Quantum Physics

2025

The Theory/Framework Of *Almost* Everything *Today*

But Most Likely NOT of Tomorrow

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Course Overview

Course Structure And Goals

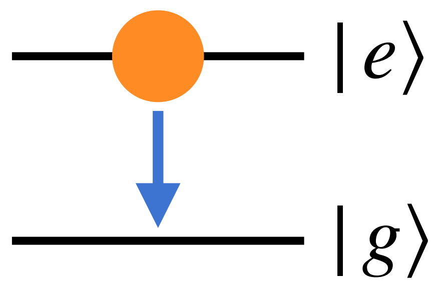
- **Part 1** : Mathematical Concepts And Tools.
- **Part 2** : Classical Physics.
- **Part 3** : Quantum Physics.

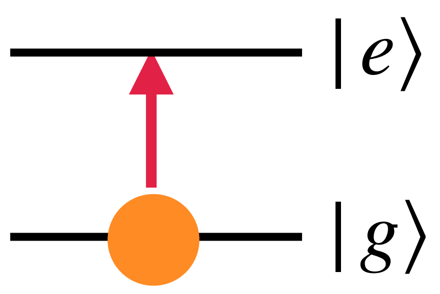
- Learn the language of quantum physics.
- Enhance the knowledge of classical physics.
- Develop modern quantum thinking.

We will focus on this one today.

Interacting Qubits

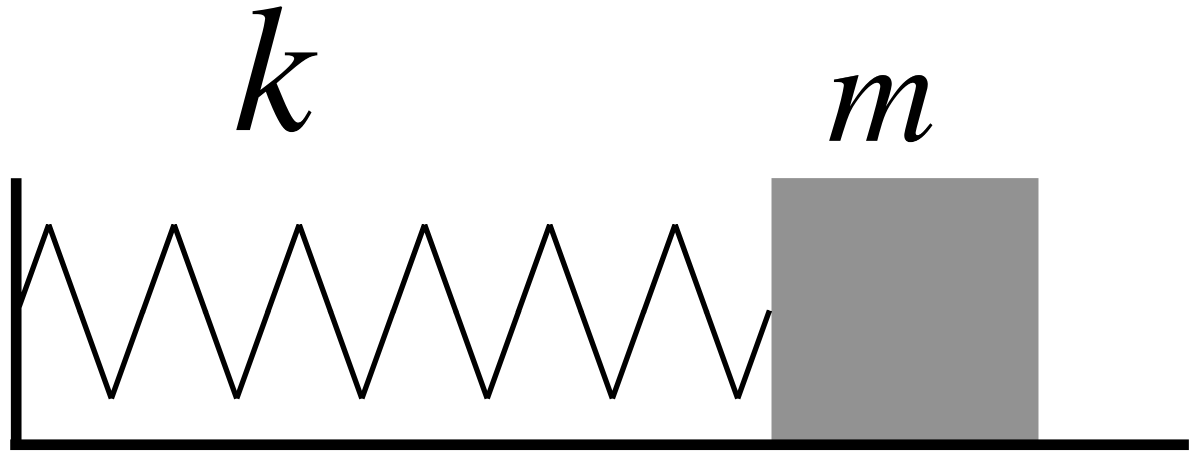
Simple. Interesting. Useful. Fully Quantum.

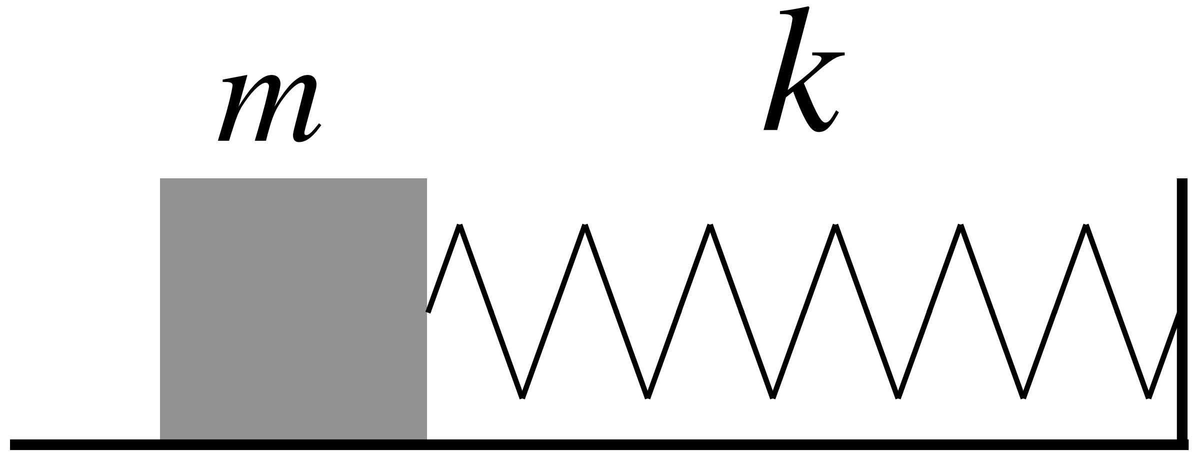




Interacting Qubits

Mechanical Analog (Almost)





$$H_a = \frac{p_1^2}{2m} + \frac{kx_1^2}{2}$$

- Two independent oscillators

$$H_b = \frac{p_2^2}{2m} + \frac{kx_2^2}{2}$$

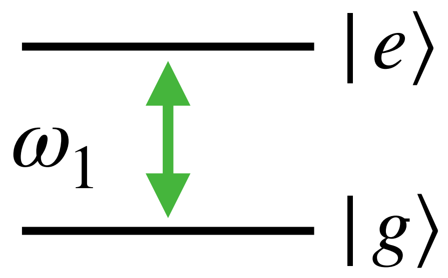
$$|\xi_1\rangle \sim (x_1, p_1)$$

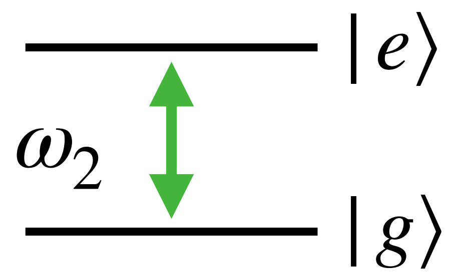
$$|\xi_2\rangle \sim (x_2, p_2)$$

$$|\xi\rangle = |\xi_1\rangle \oplus |\xi_2\rangle \sim (x_1, p_1, x_2, p_2)$$

Interacting Qubits

Composite System: Atom + Atom or Atom + Field Mode





$$\hat{H}_1 = \frac{\hbar\omega_1}{2}\left(|e\rangle\langle e| - |g\rangle\langle g|\right)$$

$$|\Psi\rangle = b_g|g\rangle + b_e|e\rangle$$

$$i\hbar\partial_t|\Psi\rangle = \hat{H}_1|\Psi\rangle$$

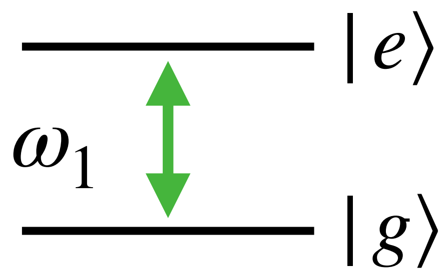
$$\hat{H}_2 = \frac{\hbar\omega_2}{2}\left(|e\rangle\langle e| - |g\rangle\langle g|\right)$$

$$|\Phi\rangle = c_g|g\rangle + c_e|e\rangle$$

$$i\hbar\partial_t|\Phi\rangle = \hat{H}_2|\Phi\rangle$$

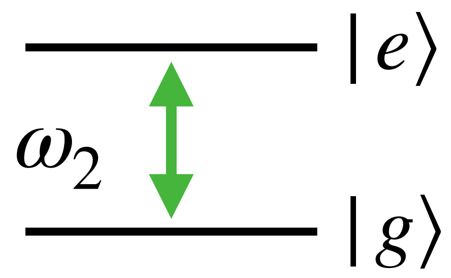
Interacting Qubits

Composite System: Atom + Atom or Atom + Field Mode



$$|\Upsilon\rangle = |\Psi\rangle \otimes |\Phi\rangle$$

$$\hat{H} = \hat{H}_1 \otimes \hat{H}_2$$



$$i\hbar\partial_t |\Upsilon\rangle = \hat{H}|\Upsilon\rangle$$

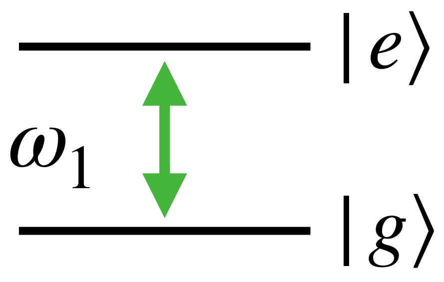
Bundle them together into single system.

Interacting Qubits

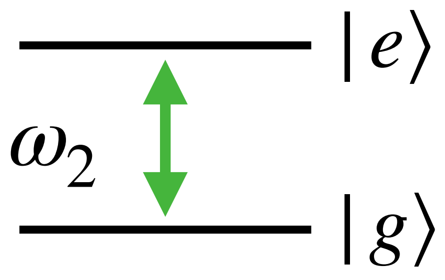
Composite System: Atom + Atom or Atom + Field Mode

$$|\Upsilon\rangle = |\Psi\rangle \otimes |\Phi\rangle$$

NOT always!



$$\hat{H} = \hat{H}_1 \otimes \hat{H}_2$$



$$i\hbar\partial_t|\Upsilon\rangle = \hat{H}|\Upsilon\rangle$$

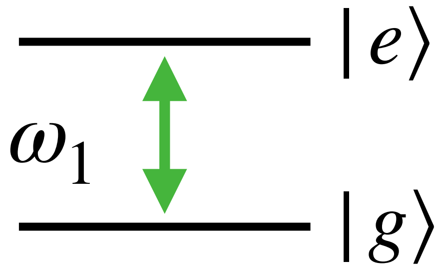
$$\boxed{|\Upsilon_1\rangle = |g\rangle \otimes |g\rangle \qquad |\Upsilon_2\rangle = |g\rangle \otimes |e\rangle \qquad |\Upsilon_3\rangle = |e\rangle \otimes |g\rangle \qquad |\Upsilon_4\rangle = |e\rangle \otimes |e\rangle}$$

Most important when $\omega_a = \omega_m$ (resonance).

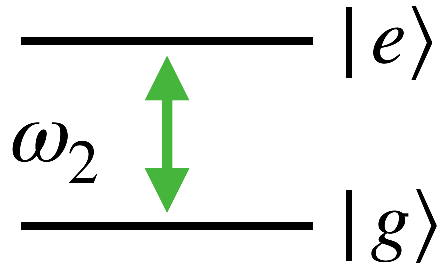
Interacting Qubits

Composite System: Atom + Atom or Atom + Field Mode

$$|\Upsilon\rangle = |\Psi\rangle \otimes |\Phi\rangle = |\Psi\rangle|\Phi\rangle$$



$$\hat{H} = \hat{H}_1 \otimes \hat{H}_2$$



$$\hat{H}|\Upsilon\rangle = (\hat{H}_1 \otimes \hat{H}_2)(|\Psi\rangle \otimes |\Phi\rangle) = (\hat{H}_1|\Psi\rangle) \otimes (\hat{H}_2|\Phi\rangle)$$

$$i\hbar\partial_t|\Upsilon\rangle = \hat{H}|\Upsilon\rangle$$

Schrödinger equation for the composite system

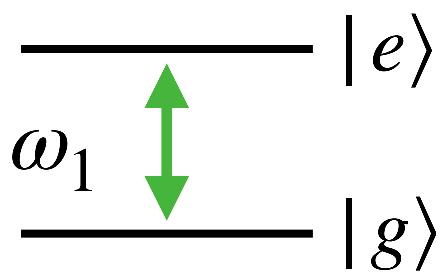
$$|\Upsilon_1\rangle = |g\rangle \otimes |g\rangle \qquad |\Upsilon_2\rangle = |g\rangle \otimes |e\rangle \qquad |\Upsilon_3\rangle = |e\rangle \otimes |g\rangle \qquad |\Upsilon_4\rangle = |e\rangle \otimes |e\rangle$$

$$\boxed{|\Upsilon_1\rangle = |g\rangle|g\rangle \qquad |\Upsilon_2\rangle = |g\rangle|e\rangle \qquad |\Upsilon_3\rangle = |e\rangle|g\rangle \qquad |\Upsilon_4\rangle = |e\rangle|e\rangle}$$

Economical notation

Raising and Lowering Operators

Represent Transitions Due to Interaction



$$\hat{\sigma}_+ = |e\rangle\langle g|$$

$$\hat{\sigma}_- = |g\rangle\langle e|$$

$$\hat{\sigma}_+|g\rangle = |e\rangle$$

$$\hat{\sigma}_-|e\rangle = |g\rangle$$

$$\hat{\sigma}_-^2 - ?, \hat{\sigma}_+^2 - ?, \hat{\sigma}_-\hat{\sigma}_+ - ?, \hat{\sigma}_+\hat{\sigma}_- - ?,$$

Raising and Lowering Operators

Represent Transitions Due to Interaction

Transition 1

$$\hat{T}_1 = \hat{\sigma}_- \otimes \hat{\sigma}_+$$

Transition 2

$$\hat{T}_2 = \hat{\sigma}_+ \otimes \hat{\sigma}_-$$

Raising and Lowering Operators

Represent Transitions Due to Interaction

Transition 1

$$\hat{T}_1 = \hat{\sigma}_- \otimes \hat{\sigma}_+$$

$$\left(\hat{\sigma}_- \otimes \hat{\sigma}_+\right) \, |e\rangle |g\rangle = |g\rangle |e\rangle$$

Transition 2

$$\hat{T}_2 = \hat{\sigma}_+ \otimes \hat{\sigma}_-$$

$$\left(\hat{\sigma}_+ \otimes \hat{\sigma}_-\right) \, |g\rangle |e\rangle = |e\rangle |g\rangle$$

Raising and Lowering Operators

Represent Transitions Due to Interaction

Transition 1

$$\hat{T}_1 = \hat{\sigma}_- \otimes \hat{\sigma}_+$$

$$\hat{T}_1 | \Upsilon_3 \rangle = | \Upsilon_2 \rangle$$
$$\hat{T}_1 | \Upsilon_i \rangle = ?$$

Transition 2

$$\hat{T}_2 = \hat{\sigma}_+ \otimes \hat{\sigma}_-$$

$$\hat{T}_2 | \Upsilon_2 \rangle = | \Upsilon_3 \rangle$$
$$\hat{T}_2 | \Upsilon_j \rangle = ?$$

Energy Operators

In Terms of Transition Operators

$$\hat{H} = E_g |g\rangle\langle g| + E_e |e\rangle\langle e|$$

E_e ——— $|e\rangle$

E_g ——— $|g\rangle$

$$E_e - E_g = \hbar\omega$$

Rewrite $\longrightarrow$

$$\hat{H} = \frac{\hbar\omega}{2}\left(|e\rangle\langle e| - |g\rangle\langle g|\right)$$

$+\hbar\omega/2$ ——— $|e\rangle$

$-\hbar\omega/2$ ——— $|g\rangle$

$$\hat{H} = \frac{\hbar\omega}{2}\left(\hat{\sigma}_+\hat{\sigma}_- - \hat{\sigma}_-\hat{\sigma}_+\right) = \frac{\hbar\omega}{2}\left[\hat{\sigma}_+, \hat{\sigma}_-\right]$$

Only energy difference matters in physics.

Energy Operators

In Terms of Transition Operators

$$\hat{H} = E_g\,|g\rangle\langle g| + E_e\,|e\rangle\langle e|$$

$$E_e \;\text{———}\; |e\rangle$$

$$E_g \;\text{———}\; |g\rangle$$

$$E_e - E_g = \hbar\omega$$

$$\hat{H} = \frac{\hbar\omega}{2}\left(|e\rangle\langle e| - |g\rangle\langle g|\right)$$

$$+\hbar\omega/2 \;\text{———}\; |e\rangle$$

$$-\hbar\omega/2 \;\text{———}\; |g\rangle$$

$$\hat{H} = \frac{\hbar\omega}{2}\left(\hat{\sigma}_+\hat{\sigma}_- - \hat{\sigma}_-\hat{\sigma}_+\right) = \frac{\hbar\omega}{2}\left[\hat{\sigma}_+, \hat{\sigma}_-\right]$$

Energy Operators

In Terms of Transition Operators

$$\hat{H} = \frac{\hbar\omega}{2}\left(|e\rangle\langle e| - |g\rangle\langle g|\right)$$

$$\hat{\sigma}_z = \hat{\sigma}_+\hat{\sigma}_- - \hat{\sigma}_-\hat{\sigma}_+ = \left[\hat{\sigma}_+, \hat{\sigma}_-\right]$$

$+\hbar\omega/2$ ——— $|e\rangle$

$-\hbar\omega/2$ ——— $|g\rangle$

$$\hat{H} = \frac{\hbar\omega}{2}\hat{\sigma}_z$$

Energy Operators

In Terms of Transition Operators

$$\hat{H} = \frac{\hbar\omega}{2}\left(|e\rangle\langle e| - |g\rangle\langle g|\right)$$

$$\hat{\sigma}_z = \hat{\sigma}_+\hat{\sigma}_- - \hat{\sigma}_-\hat{\sigma}_+ = \left[\hat{\sigma}_+, \hat{\sigma}_-\right]$$

$+\hbar\omega/2$ —— $|e\rangle$

$-\hbar\omega/2$ —— $|g\rangle$

$$\hat{H} = \frac{\hbar\omega}{2}\hat{\sigma}_z$$

$$\boxed{\sigma = F}$$

In Notebook code we
use different notation.

Energy Operators

In Terms of Transition Operators

$$\hat{H} = \frac{\hbar\omega}{2}\left(|e\rangle\langle e| - |g\rangle\langle g| \right)$$

$$\hat{\sigma}_z = \hat{\sigma}_+\hat{\sigma}_- - \hat{\sigma}_-\hat{\sigma}_+ = \left[\hat{\sigma}_+, \hat{\sigma}_-\right]$$

$+\hbar\omega/2$ —— $|e\rangle$

$-\hbar\omega/2$ —— $|g\rangle$

$$\boxed{\sigma = F}$$

$$\hat{H} = \frac{\hbar\omega}{2}\hat{\sigma}_z$$

But σs are used in QuTiP

Interacting Qubits

Fully Quantum Hamiltonian

$$\hat{T}_1 = \hat{\sigma}_- \otimes \hat{\sigma}_+$$

$$\hat{H}_1 = \frac{\hbar\omega_1}{2}\hat{\sigma}_z$$

$$\hat{H}_2 = \frac{\hbar\omega_2}{2}\hat{\sigma}_z$$

$$\hat{T}_2 = \hat{\sigma}_+ \otimes \hat{\sigma}_-$$

$$\boxed{\hat{H} = \hat{H}_1 \otimes \hat{I}_2 + \hat{I}_1 \otimes \hat{H}_2 + \epsilon\left(\hat{T}_1 + \hat{T}_2\right)}$$

$$i\hbar\partial_t|\Upsilon\rangle = \hat{H}|\Upsilon\rangle$$

$$|\Upsilon\rangle = |\Psi\rangle \otimes |\Phi\rangle$$

Interacting Qubits

Fully Quantum Hamiltonian

$$\hat{H}_1 = \frac{\hbar\omega_1}{2}\hat{\sigma}_z$$

$$\hat{T}_1 = \hat{\sigma}_- \otimes \hat{\sigma}_+$$

$$\hat{T}_2 = \hat{\sigma}_+ \otimes \hat{\sigma}_-$$

$$\hat{H}_2 = \frac{\hbar\omega_2}{2}\hat{\sigma}_z$$

$$\hat{H} = \hat{H}_1 \otimes \hat{I}_2 + \hat{I}_1 \otimes \hat{H}_2 + \epsilon\left(\hat{T}_1 + \hat{T}_2\right)$$

$$i\hbar\partial_t|\Upsilon\rangle = \hat{H}|\Upsilon\rangle$$

$$\boxed{|\Upsilon\rangle = |\Psi\rangle \otimes |\Phi\rangle}$$

NOT always!

Interacting Qubits

Fully Quantum Hamiltonian

$$\hat{T}_1 = \hat{\sigma}_- \otimes \hat{\sigma}_+$$

$$\hat{H}_1 = \frac{\hbar\omega_1}{2}\hat{\sigma}_z$$

$$\hat{H}_2 = \frac{\hbar\omega_2}{2}\hat{\sigma}_z$$

$$\hat{T}_2 = \hat{\sigma}_+ \otimes \hat{\sigma}_-$$

$$\boxed{\hat{H} = \hat{H}_1 \otimes \hat{I}_2 + \hat{I}_1 \otimes \hat{H}_2 + \epsilon\left(\hat{T}_1 + \hat{T}_2\right)}$$

Computational basis

$|\Upsilon_1\rangle = |0\rangle|0\rangle = |00\rangle$ $|\Upsilon_2\rangle = |0\rangle|1\rangle = |01\rangle$ $|\Upsilon_3\rangle = |1\rangle|0\rangle = |01\rangle$ $|\Upsilon_4\rangle = |1\rangle|1\rangle = |11\rangle$

Interacting Qubits

Fully Quantum Hamiltonian

$$\hat{T}_1 = \hat{\sigma}_- \otimes \hat{\sigma}_+$$

$$\hat{H}_1 = \frac{\hbar\omega_1}{2}\hat{\sigma}_z$$

$$\hat{H}_2 = \frac{\hbar\omega_2}{2}\hat{\sigma}_z$$

$$\hat{T}_2 = \hat{\sigma}_+ \otimes \hat{\sigma}_-$$

$$\boxed{\hat{H} = \hat{H}_1 \otimes \hat{I}_2 + \hat{I}_1 \otimes \hat{H}_2 + \epsilon\left(\hat{T}_1 + \hat{T}_2\right)}$$

Computational basis

$|\Upsilon_1\rangle = |0\rangle|0\rangle = |00\rangle$ $\quad |\Upsilon_2\rangle = |0\rangle|1\rangle = |01\rangle$ $\quad |\Upsilon_3\rangle = |1\rangle|0\rangle = |01\rangle$ $\quad |\Upsilon_4\rangle = |1\rangle|1\rangle = |11\rangle$

Bell States basis

$|\Phi^+\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$ $\quad |\Psi^+\rangle = (|01\rangle + |10\rangle)/\sqrt{2}$

$|\Phi^-\rangle = (|00\rangle - |11\rangle)/\sqrt{2}$ $\quad |\Psi^-\rangle = (|01\rangle - |10\rangle)/\sqrt{2}$

Self-Test

Answer These Questions 1hr After Class

1. What is a qubit?
2. What is a composite system?
3. How many basis states are there for a qubit pair? For qubit trio? For N-qubits?
4. How does one know that systems are interacting?

Homework Problems

Interacting Qubits

1. Evaluate $\hat{\sigma}_-^2, \hat{\sigma}_+^2, \hat{\sigma}_-\hat{\sigma}_+, \hat{\sigma}_+\hat{\sigma}_-$.
2. Evaluate $[\hat{T}_1, \hat{T}_2]$.
3. Write the Hamiltonian of interacting qubits in terms of the operators $|\Upsilon_i\rangle\langle\Upsilon_j|$.
4. Evaluate $\hat{T}_i|\Upsilon_j\rangle$ for all i and j combinations.
5. Study the Jupyter Notebook with Qubit Interaction.

Quantum Theory

In a Nutshell

II. POSTULATES FOR QUANTUM MECHANICS

In this paper, all state vectors are supposed to be normalized, and mixed states are represented by density operators, i.e., positive operators with unit trace. Let A be an observable with a nondegenerate purely discrete spectrum. Let $\phi_1, \phi_2, \ldots$ be a complete orthonormal sequence of eigenvectors of A and $a_1, a_2, \ldots$ the corresponding eigenvalues; by assumption, all different from each other.

According to the standard formulation of quantum mechanics, on the result of a measurement of the observable A the following postulates are posed:

(A1) *If the system is in the state ψ at the time of measurement, the eigenvalue a_n is obtained as the outcome of measurement with the probability $|\langle \phi_n | \psi \rangle|^2$.*

(A2) *If the outcome of measurement is the eigenvalue a_n, the system is left in the corresponding eigenstate ϕ_n at the time just after measurement.*

The postulate (A1) is called the *statistical formula*, and (A2) the *measurement axiom*. The state change $\psi \mapsto \phi_n$ described by the measurement axiom is called the *state reduction*.

You will understand this paragraph in the end of the course.