Car Sales Time Series Analysis

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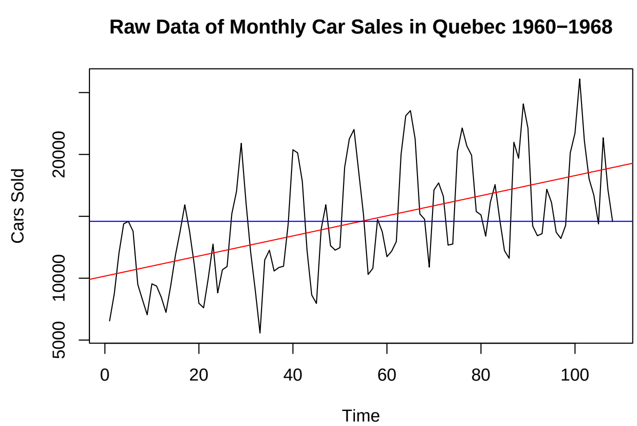
Abstract

This project focuses on the analysis and forecasting of car sales in Quebec from 1960 to 1968, utilizing the data available in the Time Series Data Library (TSDL). This dataset offers valuable insights into the historical growth of the car industry during the 1960s, a period when motorcars became more accessible and popular. Using R Studio, I've developed a time series forecasting models, including Autoregressive Integrated Moving Average (ARIMA) and Seasonal ARIMA (SARIMA), along with diagnostic tests to determine the best-fitting model for the data.

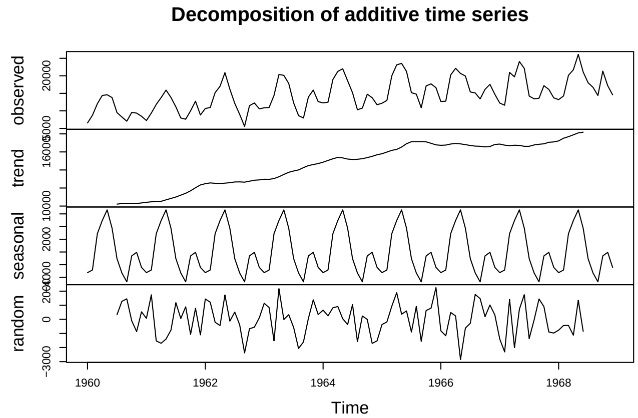
The analysis provides forecasts for future car sales trends, shedding light to a deeper understanding of the historical context of the car industry. The predicted values fell within the stated confidence intervals, demonstrating that the model is reliable in estimating the data and trends.

Analysis

To begin the analysis, I created a time series plot of the raw, untransformed data to find any visually underlying patterns. The plot shows a clear upward trend over the observed period, suggesting a general increase in demand. Addiionally, seasonality is displayed through the periodic spikes at regular intervals, suggesting cyclical variations in sales. No abrupt changes in the data's behavior can be seen, further supporting the consistency of these trends.

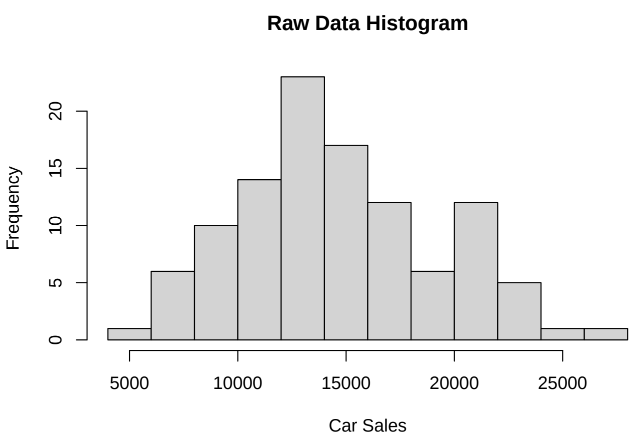


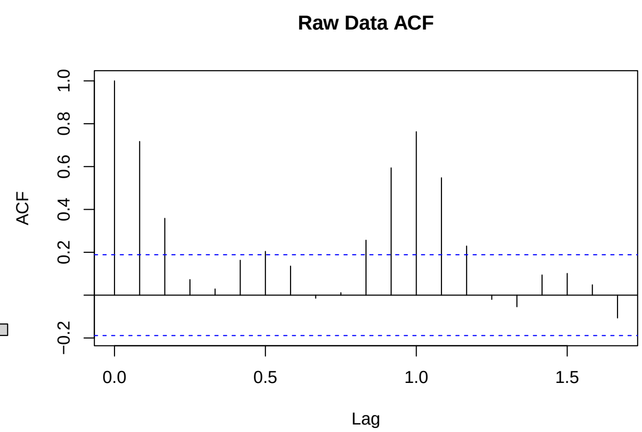
These observations align with the decomposition plot, which breaks down the time series into trend, seasonality, and residual components. The "trend" row shows a nearly linear trend, aligning with the upward trajectory seen in the initial time series plot. Not only does the consistent seasonal spikes confirm the presence of a repeating pattern, but the data also shows that the series is non-stationary, as the patterns exhibit dependency on time. This means that transformation or differencing is necessary for accurate modeling.



The time series plot of the raw data emphasizes that as the mean increases with the trend, the variability also grows, changing from smaller fluctuations at the beginning of the series to larger ones toward the end. This heteroscedasticity is a key indication of the need for variance stabilization. Additionally, the histogram shows that the data is left-skewed, with dips in frequency, while the autocorrelation function (ACF) shows multiple spikes above the confidence interval, indicating strong periodicity.

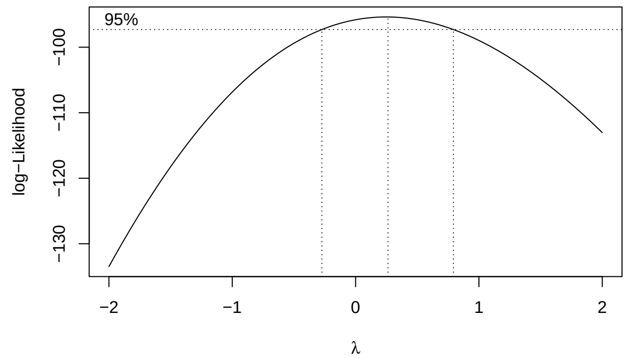
To address these issues, variance stabilization can be achieved through data transformation, such as logarithmic or power transformations. The seasonality and trend can be removed by differencing the data so the series is stationary and suitable for time series modeling.





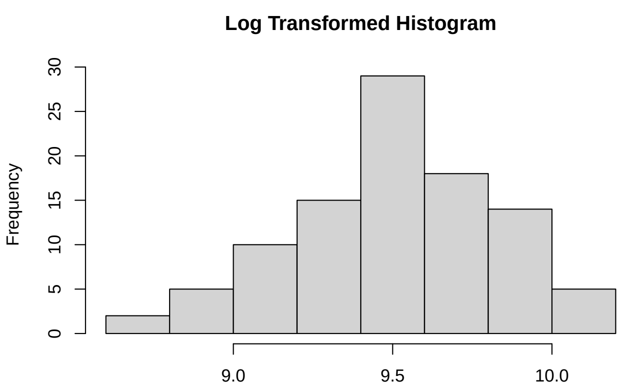
Box-Cox Transformation

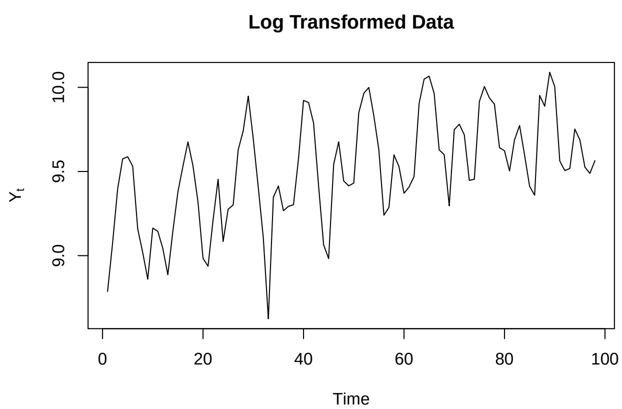
The Box-Cox transformation method allows us to stabilize the variability in the time series. The graph below reveals the optimal lambda value is 0.26. Since 0 falls within the confidence interval and the lambda value is relatively close to zero, I opted to use a lambda of 0, which corresponds to applying a logarithmic transformation. This approach stabilizes the variance while preserving the underlying patterns in the data.




```
## Lambda: 0.2626263
```

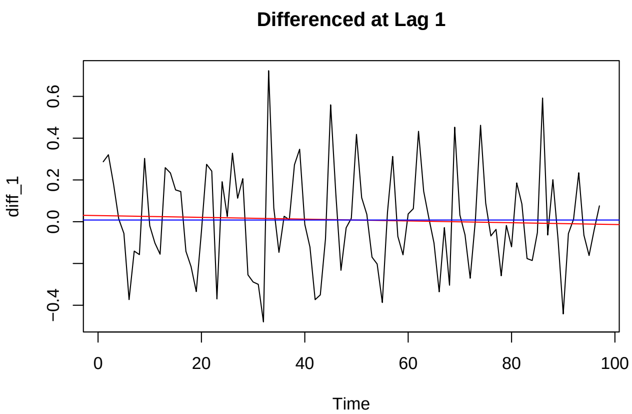
With the transformation complete, the histogram is now smoothed and approximates a normal distribution, addressing the skewness observed in the raw data. On the other hand, the line graph of the transformed data appears similar to the original, as it exhibits more stability, reduced variability, and less pronounced fluctuations. Since the Box-Cox method successfully decreased the variance, we will continue with the transformed data in the analysis, ensuring a stronger foundation for our time series modeling.

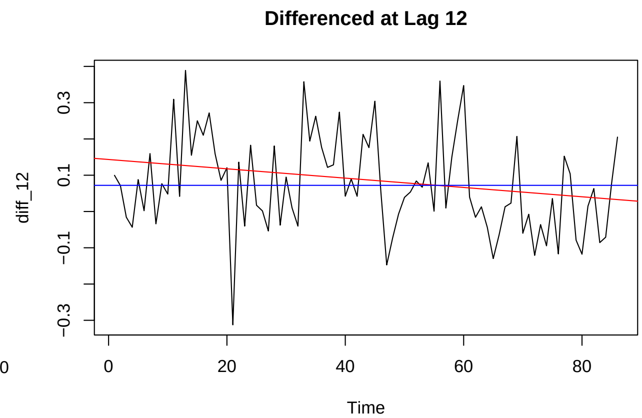




Differencing

The next step is to remove seasonality and trend by differencing the data. To achieve this, I compared differencing at lag 1 and lag 12. The differenced data at lag 1 shows a slight remaining trend and seasonality, while the differenced data at lag 12 better addresses these concerns. Additionally, the variance of the differenced data at lag 1 is lower than at lag 12, confirming that over-differencing has not occurred. Therefore, using lag 12 provides the best results for removing seasonality and trend, making it the best choice for further analysis.





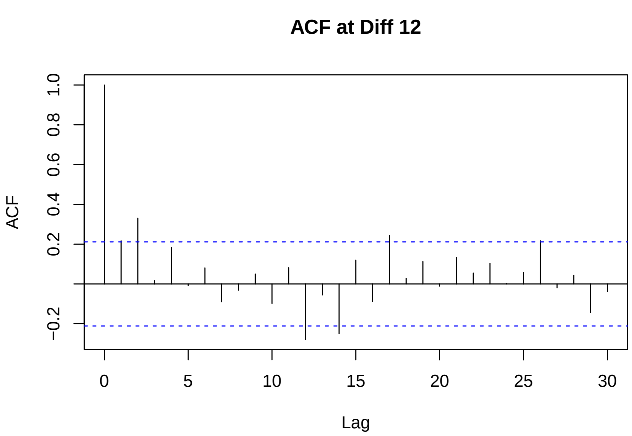

```
## Variance of Difference at Lag 1: 0.0582175
```

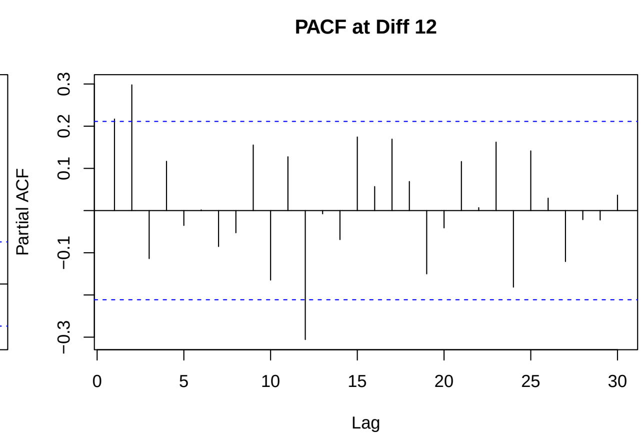
```
## Variance of Difference at Lag 12: 0.01759893
```

Model Fitting

Now that the data now de-trended, de-seasonalized, and its variance stabilized, we can proceed with building a SARIMA model. Since we applied both a seasonal differencing and a lag 12 differencing to remove the trend, it is best to set the model parameters as follows: d=1, D=1, and s=12.

The autocorrelation function (ACF) plot reveals spikes at lags 2 and 12, indicating autocorrelation at those points but no significant seasonal trend, leading to the selection of p=2, q=1, Q=0, or Q=1. Meanwhile, the partial autocorrelation function (PACF) plot shows a noticable spike at lag 1, supporting the choice of P=1. Based on these observations, this combination of parameters forms a strong foundation for constructing a SARIMA model tailored to the dataset.





Using the choices for the p, d, q, P, D, Q, and s parameters, I initially created the first model as SARIMA$(2,1,2)(1,1,1)_{12}$. However, when calling the ARIMA function, I noticed a problematic coefficient whose value was -1. To fix this, I adjusted the model by replacing Q=1 with Q=0; While this resolved the issue, some of the coefficients remained statistically insignificant, as their confidence intervals included 0. As a result, I set the coefficients to 0, refinding the model to be SARIMA$(2,1,2)(1,1,0)_{12}$.

The first candidate model equation can be expressed as: $(1-B)(1+0.8602B^{12})X_t = (1-0.3685B)Z_t$.

This equation reflects a simplified and more stable model, where the seasonal and non-seasonal components are optimized for forecasting.

```
##
```

```
## Call:
## arima(x = log(train_data), order = c(2, 1, 2), seasonal = list(order = c(1,
##     1, 0), period = 12), fixed = c(0, 0, NA, 0, NA), method = "ML")
##
## Coefficients:
##       ar1  ar2      ma1  ma2     sar1
##         0    0  -0.8602    0  -0.3685
## s.e.    0    0   0.0811    0   0.1013
##
## sigma^2 estimated as 0.01474:  log likelihood = 57.01,  aic = -108.01
```

I sought to find another suitable ARMA model using maximum likelihood estimation. From the loop, it can be shown that the most optimal model emerged with p=2 and q=4, resulting in a ARMA$(2, 4)$ model. The second candidate model equation can be expressed as:

$$(1 + 0.4002B - 0.2855B^2)(1 - B^{12})(1 + 0.3491B - 0.5472B^2)X_t = (1 + 0.2972B)Z_t$$

This equation combines both autoregressive and moving average components, while also accounting for seasonal differencing at lag 12. This model is also good for capturing the underlying patterns in the data and enhancing forecasting accuracy.

```
##    q
## p            0           1          2         3         4
##   0   0.5507344  -0.1412336  -8.705158 -20.34301 -21.84391
##   1   0.4724198   1.8770579 -15.557681 -19.40443 -24.93738
##   2  -0.8382162 -23.3628468 -26.163219 -23.80102 -26.56381
##   3 -11.8970685 -25.0152692 -22.967010 -21.57057 -25.83549
##   4 -14.5439964 -12.8567927 -25.081719 -24.55688 -22.54537
```

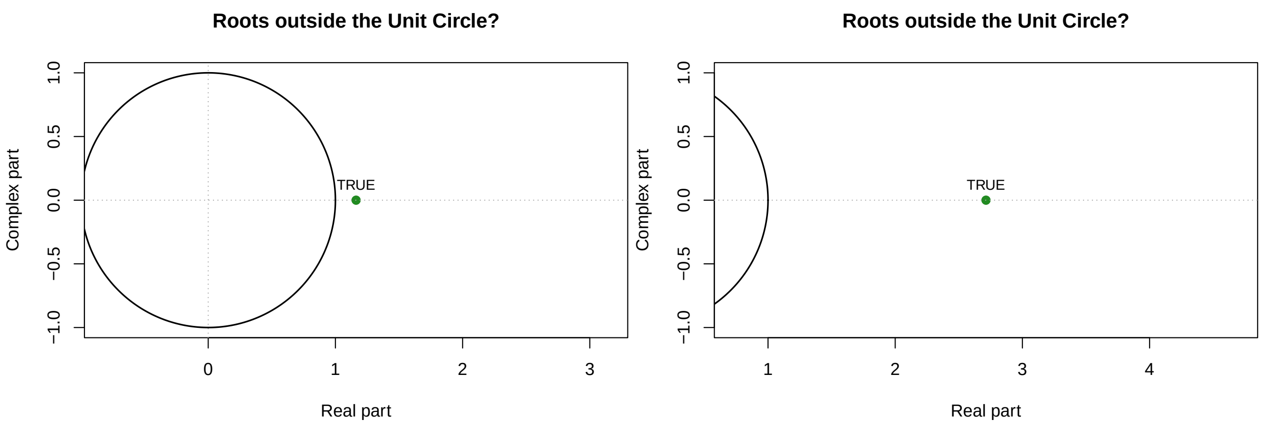
```
##
## Call:
## arima(x = log(train_data), order = c(2, 1, 4), method = "ML")
##
## Coefficients:
##           ar1      ar2     ma1     ma2      ma3      ma4
##       -0.2328  -0.4726  0.1742  0.2390  -0.6017  -0.5195
## s.e.   0.1448   0.1216  0.1491  0.0965   0.0744   0.1292
##
## sigma^2 estimated as 0.03667:  log likelihood = 20.74,  aic = -27.49
```

To select the best fitting model, I evaluated the equations' accuracy using the corrected Akaike Information Criterion (AICc), which calculates the residuals of the model to estimate the accuracy of the data. The results indicate that the first model has an AICc of -107.36, while the second model has an AICc of -26.56. Given these findings, the first model with the lower AICc value is preferred for forecasting, as it demonstrates a better fit to the data and more reliable predictive capabilities.

```
## AICc of Model 1: -107.3592

## AICc of Model 2: -26.56381
```

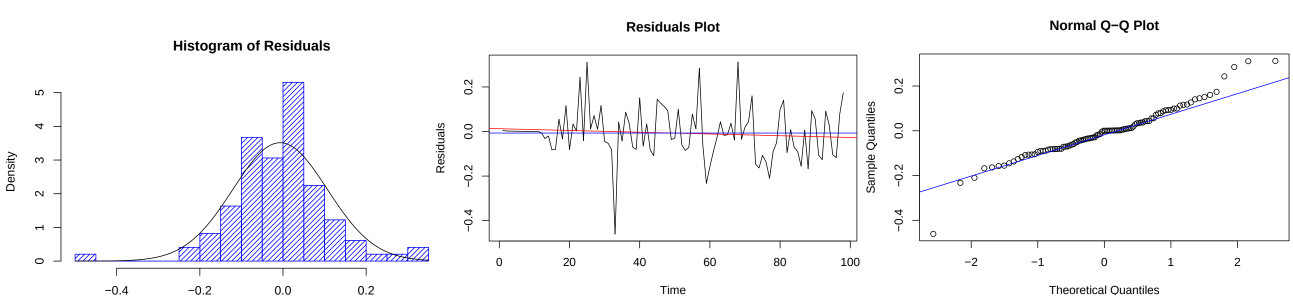
Before continuing with the forecasts, it is important to check for unit roots in the model to determine whether any additional differencing is necessary. The left graph illustrates the moving average (MA) coefficients, while the right graph displays the seasonal autoregressive (AR) coefficients. Since both coefficients are located outside the unit circle, we can conclude that the model is invertible. This suggests that no further transformations of the data is needed and that the model is ready for forecasting.



Diagnostic Check

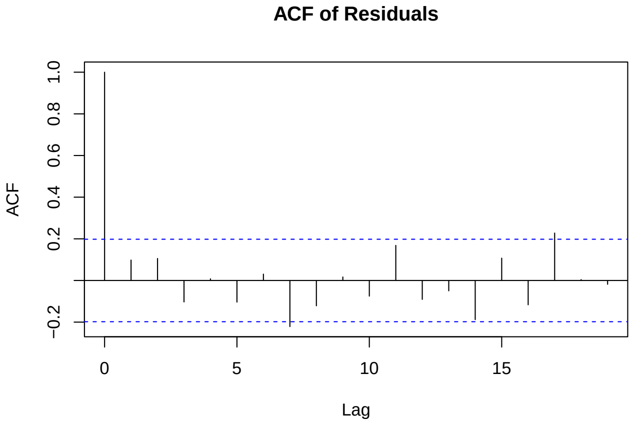
Now that it is known that the model is both stationary and invertible, we can continue with diagnostic checking for Model 1. The Q-Q plot, histogram, and residual plot indicate the residuals resembling Gaussian white noise, with a mean close to 0. However, the residuals exhibit non-constant variance, indicating some heteroscedasticity in the model.

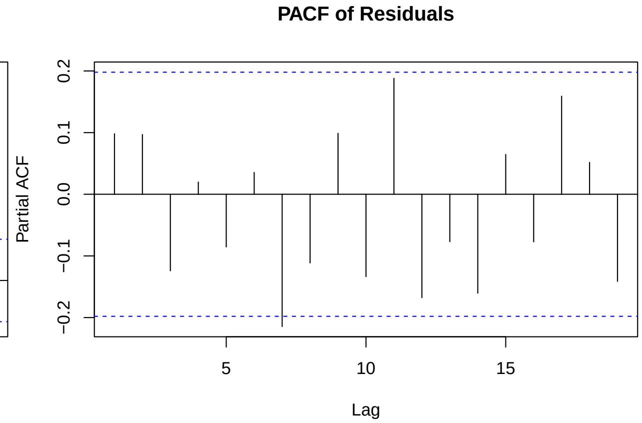
Additionally, both the histogram and Q-Q plot reveal heavy tails on either end, suggesting that the distribution of the residuals may deviate from normality and that the outliers may not be adequately captured by the model, meaning that further investigation or potential adjustments to improve the model fit is necessary.



To check whether the residuals are consistent with white noise, I plotted the ACF and PACF of the residuals. Fortunately, all lags in the ACF plot, excluding lag 0, fall within the dotted lines of the confidence interval.

This means that the residuals resemble Gaussian white noise, suggesting that there are no significant autocorrelations remaining in the residuals. This also confirms that the model captures the underlying patterns in the data and supports the validity of the chosen SARIMA model.





Lastly, I performed the Shapiro-Wilk, Box-Pierce, Ljung-Box, and McLeod-Li tests on the residuals to further confirm if they resemble white noise by checking their independence and normality. All tests passed, with p-values greater than 0.05, except for the Shapiro-Wilk test, which indicated that the residuals are not normally distributed.

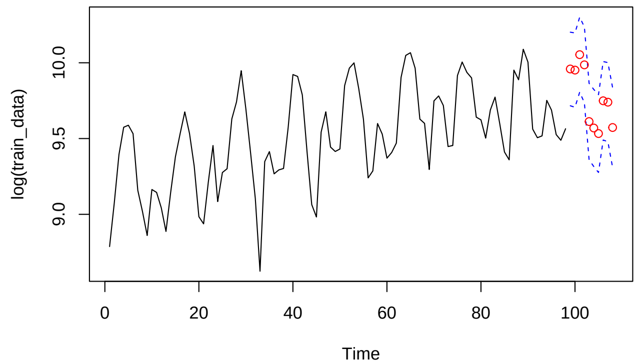
This result aligns with the earlier observation of heavy tails in the Q-Q plot and histogram. Despite the lack of normality, the residuals still resemble white noise, meaning they are independent and uncorrelated, making them acceptable for forecasting. This confirms that the model is suitable for generating reliable forecasts, even with the non-normality present.

Statistical Test	P-Value
Shapiro-Wilk	0.001742
Box-Pierce	0.1985
Ljung-Box	0.1517
McLeod Li	0.6634

Forecasting

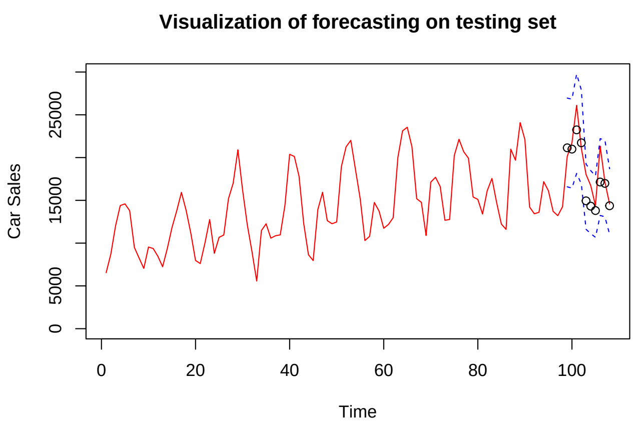
Now that the best model has been determined, I can display the forecasts based on the transformed data. The red dots representing the forecasted values fall within the blue/dotted 95% confidence intervals, indicating that the predicted values are consistent with the actual data.

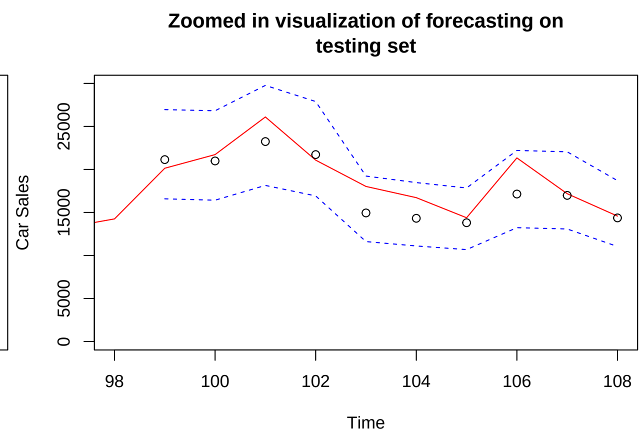
To compare the forecasts with the actual values for the last ten months, it is necessary to reverse the transformations applied to the data. This includes undoing the differencing at lag 12 and reversing the Box-Cox transformation to change the forecasts back to their original scale. This step lets the forecasted values be directly compared to the observed data in the same scale and context.



The true values are within the confidence intervals of the forecasts, which confirms that the model is performing accurately. When comparing the graph of the forecasted values to the one from the transformed data, they appear very similar, indicating consistency in the model's predictions.

In the zoomed-in view, the closeness of the black circles (which represent the true values) to the red line (forecasted values) further emphasizes the model's accuracy. Additionally, none of the true values fall outside the blue 95% confidence intervals. This alignment between the forecast and actual data illustrates that the SARIMA model is well-suited for forecasting future car sales in Quebec.





Conclusion

The goal of this project was to predict future car sales in Quebec by creating a model of the monthly data. This was achieved through the application of various time series techniques such as box-cox transformations, differencing, ARIMA, SARIMA, diagnostic checks, and unit circle tests. After going through all the procedures, the best performing model was SARIMA$(2,1,2)(1,1,0)_{12}$, with the equation $(1-B)(1+0.8602B^{12})X_t = (1-0.3685B)Z_t$. Lastly, I would like to acknowledge Professor Feldman, TAs Cosmin and Lihao, and classmates Michael Chen, Brian Ho, and James Son for helping me through this process.

References

Abraham, B., and J. Ledolter. "Statistical Methods for Forecasting." John Wiley & Sons, 1983.
Hyndman, Rob. "TSDL Library." TSDL, https://pkg.yangzhuoranyang.com/tsdl/.
Kelkar, Mihir, et al. "Time-Series Statistical Model for Forecasting Revenue and Risk Management."

Appendix

```r
library(forecast)
library(qpcR)
library(tidyr)
library(tsdl)
library(MASS)
library(UnitCircle)

#dataset 4 from TSDL library
data <- tsdl[[9]]

#plotting raw data on time series plot
plot(1:length(data), data, type = 'l', xlab='Time', ylab="Cars Sold",
     main="Raw Data of Monthly Car Sales in Quebec 1960-1968")
index = 1: length(data)
trend <- lm(data ~ index)
abline(trend, col="red")
abline(h=mean(data) , col='blue')

#decompose the data
components <- decompose(data)
plot(components)

#show the raw data on a histogram and ACF
hist(data, main="Raw Data Histogram", xlab="")
acf(data, main="Raw Data ACF")

#split the data, leave the last 10 values out
train_data <- data[1:98]
test_data <- data[99:108]

#box-cox transformation and lambda value
bcTransform <- boxcox(as.numeric(train_data)~
                        as.numeric(1:length(train_data)))
cat("Lambda:", bcTransform$x[which(bcTransform$y == max(bcTransform$y))])

#log transformed histogram and time series plot
hist(log(train_data), main="Log Transformed Histogram", xlab="")
ts.plot(log(train_data), main = "Log Transformed Data",
        ylab = expression(Y[t]))

#differencing log(data) at 1 and plot
diff_1 <- diff(log(train_data))
plot.ts(diff_1, main="Differenced at Lag 1",
        xlab='Time')
abline(lm(diff_1 ~ time(train_data)[-1]), col='red') #trend line
abline(h=mean(na.omit(diff_1)), col='blue') #mean line

#differencing log(data) at 12 and plot
diff_12 <- diff(log(train_data), 12)
plot.ts(diff_12, main="Differenced at Lag 12")
abline(lm(diff_12 ~ time(train_data)[-c(1:12)]), col='red') #trend line
abline(h=mean(na.omit(diff_12)), col='blue') #mean line
```

```r
#check the variances between both differencings
var_1 <- var(diff_1, na.rm = TRUE)
var_12 <- var(diff_12, na.rm = TRUE)
cat("Variance of Difference at Lag 1:", var_1, "\n")
cat("Variance of Difference at Lag 12:", var_12)

#diff_12 has a smaller variance, plot ACF/PACF
acf(diff_12, main='ACF at Diff 12', lag.max=30)
pacf(diff_12, main='PACF at Diff 12', lag.max=30)

#first model SARIMA(2,1,2)(1,1,1)12
m1 <- arima(x = log(train_data), order = c(2,1,2),
            seasonal = list(order = c(1,1,1), period = 12), method = "ML")

#fit 1 of model 1
fit_m1 <- arima(log(train_data), order=c(2,1,2),
                seasonal=list(order=c(1, 1, 0), period=12),
                 method="ML")

#fit 2 of model 1
fit2_m1 <- arima(log(train_data), order=c(2,1,2),
                seasonal=list(order=c(1, 1, 0), period=12),
                fixed=c(0, 0, NA, 0, NA), method="ML"); fit2_m1 #best fit

#check best ARMA fit
aiccs <- matrix(NA, nr = 5, nc = 5)
dimnames(aiccs) = list(p = 0:4, q = 0:4)

#for loop for ARMA fit
for (p in 0:4) {
  for (q in 0:4) {
    aiccs[p + 1, q + 1] = AICc(arima(log(train_data),
                                     order = c(p, 1, q), method = "ML"))
  }
}
aiccs #compare values
(aiccs==min(aiccs)) #find the minimum value
m2 <- arima(log(train_data), order = c(2,1,4), method="ML"); m2

#compare AICcs of each model
cat("AICc of Model 1:", AICc(fit2_m1))
cat("AICc of Model 2:", AICc(m2))

#check whether model 1 coefficients are outside unit circle
ma_coef <- c(1, -0.8602)
seasonal_ar_coef <- c(1, -0.3685)
uc.check(pol = ma_coef, plot_output = TRUE) #roots of MA of model A
uc.check(pol = seasonal_ar_coef, plot_output = TRUE) #roots of SAR of model A

#plot residuals to show normality
res <- residuals(fit2_m1)
hist(res,density=20,breaks=20, col="blue", xlab="",
     prob=TRUE,main="Histogram of residuals of model B") #histogram
```

```r
m <- mean(res)
std <- sqrt(var(res))
curve( dnorm(x,m,std), add=TRUE )
plot.ts(res,ylab= "residuals of model B",main="Residuals plot of model B")
fitt <- lm(res ~ as.numeric(1:length(res))) #residual plot
abline(fitt, col="red")
abline(h=mean(res), col="blue")
qqnorm(res,main= "Normal Q-Q Plot for Model B") #qqnorm plot
qqline(res,col="blue")

#residuals are within CIs in ACF/PACF
acf(res, main="ACF of Residuals")
pacf(res, main="PACF of Residuals")

#diagnostic residuals check
shapiro.test(res) #shapiro-wilk
Box.test(res, type = c("Box-Pierce"), lag = 10, fitdf=2) #box-pierce
Box.test(res, type = c("Ljung-Box"), lag = 10, fitdf=2) #ljung-box
Box.test(res^2, type = c("Ljung-Box"), lag = 10, fitdf=0) #mcleod li

#forecasting on transformed data
pred.tr <- predict(fit2_m1, n.ahead = 10)
U.tr= pred.tr$pred + 2*pred.tr$se
L.tr= pred.tr$pred - 2*pred.tr$se
ts.plot(log(train_data), xlim=c(1,length(log(train_data))+10), ylim =
          c(min(log(train_data)),max(U.tr)))
lines(U.tr, col="blue", lty="dashed")
lines(L.tr, col="blue", lty="dashed")
points((length(log(train_data))+1):(length(log(train_data))+10),
       pred.tr$pred, col="red")

pred.orig <- exp(pred.tr$pred)
U= exp(U.tr)
L= exp(L.tr)

#zoomed out forecasting
ts.plot(as.numeric(data), ylim = c(0,max(U)),col="red",
        ylab="Car Sales",main="Visualization of forecasting on testing set")
lines(U, col="blue", lty="dashed")
lines(L, col="blue", lty="dashed")
points((length(train_data)+1):(length(train_data)+10), pred.orig, col="black")

#zoomed in forecasting
ts.plot(as.numeric(data), xlim = c(98,length(train_data)+10),
        ylim = c(200,max(U)),col="red",ylab="Car Sales",
        main="Zoomed in visualization of forecasting on testing set")

lines(U, col="blue", lty="dashed")
lines(L, col="blue", lty="dashed")
points((length(train_data)+1):(length(train_data)+10), pred.orig, col="black")
```