STMC HKOI Training

Boolean arithemtic

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Boolean arithmetic

- We have just looked at conditional statements that are connected using not, or, and operators
- In fact there are algebraic structure associated with these operators called **boolean arithmetic**
- Understanding these arithmetic rules can help us simply and rewrite our conditional statements

Properties of boolean operators

- Associativity
  1. $x \vee (y \vee z) = (x \vee y) \vee z$
  2. $x \wedge (y \wedge z) = (x \wedge y) \wedge z$
- Commutativity
  1. $x \vee y = y \vee x$
  2. $x \wedge y = y \wedge x$
- Distributive of $\wedge$ over $\vee$
  1. $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$
  2. $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$

Properties of boolean operations

- Identities
  1. $x \vee F = x$
  2. $x \wedge T = x$
- Annihilators
  1. $x \vee T = T$
  2. $x \wedge F = F$
- Double negation
  1. $\neg(\neg x) = x$
- De Morgan's law
  1. $\neg(x \vee y) = \neg x \wedge \neg y$
  2. $\neg(x \wedge y) = \neg x \vee \neg y$

Exercise

Prove the following Identities:

1. $x \wedge (x \vee y) = x$
2. $\neg x \wedge y = \neg(\neg y) \wedge \neg x$
3. $a = b \wedge a$ if and only if $b = a \vee b$
4. Prove by the De Morgan's law by truth table
5. Prove by the distributive laws by truth table
6. Define $x \rightarrow y = \neg x \wedge y$. Show that $(x \wedge y) \rightarrow y = T$
7. Define $x \leftrightarrow y = (x \rightarrow y) \wedge (y \rightarrow x)$. Show that $x \leftrightarrow y = (x \wedge y) \vee (\neg x \wedge \neg y)$
8. Simplify$(x \vee y) \wedge \neg(\neg x \wedge y)$