ECE335 Summer 2019 - Lecture 10 Examples

Example 1: Prove that if $0 < a < b$, then $a^2 < b^2$. Hint: First show that $(b + a) > 0$ and $(b - a) > 0$.

Given | Goal
none | $0 < a < b \to a^2 < b^2$

Conditional

Given | Goal
$0 < a < b$ | $a^2 < b^2$

Since $a, b > 0 \Rightarrow (b + a) > 0$
Since $b > a (>0) \Rightarrow (b - a) > 0$
Hence $(b+a)(b-a) > 0 \Rightarrow b^2 - a^2 > 0$
Thus $\underline{b^2 > a^2}$ (or $a^2 < b^2$)

$a < b$
$a^2 < ab$

$a < b$
$ab < b^2$
$a^2 < ab < b^2$
$a^2 < b^2$

Example 2: Suppose $A \setminus B \subseteq C \cap D$ and $x \in A$. Prove that if $x \notin D$, then $x \in B$. Hint: Use Conditional Proof Template.

Given | Goal
$A \setminus B \subseteq C \cap D$ | $x \notin D \to x \in B$
$x \in A$ |

Conditional

Given | Goal
$A \setminus B \subseteq C \cap D$ | $x \in B$
$x \in A$ |
$x \notin D$ |

$x \in A \setminus B \Rightarrow x \in A \wedge x \notin B$ $\quad$ $x \in C \cap D \Rightarrow x \in C \wedge x \in D$

Then $\subseteq \Rightarrow \forall x\,((x \in A \wedge x \notin B) \to (x \in C \wedge x \in D))$

$\Rightarrow \forall x\,(\neg(x \in A \wedge x \notin B) \vee (x \in C \wedge x \in D))$ by conditional rule

$\Rightarrow \forall x\,((x \notin A \vee x \in B) \vee (x \in C \wedge x \in D))$ by Demorgan's law

Since $x \in A$, $\forall x\,(x \in B \vee (x \in C \wedge x \in D))$

Furthermore since $x \notin D$ means $x \in C \wedge \underline{x \in D}$ is F

Therefore $\underline{x \in B}$