# ECE335 Summer 2019 - Assignment 1

## Intro.

4. Using theorem 4 on pg. 5, we can find 5 consecutive non-prime integers by considering the formula

$$x = (n + 1)! + 2 = (5 + 1)! + 2 = 6! + 2 = 720 + 2 = 722$$

Thus 5 consecutive non-prime integers are **722** (2*361), **723** (3*241), **724** (4*181), **725** (5*145), **726** (6*121).

## Section 1.1 - Deductive Reasoning and Logical Connectives

1.1.3. Let $A =$ "Alice is in the room." and $B =$ "Bob is in the room."

a. "Alice and Bob are not both in the room."
Let $A =$ "Alice is in the room." and $B =$ "Bob is in the room."
The semantic meaning of the original statement is that they are not in the room together or equivalently "*Not* Alice is in the room *and* Bob is in the room." Thus the logical form is $\neg(P \land Q)$.

b. "Alice and Bob are both not in the room."
The semantic meaning of this statement is that neither of them is in the room or equivalently "Alice is *not* in the room *and* Bob is *not* in the room. Thus the logical form is $\neg P \land \neg Q$.

c. "Either Alice or Bob is not in the room."
The semanic meaning of this statment is that one of them is not in the room or equivalently "Alice is *not* in the room *or* Bob is *not* in the room." Thus the logical form is $\neg P \lor \neg Q$.

d. "Neither Alice nor Bob is in the room."
The semanic meaning of this statement is the same as (b) (that neither of them is in the room) so the logical form is $\neg P \land \neg Q$.

1.1.5 $P =$ "I will buy the pants." $Q =$ "I will buy the shirt."

a. $\neg(P \lor \neg Q)$
Direct substitution gives "*Not* I will buy the pants *and* I will *not* buy the shirt." This can be rewritten as "I will not both buy the pants and not buy the shirt." This means that **"I will not buy the pants without the shirt."**

b. $\neg P \land \neg Q$
Direct substitution gives "I will *not* buy the pants *and* I will *not* buy the shirt." This can be rewritten as **"I will not buy the pants or the shirt."**

c. $\neg P \lor \neg Q$
Direct substitution gives "I will *not* buy the pants *or* I will *not* buy the shirt." This can be rewritten as **"I will not buy both the pants and the shirt."**

## Section 1.2 - Truth Tables

Ex. 1.2.7. Simplify

a. $P \vee (Q \wedge \neg P)$

$$\begin{aligned} P \vee (Q \wedge \neg P) &\text{ is equivalent to } (P \vee Q) \wedge (P \vee \neg P) && \text{by the distributive law} \\ &\text{ is equivalent to } (P \vee Q) \wedge (T) && \text{by tautology} \\ &\text{ is equivalent to } (P \vee Q) && \text{by tautology law} \end{aligned}$$

b. $\neg(P \vee (Q \wedge \neg R)) \wedge Q$

$$\begin{aligned} \neg(P \vee (Q \wedge \neg R)) \wedge Q &\text{ is equivalent to } \neg((P \vee Q) \wedge (P \vee \neg R)) \wedge Q && \text{by the distributive law} \\ &\text{ is equivalent to } (\neg(P \vee Q) \vee \neg(P \vee \neg R)) \wedge Q && \text{by DeMorgan's law} \\ &\text{ is equivalent to } ((\neg P \wedge \neg Q) \vee (\neg P \wedge \neg\neg R)) \wedge Q && \text{by DeMorgan's law} \\ &\text{ is equivalent to } ((\neg P \wedge \neg Q) \vee (\neg P \wedge R)) \wedge Q && \text{by double negation law} \\ &\text{ is equivalent to } ((\neg P \wedge \neg Q) \wedge Q) \vee ((\neg P \wedge R) \wedge Q) && \text{by distributive law} \\ &\text{ is equivalent to } (\neg P \wedge (\neg Q \wedge Q)) \vee ((\neg P \wedge R) \wedge Q) && \text{by associative law} \\ &\text{ is equivalent to } (\neg P \wedge (F)) \vee ((\neg P \wedge R) \wedge Q) && \text{by contradiction} \\ &\text{ is equivalent to } (F) \vee ((\neg P \wedge R) \wedge Q) && \text{by contradiction law} \\ &\text{ is equivalent to } (\neg P \wedge R) \wedge Q && \text{by contradiction law} \end{aligned}$$

(Note this is equivalent to $\neg P \wedge (Q \wedge R)$ by the associative and commutative laws.)

1.2.12. Simplify

a. $\neg(\neg P \vee Q) \vee (P \wedge \neg R)$

$$\begin{aligned} \neg(\neg P \vee Q) \vee (P \wedge \neg R) &\text{ is equivalent to } (\neg\neg P \wedge \neg Q) \vee (P \wedge \neg R) && \text{by DeMorgan's law} \\ &\text{ is equivalent to } (P \wedge \neg Q) \vee (P \wedge \neg R) && \text{by double negation law} \\ &\text{ is equivalent to } P \wedge (\neg Q \vee \neg R) && \text{by distributive law} \\ &\text{ is equivalent to } P \wedge \neg(Q \wedge R) && \text{by DeMorgan's law} \end{aligned}$$

b. $\neg(\neg P \wedge Q) \vee (P \wedge \neg R)$

$$\begin{aligned}
\neg(\neg P \wedge Q) \vee (P \wedge \neg R) &\text{ is equivalent to } (\neg\neg P \vee \neg Q) \vee (P \wedge \neg R) && \text{by DeMorgan's law} \\
&\text{ is equivalent to } (P \vee \neg Q) \vee (P \wedge \neg R) && \text{by double negation law} \\
&\text{ is equivalent to } (\neg Q \vee P) \vee (P \wedge \neg R) && \text{by commutative law} \\
&\text{ is equivalent to } \neg Q \vee (P \vee (P \wedge \neg R)) && \text{by associative law} \\
&\text{ is equivalent to } \neg Q \vee P && \text{by absorption law} \\
&\text{ is equivalent to } P \vee \neg Q && \text{by commutative law}
\end{aligned}$$

c. $(P \wedge R) \vee (\neg R \wedge (P \vee Q))$

$$\begin{aligned}
(P \wedge R) \vee (\neg R \wedge (P \vee Q)) &\text{ is equivalent to } (P \wedge R) \vee ((P \wedge \neg R) \vee (Q \wedge \neg R)) && \text{by distributive and commutativ} \\
&\text{ is equivalent to } ((P \wedge R) \vee (P \wedge \neg R)) \vee (Q \wedge \neg R) && \text{by associative law} \\
&\text{ is equivalent to } (P \wedge (R \vee \neg R)) \vee (Q \wedge \neg R) && \text{by distributive law} \\
&\text{ is equivalent to } (P \wedge (T)) \vee (Q \wedge \neg R) && \text{by tautology} \\
&\text{ is equivalent to } P \vee (Q \wedge \neg R) && \text{by tautology law}
\end{aligned}$$

(Note that the final equivalencies can be verified using a truth table check.)

## Section 1.5 - The Conditional and Biconditional Connectives

1.5.4. Use truth tables to check the validity of the following arguments:

a. Let $S$ = "Sales will go up.", $E$ = "Expenses will go up.", and $B$ = "The boss will be happy."

Then the statements can be represented symbolically as:

"Either sales or expenses will go up." $\Rightarrow S \vee E$
"If sales go up, then the boss will be happy." $\Rightarrow S \to B$
"If expenses go up, then the boss will be unhappy." $\Rightarrow E \to \neg B$
"Therefore, sales and expenses will not both go up." $\Rightarrow \therefore \neg(S \wedge E)$

Hence the argument has the symbolic form:
$S \vee E$
$S \to B$
$\underline{E \to \neg B}$
$\therefore \neg(S \wedge E)$

Constructing the truth table for the above statements gives

| S ∨ E | S → B | E → ¬ B | ¬ ( S ∧ E ) |
|---|---|---|---|
| F F F | F T F | F T T F | T F F F |
| F F F | F T T | F T F T | T F F F |
| F T T | F T F | T T T F | T F F T |
| F T T | F T T | T F F T | T F F T |
| T T F | T F F | F T T F | T T F F |
| T T F | T T T | F T F T | T T F F |
| T T T | T F F | T T T F | F T T T |
| T T T | T T T | T F F T | F T T T |

We see that for the rows where all the premises are $T$ the conclusion is also $T$ so the argument is **valid**.

b. Let $T =$ "The tax rate will go up.", $U =$ "The unemployment rate will go up.", $G =$ "The GNP will go up.", and $R =$ "There will be a recession."

Then the statements can be represented symbolically as:

"If the tax rate and the unemployment rate both go up, then there will be a recession." $\Rightarrow (T \wedge U) \to R$
"If the GNP goes up, then there will not be a recession." $\Rightarrow G \to \neg R$
"The GNP and taxes are both going up." $\Rightarrow G \wedge T$
"Therefore, the unemployment rate is not going up." $\Rightarrow \therefore \neg U$

Hence the argument has the symbolic form:
$(T \wedge U) \to R$
$G \to \neg R$
$G \wedge T$

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$\therefore \neg U$

Constructing the truth table for the above statements gives

| $(T \wedge U) \to R$ | $G \to \neg R$ | $G \wedge T$ | $\neg U$ |
|---|---|---|---|
| F F F   T F | F T T F | F F F | T F |
| F F F   T F | T T T F | T F F | T F |
| F F F   T T | F T F T | F F F | T F |
| F F F   T T | T F F T | T F F | T F |
| F F T   T F | F T T F | F F F | F T |
| F F T   T F | T T T F | T F F | F T |
| F F T   T T | F T F T | F F F | F T |
| F F T   T T | T F F T | T F F | F T |
| T F F   T F | F T T F | F F T | T F |
| T F F   T F | T T T F | T T T | T F |
| T F F   T T | F T F T | F F T | T F |
| T F F   T T | T F F T | T T T | T F |
| T T T   F F | F T T F | F F T | F T |
| T T T   F F | T T T F | T T T | F T |
| T T T   T T | F T F T | F F T | F T |
| T T T   T T | T F F T | T T T | F T |

We see that for the rows where all the premises are $T$ the conclusion is also $T$ so the argument is **valid**.

c. Let $W =$ "The warning light will come on.", $P =$ "The pressure is too high.",and $R =$ "The relief valve is clogged."

Then the statements can be represented symbolically as:

"The warning light will come on if and only if the pressure is too high and the relief valve is clogged." $\Rightarrow W \leftrightarrow (P \wedge R)$
"The relief valve is not clogged." $\Rightarrow \neg R$
"Therefore, warning light will come on if and only if the pressure is too high." $\Rightarrow \therefore W \leftrightarrow P$

Hence the argument has the symbolic form:
$W \leftrightarrow (P \wedge R)$
$\neg R$

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$\therefore W \leftrightarrow P$

Constructing the truth table for the above statements gives

| $W \leftrightarrow (P \wedge R)$ | $\neg R$ | $W \leftrightarrow P$ |
|---|---|---|
| F T F F F | T F | F T F |
| F T F F T | F T | F T F |
| F T T F F | T F | F F T |
| F F T T T | F T | F F T |
| T F F F F | T F | T F F |
| T F F F T | F T | T F F |
| T F T F F | T F | T T T |
| T T T T T | F T | T T T |

We see that for the first row where all the premises are $T$ the conclusion is also $T$, but for the third row all the premises are also $T$ but the conclusion is $F$ so the argument is **invalid**.