BCS004 group assignment

Submission: Detailed explanation should be given to all your solutions. This part will count for 30% of your final mark. The submission deadline is 23:59:00 pm on 19.04.2024.(See Tronclass) Note that this is a harsh deadline, late submissions will NOT be accepted. Each group should submit one answer sheet. You can submit a scanned PDF copy of handwritten papers. The contribution of each group member must be declared, e.g., Mike T111000 contributed to Q1-Q2. If there exists (a) group member(s) who contributed nothing to the project, then this(these) group member(s) will get ZERO marks.
Names:
Student IDs:
Contribution of each group member:

Q1. We surveyed a group of people's preferences for watching three types of documentaries (say historical, science & nature, and art) over the last year. The survey results are shown as follows:

(a) 25% preferred watching historical documentaries
(b) 28% preferred watching science & nature documentaries
(c) 18% preferred watching art documentaries
(d) 15% preferred watching both art and science & nature documentaries
(e) 11% preferred watching both art and historical documentaries
(f) 11% preferred watching both historical and science & nature documentaries
(g) 10% preferred watching all three types of documentaries

During the last year, what is the percentage of the group that preferred watching none of the three types of documentaries? (**5 marks**)

Q2. A store issues vouchers for different sectors such as food hall, cosmetics, health care, clothing, etc. The store examines its voucher buyers and gets the following results:

(a) All the voucher buyers bought voucher(s) from at least one sector
(b) 60% of the voucher buyers bought voucher(s) from more than one sector
(c) 50% of the voucher buyers had bought voucher(s) from the food hall
(d) 80% of those voucher buyers who bought vouchers from more than one sector, have bought voucher(s) from the food hall sector

Now randomly select a voucher buyer, what is the probability he/she bought voucher(s) from exactly one sector and these/this voucher(s) are not from the food hall? (**5 marks**)

Q3. An insurance company insures burglary insurance of families from 4 districts. Here below are the statistics on the insured families of the company:

District	Probability of burglary	Portion of the insured families
A	0.05	0.20
B	0.02	0.10
C	0.03	0.48
D	0.01	0.25

Table 1: Statistics of the insured families

There is now a randomly selected family insured by the company that has suffered a burglary accident. What is the probability that the insured family is from District A? (**5 marks**)

Q4. The pleasure beach is selling the tickets. The number of tickets that include recreation facilities or aquariums is 80% of the total number of tickets sold. The number of tickets that do not include recreation facilities is 30% of the total number of tickets sold. The occurrence of including recreation facilities is independent of the occurrence of aquariums on the pleasure beach tickets. What is the probability that a randomly selected ticket includes aquariums? (**5 marks**)

Q5. Assume S be a sample space with probability P. Assume A and B be any events in the sample space S. (**10 marks in total**)

(a) Please show that $P(A \cap B^C) = P(A) - P(A \cap B)$ (**5 marks**)

(b) Please show that $P(A \cap B) \geq P(A) + P(B) - 1$ (**5 marks**)

Q6. We get two balls (without replacement) randomly from a black box which contains 10 red balls, 6 black balls, and 4 green balls. For each black ball we get, we win Mop 2; and for each red ball we get, we lose Mop 1. Assume X is a random variable presenting the amount we win. Please answer the following questions. (**20 marks in total**)

(a) Please give all the possible values of X and explain why. (**6 marks**)

(b) Please give the probability mass function (pmf) of X. (**6 marks**)

(c) Please compute the expectation of X. (**4 marks**)

(d) Please compute the variance of X. (**4 marks**)

Q7. We draw 7 balls (with replacement) randomly from a black box that contains 4 black balls and 5 white balls. Please answer the following questions. (**10 marks in total**)

(a) Please give the probability that exactly 2 white balls were drawn in the seven balls. (**4 marks**)

(b) Please give the probability that at least 3 black balls were drawn in the seven balls. (**6 marks**)

Q8. Suppose that X is continuous with the probability density function (pdf) shown in (1). (**16 marks in total**)

$$f_X(x) = \begin{cases} cx(4-x) & 0 \leq x \leq 4 \\ 0 & otherwise \end{cases} \tag{1}$$

(a) What is the value of c. (**4 marks**)

(b) Please give the mean value and the variance of X. (**4 marks**)

(c) Please give the cumulative distribution function (cdf) of X. (**4 marks**)

(d) Please compute the probability $P(2 \leq X \leq 3)$. (**4 marks**)

Q9. Assuming that you are rolling two fair dice. Let X be the largest points shown on two dice, and Y be the sum of the points on two dice. Please answer the following questions. (**24 marks in total**)

(a) What is the joint probability mass function (p.m.f) of (X, Y)? (**4 marks**)

(b) What is the p.m.f of X? (**2 marks**)

(c) What is the p.m.f. of Y? (**2 marks**)

(d) Are X and Y independent? (**4 marks**)

(e) Please compute $P(X \geq 3)$. (**4 marks**)

(f) Please compute $P(X \geq 3 | Y \geq 6)$. (**4 marks**)

(g) Please compute $cov(X, Y)$. (**4 marks**)