

```
syms u mu l lambda h x;
syms fh_l(x);
fh_l(lambda,u)=1/2*asinh(u*sin(2*lambda));
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m4id=zeros(2,2,2,2);
m4id(1,1,1,1)=1;
m4id(1,2,1,2)=1;
m4id(2,1,2,1)=1;
m4id(2,2,2,2)=1;

m4p=zeros(2,2,2,2);
m4p(1,1,1,1)=1;
m4p(1,2,2,1)=1;
m4p(2,1,1,2)=1;
m4p(2,2,2,2)=1;

m4Q=zeros(2,2,2,2);
m4Q(1,2,2,1)=1;
m4Q(2,1,1,2)=1;

m4Q2=fn_contract(m4Q,4,[3,4],m4Q,4,[1,2]);

m2C=zeros(2,2);
m2C(1,1)=1;
m2C(2,2)=-1;

syms fRt(x);
fRt(x)=(1-cos(x))*m4Q2+cos(x)*m4id+sin(x)*m4Q;
```

```
reshape(fRt(lambda),4,4)
```

ans =

$$\begin{pmatrix} \cos(\lambda) & 0 & 0 & 0 \\ 0 & 1 & \sin(\lambda) & 0 \\ 0 & \sin(\lambda) & 1 & 0 \\ 0 & 0 & 0 & \cos(\lambda) \end{pmatrix}$$

```
Rt_1=fRt(lambda-mu);
Rt_2=fRt(lambda+mu);
rt_1=fn_contract(Rt_1,5,5,reshape(Rt_1,[1,2,2,2,2]),5,1);
rt_2=fn_contract(Rt_2,5,5,reshape(Rt_2,[1,2,2,2,2]),5,1);
rt_1=permute(rt_1,[1,5,2,6,3,7,4,8]);
rt_2=permute(rt_2,[1,5,2,6,3,7,4,8]);

m4CC=fn_contract(m2C,3,3,reshape(m2C,[1,2,2]),3,1);
m4CC=permute(m4CC,[1,3,2,4]);

rt_1=simplify(rt_1);
rt_2=simplify(rt_2);
```

```

tm8=cosh(l/2)*rt_1+sinh(l/2)*fn_contract(m4CC,4,[3,4],rt_1,8,[1,2]);
tm8=cosh(h/2)*tm8 ...
    +sinh(h/2)*permute(fn_contract(m4CC,4,[3,4],tm8,8,[3,4]),...
        [3,4,1,2,5,6,7,8]);
tm8=cosh(-h/2)*tm8 ...
    +sinh(-h/2)*permute(fn_contract(m4CC,4,[3,4],tm8,8,[5,6]),...
        [3,4,5,6,1,2,7,8]);
tm8=cosh(-l/2)*tm8 ...
    +sinh(-l/2)*permute(fn_contract(m4CC,4,[3,4],tm8,8,[7,8]),...
        [3,4,5,6,7,8,1,2]);
rt1=tm8;

rt1=simplify(rt1);

```

```

tm8=cosh(l/2)*rt_2+sinh(l/2)*fn_contract(m4CC,4,[3,4],rt_2,8,[1,2]);
tm8=cosh(h/2)*tm8 ...
    +sinh(h/2)*permute(fn_contract(m4CC,4,[3,4],tm8,8,[3,4]),...
        [3,4,1,2,5,6,7,8]);
tm8=sinh(-h/2)*tm8 ...
    +cosh(-h/2)*permute(fn_contract(m4CC,4,[3,4],tm8,8,[5,6]),...
        [3,4,5,6,1,2,7,8]);
tm8=cosh(-l/2)*tm8 ...
    +sinh(-l/2)*permute(fn_contract(m4CC,4,[3,4],tm8,8,[7,8]),...
        [3,4,5,6,7,8,1,2]);
rt2=tm8;

rt2=simplify(rt2);

```

```

beta=cosh(h-l)/cos(lambda-mu)/(cos(lambda-mu)*cosh(h-l)-cos(lambda+mu)*sinh(h-l))

```

beta =

$$-\frac{\cosh(h-l)}{\cos(\lambda-\mu) (\sinh(h-l) \cos(\lambda+\mu) - \cosh(h-l) \cos(\lambda-\mu))}$$

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alpha=beta*cos(lambda-mu)/cos(lambda+mu)*sinh(h-l)/cosh(h-l)

```

alpha =

$$-\frac{\sinh(h-l)}{\cos(\lambda+\mu) (\sinh(h-l) \cos(\lambda+\mu) - \cosh(h-l) \cos(\lambda-\mu))}$$

```

tm16=beta*rt1+alpha*rt2;
tm16=simplify(tm16);
reshape(tm16,16,16)

```

where

$$\sigma_2 = -\frac{e^{h+l} \left( \frac{\sigma_{12}}{2} - \frac{\sigma_{13}}{2} + \frac{\sigma_{11}}{2} + \frac{\sigma_{10}}{2} \right)}{\sigma_{14}}$$

```
16=permute(tm16,[1,4,3,2,5,8,7,6]);
ble(subs(reshape(tm16,16,16),[lambda,mu,h,l],[2,0,1,0]))
```

```
ans =
  7.3891      0      0      0      0      0      0      0 ...
      0 -6.5320      0      0 -16.1454      0      0      0
      0      0 -6.5320      0      0      0      0      0
      0      0      0 42.6674      0      0 14.2727      0
      0 -2.1850      0      0 -6.5320      0      0      0
      0      0      0      0      0 1.0000      0      0
      0      0      0 14.2727      0      0 5.7744      0
      0      0      0      0      0      0      0 -6.5320
      0      0 -2.1850      0      0      0      0      0
      0      0      0 14.2727      0      0 4.7744      0
      .
      .
      .
      .
```