Quantum Physics

2024

Yury Deshko

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The Theory/Framework Of *Almost* Everything *Today*

Yury Deshko

Course Overview

Course Structure And Goals

- Part 1 : Mathematical Concepts And Tools
- Part 2 : Classical Physics
- Part 3 : Quantum Physics

- Learn the language of quantum physics
- Enhance the knowledge of classical physics

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Iran Admits Its Quantum Computer Had Zero Quantum in It

By Francisco Pires published 11 days ago

The quantum equivalent to shooting your own feet.

Comments (14)

Important to know what quantum physics **IS** and what it **IS NOT**. Otherwise it is easy to get *confused by hype*.

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Course Structure And Goals

- Part 1 : Mathematical Concepts And Tools
- Part 2 : Classical Physics
- Part 3 : Quantum Physics

- Learn the language of quantum physics
- Enhance the knowledge of classical physics
- Understand $i\hbar\frac{d\Psi}{dt} = \hat{H}\Psi$ — Newton's second law in disguise.
- Appreciate the importance of quantum physics in applications

Approach

Know What Quantum Physics IS and What It IS NOT

- There is a lot of **overlap** in language and mathematical tools between classical and quantum physics.
- The overlap is both **helpful** and **detrimental**.
- When you have **solid** knowledge of both classical and quantum physics — they help each other.
- If you have **poor** knowledge of both classical and quantum physics — you can get **confused**.

Required Knowledge

What You Should Know To Get A Smooth Start

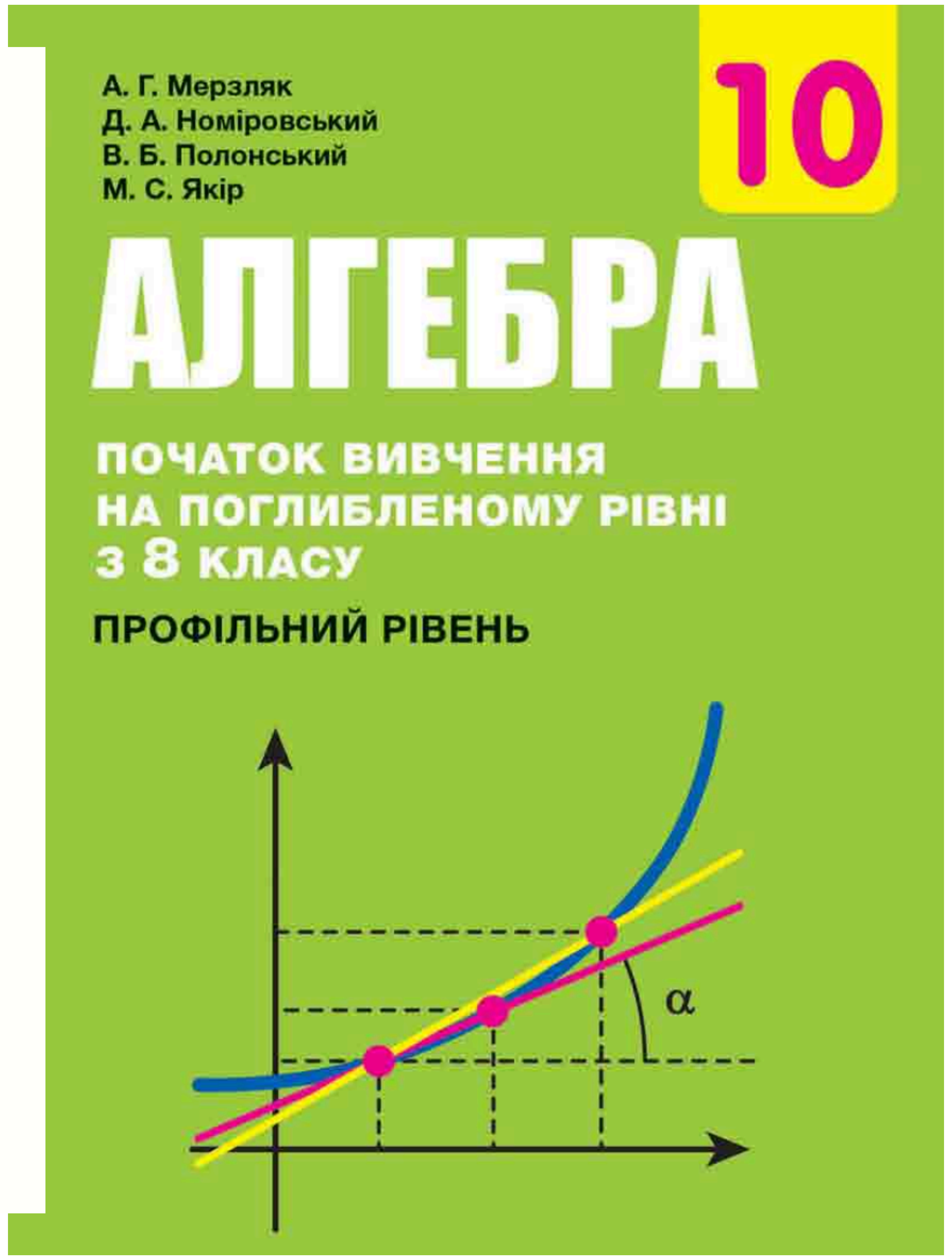
- Solid algebra, Trigonometry.
- Arithmetic and geometric series formulas:
  $1 + 2 + 3 + \ldots + n = n(n+1)/2$ and
  $1 + q + q^2 + q^3 + \ldots + q^n + \ldots = \dfrac{1}{1-q}$
- Ellipsis equation $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1.$
- Exponential function and its main properties
  $e^{x+y} = e^x e^y, e^x \approx (1 + x/N)^N, N \gg 1$

8-11 grades. More than enough.

Required Knowledge

What You Should Know To Get A Smooth Start

- Newton's second law $F = ma$
- Momentum $p = mv$ and kinetic energy $E_k = \dfrac{mv^2}{2}$.
- Hooke's law $F = kx$ and potential energy of a stretched spring $E_p = \dfrac{kx^2}{2}$.
- Newton's law of universal gravitation $F = G\dfrac{Mm}{r^2}$ and Coulomb's law $F = k\dfrac{Qq}{r^2}$.
- Conservation of energy.
- Waves and their basic properties.



8-11 grades. More than enough.

Important Ability

Be Flexible And Open-Minded

Be ready to learn your familiar concepts from *a new point of view*. Learn *new notation*.

A lot of what we will discuss will be rather *trivial* and *simple*. But often *accumulation of trivial and simple things* might appear as complicated, especially due to novelty and the lack of experience.

It is important to do exercises and try to think of your own examples.

What Comes To Mind

When You Hear Quantum Physics

- Small particles. Subatomic. Nuclear physics.
- Schrödinger equation.
- Wave-particle duality.
- Superposition.
- Entanglement.
- Probability.
- Quantization.
- Uncertainty.

What Comes To Mind

When You Hear Quantum Physics

- ~~Small particles. Subatomic. Nuclear physics.~~
- ~~Schrödinger equation.~~
- Wave-particle duality. (!?!) misunderstood. Niels Bohr’s complementarity is better.
- ~~Superposition~~.
- Entanglement.
- ~~Probability~~.
- ~~Quantization~~.
- Uncertainty. (!?!) misunderstood. Niels Bohr’s complementarity is better.

Quantum Physics

Experiment - Theory - Experiment

Experiment

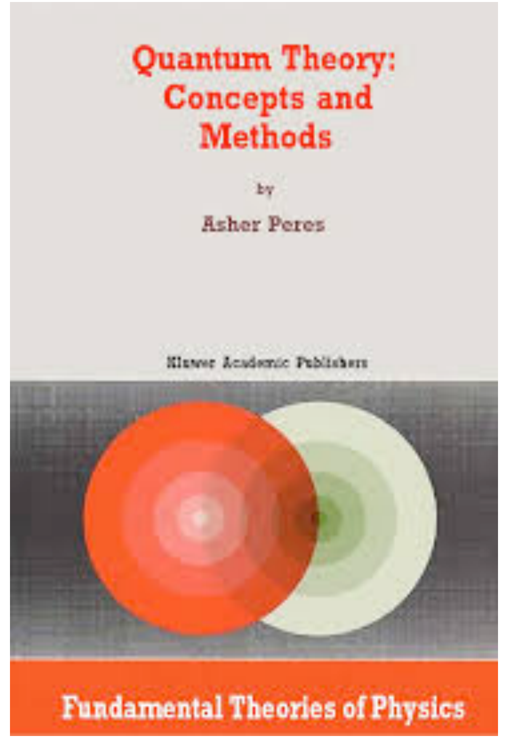
- What we do in the lab: use equipment to *prepare, measure, observe, record*.

Theory

- Words we say, concepts we use, symbols we write: *system, property, momentum, energy, state,* $|\Psi\rangle$ *etc.*

"Quantum phenomena do not occur in a Hilbert space. They occur in a laboratory."

— Asher Peres, ***Quantum Theory: Concepts and Methods***



Language Of Quantum Physics

And Much Of Classical!

- Operators. $\hat{A}, \hat{B}, \ldots, \hat{H}, \hat{L}$
- State vectors. $|\Psi\rangle, |\Phi\rangle, \ldots$
- Evolution. $|\Psi_t\rangle = \hat{U}_t|\Psi_0\rangle$
- Dynamical equations. $\partial_t|\Psi_t\rangle = \hat{V}_t|\Psi_t\rangle$
- Hamiltonian. $H, \hat{H}$
- Eigen-problem. $\hat{H}|\Psi_E\rangle = E|\Psi_E\rangle.$
- Commutators. $[\hat{A}, \hat{B}] = \hat{C}$
- And more… $|\Psi\rangle\langle\Psi|, \hat{a}^\dagger$

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Nothing is more impressive than the fact that as mathematics withdrew increasingly into the upper regions of ever greater extremes of abstract thought, it returned back to earth with a corresponding growth of importance for the analysis of concrete fact. …The paradox is now fully established that the utmost abstractions are the true weapons with which to control our thought of concrete fact.

Alfred North Whitehead

Ch. 2: "*Mathematics as an Element in the History of Thought*", p. 46

Language And Tools

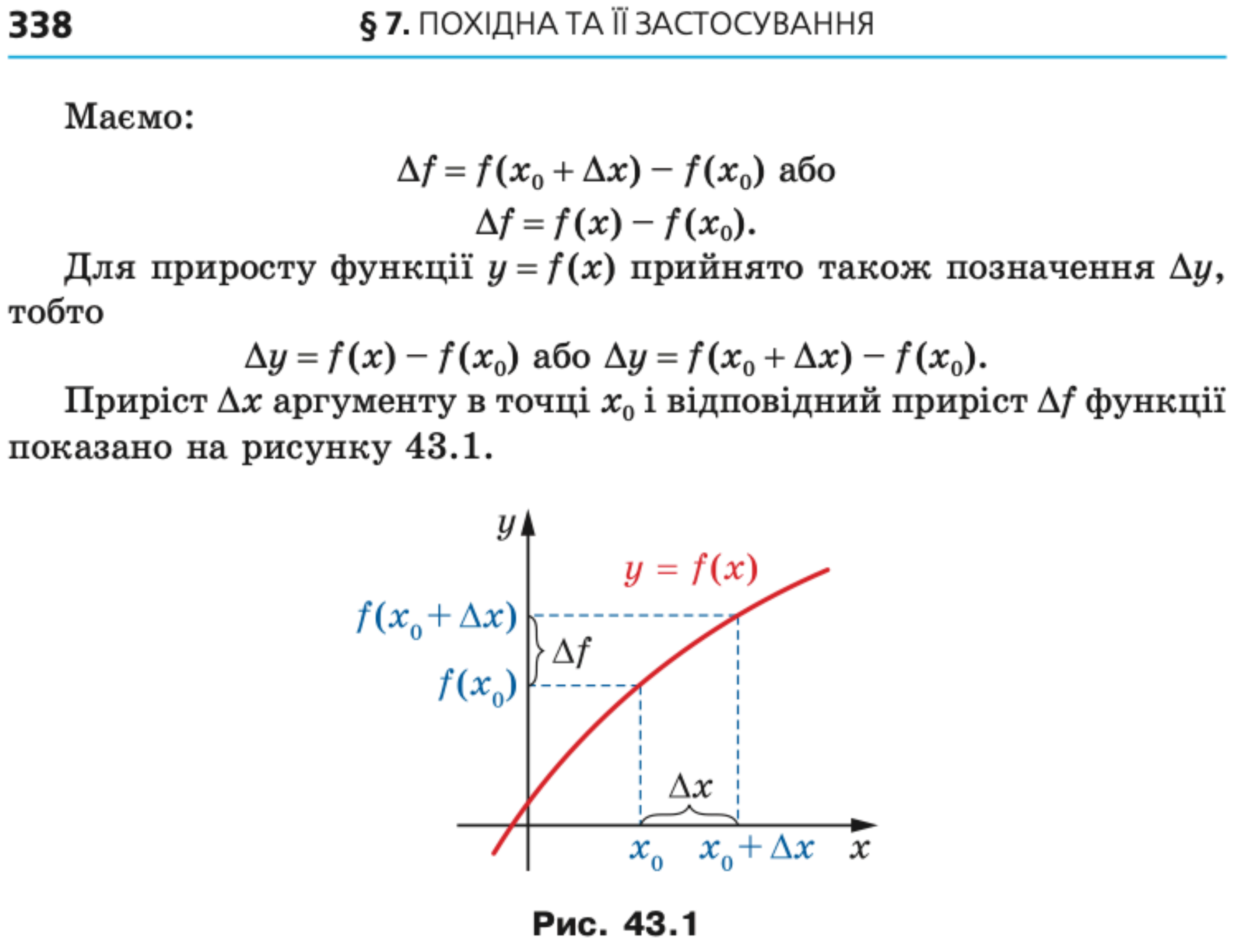
Reviewing Main Results In Math & Physics

- Kalkoolus (aka pre-Calculus)
- Game of Arrows (aka Linear Algebra)
- Functions of Operators
- New Concepts & Notation

Kalkoolus

Physicist Approach To Integrals And Derivatives

- $\Delta - \delta - \partial$ Notation
- Argument Free Expressions
- Partial Application
- Operators
- Taylor Series
- Hilbert Space



338 § 7. ПОХІДНА ТА ЇЇ ЗАСТОСУВАННЯ

Маємо:

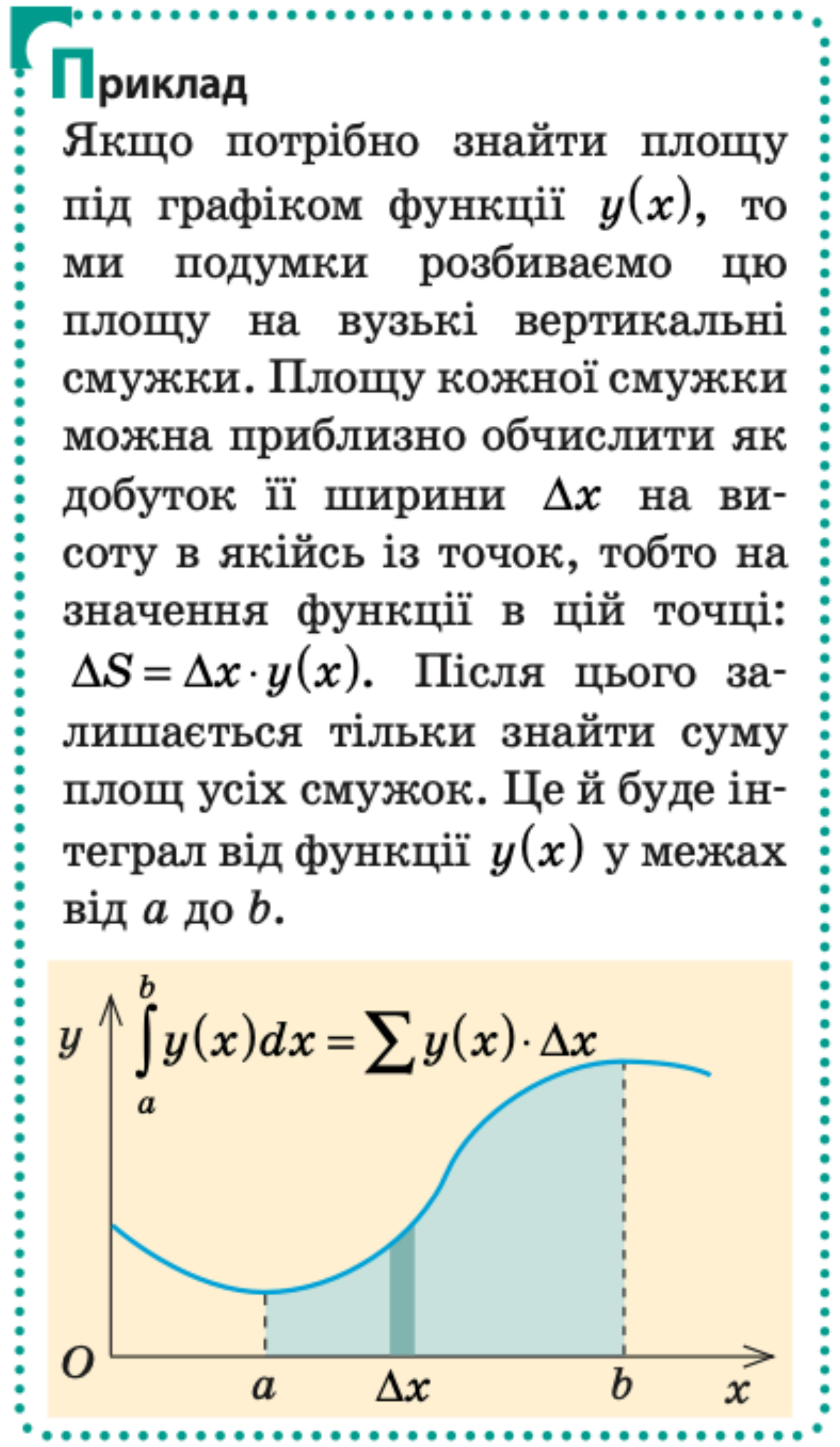
$$\Delta f = f(x_0 + \Delta x) - f(x_0) \text{ або}$$
$$\Delta f = f(x) - f(x_0).$$

Для приросту функції $y = f(x)$ прийнято також позначення Δy, тобто

$$\Delta y = f(x) - f(x_0) \text{ або } \Delta y = f(x_0 + \Delta x) - f(x_0).$$

Приріст Δx аргументу в точці x_0 і відповідний приріст Δf функції показано на рисунку 43.1.

Рис. 43.1



Приклад

Якщо потрібно знайти площу під графіком функції $y(x)$, то ми подумки розбиваємо цю площу на вузькі вертикальні смужки. Площу кожної смужки можна приблизно обчислити як добуток її ширини Δx на висоту в якійсь із точок, тобто на значення функції в цій точці: $\Delta S = \Delta x \cdot y(x)$. Після цього залишається тільки знайти суму площ усіх смужок. Це й буде інтеграл від функції $y(x)$ у межах від a до b.

More than enough.

Kalkoolus

Physicist Approach

- $\Delta - \delta - \partial$ Notation
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What we will discuss is rather trivial and simple. But often accumulation of trivial and simple things might appear as complicated, especially due to novelty and the lack of experience.

It is important to do exercises and try to think of your own examples.

Game Of Arrows 101

Physicist Approach

- Arrows - Directed Line Segments - Vectors.
- Scalar Product.
- Operators. Linear Operators.
- $\hat{J}$-operator. Circular Motion.
- Compound Numbers
- Projectors. (Optional)

Kalkoolus

Physicist/Engineer Approach

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Kalkoolus — Arithmetic with tiny quantities.

Say t is time in seconds since the start of the year. Then $\Delta t = t_2 - t_1$ can denote month, while δt will denote 1 second. δt is tiny change on the scale of the year.

Say m is the mass of a bucket almost full of water. Then Δm is the change of the mass when we pour one more cup, while δm is the increase of the mass when one single drop is added.

That's all there is to $\Delta - \delta$ notation.

Kalkoolus

Physicist/Engineer Approach

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WARNING

Mathematicians will talk about *limits, differentials,* and so on. We do not care. We will NOT use dx for tiny change because it has special meaning in math.

Forget about dx, dy, df

Remember δx, δy, δf (in this course only!)

Kalkoolus

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Rate of change of A with respect to B

Consider we track an asteroid. During 1 second (δt) it changes its position a tiny bit (δx).

Rate of change of position **with respect to** time:

$$\frac{\delta x}{\delta t}$$

∂-notation:

“Rate of change” is denoted as ∂.

Rate of change of x is denoted as ∂x

Rate of change of x with respect to t is denoted $\partial_t x$

$$\partial_t x = \frac{\delta x}{\delta t}$$

Kalkoolus

Physicist/Engineer Approach

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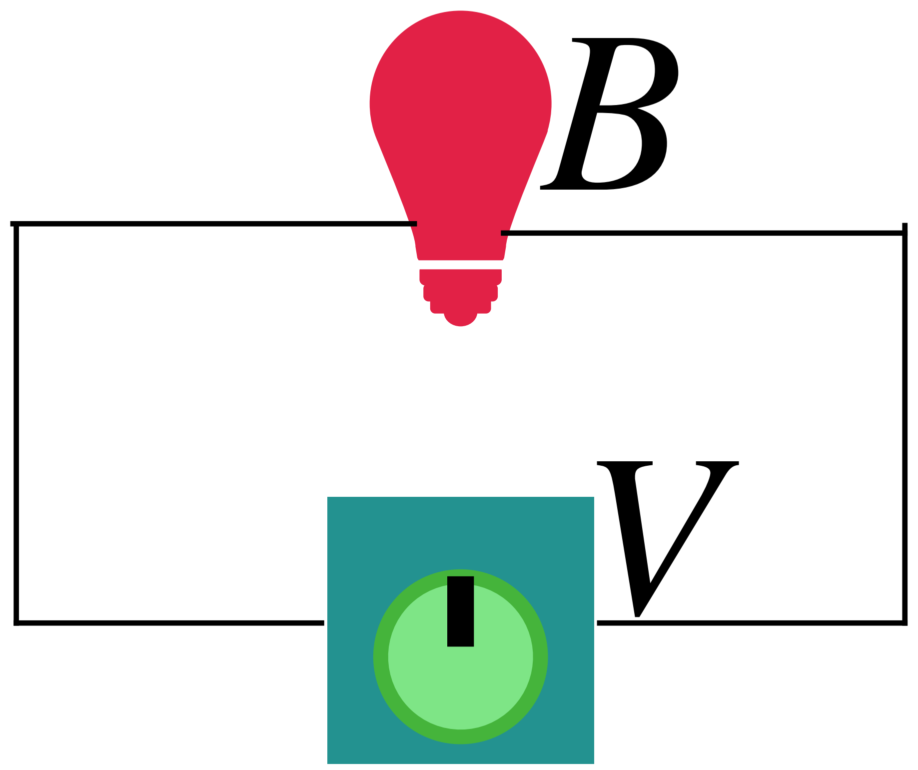
Rate of change of A with respect to B

Consider we study brightness B of a lightbulb as a function of applied voltage V. Adjust voltage a tiny bit (δV) and observe tiny change in brightness (δB).

Rate of change of brightness **with respect to** voltage:

$$\partial_V B = \frac{\delta B}{\delta V}$$



Kalkoolus

Del-Notation In Modern Math and Physics

$$0 = \partial_i A_j - \partial_j A_i .$$

Elements of Numerical Relativity, C. Bona and C. Palenzuela-Luque

$$\frac{1}{4\pi} \partial^\beta F_{\beta\alpha} = \frac{1}{c} J_\alpha$$

Classical Electrodynamics, J. D. Jackson

$$\exp(t\partial_\xi) = 1 + t\partial_\xi + \frac{t^2}{2}(\partial_\xi)^2 + \ldots$$

Modern Geometry, Novikov and Fomenko

$$\partial_k g_{ij} = -L_{kij} - L_{kji}.$$

Space-Time Algebra, D. Hestenes

$$R^\theta{}_{\phi\theta\phi} = \partial_\theta \Gamma^\theta_{\phi\phi} - \partial_\phi \Gamma^\theta_{\theta\phi} + \Gamma^\theta_{\theta\lambda}\Gamma^\lambda_{\phi\phi} - \Gamma^\theta_{\phi\lambda}\Gamma^\lambda_{\theta\phi}$$

Lecture Notes on General Relativity, Sean Carroll

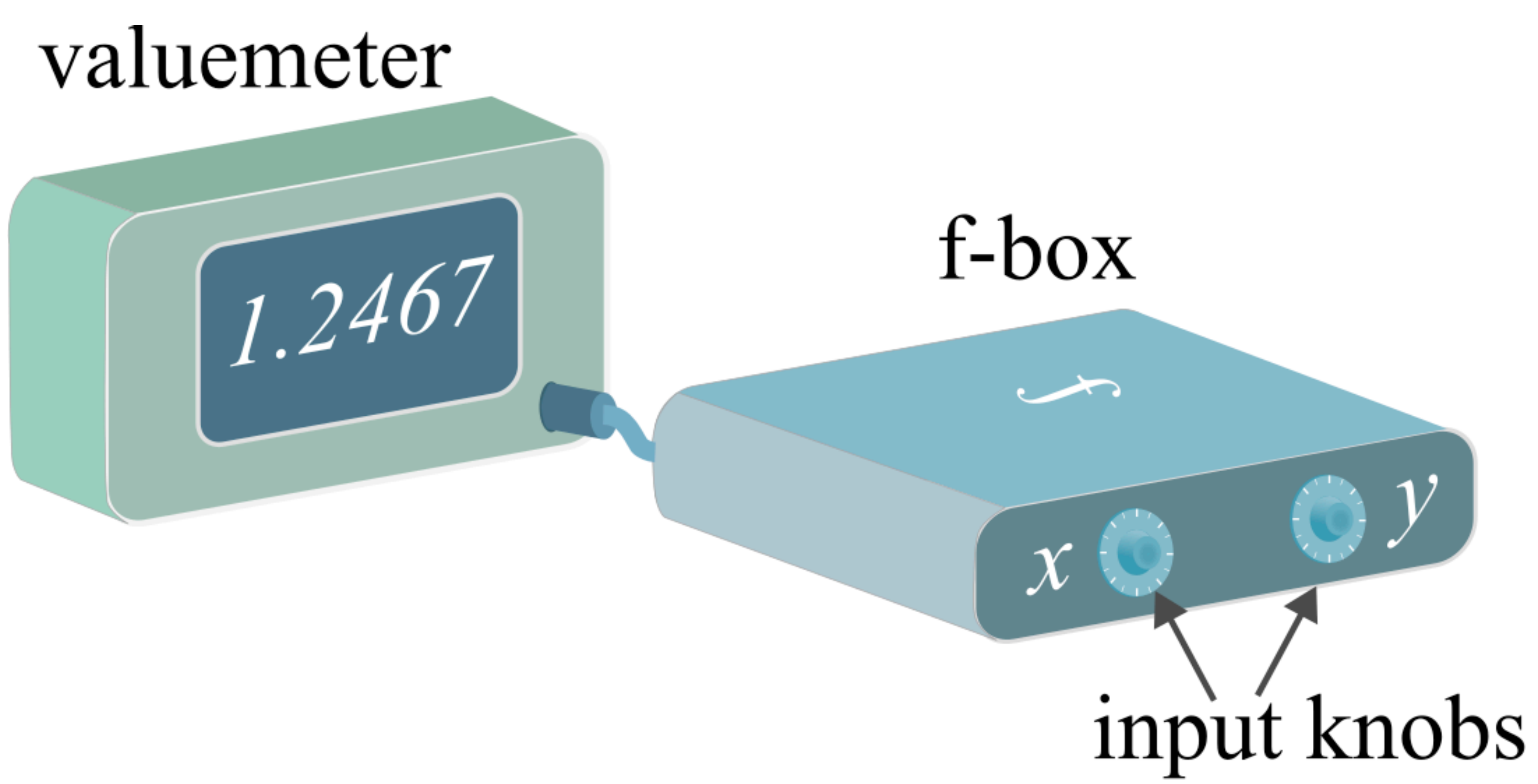
$$\mathrm{D}_\alpha \equiv \partial_\alpha + \mathrm{i}qA_\alpha(x)$$

Explorations in Mathematical Physics, D. Koks

And many more...

Kalkoolus

Physicist/Engineer Approach



Function as a box with input(s) and output(s)

Basic and simple idea. Can be extended in many ways:

* More than one input
* More than one output
* Inputs/outputs can be strings, pairs, arrows, even functions, and more!

Kalkoolus

Physicist/Engineer Approach

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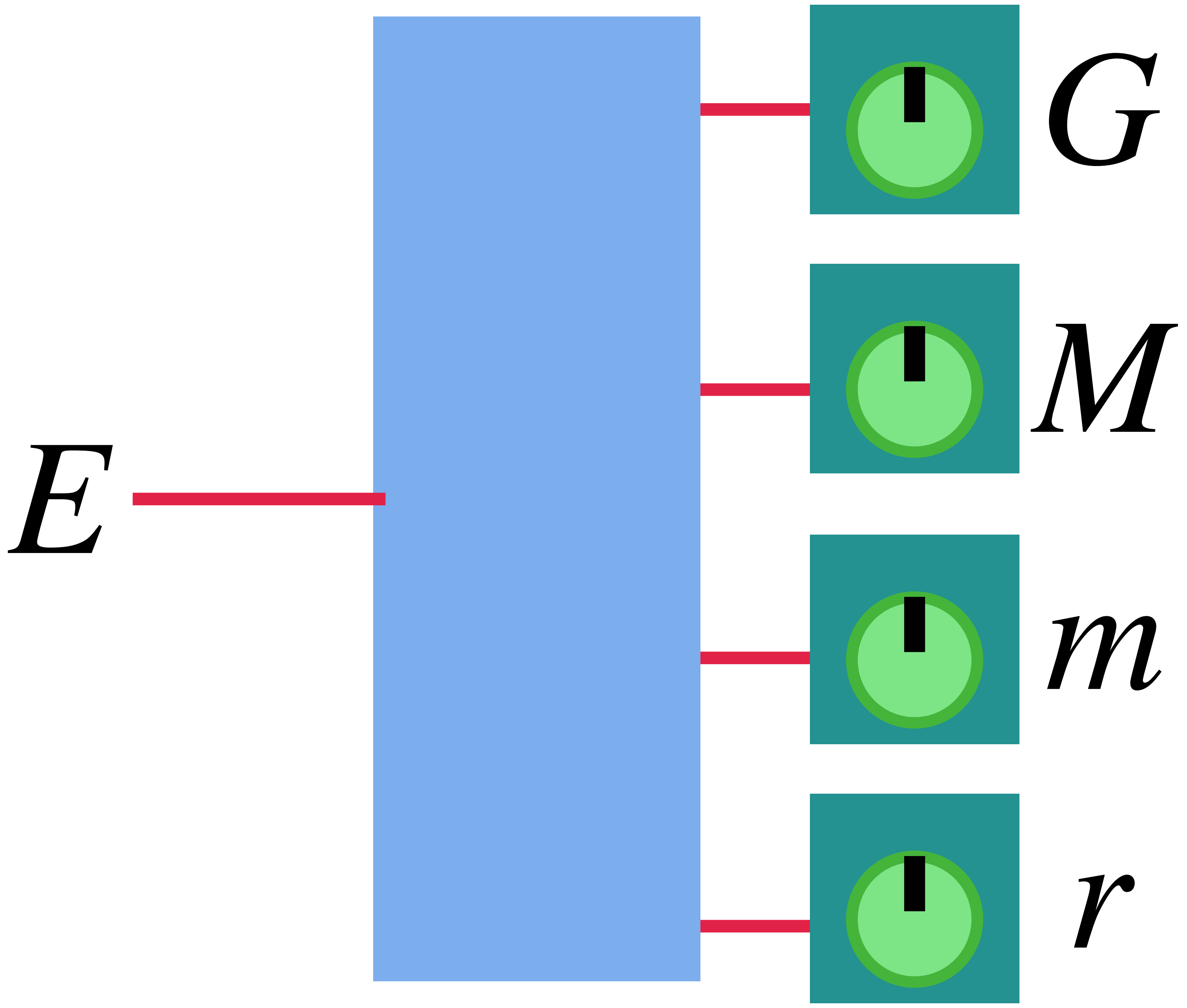
Exercise: Calculate

$$\partial_r E = \frac{\delta E}{\delta r}$$

Rate of change of A with respect to B

Generally, a function might depend on many inputs:

$E = G\dfrac{Mm}{r}$ — depends on 4 symbols!



$\partial_G E = \dfrac{\delta E}{\delta G}$ turn G-knob only

$\partial_M E = \dfrac{\delta E}{\delta M}$ turn M-knob onl

$\partial_m E = \dfrac{\delta E}{\delta m}$ turn m-knob onl

$\partial_r E = \dfrac{\delta E}{\delta r}$ turn r-knob only

Kalkoolus

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In the expression $\sin\ x$ the variable x is the argument.

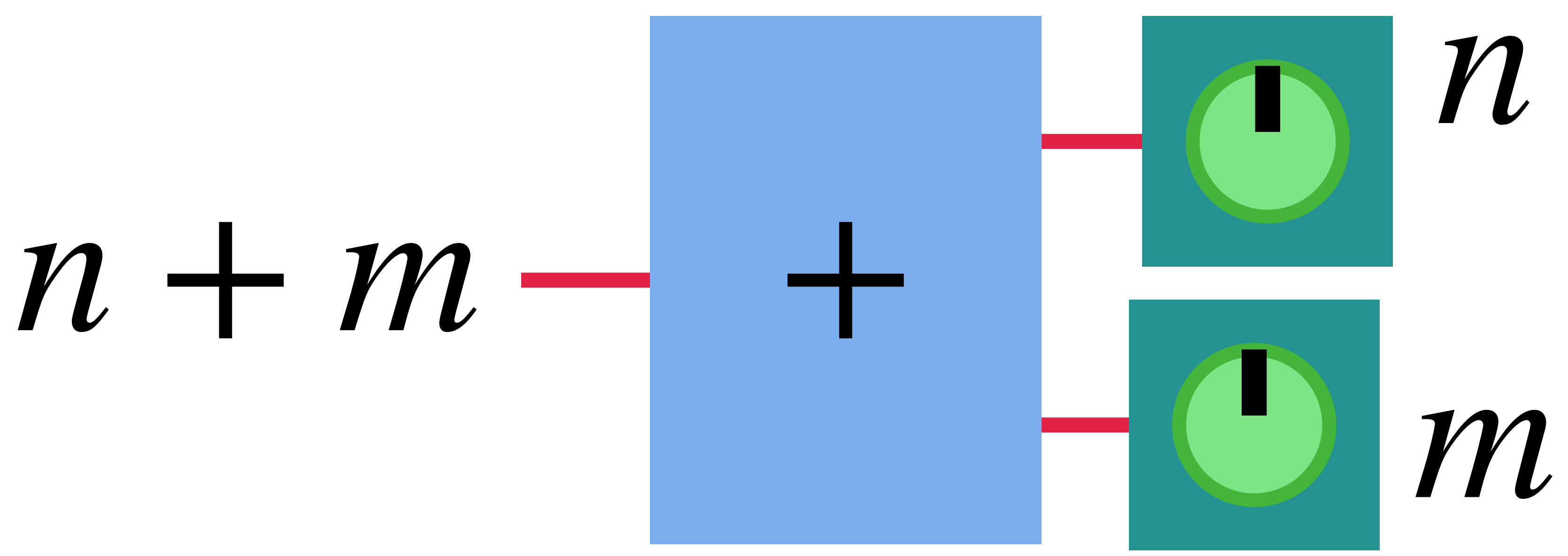
We can discuss the **function** sin **itself**, **not** the value $y = \sin\ x$.

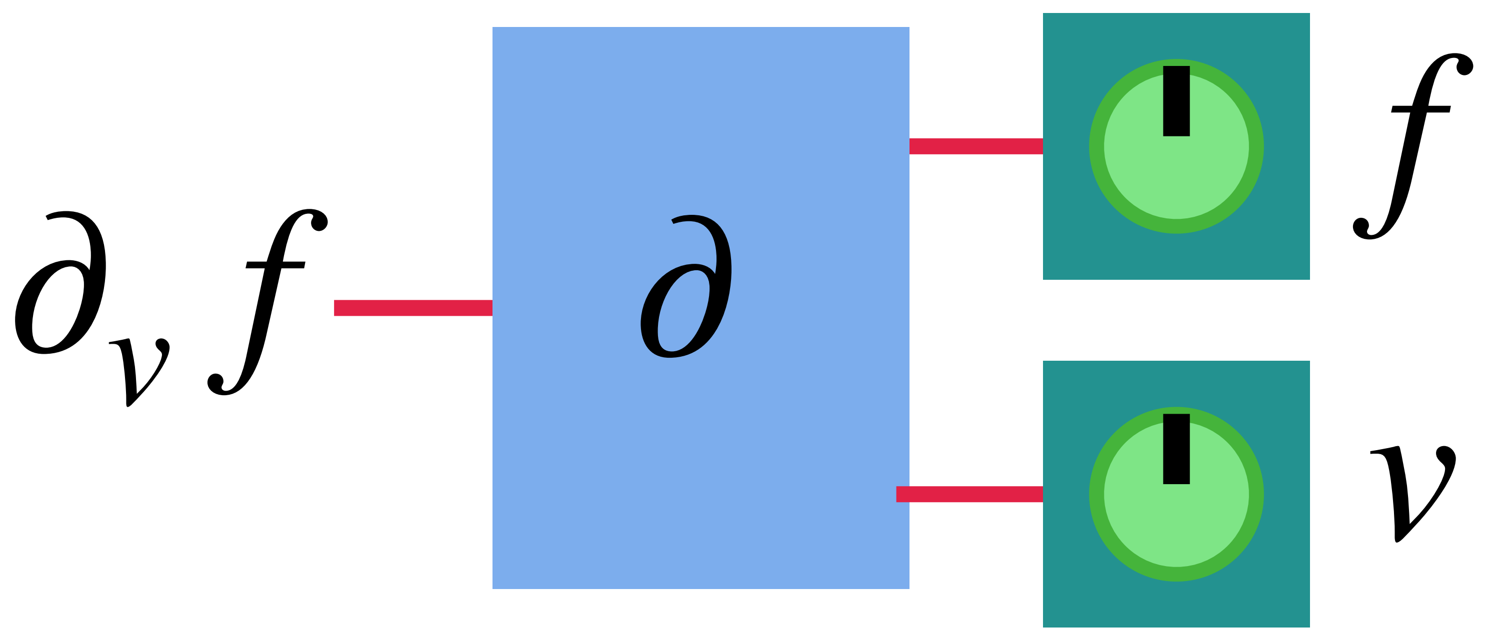
sin is the argument-free expression. Other examples:

$\tan,\ \log, \sqrt{\ }$

More "advanced":

$+,\ *\ , \partial$





Think in terms of "team" without "team **of players**"

Kalkoolus

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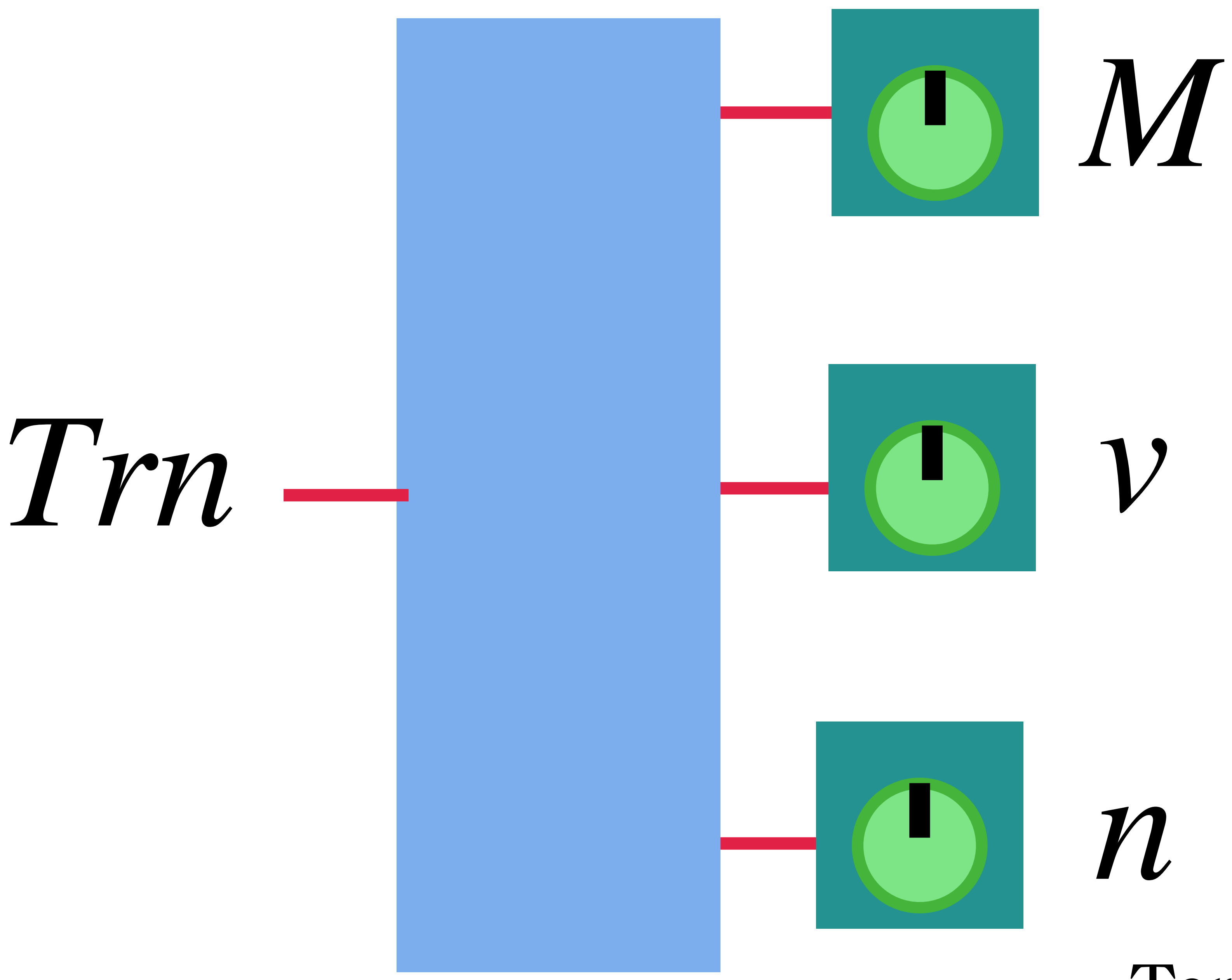
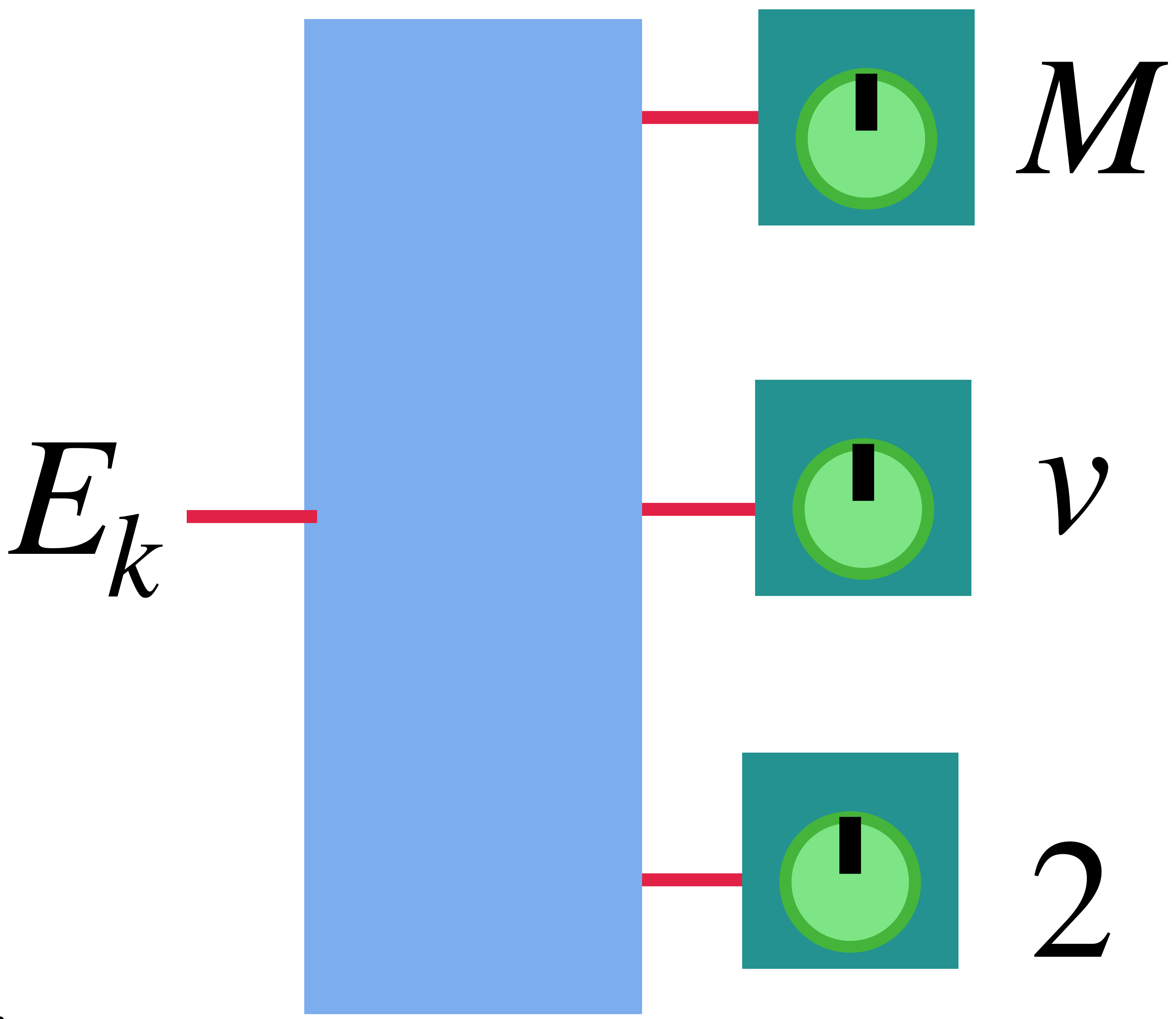
Function with several inputs, but some are fixed

Consider

$Trn = \dfrac{Mv^n}{n}$ — depends on 3 symbols! (Ternary function)





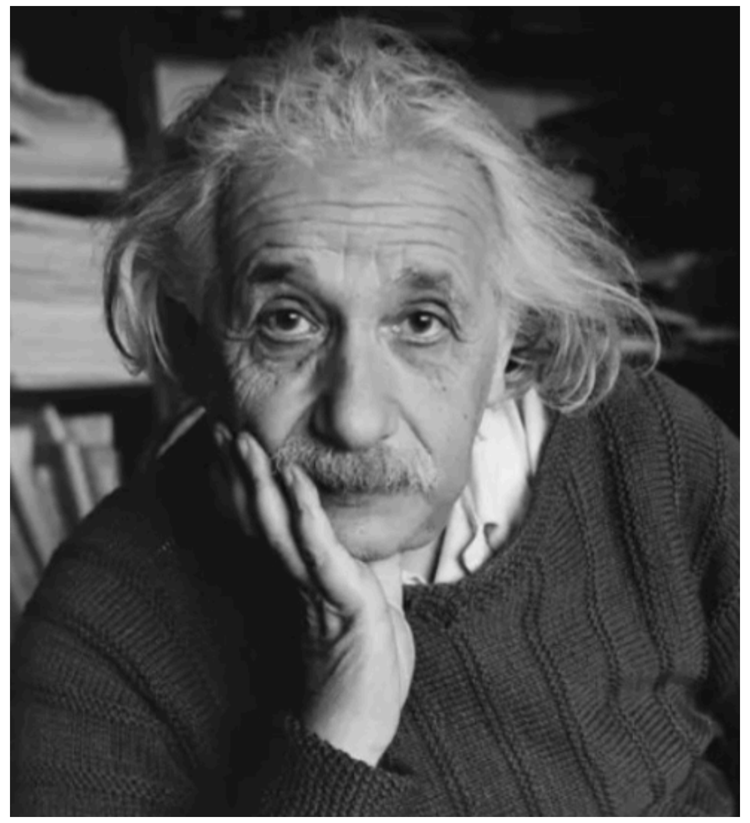
Ternary becomes binary

A Word From Our Sponsor

How To Learn According To Einstein

But personalities are not formed by what is heard and said but by labor and activity. The most important method of education accordingly always has consisted of the where pupil was urged to actual performance.

Albert Einstein, *"On Education"*



Self-Test

Answer These Questions 1hr After Class

1. Name three components of Quantum Physics.
2. What makes quantum theory difficult?
3. How can we represent functions besides formulas?
4. How do we call the method of writing functions without mentioning their arguments.
5. What other names are used for functions or function-like mathematical concepts?

Homework Problems

Mathematical Concepts and Notation Day 2

- Learn how to derive the formulas for arithmetic and geometric series.
- Using box-representation of addition, multiplication, and finding inverse (*add, mul, inv*), show how such boxes must be combined to obtain the function $E = \dfrac{kx^2}{2}$
- Calculate $\partial_k E$ and $\partial_x E$.
- Generalize the formula for E to make it a function of four variables k, x, m, n. Call this function F.
- Calculate $\partial_n F$ and $\partial_m F$.
- What is the action of $\hat{J}^3$?
- What is the meaning (action) of $\hat{L} = \sqrt{\hat{J}}$?

Summation

Very Big Sum of Very Small Quantities

... Atque si porrò ad altiores gradatim potestates pergere, levique negotio sequentem adornare laterculum licet :

Summae Potestatum

$\int n = \frac{1}{2}nn + \frac{1}{2}n$

$\int nn = \frac{1}{3}n^3 + \frac{1}{2}nn + \frac{1}{6}n$

$\int n^3 = \frac{1}{4}n^4 + \frac{1}{2}n^3 + \frac{1}{4}nn$

$\int n^4 = \frac{1}{5}n^5 + \frac{1}{2}n^4 + \frac{1}{3}n^3 - \frac{1}{30}n$

$\int n^5 = \frac{1}{6}n^6 + \frac{1}{2}n^5 + \frac{5}{12}n^4 - \frac{1}{12}nn$

$\int n^6 = \frac{1}{7}n^7 + \frac{1}{2}n^6 + \frac{1}{2}n^5 - \frac{1}{6}n^3 + \frac{1}{42}n$

$\int n^7 = \frac{1}{8}n^8 + \frac{1}{2}n^7 + \frac{7}{12}n^6 - \frac{7}{24}n^4 + \frac{1}{12}nn$

$\int n^8 = \frac{1}{9}n^9 + \frac{1}{2}n^8 + \frac{2}{3}n^7 - \frac{7}{15}n^5 + \frac{2}{9}n^3 - \frac{1}{30}n$

$\int n^9 = \frac{1}{10}n^{10} + \frac{1}{2}n^9 + \frac{3}{4}n^8 - \frac{7}{10}n^6 + \frac{1}{2}n^4 - \frac{1}{12}nn$

$\int n^{10} = \frac{1}{11}n^{11} + \frac{1}{2}n^{10} + \frac{5}{6}n^9 - 1n^7 + 1n^5 - \frac{1}{2}n^3 + \frac{5}{66}n$

Quin imò qui legem progressionis inibi attentuis ensperexit, eundem etiam continuare poterit absque his ratiociniorum ambabimus : Sumtâ enim c pro potestatis cujuslibet exponente, fit summa omnium n^c seu

$$\int n^c = \frac{1}{c+1}n^{c+1} + \frac{1}{2}n^c + \frac{c}{2}An^{c-1} + \frac{c \cdot c-1 \cdot c-2}{2 \cdot 3 \cdot 4}Bn^{c-3}$$

$$+\frac{c \cdot c-1 \cdot c-2 \cdot c-3 \cdot c-4}{2 \cdot 3 \cdot 4 \cdot 5 \cdot 6}Cn^{c-5}$$

$$+\frac{c \cdot c-1 \cdot c-2 \cdot c-3 \cdot c-4 \cdot c-5 \cdot c-6}{2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8}Dn^{c-7} \ldots \text{\& ita deinceps,}$$

exponentem potestatis ipsius n continué minuendo binario, quosque perveniatur ad n vel nn. Literae capitales A, B, C, D & c. ordine denotant coëfficientes ultimorum terminorum pro $\int nn$, $\int n^4$, $\int n^6$, $\int n^8$, & c. nempe

$$A = \frac{1}{6}, B = -\frac{1}{30}, C = \frac{1}{42}, D = -\frac{1}{30}.$$