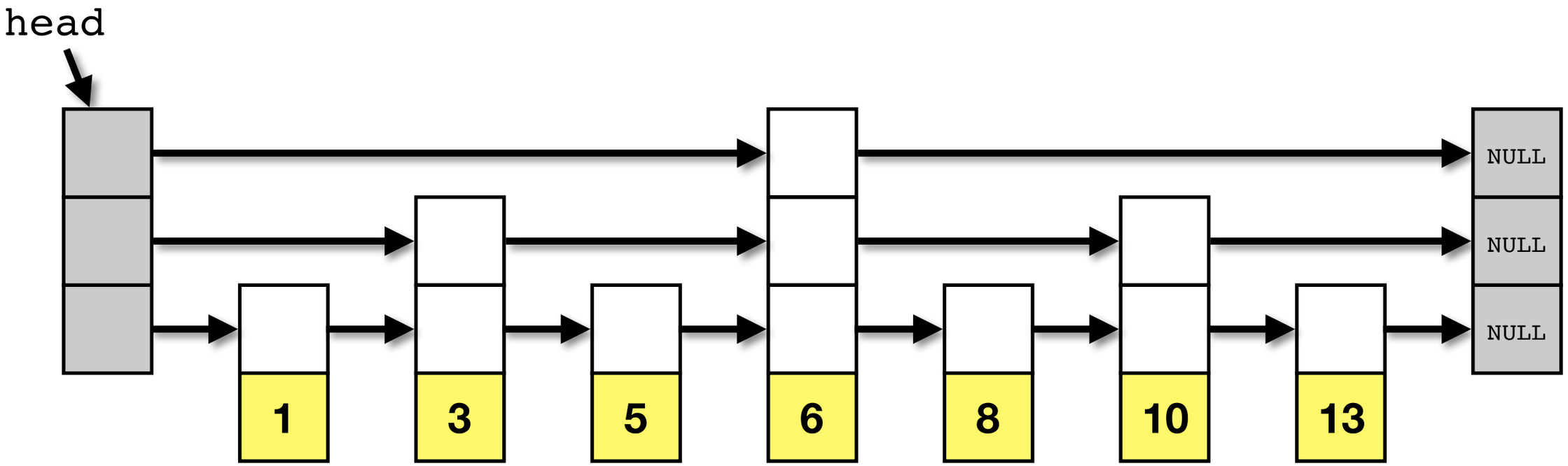
CS350: Data Structures

Skip Lists

James Moscola
Department of Engineering & Computer Science
York College of Pennsylvania

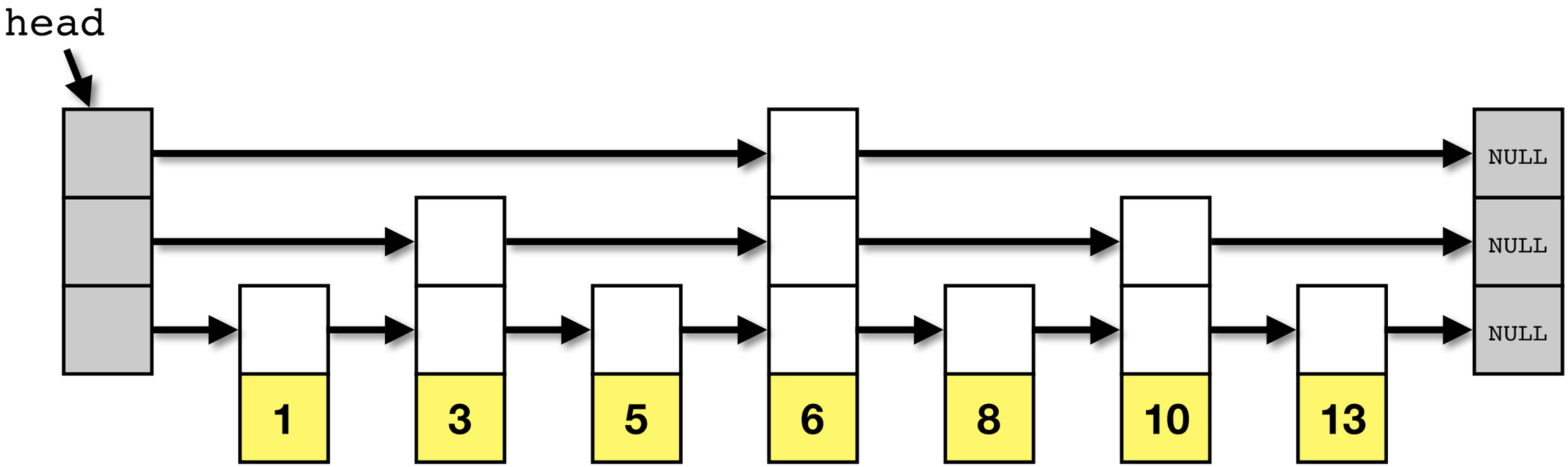
Skip List Introduction

- **A data structure used for storing a sorted list of elements using layers of linked lists**
  - Bottom layer is a standard, ordered linked list
  - Upper layer linked lists create ‘shortcuts’ or ‘fast lanes’ from one location of the list to another
  - Higher levels traverse greater portions of the list than lower levels



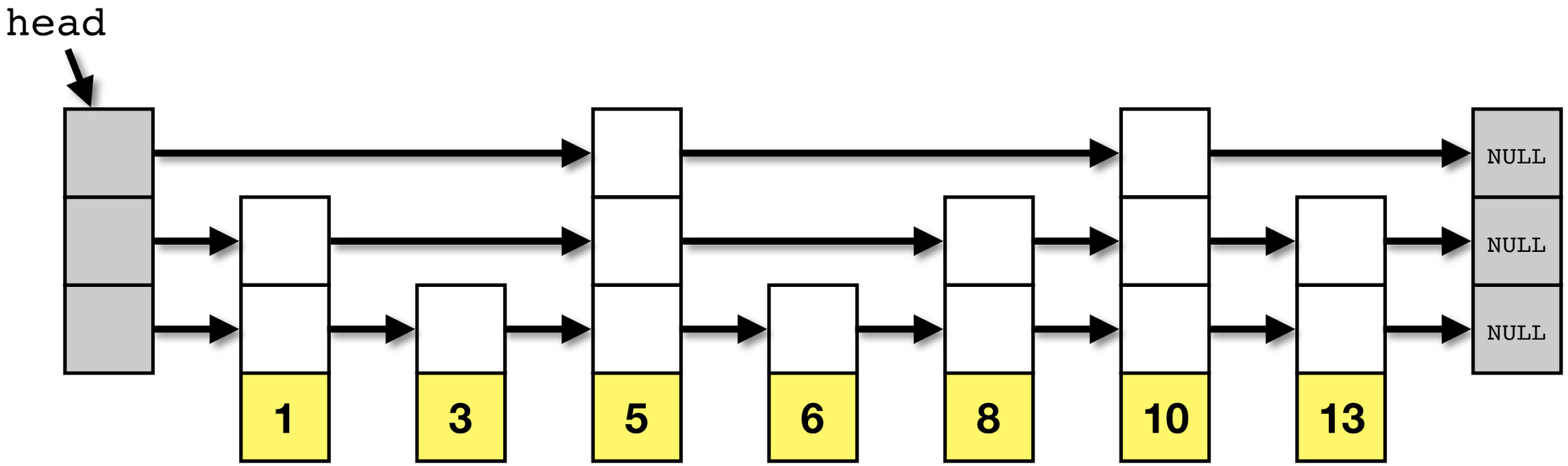
Skip List Introduction

- **The example skip list below shows the skip list divided perfectly in halves, quarters, etc.**



Skip List Introduction

- **In practice, skip list node heights are distributed randomly throughout the skip list**
  - Helps us avoid the need to ‘rebalance’ the list when insertions or deletions occur



Skip Lists

- **Operations on skip lists are comparable in efficiency to balanced binary search trees (e.g. AVL trees, Red-Black trees)**
  - Insertion, Deletion, and Search operations all run in O(log N) time

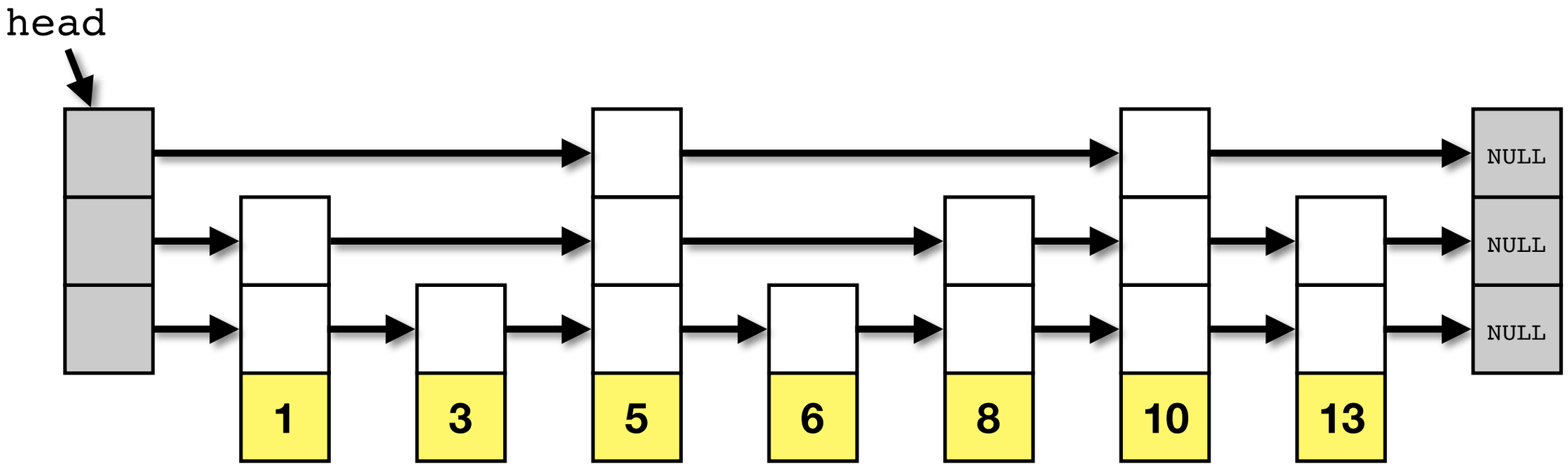
Skip List Search

Skip List Searching

- **To search for an element in a skip list:**
  - Start at the highest level at the head of the skip list and begin moving horizontally through the list
    - If the next node in the list has the desired data then done
    - If the data in the next node is less than the desired value, move to the next node and continue the search
    - If the data in the next node is greater than the desired value, drop down one level and continue the search

Skip List Searching

- **Example -- `find` the node with a key value 6**

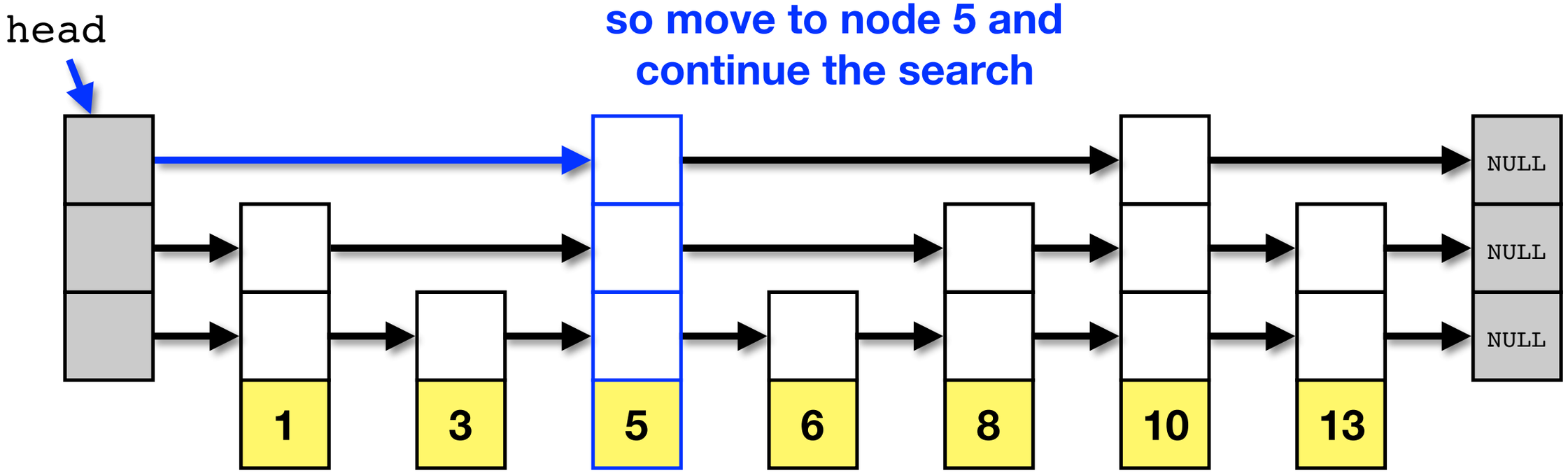


Skip List Searching

- **Example -- `find` the node with a key value 6**

(1) Start at highest level of the head node and check data value at node.next

(2) Compare node.next.data to the desired value, 5 < 6 so move to node 5 and continue the search

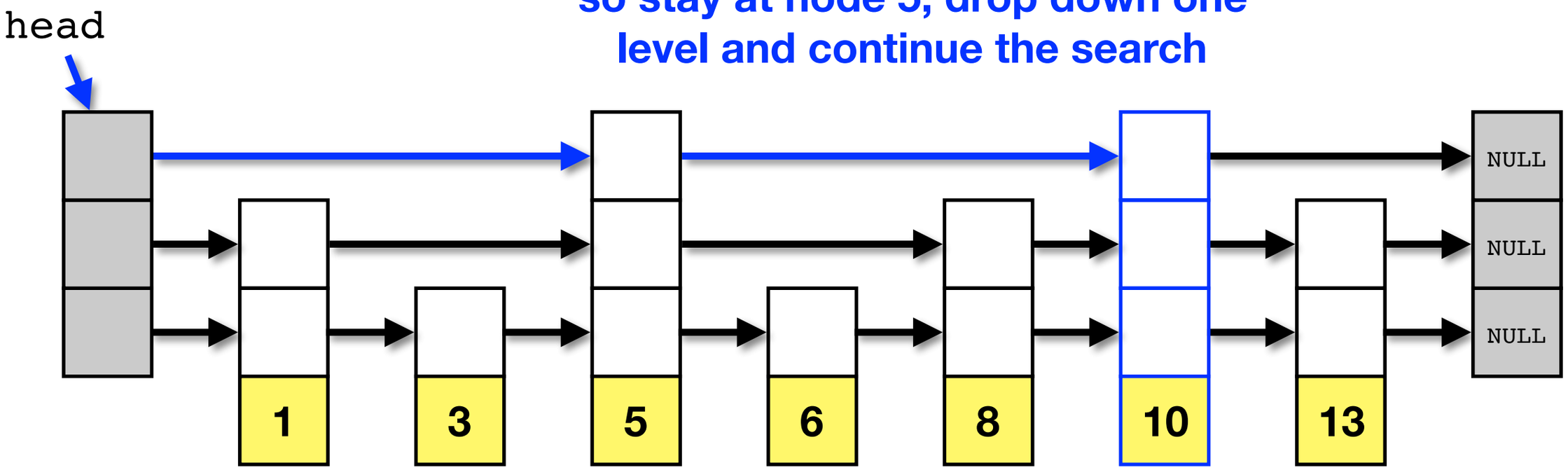


Skip List Searching

- **Example -- `find` the node with a key value 6**

(3) Compare node.next.data to the desired value, 10 > 6

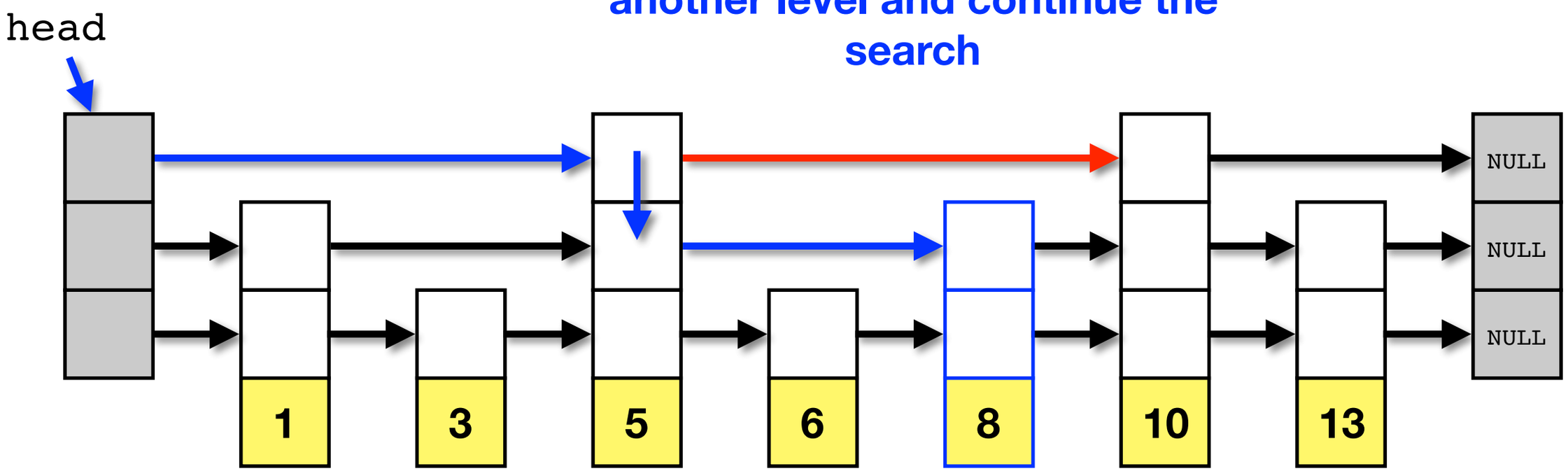
so stay at node 5, drop down one level and continue the search



Skip List Searching

- **Example -- `find` the node with a key value 6**

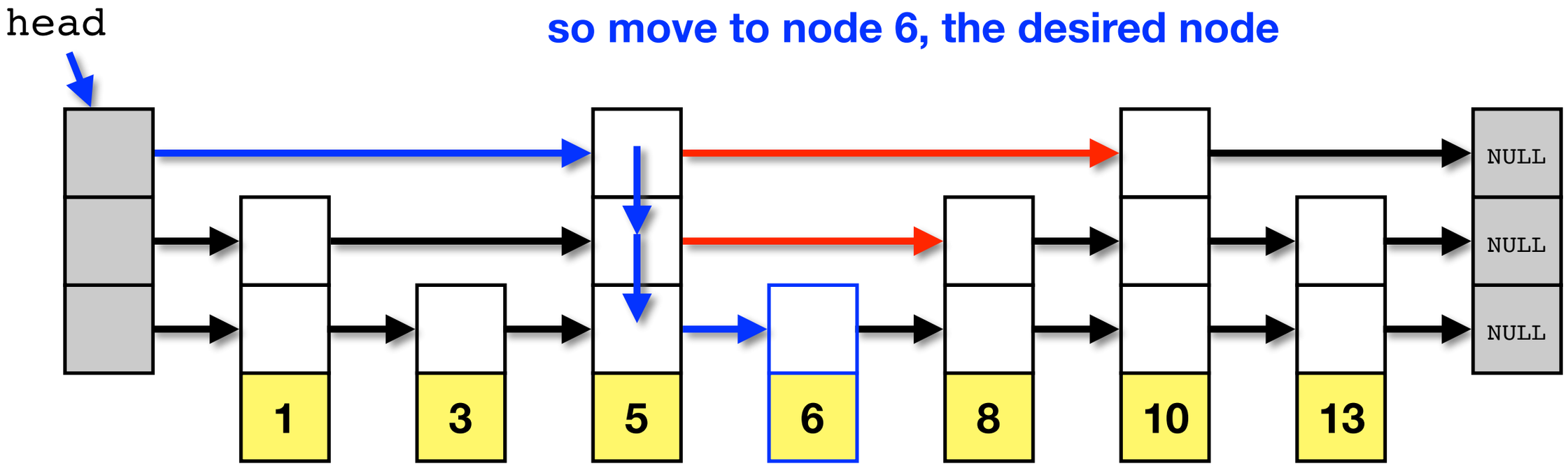
(4) Compare node.next.data to the desired value,
8 > 6
so stay at node 5, drop down another level and continue the search



Skip List Searching

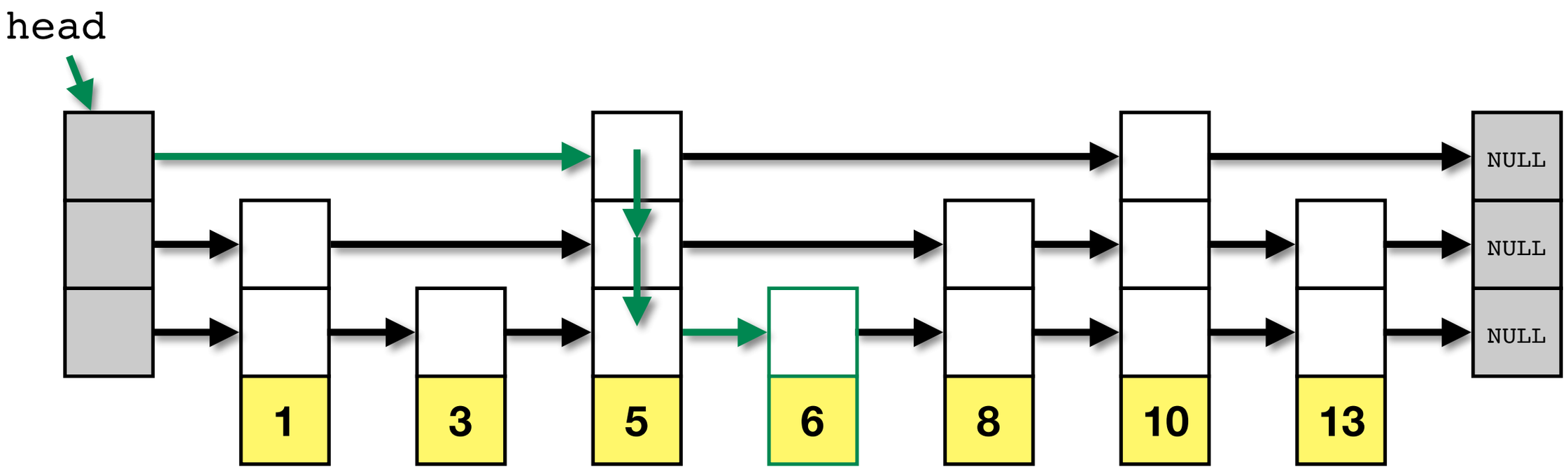
- **Example -- `find` the node with a key value 6**

(5) Compare node.next.data to the desired value,
6 == 6
so move to node 6, the desired node



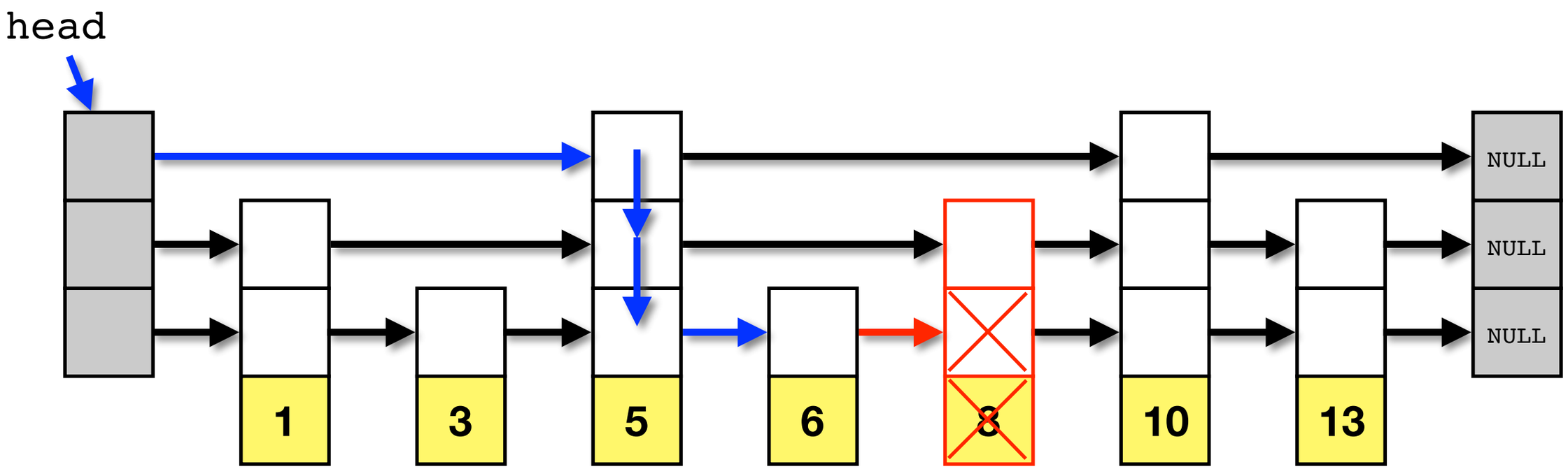
Skip List Searching

- **Example -- `find` the node with a key value 6**



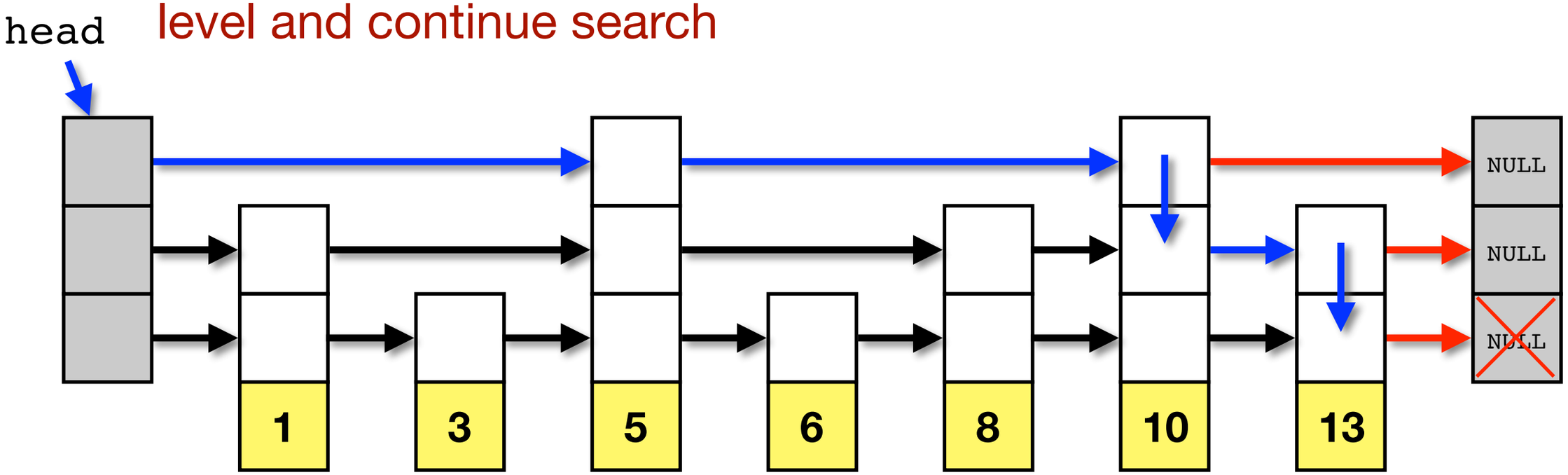
Skip List Searching

- **Example -- `find` the node with a key value 7**
  - In this example, the desired element does not exist in the list
  - Search the list just as before, but when a value is found that exceed that desired value, AND there are no more levels to drop, then the desired element doesn't exist



Skip List Searching

- **Example -- `find` the node with a key value 15**
  - Another example where the desired element does not exist in the list
  - Start with a normal search, if search reaches bottom-most level and next node is null, then element does not exist in list
    - If not at bottom-most level and node.next == NULL, then drop a level and continue search



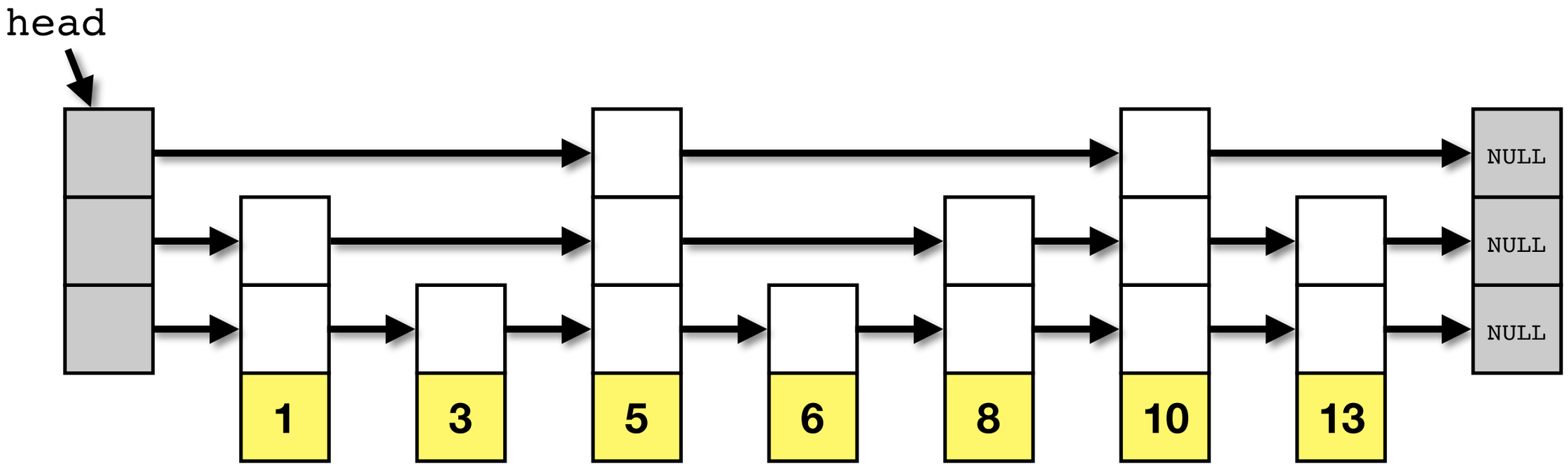
Skip List Insertion

Skip List Insertion

- **When inserting a node into a skip list, first perform a search to determine where the new node must be inserted**
- **The height of a newly inserted node is chosen at random**
  - Flip a coin, if heads, then add an additional level to the newly inserted node, if tails then don't add anymore levels
    - Probability of 1/2 that newly inserted node will only be at the bottom-most level
    - Probability of 1/2 that newly inserted node will have one additional level
    - Probability of 1/4 that newly inserted node will have two additional levels
    - Probability of 1/8 that newly inserted node will have three additional levels
    - etc.
- **During the insert procedure, maintain an array that remembers the rightmost node that is to the left of the current location for each level**

Skip List Insertion

- **Example -- `insert` the node with a key value 7**
- **Start with a search to determine where the node should be inserted**



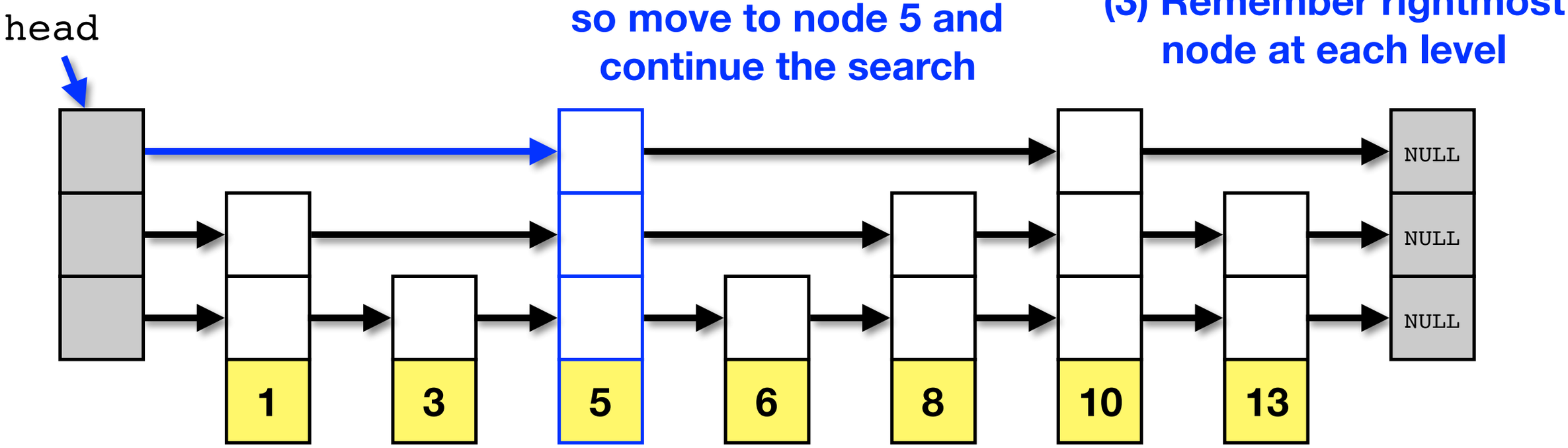
Skip List Insertion

- **Example -- `insert` the node with a key value 7**

(1) Start at highest level of the head node and check data value at node.next

(2) Compare node.next.data to the new value, 5 < 7 so move to node 5 and continue the search

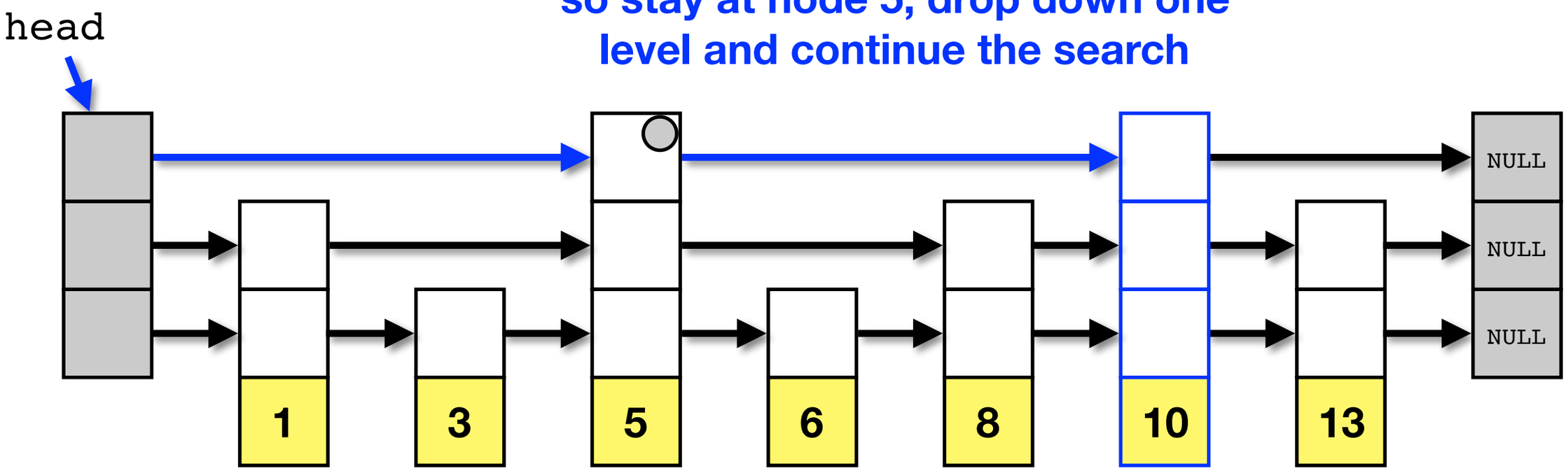
(3) Remember rightmost node at each level



Skip List Insertion

- **Example -- `insert` the node with a key value 7**

(4) Compare node.next.data to the new value,
10 > 7
so stay at node 5, drop down one level and continue the search



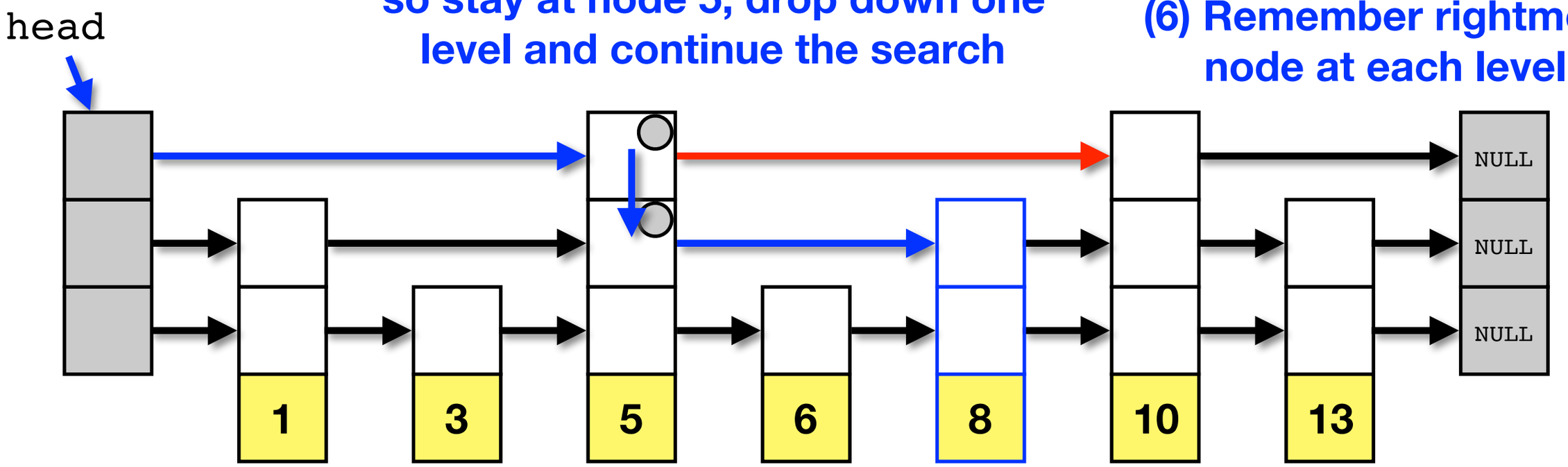
Skip List Insertion

- **Example -- `insert` the node with a key value 7**

(5) Compare node.next.data to the new value, 8 > 7

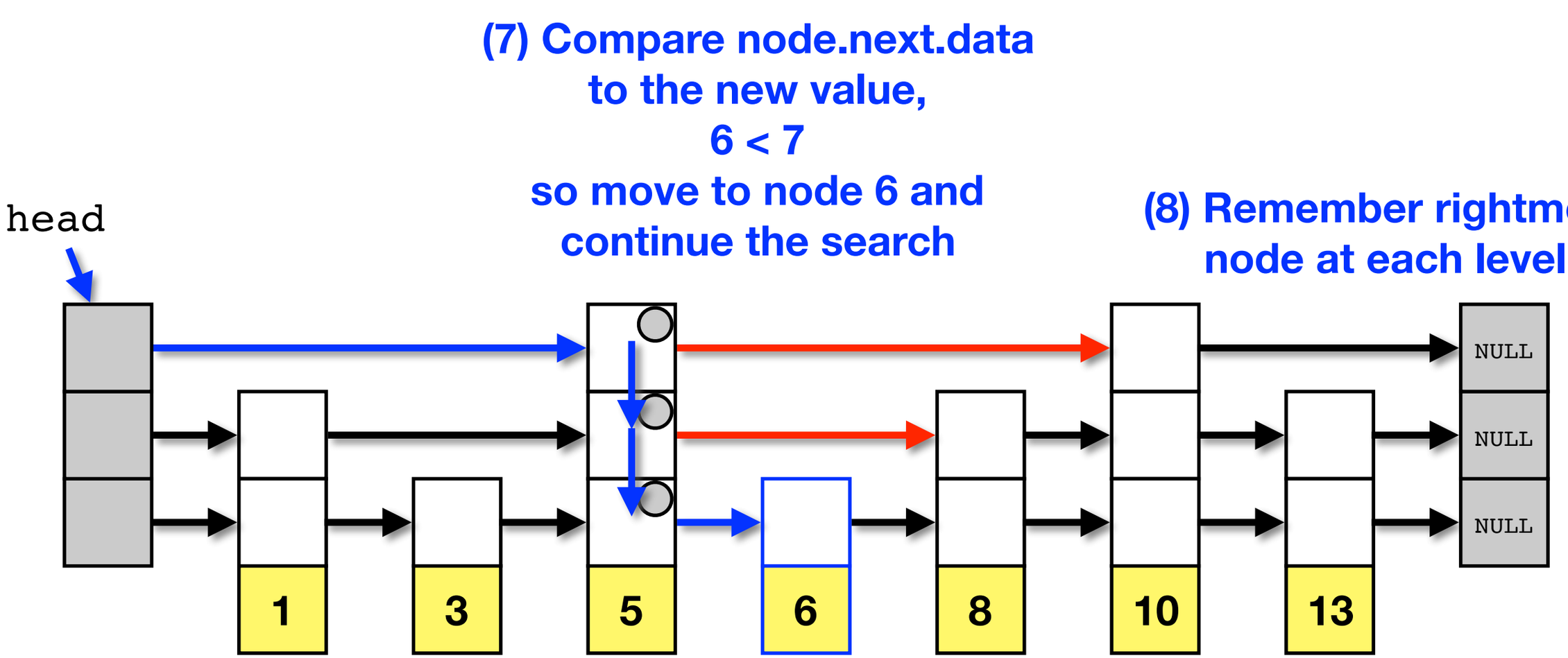
so stay at node 5, drop down one level and continue the search

(6) Remember rightmost node at each level



Skip List Insertion

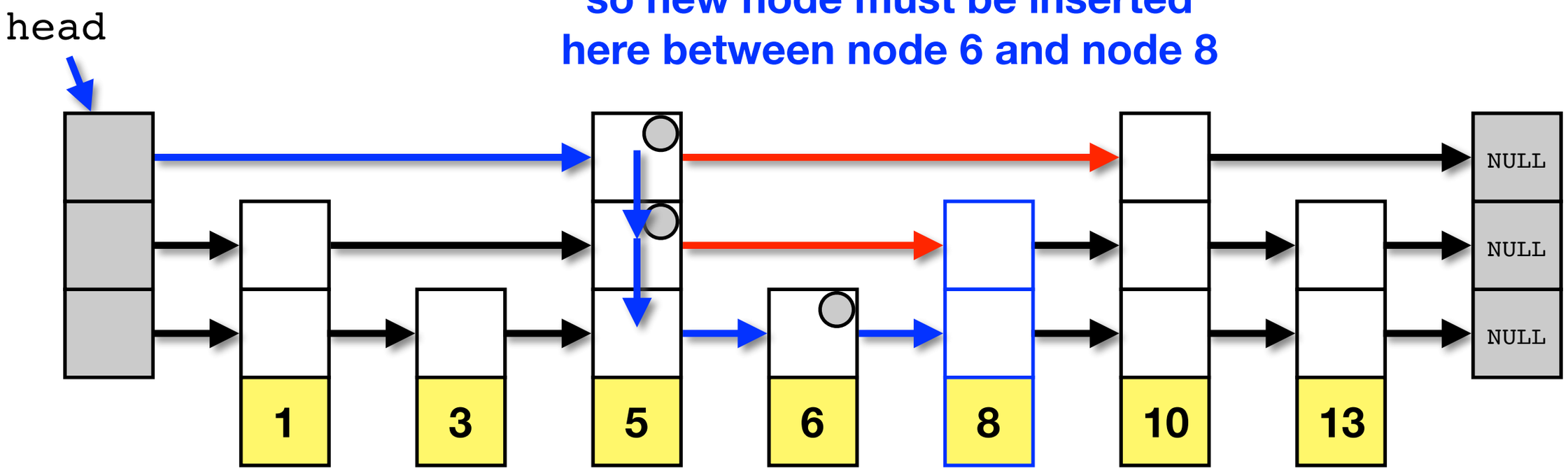
- **Example -- `insert` the node with a key value 7**



Skip List Insertion

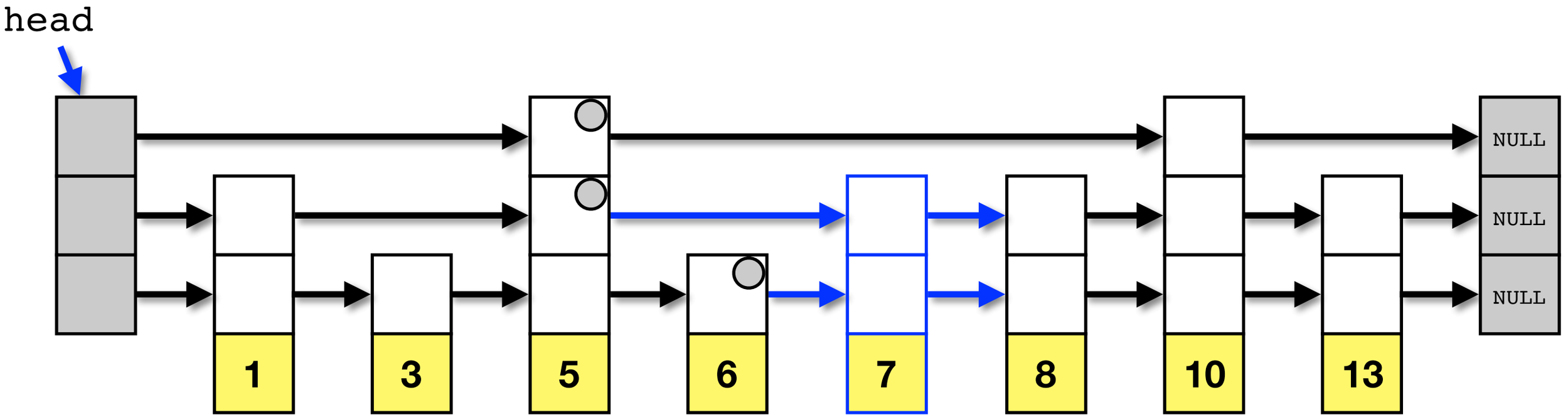
- **Example -- `insert` the node with a key value 7**

(9) Compare node.next.data to the new value,
8 > 7
so new node must be inserted here between node 6 and node 8



Skip List Insertion

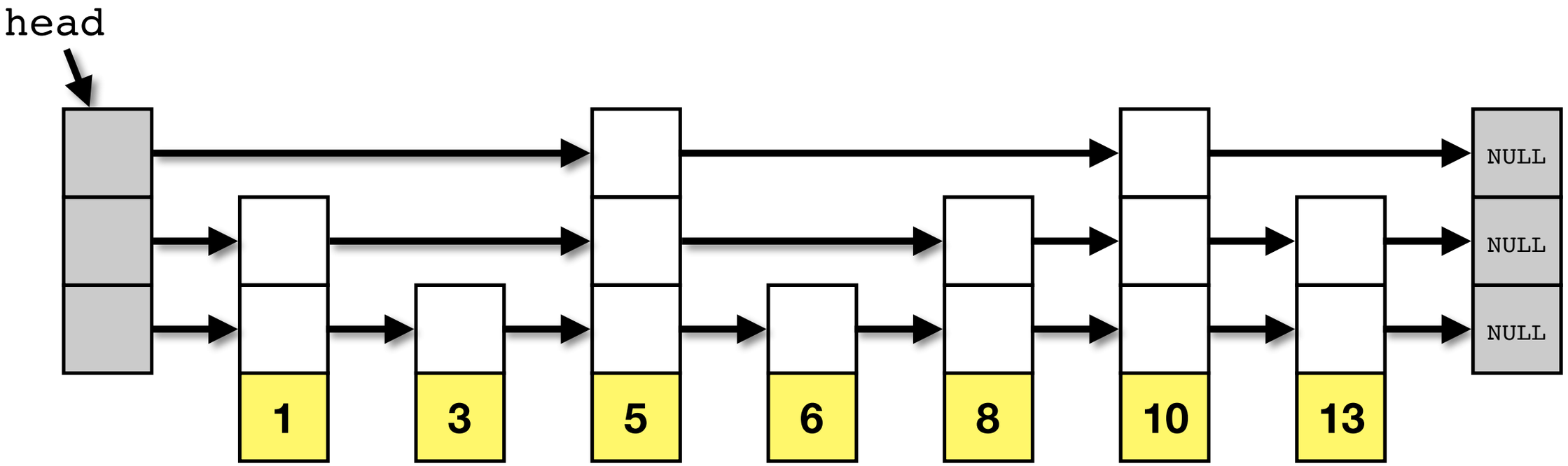
- **Example -- `insert` the node with a key value 7**
  - To **`insert`** node 7:
    - Randomly pick height for node 7
    - Markers were left behind to remember the rightmost node at each level to the left of the insertion location ... use them to update next pointers



Skip List Deletion

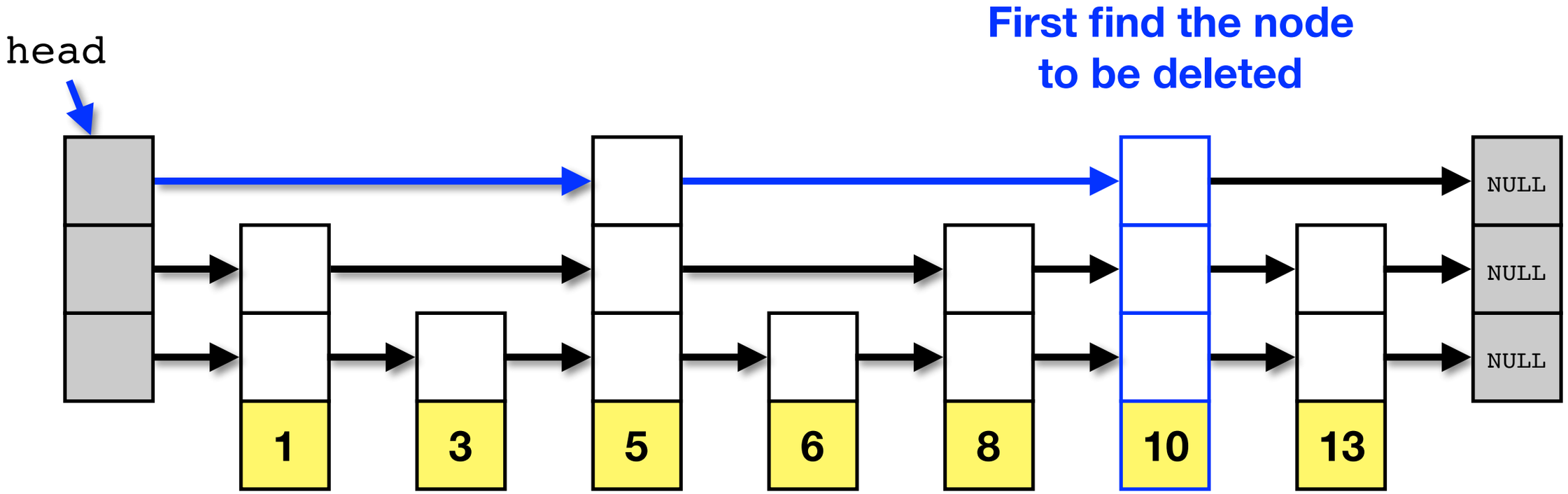
Skip List Deletion

- **For each level of which a skip list node is a part, splice the node out in the same fashion as a standard linked list**
- **Example -- `delete` the node with a key value 10**



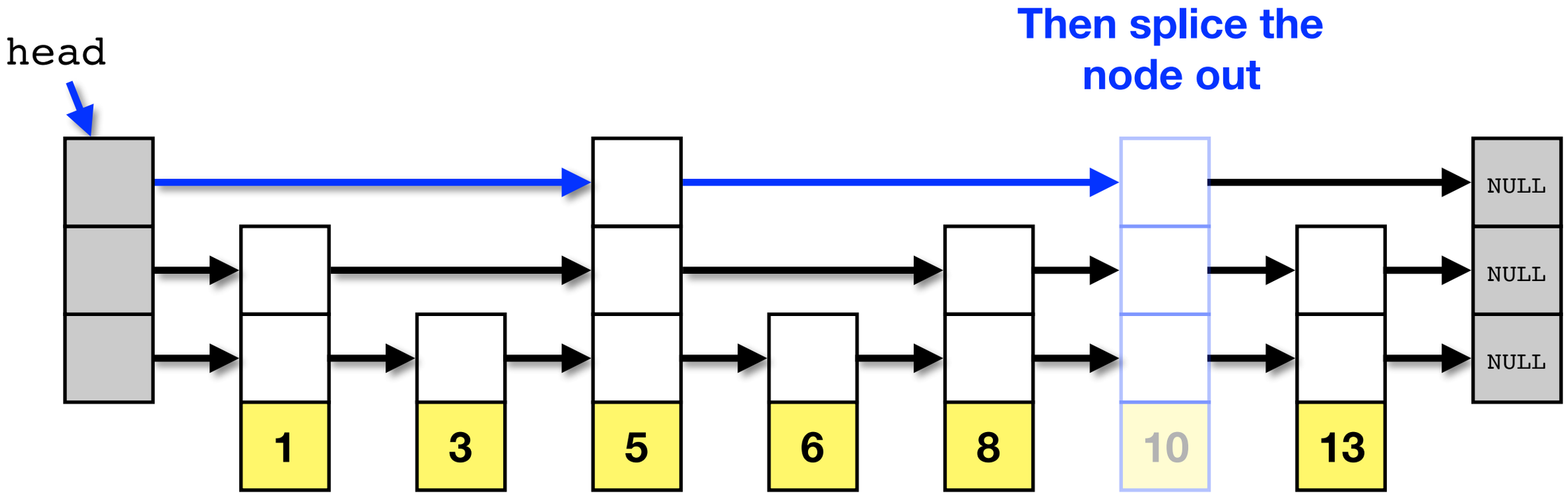
Skip List Deletion

- **For each level of which a skip list node is a part, splice the node out in the same fashion as a standard linked list**
- **Example -- `delete` the node with a key value 10**



Skip List Deletion

- **For each level of which a skip list node is a part, splice the node out in the same fashion as a standard linked list**
- **Example -- `delete` the node with a key value 10**



Skip List Deletion

- **For each level of which a skip list node is a part, splice the node out in the same fashion as a standard linked list**
- **Example -- `delete` the node with a key value 10**