Quantum Physics

2025

The Theory/Framework Of *Almost* Everything *Today*

But Most Likely NOT of Tomorrow

Yury Deshko

Course Overview

Course Structure And Goals

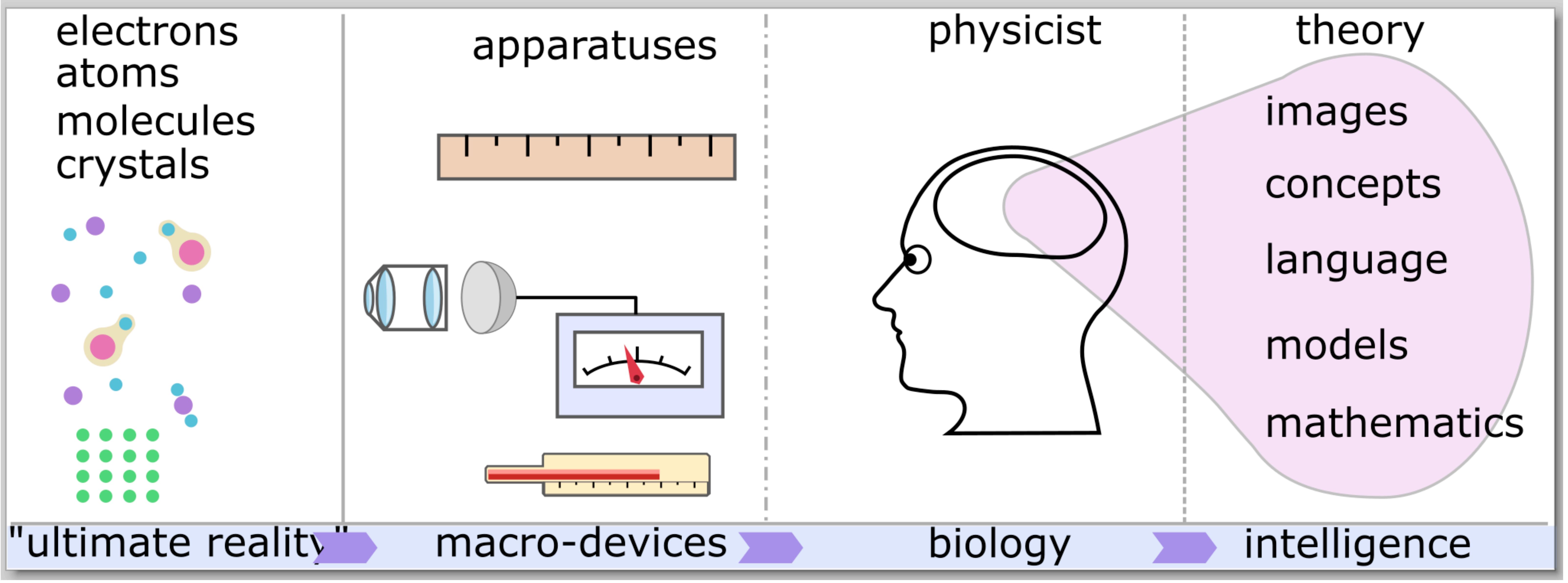
- **Part 1** : Mathematical Concepts And Tools.
- **Part 2** : Classical Physics.
- **Part 3** : Quantum Physics.

- Learn the language of quantum physics.
- Enhance the knowledge of classical physics.
- Develop modern quantum thinking.

We will focus on this one today.

Quantum Physics

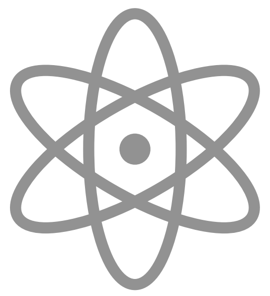
Quantum vs Classical Views

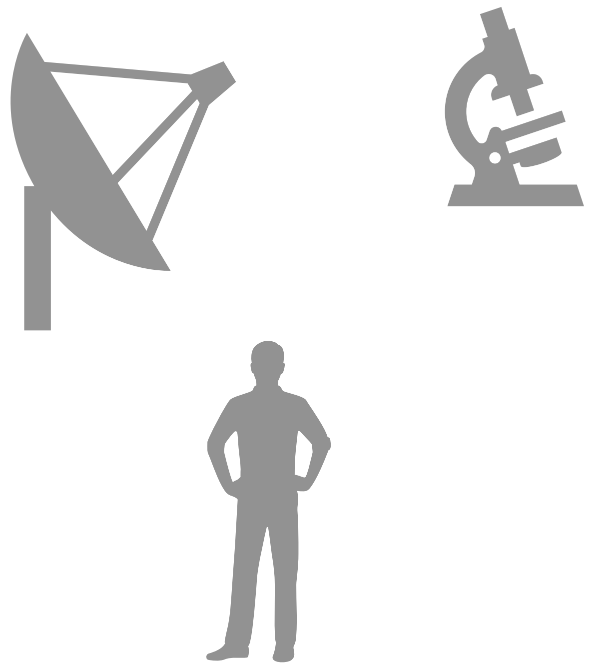


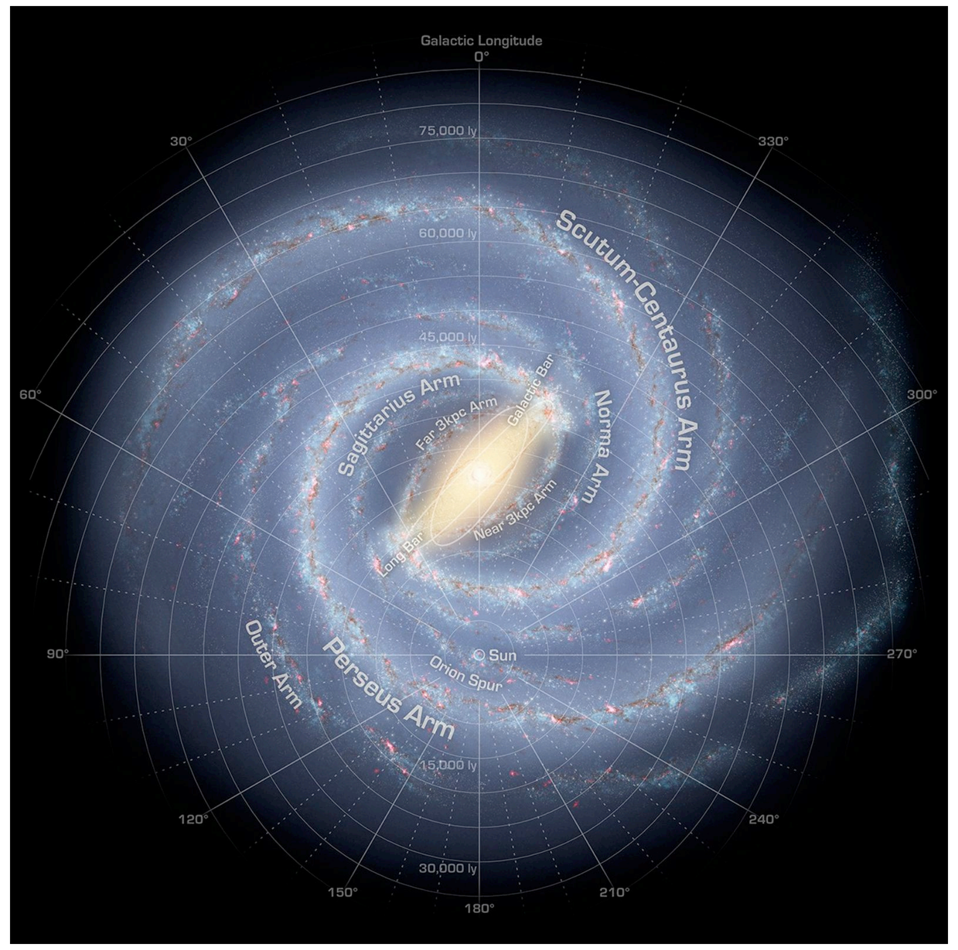
- All our knowledge of microword is **indirect** to various degrees. No one ever “saw an electron”.
- They language we use to summarize our experience/experiments/observations is heavily based on common sense/intuitive/macroscopic concepts.

Quantum Physics

Quantum vs Classical Views





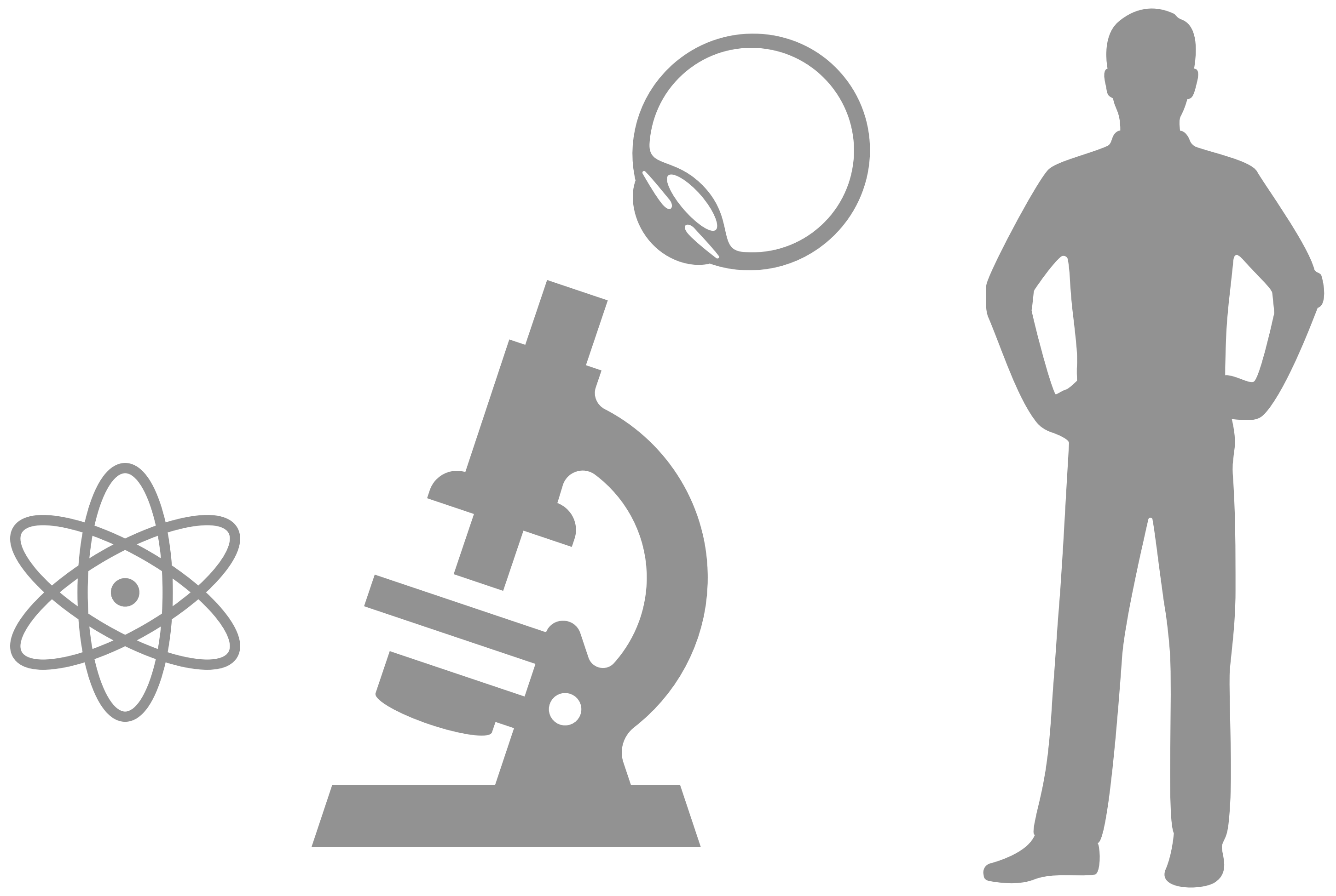


Atomic and subatomic | Macroscopic/Classical world. | "Megascopic", astronomical scale

- The concepts and language, and *philosophy*, developed for common sense/ intuitive world around us is **extrapolated** up and down the scale (e.g. particle, force, trajectory, velocity)
- Extrapolation up the scale works great. Extrapolation down fails.

Quantum Physics

Quantum vs Classical Views

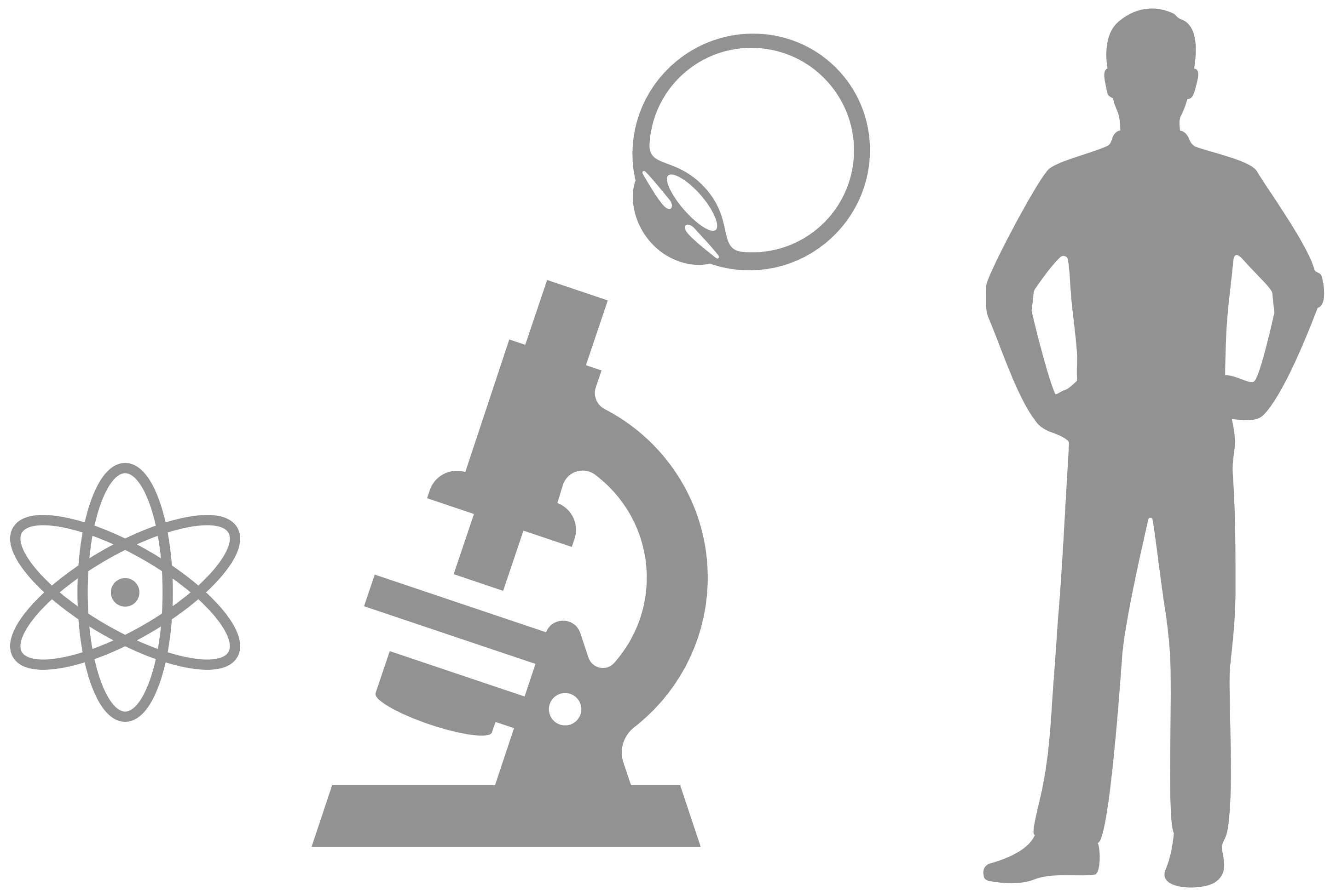


Result = Object's property + Method.

- "Obviously", *objects* (e.g. electrons) have *properties* (mass, charge, spin, velocity). We just need to determine their numerical values through careful observation, perhaps followed by some calculations.
- Observation can *affect the object*, but the goal is to be smart and take that into account and find the "true"/unperturbed/*real value* of a property.

Quantum Physics

Quantum vs Classical Views



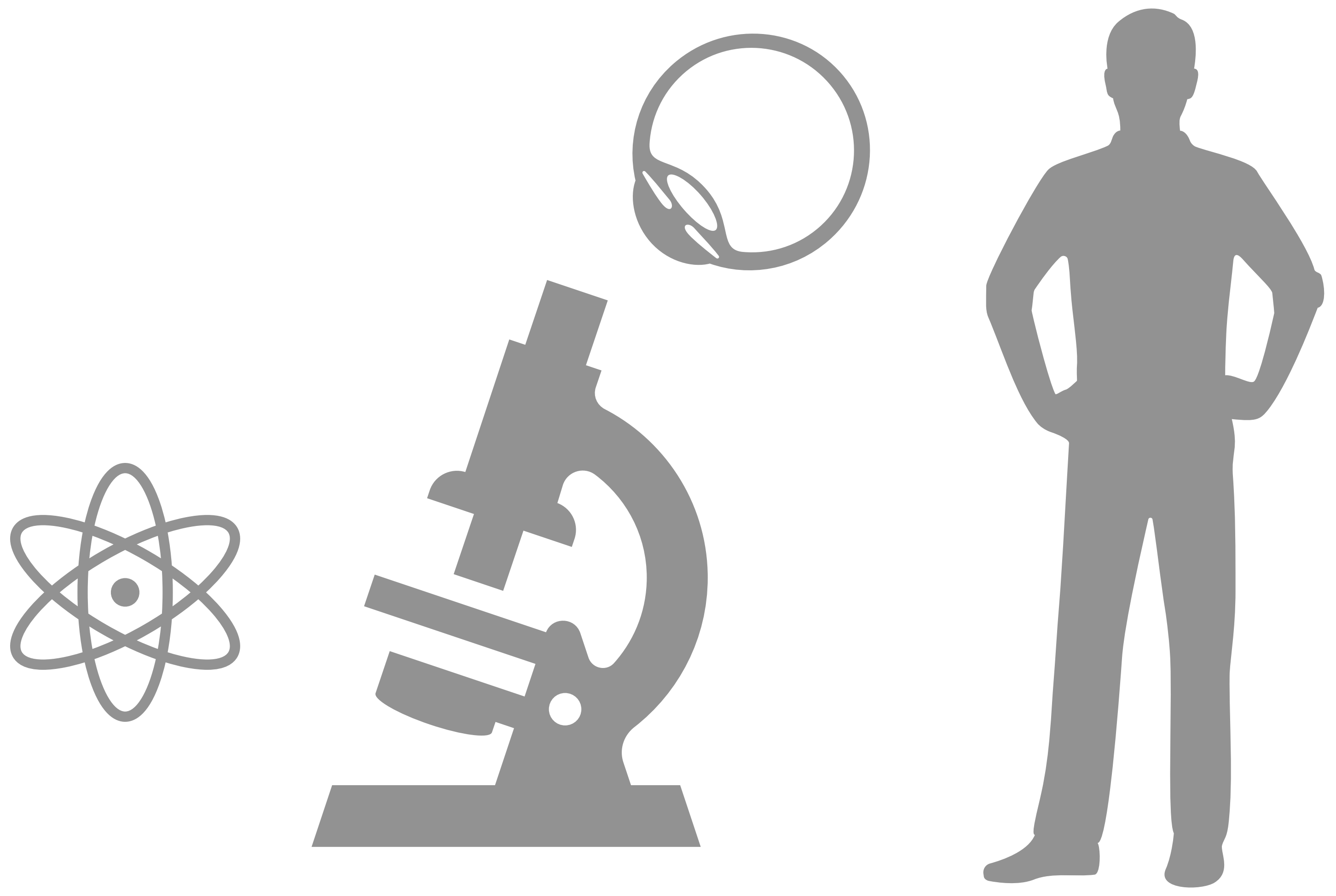
$$\text{Result} = \overset{??}{\boxed{\text{Object's property}}} + \boxed{\text{Method.}}$$

Human idea(s)?

- “Obviously”, *objects* (e.g. electrons) have *properties* (mass, charge, spin, velocity). We just need to determine their numerical values through careful observation, perhaps followed by some calculations.
- Observation can *affect the object*, but the goal is to be smart and take that into account and find the “true”/unperturbed/*real value* of a property.

Quantum Physics

Quantum vs Classical Views



!? Electron has property X, but you can never measure it.

?? Electron has property X, but you can never measure it exactly.

Electron is particle with mass m.

Electron goes through both slits.

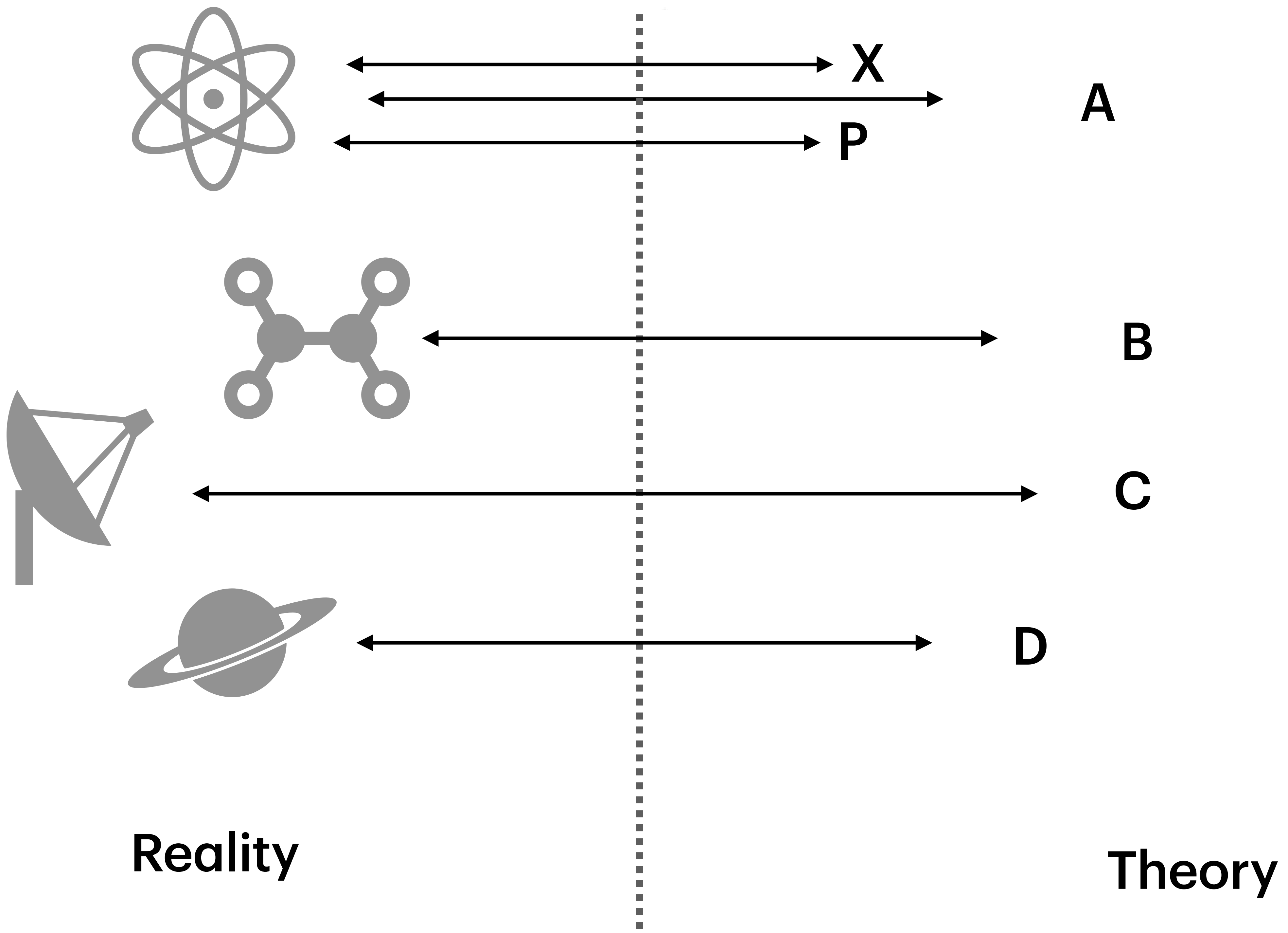
Electron is afraid.

- Quantum physics makes humans face the limits of their abilities (language and thought) and the limits of reality.
- It affects two big philosophical areas: **Ontology** (what “really exists”) and **Epistemology** (what does it mean to “know” and what can be known).

EPR

“Quantum Mechanics is Not Complete”

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

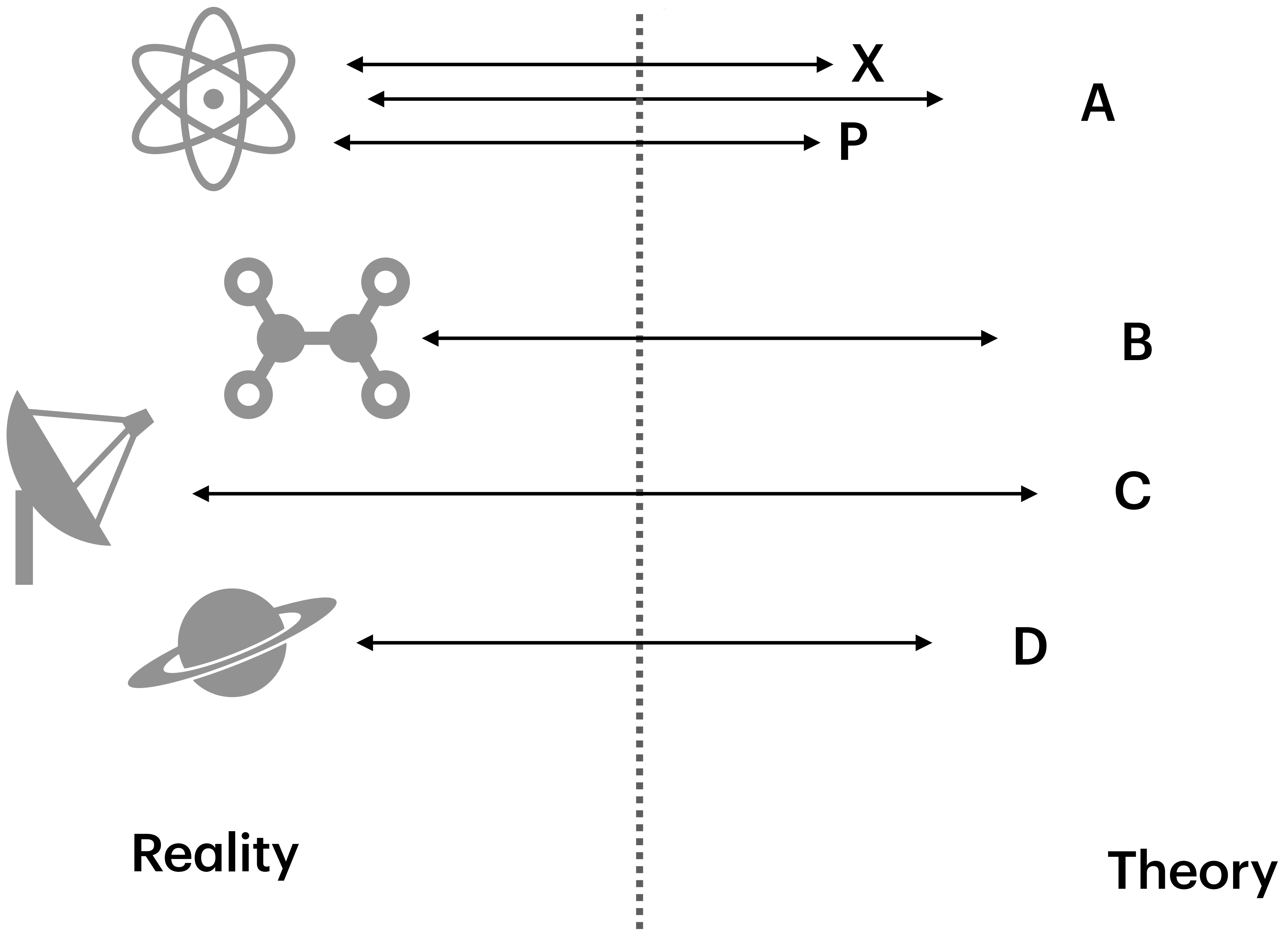


- Quantity is **real** (according to EPR) if you can predict it (its value) **with certainty** without interrogating the object.
- **Example**: Position of a particle is real (i.e. particle has position) if you can predict its value 100% definitely without even measuring it.

EPR

"Quantum Mechanics is Not Complete"

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.



Whatever the meaning assigned to the term *complete*, the following requirement for a complete theory seems to be a necessary one: *every element of the physical reality must have a counterpart in the physical theory*. We shall call this the condition of completeness. The second question

results of experiments and measurements. A comprehensive definition of reality is, however, unnecessary for our purpose. We shall be satisfied with the following criterion, which we regard as reasonable. *If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.* It

EPR

"Quantum Mechanics is Not Complete"

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

$\hat{x}\,|\psi\rangle = x\,|\psi\rangle \longrightarrow$ **Measure value x with certainty**

$\hat{p}\,|\psi\rangle = p\,|\psi\rangle \longrightarrow$ **Measure value p with certainty**

$$\hat{p}\hat{x}\,|\psi\rangle = px\,|\psi\rangle$$
$$\hat{x}\hat{p}\,|\psi\rangle = xp\,|\psi\rangle$$

$$\hat{x}\hat{p} = \hat{p}\hat{x} \leftrightarrow \hat{x}\hat{p} - \hat{p}\hat{x} = \hat{0}$$

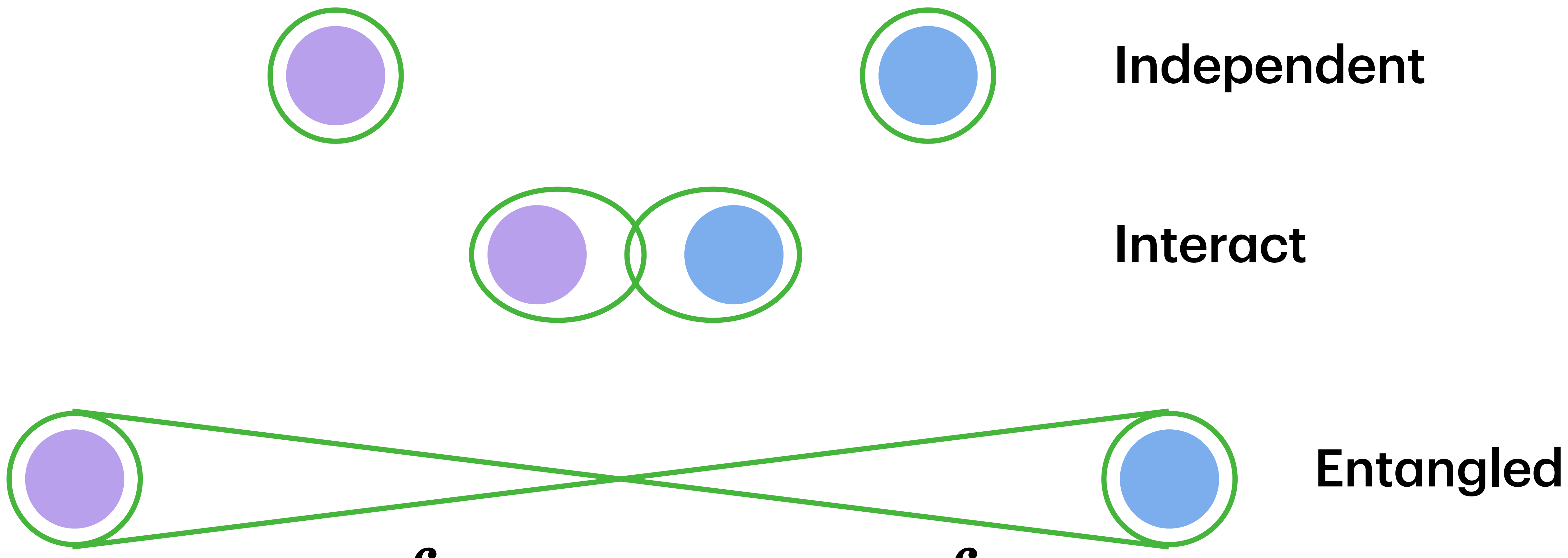
But commute, but we showed/proved that $\hat{x}\hat{p} - \hat{p}\hat{x} = i\hbar$

- Suppose $|\psi\rangle$ is the Eigen-state of both operators $\hat{x}$ and $\hat{p}$.
- Then the operators must commute.
- They don't in quantum theory.
- Thus, there are no common eigen-states of $\hat{x}$ and $\hat{p}$ — can't predict with certainty both position and momentum.

EPR

“Quantum Mechanics is Not Complete”

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.



$$|\Psi\rangle = \int |x\rangle\,|x-D\rangle = \int |p\rangle\,|-p\rangle$$

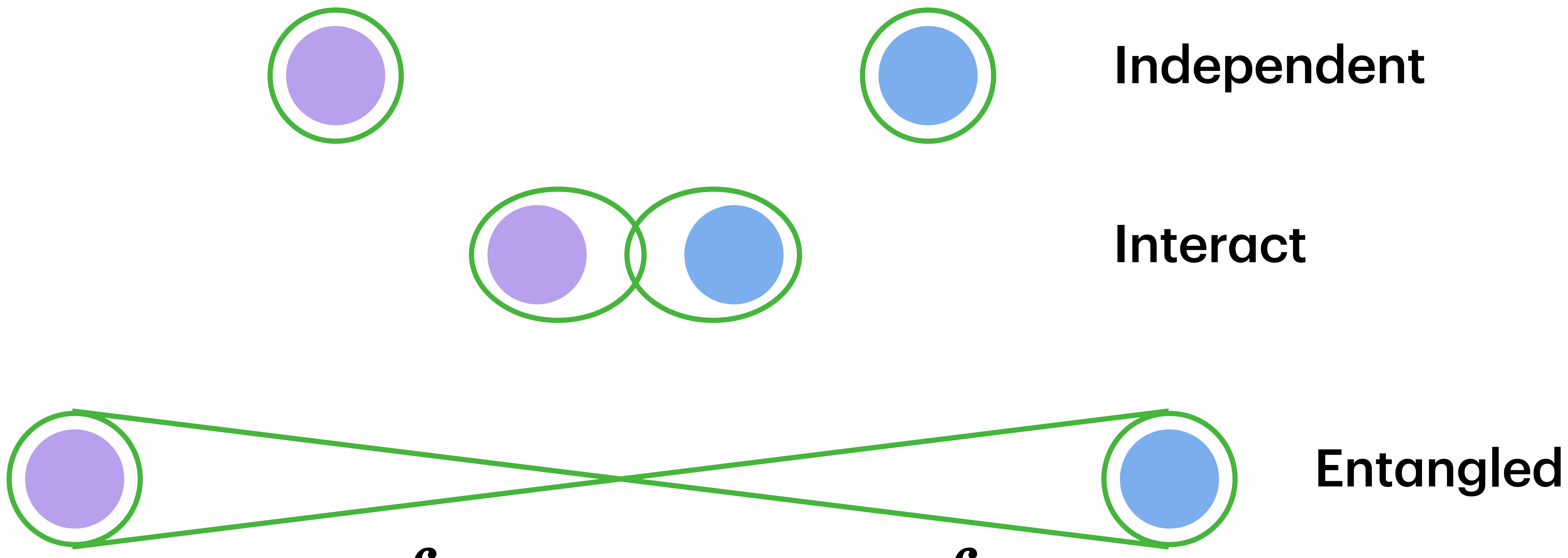
- EPR find a clever example of a two-particle state (entangled!) such that: 1) You can measure position of the left particle and **deduce with certainty** the *location* of the second OR 2) You can measure the momentum of the left particle and **deduce with certainty** the *momentum* of the right particle.

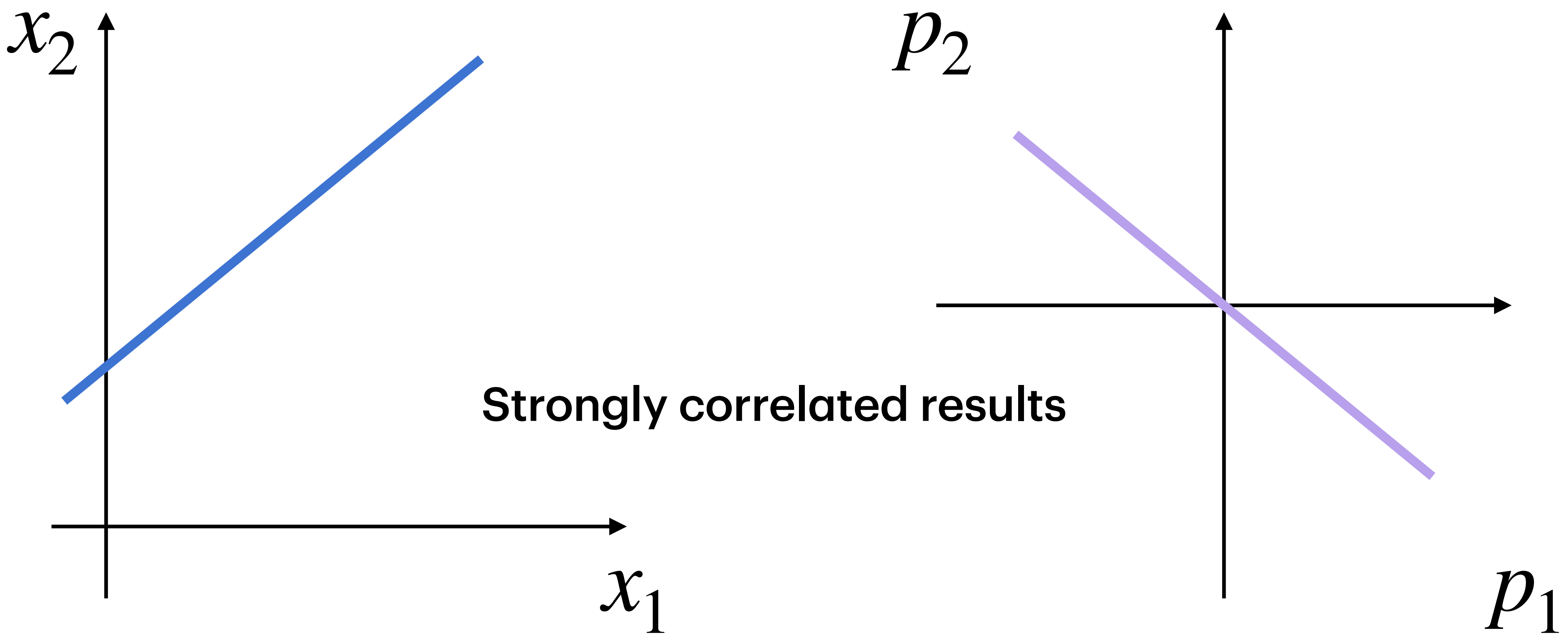
$$\Psi(x_1, x_2) = \int_{-\infty}^{\infty} e^{(2\pi i/h)(x_1 - x_2 + x_0)p}\,dp,$$

EPR

"Quantum Mechanics is Not Complete"

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.





$$\Psi(x_1, x_2) = \int_{-\infty}^{\infty} e^{(2\pi i/h)(x_1 - x_2 + x_0)p} dp = h\delta(x - x_2 + x_0).$$

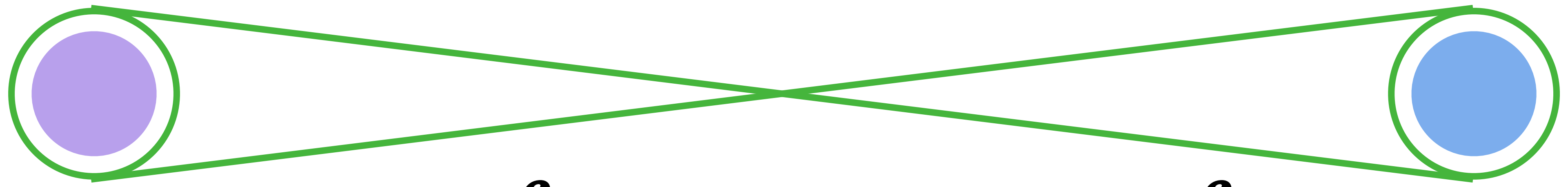
$$|\Psi\rangle = \int |x\rangle |x - D\rangle = \int |p\rangle | -p\rangle$$

No definite x or p. Random, equally probable. But correlated!

EPR

"Quantum Mechanics is Not Complete"

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

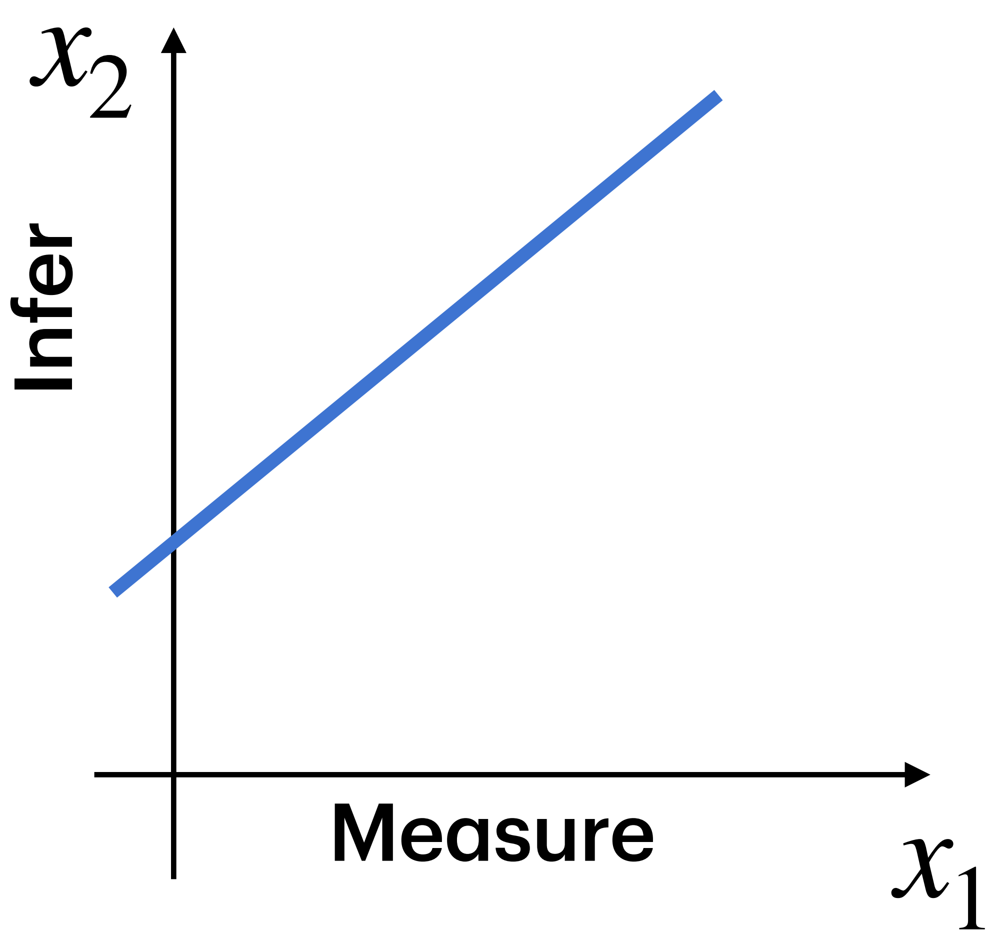


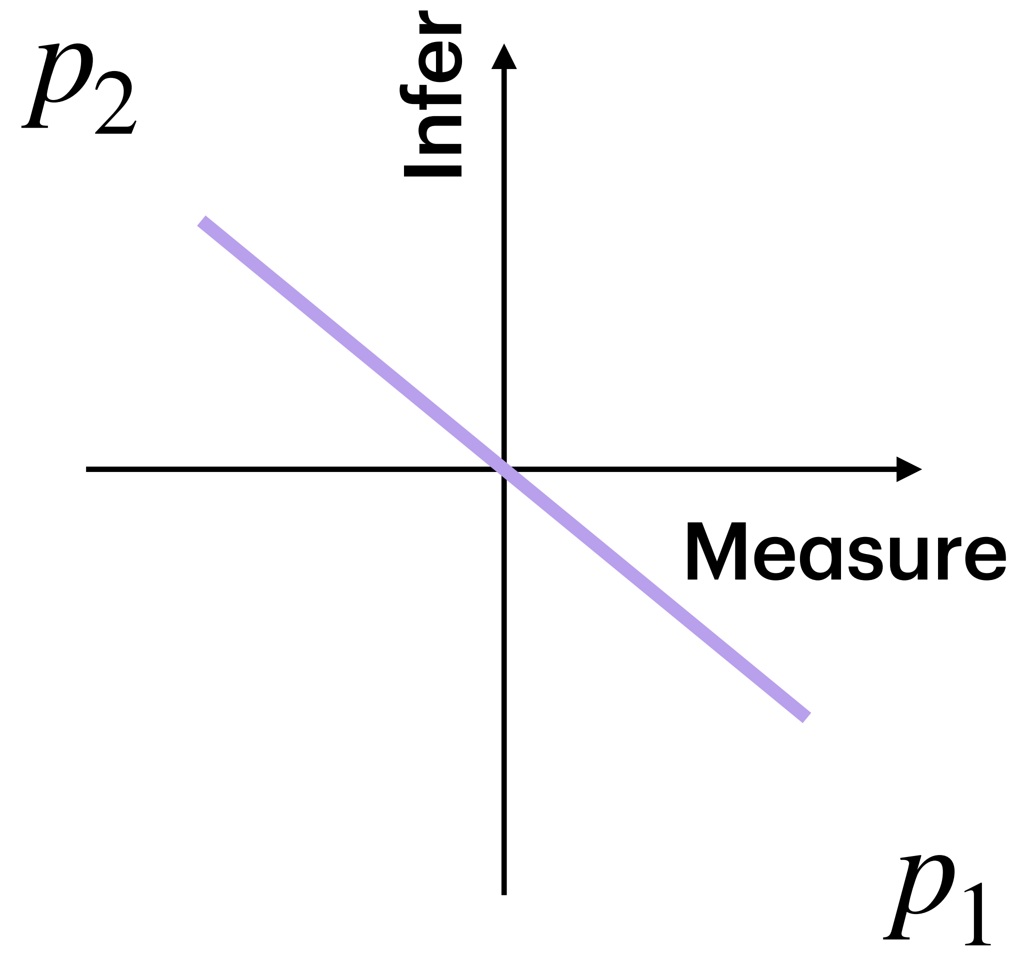
Entangled

$$|\Psi\rangle = \int |x\rangle\,|x-D\rangle = \int |p\rangle\,|-p\rangle$$

- You can measure x of the left particle — will know **with certainty** the position of the second without interrogating it — position of the second particle must be real.

- You can measure p of the left particle — will know **with certainty** the momentum of the second without interrogating it — momentum of the second particle must be real.



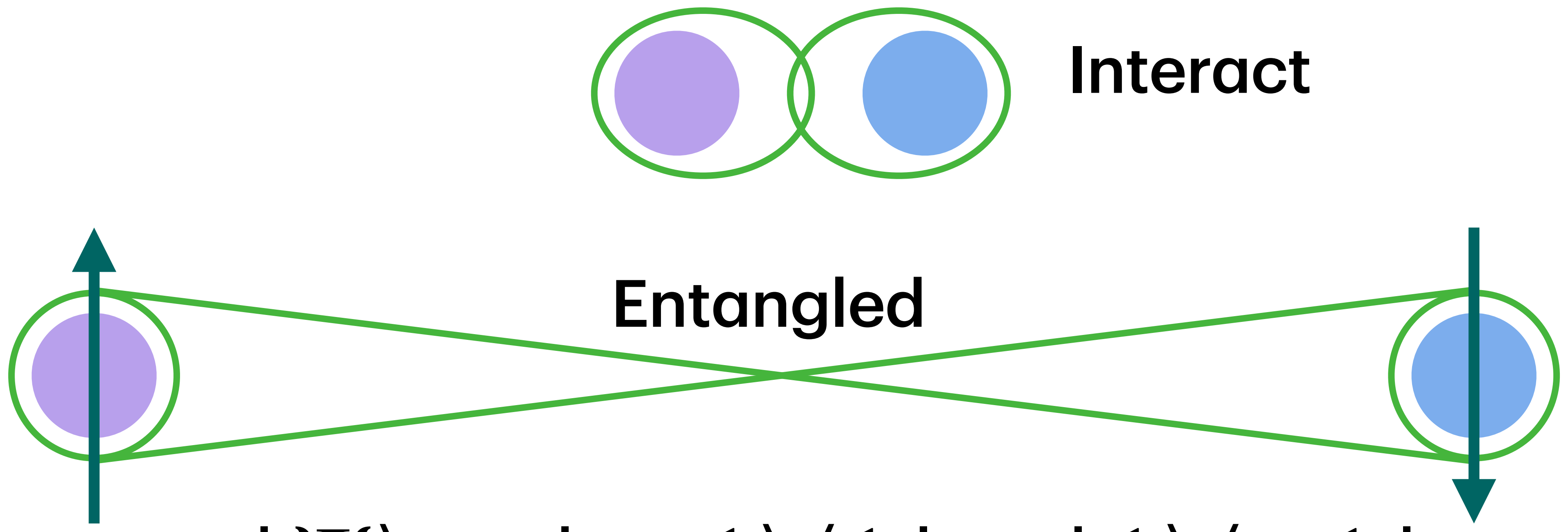


EPR: Position and momentum of the second (untouched) particle must be both real at the same time for our clevel quantum state. But quantum mechanics does not allow that. Hence — QM can not describe reality fully and is incomplete.

EPR

"Quantum Mechanics is Not Complete"

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

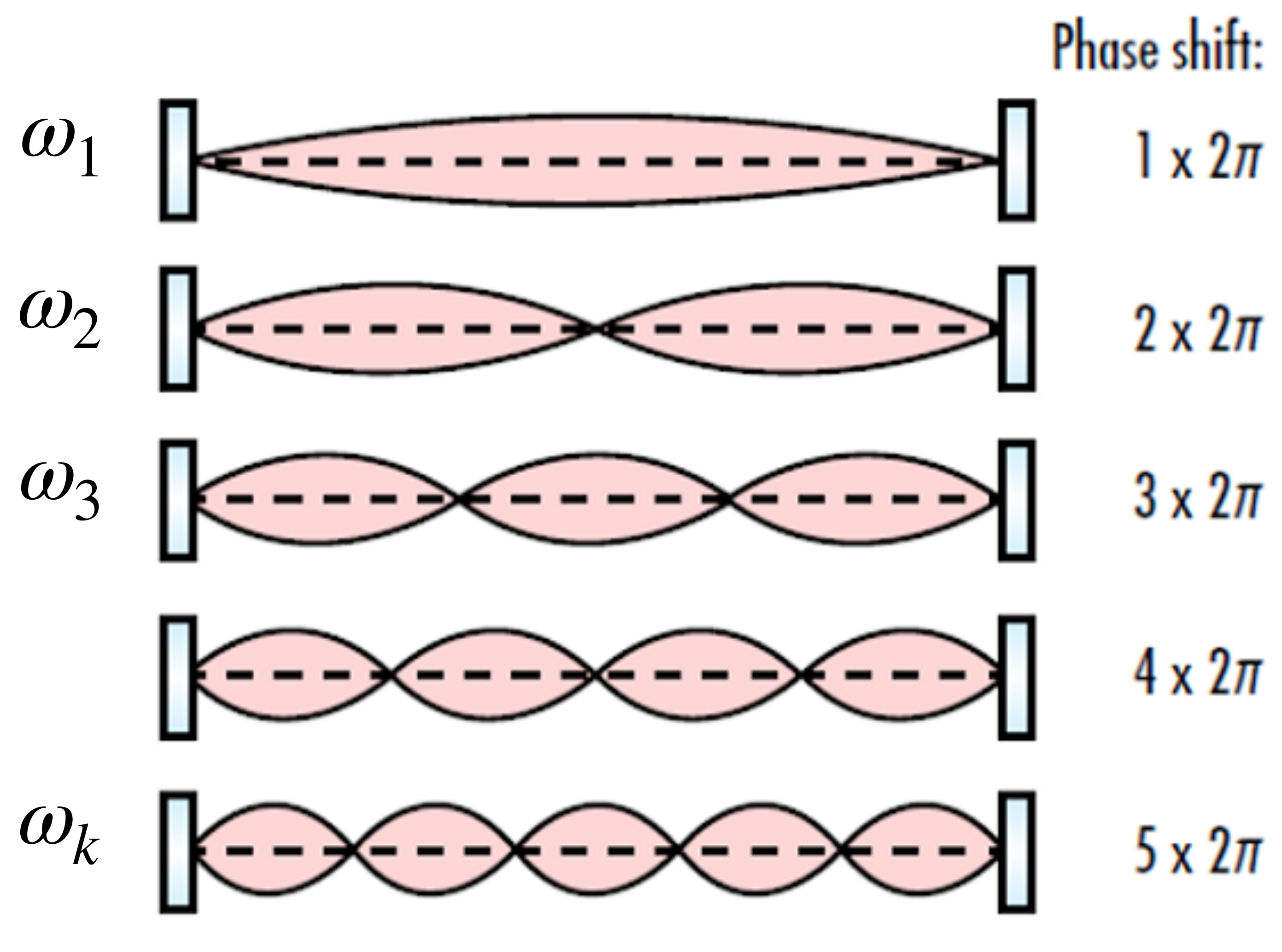


$$|\Psi\rangle = |-1\rangle\langle 1| + |1\rangle\langle -1|$$

- The example EPR chose uses values with infinite and continuous range, making the "sum" into integral.
- David Bohm made their argument much simpler by considering non-commuting quantities with values -1 and 1. (Spin projection or polarization projection)

GHZ State

Quantum vs Classical Views



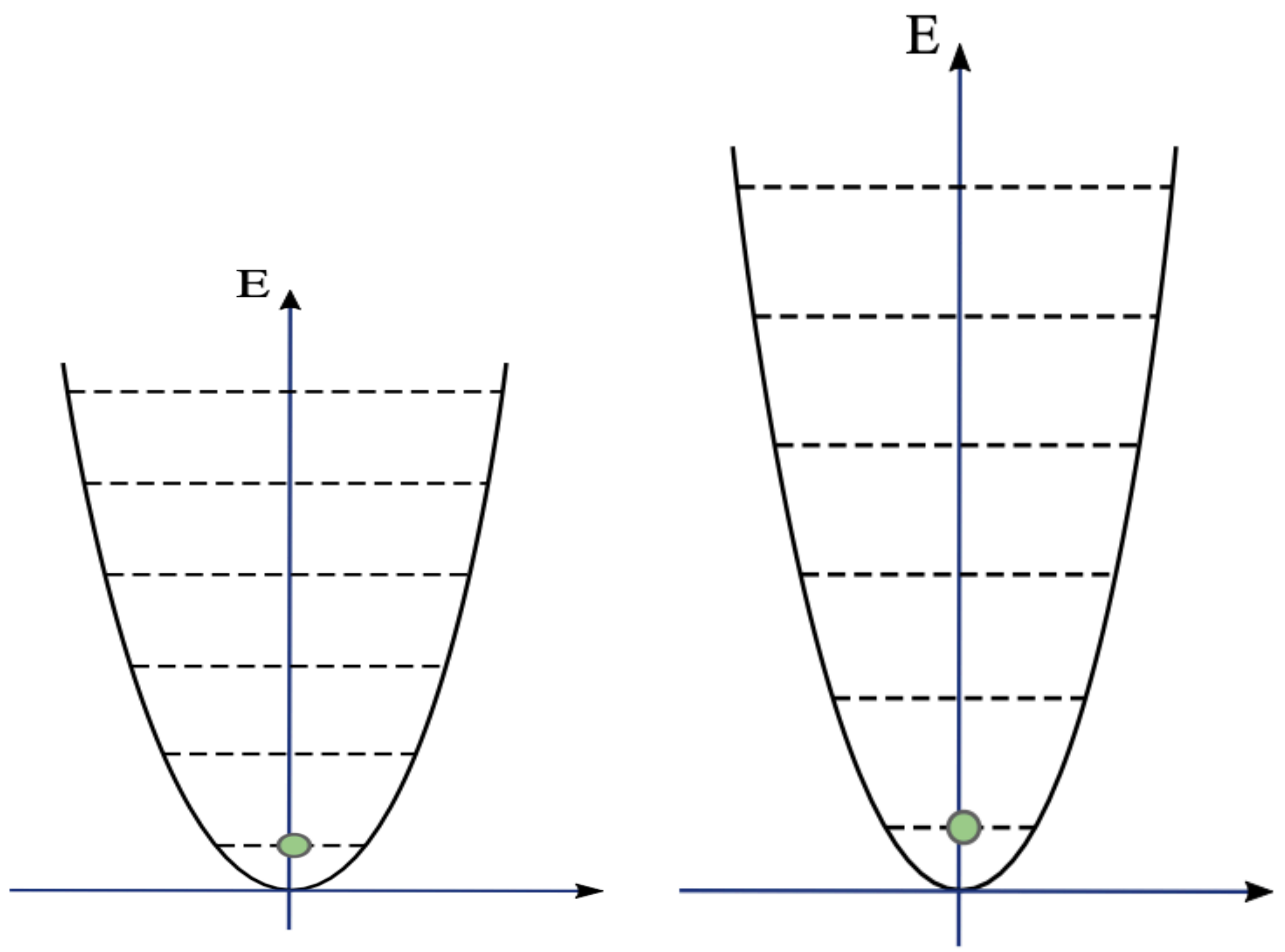
- **Quantum system**: Any physical entity or its features that exhibit quantum behavior. (E.g. mode, spin, or polarization)
- A mode of EMF is a quantum system — mathematically equivalent to a **harmonic oscillator**.
- A single cavity can support different modes — different "quantum systems" inside the same cavity.

$$|\Psi\rangle = |n_1\rangle|n_2\rangle|n_3\rangle\ldots|n_k\rangle\ldots$$

First mode

K-th mode

n_k Number of excitations in the k-th mode



GHZ State

Quantum vs Classical Views

$$|\Psi_1\rangle = \overset{\substack{\omega_1\\ \uparrow}}{|1\rangle}\overset{\substack{\omega_2\\ \uparrow}}{|0\rangle}\overset{\substack{\omega_3\\ \uparrow}}{|0\rangle}\ldots\overset{\substack{\omega_k\\ \uparrow}}{|0\rangle}\ldots$$

$$|\Psi_2\rangle = |0\rangle|1\rangle|0\rangle\ldots|0\rangle\ldots$$

$$|\Psi_3\rangle = |0\rangle|0\rangle|1\rangle\ldots|0\rangle\ldots$$

$$|\Psi_k\rangle = |0\rangle|0\rangle|0\rangle\ldots|k\rangle\ldots$$

Single photon states of EMF

$$|\Psi_4\rangle = |1\rangle|1\rangle|0\rangle\ldots|0\rangle\ldots$$

$$|\Psi_5\rangle = |1\rangle|0\rangle|1\rangle\ldots|0\rangle\ldots$$

$$|\Psi_6\rangle = |1\rangle|0\rangle|0\rangle\ldots|1\rangle\ldots$$

$$|\Psi_l\rangle = |0\rangle|1\rangle|1\rangle\ldots|0\rangle\ldots$$

Two photons states of EMF.
Photons in different modes
— in different systems.

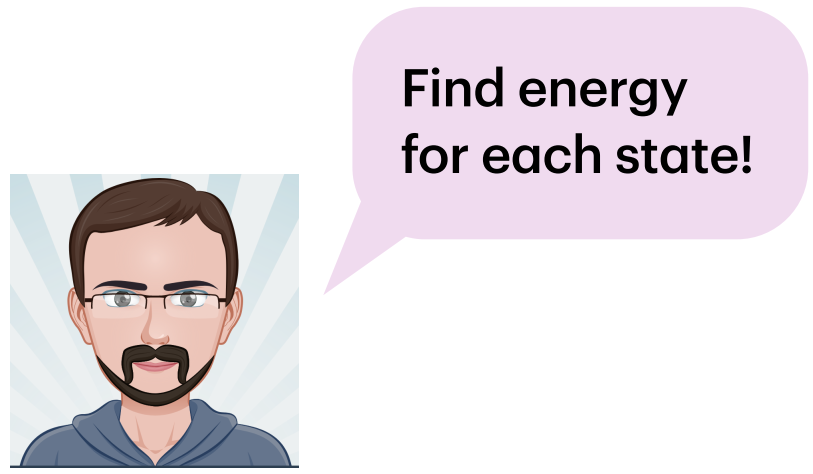
$$|\Psi_7\rangle = |2\rangle|0\rangle|0\rangle\ldots|0\rangle\ldots$$

$$|\Psi_8\rangle = |0\rangle|2\rangle|0\rangle\ldots|0\rangle\ldots$$

$$|\Psi_9\rangle = |0\rangle|0\rangle|2\rangle\ldots|0\rangle\ldots$$

$$|\Psi_m\rangle = |0\rangle|0\rangle|0\rangle\ldots|2\rangle\ldots$$

Two photons states of EMF.
Photons in the same mode
— in the same system.



GHZ State

Quantum vs Classical Views

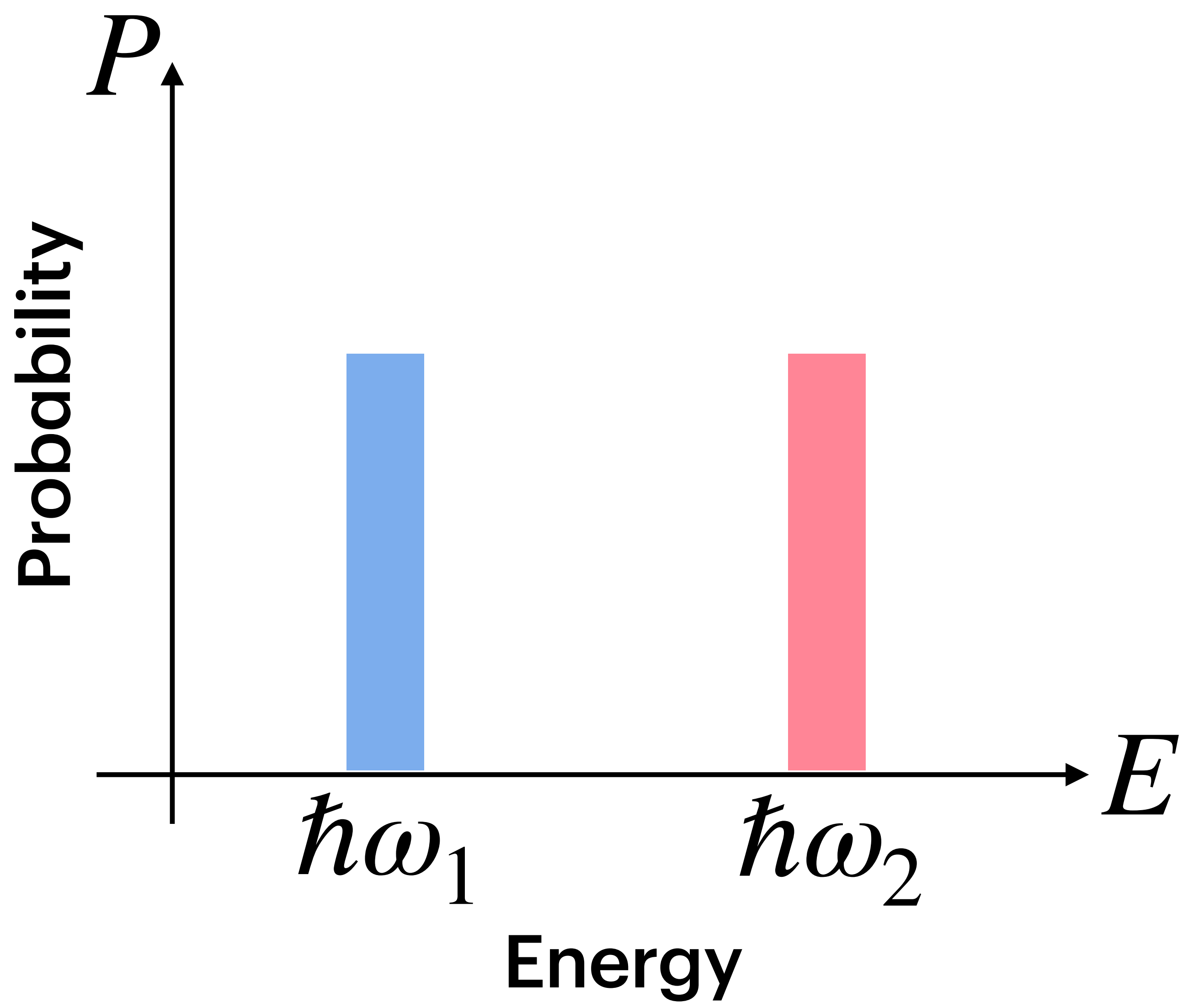
$$|\Psi_1\rangle = \underset{}{\overset{\omega_1}{\overset{\uparrow}{|1\rangle}}}\,\overset{\omega_2}{\overset{\uparrow}{|0\rangle}}\,|0\rangle\ldots|0\rangle\ldots$$

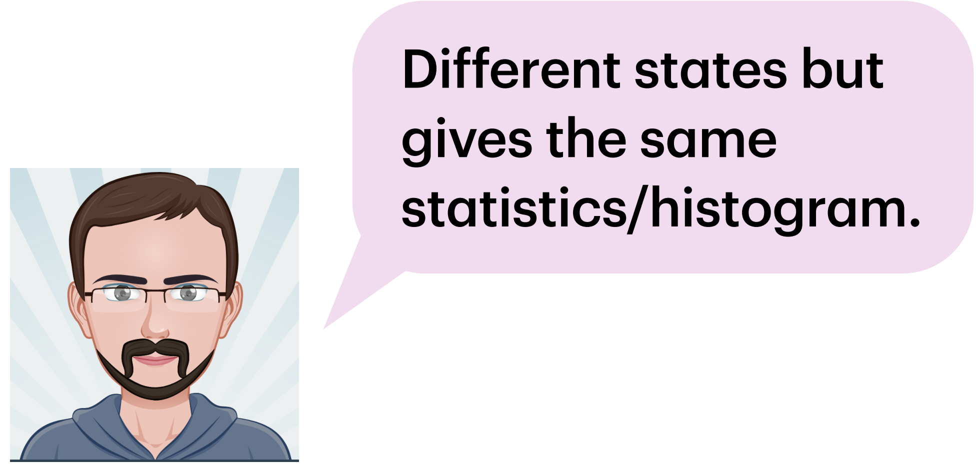
$$|\Psi_2\rangle = |0\rangle|1\rangle|0\rangle\ldots|0\rangle\ldots$$

$$|\Psi_+\rangle = (|\Psi_1\rangle + |\Psi_2\rangle)/\sqrt{2} = \big(|0\rangle|1\rangle + |1\rangle|0\rangle\big)/\sqrt{2}$$

Quantum theory allows this! State without definite energy.

$$|\Psi_-\rangle = (|\Psi_1\rangle - |\Psi_2\rangle)/\sqrt{2}$$





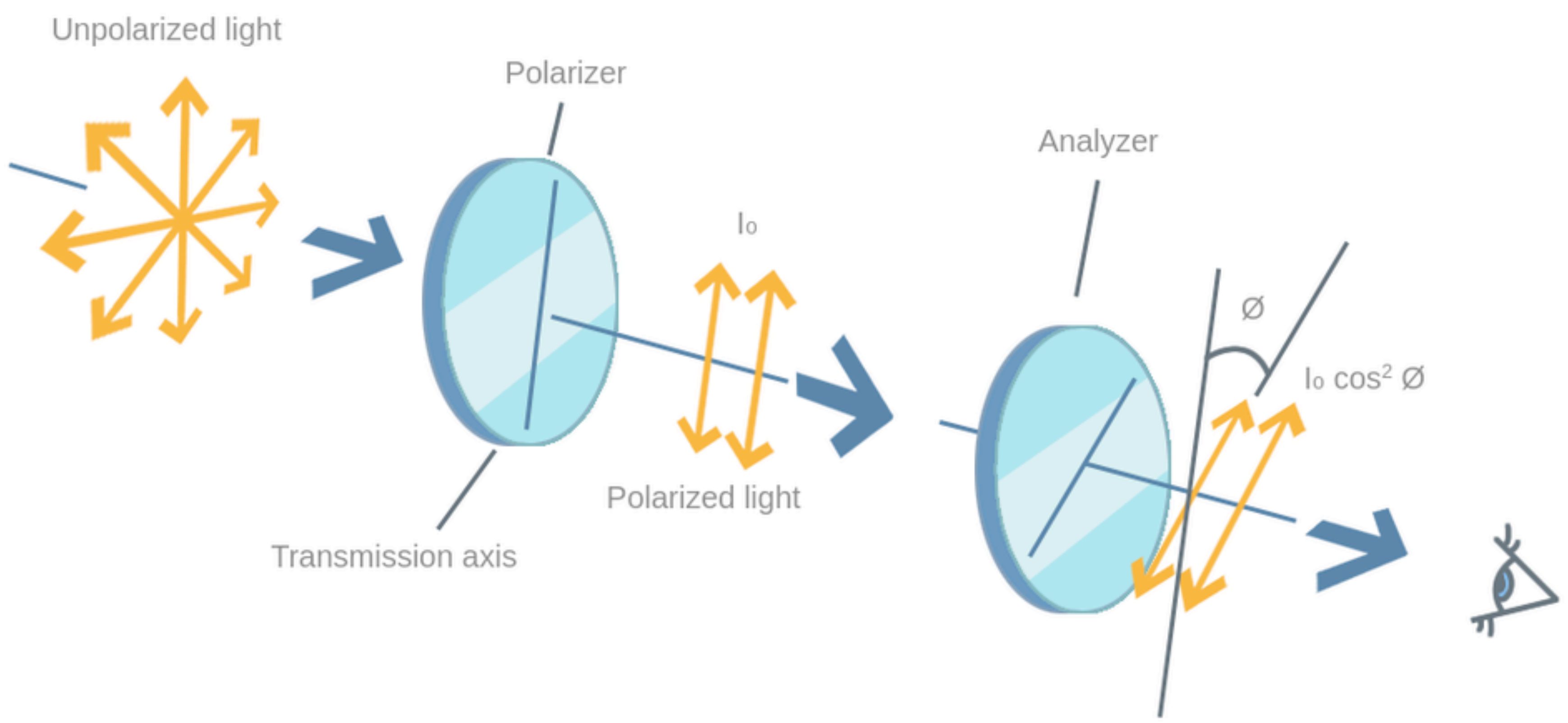
Photon Polarization

As Degree of Freedom and Quantum System

$$|\Psi_1\rangle = |1\rangle|0\rangle|0\rangle\ldots|0\rangle\ldots$$

$$|\leftrightarrow\rangle \qquad |\updownarrow\rangle$$

The Law of Malus



$$|\leftrightarrow\rangle \qquad |\updownarrow\rangle$$

$$|H\rangle \qquad |V\rangle$$ Horizontal / Vertical direction

$$|\parallel\rangle \qquad |\perp\rangle$$ Parallel / Perpendicular to axis

Single photon can be distinguished using its “alignment” relative to device.

GHZ State

Quantum vs Classical Views

$$\omega_1 \quad \omega_2 \quad \omega_3 \quad \omega_k$$

$$|\Phi_1\rangle = |\updownarrow\rangle|0\rangle|0\rangle\ldots|0\rangle\ldots \qquad |\Phi_4\rangle = |\updownarrow\rangle|\updownarrow\rangle|0\rangle\ldots|0\rangle\ldots \qquad |\Phi_7\rangle = |2\rangle|0\rangle|0\rangle\ldots|0\rangle\ldots$$

$$|\Phi_2\rangle = |\leftrightarrow\rangle|0\rangle|0\rangle\ldots|0\rangle\ldots \qquad |\Phi_5\rangle = |\updownarrow\rangle|\leftrightarrow\rangle|0\rangle\ldots|0\rangle\ldots \qquad |\Phi_8\rangle = |0\rangle|2\rangle|0\rangle\ldots|0\rangle\ldots$$

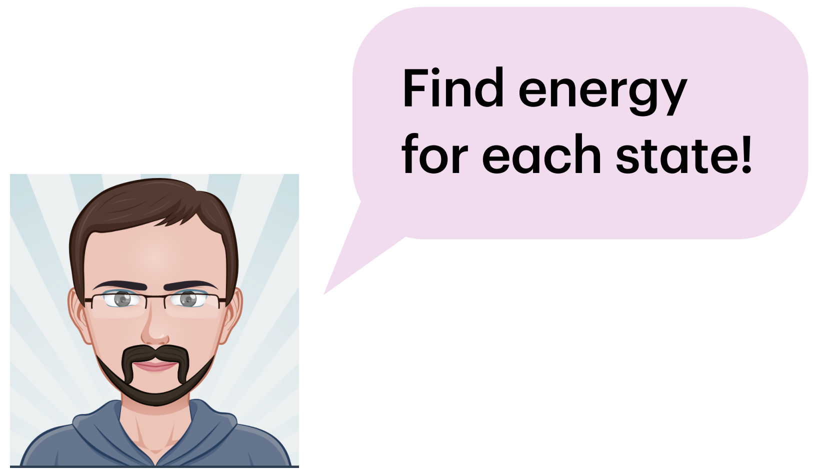
$$|\Phi_3\rangle = |0\rangle|\updownarrow\rangle|0\rangle\ldots|0\rangle\ldots \qquad |\Phi_6\rangle = |\leftrightarrow\rangle|\updownarrow\rangle|0\rangle\ldots|0\rangle\ldots \qquad |\Phi_9\rangle = |0\rangle|0\rangle|2\rangle\ldots|0\rangle\ldots$$

$$|\Phi_k\rangle = |0\rangle|0\rangle|0\rangle\ldots|\leftrightarrow\rangle\ldots \qquad |\Phi_l\rangle = |0\rangle|\leftrightarrow\rangle|\leftrightarrow\rangle\ldots|0\rangle\ldots \qquad |\Phi_m\rangle = |0\rangle|0\rangle|0\rangle\ldots|2\rangle\ldots$$

Single photon states of EMF

Two photons states of EMF.
Photons in different modes
— in different systems.

Two photons states of EMF.
Photons in the same mode
— in the same system.



GHZ State

Three "Particle" State

$$|GHZ\rangle = \left(|\updownarrow\rangle|\updownarrow\rangle|\updownarrow\rangle - |\leftrightarrow\rangle|\leftrightarrow\rangle|\leftrightarrow\rangle\right)/\sqrt{2}$$

Entangled state

$$|GHZ\rangle = \left(|1\rangle|1\rangle|1\rangle - |0\rangle|0\rangle|0\rangle\right)/\sqrt{2}$$

Entangled = strong correlations!

Self-Test

Answer These Questions 1hr After Class

1. What is realism?
2. What is locality?
3. What is EPR definition of reality?
4. What are raising and lowering operators?

Homework Problems

Jaynes-Cummings Model

1. Show that the raising operator $\hat{a}^\dagger$ can be written as follows: $\hat{a}^\dagger = \int_k \sqrt{k+1}\,|k+1\rangle\langle k|\,, k = 0,1,2,\ldots$

2. Evaluate $\left(\hat{\sigma}_- \otimes \hat{a}^\dagger\right)\ |g\rangle\langle n|$.

3. Show that $\hat{H}_m = E_0\hat{I} + \hbar\omega_m\hat{n}$ follows from $\hat{H}_m = \int_k E_k|k\rangle\langle k|$ and $E_n = E_0 + n\hbar\omega_m$. Use the important relation (easy to prove): $\hat{I} = \int_k |k\rangle\langle k|$.

4. Study the Jupyter Notebook with Jaynes-Cummings Model.

5. Show that for harmonic oscillator $\hat{x} = \sqrt{\frac{\hbar}{2m\omega}}\left(\hat{a}^\dagger + \hat{a}\right)$ and $\hat{p} = i\sqrt{\frac{\hbar m\omega}{2}}\left(\hat{a}^\dagger - \hat{a}\right)$.

6. Using the fact that $\hat{a}\hat{a}^\dagger - \hat{a}^\dagger\hat{a} = \hat{I},$ calculate $\hat{x}\hat{p} - \hat{p}\hat{x}$.

Homework Problems

Hint

$$\hat{H}|n\rangle = E_n|n\rangle = (E_0 + n\hbar\omega_m)|n\rangle \longrightarrow (\hat{H} - E_0\hat{I})|n\rangle = n\hbar\omega_m|n\rangle \longrightarrow (\hat{H} - E_0\hat{I})/\hbar\omega_m|n\rangle = n|n\rangle$$

$$(\hat{H} - E_0\hat{I})/\hbar\omega_m = \hat{n}$$

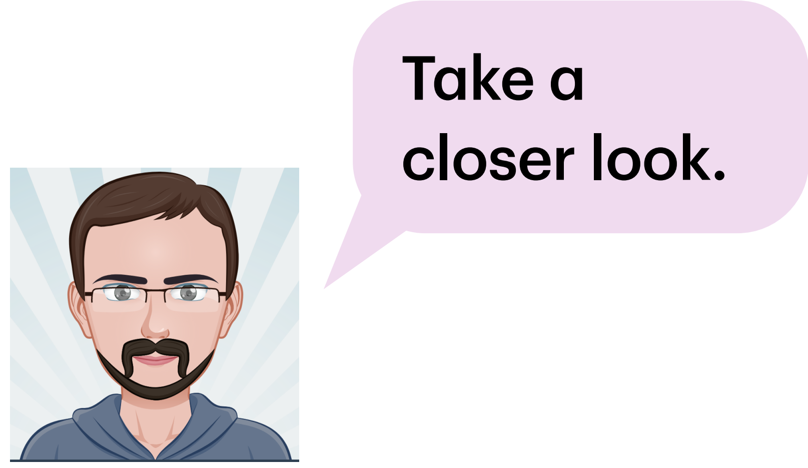
$$\hat{H} = E_0|0\rangle\langle 0| + \int_n E_n|n\rangle\langle n| = E_0|0\rangle\langle 0| + \int_n (E_0 + n\hbar\omega)|n\rangle\langle n| = E_0 \int_{m=0} |m\rangle\langle m| + \hbar\omega \int n|n\rangle\langle n|$$

$$\hat{I} = \int_{m=0} |m\rangle\langle m|$$

$$\hat{H} = E_0\hat{I} + \hbar\omega \int n|n\rangle\langle n| \longrightarrow \hat{n} = (\hat{H} - E_0\hat{I})/\hbar\omega_m = \int_{n=1} n|n\rangle\langle n| = \int_k k|k\rangle\langle k|$$

$$\boxed{\hat{H} = E_0\hat{I} + \hbar\omega\hat{n}}$$

Homework Problems



Hint

$$\hat{\sigma}_+|0\rangle = 1|1\rangle \qquad \hat{\sigma}_+|k\rangle = \sqrt{k+1}\,|k+1\rangle$$

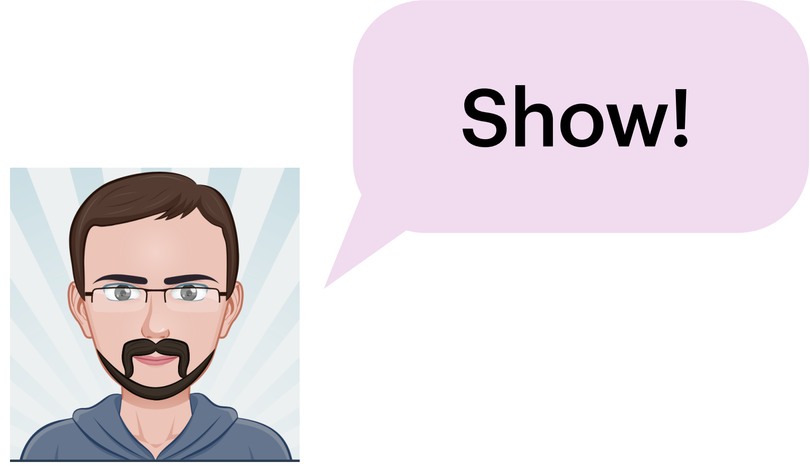
$$\hat{\sigma}_-|1\rangle = 1|0\rangle \qquad \hat{\sigma}_-|k\rangle = \sqrt{k}\,|k-1\rangle$$

$$\hat{a}^\dagger|k\rangle = \sqrt{k+1}\,|k+1\rangle$$

$$\hat{a}|k\rangle = \sqrt{k}\,|k-1\rangle$$

$$\hat{\sigma}_+\hat{\sigma}_-|k\rangle = \sqrt{k}\hat{\sigma}_+|k-1\rangle = \sqrt{k}\sqrt{k}\,|k\rangle = k|k\rangle$$

$$\hat{a}^\dagger\hat{a}|k\rangle = k|k\rangle \qquad \hat{a}^\dagger\hat{a} = \hat{n}$$



$$\hat{a}\hat{a}^\dagger|k\rangle = (k+1)|k\rangle$$

Homework Problems

Hint

$$\hat{H}_m = E_0\hat{I} + \hbar\omega_m\hat{a}^\dagger\hat{a} \qquad \hat{a}\hat{a}^\dagger - \hat{a}^\dagger\hat{a} = \hat{I}$$

$$\updownarrow$$

$$\hat{H}_m = \frac{\hat{p}^2}{2m} + \frac{k\hat{x}^2}{2} \qquad \hat{H}_m = E_0\hat{a}\hat{a}^\dagger + (\hbar\omega_m - E_0)\hat{a}^\dagger\hat{a}$$

$$\hat{x} = A\hat{a} + B\hat{a}^\dagger \qquad \hat{x}^2 = ?$$

$$\hat{p} = C\hat{a} + D\hat{a}^\dagger \qquad \hat{p}^2 = ?$$

Quantum Theory

In a Nutshell

II. POSTULATES FOR QUANTUM MECHANICS

In this paper, all state vectors are supposed to be normalized, and mixed states are represented by density operators, i.e., positive operators with unit trace. Let A be an observable with a nondegenerate purely discrete spectrum. Let $\phi_1, \phi_2, \ldots$ be a complete orthonormal sequence of eigenvectors of A and $a_1, a_2, \ldots$ the corresponding eigenvalues; by assumption, all different from each other.

According to the standard formulation of quantum mechanics, on the result of a measurement of the observable A the following postulates are posed:

(A1) *If the system is in the state ψ at the time of measurement, the eigenvalue a_n is obtained as the outcome of measurement with the probability $|\langle\phi_n|\psi\rangle|^2$.*

(A2) *If the outcome of measurement is the eigenvalue a_n, the system is left in the corresponding eigenstate ϕ_n at the time just after measurement.*

The postulate (A1) is called the *statistical formula*, and (A2) the *measurement axiom*. The state change $\psi \mapsto \phi_n$ described by the measurement axiom is called the *state reduction*.

You will understand this paragraph in the end of the course.