Sample answers

Session 8

Q1. In the same situation as above example, what does Markov's Inequality tell you about the probability it takes 2 or more attempts?

Answer. This time we have

$$P(X \geq 2) \leq \frac{E(X)}{2} = \frac{2.5}{2} = 1.25.$$

This should not be confusing. The probability must be always less than 1, so in this case, Markov's Inequality is true, but not helpful. $\square$

Q2: How to proof Lemma 5.3?

$X_1, \cdots, X_n$ independent, so, $\forall\, i, j = 1, \cdots, n,\ i \neq j$,

we have $\operatorname{cov}(X_i, X_j) = 0$

$$
\begin{aligned}
Var\left(\sum_{j=1}^{n} X_j\right) &= E\left[\left(\sum_{j=1}^{n} X_j\right)^2\right] - \left[E\left(\sum_{j=1}^{n} X_j\right)\right]^2 \\
&= E\left[\sum_{j=1}^{n} X_j^2 + 2\sum_{j=1}^{n-1}\sum_{i=j+1}^{n} X_j X_i\right] - \left(\sum_{j=1}^{n} EX_j\right)^2 \\
&= \sum_{j=1}^{n} EX_j^2 + 2\sum_{j=1}^{n-1}\sum_{i=j+1}^{n} E(X_j X_i) - \sum_{j=1}^{n} (EX_j)^2 \\
&\quad - 2\sum_{j=1}^{n-1}\sum_{i=j+1}^{n} EX_j \cdot EX_i \\
&= \sum_{j=1}^{n}\left[EX_j^2 - (EX_j)^2\right] + 2\sum_{j=1}^{n-1}\sum_{i=j+1}^{n}\left\{E(X_j X_i) - EX_j \cdot EX_i\right\} \\
&= \sum_{j=1}^{n} Var(X_j) + 2\sum_{j=1}^{n-1}\sum_{i=j+1}^{n} \operatorname{cov}(X_i, X_j) \\
&= \sum_{j=1}^{n} Var(X_j) = \sum_{j=1}^{n} \sigma_j^2
\end{aligned}
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