Quiz. 3

(Deadline March 21, 2024)

For the following selection questions, each answer must be fully explained in your own words for clarity. Merely copying answers from the internet without proper explanation will not be awarded any points.
† indicates that the question is multiple-choice.

Problem 1

† Data compression is often used in data storage and transmission. Suppose you want to use data compression in conjunction with encryption. Does it make more sense to:

☑ Compress then encrypt.

☑ Encrypt then compress. 了現有黑客加密壓縮法，也可 benefit transmission 等方面

☑ The order does not matter – either one is fine.

☐ The order does not matter – neither one will compress the data.

With compression → encryption,
first we delete the redundant data, this will make encryption more faster, also benefitting decryption and transmission.

Problem 2

† Let G: $\{0,1\}^s \to \{0,1\}^n$ be a secure PRG. Which of the following is a secure PRG:

☐ G'(k) = G(k) ‖ G(k) not random for last n bits

☑ G'(k) = G(k $\oplus$ 1^s) G is secure, so G' is secure

☐ G'(k) = G(0) Always output same text

☐ G'(k) = G(1) 同上

☐ G'(k) = G(k) ‖ 0 last 0 is not random

☑ G'(k_1, k_2) = G(k_1) ‖ G(k_2) random

☑ G'(k) = reverse(G(k)) G is secure → random

☑ G'(k) = $rotation_n$(G(k)) random, attack on G' of give attack on G

Hint:
"‖" denotes concatenation.
"reverse(x)" reverses the string x so that the first bit of x is the last bit of reverse(x), the second bit of x is the second to last bit of reverse(x), and so on.
"$rotation_n$(x)" rotates the string x by n positions. If n>0, it rotates right; if n<0, it rotates left, and characters shifted off one end reappear at the other.

A secure PRG need to be a random text.

Problem 3

Let (E, D) be a (one-time) semantically secure cipher with key space K = $\{0,1\}^k$. A bank wishes to split a decryption key k $\in \{0,1\}^k$ into two pieces p_1 and p_2 so that both are needed for decryption. The piece p_1 can be given to one executive and p_2 to another so that both must contribute their pieces for decryption to proceed.

The bank generates random k_1 in $\{0,1\}^k$ and sets k_1' $\leftarrow$ k $\oplus$ k_1. Note that $k_1 \oplus k_1$' = k. The bank can give k_1 to one executive and k_1' to another. Both must be present for decryption to proceed since, by itself, each piece contains no information about the secret key k (note that each piece is a one-time pad encryption of k).

Now, suppose the bank wants to split k into three pieces p_1, p_2, p_3 so that any two of the pieces enable decryption using k. This ensures that even if one executive is out sick, decryption can still succeed. To do so the bank generates two random pairs (k_1, k_1') and (k_2, k_2') as in the previous paragraph so that $k_1 \oplus k_1$' = $k_2 \oplus k_2$' = k. How should the bank assign pieces so that any two pieces enable decryption using k, but no single piece can decrypt?

- ☐ $p_1 = (k_1, k_2)$, $p_2 = (k_1, k_2)$, $p_3 = (k_2')$ P_1+P_2 cannot decrypt → X
- ☐ $p_1 = (k_1, k_2)$, $p_2 = (k_1', k_2')$, $p_3 = (k_2')$ P_2+P_3 cannot → X
- ☑ $p_1 = (k_1, k_2)$, $p_2 = (k_1', k_2)$, $p_3 = (k_2')$ ✓
- ☐ $p_1 = (k_1, k_2)$, $p_2 = (k_2, k_2')$, $p_3 = (k_2')$ P_2 can decrypt itself → X
- ☐ $p_1 = (k_1, k_2)$, $p_2 = (k_1')$, $p_3 = (k_2')$ P_2+P_3 cannot → X

Problem 4

Let M = C = K = { 0, 1, 2, . . . , 255 } and consider the following cipher defined over (K, M, C):
E(k, m) = m + k (mod 256); D(k, c) = c - k (mod 256)
Does this cipher has perfect secrecy?

- ☐ No, there is a simple attack on this cipher.
- ☑ Yes
- ☐ No, only the One Time Pad has perfect secrecy.

If k is used one-time
If k is unknown, there are 256 possible plaintext of it
⇒ perfect secrecy
also ∵ the key space is as large as the message space and is used only once.

Problem 5

† Let (E, D) be a (one-time) semantically secure cipher where the message and ciphertext space is $\{0, 1\}^n$. Which of the following encryption schemes are (one-time) semantically secure?

- ☐ E'(k, m) = E(0^n, m) Key is not OTP
- ☑ E'((k, k'), m) = E(k, m) ∥ E(k', m) an attack on E' give an attack on E
- ☐ E'(k, m) = E(k, m) ∥ MSB(m) If Attacker ask for the encryption of 0^n and $0^{n-1}1$, then they can distinguish the differences.
- ☑ E'(k, m) = 0 ∥ E(k, m) (i.e. prepend 0 to the ciphertext) an attack on E' give an attack on E
- ☐ E'(k, m) = E(k, m) ∥ k Attacker will read the secret key from ciphertext to decrypt it. ↓ reveals the key
- ☑ E'(k, m) = reverse(E(k, m)) an attack on E' give an attack on E
- ☑ E'(k, m) = $rotation_n$(E(k, m)) an attack on E' give an attack on E

Problem 6

61747461636b206174206461776e

646566656e6420617420 6e6f6f6e

Suppose you are told that the one time pad encryption of the message "attack at dawn" is 6c73d5240a948c86981bc294814d (the plaintext letters are encoded as 8-bit ASCII and the given ciphertext is written in hex). What would be the one time pad encryption of the message "defend at noon" under the same OTP key?

"attack at dawn" → 用ASCII轉換成8-bits binary形式 → 將ciphertext由hex → binary → 將兩者做XOR得OTP → 使用此OTP對"defend at noon"加密

Ans: 6962c720079b8c86981bc89a994d

Problem 7

† The movie industry wants to protect digital content distributed on DVD's. We develop a variant of a method used to protect Blu-ray disks called AACS.

Suppose there are at most a total of n DVD players in the world (e.g. n = 2^{32}). We view these n players as the leaves of a binary tree of height $log_2 n$. Each node in this binary tree contains an AES key k^i. These keys are kept secret from consumers and are fixed for all time. At manufacturing time each DVD player is assigned a serial number i ∈ [0, n-1]. Consider the set of nodes S_i along the path from the root to leaf number i in the binary tree. The manufacturer of the DVD player embeds in player number i the keys associated with the nodes in the set S_i. A DVD movie m is encrypted as

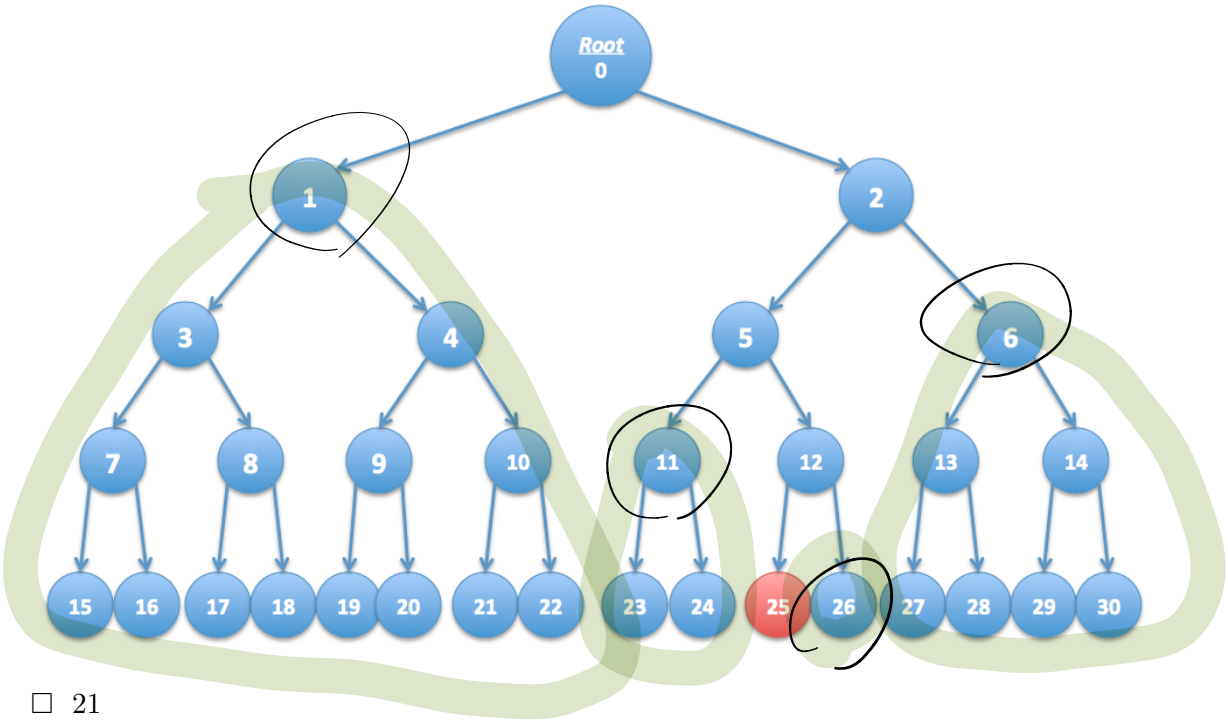
E(k_{root}, k) ∥ E(k, m)

where k is a random AES key called a content-key and k_{root} is the key associated with the root of the tree. Since all DVD players have the key k_{root} all players can decrypt the movie m. We refer to E(k_{root}, k) as the header and E(k, m) as the body. In what follows

the DVD header may contain multiple ciphertexts where each ciphertext is the encryption of the content-key k under some key k_i in the binary tree.

Suppose the keys embedded in DVD player number r are exposed by hackers and published on the Internet. In this problem we show that when the movie industry distributes a new DVD movie, they can encrypt the contents of the DVD using a slightly larger header (containing about $log_2 n$ keys) so that all DVD players, except for player number r, can decrypt the movie. In effect, the movie industry disables player number r without affecting other players.

As shown below, consider a tree with n = 16 leaves. Suppose the leaf node labeled 25 corresponds to an exposed DVD player key. Check the set of keys below under which to encrypt the key k so that every player other than player 25 can decrypt the DVD. Only four keys are needed.



- ☐ 21
- ☐ 17
- ☐ 5
- ☑ 26
- ☑ 6
- ☑ 1
- ☑ 11
- ☐ 24

choose 1 ∵ 25 is under 2 not 1
" 6 ∵ 25 is under 5 not 6
" 11 ∵ 25 is under 12 not 11
" 26 ∵ 26 is under 12 which is same as 25. so to make 26 able to decrypt. we should select 26.

Extra Credit

Did SHA-256 and SHA-512-truncated-to-256-bits have the same security properties? Which one is better? Please explain in detail.

No, though SHA-256 and SHA-512 have similiar designs, and SHA-512-truncated-to-256 is a variant of SHA-512, SHA-512-truncated-to-256 is widely believed to be more secure. ∵ when the hash function is used for properties stronger than a hash function, SHA-512 and SHA-256 are vulnerable to length extensions attacks. But this will not apply if it used to attack the hash that is truncated. Additionaly, SHA-256 is designed based on 32-bit. SHA-512 is designed based on 64-bit. So it might run faster due to different computer you use.

What to turn in:

<student_id>.pdf