

The building block of simulation  
is being able to sample an iid sequence  
of  $\text{Unif}(0,1)$

$$U_1, \dots, U_n$$

How do we sample  $\text{Unif}(0,1)$  random numbers  
on a computer?

→ approximate  $\text{unif}(0,1)$  by first  
generating a random integer

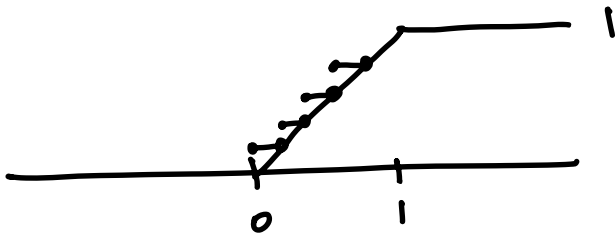
$$x_n = 0, 1, \dots, m-1$$

where  $m$  is  
large  
integer

$$U_n = \frac{x_n}{m}$$

$$\hookrightarrow p(x_n = i) = \frac{1}{m} \quad \text{for } i=0, \dots, m-1$$

cdf of  $U_n$  converges to the  
cdf of  $\text{Unif}(0,1)$  for  $m$  large



→ true random numbers are hard to generate

→ pseudo random numbers  
sequence of numbers a pseudo-random  
number generator

→ way of generating a deterministic sequence that  
looks random

simple one: (multiplicative congruential  
generator)

$x_0 = \text{seed value}$

$\vdots$

$$x_n = a \cdot x_{n-1} \bmod m$$

period of MCG is at most  $n$

$m$  is going to be based on the bit size of  
computer

$$m = 2^{31} - 1$$

$$a = 7^5$$

period 16,807

linear congruential generator

$$x_n = (a x_{n-1} + b) \bmod m$$

Mersenne Twister  
period of  $2^{19,937} - 1$