ECE335 Summer 2019 - Lecture 12 Examples

Example 1: Prove that if $A \subseteq B \setminus C$, then A and C are disjoint. Hint: Use proof by contradiction and quantifiers.

Given

$A \subseteq B \setminus C$

Goal

A & C disjoint

Contradiction

Given

$A \subseteq B \setminus C$

A & C not disjoint

Goal

contradiction

Suppose A & C are not disjoint $\Rightarrow \exists x\,(x \in A \cap C) \Rightarrow \exists x (x \in A \wedge x \in C)$

Let $x = x_0 \Rightarrow x_0 \in A$ and $x_0 \in C$

$A \subseteq B \setminus C \Rightarrow \forall x (x \in A \rightarrow x \in B \setminus C)$

Therefore since $x_0 \in A \Rightarrow x_0 \in B \setminus C$

$\Rightarrow x_0 \in B \wedge x_0 \notin C$

But since $x_0 \in C$ there is contradiction.

Hence A & C are disjoint