Proof of the Derivative of Inverse Trig. Functions

arcsin, arccos, arctan

1. Prove that $\dfrac{d}{dx}(\sin^{-1} x) = \dfrac{1}{\sqrt{1-x^2}}$
   Let $y = \sin^{-1} x$, then $\sin y = x$

$$\begin{aligned}
\frac{d}{dx}(\sin y) &= \frac{d}{dx}(x) \\
\cos y \cdot \frac{dy}{dx} &= 1 \\
\therefore \frac{dy}{dx} &= \frac{1}{\cos y} \\
&= \frac{1}{\sqrt{1-(\sin y)^2}} \quad (\because \sin^2 y + \cos^2 y = 1) \\
&= \frac{1}{\sqrt{1-x^2}}
\end{aligned}$$

2. Prove that $\dfrac{d}{dx}(\cos^{-1} x) = -\dfrac{1}{\sqrt{1-x^2}}$
   Let $y = \cos^{-1} x$, then $\cos y = x$

$$\begin{aligned}
\frac{d}{dx}(\cos y) &= \frac{d}{dx}(x) \\
-\sin y \cdot \frac{dy}{dx} &= 1 \\
\therefore \frac{dy}{dx} &= -\frac{1}{\sin y} \\
&= -\frac{1}{\sqrt{1-(\cos y)^2}} \quad (\because \sin^2 y + \cos^2 y = 1) \\
&= -\frac{1}{\sqrt{1-x^2}}
\end{aligned}$$

3. Prove that $\dfrac{d}{dx}(\tan^{-1} x) = \dfrac{1}{1+x^2}$
   Let $y = \tan^{-1} x$, then $\tan y = x$

$$\begin{aligned}
\frac{d}{dx}(\tan y) &= \frac{d}{dx}(x) \\
\sec^2 y \cdot \frac{dy}{dx} &= 1 \\
\therefore \frac{dy}{dx} &= \frac{1}{\sec^2 y} \\
&= \frac{1}{1+\tan^2 y} \quad (\because 1 + \tan^2 y = \sec^2 y) \\
&= \frac{1}{1+x^2}
\end{aligned}$$