Quantum Physics

2024

The Theory/Framework Of *Almost* Everything *Today*

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Course Overview

Course Structure And Goals

- Part 1 : Mathematical Concepts And Tools
- Part 2 : Classical Physics
- Part 3 : Quantum Physics

We want to understand SchrEq

Operator

$$\underbrace{i\hbar\partial_t}_{\text{Rate of change with respect to time}}|\Psi\rangle = \underbrace{\hat{H}}_{\textbf{Hamiltonian}}|\Psi\rangle$$

vector, but can be also made an operator

Today we will understand $\hat{H}$ better.

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We want to understand SchrEq

Operator ← $i\hbar\partial_t|\Psi\rangle = \hat{H}|\Psi\rangle$ → Vector, but can be also made an operator

Rate of change with respect to time

Hamiltonian

SchrEq is *dynamical equation* for state $|\Psi\rangle$.

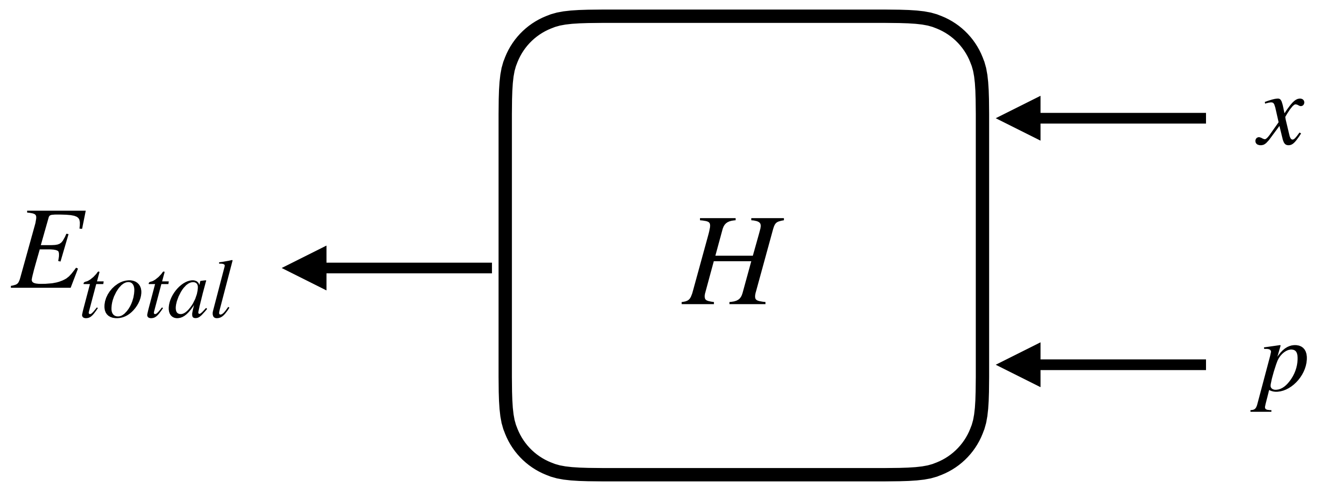
Physics (interaction and “forces”) is encoded in Hamiltonian $\hat{H}$.

Today we will understand $\hat{H}$ better.

Hamiltonian

Hamiltonian Dynamics

Hamiltonian is the total *energy* expressed in terms of *position* and *momentum*.



H**: state (x,p) —> total energy**

$$H = \frac{p^2}{2m} + \frac{kx^2}{2}$$

What can energy tell us about equations of motions?

$$F = ma$$

$$\downarrow$$

$$\partial_t x = v$$

$$\partial_t v = F/m$$

Hamiltonian

Hamiltonian Dynamics

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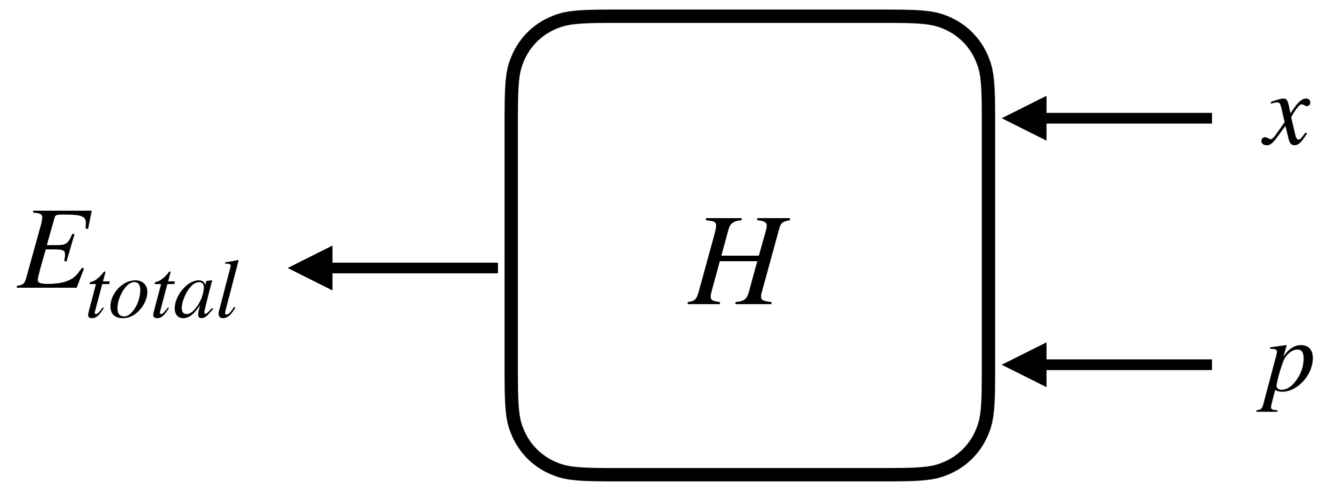


***H*: state (x,p) —> total energy**

$$\boxed{E = \frac{mv^2}{2} + \frac{kx^2}{2} = \frac{m(v+\delta v)^2}{2} + \frac{k(x+\delta x)^2}{2}}$$

$$\downarrow$$

$$m[(v+\delta v)^2 - v^2] = -k[(x+\delta x)^2 - x^2]$$

$$\downarrow$$

$$m[2va\delta t + a^2(\delta t)^2] \approx -k[2xv\delta t + v^2(\delta t)^2]$$

$$\downarrow$$

$$m[vat + a^2\delta t] \approx -k[xv + v^2\delta t]$$

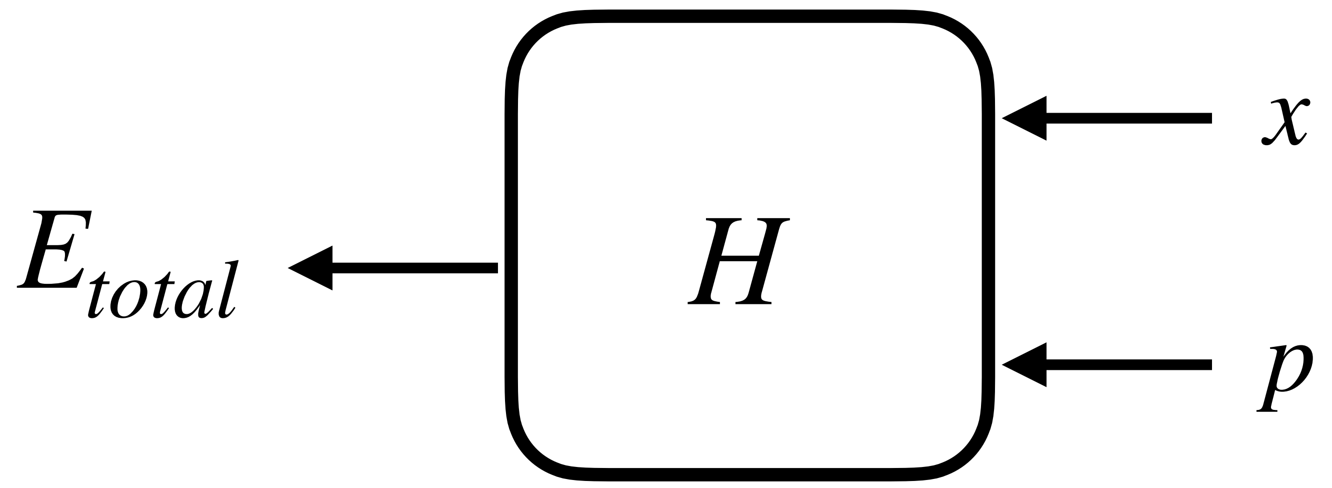
$$\downarrow$$

$$mva = -kxv \quad \rightarrow \quad \boxed{ma = -kx}$$

Hamiltonian

Hamiltonian Equations

Hamiltonian is used to obtain equations of motion in special but simple form.



H: state (x,p) —> total energy

$$\partial_t x = \hat{X} H$$

$$\partial_t p = \hat{P} H$$

Example: Oscillator $\quad H = \frac{p^2}{2m} + \frac{kx^2}{2}$

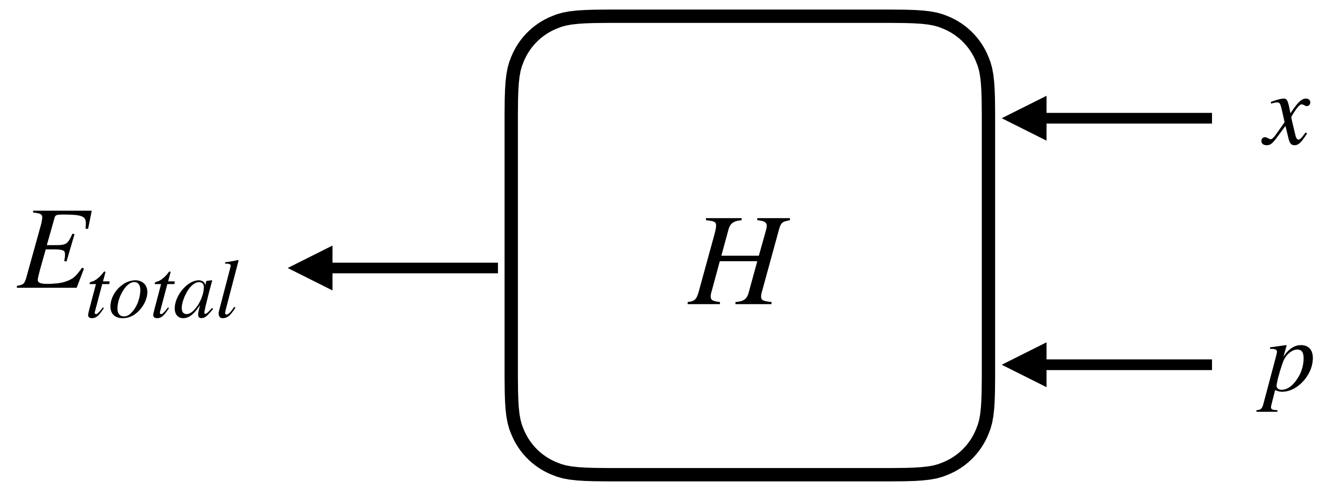
$$\partial_t x = v = \frac{p}{m} = \partial_p H$$

$$\partial_t p = F = -kx = -\partial_x H$$

Hamiltonian

Hamiltonian Equations

Hamiltonian is used to obtain equations of motion in special but simple form.



H: **state (x,p) —> total energy**

$$\boxed{\begin{aligned} \partial_t x &= \partial_p H \\ \partial_t p &= -\partial_x H \end{aligned}}$$

Hamiltonian Equations

Example: Oscillator $\quad H = \dfrac{p^2}{2m} + \dfrac{kx^2}{2}$

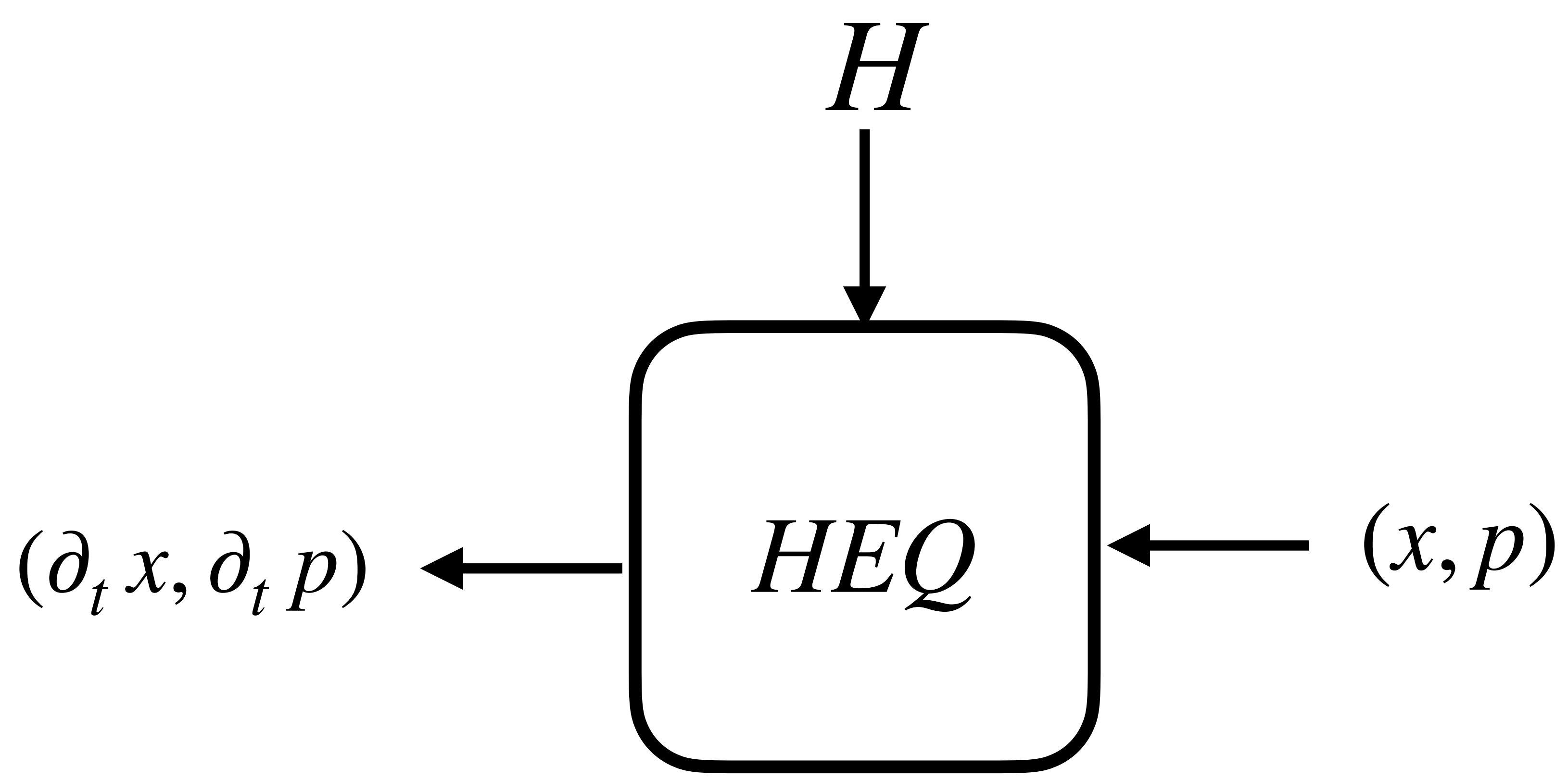
$$\partial_t x = v = \frac{p}{m} = \partial_p H$$

$$\partial_t p = F = -kx = -\partial_x H$$

Hamiltonian

Hamiltonian Approach

What is the big deal? What is the advantage?



HEQ: **state** $(x, p) \longrightarrow \partial_t (x, p)$

$$\boxed{\begin{aligned} \partial_t x &= \partial_p H \\ \partial_t p &= -\partial_x H \end{aligned}}$$

Hamiltonian Equations (HEQ)

- Hamiltonian Dynamics is **not** the best or universal solution to problems.
- It is just *another, alternative* approach to the problem of motion.
- It focuses on *energy* as more fundamental concept than force.
- HEQ are useful for numerical calculations.
- Hamiltonian Approach often helps answer general questions, like stability of motion.
- Works well in Quantum Physics. There H becomes an operator $\hat{H}$.

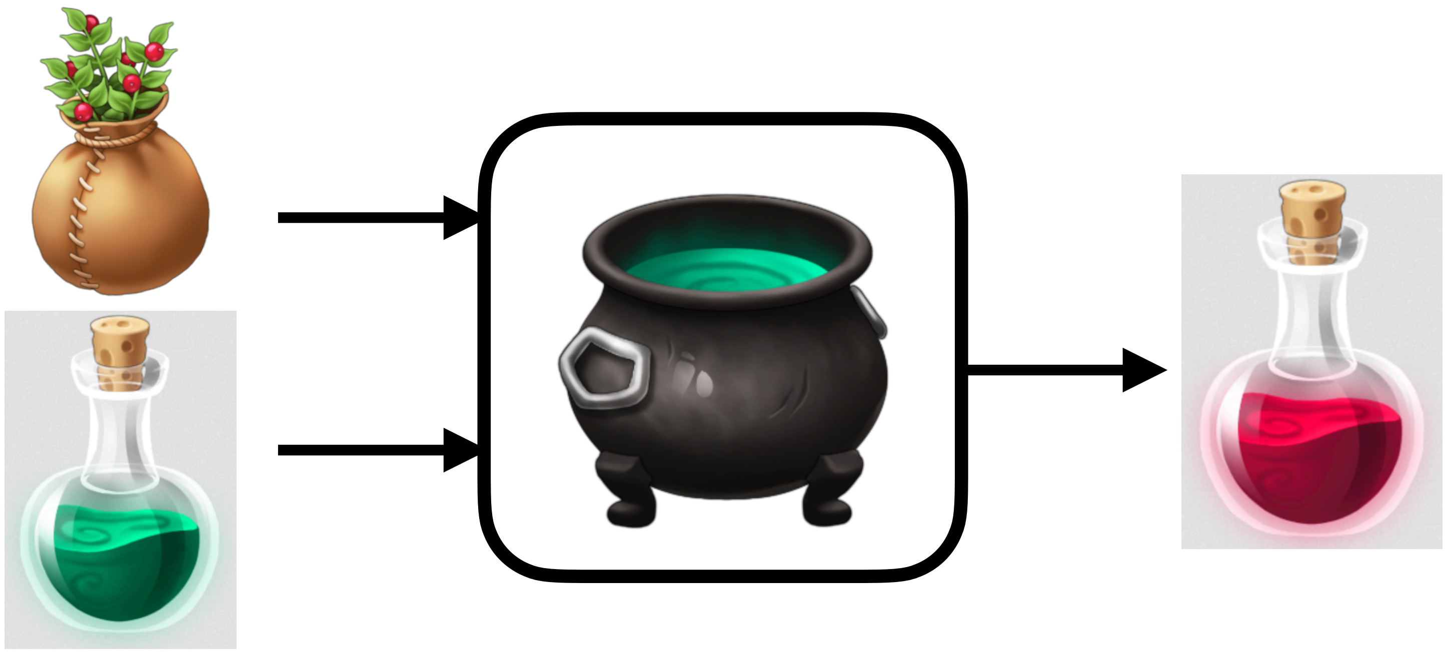
Game of Arrows and Operators

A.k.a. Vector Algebra and Operator Algebra

We know what *operators* are. We need to learn what are *vectors* and what is *algebra*.

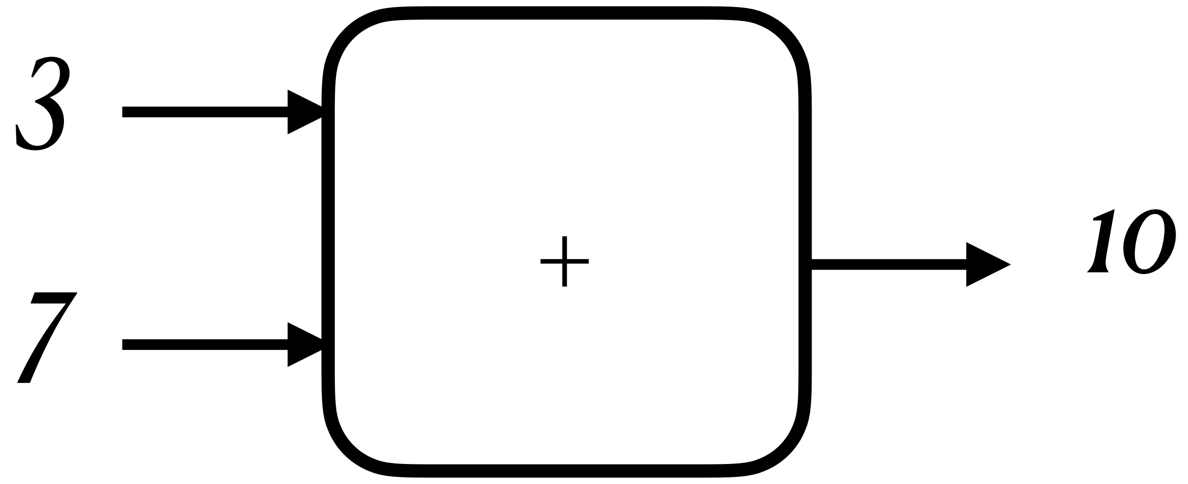
Game

- Elements
- Interaction of elements
- Rules



Algebra

- Elements
- Operations with elements
- Rules



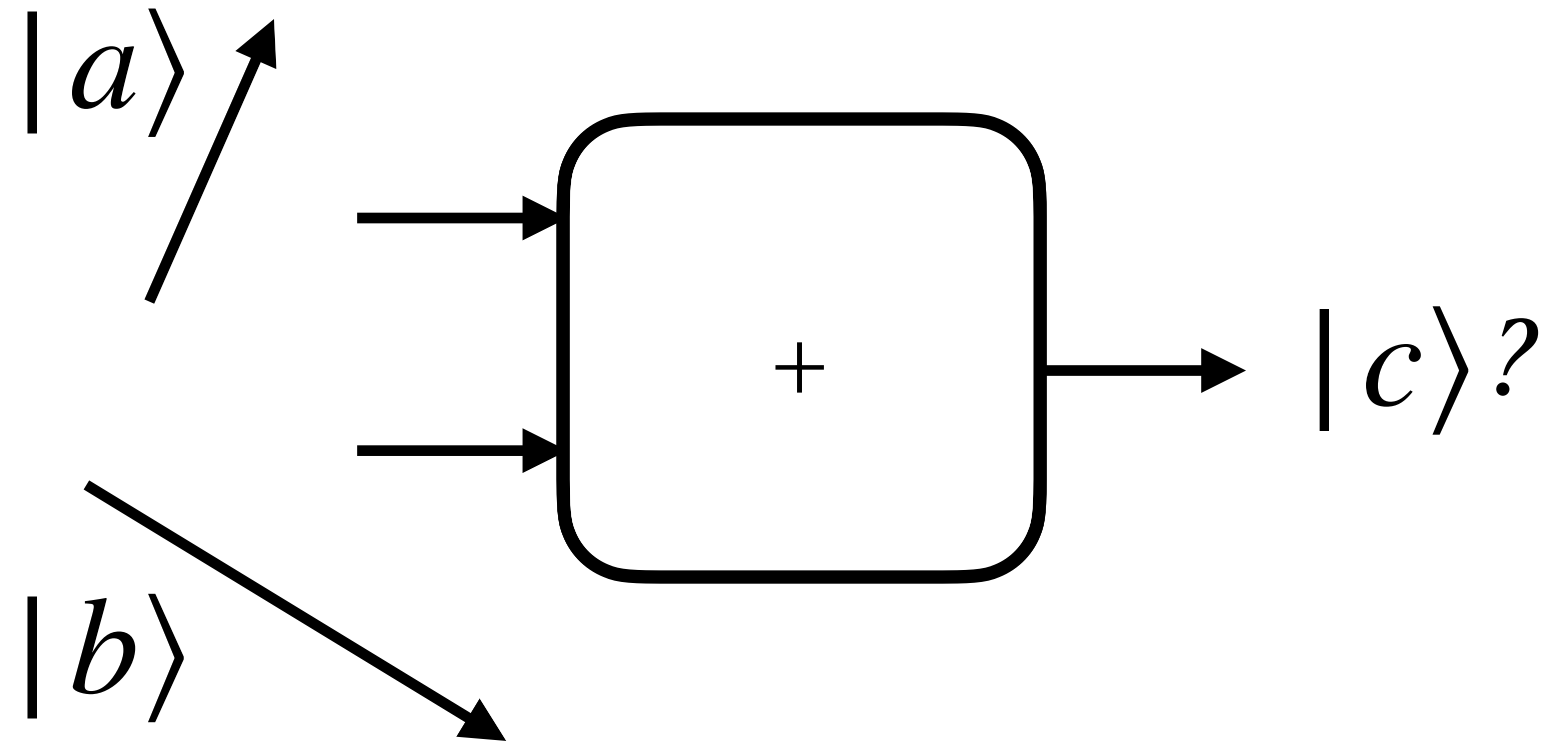
Game of Arrows and Operators

A.k.a. Vector Algebra and Operator Algebra

How to combine arrows?

How to combine operators?

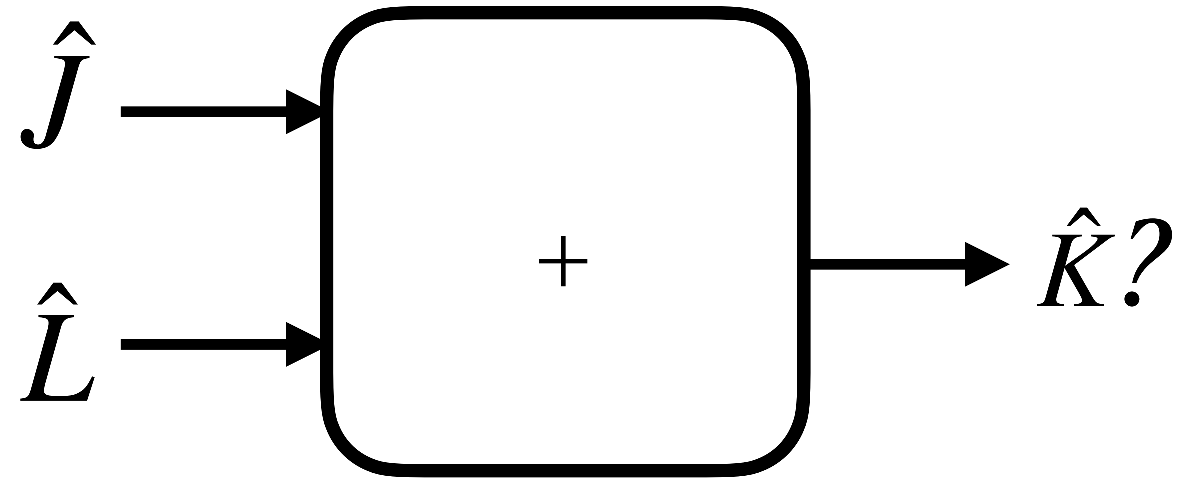
Arrows



$$|a\rangle + |b\rangle = |c\rangle$$

Operators

$$\hat{L} = \sqrt{\hat{J}}$$



$$\hat{J} + \hat{L} = \hat{K}$$

Same symbol "+" used in three different contexts. OK if clear. But must be careful!

Game of Arrows and Operators

A.k.a. Vector Algebra and Operator Algebra

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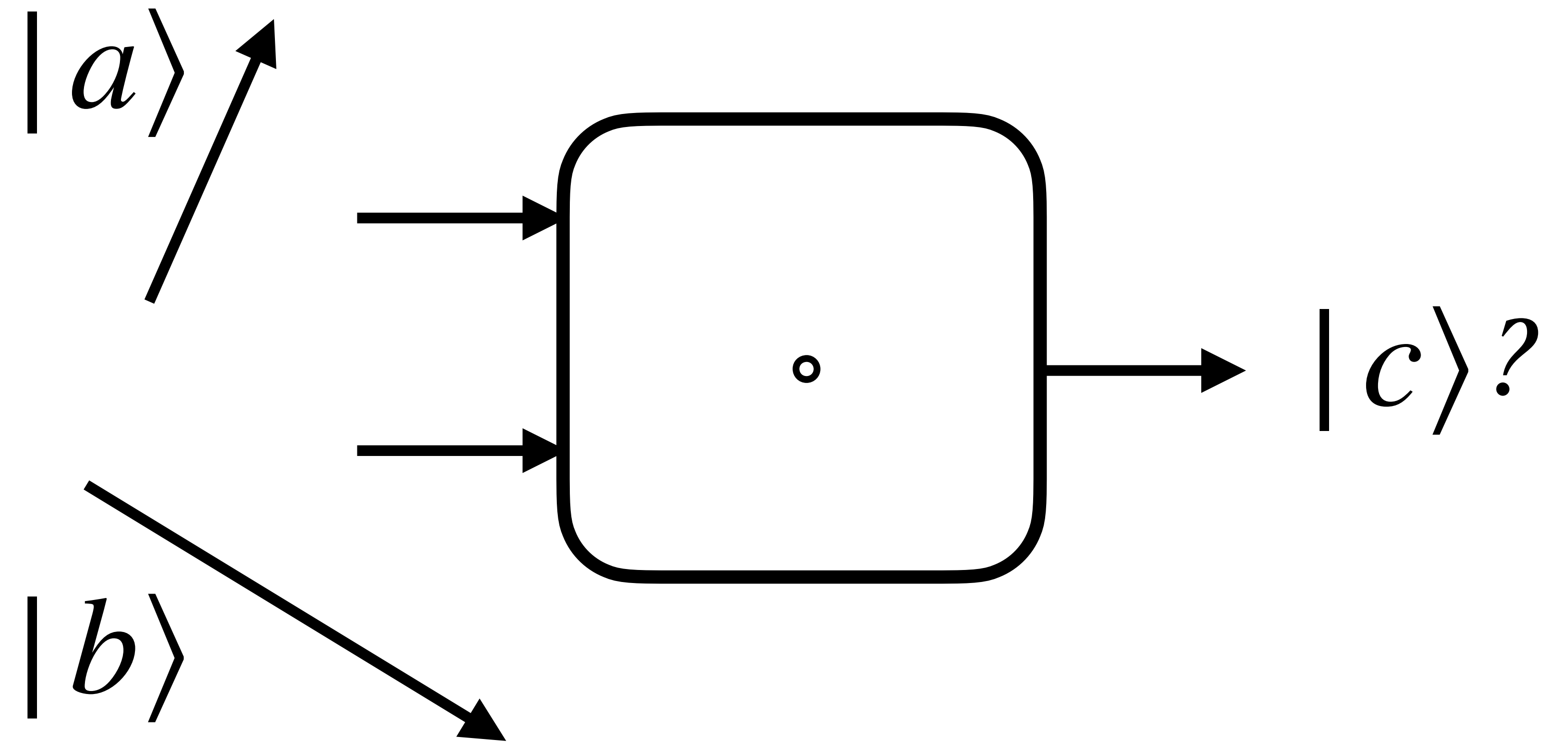
How to combine operators?

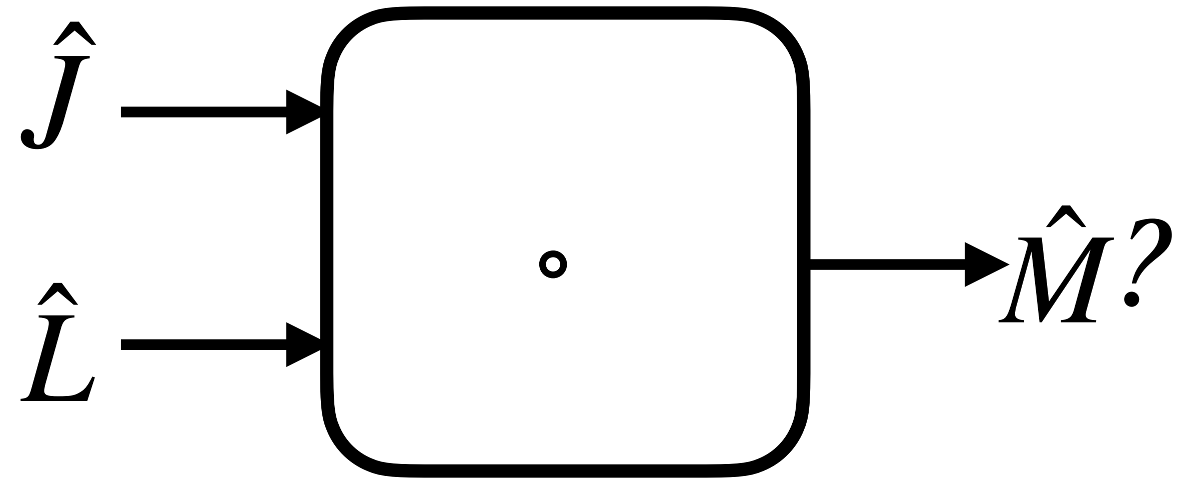
Arrows



$$|a\rangle \circ |b\rangle \stackrel{?}{=} |c\rangle$$

Operators

$$\hat{L} = \sqrt{\hat{J}}$$



$$\hat{J} \circ \hat{L} = \hat{M}$$

Operators, like functions, can also be composed. Can arrows be composed?

Self-Test

Answer These Questions 1hr After Class

1. What is the meaning of Schrödinger equation?
2. What is Hamiltonian?
3. What is role of Hamiltonian in mechanics?
4. What equations in Hamiltonian approach play the role of Newton's second law?
5. Which approach is better: Newtonian or Hamiltonian?
6. What is algebra?
7. In what sense *arrows* form an *algebra*?
8. In what sense *operators* form an *algebra*?

Homework Problems

Homework 4

- Review the properties of the function a^x. (We will need it very soon)
- For the operator $\hat{L} = \hat{I}/\sqrt{2} + \hat{J}/\sqrt{2}$, calculate $\hat{L}^2$. What does $\hat{L}$ do to any arrow?
- Potential energy of a body with mass m lifted above the ground to the height x is $E_p = mgx$. Write down the Hamiltonian for this system. Write down Hamiltonian equations, fully evaluating their right-hand side ($\partial_p H$ and $\partial_x H$)
- Write down the Hamiltonian for the following system: An asteroid with very small mass m falling down radially towards a massive star with the mass $M \gg m$. Write down Hamiltonian equations, fully evaluating their right-hand side ($\partial_p H$ and $\partial_x H$)
- Suppose the Hamiltonian of a fast moving particle is given by $H^2 = p^2 + m^2$. Show that the momentum is related to speed of the particle as follows: $p = \dfrac{mv}{\sqrt{1 - v^2}}$.

Quantum Theory

In a Nutshell

II. POSTULATES FOR QUANTUM MECHANICS

In this paper, all state vectors are supposed to be normalized, and mixed states are represented by density operators, i.e., positive operators with unit trace. Let A be an observable with a nondegenerate purely discrete spectrum. Let $\phi_1, \phi_2, \ldots$ be a complete orthonormal sequence of eigenvectors of A and $a_1, a_2, \ldots$ the corresponding eigenvalues; by assumption, all different from each other.

According to the standard formulation of quantum mechanics, on the result of a measurement of the observable A the following postulates are posed:

(A1) *If the system is in the state ψ at the time of measurement, the eigenvalue a_n is obtained as the outcome of measurement with the probability $|\langle\phi_n|\psi\rangle|^2$.*

(A2) *If the outcome of measurement is the eigenvalue a_n, the system is left in the corresponding eigenstate ϕ_n at the time just after measurement.*

The postulate (A1) is called the *statistical formula*, and (A2) the *measurement axiom*. The state change $\psi \mapsto \phi_n$ described by the measurement axiom is called the *state reduction*.

You will understand this paragraph in the end of the course.