ECE335 Summer 2019 - Lecture 16 Examples

Example 1: Prove that for $x \neq 1$, $1 + x + x^2 + \ldots + x^n = \frac{x^{n+1}-1}{x-1}$.

Step 1: Write the induction given closed form for n

$$1+x+x^2+\ldots+x^n = \sum_{i=0}^{n} x^i = \frac{x^{n+1}-1}{x-1}$$

Step 2: Show the induction given is true for a base case

For $n=0$ $\quad \sum_{i=0}^{0} x^i = x^0 = 1 \qquad \frac{x^{0+1}-1}{x-1} = \frac{x-1}{x-1} = 1$ (for $x \neq 1$)

Step 3: Write the induction goal closed form for $n+1$

$$1+x+x^2+\ldots+x^n+x^{n+1} = (1+x+x^2+\ldots+x^n)+x^{n+1} = \sum_{i=0}^{n} x^i + x^{n+1} = \frac{x^{(n+1)+1}-1}{x-1}$$

$$= \frac{x^{n+2}-1}{x-1}$$

Step 4: Assume the induction formula from step 1 is true and substitute the closed form solution into step 3

$$\sum_{i=0}^{n} x^i + x^{n+1} = \frac{x^{n+1}-1}{x-1} + x^{n+1}$$

Step 5: Perform any necessary algebra to show the closed form solution from step 3 for $n+1$

$$\frac{x^{n+1}-1}{x-1} + x^{n+1} = \frac{\cancel{x^{n+1}} - 1 + x^{n+2} - \cancel{x^{n+1}}}{x-1}$$

$$= \frac{x^{n+2}-1}{x-1}$$

Example 2: Prove that for $n \geq 1$, $1 + 3 + 5 + \dots (2n-1) = \sum_{i=1}^{n}(2i-1) = n^2$.

Step 1: Write the induction given closed form for n

$$1+3+5+\dots+(2n-1) = \sum_{i=1}^{n}(2i-1) = n^2$$

Step 2: Show the induction given is true for a base case

For $n=1$

$$\sum_{i=1}^{1}(2i-1) = 2(1)-1 = 1 \qquad 1^2 = 1 \qquad ✓$$

Step 3: Write the induction goal closed form for $n+1$

$$1+3+5+\dots+(2n-1)+(2(n+1)-1) = \sum_{i=1}^{n}(2i-1)+(2n+1) = (n+1)^2$$

Step 4: Assume the induction formula from step 1 is true and substitute the closed form solution into step 3

$$\sum_{i=1}^{n}(2i-1) + (2n+1) = n^2 + (2n+1) = (n+1)^2$$

Step 5: Perform any necessary algebra to show the closed form solution from step 3 for $n+1$

$$n^2 + (2n+1) = n^2 + 2n + 1$$

$$= \underline{(n+1)^2}$$