Quantum Physics

2025

The Theory/Framework Of *Almost* Everything *Today*

But Most Likely NOT of Tomorrow

Yury Deshko

Course Overview

Course Structure And Goals

- **Part 1** : Mathematical Concepts And Tools.
- **Part 2** : Classical Physics.
- **Part 3** : Quantum Physics.

- Learn the language of quantum physics.
- Enhance the knowledge of classical physics.
- Develop modern quantum thinking.

We will focus on this one today.

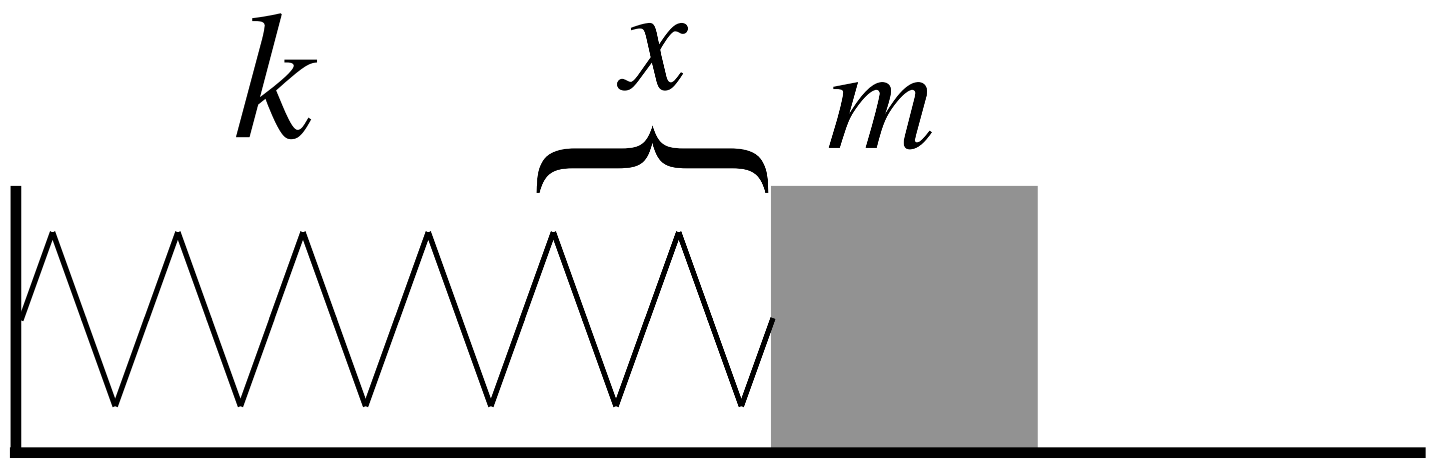
Focus Concepts

Used in Classical And Quantum Physics

- **Modes**.
- (Quantum) Field.
- (Quantum) Vacuum.

Oscillator

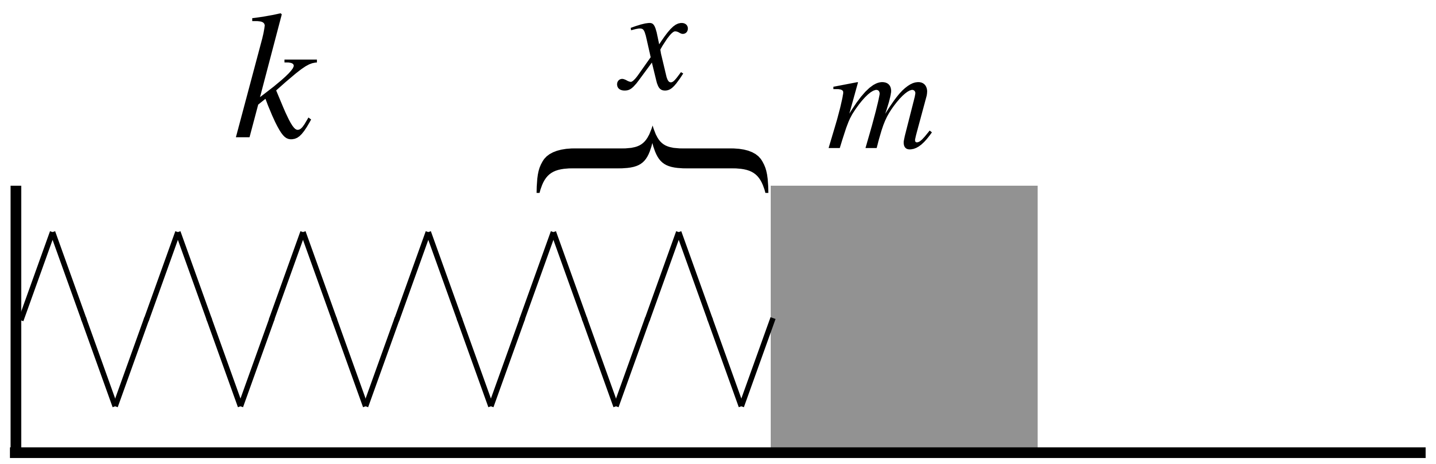
Quick Recap



$$\omega = \sqrt{\frac{k}{m}}$$

Oscillator

Quick Recap



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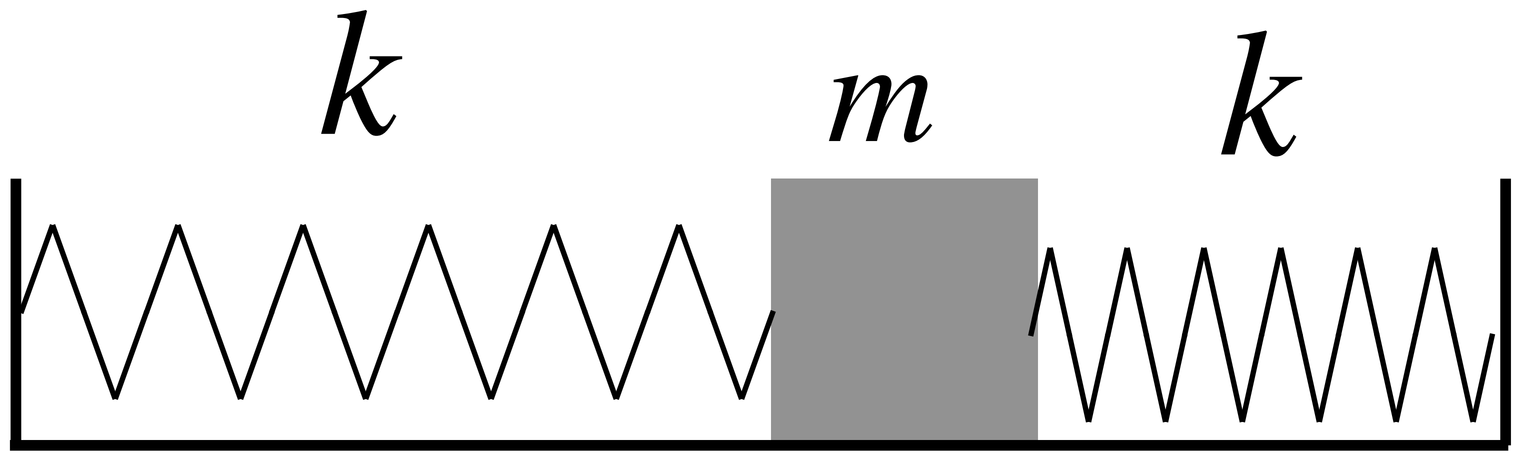


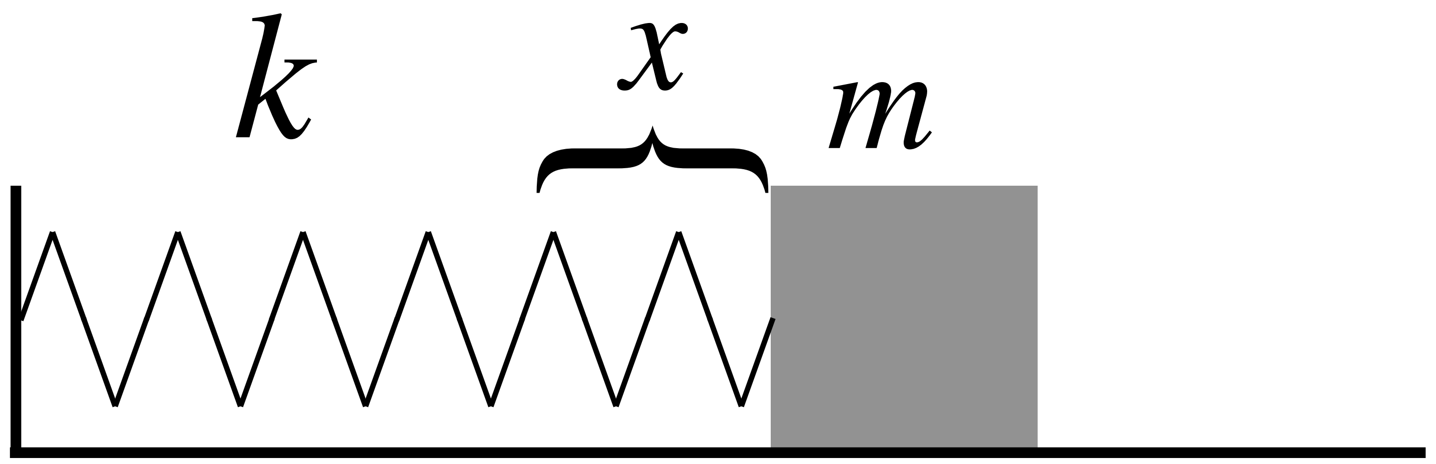
$$\omega = \sqrt{\frac{2k}{m}}$$

$$F = F_l + F_r = 2kx = k'x$$

Placing spring on opposite side — doubles the effect.

Oscillator

Quick Recap



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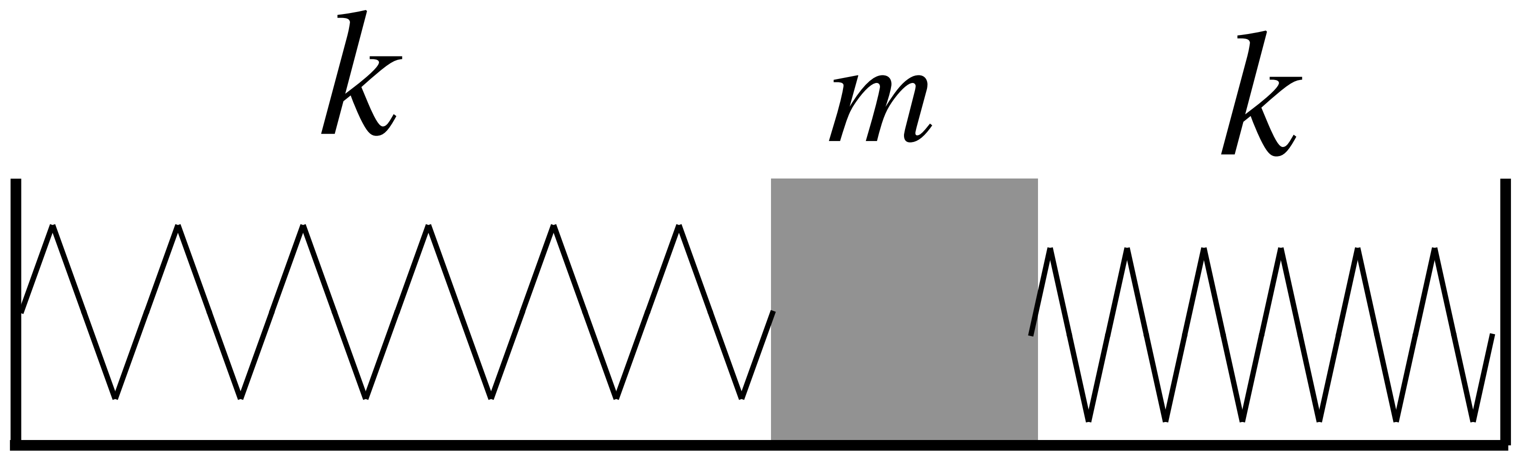

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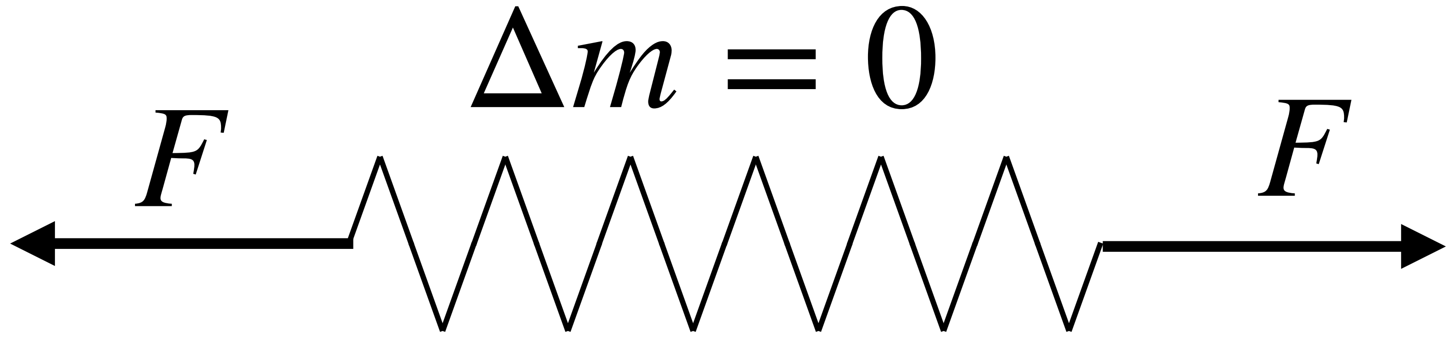
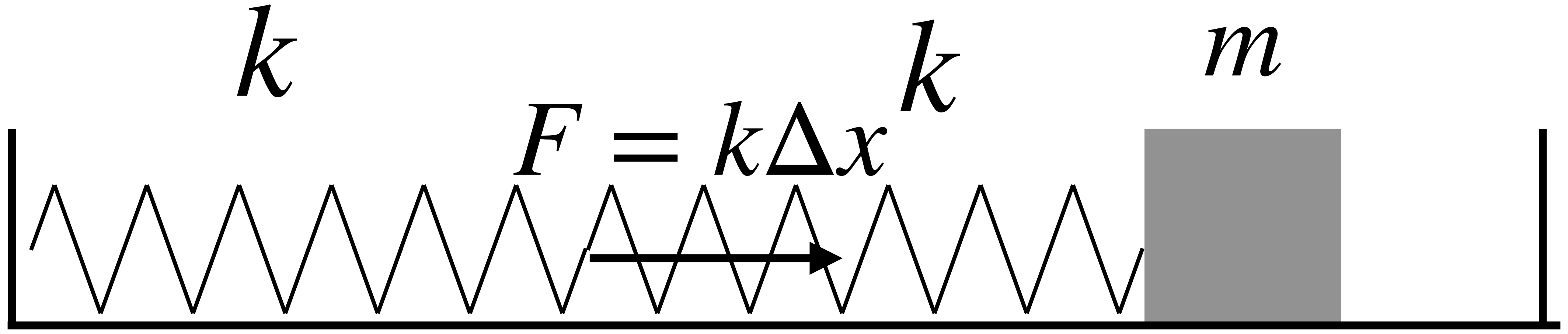


$$F_1 = F_2 = k\Delta x \qquad x = 2\Delta x$$



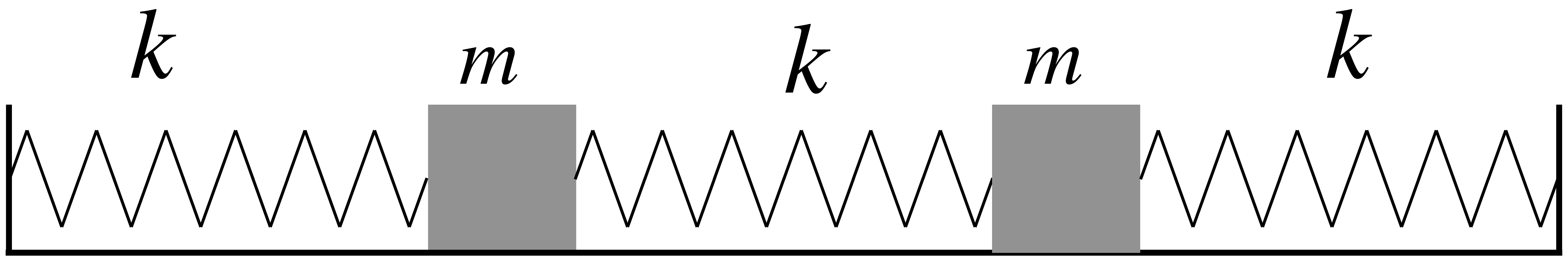
$$F = kx/2 = k'x \qquad \omega = \sqrt{\frac{k}{2m}}$$

Connecting springs in series — halves the spring's constant —> Cutting the spring in half doubles it!

Oscillators

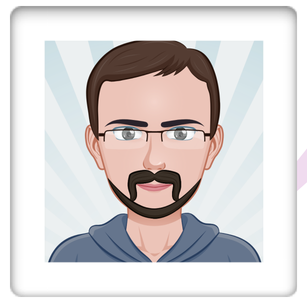
And Their Interaction/Coupling

- Two independent oscillators,
- Periodic motion with well-defined frequency.



- Also two independent oscillators! (If you look right).

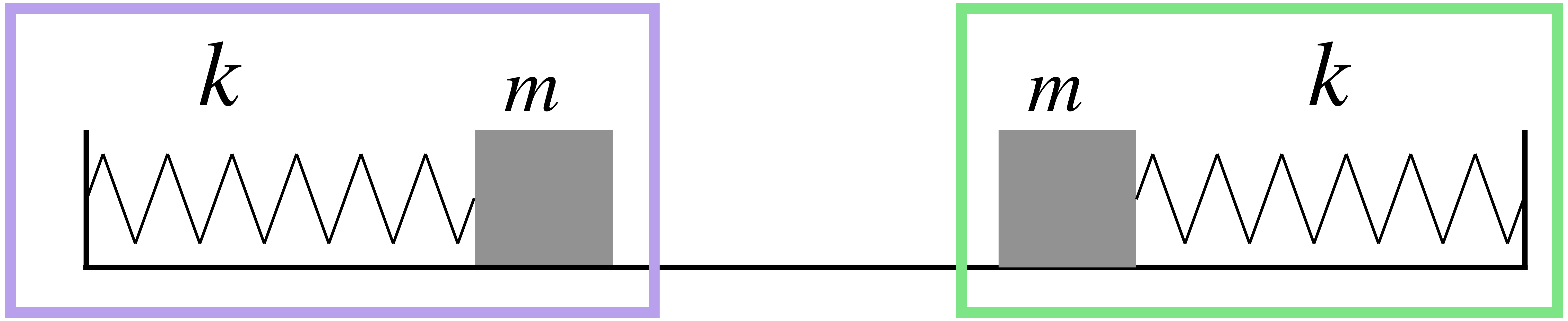
You need to “rotate the oscillator coordinates” 45 degrees



Oscillators

And Their Interaction/Coupling

- Two independent oscillators



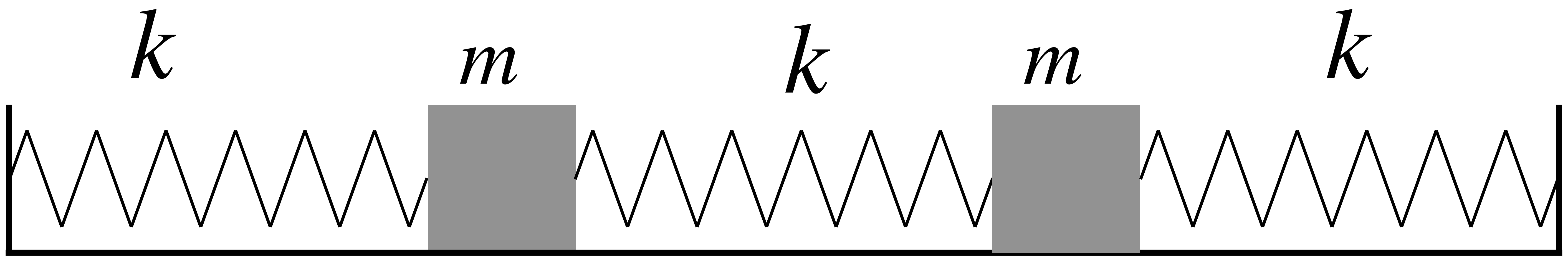
$$E = E_1 + E_2 = \boxed{\frac{mv_1^2}{2} + \frac{kx_1^2}{2}} + \boxed{\frac{mv_2^2}{2} + \frac{kx_2^2}{2}} \qquad v_i = \partial_t x_i$$

Independent = Energy imparted to one, stays there, does not go to the other one.

Oscillators

And Their Interaction/Coupling

$$E = E_1' + E_2' = \boxed{\frac{M_1 u_1^2}{2} + \frac{k_1 q_1^2}{2}} + \boxed{\frac{M_2 u_2^2}{2} + \frac{k_2 q_2^2}{2}} \qquad u_i = \partial_t q_i$$



- Also two independent oscillators! (If you look right).

There is nothing directly apparent in the oscillators that would correspond to q_i, k_i, M_i.

Oscillators

And Their Interaction/Coupling

$$E = E_1' + E_2' = \boxed{\frac{M_1 u_1^2}{2} + \frac{k_1 q_1^2}{2}} + \boxed{\frac{M_2 u_2^2}{2} + \frac{k_2 q_2^2}{2}} \qquad u_i = \partial_t q_i$$

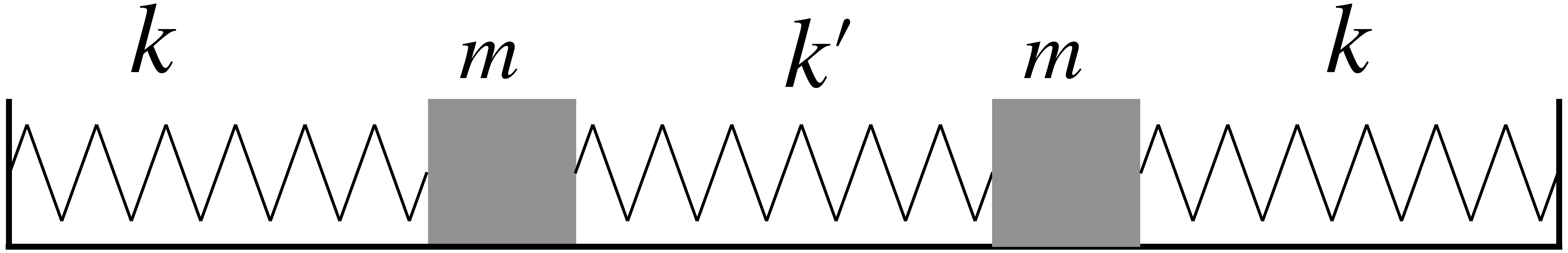
- Also two independent oscillators! (If you look right).

There is nothing directly apparent in the oscillators that would correspond to q_i, k_i, M_i.

Oscillators

And Their Interaction/Coupling

$$E = E_1 + E_2 + E_{int} = \frac{mv_1^2}{2} + \boxed{\frac{kx_1^2}{2}} + \frac{mv_2^2}{m} + \boxed{\frac{kx_2^2}{2}} + \boxed{\frac{k'(x_1 - x_2)^2}{2}}$$



- Also two independent oscillators! (If you look right).

"Rotation"

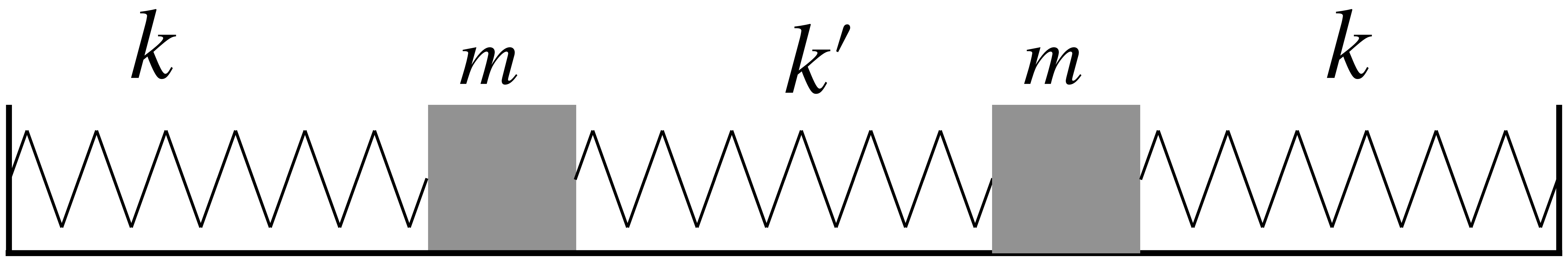
$$\left.\begin{aligned} q_1 &= x_1 - x_2 \\ q_2 &= x_1 + x_2 \end{aligned}\right\} \quad \begin{aligned} x_1 &= (q_2 + q_1)/2 \\ x_2 &= (q_2 - q_1)/2 \end{aligned}$$

$$\frac{kx_1^2}{2} + \frac{kx_2^2}{2} + \frac{k'(x_1 - x_2)^2}{2} = ?$$

Oscillators

And Their Interaction/Coupling

$$E = E_1 + E_2 + E_{int} = \frac{mv_1^2}{2} + \boxed{\frac{kx_1^2}{2}} + \frac{mv_2^2}{m} + \boxed{\frac{kx_2^2}{2}} + \boxed{\frac{k'(x_1 - x_2)^2}{2}}$$



- Also two independent oscillators! (If you look right).

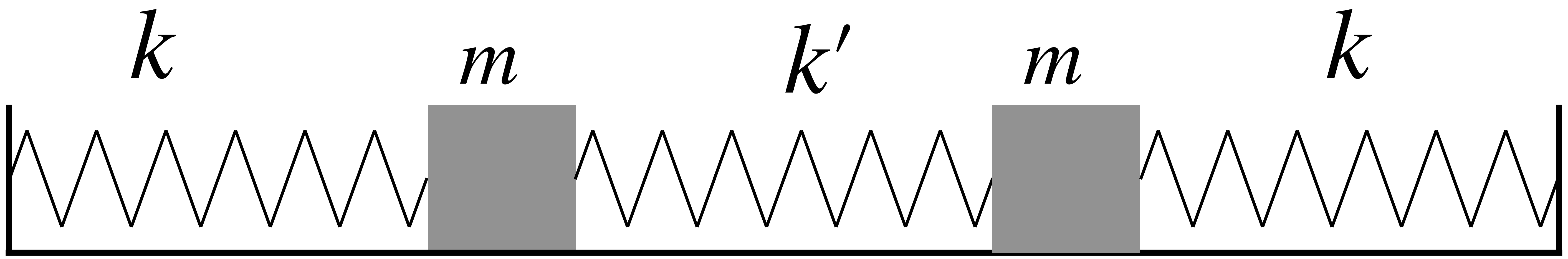
$$\left.\begin{aligned} q_1 &= x_1 - x_2 \\ q_2 &= x_1 + x_2 \end{aligned}\right\} \begin{aligned} x_1 &= (q_2 + q_1)/2 \\ x_2 &= (q_2 - q_1)/2 \end{aligned} \qquad \frac{kx_1^2}{2} + \frac{kx_2^2}{2} + \frac{k'(x_1 - x_2)^2}{2} = \frac{\left(k/2 + k'\right) q_1^2}{2} + \frac{(k/2)q_2^2}{2}$$

Oscillators

And Their Interaction/Coupling

$$E = E_1 + E_2 + E_{int} = \boxed{\frac{mv_1^2}{2}} + \frac{kx_1^2}{2} + \boxed{\frac{mv_2^2}{m}} + \frac{kx_2^2}{2} + \frac{k'(x_1 - x_2)^2}{2}$$

$$u_i = \partial_t q_i$$



- Also two independent oscillators! (If you look right).

$$x_1 = (q_1 + q_2)/2 \quad v_1 = \partial_t x_1 = (u_1 + u_2)/2$$

$$x_2 = (q_1 - q_2)/2 \quad v_2 = \partial_t x_2 = (u_1 - u_2)/2$$

$$\frac{v_1^2}{2} + \frac{v_2^2}{2} = ?$$

Oscillators

And Their Interaction/Coupling

$$E = E_1 + E_2 + E_{int} = \boxed{\frac{mv_1^2}{2}} + \frac{kx_1^2}{2} + \boxed{\frac{mv_2^2}{m}} + \frac{kx_2^2}{2} + \frac{k'(x_1 - x_2)^2}{2}$$

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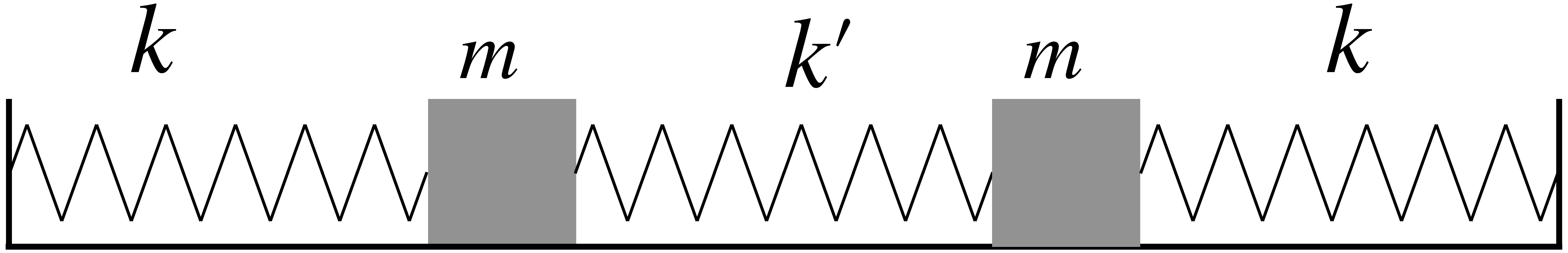


- Also two independent oscillators! (If you look right).

$$x_1 = (q_1 + q_2)/2 \quad v_1 = \partial_t x_1 = (u_1 + u_2)/2$$

$$x_2 = (q_2 - q_1)/2 \quad v_2 = \partial_t x_2 = (u_1 - u_2)/2$$

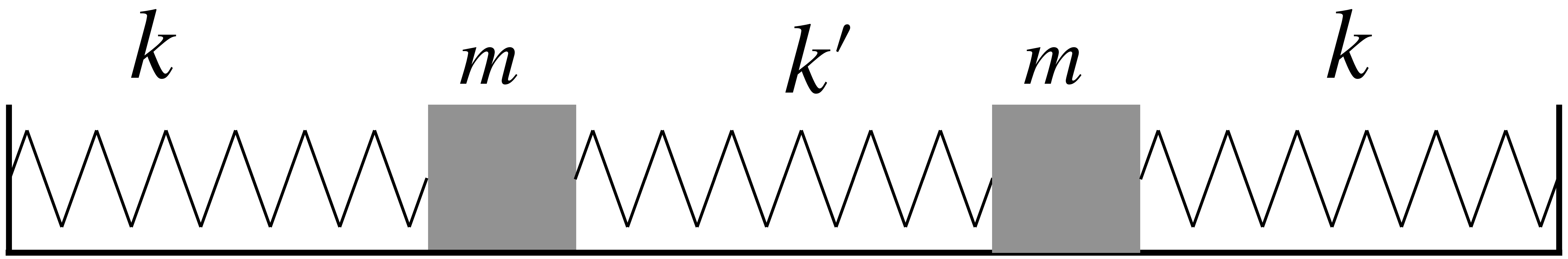
$$\frac{v_1^2}{2} + \frac{v_2^2}{2} = \frac{u_1^2}{4} + \frac{u_2^2}{4}$$

Oscillators

And Their Interaction/Coupling

$$E = \boxed{\frac{(m/2)u_1^2}{2} + \frac{(k/2 + k')q_1^2}{2}} + \boxed{\frac{(m/2)u_2^2}{m} + \frac{(k/2)q_2^2}{2}}$$

$$u_i = \partial_t q_i$$



$$q_1 = x_1 - x_2$$

$$q_2 = x_1 + x_2$$

- When q_1 is constant ($x_1 = x_2$) and q_2 is changing, both bodies move "in sync".

Oscillators

And Their Interaction/Coupling

$$E = \boxed{\frac{(m/2)u_1^2}{2} + \frac{(k/2 + k')q_1^2}{2}} + \boxed{\frac{(m/2)u_2^2}{m} + \frac{(k/2)q_2^2}{2}}$$

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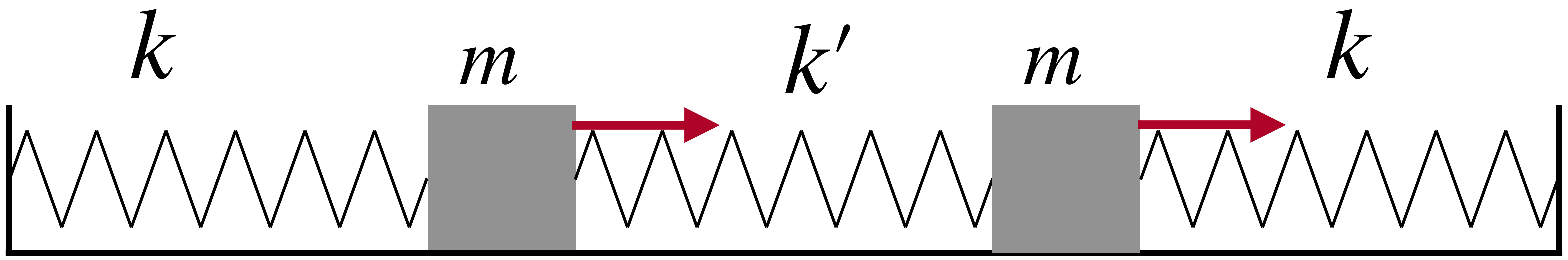


$$q_1 = x_1 - x_2$$

$$q_2 = x_1 + x_2$$

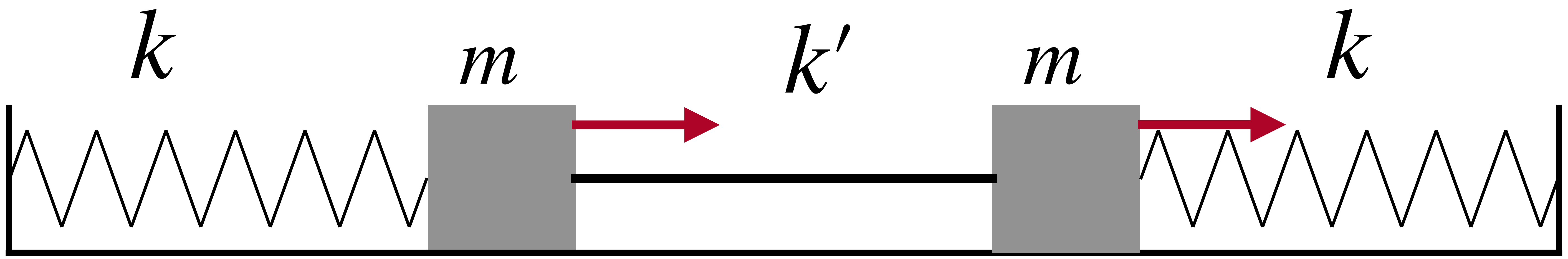
- When q_1 is zero ($x_1 = x_2$) and q_2 is changing, both bodies move “in sync”.

$$\boxed{\omega_2 = \sqrt{k_2/m_2} = \sqrt{k/m}}$$

Oscillators

And Their Interaction/Coupling

We can solve this simply by noticing that this is combined mass $M = 2m$ attached to two springs.



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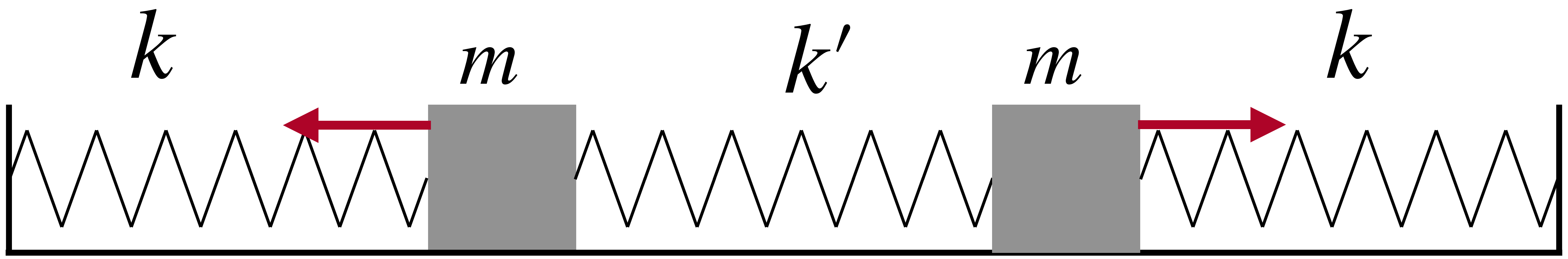
Oscillators

And Their Interaction/Coupling

$$E = \boxed{\frac{(m/2)u_1^2}{2} + \frac{(k/2 + k')q_1^2}{2}} + \boxed{\frac{(m/2)u_2^2}{m} + \frac{(k/2)q_2^2}{2}}$$

$$u_i = \partial_t q_i$$



$$q_1 = x_1 - x_2$$

$$q_2 = x_1 + x_2$$

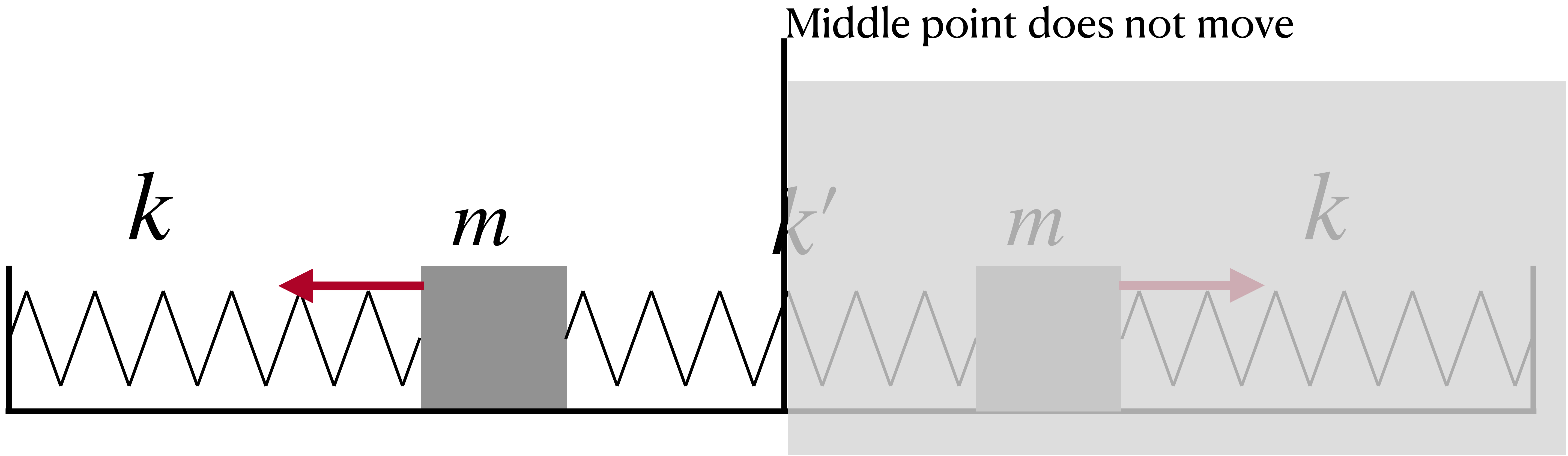
- When q_2 is zero ($x_1 = -x_2$) and q_1 is changing, the bodies move in opposite directions.

$$\boxed{\omega_1 = \sqrt{k_1/m_1} = \sqrt{(k + 2k')/m}}$$

Oscillators

And Their Interaction/Coupling

We can solve this simply by noticing that this is two identical systems: mass $M = m$ connected to springs with stiffness k and $2k'$.



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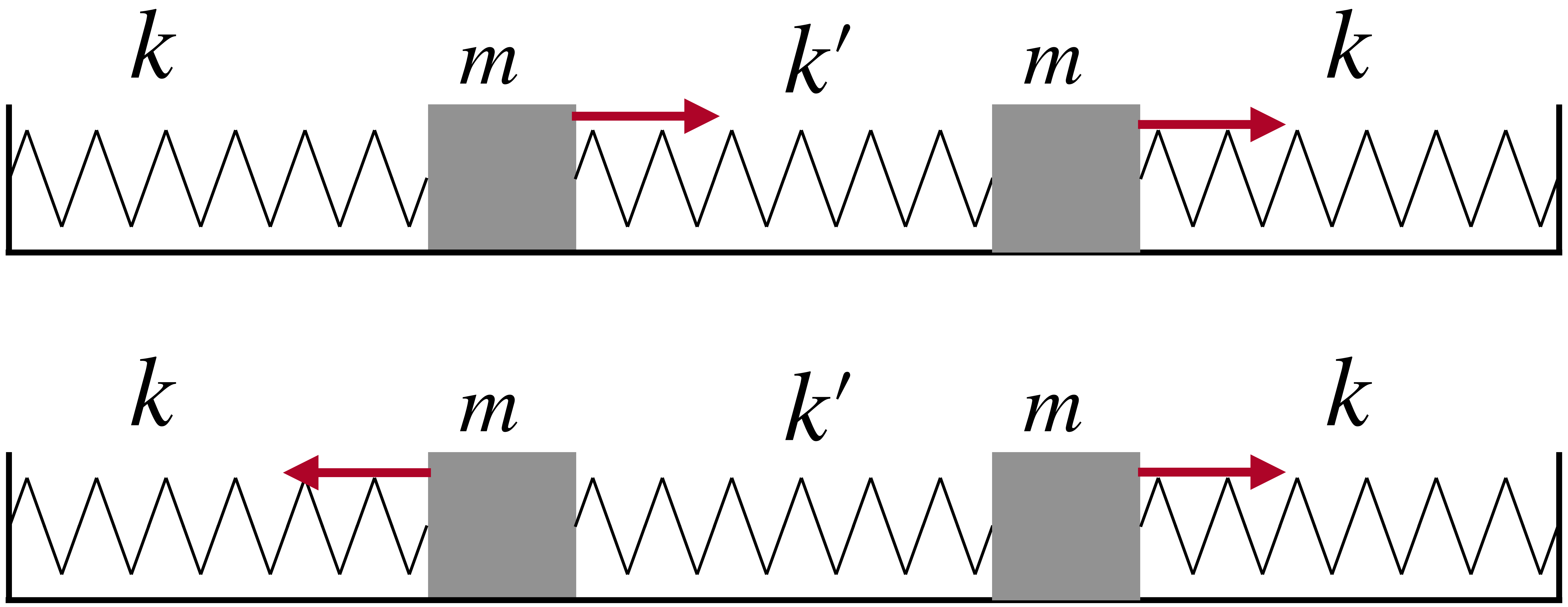
$$\boxed{\omega_1 = \sqrt{k_1/m_1} = \sqrt{(k+2k')/m}}$$

Oscillators

Two Modes/Regimes/Types of Motion/Ways of Motion

$$\omega_2 = \sqrt{k/m}$$

$$\omega_1 = \sqrt{(k+2k')/m}$$



Simple motion of an oscillator with
<u>well-defined frequency</u>

$$\boxed{\begin{aligned} q_1 &= Q_1 \cos\left(\omega_1 t + \phi_1\right) \\ q_2 &= Q_2 \cos\left(\omega_2 t + \phi_2\right) \end{aligned}}$$

Complicated motion of without well-defined frequency.
Not even periodic

$$\boxed{\begin{aligned} x_1 &= [Q_1 \cos\left(\omega_1 t + \phi_1\right) + Q_2 \cos\left(\omega_2 t + \phi_2\right)]/2 \\ x_2 &= [Q_1 \cos\left(\omega_1 t + \phi_1\right) - Q_2 \cos\left(\omega_2 t + \phi_2\right)]/2 \end{aligned}}$$

Oscillators

Two Modes/Regimes/Types of Motion/Ways of Motion

$$x_1 = [Q_1 \cos\left(\omega_1 t + \phi_1\right) + Q_2 \cos\left(\omega_2 t + \phi_2\right)]/2$$

$$Q_1 \cos\left(\omega_1 t + \phi_1\right) = \underbrace{Q_1 \cos\phi_1}_{A_1} \cos\omega_1 t - \underbrace{Q_1 \sin\phi_1}_{B_1} \sin\omega_1 t = A_1 \cos\omega_1 t + B_1 \sin\omega_1 t$$

$$x_1 \propto A_1 \cos\omega_1 t + B_1 \sin\omega_1 t + A_2 \cos\omega_2 t + B_2 \sin\omega_2 t$$

The first (left) oscillator moves like this.

$$x_1 = \int_i \left(A_i \cos\omega_i t + B_i \sin\omega_i t\right)$$

More general case of many coupled oscillators.
This is Fourier Transform!

Coupled Oscillators

As The Model For Solids And Fields

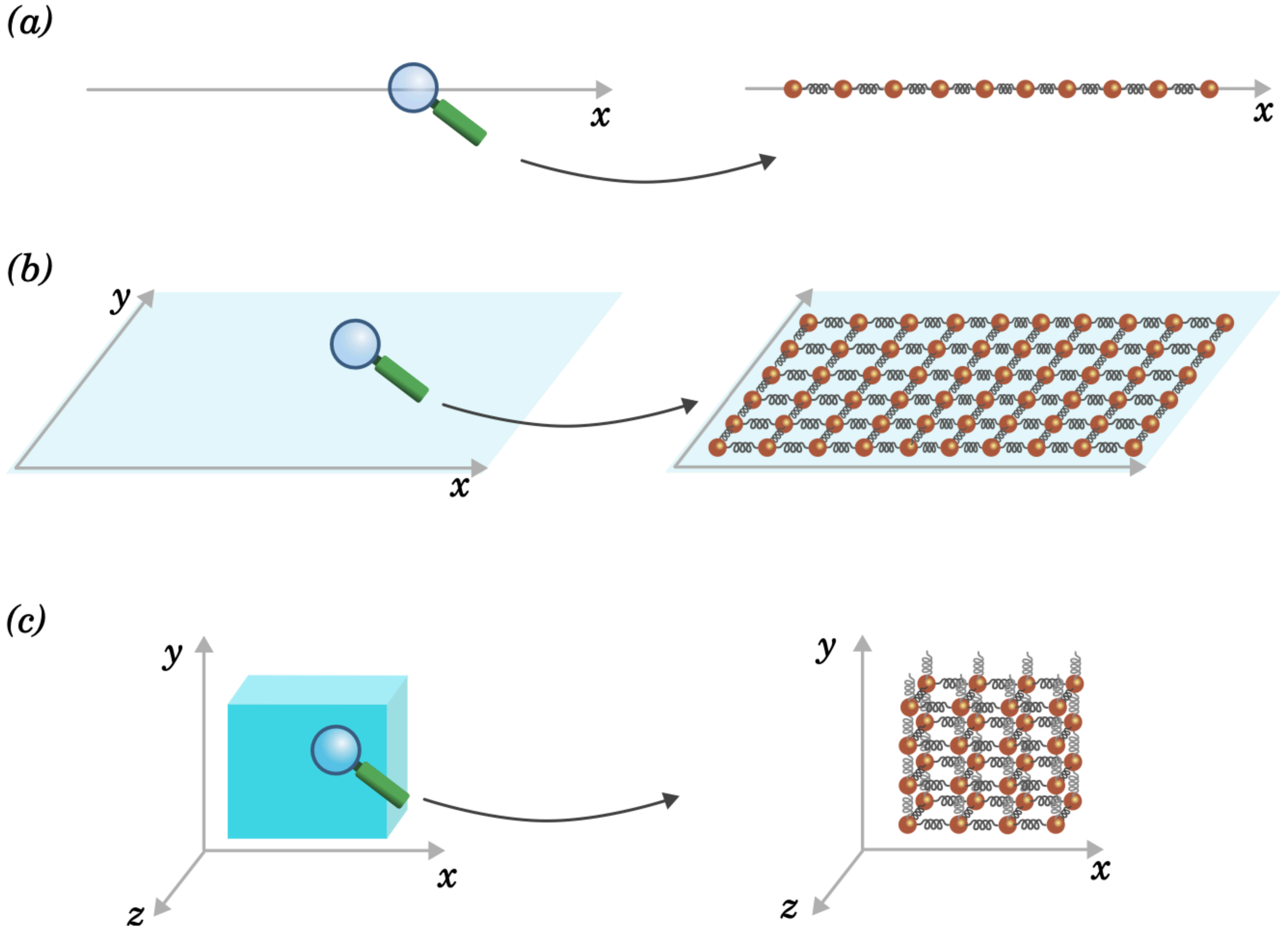


Figure 4.7: Field as a mechanical system with infinite (but countable) number of degrees of freedom.

$$x_1 = \int_i \left(A_i \cos \omega_i t + B_i \sin \omega_i t\right)$$

Coupled Oscillators

As The Model For Solids And Fields

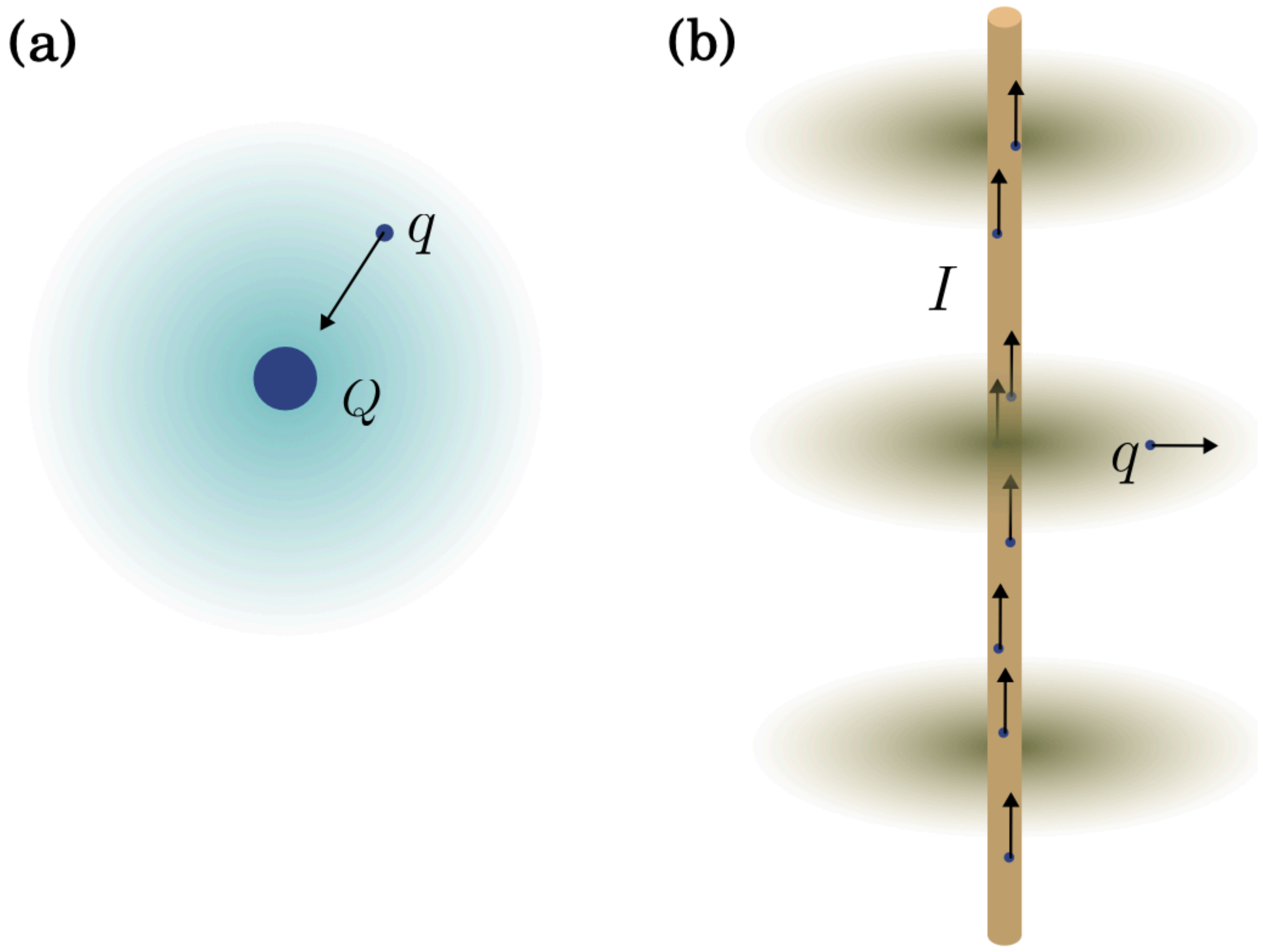


Figure 4.1: (a) Electrostatic field. (b) Magnetostatic field.

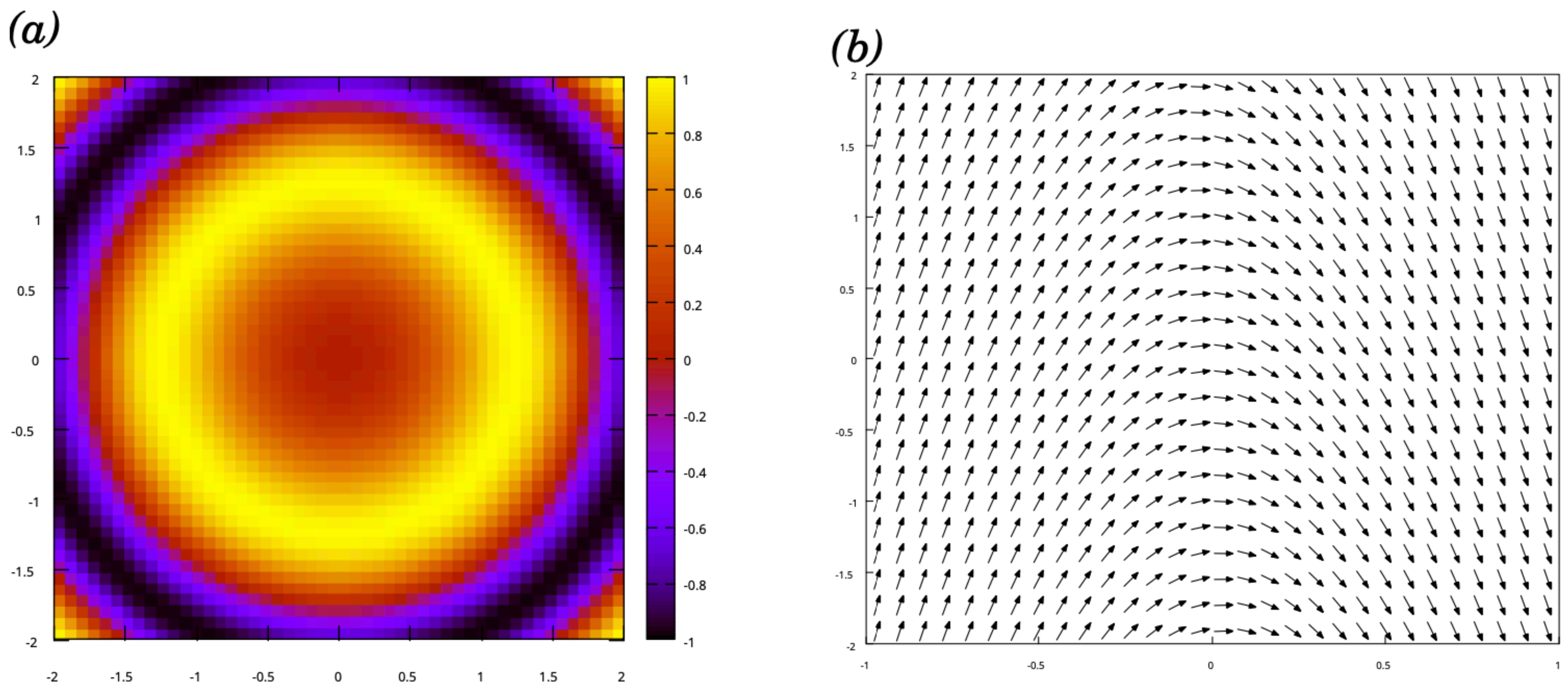


Figure 4.3: (a) Scalar field – number assigned to each point in space. (b) Vector field – vector assigned to each point in space.

Coupled Oscillators

As The Model For Solids And Fields

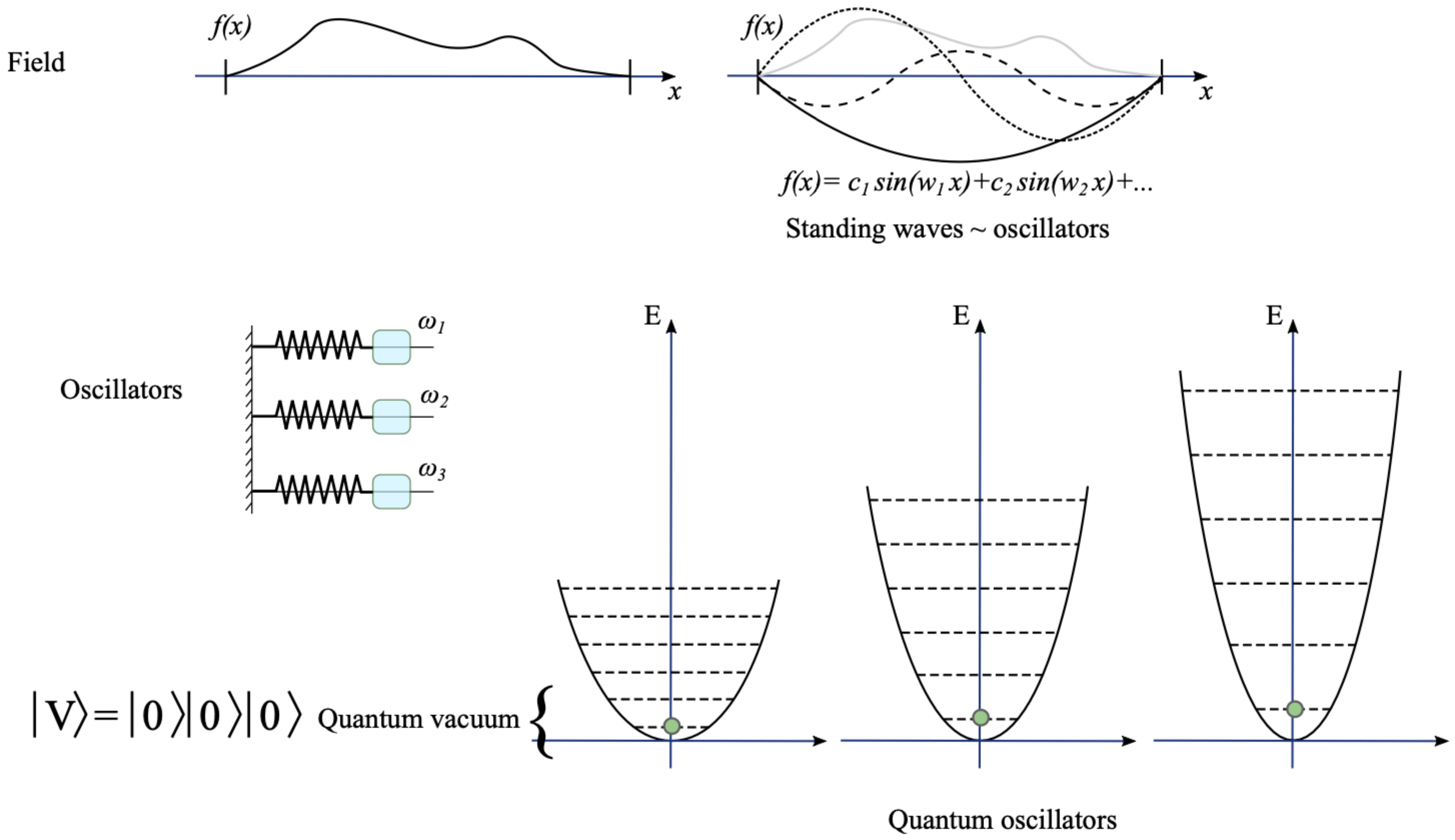
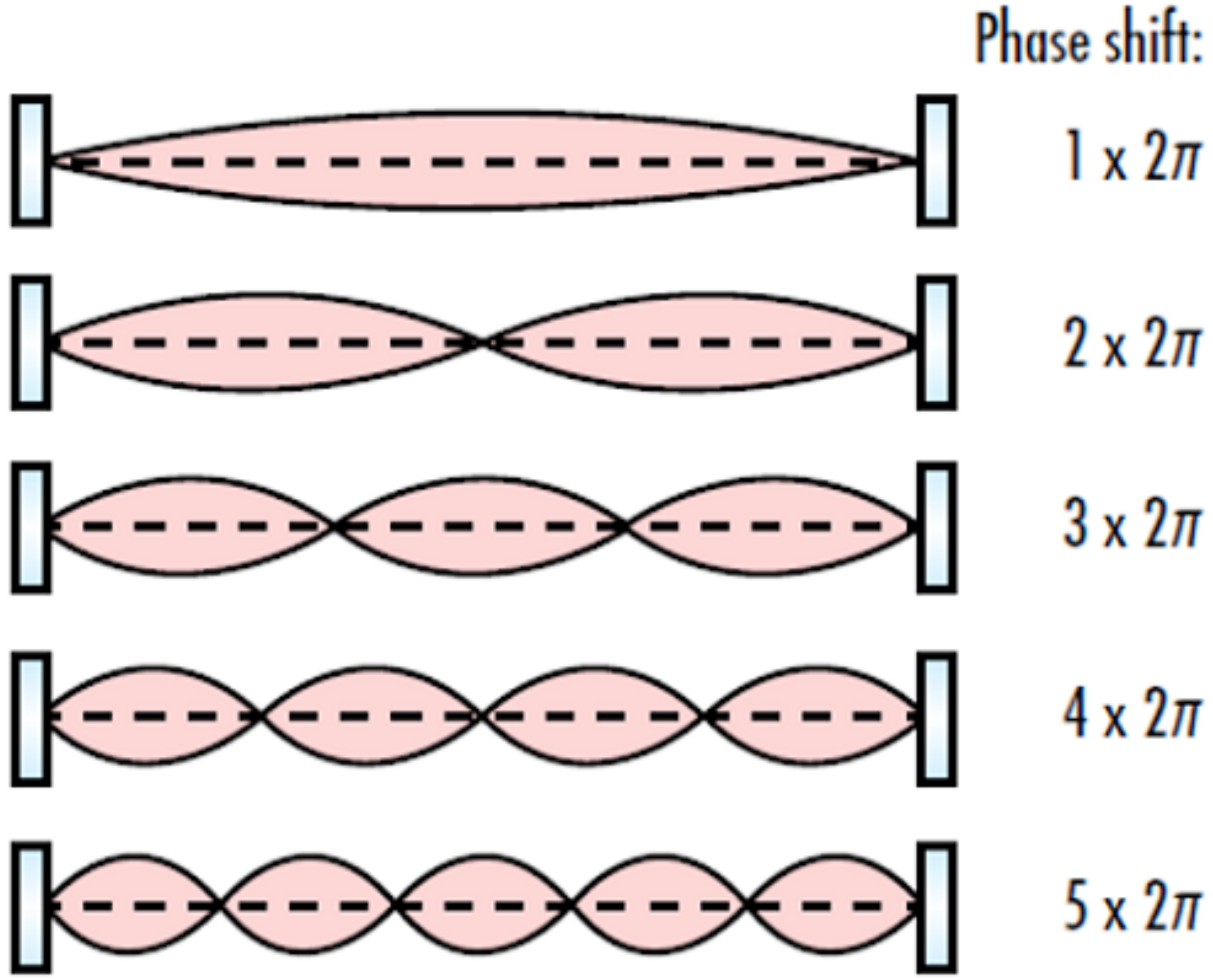


Figure 4.9: Fields can be described as fucntions over space $f(x)$. Sunch functions can be expressed as the sum over standing wafers (Fourier series), each standing wave mathematically equivalent to an oscillator. Quantum oscillators correspond to quantum description of the field.

- Field is mathematically described as value in each point of space.
- Mathematically it is a sum of independent oscillations with fixed frequency (*modes*).
- Each mode can be quantized like normal.
- Quantum Field = Set of Quantized Modes.
- Vacuum = State of the field when all modes are at the bottom level.

Cavity Modes

Of Electromagnetic Field



- Electromagnetic field can be trapped between two mirrors (polished metals) — cavity.
- It supports oscillations of various frequencies — is composed of different oscillators (modes)
- Each oscillator/mode behaves like a quantum oscillator.
- You get many quantum oscillators to do quantum physics with — Cavity Quantum Electrodynamics (Cavity QED).

Cavity Modes

Of Electromagnetic Field

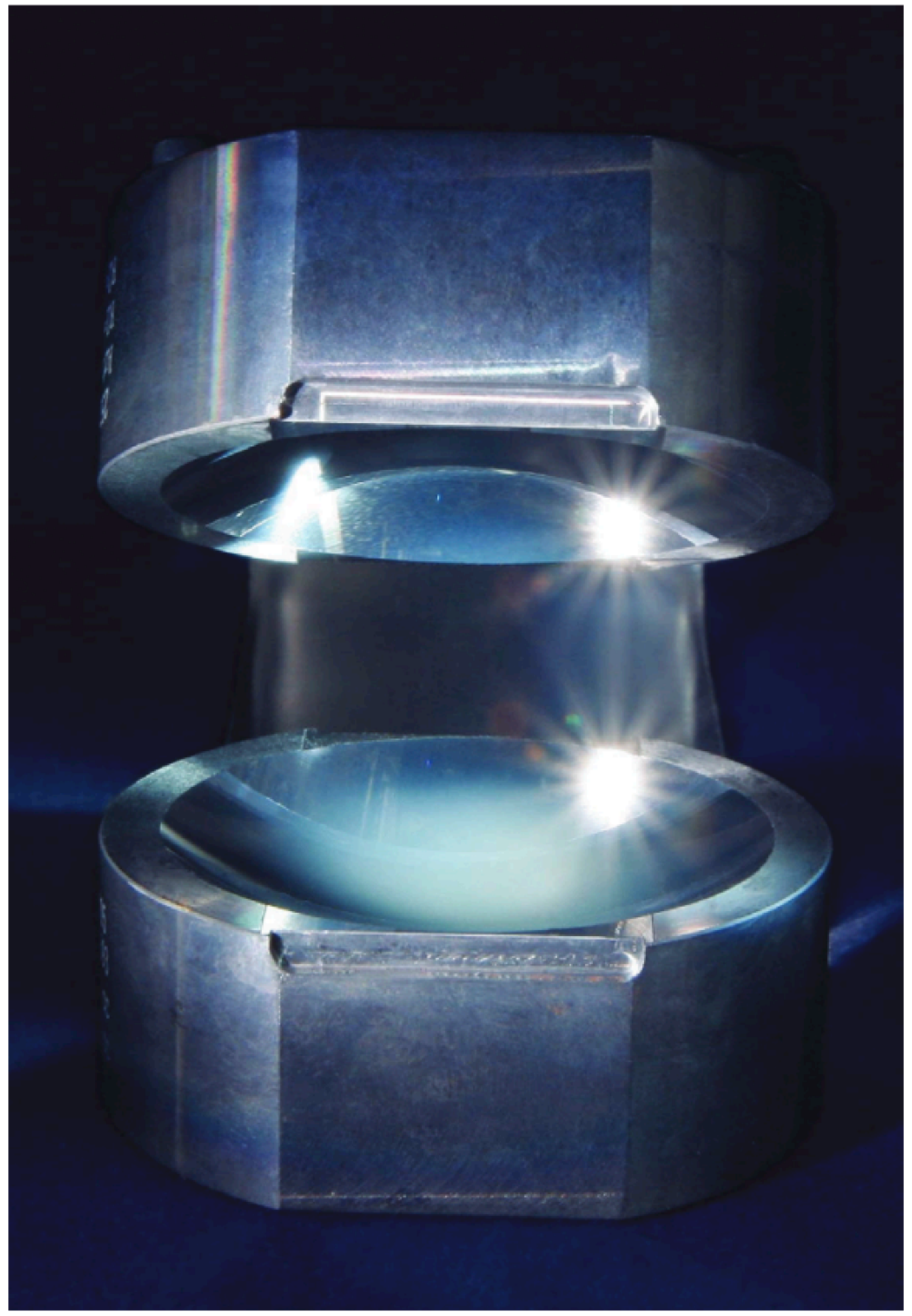
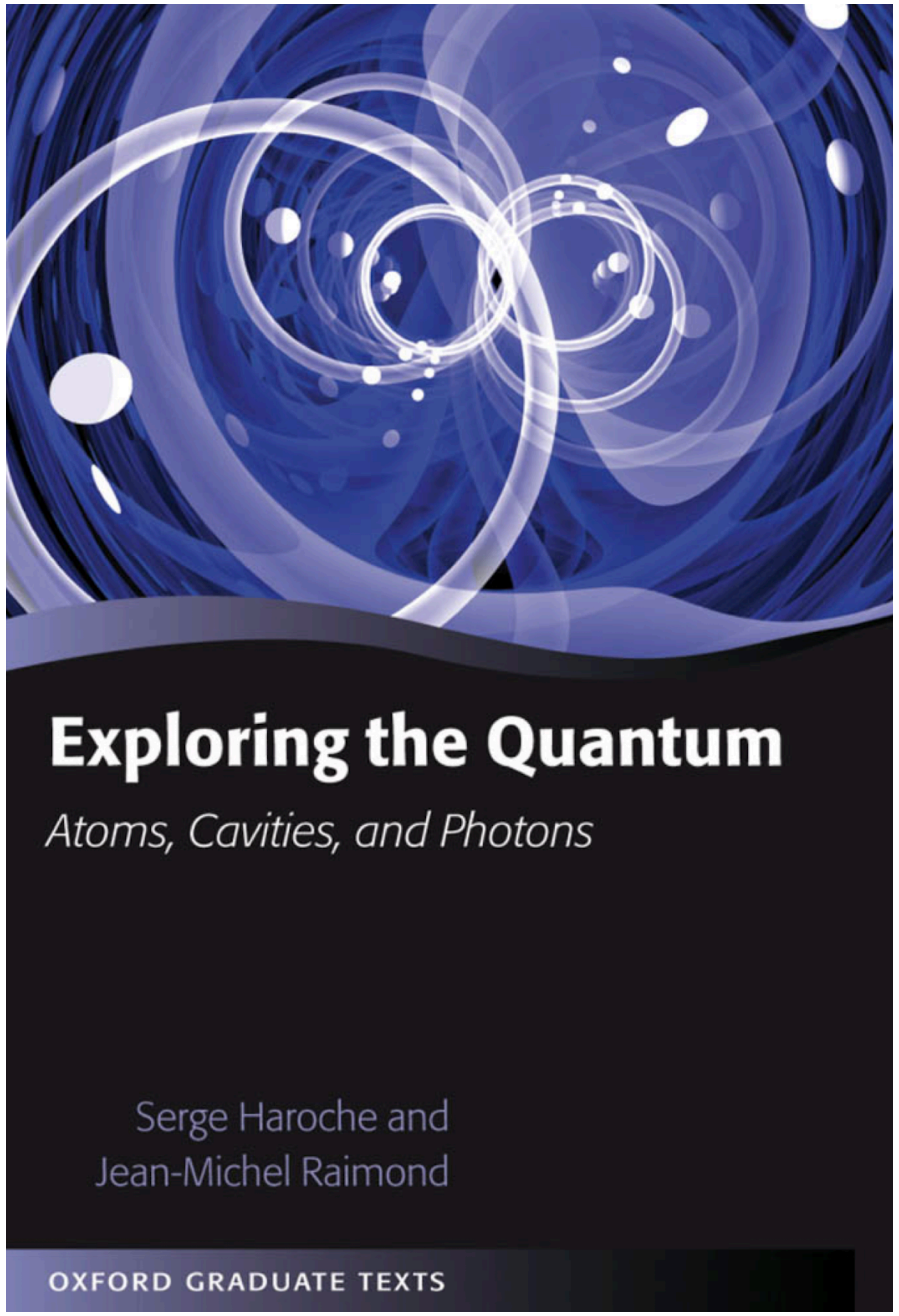


FIGURE 8. The ENS photon box (Photograph by Michel Brune). The mirrors have a diameter of 5 cm and are 2.7 cm apart (for a clear view, they are more separated in this picture than in the actual experimental set-up).



Self-Test

Answer These Questions 1hr After Class

1. How is coupling expressed in Hamiltonian or energy expressions?
2. What is a mode?
3. What is a field?
4. How is quantum field equivalent to a set of quantum oscillators?
5. What is quantum vacuum? How is it different from classical vacuum?
6. What does Cavity Quantum Electrodynamics study?

Homework Problems

Oscillators

1. Show that the Hamiltonian of two coupled oscillators can be written in the following form: $H = H_1 + H_2 + H_{int} = \frac{p_1^2}{2m} + \frac{k_1 x_1^2}{2} + \frac{p_2^2}{2m} + \frac{k_2 x_2^2}{2} - k' x_1 x_2$. Notice that "coupling term" $k' x_1 x_2$.
2. Repeat the derivations from the slides. Make sure you understand how we arrive at "uncoupled" oscillators.
3. Play with the simulations: https://mathlets.org/mathlets/coupled-oscillators/
4. Watch the videos:
    1. https://www.youtube.com/watch?v=J6UqNHq0Nts
    2. https://www.youtube.com/watch?v=UpseJGf_l2w&pp=ygUTY291cGxlZCBvc2NpbGxhdG9ycw%3D%3D
    3. https://youtu.be/KrTmMYUvNz4
    4. https://www.youtube.com/watch?v=Hm_nZ-0Amdo&pp=ygUTY291cGxlZCBvc2NpbGxhdG9ycw%3D%3D

Quantum Theory

In a Nutshell

II. POSTULATES FOR QUANTUM MECHANICS

In this paper, all state vectors are supposed to be normalized, and mixed states are represented by density operators, i.e., positive operators with unit trace. Let A be an observable with a nondegenerate purely discrete spectrum. Let $\phi_1, \phi_2, \ldots$ be a complete orthonormal sequence of eigenvectors of A and $a_1, a_2, \ldots$ the corresponding eigenvalues; by assumption, all different from each other.

According to the standard formulation of quantum mechanics, on the result of a measurement of the observable A the following postulates are posed:

(A1) *If the system is in the state ψ at the time of measurement, the eigenvalue a_n is obtained as the outcome of measurement with the probability $|\langle\phi_n|\psi\rangle|^2$.*

(A2) *If the outcome of measurement is the eigenvalue a_n, the system is left in the corresponding eigenstate ϕ_n at the time just after measurement.*

The postulate (A1) is called the *statistical formula*, and (A2) the *measurement axiom*. The state change $\psi \mapsto \phi_n$ described by the measurement axiom is called the *state reduction*.

You will understand this paragraph in the end of the course.