Week04

Problem1

1.Results

・Classical Brownian Motion:

Simulated Mean: 100.000748

Simulated Standard Deviation: 0.099043

Expected Mean: 100.0

Expected Standard Deviation: 0.1

The simulated results are very close to the expected theoretical values. The mean is nearly identical, and the standard deviation is also very close to the expected value, which aligns well with the theory.

・Arithmetic Return System:

Simulated Mean: 100.074777

Simulated Standard Deviation: 9.904342

Expected Mean: 100.0

Expected Standard Deviation: 10.0

The simulated mean is slightly higher than expected, but still very close to the theoretical mean of 100. The standard deviation is also close to the expected value of 10. This small deviation is within an acceptable range given the randomness in simulations.

・Geometric Brownian Motion:

Simulated Mean: 100.567049

Simulated Standard Deviation: 9.992300

Expected Mean: 100.0

Expected Standard Deviation: 10.0

The simulated mean here is slightly higher than expected, with a value of around 100.57 compared to the theoretical 100. The standard deviation, however, matches almost exactly with the expected value of 10. This slight discrepancy in the mean can be attributed to the exponential nature of this model, which tends to introduce a small bias upwards.

2.Conclusion:

Overall, the simulations are consistent with the theoretical expectations, with only minor deviations that are typical in numerical simulations. The differences are minor and do not significantly impact the validity of the models. This shows that the implementation of the Classical Brownian Motion, Arithmetic Return System, and Geometric Brownian Motion is functioning as expected.

Model	Mean	Standard Deviation	Expected Mean	Expected Std Dev
Classical Brownian Motion	100.000748	0.099043	100.0	0.1
Arithmetic Return System	100.074777	9.904342	100.0	10.0
Geometric Brownian Motion	100.567049	9.992300	100.0	10.0

Problem 2

1.Compare and Analyze

・Normal Distribution VaR: 0.0382

The result represents the potential loss under normal conditions.

・Exponentially Weighted Moving Average (EWMA) VaR: 0.0310

EWMA places more weight on recent data, meaning the volatility of the recent market conditions is more strongly reflected. The VaR is slightly lower than the normal distribution method, likely due to reduced recent volatility.

・MLE T-Distribution VaR: 0.0324

The T-distribution model accounts for the potential of more extreme outcomes due to fatter tails. This model yields a VaR of 0.0324, which is higher than the EWMA method but lower than the normal distribution method, indicating it accounts for extreme market movements but not as extreme as expected by the normal distribution model.

・AR(1) VaR: 0.0384

The AR(1) model incorporates autocorrelation in returns. It produced a VaR of 0.0384, which is very close to the normal distribution VaR, suggesting that recent returns have a strong correlation with previous returns.

・Historical Simulation VaR: 0.0295

This method directly uses historical data without making distributional assumptions. The resulting VaR is 0.0295, lower than other methods, indicating that the historical data may have had fewer extreme movements.

2.Conclusion:

The different methods provide a range of VaR values, from 0.0295 to 0.0384. The normal and AR(1) models resulted in the highest VaR values, implying that they expect more volatility based on the given assumptions. The historical simulation, on the other hand, provided the lowest VaR, as it is more reflective of actual historical data, which may not include extreme events. MLE T-distribution captures some of the risk from fat tails but still remains lower than the normal distribution estimate.

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VaR Calculations:
Normal VaR: 0.03824983872458302
EWMA VaR: 0.030991424222976006
MLE T-Distribution VaR: 0.03242585900409047
AR(1) VaR: 0.03837900413254758
Historical VaR: 0.029464221305067637
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Problem3:

1. Arithmetic Model

・Method

I first calculated Discrete Returns (Arithmetic Returns) as the percentage change between two consecutive days. The assumption here is that prices are proportionally related to the previous day's price. The returns are then used to calculate the exponentially weighted covariance matrix with $\lambda = 0.97$.

Based on this covariance matrix, I compute the VaR for each portfolio and the total VaR of all holdings.

・Results:

Portfolio A EW VaR (Discrete): $17,565.28
Portfolio B EW VaR (Discrete): $10,952.34
Portfolio C EW VaR (Discrete): $17,466.23
Total Portfolio EW VaR (Discrete): $44,415.00

The results show that Portfolio A and Portfolio C have higher risk exposure compared to Portfolio B. The total portfolio VaR reflects the combined risk of holding all portfolios, which is consistent with the individual portfolio risks.

2. Log Returns Model

・The reason for choosing this model:

The second approach uses Log Returns instead of discrete returns. Log returns are defined as the natural logarithm of the ratio of consecutive prices. This method accounts for compounding and provides a symmetrical measure of returns, which is particularly useful for long-term analysis and extreme market movements.

・Method

I applied the same EWMA method with $\lambda = 0.97$ to the log returns and computed the VaR for each portfolio and total holdings.

・Results:

Portfolio A EW VaR (Log): $17,623.72
Portfolio B EW VaR (Log): $11,018.22
Portfolio C EW VaR (Log): $17,380.79
Total Portfolio EW VaR (Log): $44,532.72

The results are similar to those obtained using discrete returns, but the log-based VaR estimates are slightly higher for each portfolio and for the total holdings.

3.Comparison

Both methods provided similar results, but the log return model offers a more robust and reliable approach, particularly for long-term analysis and risk management. Its ability to handle compounding makes it superior in cases of extreme market events. Thus, the log return model is often chosen over the discrete return model, especially when aiming for more accurate, long-term risk estimation.

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Portfolio A EW VaR (Discrete) in $: 17565.28
Portfolio A EW VaR (Log) in $: 17623.72
Portfolio B EW VaR (Discrete) in $: 10952.34
Portfolio B EW VaR (Log) in $: 11018.22
Portfolio C EW VaR (Discrete) in $: 17466.23
Portfolio C EW VaR (Log) in $: 17380.79
Total Portfolio EW VaR (Discrete) in $: 44415.00
Total Portfolio EW VaR (Log) in $: 44532.72
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