Problem 1: Compton effect & electron recoil

$E'^2 = p'^2c^2 + m_e^2c^4$ ← electron has rest mass

8.04 Pset 2
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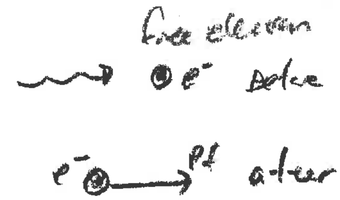
a) Assumption: photon is 100% absorbed by electron

$P_1 + P_\nu = P_2$ where $P_\nu \Rightarrow$ momentum of photon



$\left(\sqrt{P_1^2c^2 + m_e^2c^4} + P_\nu c = \sqrt{P_2c^2 + m_e^2c^4}\right)^2$ & substitute $P_1 + P_\nu = P_2$

$\Rightarrow (P_2-P_1)^2c^2 - (P_2^2-P_1^2)c^2 + 2(P_2-P_1)c\sqrt{P_1^2c^2+m_e^2c^4} = 0$

$P_2 - P_1 = 0$ ← but not possible under conservation of momentum

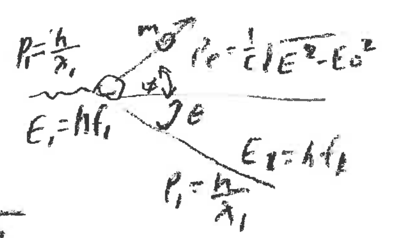
or $\sqrt{P_1^2c^2+m_e^2c^4} - P_1c = 0$, but electron have nonzero mass

Conclusion: photon cannot be 100% absorbed by electron or else you wouldn't have conservation of momentum & mass (given conservation of energy)

b) $\Delta\lambda = \lambda_1 - \lambda_0 = \lambda_c(1-\cos\theta)$

$P_0 = \frac{E_0}{c} = \frac{hf_0}{c} = \frac{h}{\lambda_0}$ $\quad P_1 = \frac{h}{\lambda_2}$ $\quad P_1 - P_0 = P_e$

$P_e^2 = P_0^2 + P_1^2 - 2P_1P_0$
$= P_0^2 + P_1^2 - 2P_1P_0\cos\theta$



conservation of energy

$P_0c + E_0 = P_1c + \sqrt{E_0^2 + P_e^2c^2}$

$E_{\text{electron after collision}} = \sqrt{E_0^2 + P_e^2c^2}$

$(P_0c - P_1c + E_0)^2 = E_0^2 + P_e^2c^2$

$= c^2(P_0-P_1)^2 + 2c(P_0-P_1)E_0 + \cancel{E_0^2} = \cancel{E_0^2} + P_e^2c^2$

$= c(P_0-P_1)^2 + 2(P_0-P_1)E_0 = P_e^2c$

$P_e^2 = \underbrace{(P_0-P_1)^2}_{= P_0^2 + P_1^2 - 2P_1P_0} + \frac{2E_0(P_0-P_1)}{c} = \cancel{P_0^2 + P_1^2} - 2P_1P_0\cos\theta$

$\Rightarrow 2P_1P_0 - 2P_1P_0\cos\theta = \frac{2E_0(P_0-P_1)}{c}$

$\frac{hc}{P_1P_0E_0}\left[P_1P_0(1-\cos\theta) = \frac{E_0(P_0-P_1)}{c}\right] = \frac{hc}{E_0}(1-\cos\theta) = \frac{h}{P_1P_0}(P_0-P_1)$

$= \boxed{\lambda_1 - \lambda_0 = \lambda_c(1-\cos\theta)}$

$\lambda_c = \frac{h}{m_ec} = \frac{[6.626\times10^{-34}\ \cancel{kg}\cdot m^2/s]}{[9.11\times10^{-31}\ \cancel{kg}][3\times10^8\ m/s]} = \boxed{2.424\times10^{-12}\ m}$

c) Compton shift: $\Delta\lambda$ $\quad E_0 = h\nu_0$ $\quad \Delta E = E_1 - E_0$ $\quad$ Conservation of energy $h\nu_i + m_ec^2 = h\nu_f + \sqrt{P_e^2c^2 + m_e^2c^4}$

$\Delta E = \frac{hc}{\lambda_1} - \frac{hc}{\lambda_0} = hc\left[\frac{1}{\lambda_1} - \frac{1}{\lambda_0}\right]$ $\quad \cos\left(\frac{\pi}{2}\right) = 0$; so angle doesn't matter in this case because $\lambda_1 - \lambda_0 = \lambda_c$

$= [6.626\times10^{-34}\ J\cdot s]\left[3\times10^8\ \frac{m}{s}\right]\left[\frac{1}{10\ keV\left[\frac{1\ eV}{0.001\ keV}\right]} - \frac{1}{10\ \cancel{MeV}\left[\frac{1000000\ eV}{1\ MeV}\right]}\right]$

$= \boxed{1.986\times10^{-29}\ J}$

Problem 1 (continued)

d) X-rays because it has more energy so it's easier to measure the momentum change of the electron.

e) $\cot\frac{\theta}{2} = \left[1 + \frac{h\nu_0}{M_e c^2}\right]\tan\phi$ $\qquad \lambda_1 - \lambda_0 = \lambda_c(1-\cos\theta)$

$\lambda_1 - \lambda_0 = \frac{h}{mc}(1-\cos\theta)$ $\qquad \frac{\lambda_1 = \lambda_0 + \frac{h}{mc}(1-\cos\theta)}{\lambda_0} \div \frac{\lambda_1}{\lambda_0} = 1 + \frac{h}{mc\lambda_1}(1-\cos\theta) = \frac{P_i}{P_f}$

$\rightarrow P_e \sin\phi = P_f \sin\theta$ / $P_e\cos\phi = P_1 - P_f\cos\theta$

$P_e\cos\phi + P_f\cos\theta = P_1$

$= \tan\phi = \frac{P_f \sin\theta}{P_1 - P_f\cos\theta} = \frac{\sin\theta}{\frac{P_1}{P_f} - \cos\theta} = \frac{\sin\theta}{\left[1 + \frac{h\nu_0}{mc^2}(1-\cos\theta)\right] - \cos\theta}$

trig identity $\frac{1-\cos\theta}{\sin\theta} = \tan\frac{\theta}{2}$

$= \frac{\sin\theta}{\left[1+\frac{h\nu_0}{mc^2}\right]\left[1-\cos\theta\right]}$ $\qquad \tan\phi\left(1+\frac{h\nu_0}{mc^2}\right) = \frac{\sin\theta}{1-\cos\theta} = \cot\frac{\theta}{2}$

$\Rightarrow \cot\frac{\theta}{2} = \left(1 + \frac{h\nu_0}{M_e c^2}\right)\tan\phi$

Problem 5: X-ray production

$V = 30 kV$ $\quad \lambda_{min} = \frac{12375}{V(V)} \approx 0.41 Å$ ✓

$$\lambda_{min} = \frac{hc}{q_e V_0} = \frac{1.24 Å}{V_0 / 10 kV} = \frac{1.24 Å \frac{1m}{1\times10^{10} Å}}{30 kV / 10 kV}$$

$$= \boxed{4.133 \times 10^{-11} m}$$

Work function does not matter.

Problem 2: Photons interacting with electrons

If energy imparted on electron > work function W, electron ejected w/ KE $E_{e^-} = \frac{1}{2} m_e v^2 = E_r - W = h\nu - W$

a) $\lambda_{uv} = 350 nm$ $\quad E_{max} = 1.6 eV$

$E_{max} = h\nu - W$ $\quad W = h\nu - E_{max}$ $\quad \frac{hc}{\lambda_{uv}} = h\nu$

$$\frac{[6.626\times10^{-34} J\cdot s][3\times10^8 m/s]}{[350\times10^{-9} m]} - 1.6 eV \frac{1.602\times10^{-19} J}{1 eV}$$

$$= \boxed{3.11623\times10^{-19} J}$$

$$\text{Solve}\left[\frac{hc}{\lambda_{min}} - W == 0, \lambda_{min}\right]$$

$$= \left[\frac{(6.626\times10^{-34} J\cdot s)(3\times10^8 m/s)}{\lambda_{min}} - \frac{(1.6 eV)(1.602\times10^{-19} J)}{1 eV} == 0, \lambda_{min}\right]$$

$$\lambda = 7.755\times10^{-7} m$$

b) $\lambda_{\text{scattered x-rays}} = 0.035 Å$

$$\lambda' - \lambda = \frac{h}{m_e c}(1-\cos\theta) \quad \theta = 60°$$

initial wavelength (λ); wavelength after scattering (λ')

$\lambda_{de Broglie} = \frac{h}{p} = \frac{h}{mv}$

Compton wavelength $= \lambda_C = \frac{h}{mc}$

$$\lambda = \lambda' - \frac{h}{m_e c}(1-\cos\theta) + \lambda \qquad \frac{hc}{\lambda_{uv}}$$

$$0.035 Å \frac{1\times10^{-10} m}{1 Å} - \frac{\left[6.626\times10^{-34} \frac{kg\, m^2}{s}\right]}{[9.11\times10^{-31} kg][3\times10^8 m/s]}(1-\cos(60°)) = 2.28778\times10^{-12} m$$

$$E_{xrays} = \frac{hc}{\lambda_{xrays}} = \frac{[6.626\times10^{-34} J\cdot s][3\times10^8 m/s]}{2.28778\times10^{-12} m}$$

$$= \boxed{8.689\times10^{-14} J}$$

Problem 4: The de Broglie wavelength of macroscopic objects

$M_{automobile} = 2000 kg$ 25 mph

$$\lambda_{de\ Broglie} = \frac{h}{mv} = \frac{[6.626\times10^{-34}\ \frac{kg\cdot m^2}{s}]}{[2000\ kg][25\ mph\ \frac{0.44704\ \frac{m}{s}}{1\ mph}]}$$

$$= \boxed{2.96439\times10^{-38}\ m}$$

$\frac{1\ miles}{hr} = 0.44704\ \frac{m}{s}$

$r = 1\mu m$ $\rho = \frac{200\ kg}{m^3}$ $T = 300K$ $V = \frac{4}{3}\pi r^3$

$$M_{dust\ particle} = \rho V = \frac{200\ kg}{m^3}\cdot\frac{4}{3}\pi(1\times10^{-6}m)^3 = 8.3776\times10^{-16}\ kg$$

$$KE = \frac{3}{2}k_B T = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{3k_BT}{m}} = \sqrt{\frac{3(1.38\times10^{-23}\ \frac{m^2 kg}{s^2 K})(300K)}{(8.3776\times10^{-16}\ kg)}} = 0.0039\ m/s$$

$$\lambda_{\substack{dust\ particle\\300K}} = \frac{h}{mv} = \frac{[6.626\times10^{-34}\ \frac{kg\cdot m^2}{s}]}{[8.3776\times10^{-16}\ kg][0.0039\ m/s]} = \boxed{2.03321\times10^{-16}\ m}$$

$M_{Rb} = 1.41923\times10^{-25} kg$

$$V_{100\mu K} = \sqrt{\frac{3(1.38\times10^{-23}\ \frac{m^2 kg}{s^2 K})(100\times10^{-6}K)}{1.41923\times10^{-25}\ kg}} = 0.171 m/s$$

$$\lambda_{\substack{Rb\\100\mu K}} = \frac{h}{mv} = \frac{[6.626\times10^{-34}\ \frac{kg\cdot m^2}{s}]}{[1.41923\times10^{-25}\ kg][0.171\ m/s]} = \boxed{2.73025\times10^{-8} m}$$

Problem 5: Time delay in photoelectric delay

$\lambda = 121 nm$ Intensity $\frac{10 nW}{cm^2}$ $A = 1cm^2$ $W = eV$ πr^2

a) Intensity $= \frac{energy/time}{area}$ time $= \frac{energy}{area\ (intensity)}$

$$time = \frac{[5eV]}{[\pi\cdot[\frac{1\text{Å}\ 10^{-8}cm}{1\text{Å}}]^2][\frac{10nW}{cm^2}]} = \frac{[5eV\ \frac{1.602\times10^{-19}J}{1eV}]}{[\pi\times(10^{-8})^2][\frac{10\times10^{-9}J/s}{cm^2}]} = \boxed{2.5497\times10^5\ s}$$

b) $$time = \frac{[5eV\ \frac{1.602\times10^{-19}J}{1eV}]}{[(\frac{3}{2}\pi)[121\times10^{-9}m\ \frac{100cm}{1m}]^2][\frac{10\times10^{-9}J/s}{cm^2}]} = \boxed{0.1161\ s}$$

5c (continued)

$$E = \frac{hc}{\lambda} = \frac{[6.626\times10^{-34}\,J\cdot s][3\times10^{8}\,m/s]}{121\times10^{-9}\,m} = 1.643\times10^{-18}\,J$$

$$\underset{\times A}{\text{Intensity}} = \frac{10\times10^{-9}\,J/s}{1\,cm^2}(1\,cm^2) = 10\times10^{-9}\,J/s$$

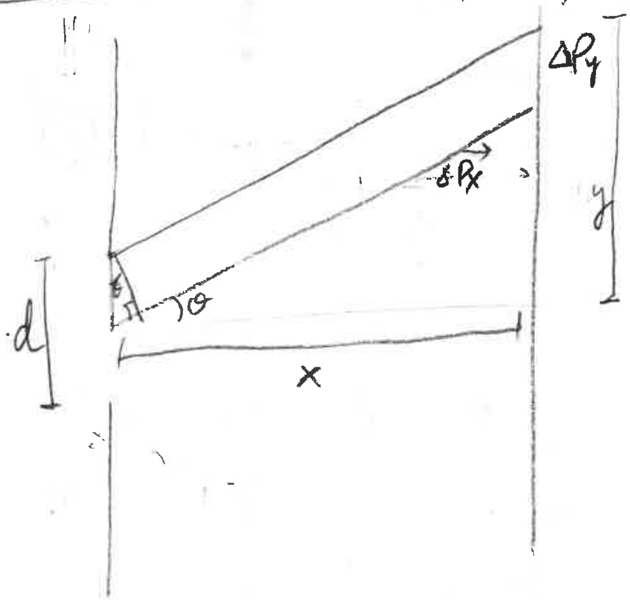
$$\frac{10\times10^{-9}\,J/s}{1.643\times10^{-18}\,J} = 6.086\times10^{9}\,s^{-1}$$

avg time delay between photons

$$1/6.086\times10^{9} = \boxed{1.643\times10^{-10}\,s}$$

$$\text{ratio}\ \frac{\text{classical}}{\text{quantum}} = \frac{0.1161\,\cancel{s}}{1.643\times10^{-10}\,\cancel{s}} = \boxed{7.066\times10^{8}}$$

Problem 6: Single-Slit Diffraction and the Uncertainty Principle



$$\frac{d}{2}\sin\theta = \frac{\lambda}{2} \Rightarrow \underset{\text{approx.}}{\text{small angle}} \ \boxed{\theta = \frac{\lambda}{d}}$$

$$p = \frac{h}{\lambda} \qquad \underbrace{p\lambda}_{} = h$$

characterize in p & λ for system.

$$p = \frac{y}{x}\Delta p_y \qquad \lambda = \frac{x}{y}\Delta y$$

$$p\lambda = \left(\frac{y}{x}\right)\left(\frac{x}{y}\right)\Delta p_y\,\Delta y$$

$$\Rightarrow \boxed{h \sim \Delta p_y \Delta y}$$