APPENDIX A
PROOF OF PROPOSITION 1

The objective function of Problem (7) can be rewritten as

$$\delta(x_i) = \lambda_i \ln(x_i+1) - bx_i, \tag{19}$$

and its second derivative is

$$\delta^{''}(x_i) = -\frac{\lambda_i}{(x_i+1)^2} \leq 0. \tag{20}$$

Therefore, the objective function of Problem (7) is a concave function to maximize and there exists an x_i^* that maximizes the payoff and also makes the following first derivative $\delta^{'}(x_i^*)$ equal to zero. By solving

$$\delta^{'}(x_i^*) = \frac{\lambda_i}{x_i^*+1} - b = 0, \tag{21}$$

we can obtain x_i^* in closed-form. Further, we constrain the solution within $[0, d_i]$ to ensure x_i^* lies in the valid range and then we can obtain Proposition 1.

APPENDIX B
PROOF OF THEOREM 1

Given the definition of three types users in Corollary 1, we have $b \leq \lambda_i, \forall i \in \mathcal{I}_3(b)$, thus we can derive that

$$\frac{\sum_{i\in\mathcal{I}_3(b)} \lambda_i}{b} - |\mathcal{I}_3(b)| \geq 0 \tag{22}$$

and further derive that

$$e^{A_2\gamma(\sum_{i\in\mathcal{I}_2(b)} d_i + \frac{\sum_{i\in\mathcal{I}_3(b)}\lambda_i}{b} - |\mathcal{I}_3(b)|)} \geq 1. \tag{23}$$

Combining Condition (10) and (23), we can get

$$-\alpha A_1 A_2 e^{A_2\gamma(\sum_{i\in\mathcal{I}_2(b)} d_i + \frac{\sum_{i\in\mathcal{I}_3(b)}\lambda_i}{b} - |\mathcal{I}_3(b)|)} + \beta T_0 \leq 0. \tag{24}$$

Multiplying a positive term $\frac{2\gamma\sum_{i\in\mathcal{I}_3(b)}\lambda_i}{b^3}$ on both sides of Condition (24), we can get

$$\begin{aligned} &-\tfrac{2\alpha A_1 A_2\gamma\sum_{i\in\mathcal{I}_3(b)}\lambda_i}{b^3} e^{A_2\gamma(\sum_{i\in\mathcal{I}_2(b)} d_i + \frac{\sum_{i\in\mathcal{I}_3(b)}\lambda_i}{b} - |\mathcal{I}_3(b)|)} \\ &+\tfrac{2\beta T_0\gamma\sum_{i\in\mathcal{I}_3(b)}\lambda_i}{b^3} \leq 0. \end{aligned} \tag{25}$$

The second derivative of the objective function of Problem (6) can be expressed as

$$\begin{aligned} \phi^{''}(b) \quad &= -\alpha A_1 (\tfrac{\gamma A_2\sum_{i\in\mathcal{I}_3(b)}\lambda_i}{b^2})^2 \cdot \\ &e^{A_2\gamma(\sum_{i\in\mathcal{I}_2(b)} d_i + \frac{\sum_{i\in\mathcal{I}_3(b)}\lambda_i}{b} - |\mathcal{I}_3(b)|)} \\ &-\tfrac{2\alpha A_1 A_2\gamma\sum_{i\in\mathcal{I}_3(b)}\lambda_i}{b^3} \cdot \\ &e^{A_2\gamma(\sum_{i\in\mathcal{I}_2(b)} d_i + \frac{\sum_{i\in\mathcal{I}_3(b)}\lambda_i}{b} - |\mathcal{I}_3(b)|)} \\ &+\tfrac{2\gamma\beta T_0\sum_{i\in\mathcal{I}_3(b)}\lambda_i}{b^3}, \end{aligned} \tag{26}$$

where the first term is negative and the sum of the last two terms is also negative according to Condition (25). Therefore, we can get $\phi^{''}(b) \leq 0$, which means the objective function of Problem (6) is concave. Since Problem (6) only has linear constraints, Problem (6) is a *convex* optimization problem [19].

APPENDIX C
PROOF OF THEOREM 2

The left side of Equation (13) equals the sum of all users' redeemed data amount

$$\begin{aligned} &\textstyle\sum_{i\in\mathcal{I}_2(\tilde{b})} d_i + \frac{\sum_{i\in\mathcal{I}_3(\tilde{b})}\lambda_i}{\tilde{b}} - |\mathcal{I}_3(\tilde{b})| \\ &= \textstyle\sum_{i\in\mathcal{I}_1(\tilde{b})} x_i + \sum_{i\in\mathcal{I}_2(\tilde{b})} x_i + \sum_{i\in\mathcal{I}_3(\tilde{b})} x_i, \end{aligned} \tag{27}$$

and it is a decreasing function in b. To prove the right side of Equation (27) is a decreasing function in b, we separately analyze how the data redemption amount change if b increases according to Corollary 1.

- For those who were categorized as $\mathcal{I}_1$, they will still be $\mathcal{I}_1$ with the same data redemption amount.
- For those who were categorized as $\mathcal{I}_2$, no matter which group they belong to given the increased b, the data redemption amount can only decrease or remain unchanged since they chose the maximum amount before.
- For those who were categorized as $\mathcal{I}_3$, some of them will switch to $\mathcal{I}_1$ and the rest of them still belong to $\mathcal{I}_3$, it means some users choose to decrease their data redemption amount.

In summary, the total data redemption amount will decrease when the unit price increases. Thus, the right side of Equation (27) is a decreasing function in b, so as its left side.

Therefore, there exists a threshold $\tilde{b}$ that makes

$$\begin{aligned} \textstyle\sum_{i\in\mathcal{I}_2(\tilde{b})} d_i + \frac{\sum_{i\in\mathcal{I}_3(\tilde{b})}\lambda_i}{\tilde{b}} - |\mathcal{I}_3(\tilde{b})| \geq \frac{1}{A_2\gamma}\ln\frac{\beta T_0}{\alpha A_1 A_2}, \\ \forall b \in [0, \tilde{b}], \end{aligned} \tag{28}$$

and

$$\begin{aligned} \textstyle\sum_{i\in\mathcal{I}_2(\tilde{b})} d_i + \frac{\sum_{i\in\mathcal{I}_3(\tilde{b})}\lambda_i}{\tilde{b}} - |\mathcal{I}_3(\tilde{b})| < \frac{1}{A_2\gamma}\ln\frac{\beta T_0}{\alpha A_1 A_2}, \\ \forall b \in [\tilde{b}, b_{\max}]. \end{aligned} \tag{29}$$

Combining with Condition (12), the above Condition (28) and (29) can be rewritten as

$$\begin{aligned} e^{A_2\gamma(\sum_{i\in\mathcal{I}_2(b)} d_i + \frac{\sum_{i\in\mathcal{I}_3(b)}\lambda_i}{b} - |\mathcal{I}_3(b)|)}\alpha A_1 A_2 - \beta T_0 \geq 0, \\ \forall b \in [0, \tilde{b}], \end{aligned} \tag{30}$$

and

$$\begin{aligned} e^{A_2\gamma(\sum_{i\in\mathcal{I}_2(b)} d_i + \frac{\sum_{i\in\mathcal{I}_3(b)}\lambda_i}{b} - |\mathcal{I}_3(b)|)}\alpha A_1 A_2 - \beta T_0 < 0, \\ \forall b \in [\tilde{b}, b_{\max}]. \end{aligned} \tag{31}$$

We can find that Condition (30) is the same as Condition (24). Therefore, according to Online Appendix [29] Section B, $\phi^{''}(b) \leq 0$ still hold and Problem (6) is a *convex* optimization problem when $b \in [0, \tilde{b}]$.

APPENDIX D
PROOF OF PROPOSITION 2

By substituting the exponential term in Equation (14) with Condition (31), we can get

$$\phi^{'}(b) \leq \gamma\left(\sum_{i\in\mathcal{I}_2(b)} d_i - |\mathcal{I}_3(b)|\right), \forall b \in [\tilde{b}, b_{\max}]. \tag{32}$$

By substituting the term βT_0 in Equation (14) with Condition (12), we can get

$$\begin{aligned}\phi^{'}(b) &> \tfrac{\alpha A_1 A_2 \gamma \sum_{i\in\mathcal{I}_3(b)}\lambda_i}{b^2} e^{A_2\gamma(\sum_{i\in\mathcal{I}_2(b)} d_i + \frac{\sum_{i\in\mathcal{I}_3}\lambda_i}{b} - |\mathcal{I}_3(b)|)} \\ &+\gamma \textstyle\sum_{i\in\mathcal{I}_2(b)} d_i - \gamma|\mathcal{I}_3(b)| - \tfrac{\alpha A_1 A_2 \gamma \sum_{i\in\mathcal{I}_3(b)}\lambda_i}{b^2} \\ &= \gamma \textstyle\sum_{i\in\mathcal{I}_2(b)} d_i - \gamma|\mathcal{I}_3(b)| + \tfrac{\alpha A_1 A_2 \gamma \sum_{i\in\mathcal{I}_3(b)}\lambda_i}{b^2} \cdot \\ &(e^{A_2\gamma(\sum_{i\in\mathcal{I}_2(b)} d_i + \frac{\sum_{i\in\mathcal{I}_3(b)}\lambda_i}{b} - |\mathcal{I}_3(b)|)} - 1) \\ &\geq \gamma\left(\textstyle\sum_{i\in\mathcal{I}_2(b)} d_i - |\mathcal{I}_3(b)|\right), \forall b \in [\tilde{b}, b_{\max}].\end{aligned} \tag{33}$$

Combining Condition (32) and Condition (33), we can get

$$\phi^{'}(b) = \gamma\left(\sum_{i\in\mathcal{I}_2(b)} d_i - |\mathcal{I}_3(b)|\right), \forall b \in [\tilde{b}, b_{\max}]. \tag{34}$$

Appendix E
Proof of Theorem 3

When $\{\lambda_i\}_{\forall i\in\mathcal{I}_r}$ follows a uniform distribution $\mathbb{U}(\lambda_{\min}, \lambda_{\max})$ and $d_i = d, \forall i \in \mathcal{I}$, we can derive that the number of users in $\mathcal{I}_1(b)$, $\mathcal{I}_2(b)$, and $\mathcal{I}_3(b)$ are

$$|\mathcal{I}_1(b)| = \frac{b-\lambda_{\min}}{\lambda_{\max}-\lambda_{\min}}\gamma|\mathcal{I}|, \tag{35}$$

$$|\mathcal{I}_2(b)| = \frac{\lambda_{\max}-b(d+1)}{\lambda_{\max}-\lambda_{\min}}\gamma|\mathcal{I}|, \tag{36}$$

and

$$|\mathcal{I}_3(b)| = |\mathcal{I}| - |\mathcal{I}_1(b)| - |\mathcal{I}_2(b)| = \frac{bd}{\lambda_{\max}-\lambda_{\min}}\gamma|\mathcal{I}|. \tag{37}$$

Thus, $\sum_{i\in\mathcal{I}_t(b)} d_i = d|\mathcal{I}_t|, \forall t \in \{1,2,3\}$.

Further, we have

$$\sum_{i\in\mathcal{I}_1(b)}\lambda_i = \int_{\lambda_{\min}}^{b}\lambda d\lambda = \frac{1}{2}b^2 - \frac{1}{2}\lambda_{\min}^2, \tag{38}$$

$$\sum_{i\in\mathcal{I}_2(b)}\lambda_i = \int_{b(d+1)}^{\lambda_{\max}}\lambda d\lambda = \frac{1}{2}\lambda_{\max}^2 - \frac{1}{2}b^2(d+1)^2, \tag{39}$$

and

$$\begin{aligned}\textstyle\sum_{i\in\mathcal{I}_3(b)}\lambda_i &= \textstyle\sum_{i\in\mathcal{I}}\lambda_i - \sum_{i\in\mathcal{I}_1(b)}\lambda_i - \sum_{i\in\mathcal{I}_2(b)}\lambda_i \\ &= \textstyle\int_{\lambda_{\min}}^{\lambda_{\max}}\lambda d\lambda - \sum_{i\in\mathcal{I}_1(b)}\lambda_i - \sum_{i\in\mathcal{I}_2(b)}\lambda_i \\ &= \tfrac{1}{2}d(d+2)b^2.\end{aligned} \tag{40}$$

In this case, Theorem 1, Theorem 2 and Proposition 2 still hold. Therefore, when Condition (10) is satisfied, Problem (6) is a convex optimization problem. By letting Equation (14) equal to zero and substituting it with Equation (35) to (40), we can obtain b_1 by solving

$$\begin{aligned}\phi^{'}(b_1) &= \alpha\gamma A_1 A_2 \tfrac{1}{2}d(d+2)e^{\mathcal{M}} + \gamma^2\tfrac{\lambda_{\max}-b_1(d+1)}{\lambda_{\max}-\lambda_{\min}}|\mathcal{I}|d \\ &-\gamma^2\tfrac{b_1 d}{\lambda_{\max}-\lambda_{\min}}|\mathcal{I}| - \beta T_0\gamma\tfrac{1}{2}d(d+2) = 0,\end{aligned} \tag{41}$$

where $\mathcal{M} = A_2\gamma(\frac{\lambda_{\max}-b_1(d+1)}{\lambda_{\max}-\lambda_{\min}}\gamma|\mathcal{I}|d + \frac{\frac{1}{2}d(d+2)b_1^2}{b_1} - \frac{b_1 d}{\lambda_{\max}-\lambda_{\min}}\gamma|\mathcal{I}|)$ and the result is shown Equation (16). Thus, a unique and optimal solution when Condition (10) is satisfied is $b^* = [b_1]_0^{b_{\max}}$. Otherwise, when Condition (10) is not satisfied, we can compute a threshold $\tilde{b}$ in closed-form as Equation (17) by substituting Equation (13) with Equation (35) to (40) as

$$\begin{aligned}&\tfrac{\lambda_{\max}-\tilde{b}(d+1)}{\lambda_{\max}-\lambda_{\min}}\gamma|\mathcal{I}|d + \tfrac{\frac{1}{2}d(d+2)\tilde{b}^2}{\tilde{b}} - \tfrac{\tilde{b}d}{\lambda_{\max}-\lambda_{\min}}\gamma|\mathcal{I}| \\ &= \tfrac{1}{A_2\gamma}\ln\tfrac{\beta T_0}{\alpha A_1 A_2}.\end{aligned} \tag{42}$$

Then, based on Theorem 2, when $b \in [0, \tilde{b}]$, Problem (6) is a convex optimization problem, and we can obtain the optimal solution by letting Equation (14) equal to zero as Equation (41) and limiting its domain (i.e., $[b_1]_0^{\tilde{b}}$). When $b \in [\tilde{b}, b_{\max}]$, Problem (6) becomes a convex optimization problem in this special case since Problem (6)'s first derivative is a decreasing function in b when we substitute Equation (14) with Equation (35) to (40) as

$$\begin{aligned}\phi^{'}(b) &= \tfrac{\alpha\gamma A_1 A_2 \sum_{i\in\mathcal{I}_3(b)}\lambda_i}{b^2} e^{A_2\gamma(\sum_{i\in\mathcal{I}_2(b)} d_i + \frac{\sum_{i\in\mathcal{I}_3(b)}\lambda_i}{b} - |\mathcal{I}_3(b)|)} \\ &+\gamma\textstyle\sum_{i\in\mathcal{I}_2(b)} d_i - \gamma|\mathcal{I}_3(b)| - \tfrac{\beta T_0\gamma\sum_{i\in\mathcal{I}_3(b)}\lambda_i}{b^2} \\ &= \tfrac{d|\mathcal{I}|}{\lambda_{\max}-\lambda_{\min}}(\lambda_{\max} - b(d+2)).\end{aligned} \tag{43}$$

Thus, we can obtain the closed-form solution (i.e., $[\frac{\lambda_{\max}}{d+2}]_{\tilde{b}}^{b_{\max}}$) by solving Equation (15) equals to zero, integrating it with Equation (35) to (40) as

$$\gamma\left(\frac{\lambda_{\max}-b(d+1)}{\lambda_{\max}-\lambda_{\min}}\gamma|\mathcal{I}|d - \frac{bd}{\lambda_{\max}-\lambda_{\min}}\gamma|\mathcal{I}|\right) = 0, \tag{44}$$

and limiting its domain. Lastly, we should choose one of the solutions from the two sub-problems that maximizes the server's profit as Problem (11).