Homework 2: Gabor transforms

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AMATH 482

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Part I

1. Abstract

In this part of the homework, we will analyze a portion of Handel's Messiah with time-frequency analysis. We will use the Gabor filtering method at first with simple Gaussian. Then we test for different window width and window type of Gabor transform. Oversampling and Under-sampling will be included in the discussion. Then we will analyze two music and find their music scale.

2. Introduction and Overview

First, we import a portion of Handel's Messiah directly form MATLAB. We use Gabor transformation with different time steps on this signal to create spectrograms. By altering the Gabor filter's window width and using two different Gabor filter wavelets, the Mexican hat wavelet and a step-function (Shannon) window, we can see the variation of the spectrogram related to the time and frequency domain.

3. Theoretical Background

Gabor transformation:

The Gabor transform, named after Dennis Gabor, is a special case of the short-time Fourier transform. It is used to determine the sinusoidal frequency and phase content of local sections of a signal as it changes over time. The function to be transformed is first multiplied by a Gaussian function, which can be regarded as a window function, and the resulting function is then transformed with a Fourier transform to derive the time-frequency analysis. (wiki)

Gaussian window function:

$$g(t) = e^{-a(t-b)^2}$$

Here, a is the window width and b is the translation parameter.

the Mexican hat wavelet:

$$\frac{2}{\sqrt{3\sigma}\pi^{\frac{1}{4}}}\left(1-\left(\frac{t}{\sigma}\right)^{2}\right)e^{\frac{t^{2}}{2\sigma^{2}}}$$

In mathematics and numerical analysis, the Ricker wavelet, also known as **the Mexican hat wavelet** is the negative normalized second derivative of a Gaussian function.

σ here controlling the wavelet window width.

Step-function (Shannon) window:

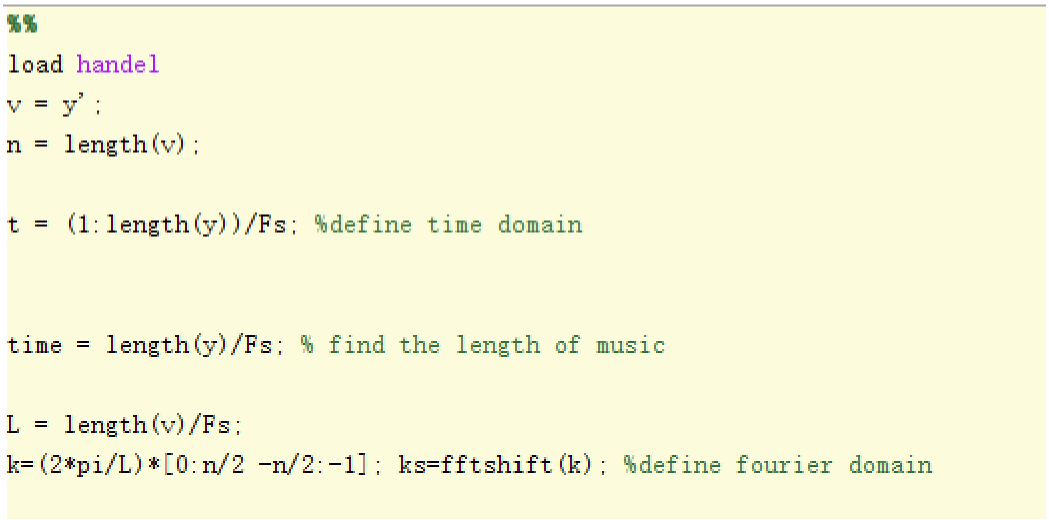
It is the function of real intervals.

$$f(x) = \begin{cases} 1 & if\ x \in A \\ 0 & if\ x \notin A \end{cases}$$

A here can control the wavelet window width.

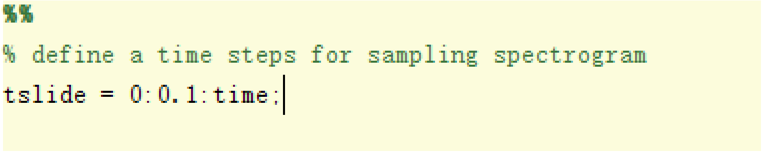
4. Algorithm Implementation and Development

First, load Handel music signal and create a Fourier domain for its frequency.



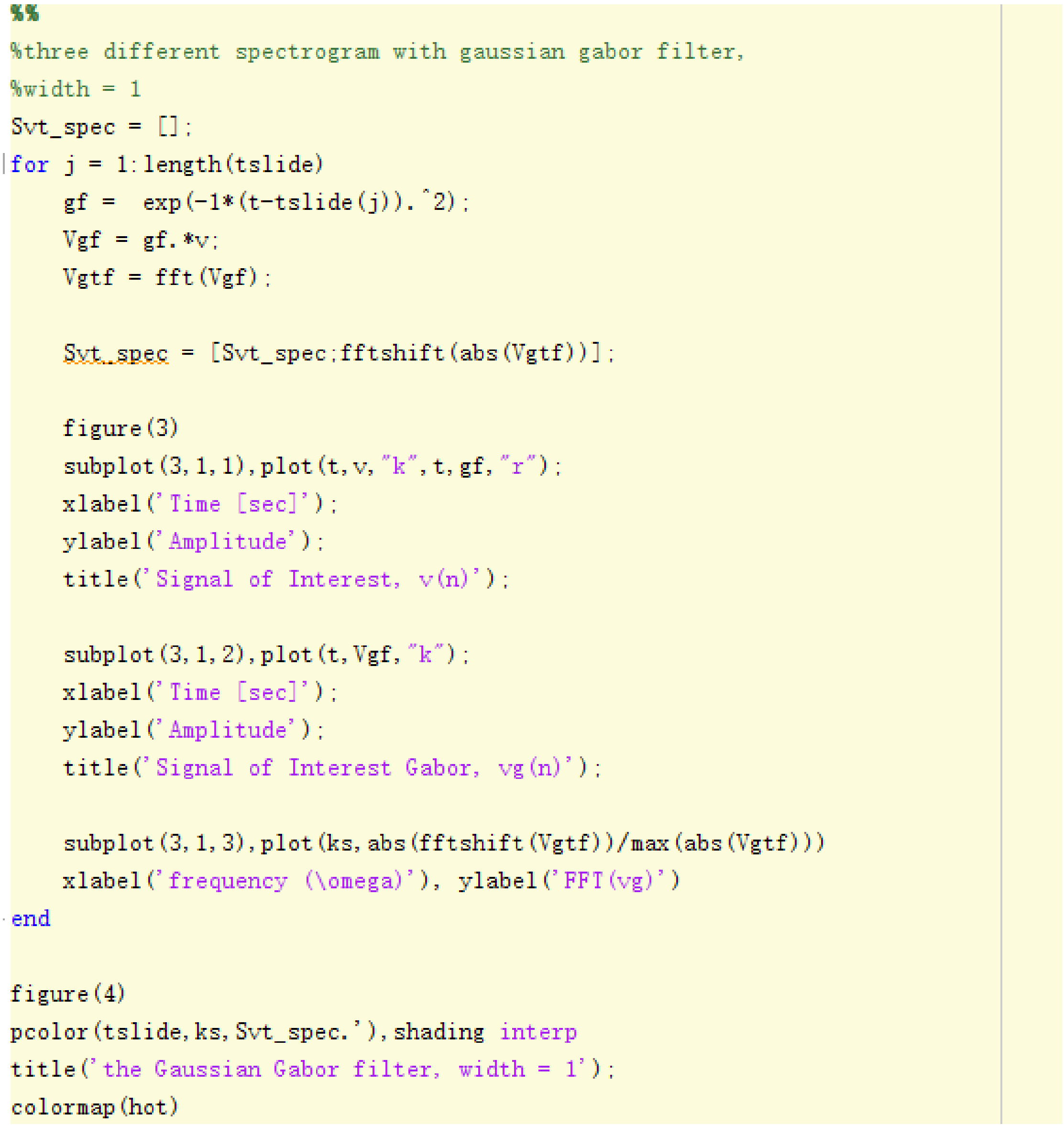
```
%%
load handel
v = y';
n = length(v);

t = (1:length(y))/Fs; %define time domain


time = length(y)/Fs; % find the length of music

L = length(v)/Fs;
k=(2*pi/L)*[0:n/2 -n/2:-1]; ks=fftshift(k); %define fourier domain
```

Define the time steps for spectrogram sampling.



```
%%
% define a time steps for sampling spectrogram
tslide = 0:0.1:time;
```

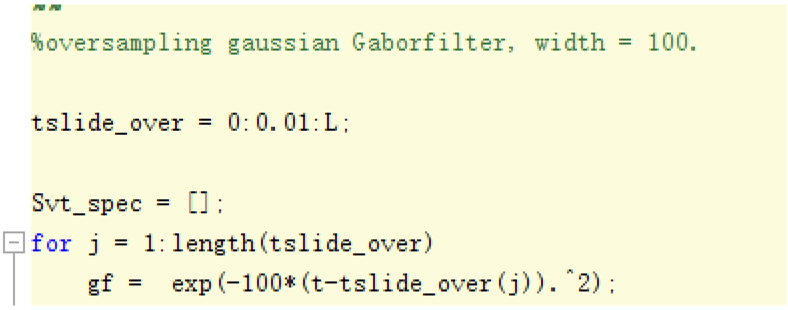
We use the Gaussian Gabor filter with width = 1 to sampling the spectrogram.



```
%%
%three different spectrogram with gaussian gabor filter,
%width = 1
Svt_spec = [];
for j = 1:length(tslide)
    gf =  exp(-1*(t-tslide(j)).^2);
    Vgf = gf.*v;
    Vgtf = fft(Vgf);

    Svt_spec = [Svt_spec;fftshift(abs(Vgtf))];

    figure(3)
    subplot(3,1,1),plot(t,v,"k",t,gf,"r");
    xlabel('Time [sec]');
    ylabel('Amplitude');
    title('Signal of Interest, v(n)');

    subplot(3,1,2),plot(t,Vgf,"k");
    xlabel('Time [sec]');
    ylabel('Amplitude');
    title('Signal of Interest Gabor, vg(n)');

    subplot(3,1,3),plot(ks,abs(fftshift(Vgtf))/max(abs(Vgtf)))
    xlabel('frequency (\omega)'), ylabel('FFT(vg)')
end

figure(4)
pcolor(tslide,ks,Svt_spec.'),shading interp
title('the Gaussian Gabor filter, width = 1');
colormap(hot)
```

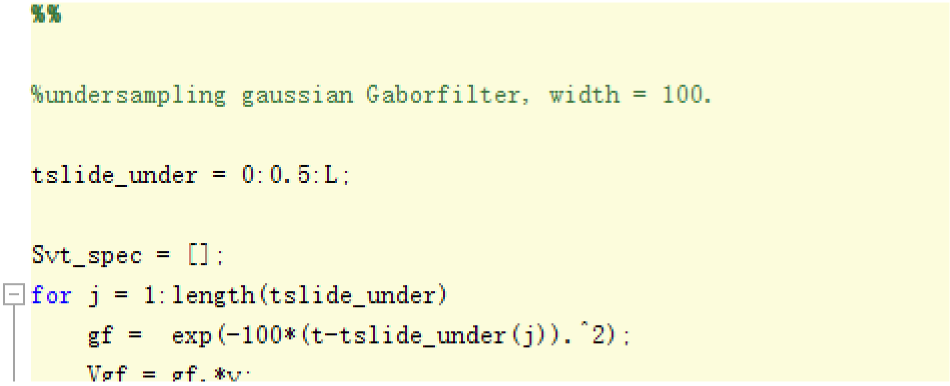
Then we change the width of the Gaussian Gabor filter to 100 and 1000 respectively.

Now we oversample the Gaussian Gabor Filter by decreasing the time interval of time steps to 0.01, $\frac{1}{10}$ of the normal value(0.1).



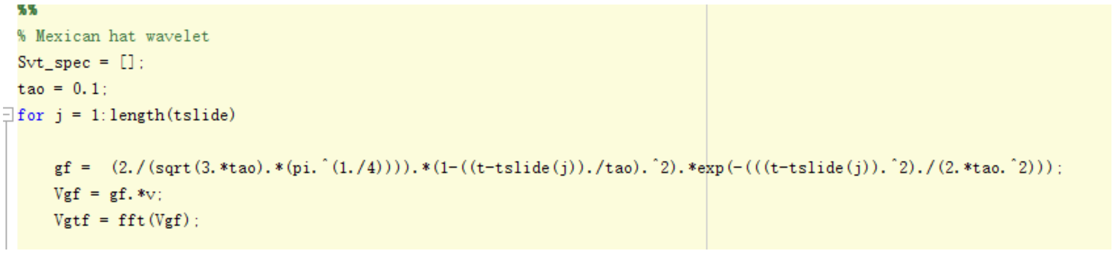
```
%%
%oversampling gaussian Gaborfilter, width = 100.

tslide_over = 0:0.01:L;

Svt_spec = [];
for j = 1:length(tslide_over)
    gf =  exp(-100*(t-tslide_over(j)).^2);
```

Then we undersample the Gaussian Gabor Filter by increasing the time interval of time steps to 0.5, 5 times of the normal value(0.1).



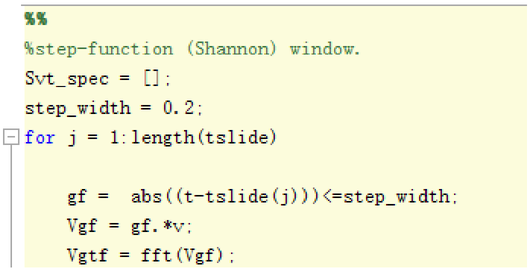
```
%%

%undersampling gaussian Gaborfilter, width = 100.

tslide_under = 0:0.5:L;

Svt_spec = [];
for j = 1:length(tslide_under)
    gf =  exp(-100*(t-tslide_under(j)).^2);
    Vgf = gf.*v;
```

We use a new wavelet, the Mexican hat wavelet, to sample the spectrogram. Using σ to control the width.



```
%%
% Mexican hat wavelet
Svt_spec = [];
tao = 0.1;
for j = 1:length(tslide)

    gf =  (2./(sqrt(3.*tao).*(pi.^(1./4)))).*(1-((t-tslide(j))./tao).^2).*exp(-(((t-tslide(j)).^2)./(2.*tao.^2)));
    Vgf = gf.*v;
    Vgtf = fft(Vgf);
```

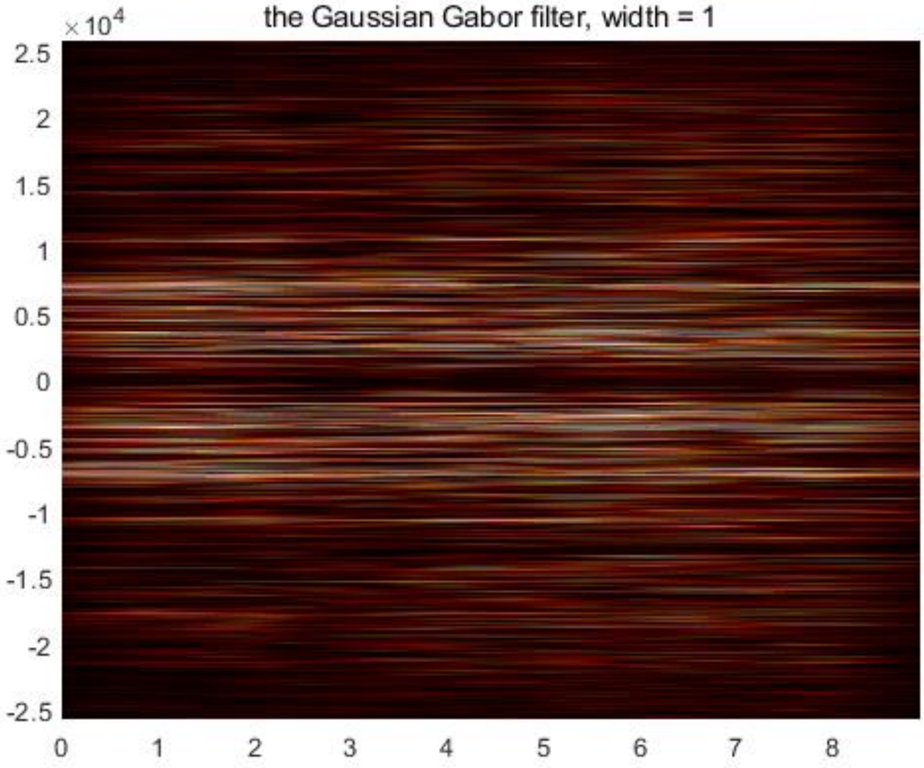
Then, we use a step-function (Shannon) window to sample the spectrogram.

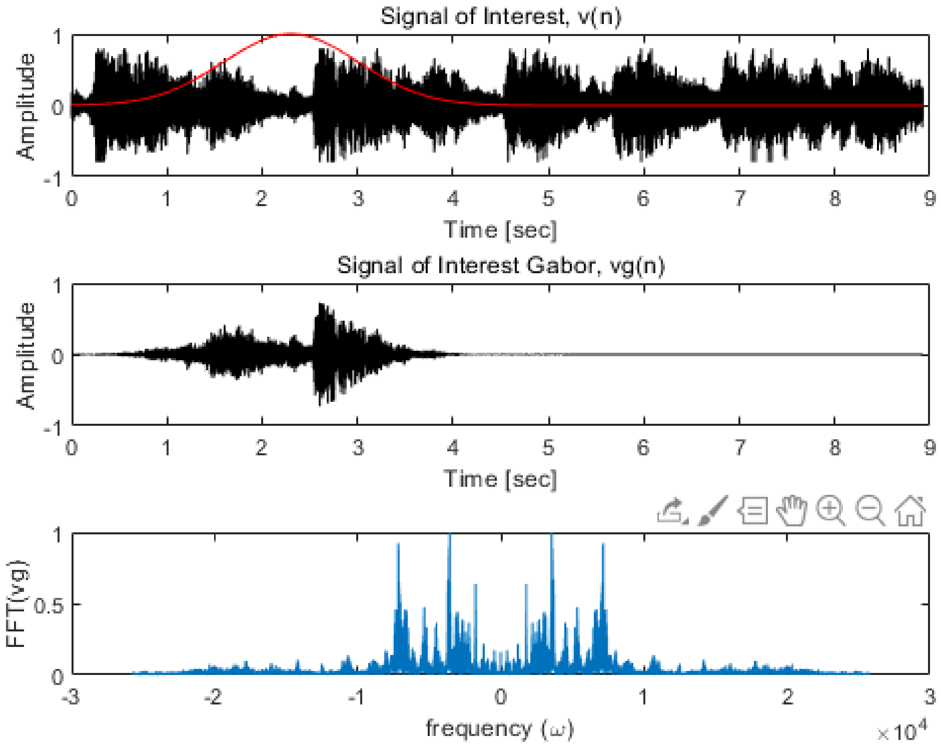


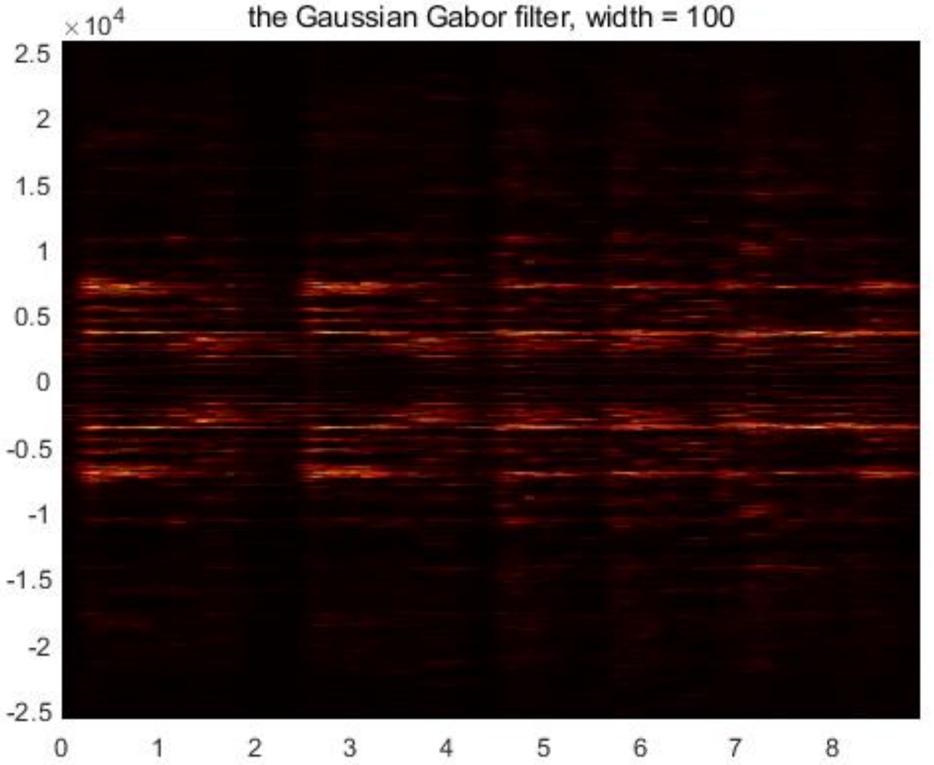
```
%%
%step-function (Shannon) window.
Svt_spec = [];
step_width = 0.2;
for j = 1:length(tslide)

    gf =  abs((t-tslide(j)))<=step_width;
    Vgf = gf.*v;
    Vgtf = fft(Vgf);
```

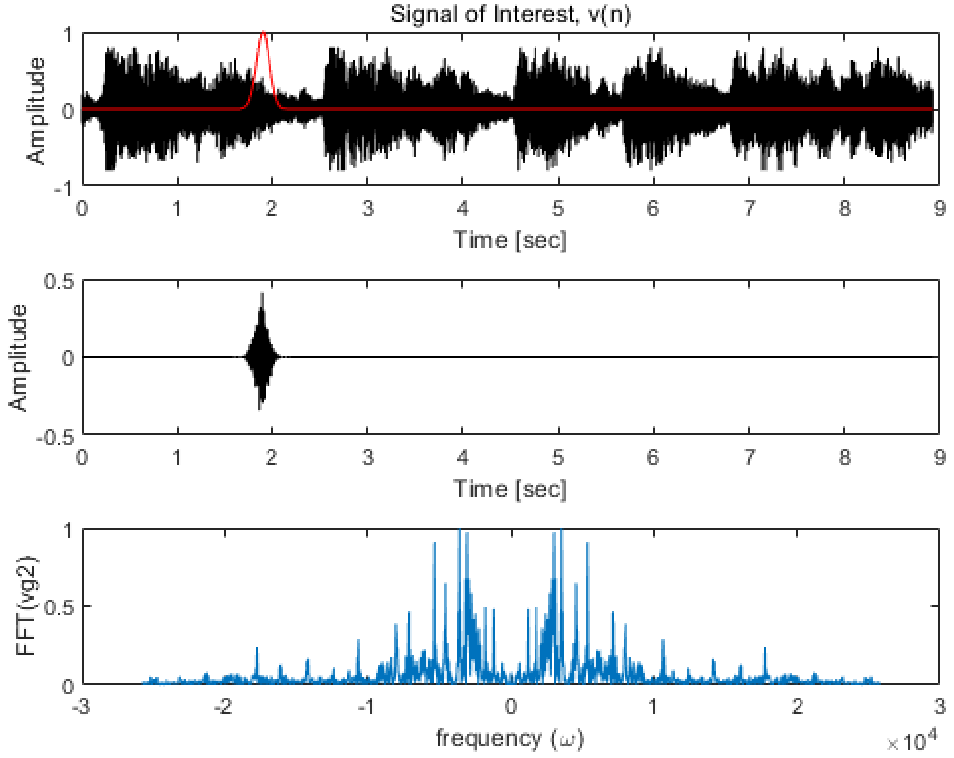
5. Computational Results

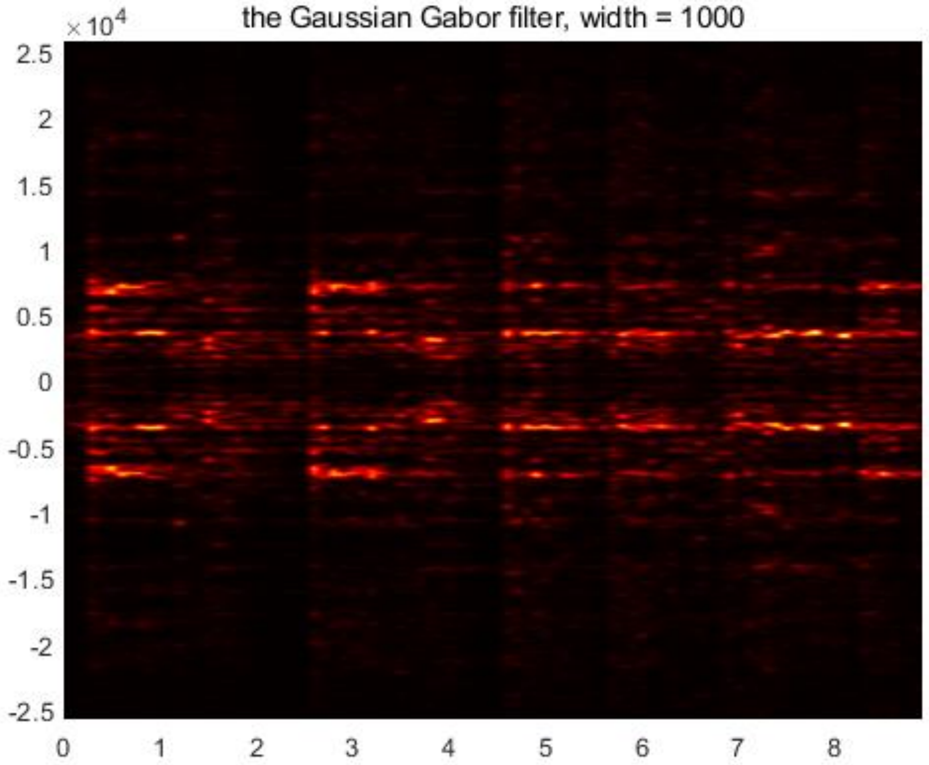
For three different width of the Gaussian Gabor Filter:

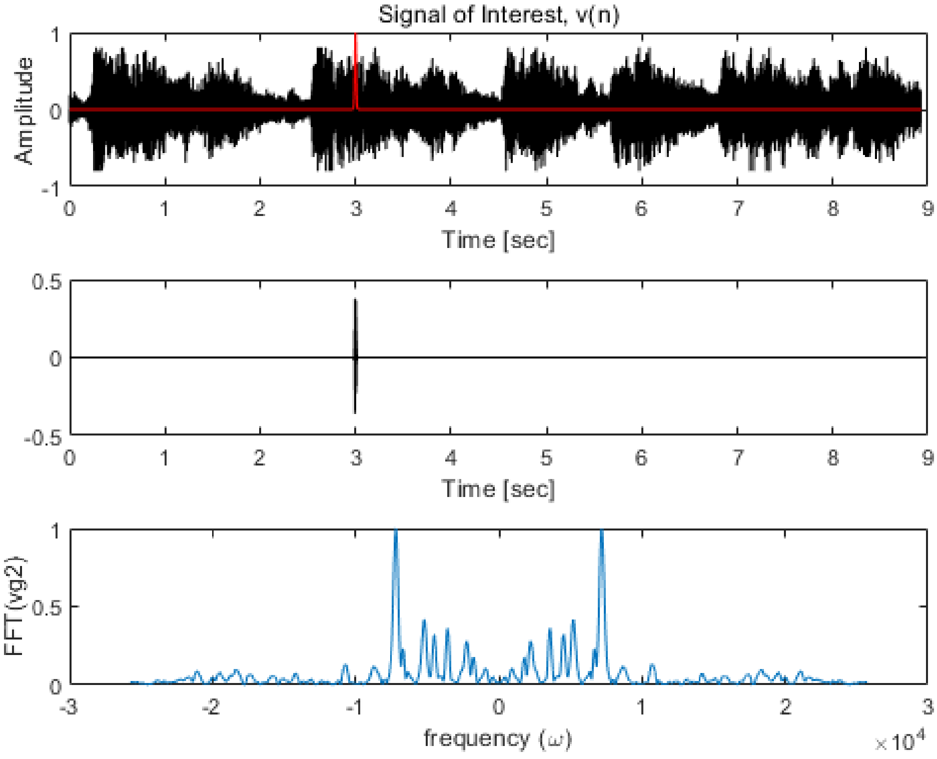




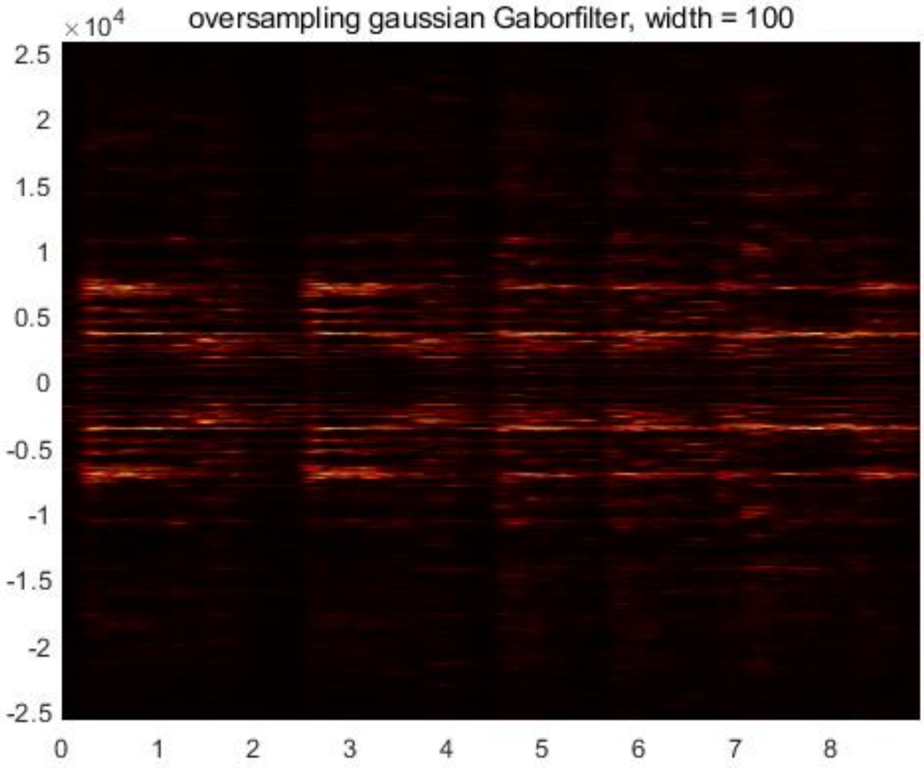

the Gaussian Gabor filter, width = 100
×10^4
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
0
1
2
3
4
5
6
7
8

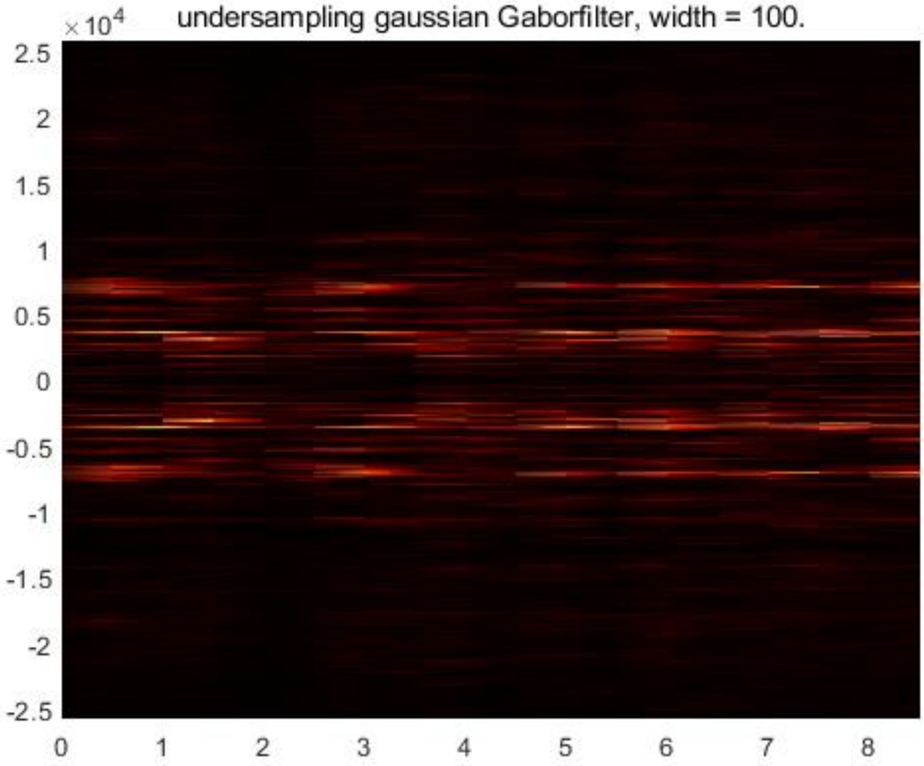

Signal of Interest, v(n)
Amplitude
1
0
-1
Time [sec]
0
1
2
3
4
5
6
7
8
9
Amplitude
0.5
0
-0.5
Time [sec]
FFT(vg2)
1
0.5
0
-3
-2
-1
0
1
2
3
frequency (ω)
×10^4


the Gaussian Gabor filter, width = 1000
×10^4
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
0
1
2
3
4
5
6
7
8

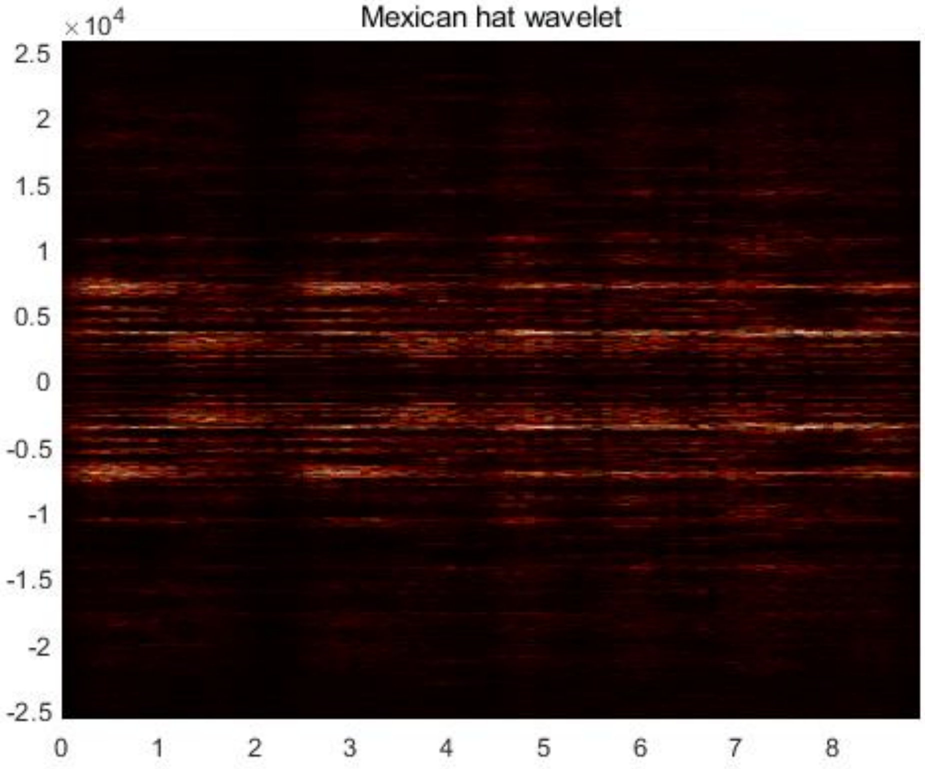

Signal of Interest, v(n)
Amplitude
1
0
-1
0
1
2
3
4
5
6
7
8
9
Time [sec]
0.5
0
-0.5
0
1
2
3
4
5
6
7
8
9
Time [sec]
FFT(vg2)
1
0.5
0
-3
-2
-1
0
1
2
3
frequency (ω)
×10^4

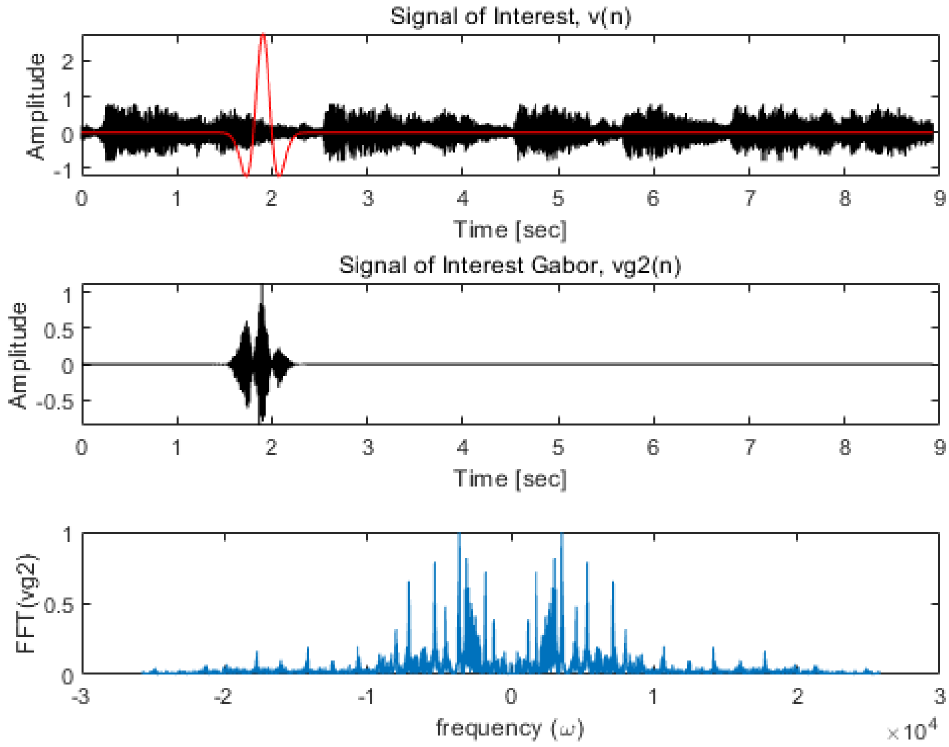
For oversampling and undersampling the Gaussian Gabor Filter, width = 100:

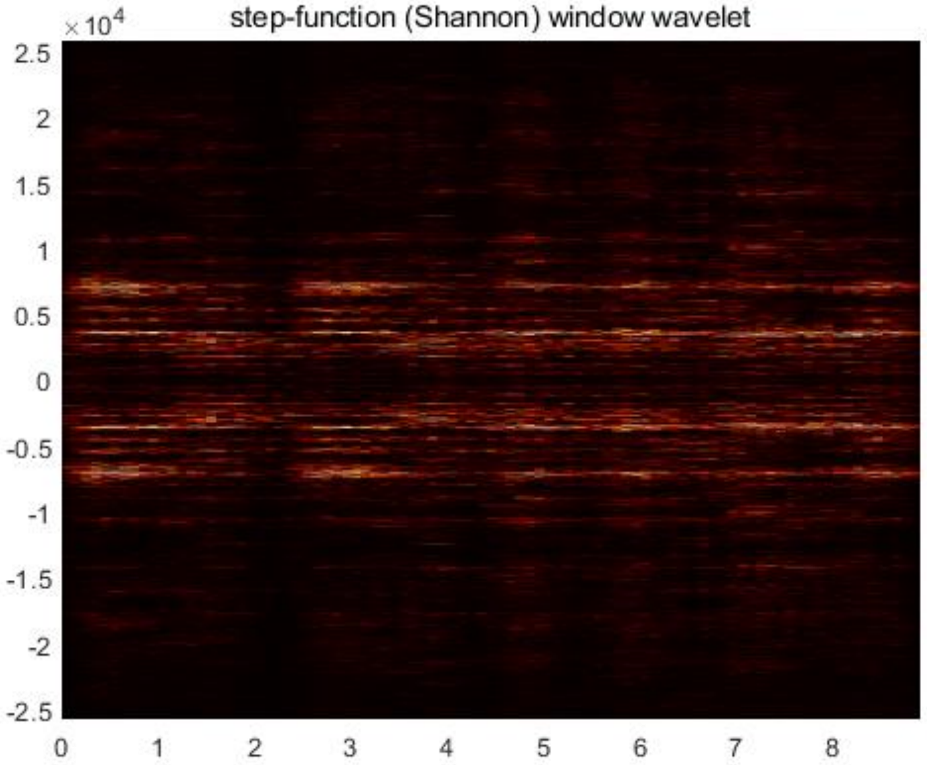


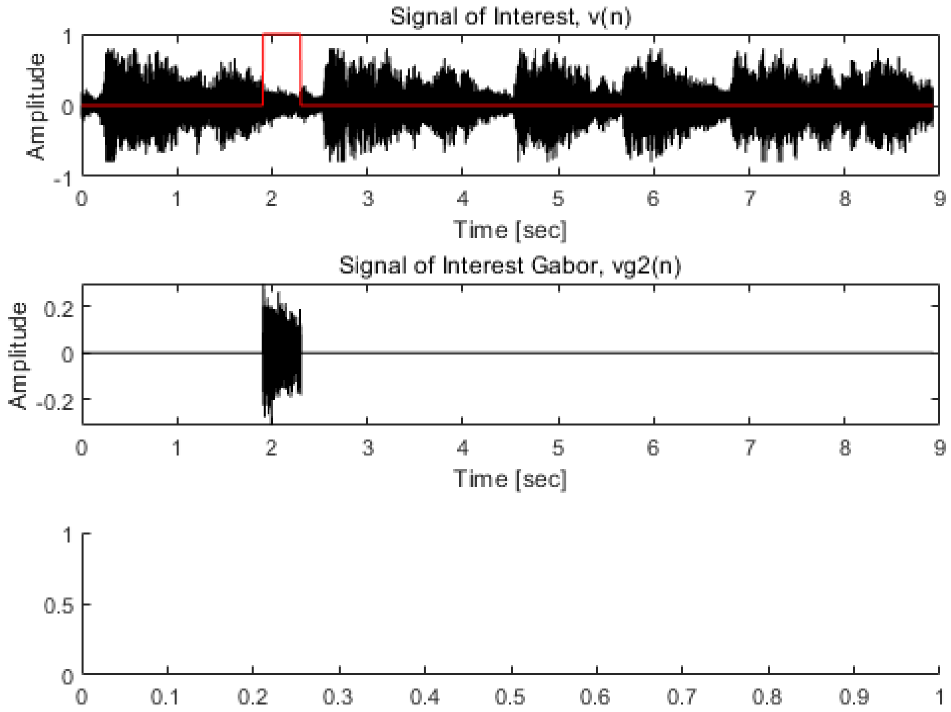


Mexican hat wavelet and step-function wavelet:






step-function (Shannon) window wavelet
× 10^4
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
0
1
2
3
4
5
6
7
8


Signal of Interest, v(n)
Amplitude
1
0
-1
0
1
2
3
4
5
6
7
8
9
Time [sec]
Signal of Interest Gabor, vg2(n)
Amplitude
0.2
0
-0.2
0
1
2
3
4
5
6
7
8
9
Time [sec]
1
0.5
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

6. Summary and Conclusions

When the only variable of the Gaussian Gabor Filter is width, the spectrogram will tell us many things. When we see a huge wide window Gaussian Gabor Filter, we lose much information on the time domain, however, it sampled more information on the frequency domain. On the contrary, when we see a small window width Gaussian Gabor Filter, we lose much information on the frequency domain, however, it sampled more information on the time domain. So, a proper window width is critical to get a clear spectrogram and analyze information. Oversampling will create more accuracy in both time and frequency domain.

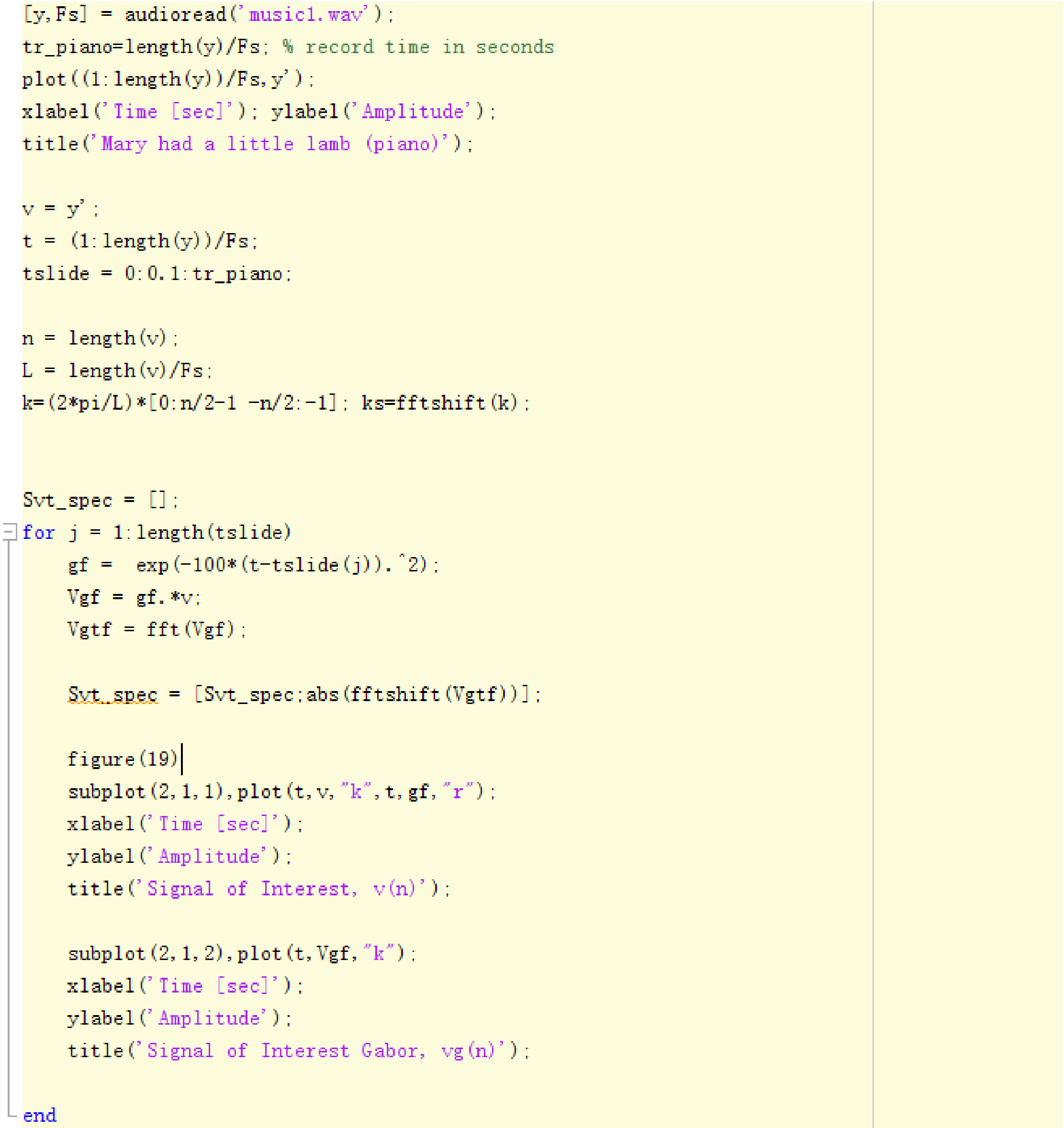
Using different wavelets can acquire more information in the frequency domain.

Part II

1. Introduction and Overview

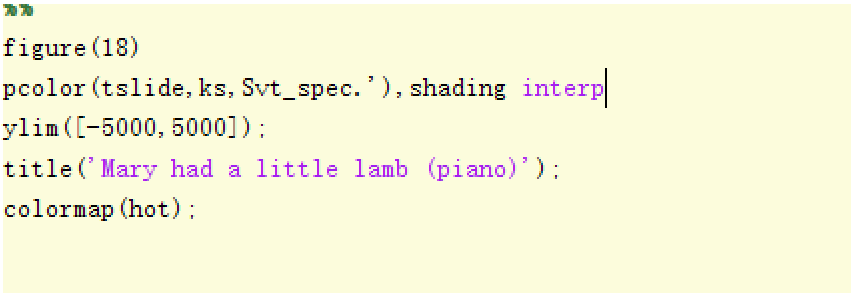
We will use the same sampling skill from part I to find two music, which the same pattern played by two different musical instruments. By analyzing their spectrogram, we will see each musical scale lying on their frequency. Then, determined the scale by checking the datasheet

2. Algorithm Implementation and Development



```
[y,Fs] = audioread('music1.wav');
tr_piano=length(y)/Fs; % record time in seconds
plot((1:length(y))/Fs,y');
xlabel('Time [sec]'); ylabel('Amplitude');
title('Mary had a little lamb (piano)');

v = y';
t = (1:length(y))/Fs;
tslide = 0:0.1:tr_piano;

n = length(v);
L = length(v)/Fs;
k=(2*pi/L)*[0:n/2-1 -n/2:-1]; ks=fftshift(k);


Svt_spec = [];
for j = 1:length(tslide)
    gf =  exp(-100*(t-tslide(j)).^2);
    Vgf = gf.*v;
    Vgtf = fft(Vgf);

    Svt_spec = [Svt_spec;abs(fftshift(Vgtf))];

    figure(19)
    subplot(2,1,1),plot(t,v,"k",t,gf,"r");
    xlabel('Time [sec]');
    ylabel('Amplitude');
    title('Signal of Interest, v(n)');

    subplot(2,1,2),plot(t,Vgf,"k");
    xlabel('Time [sec]');
    ylabel('Amplitude');
    title('Signal of Interest Gabor, vg(n)');

end
```

We first import our music to Matlab by using function audioread. Then we analyze it by using the Gaussian Gabor filter with width 100.

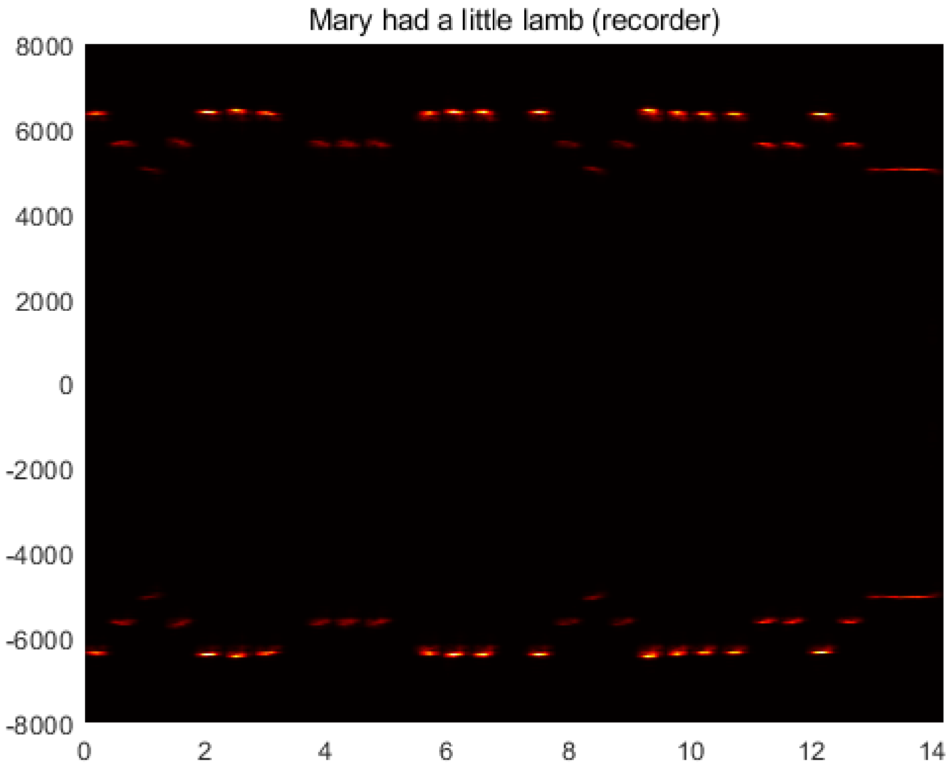


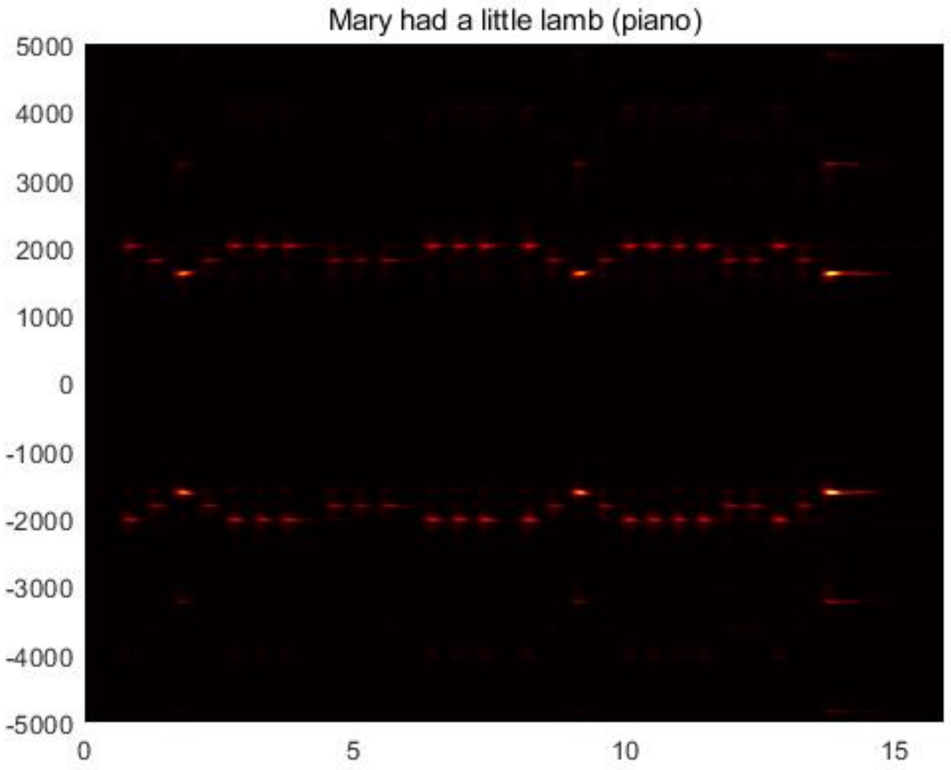
```
%%
figure(18)
pcolor(tslide,ks,Svt_spec.'),shading interp
ylim([-5000,5000]);
title('Mary had a little lamb (piano)');
colormap(hot);
```

We draw a spectrogram of this music.

Same procedural for music2.

3. Computational Results





4. Summary and Conclusions

We see that the timbre of that two pieces of music are different since the frequency range of mucic1 is much smaller than the one of music2.

$$\omega_{piano} = \frac{1}{3}\omega_{recorder}$$

Appendix A. MATLAB functions used and brief implementation explanation

length(v)	Find the length of variable v, in the float.
fftshift(v)	rearranges a Fourier transform v by shifting the zero-frequency component to the center of the array.
pcolor(v), shading	creates a pseudocolor plot using the values in matrix C. shading changes the shading model.
plot(X,Y)	plot(X,Y) creates a 2-D line plot of the data in Y versus the corresponding values in X.
colormap(map)	Colormap(map) sets the colormap for the current figure to one of the predefined colormaps.
[y,Fs] = audioread(filename)	[y,Fs] = audioread(filename) reads data from the file named filename, and returns sampled data, y, and a sample rate for that data, Fs.

Appendix B. MATLAB codes

```
clear all; close all; clc

load handel

v = y’;

n = length(v);

t = (1:length(y))/Fs; %define time domain

time = length(y)/Fs; % find the length of music

L = length(v)/Fs;

k=(2*pi/L)*[0:n/2 -n/2:-1]; ks=fftshift(k); %define fourier domain
```

define a time steps for sampling spectrogram

```
tslide = 0:0.1:time;

%three different spectrogram with gaussian gabor filter,

%width = 1

Svt_spec = [];

for j = 1:length(tslide)

    gf =  exp(-1*(t-tslide(j)).^2);

    Vgf = gf.*v;

    Vgtf = fft(Vgf);
    Svt_spec = [Svt_spec;fftshift(abs(Vgtf))];

    figure(3)

    subplot(3,1,1),plot(t,v,"k",t,gf,"r");

    xlabel('Time [sec]');

    ylabel('Amplitude');

    title('Signal of Interest, v(n)');

    subplot(3,1,2),plot(t,Vgf,"k");

    xlabel('Time [sec]');

    ylabel('Amplitude');

    title('Signal of Interest Gabor, vg(n)');

    subplot(3,1,3),plot(ks,abs(fftshift(Vgtf))/max(abs(Vgtf)))

    xlabel('frequency (\omega)'), ylabel('FFT(vg)')

end



figure(4)

pcolor(tslide,ks,Svt_spec.'),shading interp

title('the Gaussian Gabor filter, width = 1');

colormap(hot)
```

```
%width = 100

Svt_spec2 = [];

for j = 1:length(tslide)

    gf =  exp(-100*(t-tslide(j)).^2);

    Vgf = gf.*v;

    Vgtf = fft(Vgf);

    Svt_spec2 = [Svt_spec2;abs(fftshift(Vgtf))];

    figure(5)

    subplot(3,1,1),plot(t,v,"k",t,gf,"r");

    xlabel('Time [sec]');

    ylabel('Amplitude');

    title('Signal of Interest, v(n)');

    subplot(3,1,2),plot(t,Vgf,"k");

    xlabel('Time [sec]');

    ylabel('Amplitude');

    title('Signal of Interest Gabor, vg2(n)');

    subplot(3,1,3),plot(ks,abs(fftshift(Vgtf))/max(abs(Vgtf)))

    xlabel('frequency (\omega)'), ylabel('FFT(vg2)')
```

```
end
figure(6)
pcolor(tslide,ks,Svt_spec2.’),shading interp
title(’the Gaussian Gabor filter, width = 100’);
colormap(hot)
```

```
%width = 1000
Svt_spec3 = [];
for j = 1:length(tslide)
    gf =  exp(-10000*(t-tslide(j)).^2);
    Vgf = gf.*v;
    Vgtf = fft(Vgf);
    Svt_spec3 = [Svt_spec3;abs(fftshift(Vgtf))];
    figure(7)
    subplot(3,1,1),plot(t,v,″k″,t,gf,″r″);
    xlabel(’Time [sec]’);
    ylabel(’Amplitude’);
    title(’Signal of Interest, v(n)’);
    subplot(3,1,2),plot(t,Vgf,″k″);
    xlabel(’Time [sec]’);
    ylabel(’Amplitude’);
    title(’Signal of Interest Gabor, vg2(n)’);
    subplot(3,1,3),plot(ks,abs(fftshift(Vgtf))/max(abs(Vgtf)))
    xlabel(’frequency (\omega)’), ylabel(’FFT(vg2)’)
end
figure(8)
pcolor(tslide,ks,Svt_spec3.’),shading interp
title(’the Gaussian Gabor filter, width = 1000’);
colormap(hot);
```

```
%oversampling gaussian Gaborfilter, width = 100.
tslide_over = 0:0.01:L;
Svt_spec = [];
for j = 1:length(tslide_over)
    gf =  exp(-100*(t-tslide_over(j)).^2);
    Vgf = gf.*v;
    Vgtf = fft(Vgf);
    Svt_spec = [Svt_spec;abs(fftshift(Vgtf))];
    figure(9)
    subplot(3,1,1),plot(t,v,"k",t,gf,"r");
    xlabel(’Time [sec]’);
    ylabel(’Amplitude’);
    title(’Signal of Interest, v(n)’);
    subplot(3,1,2),plot(t,Vgf,"k");
    xlabel(’Time [sec]’);
    ylabel(’Amplitude’);
    title(’Signal of Interest Gabor, vg(n)’);
    subplot(3,1,3),plot(ks,abs(fftshift(Vgtf))/max(abs(Vgtf)))
    xlabel(’frequency (\omega)’), ylabel(’FFT(vg2)’)
end
figure(10)
pcolor(tslide_over,ks,Svt_spec.’),shading interp
title(’oversampling gaussian Gaborfilter, width = 100’);
colormap(hot);
```

```
%undersampling gaussian Gaborfilter, width = 100.
tslide_under = 0:0.5:L;
```

```
Svt_spec = [];
for j = 1:length(tslide_under)
    gf =  exp(-100*(t-tslide_under(j)).^2);
    Vgf = gf.*v;
    Vgtf = fft(Vgf);
    Svt_spec = [Svt_spec;abs(fftshift(Vgtf))];
    figure(11)
    subplot(3,1,1),plot(t,v,"k",t,gf,"r");
    xlabel('Time [sec]');
    ylabel('Amplitude');
    title('Signal of Interest, v(n)');
    subplot(3,1,2),plot(t,Vgf,"k");
    xlabel('Time [sec]');
    ylabel('Amplitude');
    title('Signal of Interest Gabor, vg(n)');
    subplot(3,1,3),plot(ks,abs(fftshift(Vgtf))/max(abs(Vgtf)))
    xlabel('frequency (\omega)'), ylabel('FFT(vg2)')
end
figure(12)
```

```
pcolor(tslide_under,ks,Svt_spec.'),shading interp

title('undersampling gaussian Gaborfilter, width = 100.');

colormap(hot);
```

Mexican hat wavelet

```
Svt_spec = [];

tao = 0.1;

for j = 1:length(tslide)

    gf =  (2./(sqrt(3.*tao).*(pi.^(1./4)))).*(1-((t-tslide(j))./tao).^2).*exp(-(((t-
tslide(j)).^2)./(2.*tao.^2)));

    Vgf = gf.*v;

    Vgtf = fft(Vgf);

    Svt_spec = [Svt_spec;abs(fftshift(Vgtf))];

    figure(13)

    subplot(3,1,1),plot(t,v,"k",t,gf,"r");

    xlabel('Time [sec]');

    ylabel('Amplitude');

    title('Signal of Interest, v(n)');

    subplot(3,1,2),plot(t,Vgf,"k");
    xlabel(’Time [sec]’);
    ylabel(’Amplitude’);
    title(’Signal of Interest Gabor, vg2(n)’);
    subplot(3,1,3),plot(ks,abs(fftshift(Vgtf))/max(abs(Vgtf)))
    xlabel(’frequency (\omega)’), ylabel(’FFT(vg2)’)
end
figure(14)
pcolor(tslide,ks,Svt_spec.’),shading interp
title(’Mexican hat wavelet’);
colormap(hot);
```

```
%step-function (Shannon) window.
Svt_spec = [];
step_width = 0.2;
for j = 1:length(tslide)
    gf =  abs((t-tslide(j)))<=step_width;
    Vgf = gf.*v;
    Vgtf = fft(Vgf);
    Svt_spec = [Svt_spec;abs(fftshift(Vgtf))];
    figure(15)
    subplot(3,1,1),plot(t,v,"k",t,gf,"r");
    xlabel(’Time [sec]’);
    ylabel(’Amplitude’);
    title(’Signal of Interest, v(n)’);
    subplot(3,1,2),plot(t,Vgf,"k");
    xlabel(’Time [sec]’);
    ylabel(’Amplitude’);
    title(’Signal of Interest Gabor, vg2(n)’);
    subplot(3,1,3),plot(ks,abs(fftshift(Vgtf))/max(abs(Vgtf)))
    xlabel(’frequency (\omega)’), ylabel(’FFT(vg2)’)
end
figure(16)
pcolor(tslide,ks,Svt_spec.’),shading interp
title(’step-function (Shannon) window wavelet’);
colormap(hot);
```

```
clear all; close all; clc
%Part II
```

```
figure(17)
[y,Fs] = audioread(’music1.wav’);
tr_piano=length(y)/Fs; % record time in seconds
plot((1:length(y))/Fs,y’);
xlabel(’Time [sec]’); ylabel(’Amplitude’);
title(’Mary had a little lamb (piano)’);
v = y’;
t = (1:length(y))/Fs;
tslide = 0:0.1:tr_piano;
n = length(v);
L = length(v)/Fs;
k=(2*pi/L)*[0:n/2-1 -n/2:-1]; ks=fftshift(k);
Svt_spec = [];
for j = 1:length(tslide)
    gf =  exp(-100*(t-tslide(j)).ˆ2);
    Vgf = gf.*v;
    Vgtf = fft(Vgf);
    Svt_spec = [Svt_spec;abs(fftshift(Vgtf))];
    figure(19)
    subplot(2,1,1),plot(t,v,″k″,t,gf,″r″);
    xlabel(’Time [sec]’);
    ylabel(’Amplitude’);
    title(’Signal of Interest, v(n)’);
    subplot(2,1,2),plot(t,Vgf,″k″);
    xlabel(’Time [sec]’);
    ylabel(’Amplitude’);
    title(’Signal of Interest Gabor, vg(n)’);
end
```

```
figure(18)
pcolor(tslide,ks,Svt_spec.’),shading interp
ylim([-5000,5000]);
title(’Mary had a little lamb (piano)’);
colormap(hot);
```

```
figure(19)
[y,Fs] = audioread(’music2.wav’);
```

```
tr_rec=length(y)/Fs; % record time in seconds

plot((1:length(y))/Fs,y);

xlabel('Time [sec]'); ylabel('Amplitude');

title('Mary had a little lamb (recorder)');

v = y';

t = (1:length(y))/Fs;

tslide = 0:0.1:tr_rec;

n = length(v);

L = length(v)/Fs;

k=(2*pi/L)*[0:n/2-1 -n/2:-1]; ks=fftshift(k);

Svt_spec = [];

for j = 1:length(tslide)

    gf =  exp(-100*(t-tslide(j)).^2);

    Vgf = gf.*v;

    Vgtf = fft(Vgf);

    Svt_spec = [Svt_spec;abs(fftshift(Vgtf))];

    figure(20)

    subplot(2,1,1),plot(t,v,"k",t,gf,"r");

    xlabel('Time [sec]');
    ylabel('Amplitude');
    title('Signal of Interest, v(n)');
    subplot(2,1,2),plot(t,Vgf,"k");
    xlabel('Time [sec]');
    ylabel('Amplitude');
    title('Signal of Interest Gabor, vg(n)');
end
```

```
figure(21)
pcolor(tslide,ks,Svt_spec.'),shading interp
ylim([-8000,8000]);
title('Mary had a little lamb (recorder)');
colormap(hot);
```

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