ECE335 Summer 2019 - Assignment 2

Section 1.5 - The Conditional and Biconditional Connectives

1.5.5. Verify the following equivalences

a. $P \leftrightarrow Q$ is equivalent to $(P \wedge Q) \vee (\neg P \wedge \neg Q)$

Constructing the truth table for the above statements gives

P↔Q	(P^Q)	∨	(¬P^¬Q)
F**T**F	F**F**F	**T**	TF**T**TF
F**F**T	F**F**T	**F**	TF**F**FT
T**F**F	T**F**F	**F**	FT**F**TF
T**T**T	T**T**T	**T**	FT**F**FT

We see that the truth tables are the same for all rows and thus the statements are logically equivalent.

b. $(P \to Q) \vee (P \to R)$ is equivalent to $P \to (Q \vee R)$

$(P \to Q) \vee (P \to R)$	is equivalent to $(\neg P \vee Q) \vee (\neg P \vee R)$	by the conditional law
	is equivalent to $(\neg P \vee \neg P) \vee (Q \vee R)$	by commutative and associative laws
	is equivalent to $\neg P \vee (Q \vee R)$	by idempotent law
	is equivalent to $P \to (Q \vee R)$	by conditional law

Section 2.1 - Quantifiers

2.1.1. Analyze the logical forms of the following statements.

a. "Anyone who has forgiven at least one person is a saint."

"Anyone" represents a universal quantifier (since it implies for all), thus we can rewrite the original statement as

$\forall x$ (if x has forgiven someone then x is a saint).

"Someone" represents an existential quantifier (since it implies there exists), thus we can further rewrite the statement as

$\forall x$ ($\exists y$ if x has forgiven y then x is a saint).

Now we create symbolic relationships $F(x, y)$ for "x has forgiven y" and $S(x)$ for "x is a saint" and use the conditional symbol giving the final logical form

$\forall x(\exists y F(x, y) \to S(x))$

b. "Nobody in the calculus class is smarter than everybody in the discrete math class."

"Nobody" represents a negated existential quantifier (since it implies there does not exist), thus we can rewrite the original statement as

$\neg\exists x$ (if x is in the calculus class, then they are smarter than everybody in the discrete math class).

"Everybody" is a universal quantifier (since it implies for all), thus we can further rewrite the statement as

$\neg\exists x$ (if x is in the calculus class, then $\forall y$ x is smarter than y in the discrete math class).

Creating symbolic relationships $C(x)$ for "x is in the calculus class", $D(y)$ for "y is in the discrete math class, and $S(x, y)$ for "x is smarter than y" gives

$\neg\exists x$ (if $C(x)$ then $\forall y$ (if $D(y)$ then $S(x, y)$))).

Finally replacing the connective and conditional gives the logical form

$\neg\exists x(C(x) \to \forall y(D(y) \to S(x, y)))$

c. "Everyone likes Mary, except Mary herself."

"Everyone" represents a universal quantifier (since it implies for all), thus we can rewrite the original statement as

$\forall x$ (x likes Mary, except Mary herself).

Letting m represent Mary and creating a symbolic relationship $L(x, y)$ for "x likes y" we can further rewrite the statement as

$\forall x$ ($L(x, m)$, except $x = m$).

"Except" indicates a negation (true if a value does not equal a specific one) thus the logical form is

$\forall x(\neg(x = m) \to L(x, m))$

d. "Jane saw a police officer, and Roger saw one too."

"A police officer" represents an existential quantifier (since it implies one from a set). "...saw one" implies that Jane and Roger could have seen different officers, so we need two different variables allowing the statement to be rewritten as

$\exists x$ (Jane saw a police officer x) and $\exists y$ (Roger saw a police officer y).

Creating the symbolic relationships $P(x)$ for "x is a police officer" and $S(x, y)$ for "x saw y", if we let j represent Jane and r represent Roger the statement can be rewritten as

$\exists x$ ($P(x)$ and $S(j, x)$) and $\exists y$ ($P(y)$ and $S(r, y)$)

Note that we reuse the function $P()$ with the different variables x, y since each person may have seen different people both of which are police officers. Finally replacing the connectives gives the logical form

$\exists x(P(x) \wedge S(j, x)) \wedge \exists y(P(y) \wedge S(r, y))$

e. "Jane saw a police officer, and Roger saw him too."

"A police officer" represents an existential quantifier (since it implies one from a set). "...saw him" implies that Jane and Roger saw the same officer, thus we only need one variable allowing the statement to be rewritten as

$\exists x$ (x is a police officer and Jane saw x and Roger saw x).

Creating the same symbolic relationships as before $P(x)$ for "x is a police officer" and $S(x, y)$ for "x saw y", if we let j represent Jane and r represent Roger the statement can be rewritten as

$\exists x(P(x) \wedge (S(j, x) \wedge S(r, x)))$

2.1.1. Analyze the logical forms of the following statements.

1. "Everybody in the dorm has a roommate he doesn't like."

   "Everybody" indicates a universal quantifier (since it implies for all) and "a roommate" indicates an existential quantifier (since it implies there exists) thus we can rewrite the original statement as

   $\forall x$ (if x lives in the dorm then $\exists y$ (x and y are roommates and x doesn't like y)

   Creating the symbolic relationships $D(x)$ for "x lives in the dorm", $R(x, y)$ for "x and y are roommates", and $L(x, y)$ for "x likes y" then the logical form is

   $\forall x(D(x) \rightarrow \exists y(R(y, x) \wedge \neg L(x, y)))$

2. "Nobody likes a sore loser."

   "Nobody" indicates a negated existential quantifier (since it implies not there exists) and "a sore loser" indicates a *universal* quantifier (since it implies for all losers) thus we can rewrite the original statement as

   $\forall x$ (if x is a sore loser then $\neg\exists y$ (y likes x)

   Creating the symbolic relationships $S(x)$ for "x is a sore loser" and $L(y, x)$ for "y likes x" the logical form is

   $\forall x(S(x) \rightarrow \neg\exists \text{y}(L(y, x)))$

3. "Anyone who has a friend who has the measles will have to be quarantined."

   "Anyone" indicates a universal quantifier (since it implies for all) and "a friend" indicates an existential quantifier (since it indicates there exists) thus we can rewrite the original statement as

$\forall x(\exists y$ (if x is a friend of y and y has the measles, then x will have to be quarantined))

Creating the symbolic relationships $F(x,y)$ for "x and y are friends", $M(y)$ for "y has the measles", and $Q(x)$ for "x will have to be quarantined" the logical form is

$\forall x\exists y(F(x,y) \land M(y) \to Q(x))$

4. "If anyone in the dorm has a friend who has the measles, then everyone in the dorm will have to be quarantined."

   Here "Anyone" indicates an existential quantifier (since it implies there exists), "a friend" indicates an existential quantifier (since it indicates there exists), and "everyone" indicates a universal quantifier (since it indicates for all) thus we can rewrite the original statement as

   $(\exists x$ (x lives in the dorm and $\exists y$ (x and y are friends and y has the measles)) $\to$ $(\forall z$ (if z lives in the dorm, then z will have to be quarantined)

   Creating the symbolic relationships $D(x)$ for "x lives in the dorm", $F(x,y)$ for "x and y are friends", $M(y)$ for "y has the measles", and $Q(z)$ for "z will have to be quarantined" then the logical form is

   $(\exists x(D(x) \land \exists y(F(x,y) \land M(y)))) \to (\forall z(D(z) \to Q(z)))$

2.1.4. Translate into English

a. $H(x)$ for "x is a man", $M(x,y)$ for "x is married to y", and $U(x)$ for "x is unhappy"

   $\forall x[(H(x) \land \neg\exists y M(x,y)) \to U(x)]$

   Literal translation

   "For all x, if x is a man and there is not someone y that x is married to, then x is unhappy."

   Simplifying gives

   "Every unmarried man is unhappy."

b. $P(z,x)$ for "z is a parent of x", $S(z,y)$ for "z and y are siblings", and $W(y)$ for "y is a woman"

   $\exists z(P(z,x) \land S(z,y) \land W(y))$

   Literal translation

   "There exists a person that is a parent of x and is a sibling of y who is a woman."

This means that y is the sister of the parent of x or in otherwords "y is x's aunt."

Section 2.2 - Equivalences Involving Quantifiers

2.2.1. Negate the following statements and express the result as a positive statement.

a. "Everyone who is majoring in math has a friend who needs help with his homework."

"Everyone" indicates a universal quantifier (since it implies for all) and "a friend" indicates an existential quantifier (since it means there exists) thus we can rewrite the original statement as

$\forall x$ (if x is majoring in math, then $\exists y$ (if y is a friend of x, then y needs help with his homework.))

Creating the symbolic relationships $M(x)$ for "x is majoring in math", $F(y, x)$ for "y is a friend of x", and $H(y)$ for "y needs help with his homework" then the logical form is

$\forall x(M(x) \to \exists y(F(y, x) \to H(y)))$

Thus the negation of this statment is

$$\begin{aligned}
\neg\forall x(M(x) \to \exists y(F(y,x) \to H(y))) & \quad \text{is equivalent to} \\
\exists x\neg(M(x) \to \exists y(F(y,x) \to H(y))) & \quad \text{by the quantifier negation law} \\
\exists x\neg(\neg M(x) \lor \exists y(F(y,x) \to H(y))) & \quad \text{by the conditional law} \\
\exists x(M(x) \land \neg\exists y(F(y,x) \to H(y))) & \quad \text{by DeMorgan's law} \\
\exists x(M(x) \land \forall y\neg(F(y,x) \to H(y))) & \quad \text{by quantifier negation law} \\
\exists x(M(x) \land \forall y\neg(\neg F(y,x) \lor H(y))) & \quad \text{by conditional law} \\
\exists x(M(x) \land \forall y(F(y,x) \land \neg H(y))) & \quad \text{by DeMorgan's law}
\end{aligned}$$

Translating back to English gives

"There is a math major all of whose friends do not need help with their homework."

b. "Everyone has a roomate who dislikes everyone."

In both cases "everyone" indicates a universal quantifier (since both imply for all) and "a roommate" indicates an existential quantifier (since it implies there exists) thus we can rewrite the original statement as

$\forall x\exists y$ (if x and y are roomates, then $\forall z$ (y dislikes z)

Creating the symbolic relationships $R(x, y)$ for "x and y are roommates" and $L(y, z)$ for "y likes z" then the logical form is

$\forall x \exists y(R(x,y) \rightarrow \forall z(\neg L(y,z)))$

Thus the negation of this statment is

$$
\begin{aligned}
\neg\forall x \exists y(R(x,y) \rightarrow \forall z(\neg L(y,z))) &\quad \text{is equivalent to} \\
\exists x \neg\exists y(R(x,y) \rightarrow \forall z(\neg L(y,z))) &\quad \text{by the quantifier negation law} \\
\exists x \forall y \neg(R(x,y) \rightarrow \forall z(\neg L(y,z))) &\quad \text{by the quantifier negation law} \\
\exists x \forall y \neg(\neg R(x,y) \vee \forall z(\neg L(y,z))) &\quad \text{by the conditional law} \\
\exists x \forall y(R(x,y) \wedge \neg\forall z(\neg L(y,z))) &\quad \text{by DeMorgan's law} \\
\exists x \forall y(R(x,y) \wedge \exists z \neg(\neg L(y,z))) &\quad \text{by the quantifier negation law} \\
\exists x \forall y(R(x,y) \wedge \exists z(L(y,z))) &\quad \text{by double negation law}
\end{aligned}
$$

Translating back to English gives

"There is a person with all their roommates that like someone."

2.2.12. Translate into English using $T(x,y)$ for "x is a teacher of y"

a. $\exists! y T(x,y)$

This would mean "x has only one student."

b. $\exists x \exists! y T(x,y)$

This would mean "There is a teacher with only one student."

c. $\exists! x \exists y T(x,y)$

This would mean "There is only one teacher that has students."