

CS350: Data Structures

Red-Black Trees

James Moscola

Department of Engineering & Computer Science
York College of Pennsylvania



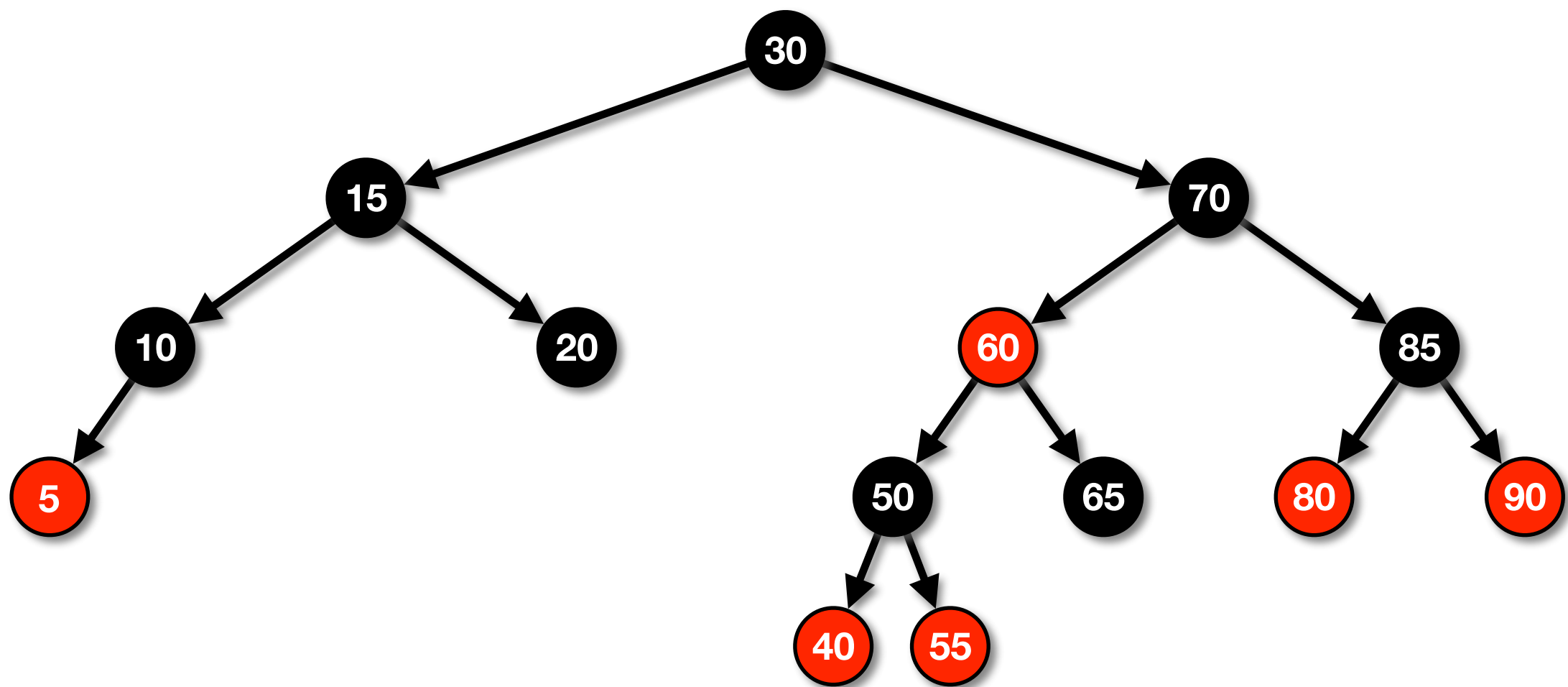
Red-Black Tree

- **An alternative to AVL trees**
- **Insertion can be done in a bottom-up or a top-down fashion**
 - An AVL tree uses a pass down the tree for an insertion and a second pass back up the tree to update node heights and potentially rebalance the tree
 - A top-down insertion into a **red**-black tree requires only a single pass down the tree
 - **We'll focus on bottom-up insertion**

Red-Black Tree (Cont.)

- **A red-black tree is a binary search tree that has the following properties:**
 - (1) Every node is colored either red or black
 - (2) The root node is black
 - (3) If a node is red, its children must be black
 - (4) Every path from a node to a null node must contain the same number of black nodes
- **These properties must be maintained after each insertion or deletion operation**
- **A null node is considered black**

Example Red-Black Tree

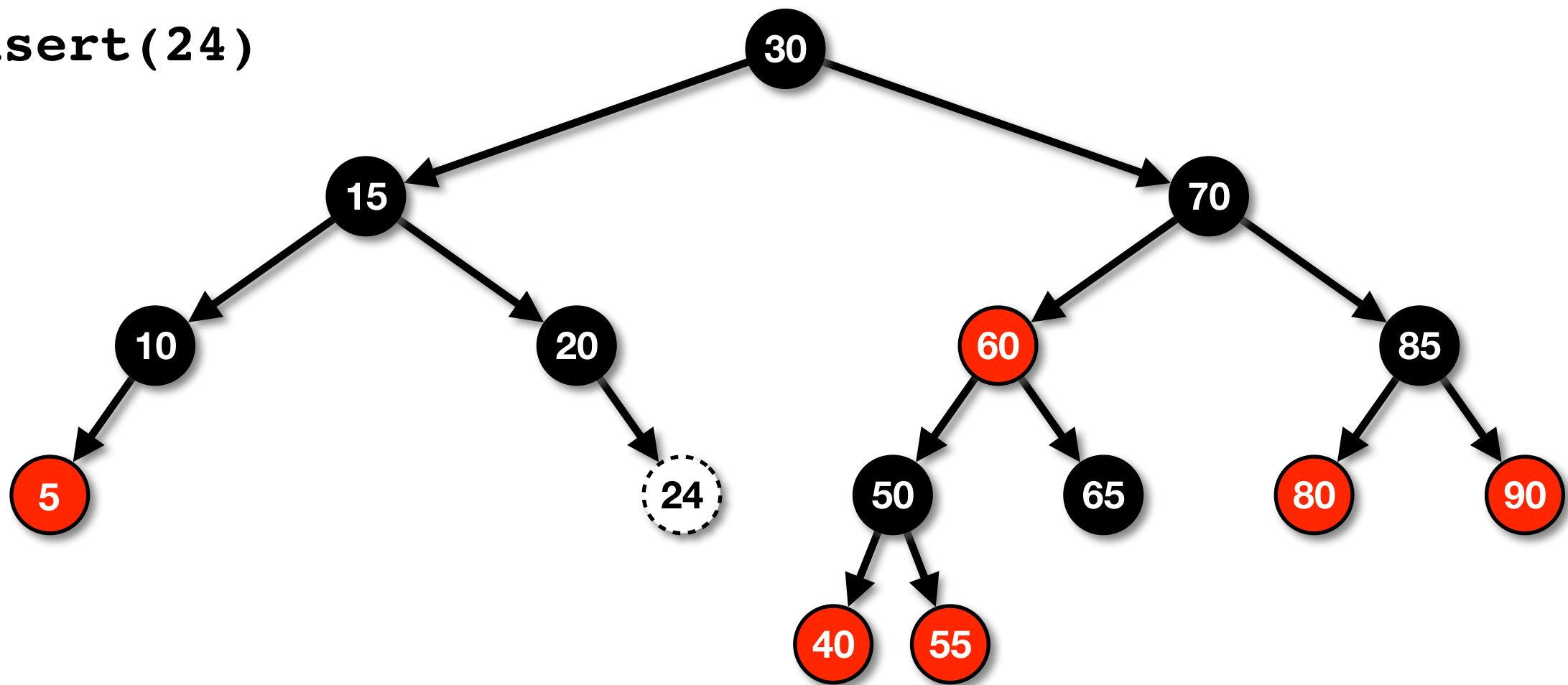


- (1) Every node is colored either red or black
- (2) The root node is black
- (3) If a node is red, its children must be black
- (4) Every path from a node to a null link must contain the same number of black nodes

Red-Black Tree Insertion

- When inserting a new leaf node, what color should it be?

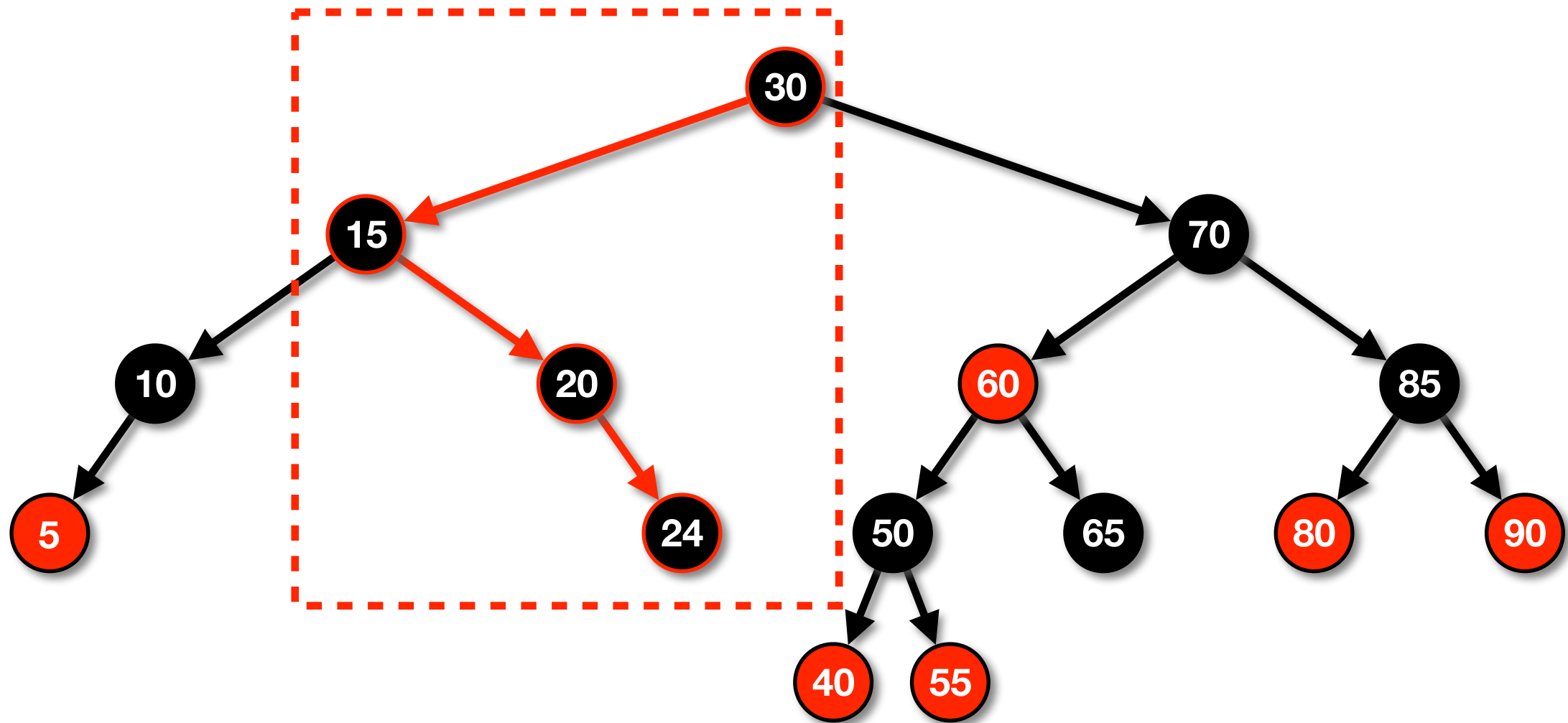
insert(24)



- (1) Every node is colored either red or black
- (2) The root node is black
- (3) If a node is red, its children must be black
- (4) Every path from a node to a null link must contain the same number of black nodes

Red-Black Tree Insertion

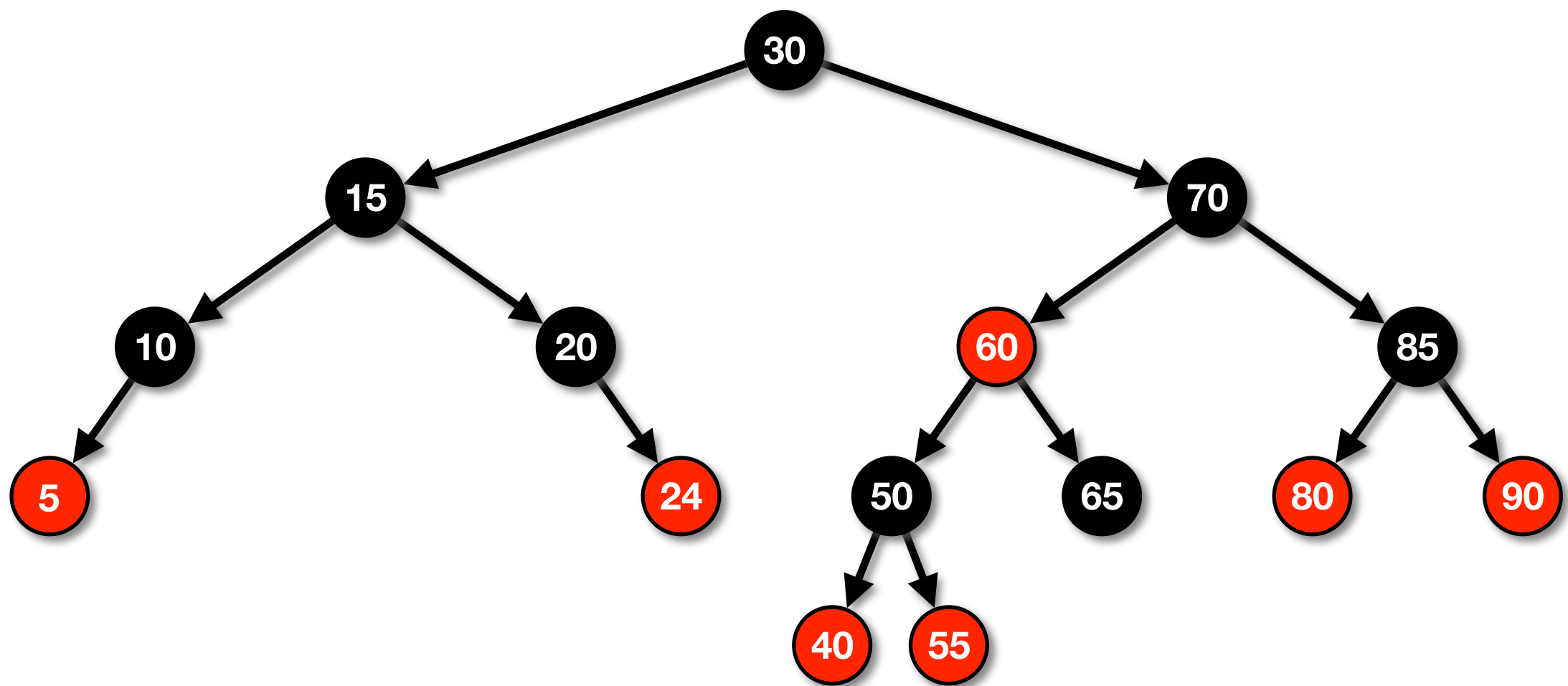
- Inserting as a black node violates property #4



- (1) Every node is colored either red or black
- (2) The root node is black
- (3) If a node is red, its children must be black
- ✗ (4) Every path from a node to a null link must contain the same number of black nodes

Red-Black Tree Insertion

- Inserting as a **red** node satisfies all four properties (in this case)

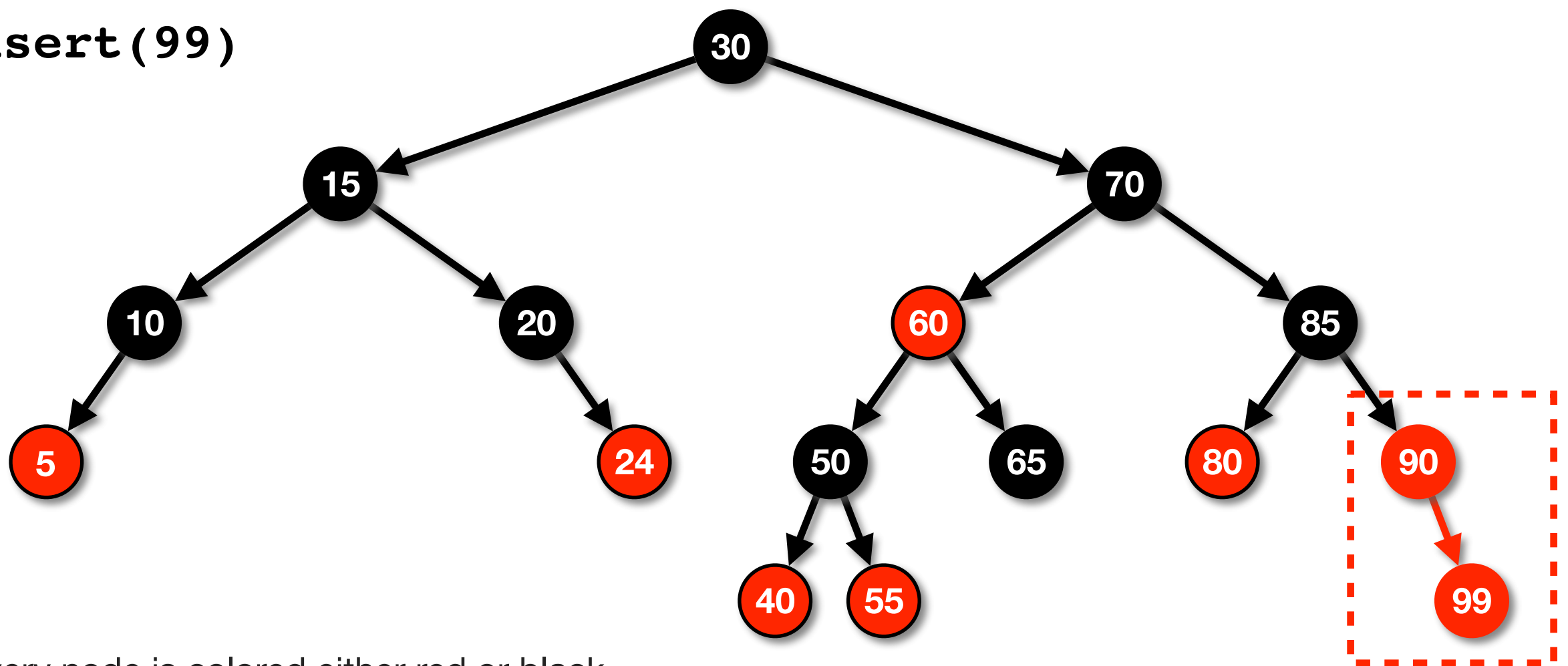


- (1) Every node is colored either red or black
- (2) The root node is black
- (3) If a node is red, its children must be black
- (4) Every path from a node to a null link must contain the same number of black nodes

Red-Black Tree Insertion

- Inserting as a **red** may not always satisfy the four properties

insert(99)



(1) Every node is colored either red or black

(2) The root node is black

✗ (3) If a node is red, its children must be black

(4) Every path from a node to a null link must contain the same number of black nodes

Red-Black Tree Insertion

- **Easier to fix a violation of property #3 than it is to fix a violation of property #4**
 - So, insert all nodes as **red** nodes
- **Must repair violations of property #3 and any new violations that occur as a result of the tree modification**
 - Operations for repairing the tree include:
 - **Single and Double Rotations (similar to AVL trees)**
 - **Color changes**

Red-Black Tree Insertion (Bottom-Up)

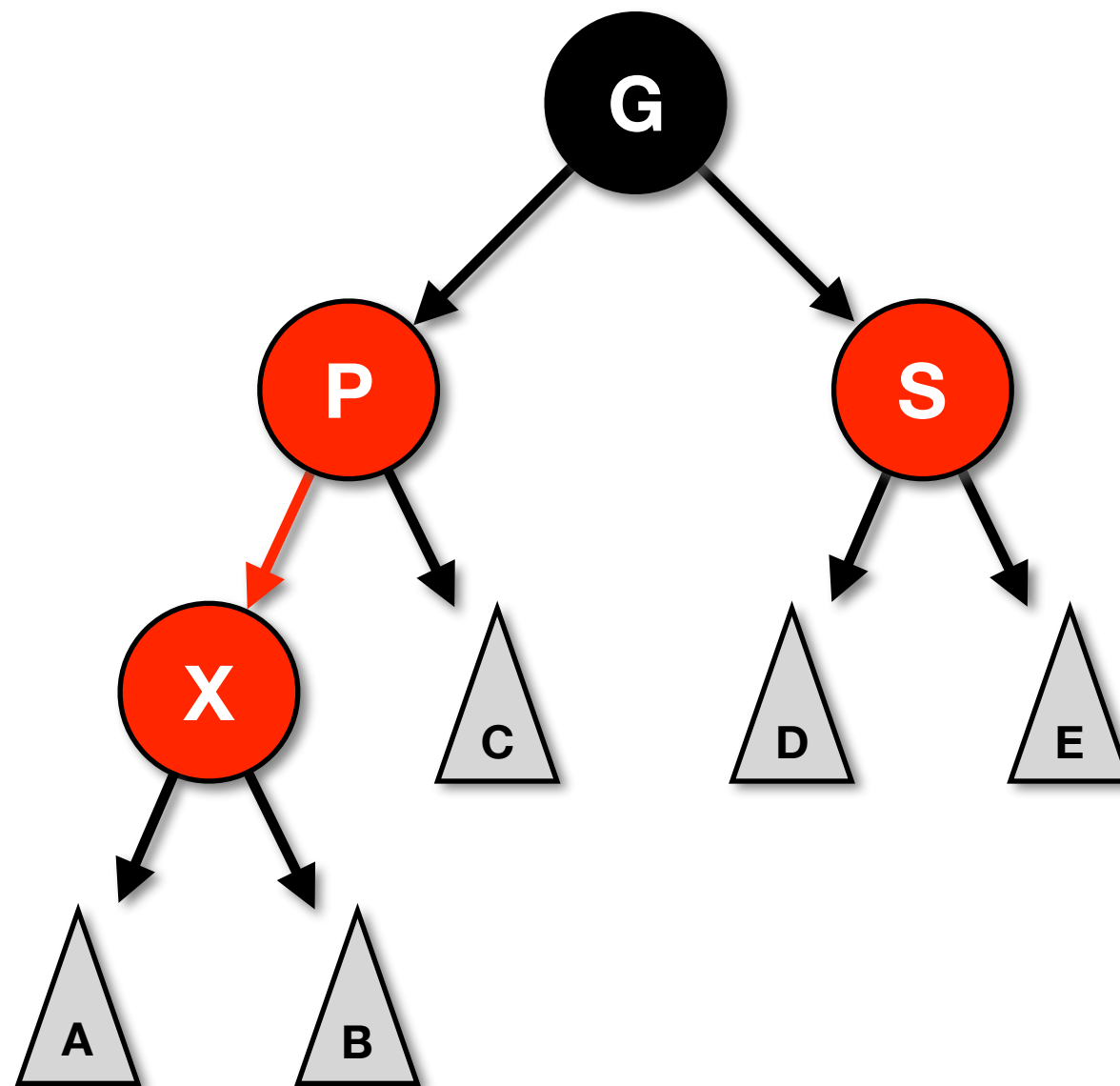
- **Insert nodes into a red-black tree using the standard binary search tree insertion**
 - Make the newly inserted node **red**
 - If the parent of the newly inserted node is black, then no violations have occurred and the insertion is complete
 - If the parent of the newly inserted node is **red**, then property #3 has been violated and must be fixed through rotations and recoloring
 - **Four different cases must be considered**

Red-Black Tree Insertion -- Violations

- The four cases to consider when property #3 is violated (i.e. when a **red** node is inserted as the child of another **red** node)
 - (1) Parent's sibling is **red** and new node is inserted as an outside grandchild
 - (2) Parent's sibling is **red** and new node is inserted as an inside grandchild
 - (3) Parent's sibling is **black** and new node is inserted as an outside grandchild
 - (4) Parent's sibling is **black** and new node is inserted as an inside grandchild
- There is a different approach to fix each of these cases

Insert Violation -- Case #1

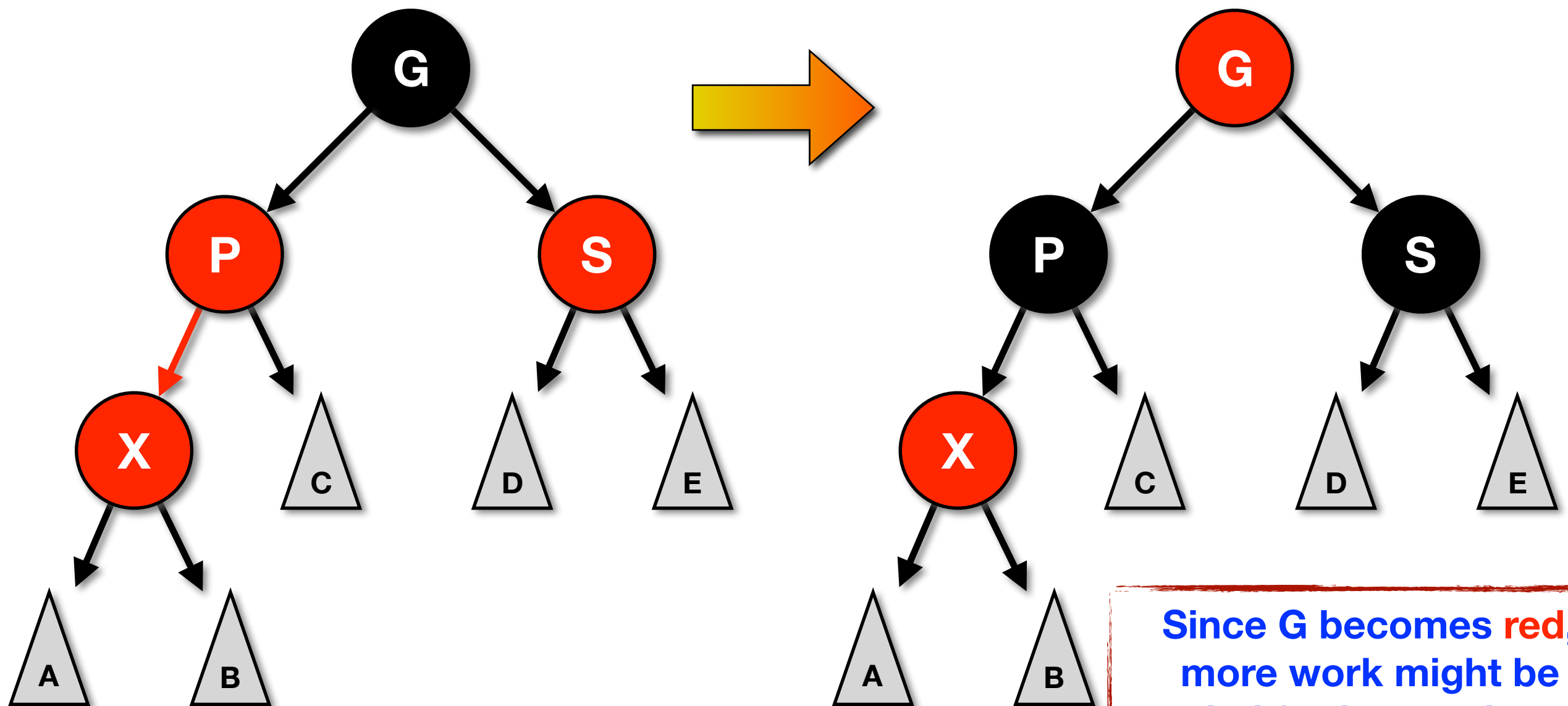
(1) Parent's sibling is **red** and new node is inserted as an outside grandchild



Fixing the Insertion Violation -- Case #1

No rotation necessary

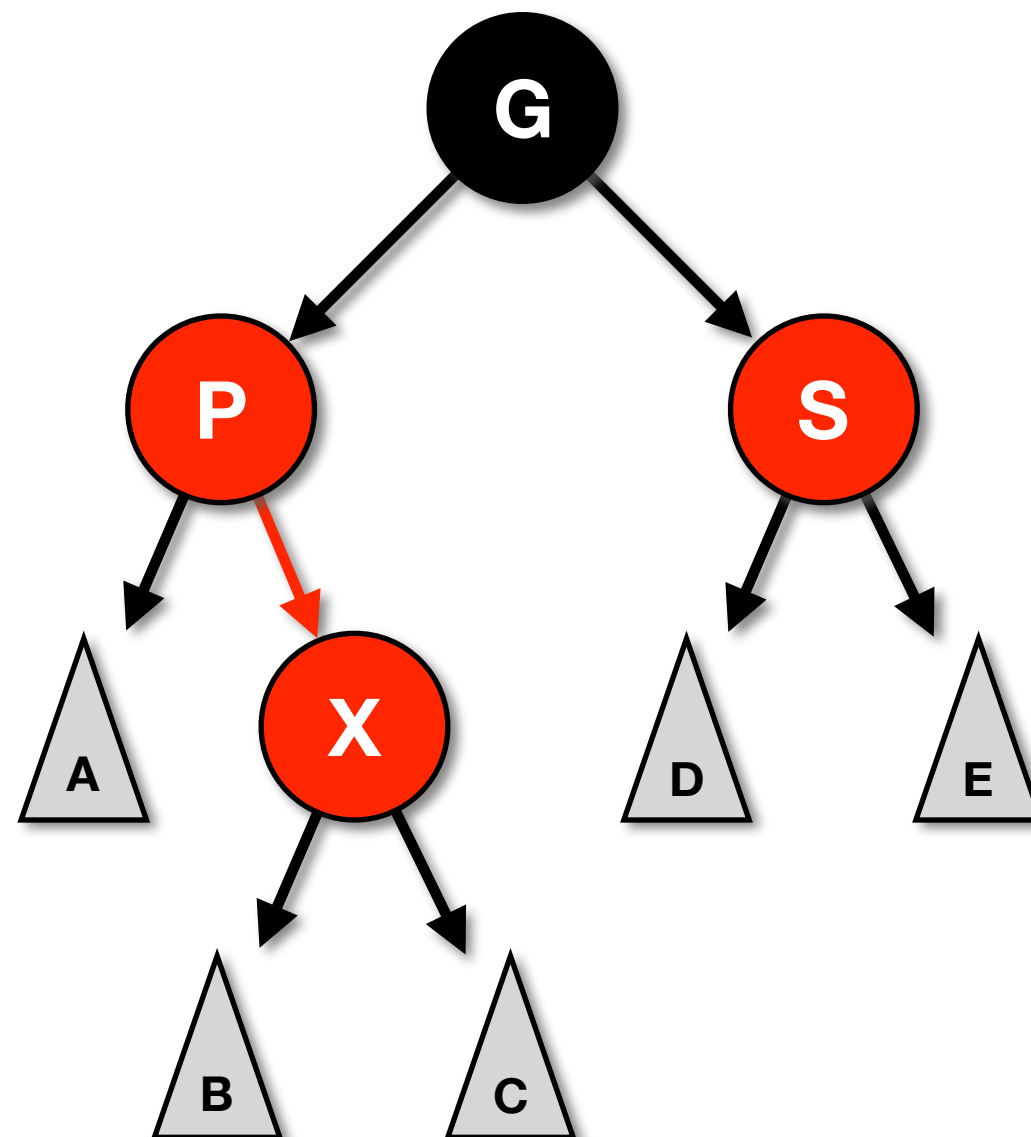
Make Parent and Sibling **black**,
Grandparent becomes **red**



Since **G** becomes **red**,
more work might be
needed further up the tree

Insert Violation -- Case #2

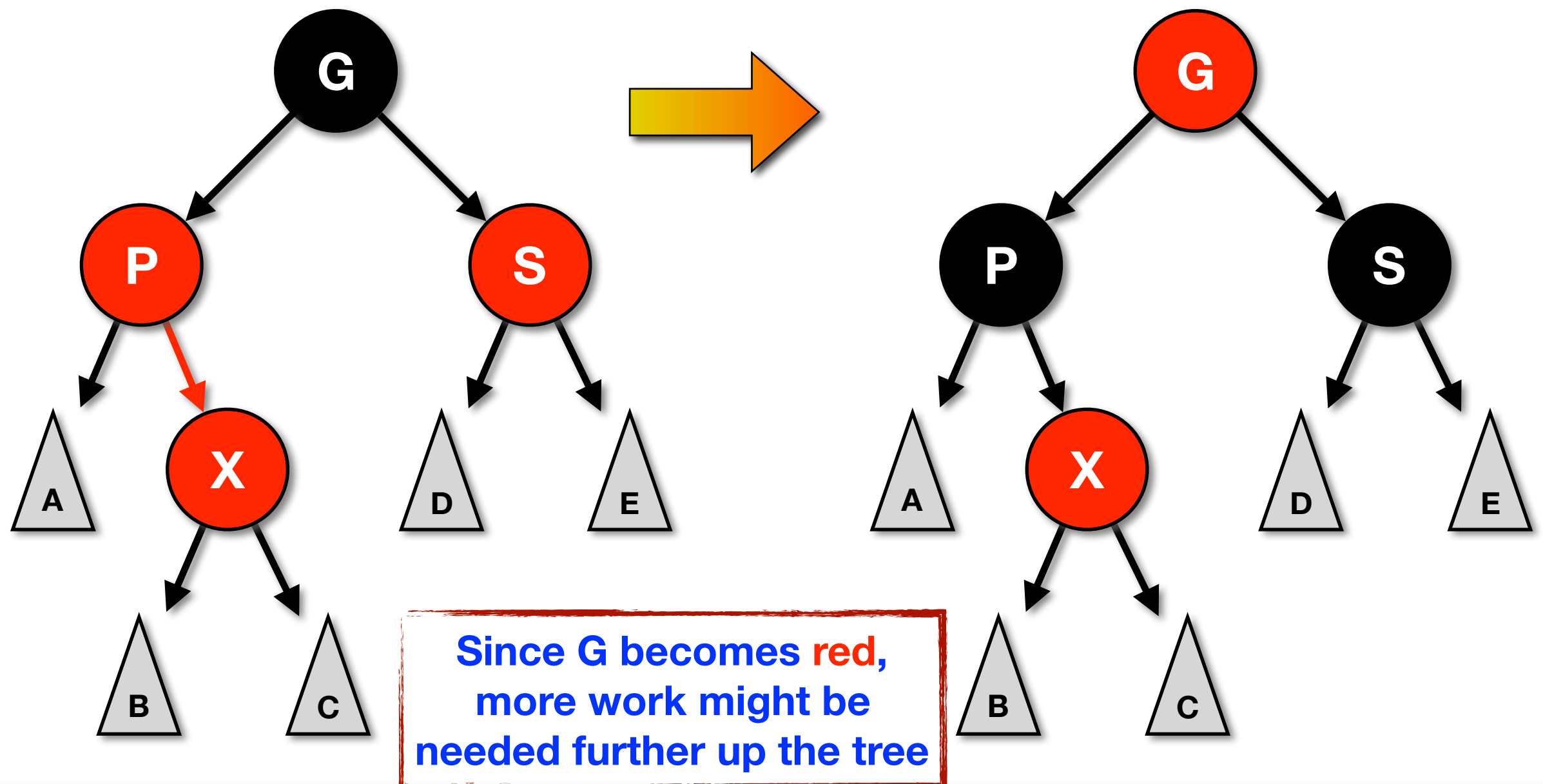
(2) Parent's sibling is **red** and new node is inserted as an inside grandchild



Fixing the Insertion Violation -- Case #2

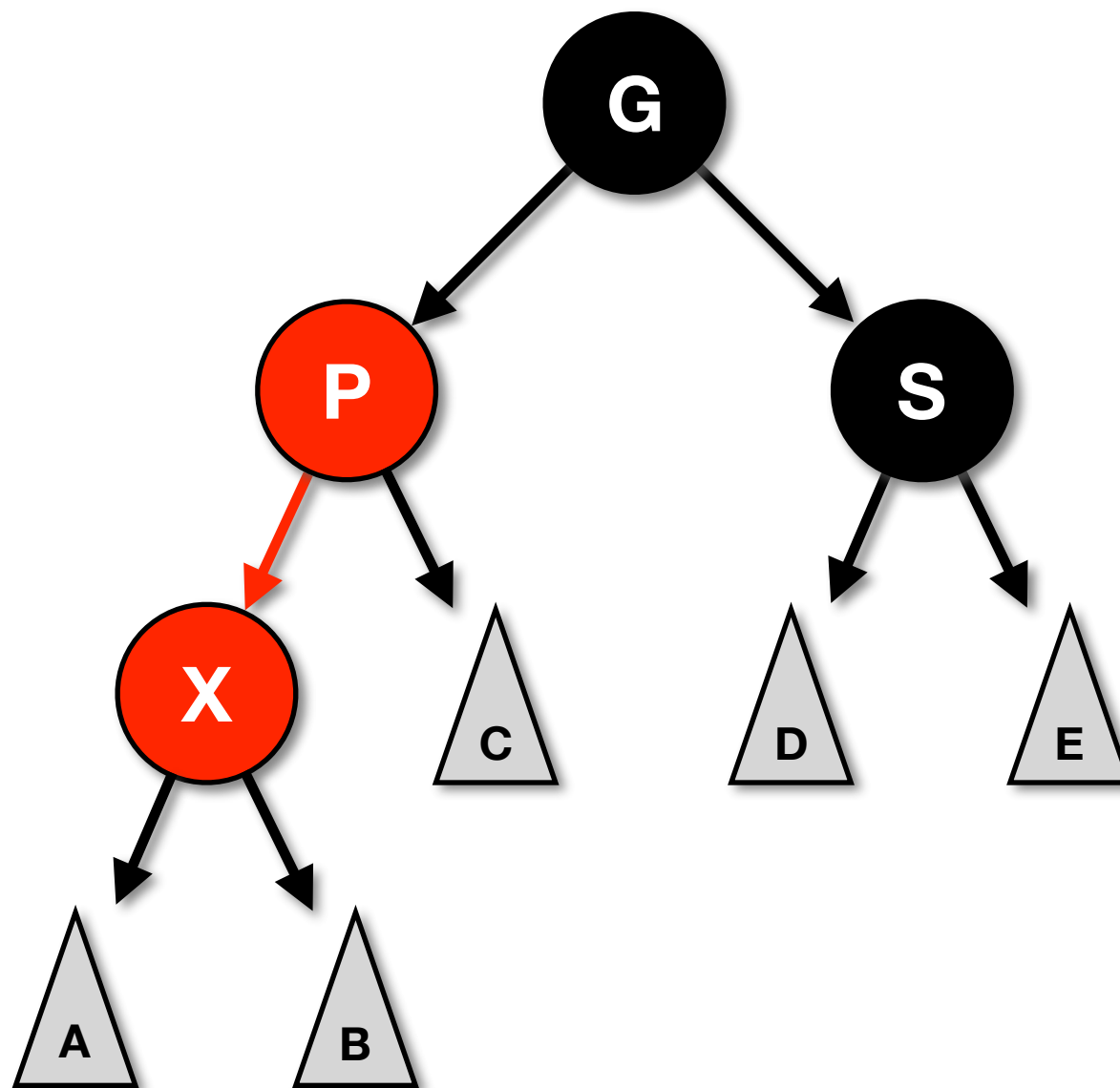
No rotation necessary

Make Parent and Sibling black,
Grandparent becomes red



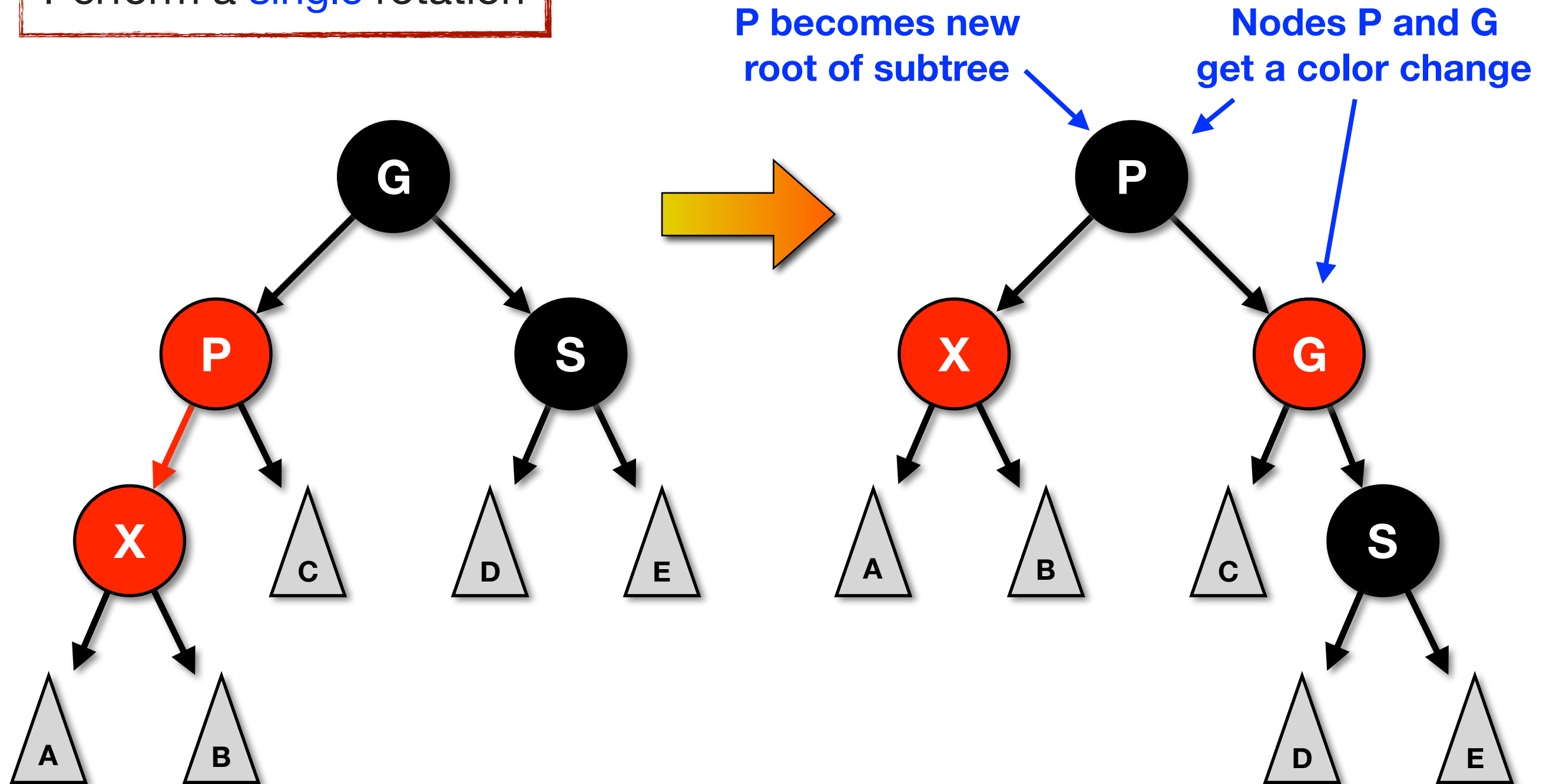
Insert Violation -- Case #3

(3) Parent's sibling is **black** and new node is inserted as an **outside grandchild**



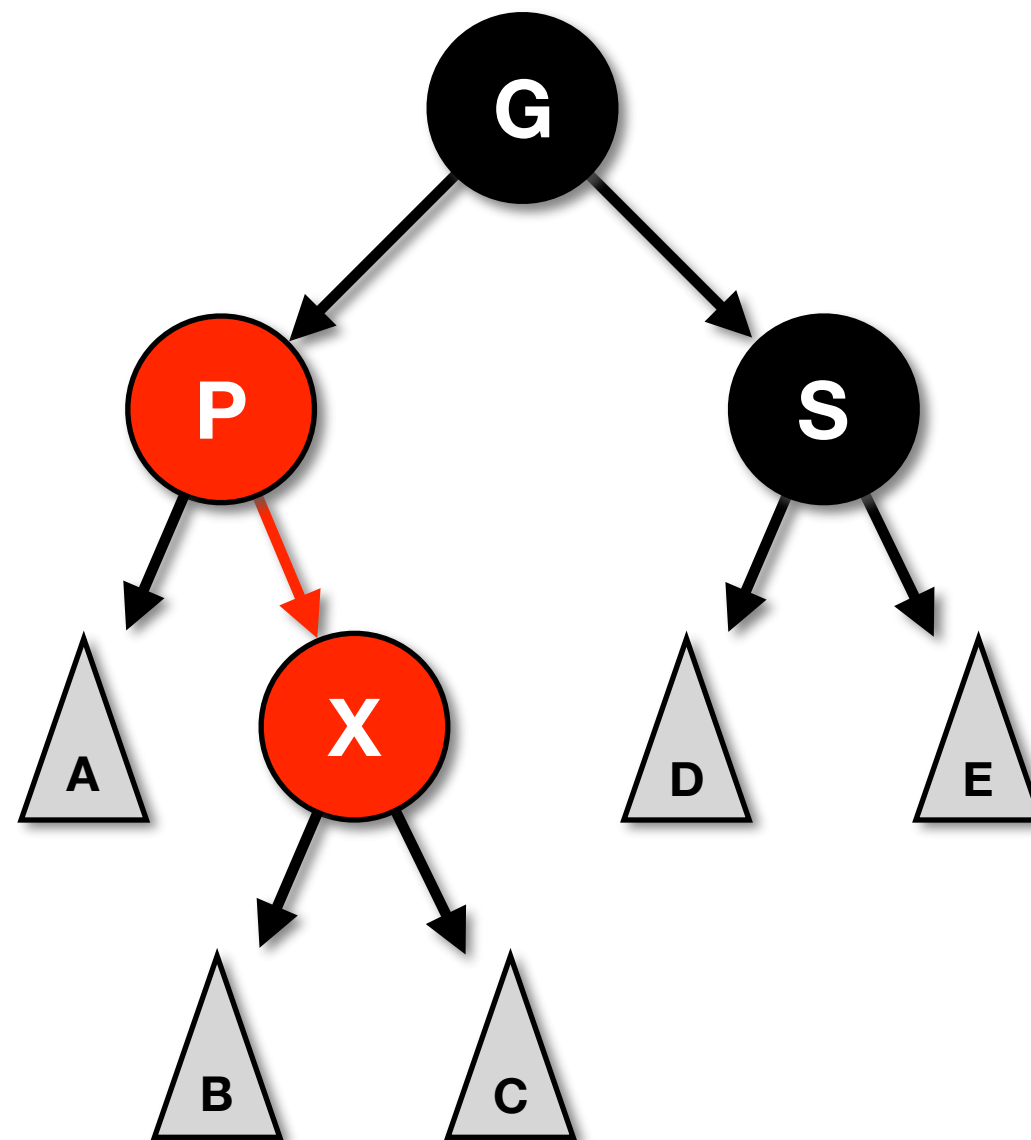
Fixing the Insertion Violation -- Case #3

Perform a **single** rotation



Insert Violation -- Case #4

(4) Parent's sibling is **black** and new node is inserted as an **inside grandchild**

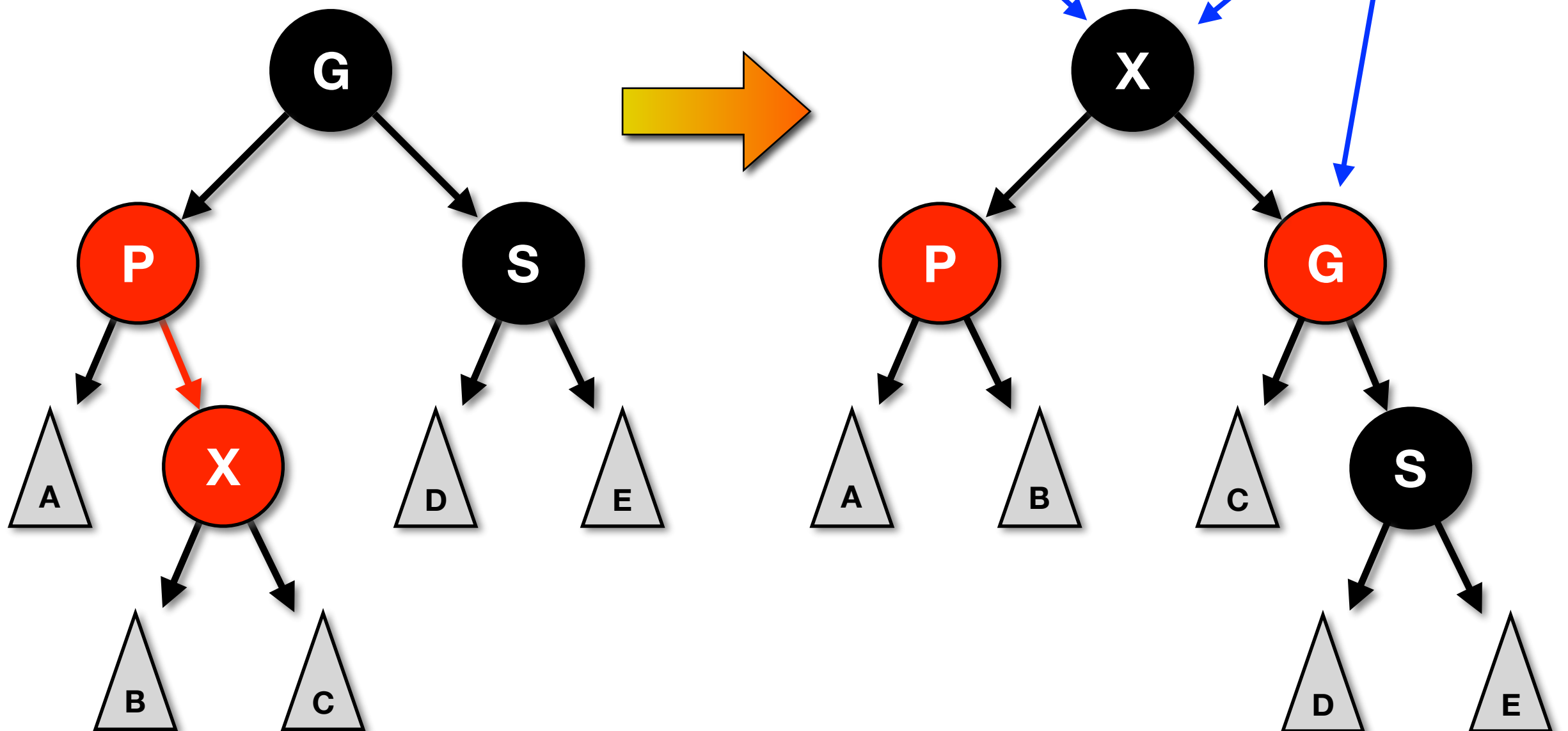


Fixing the Insertion Violation -- Case #4

Perform a **double** rotation

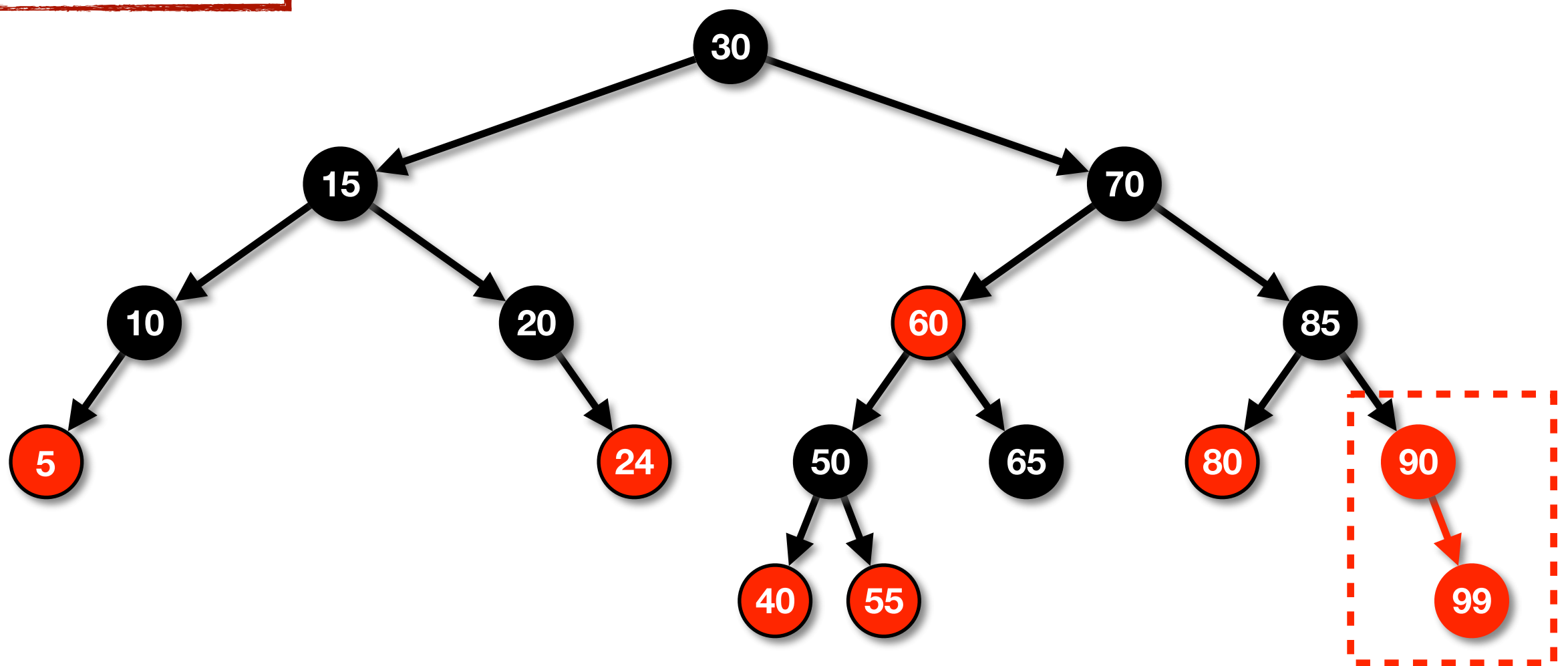
X becomes new
root of subtree

Nodes X and G
get a color change



Red-Black Tree Insertion Example

insert(99)



(1) Every node is colored either red or black

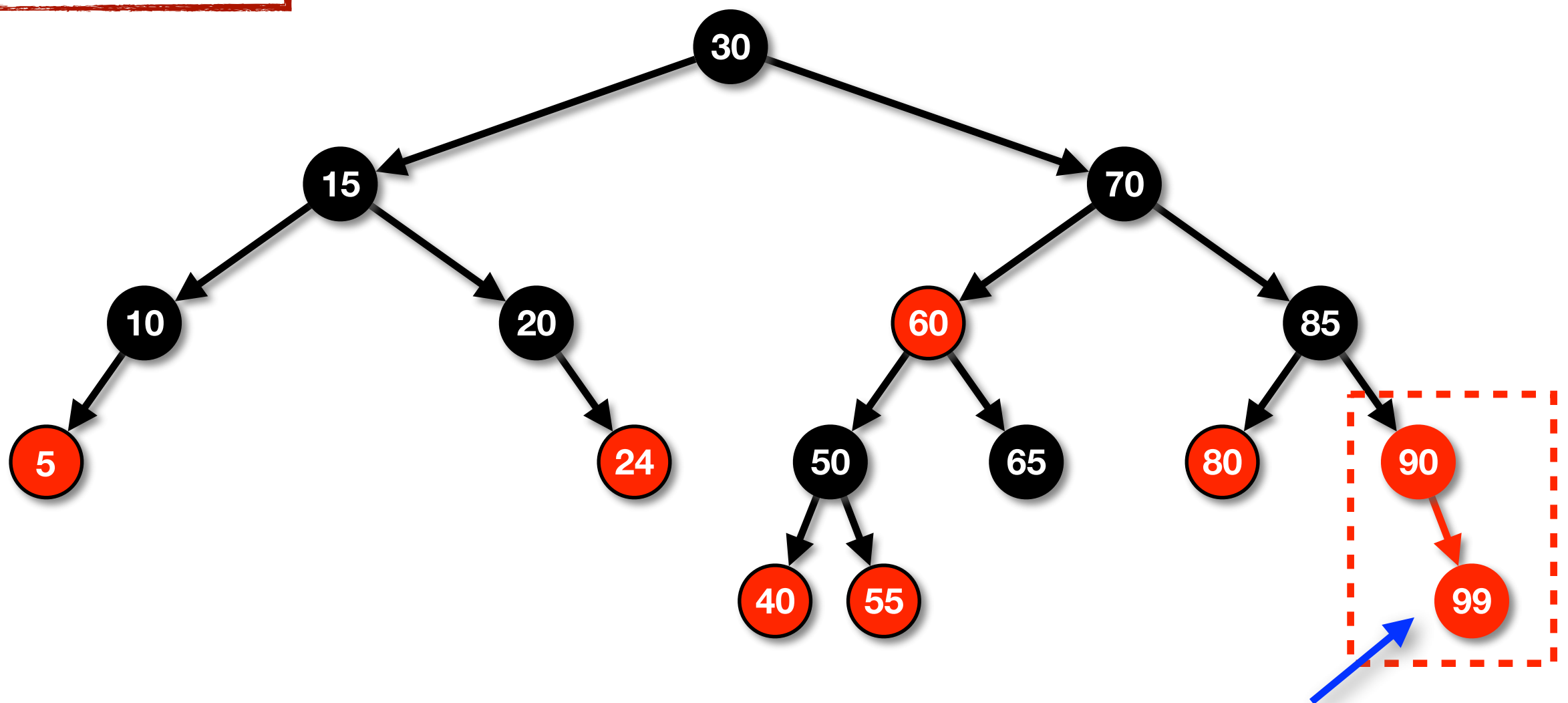
(2) The root node is black

✗ (3) If a node is red, its children must be black

(4) Every path from a node to a null link must contain the same number of black nodes

Red-Black Tree Insertion Example

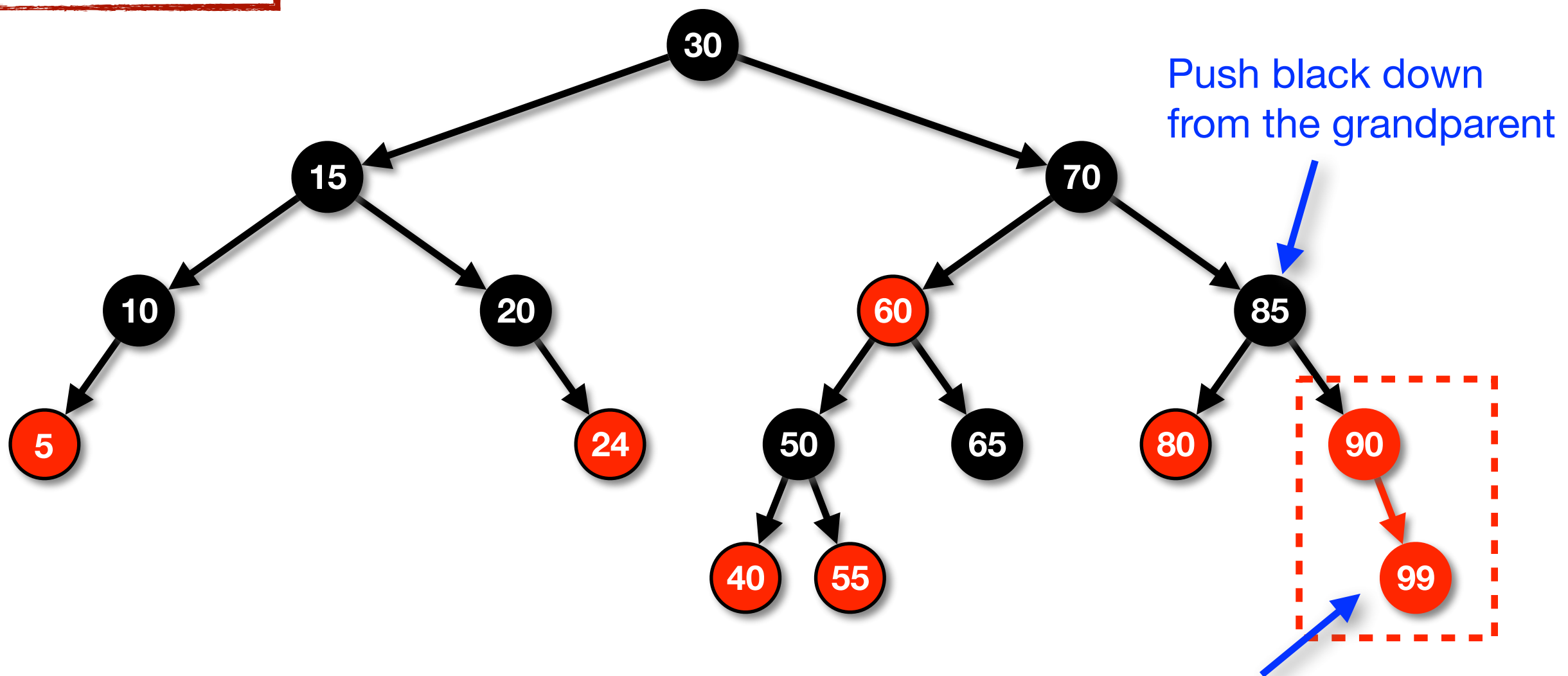
`insert(99)`



Case #1: Parent's sibling is **red** and new node is inserted as an outside grandchild

Red-Black Tree Insertion Example

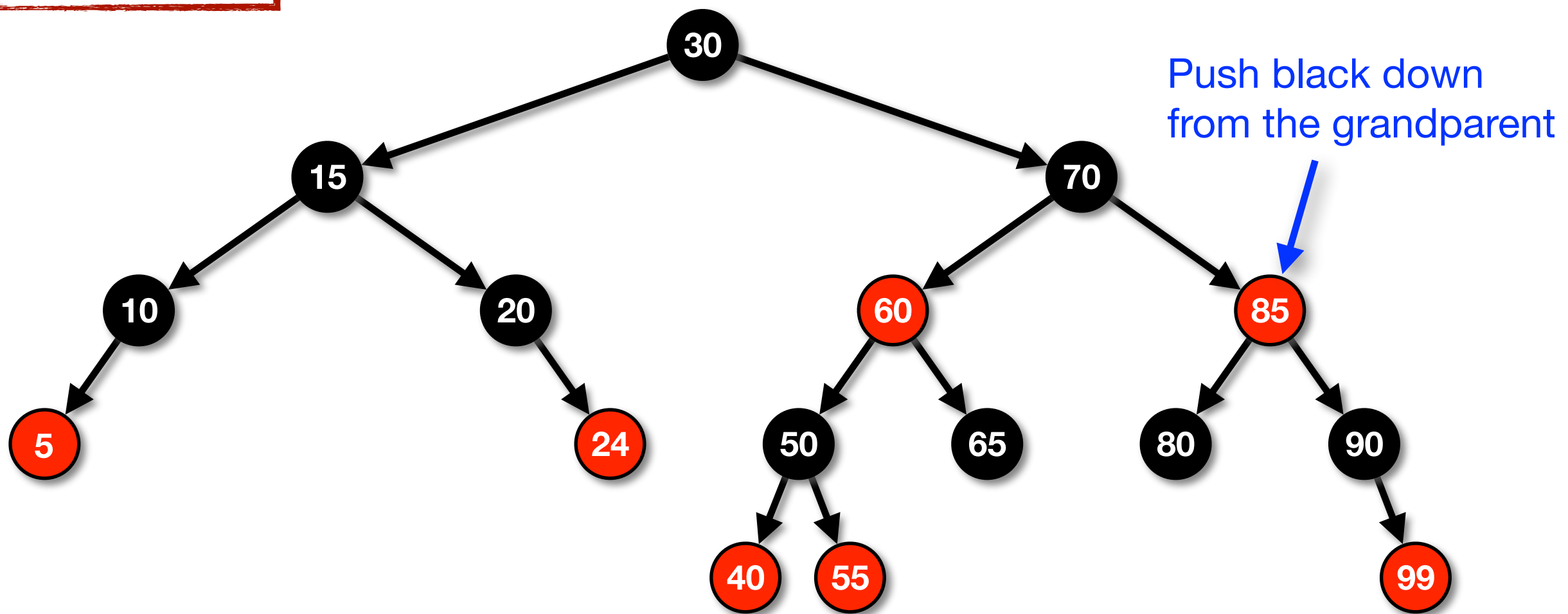
`insert(99)`



Case #1: Parent's sibling is **red** and new node is inserted as an outside grandchild

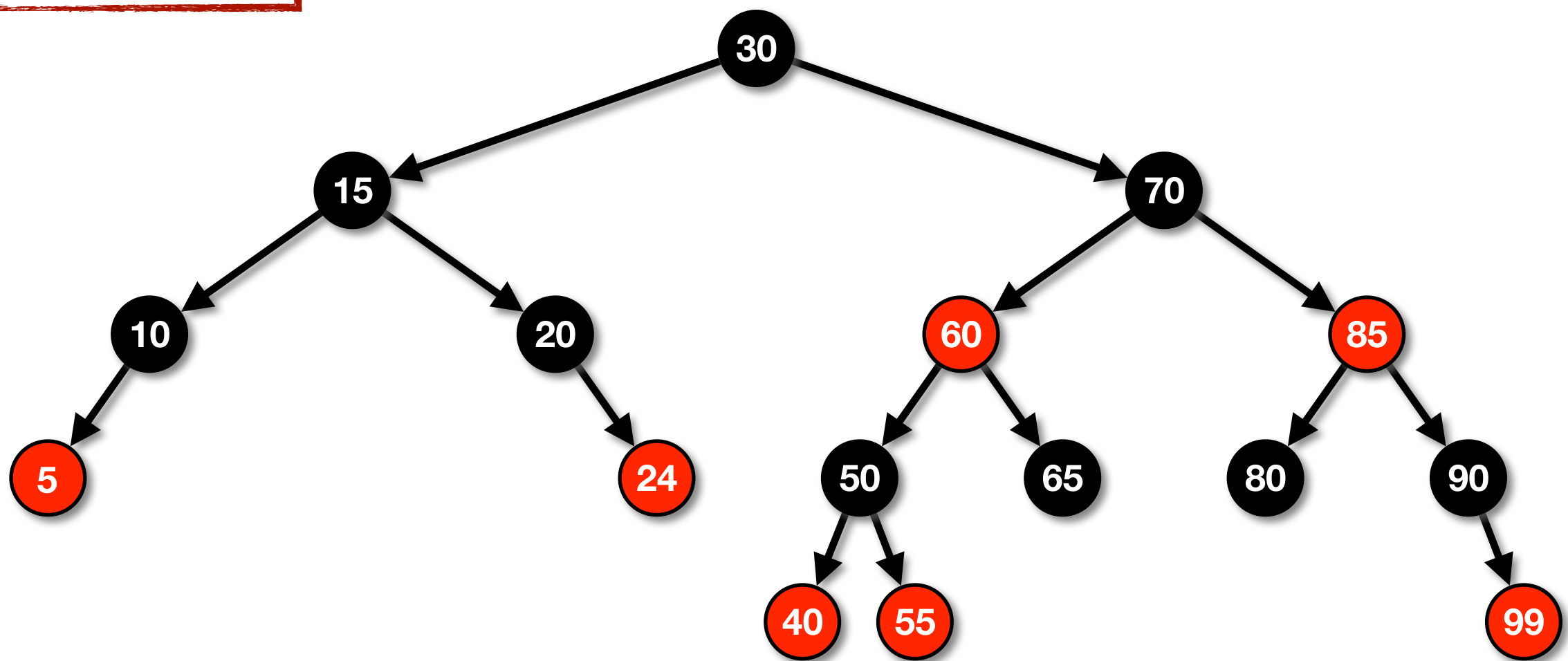
Red-Black Tree Insertion Example

`insert(99)`



Red-Black Tree Insertion Example

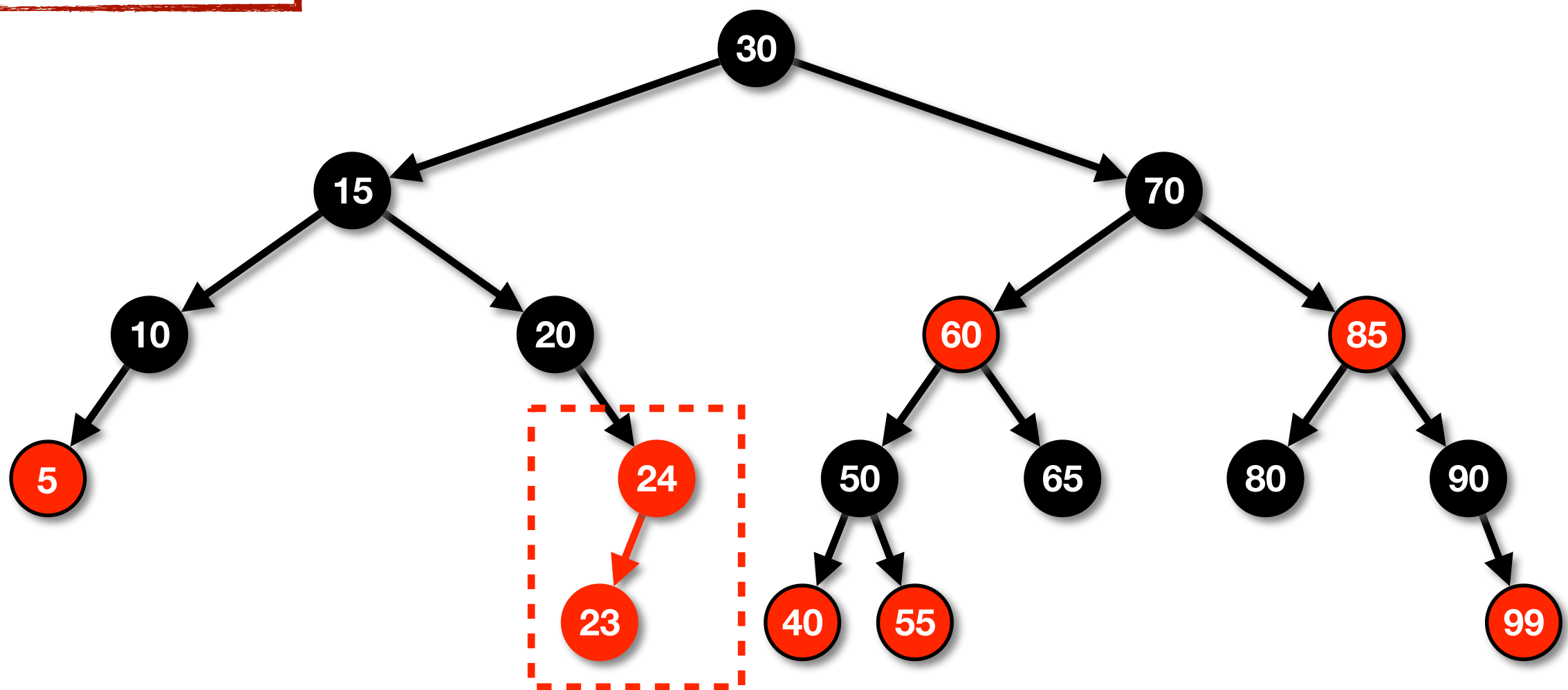
`insert(99)`



Balanced

Red-Black Tree Insertion Example

insert(23)



(1) Every node is colored either red or black

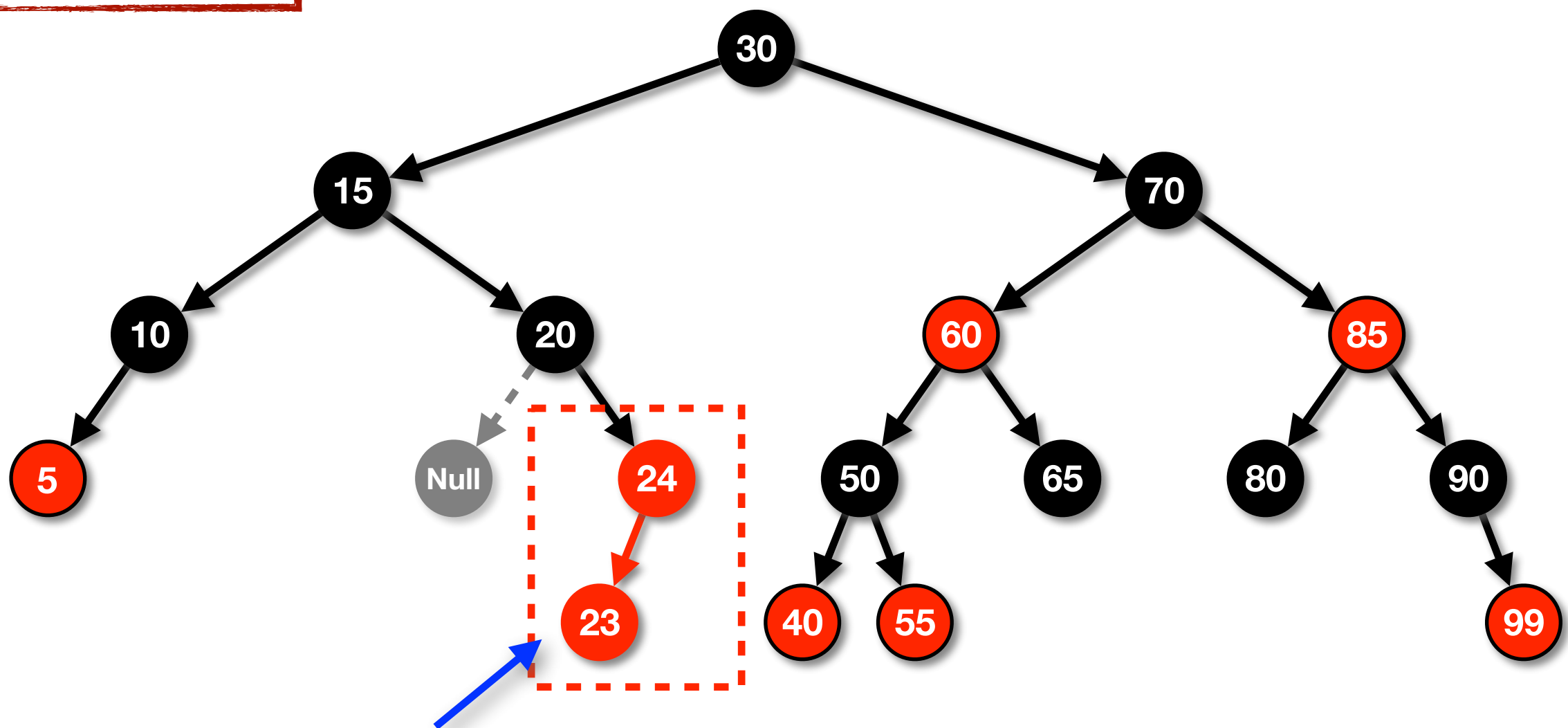
(2) The root node is black

✗ (3) If a node is red, its children must be black

(4) Every path from a node to a null link must contain the same number of black nodes

Red-Black Tree Insertion Example

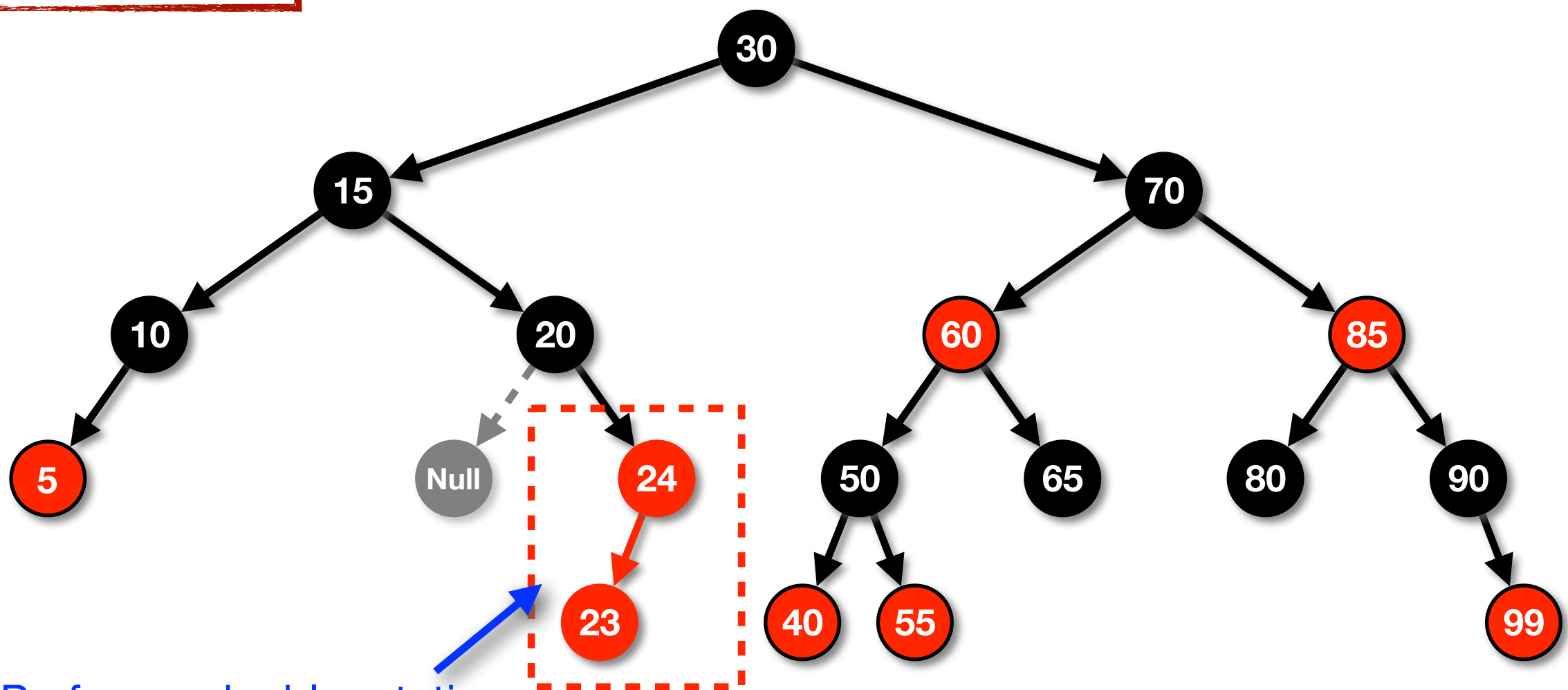
`insert(23)`



Case #4: Parent's sibling is **black** and new node is inserted as an inside grandchild

Red-Black Tree Insertion Example

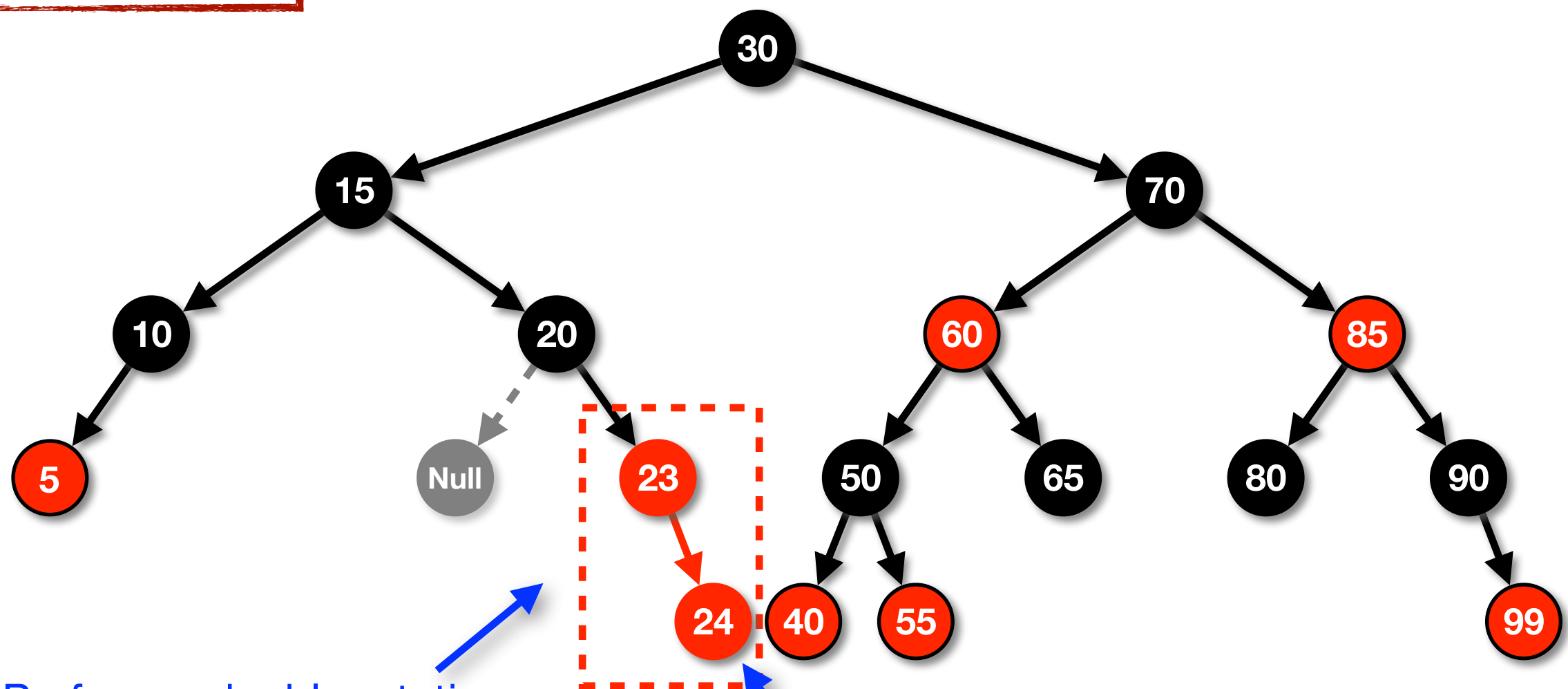
insert(23)



Perform a double rotation:
First, rotate between nodes 23 & 24,
then, rotate between nodes 23 & 20
and change their colors

Red-Black Tree Insertion Example

`insert(23)`

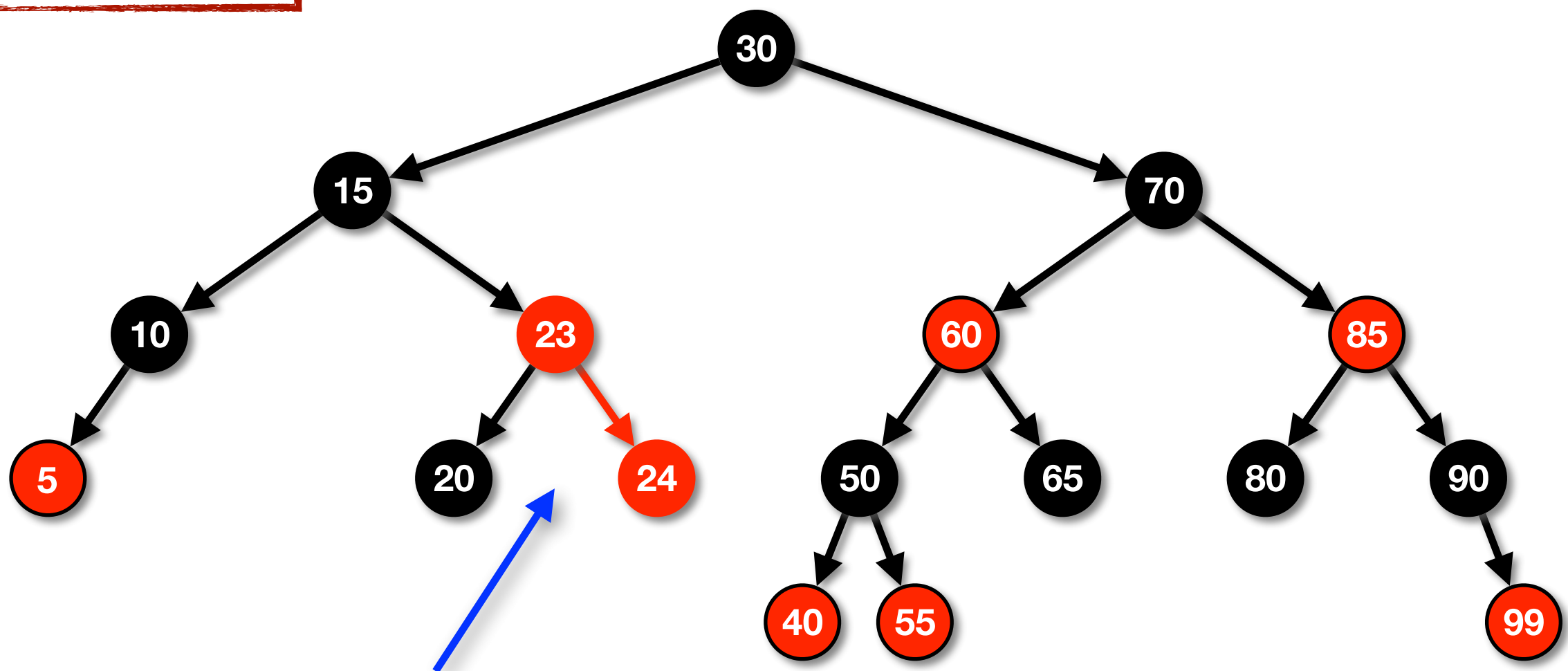


Perform a double rotation:
First, rotate between nodes 23 & 24,
then, rotate between nodes 23 & 20
and change their colors

HEY, LOOK who showed up after the first rotation!! It's case #3: Parent's sibling is **black** and new node is inserted as an outside grandchild

Red-Black Tree Insertion Example

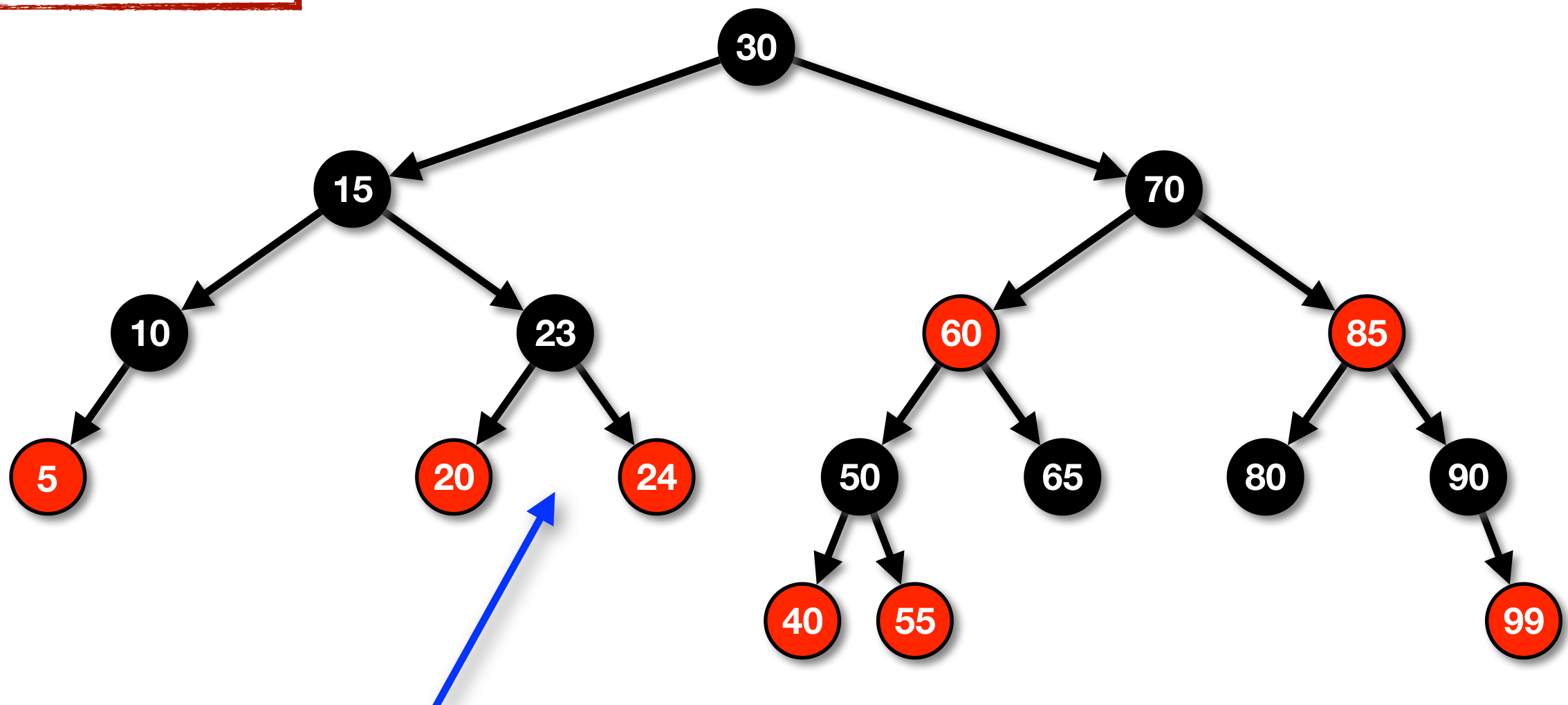
insert(23)



Perform a double rotation:
First, rotate between nodes 23 & 24,
then, rotate between nodes 23 & 20
and change their colors

Red-Black Tree Insertion Example

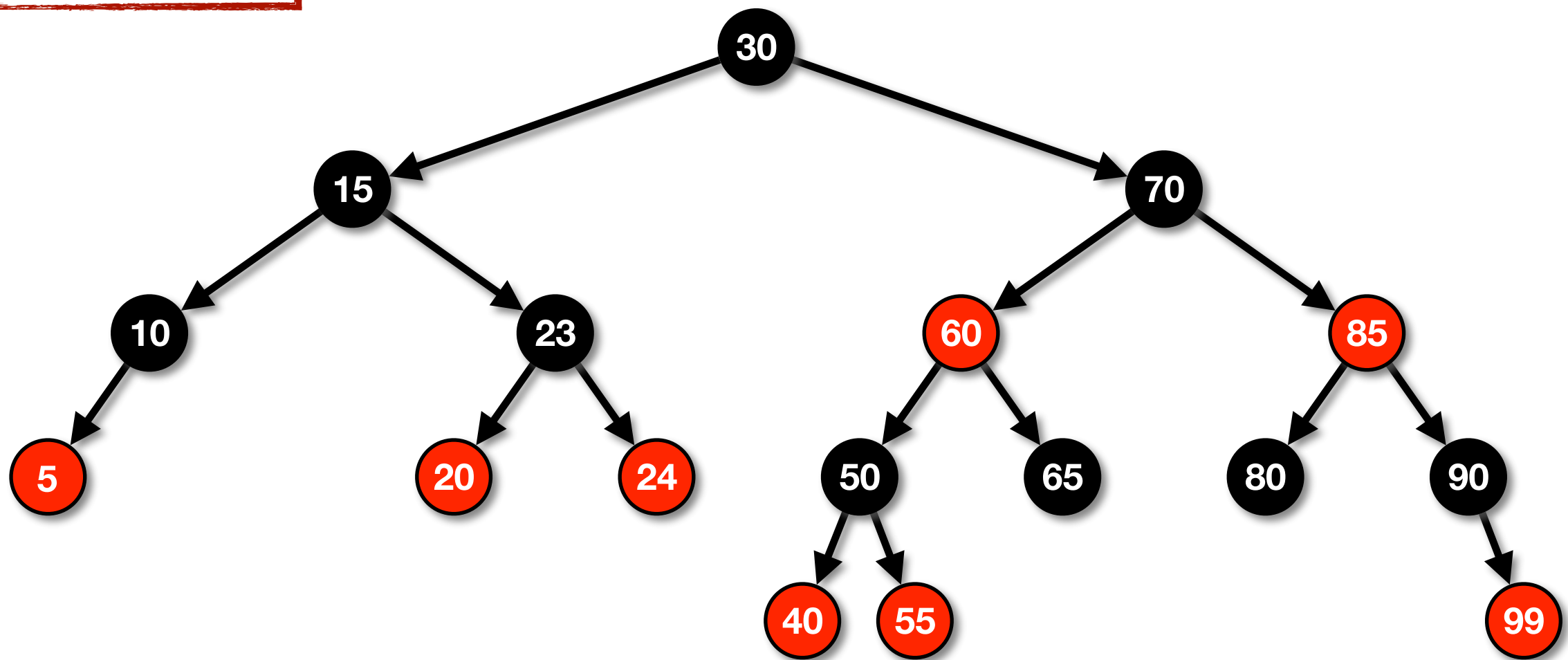
`insert(23)`



Finally, recolor
nodes 23 and 20

Red-Black Tree Insertion Example

`insert(23)`



Balanced

Red-Black Tree Deletion

- **Start with standard BST deletion**

- In BST deletion, when deleting a node with two children, the node was not actually deleted -- its contents were replaced
 - (1) Contents replaced with contents from successor or predecessor
 - (2) Then the successor/predecessor node was deleted
 - Successor/predecessor can have 0 or 1 child (if the node had two children, then one of its children would be the successor/predecessor)
- Replacing the contents of a node do not affect the coloring of the node, therefore the properties of the red-black tree are not altered
- Removal of the node with 0 or 1 child needs consideration as it can potentially cause violations in the red-black tree

Red-Black Tree Deletion

- If the node removed is **red**, then there are no problems and the 4 properties of the **red-black** tree will still be true -- **there is nothing to do in this case**
- If the node removed is **black**, then there is a new violation in the **red-black** tree -- violation of property #4

(4) Every path from a node to a null node must contain the same number of black nodes

- It is necessary to work back up the tree and account for the missing black node