CS350: Data Structures

Heap Sort

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Heapsort

- **The priority queue can be used to sort N items as follows**
  - Insert N elements to be sorted into heap
  - Heapify the heap to put it into heap-order (minHeap)
  - Call deleteMin N times and the elements will get removed from the heap in ascending order

This same procedure can be used with a maxHeap and deleteMax to produce output in descending order

Heapsort

- **Build the heap so that you extract the values in the opposite order that you want them sorted**
  - Build a minHeap if descending sort order is desired
  - Build a maxHeap if ascending sort order is desired

Heapsort

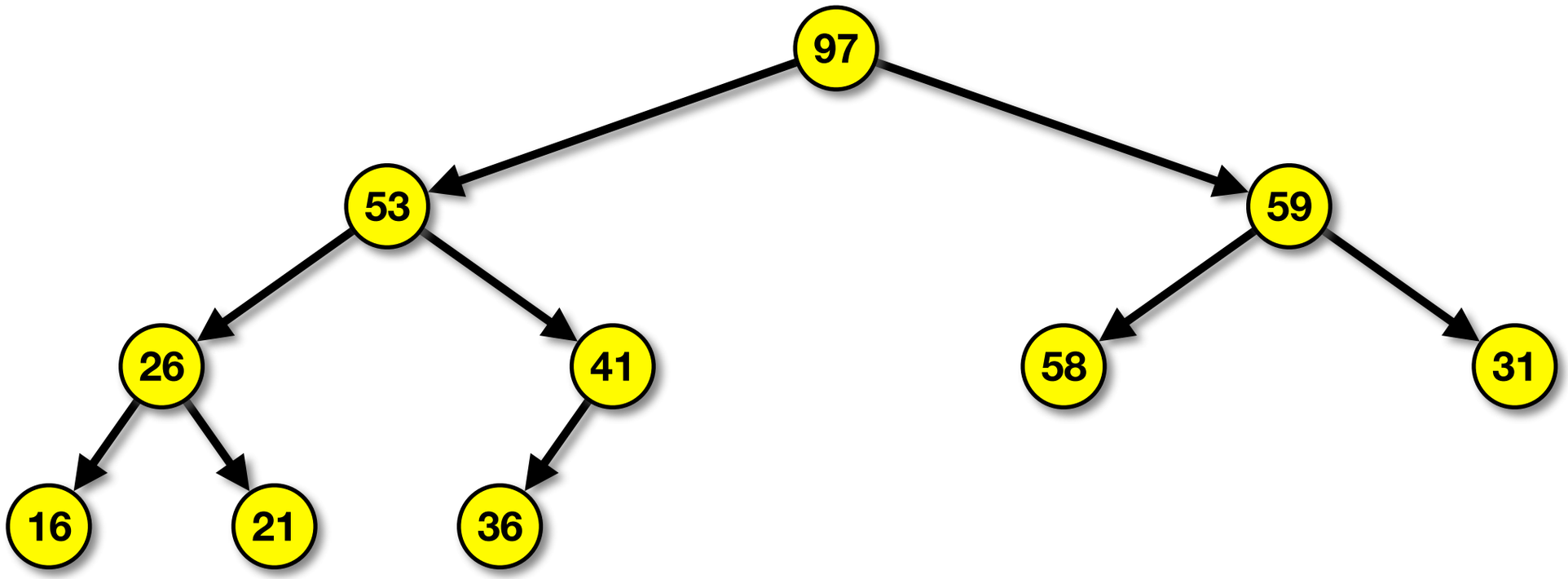
- **Run time of heap sort is broken down into each step**
  - Initially inserting N elements into the heap takes O(N)
  - The heapify step runs in O(N) time
  - The sorting step runs in O(N log N) time
    - Each deleteMin takes O(log N) time
    - Must call deleteMin N times
  - Total time complexity = O(N) + O(N) + O(N log N) = O(N log N)

Heapsort

- **When calling deleteMin or deleteMax on the heap, the sorted values must be stored somewhere**
  - One option is to create a second array that stores the sorted values
    - Doubles the memory requirements for the sort
  - Another option is to reuse heap space in a clever manner so as not to need a secondary array
    - As heap shrinks, store sorted values in array positions that are no longer part of the heap

Heapsort Example

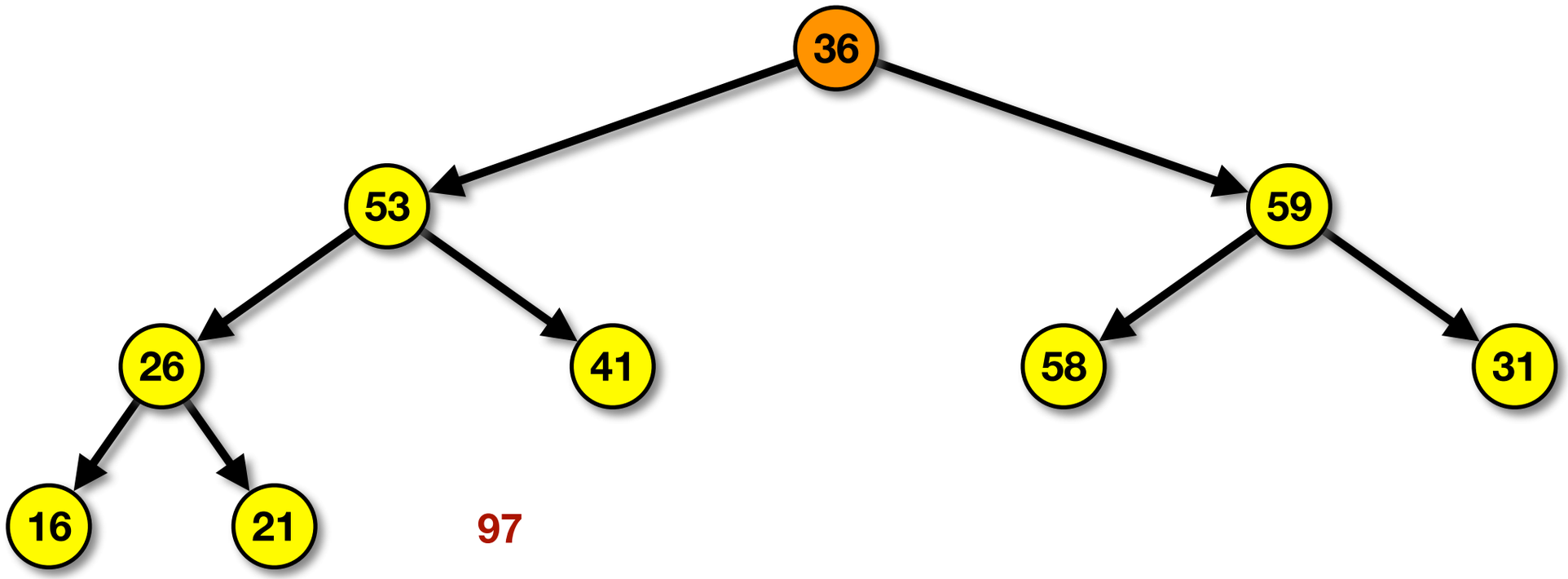
- **Start with a maxHeap**
- **Repeatedly call deleteMax and place the max element in the newly open array position at the end of the heap**

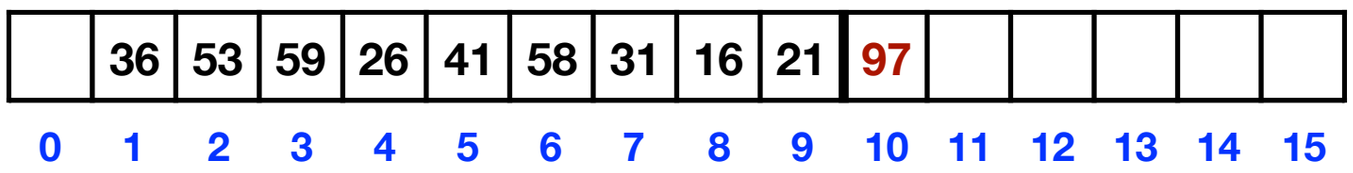


	97	53	59	26	41	58	31	16	21	36					
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Heapsort Example

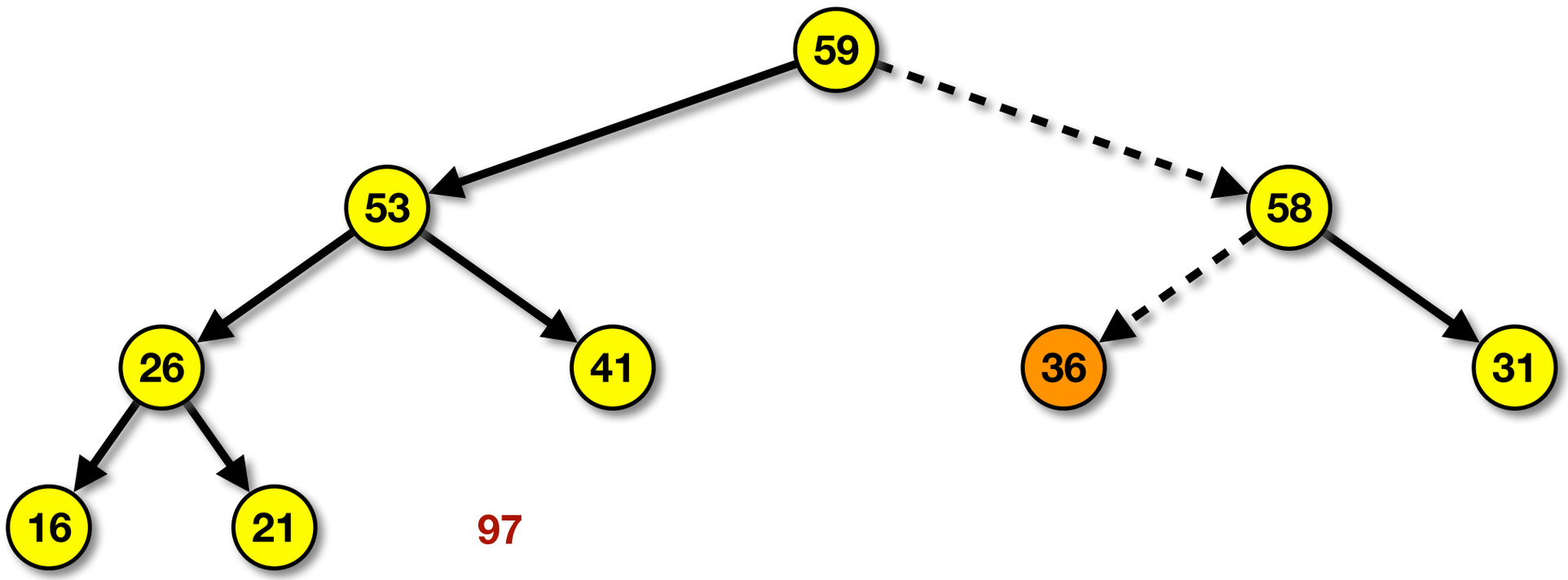
- **Call deleteMax method**
- **Swap node 36 with node 97 and shrink the size of the heap**





Heapsort Example

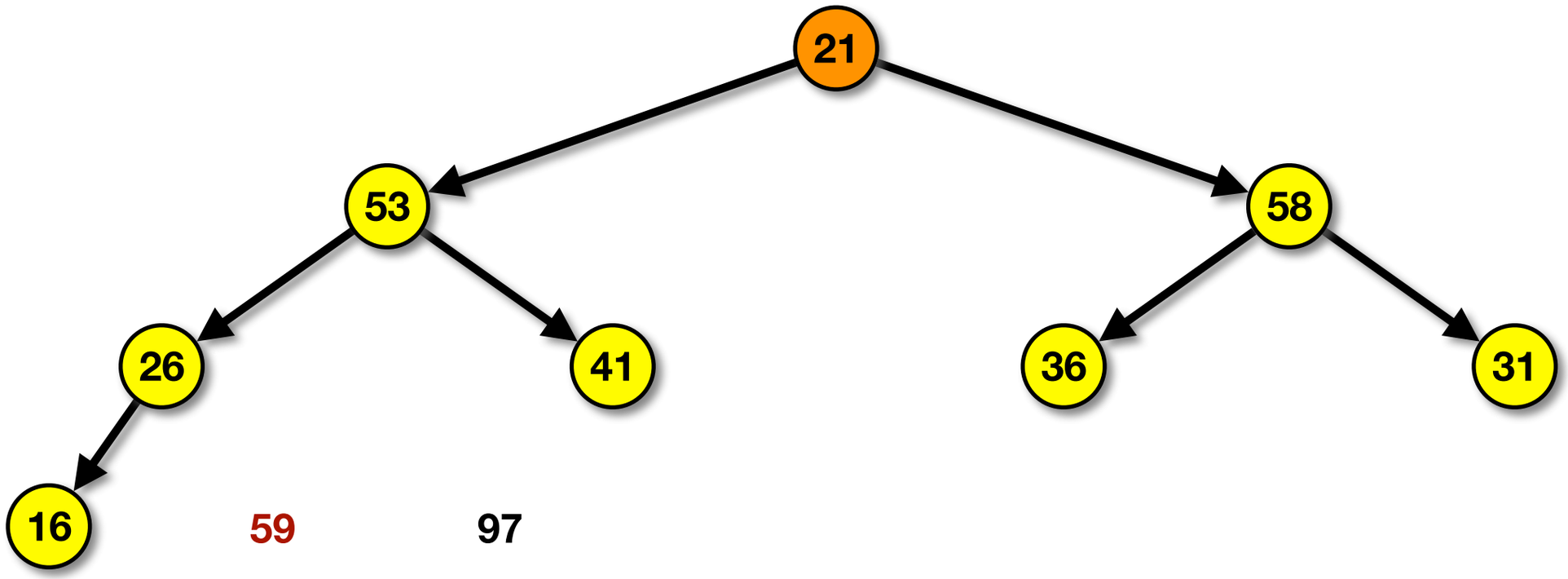
- **Call percolateDown on node 36**

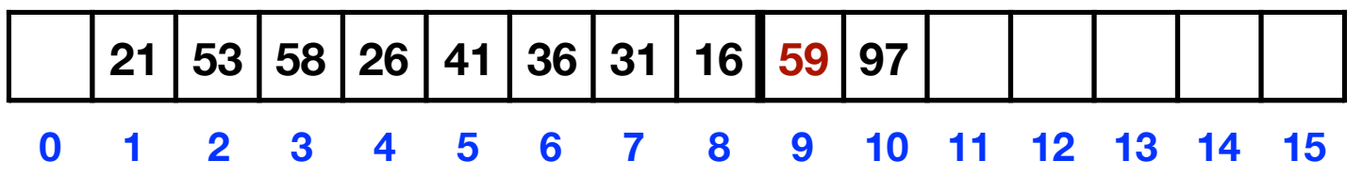


	59	53	58	26	41	36	31	16	21	97					
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Heapsort Example

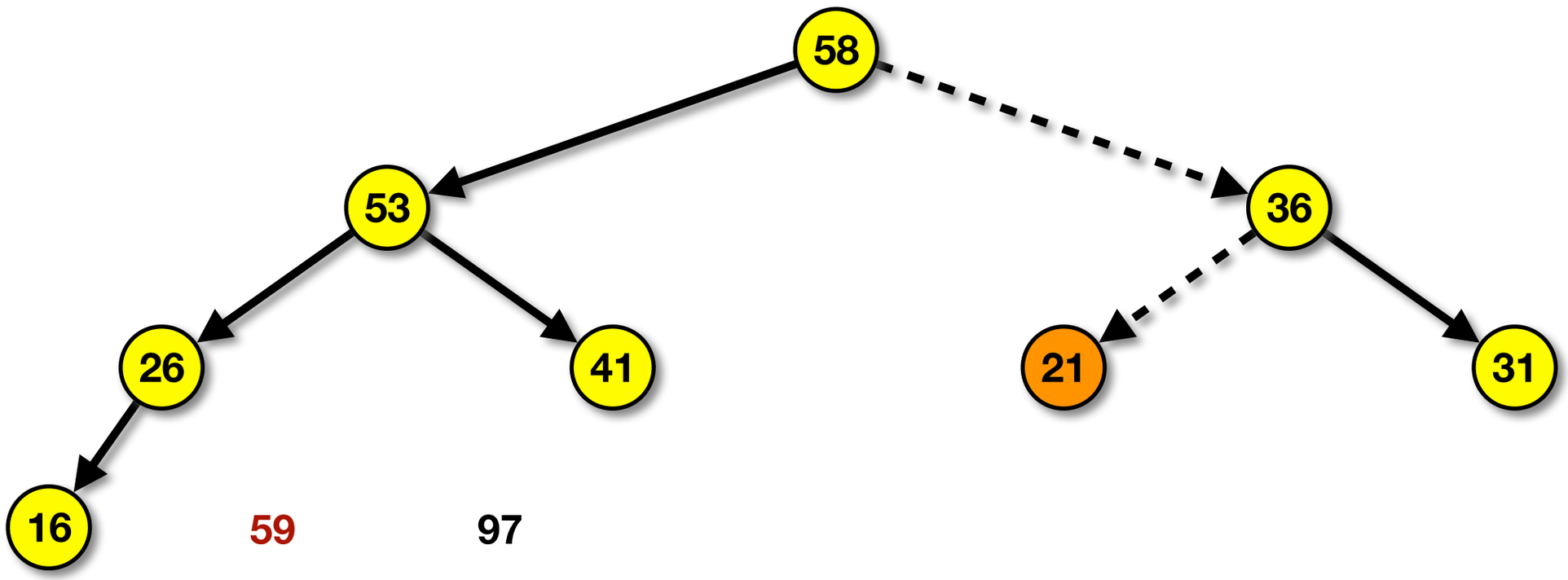
- **Call deleteMax method**
- **Swap node 21 with node 59 and shrink the size of the heap**





Heapsort Example

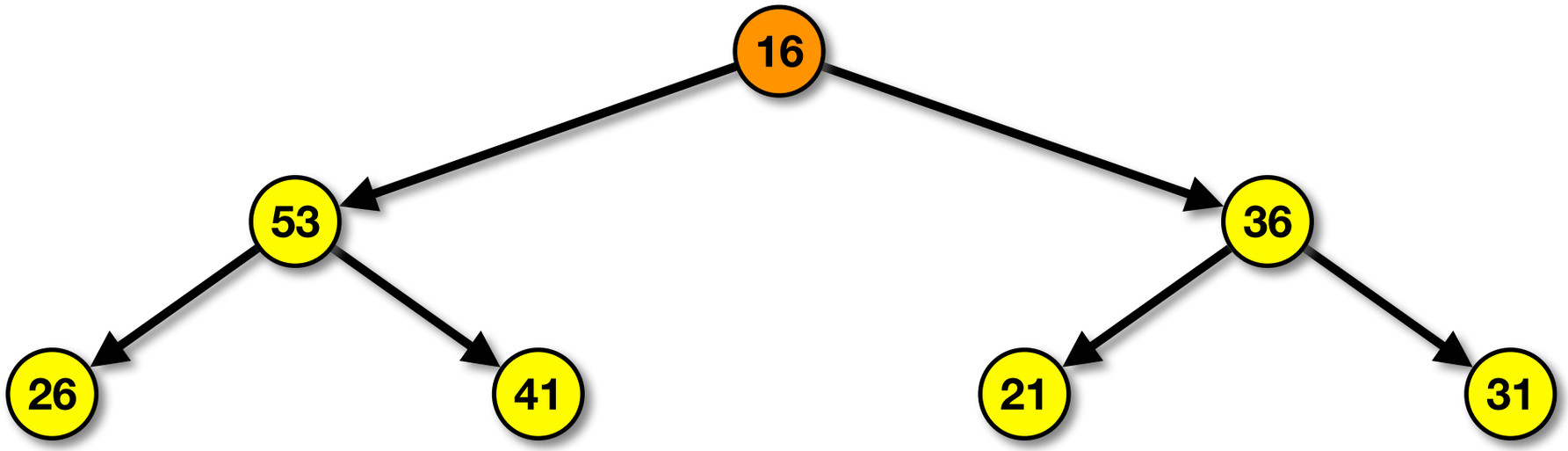
- **Call percolateDown on node 21**



	58	53	36	26	41	21	31	16	59	97					
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Heapsort Example

- **Call deleteMax method**
- **Swap node 16 with node 58 and shrink the size of the heap**

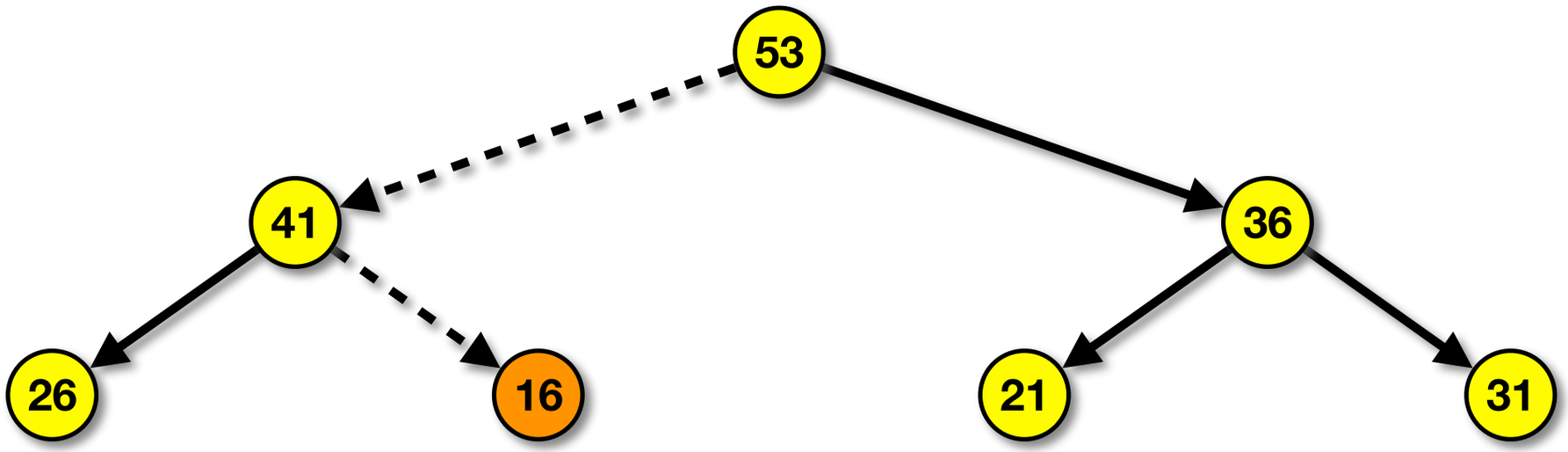


58 59 97

	16	53	36	26	41	21	31	58	59	97					
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Heapsort Example

- **Call percolateDown on node 16**

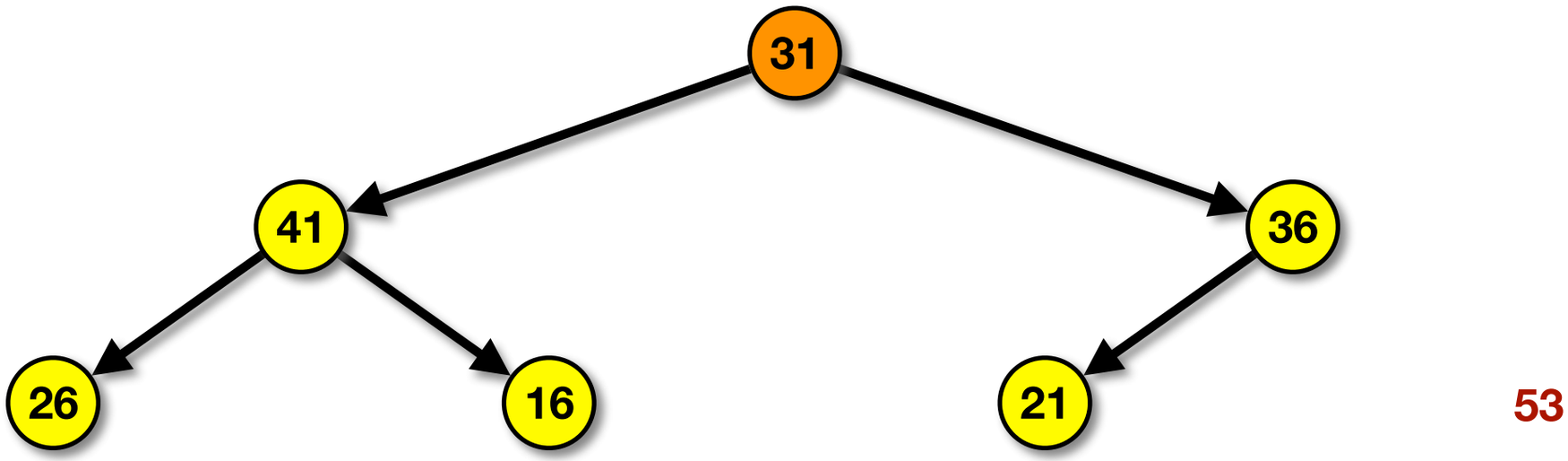


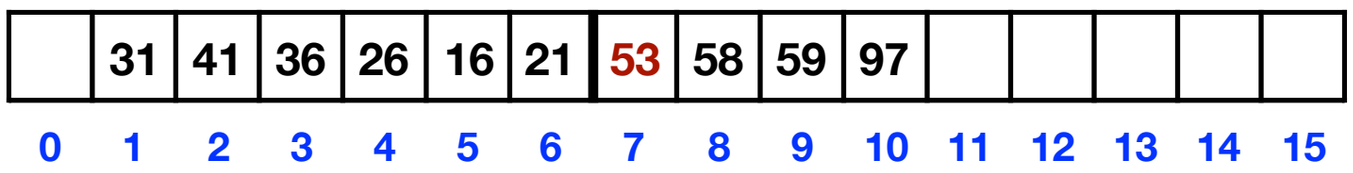
58 59 97

	53	41	36	26	16	21	31	58	59	97					
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Heapsort Example

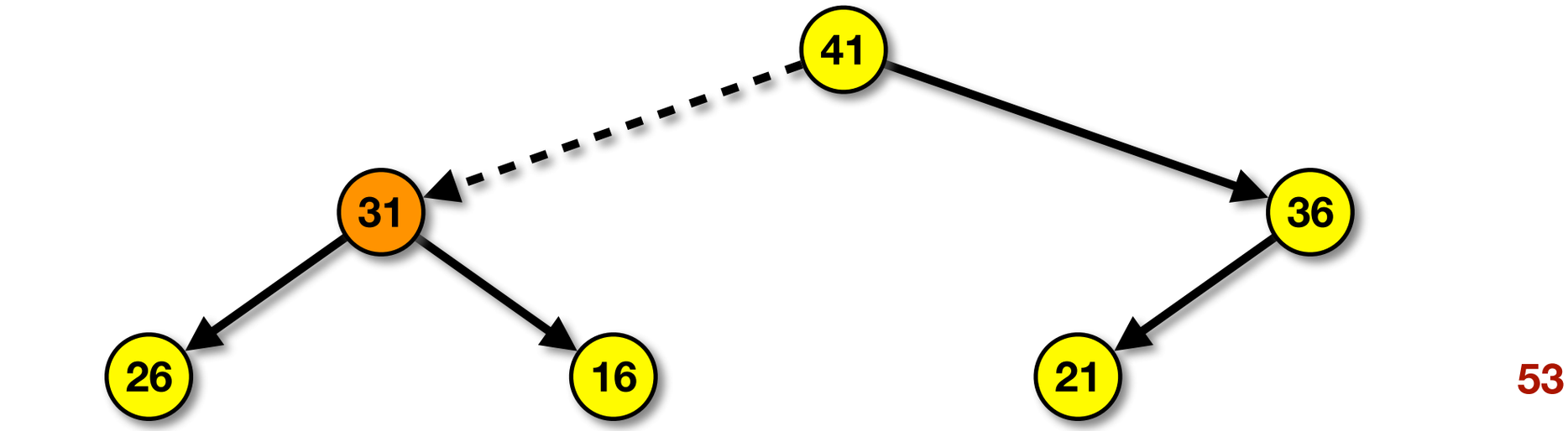
- **Call deleteMax method**
- **Swap node 31 with node 53 and shrink the size of the heap**





Heapsort Example

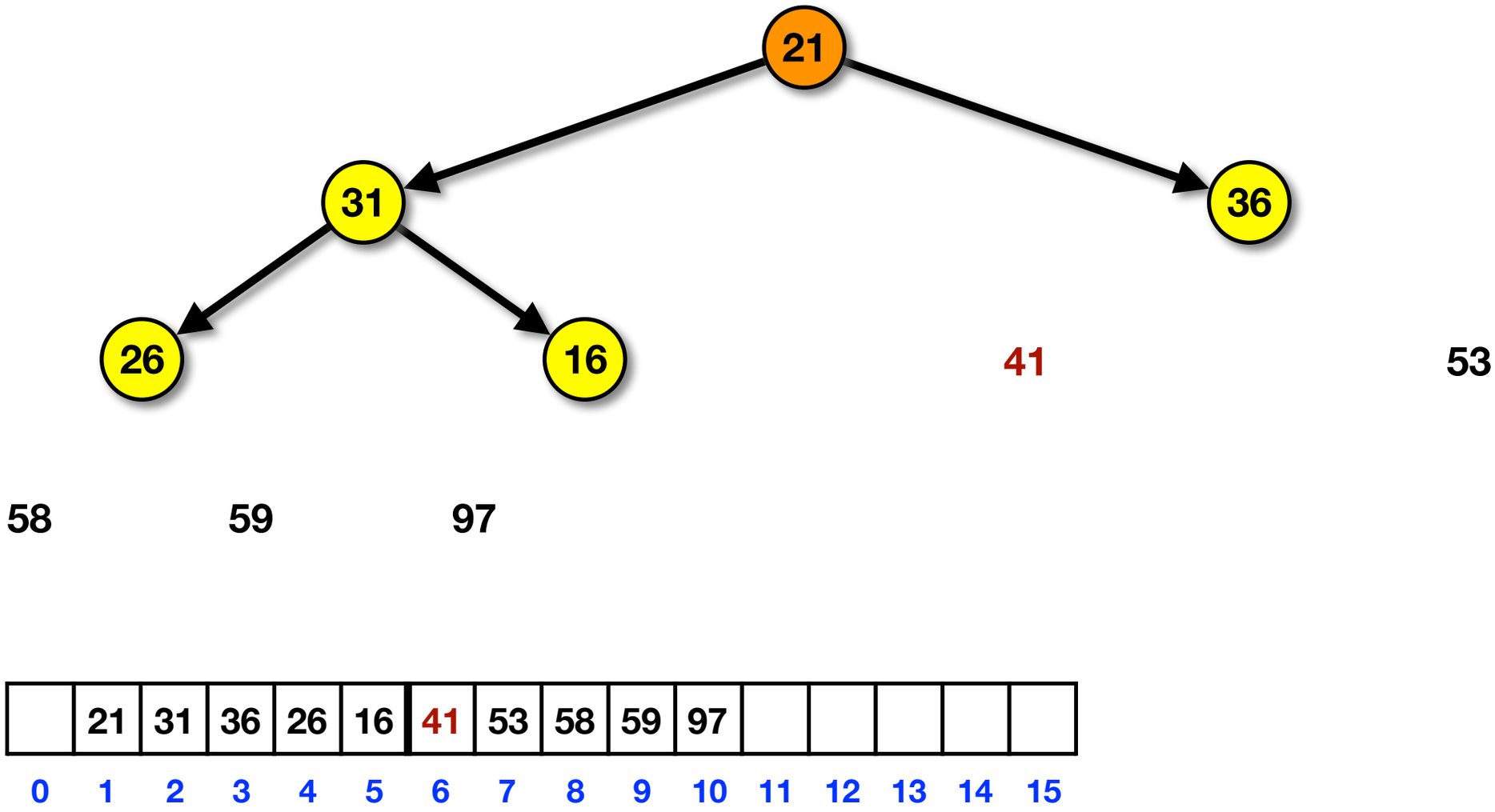
- **Call percolateDown on node 31**



	41	31	36	26	16	21	53	58	59	97					
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Heapsort Example

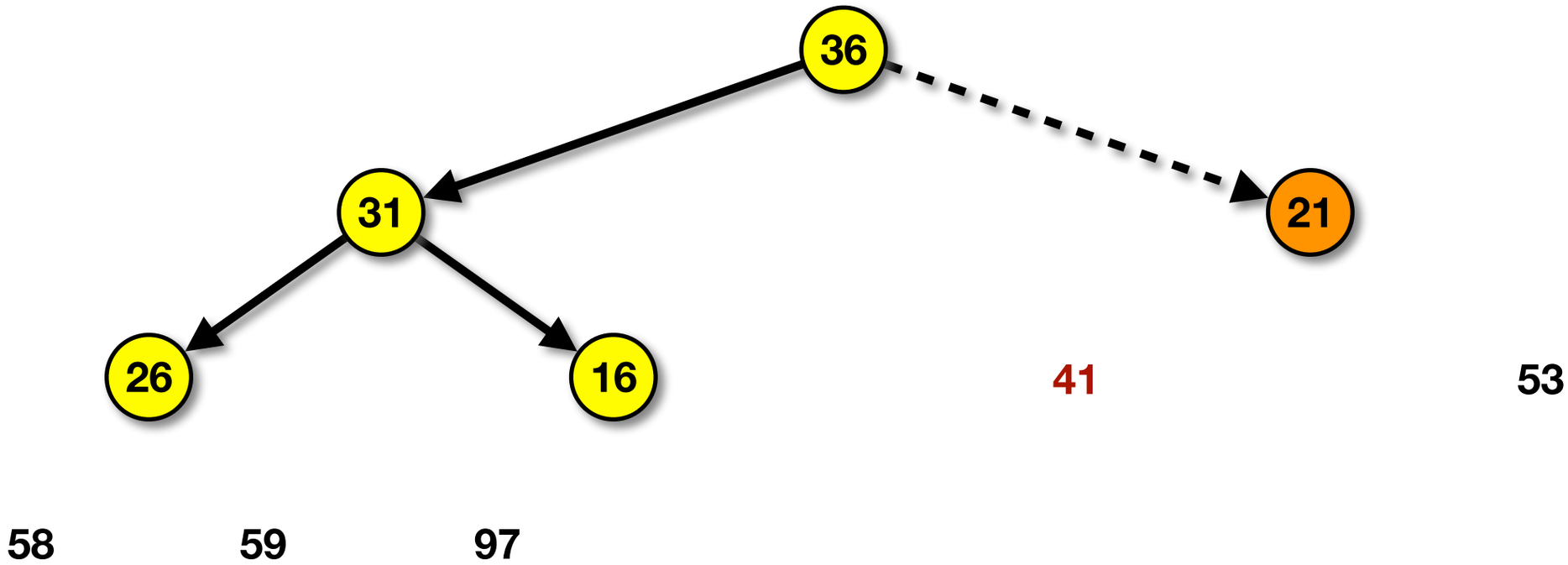
- **Call deleteMax method**
- **Swap node 21 with node 41 and shrink the size of the heap**



	21	31	36	26	16	41	53	58	59	97					
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Heapsort Example

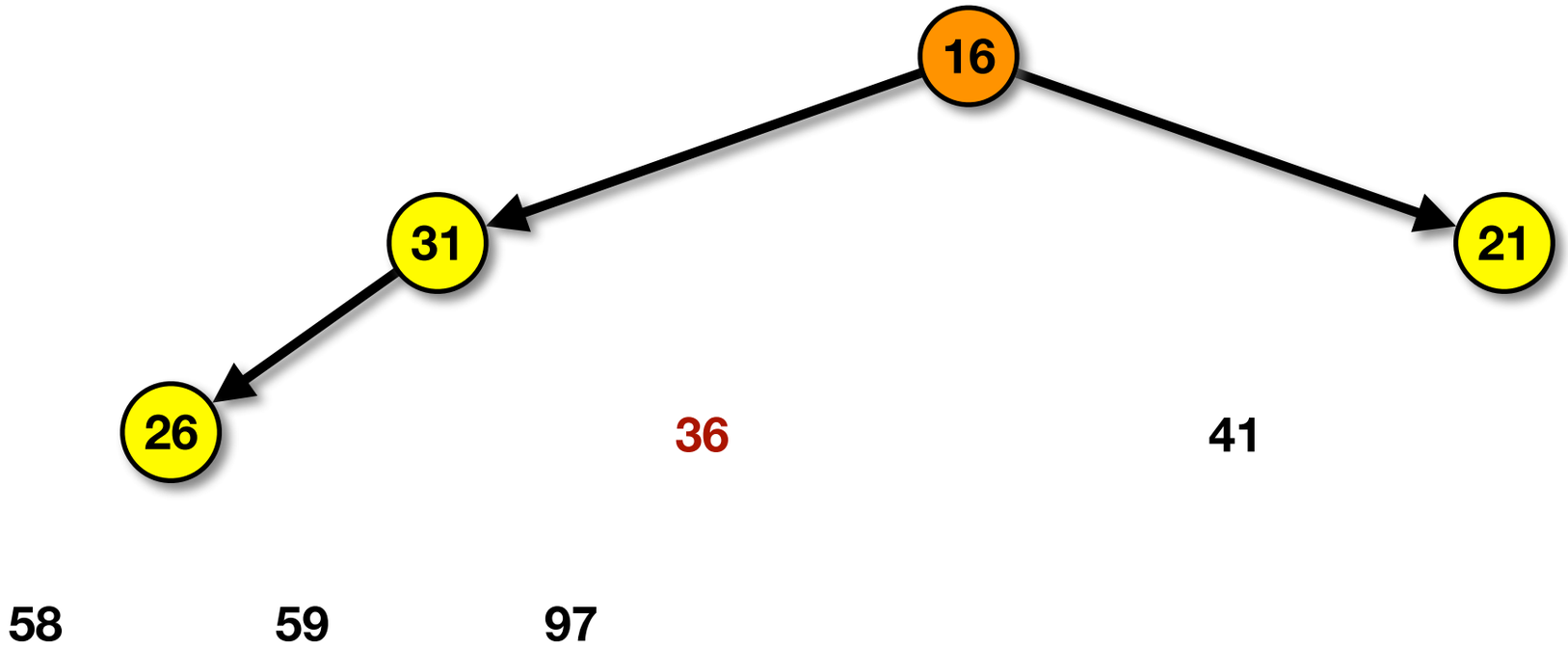
- **Call percolateDown on node 21**

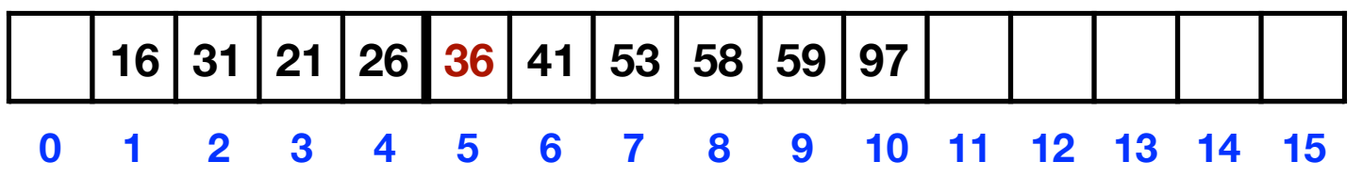


	36	31	21	26	16	41	53	58	59	97					
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Heapsort Example

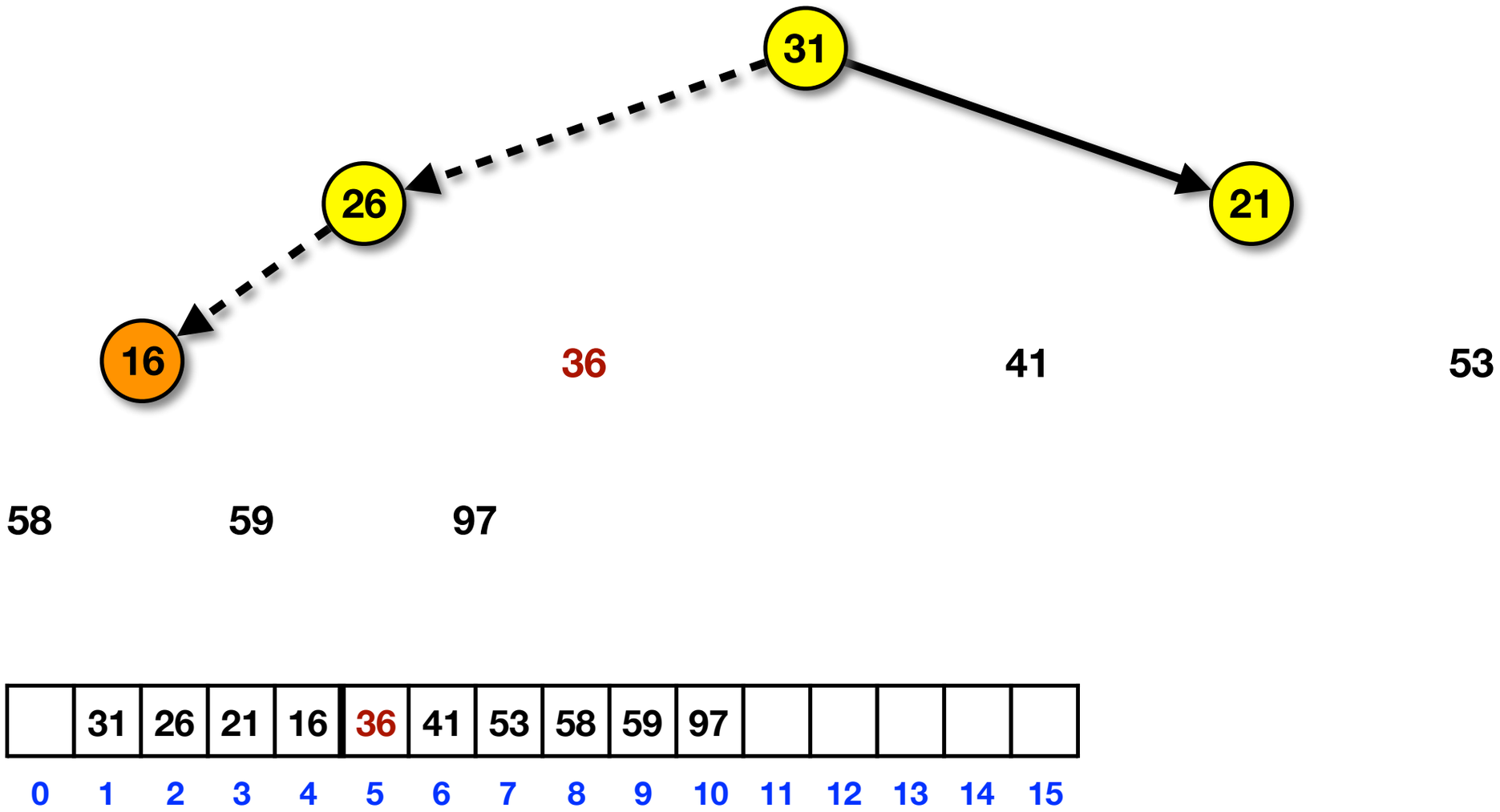
- **Call deleteMax method**
- **Swap node 16 with node 36 and shrink the size of the heap**





Heapsort Example

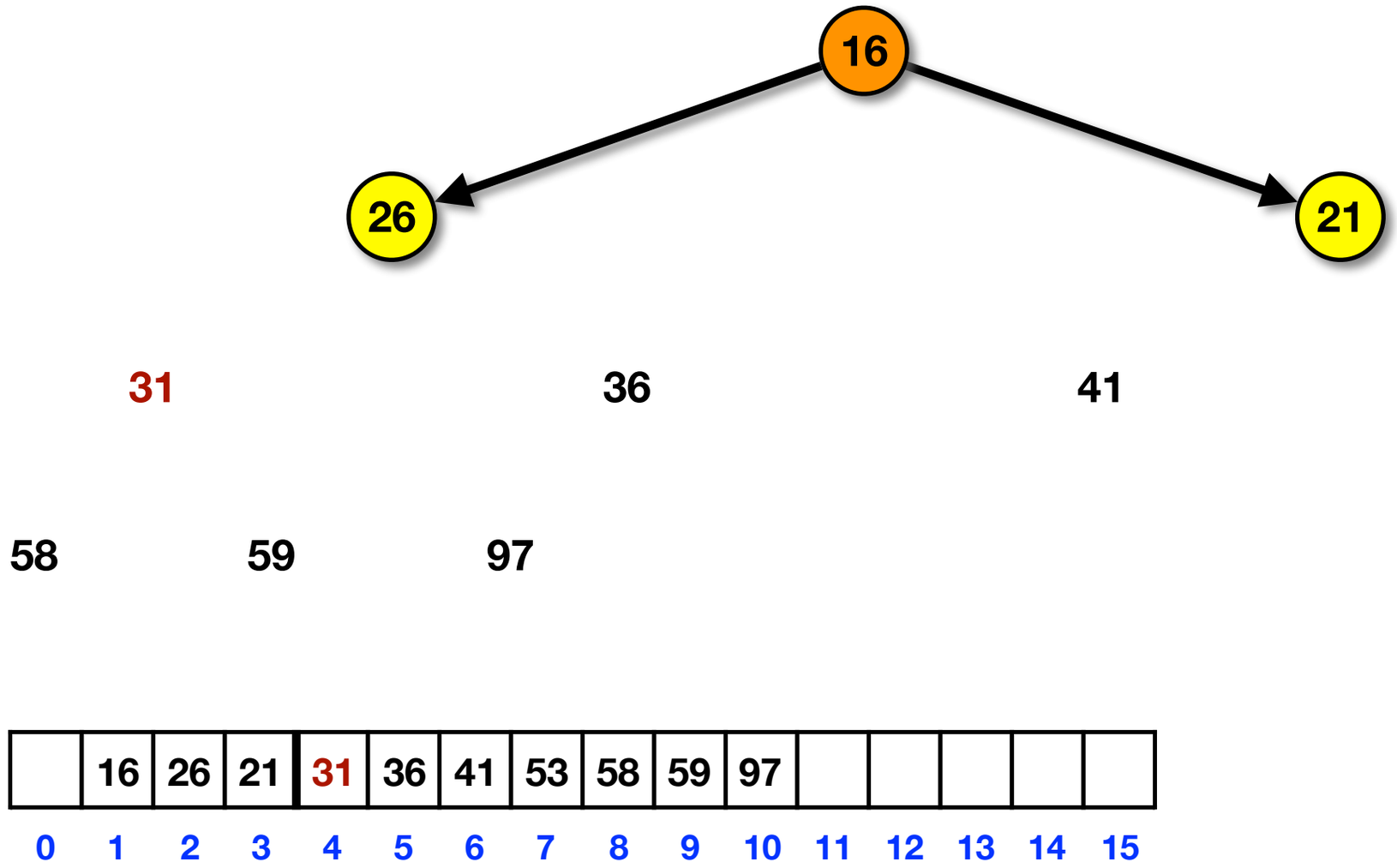
- **Call percolateDown on node 16**



	31	26	21	16	36	41	53	58	59	97					
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

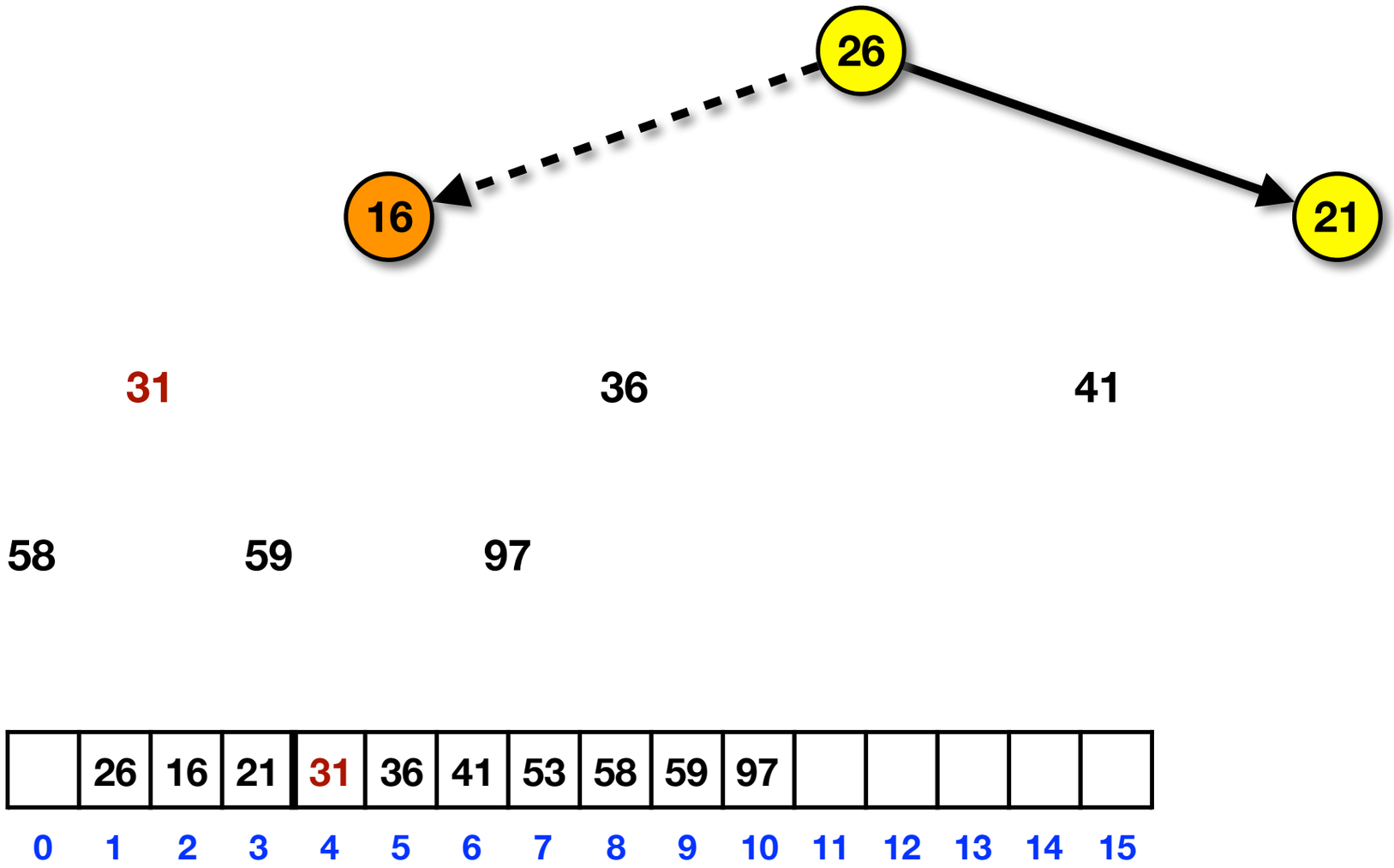
Heapsort Example

- **Call deleteMax method**
- **Swap node 16 with node 31 and shrink the size of the heap**



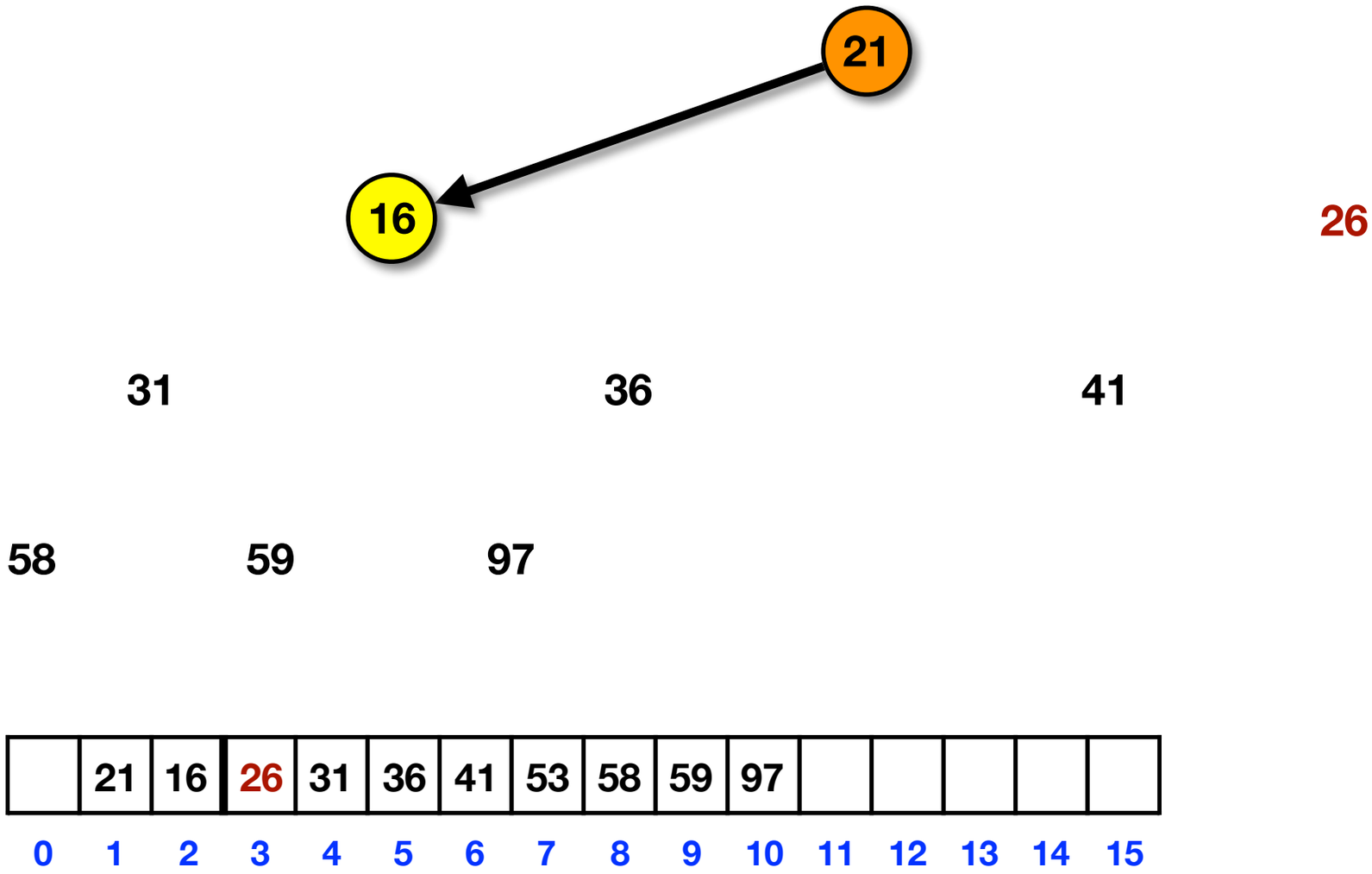
Heapsort Example

- **Call percolateDown on node 16**



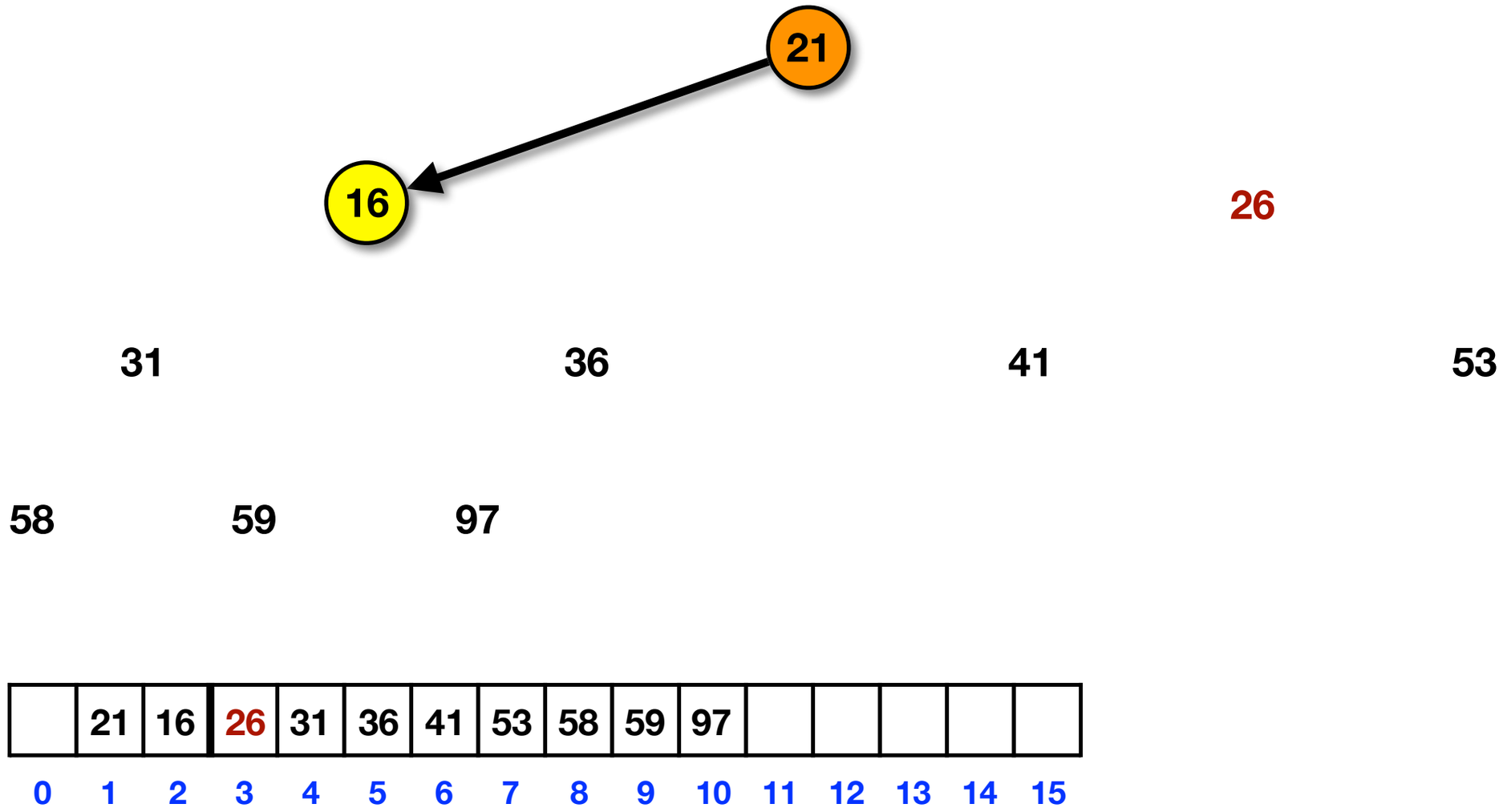
Heapsort Example

- **Call deleteMax method**
- **Swap node 21 with node 26 and shrink the size of the heap**



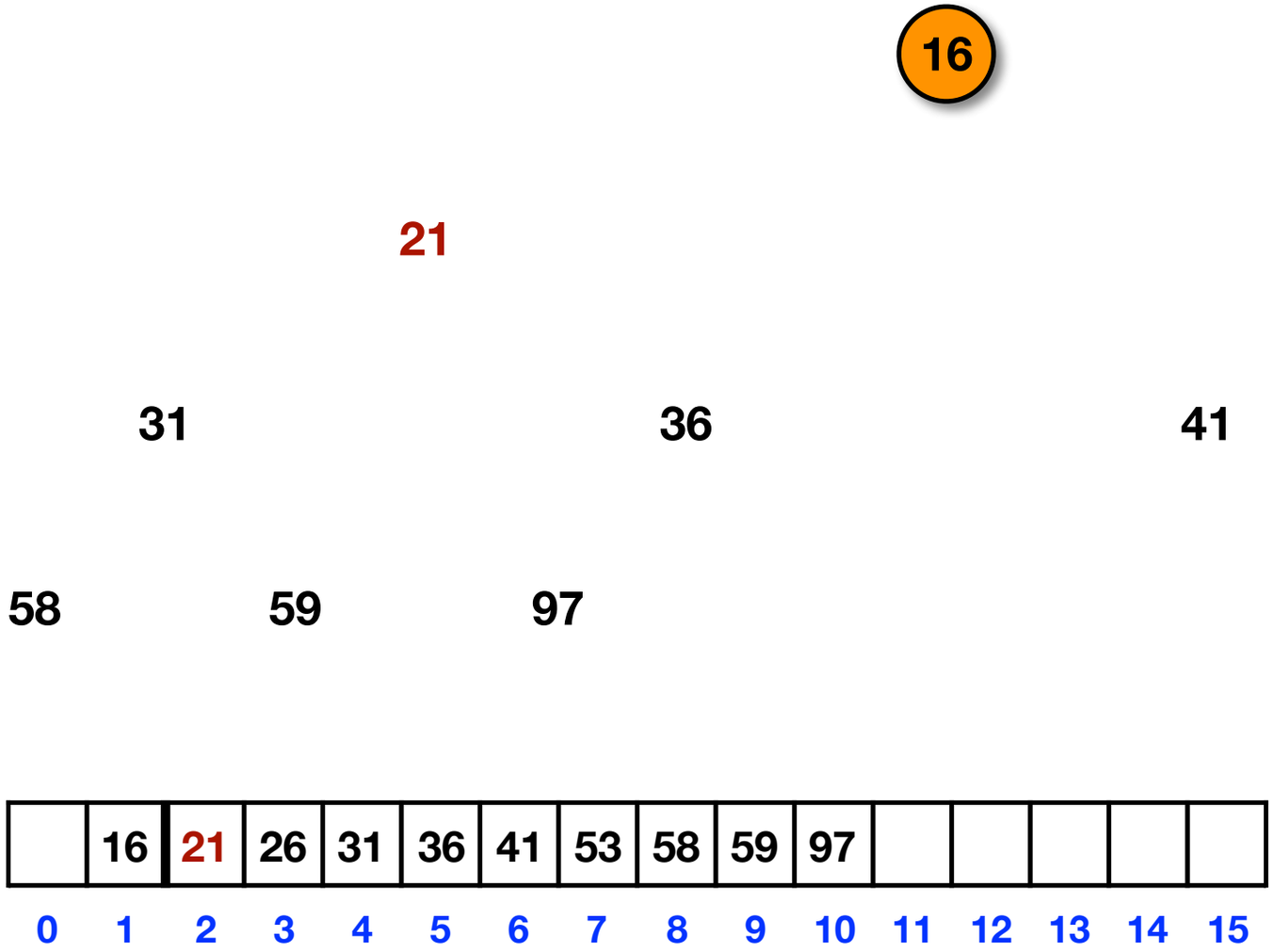
Heapsort Example

- **Call percolateDown on node 21**



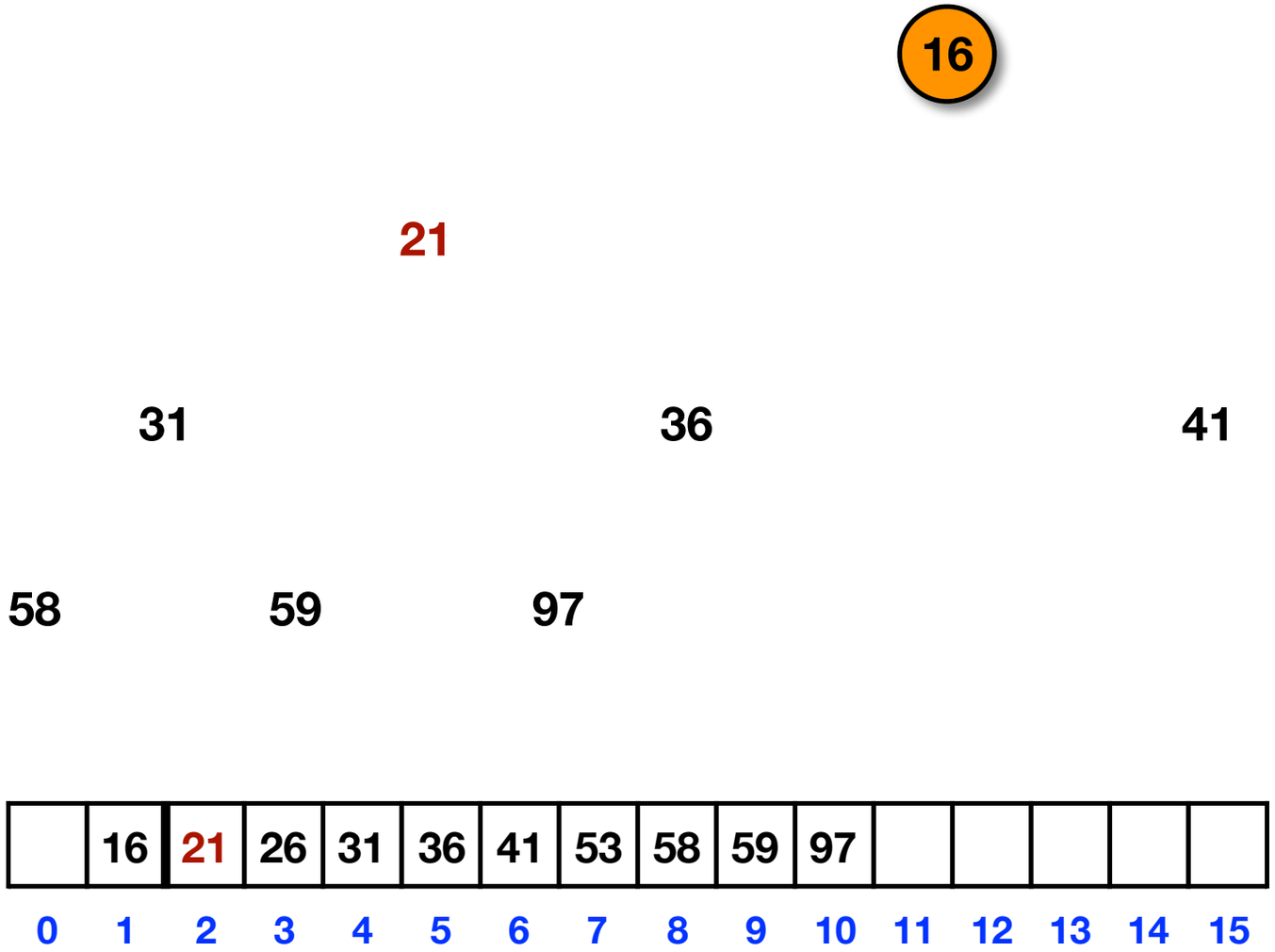
Heapsort Example

- **Call deleteMax method**
- **Swap node 16 with node 21 and shrink the size of the heap**



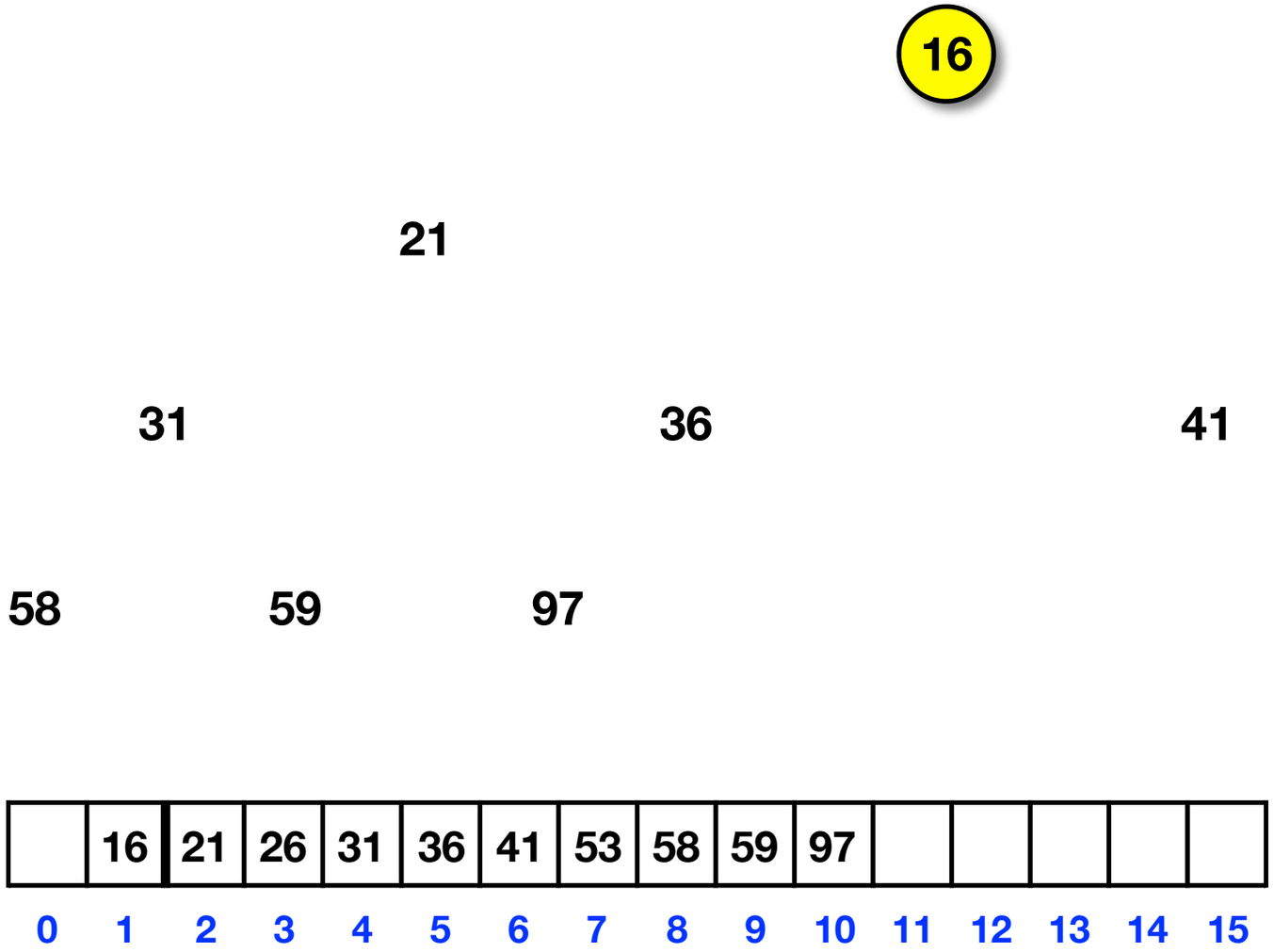
Heapsort Example

- **Call percolateDown on node 16**



Heapsort Example

- **Since node 16 is the last node, no need to call deleteMax again**
- **Heapsort is complete**



Heapsort Example