Quantum Physics

2025

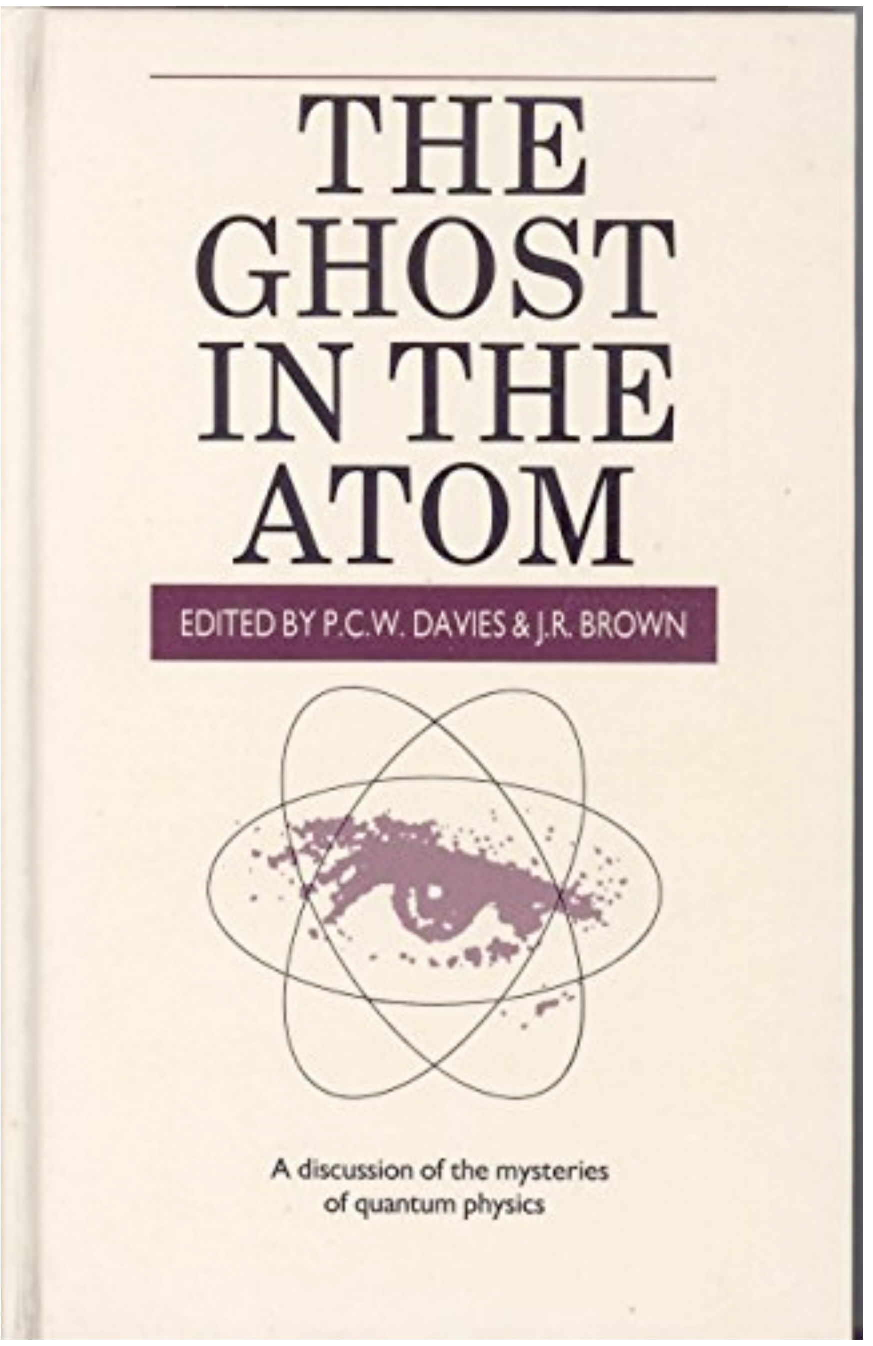
The Theory/Framework Of *Almost* Everything *Today*

But Most Likely NOT of Tomorrow

Yury Deshko

"I'm quite convinced of that: quantum theory is only a temporary expedient."

John Bell in "*The Ghost In The Atom*".



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Course Overview

Course Structure And Goals

- **Part 1** : Mathematical Concepts And Tools.
- **Part 2** : Classical Physics.
- **Part 3** : Quantum Physics.

- Learn the language of quantum physics.
- Enhance the knowledge of classical physics.
- Develop modern quantum thinking.

“Bundling”

Of Mathematical Objects

1. Simple and powerful method of creating new mathematical tools.
2. Comes in different forms, most of which have the names of “*product*” or “*sum*”.
3. Works on mathematical objects of various kinds: can “bundle” not only “apples to apples”, but also “apples to oranges”!

Simple Example

Bundling Of Numbers

- Rational numbers are essentially bundles: $\dfrac{2}{3} = (2,3), \dfrac{p}{q} = (p,q)$ — bundling integers.
- Algebraic way of representing arrow-vectors using components uses bundling: $|a\rangle = a_1 |e_1\rangle + a_2 |e_2\rangle \sim (a_1, a_2)$.
- Complex numbers are essentially arrow-vectors in a plane, so they are bundles too: $z = x + iy = x + \hat{J}y \sim (x, y)$

NOTE: Both parts of the bundle are equally important. They do not "mix" and do not disappear. They can be "extracted" from the bundle.

$$a_i = \langle e_i | a \rangle \qquad x = \operatorname{Re} z \qquad y = \operatorname{Im} z$$

Compare this to $|a\rangle \bullet |b\rangle = s$. Two vectors are replaced with a single number. You can't reconstruct original vectors anymore!

Simple Example

Bundling Of Vectors

- We can "combine" ket-vectors ($|a\rangle, |b\rangle$) — "apples to apples."
- We can "combine" bra-vectors ($\langle a|, \langle b|$) — "oranges to oranges."
- We can "combine" bra-vectors and ket-vectors ($\langle a|, |b\rangle$) — "oranges to apples."
- We can "combine" ket-vectors and bra-vectors ($|a\rangle, \langle b|$) — "oranges to apples.

BENEFIT: This way we can keep information about different quantum systems in one place

Think of this like bundling two different books
($book_1$, $book_2$)

Simple Example Extended

Bundling Of Operators

- We can even “combine” operators $(\hat{H}, \hat{J})$ — “spoons to spoons.”
- (**BUT**) We **don’t** usually “combine” vectors and operators $(\langle a \,|\,, \hat{J})$ — “oranges to spoons.”

Sum

Direct Sum $\oplus$

- We can "combine" vectors of the same kind ("apples to apples", ket-with-ket) in a straightforward manner: $|C\rangle = |a\rangle \oplus |b\rangle$. The new object is also a ket vector but of *higher dimension*. In components, if $|a\rangle \sim (a_1, a_2)$ and $|b\rangle \sim (b_1, b_2)$, then $|C\rangle \sim (a_1, a_2, b_1, b_2) = (c_1, c_2, c_3, c_4)$
- Dimensions simply **add**, hence **sum**.

$book_1 \oplus book_2$

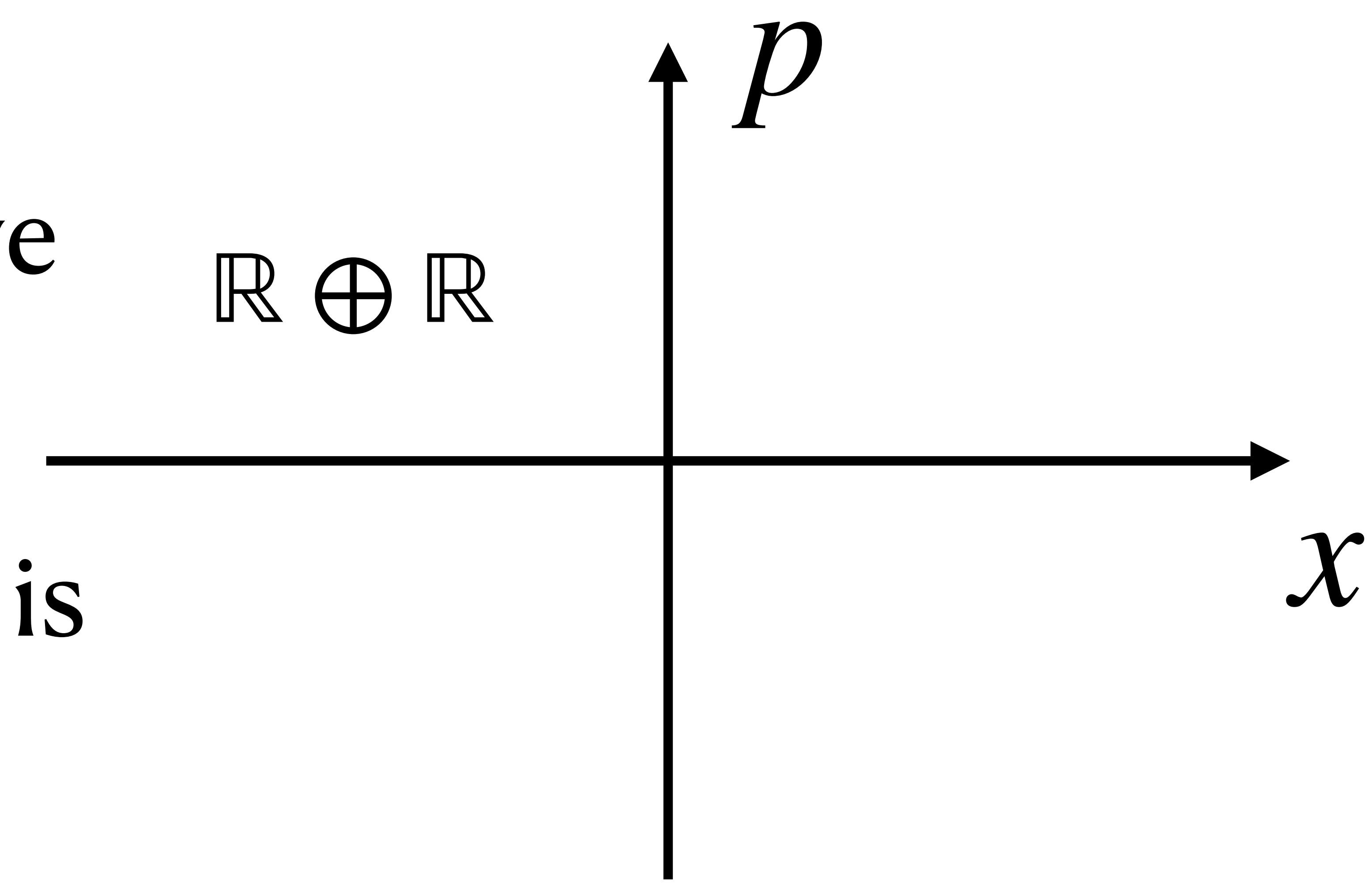
Direct sum of books — new longer book. Number of pages added.

$book_1 \oplus book_2 \oplus book_3 \oplus book_4$

Direct Sum Example

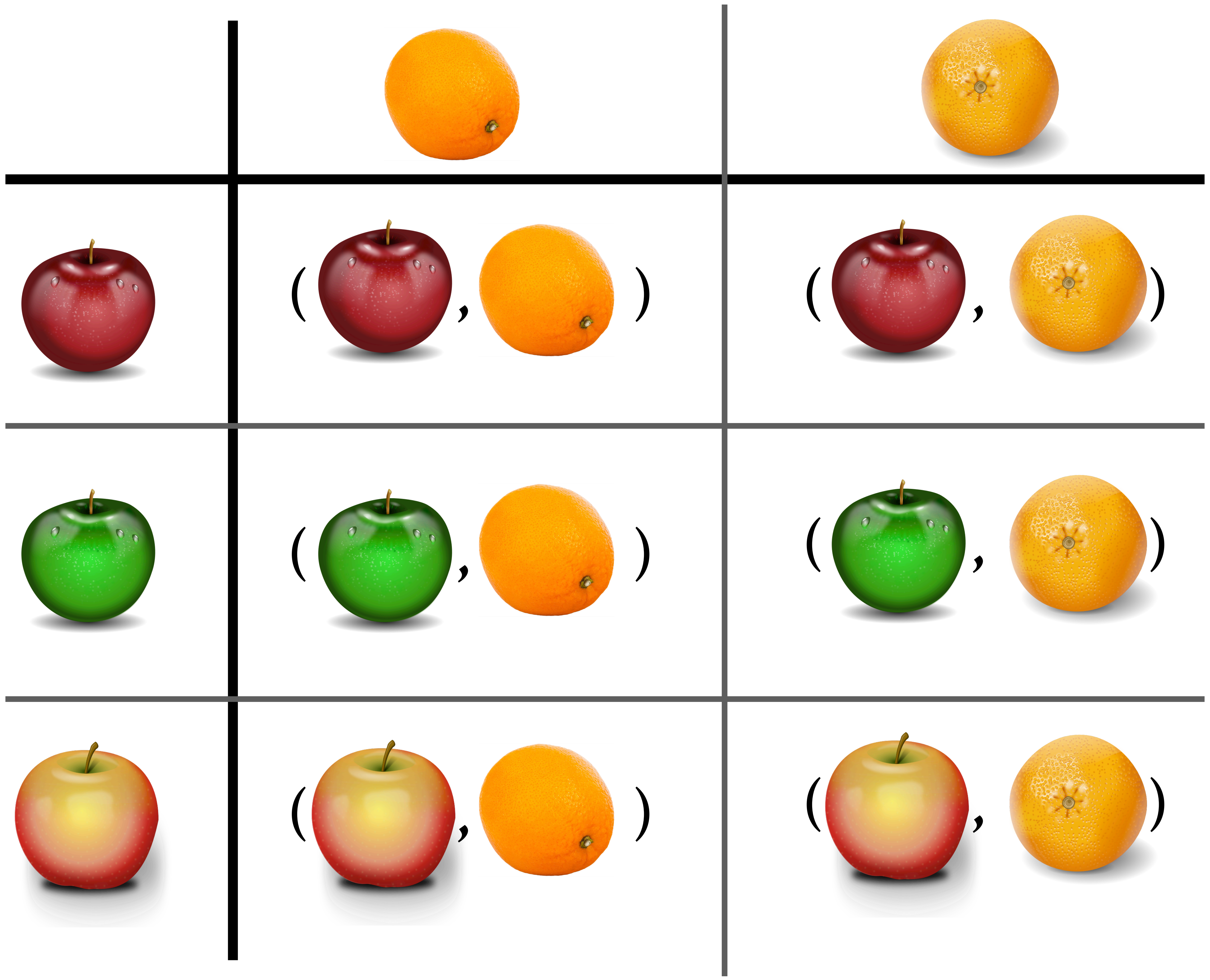
Phase Space

- We have position x and momentum p. Values of x come from the real number line $\mathbb{R}$, and values of p come from the real number line $\mathbb{R}$. We form direct sum $|\xi\rangle = |x\rangle \oplus |p\rangle \sim (x, p)$.
- For simple systems like oscillator moving along a single axis, we obtain *state space* $\mathbb{R} \oplus \mathbb{R}$.
- For more complicated systems, the idea is the same, the result is more complicated. Single particle moving in three dimensions: $|\xi\rangle \sim (x, y, z, p_x, p_y, p_z)$ with state space $\mathbb{R} \oplus \mathbb{R} \oplus \mathbb{R} \oplus \mathbb{R} \oplus \mathbb{R} \oplus \mathbb{R}$



Product

Cartesian Product $\times$

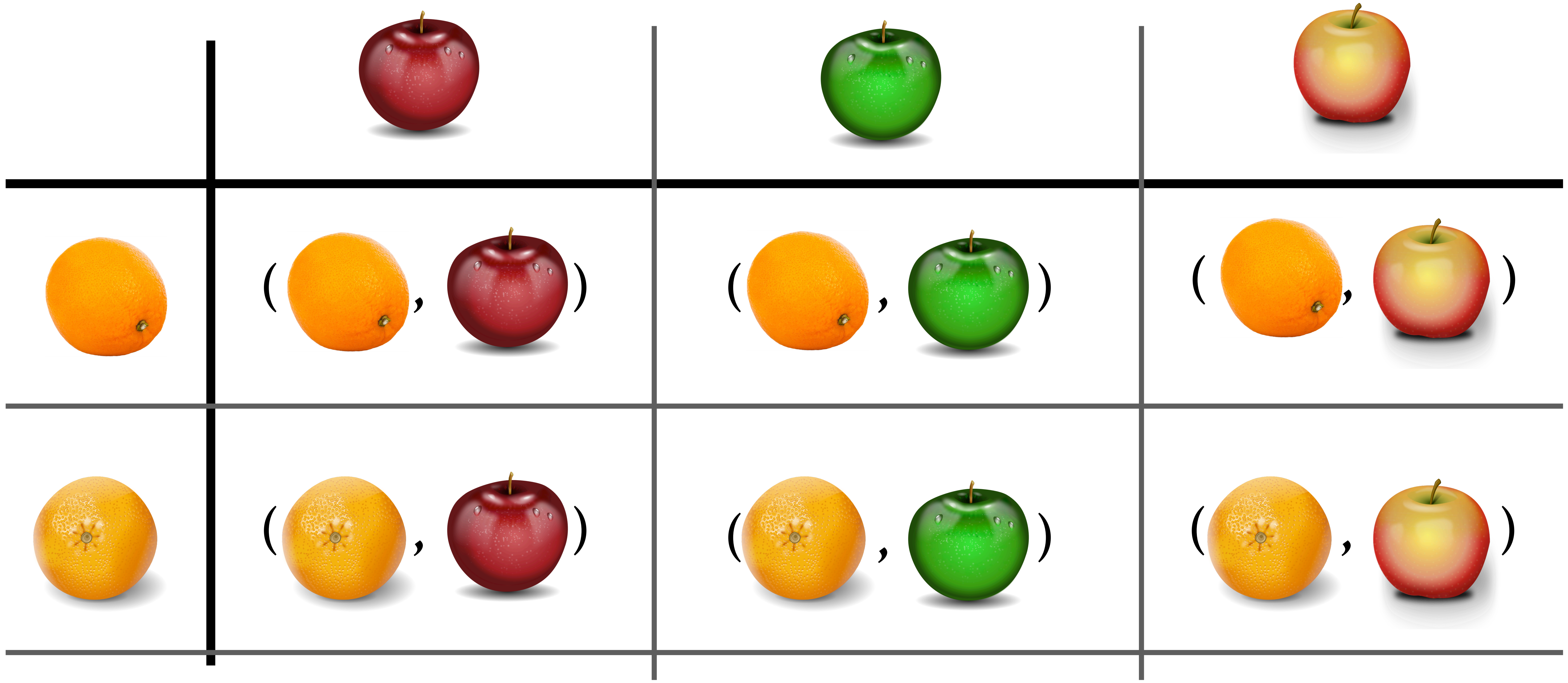


- Very general idea of bundling objects.
- Applies to sets that have no clear mathematical structure. For example, if $\mathbb{A}$ is the set of apples, and $\mathbb{O}$ is the set of oranges, we have *Cartesian* product $\mathbb{A} \times \mathbb{O}$ — the set of *ordered* pairs.

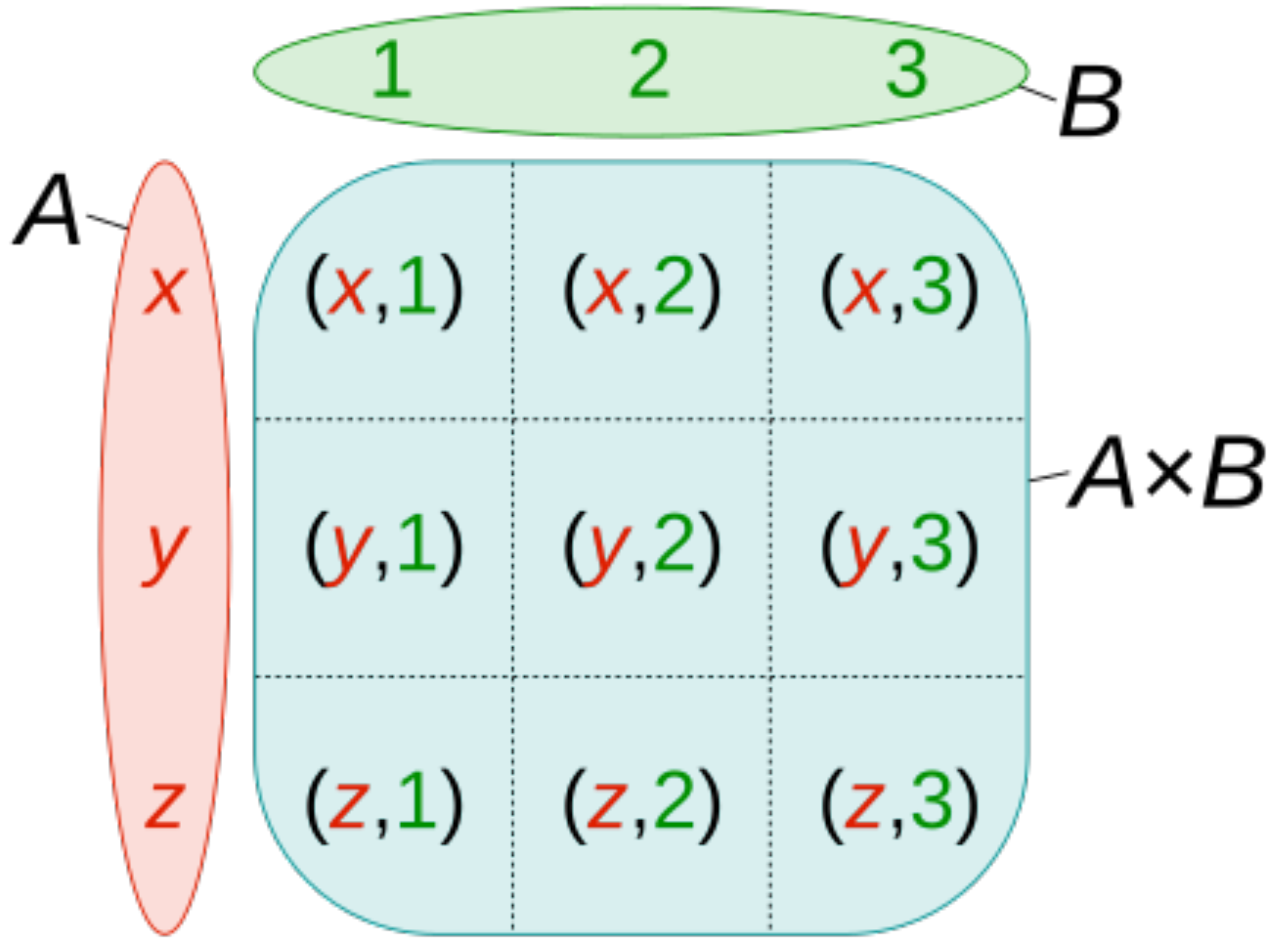
The size (number of elements) of the final set grows as the product of size of factors — hence **product**.

Product

Cartesian Product $\times$



If $\mathbb{A}$ is the set of apples, and $\mathbb{O}$ is the set of oranges, we have *Cartesian* product $\mathbb{O} \times \mathbb{A}$ — the set of *ordered* pairs.



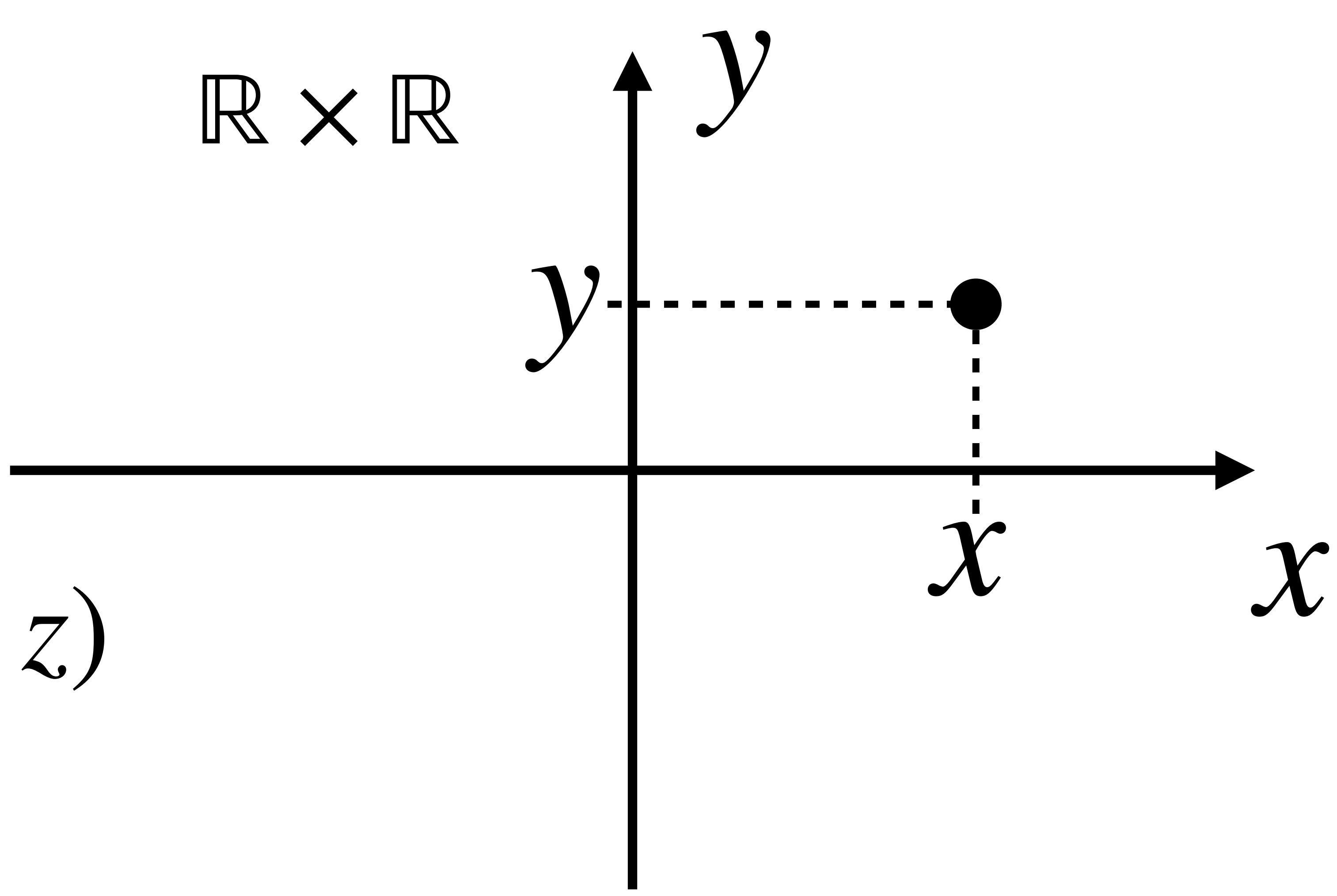
$$\mathbb{O} \times \mathbb{A} \neq \mathbb{A} \times \mathbb{O}$$

First eating an orange and then an apple tastes differently that eating an apple first, and then eating an orange.

Cartesian Product Example

Coordinates In Plane

- Each point in a plane has position x and y. Values of x come from the real number line $\mathbb{R}$, and values of y come from the real number line $\mathbb{R}$. We form Cartesian product $|a\rangle \sim (x, y)$.
- For simple plane we obtain *space* $\mathbb{R} \times \mathbb{R}$.
- For more complicated spaces, the idea is the same, the result is more complicated. For points in three dimensions: $|a\rangle \sim (x, y, z)$ with state space $\mathbb{R} \times \mathbb{R} \times \mathbb{R}$

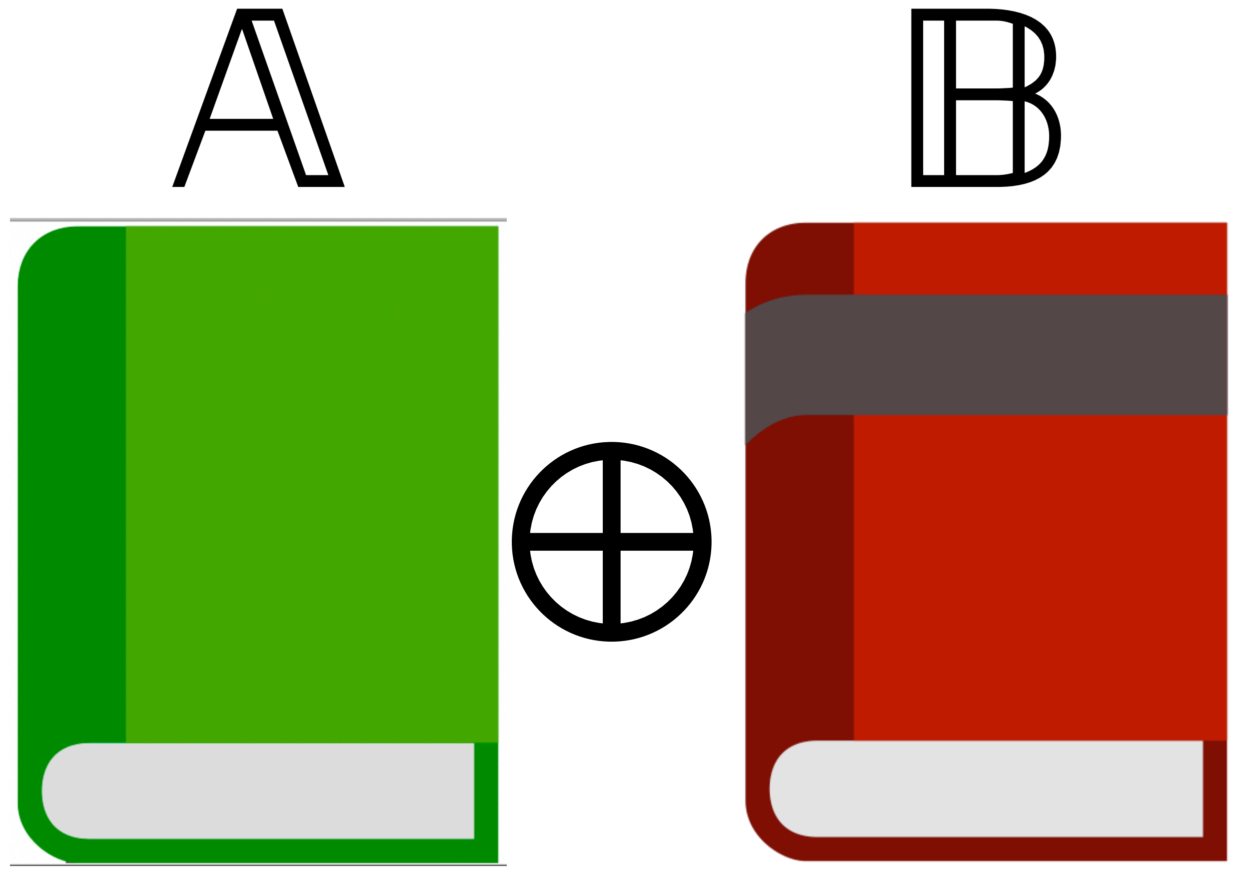


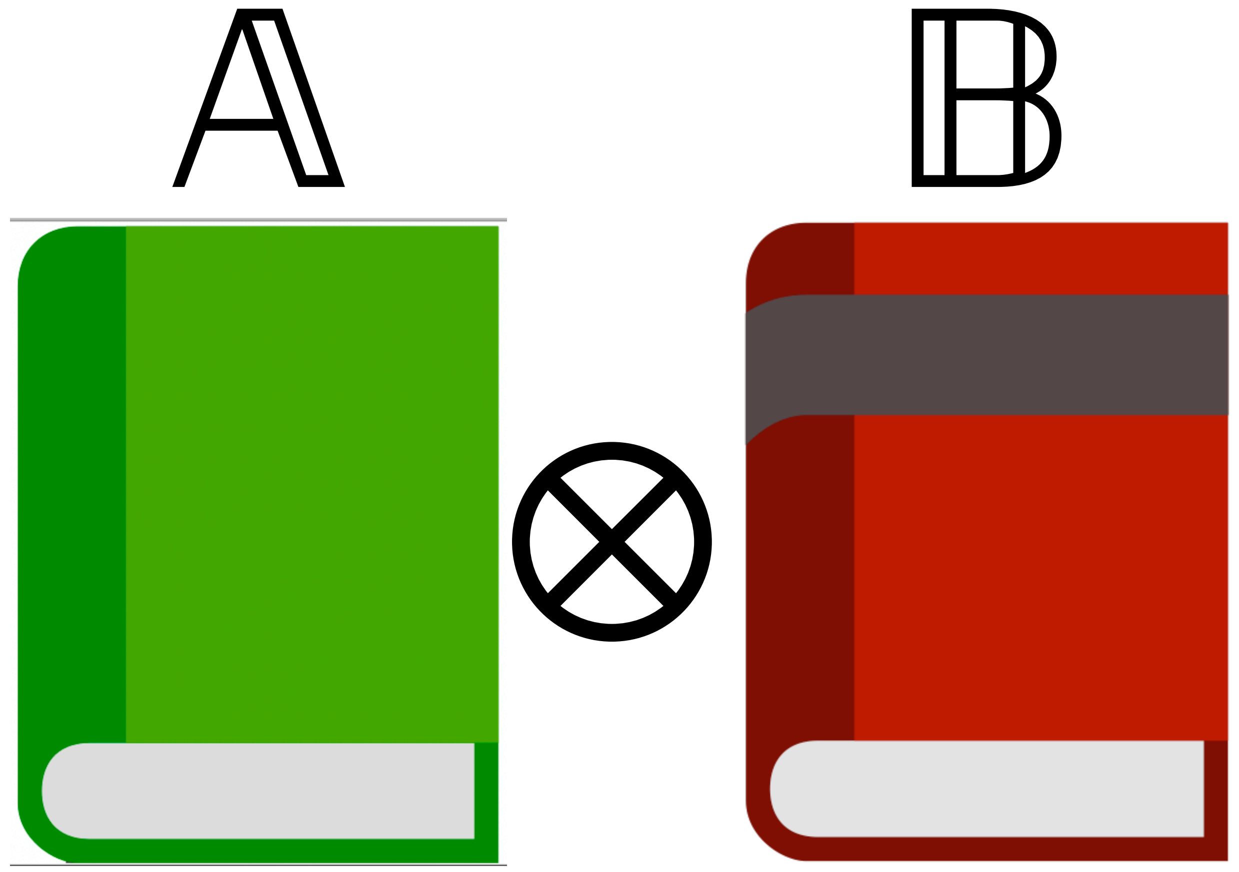
$$\mathbb{R} \times \mathbb{R} = \mathbb{R}^2$$
$$\mathbb{R} \times \mathbb{R} \times \mathbb{R} = \mathbb{R}^3$$
$$\mathbb{R} \times \mathbb{R} \times \cdots \times \mathbb{R} = \mathbb{R}^n$$

Cartesian Product Vs Direct Sum

Look Too Similar? What's The Difference?

To better understand the difference, use book example. If $\mathbb{A}$ is the first book (a set of N pages), and $\mathbb{B}$ is the second book (a set of M pages) then:





Your read $\mathbb{A}$ page by page and then read $\mathbb{B}$ page by page. Total number of pages read $P = N + M$

Your read $\mathbb{A}$. Then you read $\mathbb{B}$ page by page, but information from each page of $\mathbb{B}$ changes how you understand the whole book $\mathbb{A}$ and you re-read whole $\mathbb{A}$ again. The total number of pages read $P = N \cdot M$

Tensor Product

Of Vector Spaces

What we need is to bundle all kinds of vectors — ket-with-ket, bra-with-bra, ket-with-bra, bra-with-ket, ket-with-ket-with-ket, and so on.

Let's understand ket-with-ket first, other variants are very similar.

Vector.
N-dimensional.
$(a_1, a_2, \dots\ a_n)$

Vector.
N-dimensional.
$(a_1, a_2, \dots\ a_n)$

$$\underbrace{|a\rangle \otimes |b\rangle}$$

"Bundle" of vectors (tensor).
N*N-dimensional.
$(a_1b_1, a_1b_2, \dots\ a_2b_1, a_2b_2, \dots, a_nb_1, a_nb_2, \dots, a_nb_n)$

	b_1	b_2	b_3	$\cdots$	b_n
a_1	a_1b_1	a_1b_2	a_1b_3	$\cdots$	a_1b_n
a_2	a_2b_1	a_2b_2	a_2b_3	$\cdots$	a_2b_n
a_3	a_3b_1	a_3b_2	a_3b_3	$\cdots$	a_3b_n
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
a_n	a_nb_1	a_nb_2	a_nb_3	$\cdots$	a_nb_n

Tensor Product

Of Vector Spaces

Now, $|a\rangle \otimes |b\rangle$ is not the same type of vector as $|a\rangle$ or $|b\rangle$. It is a new "creature." It is a "super-vector" $|C\rangle$ with components $C_{ij} = a_i b_j$.

Although C is technically called *tensor*, there is *no fundamental difference* between tensors and vectors, like no real difference between operators and vectors.

In fact, in quantum physics tensor product is used to describe quantum systems with many "parts".

State of hydrogen atom $\longleftarrow |\Psi_H\rangle = |\psi_e\rangle \otimes |\psi_{ph}\rangle$

State of electron

State of proton

Tensor Product

Of Vector Spaces

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In fact, in quantum physics tensor product is used to describe quantum systems with many "parts".

$$\text{State of hydrogen atom} \longleftarrow |\Psi_H\rangle = \underbrace{|\psi_e\rangle}_{\text{State of electron}} \otimes \underbrace{|\psi_{ph}\rangle}_{\text{State of proton}}$$

The state vector $|\Psi_H\rangle$ is said to be factorized.
NOT EVERY STATE CAN BE FACTORIZED!

Tensor Product

Use For Systems of Qubits

Qubit is *any* quantum system with two distinct states available for manipulation.

The states are traditionally labeled as $|0\rangle$ for the lower energy, and $|1\rangle$ for the higher energy. This agrees with binary digits used in digital computing, “010110”.

$|0\rangle \quad |1\rangle \quad |0\rangle \quad |1\rangle \quad |1\rangle \quad |0\rangle$

$$|\Psi\rangle = |0\rangle \otimes |1\rangle \otimes |0\rangle \otimes |1\rangle \otimes |1\rangle \otimes |0\rangle$$

$\longrightarrow$ Full form.

$$|\Psi\rangle = |0\rangle|1\rangle|0\rangle|1\rangle|1\rangle|0\rangle$$

$\longrightarrow$ Economical form.

$$|\Psi\rangle = |010110\rangle$$

$\longrightarrow$ Lazy form.

Self-Test

Answer These Questions 1hr After Class

1. What are two basic forms of “bundling” of mathematical objects?
2. Give three examples of useful “bundling”.
3. What’s the big difference between “bundling” and scalar product?
4. What is the difference between a direct sum and a direct/tensor product of vectors?
5. What is a “factorized” state?
6. What are three way to write the state vector of a composite system made of 3 qubits?

Homework Problems

Tensor Product

1. Compare the "bundling" of numbers to create rationals $r = (q, p)$ and the "bundling" of vector components (a_1, a_2). What are the similarities and differences?

2. Explain the difference between the sum of two vectors $|a\rangle$ and $|b\rangle$ and their *direct sum*.

3. Explain the difference between the product of two vectors $|a\rangle$ and $|b\rangle$ and their *tensor product*.

4. Given two sets $\mathbb{A} = \{1,2,3\}$ and $\mathbb{B} = \{t, f\}$, form Cartesian products $\mathbb{A} \times \mathbb{B}$ and $\mathbb{B} \times \mathbb{A}$. Can we form *direct sum* or *tensor product* of these sets?

5. ***challenging*** To be called *sum* (or *product),* mathematical operation must have properties of sum (or product). One familiar property of product is *distributivity* (another aspect of linearity): $|\psi\rangle \otimes (|\phi\rangle + |\chi\rangle) = |\psi\rangle \otimes |\phi\rangle + |\psi\rangle \otimes |\chi\rangle$. Consider the state of a qubit pair $|\psi\rangle = \left(|0\rangle|0\rangle + |1\rangle|1\rangle\right)/\sqrt{2}$. Prove that there are no states of individual qubits $|\phi\rangle$ and $|\chi\rangle$ such that $|\psi\rangle = |\phi\rangle|\chi\rangle$.

Quantum Theory

In a Nutshell

II. POSTULATES FOR QUANTUM MECHANICS

In this paper, all state vectors are supposed to be normalized, and mixed states are represented by density operators, i.e., positive operators with unit trace. Let A be an observable with a nondegenerate purely discrete spectrum. Let $\phi_1, \phi_2, \ldots$ be a complete orthonormal sequence of eigenvectors of A and $a_1, a_2, \ldots$ the corresponding eigenvalues; by assumption, all different from each other.

According to the standard formulation of quantum mechanics, on the result of a measurement of the observable A the following postulates are posed:

(A1) *If the system is in the state ψ at the time of measurement, the eigenvalue a_n is obtained as the outcome of measurement with the probability $|\langle\phi_n|\psi\rangle|^2$.*

(A2) *If the outcome of measurement is the eigenvalue a_n, the system is left in the corresponding eigenstate ϕ_n at the time just after measurement.*

The postulate (A1) is called the *statistical formula*, and (A2) the *measurement axiom*. The state change $\psi \mapsto \phi_n$ described by the measurement axiom is called the *state reduction*.

You will understand this paragraph in the end of the course.