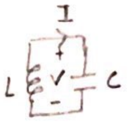
Catherine Zeng Pset 1

1) a. $V_L = -L\frac{di}{dt}$ $\quad i_c = C\frac{dV}{dt}$



$\frac{dV_L}{dt} = -L\frac{d^2 i}{dt^2}$ $\quad \frac{i_c}{C} = -L\frac{d^2 i}{dt^2} \Rightarrow i = -LC\frac{d^2 i}{dt^2}$ $\quad w_0 = \frac{1}{\sqrt{LC}}$

$$\langle\langle \tfrac{1}{2}CV^2\rangle\rangle = \frac{\iint \frac{1}{2}CV^2 e^{-U/(K_BT)}\,di\,dv}{\iint e^{-U/(K_BT)}\,di\,dv}$$ where $U = \frac{1}{2}Li^2 + \frac{1}{2}CV^2$

$$= \boxed{\frac{K_BT}{2}}$$

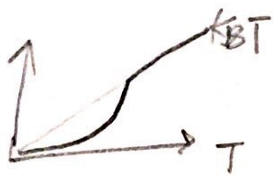
b. $E = n\hbar w$

$$\langle\langle E\rangle\rangle = \frac{\sum_n n\hbar w\, e^{-n\hbar w/(K_BT)}}{\sum_n e^{-n\hbar w/(K_BT)}} = \boxed{\frac{\hbar}{e^{\hbar w/(K_BT)}-1}}$$

$$-\frac{d}{d\beta}\sum e^{-\beta n} = \ldots -\frac{d}{d\beta}\left(\frac{1}{1-e^{-\beta}}\right) = \frac{1}{(1-e^{-\beta})^2}e^{-\beta}$$

$$= \frac{e^{-\beta}/(1-e^{-\beta})^2}{1/(1-e^{-\beta})} = \frac{e^{-\beta}}{1-e^{-\beta}} = \frac{1}{e^{\beta}-1} = \frac{1}{(1-e^{-\beta})^2}e^{-\beta}$$

c) $\frac{\hbar w_0}{e^{\hbar w_0/K_BT}-1} = \frac{K_BT}{2}$



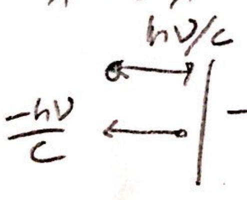
at exactly 0 temperature or high temperature the LC circuit behaves like a classical system

when $T = \infty$ $\quad E \sim K_BT$
when $T = 0$ $\quad E \sim 0$

valid classical temperature when $\frac{\hbar w_0}{K_BT} \ll 1$

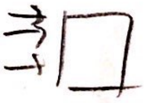
2) a. $p = \frac{h}{\lambda}$ $\quad \nu = \frac{c}{\lambda} \Rightarrow \lambda = \frac{c}{\nu}$

$\frac{h}{c/\nu} = \frac{h\nu}{c}$



$\rightarrow \boxed{\frac{2h\nu}{c}}$

b.



$F = \frac{\Delta P}{\Delta t} = \frac{2\hbar\nu}{c\Delta t}$ $\quad I_A = \frac{\text{energy}}{\text{unit time}}$

$I_A = \frac{n\hbar\nu}{\Delta t A} \Rightarrow n = \frac{\Delta t\, I_A A}{\hbar\nu}$

$F_{total} = nF = \frac{\Delta t\, I_A A}{\hbar\nu} \times \frac{2\hbar\nu}{c\Delta t} = \frac{2IA}{c}$

$\frac{F_{total}}{A} = \boxed{\frac{2I}{c}}$

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c. $I = \int_0^\infty S(\omega)\, d\omega$

$F_{total} = \frac{2}{c} \int_0^\infty S(\omega)\, d\omega A$

$\frac{F_{total}}{A} = \frac{2}{c} \int_0^\infty f(\omega)\, d\omega = \frac{2}{c} \frac{L}{4\pi r^2} = \frac{2}{c} \frac{(3.83\times10^{26} W)}{4\pi (1.5\times10^{11} m)} = 1.36\times10^{6}$

d. $F = ma \quad a = \frac{F}{m_{total}} = \frac{2I/c\, \cancel{A}}{\rho \cancel{A} + \cancel{M_{everything\ else}}^{\,0}} = \frac{2I}{\rho c}$

$\rho = \frac{1.4g}{cm^3} \frac{10^6 cm^3}{m^3} \frac{1kg}{1000g} = \frac{1.4\times10^3 kg}{m^3}$

$\frac{2I}{\rho c} = \frac{\left[\frac{kg \cdot m^2}{s^3}\right]}{\left[\frac{kg}{m^3}\right]\left[\frac{m}{s}\right]} = \frac{m^2\, m^3\, s}{s^3\, m} = \frac{m^4}{s^2} / m^3 = m/s^2$ ← for 1 micron thick

$\frac{2I}{\rho c} = \frac{2(1500)}{(1.4\times10^3)(3\times10^8)} = 7.14\times10^{-9}\, m/s^2$

3) a. $\phi(x) = De^{-x^2/2}$ $\quad \int_{-\infty}^{\infty} |\phi(x)|^2 = 1 = \int_{-\infty}^{\infty} D^2 e^{-x^2} = 1$

$\int_{-\infty}^{\infty} e^{-x^2} = \sqrt{\pi} \quad D^2\sqrt{\pi} = 1 \Rightarrow \boxed{D = \frac{1}{\pi^{\frac{1}{4}}}}$

b. $(\Delta x)^2 = \int_{-\infty}^{\infty} P(x)\,(x - \langle x\rangle)^2 dx = \int_{-\infty}^{\infty} P(x)\,(x^2 - 2x\langle x\rangle + \langle x\rangle^2)\, dx$

$(\Delta x)^2 = \langle x^2\rangle - 2\langle x\rangle^2 + \langle x\rangle^2 = \langle x^2\rangle - \langle x\rangle^2$ 0 ← this is 0 because it's an odd function

odd function → 0

$= \int_{-\infty}^{\infty} x^2 \frac{1}{\pi^{\frac{1}{4}}} e^{-x^2/2} = \frac{1}{\pi^{\frac{1}{4}}} \sqrt{\frac{\pi}{\frac{1}{2}}} \frac{1}{2\left(\frac{1}{2}\right)} = \frac{\sqrt{\pi}}{\frac{1}{4}\pi^{\frac{1}{4}}} = \frac{4\sqrt{\pi}}{\pi^{\frac{1}{4}}}$ $\quad \int_{-\infty}^{\infty} x^2 \frac{1}{\pi^{\frac{1}{4}}} e^{-x^2/2}$

$a = \frac{1}{2}$

$\Rightarrow \Delta x = \boxed{\frac{2\pi^{\frac{1}{4}}}{\pi^{\frac{1}{8}}}}$

c. $\Phi(k) = Ce^{-k^2/2}$ $\quad \int_{-\infty}^{\infty} \frac{|\Phi(k)|^2}{2\pi} = 1 = \int_{-\infty}^{\infty} \frac{C^2 e^{-x^2}}{2\pi} = 1 \quad \int_{-\infty}^{\infty} e^{-x^2} = \sqrt{\pi}$

$\frac{C^2}{2\pi}\sqrt{\pi} = 1 \Rightarrow C = \sqrt{\frac{2\pi}{\sqrt{\pi}}} = \boxed{\frac{\sqrt{2\pi}}{\pi^{\frac{1}{4}}}}$

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Catherine Zey (3)

d. $(\Delta k)^2 = \int_{-\infty}^{\infty} P(k)(k-\langle k\rangle)^2 dk$

$= \int_{-\infty}^{\infty} P(k)(k^2 - 2k\langle k\rangle + \langle k\rangle^2) dk$

odd $\Rightarrow 0$

$= \langle k^2\rangle - 2\langle k\rangle^2 + \langle k\rangle^2 = \langle k^2\rangle - \langle k\rangle^2 = \int_{-\infty}^{\infty} k^2 \frac{\sqrt{2\pi}}{\pi^{\frac14}} e^{-k^2/2} - \int_{-\infty}^{\infty} k \frac{\pi^{\frac14}}{(2\pi)^{\frac12}} e^{-k^2/2}$

$= \frac{\sqrt{2\pi}}{\pi^{\frac14}} \int_{-\infty}^{\infty} k^2 e^{-\frac12 k^2} = \frac{\sqrt{2\pi}}{\pi^{\frac14}} \sqrt{\frac{\pi}{\frac12}} \cancel{\frac{1}{1}} = \frac{\sqrt2\,\pi}{\sqrt{\frac12}\,\pi^{\frac14}}$ $\boxed{\Delta k = \frac{2^{\frac14}\sqrt{\pi}}{(\frac12)^{\frac14}\pi^{\frac18}}}$

e. $\Delta k = \frac{2^{\frac14}\sqrt{\pi}}{(\frac12)^{\frac14}\pi^{\frac18}}$ $\quad \Delta x = \frac{2\pi^{\frac14}}{\pi^{\frac18}}$

$\Delta x\,\Delta k = \frac{2^{\frac14}\sqrt{\pi}\,2\pi^{\frac14}}{(\frac12)^{\frac14}\pi^{\frac18}\pi^{\frac18}} = \boxed{2\sqrt{2\pi}} \approx 5.013$

f. selected $\phi(x) = Ce^{-2x^2}$ instead of $\phi(x) = Ce^{-x^2/2}$

$\phi(x) = De^{-2x^2}$ $\quad \int_{-\infty}^{\infty} |\phi(x)|^2 = 1 = \int_{-\infty}^{\infty} D^2 e^{-4x^2} = 1$

$\int_{-\infty}^{\infty} e^{-4x^2} = \sqrt{\frac{\pi}{4}}$ $\quad D^2\sqrt{\frac{\pi}{4}} = 1$ $\quad D = \sqrt{\frac{1}{\sqrt{\pi/4}}} = \frac{1}{(\frac{\pi}{4})^{\frac14}}$

so then $\langle x^2\rangle - \langle x\rangle^2 = \int_{-\infty}^{\infty} x^2 \frac{1}{(\frac{\pi}{4})^{\frac14}} e^{-2x^2} = \frac{1}{(\frac{\pi}{4})^{\frac14}} \int_{-\infty}^{\infty} x^2 e^{-2x^2}$

$(\Delta x)^2 = \frac{1}{(\frac{\pi}{4})^{\frac14}} \sqrt{\frac{\pi}{2}} \frac{1}{4} = \frac{1}{4(\frac{\pi}{4})^{\frac14}} \sqrt{\frac{\pi}{2}}$

$\Rightarrow \boxed{\Delta x = \frac{1}{2(\frac{\pi}{4})^{\frac18}} \left(\frac{\pi}{4}\right)^{\frac14}}$

$\Phi(k) = Ce^{-2x^2}$ $\quad \int_{-\infty}^{\infty} \frac{C^2 e^{-4x^2}}{2\pi} = 1$ $\quad \int_{-\infty}^{\infty} e^{-4x^2} = \sqrt{\frac{\pi}{4}}$ $\quad \frac{C^2}{2\pi}\sqrt{\frac{\pi}{4}} = 1$

$\Rightarrow C = \sqrt{\frac{2\pi}{\sqrt{\frac{\pi}{4}}}} = \frac{\sqrt{2\pi}}{(\frac{\pi}{4})^{\frac14}}$

$\langle k^2\rangle - \langle k\rangle^2$

$\Delta k^2 = \int_{-\infty}^{\infty} k^2 \frac{\sqrt{2\pi}}{(\frac{\pi}{4})^{\frac14}} e^{-2x^2} = \frac{\sqrt{2\pi}}{(\frac{\pi}{4})^{\frac14}} \underbrace{\int_{-\infty}^{\infty} k^2 e^{-2x^2}}_{=\sqrt{\frac{\pi}{2}}\frac14} = \frac{\sqrt{2\pi}}{(\frac{\pi}{4})^{\frac14}} \sqrt{\frac{\pi}{2}}\frac{1}{4}$

$\Delta k = \frac{\sqrt{\pi}}{2(\frac{\pi}{4})^{\frac18}}$ $\quad \Rightarrow \Delta x\,\Delta k = \frac{1}{2(\frac{\pi}{4})^{\frac18}} \left(\frac{\pi}{4}\right)^{\frac14} \left(\frac{\sqrt{\pi}}{2(\frac{\pi}{4})^{\frac18}}\right) = \frac{\cancel{\sqrt2}\,\pi}{(\frac{\pi}{4})^{\frac14}\cancel{\sqrt2}\,4} = \frac{\pi}{4(\frac{\pi}{4})^{\frac14}}$

$= \boxed{\frac{\sqrt{\pi}}{4}}$

See page 4 for question 4

4) $\phi(x) = C\,\text{sech}(x)$ $\Phi(k) = D\,\text{sech}(\pi k/2)$ Catherine Zey (4)

Finding C:

$\phi(x) = C\,\text{sech}(x)$ $\int_{-\infty}^{\infty} |\phi(x)|^2 = 1 = \int_{-\infty}^{\infty} C^2\,\text{sech}^2(x) = 1$ $\int_{-\infty}^{\infty} \text{sech}^2(x)\,dx = \frac{2}{1} = 2$

$\therefore 2C^2 = 1 \Rightarrow C = \sqrt{\tfrac{1}{2}} \Rightarrow \boxed{\phi(x) = \sqrt{\tfrac{1}{2}}\,\text{sech}(x)}$

Finding Δx:

$(\Delta x)^2 = \int_{-\infty}^{\infty} P(x)\,(x - \langle x\rangle)^2\,dx = \int_{-\infty}^{\infty} P(x)\left[\langle x^2\rangle - \cancel{\langle x\rangle^2}^{\,0}\right]$ → odd function

$= \int_{-\infty}^{\infty} x^2 \sqrt{\tfrac{1}{2}}\,\text{sech}(x) -$

$\sqrt{\tfrac{1}{2}} \int_{-\infty}^{\infty} x^2\,\text{sech}^2(x)\,dx = \frac{\pi^2}{6}\sqrt{\tfrac{1}{2}}$ $\boxed{\Delta x = \frac{\pi}{\sqrt{6}}\left(\tfrac{1}{2}\right)^{\frac{1}{4}}}$

Finding D:

$\Phi(k) = D\,\text{sech}\left(\frac{\pi k}{2}\right)$ $\int_{-\infty}^{\infty} \frac{|\Phi(k)|^2}{2\pi} = 1 = \frac{D^2}{2\pi} \int_{-\infty}^{\infty} \text{sech}^2\left(\frac{\pi k}{2}\right) = 1 = \frac{D^2}{\cancel{2}\pi} \frac{\cancel{2}}{(\pi/2)} = \frac{2D^2}{\pi^2}$

$a = \frac{\pi}{2}$

$\frac{2D^2}{\pi^2} = 1 \Rightarrow \sqrt{\frac{\pi^2}{2}} = D = \frac{\pi}{\sqrt{2}}$

$\Rightarrow \Phi(k) = \boxed{\frac{\pi}{\sqrt{2}}\,\text{sec}\left(\frac{\pi k}{2}\right)}$

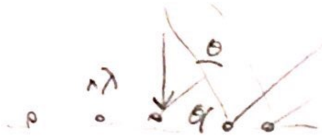
$(\Delta k)^2 = \int_{-\infty}^{\infty} P(k)\,(k - \langle k\rangle)^2\,dk = \int_{-\infty}^{\infty} P(k)\left[\langle k^2\rangle - \langle k\rangle^2\right]$

$= \int_{-\infty}^{\infty} k^2 \frac{\pi}{\sqrt{2}}\,\text{sech}\left(\frac{\pi k}{2}\right)$

$\frac{\pi}{\sqrt{2}} \int_{-\infty}^{\infty} k^2\,\text{sech}^2\left(\frac{\pi k}{2}\right) = \frac{\pi^2}{6\left(\frac{\pi}{2}\right)^3}\,\frac{\pi}{\sqrt{2}}$ $\boxed{\Delta k = \frac{\pi\sqrt{\pi}}{\sqrt{6}\left(\frac{\pi}{2}\right)^{3/2}(2)^{\frac{1}{4}}}}$

$\Delta k\,\Delta x = \frac{\pi^2 \sqrt{\pi}\left(\frac{1}{2}\right)^{\frac{1}{4}}}{6\left(\frac{\pi}{2}\right)^{3/2}(2)^{\frac{1}{4}}} = \boxed{\frac{\pi}{3}}$

5) a.

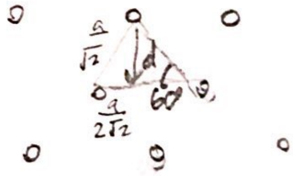


Can we relate nλ to d

$d\sin\theta = n\lambda$ at first order, n=1

$\Rightarrow \boxed{d\sin\theta = \lambda}$

b.



$\boxed{d = \frac{a}{2\sqrt{2}}\sqrt{3}}$

c) $\frac{a}{2\sqrt{2}}\sqrt{3}\sin\theta = \lambda \Rightarrow \frac{a}{2\sqrt{2}}\sqrt{3}\sin(60°) = \frac{a}{2\sqrt{2}}\sqrt{3}\,\frac{\sqrt{3}}{2} = \boxed{\frac{3}{4\sqrt{2}}a}$

d) $\lambda = \frac{h}{mv}$ $h = 6.626\times10^{-34}$ Js

$KE = \frac{1}{2}mv^2 \times \frac{m}{m} = \frac{1}{2}\frac{m^2v^2}{m}$

$mv = \sqrt{2KEm}$ $\Rightarrow \lambda = \frac{h}{\sqrt{2KEm}}$

$54\,eV = 54(1.6\times10^{-19}) = 8.65\times10^{-18}$ J

$m_e = 9.1\times10^{-31}$ kg

$$\lambda = \frac{6.626\times10^{-34}}{\sqrt{2(8.65\times10^{-18})(9.1\times10^{-31})}} = \boxed{1.67\times10^{-10}\,m}$$

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