Master Theorem

For recursive equations of the form:

$$T(n) = aT(n/b) + f(n)$$

Case 1: (Recursion dominates) If there exists a constant $\epsilon > 0$ such that $f(n) = O\left(n^{\log_b a - \epsilon}\right)$

Solution: $T(n) = \Theta\left(n^{\log_b a}\right)$

Case 2: (Equal) If there exists a constant $k > 0$ such that $f(n) = \Theta\left(n^{\log_b a} \lg^k n\right)$

Solution: $T(n) = \Theta\left(n^{\log_b a} \lg^{k+1} n\right)$

Case 2: (Combine dominates) If there exists a constant $\epsilon > 0$ such that $f(n) = \Omega\left(n^{\log_b a + \epsilon}\right)$ **AND** $f(n)$ satisfies the *regularity condition*

$af(n/b) \leq cf(n)$ for some $c < 1$ and all *sufficiently large* n

Solution: $T(n) = \Theta\left(f(n)\right)$

Note: $\log_b a = \frac{\ln a}{\ln b}$ (or $\log_b a = \frac{\log_{10} a}{\log_{10} b}$)