CS350: Data Structures

Splay Trees

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Splay Trees

- **Another variation of the binary tree**
- **Does NOT guarantee O(log N) worst case operations**
- **Splay tree may exhibit O(N) worst case for any single operation**
  - Time for operations is amortized
  - On average, splay tree operations will exhibit O(log N) worst case time
- **Does not require any accounting information (i.e. color, level, height, etc.)**
- **Key Feature: Frequently accessed keys are moved up so that they are near the root**
  - 90/10 rule - 90% of data access in data structure are to only 10% of the data

`splay` Operation

- **Operation used to rearrange tree to make a specified node the root node of the tree**
  - Has a nice side-effect that any imbalances in the tree are corrected
- **Splay operation is called whenever a node is accessed**
  - After a node is inserted, a `splay` operation is called on that node
    - A node inserted into the tree will always end up as the root when the insertion completes
  - After searching for a node, a `splay` operation is called on it
    - Subsequent searches for that node should execute more quickly
  - To `delete` a node, first `splay` it to the top of tree, then `delete` it

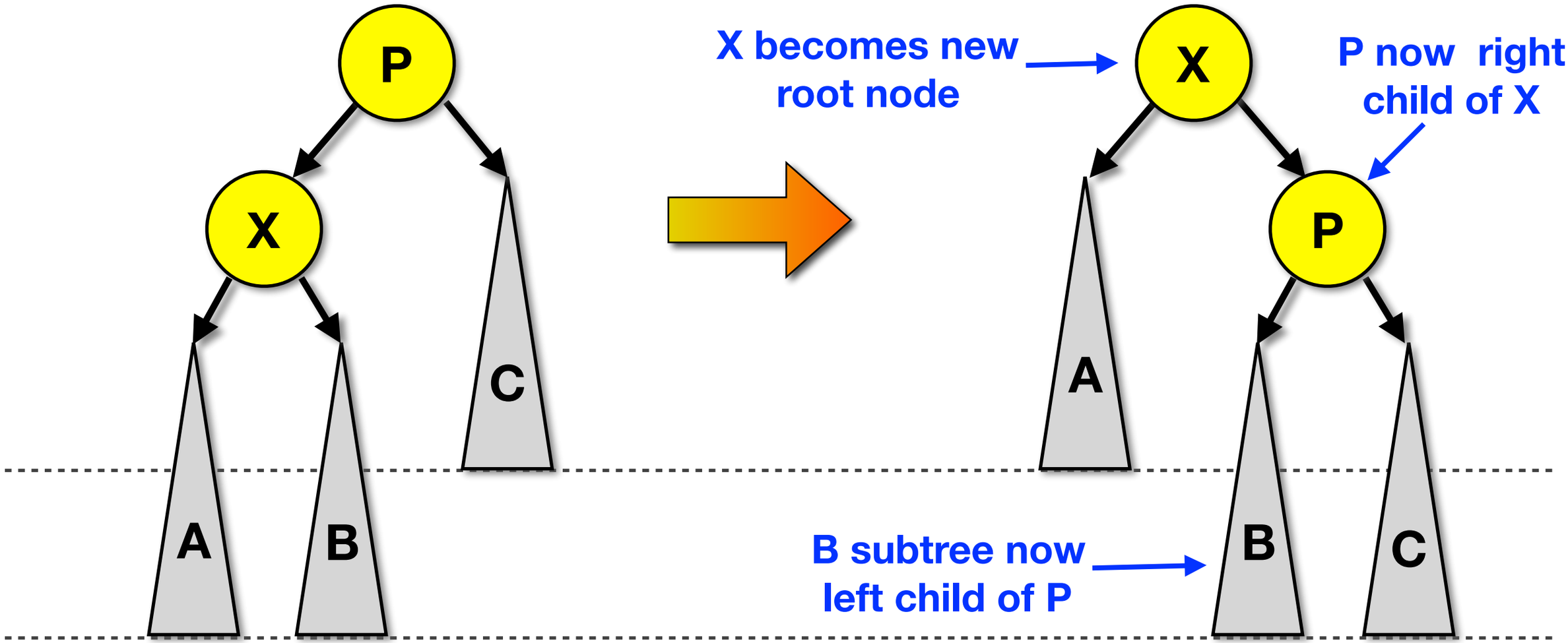
`splay` Operation

- **When `splay` is called on a node, continue to `splay` that node until it becomes the root of the entire tree**

- **There are three cases to consider when splaying a node**
  - Case #1 - a zig - a node X is the left child of a node P
  - Case #2 - a zig-zag - a node X is the inside grandchild of a node G
  - Case #3 - a zig-zig - a node X is the outside grandchild of a node G

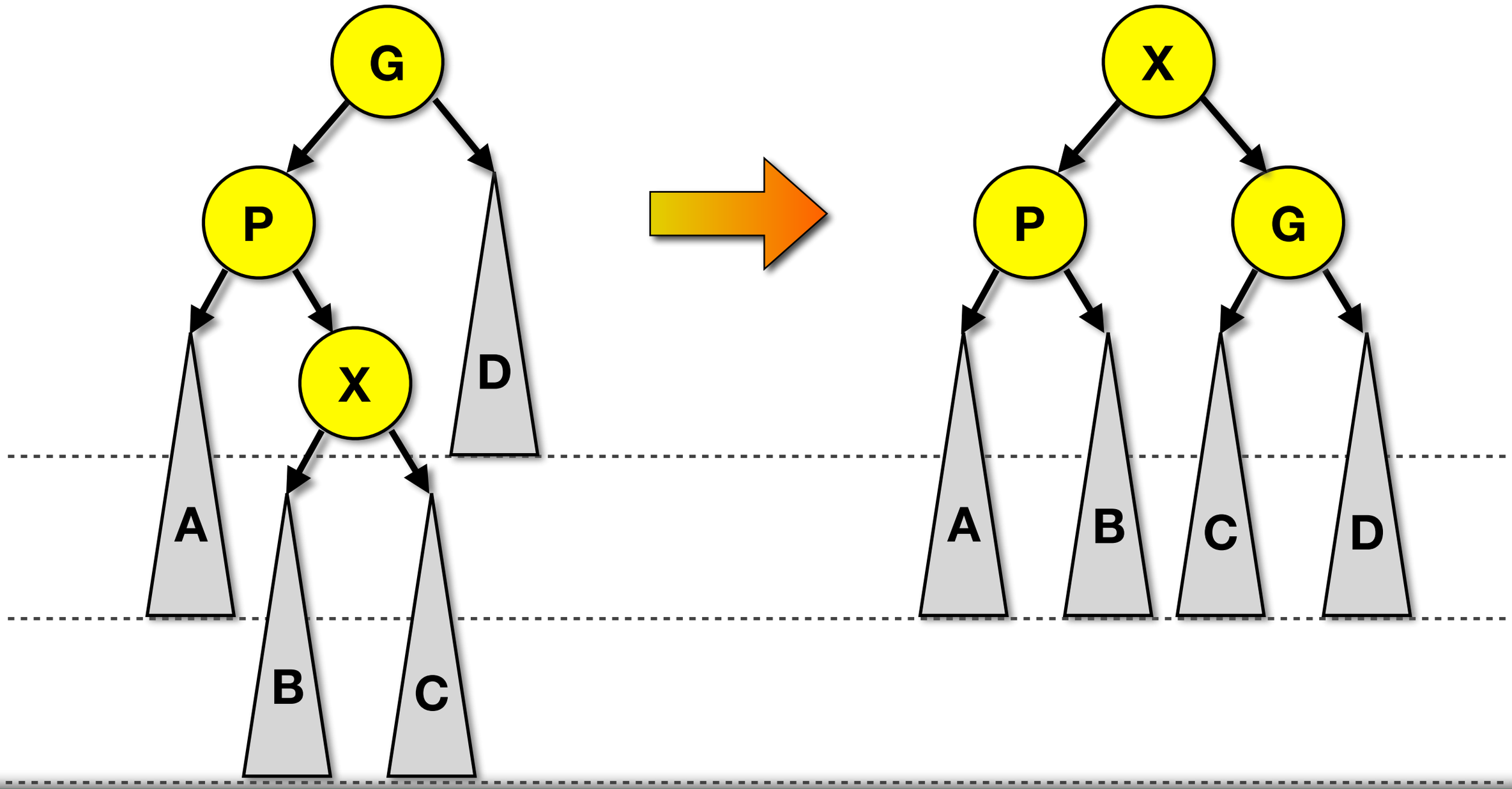
`splay` Operation -- **Case #1 (a Zig)**

Splaying a node X whose parent P is the root of the subtree.
Same as a single rotation of the node X with the root P.



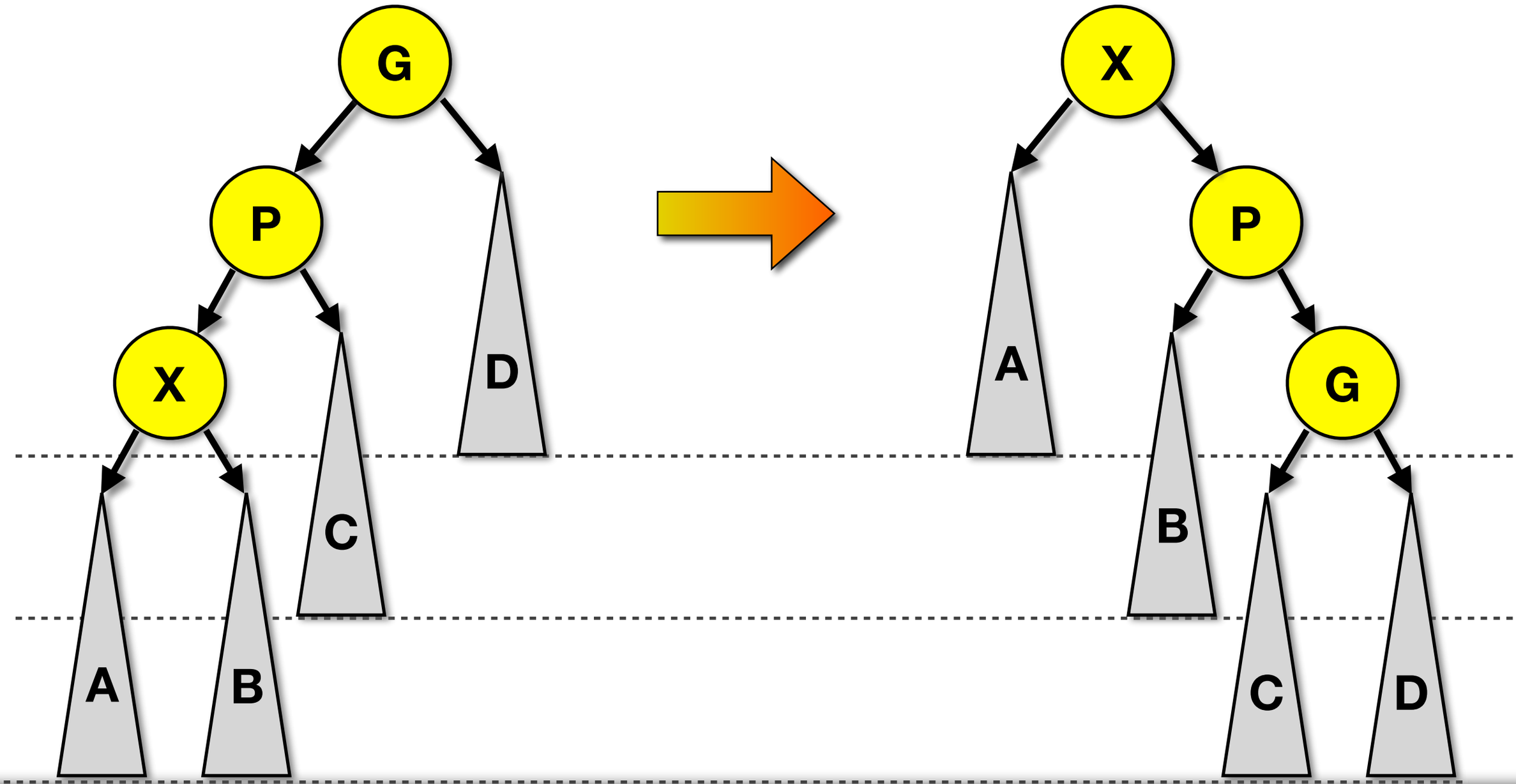
`splay` Operation -- **Case #2 (a Zig-Zag)**

Splaying a node X which is the inside grandchild of G.
Same as a double rotation of the node X.



`splay` Operation -- **Case #3 (a Zig-Zig)**

Splaying a node X which is the outside grandchild of G.
A single rotation between P and G followed by single rotation between X and P.



Splay Tree Operations

- **The Splay tree supports same operations as other binary search trees**
  - **`find`** - starts with the standard BST Find algorithm; then performs the **`splay`** operation on the node after it is found to bring it up to the root of the tree
    - If node is not found, then the **`splay`** operation is performed on the last node in the tree that was visited while performing the search
  - **`insert`** - executes a standard BST **`insert`** algorithm; then performs the **`splay`** operation on the newly inserted node
    - A node inserted into the tree will always end up as the root when the insertion completes

Splay Tree Operations (Cont.)

- **The Splay tree supports same operations as other binary search trees**
  - **`delete`**
    (1) Performs a **`find`** to locate the node to be deleted, this also brings the node to be deleted to the root of the tree;

    (2) **`delete`** the root of the tree leaving two unattached subtrees

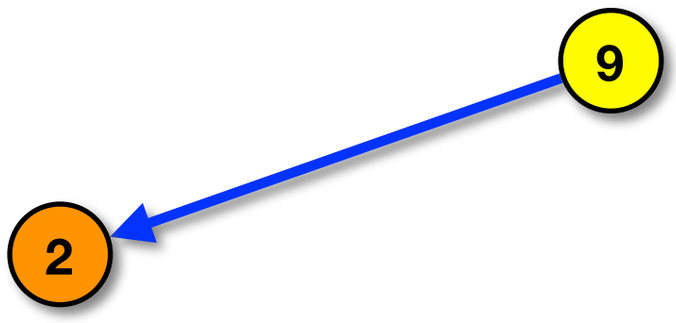
    (3) Perform a **`findMax`** on the left subtree to find the maximum node and **`splay`** that maximum node to the root of the left subtree (it will not have a right child, even after the **`splay`**)

    (4) The maximum node from step 3 becomes the new root node for the entire tree; attach the unattached right subtree from step 2 as a right child of this new root node

Insertion Example

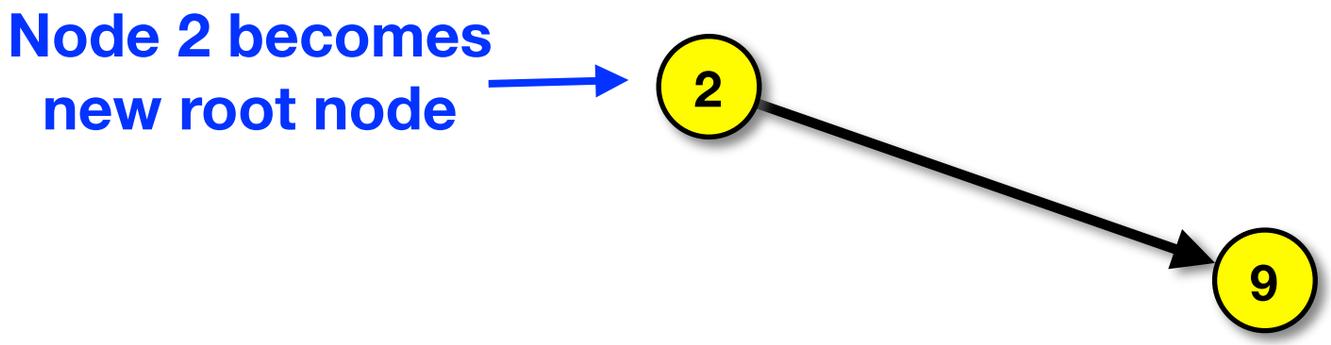
Insert node 9
Calling splay on node 9 doesn’t do anything

Insertion Example



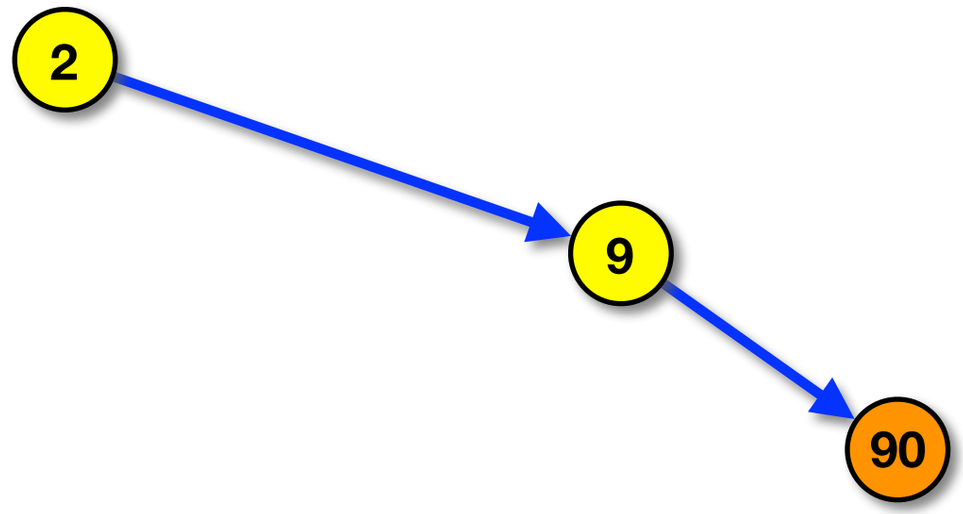
Insert node 2
Case #1 - perform a zig operation

Insertion Example



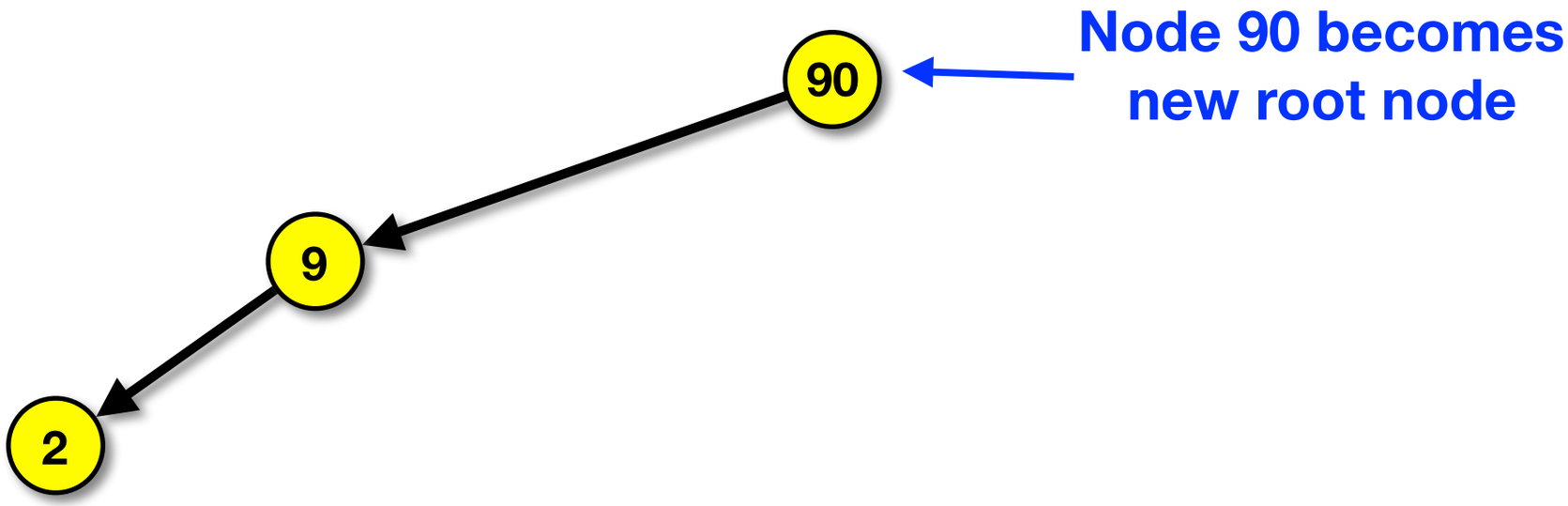
Insert node 2
Result of zig operation

Insertion Example



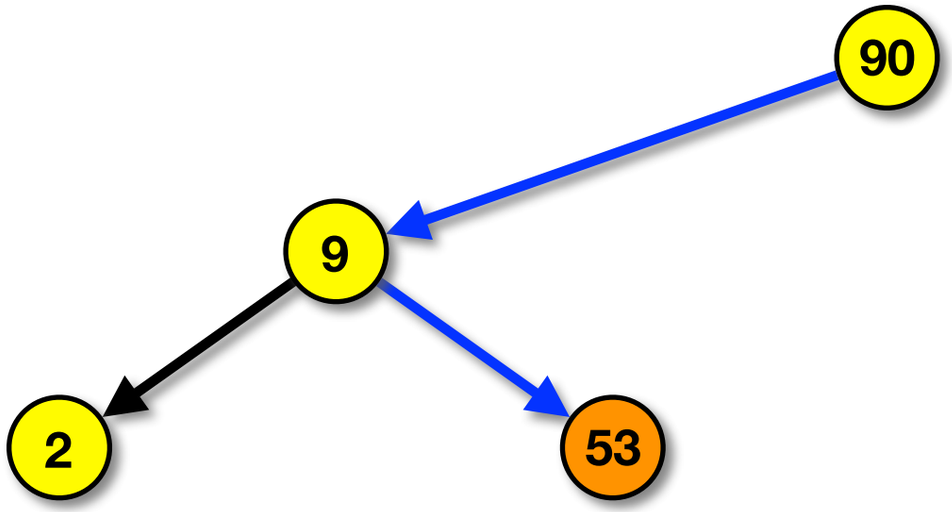
Insert node 90
Case #3 - perform a zig-zig operation

Insertion Example



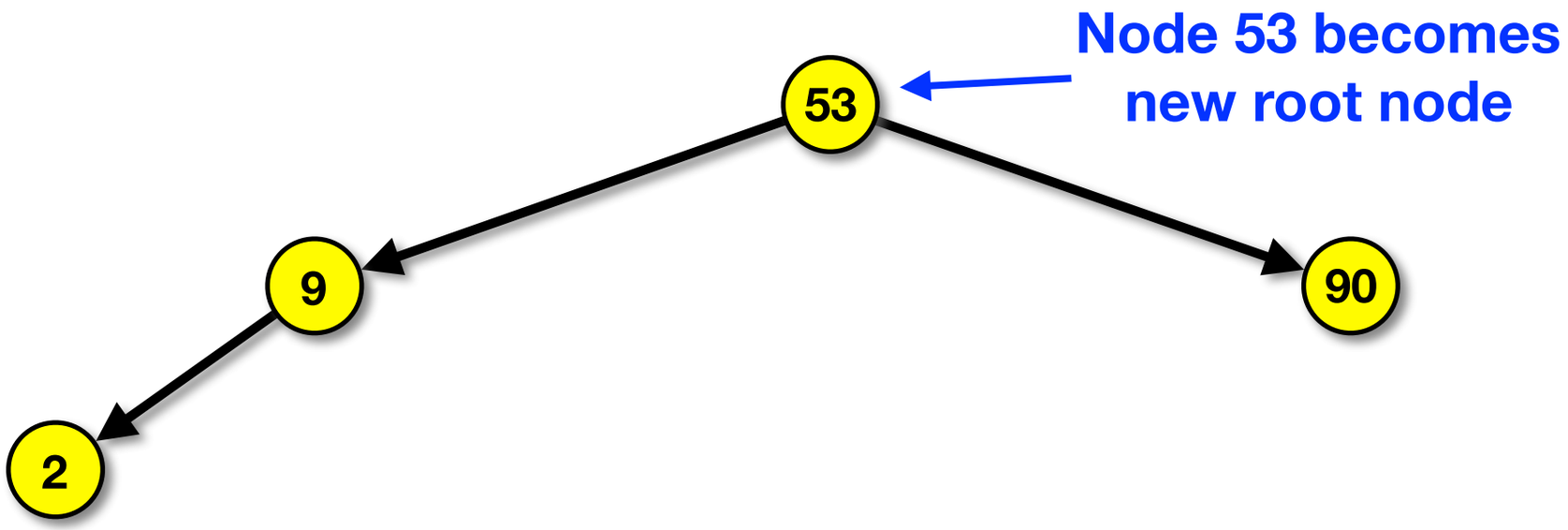
Insert node 90
Result of zig-zig operation

Insertion Example



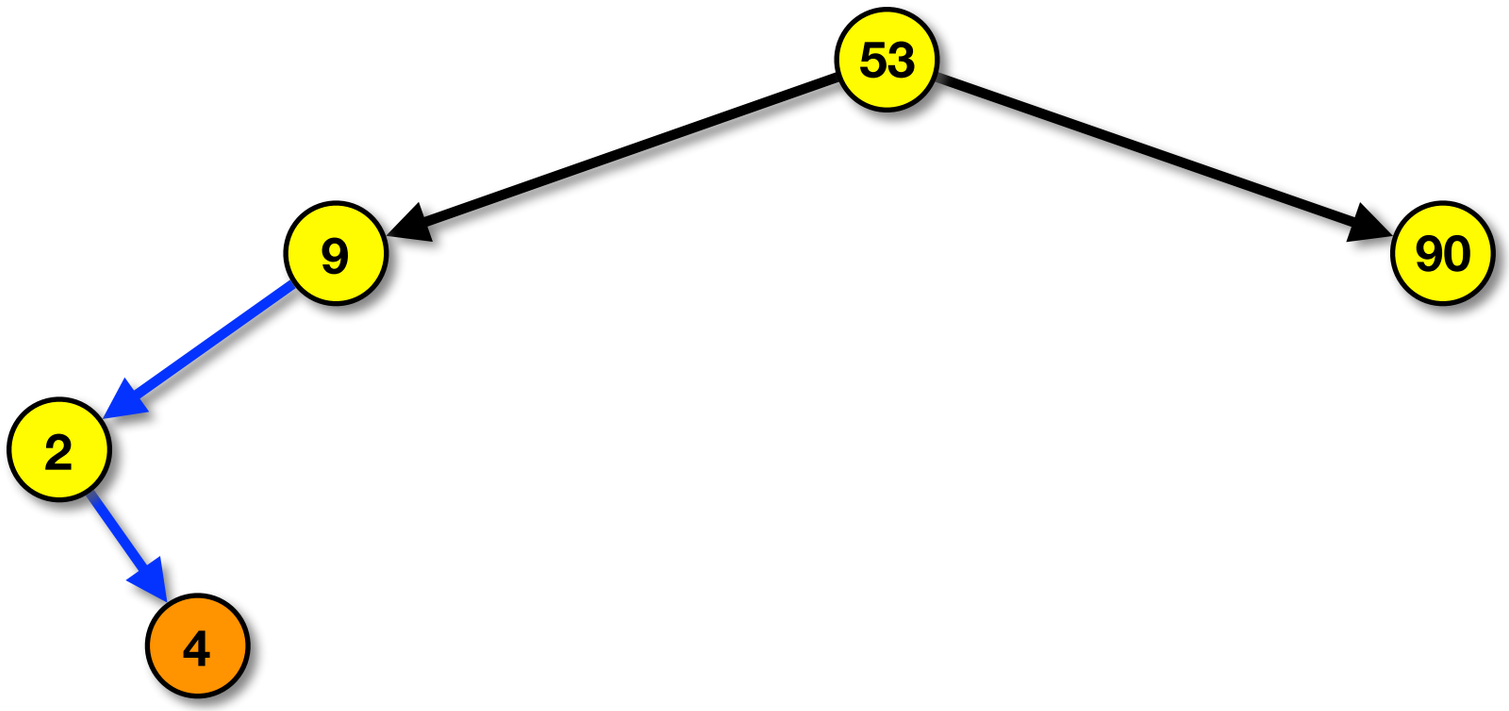
Insert node 53
Case #2 - perform a zig-zag operation

Insertion Example



Insert node 53
Result of zig-zag operation

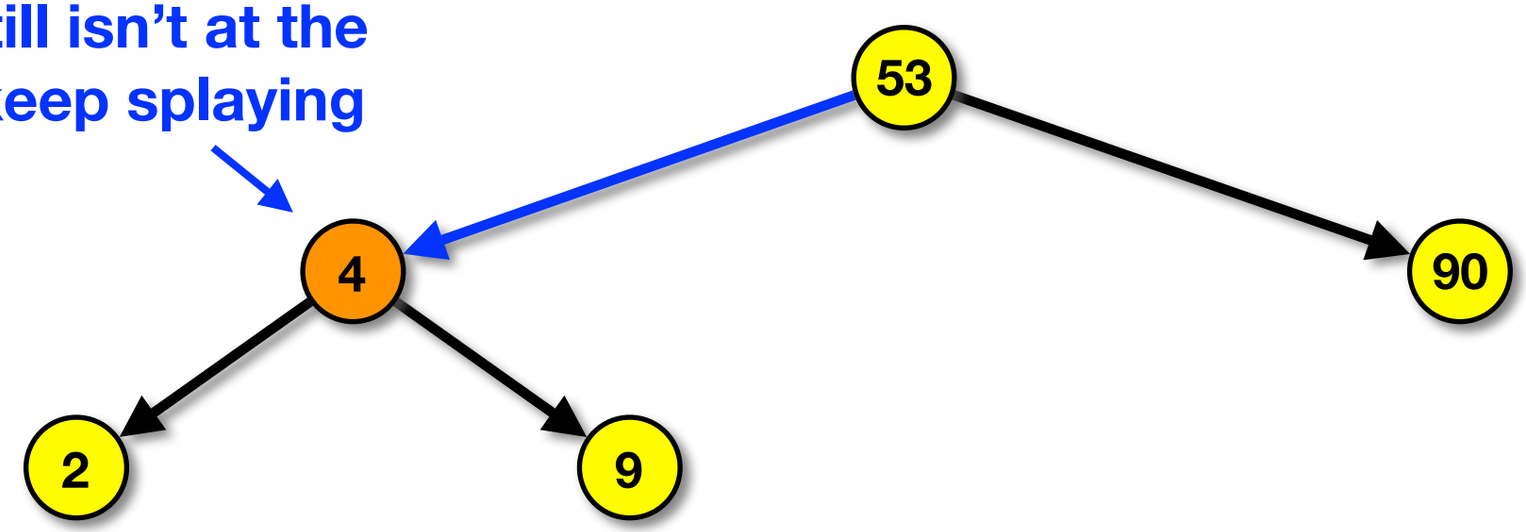
Insertion Example



Insert node 4
Case #2 - perform a zig-zag operation

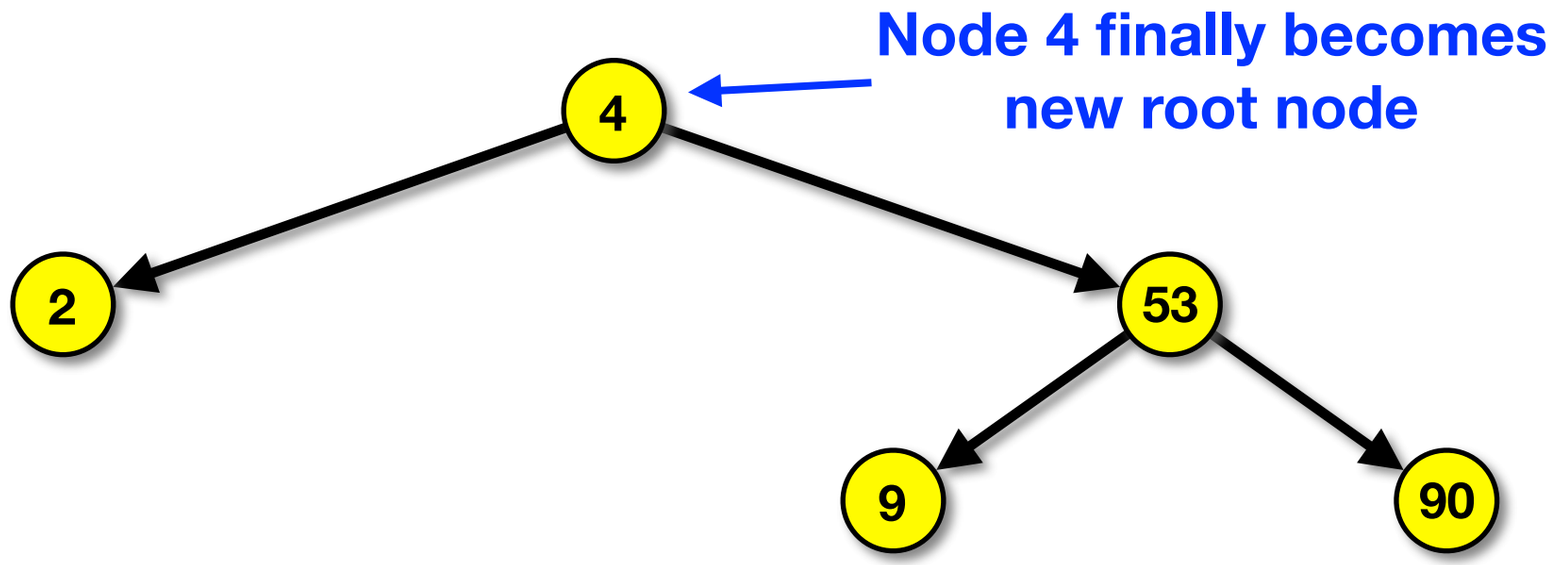
Insertion Example

Node 4 still isn't at the root, so keep splaying



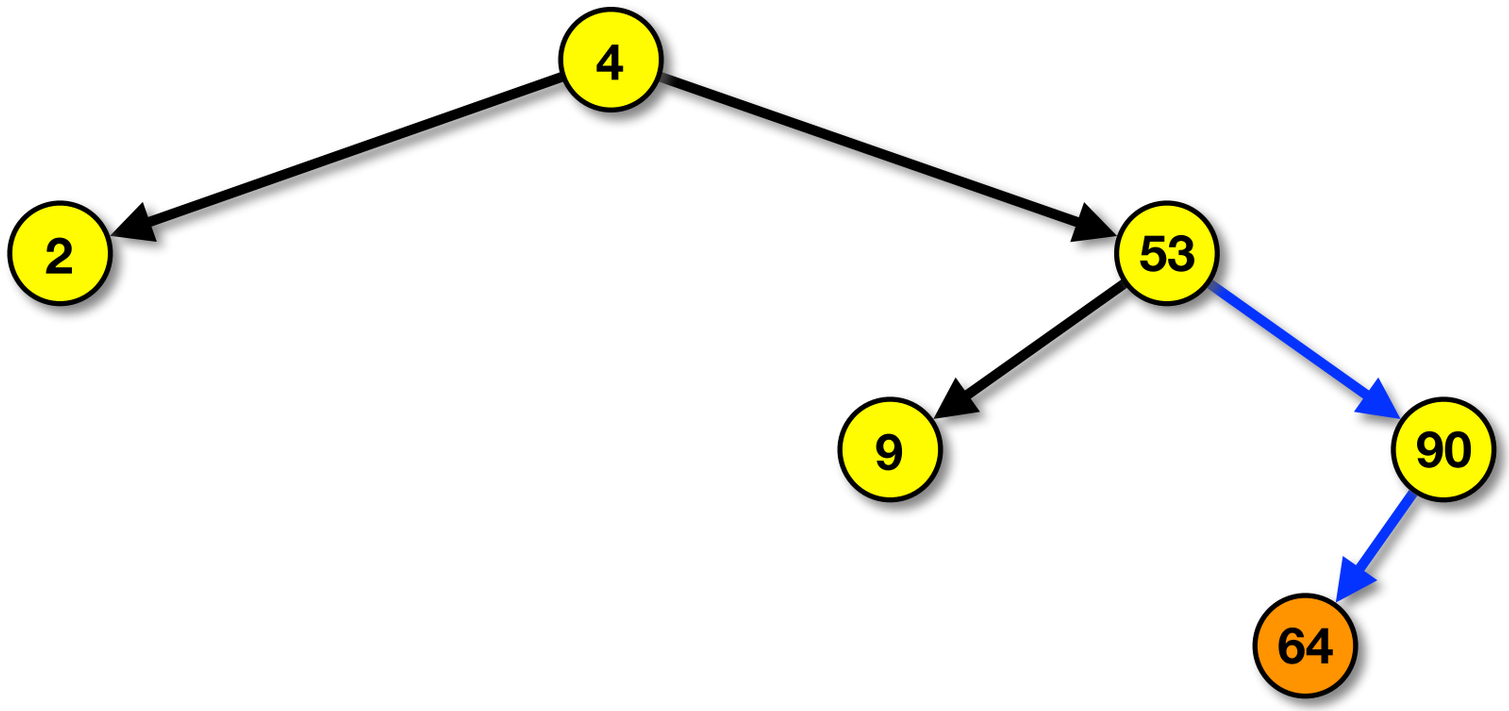
Insert node 4 (Cont.)
Case #1 - perform a zig operation

Insertion Example



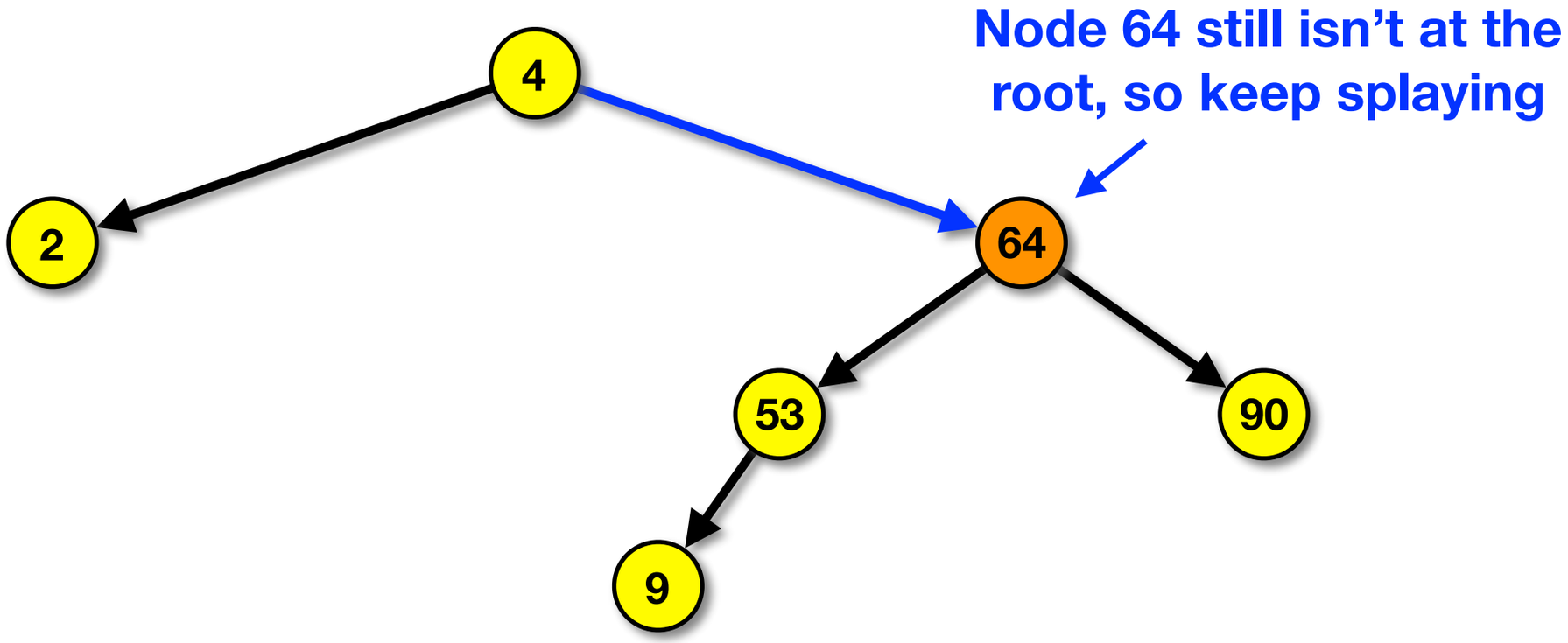
Insert node 4
Result of zig-zag operation
followed by a zig operation

Insertion Example



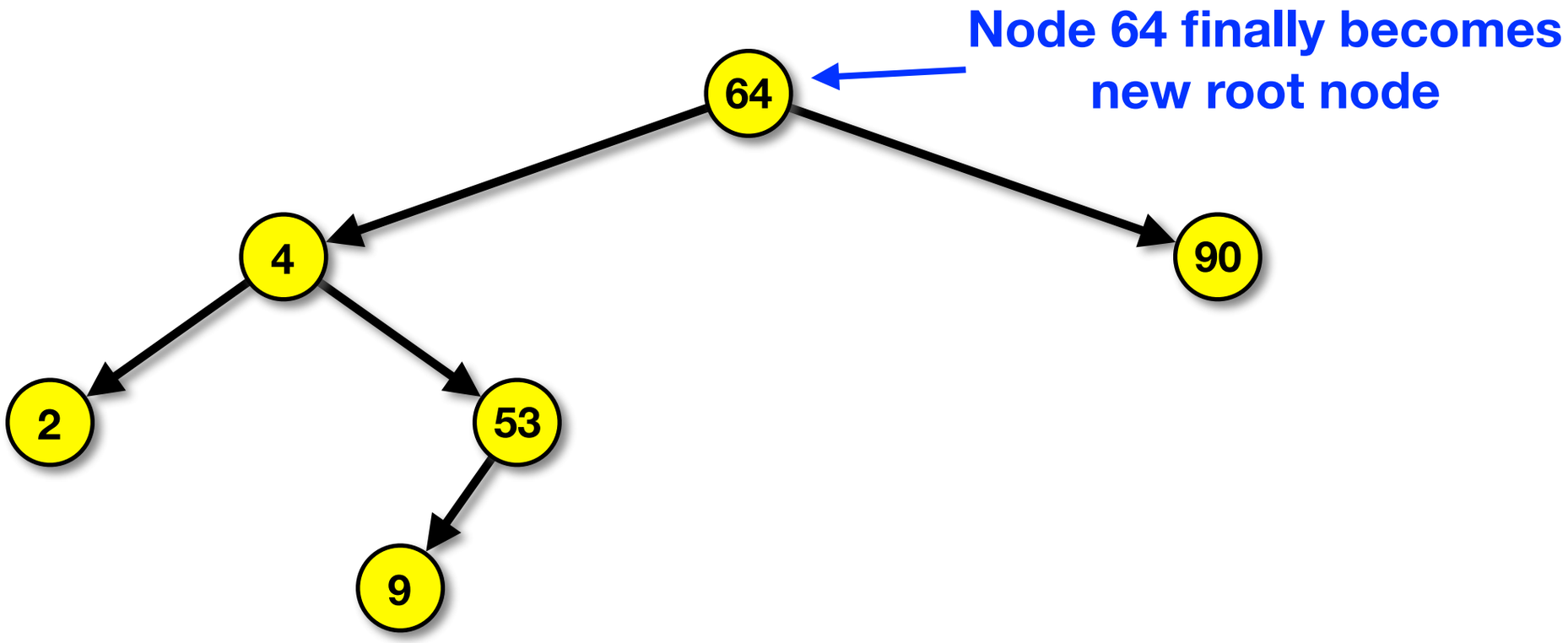
Insert node 64
Case #2 - perform a zig-zag operation

Insertion Example



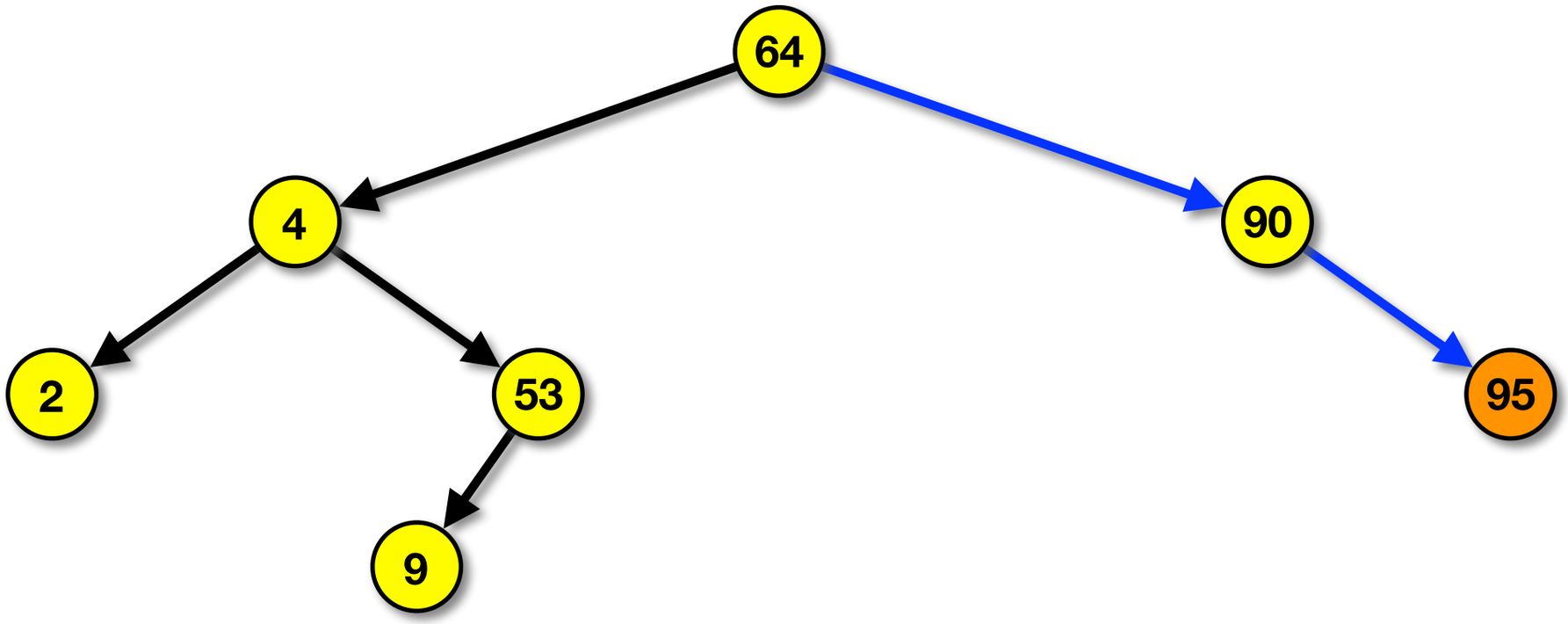
Insert node 4 (Cont.)
Case #1 - perform a zig operation

Insertion Example



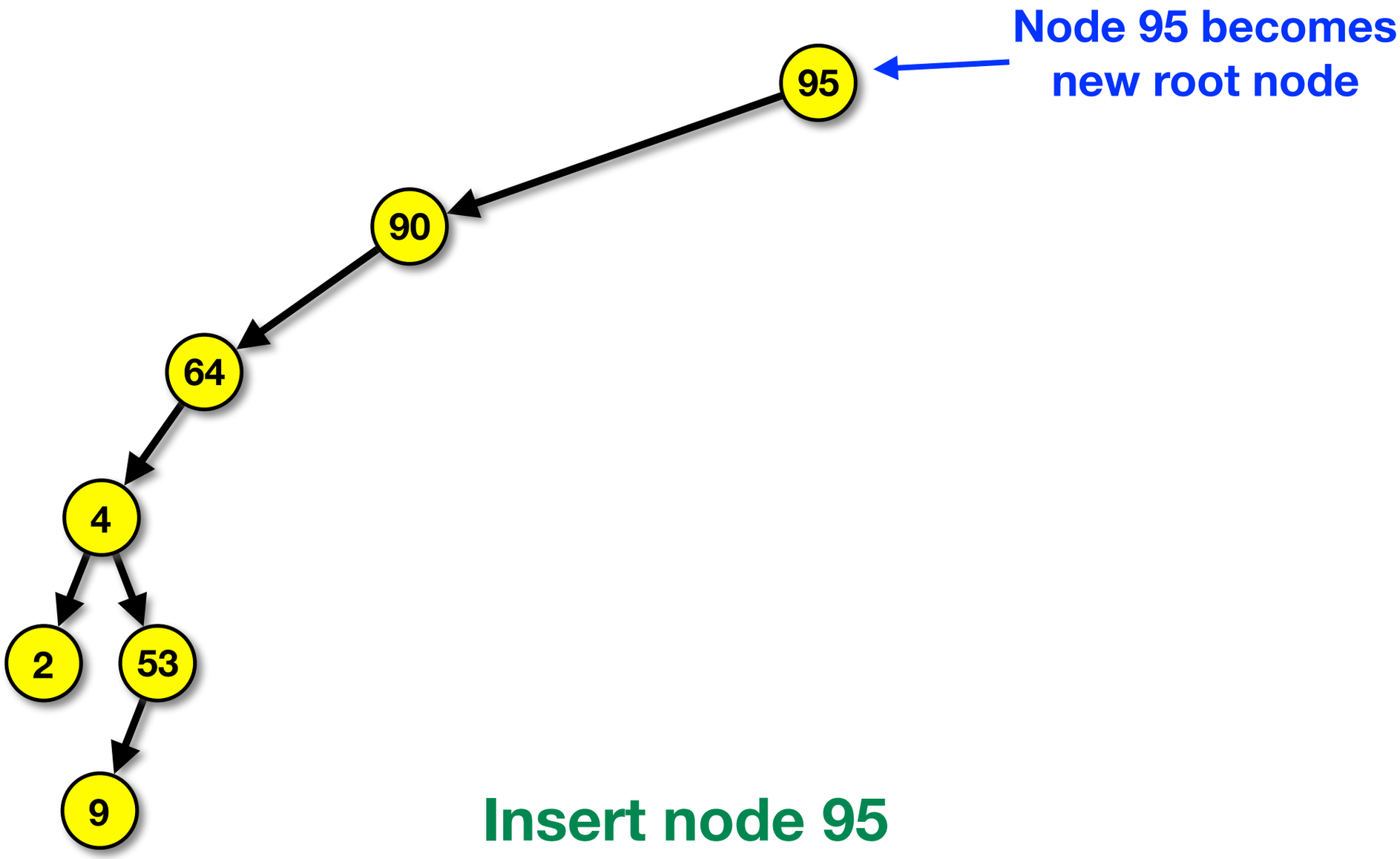
Insert node 64
Result of zig-zag operation
followed by a zig operation

Insertion Example



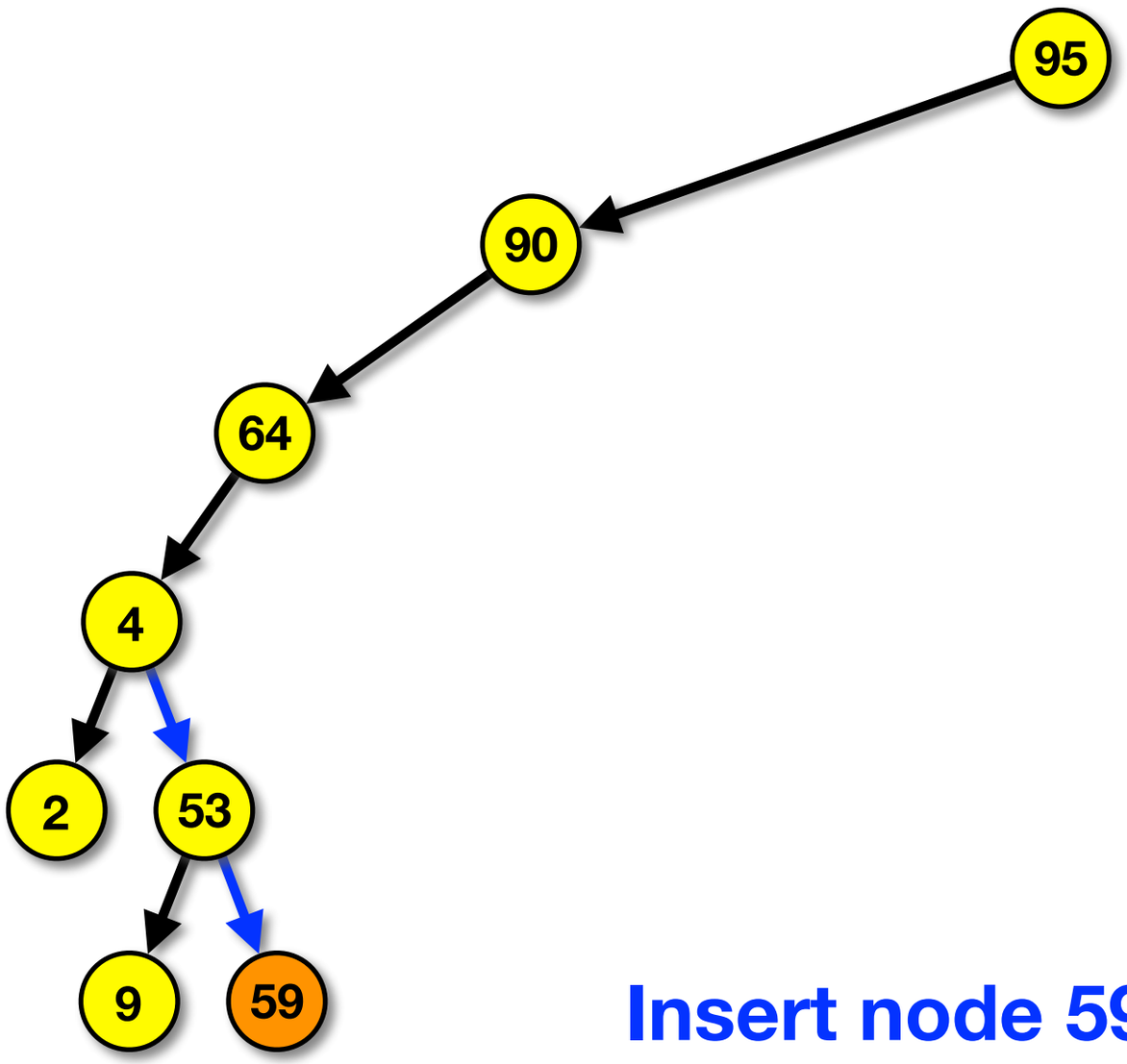
Insert node 95
Case #3 - perform a zig-zig operation

Insertion Example



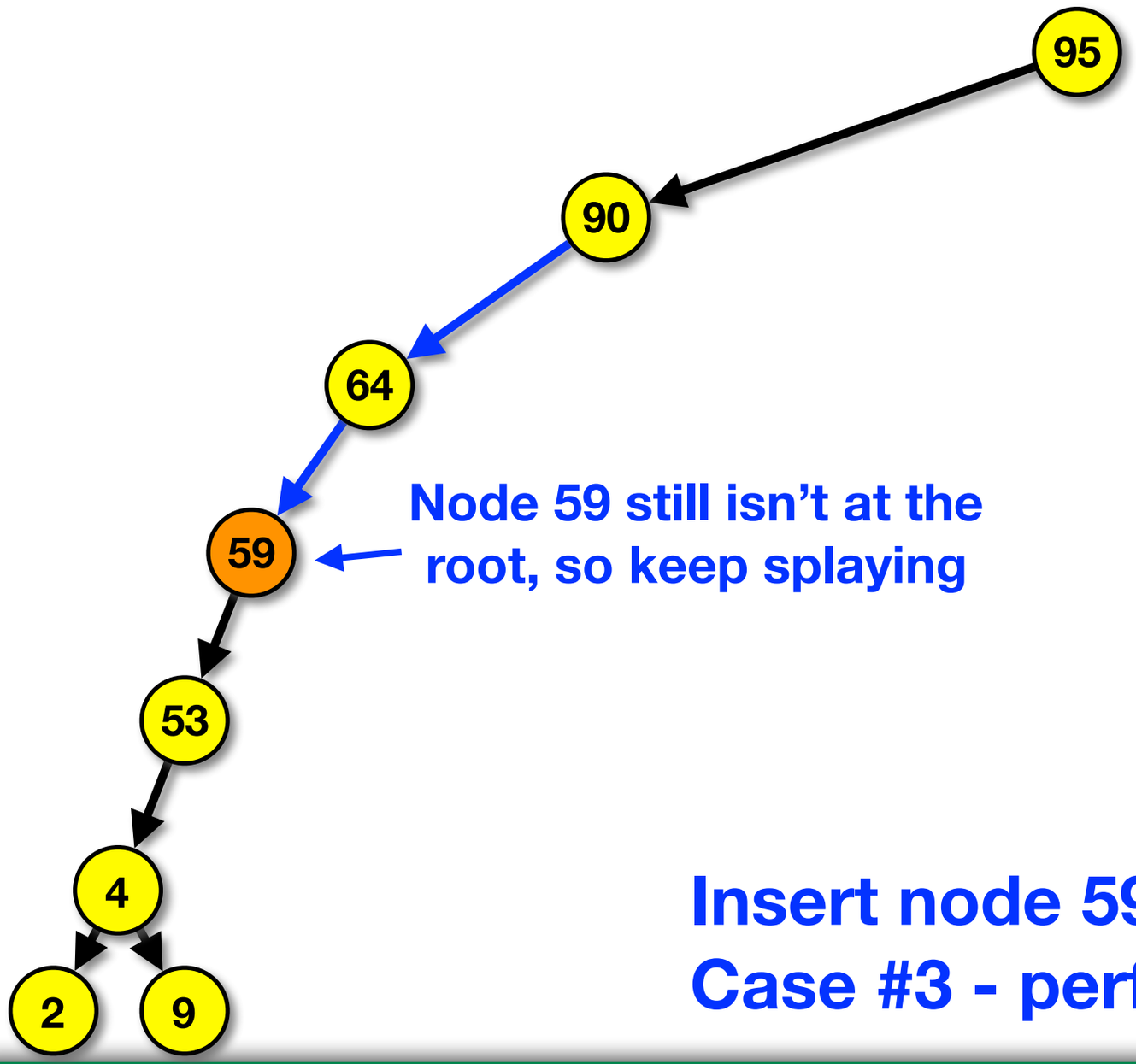
Insert node 95
Result of zig-zig operation

Insertion Example



Insert node 59
Case #3 - perform a zig-zig operation

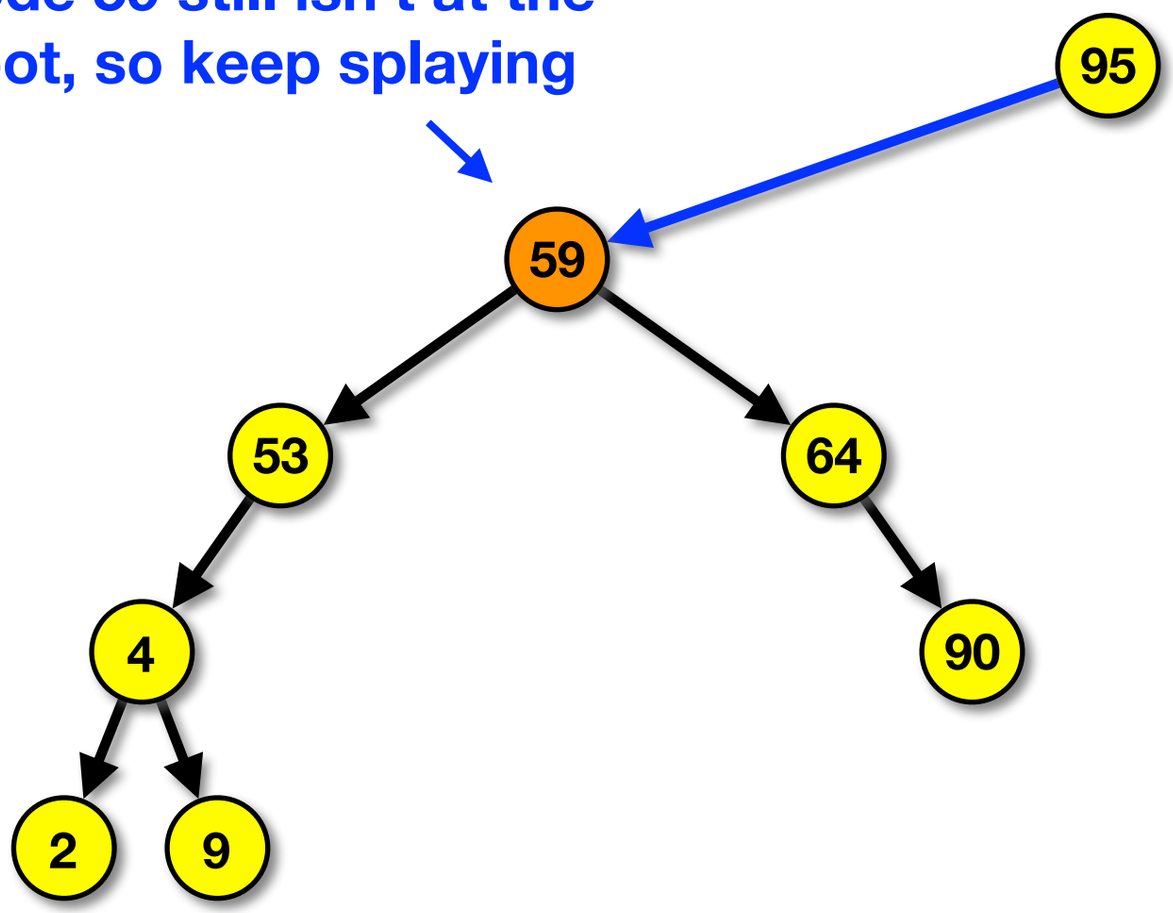
Insertion Example



Insert node 59 (Cont.)
Case #3 - perform a zig-zig operation

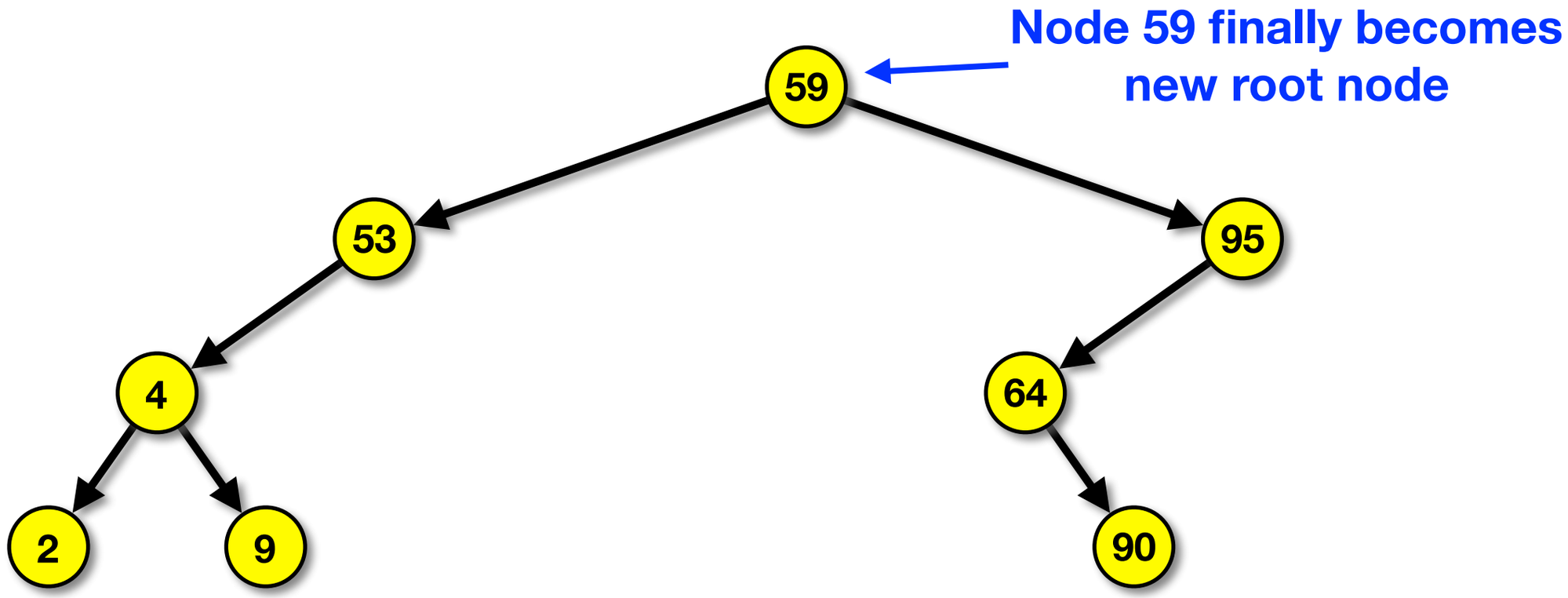
Insertion Example

Node 59 still isn't at the root, so keep splaying



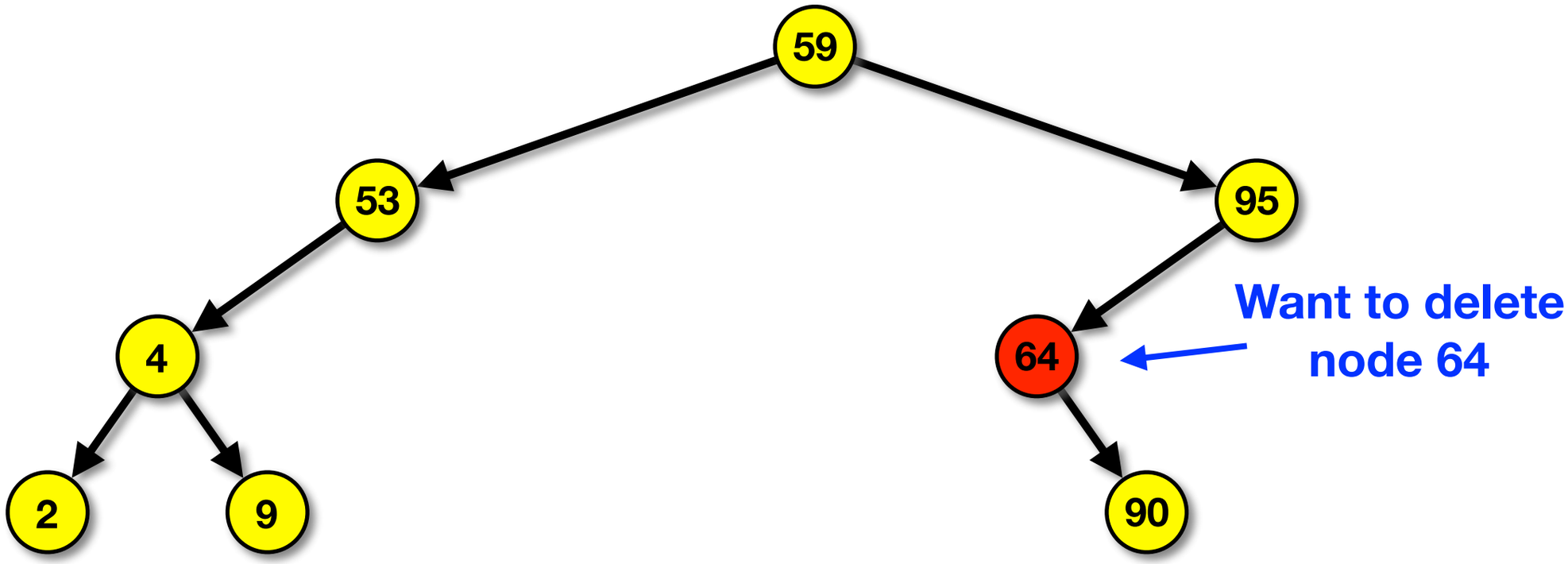
Insert node 59 (Cont.)
Case #1 - perform a zig operation

Insertion Example



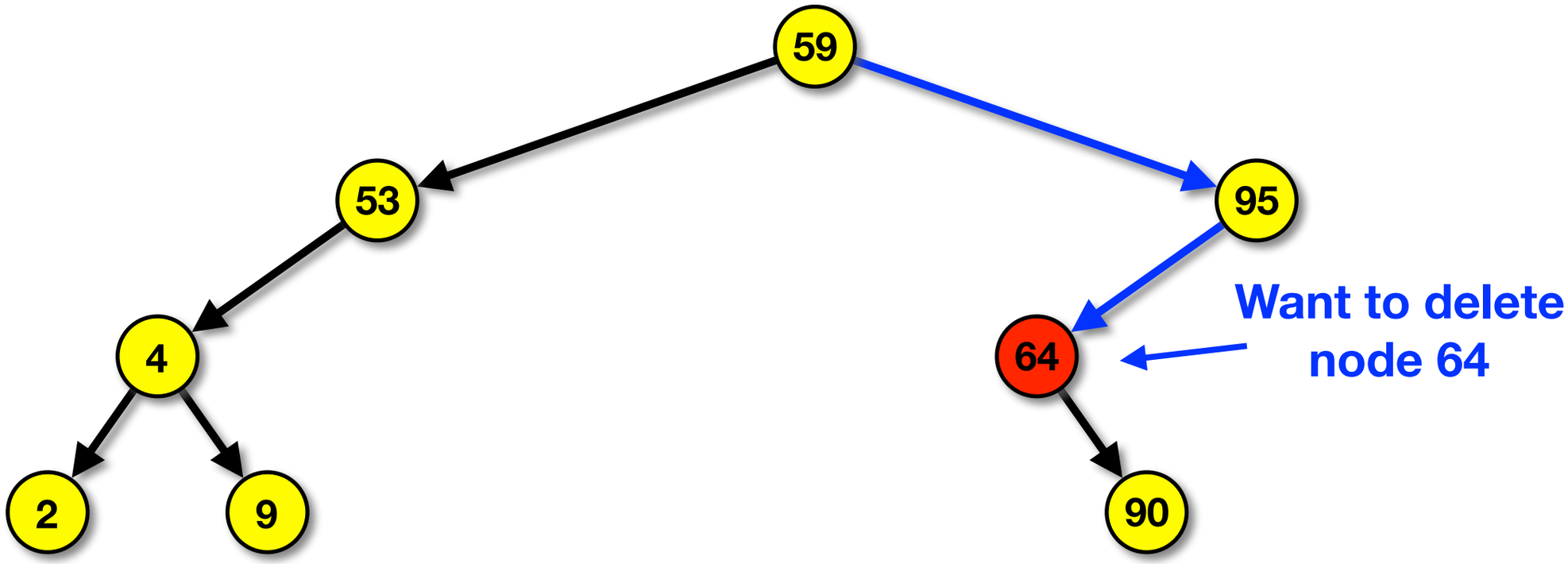
Insert node 59
Result of zig-zig operation
followed by a zig-zig operation
followed by a zig operation

Deletion Example



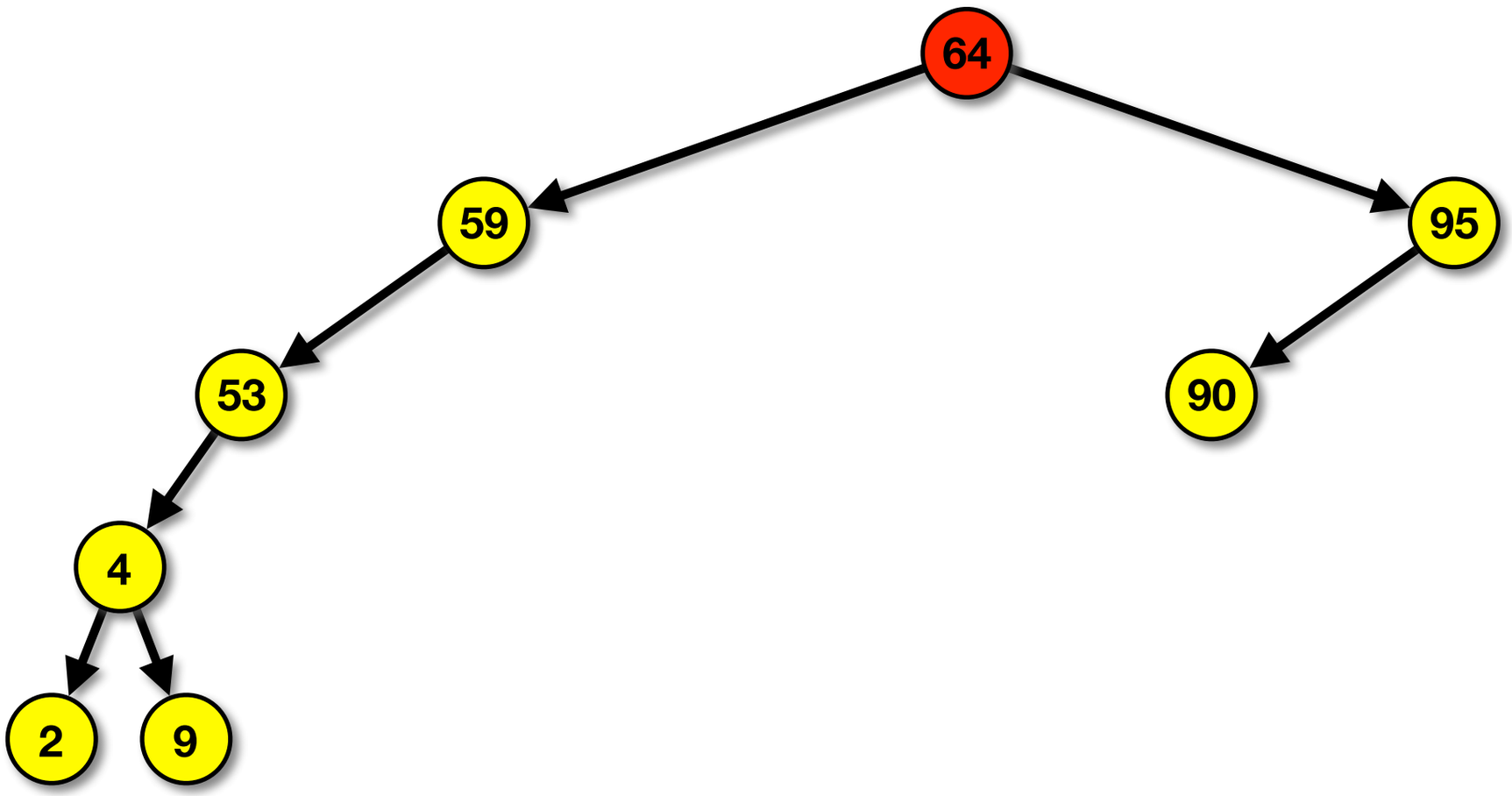
Delete node 64
Step #1 - perform a Find operation to move node 64 to the root

Deletion Example



Delete node 64
Step #1 - perform a Find operation to move node 64 to the root
- a zig-zag operation is needed

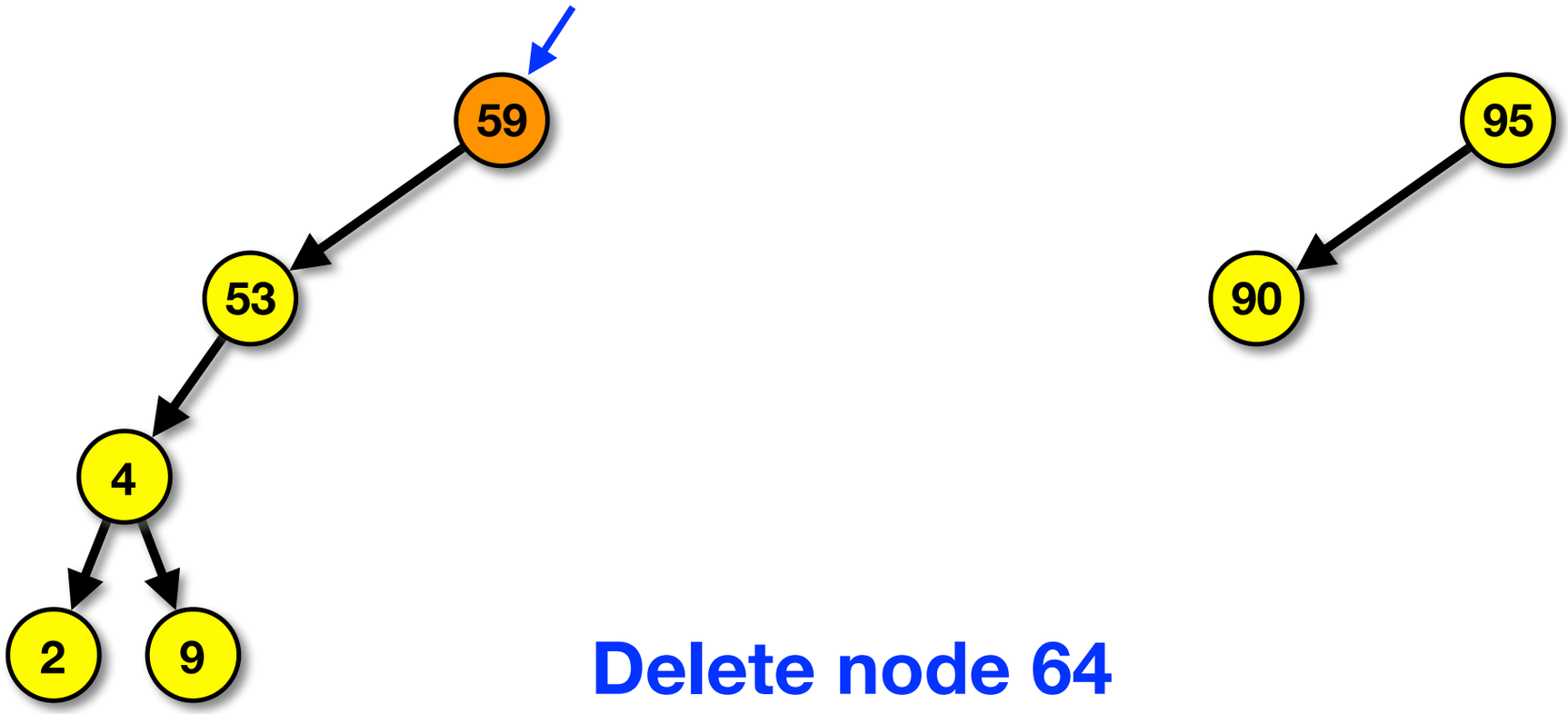
Deletion Example



Delete node 64
Step #2 - Now that node 64 is at the root, delete it leaving two unattached subtrees

Deletion Example

FindMax returns 59, since 59 is already root of left subtree, splay doesn't do anything

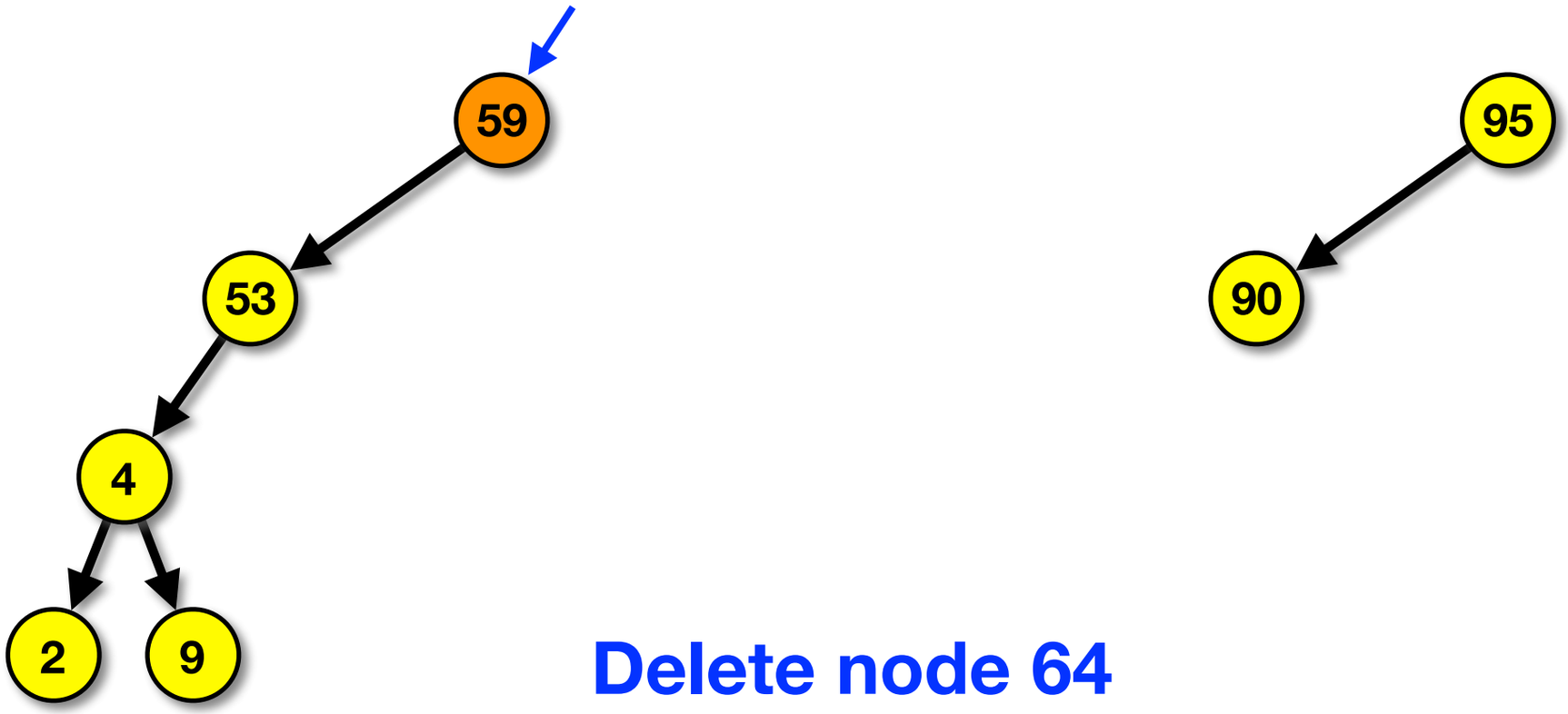


Delete node 64

Step #3 - perform a FindMax on the left subtree and splay that max node to the root of the left subtree

Deletion Example

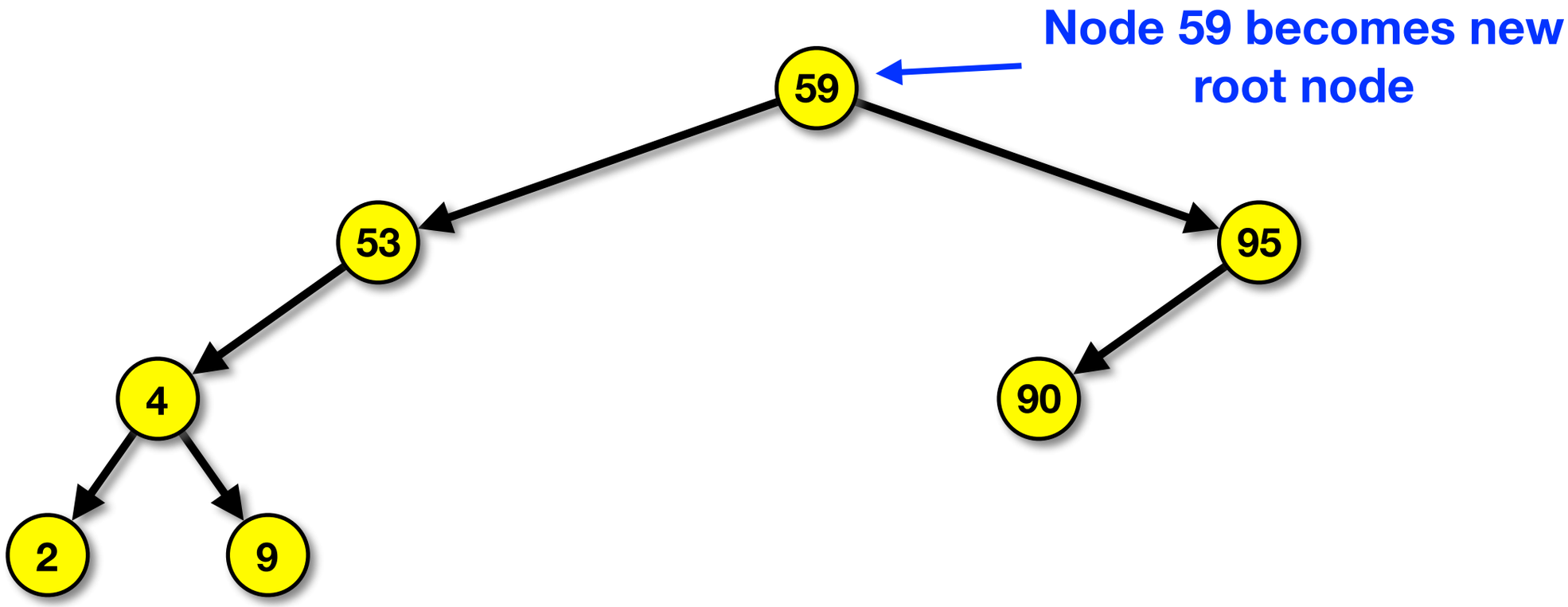
FindMax returns 59, since 59 is already root of left subtree, splay doesn't do anything



Delete node 64

Step #4 - node 59 will become the new root of the tree and the right subtree will become the right child of node 59

Deletion Example



Result of deleting node 64