RV X is mapping from S to $\mathbb{R}$

S_X is the collection of vals a / is the subset of $\mathbb{R}$

s/t $\exists\ \omega \in S$

$X(\omega) = a$

discrete rv has support that is either finite or countable

$$S_X = \{x_1, x_2, \dots, \}$$

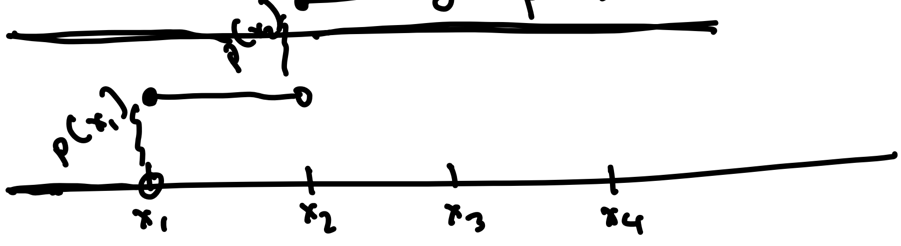
pmf $\quad p_X(x) = P(X = x)$

$$\sum_{x \in S_X} p(x) = 1$$

$$F_X(x) = P(X \le x) \qquad S_X = \{x_1 < x_2 < x_3 < \dots\}$$

$$= \sum_{\substack{y \in S_X: \\ y \le x}} p(y)$$

relationship btwn F and p



$$E[X] = \sum_{x \in S_X} p(x) \cdot x$$

$$\mathrm{Var}(X) = E[(X - E[X])^2]$$

$$= E[X^2] - E[X]^2$$

Ex $X \sim \mathrm{Bern}(q)$

$p(0) = 1-q$

$p(1) = q$

1. $U \sim \mathrm{Unif}(0,1)$
2. If $U \leq 1-q \Rightarrow X = 0$

   Else $X = 1$

Ex $S_X = \{1, 2, 3, 4\}$

$p(1) = 0.2$

$p(2) = 0.15$

$p(3) = 0.25$

$p(4) = 0.4$

1. $U \sim \mathrm{Unif}(0,1)$
2. $X(U) = \begin{cases} 1, & U \leq 0.2 \\ 2, & 0.2 < U \leq 0.35 \\ 3, & 0.35 < U \leq 0.6 \\ 4, & 0.6 < U \leq 1.0 \end{cases}$

$U \leftarrow \mathrm{Unif}(0,1)$

If $U \leq 0.2$

return 1

$N = \#$ of comparisons

$P(N=1) = 0.2$

Elif $U \le 0.35$
return 2

Elif $U \le 0.6$
return 3

Else return 4

$P(N=2) = 0.15$

$P(N=3) = 0.65$
(proc of elimination)

$E[N] = 0.2 + (2)(0.15) + (3)(0.65)$
$= 2.45$

Inverse Transform Method

$S_X = \{x_1, x_2, \dots\}$

pmf p $\quad$ $p(x_j) = p_j$

$U \sim \text{Unif}(0,1)$

$$X(U) = \begin{cases} x_1 & \text{if } \underbrace{U \le p_1} \\ x_2 & \text{if } p_1 < U \le p_1 + p_2 \\ x_3 & \text{if } p_1 + p_2 < U \le p_1 + p_2 + p_3 \\ \vdots & \\ x_j & \text{if } \underline{\sum_{i=1}^{j-1} p_i} < U \le \underline{\sum_{i=1}^{j} p_i} \\ \vdots & \end{cases}$$

$$P(X = x_j) = P\left(\sum_{i=1}^{j-1} p_i < U \le \sum_{i=1}^{j} p_i\right)$$

$$= p_j$$

$\Longrightarrow$ can .beat performance

$\rightarrow$ quantified a expected run-time given that run-time is random

Ex (prev example con'd)

$p(1) = 0.2$
$p(2) = 0.15$
$p(3) = 0.25$
$p(4) = 0.4$

$$X(U) = \begin{cases} 4, & 0 < U \leq 0.4 \\ 3, & 0.4 < U \leq 0.65 \\ 1, & 0.65 \; U \leq 0.85 \\ 2, & U > 0.85 \end{cases}$$

$$E[N] = (0.4)(1) + (0.25)2 + (0.35)(3)$$

$$= 1.95$$

Ex Discrete Uniform (n)

$S_x = \{1, \dots, n\}$

$p(i) = 1/n \quad , \quad i = 1, \dots, n$

① $U \sim Unif(0,1)$

② If $U \le \frac{1}{n}$ $\qquad nU \le 1$

return 1

Elif $U \le \frac{2}{n}$ $\qquad 1 < nU \le 2$

return 2

Elif $U \le \frac{3}{n}$ $\qquad 2 < nU \le 3$

return 3

$\Longrightarrow$ replace with $\lceil nU \rceil$

Ex Geometric random variable

$X \sim geo(p)$

$$S_x = \{1, 2, 3, \ldots\}$$

$$P(X=i) = p(1-p)^{i-1}$$

$$F(n) = p\sum_{i=1}^{n} (1-p)^{i-1}$$

$$= p\underbrace{\sum_{i=0}^{n-1} (1-p)^{i}}$$

$$= p \cdot \frac{(1-(1-p)^n)}{1-(1-p)}$$

$$= 1-(1-p)^n$$

$$\left[\begin{array}{l} |x|<1 \\ \displaystyle\sum_{i=0}^{\infty} x^i = \frac{1}{1-x} \\ \displaystyle\sum_{i=0}^{n} x^i = \frac{1-x^{n+1}}{1-x} \end{array}\right]$$