Sample answers

Session 11

Exercises:

1. We assume X presenting the pizza cost, and Y presenting the delivery fee. X and Y follows an linear relationship $Y = \beta_0 + \beta_1 X$. We now have four observations of $\{(x_i, y_i)\}_{i=1}^{4}$ shown in the Table below. Please estimate the values of β_0 and β_1.

Pizza cost (MOP) x_i	Delivery fee (MOP) y_i
50	8
60	9
100	12
120	13

$$\bar{x} = \frac{1}{4}(50 + 60 + 100 + 120) = 82.5$$

$$\bar{y} = \frac{1}{4}(8 + 9 + 12 + 13) = 10.5$$

$$\sum_{i=1}^{n}(x_i - \bar{x})^2 = (50 - 82.5)^2 + (60 - 82.5)^2 + (100 - 82.5)^2$$

$$+ (120 - 82.5)^2$$

$$= 1056.25 + 506.25 + 306.25 + 1406.25$$

$$= 3275$$

$$\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y}) = (50 - 82.5)\cdot(8 - 10.5) + (60 - 82.5)\cdot(9 - 10.5)$$

$$+ (100 - 82.5)\cdot(12 - 10.5) + (120 - 82.5)\cdot(13 - 10.5)$$

$$= (-32.5)(-2.5) + (-22.5)\cdot(-1.5)$$

$$+ (17.5)\cdot(1.5) + (37.5)\cdot(2.5)$$

$$= 235$$

$$\therefore \hat{\beta_1} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2} = \frac{3275}{235} \approx 13.94 \quad (\text{OLS estimation})$$

$$\hat{\beta_0} = \bar{y} - \hat{\beta_1}\bar{x} = 10.5 - 13.94 \cdot 82.5 = -1139.55$$