ECE335 Summer 2019 - Lecture 14 Examples

Example 1: Suppose $A \subseteq C$ and B and C are disjoint. Prove that either $x \notin A$ or $x \notin B$. Hint: Use disjunction. (Note: This is an alternative version of the example from lecture 11.)

Given

$A \subseteq C$

$B + C$ disjoint

Goal

$x \notin A \vee x \notin B$

Given

[Def of $\subseteq$]

Let x be arbitrary

$x \in A \to x \in C$

$B + C$ disjoint

Goal

$x \notin A \vee x \notin B$

Given

Let x be arbitrary

$x \in A \to x \in C$

[disjoint] $x \in B \to x \notin C$

$x \in C \to x \notin B$

Goal

$x \notin A \vee x \notin B$

Case 1: Suppose $x \in A$

Then since $x \in A \Rightarrow x \in C$

But if $x \in C \Rightarrow \underline{x \notin B}$

Case 2: Suppose $x \in B$

Then since $x \in B \Rightarrow x \notin C$

By the contrapositive of the second given $(x \notin C \to x \notin A)$

$\Rightarrow \underline{x \notin A}$

Therefore either $x \notin A \vee x \notin B$

Example 2: Suppose A, B, and C are sets. Prove that if $A \subseteq C$ and $B \subseteq C$, then $A \cup B \subseteq C$. Hint: Use quantifiers and disjunction.

Given

A, B, C sets

Goal

$A \subseteq C$ and $B \subseteq C \rightarrow A \cup B \subseteq C$

[conditional + definition of $\subseteq$]

Given

Let x be arbitrary

$x \in A \rightarrow x \in C$

$x \in B \rightarrow x \in C$

Goal

$(x \in A \vee x \in B) \rightarrow x \in C$

[conditional]

Given

Let x be arbitrary

$x \in A \rightarrow x \in C$

$x \in B \rightarrow x \in C$

$x \in A \vee x \in B$

Goal

$x \in C$

Case 1: Suppose $x \in A$

Since $x \in A \Rightarrow \underline{x \in C}$

Case 2: Suppose $x \in B$

Since $x \in B \Rightarrow \underline{x \in C}$

Therefore since x was arbitrary, in either case $(A \cup B) \subseteq C$