CS350: Data Structures

Splay Trees

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Splay Trees

- **Another variation of the binary tree**
- **Does NOT guarantee O(log N) worst case operations**
- **Splay tree may exhibit O(N) worst case for any single operation**
  - Time for operations is amortized
  - On average, splay tree operations will exhibit O(log N) worst case time
- **Does not require any accounting information (i.e. color, level, height, etc.)**
- **Key Feature: Frequently accessed keys are moved up so that they are near the root**
  - 90/10 rule - 90% of data access in data structure are to only 10% of the data

Splay Operation

- **Operation used to rearrange tree to make a specified node the root node of the tree**
  - Has a nice side-effect that any imbalances in the tree are corrected
- **Splay operation is called whenever a node is accessed**
  - After a node is inserted, a splay operation is called on that node
    - A node inserted into the tree will always end up as the root when the insertion completes
  - After searching for a node, a splay operation is called on it
    - Subsequent searches for that node should execute more quickly
  - To delete a node, first splay it to the top of the tree, then delete it

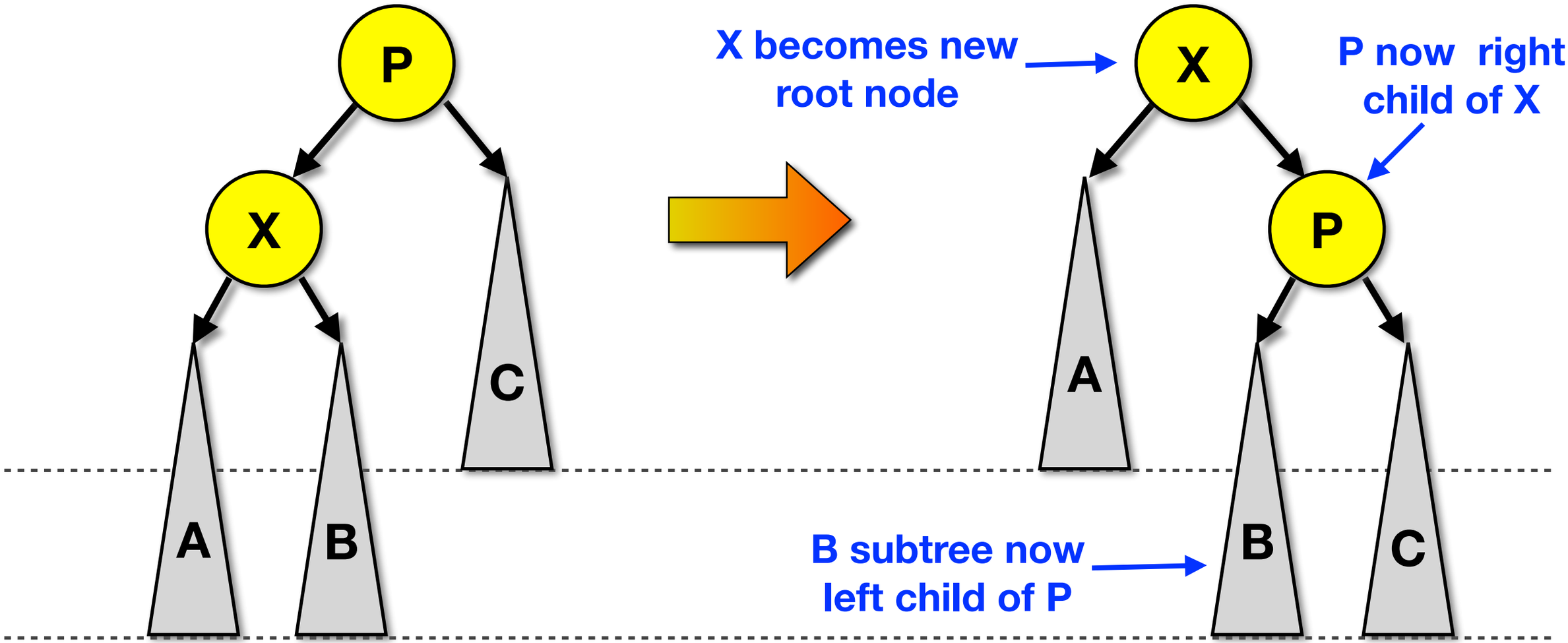
Splay Operation

- **When splay is called on a node, continue to splay that node until it becomes the root of the entire tree**

- **There are three cases to consider when splaying a node**
  - Case #1 - a zig - a node X is the left child of a node P
  - Case #2 - a zig-zag - a node X is the inside grandchild of a node G
  - Case #3 - a zig-zig - a node X is the outside grandchild of a node G

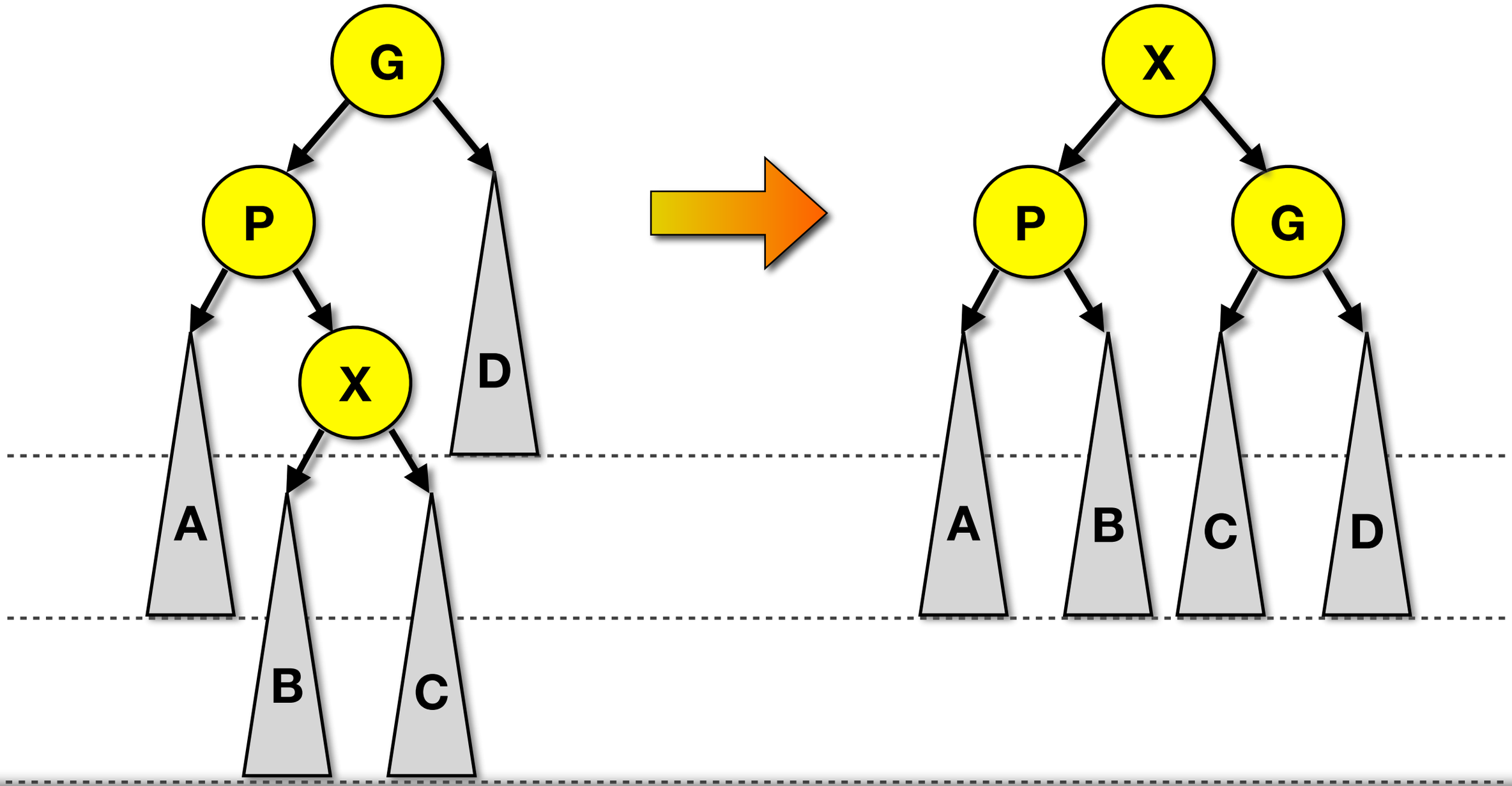
Splay Operation -- **Case #1 (a Zig)**

Splaying a node X whose parent P is the root of the subtree.
Same as a single rotation of the node X with the root P.



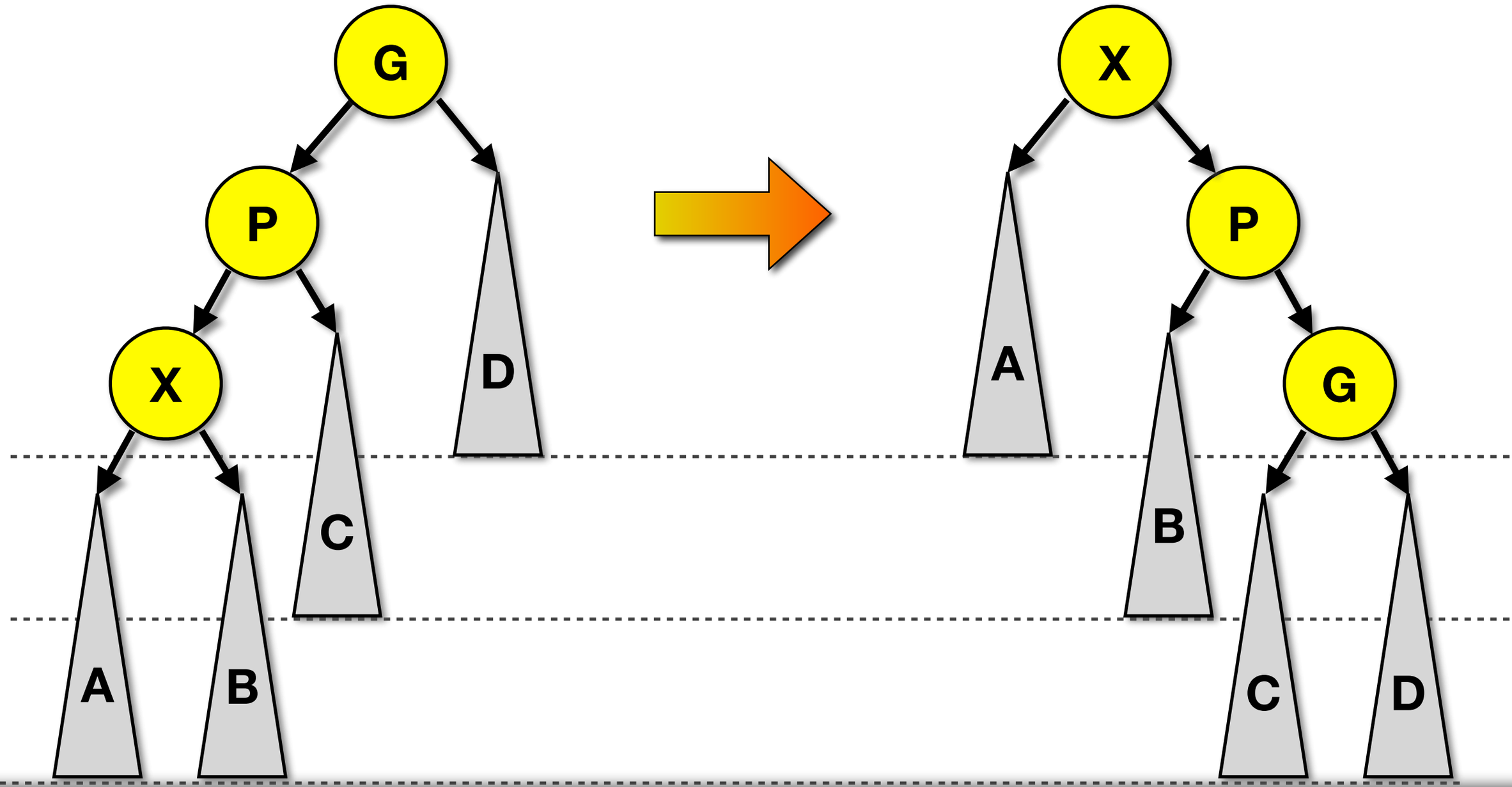
Splay Operation -- **Case #2 (a Zig-Zag)**

Splaying a node X which is the inside grandchild of G.
Same as a double rotation of the node X.



Splay Operation -- **Case #3 (a Zig-Zig)**

Splaying a node X which is the outside grandchild of G.
A single rotation between P and G followed by single rotation between X and P.



Splay Tree Operations

- **The Splay tree supports same operations as other binary search trees**
  - `Find` - starts with the standard BST `Find` algorithm; then performs the splay operation on the node after it is found to bring it up to the root of the tree
    - If node is not found, then the splay operation is performed on the last node in the tree that was visited while performing the search
  - `Insert` - executes a standard BST `Insert` algorithm; then performs the splay operation on the newly inserted node
    - A node inserted into the tree will always end up as the root when the insertion completes

Splay Tree Operations (Cont.)

- **The Splay tree supports same operations as other binary search trees**
  - `Delete`
    (1) Performs a `Find` to locate the node to be deleted, this also brings the node to be deleted to the root of the tree;

    (2) Delete the root of the tree leaving two unattached subtrees

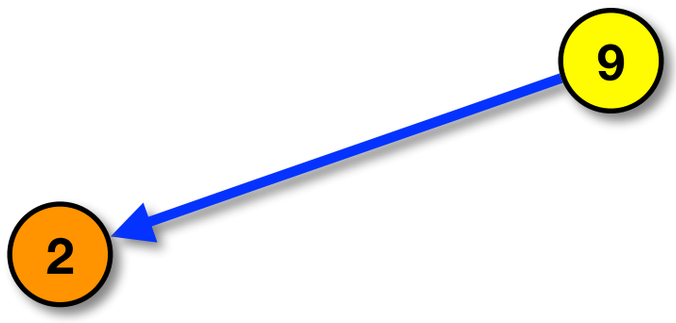
    (3) Perform a `FindMax` on the left subtree to find the maximum node and splay that maximum node to the root of the left subtree (it will not have a right child, even after the splay)

    (4) The maximum node from step 3 becomes the new root node for the entire tree; attach the unattached right subtree from step 2 as a right child of this new root node

Insertion Example

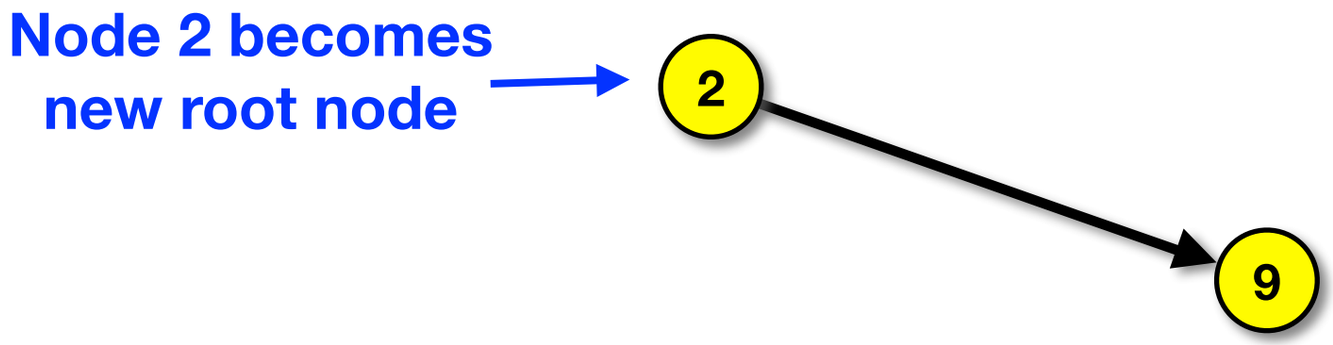
Insert node 9
Calling splay on node 9 doesn’t do anything

Insertion Example



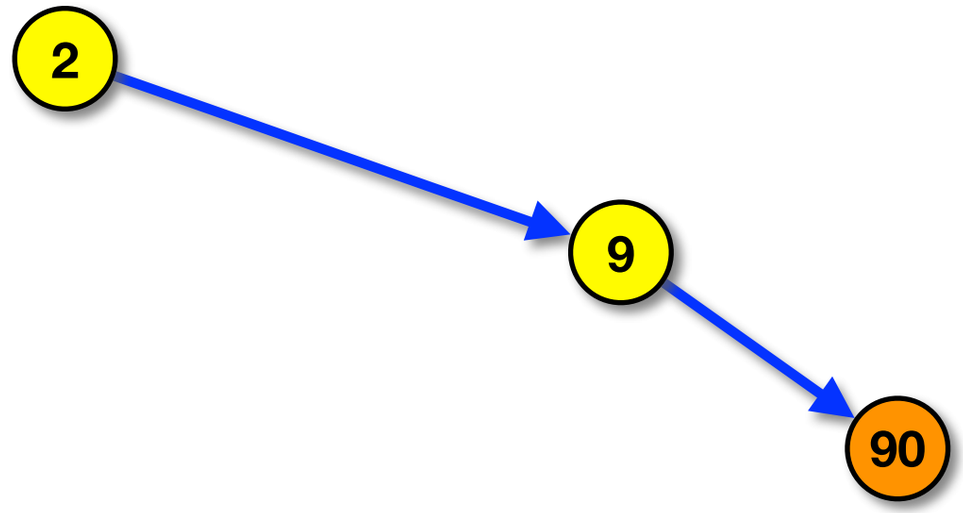
Insert node 2
Case #1 - perform a zig operation

Insertion Example



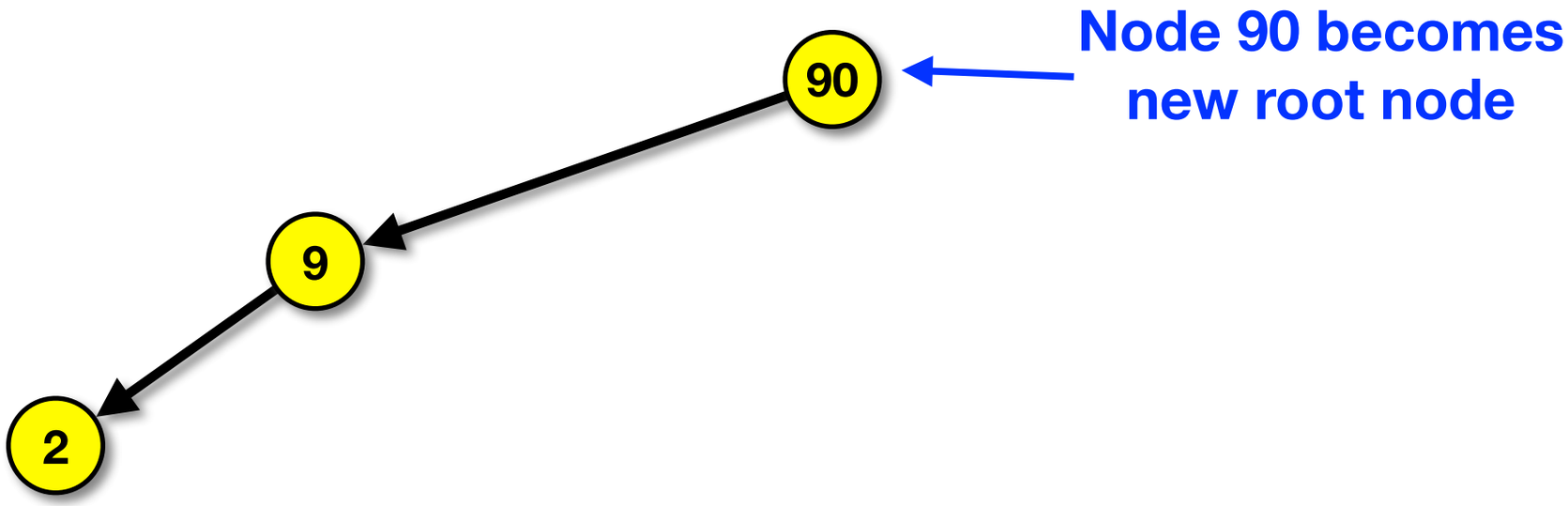
Insert node 2
Result of zig operation

Insertion Example



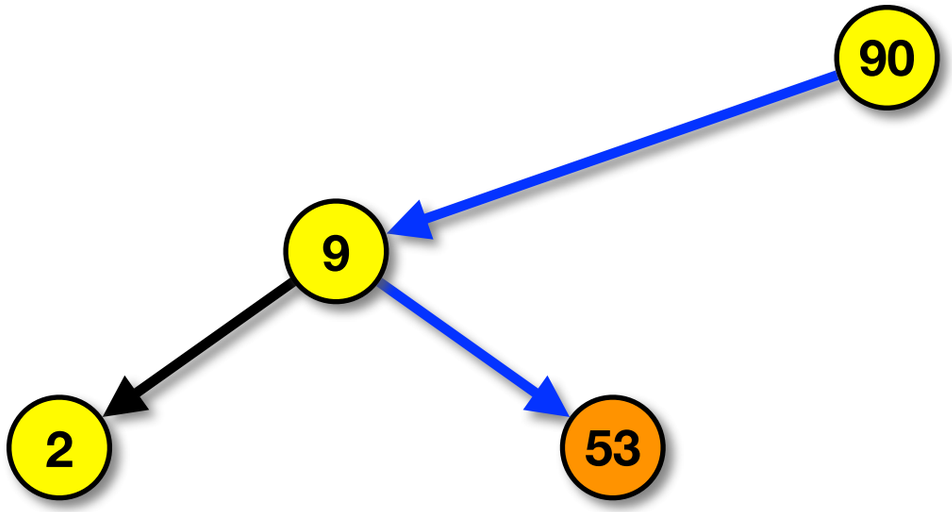
Insert node 90
Case #3 - perform a zig-zig operation

Insertion Example



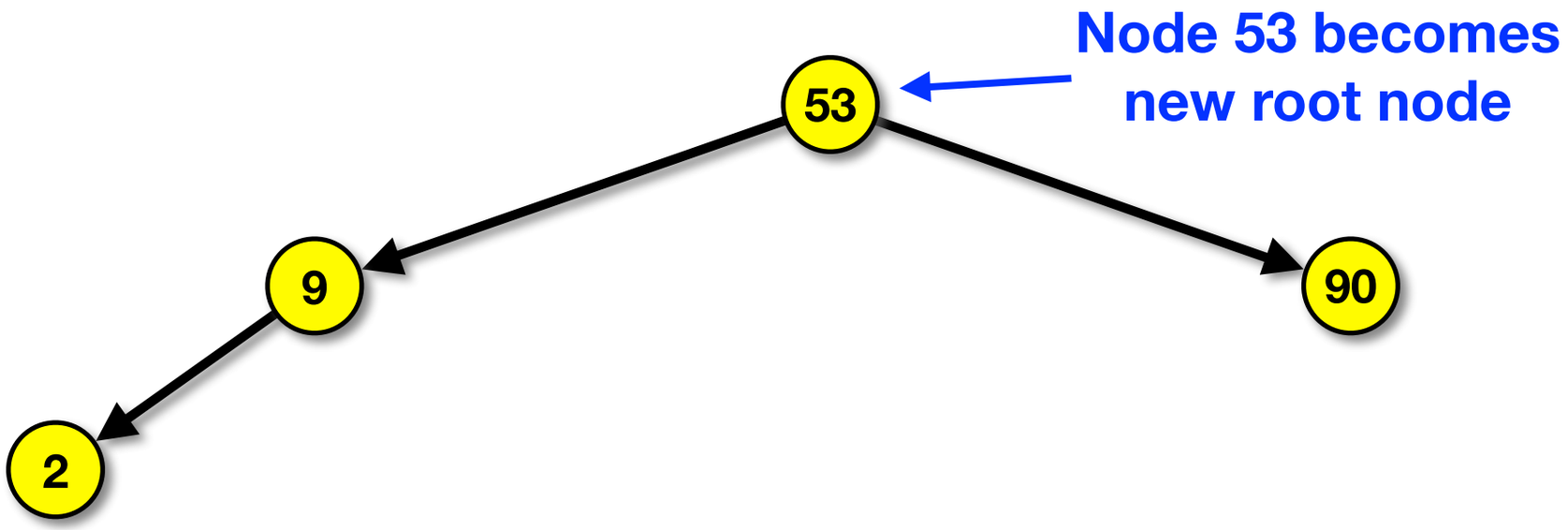
Insert node 90
Result of zig-zig operation

Insertion Example



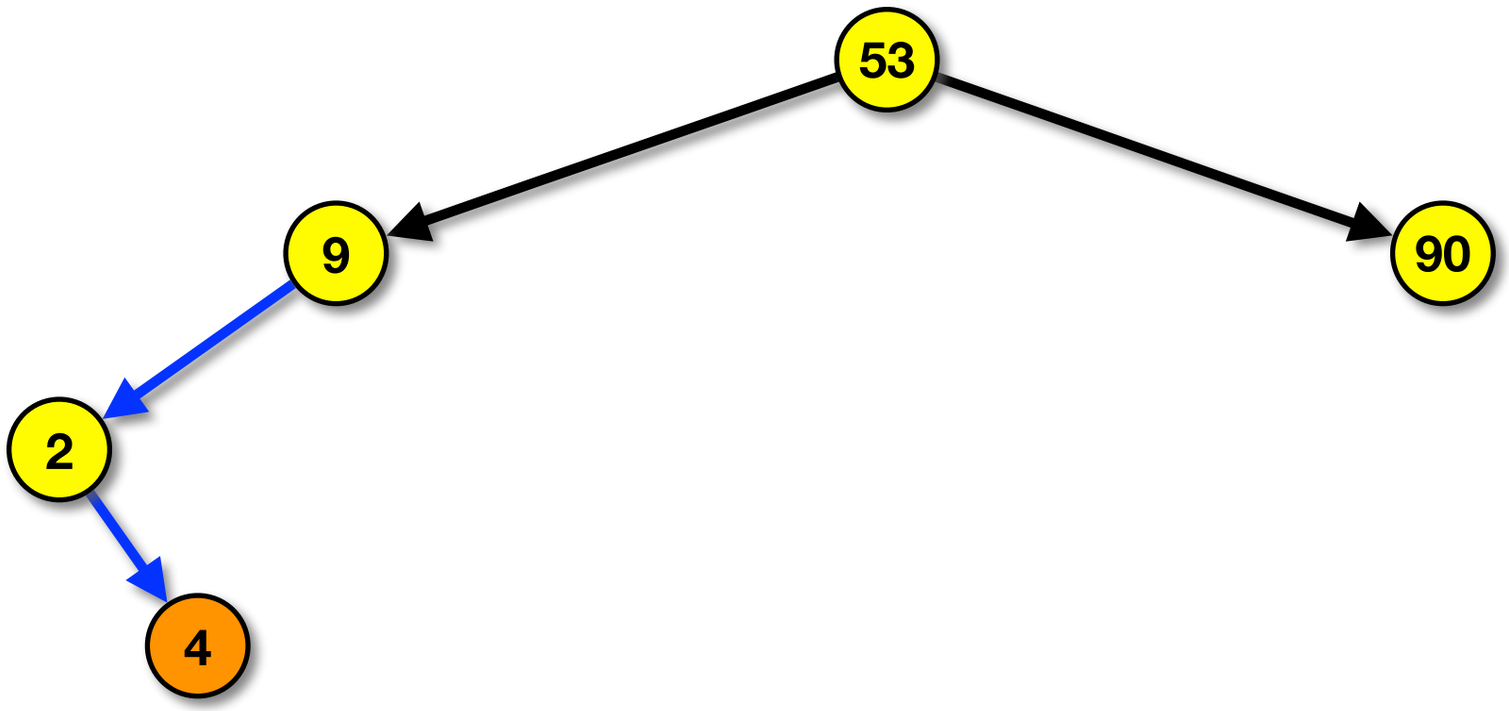
Insert node 53
Case #2 - perform a zig-zag operation

Insertion Example



Insert node 53
Result of zig-zag operation

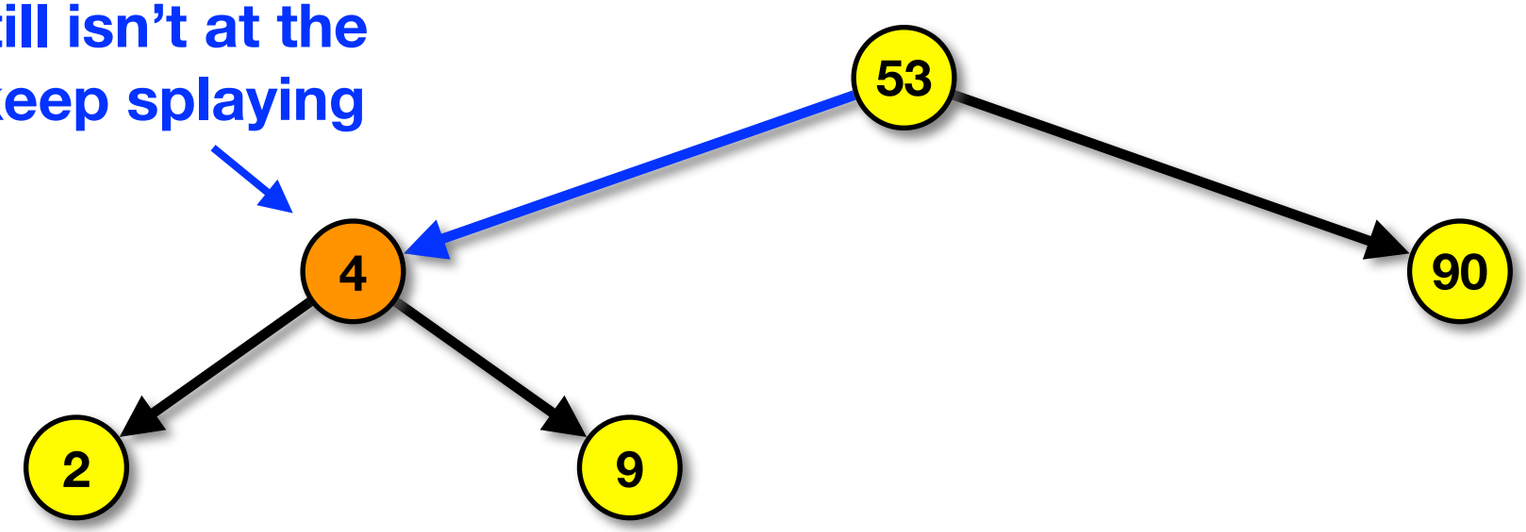
Insertion Example



Insert node 4
Case #2 - perform a zig-zag operation

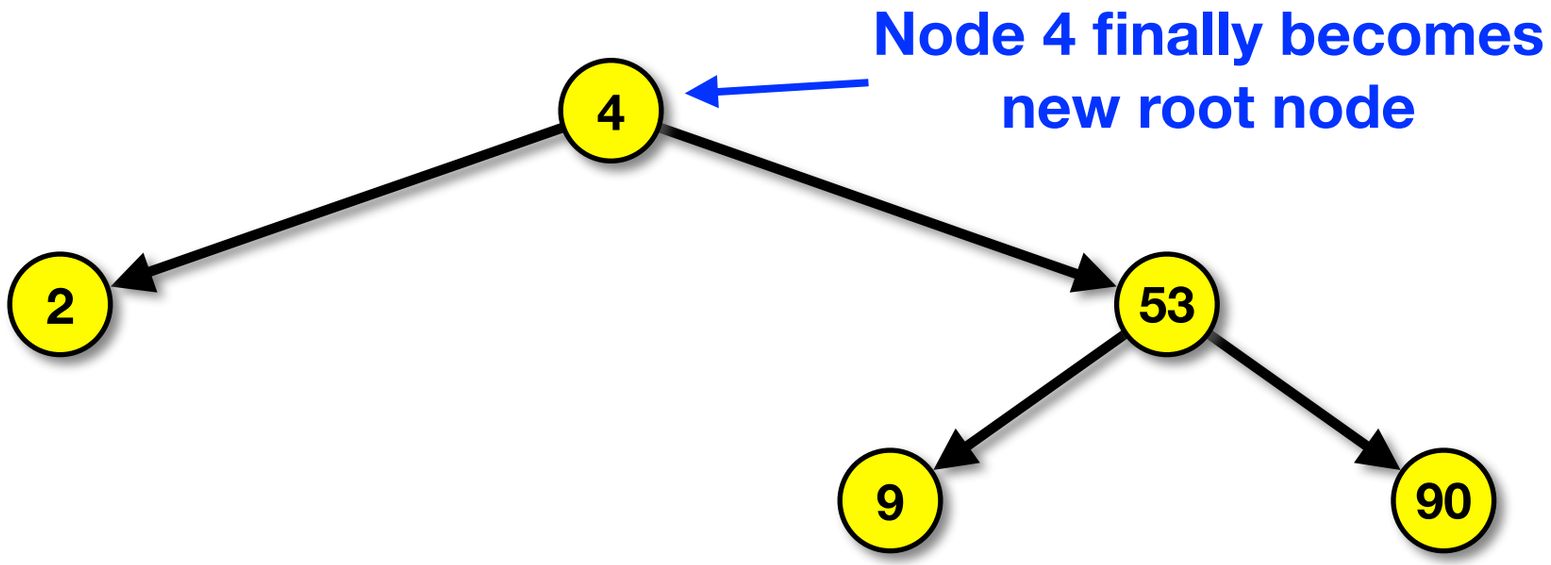
Insertion Example

Node 4 still isn't at the root, so keep splaying



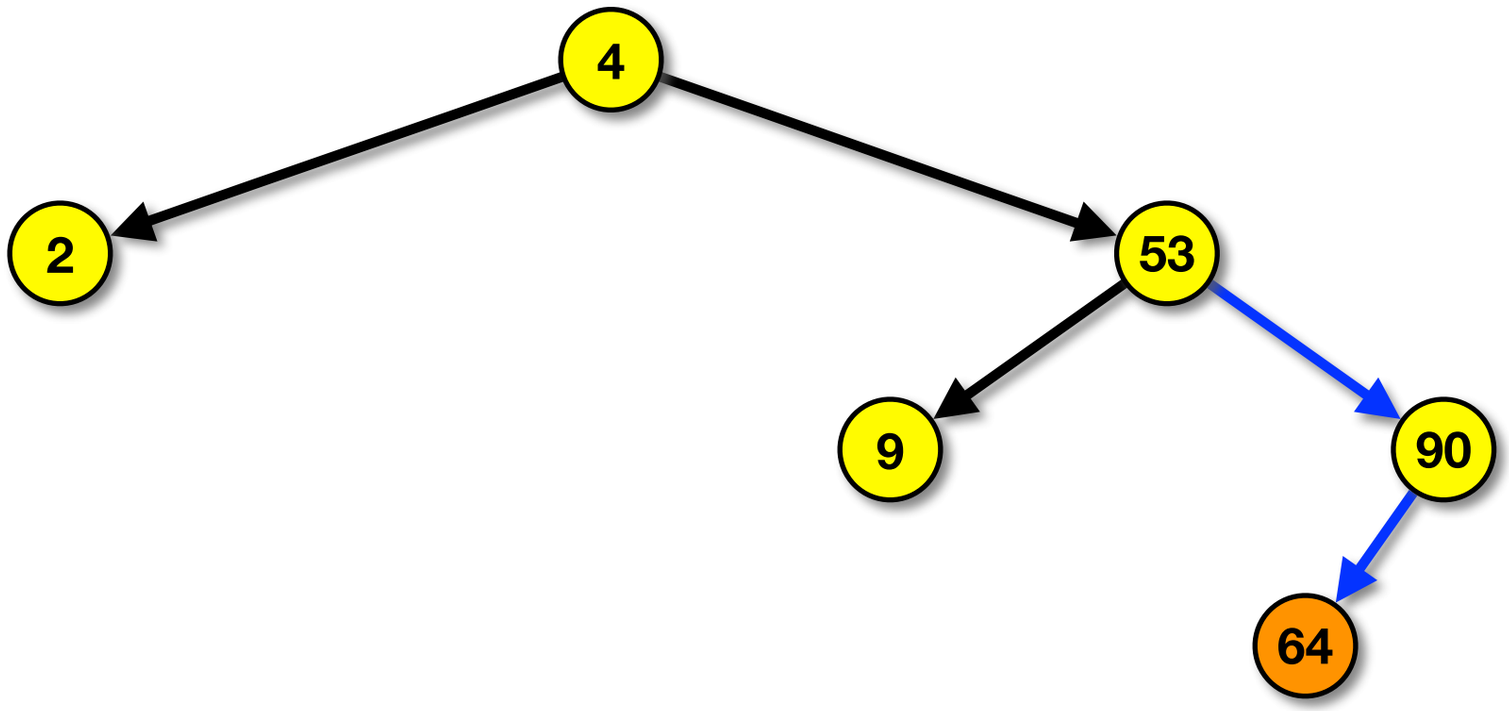
Insert node 4 (Cont.)
Case #1 - perform a zig operation

Insertion Example



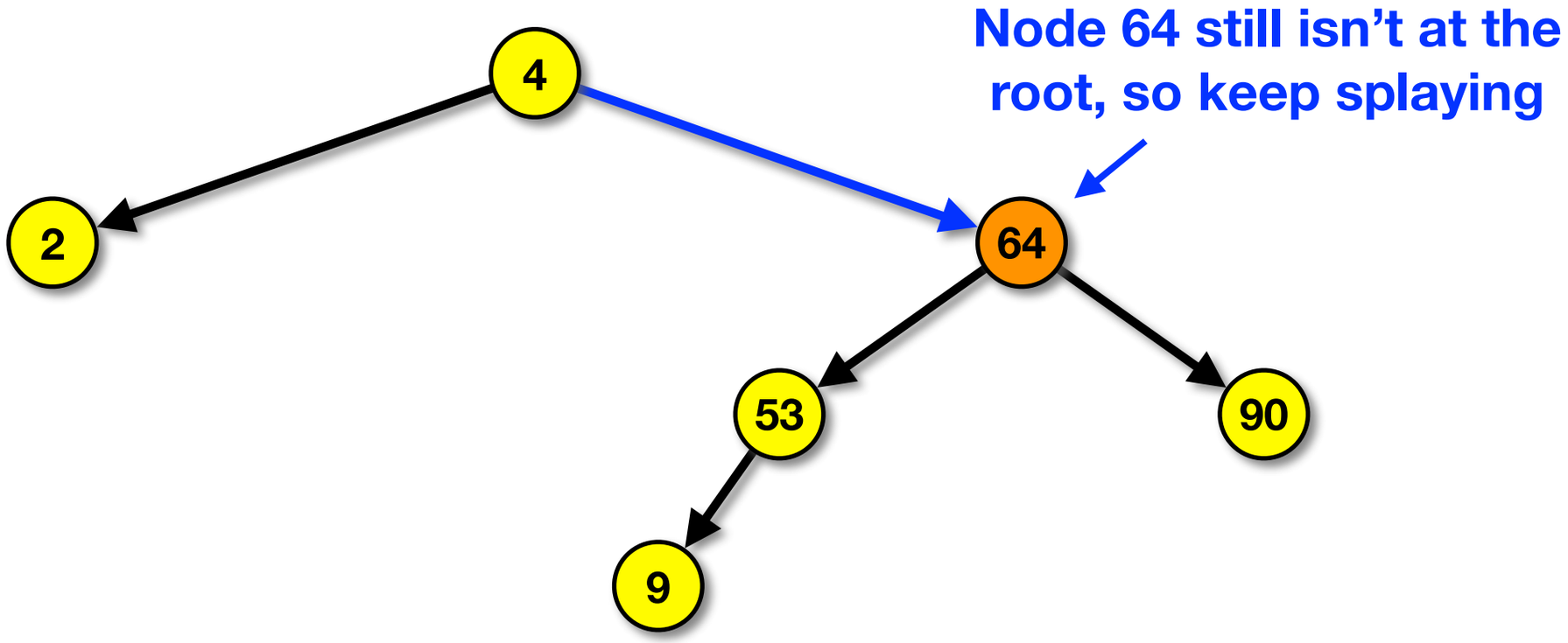
Insert node 4
Result of zig-zag operation
followed by a zig operation

Insertion Example



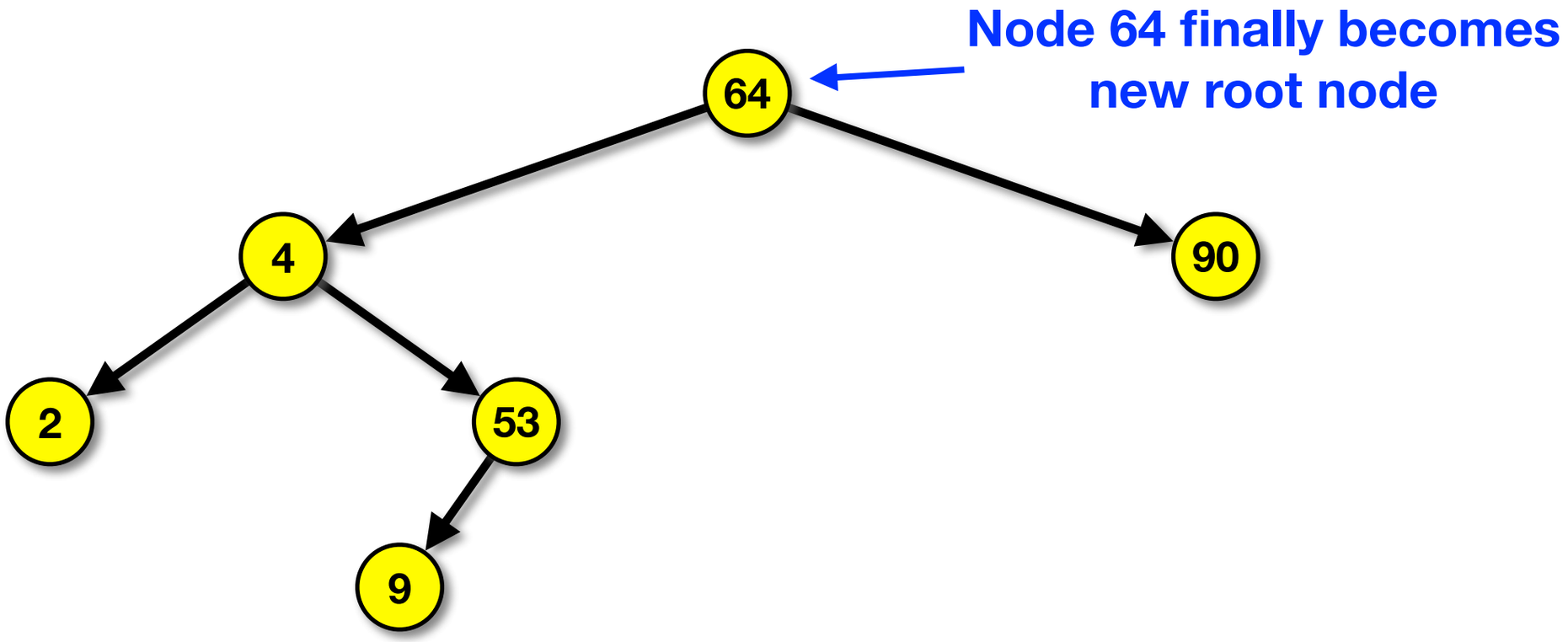
Insert node 64
Case #2 - perform a zig-zag operation

Insertion Example



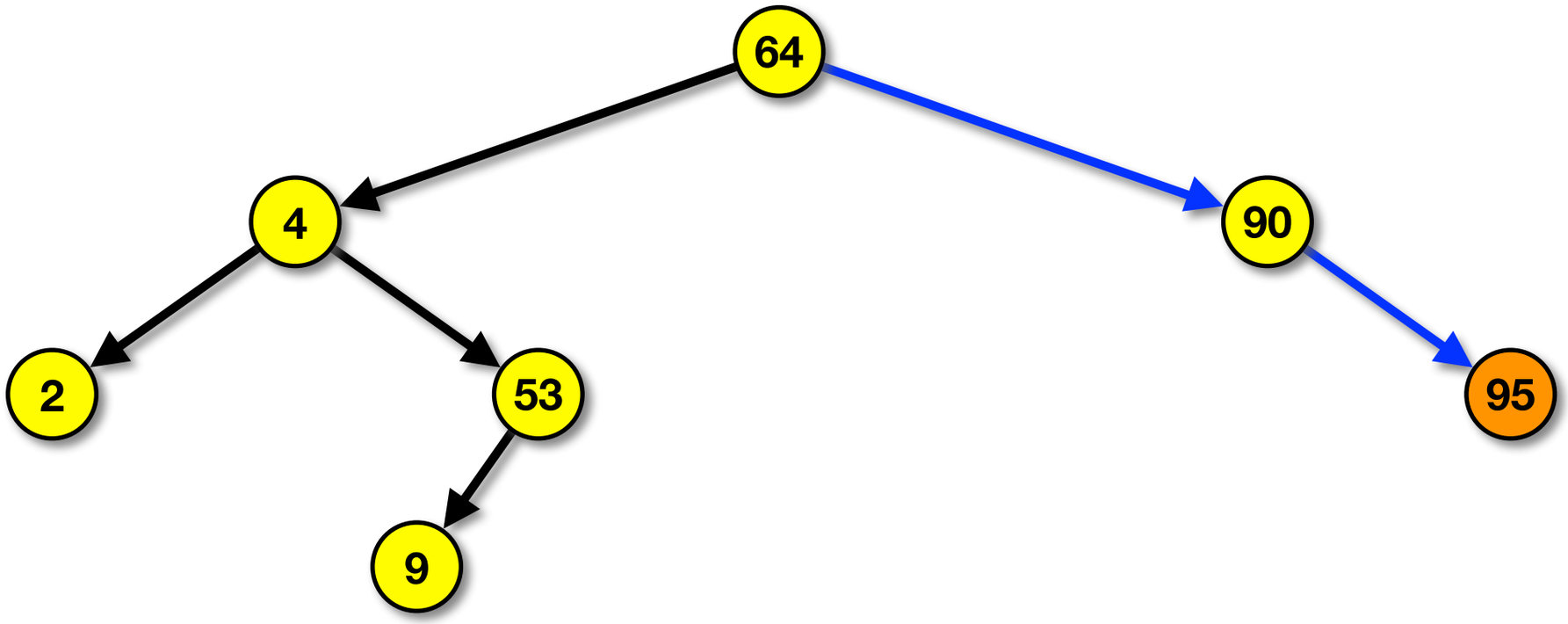
Insert node 4 (Cont.)
Case #1 - perform a zig operation

Insertion Example



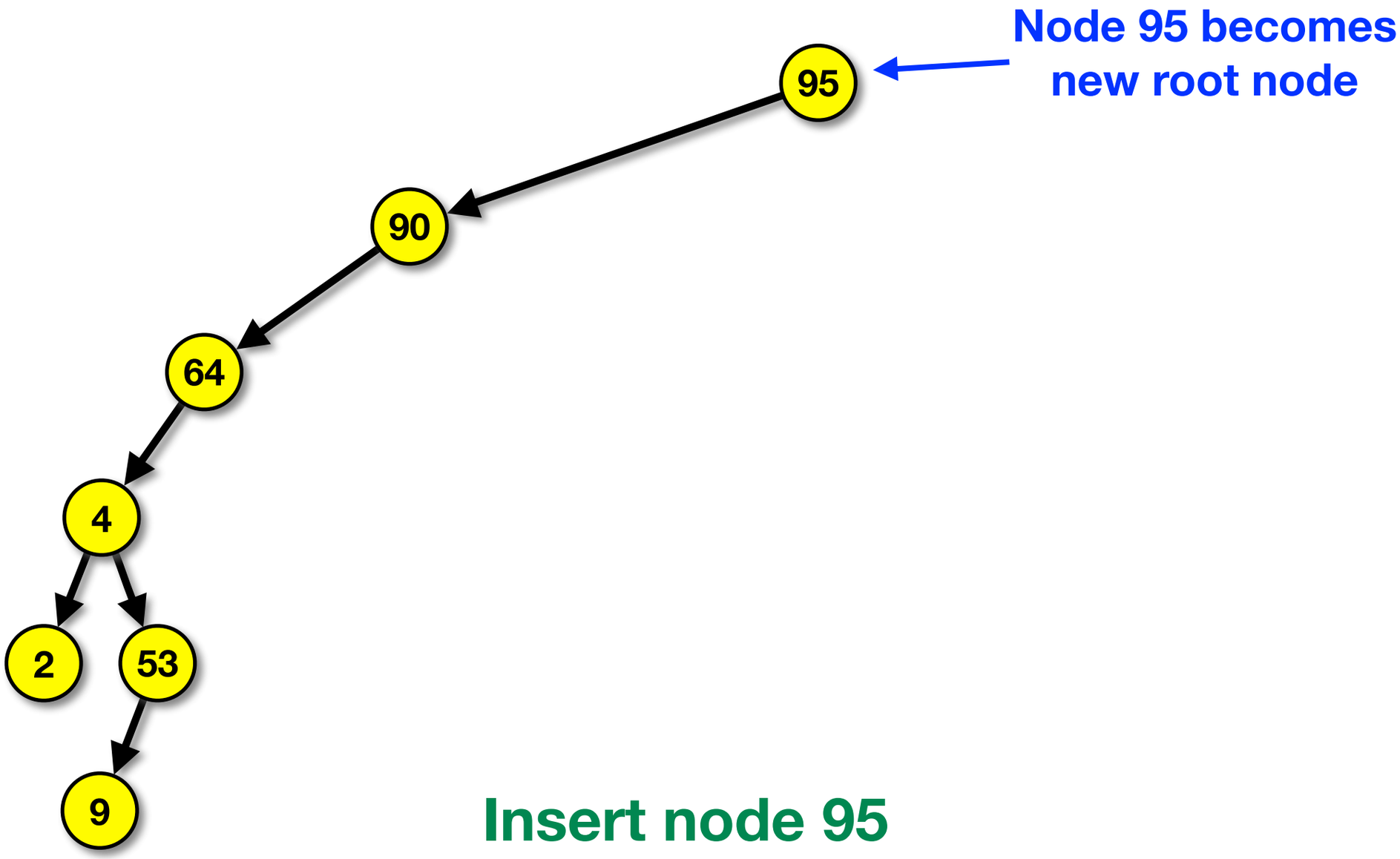
Insert node 64
Result of zig-zag operation
followed by a zig operation

Insertion Example



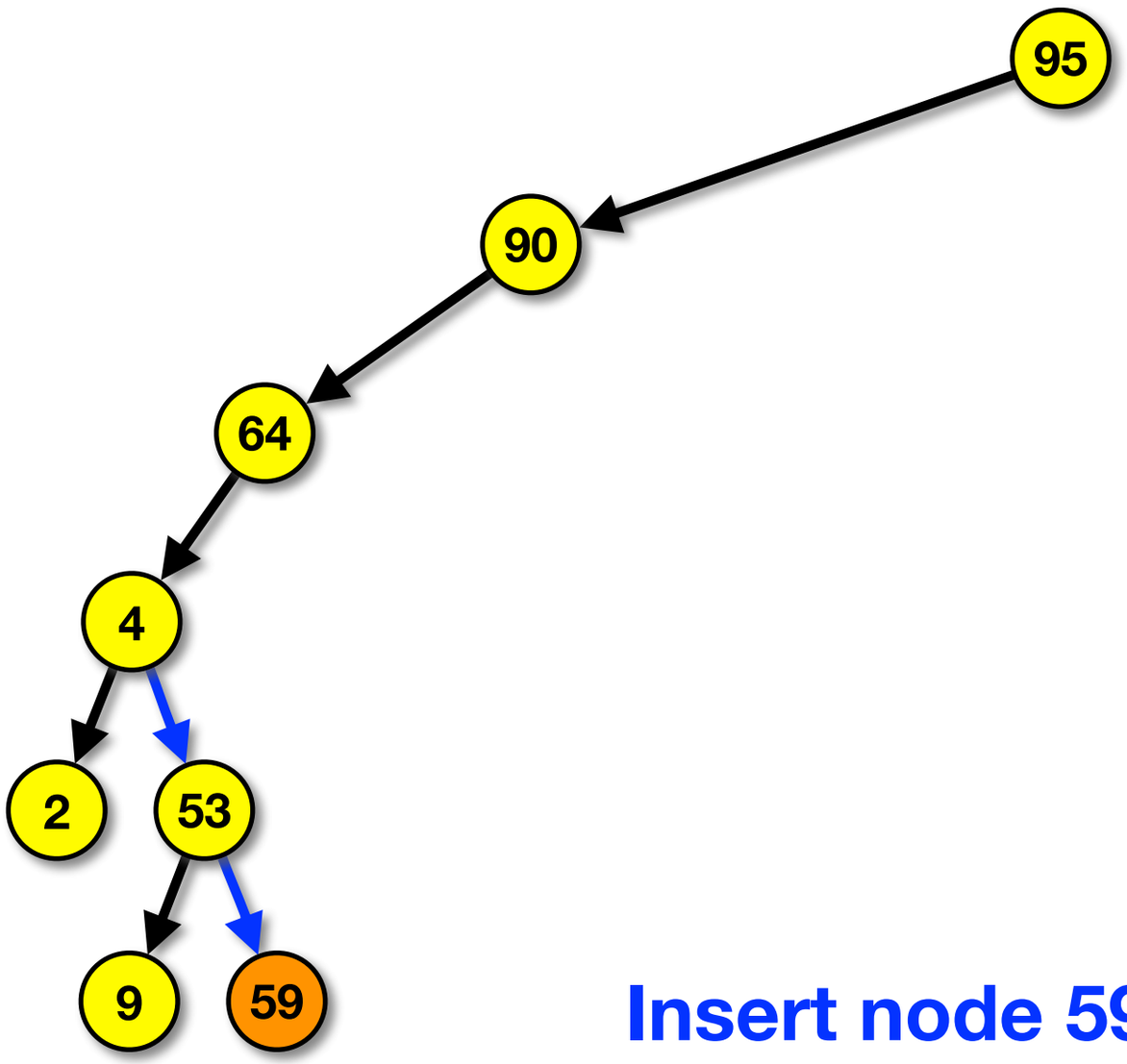
Insert node 95
Case #3 - perform a zig-zig operation

Insertion Example



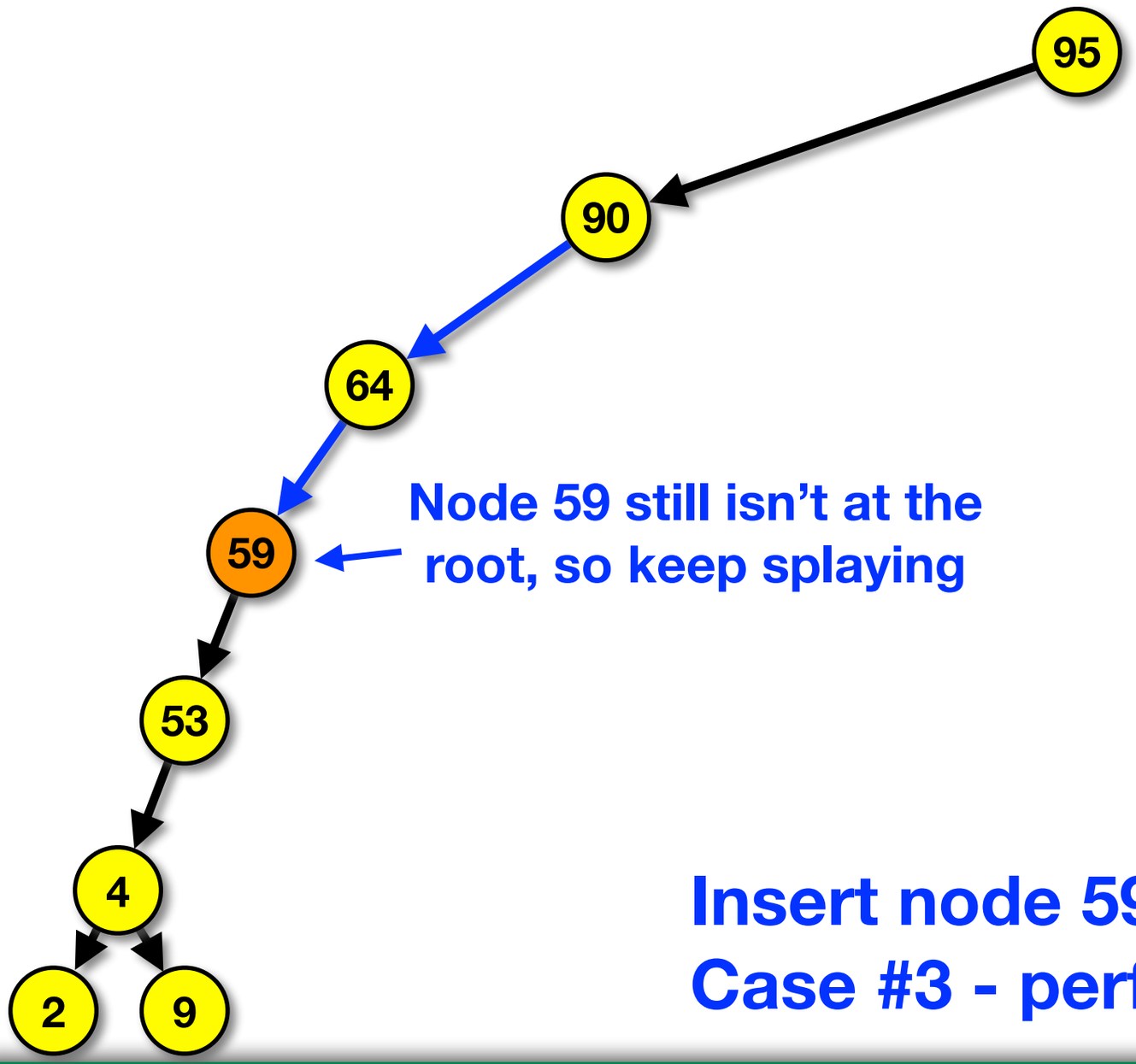
Insert node 95
Result of zig-zig operation

Insertion Example



Insert node 59
Case #3 - perform a zig-zig operation

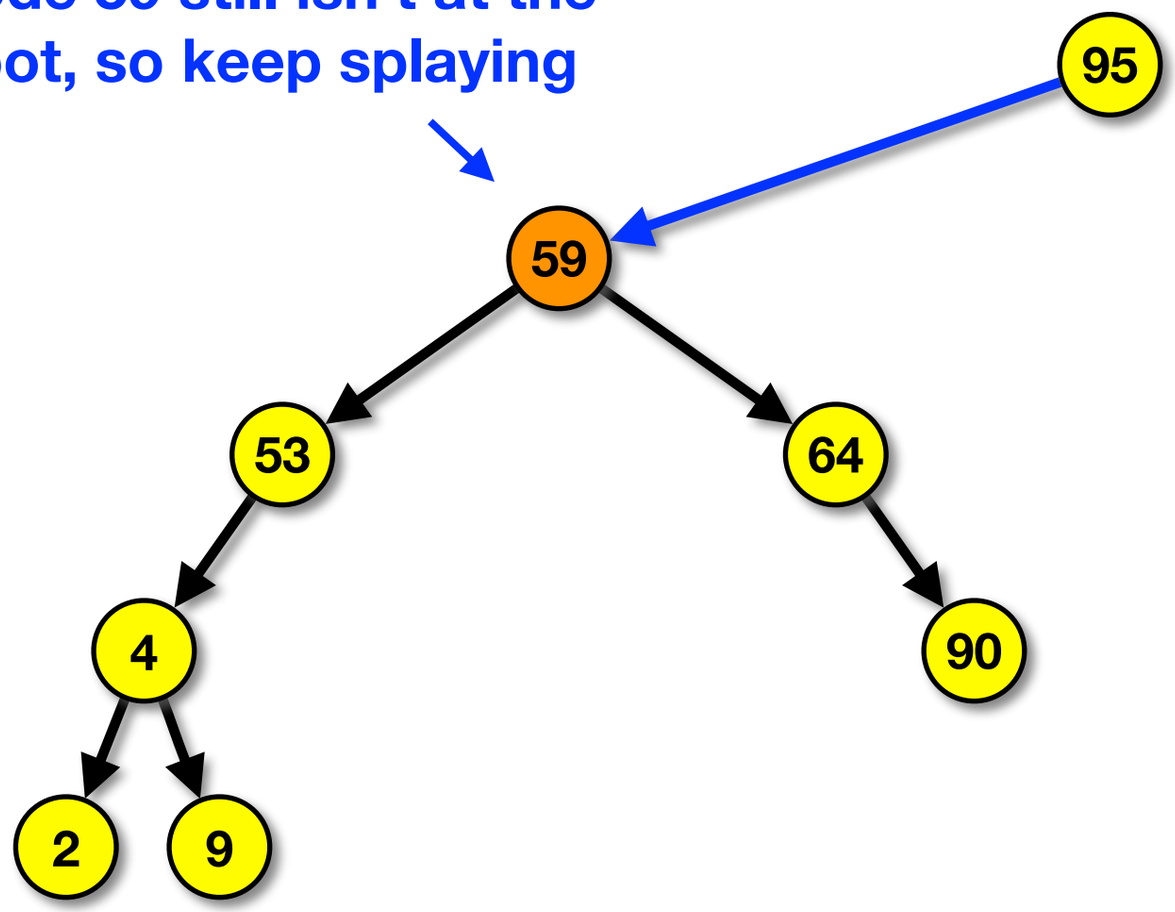
Insertion Example



Insert node 59 (Cont.)
Case #3 - perform a zig-zig operation

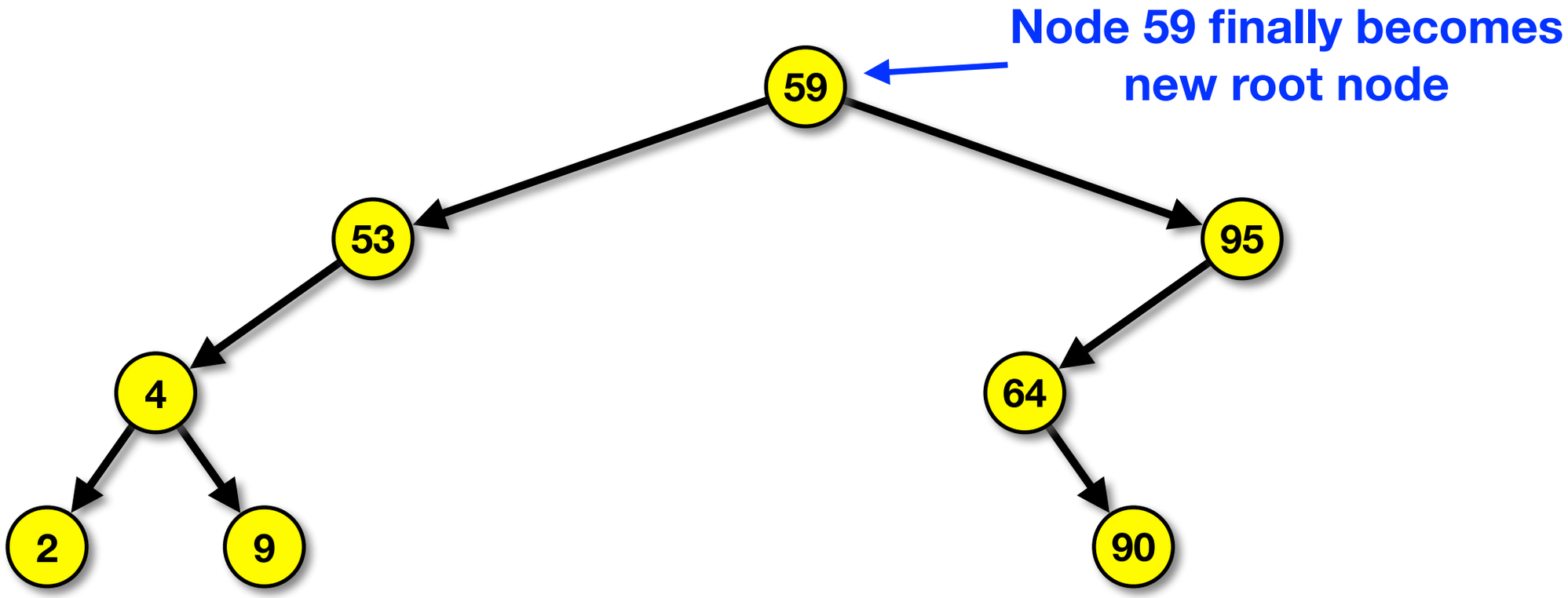
Insertion Example

Node 59 still isn't at the root, so keep splaying



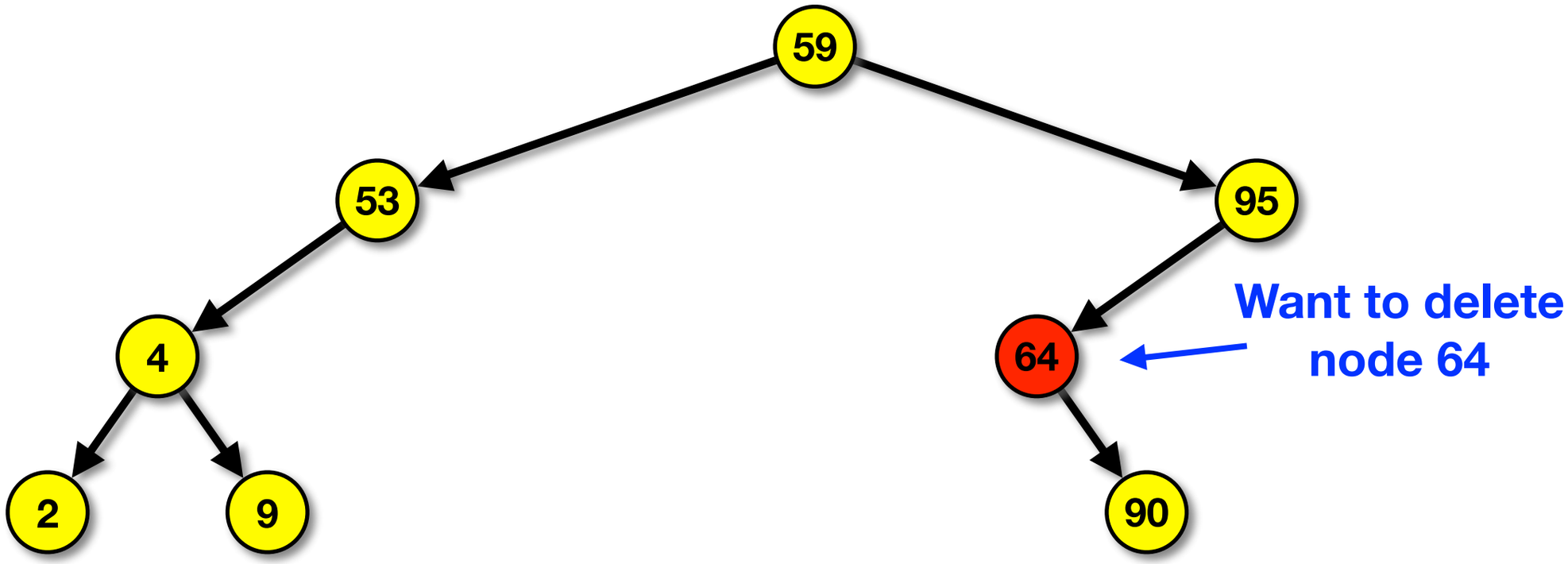
Insert node 59 (Cont.)
Case #1 - perform a zig operation

Insertion Example



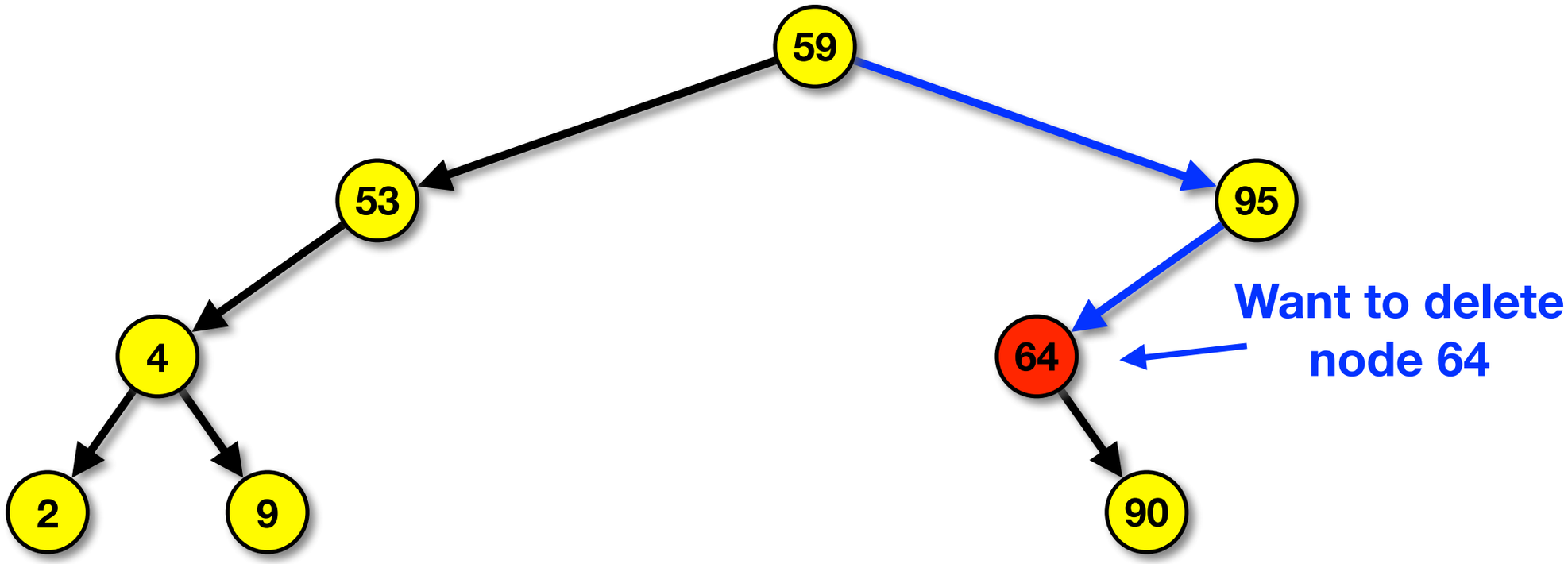
Insert node 59
Result of zig-zig operation
followed by a zig-zig operation
followed by a zig operation

Deletion Example



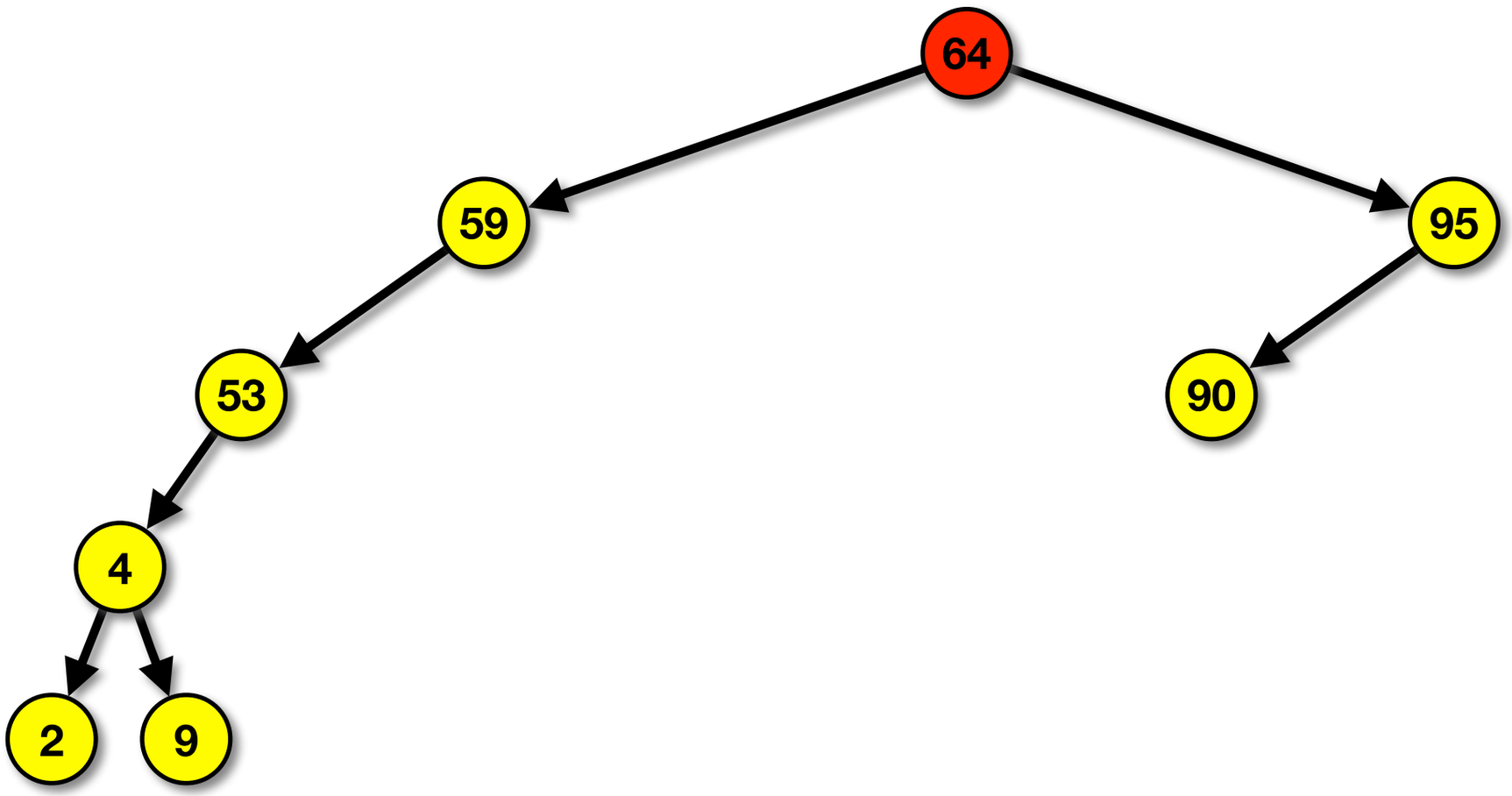
Delete node 64
Step #1 - perform a `Find` operation to move node 64 to the root

Deletion Example



Delete node 64
Step #1 - perform a `Find` operation to move node 64 to the root
- a zig-zag operation is needed

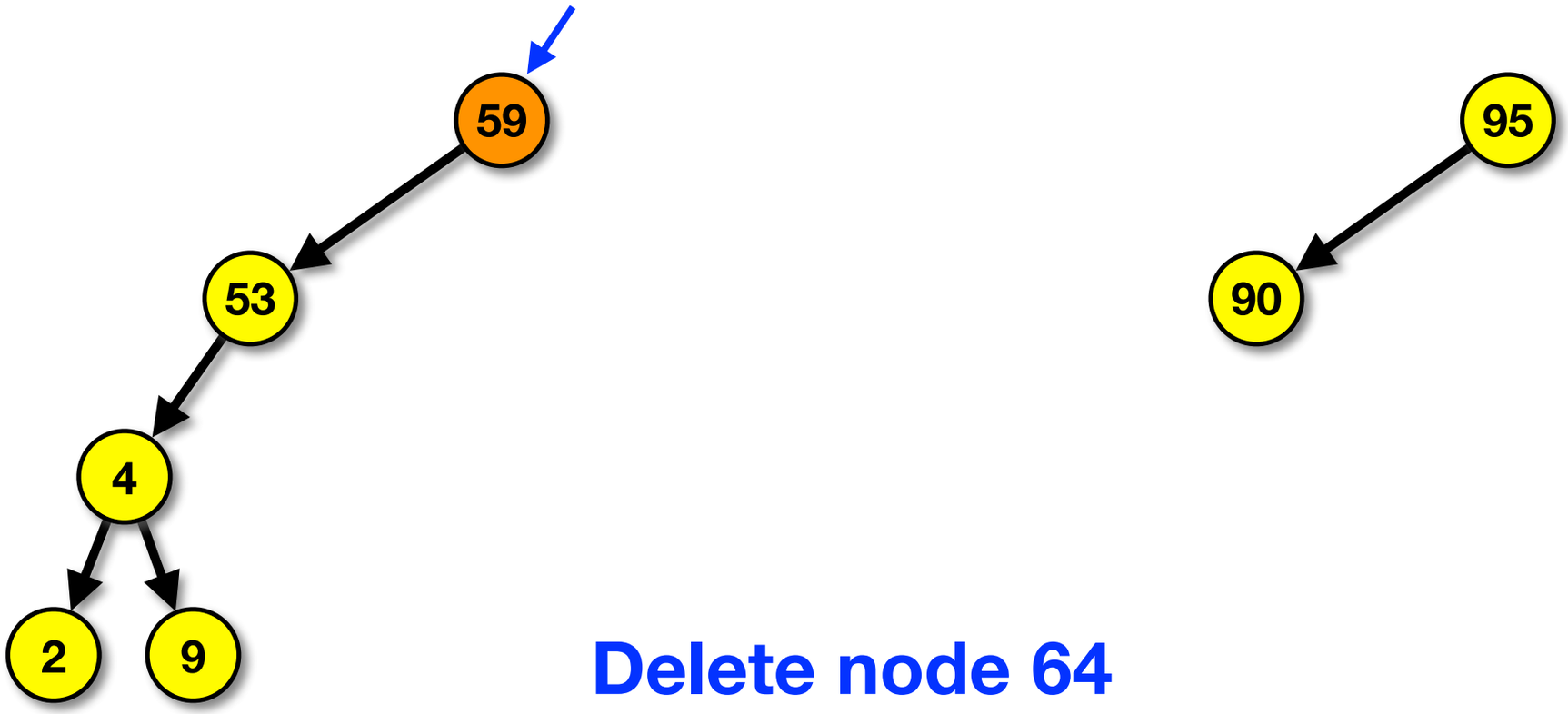
Deletion Example



Delete node 64
Step #2 - Now that node 64 is at the root, delete it leaving two unattached subtrees

Deletion Example

`FindMax` **returns 59, since 59 is already root of left subtree, splay doesn't do anything**

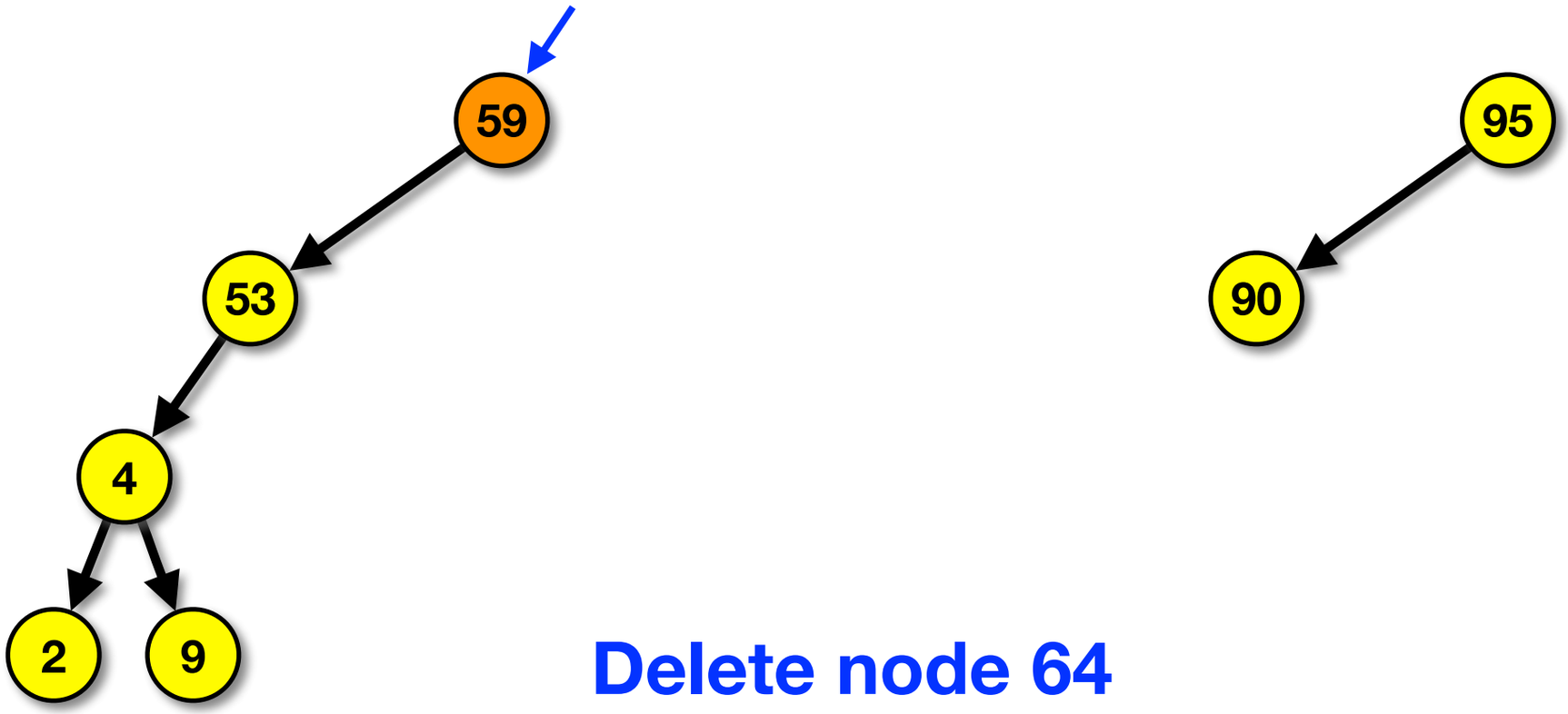


Delete node 64

Step #3 - perform a `FindMax` on the left subtree and splay that max node to the root of the left subtree

Deletion Example

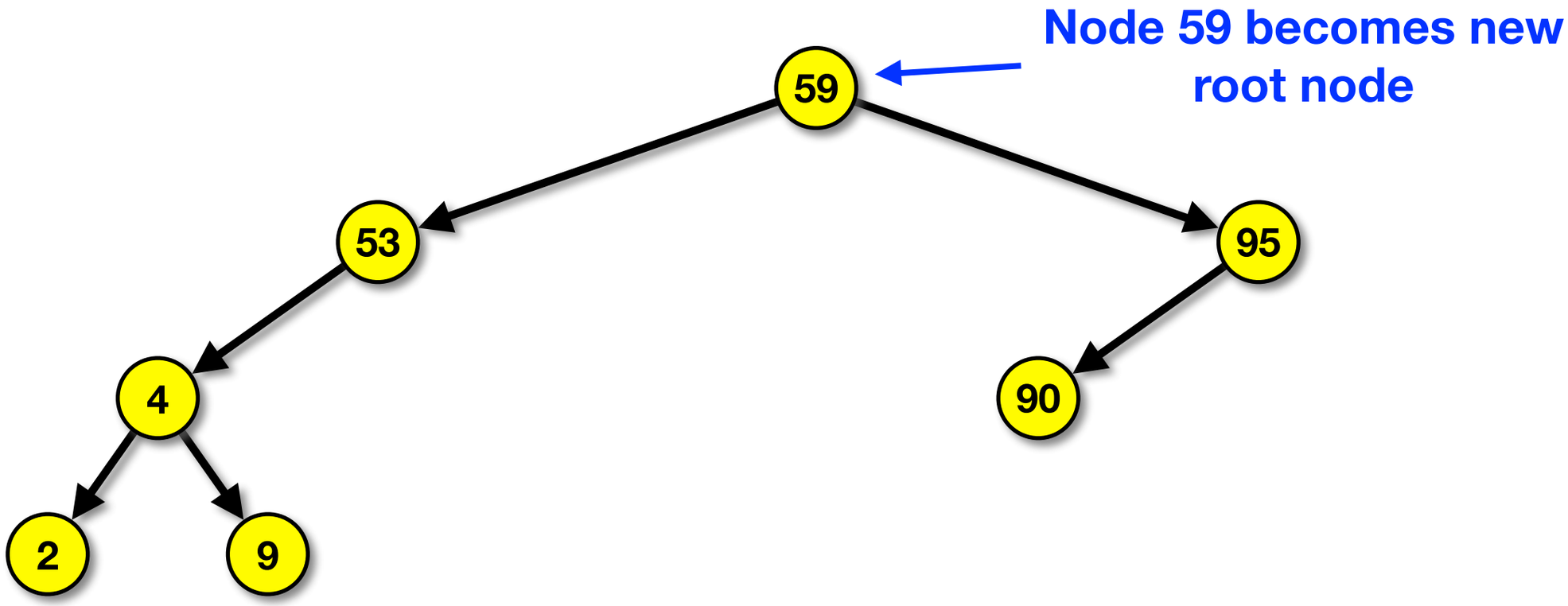
`FindMax` **returns 59, since 59 is already root of left subtree, splay doesn't do anything**



Delete node 64

Step #4 - node 59 will become the new root of the tree and the right subtree will become the right child of node 59

Deletion Example



Result of deleting node 64