SoK: Systematically Evaluating and Constructing zkSNARKs

YUNCONG ZHANG

1 INTRODUCTION

Zero-Knowledge Succinct Non-interactive ARgument of Knowledge (zkSNARK) [1] enables constant-or-logarithmic-time verification of computation outputs without knowing the inputs. The zkSNARKs are particularly useful in blockchains [2, 3], where zkSNARKs facilitate creating confidential transactions that conceal part or all of the transaction details. Currently, zkSNARKs are under active research, and recent years have seen an explosion of zkSNARK constructions enjoying different properties, including constant-size proofs [4–8], universal or trustless setups [9–14], and post-quantum security [12, 13].

However, the rapid development of zkSNARK poses considerable challenges for researchers to keep up with the state-of-the-art. Existing zkSNARK constructions rely on a large and growing number of underlying tools, examples shown in Table 1. Existing reviews of literature [15–17] only introduced these tools separately instead of in a unified perspective. It is also tricky to assess and compare existing schemes due to the high-dimensionality of measurement metrics, summarized in Table 2. Most of the studies that propose new constructions include an efficiency analysis and compare their products with previous ones. But these analyses are often incomplete and, to make things worse, diverse in notations, metrics, and parameters. A comprehensive survey of literature in zkSNARKs can serve as an anchor of knowledge in this field, to enhance understanding of why and how zkSNARKs appeared and thrived, to help to select appropriate zkSNARKs for different application scenarios, to reveal the insights behind current constructions, and to uncover the full potential of the existing techniques to construct more efficient and secure zkSNARKs.

In this paper, we present a survey that summarizes the knowledge of zkSNARKs systematically. To begin the story, we recall the history of zkSNARKs to understand the role zkSNARK plays in a broader field called "proof systems". For most of this survey, we present the knowledge of zkSNARKs in three levels:

(1) In the theoretical level, we define zkSNARK as a special case of proof systems. Based on this definition, we illustrate current

Author's address: Yuncong Zhang.

Table 1. Examples of zkSNARK building tools

Proof Model	Computation Model	Cryptography
IP [18]	Boolean circuit	Polynomial commitments [19]
PCP/PCPP [20]	Arithmetic circuit	CRH/ECRH
IPCP	Structured circuit	Bilinear pairing [21]
IOP/IOPP [22]	Layered circuit	Fiat-Shamir [23]
LPCP/LIP [24]	Random access machine	Accumulator
	QSP/QAP/SSP [5]	Multi-party computation
	AIR/ACSP [12]	Lattic-based
	R1CS	Group of unknown order [11]

Table 2. Measurement metrics for zkSNARKs

Efficiency	Security	Functionality
Prover complexity	Soundness	Expressiveness
Verifier complexity	Zero-knowledgeness	Public verifiability
Setup complexity	Trusted setup	Universality
Proof length	Post-quantum security	(Non)Preprocessing
CRS length	Cryptographic assumption	

theoretic results on zkSNARKs, e.g. the best efficiency we can achieve under certain security assumptions.

(2) In the technical level, we review and unify the existing frameworks of zkSNARK constructions. In this unified framework, we classify and examine the existing zkSNARKs to reveal the insights behind the construction techniques.

(3) In the application level, we propose an analysis system for measuring and comparing zkSNARKs. Instead of the traditional way that emphasizes the asymptotic complexities and practical performance in "toy examples", our system focus on evaluating the practicality of zkSNARKs in different application scenarios. Using this system, we conduct a comprehensive comparison of existing constructions.

2 RELATED WORKS

The zkSNARKs are variations of zero-knowledge proofs that originated from the ground-breaking work by Goldwasser, Micali, and Rackoff [18]. Zero-knowledge proofs are protocols allowing a party, called the prover, to prove a statement to another party, called the verifier, interactively without leaking secret information. This work also introduced the broader concept of *interactive proofs*, which was simultaneously and independently proposed in the work of Babai [25] in the name of Arthur-Merlin (AM) proofs. The major difference between AM proofs and interactive proofs is that AM proofs are public-coin proofs, which means all randomnesses the verifier uses must be revealed later to the prover.

All the precursors of zkSNARKs, including zkSNARKs themselves, can be viewed as special cases of *interactive proofs* with different

property combinations. Particularly, non-interactive proofs are special interactive proofs where the interaction consists of a single message. For clarity, we prefer to use the term *proof systems* for this broader concept, and refer to interactive proofs only when the interaction contains at least two messages from different parties.

Zero-knowledge proofs demonstrated wide applications [26, 27] soon after proposition. However, the zero-knowledge proof scheme provided by Goldwasser et al. is only of theoretical interest and not practical for real-world applications. Followup works focused on two goals: improving the efficiency to achieve *succinct* interactive proofs, and constructing non-interactive zero-knowledge (NIZK) schemes. Succinctness means the communication cost and optionally the verifier computation cost are lower than the case when the prover directly reveals the witness to the verifier.

The first breakthrough in improving the efficiency is the proposition of the *Probabilistically Checkable Proof (PCP)* model by Babai et al [20]. The PCP model allows the prover to send to the verifier a proof oracle that the verifier can query probabilistically, instead of a complete proof string that the verifier must read in the whole. This additional power makes constructing succinct PCP easier than constructing succinct interactive proofs. The PCP theorem [28] later proved by Arora et al. indicates the existence of succinct PCPs for any NP languages.

However, the PCP is only an ideal model as the proof oracle is unrealistic to implement. Regarding this issue, Kilian proposed a method to transform any PCP into a four-message interactive proof [29] using cryptographic tools, including collision-resistant hash functions and Merkle-trees. Kilian's construction is zero-knowledge if the underlying PCP is zero-knowledge. Meanwhile, the introduction of cryptography restricts the security to hold only against computationally bounded provers, since all-powerful provers can successfully cheat the verifier with false statements by breaking the hash function. Proof systems with this relaxed security requirement are called *argument systems* [30].

For the other goal regarding non-interactivity, Blum et al. [31] showed that NIZK is possible if the prover and the verifier are assumed to share a common random string before the prover generates any proofs. They also proposed the first NIZK, under the computational assumption that products of two large primes are indifferentiable from products of three large primes. Furthermore...

3 PRELIMINARIES

Designing zkSNARKs requires systematic use of cryptography and computational complexity theory. We will illustrate the concepts in these fields that are most relevant to zkSNARKs.

3.1 Notations

We use $\mathcal{R}$ for an NP relationship, i.e. a set of pairs $\{(x, w) : f(x, w) = 0\}$ where x, w are bit strings of size n and f is a deterministic polynomial-time algorithm. The NP language $\mathcal{L}$ induced from $\mathcal{R}$ is the set $\{x : \exists(x, w) \in \mathcal{R}\}$. For $(x, w) \in \mathcal{R}$, we say x is an instance of $\mathcal{L}$ and string w is a witness of the fact $x \in \mathcal{L}$.

We say a system has λ-bit security if breaking this system costs at least 2^λ units of computation power. We use n for the bit-lengths of algorithms inputs. We use $\eta(n)$ for a negligible function in n,

polylog(n) for a poly-logarithmic function, and poly(n) for a polynomial function. We say an algorithm is p.p.t. if the algorithm is probabilistic and can only execute for a period of length poly(n).

3.2 Commitment scheme

A cryptographic commitment scheme enables ... A commitment scheme is a tuple $\Gamma = $ (Setup, Comm, Open) of p.p.t. algorithms.

(1) Setup(1^λ) $\to$ pp generates public parameters given the security bit number
(2) Comm(pp, m) $\to$ (cm, r) takes a message m and generates a commitment cm, together with an opening hint r
(3) Open(pp, cm, m, r) $\to b \in \{0, 1\}$ takes a message m, a commitment cm and an opening hint r, and verifies if cm is a valid commitment of m. If $b = 1$, we say (m, r) is a correct opening of cm.

A commitment scheme is expected to be *binding*.

Definition 3.1 (Binding). A commitment scheme $\Gamma = $ (Setup, Comm, Open) is binding if ...

A commitment scheme can optionally be *hiding*.

Definition 3.2 (Hiding). A commitment scheme $\Gamma = $ (Setup, Comm, Open) is hiding if ...

Polynomial commitment. Polynomial commitment schemes are variants of commitment schemes that have message space $R[X]$, i.e. polynomials over ring R, and allow opening a single evaluation of the committed polynomial [19]. A polynomial commitment scheme is a tuple $\Gamma = $ (Setup, Comm, Open, Eval) where (Setup, Comm, Open) is a commitment scheme and Eval(pp, cm, x, y, d) $\to b \in \{0, 1\}$ is a protocol ... Except for the binding and hiding properties, a polynomial commitment scheme Γ is also expected to be *correct* and *evaluation binding*.

Definition 3.3 (Correct). A polynomial commitment sheme $\Gamma = $ (Setup, Comm, Open, Eval) is correct if ...

Definition 3.4 (Evaluation binding). A polynomial commitment sheme $\Gamma = $ (Setup, Comm, Open, Eval) is evaluation binding if ...

Sometimes we need a stronger property than the evaluation binding called *knowledge soundness* that requires the prover to "know" the committed polynomial [11].

Definition 3.5 (Knowledge soundness). A polynomial commitment sheme $\Gamma = $ (Setup, Comm, Open, Eval) has knowledge soundness if ...

Accumulator. Accumulators are another kind of variations of commitment schemes. Instead of committing a single element, an accumulator allows committing to a list of elements, and ...

Merkle-tree is an example of an accumulator ...

3.3 Fiat-Shamir transformation and random oracle

Fiat-Shamir transformation is the standard way to transform a public-coin interactive protocol to a non-interactive scheme. The Fiat-Shamir transformation works by simulating the verifier challenges by the hash value of prover messages. However, the security of the resulting non-interactive scheme is established only in the

random oracle model. Current security proving techniques require modeling the hash function by a random oracle, which is an ideal functionality that does not exist in reality.

A random oracle is ...

3.4 Bilinear pairing

Given groups $\mathbb{G}_1$, $\mathbb{G}_2$ and $\mathbb{G}_T$, with $|\mathbb{G}_1| = |\mathbb{G}_2| = |\mathbb{G}_T| = q$, a bilinear pairing [21] is a mapping $e : \mathbb{G}_1 \times \mathbb{G}_2 \to \mathbb{G}_T$ that satisfies...

4 DEFINITIONS AND THEORETICAL RESULTS

The zkSNARKs are special proof systems satisfying a specific set of properties. We first give the definitions of proof systems and several properties that a proof system may have. Then we define zkSNARKs by referring to these definitions. Finally, we discuss the theoretical results about different properties of proof systems and demonstrate the minimum assumptions underlying desired combinations of properties.

4.1 Proof systems

A proof system is a protocol that allows a party, namely the prover, to prove a statement to another party, namely the verifier.

Statements. Proof systems usually deal with two types of statements related to an NP language $\mathcal{L}$:

(1) given string x, the statement claims that $x \in \mathcal{L}$;
(2) given string x, the statement claims knowledge of w such that $(x, w) \in \mathcal{R}$.

Syntax. Given an NP language $\mathcal{L}$, a proof system $\Pi_{\mathcal{L}}$ consists of three algorithms (Setup, Prove, Verify).

(1) Setup(pp) $\to (\sigma_P, \sigma_V)$. The Setup algorithm takes public parameters pp as inputs and generates reference strings σ_P and σ_V for the prover and the verifier respectively. The reference strings σ_P and σ_V may intersect with each other, in which case, the intersection is called a *common reference string (CRS)*. The Setup algorithm must be executed before any instance of Π can start.
(2) $\langle \text{Prove}(\sigma_P, x, w) \rightleftharpoons \text{Verify}(\sigma_V, x) \rangle \to 0/1$. Given a pair $(x, w) \in \mathcal{R}$, the Prove algorithm takes a pair (x, w) and the reference string σ_P as inputs. The Verify algorithm takes x and the reference string σ_V as inputs. When being executed, the algorithms may exchange messages between each other. Finally, the Verify algorithm outputs 0 or 1, which is regarded as the output of this protocol.

The execution of a proof system is illustrated in Fig. 1.

Properties. A proof system $\Pi_{\mathcal{L}} = (\text{Setup}, \text{Prove}, \text{Verify})$ is expected to satisfy the following properties:

Definition 4.1 (Completeness). A proof system $\Pi_{\mathcal{L}} = (\text{Setup}, \text{Prove}, \text{Verify})$ has *completeness* if for any $(x, w) \in \mathcal{R}$, correctly executing the protocol always outputs 1, i.e.

$$\Pr\left[b = 1 \middle| \begin{array}{c} \text{Setup(pp)} \to (\sigma_P, \sigma_V); \\ \langle \text{Prove}(\sigma_P, x, w) \rightleftharpoons \text{Verify}(\sigma_V, x) \rangle \to b \end{array} \right] = 1 \qquad (1)$$

Definition 4.2 (Soundness). A proof system $\Pi_{\mathcal{L}} = (\text{Setup}, \text{Prove}, \text{Verify})$ has *soundness* if for any $x \notin \mathcal{L}$, for any adversary $\mathcal{A}$ acting as the
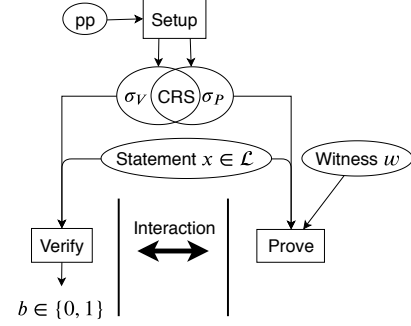

Fig. 1. Proof System

An example of a proof system execution. The rectangles represent algorithms. The ovals represent inputs to these algorithms.

prover, the protocol outputs 1 with probability $\epsilon < 1/2$, where ϵ is called the soundness error of $\Pi_{\mathcal{L}}$.

$$\Pr\left[b = 1 \middle| \begin{array}{c} \text{Setup(pp)} \to (\sigma_P, \sigma_V); \\ \langle \mathcal{A}(\sigma_P, x) \rightleftharpoons \text{Verify}(\sigma_V, x) \rangle \to b \end{array} \right] = \epsilon < 1/2 \qquad (2)$$

If equation (2) holds only for p.p.t. adversaries $\mathcal{A}$, we say $\Pi_{\mathcal{L}}$ has *computational soundness*. In this case, we say $\Pi_{\mathcal{L}}$ is an *argument system*.

Note that given $\Pi_{\mathcal{L}}$ with non-negligible soundness error $\epsilon < 1/2$, we can always construct $\Pi'_{\mathcal{L}}$ with exponentially small soundness error ϵ', by repeating the protocol a polynomial number of times. Therefore, requiring the soundness error to be smaller than $1/2$ is sufficient for a proof system to be useful in practice.

Variations. Based on the standard definitions described above, proof systems may take variations in many aspects.

(1) *Proof-of-knowledge.* Informally, a proof system is said to have proof-of-knowledge, if Definition 4.2 holds not only for $x \notin \mathcal{L}$, but also for $x \in \mathcal{L}$ if the adversary does not "know" any witness of x. The notion "knowledge" is formally defined by an extractor algorithm.

Definition 4.3 (Proof-of-Knowledge). A proof system $\Pi_{\mathcal{L}} = (\text{Setup}, \text{Prove}, \text{Verify})$ has *proof-of-knowledge* if for any $x \in \{0, 1\}^n$, for any adversary $\mathcal{A}$ acting as the prover, there exists a negligible function $\eta(n)$ and an extractor $\text{Ext}^{\mathcal{A}}$ which has non-blackbox access to the adversary, such that whenever the adversary successfully cheats the verifier into outputting 1, $\text{Ext}^{\mathcal{A}}$ outputs a witness w s.t. $(x, w) \in \mathcal{R}$ with overwhelming probability, i.e.

$$\Pr\left[\begin{array}{c} b = 1 \wedge \\ (x, w) \notin \mathcal{R} \end{array} \middle| \begin{array}{c} \text{Setup(pp)} \to (\sigma_P, \sigma_V); \\ \langle \mathcal{A}(\sigma_P, x) \rightleftharpoons \text{Verify}(\sigma_V, x) \rangle \to b; \\ \text{Ext}^{\mathcal{A}}(\sigma_P, x) \to w \end{array} \right] = \eta(n) \qquad (3)$$

If equation (3) only holds for computationally bounded adversaries, we say the proof system has *argument-of-knowledge*.

(2) *Zero-knowledge.* A proof system is said to be zero-knowledge if the verifier cannot extract any "knowledge" from the interaction transcripts, i.e. the collection of all the messages sent

during the protocol execution. The zero-knowledge property is formally defined by a simulator algorithm.

Definition 4.4 (Zero-knowledgeness). Let $\text{tr} = [\text{Prove}(\sigma_P, x) \rightleftharpoons \text{Verify}(\sigma_V, x)]$ be the interaction transcript between the prover and the verifier, and call (σ_V, tr) the view of the verifier. A proof system $\Pi_{\mathcal{L}} = (\text{Setup}, \text{Prove}, \text{Verify})$ is zero-knowledge if for any adversary $\mathcal{A}$ acting as the verifier, there exists a simulator $\text{Sim}(\cdot)$ such that for any $(x, w) \in \mathcal{R}$, $\text{Sim}(x)$ is indifferentiable from the view of $\mathcal{A}$ during the protocol execution, i.e. there exists negligible function $\eta(n)$, s.t. for any differentiator $\mathcal{D}$:

$$
\begin{vmatrix}
\Pr\left[\mathcal{D}(\sigma_V, tr) = 1 \middle| \begin{array}{l} \text{Setup}(\text{pp}) \to (\sigma_P, \sigma_V); \\ [\text{Prove}(\sigma_P, x) \rightleftharpoons \mathcal{A}(\sigma_V, x)] \to tr \end{array}\right] \\
- \Pr\left[\mathcal{D}(\sigma_V, tr) = 1 \middle| \text{Sim}(x) \to (\sigma_V, tr)\right]
\end{vmatrix} = \eta(n) \quad (4)
$$

The definition of zero-knowledge has some variations. *Honest-verifier zero-knowledge ... Unconditional zero-knowledge* or *perfect zero-knowledge ... Statistical zero-knowledge ... Computational zero-knowledge ...*

(3) *Interaction models.* A standard proof system only admits normal interactions, i.e. the prover and the verifier send messages to each other, and each message is read in the whole by the receiver...
Non-interactive ... Probabilistically Checkable Proof (PCP) ... Interactive PCP (IPCP) ... Linear PCP (LPCP) ... Interactive Oracle Proof (IOP) ...

Efficiency. The efficiency of a proof system is measured by the efficiency of each of the algorithms (Setup, Prove, Verify), sizes of the reference strings (σ_P, σ_V), and the communication cost...

Definition 4.5 (Succinctness). A proof system $\Pi_{\mathcal{L}}$ is succinct if the communication cost of $\Pi_{\mathcal{L}}$ is at most polylogarithmic to the witness size.

Definition 4.6 (Scalability). A proof system $\Pi_{\mathcal{L}}$ is scalable if...

4.2 zkSNARK

With the above definitions on proof systems and their properties, we can now define zkSNARKs.

Definition 4.7 (zkSNARK). A *zkSNARK* is a proof system that is zero-knowledge (Definition 4.4), succinct (Definition 4.5), non-interactive, and argument-of-knowledge (Definition 4.3).

Depending on how Setup works, a zkSNARK may be in one or more of the following models.

Preprocessing. A zkSNARK is called *preprocessing* if ...

Universal setup. A zkSNARK has *universal setup* if ...

Transparent setup. A zkSNARK has *transparent setup* if ...

4.3 Security Assumptions

After the introduction of zero-knowledge proofs by Goldwasser et al. [18], Goldreich, Micali, and Wigderson showed that zero-knowledge proofs exist for any NP languages, under the assumption

Table 3. Security Assumptions for Different Proof Systems

Name	Inter.	ZK	Succ.	Sound	Assum.	Model
NP	×	×	×	Uncond.	None	Std.
IP	✓	×	×	Stat.	None	Std.
ZKP	✓	✓	×	Stat.	OWF	Std.
NIZK	×	✓	×	Stat.	TDP	CRS
sARG	✓	×	✓	Comp.	CRH	Std.
sZKA	✓	✓	✓	Comp.	CRH	Std.
sNIZK	×	✓	✓	Comp.	CRH	CRS / RO
SNARG	×	×	✓	Comp.	nonf.	CRS / RO
SNARK	×	×	✓	AoK	nonf.	CRS / RO
zkSNARK	×	✓	✓	AoK	nonf.	CRS / RO

that one-way function exists [26]. The existence of one-way functions is almost the weakest assumption in cryptography. The only weaker one is P≠NP. It is also proved that statistical ZK proofs (i.e., both ZK and soundness are statistical) are impossible for NP-complete languages unless P=NP [32–34]. Therefore, considering general NP languages, at least one side—ZK or soundness—must be computational. In the interactive case, it is demonstrated that both flavors exist: statistical ZK with computational soundness, or statistical soundness with computational ZK [26, 30, 35].

In the non-interactive case, NIZK constructions only had computational ZK until ...

Succinctness is another property that poses stronger security assumptions on the proof system. Succinctness is only possible for argument systems, i.e. proof systems with only computational soundness [36–39]. We provide an intuitive proof for the nonexistence of succinct statistically-sound proof systems for NP-complete language $\mathcal{L}$. Recall that succinct proof systems have only $O(\log n)$ computation cost where n is the size of NP witness w. If a succinct proof system is also statistically-sound, i.e. it is secure against unbounded prover, the prover should not be able to find valid responses with probability over $1/2$. Since the prover can search through the entire response space, this means valid responses do not exist at all. However, that would facilitate a probabilistic $O(n)$ algorithm $\mathcal{A}$ deciding the language $\mathcal{L}$ with error $1/2$: for any $x \in \mathcal{L}$, $\mathcal{A}$ randomly sample the verifier challenges, loop through the response space in $O(n)$ time, and if $\mathcal{A}$ does not find any valid response, $\mathcal{A}$ outputs 0. Therefore, $\mathcal{L}$ is a degenerated language in BPP, i.e. languages having p.p.t. deciding algorithms, which would indicate the collapse of the polynomial hierarchy. Hence we proved succinct statistically-sound proof systems should not exist for any interesting NP problems.

Furthermore, ...

Table 3 summarizes the minimum security assumptions underlying different proof systems.

5 ZKSNARK CONSTRUCTION TECHNIQUES

Having learned about the theoretical definitions, we are ready to investigate the details of current constructions.

5.1 Construction Framework

Before explaining the details, it would be convenient to put the various constructions into a common framework. We will briefly review the existing frameworks and demonstrate our unified framework.

Bünz et al. [11] proposed a framework by noting that many SNARKs can be obtained by compiling a polynomial IOP with a polynomial commitment scheme and the Fiat-Shamir protocol. A polynomial IOP is an IOP where the proof oracles are restricted to a polynomial, i.e. an oracle that responses with $f(x)$ at query x for some polynomial f of bounded degree. The polynomial commitment scheme is responsible for ensuring the prover to behave like a polynomial oracle.

The ZKProof Community in a reference document [15] presents a more general framework saying that all the ZKPs are obtained by applying a cryptographic transformation to an information-theoretic proof system.

Ben-Sasson recently showed another perspective to view all cryptographic proofs, not just zkSNARKs, as a combination of an *arithmetization* method and a polynomial *low degree checking (LDC)* scheme [40]. The arithmetization step transforms a computation satisfaction problem into an algebraic problem that is more friendly with proof systems. The algebraic problem is usually described as a polynomial equation. The low degree checking step verifies that the polynomials provided by the prover are within appropriate degree limits.

These different frameworks focus on different parts of the same zkSNARK construction pipeline, illustrated in Fig. 2. Bünz et al. [11] focuses on abstracting all the proof systems with polynomial IOP and the first compilation into argument systems. Ben-Sasson [40] focuses on arithmetization and views the other three steps in the whole as an LDC scheme.
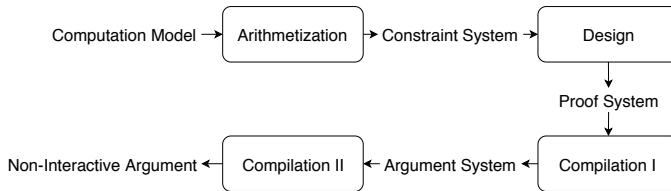

Fig. 2. zkSNARK Construction Pipeline

We propose a new framework based on this pipeline, ...

We will introduce the current zkSNARK constructions grouped by the research lines. In each group, we will extract the common techniques they share and dive into the details of their differentiations ...

5.2 PCP-based zkSNARKs

The PCP-based approach is the result of the series of works of Blum et al. [20], Kilian [29], and Micali [41]. The idea is to design a ZK-PCP for the given language, then compile it into a succinct interactive argument using a commitment scheme [29], and finally apply the Fiat-Shamir transformation to obtain a zkSNARK [41]. For those zkSNARKs based on generalizations of ZK-PCP, e.g. ZK-IPCP, ZK-IOP, we also categorize them into this group.

Most of the researches in this category focus on the design of ZK-PCP. Therefore, they mainly differ in the arithmetization and proof system design. Another two lines of work fall into this category: the Multi-Party Computation (MPC)-based, and the Polynomial IOP (PIOP)-based zkSNARKs.

STARK..

Aurora.

Fractal.

5.2.1 MPC-based zkSNARKs.
The MPC-based approach presents a way to transform a secure multi-party computation for any NP language into a ZK-IPCP. This line of work originated from Ishai, Kushilevitz, Ostrovsky, and Sahai (IKOS) [42], ...

A multi-party computation scheme is ...

ZKBoo.

ZKB++.

Ligero.

5.2.2 PIOP-based zkSNARKs.

Sonic.

PLONK.

Supersonic.

5.3 Pairing-based zkSNARKs

The pairing-based zkSNARKs have one of the most efficient verifier and the smallest proof size. As a sacrifice, they rely on an expensive per-circuit trusted setup. This line of research originated from the series of works by Groth, Ostrovsky, and Sahai [43–45], after the introduction of bilinear-pairing by Boneh et al. [21].

GGRP13.

Pinocchio.

BCGTV13.

Groth16.

5.4 IP-based zkSNARKs

The interactive proof (IP)-based line of research started from the Muggles paper [46] by Goldwasser et al. that brings the GKR protocol. However, these studies did not produce a zkSNARK until T. Xie et al. proposed Libra [47] which introduces the zero-knowledge property to GKR.

To learn about the Libra zkSNARK, it is inevitable to first introduce the famous sum-check protocol, which is the foundation of GKR protocol.

The Sumcheck protocol.

The GKR protocol.

Libra.

5.5 Recursive zkSNARKs

The recursive technique is based on the insight that we can further verify the zkSNARK verification computation recursively by another zkSNARK. Recursive zkSNARKs work with two primitives: proof-carrying data (PCD) and bootstrapping...

6 APPLICATION-ORIENTED ANALYSIS

Evaluating and comparing the zkSNARK constructions is challenging because of the high dimensionality of metrics. To mitigate this problem, we categorize these metrics into three groups: efficiency, security, and functionality. Instead of presenting all the dimensions simultaneously as efficiency analysis usually does, we design several real-world application scenarios. Each scenario specifies two sets of criteria: the hard criteria filter for zkSNARKs that satisfy given conditions, and the soft criteria define an order over these filtered zkSNARKs.

6.1 Computation Delegation

A common application of zkSNARKs is computation delegation. In this scenario, the verifier is usually a device with weak computation power, e.g. a smartphone, while the prover has much more computational resources, e.g. a cloud server. The verifier may delegate computation tasks to the prover, but the prover is not completely trustable. In this case...

6.2 Organization with Authorized Leader

6.3 Value-transfer-only Permissionless Blockchain

In a permissionless blockchain, if most activities are transfers of currency ownerships, e.g. in Bitcoin or Zerocash, the verifier efficiency, the proof size, and the public verifiability are crucial. Meanwhile, other properties e.g. high prover efficiency, post-quantum security, transparency are nice but not fate-determining. Expressiveness and universality are less interesting as the blockchain functionality is simple and stable. In this case...

6.4 Permissionless Blockchain with Smart Contract

In permissionless blockchains supporting Turing complete smart contracts, e.g. Ethereum, Nervos, the verifier efficiency, the proof size, and the public verifiability are as important as in value-transfer-only blockchains. Meanwhile, expressiveness and universality are as significant as efficiency...

7 CONCLUSION

We recalled the history of zkSNARKs and summarized the knowledge of zkSNARKs in three levels: the theoretical level, the technical level, and the application level. At the theoretical level, by recalling the history, we introduced the proof systems and presented a definition for zkSNARKs in this broader context. To better understand the zkSNARK constructions, we demonstrated the theoretical results about proof systems and zkSNARKs, e.g. the security assumptions required for different combinations of efficiency and functionality. At the technical level, we introduced existing zkSNARK construction frameworks and presented a unified framework. We provided the details of different zkSNARK research lines in the perspective of this framework. Finally, at the application level, we analyzed

and compared the zkSNARKs in terms of efficiency, security, and functionality, in different application scenarios.

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