CS350: Data Structures

Binary Search Trees

James Moscola
Department of Engineering & Computer Science
York College of Pennsylvania

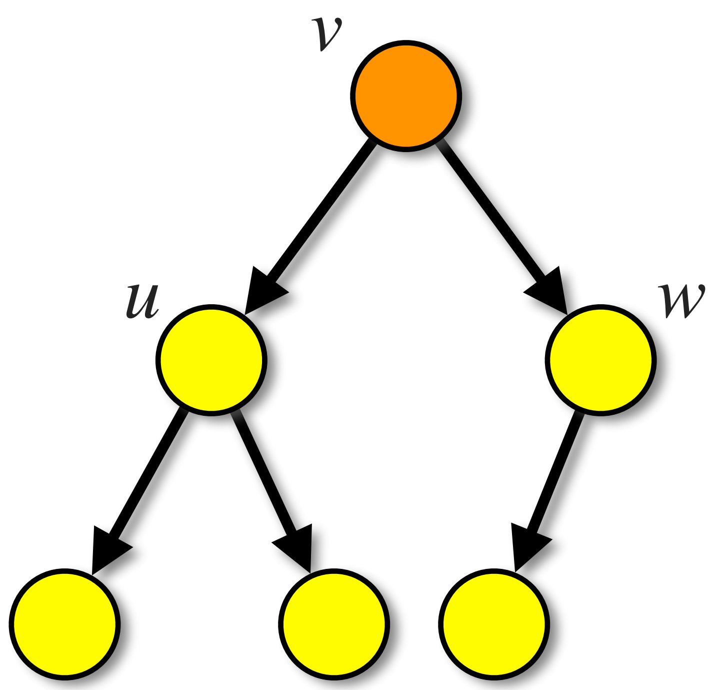
YORK COLLEGE OF PENNSYLVANIA

Introduction to Binary Search Trees

- **A binary search tree is a binary tree that stores keys (or key-element pairs) in such a way as to satisfy the following:**
  - For every node *X* in the tree, the values of all the keys in the left subtree are smaller than the key in *X*
  - For every node *X* in the tree, the values of all the keys in the right subtree are larger than the key in *X*

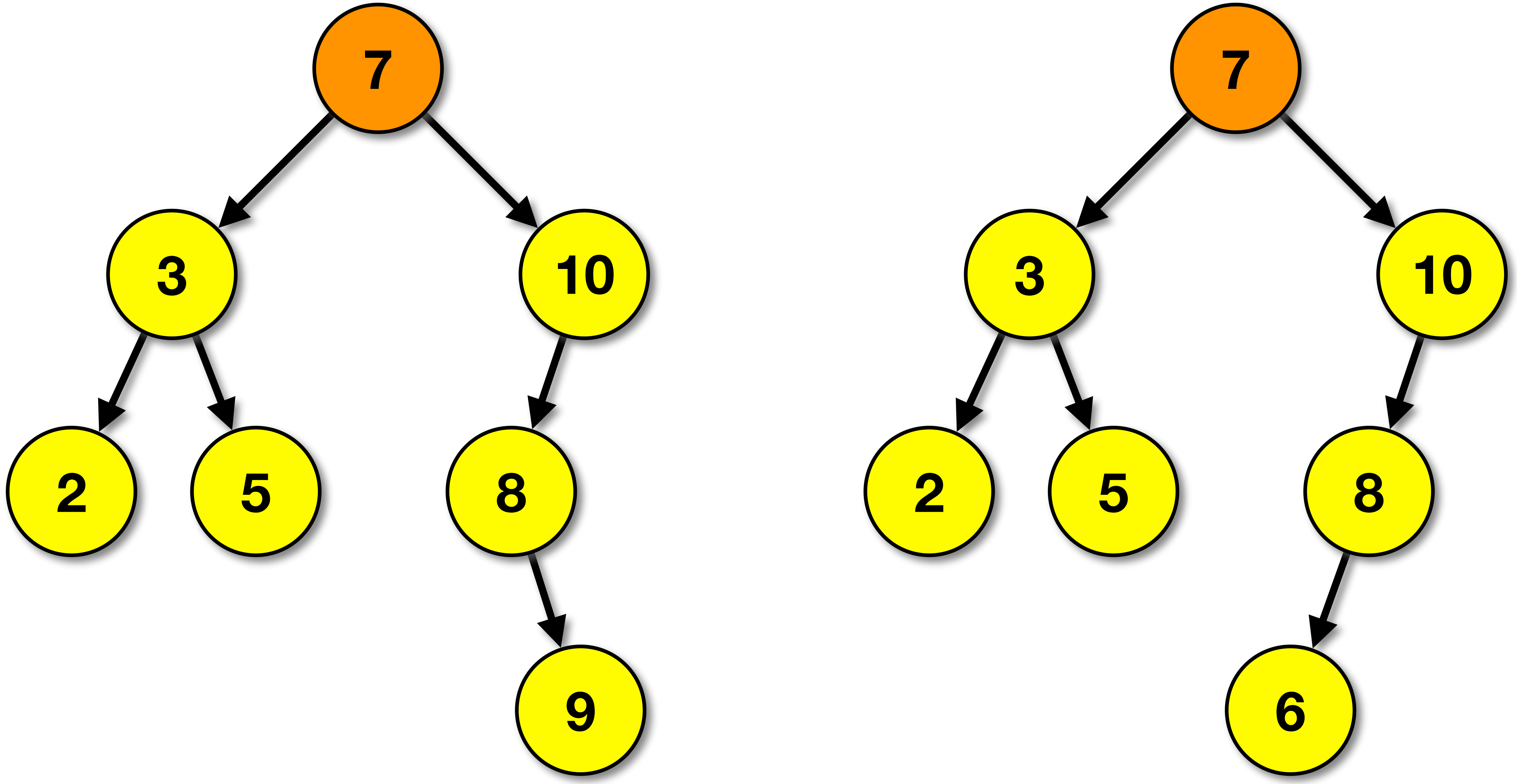
$$\forall \text{ nodes } n \text{ in subtree } u,\ key(n) < key(v)$$

$$\forall \text{ nodes } n \text{ in subtree } w,\ key(n) > key(v)$$



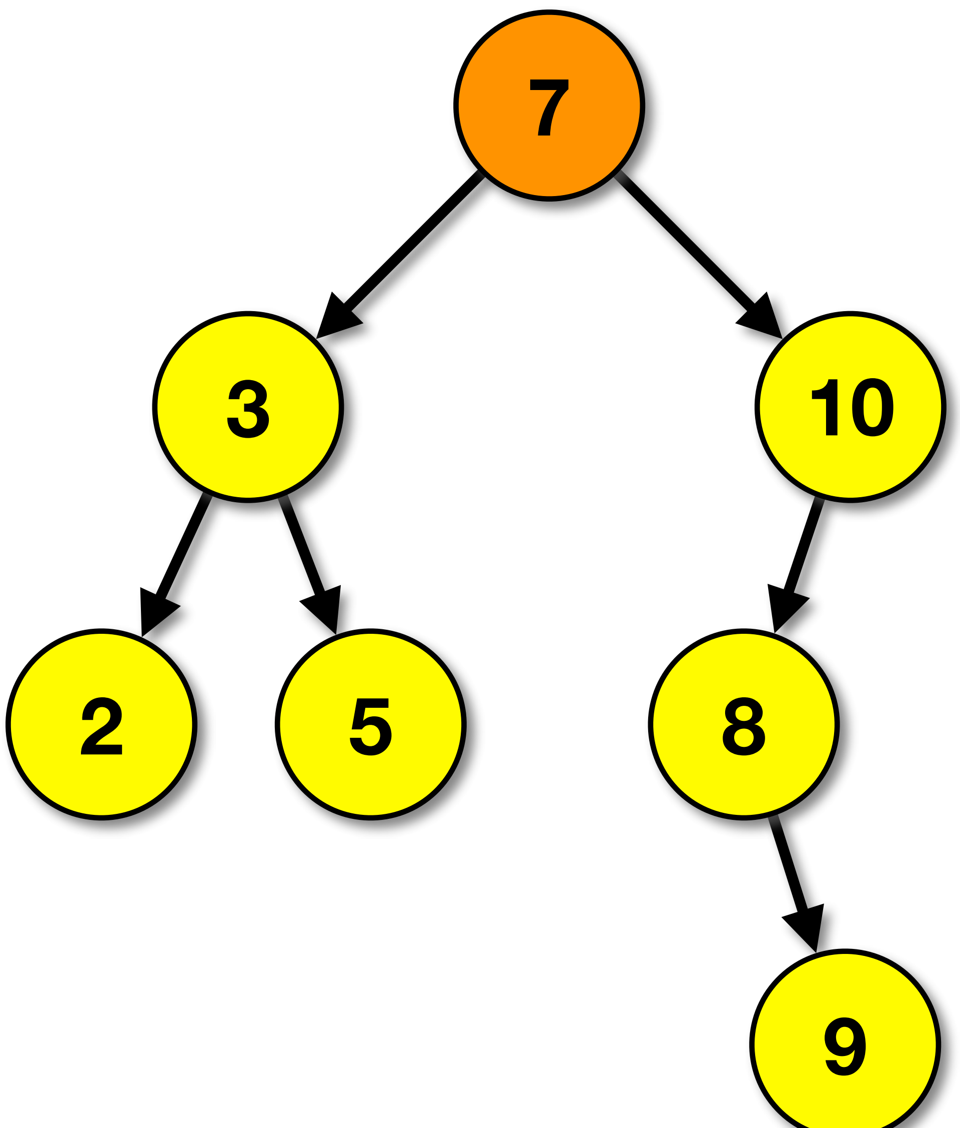
Binary Search Trees

- **Which of these is not a BST? Why?**

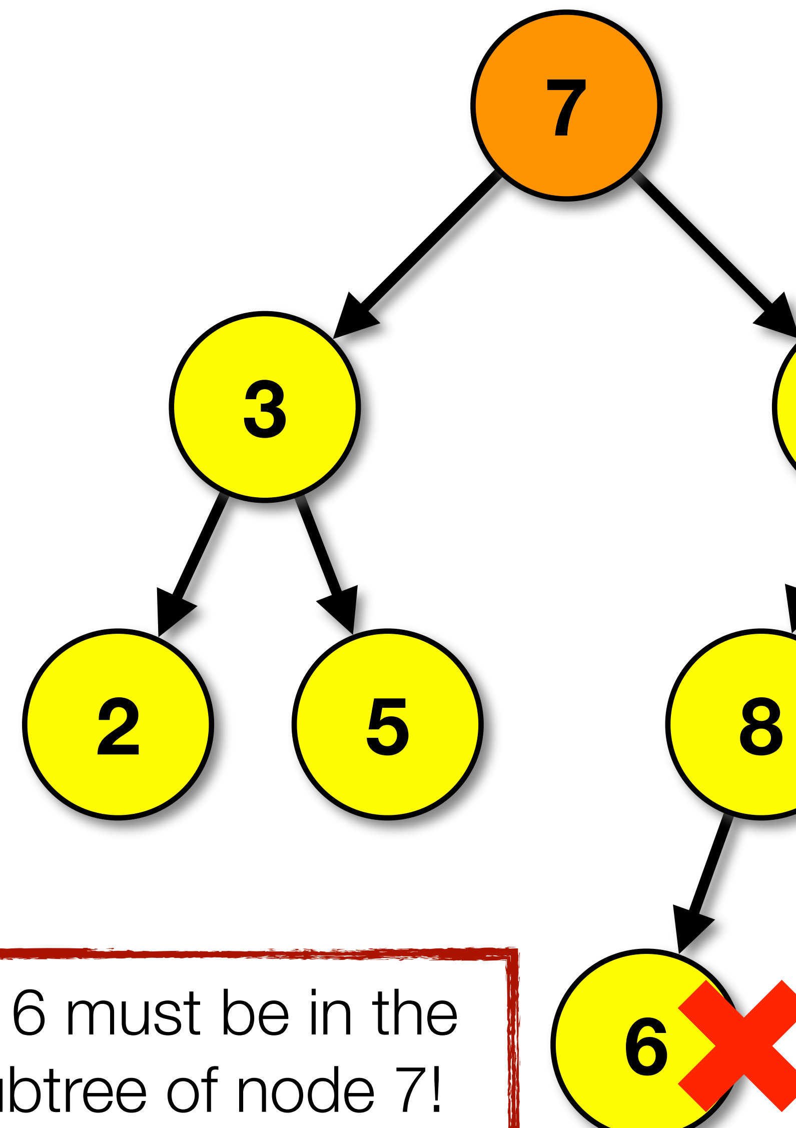


Binary Search Trees

- **Which of these is not a BST? Why?**



This tree is NOT a BST



Node 6 must be in the left subtree of node 7!

Typical Binary Search Tree Operations

- **`isEmpty`** - checks to see if the BST is empty

- **`makeEmpty`** - empties the BST

- **`find`** - searches for and returns the node with a specified key in the BST

- **`findMin`** - returns the node with the smallest key (or the key itself)

- **`findMax`** - returns the node with the largest key (or the key itself)

- **`insert`** - inserts a new node into the tree while maintaining the properties of a BST; all nodes are inserted as leaves

- **`remove`** - removes a node from the tree while maintaining the properties of a BST

Implementation of a Binary Search Tree

- **BST nodes are implemented similarly to other tree nodes**
  - Contains pointers to left and right subtrees
  - Contains a data element
    - Some implementations may contain a key-element pair where the key is used to determine where to insert the node in the tree

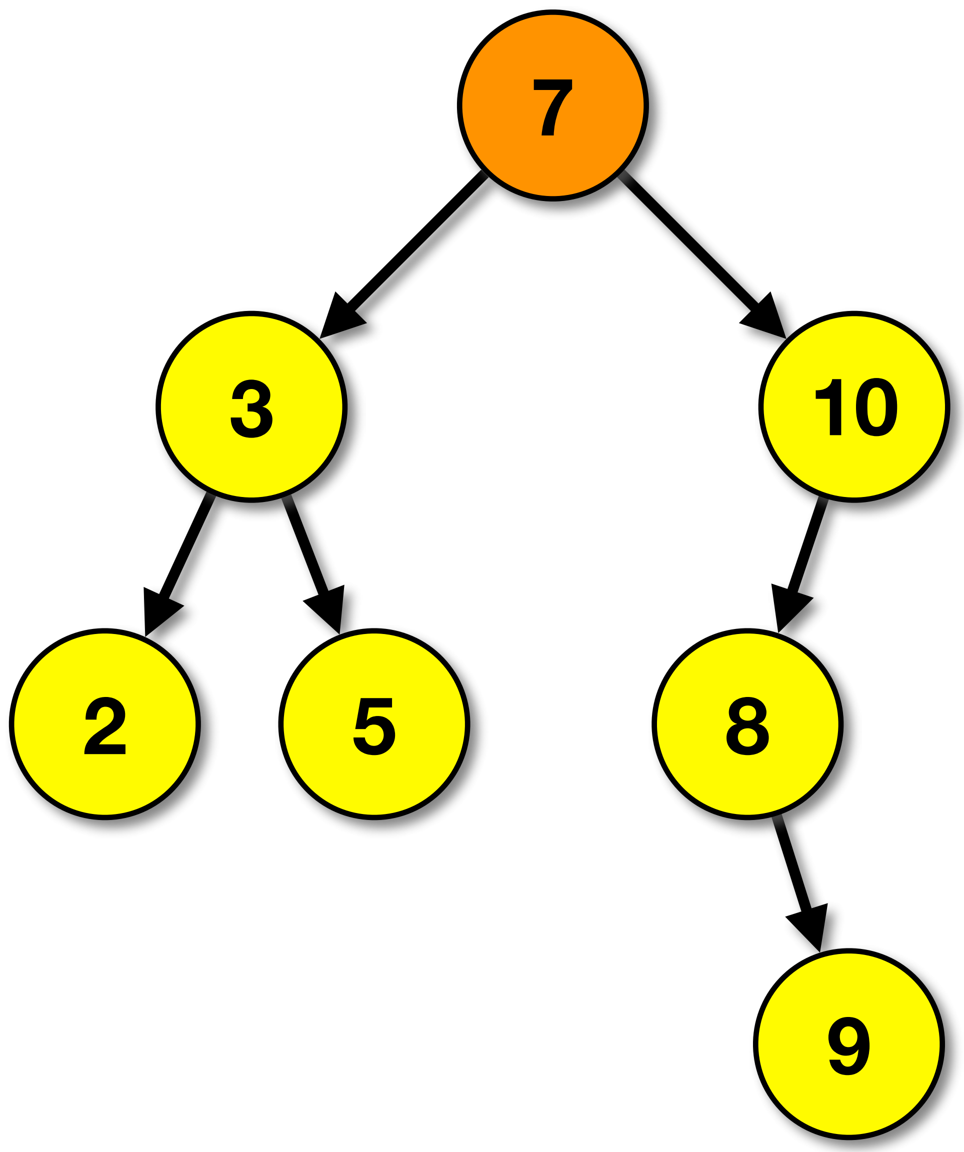
```
class BinTreeNode<E> {
   E element;
   BinTreeNode<E> left, right;

   BinTreeNode(E newElement) {
      element = newElement;
   }
}
```

The `find` Operation

- **To find a node with key `k`**
  - Start at the root node
  - Compare `k` with the key at the node
    - Move to left child if `k` is < the key at the node
    - Move to right child if `k` is > the key at the node
    - Repeat until either the desired key is found or until a leaf node is found
  - If a leaf node is found and the desired key has not been found, then the desired key does not exist in the tree, return `null`

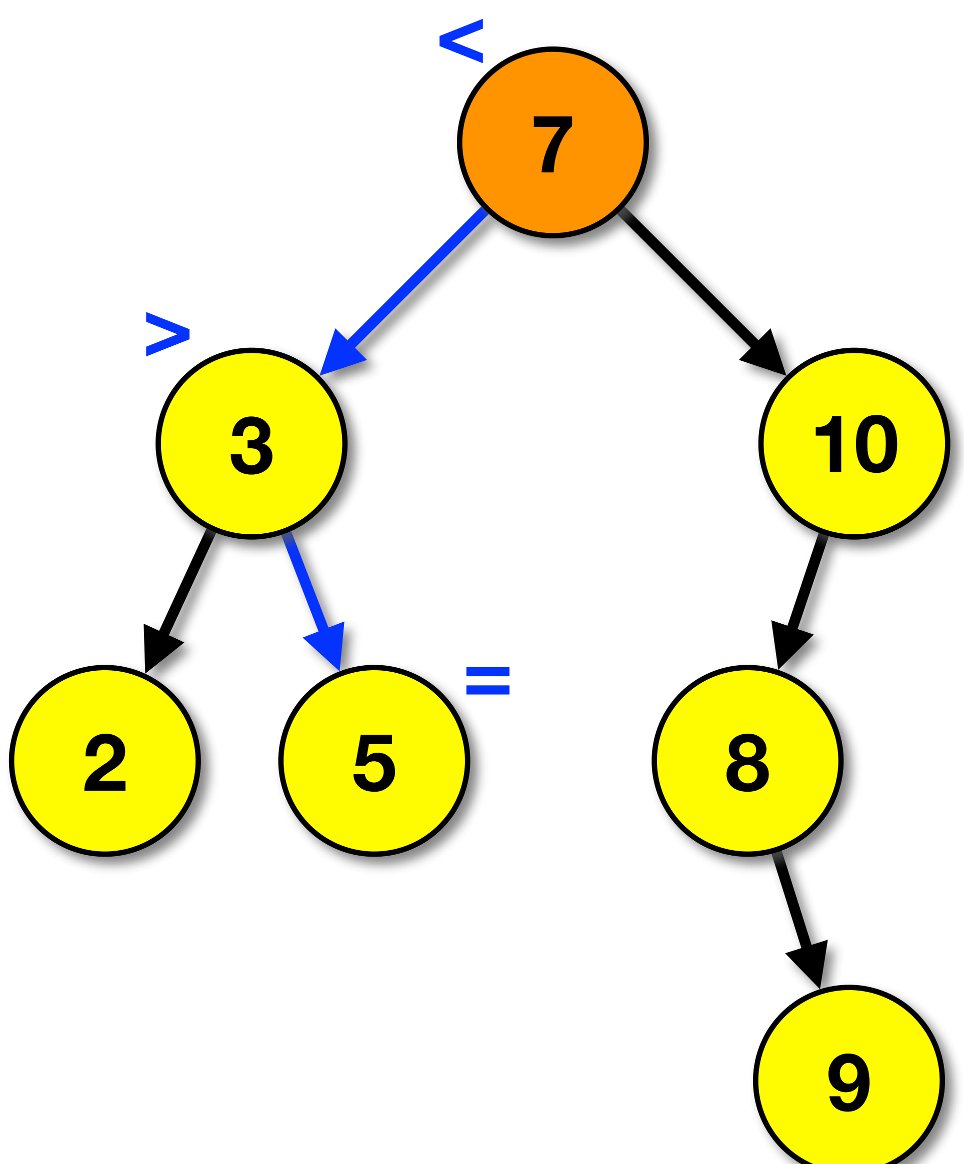
The `find` Operation

- **Example of `find` operation - `find(5)`**



The **`find`** Operation

- **Example of `find` operation - `find(5)`**



(1) start at root node (node 7)
(2) compare 5 to 7
(3) 5 < 7 so move left
(4) compare 5 to 3
(5) 5 > 3 so move right
(6) compare 5 to 5
(7) found node

A Recursive Implementation of **`find`**

```
BinTreeNode<E> find(BinTreeNode<E> node, E element) {
   if (node == null) {
      return node;
   } else if (element == node.element) {
      return node;
   } else if (element < node.element) {
      return find(node.left, element);
   } else {
      return find(node.right, element);
   }
}
```

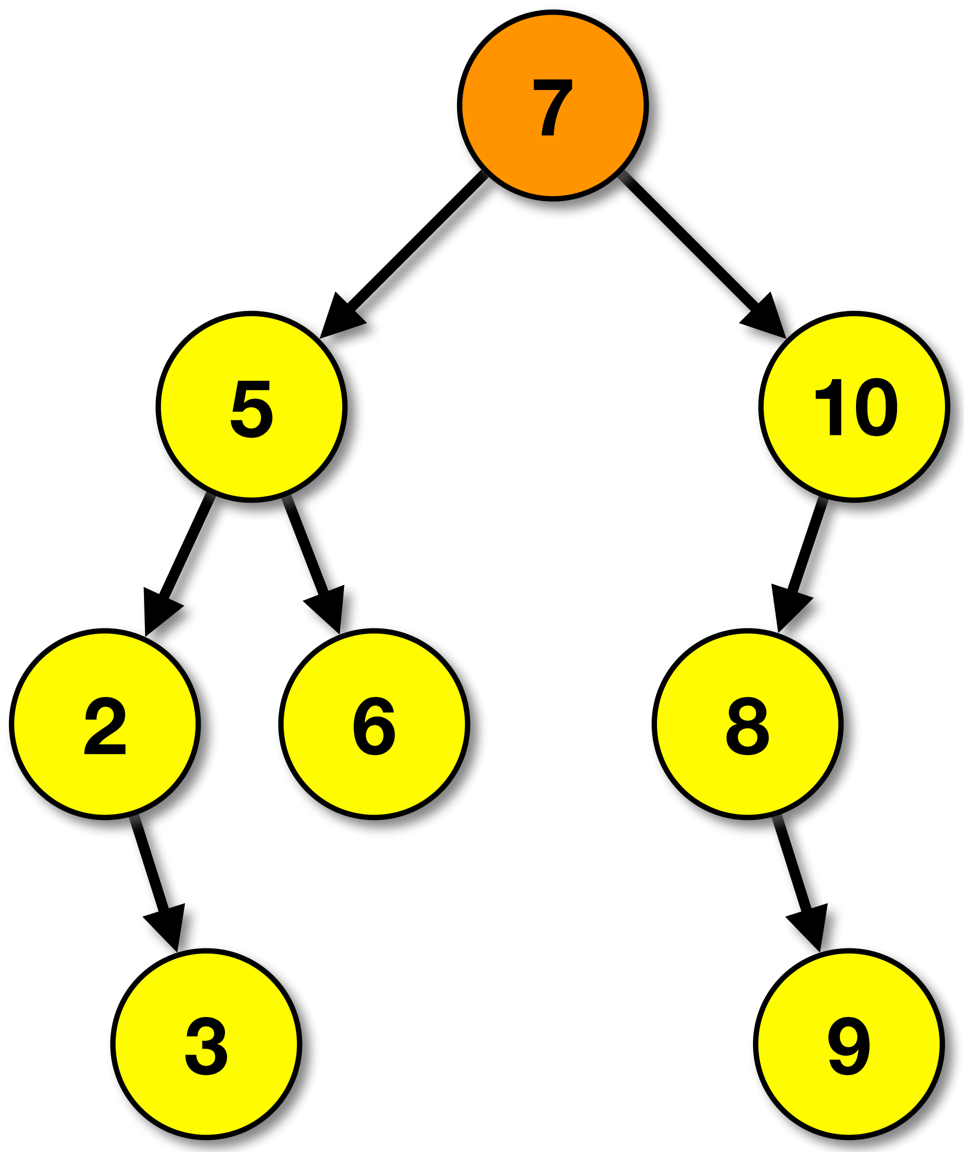
The `findMin` Operation

- **How can the node with the smallest key be found?**

- **To find the node in the tree with the smallest key `k`**
  - Start at the root node
  - Repeatedly move to the left child until there are no more left children
  - Return the node

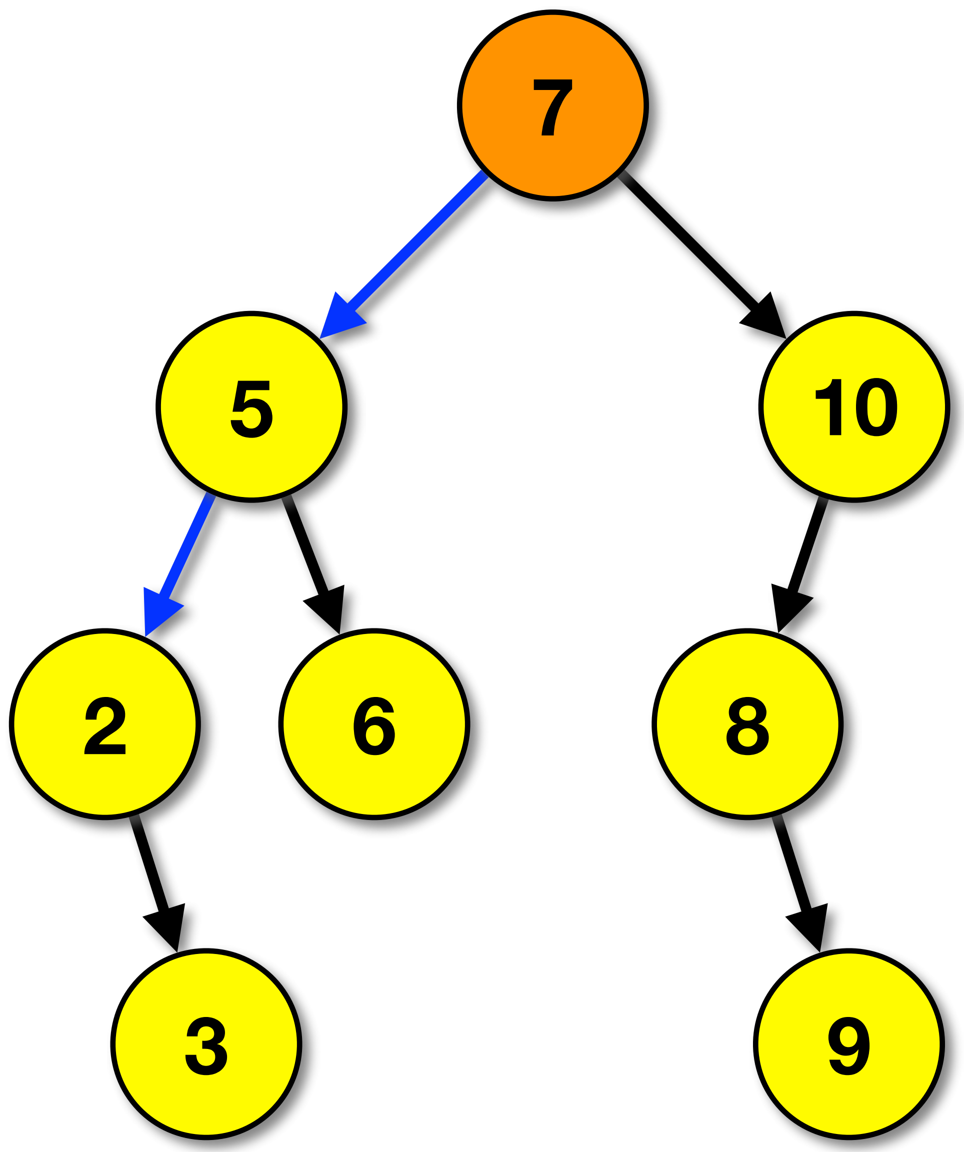
The **`findMin`** Operation

- **Example of `findMin` operation - `findMin( )`**



The `findMin` Operation

- **Example of `findMin` operation - `findMin( )`**



(1) start at root node (node 7)
(2) move to left child (node 5)
(3) move to left child (node 2)
(4) no more left children, so return node 2 as minimum key in tree

A Recursive Implementation of `findMin`

```
BinTreeNode<E> findMin(BinTreeNode<E> node) {
   if (node.left == null) {
      return node;
   } else {
      return findMin(node.left);
   }
}
```

The `findMax` Operation

- **How can the node with the largest key be found?**

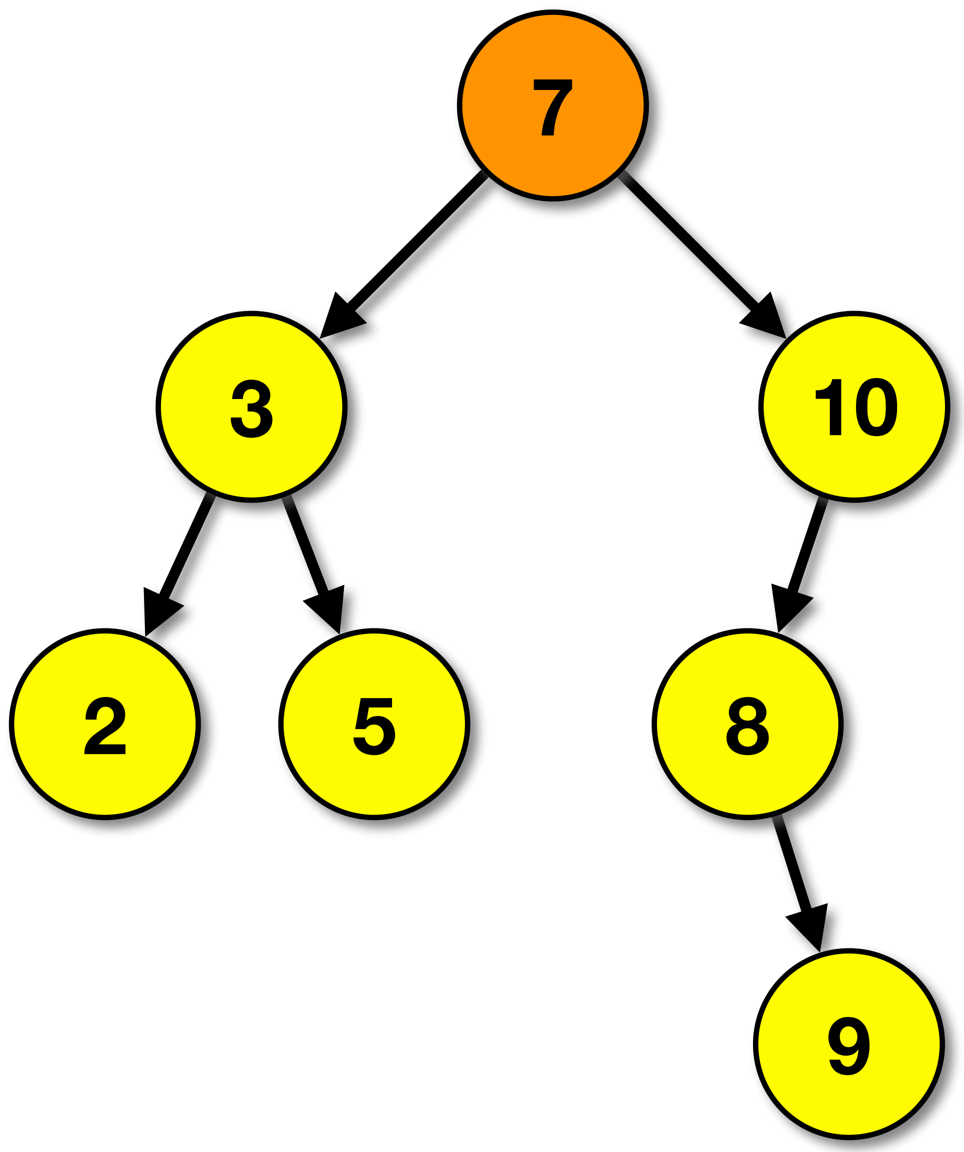
- **To find the node in the tree with the largest key `k`**
  - Start at the root node
  - Repeatedly move to the right child until there are no more right children
  - Return the node

The `insert` Operation

- **To insert a node with key `k`**
  - Start at the root node
  - Compare `k` with the key at the node
    - If the node is null, insert new node at current location in BST
    - Move to left child if `k` is < the key at the node
    - Move to right child if `k` is > the key at the node
    - Repeat until a null location is found in which to insert the new node
  - All insertions create a new leaf node

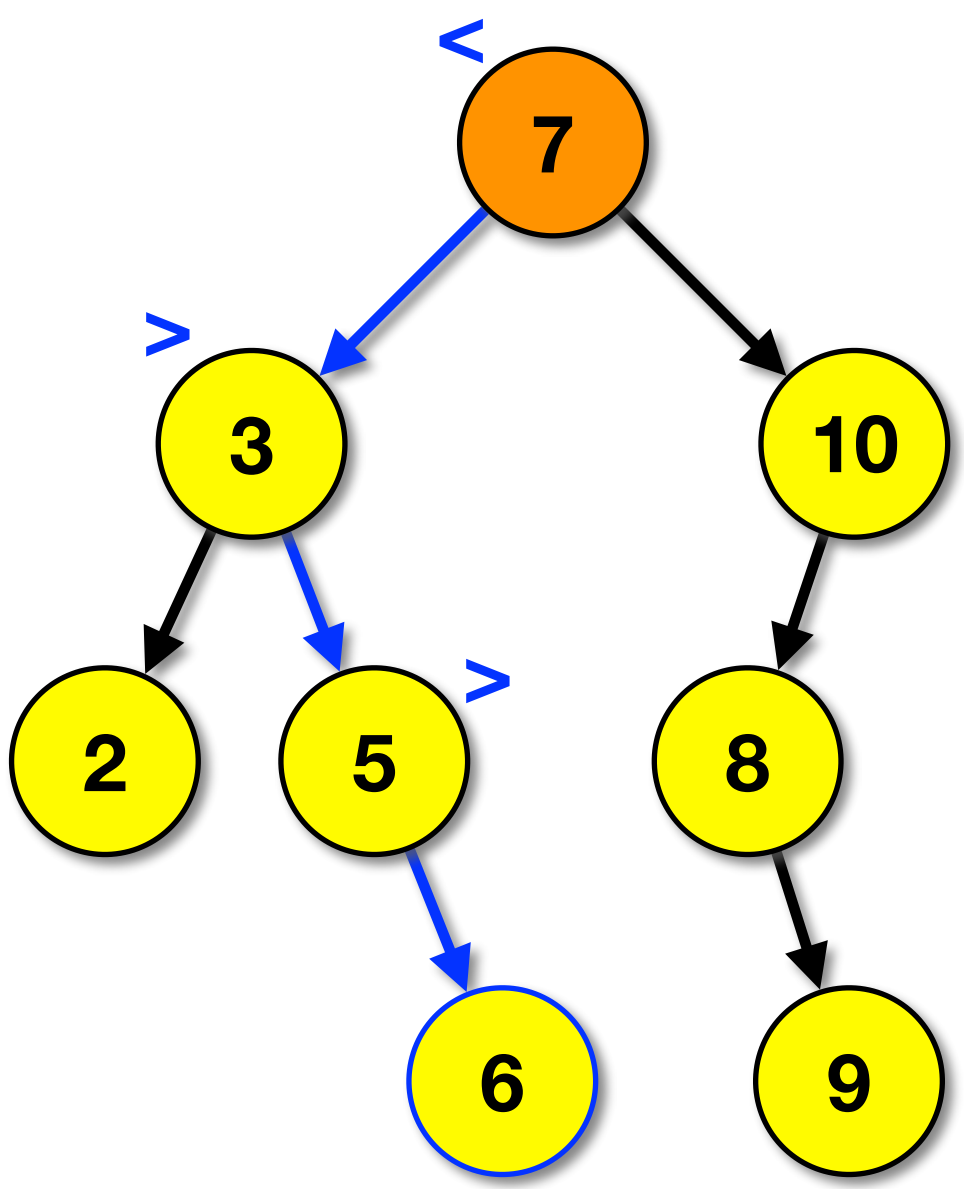
The `insert` Operation

- **Example of `insert` operation - `insert(6)`**



The `insert` Operation

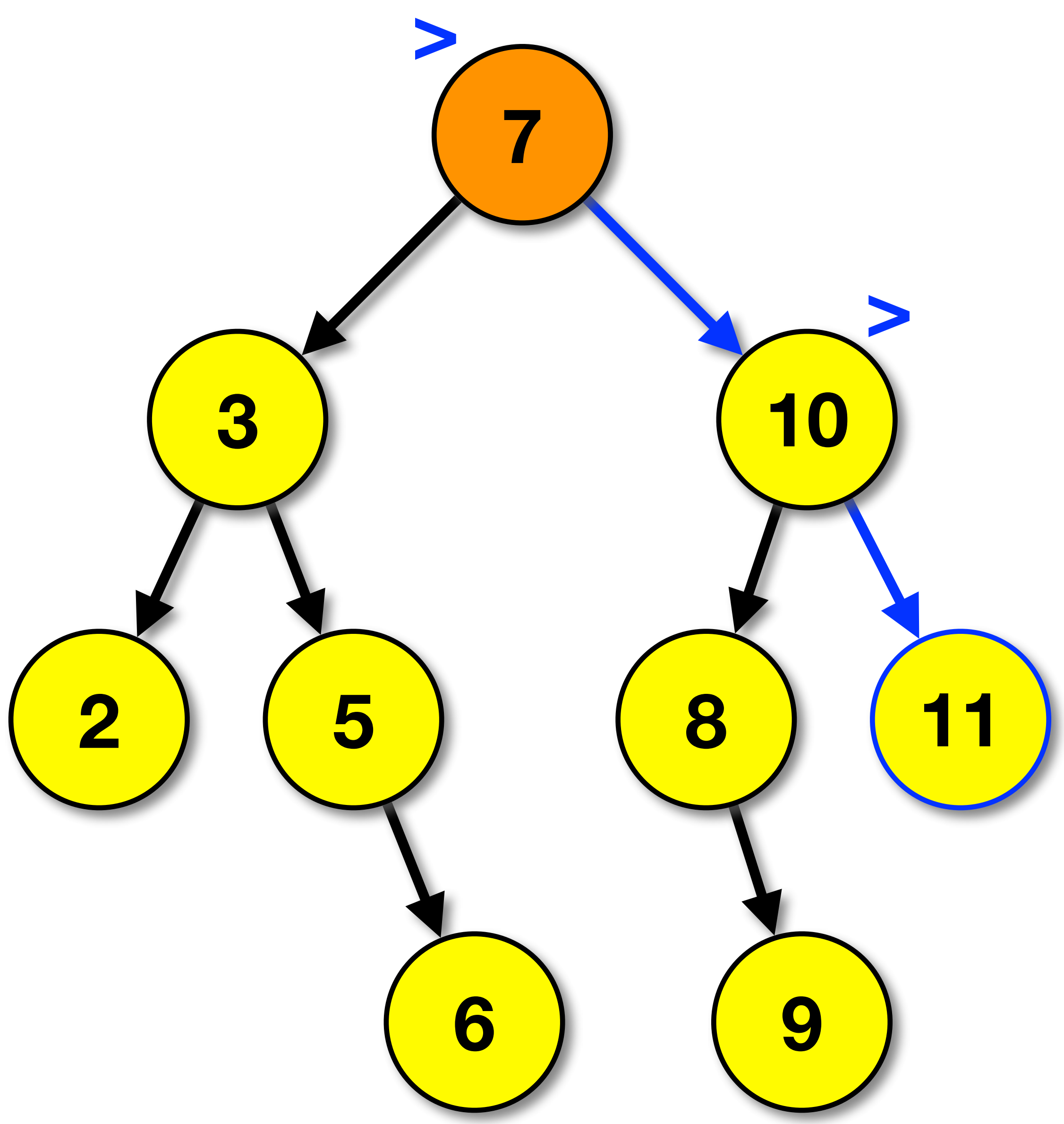
- **Example of `insert` operation - `insert(6)`**



(1) start at root node (node 7)
(2) compare 6 to 7
(3) 6 < 7 so move left
(4) compare 6 to 3
(5) 6 > 3 so move right
(6) compare 6 to 5
(7) 6 > 5 so move right
(8) found null location, so add new node 6 there

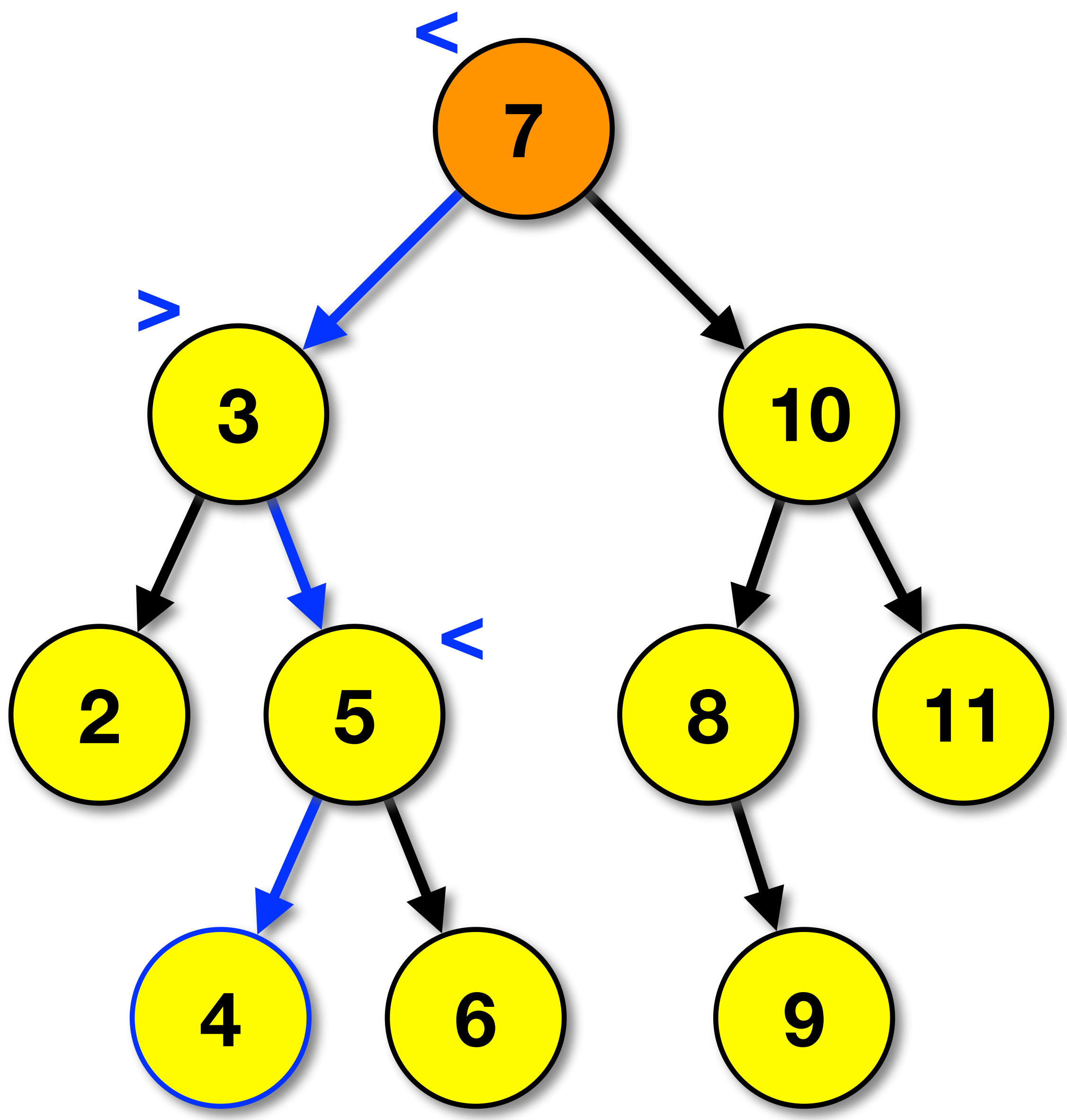
The `insert` Operation

- **Example of `insert` operation - `insert(11)`**



The `insert` Operation

- **Example of `insert` operation - `insert(4)`**



A Recursive Implementation of **`insert`**

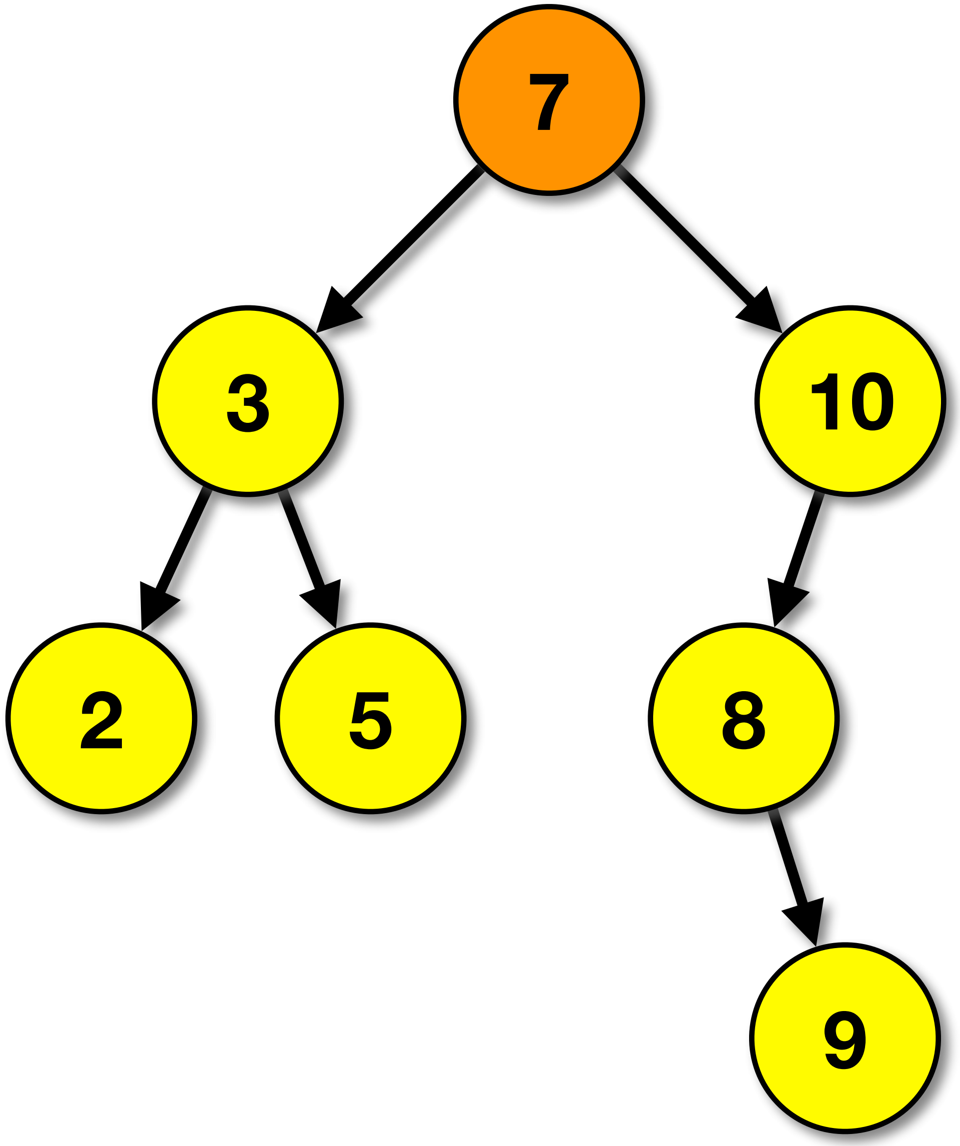
```
BinTreeNode<E> insert(BinTreeNode<E> node, E element) {
   if (node == null) {
      node = new BinTreeNode(element);
   } else if (element < node.element) {
      node.left = insert(node.left, element);
   } else {
      node.right = insert(node.right, element);
   }
   return node;
}
```

The `remove` Operation

- **Find a node N with key `k` and remove it from the tree**
  - Start at the root node
  - Find the node requested for removal
  - Remove the node (there are several different cases that need to be considered when removing a node):
    1) Node N is not found -- do nothing
    2) Node N is a leaf node -- simply remove node N from tree
    3) Node N has only a single child -- remove node N and replace it with its child
    4) Node N has two children -- do not delete node N, instead find its in-order successor node (or in-order predecessor) (node R) and replace the values in the node N with those from node R. Then delete node R.

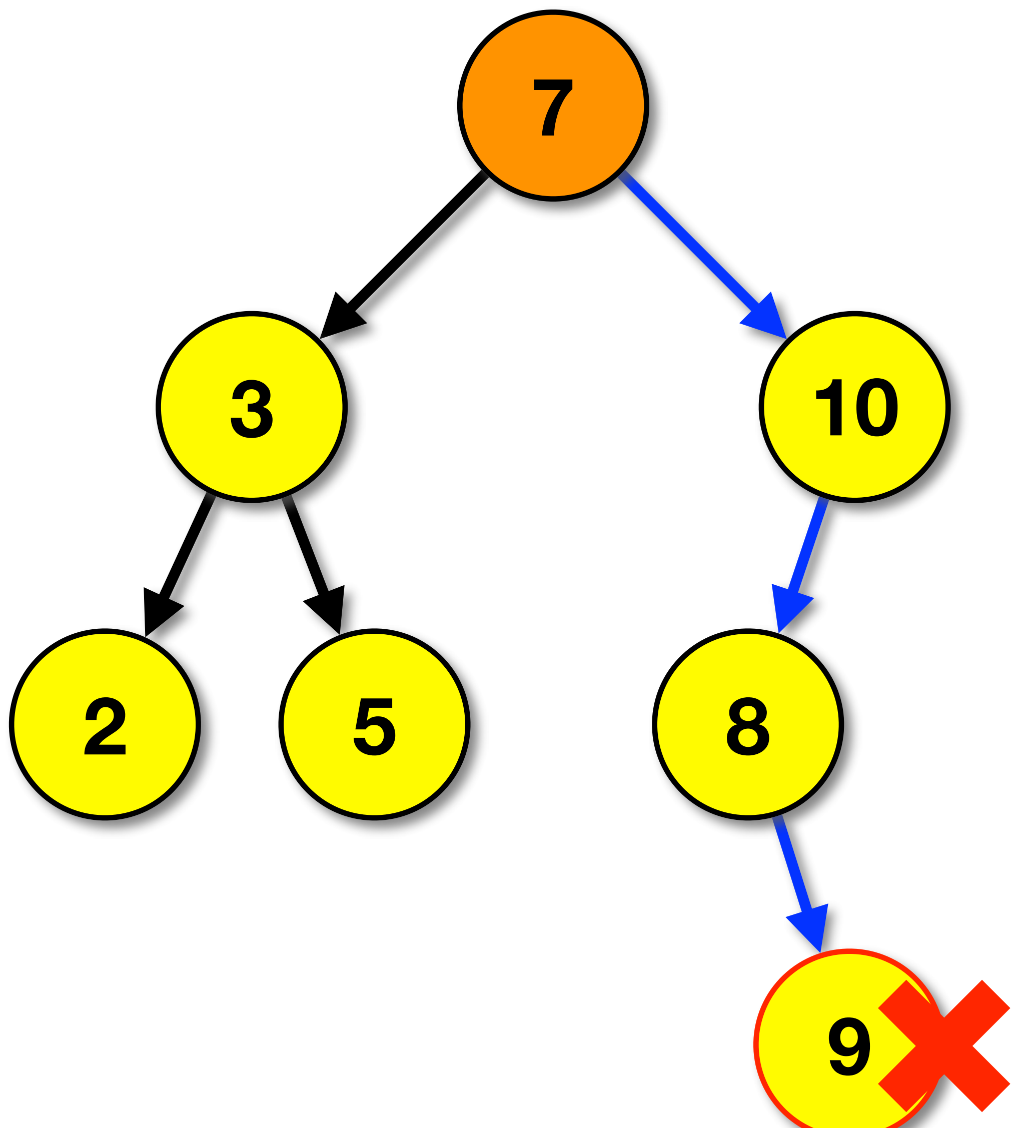
The `remove` Operation: Case #2 - Leaf Node

- **Example of `remove` operation on leaf node - `remove(9)`**



The `remove` Operation: Case #2 - Leaf Node

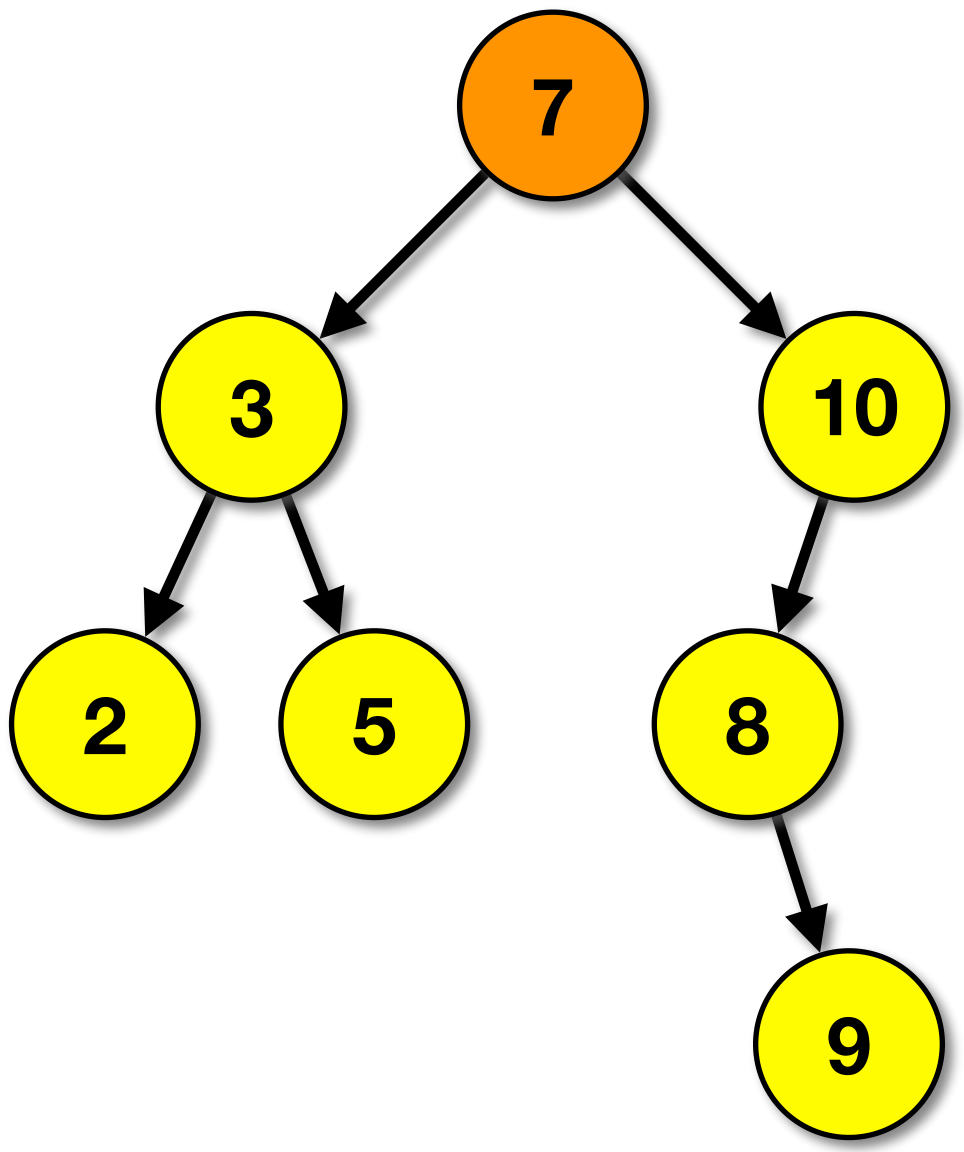
- **Example of `remove` operation on leaf node - `remove(9)`**



(1) start at root node (node 7)
(2) find node 9
(3) node 9 is a leaf node
(4) simply delete node 9 from the tree (i.e. set node 8's right child to null)

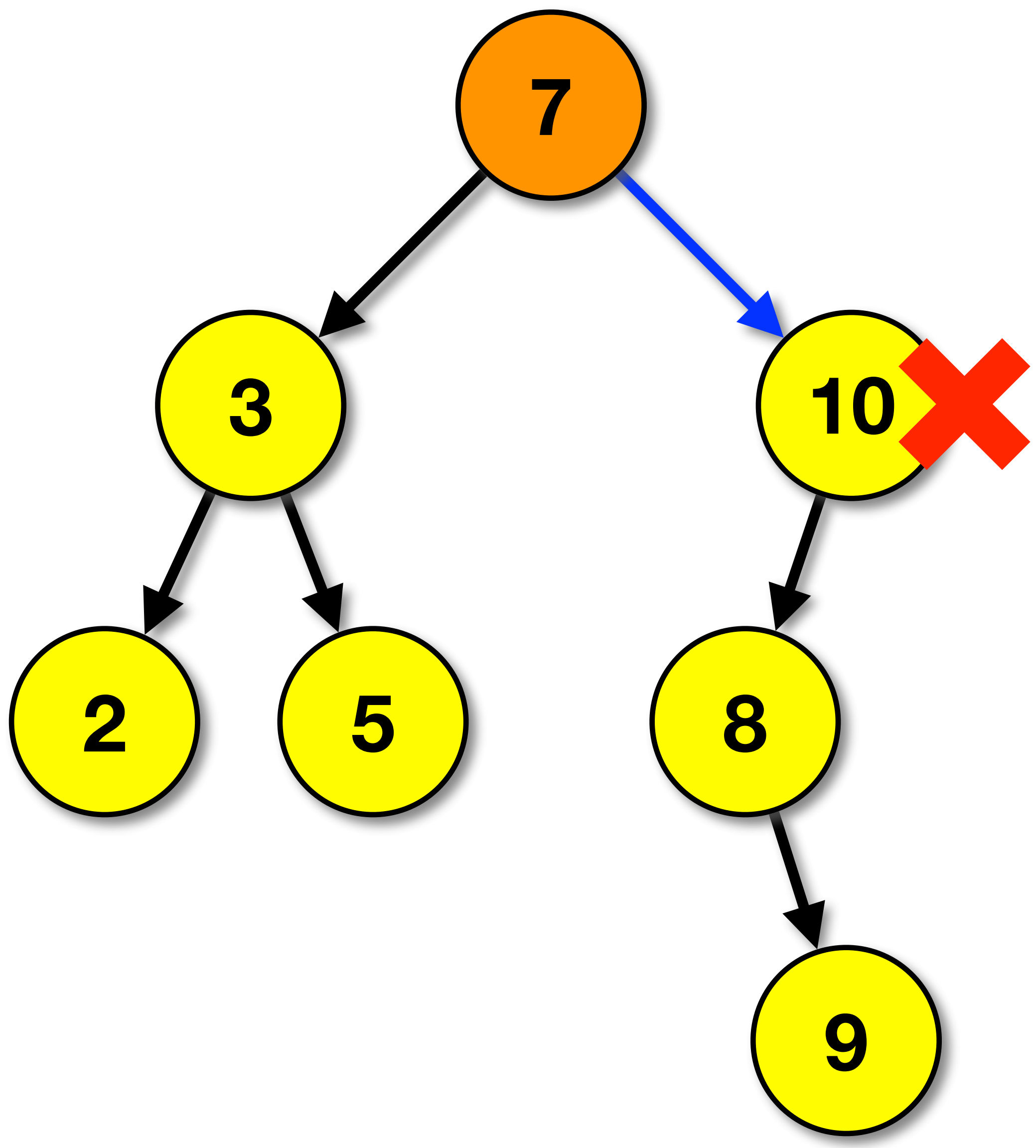
The `remove` Operation: Case #3 - Single Child

- **Example of `remove` operation with a single child - `remove(10)`**



The `remove` Operation: Case #3 - Single Child

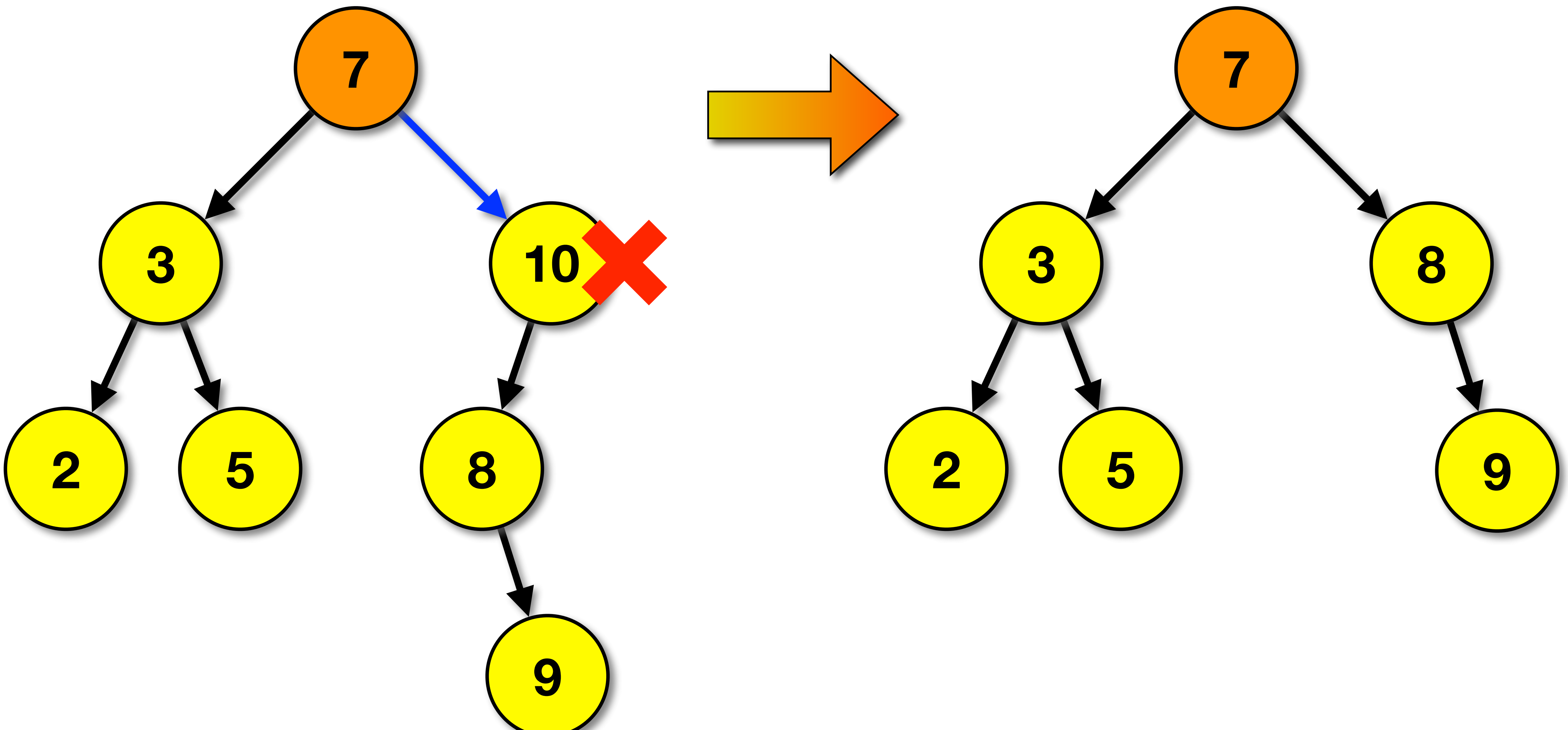
- **Example of `remove` operation with a single child - `remove(10)`**



(1) start at root node (node 7)
(2) find node 10
(3) node 10 has a single child
(4) remove node 10, and replace it with its only child

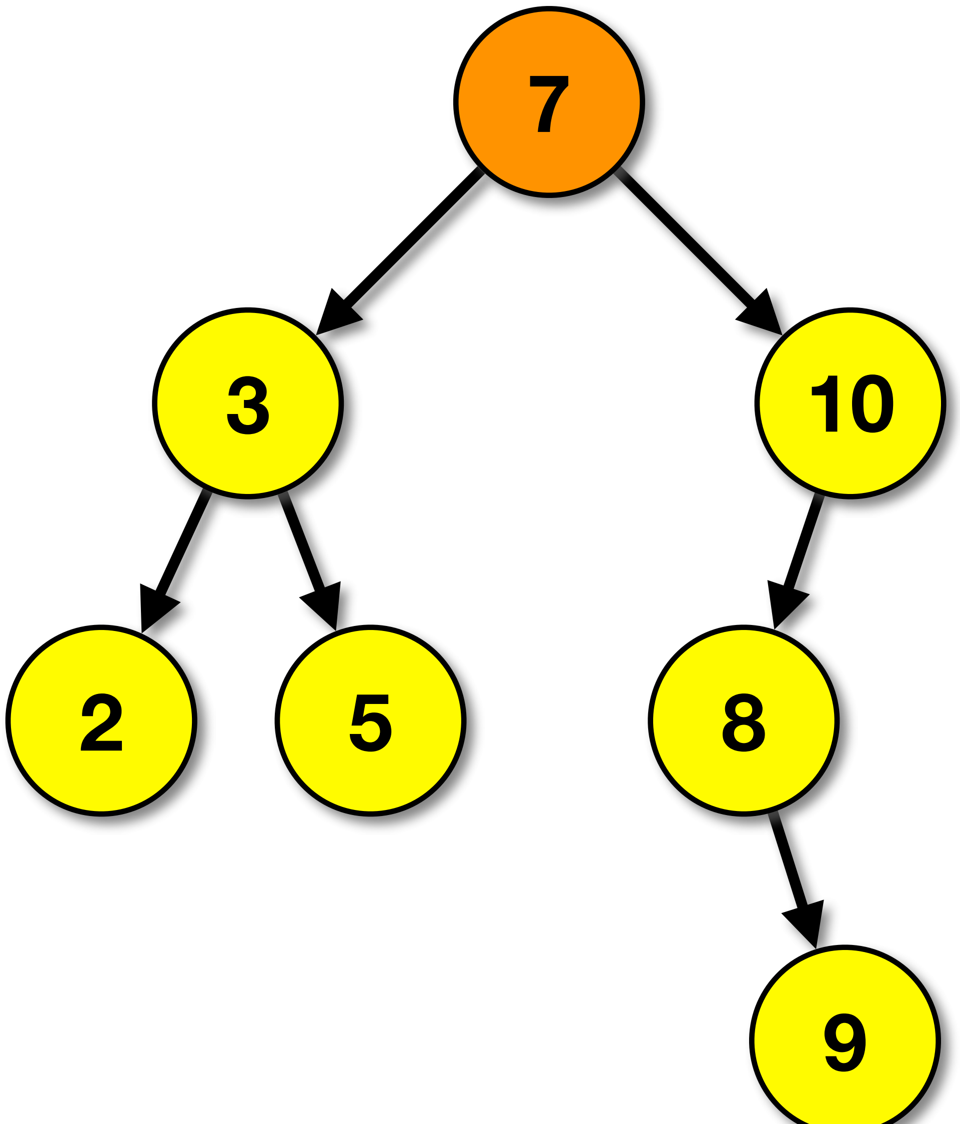
The `remove` Operation: Case #3 - Single Child

- **Example of `remove` operation with a single child - `remove(10)`**



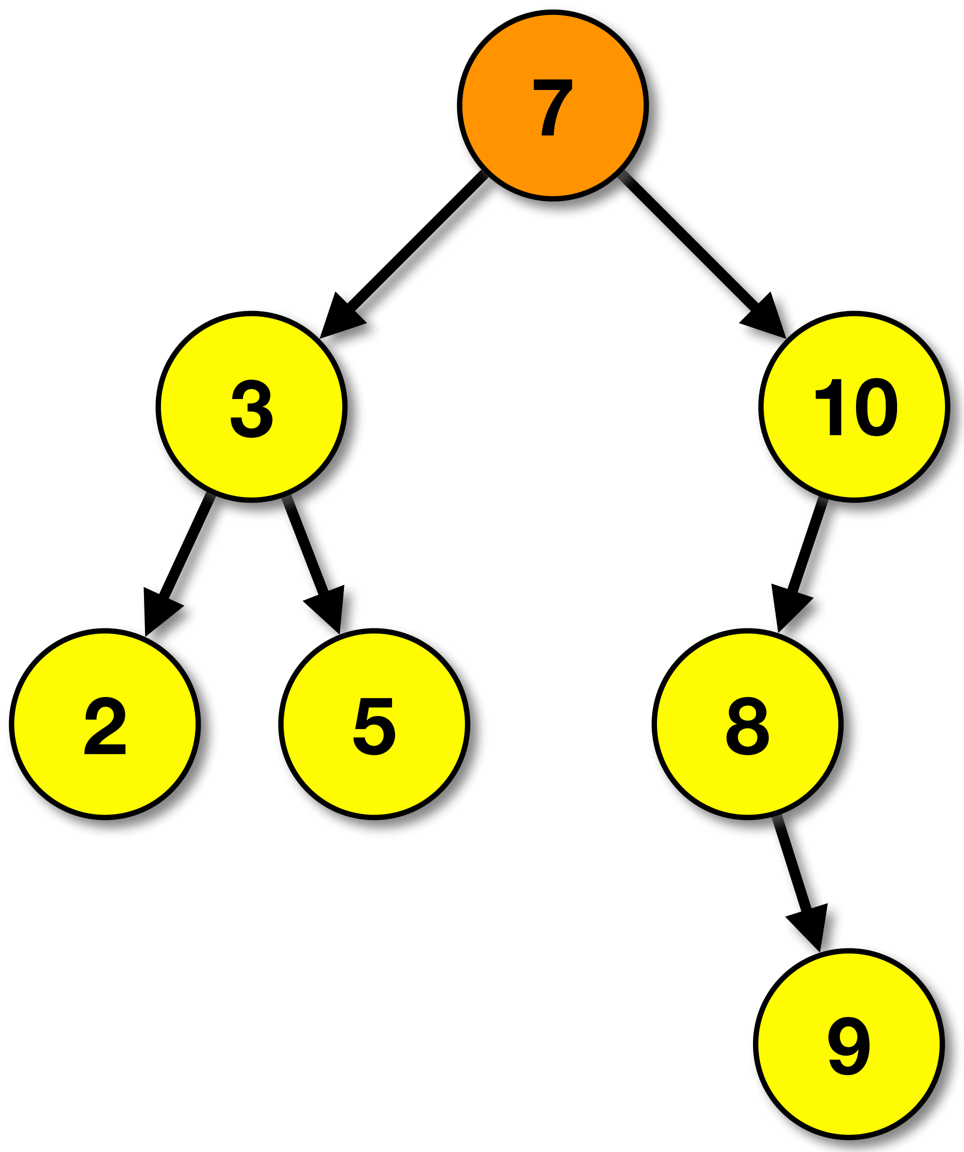
The `remove` Operation: Case #4 - Two Children

- **Example of `remove` operation with two children - `remove(3)`**



The `remove` Operation: Case #4 - Two Children

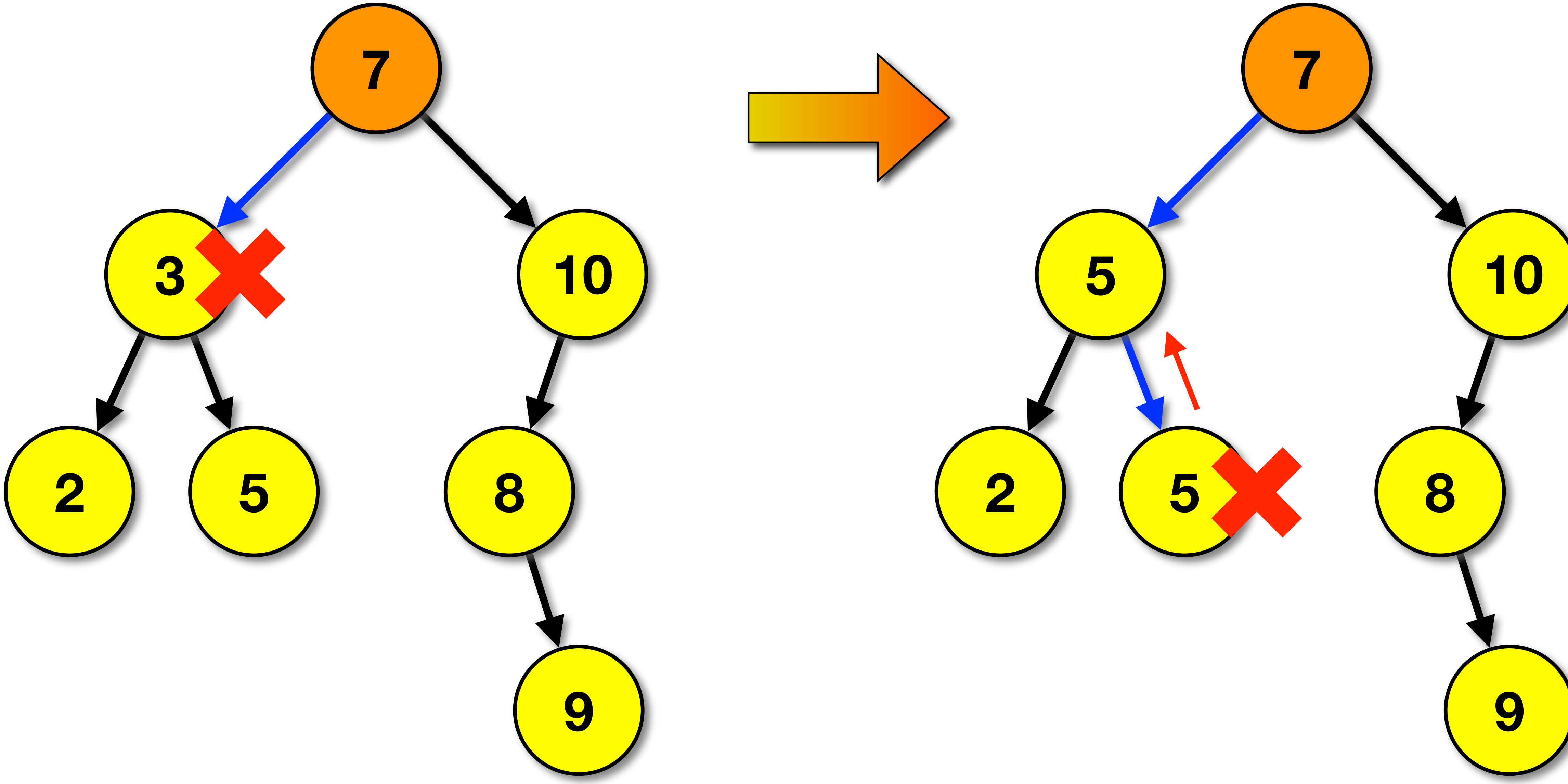
- **Example of `remove` operation with two children - `remove(3)`**



(1) start at root node (node 7)
(2) find node 3
(3) node 3 is has two children
(4) find the successor of node 3 (i.e. the min value from its right subtree)
(5) replace values of node 3 with those from successor
(6) remove the successor (may require additional modifications to the tree)

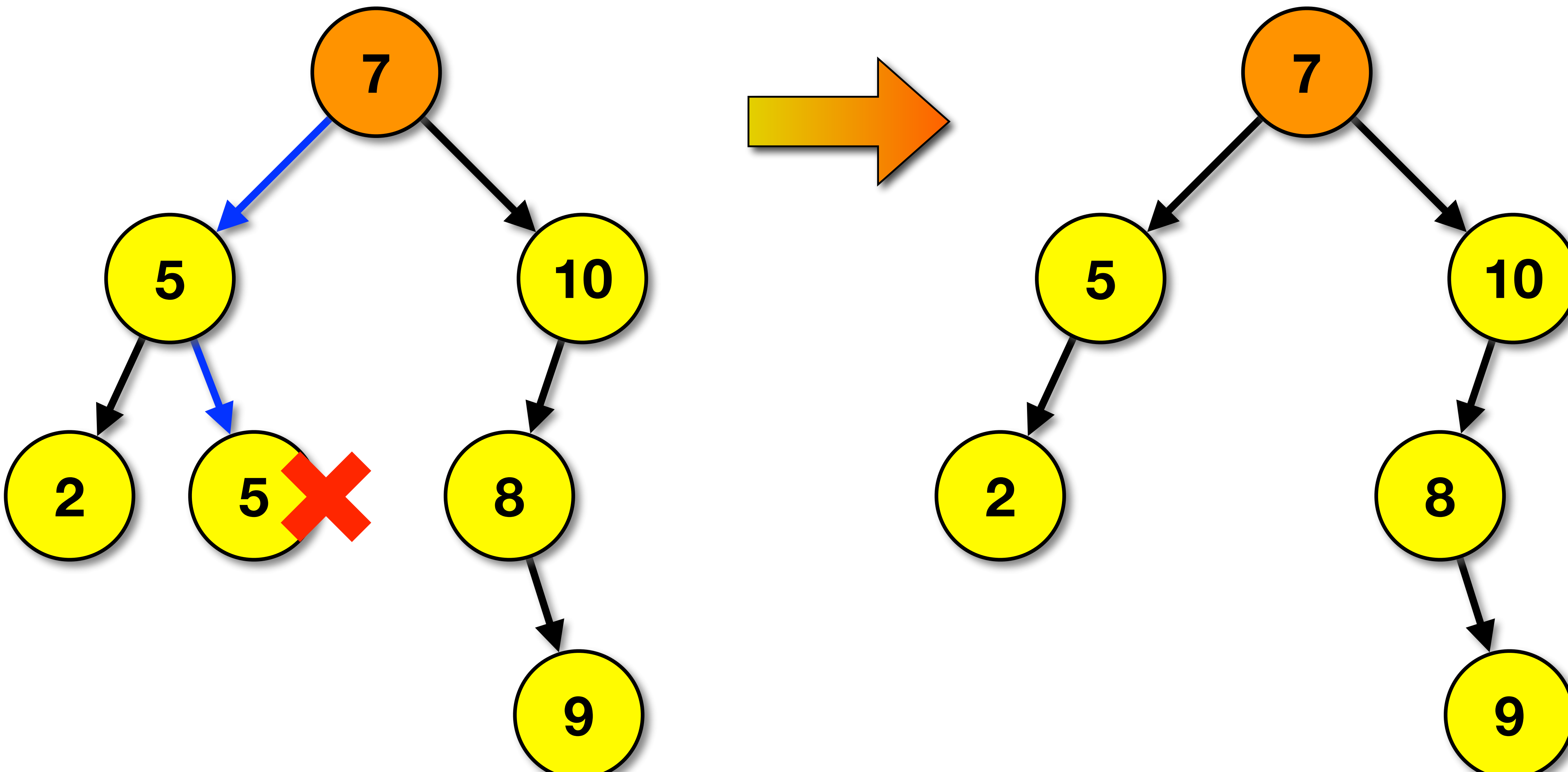
The `remove` Operation: Case #4 - Two Children

- **Example of `remove` operation with two children - `remove(3)`**



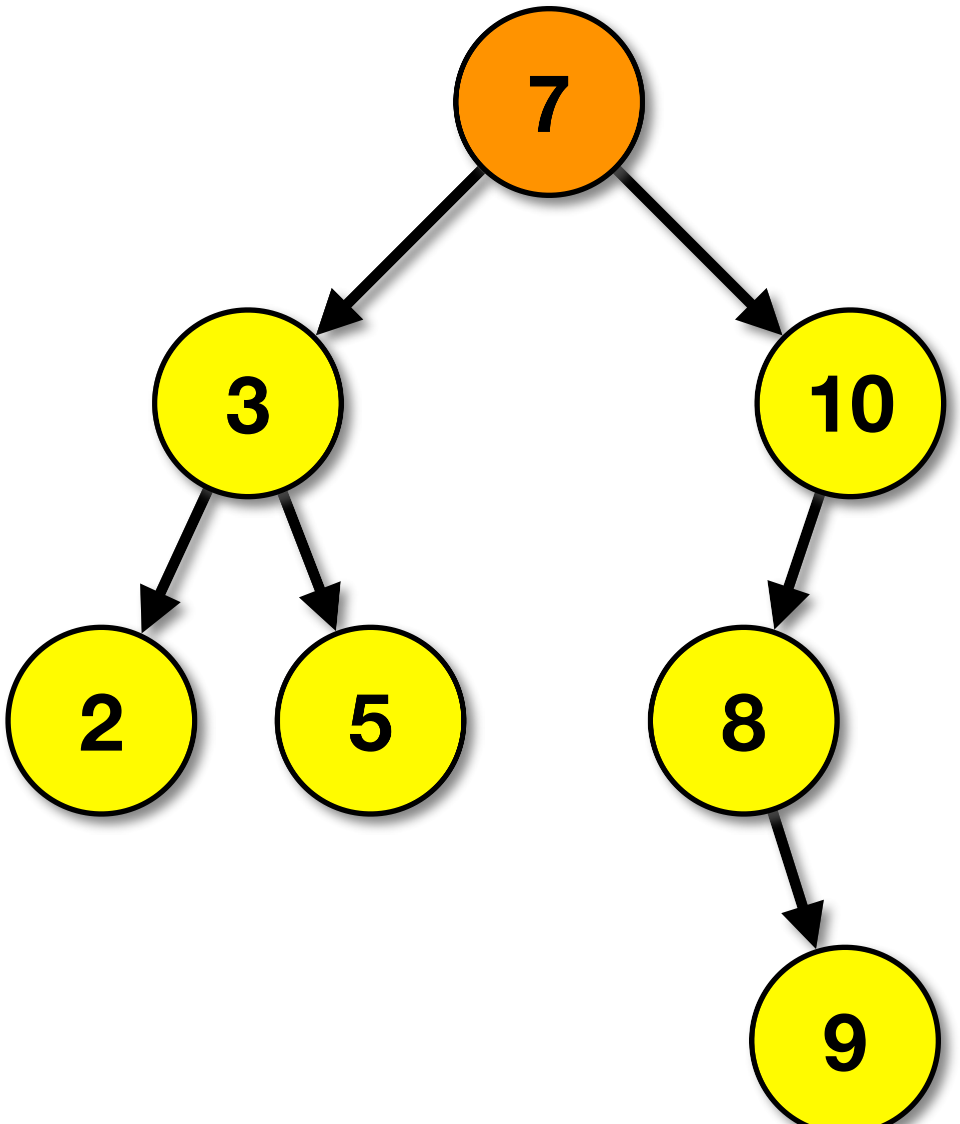
The `remove` Operation: Case #4 - Two Children

- **Example of `remove` operation with two children - `remove(3)`**



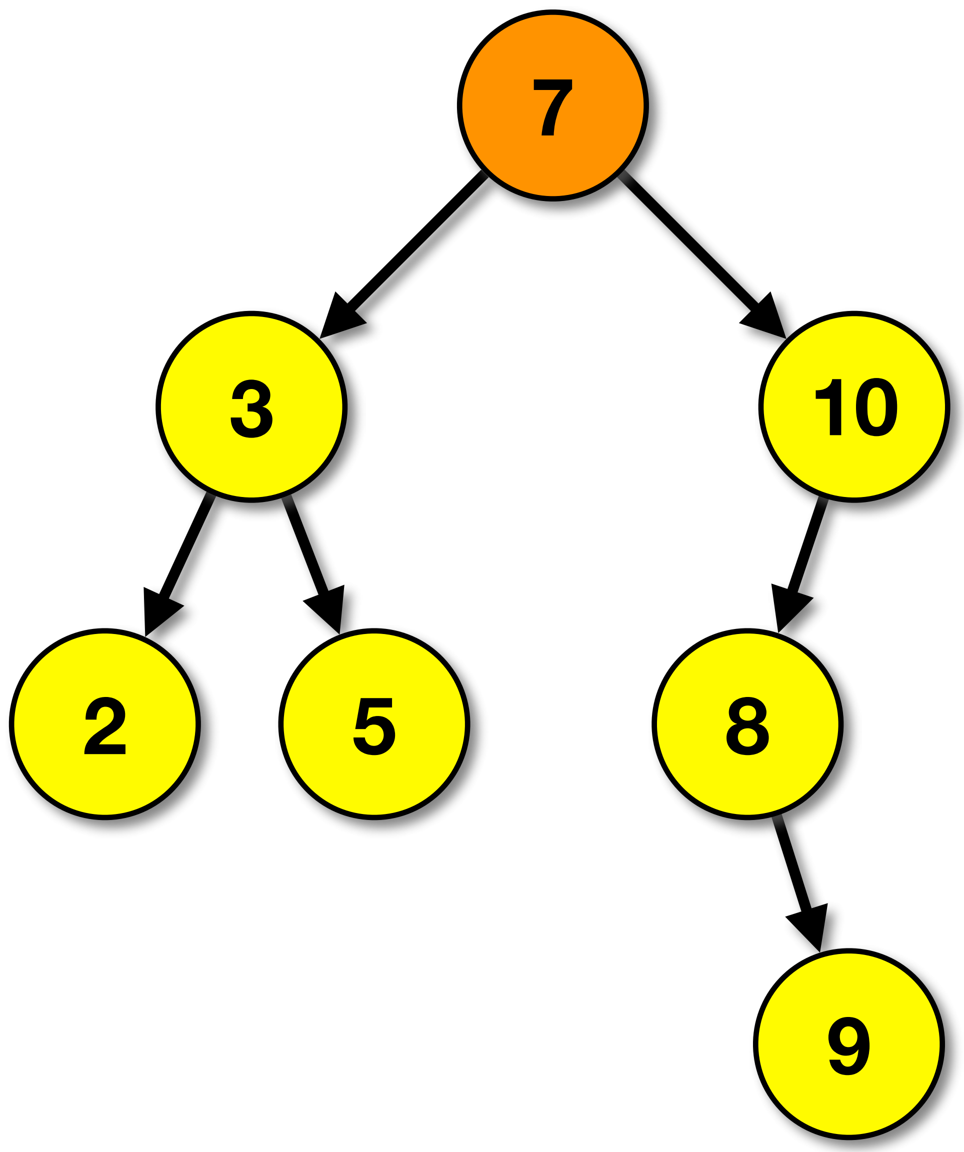
The `remove` Operation: Case #4 - Two Children

- **Example of `remove` operation with two children - `remove(7)`**



The `remove` Operation: Case #4 - Two Children

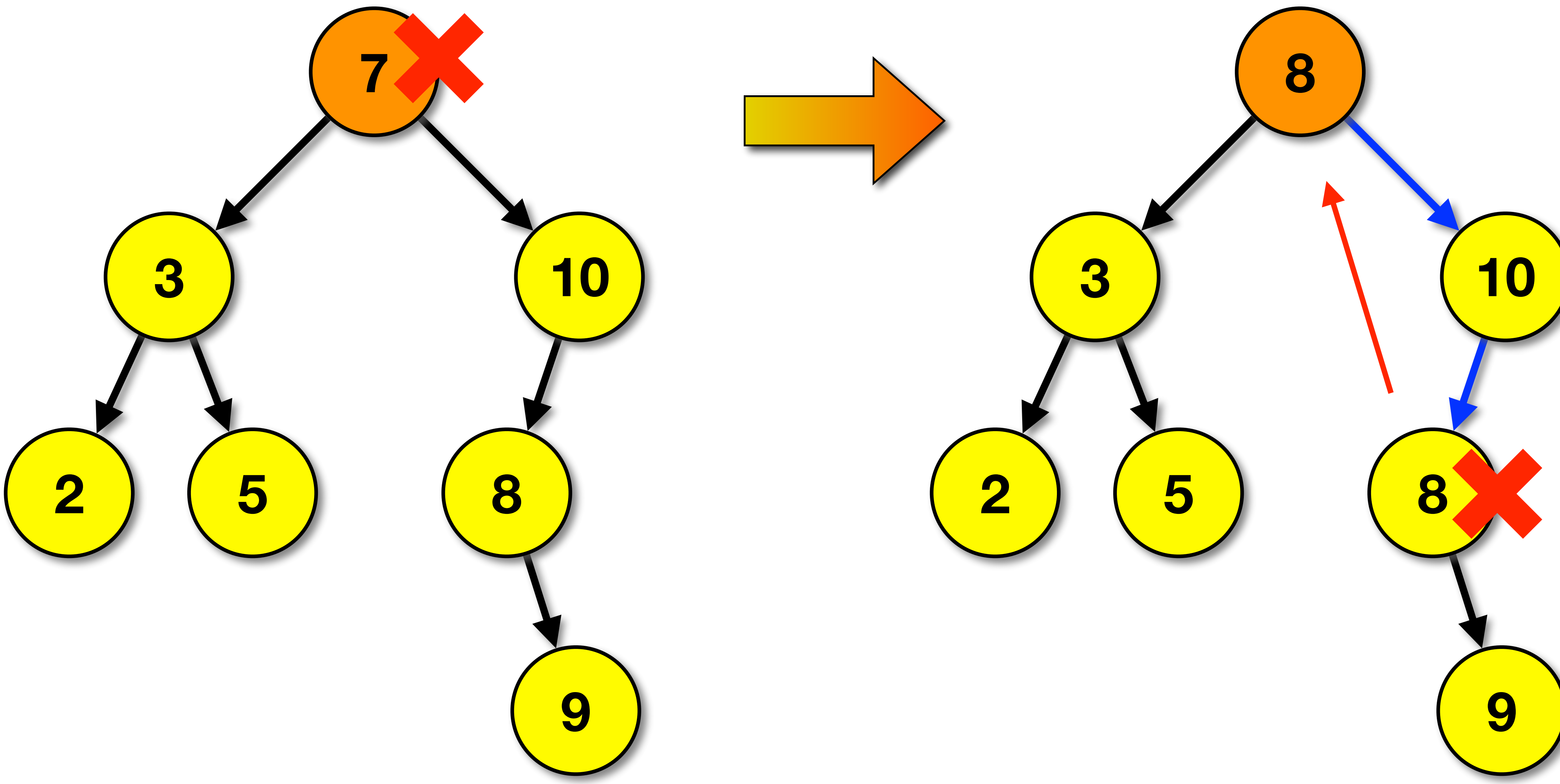
- **Example of `remove` operation with two children - `remove(7)`**



(1) start at root node (node 7)
(2) find node 7
(3) node 7 is has two children
(4) find the successor of node 7 (i.e. the min value from its right subtree)
(5) replace values of node 7 with those from successor
(6) remove the successor (may require additional modifications to the tree)

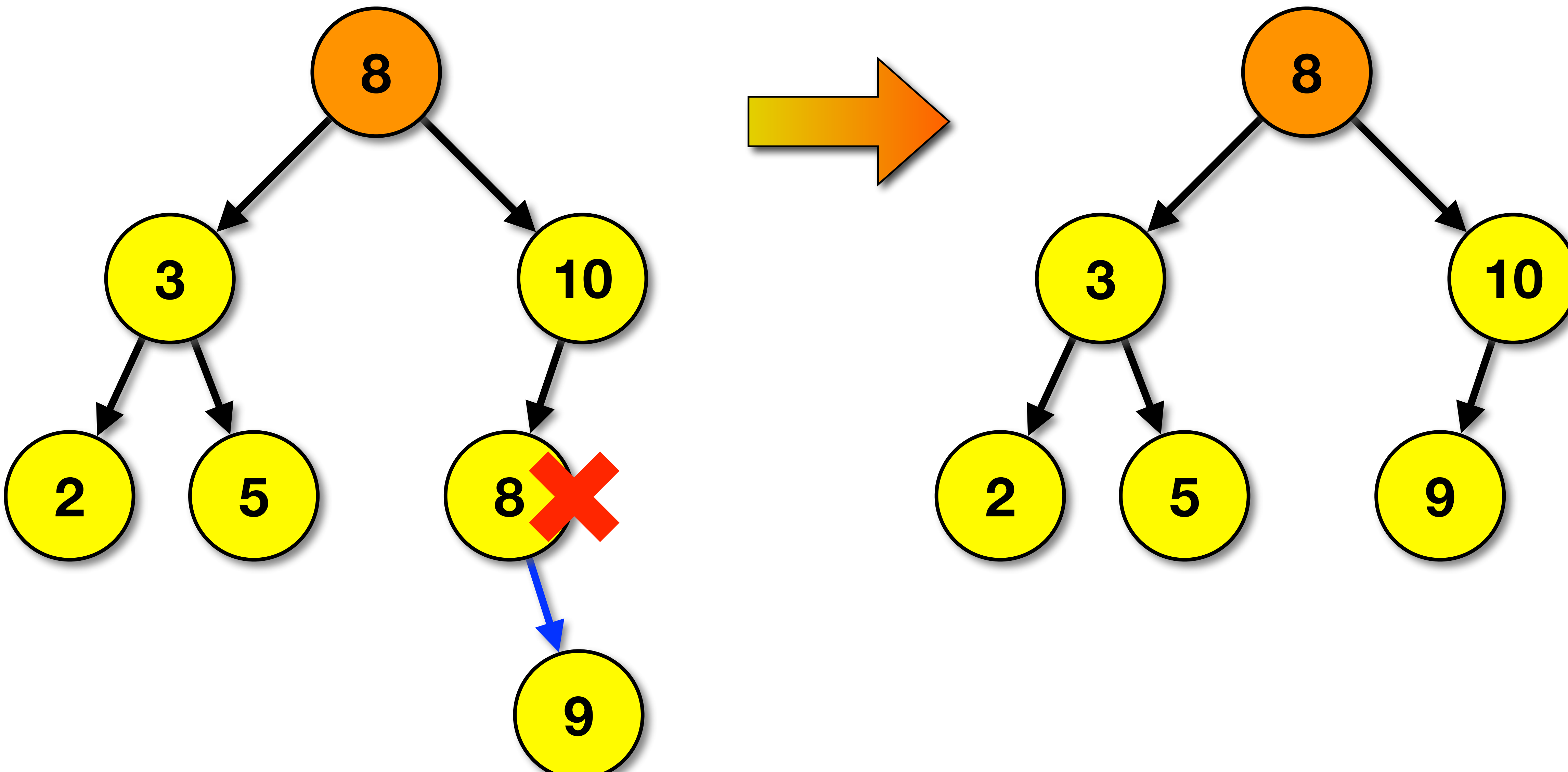
The `remove` Operation: Case #4 - Two Children

- **Example of `remove` operation with two children - `remove(7)`**



The `remove` Operation: Case #4 - Two Children

- **Example of `remove` operation with two children - `remove(7)`**



Analysis of BST Operations

- **Time complexity of BST operations**

	worst case	average
`find`	O(N)	O(log N)
`insert`	O(N)	O(log N)
`remove`	O(N)	O(log N)