Sample answers

Session 5

EX1 Denote by d the dominant gene and by r the recessive gene at a single locus. Then dd is called the pure dominant genotype, dr is called the hybrid, and rr the pure recessive genotype. The two genotypes with at least one dominant gene, dd and dr, result in the phenotype of the dominant gene, while rr results in a recessive phenotype. Assuming that both parents are hybrid and have n children, what is the probability that at least two will have the recessive phenotype? Each child, independently, gets one of the genes at random from each parent.

For each child, independently, the probability of the rr genotype is $\frac{1}{4}$. If X is the number of rr children, then X is Binomial(n, $\frac{1}{4}$). Therefore,

$$P(X \geq 2) = 1 - P(X = 0) - P(X = 1) = 1 - \left(\frac{3}{4}\right)^n - n \cdot \frac{1}{4}\left(\frac{3}{4}\right)^{n-1}.$$

EX2 You roll a fair die, your opponent tosses a fair coin independently. If you roll 6 you win; if you do not roll 6 and your opponent tosses Heads you lose; otherwise, this round ends and the game repeats. On the average, how many rounds does the game last?

Game decided on round 1: roll a 6 --- the probability is 1/6; roll 1-5 and your opponent tosses a head --- the probability is P(roll 1-5 and your opponent tosses a head)=P(roll 1-5)*P(your opponent tosses a head)=(5/6)*(1/2)

$$P(\text{game decided on round 1}) = \frac{1}{6} + \frac{5}{6} \cdot \frac{1}{2} = \frac{7}{12},$$

and so the number of rounds N is Geometric($\frac{7}{12}$), and

$$EN = \frac{12}{7}.$$

EX3 Suppose that the probability that a person is killed by lightning in a year is, independently, 1/(500 million). Assume that the US population is 300 million.
(1) Compute P(3 or more people will be killed by lightning next year) exactly.
(2) Approximate the above probability.

1. Compute P(3 or more people will be killed by lightning next year) exactly.

   If X is the number of people killed by lightning, then X is Binomial(n, p), where $n = 300$ million and $p =$1/(500 million), and the answer is

$$1 - (1-p)^n - np(1-p)^{n-1} - \binom{n}{2} p^2 (1-p)^{n-2} \approx 0.02311530.$$

2. Approximate the above probability.

   As $np = \frac{3}{5}$, X is approximately Poisson$(\frac{3}{5})$, and the answer is

$$1 - e^{-\lambda} - \lambda e^{-\lambda} - \frac{\lambda^2}{2} e^{-\lambda} \approx 0.02311529.$$