D SOLUTION OF CONSTANT PRICE BASELINE

In the *Constant Price* baseline, let $P(x_i) \triangleq k, k \geq 0$ be the unit price function. Similarly, we also solve the profit maximization problem of the server and the payoff maximization problem of users individually.

D.1 Solution of Stage II in Constant Price

We can write the objective function of Stage II in the Constant Price baseline as

$$\begin{aligned} \underset{x_i}{\text{minimize}} \quad h(x_i) &= x_i P(x_i) - U_i(x_i) \\ &= x_i^2 + (k - 2d_i)x_i, \\ \text{subject to} \quad & 0 \leq x_i \leq d_i. \end{aligned} \tag{28}$$

We can find that $h(x_i)$ is a convex problem by computing its secondary derivate $h^{''}(x_i) = 2 > 0$. Also, we can find that $h(x_i)$ is a quadratic function, thus we can derive the solution of the optimal data redemption amount in constant price baseline as

$$\bar{x}_i = min\{max\{0, \frac{2d_i - k}{2}\}, d_i\}. \tag{29}$$

Since the constraint of the unit price function is $k \geq 0$, we can derive that $\frac{2d_i - k}{2} \leq d_i$. Then we can rewrite the solution of Stage II in a simpler way that

$$\bar{x}_i = max\{0, \frac{2d_i - k}{2}\}. \tag{30}$$

D.2 Solution of Stage I in Constant Price

The objective function of Stage I can be rewritten as

$$\begin{aligned} \underset{k}{\text{minimize}} \quad z(k) &= C(\textstyle\sum_{i\in\mathcal{I}} x_i) - \sum_{i\in\mathcal{I}} x_i P(x_i) \\ &= (a\theta + t)(\textstyle\sum_{i\in\mathcal{I}} x_i)^2 - 2t \sum_{i\in\mathcal{I}} d_i \sum_{i\in\mathcal{I}} x_i \\ &\quad + t(\textstyle\sum_{i\in\mathcal{I}} d_i)^2 - k(\sum_{i\in\mathcal{I}} x_i^2 \\ \text{subject to} \quad & k \geq 0. \end{aligned} \tag{31}$$

We replace the x_i in Equation (31) by Equation (30), and similarly, we also explore two sub-problem by different ranges of k.

When $k \geq 2d_i$, the optimal solution of Stage II is $\bar{x}_i = 0$. Then the objective function of Stage I is a constant where

$$z(k) = t(\sum_{i\in\mathcal{I}} d_i)^2, if\ k \geq 2d_i. \tag{32}$$

In such a case, the value of k is meaningless because no data will be redeemed.

When $0 \leq k < 2d_i$, the optimal solution of Stage II is $\bar{x}_i = \frac{2d_i - k}{2}$. Then the objective function of Stage I can be rewritten as

$$\begin{aligned} z(k) = \quad & (\tfrac{1}{4}(a\theta + t)I^2 + \tfrac{1}{2}I)k^2 - (((a\theta + 2t)I - 1) \textstyle\sum_{i\in\mathcal{I}} d_i)k \\ & + a\theta(\textstyle\sum_{i\in\mathcal{I}} d_i)^2,\ if\ 0 \leq k < 2d_i. \end{aligned} \tag{33}$$

The Equation (33) can be considered as a quadratic function

$$z(k) = uk^2 + vk + w, \tag{34}$$

where $u > 0, v < 0, w > 0$. Given the symmetry of Equation (33), let $\bar{k} = \frac{v}{-2u} > 0$ be the symmetry axis that minimizes $z(k)$. Considering the positional relationship between $\bar{k}$ and its boundary $2d_i$ in this situation, we can derive the optimal constant price parameter as

$$\bar{k} = min\{\frac{v}{-2u}, 2d_i\}. \tag{35}$$

Combing the above two situations, the optimal constant price parameter is

$$\bar{k} = min\{\frac{[(a\theta + 2t)I - 1] \sum_{i\in\mathcal{I}} d_i}{\frac{1}{2}(a\theta + t)I^2 + I}, 2d_i\}. \tag{36}$$

E PROOF OF OBSERVATIONS

E.1 Proof of Observation 1&2

In this section, we provide a case study to prove two observations: 1) *Observations 1 (μ and the value of k)*: the changes in the mean amount of sold data μ do not affect the value of k; and 2) *Observations 2 (μ and the server profit)*: the mean amount of sold data μ has positive influences on the server's profit $(-g(k))$.

Consider a case where $\mu = d$. Following the setting in Section 4.1.3, we have $\sigma = 0$, $D = 3d$ and $\theta = 1$. Thus, we can derive that $\sum_{i\in\mathcal{I}} d_i = Id$, $\sum_{i\in\mathcal{I}} d_i^2 = Id^2$ and $D^2 = 9d^2$.

Using the solution of k in Theorem 2, we can derive that

$$\begin{aligned} k^* = \quad & \frac{2[(a\theta+t)(\sum_{i\in\mathcal{I}} d_i)^2+\frac{1}{4}(\theta+1)I^2D^2-\frac{1}{2}D\sum_{i\in\mathcal{I}} d_i+ID^2+\sum_{i\in\mathcal{I}} d_i^2]}{(-\theta I-1)D\sum_{i\in\mathcal{I}} d_i-\frac{1}{2}(\theta+1)I^2D^2-2t(\sum_{i\in\mathcal{I}} d_i)^2-2ID^2-\sum_{i\in\mathcal{I}} d_i^2} + 1 \\ = \quad & \frac{2[2I^2d^2+13Id^2]}{-14I^2d^2-22Id^2} = \frac{-10I+4}{-14I-22}, \end{aligned} \tag{37}$$

which demonstrates that the value of k^* is not affected by μ.

Since the average requested data ration $\frac{x_i}{d_i}$ is stable with μ, we can simply set $x_i = \gamma d_i$ and therefore, $\sum_{i\in\mathcal{I}} x_i = \gamma \sum_{i\in\mathcal{I}} d_i$. Using Equation (21), we can derive that the server profit is

$$-g(k) = 2\gamma^2 I^2 d^2 - 2\gamma I^2 d^2 + k(3\gamma Id^2 + \gamma^2 Id^2), \tag{38}$$

which is a monotonically increasing function of d (i.e., μ) when the value of k is stable.

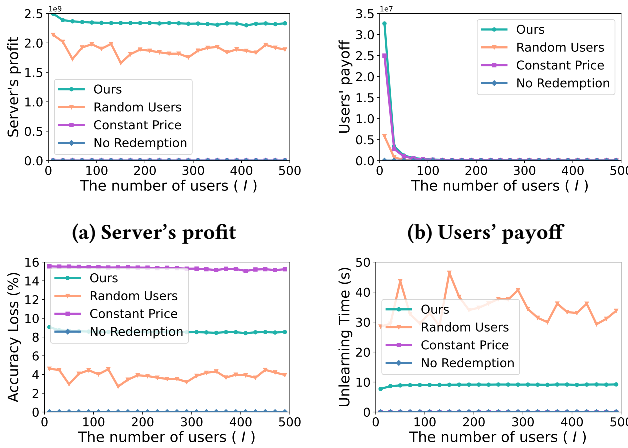
E.2 Proof of Observation 3&4

In this section, we provide a case study to prove two observations: 1) *Observations 3 (σ and the value of k)*: the increment of the standard deviation of the sold data σ results in an increasing and then decreasing trend on the value of k; and 2) *Observations 4 (σ and the requested ratio $\frac{x_i}{d_i}$)*: the increment of the standard deviation of the sold data σ monotonically decreases the requested data ratio.

Similar to Online Appendix [4] Section E.1, consider a case where $\mu = d$. Following the setting in Section 4.1.3, we have $\sigma = 0.1d$, $a = 1$, and $t = 1$. Thus, we can derive that $\sum_{i\in\mathcal{I}} d_i = Id$ and $\sum_{i\in\mathcal{I}} d_i^2 = Id^2$.

Using the solution of k in Theorem 2, we can derive that

$$\begin{aligned} k = \quad & \frac{2[(a\theta+t)(\sum_{i\in\mathcal{I}} d_i)^2+\frac{1}{4}(\theta+1)I^2D^2-\frac{1}{2}D\sum_{i\in\mathcal{I}} d_i+ID^2+\sum_{i\in\mathcal{I}} d_i^2]}{(-\theta I-1)D\sum_{i\in\mathcal{I}} d_i-\frac{1}{2}(\theta+1)I^2D^2-2t(\sum_{i\in\mathcal{I}} d_i)^2-2ID^2-\sum_{i\in\mathcal{I}} d_i^2} + 1 \\ = \quad & \frac{D(3D-d)+4Id^2+2d^2}{-D(Id+d+3ID)-2Id^2-d^2}. \end{aligned} \tag{39}$$



(a) Server's profit **(b) Users' payoff**

(c) Accuracy Loss (%) **(d) Unlearning Time (s)**

Figure 8: Impact of the number of users (I) on Cifar-10 dataset.

Let $A = Id + d > 0$, $B = 4Id^2 + 2d^2 > 0$, and $C = -2Id^2 - d^2 < 0$, which are all constants. We can rewrite the value of k by

$$k = \frac{D(3D - d) + B}{-D(3ID + A) + C}, \tag{40}$$

which is related to D. And the increment of σ results in a monotonical increment of D. Therefore, the relationship between σ and k is equalized to the relationship between D and k.

To further analyze k, we compute its first-order derivative

$$k^{'} = \frac{(-3Id - 3A)D^2 + (Ad + 6C + 6IB)D - Cd + AB - Ad}{(-D(3ID + A) + C)^2}, \tag{41}$$

where $-3Id - 3A < 0$, $Ad + 6C + 6IB > 0$, and $-Cd + AB - Ad > 0$. Therefore, we can find that $k^{'}$ is positive and then becomes negative according to the properties of quadratic functions. Then we can derive that the value of k is first increasing and then decreasing as the increment of σ.

Then, the analysis of x_i can be divided into two situations: 1) k increases, σ increases and 2) k decreases, σ increases.

In the first situation, using Equation (16), we can derive

$$x_i = \frac{2d_i - kD}{2(1-k)} = \frac{d_i}{1-k} - \frac{D}{2(\frac{1}{k} - 1)}, \tag{42}$$

which is simply decreasing in this situation.

While in the second situation, the increment of D is much more significant than the decrement of k, because any slight increment of σ results in a dramatical increment of $max\{d_1, ..., d_I\}$. Thus, kD is still increasing when σ increases and $x_i = \frac{2d_i - kD}{2(1-k)}$ is also decreasing.

F RESULTS ON CIFAR-10 DATASET

We also investigate the impact of the number of users I on the Cifar-10 dataset. The results are shown in Figure 8. We can observe that the results are similar to the MNIST dataset, thus related analysis can be found in Section 4.2.