

ECE335 Summer 2019 - Lecture 13 Examples

Example 1: Prove that if $A \subseteq B$ and $A \not\subseteq C$, then $B \not\subseteq C$. Hint: Use quantifiers and conjunction.

Given

$$A \subseteq B$$

$$A \not\subseteq C$$

Goal

$$B \not\subseteq C$$

Given

$$\forall x (x \in A \rightarrow x \in B)$$

$$\exists y (y \in A \wedge y \notin C)$$

Goal

$$\exists z (z \in B \wedge z \notin C)$$

Let x be arb. thing + $y = y_0$ $z = z_0$

Given

$$x \in A \rightarrow x \in B$$

$$y_0 \in A$$

$$y_0 \notin C$$

Goal

$$z_0 \in B \wedge z_0 \notin C$$

Since x is arb. thing, let $x = y_0$

Then since $y_0 \in A \rightarrow y_0 \in B$

But since $y_0 \notin C$

$$\Rightarrow y_0 \in B \text{ but } y_0 \notin C \Rightarrow B \not\subseteq C$$