MNIST digits

May 7, 2019

Import statements:

```
In [1]: import pickle, gzip, numpy as np
        import matplotlib.pyplot as plt
        import matplotlib.cm as cm
        import math
        import random
```

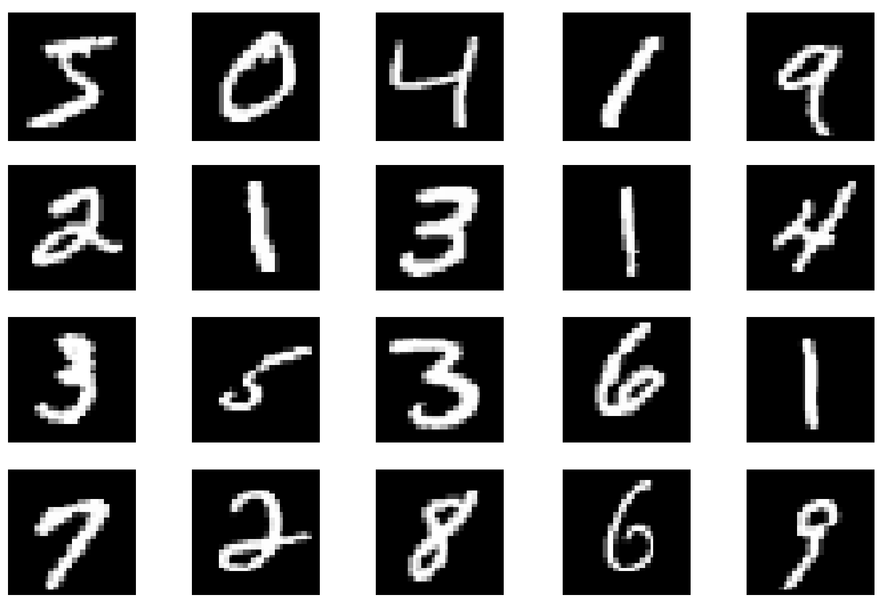
Importing the MNIST dataset:

```
In [2]: def plotImages(X):
            if X.ndim == 1:
                X = np.array([X])
            numImages = X.shape[0]
            numRows = math.floor(math.sqrt(numImages))
            numCols = math.ceil(numImages/numRows)
            for i in range(numImages):
                reshapedImage = X[i,:].reshape(28,28)
                plt.subplot(numRows, numCols, i+1)
                plt.imshow(reshapedImage, cmap = cm.Greys_r)
                plt.axis('off')
            plt.show()

        def readPickleData(fileName):
            f = gzip.open(fileName, 'rb')
            data = pickle.load(f, encoding='latin1')
            f.close()
            return data

        def getMNISTData():
            trainSet, validSet, testSet = readPickleData('mnist.pkl.gz')
            trainX, trainY = trainSet
            validX, validY = validSet
            trainX = np.vstack((trainX, validX))
            trainY = np.append(trainY, validY)
            testX, testY = testSet
            return (trainX, trainY, testX, testY)
```

```
def plotImagesHorizontal(X):
    if X.ndim == 1:
        X = np.array([X])
    numImages = X.shape[0]
    numRows = 1
    numCols = numImages
    for i in range(numImages):
        reshapedImage = X[i,:].reshape(28,28)
        plt.subplot(numRows, numCols, i+1)
        plt.imshow(reshapedImage, cmap = cm.Greys_r)
        plt.axis('off')
    plt.show()

(trainX, trainY, testX, testY) = getMNISTData()
```

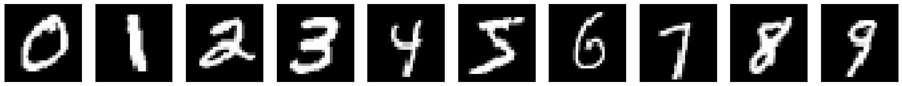
We print out the first 20 digits of the MNIST training dataset to get a feel for the dataset:

```
In [3]: plotImages(trainX[0:20,:])
```



For our first proof of concept, we will store one of each of the 9 digits in associative memory:

```
In [106]: digits = [1, 6, 5, 7, 26, 0, 18, 42, 17, 43]

          plotImagesHorizontal(np.array([trainX[i] for i in digits]))
```



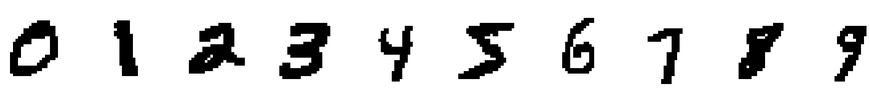
Next, we generate the coupling coefficients according to the following equation:

$$s_{ij} = \frac{1}{N}\sum_{m=1}^{M}\sigma_i^m\sigma_j^m \tag{1}$$

In [142]:
```
N = 784 # the mnist digits are 28 by 28 pixels and 28^2 = 784
s = np.zeros(shape=(784,784))

# converts digits to 1s and -1s
return_abs = lambda x: -1 if x > 0 else 1

# grabs the absolute values for each item in the array for mnist digit
get_abs = lambda x: np.array([return_abs(i) for i in trainX[x]])

# stores one of each digit
data = np.array([get_abs(i) for i in digits])

# grabs the coupling coefficients
for i in range(784):
    for j in range(784):
        total = 0
        for k in range(len(data)):
            total += data[k][i] * data[k][j]
        s[i][j] = 1/784 * total
```

We plot the images to see what they look like after processing:

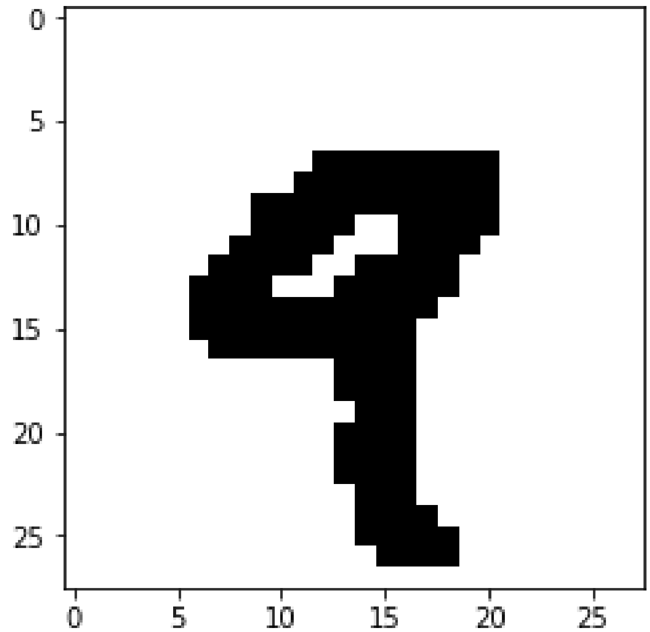
In [143]:
```
plotImagesHorizontal(data)
```



We run the first digit in the test set and see what item in associative memory it matches with from the 9 digits stored from the training set:

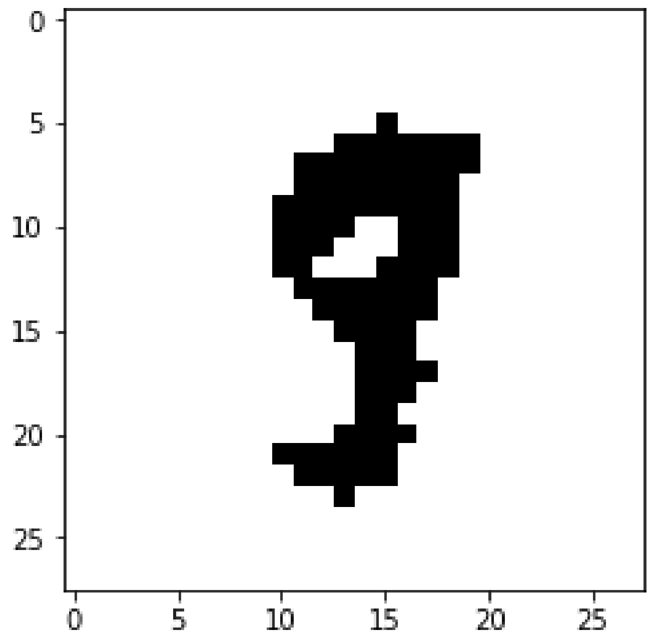
In [144]:
```
test = np.array([return_abs(i) for i in trainX[4]])
test_resized = np.array(test).reshape(28, 28)
plt.imshow(test_resized, cmap = cm.Greys_r)
```

Out[144]: <matplotlib.image.AxesImage at 0x7f41ba78b438>



In [145]:
```
V = test # this is the initial state (where the tested image goes)
new_V = np.ones(784)

for i in range(784):
    total_sV = 0
    for j in range(784):
        total_sV += s[i][j] * V[j]

    if total_sV >= 0:
        new_V[i] = 1
    else:
        new_V[i] = -1

V = new_V
new = np.array(V).reshape(28, 28)
plt.imshow(new, cmap = cm.Greys_r)
```

Out[145]: <matplotlib.image.AxesImage at 0x7f41ba6f1be0>



Here, we can see that the associative memory returned a 9 when the a 9 input digit was inserted.