Homework 1: An ultrasound problem

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1. Abstract

In this homework, we want to get information from a noisy data signal set by using averaging and filtering. I prefer to use Fourier transforms to translate a signal to the frequency domain and averaged over time. After that, I prefer to use the Gaussian function to remover noise. Finally, a clear data set will be remained and available for us to use.

2. In Introduction and Overview

Your dog fluffy swallowed a marble. The vet suspects that it has now worked its way into the intestines. Using ultrasound, data is obtained concerning the spatial variations in a small area of the intestines where the marble is suspected to be. Unfortunately, fluffy keeps moving and the internal fluid movement through the intestines generates highly noisy data.

We can assume the frequency in ultrasound data is random since it comes from fluid movement – a random movement. So, we can use averaging to clear the random frequencies and leaving marble frequency only. Also, a filter can help us to get more precise marble data. Finally, we can convert it back to time base to see the movement of marble in time.

3. Theoretical Background.

In this homework, we use two main concepts: Fourier Transform and Gaussian filtering.

3.1 Fourier Transform

The Fourier transform (FT) decomposes a function of time (a signal) into its constituent frequencies. The Fourier transform of a function of time is itself a complex-valued function of frequency, whose magnitude (modulus) represents the amount of that frequency present in the original function, and whose argument is the phase offset of the basic sinusoid in that frequency.(from wiki)

For example:

We have a function $F(x)$ in time base and converting it to a function $F(k)$ in frequency base.

$$F(k) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-ikx} f(x)dx \quad (eq. 3.1.1)$$

Also, the Inverse Fourier Transform dose:

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{ikx} F(k)dk \quad (eq. 3.1.2)$$

In practice, we use Fast Fourier Transform, since it is fast in actual computation. It computes in $O(NlogN)$ order rather than $O(N^2)$. In the presence of round-off error, many FFT algorithms are much more accurate than evaluating the DFT definition directly or indirectly.

3.2 Gaussian filtering

A Gaussian filter is a filter whose impulse response is a Gaussian function. We can multiply our frequency signal by the Gaussian function to attenuate our signal, leaving only the data we need.

$$e^{-\tau(K-k_0)^2} \qquad (eq. 3.2)$$

We can use this function smooth our frequency effectively.

4. Algorithm Implementation and Development.

In this homework, we use MATLAB as the implementation tool.

4.1 Setup

First, import our dataset.

```
clear; close all; clc;
load Testdata
```

4.2 Define the Domain

```
L=15; % spatial domain
n=64; % Fourier modes
x2=linspace(-L,L,n+1); x=x2(1:n); y=x; z=x;
k=(2*pi/(2*L))*[0:(n/2-1) -n/2:-1]; ks=fftshift(k);

[X,Y,Z]=meshgrid(x,y,z);
[Kx,Ky,Kz]=meshgrid(ks,ks,ks)
```