Linear Algebra
Practical 1

Qi Zhong

Assistant Professor in FDS

City University of Macau

Email: qizhong@cityu.mo

Oct 2023

Question 1. 把下列矩阵化为标准型矩阵 $D = \begin{pmatrix} E_r & O \\ O & O \end{pmatrix}$.

- (1) $\begin{pmatrix} 1 & -1 & 2 \\ 3 & 2 & 1 \\ 1 & -2 & 0 \end{pmatrix}$;

- (2) $\begin{pmatrix} 1 & -1 & 2 \\ 3 & -3 & 1 \\ -2 & 2 & -4 \end{pmatrix}$;

- (3) $\begin{pmatrix} 1 & 0 & 2 & -1 \\ 2 & 0 & 3 & 1 \\ 3 & 0 & 4 & -3 \end{pmatrix}$;

- (4) $\begin{pmatrix} 1 & -1 & 3 & -4 & 3 \\ 3 & -3 & 5 & -4 & 1 \\ 2 & -2 & 3 & -2 & 0 \\ 3 & -3 & 4 & -2 & -1 \end{pmatrix}$

Solution 1:

- (1) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$;
- (2) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$;
- (3) $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$;
- (4) $\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

Question 2. 用初等变换法判定下列矩阵是否可逆，如可逆，求其逆矩阵。

- (1) $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 2 & 3 \end{pmatrix}$;
- (2) $\begin{pmatrix} 2 & 2 & -1 \\ 1 & -2 & 4 \\ 5 & 8 & 2 \end{pmatrix}$;
- (3) $\begin{pmatrix} 3 & 2 & 1 \\ 3 & 1 & 5 \\ 3 & 2 & 3 \end{pmatrix}$
- (4) $\begin{pmatrix} 3 & -2 & 0 & -1 \\ 0 & 2 & 2 & 1 \\ 1 & -2 & -3 & -2 \\ 0 & 1 & 2 & 1 \end{pmatrix}$;

Solution 2.

- (1) $\begin{pmatrix} 1 & 0 & 0 \\ -1/2 & 1/2 & 0 \\ 0 & -1/3 & 1/3 \end{pmatrix}$;

- (2) $\begin{pmatrix} 2/3 & 2/9 & -1/9 \\ -1/3 & -1/6 & 1/6 \\ -1/3 & 1/9 & 1/9 \end{pmatrix}$;

- (3) $\begin{pmatrix} 7/6 & 2/3 & -3/2 \\ -1 & -1 & 2 \\ -1/2 & 0 & 1/2 \end{pmatrix}$

- (4) $\begin{pmatrix} 1 & 1 & -2 & -4 \\ 0 & 1 & 0 & -1 \\ -1 & -1 & 3 & 6 \\ 2 & 1 & -6 & -10 \end{pmatrix}$;

Question 3. 解下列矩阵方程

- (1) 设 $A=\begin{pmatrix}4 & 1 & -2\\ 2 & 2 & 1\\ 3 & 1 & -1\end{pmatrix}, B=\begin{pmatrix}1 & -3\\ 2 & 2\\ 3 & -1\end{pmatrix}$，求 X 使 AX=B.

- (2) 设 $A=\begin{pmatrix}0 & 2 & 1\\ 2 & -1 & 3\\ -3 & 3 & -4\end{pmatrix}, B=\begin{pmatrix}1 & 2 & 3\\ 2 & -3 & 1\end{pmatrix}$，求 X 使 XA=B.

- (3) 设 $A=\begin{pmatrix}1 & -1 & 0\\ 0 & 1 & -1\\ -1 & 0 & 1\end{pmatrix}$, AX=2X+A, 求X.

- (4) 设 $\begin{pmatrix}0 & 1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 1\end{pmatrix}X\begin{pmatrix}1 & 0 & 0\\ -2 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}=\begin{pmatrix}1 & -4 & 3\\ 2 & 0 & -1\\ 0 & -2 & 1\end{pmatrix}$，求X.

Solution 3.

- (1) $\begin{pmatrix} 10 & 2 \\ -15 & -3 \\ 12 & 4 \end{pmatrix}$

- (2) $\begin{pmatrix} 2 & -1 & -1 \\ -4 & 7 & 4 \end{pmatrix}$

- (3) $\begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix}$

- (4) $\begin{pmatrix} 2 & 0 & -1 \\ -7 & -4 & 3 \\ -4 & -2 & 1 \end{pmatrix}$

Question 4. 按第三列展开下列行列式，并计算其值：

- (1) $\begin{vmatrix} 1 & 0 & a & 1 \\ 0 & -1 & b & -1 \\ -1 & -1 & c & -1 \\ -1 & 1 & d & 0 \end{vmatrix}$
- (2) $\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & 0 & 0 & 0 \\ a_{41} & a_{42} & 0 & 0 & 0 \\ a_{51} & a_{52} & 0 & 0 & 0 \end{vmatrix}$

Solution 4.

- (1) $a + b + d$
- (2) 0

Question 5. 用降阶法计算下列行列式：

- (1) $\begin{vmatrix} 1+x & 1 & 1 & 1 \\ 1 & 1-x & 1 & 1 \\ 1 & 1 & 1+y & 1 \\ 1 & 1 & 1 & 1-y \end{vmatrix}$
- (2) $\begin{vmatrix} 0 & a & b & a \\ a & 0 & a & b \\ b & a & 0 & a \\ a & b & a & 0 \end{vmatrix}$
- (3) $\begin{vmatrix} x & y & 0 & \cdots & 0 & 0 \\ 0 & x & y & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & x & y \\ y & 0 & 0 & \cdots & 0 & x \end{vmatrix}$

Solution 5.

- (1)x^2y^2
- (2) $b^2(b^2$-4 $a^2)$
- (3) $x^n + (-1)^{n+1}y^n$