NA 568 - Winter 2024

Robotic Mapping

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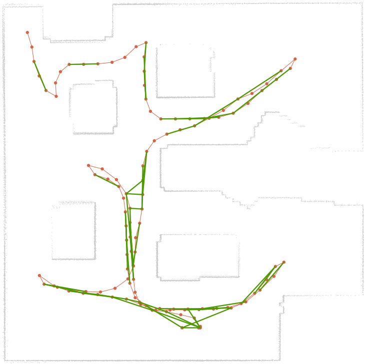
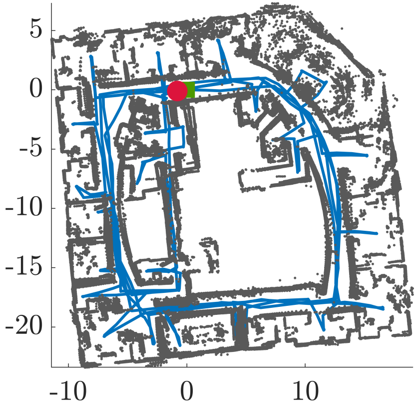
UNIVERSITY OF MICHIGAN

Pose SLAM graph and its dense occupancy grid map.

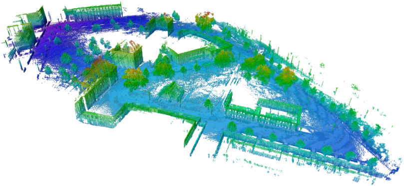


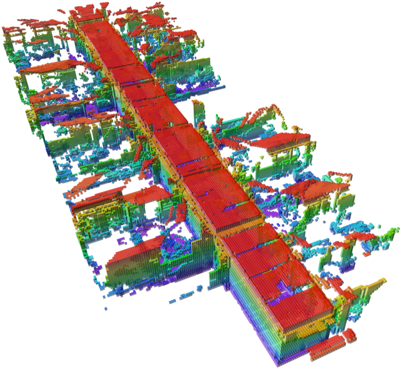
Intel dataset.



Extension to 3D Occupancy Maps: OctoMap

An Efficient Probabilistic 3D Mapping Framework Based on Octrees:





Freiburg campus (left) and Freiburg building 079 (right)

Occupancy Grid Maps Pros and Cons

Pros:

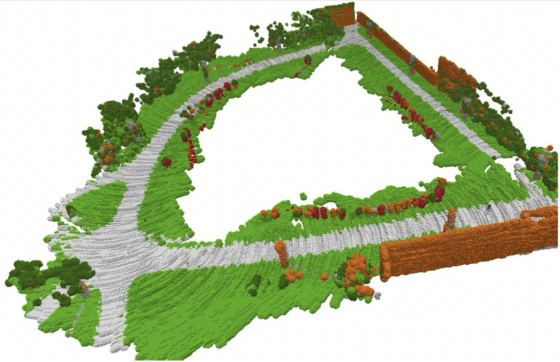
- Computationally efficient.
- Easy to impelement.
- Available software for 2D and 3D data and sensors.

Cons:

- Grids are assumed to be statistically independent whereas in reality map points have local correlations.
- No notion of map uncertainty.
- Fixed map resolution. Once the map is built, it is not possible to increase the resolution.

- Closed-form recursive Bayesian inference for dense 3D semantic (scene segmentation) occupancy mapping.

- Continuous semantic map via Bayesian kernel inference by exploiting local correlations present in the environment; queries at arbitrary resolutions.



Review of Bayes' Rule

- $$p(\text{hypothesis}|\text{data}) = \frac{p(\text{data}|\text{hypothesis})p(\text{hypothesis})}{p(\text{data})}$$

- $$\text{Posterior} = \frac{\text{Likelihood} \times \text{Prior}}{\text{Evidence (Marginal Likelihood)}}$$

Distribution of the Occupancy Probability

- In the last OGM lecture, we obtained a *point estimate* of the occupancy probability of a grid cell.
- In this lecture, we will estimate the *full distribution* of the occupancy probability of a grid cell.

Conjugate Distributions

https://en.wikipedia.org/wiki/Conjugate_prior

- If the posterior distributions are in the same probability distribution family as the prior probability distribution, the prior and posterior are then called conjugate distributions.

- The prior is called a **conjugate prior** for the likelihood function.

- A conjugate prior is an algebraic convenience, giving a closed-form expression for the posterior; otherwise numerical integration may be necessary.

- The Gaussian distribution is a conjugate prior for the likelihood that is also Gaussian. *We can use this model, for example, for the substance concentration mapping.*
- The beta distribution is a conjugate prior for the Bernoulli likelihood. *We will use this model for occupancy mapping.*
- The Dirichlet distribution is a conjugate prior for the Categorical likelihood. *We will use this model for semantic (multi-class) mapping.*

- The beta distribution is a conjugate prior for the Bernoulli likelihood.

$$p(\theta; \alpha, \beta) = \frac{1}{B(\alpha, \beta)} \theta^{\alpha-1} (1-\theta)^{\beta-1}$$

$$p(y|\theta) = \theta^{y} (1-\theta)^{1-y}$$

- The Dirichlet distribution is a conjugate prior for the Categorical likelihood.
  - Multi-category generalization of the above case.

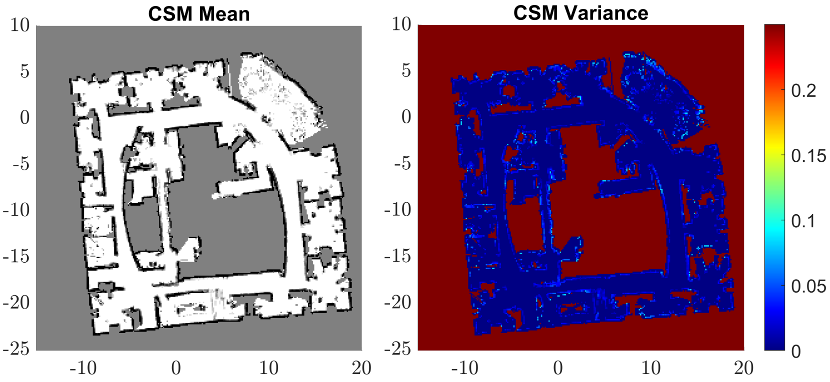
- One-hot-encoded measurement tuple $y = (y_1, ..., y_K)$, where $y_k \in \{0,1\}$ and $\sum_{k=1}^{K} y_k = 1$.
- For any map voxel, K possible categories with the probability of $\theta = (\theta_1, ..., \theta_K)$, where $\sum_{k=1}^{K} \theta_k = 1$.
- Categorical likelihood $p(y|\theta) = \prod_{k=1}^{K} \theta_k^{y_k}$.
- Dirichlet prior $p(\theta; \alpha) = \mathrm{Dir}(\alpha) = \frac{1}{B(\alpha)} \prod_{k=1}^{K} \theta_k^{\alpha_k - 1}$. $\alpha = (\alpha_1, ..., \alpha_K)$.

- Posterior is another Dirichlet distribution $p(\theta|y;\alpha^{\text{new}}) \propto p(y|\theta)p(\theta;\alpha)$.
- $p(\theta|y;\alpha^{\text{new}}) \propto \prod_{k=1}^{K} \theta_k^{y_k} \cdot \prod_{k=1}^{K} \theta_k^{\alpha_k - 1} = \prod_{k=1}^{K} \theta_k^{\alpha_k + y_k - 1}$.
- We learn that $\alpha_k^{\text{new}} = \alpha_k + y_k$. Let $\alpha_s := \sum_{k=1}^{K} \alpha_k$.
- $\mathbb{E}[\theta_k] = \frac{\alpha_k}{\alpha_s}$, $\mathbb{V}[\theta_k] = \frac{\alpha_k(\alpha_s - \alpha_k)}{\alpha_s^2(\alpha_s+1)}$.

Remark

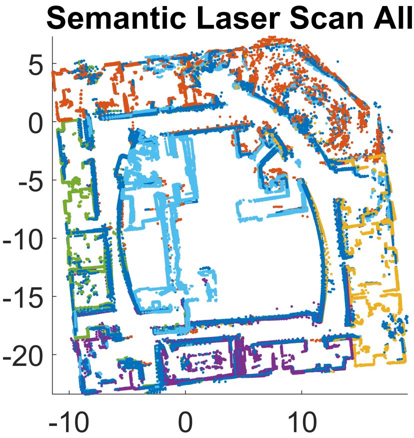
Note that if $K = 2$ the model exactly reduces to the occupancy mapping problem. Therefore, it is possible to use the general form for implementation.

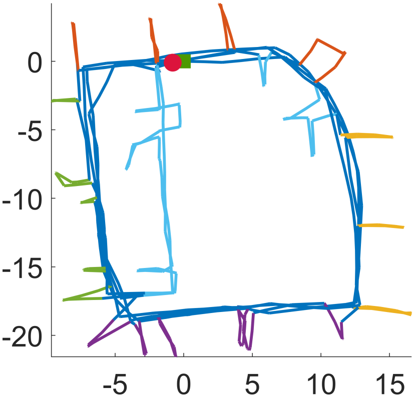
Occupancy mapping using the counting sensor model; Intel dataset.



Semantic Counting Sensor Model ($K > 2$)

Semantic occupancy mapping using the counting sensor model; Intel dataset.





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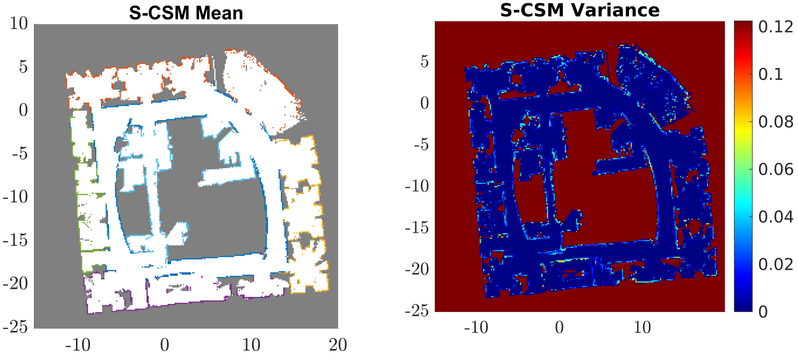


Continuous Mapping via Bayesian Generalized Kernel Inference

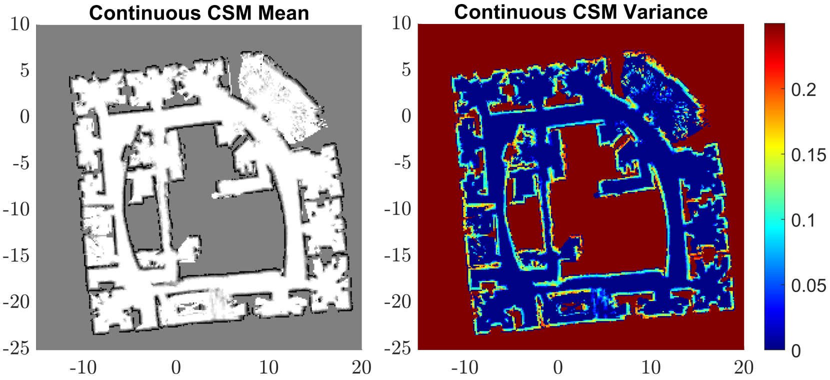
This method provides an approximation to Gaussian Processes.

- $\mathcal{X} \subset \mathbb{R}^3$ is the map spatial support.
- $k(\cdot,\cdot)$ is a kernel function operating on 3D spatial inputs $k : \mathcal{X} \times \mathcal{X} \to [0, 1]$.
- $\alpha_k^* := \alpha_k + k(x^*, x)y_k$. For any query point $x^* \in \mathcal{X}$.
- Compute $\mathbb{E}[\theta_k^*]$ and $\mathbb{V}[\theta_k^*]$ as before.

- Vega-Brown, W.R., Doniec, M. and Roy, N.G., 2014. Nonparametric Bayesian inference on multivariate exponential families. In Advances in Neural Information Processing Systems (pp. 2546-2554).
- Doherty, K., Shan, T., Wang, J. and Englot, B., 2019. Learning-Aided 3-D Occupancy Mapping With Bayesian Generalized Kernel Inference. IEEE Transactions on Robotics, 35(4), pp.953-966.

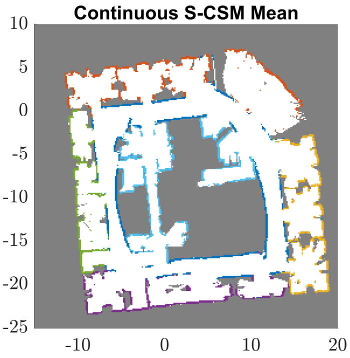
Continuous Counting Sensor Model ($K = 2$)

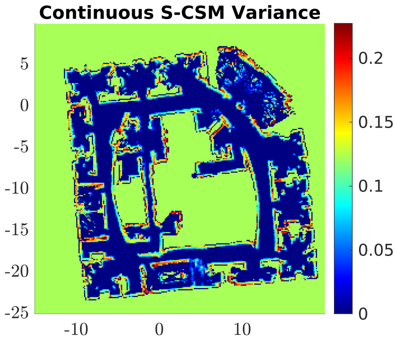
Occupancy mapping using the continuous counting sensor model; Intel dataset.



Continuous Semantic Counting Sensor Model ($K > 2$)

Semantic occupancy mapping using the continuous counting sensor model; Intel dataset.





Labeled (Segmented) Point Cloud

Q. Where do we get segmented point clouds?

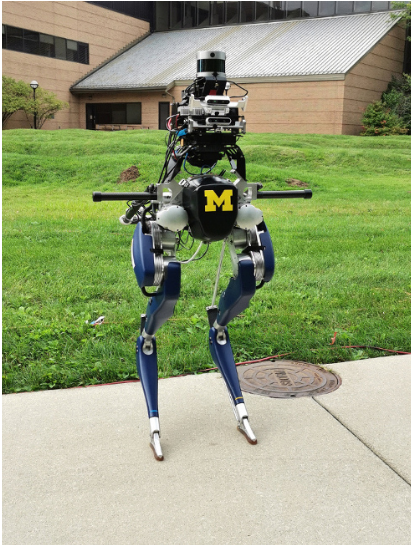
- 2D image segmentation and transferring to 3D;
- Directly labeling point clouds in 3D.

Segmented point clouds can be seen as high-level measurements or prior over a local area of the semantic map:

- Finding a "good" prior can be hard;
- Informative prior $\Longrightarrow$ better posterior or more efficient inference!
- Many advances in deep learning can be used for better initialization and prior over the state of desired random variables.

Autonomous Navigation and 3D Semantic Mapping

https://www.youtube.com/watch?v=uFyT8zCg1Kk



Software

- Continuous Semantic Occupancy Mapping:
  https://github.com/ganlumomo/BKISemanticMapping
- Continuous Occupancy Mapping:
  https://github.com/RobustFieldAutonomyLab/la3dm
- Dynamic Continuous Semantic Occupancy Mapping:
  https://github.com/UMich-CURLY/BKIDynamicSemanticMapping

The “Map in the Head”

The "Map in the Head" Metaphor

Benjamin Kuipers

First Published March 1, 1982 | Research Article
https://doi.org/10.1177/0013916584142005

Abstract

The "Map in the Head" metaphor states that knowledge of large-scale space is isomorphic to the information stored in a graphical map: That is, corresponding operations are used to store and retrieve information. The purpose of this essay is to look carefully at the "Map in the Head" metaphor to see the limits of its applicability. There are two types of experimental results that are difficult to accommodate within this metaphor. First, instead of being integrated into a single map, spatial knowledge can fall into disconnected components, with little or no relation between the components. Second, knowledge of routes (and other spatial facts) may be represented asymmetrically, so that a route can be followed in one direction but not in the other. The first set of results leads us to replace the simple "Map in the Head" with a more complex and sophisticated metaphor including separate metrical and topological components. The second set of results suggests that even the more sophisticated "Map in the Head" is built from computational structures that occasionally reveal their nonmaplike properties. A computational model is presented for assimilating observations gathered during travel, first into a description of the particular route, then into representations for the topological and metrical features of the environment.

Hybrid Spatial Semantic Hierarchy

Factoring the Mapping Problem: Mobile Robot Map-building in the Hybrid Spatial Semantic Hierarchy

Patriick Beeson Joseph Modayil Benjamin Kuipers

First Published May 19, 2009 | Research Article
https://doi.org/10.1177/0278364909100586

Abstract

We propose a factored approach to mobile robot map-building that handles qualitatively different types of uncertainty by combining the strengths of topological and metrical approaches. Our framework is based on a computational model of the human cognitive map; thus it allows robust navigation and communication within several different spatial ontologies. This paper focuses exclusively on the issue of map-building using the framework.

Our approach factors the mapping problem into natural sub-goals: building a metrical representation for local small-scale spaces; finding a topological map that represents the qualitative structure of large-scale space; and (when necessary) constructing a metrical representation for large-scale space using the skeleton provided by the topological map. We describe how to abstract a symbolic description of the robot's immediate surround from local metrical models, how to combine these local symbolic models in order to build global symbolic models, and how to create a globally consistent metrical map from a topological skeleton by connecting local frames of reference.

Keywords

mapping, localization, autonomous agents, cognitive robotics

https://www.youtube.com/watch?v=SWbofjhyPzI

3D Dynamic Scene Graphs: Actionable Spatial Perception with Places, Objects, and Humans

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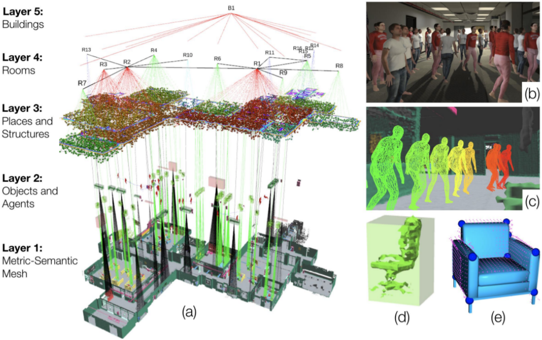

Readings & References

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