Time, Clocks, and the Ordering of Events in a Distributed System

Leslie Lamport

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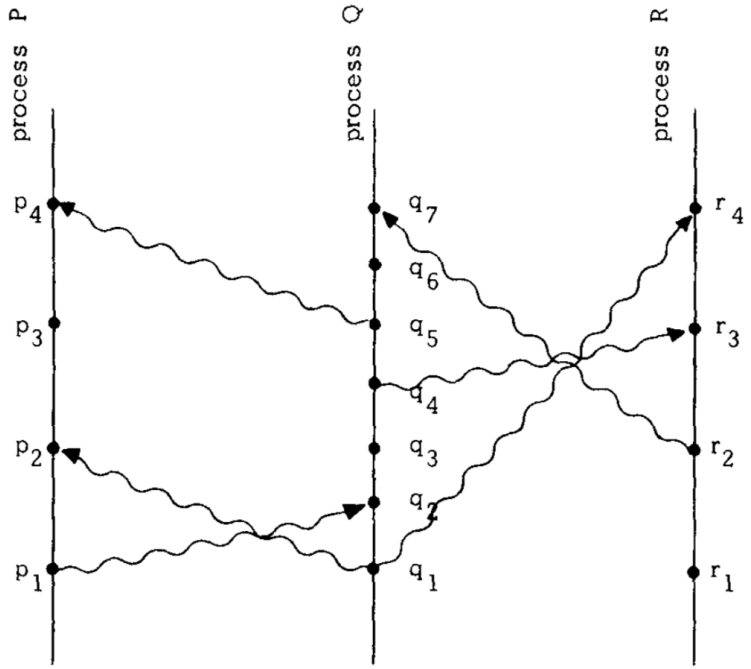
April 23, 2020

Outline

Events in Distributed System

What does it mean by "*a* happens before *b*" in distributed system?



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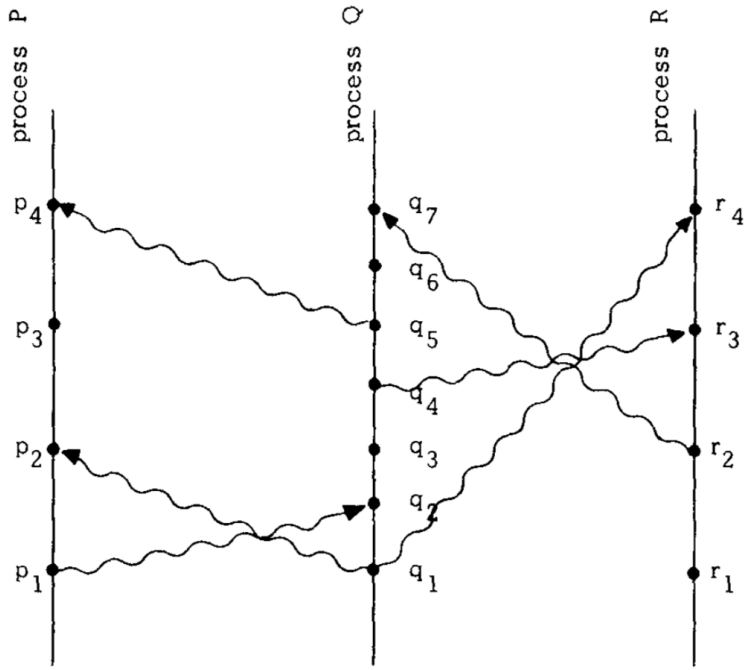

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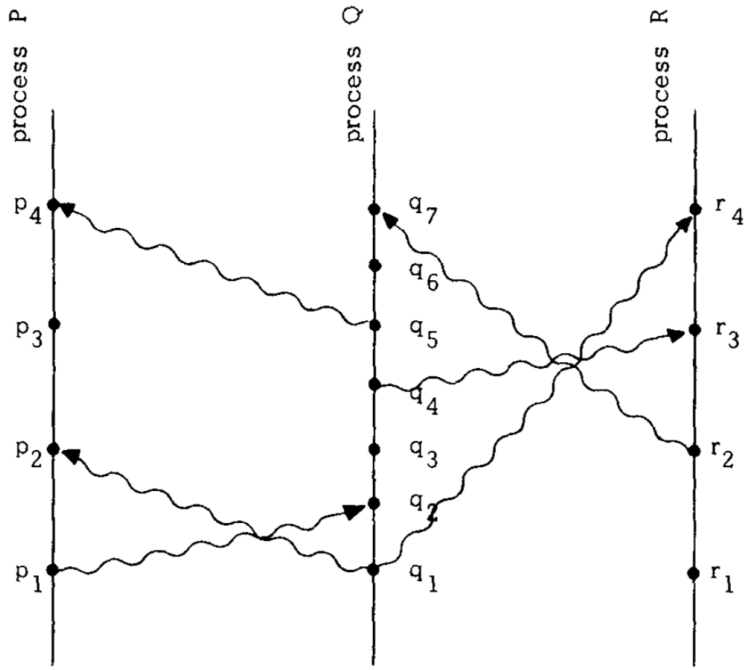

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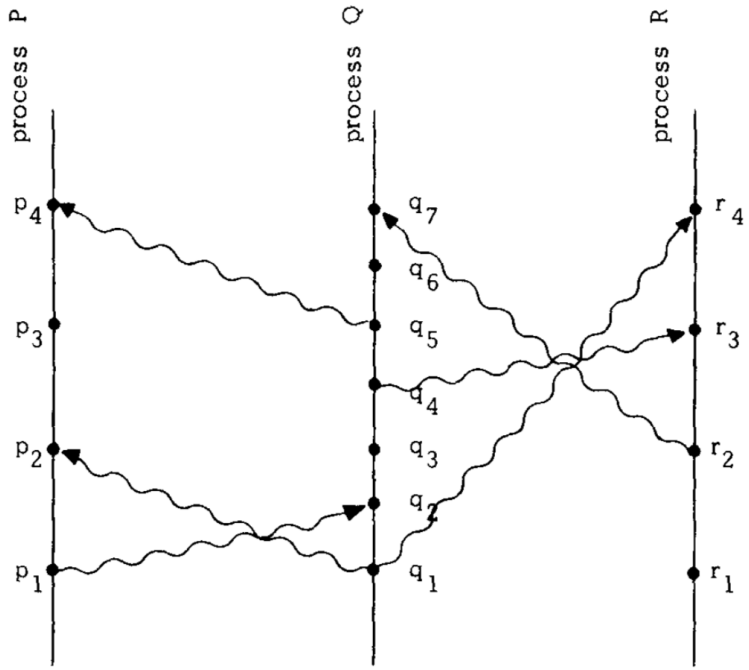

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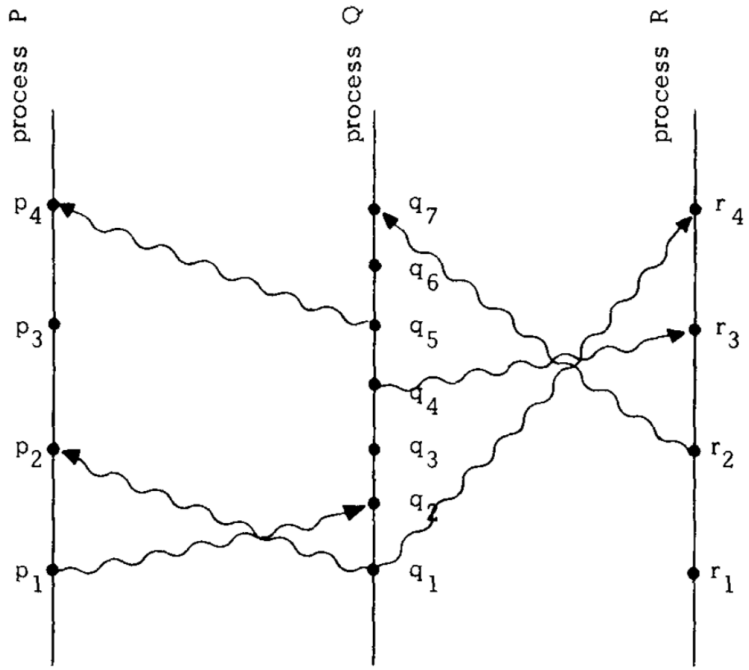
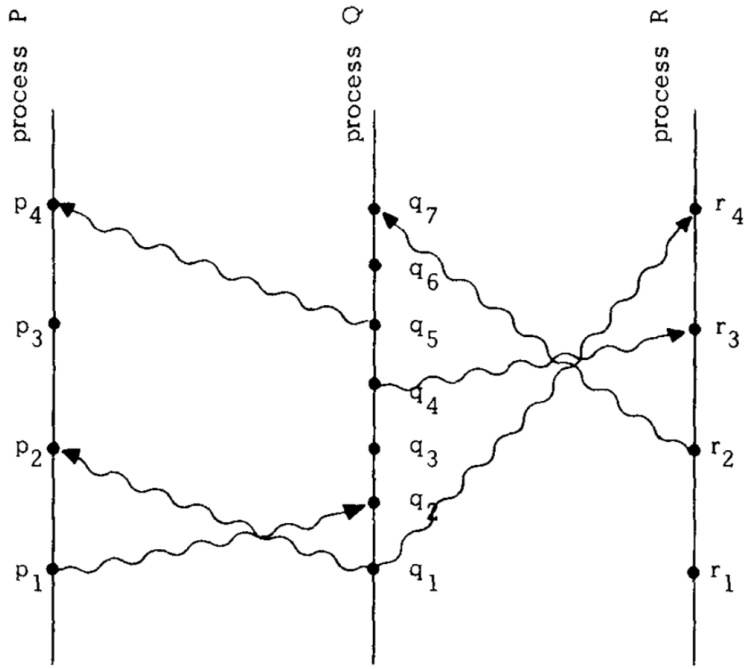
- If *a* happens before *b*, and *b* happens before *c*, then *a* happens before *c*



Partial Ordering of Events

Denote by $a \rightarrow b$ if a happens before b.



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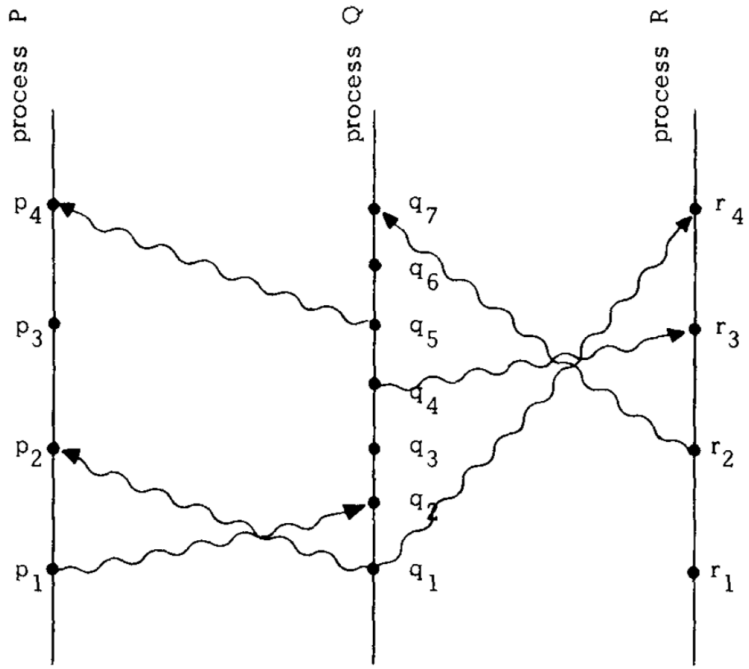

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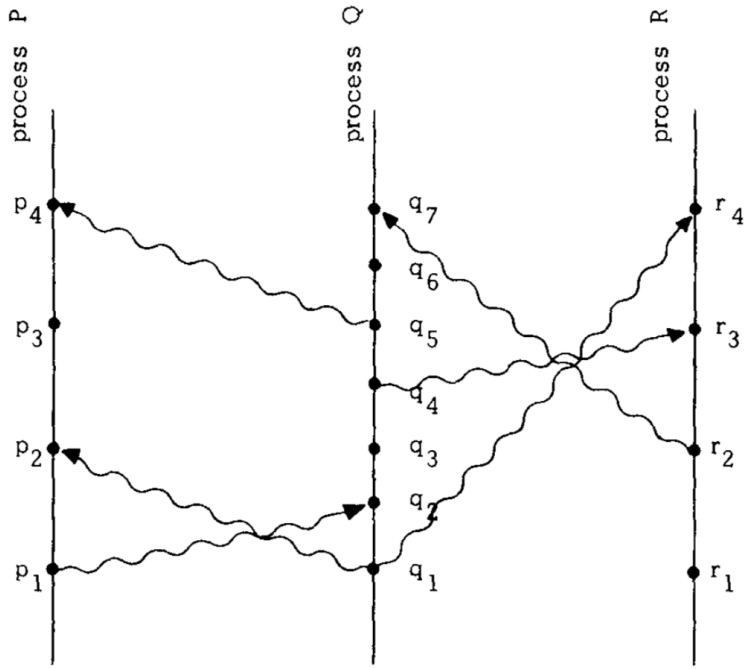

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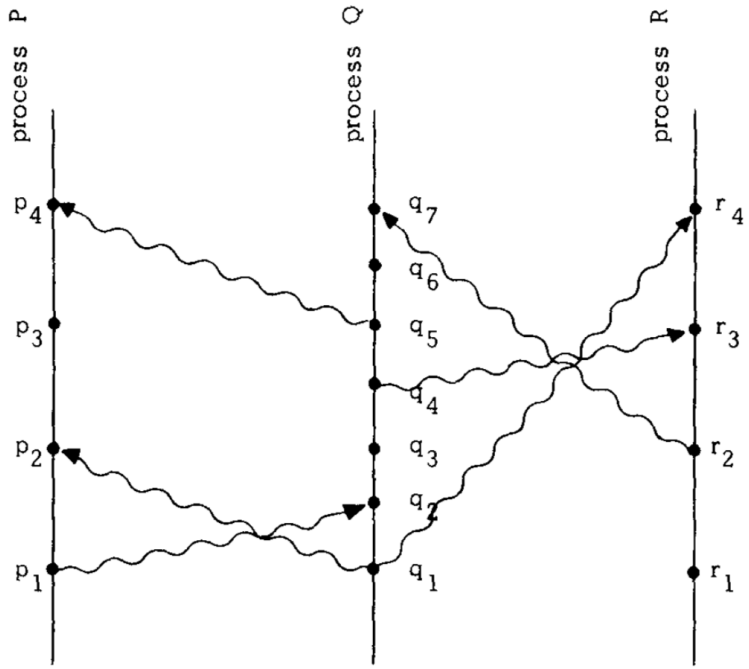
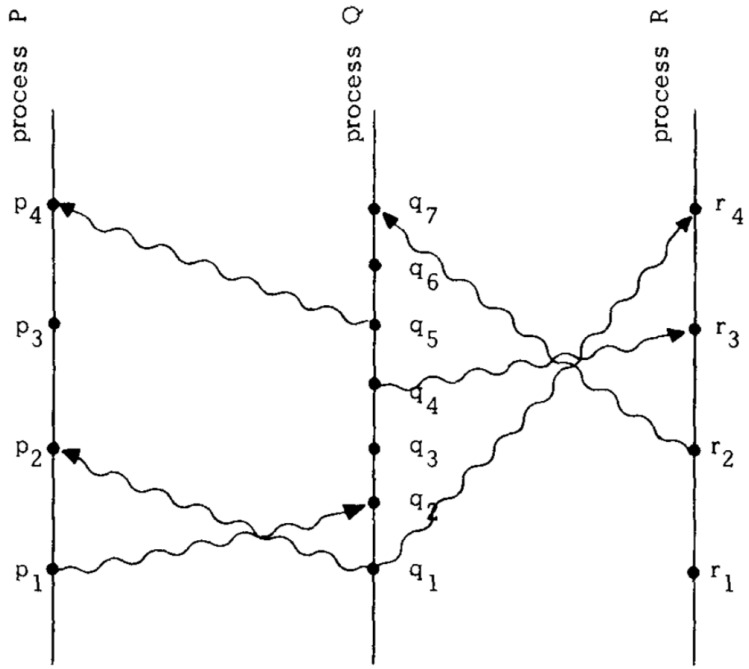
- $a \rightarrow b$ is equivalent to saying one can go from a to b in the diagram by moving forward in time along process and message lines.



Partial Ordering of Events

Example: $p_1 \rightarrow r_4$



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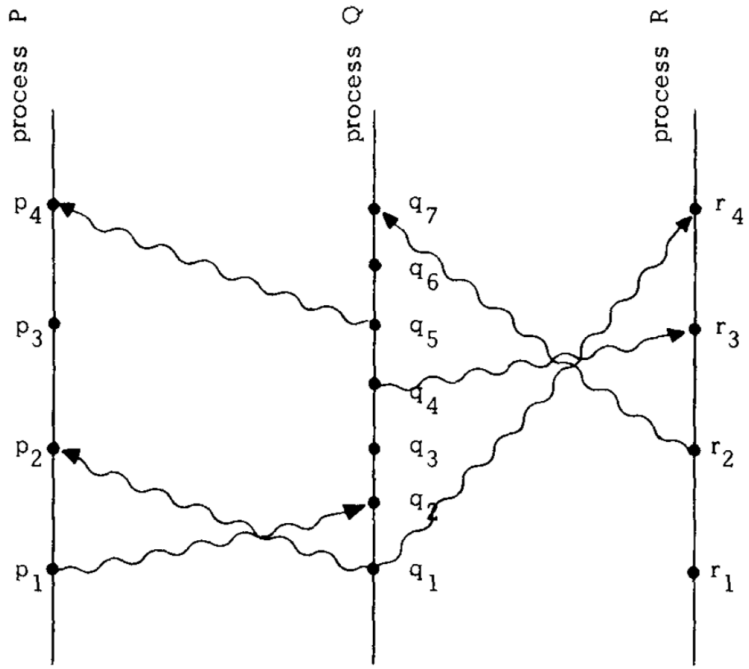

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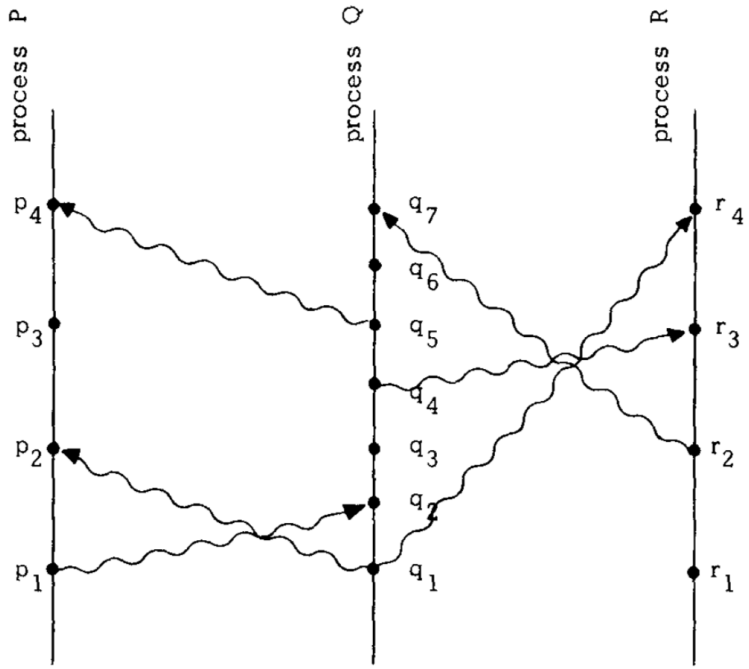

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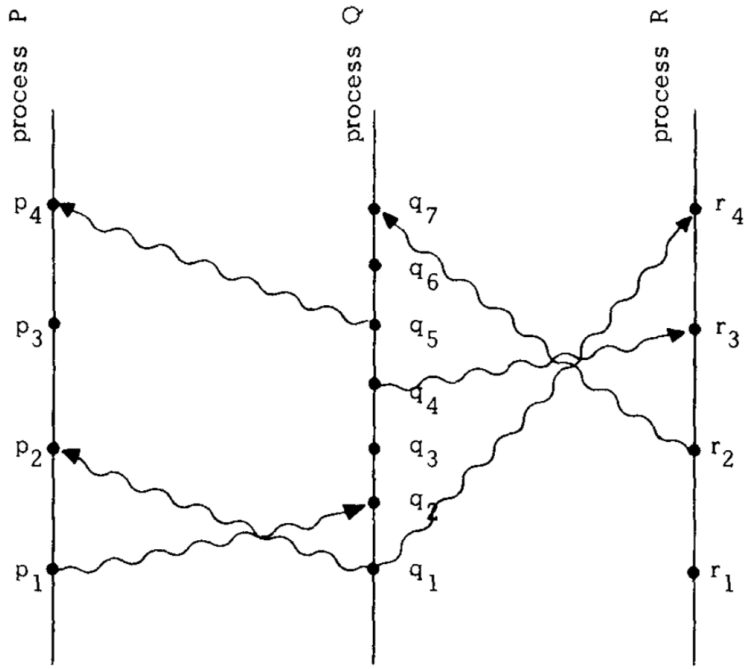
- $p_1 \rightarrow q_2$
- $q_2 \rightarrow q_4$
- $q_4 \rightarrow r_3$



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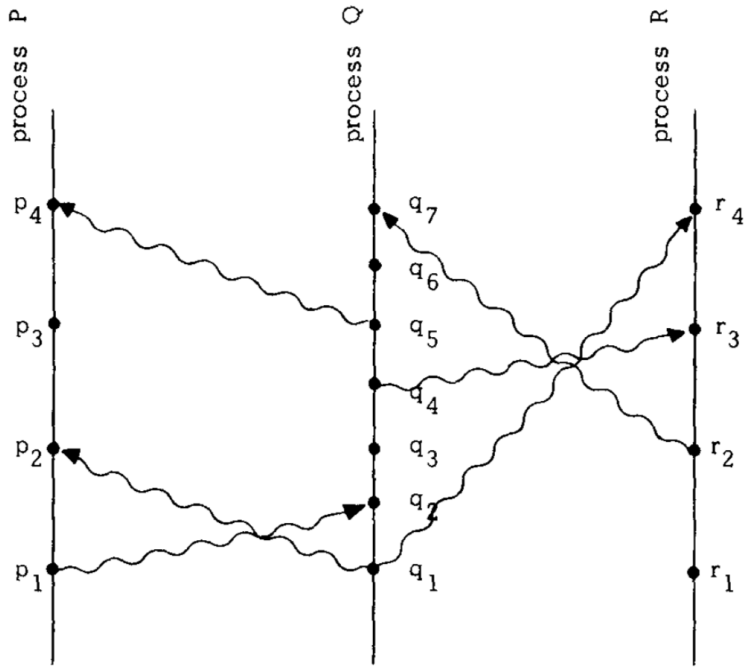
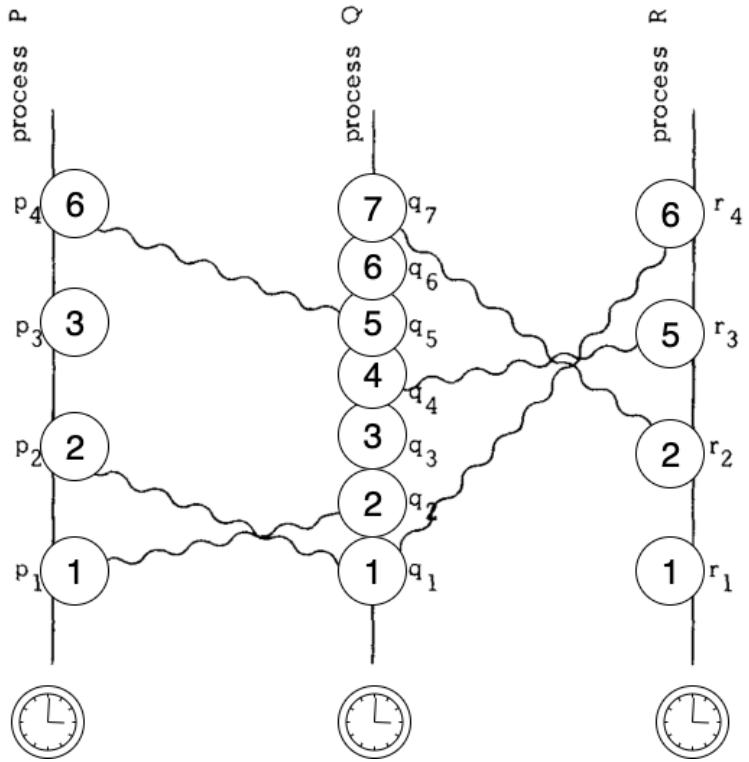


Logical Clocks

Logical clock is an assignment of numbers on events



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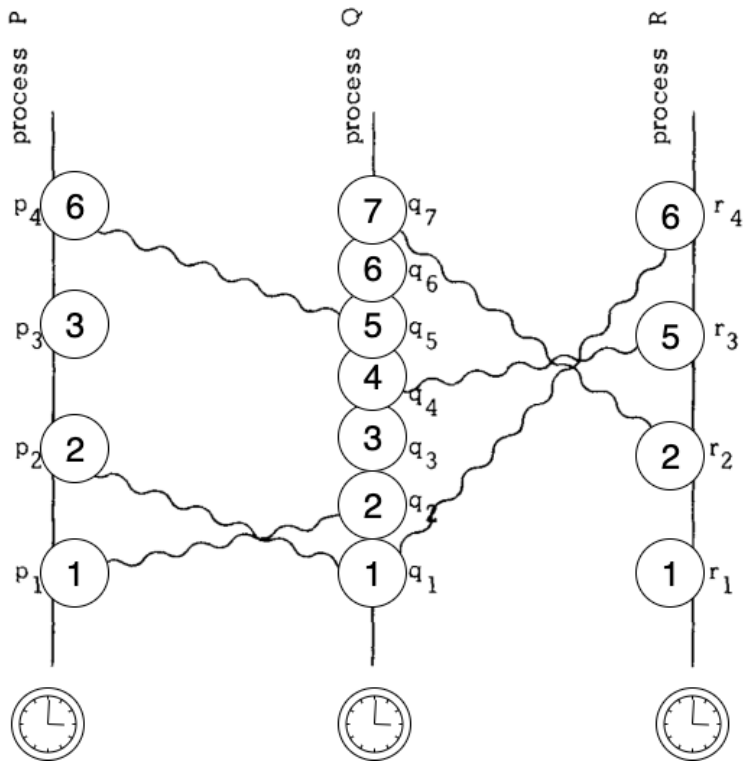
- Local clock C_i assigns number $C_i\langle a \rangle$ to event a in process P_i



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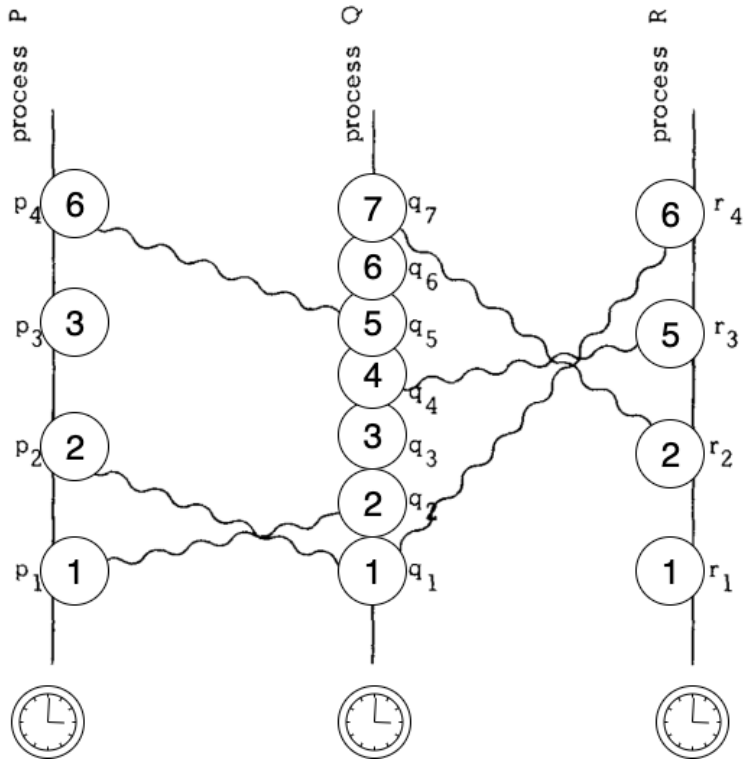
- Local clock C_i assigns number $C_i\langle a\rangle$ to event a in process P_i
- Global clock C defined by $C\langle a\rangle = C_i\langle a\rangle$ if a is in process P_i



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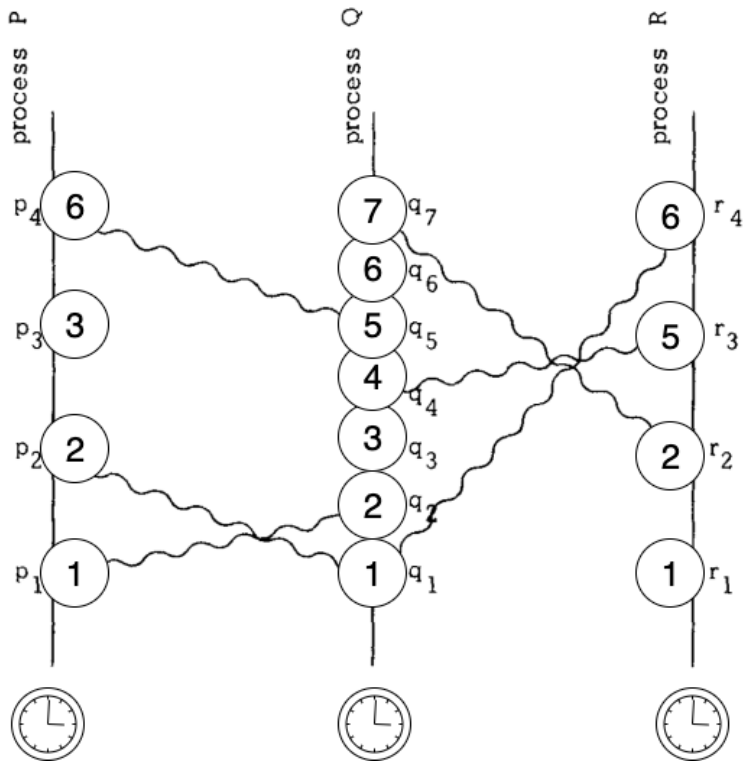
Clock Condition

For any events a, b:
if $a \rightarrow b$ then $C\langle a \rangle < C\langle b \rangle$



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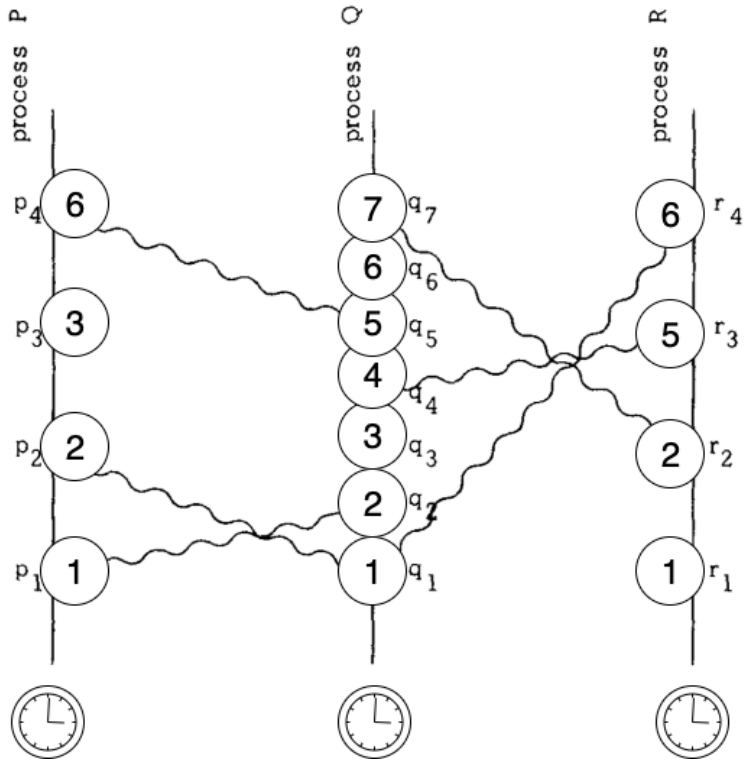
Remark

The converse is not required:
$C\langle a \rangle < C\langle b \rangle$ does not imply $a \rightarrow b$
Because that would require concurrent events to have equal clock values.



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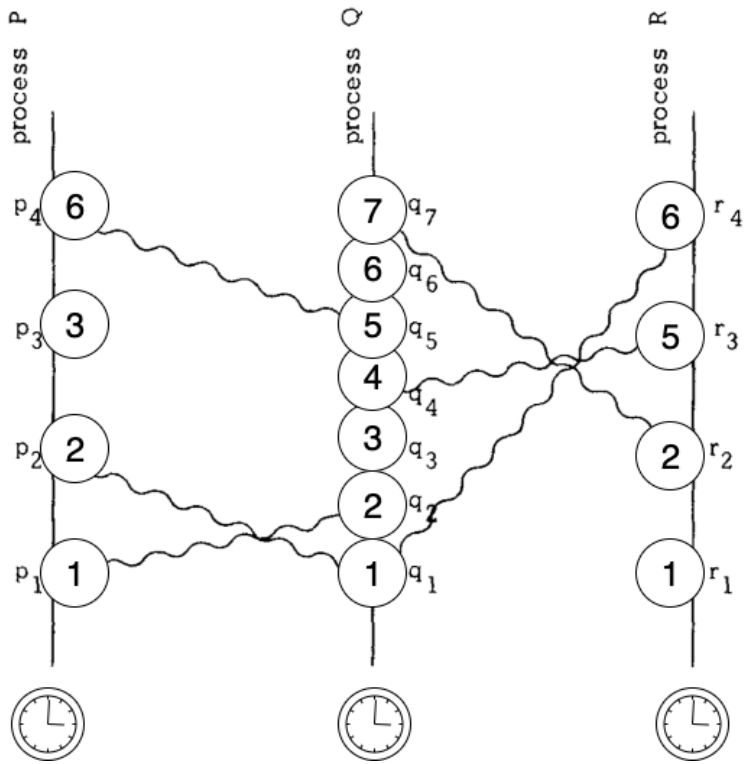
Implement the logical clock:



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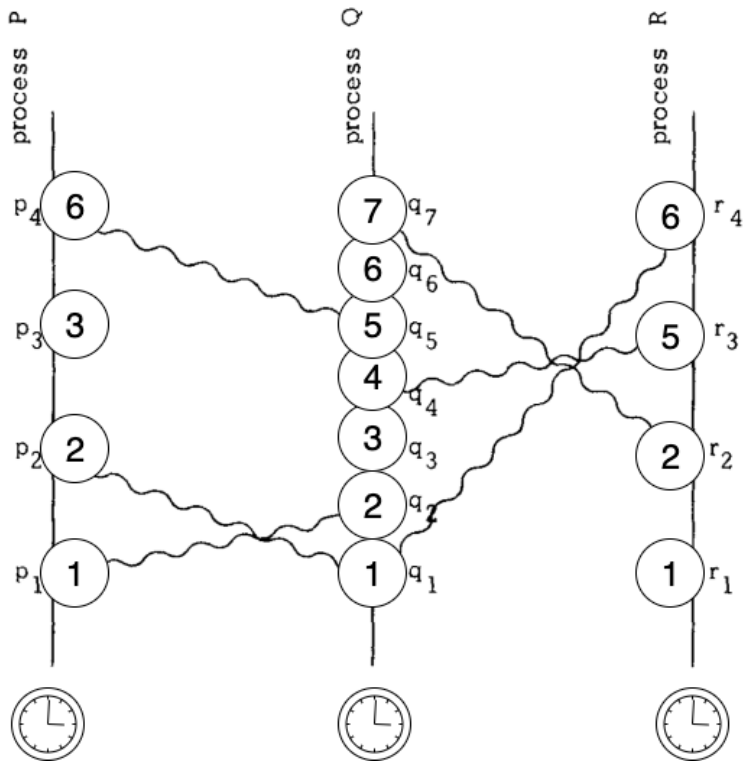

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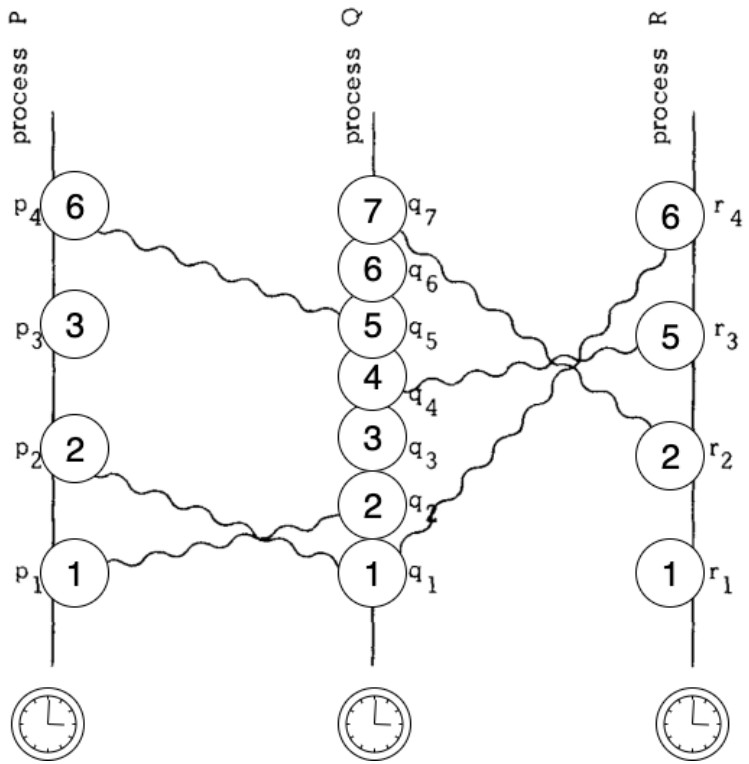

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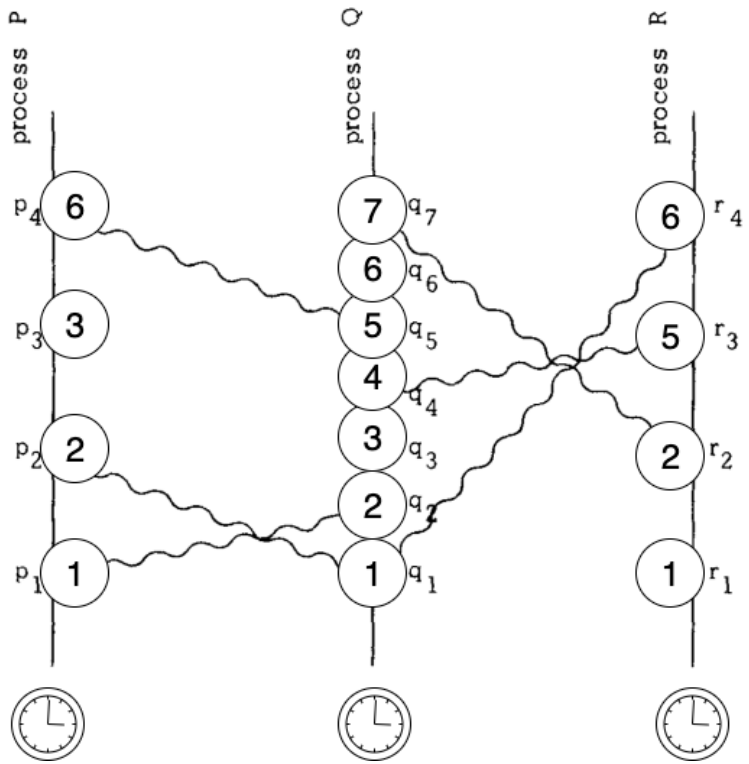

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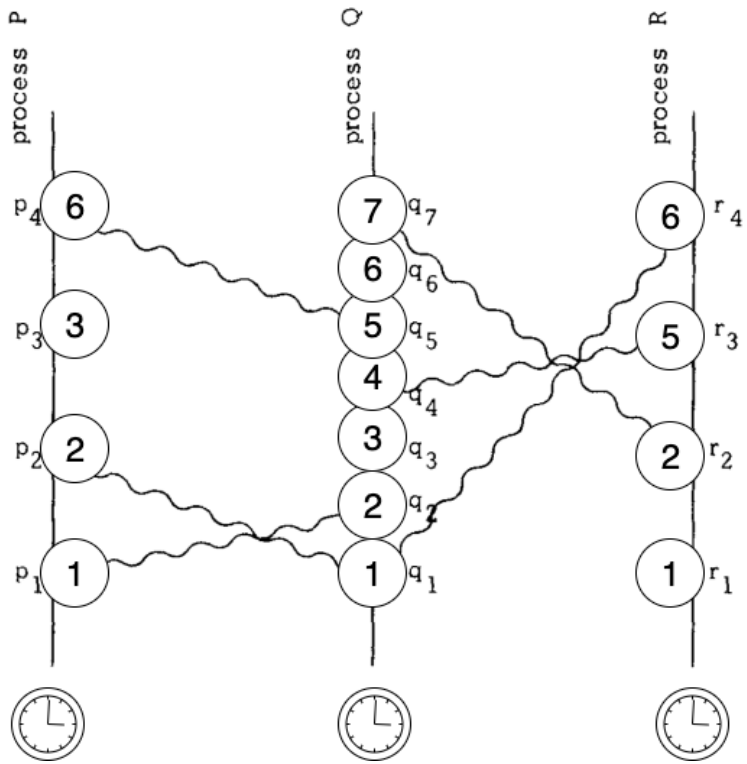
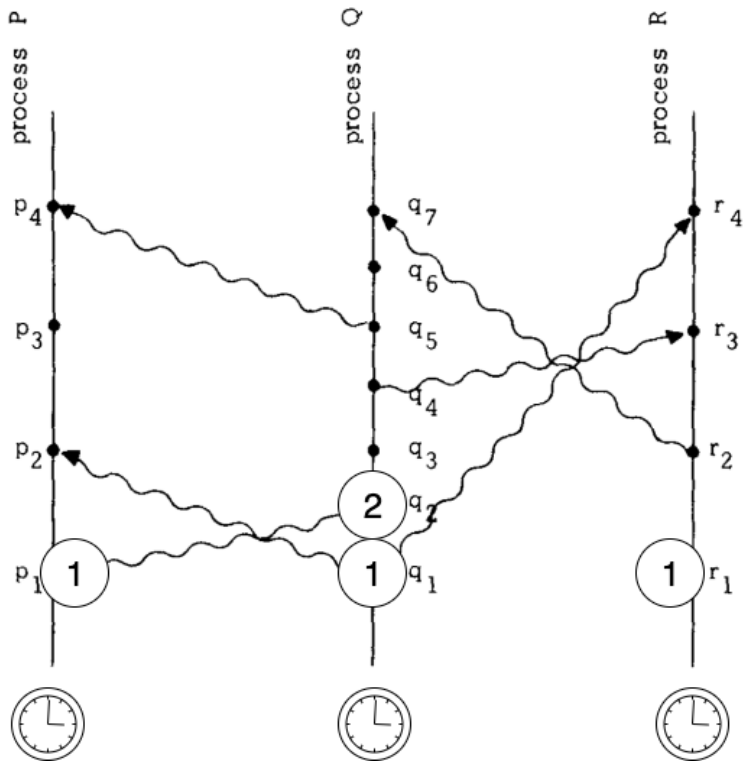
- On receiving message m, P_j sets C_j to be greater than both T_m and current clock value



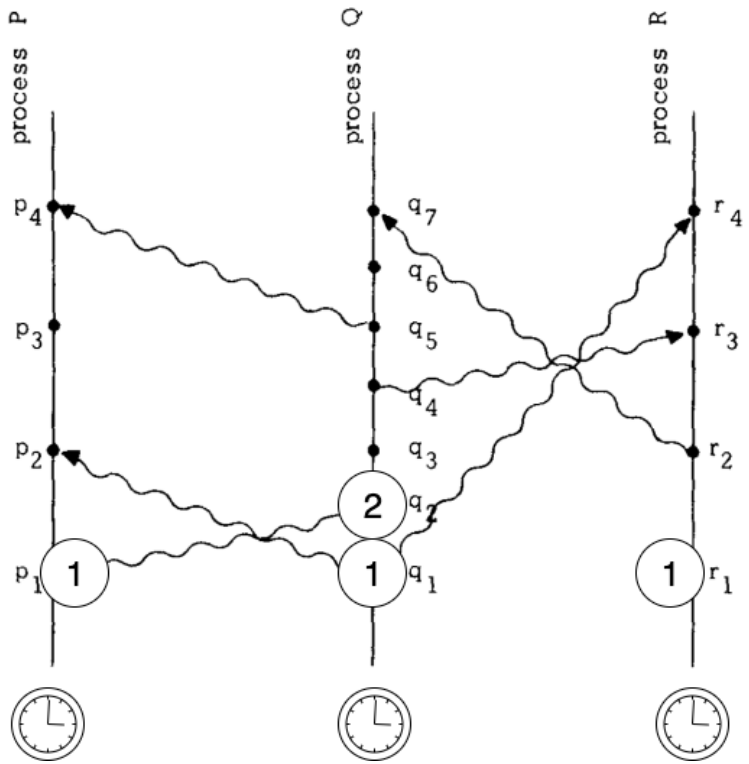
Logical Clocks

Example



Logical Clocks

Process Q receives message p_1, updates clock to 2



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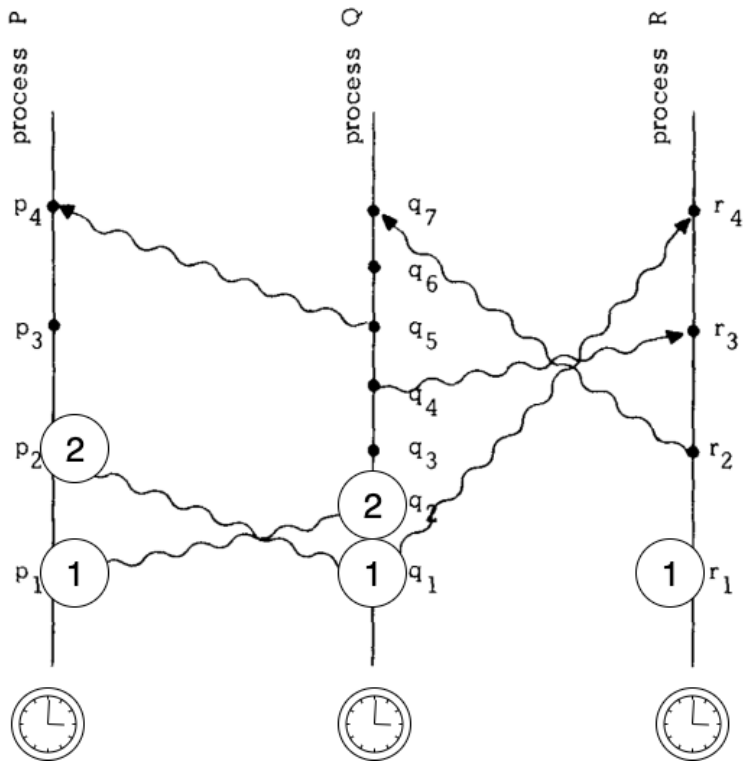
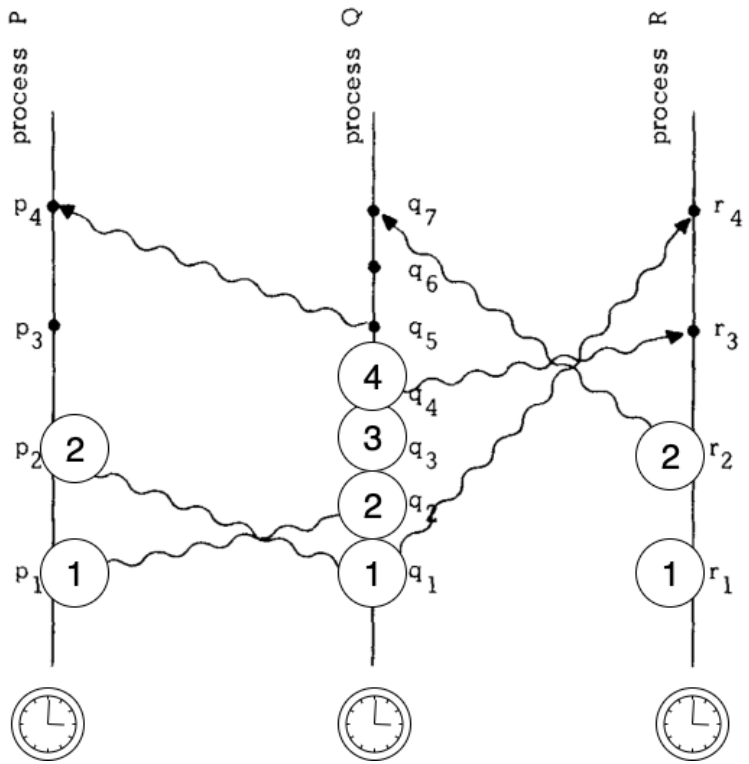
Process P receives message q_1, updates clock to 2



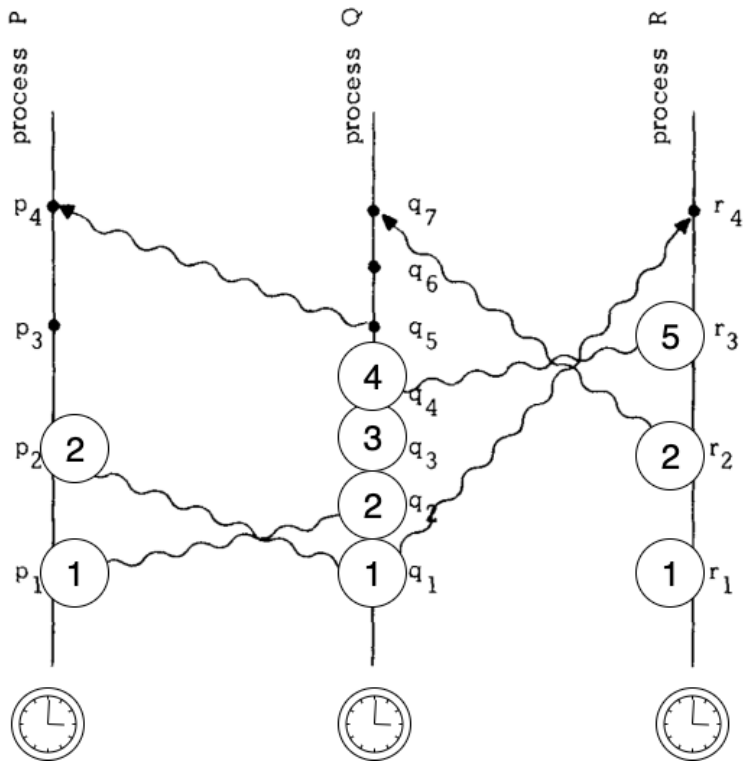
Logical Clocks

Proceeds until Q sends a message to R at event q_4 with timestamp 4



Logical Clocks

Process R receives the message with timestamp 4, and updates clock to 5



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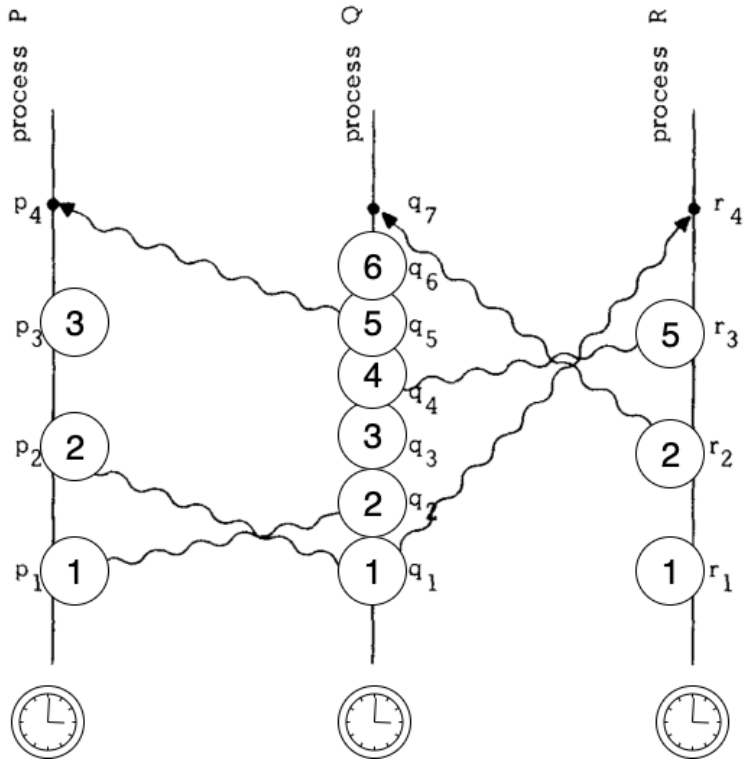
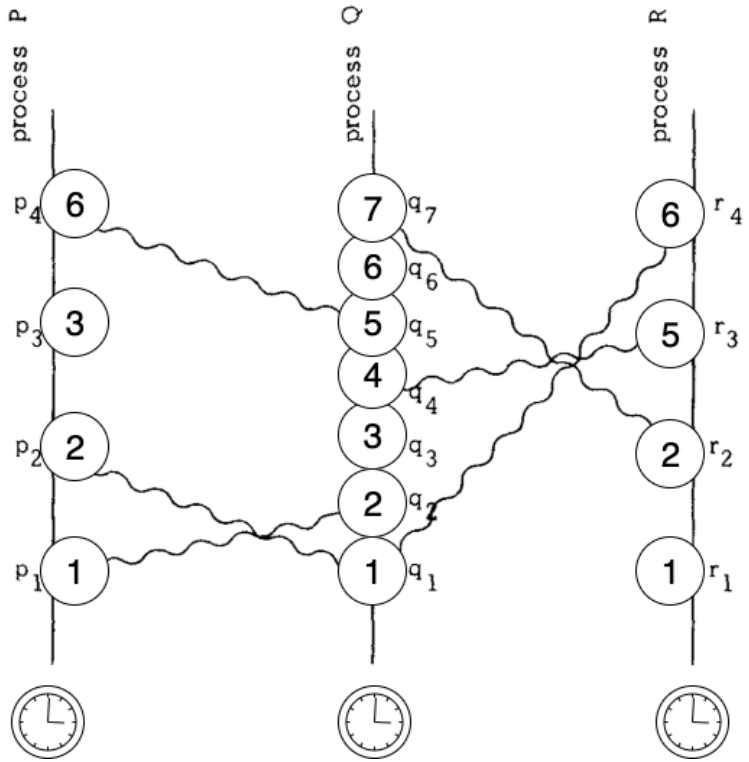
Process Q sends message to P with timestamp 5



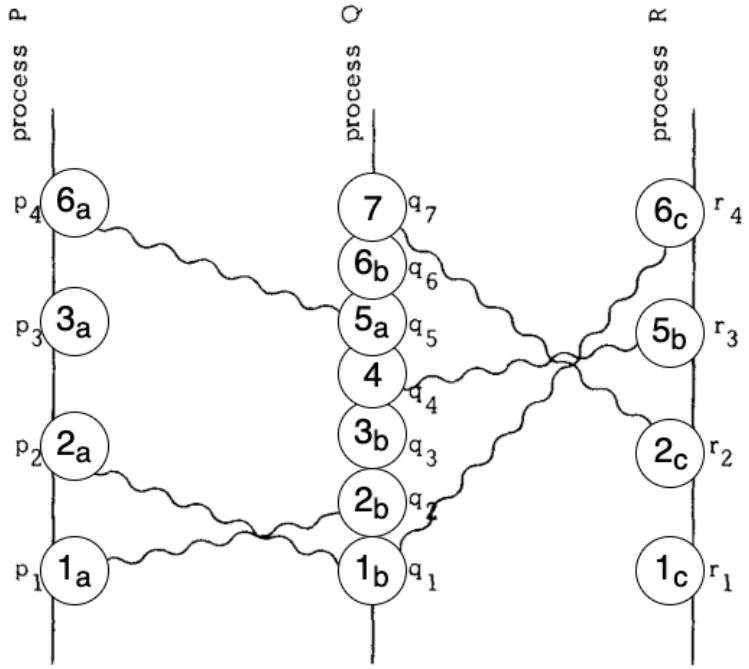
Logical Clocks

Process P updates clock on receiving message with timestamp 5. Clocks of processes Q and R are not affected by messages.



Total Ordering

With the logical clock, we are ready to define a total order "$\Rightarrow$" for all events.



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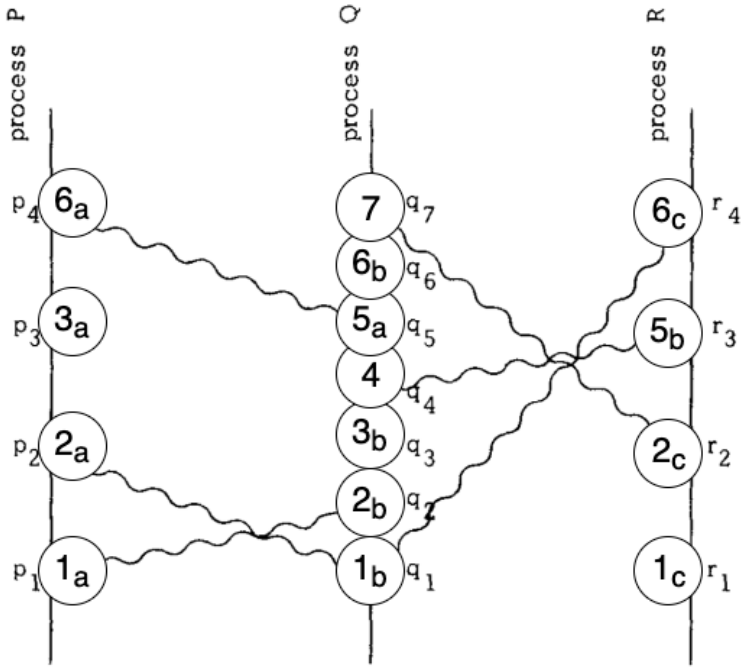

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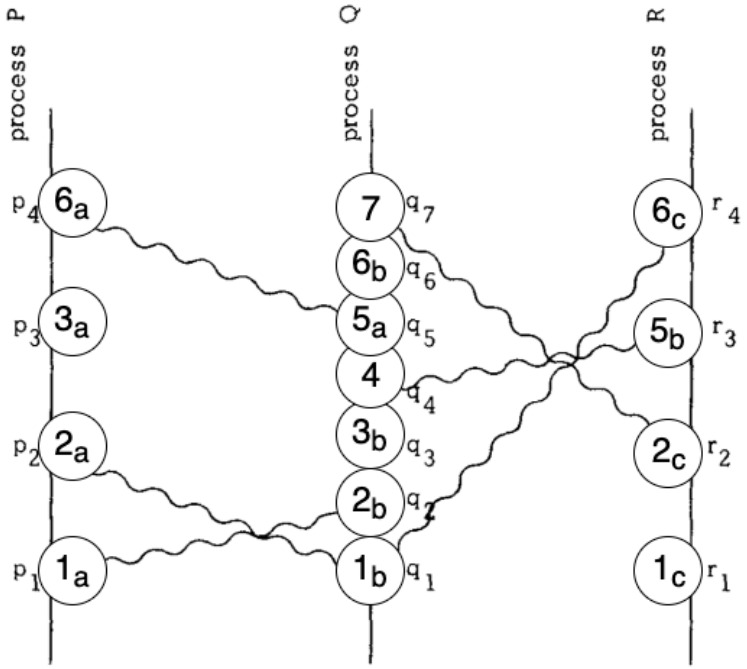

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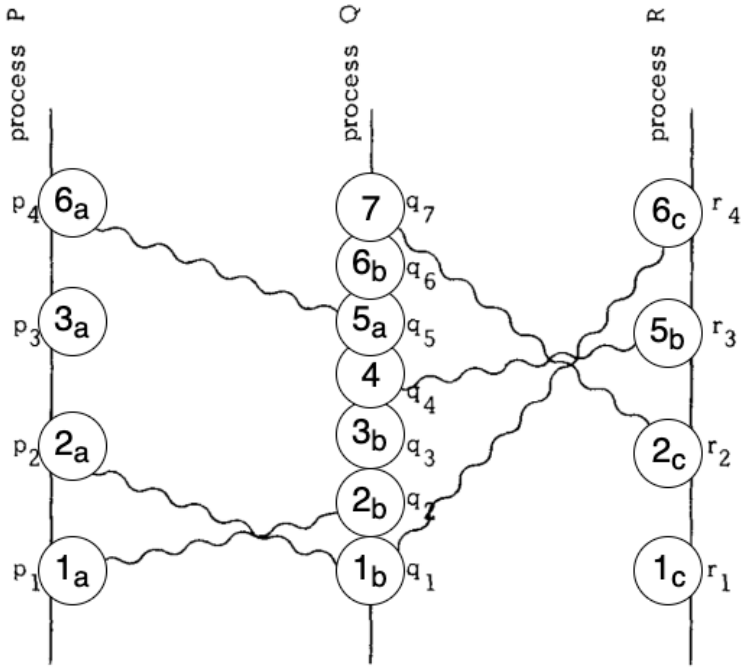
- Formally, for events a in P_i and b in P_j, $a \Rightarrow b$ if and only if either
  - $C_i\langle a\rangle < C_j\langle b\rangle$ or;
  - $C_i\langle a\rangle = C_j\langle b\rangle$ and $P_i \prec P_j$



Mutual Exclusion

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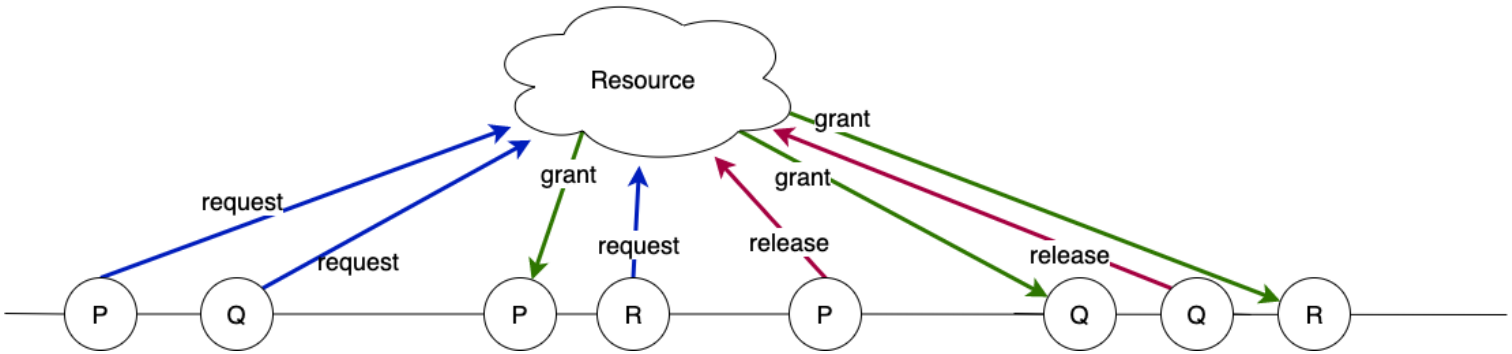

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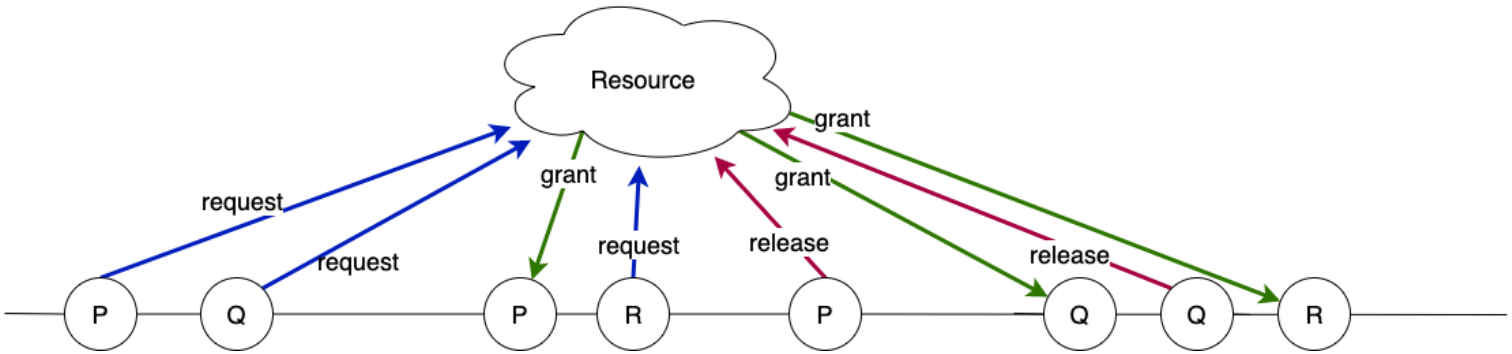

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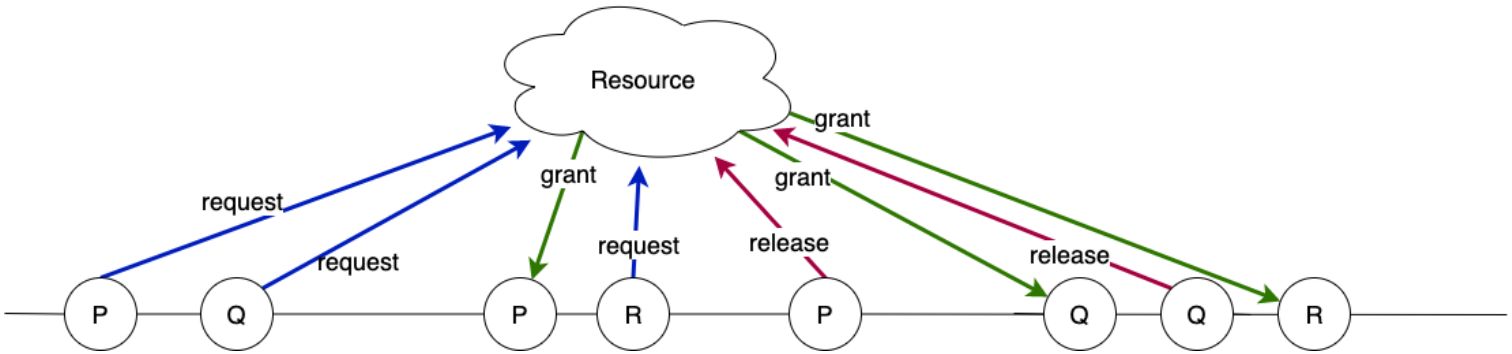

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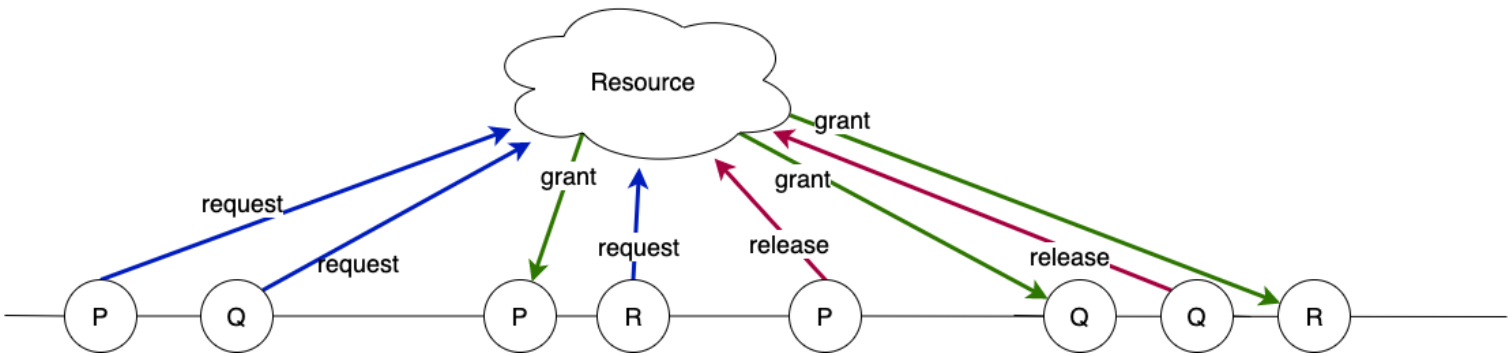

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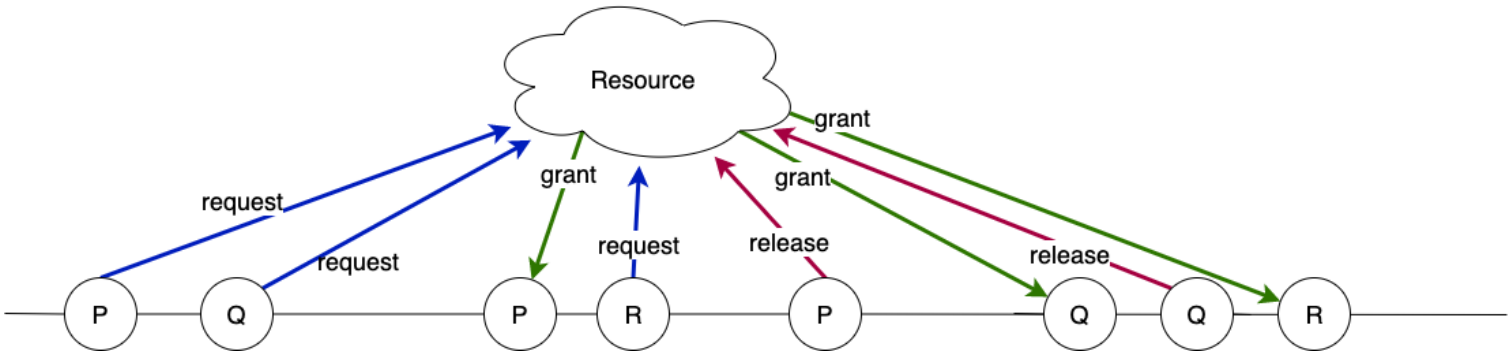

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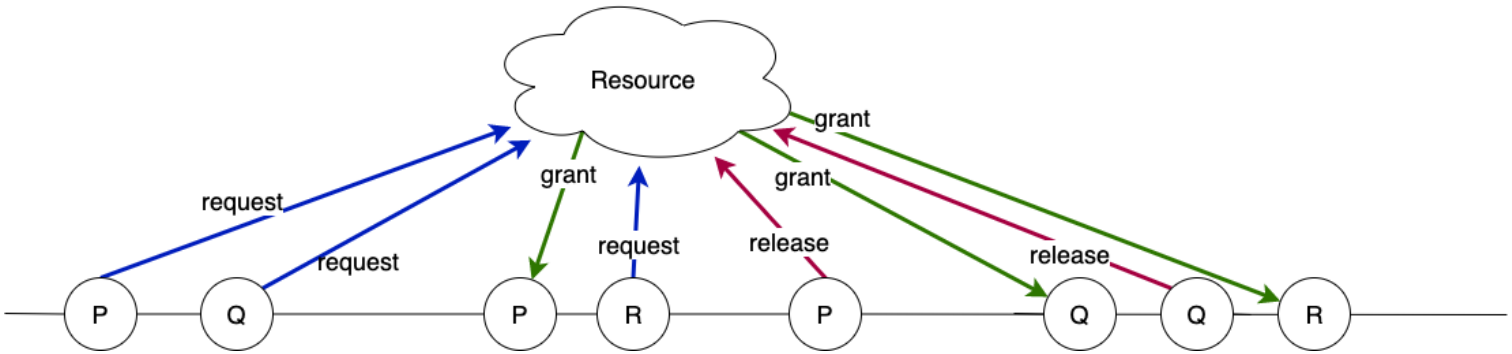

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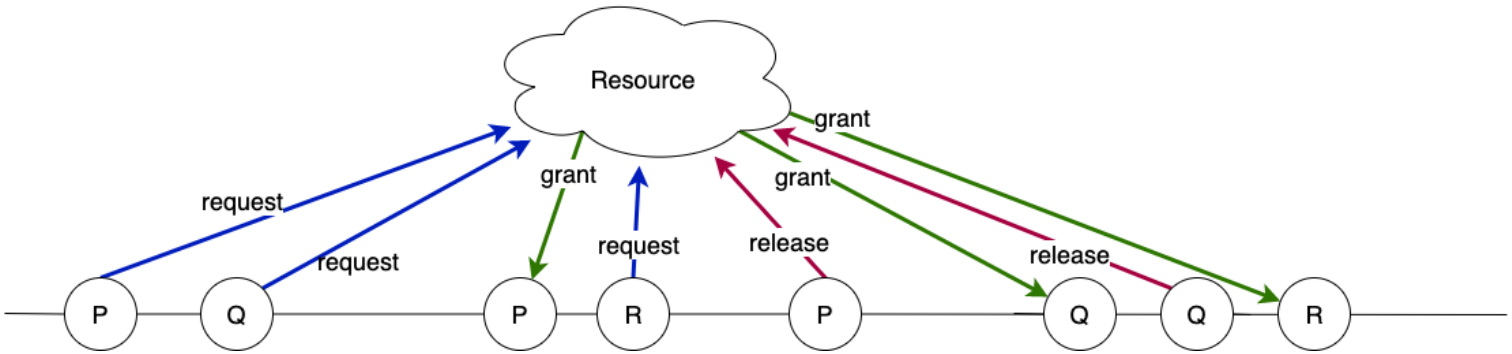
- **(II) In Order**: Earlier requests granted ealier
- **(III) Accessibility**: If every granted request eventually releases, then every request is eventually granted.



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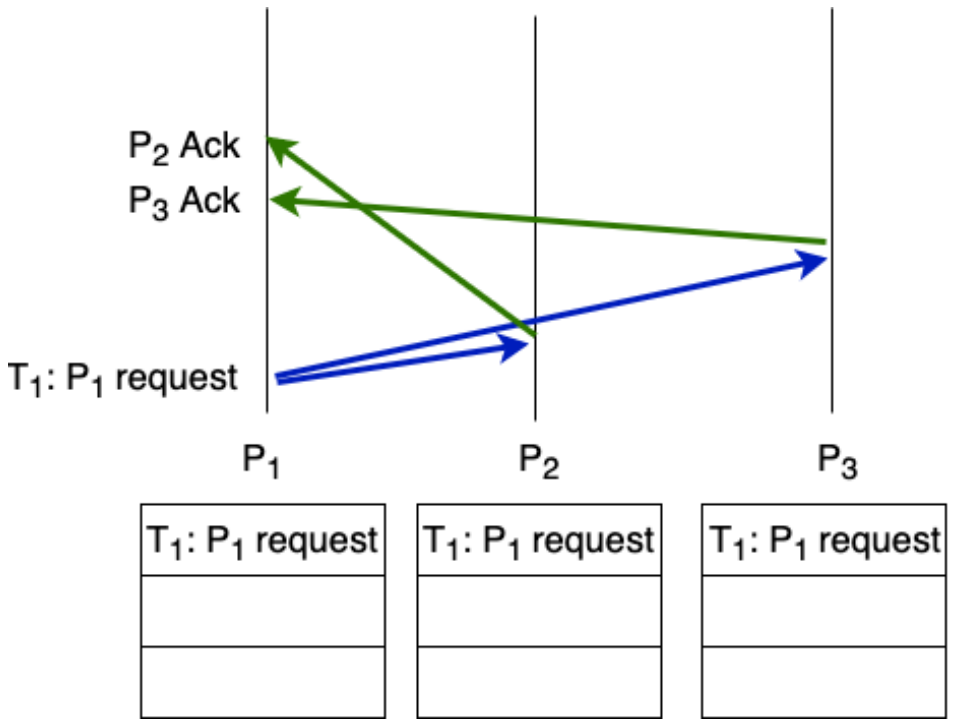
- Every message is eventually received
- A process can send messages directly to every other process

Mutual Exclusion



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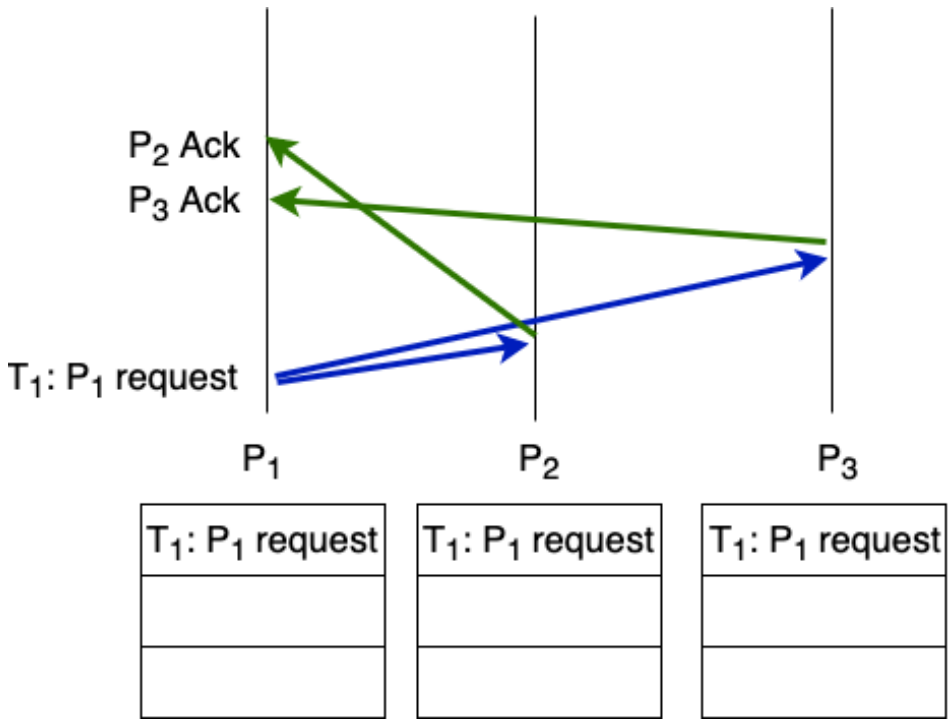
Each process maintains its own *request queue*.



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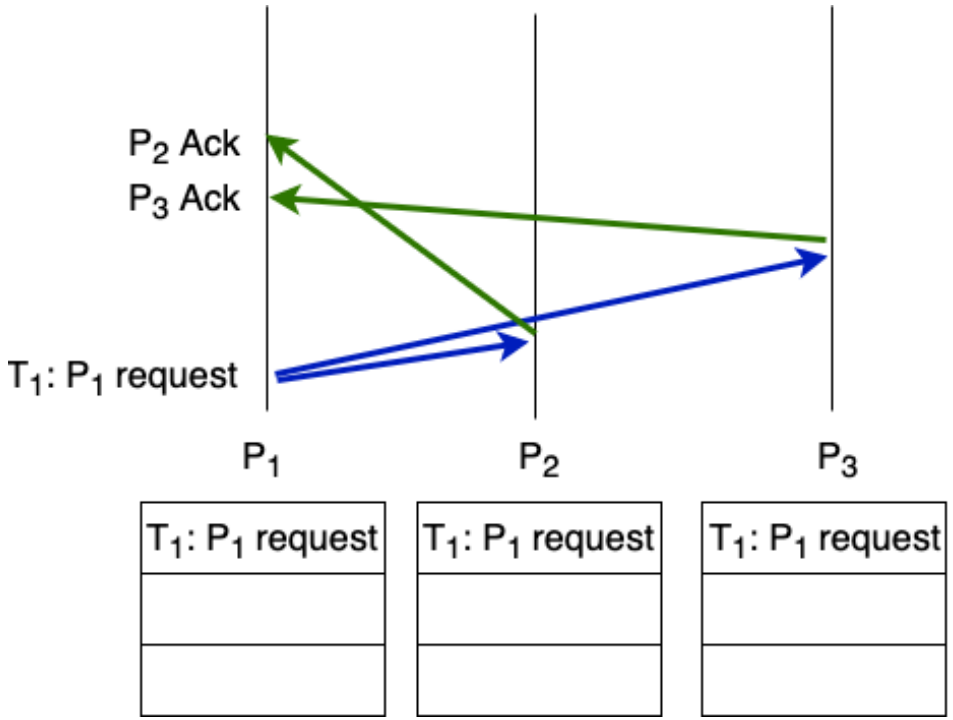
- If P_i wants the resource, it sends a message $m = \langle T_m : P_i \text{ requests resource} \rangle$ to every other process, and puts m on its request queue



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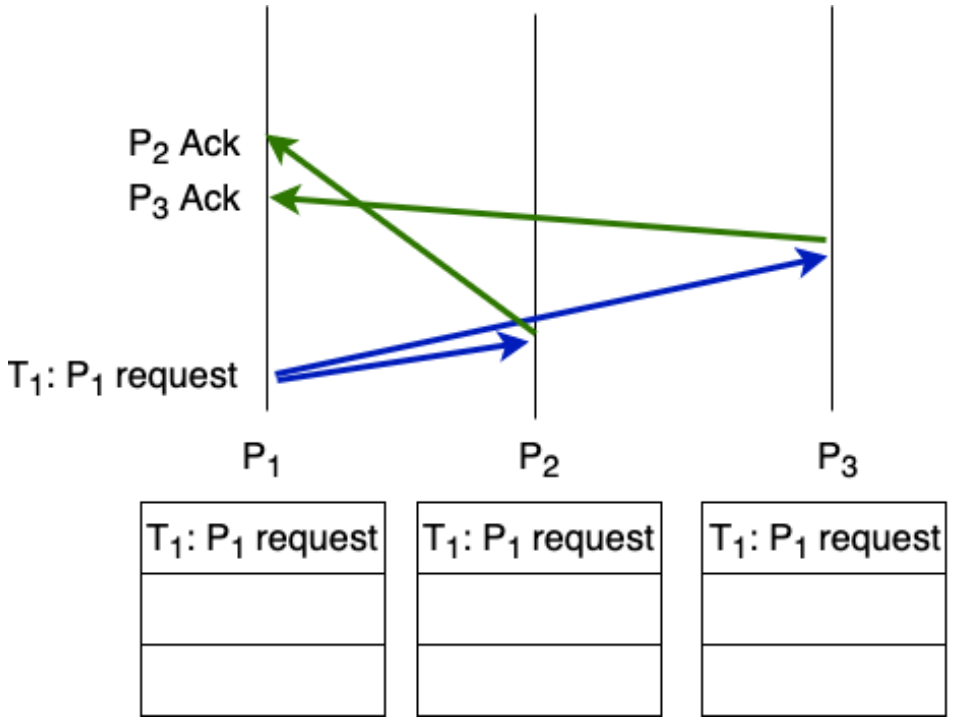

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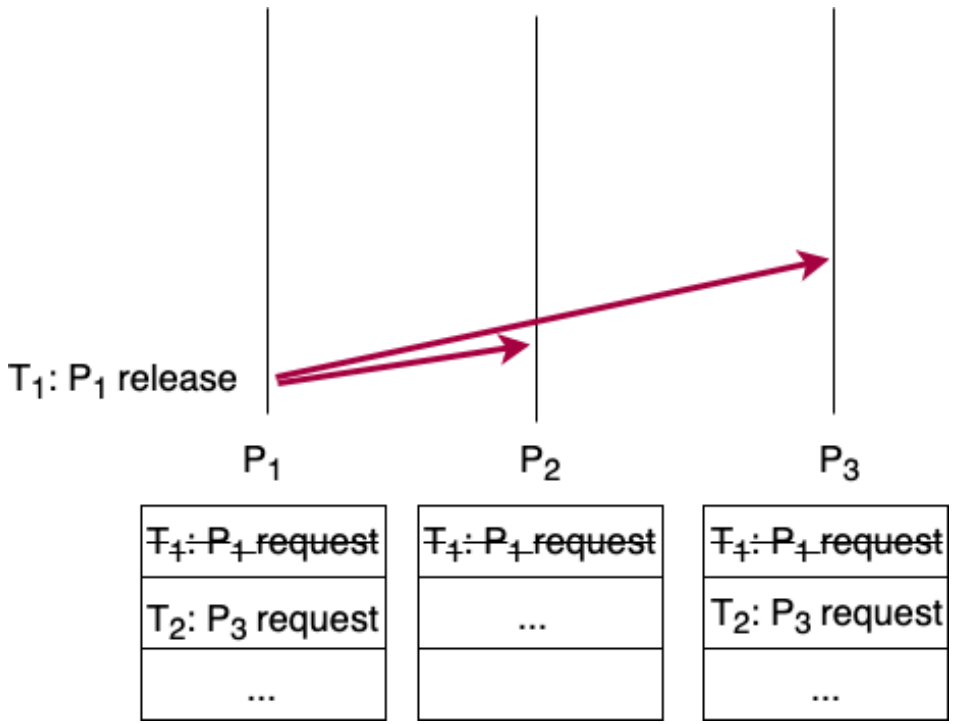

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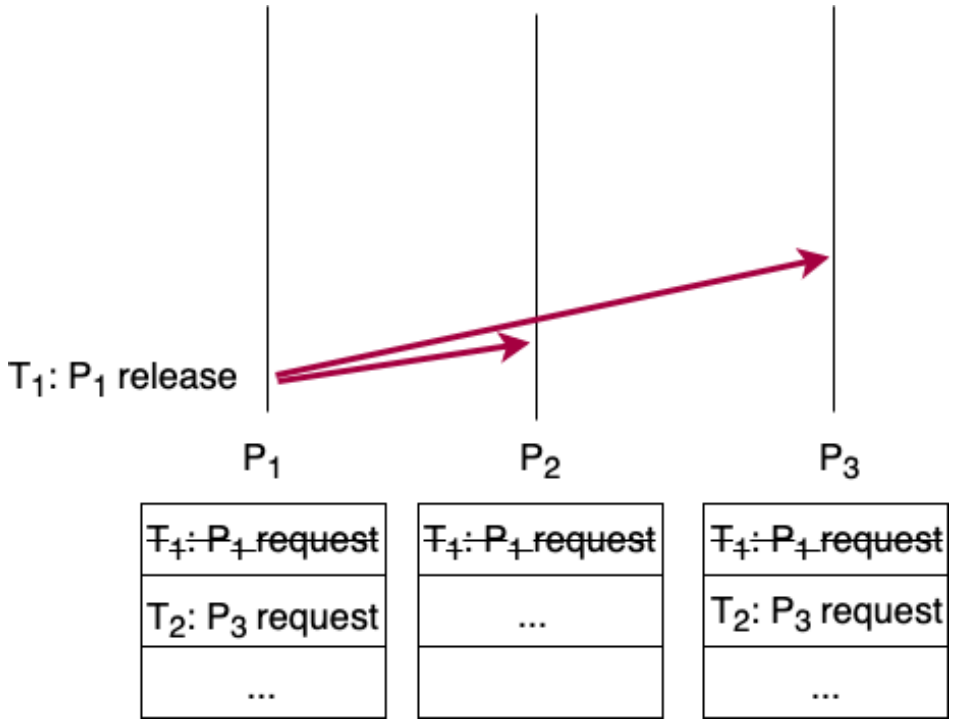

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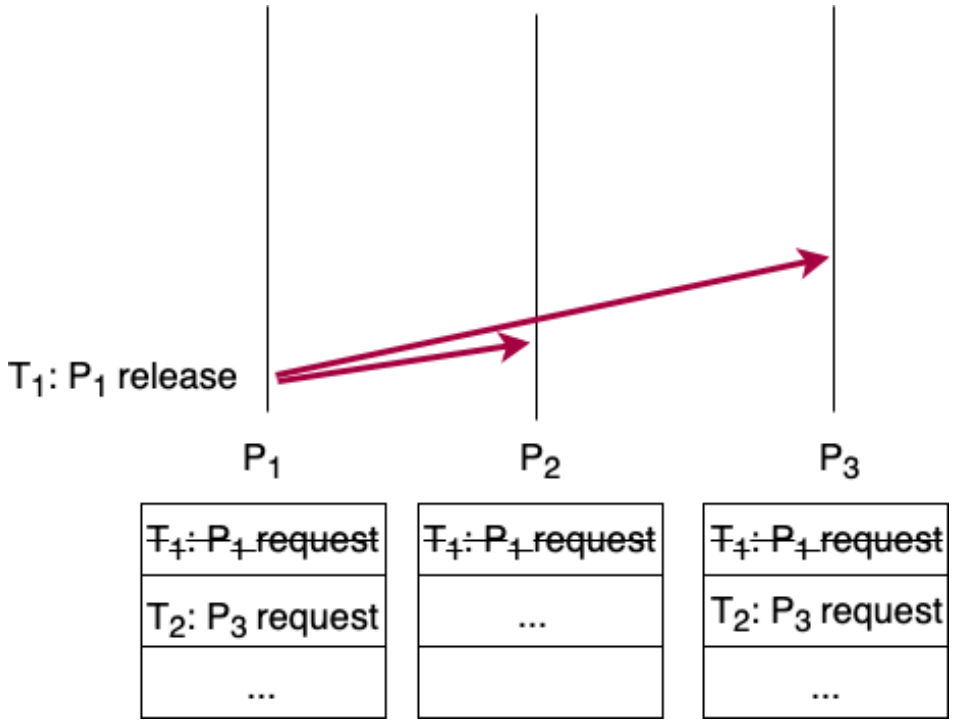

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- **Condition III:** For each message $m = \langle T_m : P_i \textit{ requests resource} \rangle$:
  - m will be the oldest in P_i's queue eventually
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- Cryptographic mechanisms for trustlessness?
- Shared resource can be: leadership, write permission, ...

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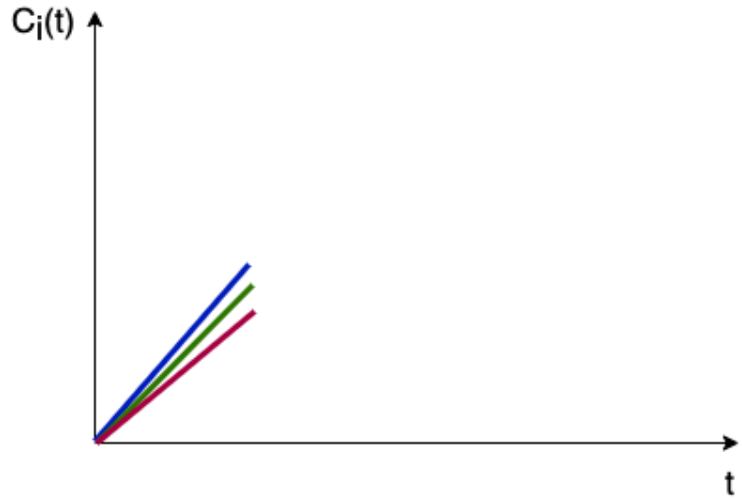
  - Frequencies of synchronization messages sent to each other
  - Accuracy of the local clocks

Physical Clock

The clocks run at different rates and start deviating from each other



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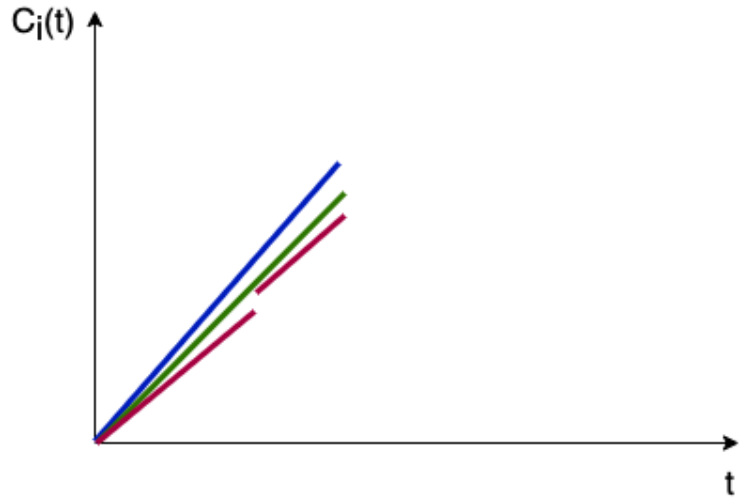
The third process receives a message from second process, and resets clock.



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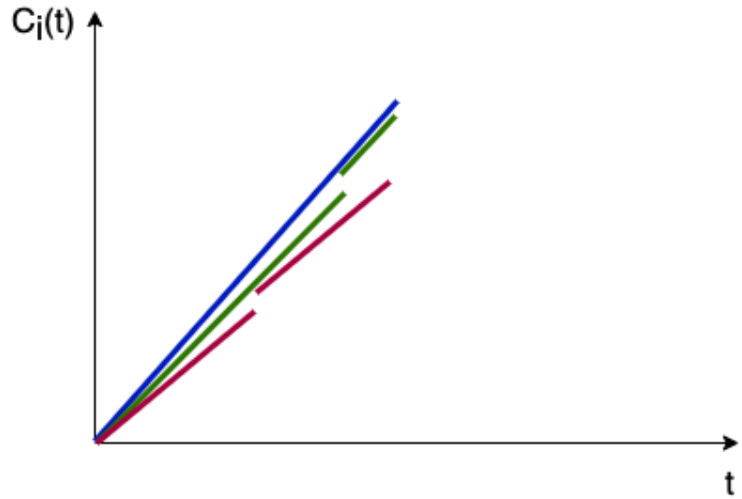

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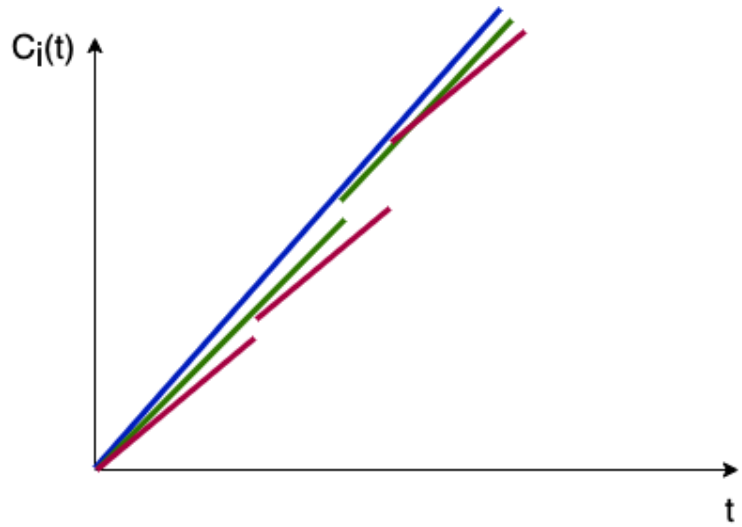
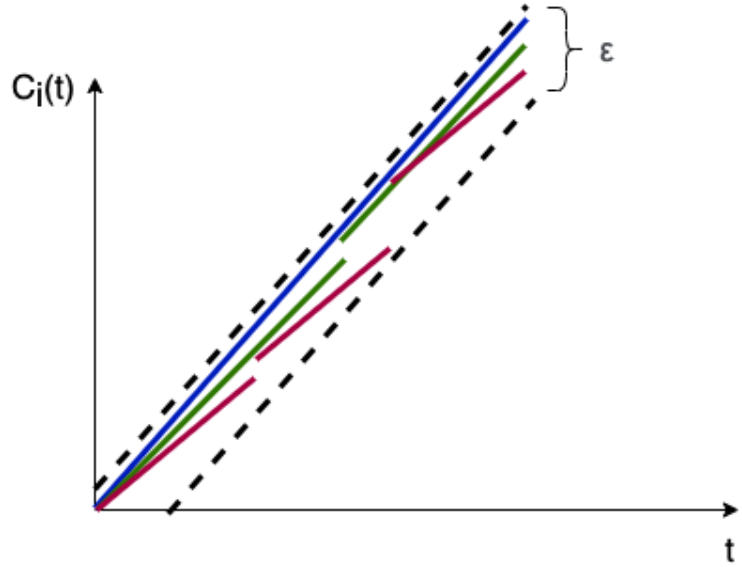


Physical Clock

Given appropriate assumptions, the clocks are syncronized within bounded error.



Summary

- Total ordering
  - Partial order "happens before" on events: $\rightarrow$
  - Logical clock that respects "$\rightarrow$"
  - Logical clock + Total order on processes $\Rightarrow$ total order on events
- Protocol that achieves mutual exclusion, and something more
- Physical clock

Q&A