ECE335 Summer 2019 - Lecture 11 Examples

Example 1: Suppose $A \subseteq C$ and B and C are disjoint. Prove that if $x \in A$, then $x \notin B$. Hint: Use proof by contradiction.

Given
$A \subseteq C$
$B + C$ disjoint

Goal
$x \in A \rightarrow x \notin B$

Conditional

Given
$A \subseteq C$
$B + C$ disjoint
$x \in A$

Goal
$x \notin B$

Contradiction

Given
$A \subseteq C$
$B + C$ disjoint
$x \in A$
$x \in B$

Goal
$\emptyset$

Goal
contradiction

Assume $x \in B$. Since $B + C$ are disjoint $(B \cap C = \emptyset) \Rightarrow x \notin C$
Since $A \subseteq C$ $(\forall x (x \in A \rightarrow x \in C)$ for contrapositive $\forall x (x \notin C \rightarrow x \notin A)$
gives $x \notin A$. But this contradicts the given that $x \in A$

Thus $x \notin B$.