MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Spring 2020 | Physics 8.04 | Vuletic

Assignment #2 (no online part this week)

Due Friday, February 21, 2020

1. **Compton effect and electron recoil.** (30 points)
   a) Show that it is impossible for a free electron to absorb all of the energy of a single photon which collides with it.
   b) Derive the Compton wavelength shift for a photon scattered from a free, initially stationary electron
   $$\Delta\lambda = \lambda_1 - \lambda_0 = \lambda_c(1-\cos\theta),$$
   where θ is the photon scattering angle, λ_0 is the wavelength of the incident, λ_1 the wavelength of the scattered photon, and λ_c the Compton wavelength. Calculate the numerical value of the Compton wavelength.
   (c) The Compton shift in wavelength, $\Delta\lambda$, is independent of the incident photon energy $E_0=h\nu_0$. However, the Compton shift in energy, $\Delta E=E_1-E_0$, is strongly dependent on E_0. Find the expression for the Compton energy shift ΔE. (Does the photon gain or lose energy in the collision?) Compute the numerical value of the fractional shift in energy for a 10 keV photon and a 10 MeV photon, assuming $\theta=\pi/2$.
   (d) Is it easier to observe the Compton effect with visible light or with X-rays? Why?
   (e) Show that the relation between the directions of motion of the scattered photon and the recoiling electron is
   $$\cot\frac{\theta}{2} = \left(1+\frac{h\nu_0}{m_e c^2}\right)\tan\phi,$$
   where the angle ϕ specifies the direction of the recoiling electron.

2. **Photons interacting with electrons. (**20 points)
   a) Ultraviolet light of wavelength 350nm falls on a potassium surface. The maximum energy of the photoelectrons is 1.6eV. What is the work function of potassium? Above what wavelength will no photoemission be observed?
   b) A beam of X-rays is scattered by electrons at rest. What is the energy of the X-rays if the wavelength of the X-rays scattered at 60^o to the beam axis is 0.035Å?

3. **X-ray production.** (5 points)

Calculate the short-wavelength limit for X-rays produced by an electron acceleration voltage of 30kV. Is the work function of the metal relevant?

4. **The de Broglie wavelength of macroscopic objects** (10 points)

What is the de Broglie wavelength of an automobile (2000kg) traveling at 25 miles per hour? A dust particle of radius 1μm and density 200 kg/m^3 being jostled around by air molecules at room temperature (T=300K)? An ^{87}Rb atom that has been laser cooled to a temperature of T=100μK? Assume that the kinetic energy of the particle is given by $(3/2)k_BT$.

5. **Time delay in photoelectric effect.** (20 points)
A beam of ultraviolet light (λ=121 nm) of intensity 10nW/cm^2 and area A=1cm^2 is turned on suddenly and falls on a metal surface, ejecting electrons through the photoelectric effect. The work function of the metal is 5 eV. How soon after the beam is turned on might one expect photoelectric emission to occur?

(a) Classically, one can estimate this as the time needed for the work-function energy (5 eV) to be accumulated over the area of one atom (radius 1Å). Calculate how long this would be, assuming the energy of the light beam to be uniformly distributed over its cross-section.

(b) Actually the estimate from part (a) is too pessimistic. An atom can represent a cross section $\sigma_{eff} = (3/2\pi)\lambda^2$ for the absorption of resonant light. Calculate a classical delay time on this basis.

(c) In the quantum picture, it is possible for photoelectric emission to begin immediately - as soon as the first photon strikes the emitting surface. However, in order to obtain a time that may be compared to the classical estimates, calculate the average time interval between the arrivals of successive photons. This would also be the average time delay between switching on the beam and observing the first photoelectron. Express the ratio of the classical to the quantum mechanical delay time in terms of quantities give above.

6. **Single-slit diffraction and the uncertainty principle**. (15 points)
A well-collimated beam of particles with sharply defined *x*-momentum p_x falls normally on a screen with a slit of width *d*. (The screen lies along the *y*-axis.) Using an argument about phases or a calculation, show that the diffraction pattern has characteristic angular width $\theta \approx \lambda/d$ (for $\lambda/d \ll 1$), where $\lambda = h/p$ is the deBroglie wavelength of the particle. Show that this diffraction spreading can also be approximately characterized in terms of the uncertainty product $\Delta y \Delta p_y$ for the direction *y* transverse to the beam.