

Project 3 Dynamic Programming

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1 Problem Analysis

1.1 Problem Description

Question 0: Teleportation in Astro haunted galaxies

You have a teleporter that can take you from galaxy i to galaxy j . Cost to teleport is given by $c(i,j)$, which can be arbitrary. Some galaxies are “astro-haunted” – this is specified by $a(i)$ which can be 0 or 1 (1 means that that galaxy is “astro-haunted”). Give a polynomial time algorithm that minimizes the cost of going from galaxy 1 to galaxy n , such that you pass through no more than k astro-haunted galaxies. (You can assume that galaxies 1 and n are not astro-haunted.)

1.2 Keys to the Solution

This question is similar to *All Pairs Shortest Path* Problem. We can use dynamic programming to solve it.

Notation We use the notation: $D(i,j,m)$ to denote the length of the shortest path from i to j passing through a maximum of m astro-haunted galaxies.

As a degenerate case, we observe that $D(i,j,0)$ signifies the *All Pairs Shortest Path* Problem between i and j .

Proof of Optimality

Recurrence Relation For the base case of $D(i,j,0)$, we can solve it as an APAS problem. Therefore, the recurrence relations can be written as follows:

$$D^{(k)}(i,j,0) = \min \left\{ D^{(k-1)}(i,j,0), D^{(k-1)}(i,k,0) + D^{(k-1)}(k,j,0) \right\}$$

For the general case of $D(i,j,m)$, we can write the recurrence relation as follows:

$$D(i,j,m) = \min \{ D(i,z,m-1) + D(z,j,0) \}, z \in \{ \text{all galaxies} \}$$

Pseudocode

2 Theoretical Analysis

3 Experimental Analysis

4 Conclusions