Project 3 Dymamic Programming

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1 Problem Analysis

1.1 Problem Description

Question 0: Teleportation in Astro haunted galaxies
You have a teleporter that can take you from galaxy i to galaxy j. Cost to teleport is given by c(i,j), which can be arbitrary. Some galaxies are "astro-haunted" – this is specified by a(i) which can be 0 or 1 (1 means that that galaxy is "astro-haunted"). Give a polynomial time algorithm that minimizes the cost of going from galaxy 1 to galaxy n, such that you pass through no more than k astro-haunted galaxies. (You can assume that galaxies 1 and n are not astro-haunted.)

1.2 Keys to the Solution

This question is similar to *All Pairs Shortest Path* Problem. We can use dynamic programming to solve it.

Notation We use the notation: $D(i, j, m)$ to denote the length of the shortest path from i to j passing through a maximum of m astro-haunted galaxies.
As a degenerate case, we observe that $D(i, j, 0)$ signifies the *App Pairs Shortest Path* Problem between i and j.

Proof of Optimality

Recurrence Relation For the base case of $D(i, j, 0)$, we can solve it as an APSP problem. Therefore, the recurrence relations can be writteen as follows:

$$D^{(k)}(i, j, 0) = \begin{cases} min\left\{D^{(k-1)}(i, j, 0), D^{(k-1)}(i, k, 0) + D^{(k-1)}(k, j, 0)\right\} & (if\ k\ is\ not\ astro - haunted) \\ D^{(k-1)}(i, j, 0) & (if\ k\ is\ astro - haunted) \end{cases}$$

For the general case of $D(i, j, m)$, we can write the recurrence relation as follows:

$$D(i, j, m) = min\{D(i, z, m - a[z]) + D(z, j, 0)\}, z \in \{all\ galaxies\}$$

Pseudocode

2 Theoretical Analysis

3 Experimental Analysis

4 Conclusion