Sample answers

Session 1

Q1 Roll two dice. What is the most likely sum?

Outcomes are ordered pairs (i, j), $1 \leq i \leq 6$, $1 \leq j \leq 6$.

sum	no. of outcomes
2	1
3	2
4	3
5	4
6	5
7	6
8	5
9	4
10	3
11	2
12	1

Our answer is 7, and $P(\text{sum} = 7) = \frac{6}{36} = \frac{1}{6}$.

Q2 A bag has 6 pieces of paper, each with one of the letters, E, E, P, P, P, and R, on it. Pull 6 pieces at random out of the bag (1) without, and (2) with replacement. What is the probability that these pieces, in order, spell PEPPER? (You should consider both with and without replacement cases.)

There are two problems to solve. For sampling without replacement:

1. An outcome is an ordering of the pieces of paper $E_1E_2P_1P_2P_3R$.
2. The number of outcomes thus is 6!.
3. The number of good outcomes is 3!2!.

The probability is $\frac{3!2!}{6!} = \frac{1}{60}$.

For sampling with replacement, the answer is $\frac{3^3 \cdot 2^2}{6^6} = \frac{1}{2 \cdot 6^3}$, quite a lot smaller.

EX1. A full deck of 52 cards contains 13 hearts. Pick 7 cards from the deck at random (a) without replacement and (b) with replacement. In each case, compute the probability that you get no hearts.

E --- getting no heart

(a) without replacement $|E| = C_{52-13}^{7}$. (Taking out all the 7 cards from those which contain no heart (52-13 cards in total).)

The size of the sample space should be $|S| = C_{52}^{7}$ --- taking 7 cards out from 52

$$P(E) = \frac{C_{52-13}^{7}}{C_{52}^{7}}$$

(b) With replacement, $|E| = (52-13)^7$ (each time we take one card from 52-13 cards, which is repeated 7 times)

The size of the sample space is $|S| = 52^7$ (each time we take one card from 52 cards, which is repeated 7 times)

$$P(E) = \frac{(52-13)^7}{52^7}$$

EX2 Compute the probability of a 2:2 boy-girl split in a four-children family. Please give the sample space too.

BBBB	BBBG	BBGB	BBGG
BGBB	BGBG	BGGB	BGGG
GBBB	GBBG	GBGB	GBGG
GGBB	GGBG	GGGB	GGGG

The above table gives all the elements in the sample space S.

$$P(2{:}2 \text{ split}) = \frac{6}{16} = \frac{3}{8}$$