NHP $\Rightarrow$ $\lambda(t)$ intensity

1. $N(0) = 0$
2. independent increments
3. $N(t) - N(s) \sim \text{Poiss}\left(\int_s^t \lambda(u)\,du\right)$

$\Lambda(t) =$ Expected number of arrivals by time t

$$= \int_0^t \lambda(s)\,ds$$

$$\Lambda(t) - \Lambda(s) = \int_s^t \lambda(u)\,du$$

1. $\lambda(t) \leq \lambda_{max} \quad 0 \leq t \leq T$

$\Rightarrow$ 1. generate the arrivals of a homogenous poisson process w/ intensity λ_{max}

$s_1, \dots, s_n$

② keep s_i w/ prob

$$p(s_i) = \frac{\lambda(s_i)}{\lambda_{max}}$$

(thinning)

② order statistic

→ sample $N(T) \sim Poiss(\Lambda(T))$

→ given $N(T) = k$

sample $Y_1, \ldots, Y_k$

$$F(t) = \frac{\Lambda(t)}{\Lambda(T)}$$ → inverse transform

$$f(t) = \frac{\lambda(t)}{\Lambda(T)}$$ → rejection sampling

③ Inverse method

$\Lambda(t)$ is invertible

$\Lambda^{-1}(t)$ exists

① sample arrival times Poisson Process (1) on $[0, \Lambda(T)]$

$s_1, \ldots, s_K$

② arrival times NHP

$\Lambda^{-1}(s_1), \ldots, \Lambda^{-1}(s_K)$

	Good	Bad
Thinning	$\lambda(t)$ is bounded	$\lambda(t)$ is not bounded $\lambda_{max} >> \lambda(t)$ for most t
Order	works nicely if computing Λ is slow	if we don't have a good proposal density for rejection sampling
Inverse	Λ^{-1} exists and easily compute	$\lambda(t) = 0$ on some interval (not invertible) computing inverse computationally

slow

Is Λ invertible and Λ^{-1} fast to compute? $\xrightarrow{Y}$ Inverse method

$\downarrow$ N

do we have a good proposal density for $\dfrac{\lambda(t)}{\Lambda(T)}$? $\xrightarrow{Y}$ order statistics

$\downarrow$ N

Is λ bounded? $\xrightarrow{Y}$ thinning

$\downarrow$ N

need to deal w/ computation Λ^{-1} being slow $\longrightarrow$ inverse method

Simulating from a NHP
using data

$t_1 = 0.25$

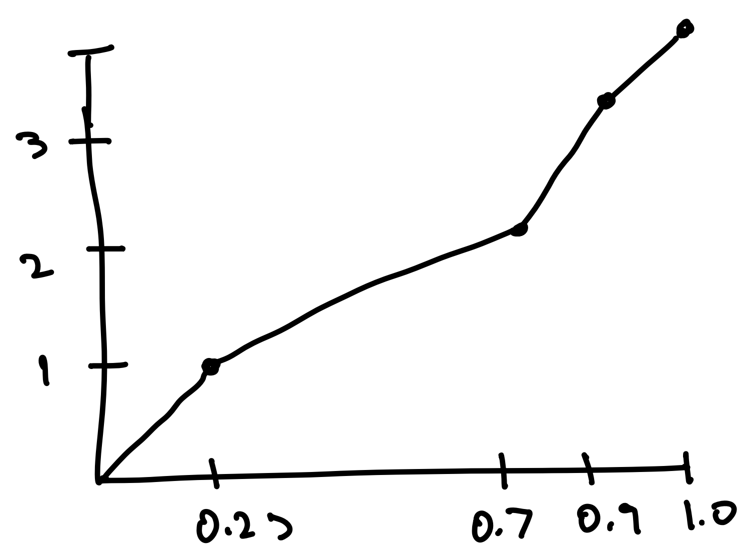
$t_2 = 0.7$

$t_3 = 0.9$

$\Lambda(0.25) = 1$ $\quad$ $\Lambda(1) = 4$

$\Lambda(0.7) = 2$

$\Lambda(0.9) = 3$



I obsere one set of arrivals
on time $[0,1]$

$t_1, \dots, t_n$

$t_0 = 0$

$t_{n+1} = 1$

$$\Lambda(t) = \begin{cases} i + \frac{1}{t_{i+1} - t_i}(t - t_i), & t \in [t_i, t_{i+1}) \end{cases}$$

day 1: $[t_{11}, \dots, t_{1n_1}]$

$$n = \sum_{i=1}^{k} n_i$$

$\vdots$

day k: $[t_{k1}, \dots, t_{kn_k}]$

$t_{(0)} = 0$

$t_{(1)}, \dots, t_{(n)}$

$t_{(n+1)} = 1$

$$\Lambda(t_{(1)}) = \frac{1}{k}$$

$$\Lambda(t_{(2)}) = \frac{2}{k}$$

$\vdots$

$$\Lambda(t_{(n)}) = \frac{n}{k}$$

$\Longrightarrow$ all we need to do is instead of linearly interpolating piecewise between $(0,0), (t_1, 1), \ldots$

$$(0,0), \left(t_{(1)}, \frac{1}{k}\right), \left(t_{(2)}, \frac{2}{k}\right), \ldots$$

x y