Quantum Physics

2025

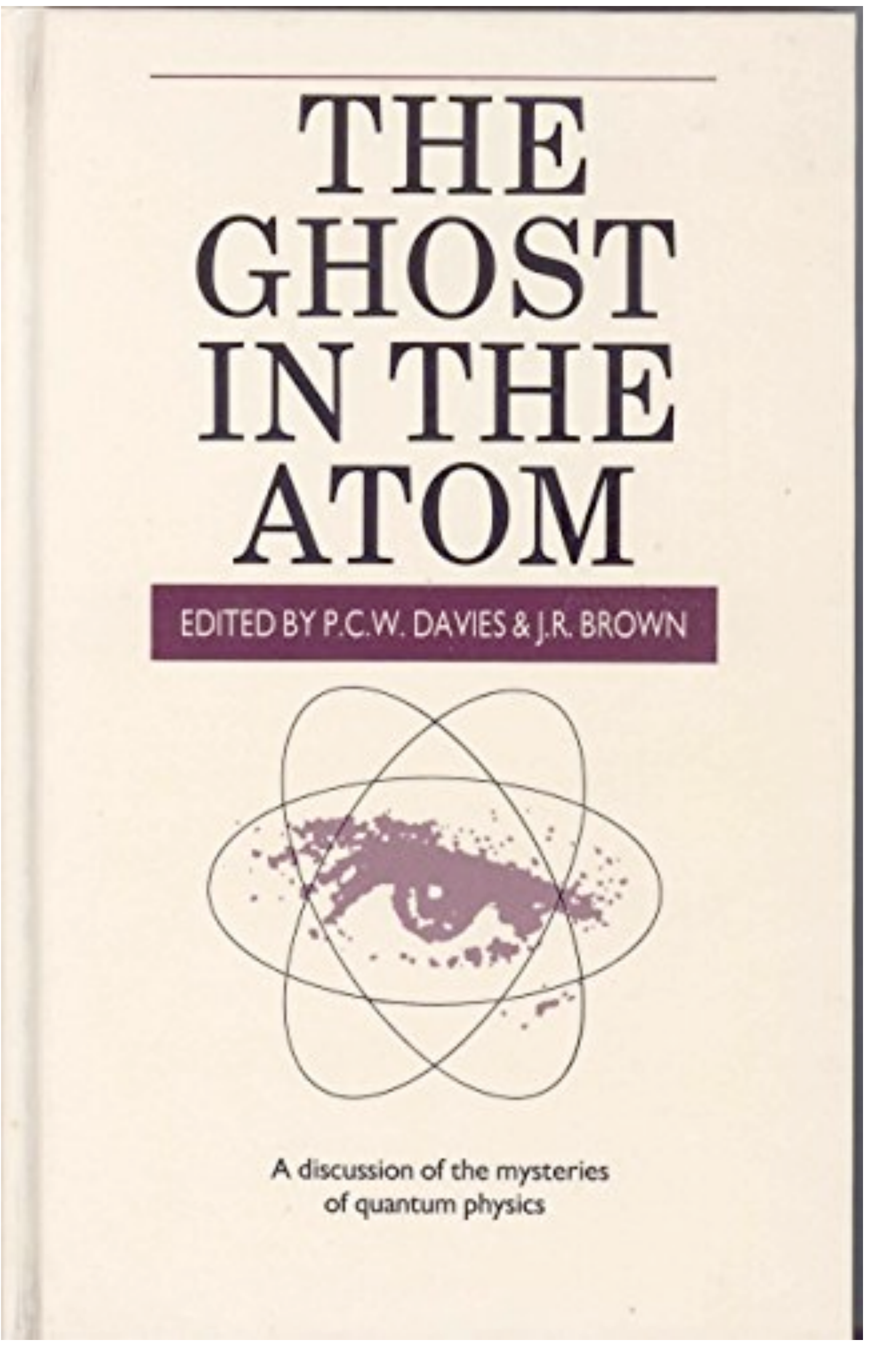
The Theory/Framework Of *Almost* Everything *Today*

But Most Likely NOT of Tomorrow

Yury Deshko

“I’m quite convinced of that: quantum theory is only a temporary expedient.”

John Bell in “*The Ghost In The Atom*”.



Quantum Physics

2025

The Theory/Framework Of *Almost* Everything *Today*

But Most Likely NOT of Tomorrow

Yury Deshko

Course Overview

Course Structure And Goals

- **Part 1** : Mathematical Concepts And Tools.
- **Part 2** : Classical Physics.
- **Part 3** : Quantum Physics.

- Learn the language of quantum physics.
- Enhance the knowledge of classical physics.
- Develop modern quantum thinking.

Language Of Quantum Physics

And Much Of Classical!

- Operators. $\hat{A}, \hat{B}, \ldots, \hat{H}, \hat{L}$
- State vectors. $|\Psi\rangle, |\Phi\rangle, \ldots$
- Evolution. $|\Psi_t\rangle = \hat{U}_t |\Psi_0\rangle$
- Dynamical equations. $\partial_t |\Psi_t\rangle = \hat{V}_t |\Psi_t\rangle$
- Hamiltonian. $H, \hat{H}$
- Eigen-problem. $\hat{H}|\Psi_E\rangle = E|\Psi_E\rangle.$
- Commutators. $[\hat{A}, \hat{B}] = \hat{C}$
- And more... $|\Psi\rangle\langle\Psi|, \hat{a}^\dagger$

Language as a Tool

Different Tools for Different Jobs

Quantum/Micro Scale ← Classical/Everyday Scale → Cosmological Scale

Language Of Quantum Physics

And Much Of Classical!

- **Operators.** $\hat{A}, \hat{B}, \ldots, \hat{H}, \hat{L}$
- State vectors. $|\Psi\rangle, |\Phi\rangle, \ldots$
- Evolution. $|\Psi_t\rangle = \hat{U}_t |\Psi_0\rangle$
- Dynamical equations. $\partial_t |\Psi_t\rangle = \hat{V}_t |\Psi_t\rangle$
- Hamiltonian. $H, \hat{H}$
- Eigen-problem. $\hat{H}|\Psi_E\rangle = E|\Psi_E\rangle.$
- Commutators. $[\hat{A}, \hat{B}] = \hat{C}$
- And more... $|\Psi\rangle\langle\Psi|, \hat{a}^\dagger$

Important Concepts

To Describe Quantum World

1. States, vectors, operators, dynamics. Reversible and irreversible processes (e.g. measurement.)
2. Composite systems and their mathematical representation.
3. Advanced products: Tensor products $|\Psi\rangle \otimes |\Phi\rangle$, projectors $|\Psi\rangle \otimes \langle\Psi|$.
4. Correlations.
5. Degrees of freedom.
6. Spin and polarization.
7. Gates and operators.
8. Pre-Quantum notions and terminology. *"Dolphin"* language.

State Vector

Mathematical Representation of Information About Measurements

$|\Psi\rangle$

$$\hat{A} \qquad |a_1\rangle, |a_2\rangle, |a_3\rangle, \ldots, |a_m\rangle$$

$$\hat{B} \qquad |b_1\rangle, |b_2\rangle, |b_3\rangle, \ldots, |b_n\rangle$$

$$\cdots$$

$$\hat{X} \qquad |x_1\rangle, |x_2\rangle, |x_3\rangle, \ldots, |x_k\rangle$$

States are “special” only relative to a selected measurement (energy, position, polarization, etc.)

$$\hat{A}\,|a_p\rangle = A_p\,|a_p\rangle$$

$$\hat{B}\,|b_p\rangle = B_p\,|b_p\rangle$$

$$\hat{X}\,|x_p\rangle = X_p\,|x_p\rangle$$

$$|\Psi\rangle = \int_i \alpha_i\,|a_i\rangle = \int_j \beta_j\,|b_j\rangle = \int_s \xi_s\,|x_s\rangle$$

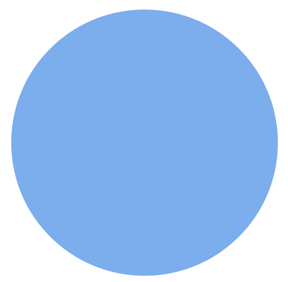
Complete knowledge about quantum system.

Coefficients α (or β, or ξ) encode the results of the experimental test — preparation of the state $|\Psi\rangle$

State Vector

Mathematical Representation of Information About Measurements

$|\Psi\rangle$



$\hat{A}$ $\quad |a_1\rangle\,, |a_2\rangle\,, |a_3\rangle\; \ldots\;, |a_m\rangle$

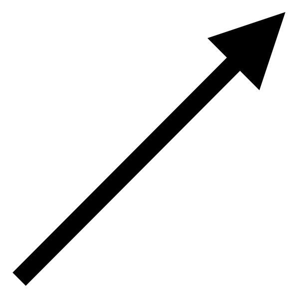
$\hat{B}$ $\quad |b_1\rangle\,, |b_2\rangle\,, |b_3\rangle\; \ldots\;, |b_n\rangle$

$\hat{X}$ $\quad |x_1\rangle\,, |x_2\rangle\,, |x_3\rangle\; \ldots\;, |x_k\rangle$

States are “special” only relative to a selected measurement (energy, position, polarization, etc.)

$$\hat{A}\,|a_p\rangle = A_p\,|a_p\rangle$$
$$\hat{B}\,|b_p\rangle = B_p\,|b_p\rangle$$
$$\hat{X}\,|x_p\rangle = X_p\,|x_p\rangle$$

Vector, but much more powerful than simple arrow.



$$|\Psi\rangle = \int_i \alpha_i\,|a_i\rangle = \int_j \beta_j\,|b_j\rangle = \int_s \xi_s\,|x_s\rangle$$

Complete knowledge about quantum system.

Eigen-problem: Eigen-states and Eigen-values.

$|\Psi\rangle$ Kets & Bras $\langle\Psi|$

Dual or Conjugate Vectors

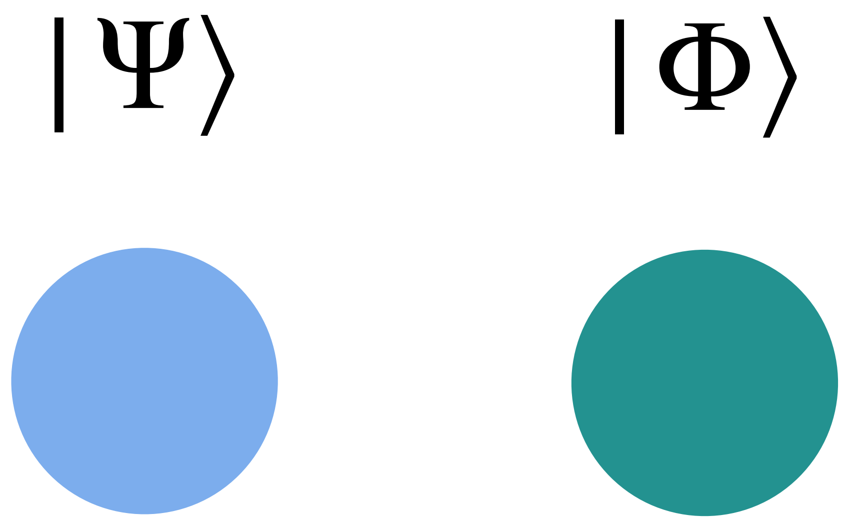
$|\Psi\rangle$ Kets & Bras $\langle\Psi|$

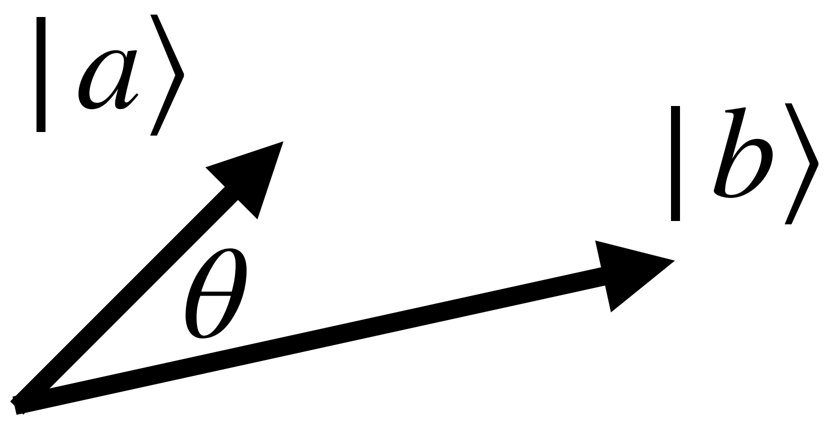
The moment you have scalar product you have dual/conjugate vectors (kets and bras)

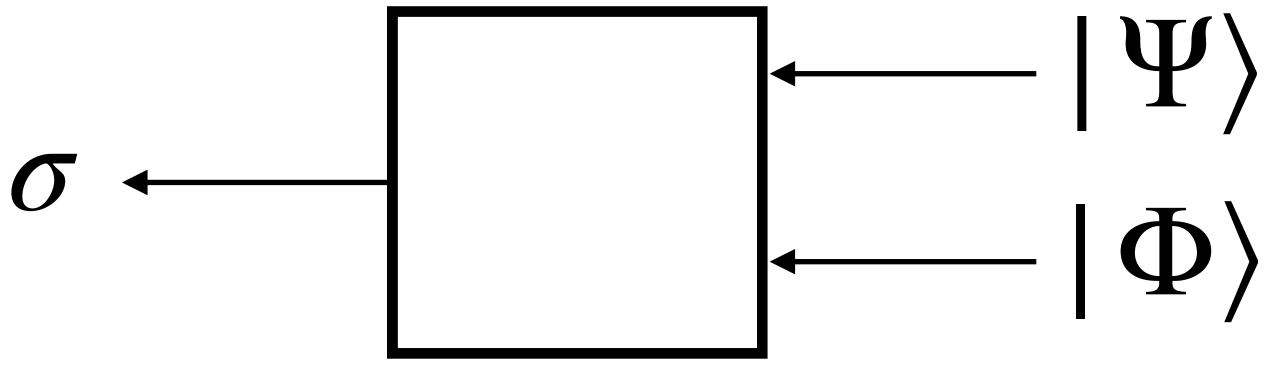
Dual or Conjugate Vectors

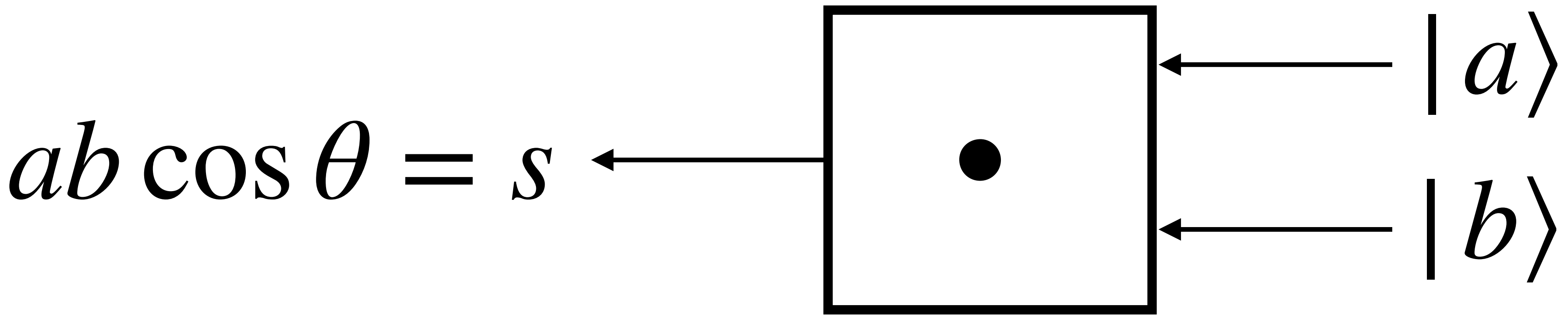
State Vector

Comparing Information Overlap









How similar are the states?

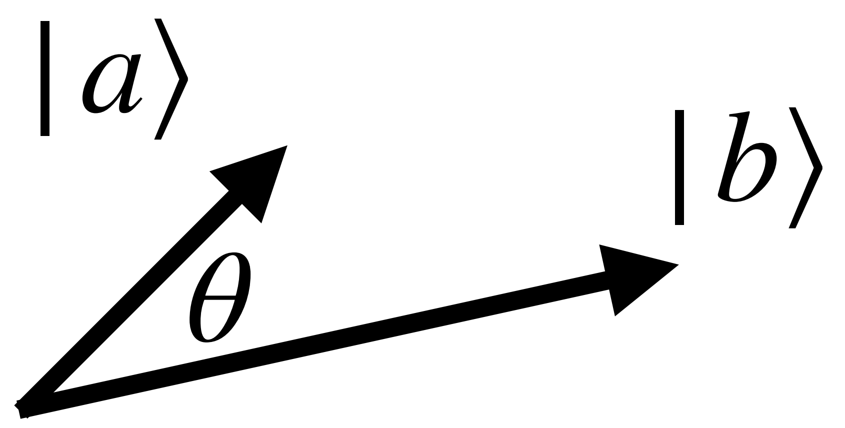
$$|a\rangle \bullet |b\rangle = a_1b_1 + a_2b_2 + \ldots + a_kb_k$$

Scalar Product

For Arrow Vectors

$$|a\rangle = a_1|e_1\rangle + a_1|e_1\rangle + \ldots + a_k|e_k\rangle$$

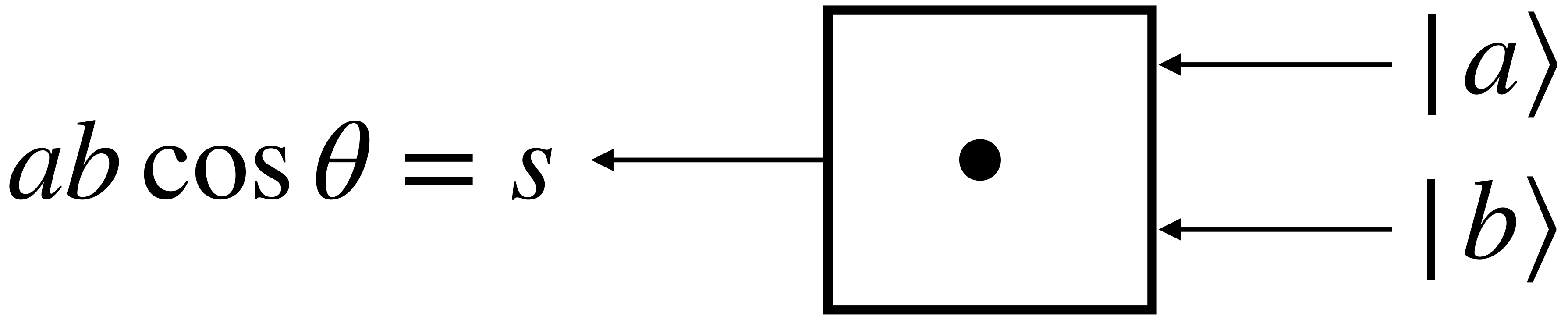
$$|b\rangle = b_1|e_1\rangle + b_1|e_1\rangle + \ldots + b_k|e_k\rangle$$



$$|a\rangle \bullet |b\rangle = \int_{ij} a_i b_j |e_i\rangle \bullet |e_j\rangle$$

$$|e_i\rangle \bullet |e_j\rangle = 0$$

$$|e_n\rangle \bullet |e_n\rangle = 1$$



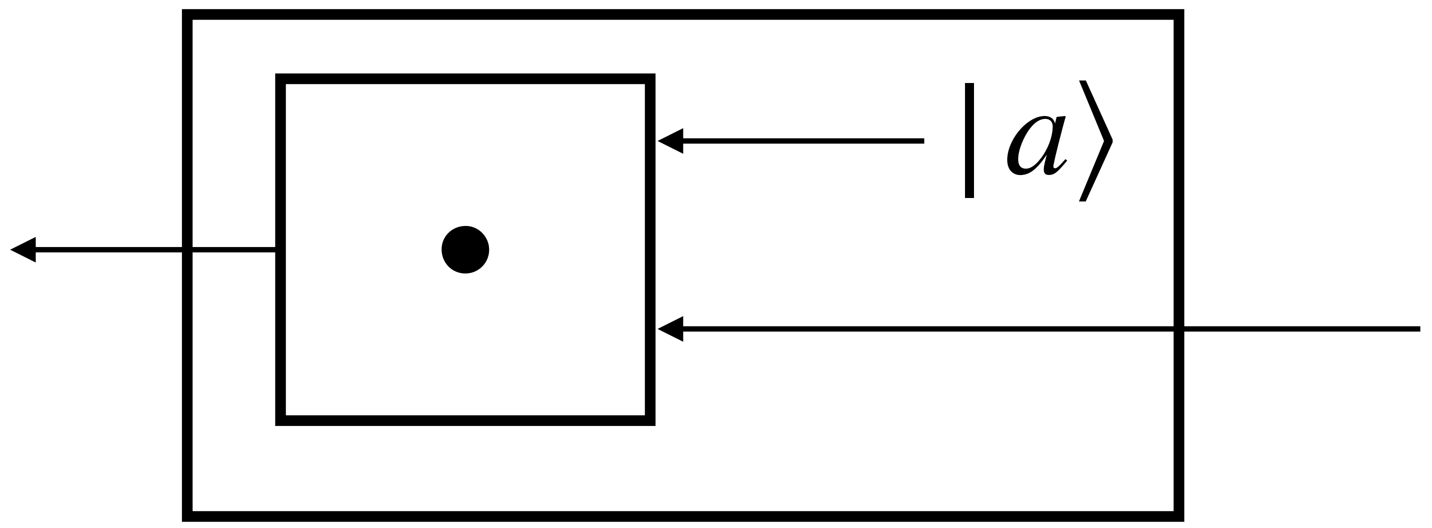
$$|a\rangle \bullet |b\rangle = a_1b_1 + a_2b_2 + \ldots + a_kb_k$$

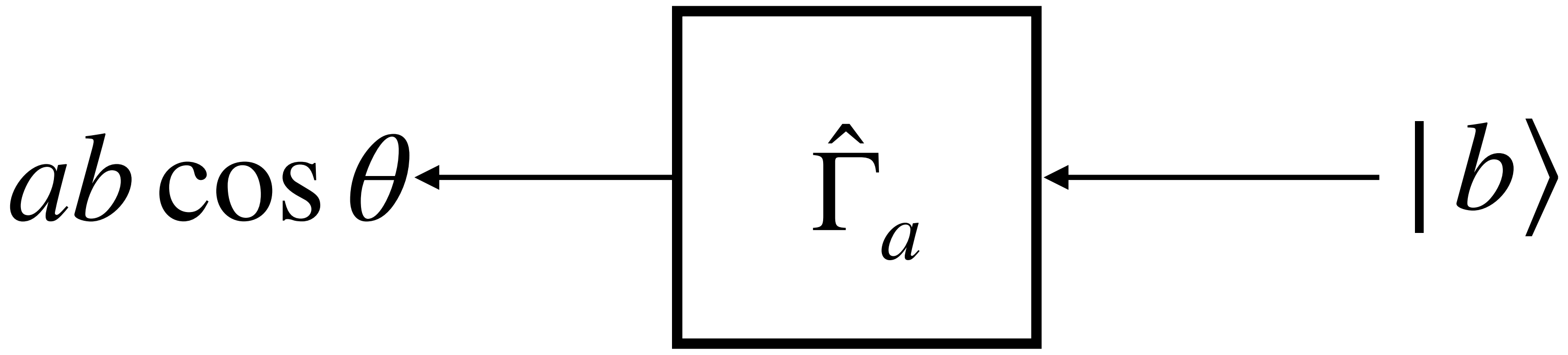
Partial Application

Of Scalar Product Binary Operator

Fix $|a\rangle$.

$$|a\rangle \bullet \square$$





We get a simple (linear) function that maps any arrow $|b\rangle$ into a number:

$$|b\rangle \quad \longrightarrow \quad s = |a\rangle \bullet |b\rangle$$

We created a simple (linear) operator of the type

"vector —> number"

$$\hat{\Gamma}_a : |b\rangle \to s$$

There are as many operators $\hat{\Gamma}$ as there are vectors $|a\rangle$.

$$\boxed{\hat{\Gamma}_a \leftrightarrow |a\rangle}$$

1-to-1 correspondence.

Partial Application

Bra-Ket Notation

Special notation exists, thanks to Paul Dirac

$$\hat{\Gamma}_a = \langle a |$$

bracket

$$\hat{\Gamma}_a | b \rangle = \langle a | | b \rangle = \langle a | b \rangle$$

bra

ket

Bra-Vectors are Operators

Operators Are Vectors

Take basis vectors $|e_i\rangle$. Each has a "mirror image" (dual):

$$\hat{\Gamma}_i = \langle e_i|$$

Exercise: Find $\langle e_i|a\rangle$ for $|a\rangle = a_1|e_1\rangle + a_2|e_2\rangle + \ldots + a_n|e_n\rangle$.

Operators can be added and multiplied by numbers (just like any function): $\hat{\Gamma} = c_1\hat{\Gamma}_1 + c_2\hat{\Gamma}_2 = c_1\langle e_1| + c_2\langle e_2|$

Such sums can be given a *simple and clear meaning*: $\hat{\Gamma}|a\rangle = c_1\langle e_1|a\rangle + c_2\langle e_2|a\rangle$

Operators $\hat{\Gamma}$ are linear, therefore to find its action on *any* vector $|a\rangle$ it is *sufficient* to know its action on a few basis vectors:

$$\hat{\Gamma}|a\rangle = \hat{\Gamma}\left(a_1|e_1\rangle + a_2|e_2\rangle + \ldots + a_n|e_n\rangle\right) = a_1(\hat{\Gamma}|e_1\rangle) + a_2(\hat{\Gamma}|e_2\rangle) + \ldots a_k(\hat{\Gamma}|e_k\rangle)$$

$$\hat{\Gamma}|a\rangle = a_1\gamma_1 + a_2\gamma_2 + \ldots + a_k\gamma_k \qquad \gamma_i = \hat{\Gamma}|e_i\rangle$$

Bra-Vectors are Operators

Operators Are Vectors

Example:

$$\hat{\Gamma}\,|e_1\rangle = 2 \qquad \hat{\Gamma}\,|e_2\rangle = -1$$

$$|a\rangle = |e_1\rangle - 3\,|e_2\rangle$$

$$\hat{\Gamma}\,|a\rangle = 1(\hat{\Gamma}\,|e_1\rangle + (-3)(\hat{\Gamma}\,|e_2\rangle = 2 \cdot 1 + (-3) \cdot (-1) = 5$$

Basis For Operators

All Operators Have "Building Blocks" — Bases

Any linear operator $\hat{\Gamma}$ can be written as linear combination (superposition) of some basic operators.

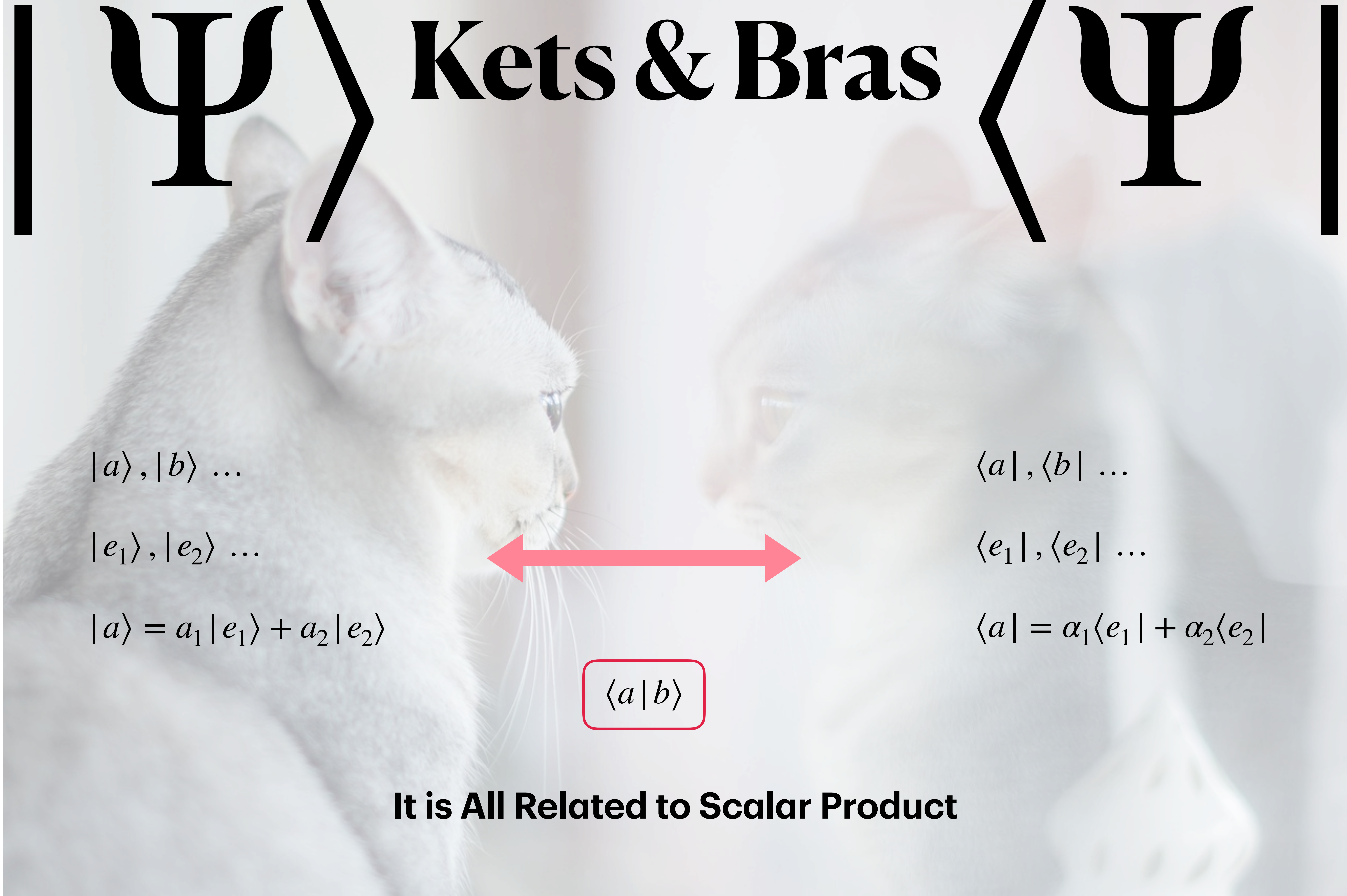
One simple way to ("natural") to get basis for $\hat{\Gamma}$'s is to *conjugate* "usual"/ket basis:

$$\hat{\Gamma}_1 = \langle e_1 | \, , \hat{\Gamma}_2 = \langle e_2 | \, , \ldots \, , \hat{\Gamma}_k = \langle e_k | \, .$$

Then we immediately get $\hat{\Gamma} = \gamma_1 \hat{\Gamma}_1 + \gamma_2 \hat{\Gamma}_2 + \ldots + \gamma_k \hat{\Gamma}_k$

Bottomline: Dual/conjugate objects are linear operators that have all the properties of vectors.

$|\Psi\rangle$ Kets & Bras $\langle\Psi|$



$|a\rangle, |b\rangle \; \dots$

$|e_1\rangle, |e_2\rangle \; \dots$

$|a\rangle = a_1|e_1\rangle + a_2|e_2\rangle$

$\langle a|, \langle b| \; \dots$

$\langle e_1|, \langle e_2| \; \dots$

$\langle a| = \alpha_1\langle e_1| + \alpha_2\langle e_2|$

$$\boxed{\langle a|b\rangle}$$

It is All Related to Scalar Product

What to do with $\hat{J}$?

$|a\rangle, |b\rangle \ \dots$ $\qquad$ $\langle a|, \langle b| \ \dots$

$|e_1\rangle, |e_2\rangle \ \dots$ $\qquad$ $\langle e_1|, \langle e_2| \ \dots$

$|a\rangle = a_1|e_1\rangle + a_2|e_2\rangle$ $\qquad$ $\langle a| = \alpha_1\langle e_1| + \alpha_2\langle e_2|$

$\boxed{\langle a|b\rangle}$

$|c\rangle = |a\rangle + \hat{J}|b\rangle$

$\langle c| = \langle a| + \hat{J}\langle b|$

$\langle c| = \langle a| - \hat{J}\langle b|$

$$\langle c|c\rangle = \Big(\langle a| + \hat{J}\langle b|\Big)\Big(|a\rangle + \hat{J}|b\rangle\Big) = a^2 - b^2 + 2\hat{J}ab\cos\phi$$

$$\langle a|a\rangle = \Big(\langle a| - \hat{J}\langle b|\Big)\Big(|a\rangle + \hat{J}|b\rangle\Big) = \boxed{a^2 + b^2 + 2ab\cos\phi}$$

Bottomline: When you conjugate — conjugate everything. Including $\hat{J}$. Replace it with $-\hat{J}$.

Example in QP

$$|\Psi\rangle = e^{\hat{J}Et/\hbar}|\Phi_0\rangle = \left(\cos\frac{Et}{\hbar} + \hat{J}\sin\frac{Et}{\hbar}\right)|\Phi_0\rangle$$

$$\langle\Psi| = e^{-\hat{J}Et/\hbar}\langle\Phi_0| = \left(\cos\frac{Et}{\hbar} - \hat{J}\sin\frac{Et}{\hbar}\right)\langle\Phi_0|$$

When “normal” functions are used — state functions $\psi(x)$ — conjugations is simply “complex conjugation”:

$$\psi(x) \quad \rightarrow \quad \psi^*(x)$$

You can find examples of this complex conjugation in nearly all textbooks.

Bottomline: When you conjugate — conjugate everything. Including $\hat{J}$. Replace it with $-\hat{J}$.

Self-Test

Answer These Questions 1hr After Class

1. Do physicists think that Quantum Theory is the final theory of microscopic world?
2. What is the most powerful generalization of the idea of function which is used in Quantum Theory?
3. How is the information about quantum system encoded in a state vector?
4. What is one way to compare vectors numerically?
5. What mathematical object do we get if we partially apply scalar product operator “•”?
6. What makes the partially applied scalar products vectors?
7. How do we conjugate the operator $\hat{J}$?

Homework Problems

Dual or Conjugate Vectors

1. Find $\langle e_i | a \rangle$ for $|a\rangle = a_1|e_1\rangle + a_2|e_2\rangle + \ldots + a_n|e_n\rangle$.
2. For the operators from the dual basis $\langle e_k|$, find $\langle e_k|e_l\rangle$.
3. Think of this: If we start with operators/bra-vectors $\hat{\Gamma}$, what do "regular" vectors $|a\rangle$ do to them? Convince yourself that $|a\rangle$ are linear operators that *map* bra-vectors $\hat{\Gamma}$ into numbers. So, the distinction is not absolute. Bras and kets are equally important vectors that "complement" each other to a normal number.
4. For $\hat{\Gamma}|e_1\rangle = 1$ and $\hat{\Gamma}|e_2\rangle = -1$, calculate $\hat{\Gamma}|a\rangle$ for $|a\rangle = |e_1\rangle + |e_2\rangle$.
5. Given $|a\rangle = |e_1\rangle + 2|e_2\rangle$ and $|b\rangle = 3|e_1\rangle - |e_2\rangle$, find the conjugate of $|c\rangle = |a\rangle + \hat{J}|b\rangle$.
6. Simply observe an example of "complex conjugation" used in real papers:

To facilitate the study of the transition from quantum to classical behavior, it is convenient to employ the Wigner transform of a wave function $\psi(x)$:

$$W(x,p) = \frac{1}{2\pi\hbar}\int_{-\infty}^{\infty} e^{ipy/\hbar}\psi^*\left(x+\frac{y}{2}\right)\,\psi\,\left(x-\frac{y}{2}\right)\,dy\ , \qquad (20)$$

which expresses quantum states as functions of position and momentum.

Decoherence and the Transition from Quantum to Classical—Revisited

By Wojciech H. Zurek

Quantum Theory

In a Nutshell

II. POSTULATES FOR QUANTUM MECHANICS

In this paper, all state vectors are supposed to be normalized, and mixed states are represented by density operators, i.e., positive operators with unit trace. Let A be an observable with a nondegenerate purely discrete spectrum. Let $\phi_1, \phi_2, \ldots$ be a complete orthonormal sequence of eigenvectors of A and $a_1, a_2, \ldots$ the corresponding eigenvalues; by assumption, all different from each other.

According to the standard formulation of quantum mechanics, on the result of a measurement of the observable A the following postulates are posed:

(A1) *If the system is in the state ψ at the time of measurement, the eigenvalue a_n is obtained as the outcome of measurement with the probability $|\langle\phi_n|\psi\rangle|^2$.*

(A2) *If the outcome of measurement is the eigenvalue a_n, the system is left in the corresponding eigenstate ϕ_n at the time just after measurement.*

The postulate (A1) is called the *statistical formula*, and (A2) the *measurement axiom*. The state change $\psi \mapsto \phi_n$ described by the measurement axiom is called the *state reduction*.

You will understand this paragraph in the end of the course.