CS350: Data Structures

Red-Black Trees

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Red-Black Tree

- **An alternative to AVL trees**

- **Insertion can be done in a bottom-up or a top-down fashion**
  - An AVL tree uses a pass down the tree for an insertion and a second pass back up the tree to update node heights and potentially rebalance the tree
  - A top-down insertion into a red-black tree requires only a single pass down the tree
    - We'll focus on bottom-up insertion

Red-Black Tree (Cont.)

- **A red-black tree is a binary search tree that has the following properties:**

  (1) Every node is colored either red or black

  (2) The root node is black

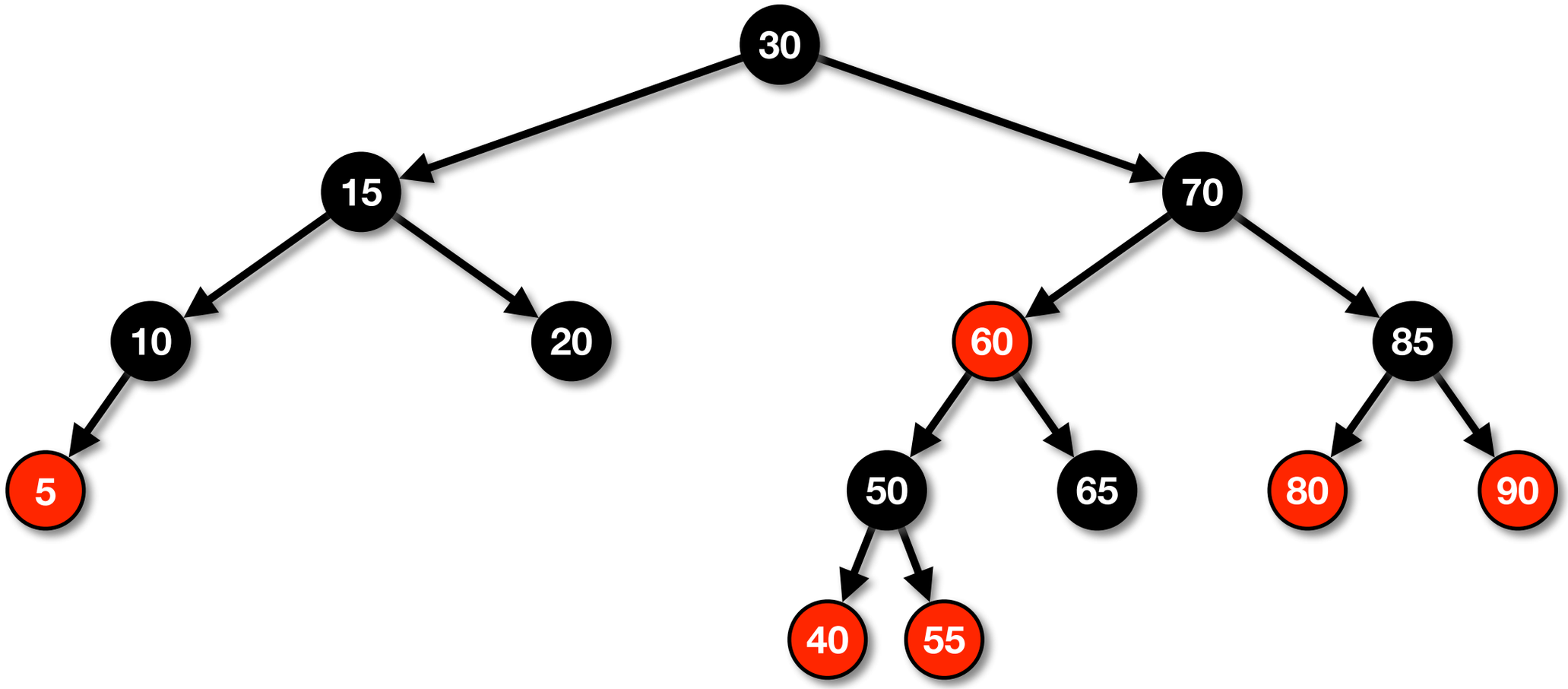
  (3) If a node is red, its children must be black

  (4) Every path from a node to a null node must contain the same number of black nodes

- **These properties must be maintained after each insertion or deletion operation**

- **A null node is considered black**

Example Red-Black Tree



(1) Every node is colored either red or black

(2) The root node is black

(3) If a node is red, its children must be black

(4) Every path from a node to a null link must contain the same number of black nodes

Red-Black Tree Insertion

- **When inserting a new leaf node, what color should it be?**

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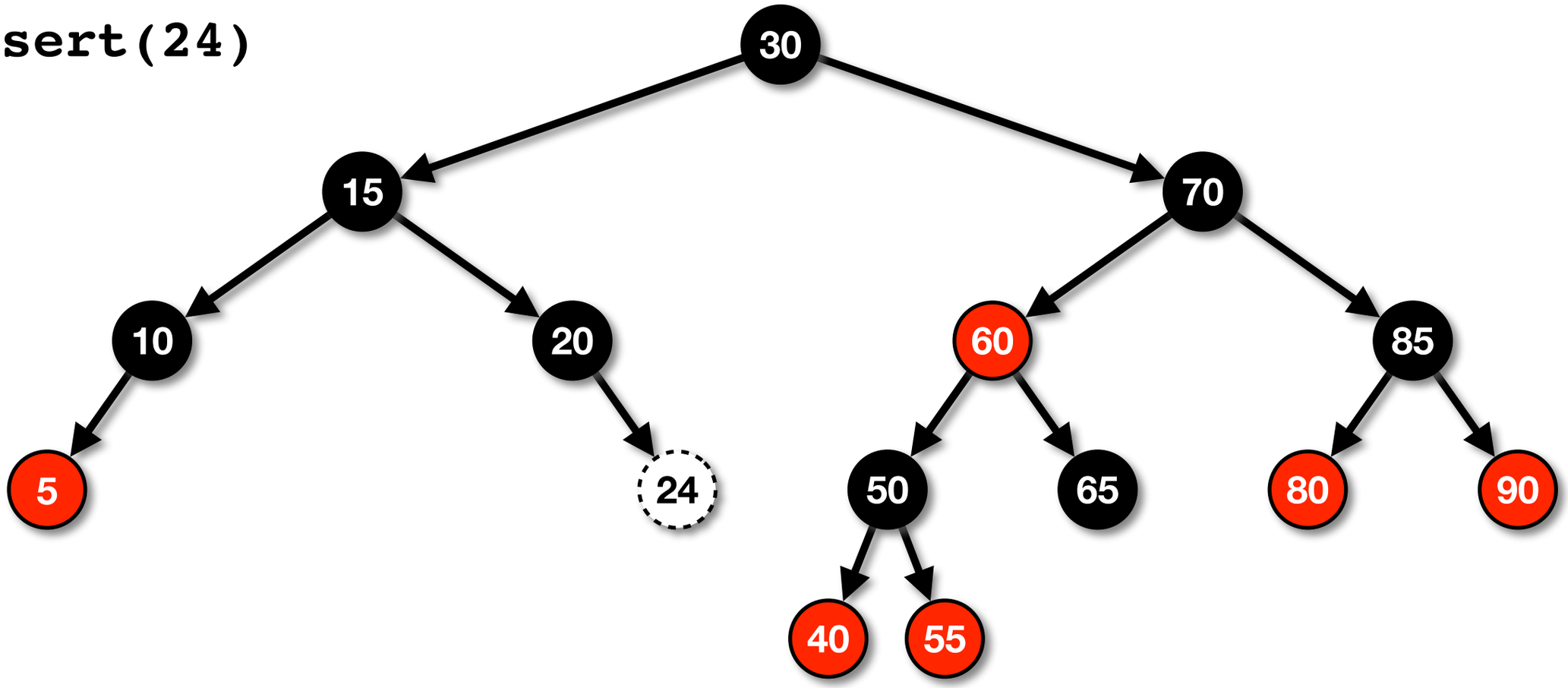
insert(24)
```



(1) Every node is colored either red or black
(2) The root node is black
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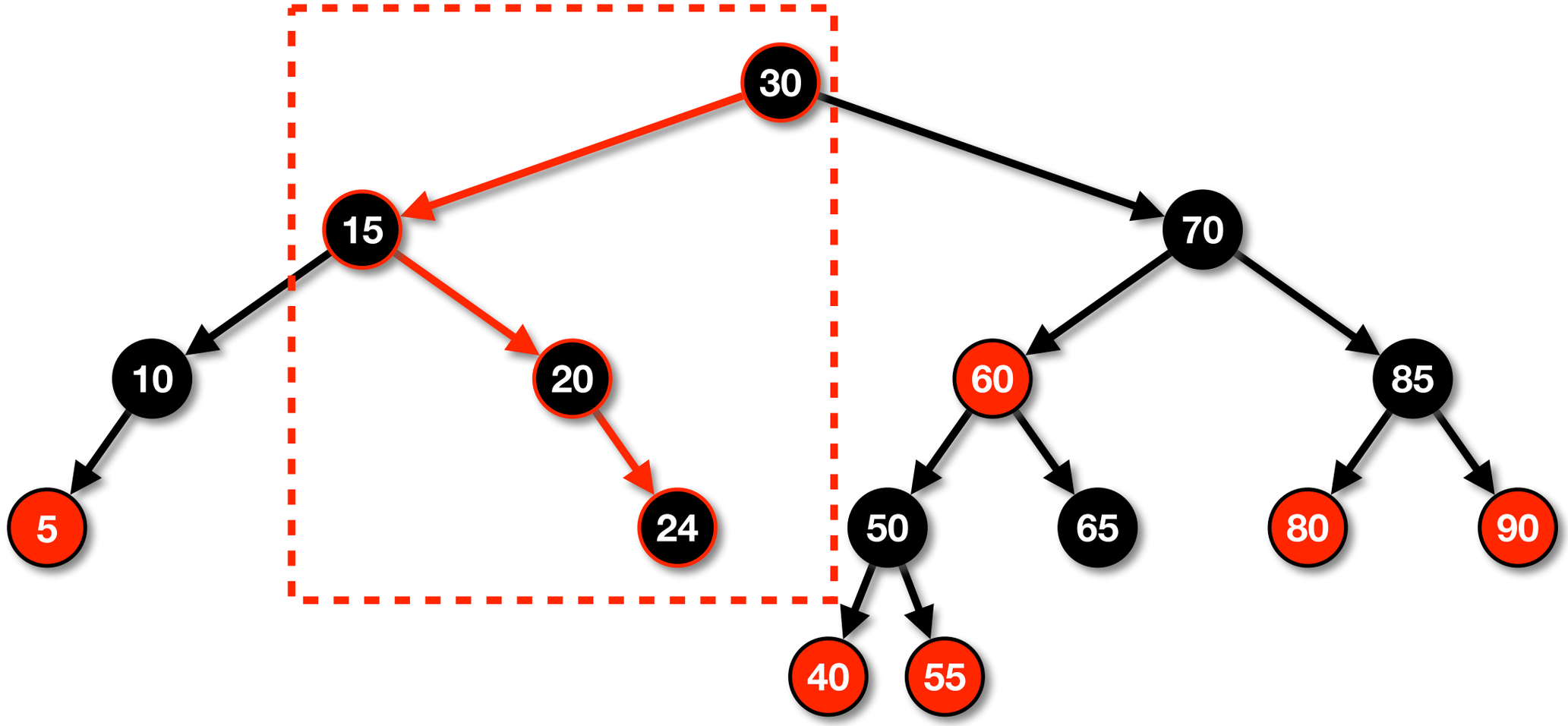
Red-Black Tree Insertion

- **Inserting as a black node violates property #4**



(1) Every node is colored either red or black

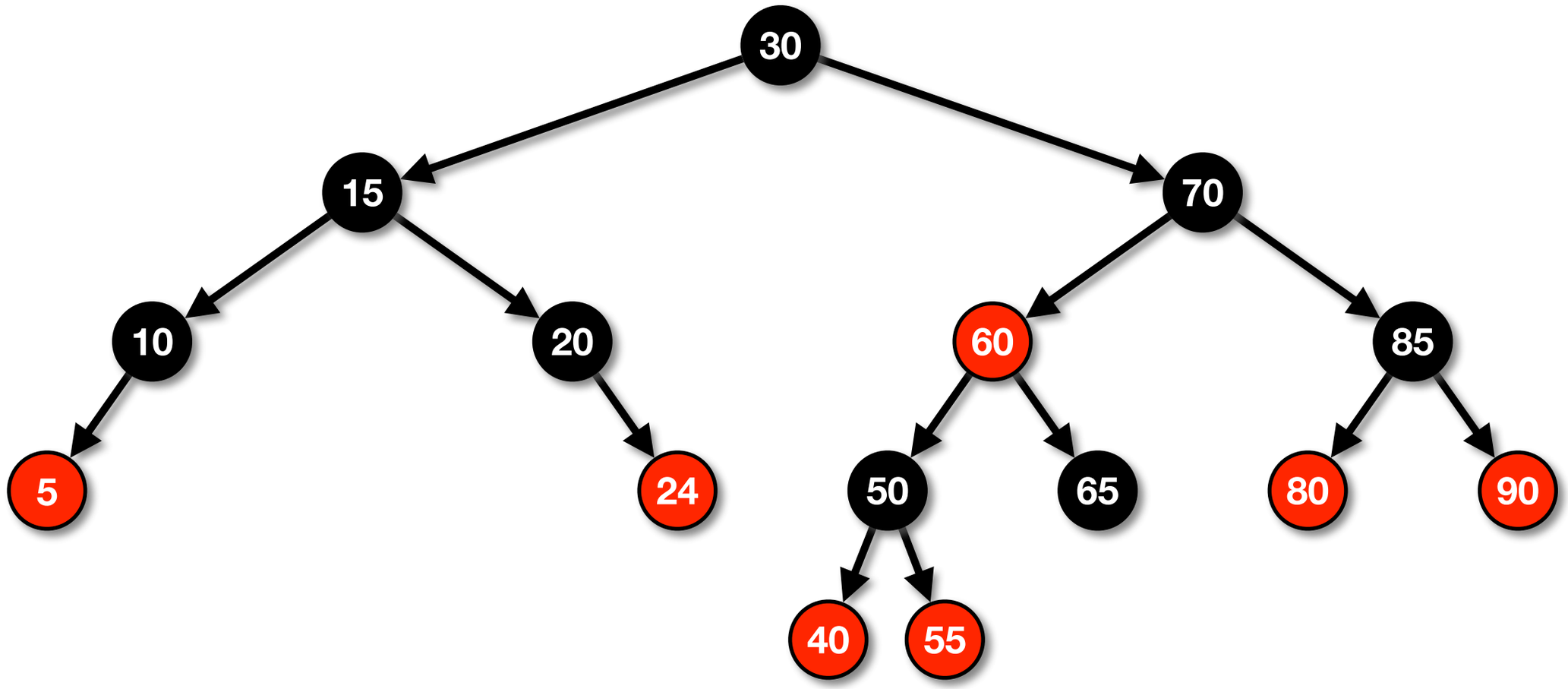
(2) The root node is black

(3) If a node is red, its children must be black

✖(4) Every path from a node to a null link must contain the same number of black nodes

Red-Black Tree Insertion

- **Inserting as a red node satisfies all four properties (in this case)**



(1) Every node is colored either red or black
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(4) Every path from a node to a null link must contain the same number of black nodes

Red-Black Tree Insertion

- **Inserting as a red may not always satisfy the four properties**

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insert(99)
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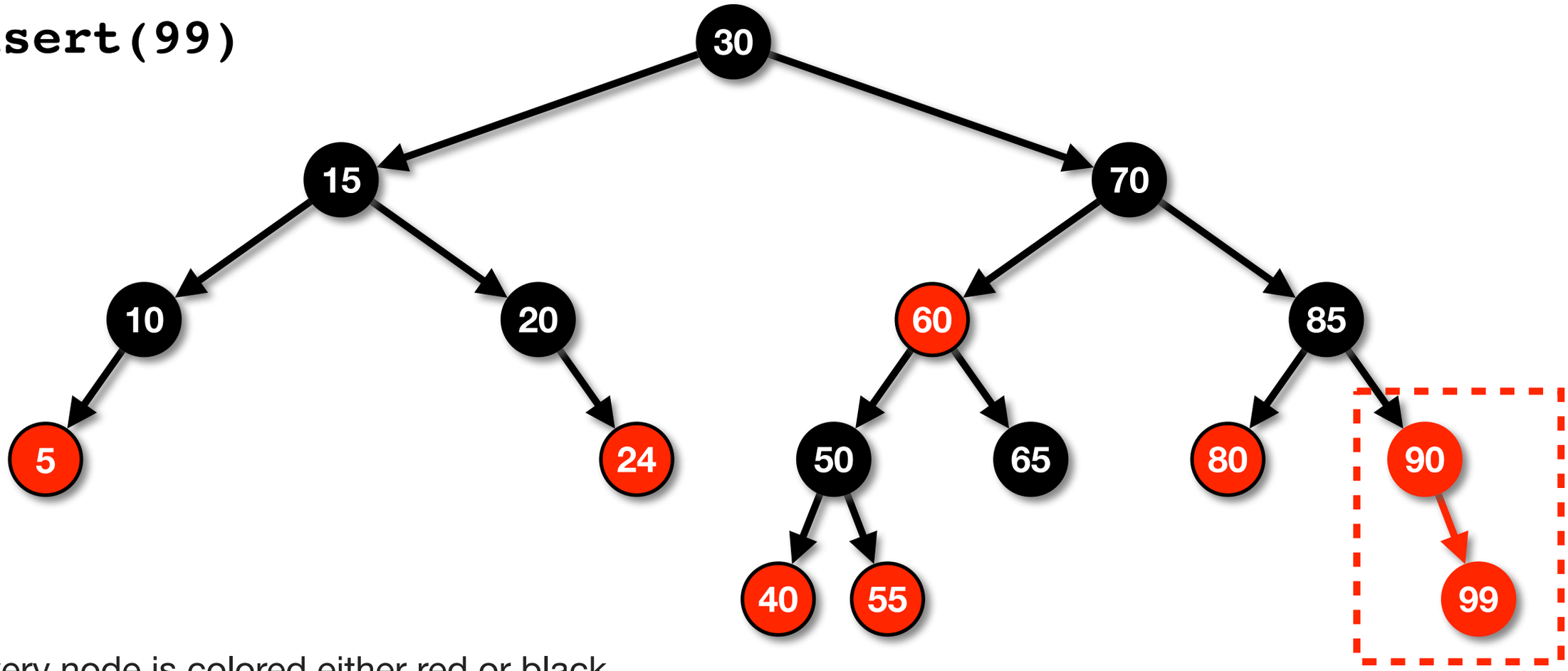

(1) Every node is colored either red or black

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Red-Black Tree Insertion

- **Easier to fix a violation of property #3 than it is to fix a violation of property #4**
  - So, **ALWAYS** insert all nodes as red nodes

- **Must repair violations of property #3 and any new violations that occur as a result of the tree modification**
  - Operations for repairing the tree include:
    - Single and Double Rotations (similar to AVL trees)
    - Color changes

Red-Black Tree Insertion (Bottom-Up)

- **Insert nodes into a red-black tree using the standard binary search tree insertion**
  - Make the newly inserted node red
  - If the parent of the newly inserted node is black, then no violations have occurred and the insertion is complete
  - If the parent of the newly inserted node is red, then property #3 has been violated and must be fixed through rotations and recoloring
    - Four different cases must be considered

Red-Black Tree Insertion -- Violations

- **The four cases to consider when property #3 is violated (i.e. when a red node is inserted as the child of another red node)**

  (1) Parent's sibling is **red** and new node is inserted as an **outside** grandchild

  (2) Parent's sibling is **red** and new node is inserted as an **inside** grandchild

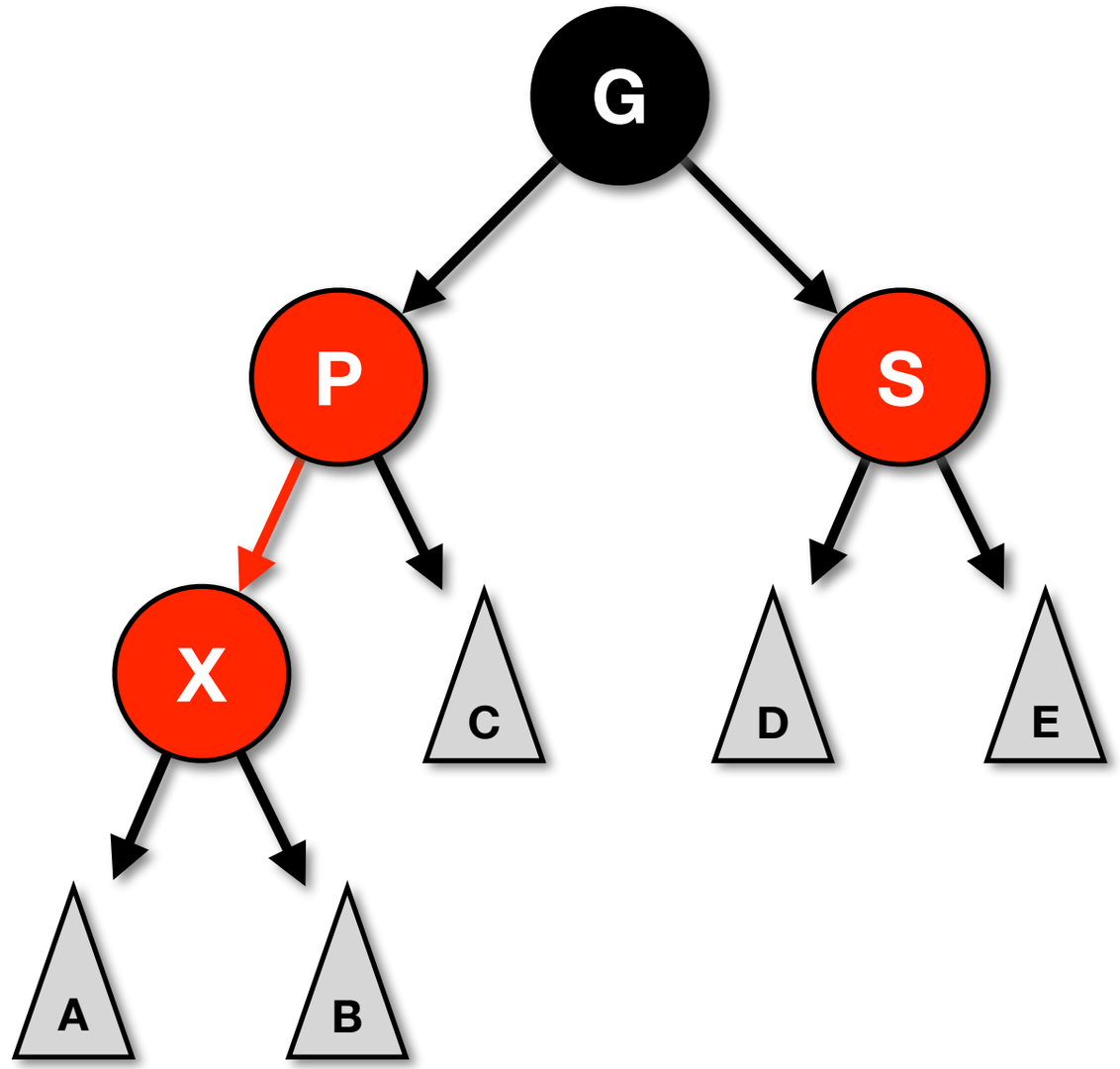
  (3) Parent's sibling is **black** and new node is inserted as an **outside** grandchild

  (4) Parent's sibling is **black** and new node is inserted as an **inside** grandchild

- **There is a different approach to fix each of these cases**

Insert Violation -- **Case #1**

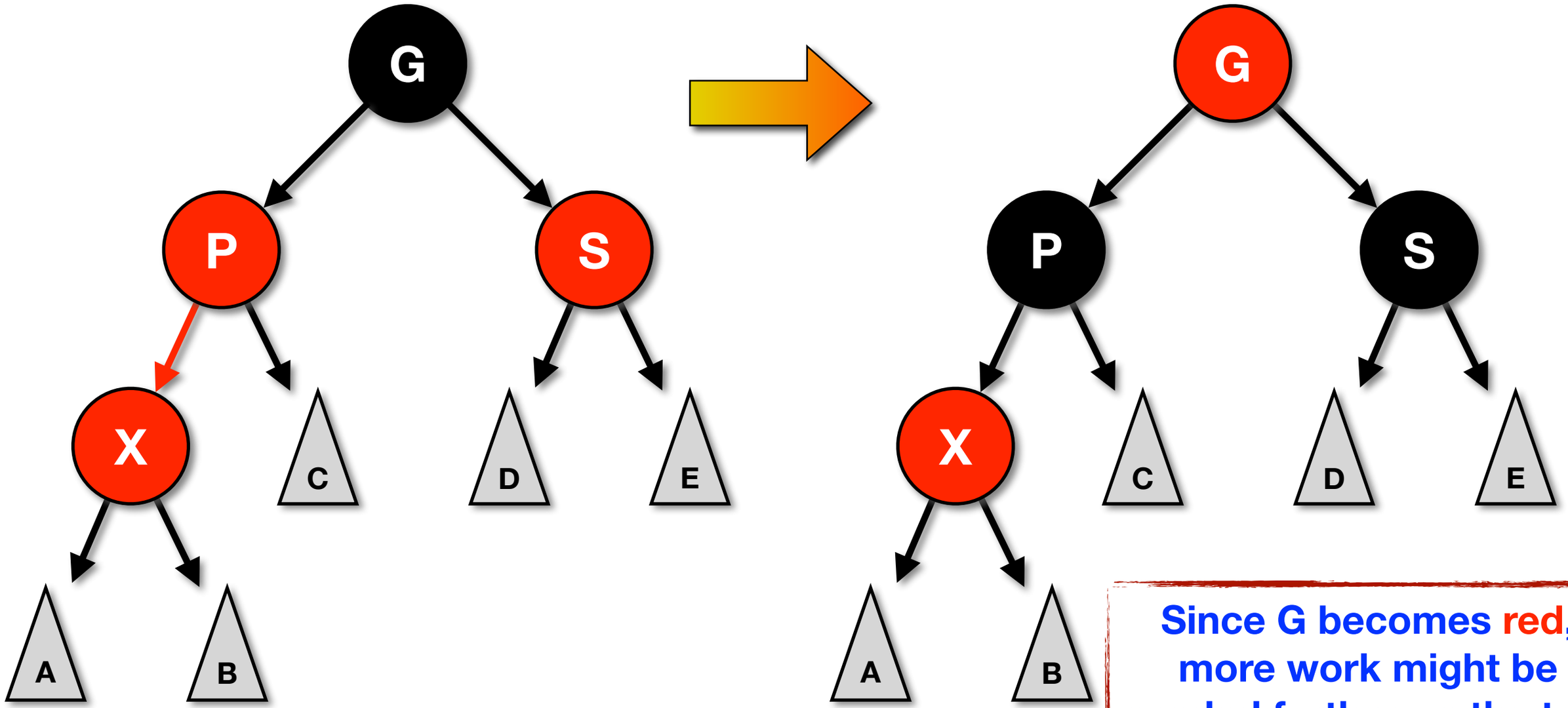
(1) Parent's sibling is **red** and new node is inserted as an **outside** grandchild



Fixing the Insertion Violation -- **Case #1**

No rotation necessary

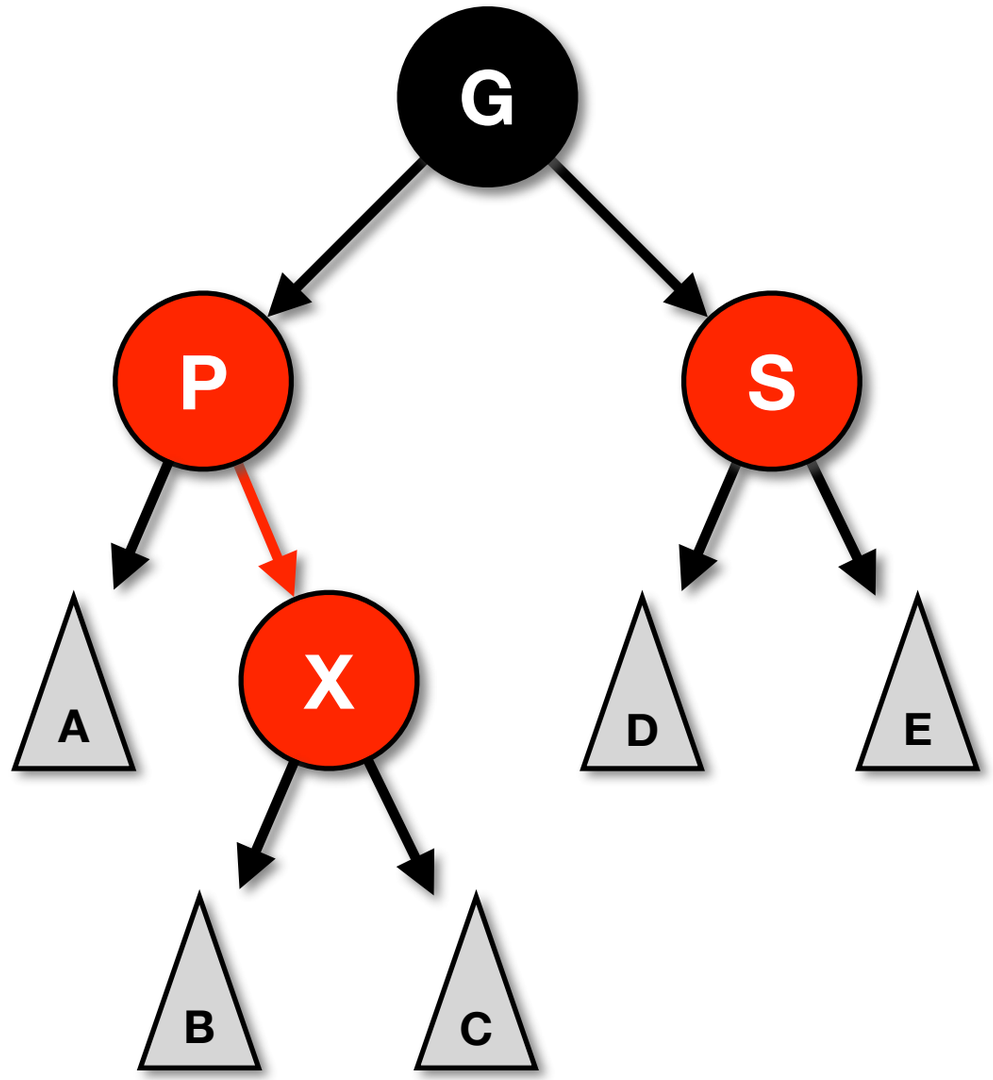
Make Parent and Sibling black, Grandparent becomes red



Since G becomes red, more work might be needed further up the tree

Insert Violation -- **Case #2**

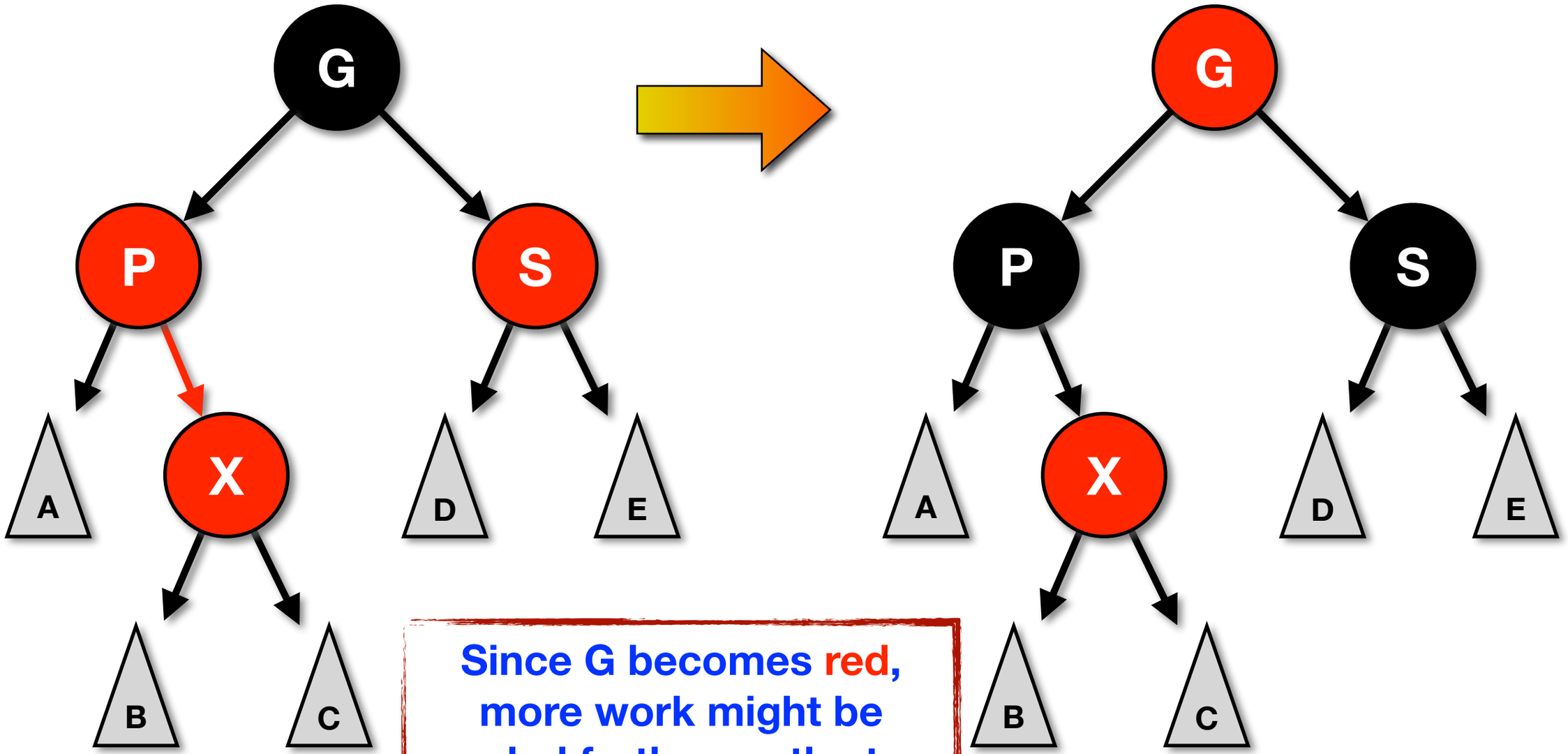
(2) Parent's sibling is **red** and new node is inserted as an **inside** grandchild



Fixing the Insertion Violation -- **Case #2**

No rotation necessary

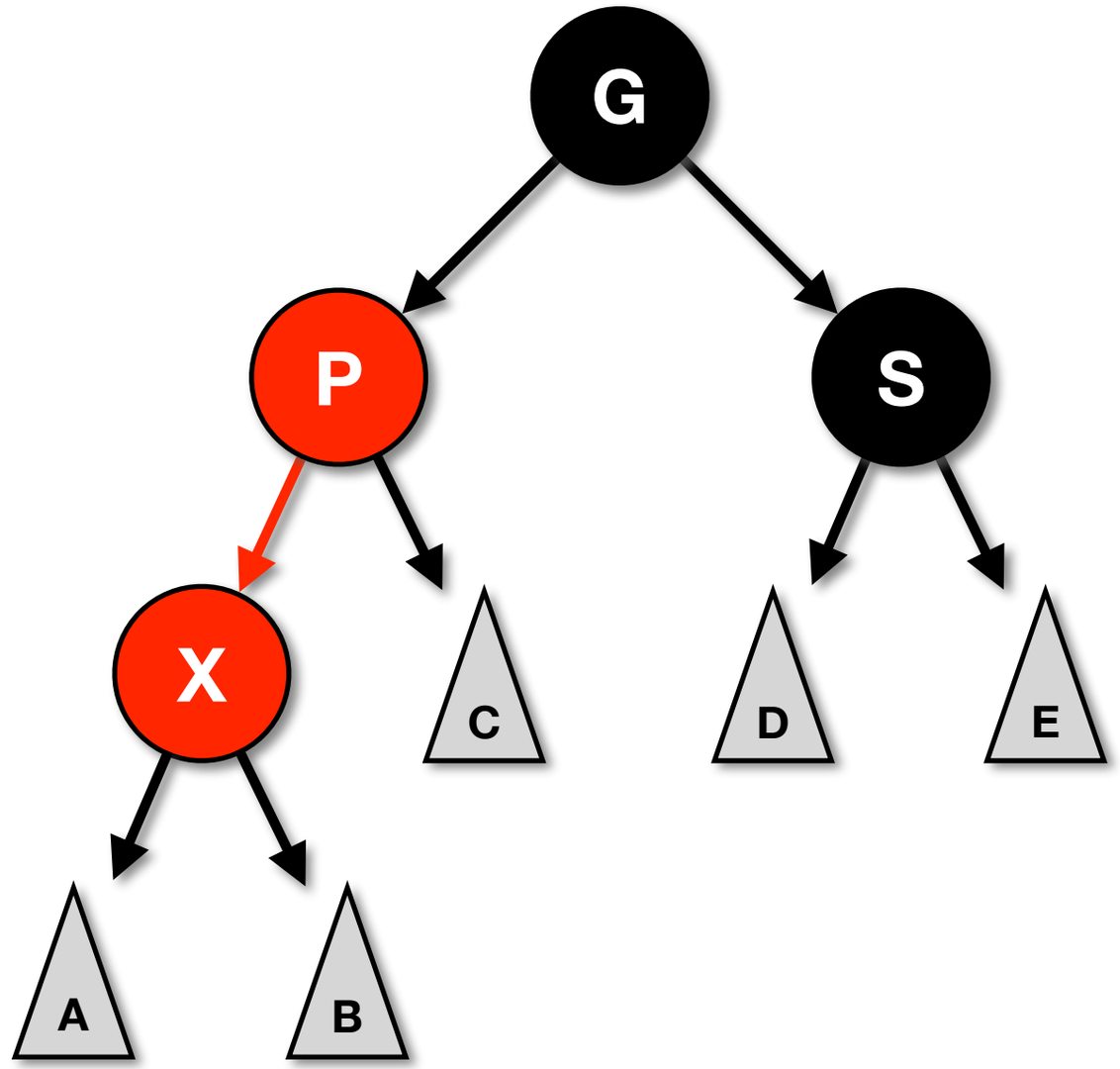
Make Parent and Sibling black, Grandparent becomes red



Since G becomes red, more work might be needed further up the tree

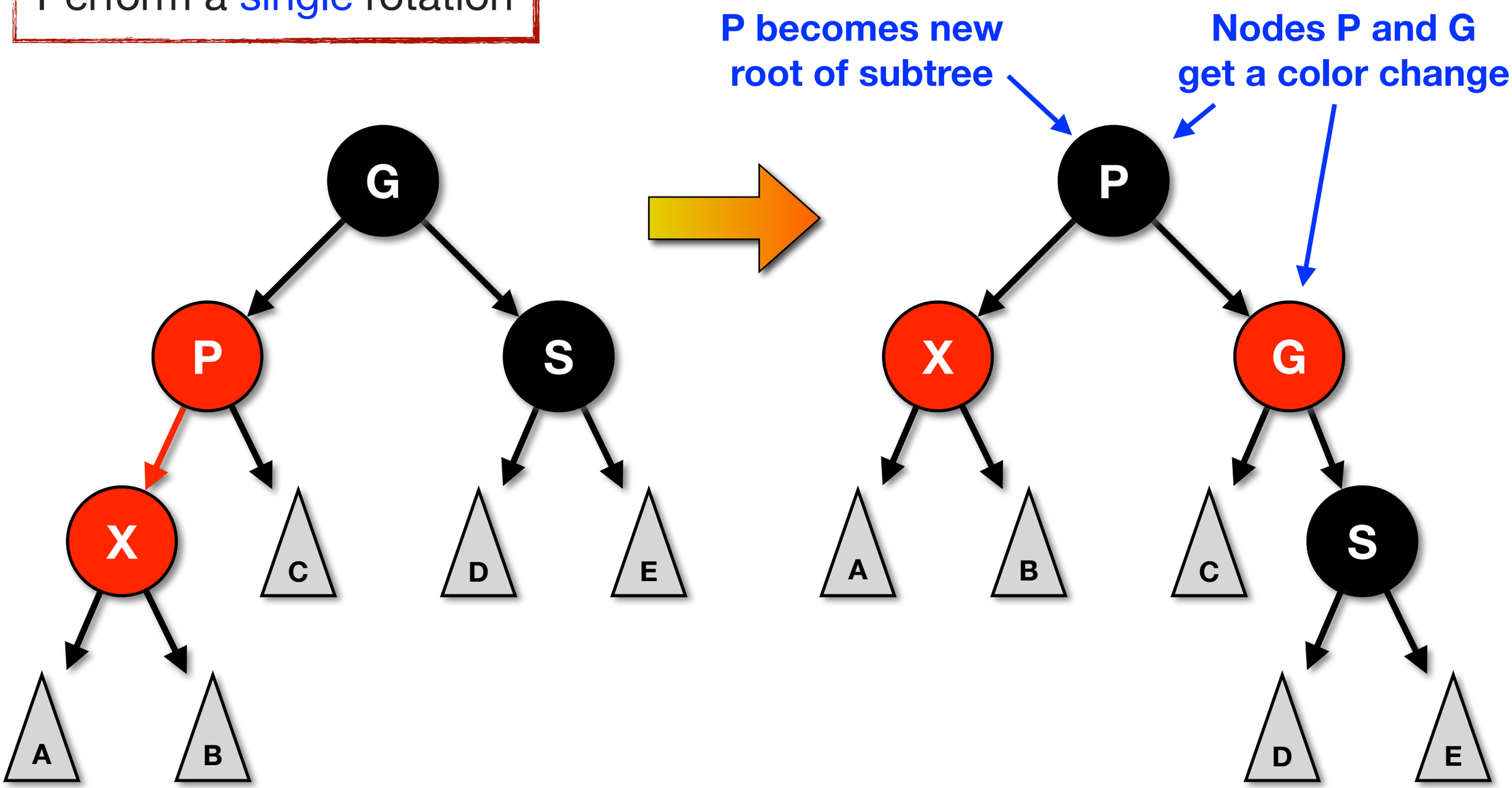
Insert Violation -- **Case #3**

(3) Parent's sibling is **black** and new node is inserted as an **outside** grandchild



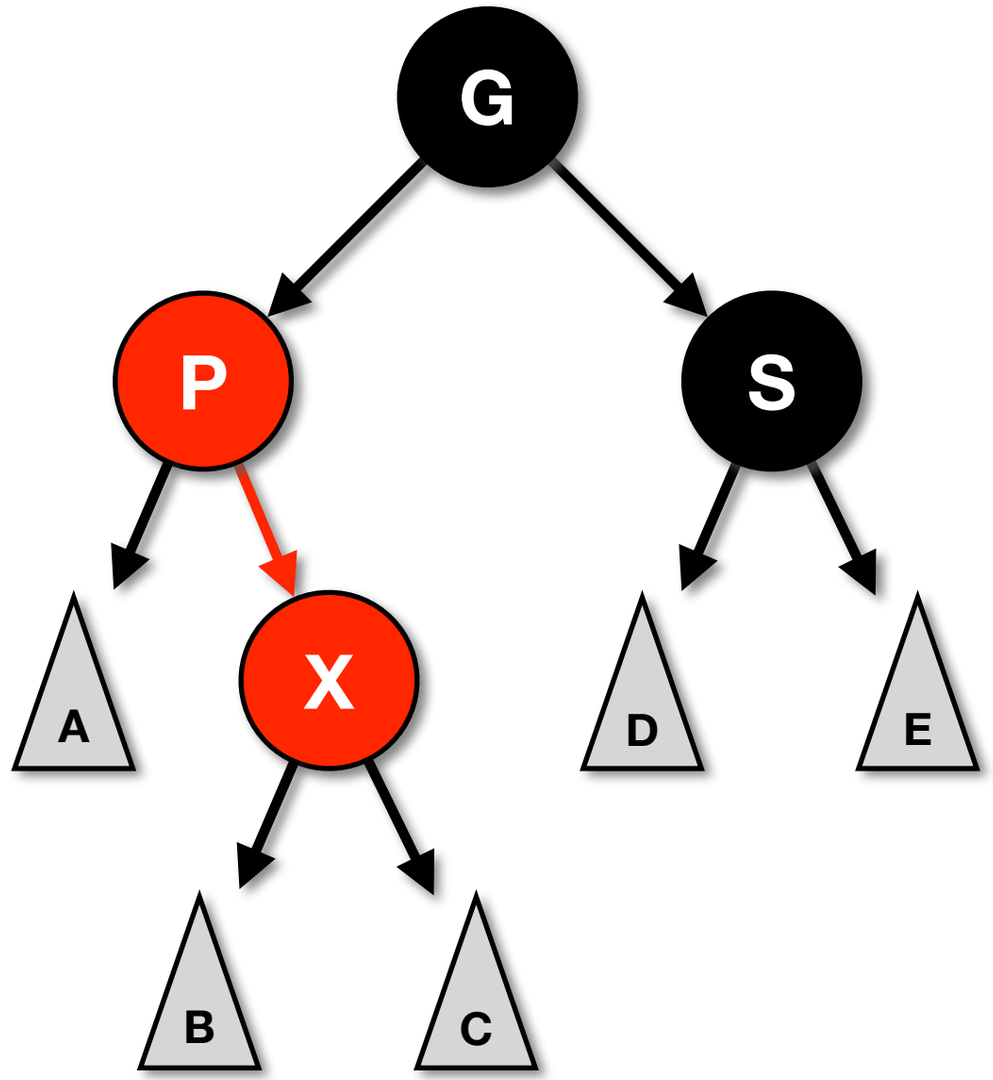
Fixing the Insertion Violation -- **Case #3**

Perform a single rotation



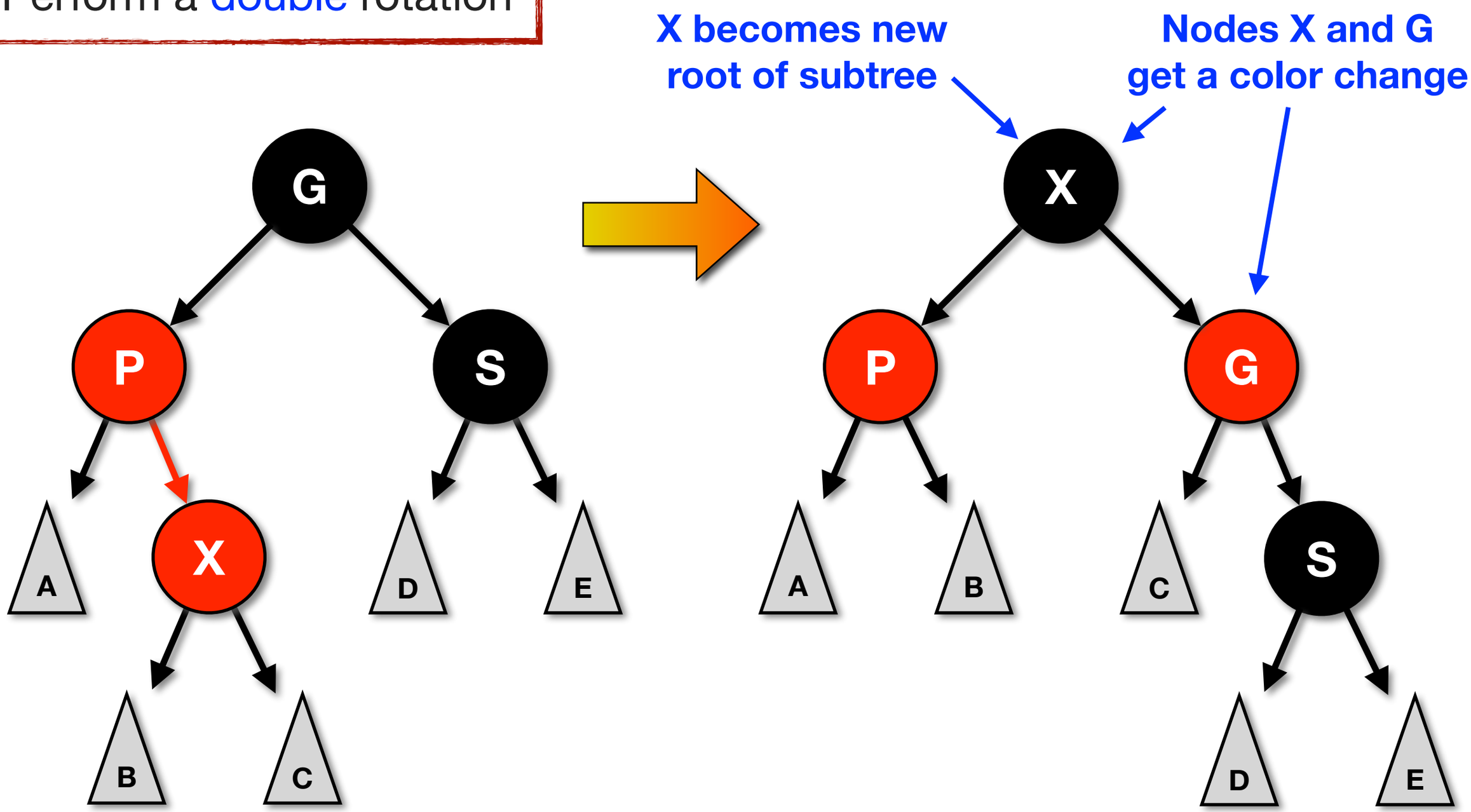
Insert Violation -- **Case #4**

(4) Parent's sibling is **black** and new node is inserted as an **inside** grandchild



Fixing the Insertion Violation -- **Case #4**

Perform a double rotation



Red-Black Tree Insertion Example

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insert(99)
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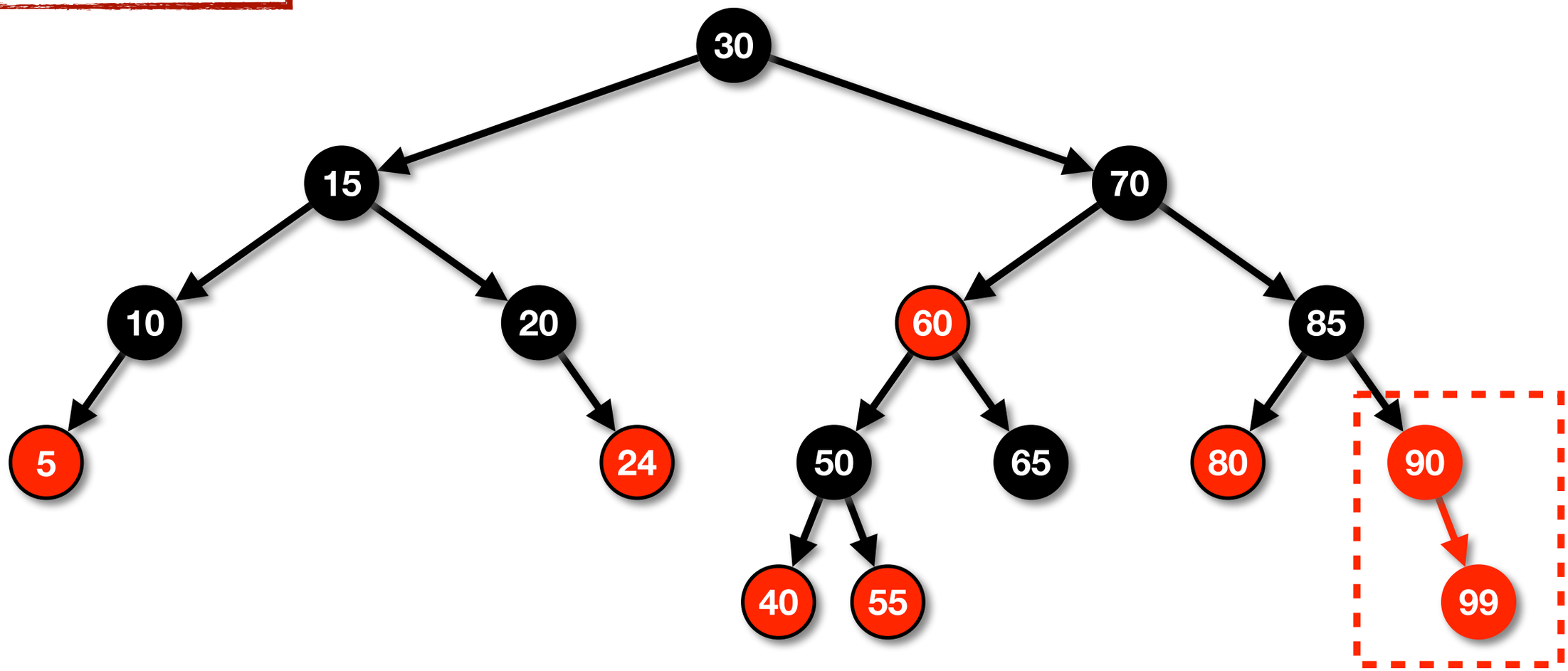

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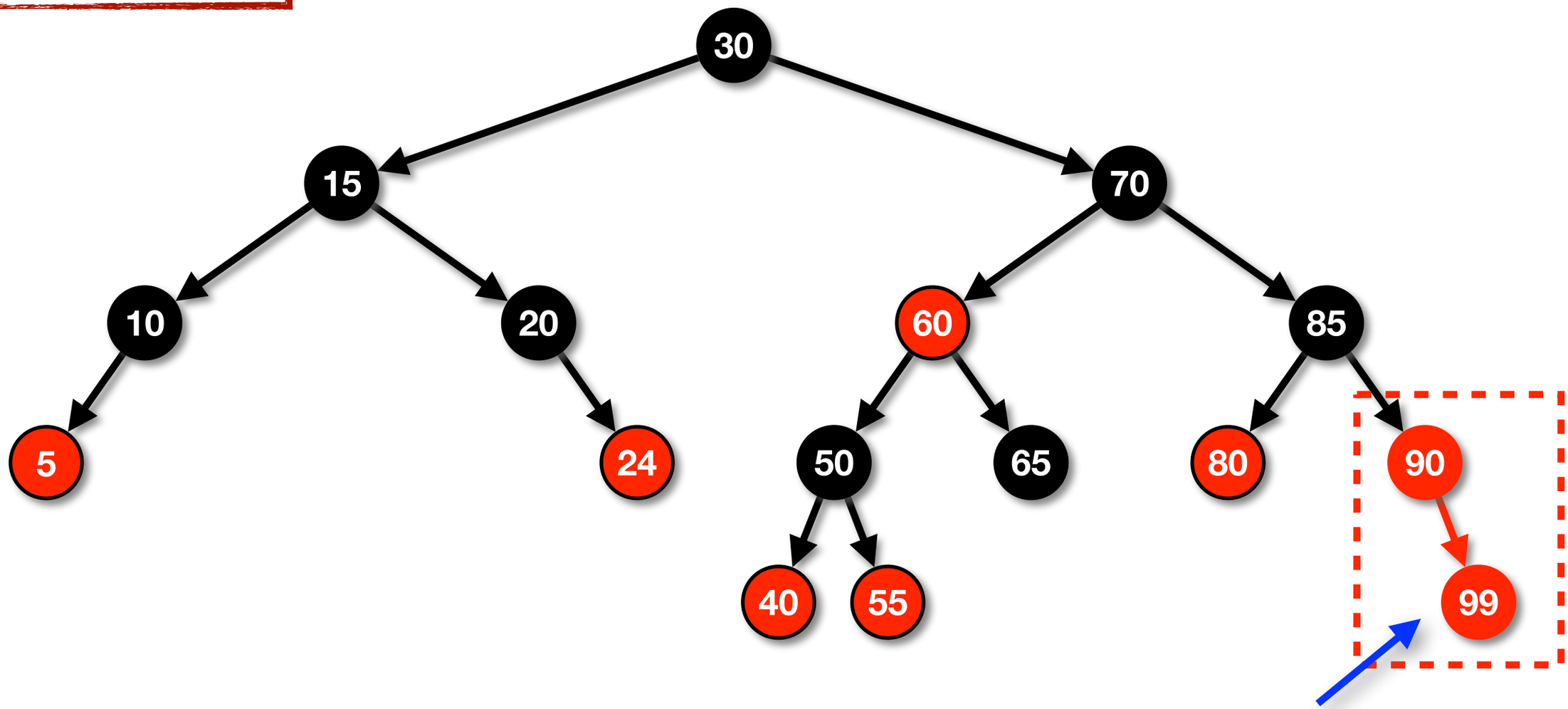
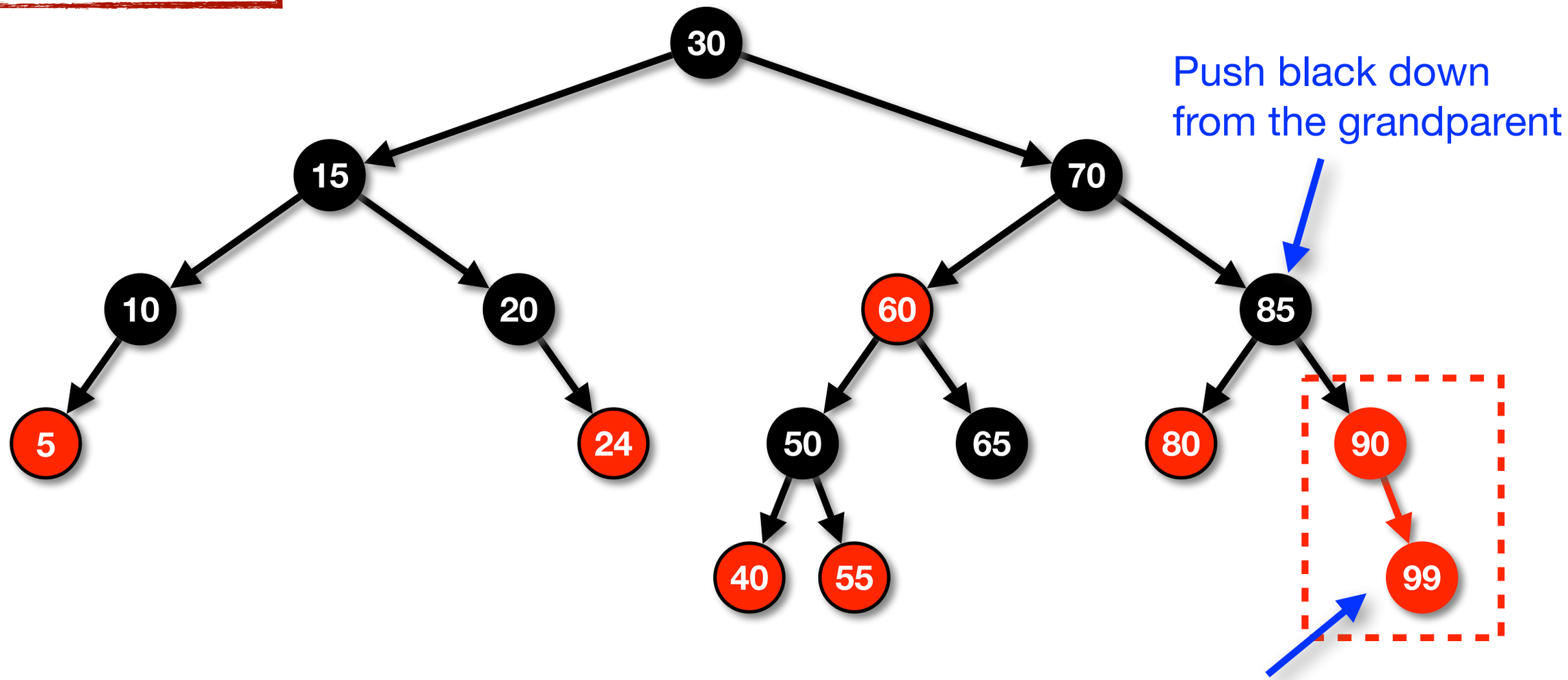


Case #1: Parent's sibling is **red** and new node is inserted as an outside grandchild

Red-Black Tree Insertion Example

```
insert(99)
```



Red-Black Tree Insertion Example

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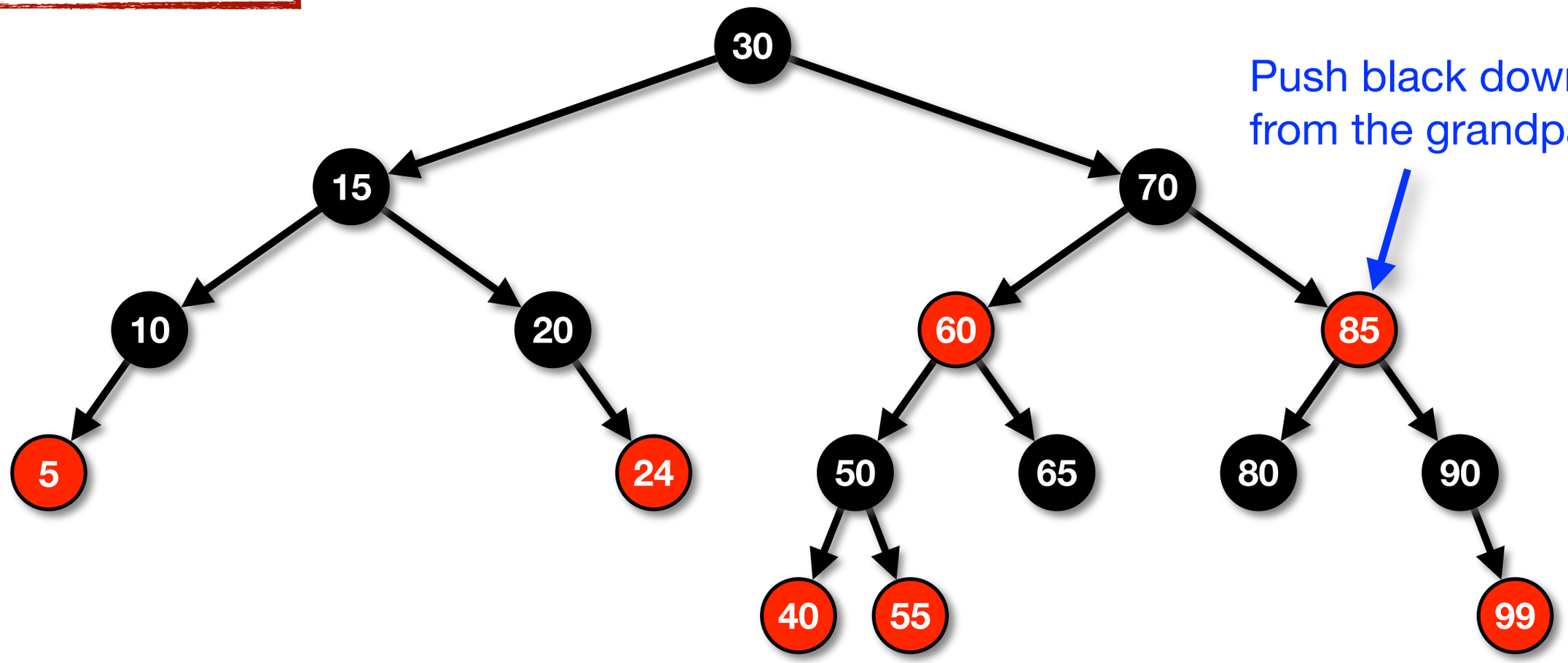

Red-Black Tree Insertion Example

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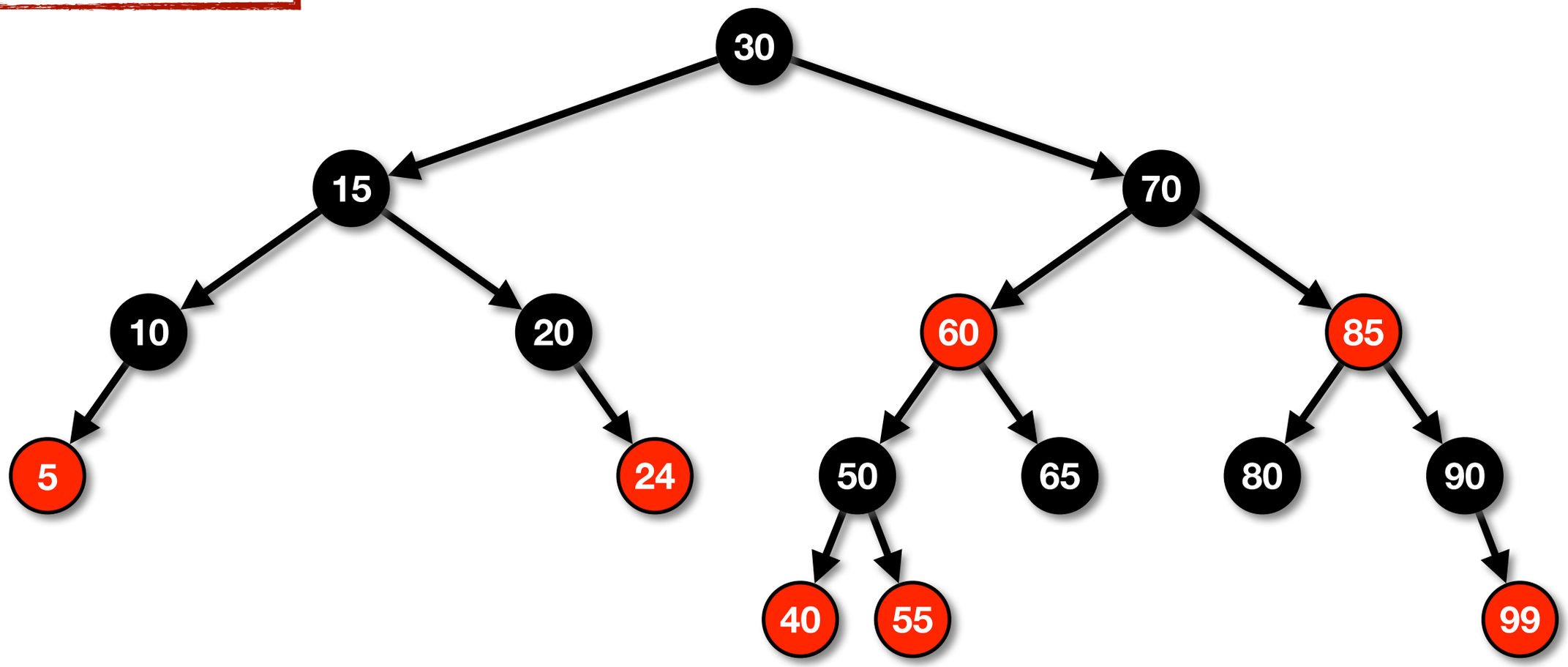

Balanced

Red-Black Tree Insertion Example

```
insert(23)
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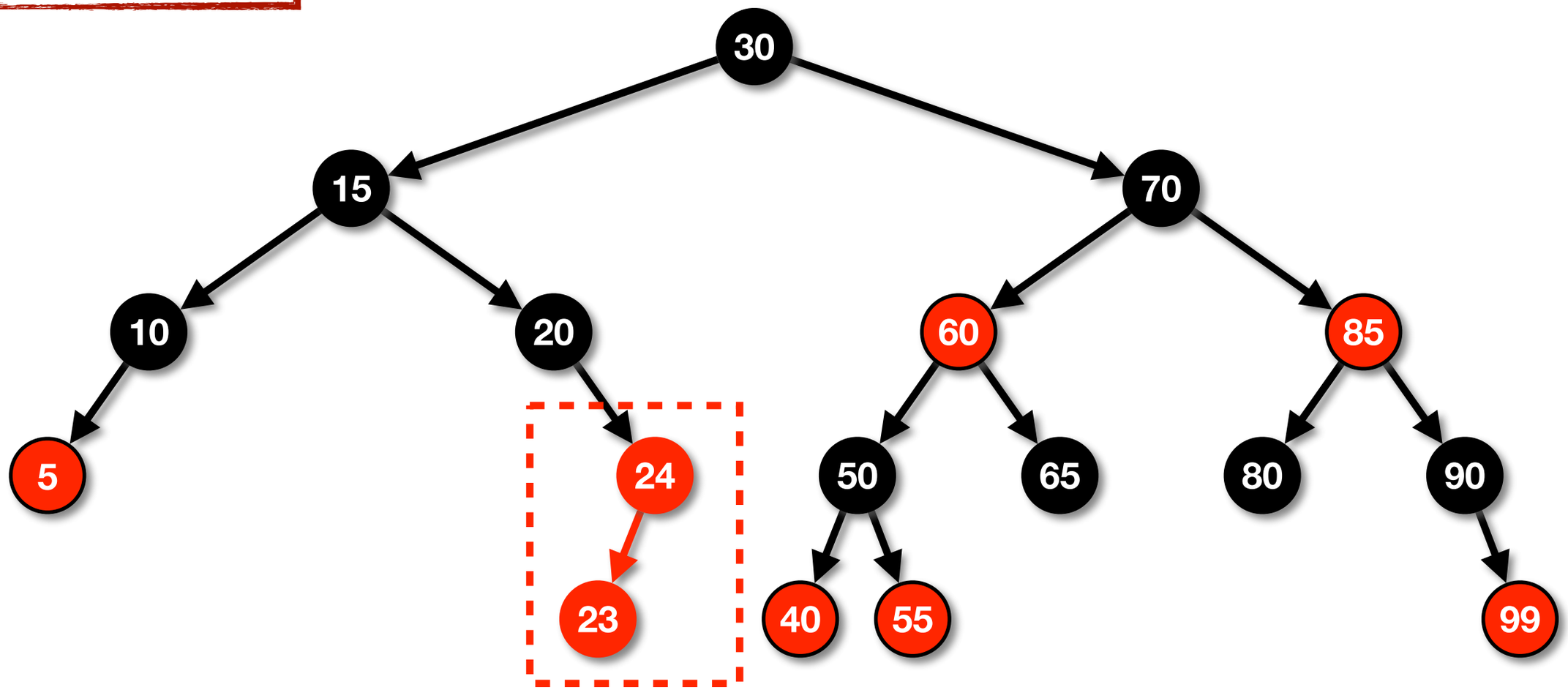

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Red-Black Tree Insertion Example

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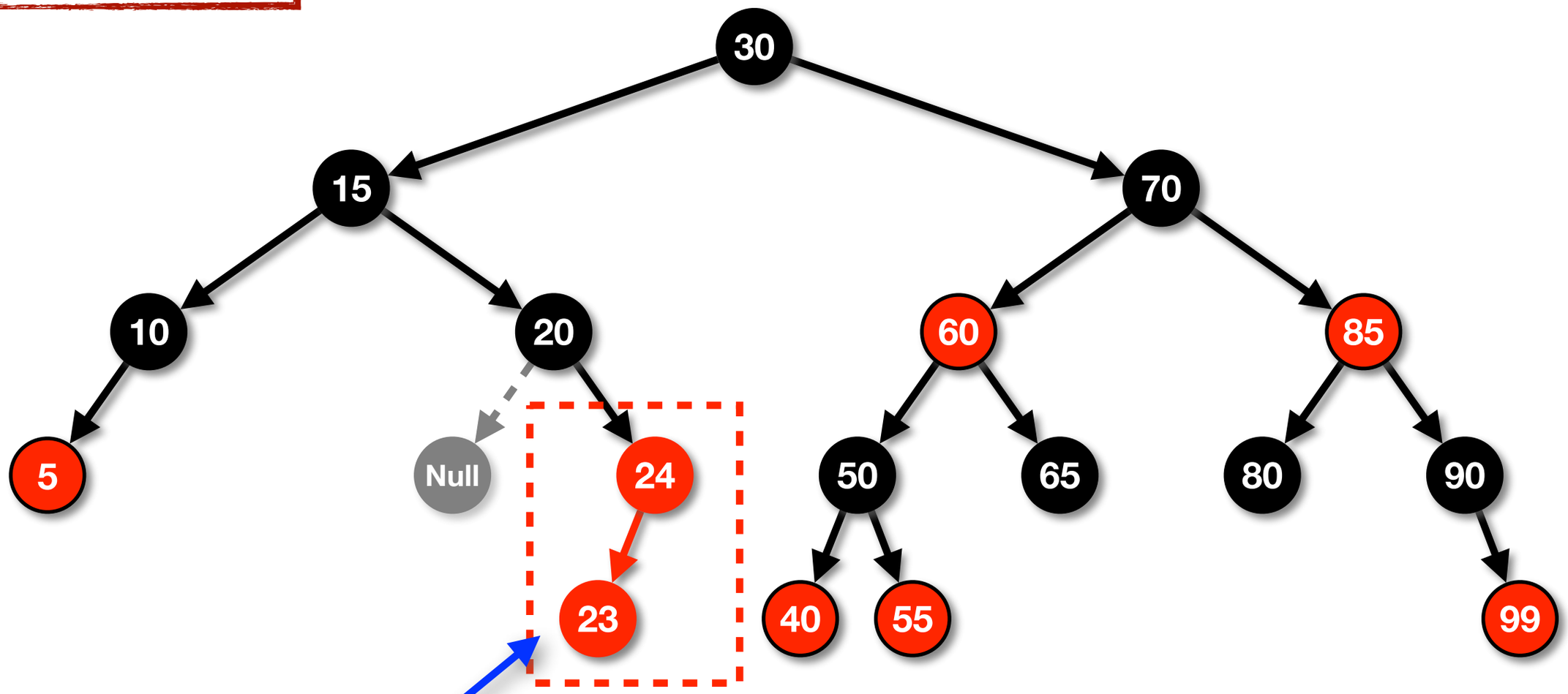
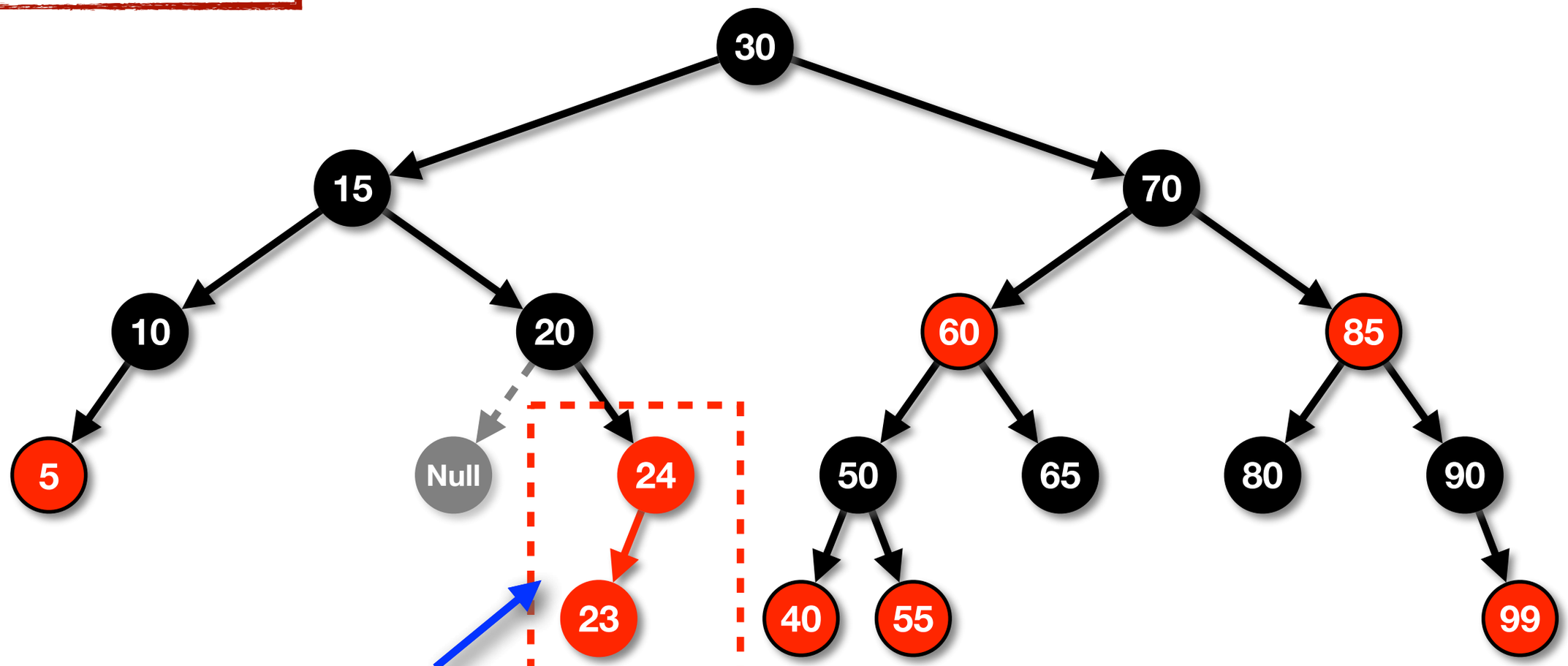


Case #4: Parent's sibling is **black** and new node is inserted as an inside grandchild

Red-Black Tree Insertion Example

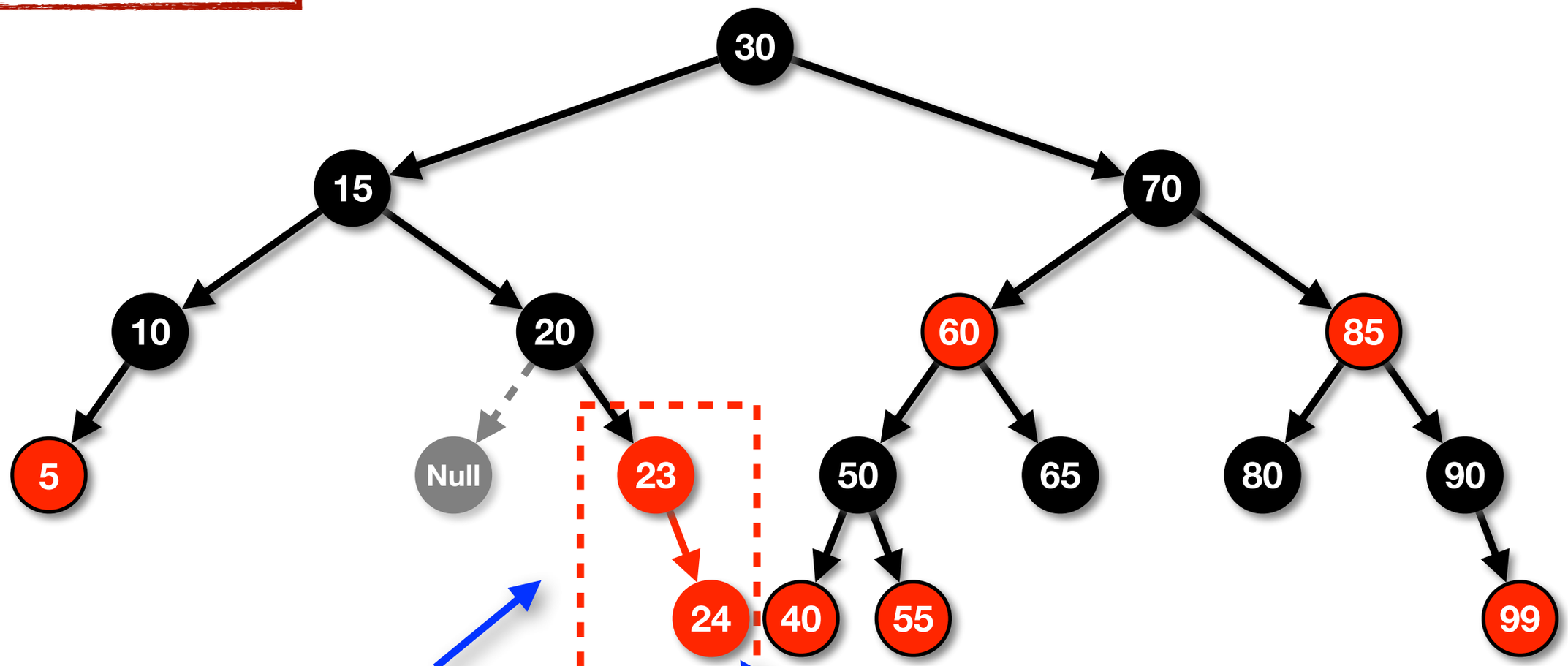
```
insert(23)
```



Perform a double rotation:
First, rotate between nodes 23 & 24,
then, rotate between nodes 23 & 20
and change their colors

Red-Black Tree Insertion Example

```
insert(23)
```

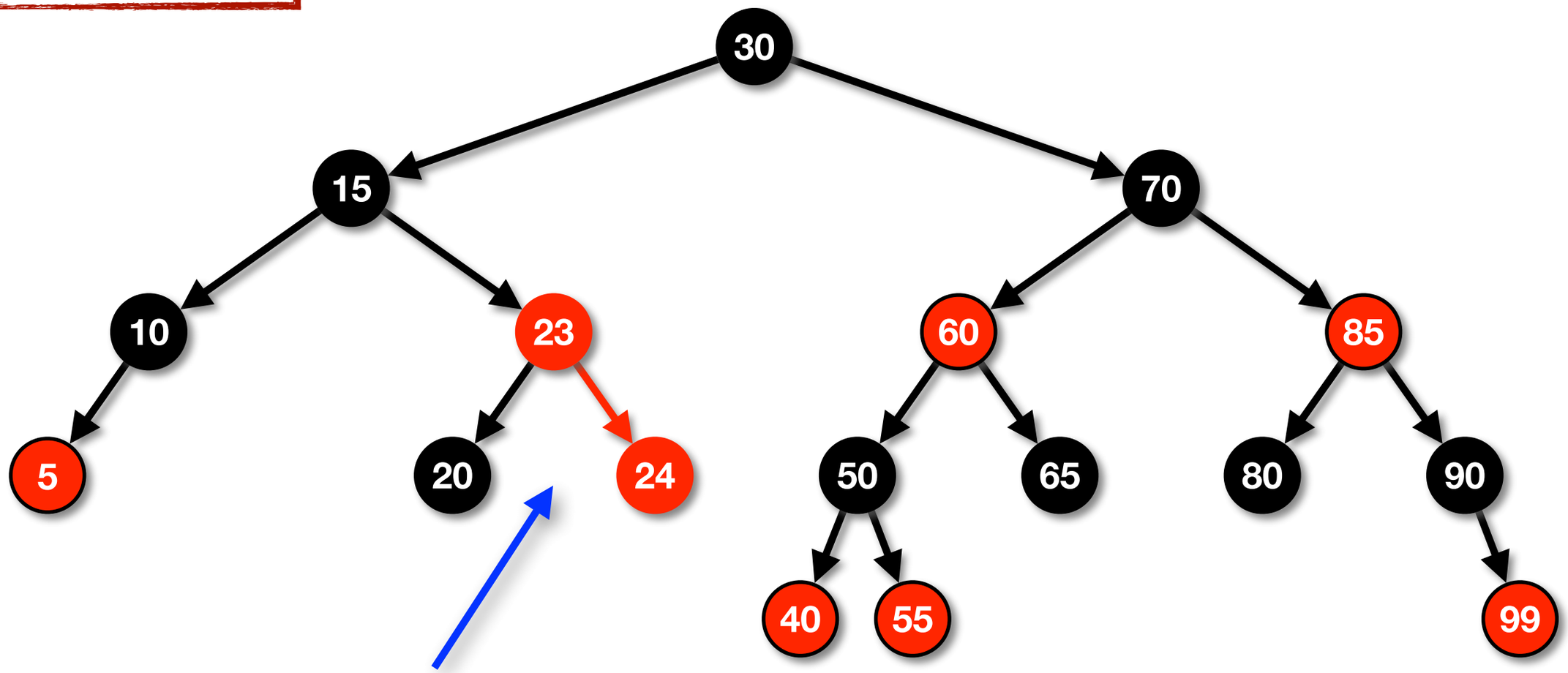


Perform a double rotation:
First, rotate between nodes 23 & 24,
then, rotate between nodes 23 & 20
and change their colors

HEY, LOOK who showed up after the first rotation!! It's case #3: Parent's sibling is **black** and new node is inserted as an outside grandchild

Red-Black Tree Insertion Example

```
insert(23)
```



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First, rotate between nodes 23 & 24,
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and change their colors

Red-Black Tree Insertion Example

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insert(23)
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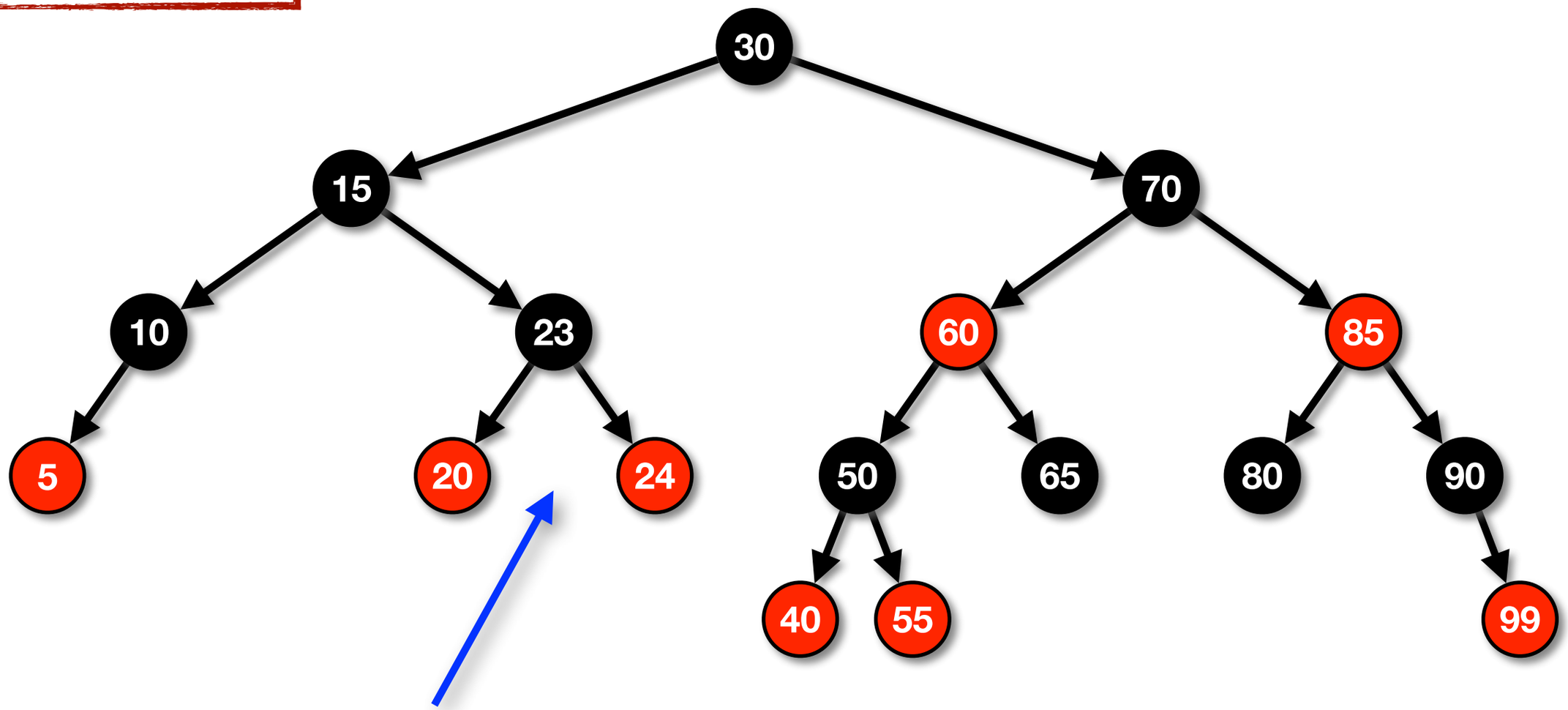

Red-Black Tree Insertion Example

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insert(23)
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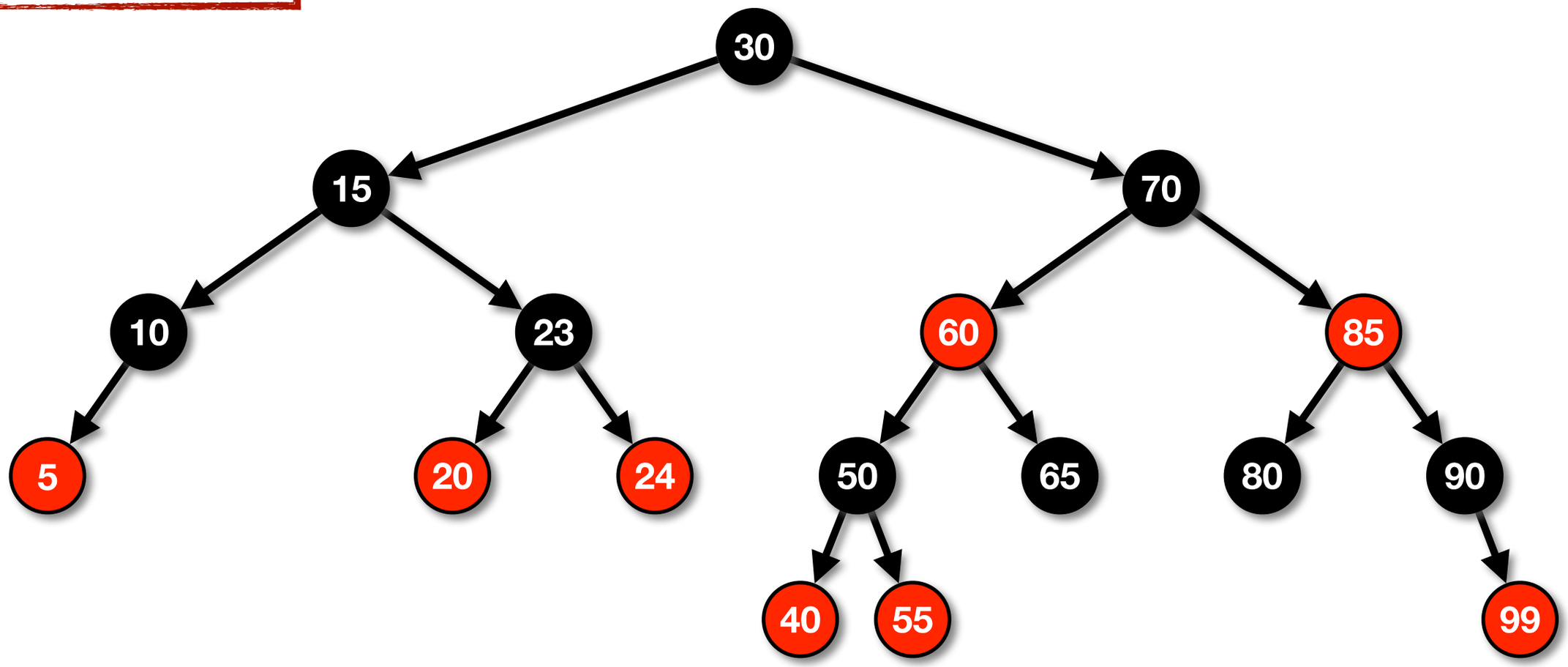

Balanced

Red-Black Tree Deletion

- **Start with standard BST deletion**
  - In BST deletion, when deleting a node with two children, the node was not actually deleted -- its contents were replaced

    (1) Contents replaced with contents from successor or predecessor

    (2) Then the successor/predecessor node was deleted

      - Successor/predecessor can have 0 or 1 child (if the node had two children, then one of its children would be the successor/predecessor)

- Replacing the contents of a node do not affect the coloring of the node, therefore the properties of the red-black tree are not altered
- Removal of the node with 0 or 1 child needs consideration as it can potentially cause violations in the red-black tree

Red-Black Tree Deletion

- **If the node removed is red, then there are no problems and the 4 properties of the red-black tree will still be true -- there is nothing to do in this case**
- **If the node removed is black, then there is a new violation in the red-black tree -- violation of property #4**

> (4) Every path from a node to a null node must contain the same number of black nodes

  - It is necessary to work back up the tree and account for the missing black node