$$\int_0^1 f(x)\,dx = \underbrace{E[f(U)]}_{=\theta}$$

$U_i \overset{iid}{\sim} \text{Unif}(0,1)$

$$\hat{\theta} = \frac{1}{n}\sum_{i=1}^{n} f(U_i)$$

$$\theta = \int_a^b f(x)\,dx = \int_0^1 \underbrace{f(h(x))h'(x)}\,dx$$

$$= E[f(h(U))h'(U)] \quad \begin{bmatrix} h(0)=a \\ h(1)=b \end{bmatrix}$$

$$\hat{\theta} = \frac{1}{n}\sum_{i=1}^{n} f(h(U_i))h'(U_i)$$

$$\int_0^1\int_0^1 \cdots \int_0^1 f(x_1,\dots,x_d)\,dx_1\cdots dx_d$$

$U_1,\dots,U_d \overset{iid}{\sim} \text{Unif}(0,1)$

$$= E[f(U_1,\dots,U_d)]$$

$$\hat{\theta} = \frac{1}{n}\sum_{i=1}^{n} f(U_{i1},\dots,U_{id})$$

$$\int_E f(x)\,dx = \int_A f(g(x))\,|\det J_g|\,dx$$

$g(A) = E$

$$\int_{a_1}^{b_1} \cdots \int_{a_d}^{b_d} f(x)\,dx =$$

what g to use

$$g(x_1, \ldots, x_d) = \begin{pmatrix} a_1 + (b_1 - a_1) x_1 \\ \vdots \\ a_d + (b_d - a_d) x_d \end{pmatrix}$$

$$\int_0^1 \cdots \int_0^1 f(g(x_1, \ldots, x_d)) \cdot \prod_{i=1}^{d} (b_i - a_i) \, dx_1 \cdots dx_d$$

$$= E\left[f(g(U_1, \ldots, U_d)) \prod_{i=1}^{d} (b_i - a_i) \right]$$

$$\hat{\theta} = \left(\prod_{i=1}^{d} (b_i - a_i) \right) \cdot \sum_{j=1}^{n} f(g(U_{j1}, \ldots, U_{jd}))$$

$\underline{Ex}$ $\quad d = 2$

$$\int_2^4 \int_1^3 e^{-x^2 y}\, dx\, dy \qquad g = \begin{pmatrix} 1 + 2x \\ 2 + 2y \end{pmatrix}$$

$$= \int_0^1 \int_0^1 4 \cdot e^{-(1+2x)^2 (2+2y)}\, dx\, dy \qquad |\det J_g| = 4$$

$$= E\left[4 \cdot e^{-(1+2U_1)^2(2+2U_2)}\right]$$

$$\hat{\theta} = \frac{1}{n}\sum_{i=1}^{n} 4 \cdot \exp\left(-(1+2U_{i1})^2(2+U_{i2})\right)$$

<u>Ex</u>

$$\int_2^4\int_0^\infty e^{-x^2y}\,dx\,dy \qquad g = \begin{pmatrix} \frac{1}{1-x} - 1 \\ 2+2y \end{pmatrix}$$

$$= \int_0^1\int_0^1 \exp\left(-\left(\frac{1}{1-x}-1\right)^2(2+2y)\right) \cdot \frac{2}{(1-x)^2}\,dx\,dy$$

$$J_g = \begin{bmatrix} \frac{1}{(1-x)^2} & 0 \\ 0 & 2 \end{bmatrix}$$

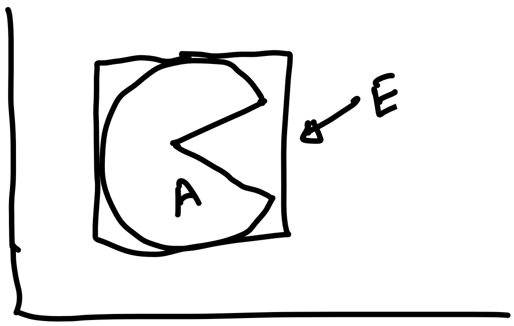
$$= E\left[\exp\left(-\left(\frac{1}{(1-U_1)}-1\right)^2(2+2U_2)\right) \cdot \frac{2}{(1-U_1)^2}\right]$$

$$|\det J_g| = \frac{2}{(1-x)^2}$$

$$\hat{\theta} = \frac{1}{n}\sum_{i=1}^{n} \frac{2}{(1-U_{i1})^2} \cdot \exp\left(-\left(\frac{1}{1-U_{i1}}-1\right)^2(2+2U_{i2})\right)$$

__Ex__

$$\int_A f(x)\,dx = \int_E f(x)\, \mathbb{1}_A(x)\,dx =$$



find g s/t

$g(I) = E$

I is the unit high-d square

$$= \int_I f(g(x_1, \dots, x_d))\, |J_g| \cdot \mathbb{1}_A(g(x_1, \dots, x_d))\, dx_1 \cdot \ldots \cdot dx_d$$