# D SOLUTION OF CONSTANT PRICE BASELINE

In the *Constant Price* baseline, let $P(x_i) \triangleq k, k \geq 0$ be the unit price function. Similarly, we also solve the profit maximization problem of the server and the payoff maximization problem of users individually.

## D.1 Solution of Stage II in Constant Price

We can write the objective function of Stage II in the Constant Price baseline as

$$\begin{aligned} \underset{x_i}{\text{minimize}} \quad h(x_i) \quad &= x_i P(x_i) - U_i(x_i) \\ &= x_i^2 + (k - 2d_i)x_i, \\ \text{subject to} \quad & 0 \leq x_i \leq d_i. \end{aligned} \tag{28}$$

We can find that $h(x_i)$ is a convex problem by computing its secondary derivate $h^{''}(x_i) = 2 > 0$. Also, we can find that $h(x_i)$ is a quadratic function, thus we can derive the solution of the optimal data redemption amount as

$$x_i^* = min\{max\{0, \frac{2d_i - k}{2}\}, d_i\}. \tag{29}$$

Since the constraint of the unit price function is $k \geq 0$, we can derive that $\frac{2d_i - k}{2} \leq d_i$. Then we can rewrite the solution of Stage II in a simpler way that

$$x_i^* = max\{0, \frac{2d_i - k}{2}\}. \tag{30}$$

## D.2 Solution of Stage I in Constant Price

The objective function of Stage I can be rewritten as

$$\begin{aligned} \underset{k}{\text{minimize}} \quad z(k) \quad &= C(\textstyle\sum_{i \in \mathcal{I}} x_i) - \sum_{i \in \mathcal{I}} x_i P(x_i) \\ &= (a\theta + t)(\textstyle\sum_{i \in \mathcal{I}} x_i)^2 - 2t \sum_{i \in \mathcal{I}} d_i \sum_{i \in \mathcal{I}} x_i \\ &\quad + t(\textstyle\sum_{i \in \mathcal{I}} d_i)^2 - k(\sum_{i \in \mathcal{I}} x_i^2 \\ \text{subject to} \quad & k \geq 0. \end{aligned} \tag{31}$$

We replace the $x_i$ in Equation (31) by Equation (30), and similarly, we also explore two sub-problem by different ranges of $k$.

When $k \geq 2d_i$, the optimal solution of Stage II is $x_i^* = 0$. Then the objective function of Stage I is a constant where

$$z(k) = t(\sum_{i \in \mathcal{I}} d_i)^2, if\ k \geq 2d_i. \tag{32}$$

In such case, the value of $k$ is meaningless because no data will be redeemed.

When $0 \leq k < 2d_i$, the optimal solution of Stage II is $x_i^* = \frac{2d_i - k}{2}$. Then the objective function of Stage I can be rewritten as

$$\begin{aligned} z(k) = \quad & (\tfrac{1}{4}(a\theta + t)I^2 + \tfrac{1}{2}I)k^2 - (((a\theta + 2t)I - 1) \textstyle\sum_{i \in \mathcal{I}} d_i)k \\ & + a\theta(\textstyle\sum_{i \in \mathcal{I}} d_i)^2,\ if\ 0 \leq k < 2d_i. \end{aligned} \tag{33}$$

The Equation (33) can be considered as a quadratic function

$$z(k) = uk^2 + vk + w, \tag{34}$$

where $u > 0, v < 0, w > 0$. Given the symmetry of Equation (33), let $\bar{k} = \frac{v}{-2u} > 0$ be the symmetry axis that minimizes $z(k)$. Considering the positional relationship between $\bar{k}$ and its boundary $2d_i$ in this situation, we can derive the optimal constant price parameter as

$$k = min\{\frac{v}{-2u}, 2d_i\}. \tag{35}$$

Combing the above two situations, the optimal constant price parameter is

$$k = min\{\frac{[(a\theta + 2t)I - 1] \sum_{i \in \mathcal{I}} d_i}{\frac{1}{2}(a\theta + t)I^2 + I}, 2d_i\}. \tag{36}$$

# E PROOF OF OBSERVATIONS

## E.1 Proof of Observation 1&2

In this section, we provide a case study to prove two observations: 1) *Observations 1 ($\mu$ and the value of $k$)*: the changes in the mean amount of sold data $\mu$ do not affect the value of $k$; and 2) *Observations 2 ($\mu$ and the server profit)*: the mean amount of sold data $\mu$ has positive influences on the server's profit $(-g(k))$.

Consider a case where $\mu = d$. Following the setting in Section 4.1.3, we have $\sigma = 0$, $D = 3d$ and $\theta = 1$. Thus, we can derive that $\sum_{i \in \mathcal{I}} d_i = Id$, $\sum_{i \in \mathcal{I}} d_i^2 = Id^2$ and $D^2 = 9d^2$.

Using the solution of $k$ in Theorem 2, we can derive that

$$\begin{aligned} k^* = \quad & \frac{2[(a\theta+t)(\sum_{i \in \mathcal{I}} d_i)^2 + \frac{1}{4}(\theta+1)I^2D^2 - \frac{1}{2}D\sum_{i \in \mathcal{I}} d_i + ID^2 + \sum_{i \in \mathcal{I}} d_i^2]}{(-\theta I - 1)D\sum_{i \in \mathcal{I}} d_i - \frac{1}{2}(\theta+1)I^2D^2 - 2t(\sum_{i \in \mathcal{I}} d_i)^2 - 2ID^2 - \sum_{i \in \mathcal{I}} d_i^2} + 1 \\ = \quad & \frac{2[2I^2d^2 + 13Id^2]}{-14I^2d^2 - 22Id^2} = \frac{-10I + 4}{-14I - 22}, \end{aligned} \tag{37}$$

which demonstrates that the value of $k^*$ is not affected by $\mu$.

Since the average requested data ration $\frac{x_i}{d_i}$ is stable with $\mu$, we can simply set $x_i = \gamma d_i$ and therefore, $\sum_{i \in \mathcal{I}} x_i = \gamma \sum_{i \in \mathcal{I}} d_i$. Using Equation (21), we can derive that the server profit is

$$-g(k) = 2\gamma^2 I^2 d^2 - 2\gamma I^2 d^2 + k(3\gamma I d^2 + \gamma^2 I d^2), \tag{38}$$

which is a monotonically increasing function of $d$ (i.e., $\mu$) when the value of $k$ is stable.

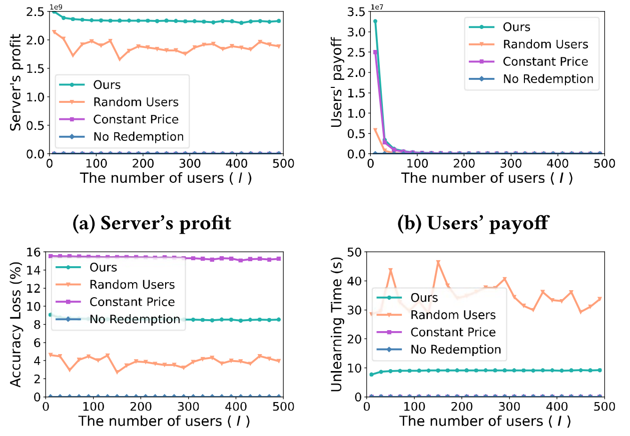
## E.2 Proof of Observation 3&4

In this section, we provide a case study to prove two observations: 1) *Observations 3 ($\sigma$ and the value of $k$)*: the increment of the standard deviation of the sold data $\sigma$ results in an increasing and then decreasing trend on the value of $k$; and 2) *Observations 4 ($\sigma$ and the requested ratio $\frac{x_i}{d_i}$)*: the increment of the standard deviation of the sold data $\sigma$ monotonically decreases the requested data ratio.

Similar to Appendix Section E.1, consider a case where $\mu = d$. Following the setting in Section 4.1.3, we have $\sigma = 0.1d$, $a = 1$, and $t = 1$. Thus, we can derive that $\sum_{i \in \mathcal{I}} d_i = Id$ and $\sum_{i \in \mathcal{I}} d_i^2 = Id^2$.

Using the solution of $k$ in Theorem 2, we can derive that

$$\begin{aligned} k = \quad & \frac{2[(a\theta+t)(\sum_{i \in \mathcal{I}} d_i)^2 + \frac{1}{4}(\theta+1)I^2D^2 - \frac{1}{2}D\sum_{i \in \mathcal{I}} d_i + ID^2 + \sum_{i \in \mathcal{I}} d_i^2]}{(-\theta I - 1)D\sum_{i \in \mathcal{I}} d_i - \frac{1}{2}(\theta+1)I^2D^2 - 2t(\sum_{i \in \mathcal{I}} d_i)^2 - 2ID^2 - \sum_{i \in \mathcal{I}} d_i^2} + 1 \\ = \quad & \frac{D(3D-d) + 4Id^2 + 2d^2}{-D(Id + d + 3ID) - 2Id^2 - d^2}. \end{aligned} \tag{39}$$




(a) Server's profit (b) Users' payoff

(c) Accuracy Loss (%) (d) Unlearning Time (s)

**Figure 8: Impact of the number of users ($I$) on Cifar-10 dataset.**

Let $A = Id + d > 0$, $B = 4Id^2 + 2d^2 > 0$, and $C = -2Id^2 - d^2 < 0$, which are all constants. We can rewrite the value of k by

$$k = \frac{D(3D - d) + B}{-D(3ID + A) + C}, \tag{40}$$

which is related to $D$. And the increment of $\sigma$ results in a monotonical increment of $D$. Therefore, the relationship between $\sigma$ and $k$ is equalized to the relationship between $D$ and $k$.

To further analyze $k$, we compute its first-order derivative

$$k^{'} = \frac{(-3Id - 3A)D^2 + (Ad + 6C + 6IB)D - Cd + AB - Ad}{(-D(3ID + A) + C)^2}, \tag{41}$$

where $-3Id - 3A < 0$, $Ad + 6C + 6IB > 0$, and $-Cd + AB - Ad > 0$. Therefore, we can find that $k^{'}$ is positive and then becomes negative according to the properties of quadratic functions. Then we can derive that the value of $k$ is first increasing and then decreasing as the increment of $\sigma$.

Then, the analysis of $x_i$ can be divided into two situations: 1) $k$ increases, $\sigma$ increases and 2) $k$ decreases, $\sigma$ increases.

In the first situation, using Equation (16), we can derive

$$x_i = \frac{2d_i - kD}{2(1-k)} = \frac{d_i}{1-k} - \frac{D}{2(\frac{1}{k} - 1)}, \tag{42}$$

which is simply decreasing in this situation.

While in the second situation, the increment of $D$ is much more significant than the decrement of $k$, because any slight increment of $\sigma$ results in a dramatical increment of $max\{d_1, ..., d_I\}$. Thus, $kD$ is still increasing when $\sigma$ increases and $x_i = \frac{2d_i - kD}{2(1-k)}$ is also decreasing.

# F RESULTS ON CIFAR-10 DATASET

We also investigate the impact of the number of users $I$ on the Cifar-10 dataset. The results are shown in Figure 8. We can observe that the results are similar to the MNIST dataset, thus related analysis can be found in Section 4.2.