ECE335 Summer 2019 - Lecture 13 Examples

Example 1: Prove that if $A \subseteq B$ and $A \not\subseteq C$, then $B \not\subseteq C$. Hint: Use quantifiers and conjunction.

Given

$A \subseteq B$

$A \not\subseteq C$

Goal

$B \not\subseteq C$

Given

$\forall x (x \in A \rightarrow x \in B)$

$\exists y (y \in A \wedge y \notin C)$

Goal

$\exists z (z \in B \wedge z \notin C)$

Let x be arb. thing + $y = y_0$ $z = z_0$

Given

$x \in A \rightarrow x \in B$

$y_0 \in A$

$y_0 \notin C$

Goal

$z_0 \in B \wedge z_0 \notin C$

Since x is arb. thing, let $x = y_0$

Then since $y_0 \in A \rightarrow y_0 \in B$

But since $y_0 \notin C$ if $z_0 = y_0$

$\Rightarrow y_0 \in B$ but $y_0 \notin C \Rightarrow B \not\subseteq C$