Project 3 Dymamic Programming

Author:

Alex Zongo (GWID)
Harsh Sharma (GWID)
He Jiaying (GWID)
Yiding Pei G28608005

1 Problem Analysis

1.1 Problem Description

Question 0: Teleportation in Astro haunted galaxies
You have a teleporter that can take you from galaxy i to galaxy j. Cost to teleport is given by c(i,j), which can be arbitrary. Some galaxies are "astro-haunted" – this is specified by a(i) which can be 0 or 1 (1 means that that galaxy is "astro-haunted"). Give a polynomial time algorithm that minimizes the cost of going from galaxy 1 to galaxy n, such that you pass through no more than k astro-haunted galaxies. (You can assume that galaxies 1 and n are not astro-haunted.)

1.2 Keys to the Solution

This question is similar to *All Pairs Shortest Path* Problem. We can use dynamic programming to solve it.

Notation We use the notation: $D(i, j, m)$ to denote the length of the shortest path from i to j passing through a maximum of m astro-haunted galaxies.
As a degenerate case, we observe that $D(i, j, 0)$ signifies the *App Pairs Shortest Path* Problem between i and j.

Proof of Optimality

Recurrence Relation For the base case of $D(i, j, 0)$, we can solve it as an APSP problem. Therefore, the recurrence relations can be writteen as follows:

$$D^{(k)}(i,j,0) = \begin{cases} min\left\{D^{(k-1)}(i,j,0), D^{(k-1)}(i,k,0) + D^{(k-1)}(k,j,0)\right\} (if\ k\ is\ not\ astro-haunted) \\ D^{(k-1)}(i,j,0)\ (if\ k\ is\ astro-haunted) \end{cases}$$

For the general case of $D(i, j, m)$, we can write the recurrence relation as follows:

$$D(i,j,m) = min\{D(i,z,m-a[z]) + D(z,j,0)\}, z \in \{all\ galaxies\}$$

Pseudocode

2 Theoretical Analysis

3 Experimental Analysis

4 Conclusions