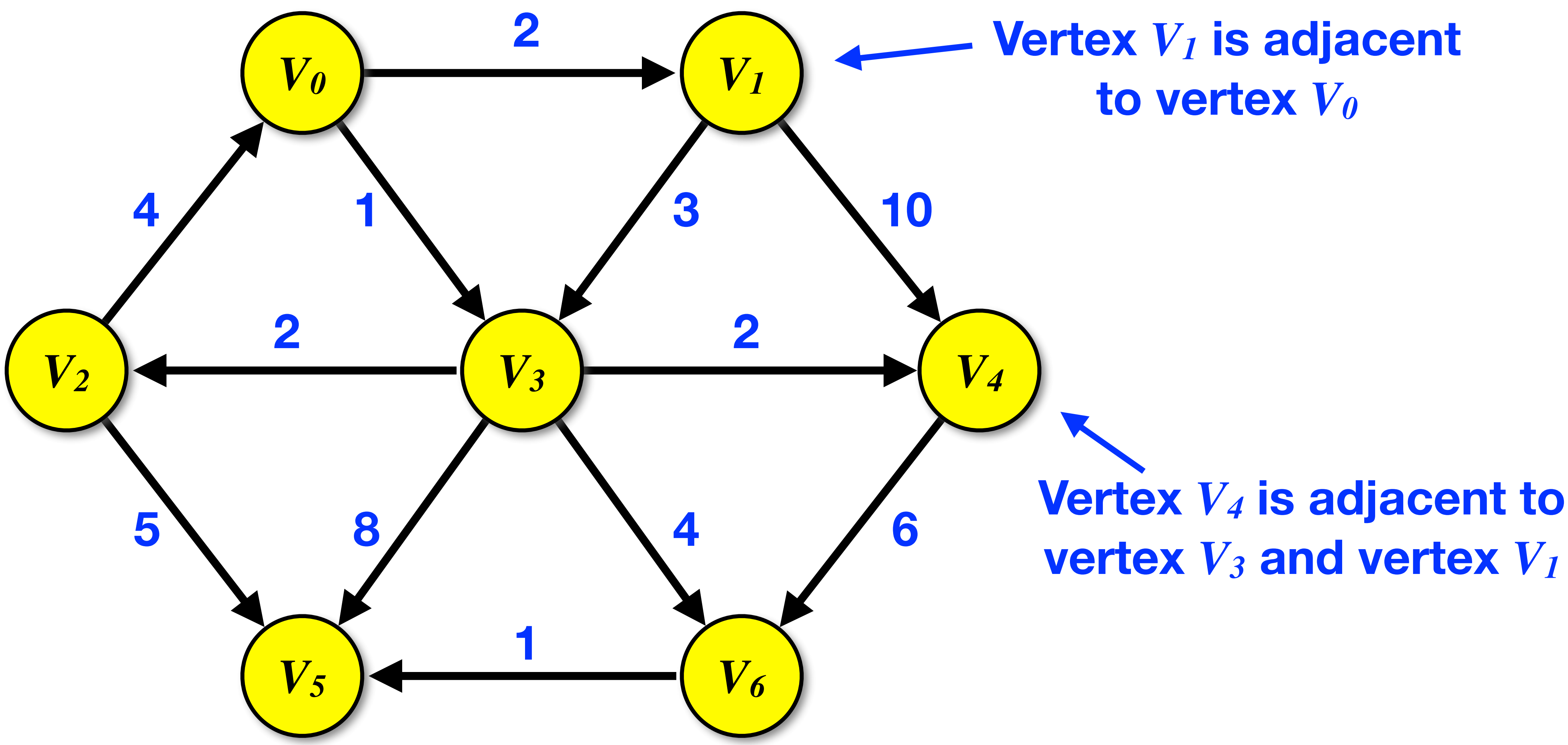
- **In a directed graph, the ordering of an edge pair matters**
  - $(v, w) \neq (w, v)$



- **In an undirected graph, the ordering of an edge pair does not matter**
  - $(v, w) = (w, v)$

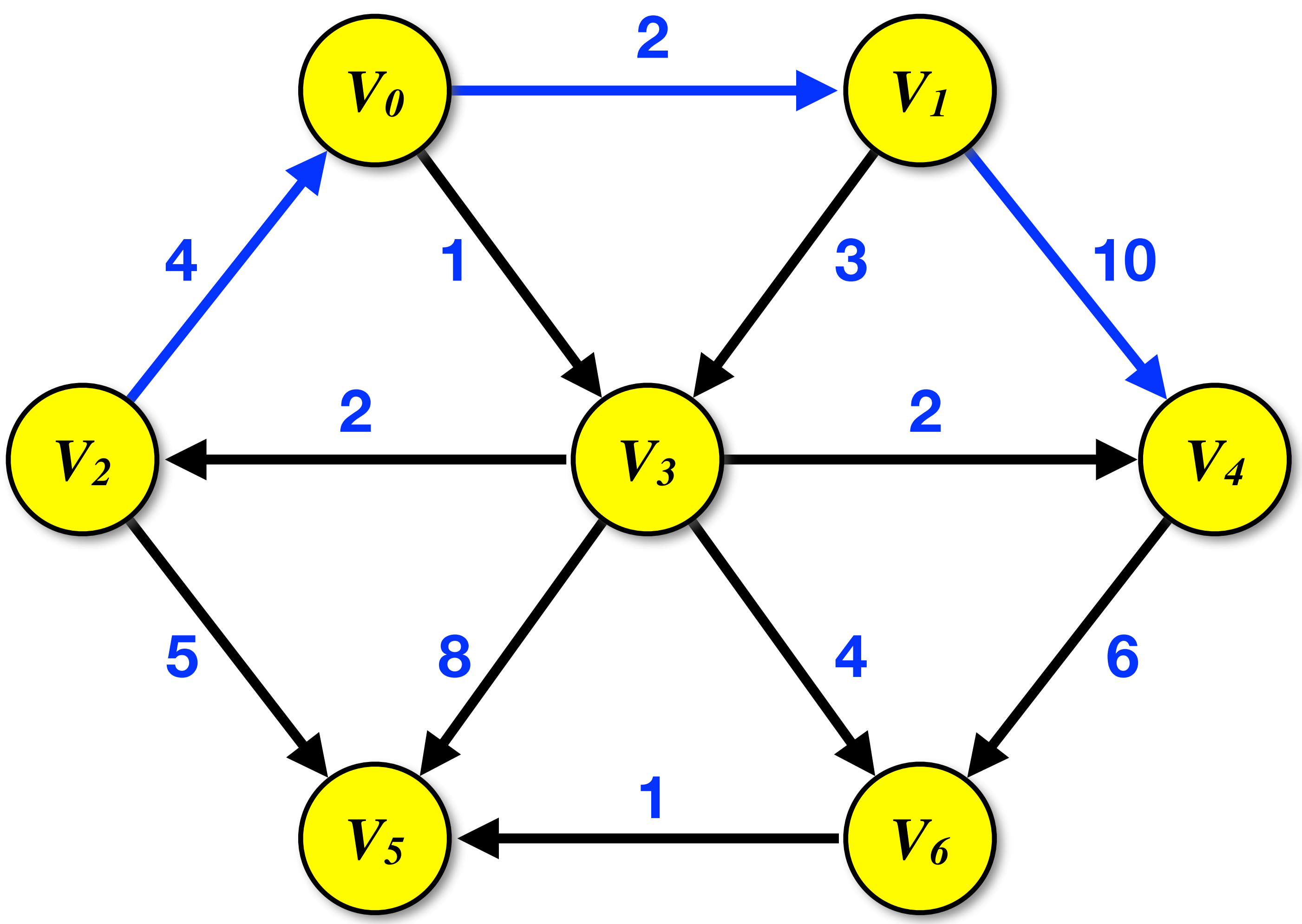


Example of a Directed Graph with Weights



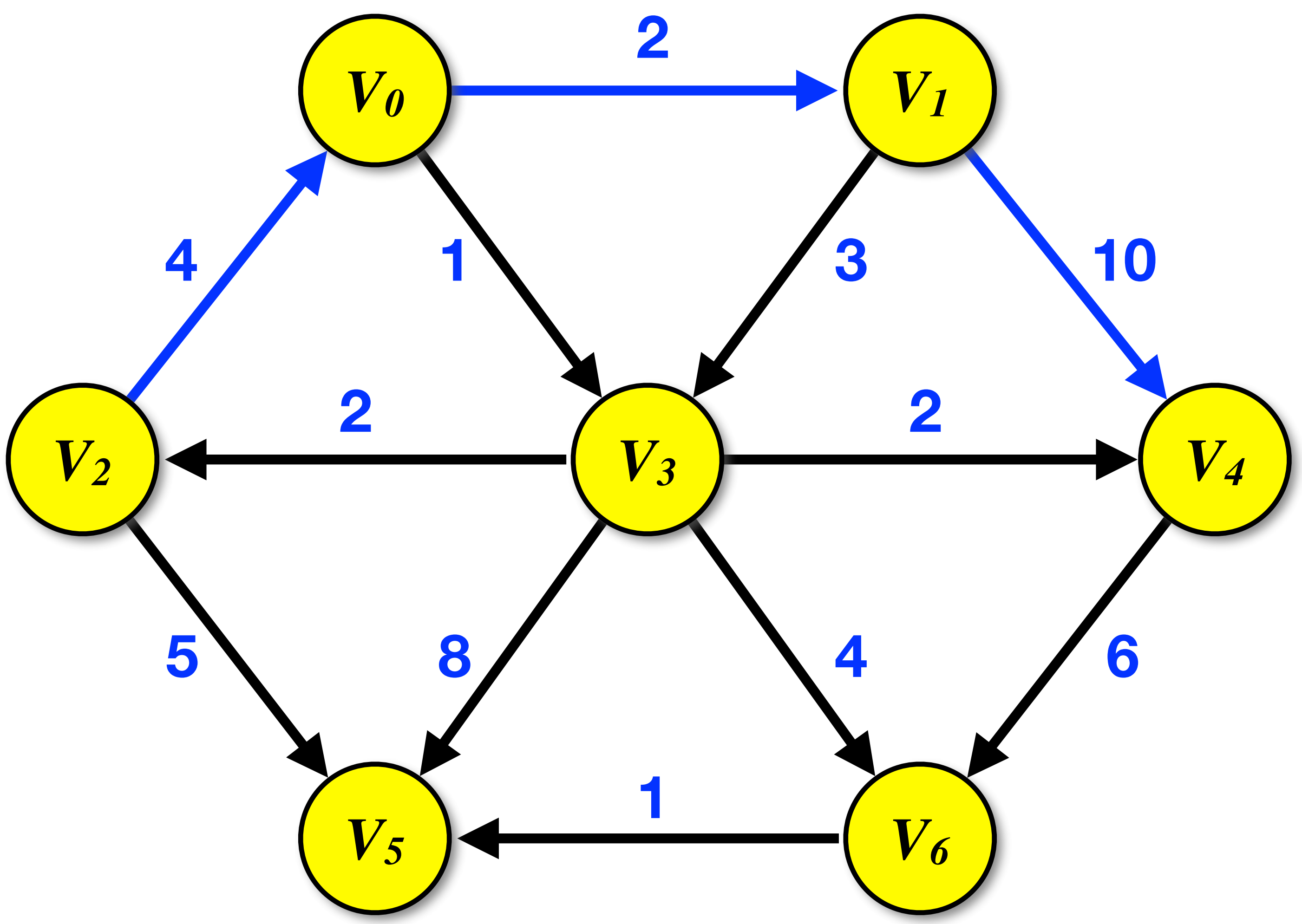
$$E = \left\{ \begin{array}{l} (V_0, V_1, 2), (V_0, V_3, 1), (V_1, V_3, 3), (V_1, V_4, 10), \\ (V_3, V_4, 2), (V_3, V_6, 4), (V_3, V_5, 8), (V_3, V_2, 2), \\ (V_2, V_0, 4), (V_2, V_5, 5), (V_4, V_6, 6), (V_6, V_5, 1) \end{array} \right\}$$

Example of a Directed Graph with Weights



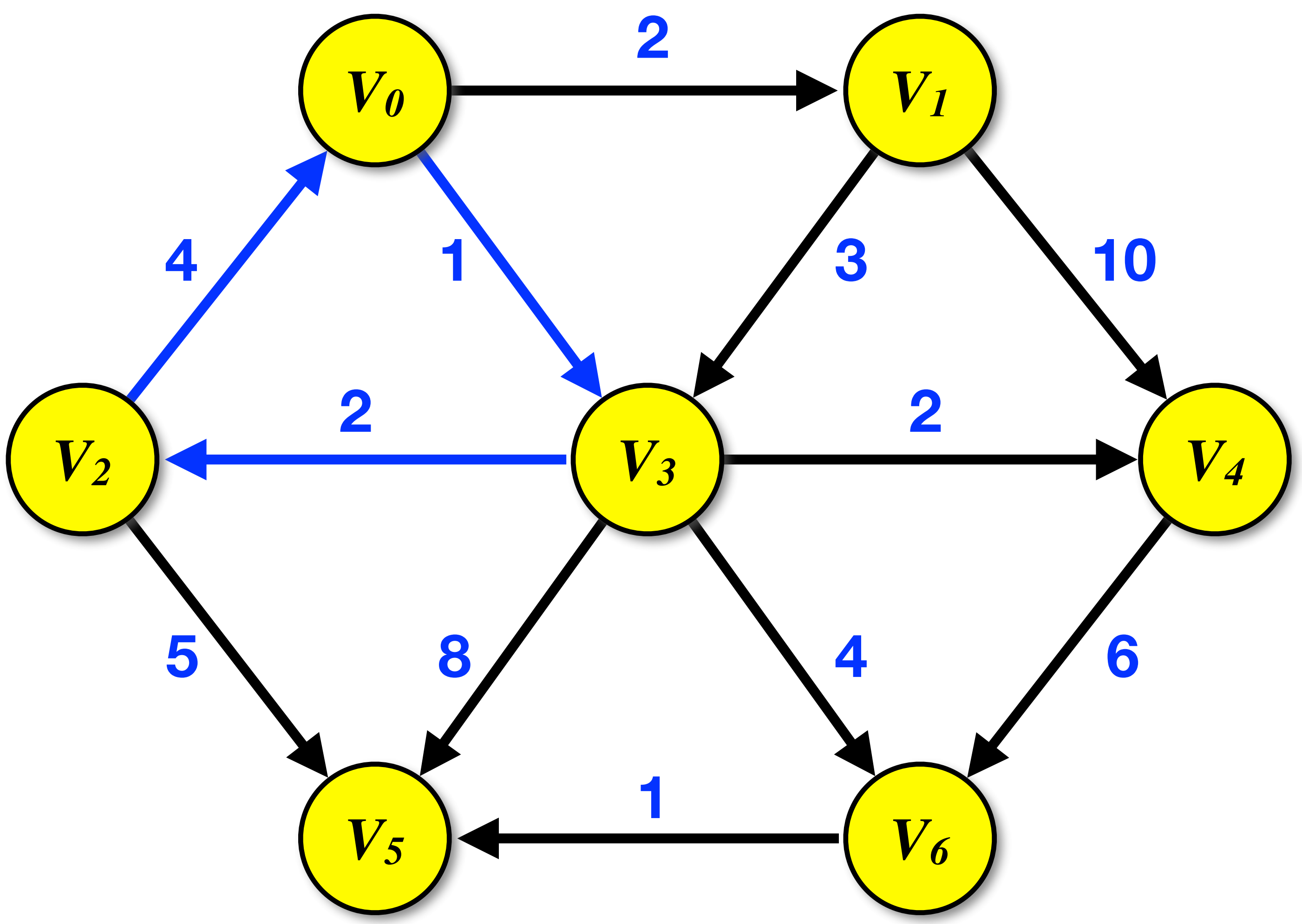
- **The path length between two vertices is the number of edges that must be traversed to get from one vertex to the other**
  - Multiple paths may exist between two vertices
  - Example: path length from V_2 to V_4 is 3 -- ($V_2 \rightarrow V_0 \rightarrow V_1 \rightarrow V_4$)

Example of a Directed Graph with Weights



- **The weighted path length between two vertices is the sum of the weights of the edges along the path**
  - Multiple paths may exist between two vertices
  - Example: weighted path length from V_2 to V_4 is 16 (or 7, or 11, ...)

Example of a Directed Graph with Weights

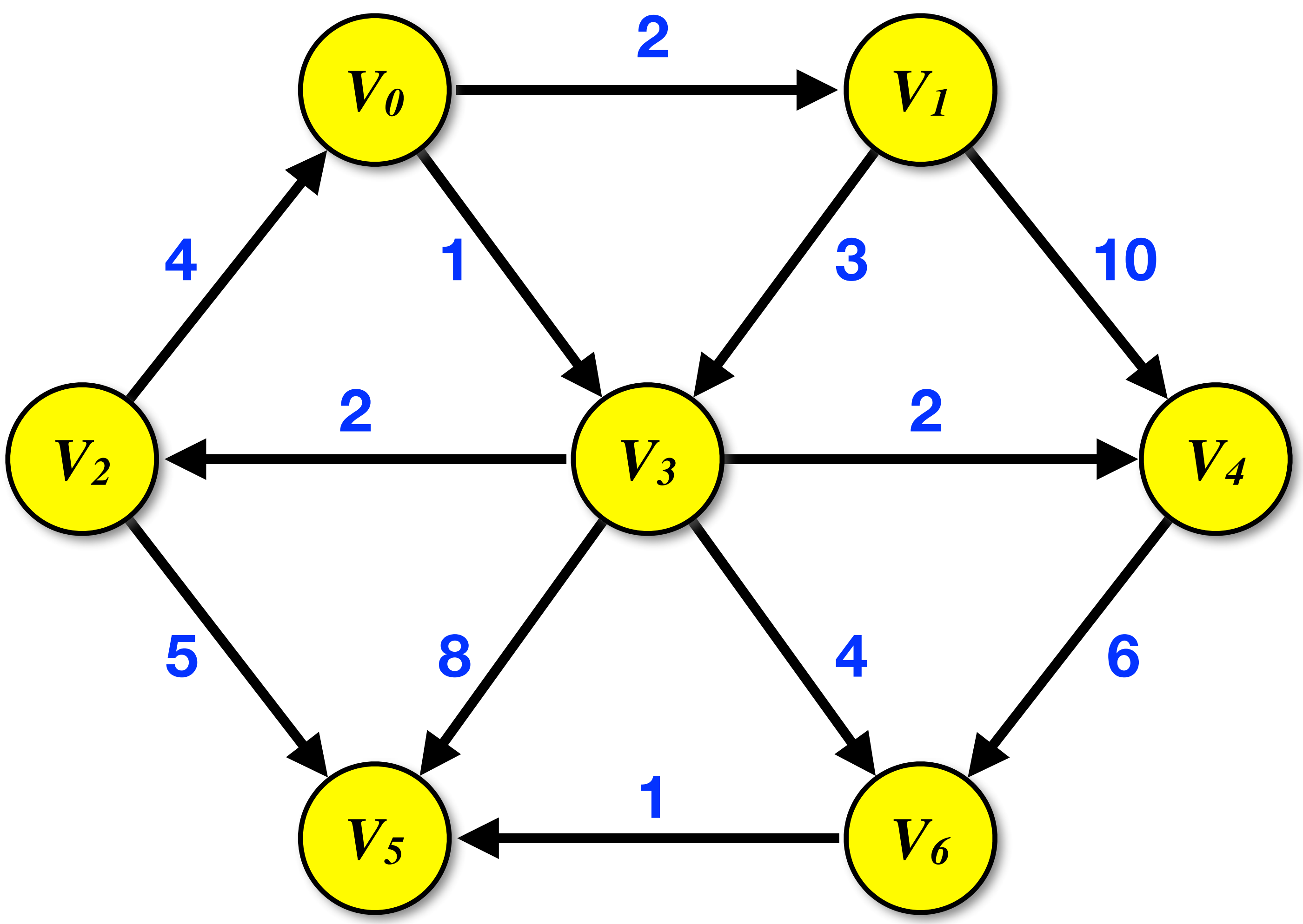


- **A cycle in a directed graph is a path that begins and ends at the same vertex and contains at least one edge**
  - A directed graph may have zero, one, or more cycles
  - A graph with cycles is said to be cyclic, whereas a graph with no cycles is said to be acyclic

Graph Representations

- **There are two common representations for graphs, the first is an adjacency matrix**
  - Utilizes a 2-dimensional matrix, where each vertex is represented by both a row and a column
  - Each location in the matrix, `mat[v][w]`, represents the weight of a directed edge between vertex v and vertex w
    - If no edge exists between a vertex v and a vertex w, then set the edge weight in the matrix to `INFINITY`
  - Constant time operation to find info about any edge
  - Requires $O(|V|^2)$ space to store matrix
    - Can be wasteful if representing a sparse graph, that is, a graph without many edge

Adjacency Matrix Representation

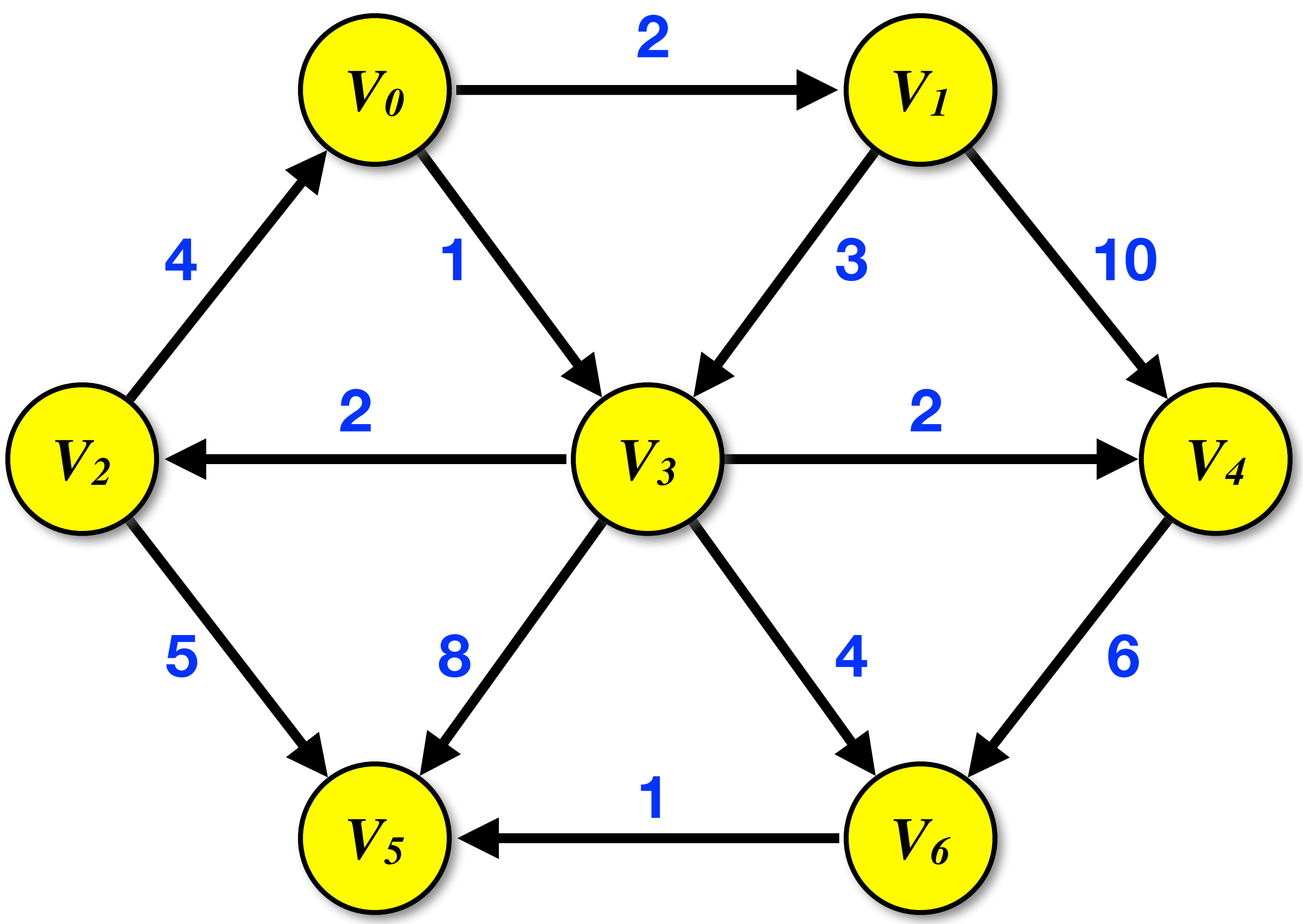


	V_0	V_1	V_2	V_3	V_4	V_5	V_6
V_0	∞	2	∞	1	∞	∞	∞
V_1	∞	∞	∞	3	10	∞	∞
V_2	4	∞	∞	∞	∞	5	∞
V_3	∞	∞	2	∞	2	8	4
V_4	∞	∞	∞	∞	∞	∞	6
V_5	∞	∞	∞	∞	∞	∞	∞
V_6	∞	∞	∞	∞	∞	1	∞

Graph Representations

- **Another graph representation is the adjacency list**
    - The graph is represented as a list of edges
        - For each vertex, keep a list of all adjacent vertices
    - Each edge appears in an adjacency list, thus the space required is $O(|E|)$
        - Better suited for sparse graphs
    - May take additional time to search lists to see if an edge exists between two vertices

Adjacency List Representation



V_0: V_1 (2) → V_3 (1)

V_1: V_4 (10) → V_3 (3)

V_2: V_0 (4) → V_5 (5)

V_3: V_4 (2) → V_6 (4) → V_5 (8) → V_2 (2)

V_4: V_6 (6)

V_5

V_6: V_5 (1)