# Quantum Physics

## 2024

The Theory/Framework Of *Almost* Everything *Today*

Yury Deshko

$$|\Psi\rangle$$

# Part B

$$|\Psi\rangle$$

# State Vector

**<u>Complete</u> knowledge <u>independent</u> of specific <u>representation</u>**

# Course Overview

## Course Structure And Goals

- Part 1 : Mathematical Concepts And Tools
- Part 2 : Classical Physics
- Part 3 : Quantum Physics

$$i\hbar\frac{\delta|\Psi\rangle}{\delta t} = \hat{H}|\Psi\rangle$$

<u>State:</u> Complete *information* about the system. Full *knowledge* of it.

$$\begin{tikzcd}
 & x \\
|\Psi\rangle \arrow[ur] \arrow[r] \arrow[dr] \arrow[ddr] & p \\
 & E \\
 & L
\end{tikzcd}$$

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$|S\rangle = (x, v)$ Newton

$|\xi\rangle = (x, p)$ Hamilton

$|\Psi\rangle = ?$ Schrödinger

$|\Psi\rangle$ is a mathematical tool used as a “container” for complete information/knowledge about a system.

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- Part 1 : Mathematical Concepts And Tools
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$$i\hbar \frac{\delta |\Psi\rangle}{\delta t} = \hat{H} |\Psi\rangle$$

In "real" mechanics we need $\rho(x, p)$

$|S\rangle = (x, v)$ Newton

$|\xi\rangle = (x, p)$ Hamilton

$|\Psi\rangle = ?$ Schrödinger

State: Complete *information* about the system. Full *knowledge* of it.

$|\Psi\rangle$ is a mathematical tool used as a "container" for complete information/knowledge about a system.

# Measurement

## Of a value of a property of a quantum system

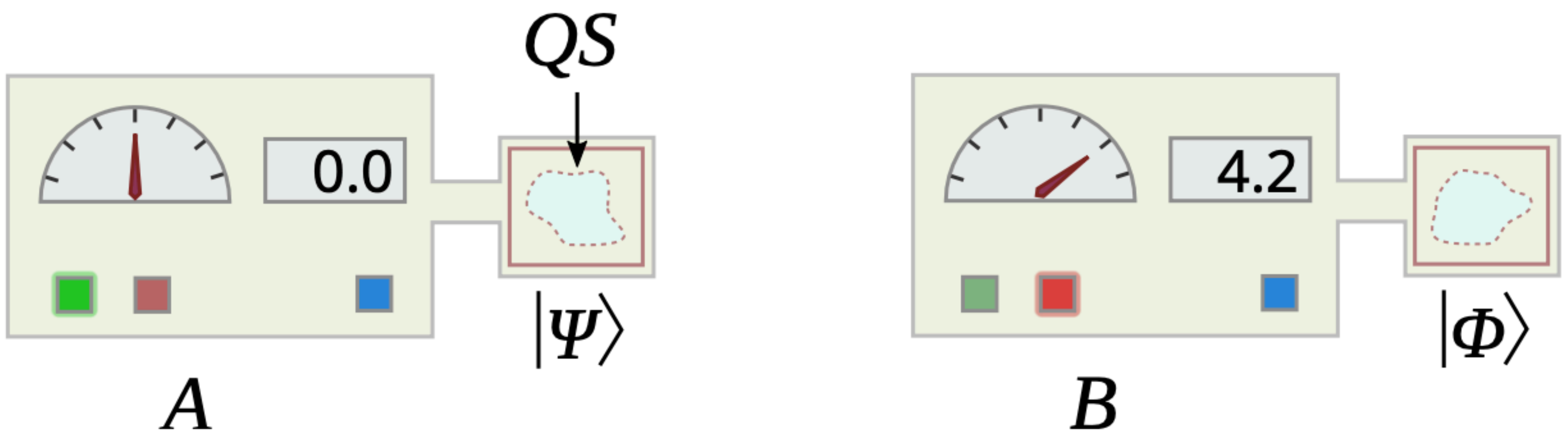



*Figure 3.2: (a) Initial state of classical apparatus and quantum system (QS). (b) After the measurements, the apparatus and the quantum system might change their state.*

$$A\,,\,|\Psi\rangle \quad \xrightarrow{\widehat{M}} \quad B\,,\,|\Phi\rangle\,;$$

Quantum physics does not work without classical physics. We use macroscopic classical apparatus, and macroscopic classical language.

# Measurement

## Apparatus and system are "aligned"

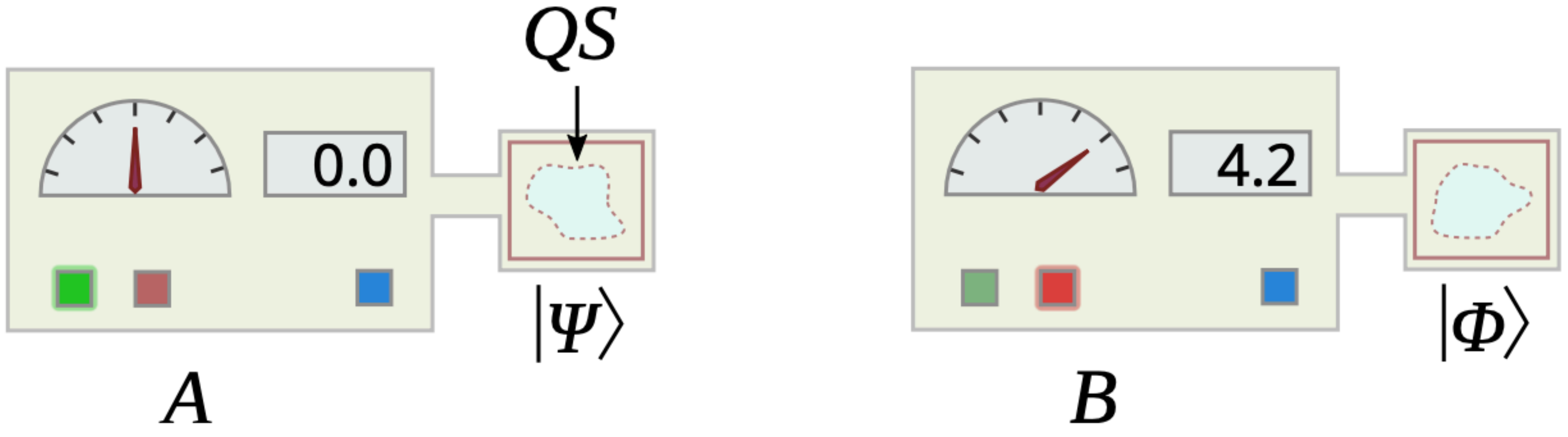



*Figure 3.2: (a) Initial state of classical apparatus and quantum system (QS). (b) After the measurements, the apparatus and the quantum system might change their state.*

$$A\,,\,|\Psi\rangle \quad \xrightarrow{\widehat{M}} \quad A\,,\,|\Psi\rangle \quad \xrightarrow{\widehat{M}} \quad A\,,\,|\Psi\rangle \quad \xrightarrow{\widehat{M}} \quad \ldots$$

$$\widehat{M}|\Psi\rangle = A|\Psi\rangle\,.$$

Eigen-problem. Eigen-value.

Eigen-vector. Eigen-state.

The state $|\Psi\rangle$ is called "proper"/own/eigen-state of the measurement operator $\hat{M}$.

Eigen - "own" in German.

# Measurement

## Apparatus and system are not "aligned"

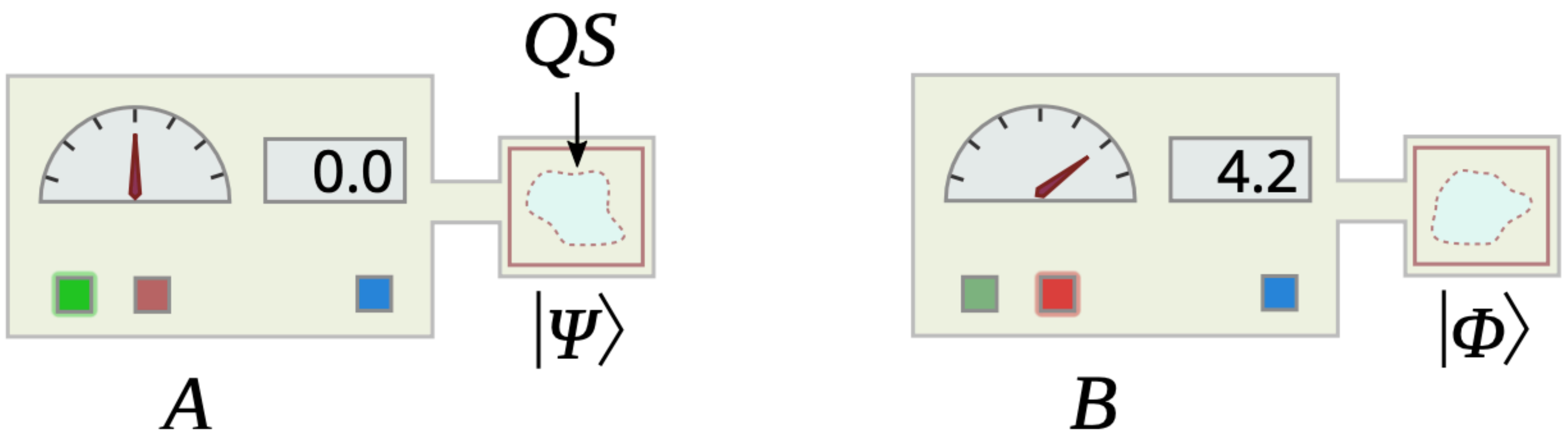



*Figure 3.2: (a) Initial state of classical apparatus and quantum system (QS). (b) After the measurements, the apparatus and the quantum system might change their state.*

$$A\,,\,|\Psi\rangle \quad \xrightarrow{\widehat{M}} \quad \begin{cases} p_1, |\Phi_1\rangle, V_1 \\ p_2, |\Phi_2\rangle, V_2 \\ \dots \\ p_n, |\Phi_n\rangle, V_n \end{cases}$$

$$p_1 = \frac{N_1}{N}\,.$$

$$|\Psi\rangle = F(|\Phi_1\rangle, |\Phi_1\rangle, \dots, |\Phi_1\rangle, p_1, p_2, \dots, p_n)\,,$$

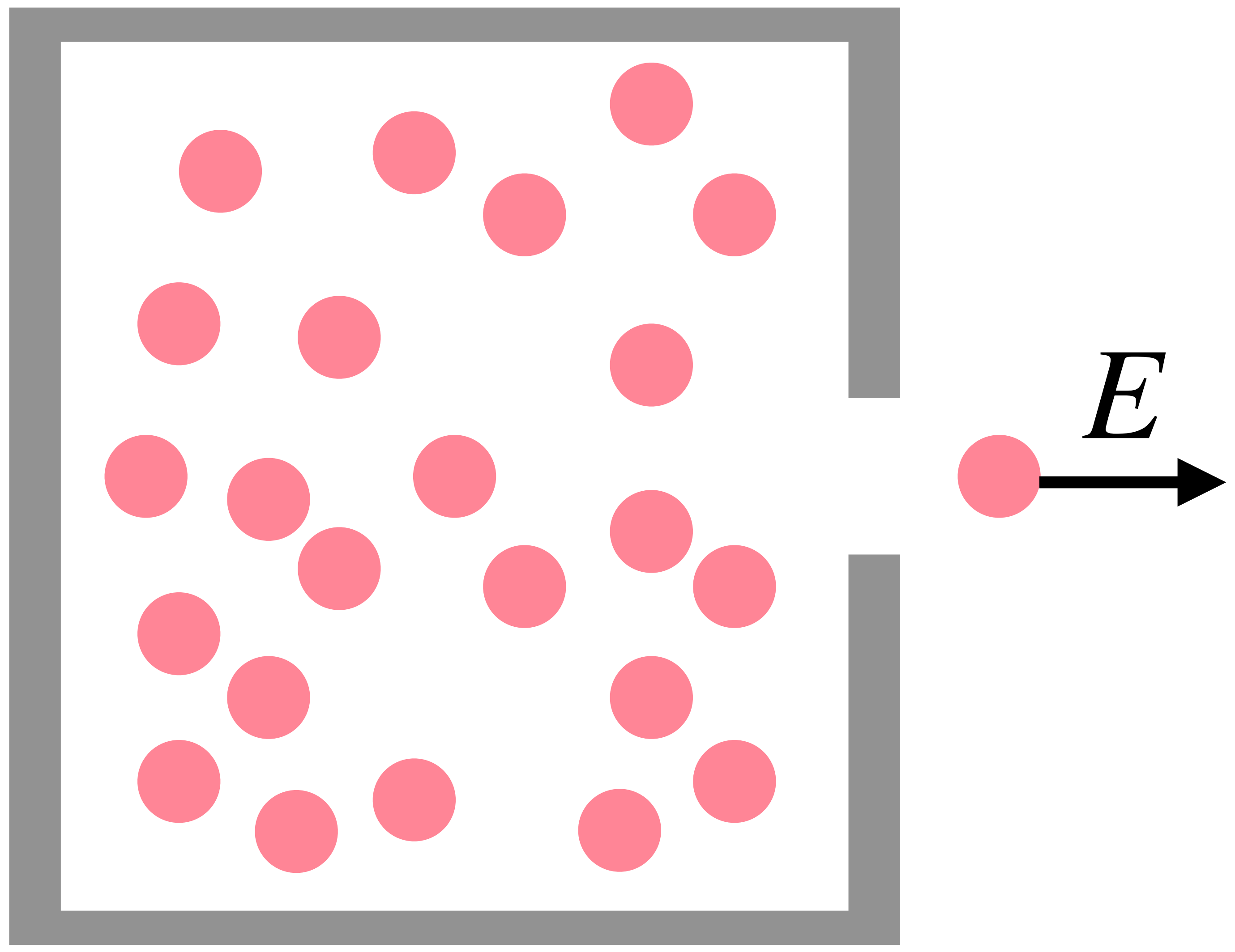
$$|\Psi\rangle = p_1|\Phi_1\rangle + p_2|\Phi_2\rangle + \dots p_n|\Phi_n\rangle\,.$$

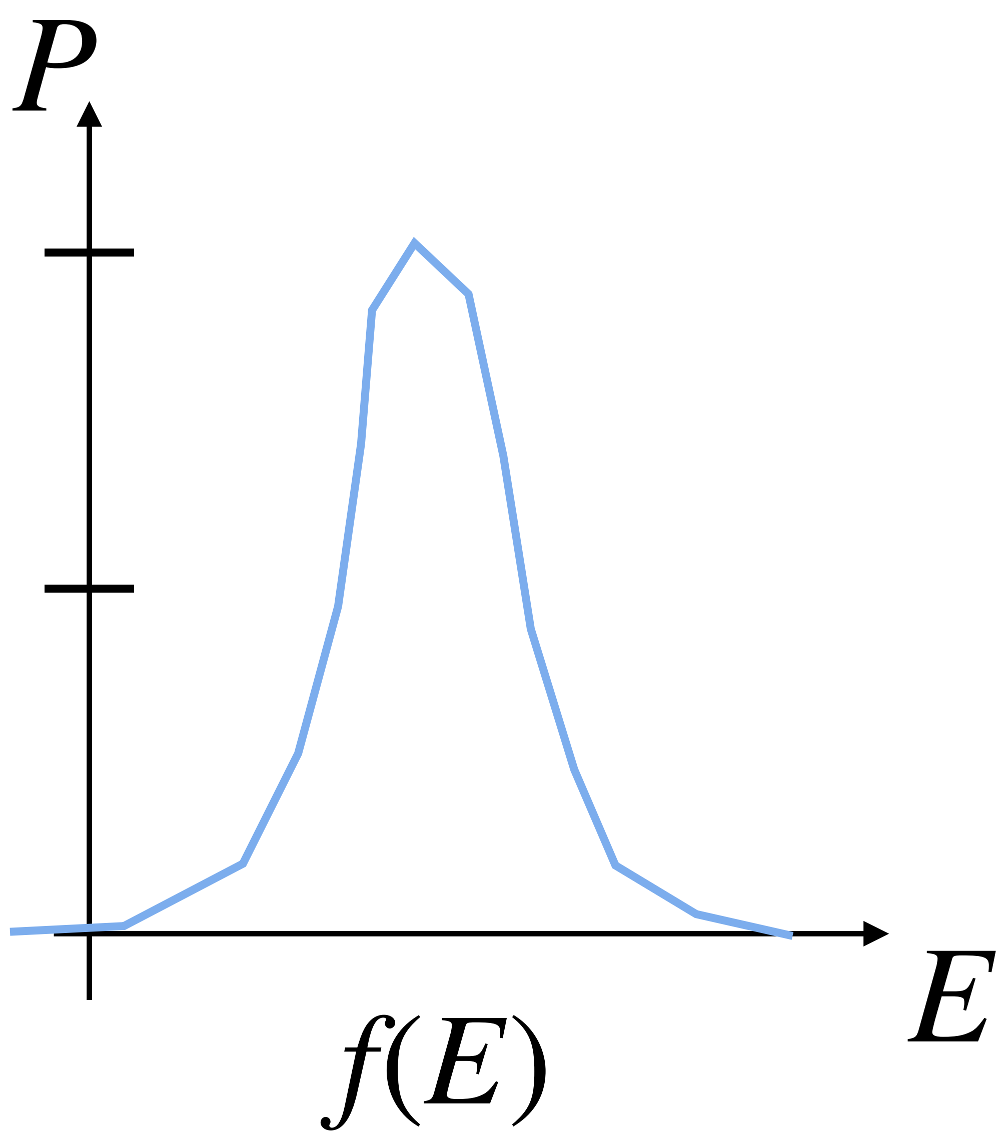
# Measurement

## Learning From “Ensemble”

Mixture of molecules.

Each with its own $E_i = mv_i^2/2$




$$E_{avg} \sim T$$

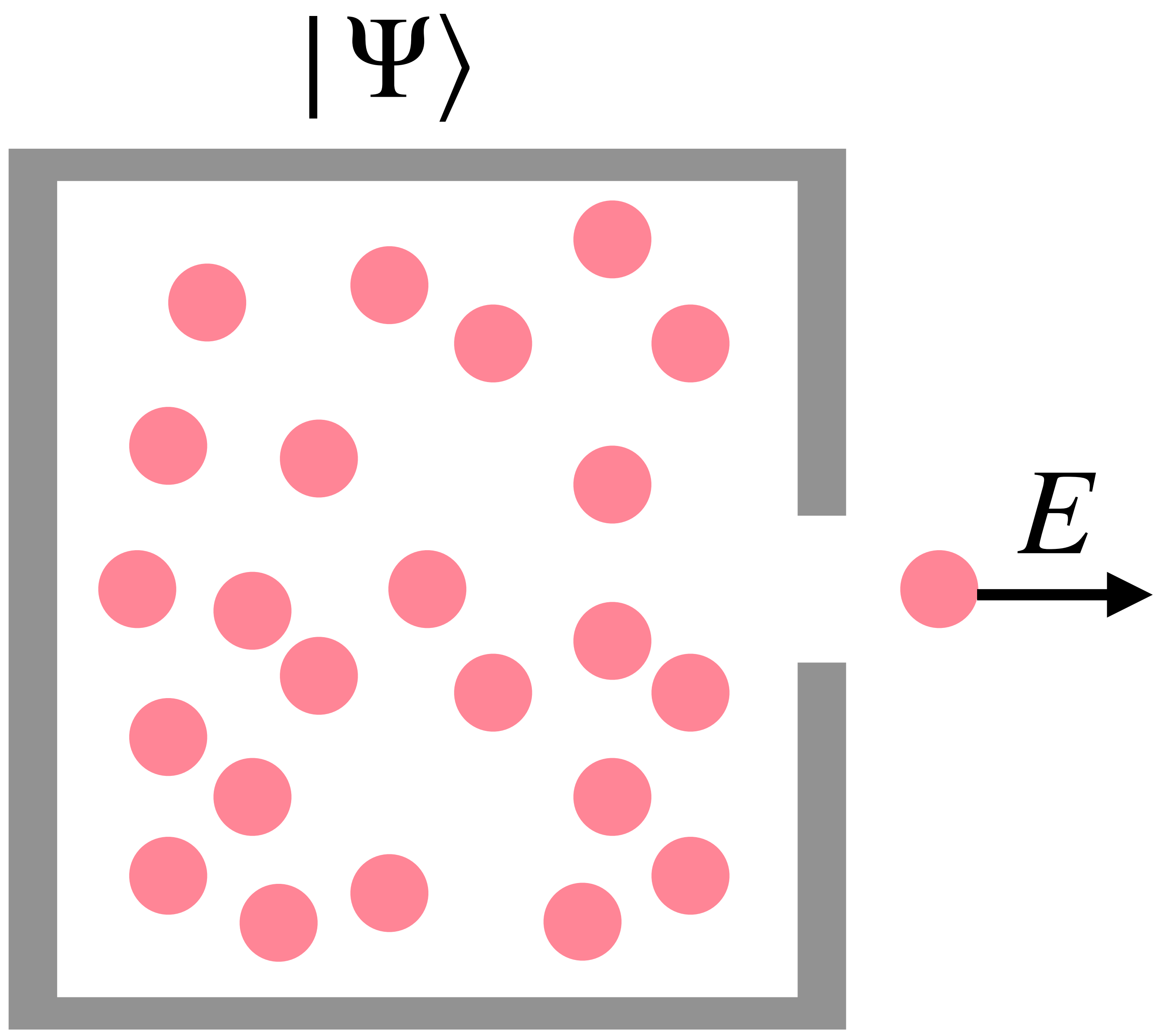
Statistical physics and kinetic theory of gases uses this idea.

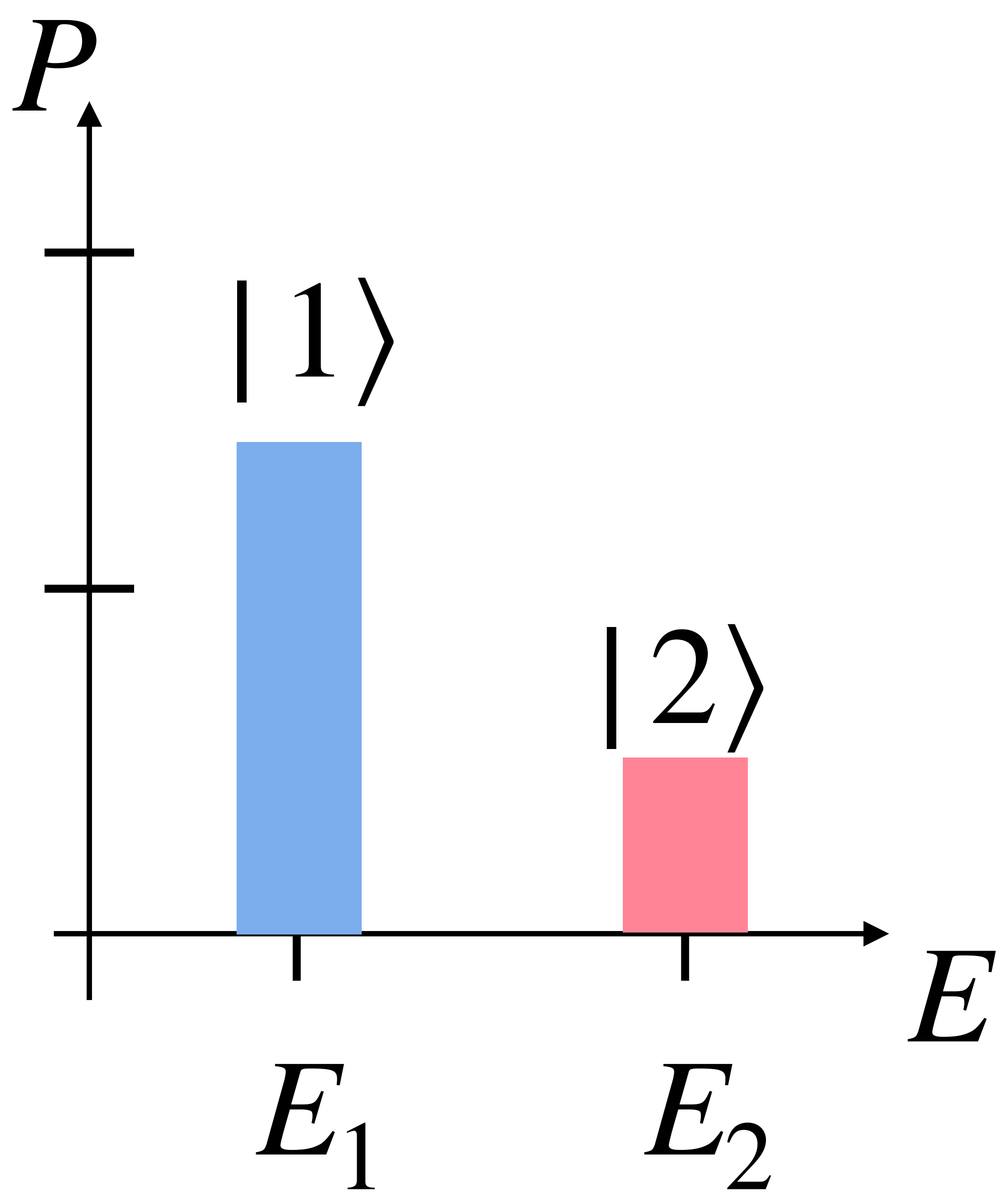
$f(E)$ describe the whole “group” of particles, all in similar/identical conditions (temperature?)

**Question**: Why is the distribution? What is the source of randomness?

# Measurement

**Learning From "Ensemble"**




Qubit = quantum system with 2 distinct states available for manipulation.

Qutrit = 3 states.

Qudit = d states

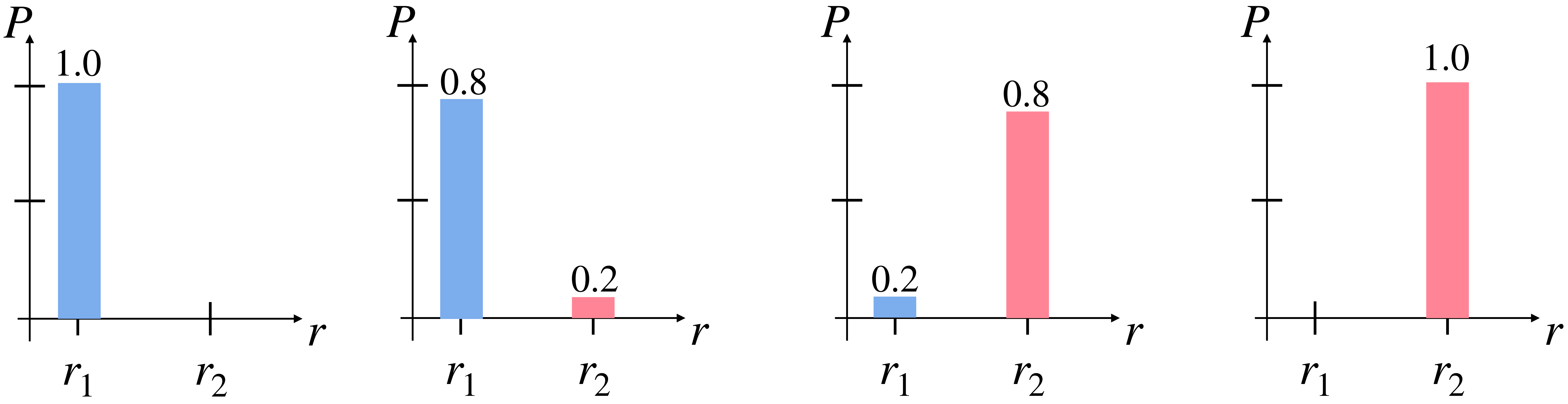
Measuring energy of a large number of qubits (e.g., harmonic oscillators with limited energy supply) reveals their state $|\Psi\rangle$.

# Probability

## And its representation

Reasonable first try. Will be improved/corrected later.

$|\Phi_1\rangle = 0.8\,|1\rangle + 0.2\,|2\rangle$ $\qquad$ $|\Phi_2\rangle = 0.2\,|1\rangle + 0.8\,|2\rangle$

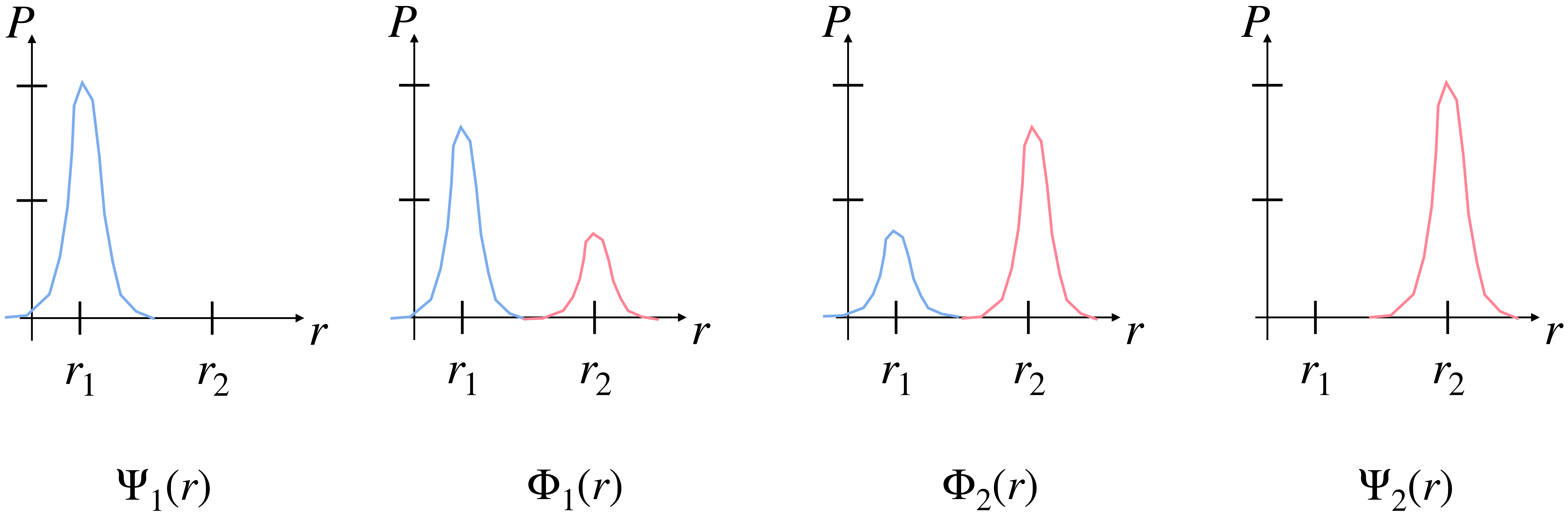



$|\Psi_1\rangle = |1\rangle$ $\qquad$ $|\Phi_1\rangle$ $\qquad$ $|\Phi_2\rangle$ $\qquad$ $|\Psi_2\rangle = |2\rangle$

# Probability

## And its representation. Continuous Case

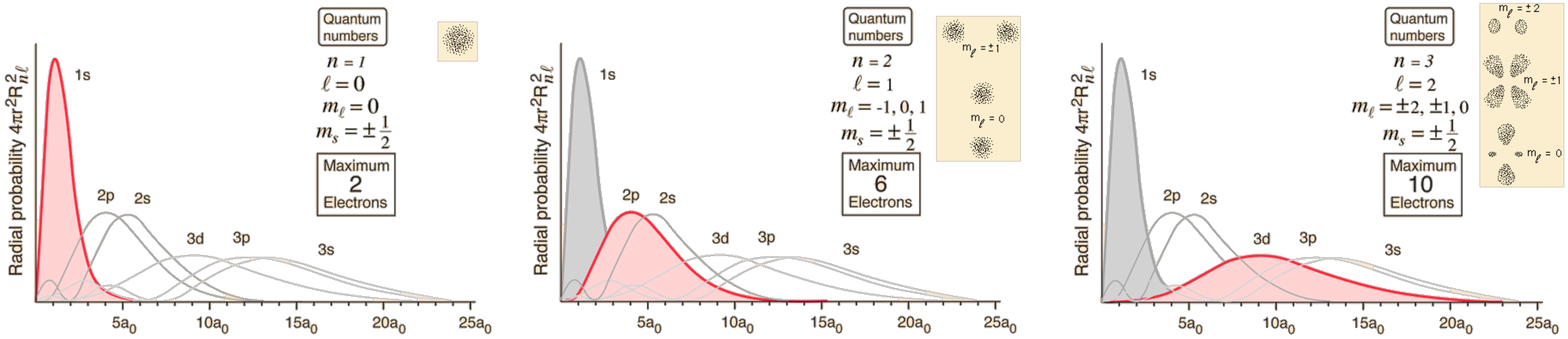
**State Functions $f(r)$. (NOT ~~Wave functions~~)**



# Probability

## Charge Density in Hydrogen Atom

**State Functions $f(r)$. (NOT ~~Wave functions~~)**

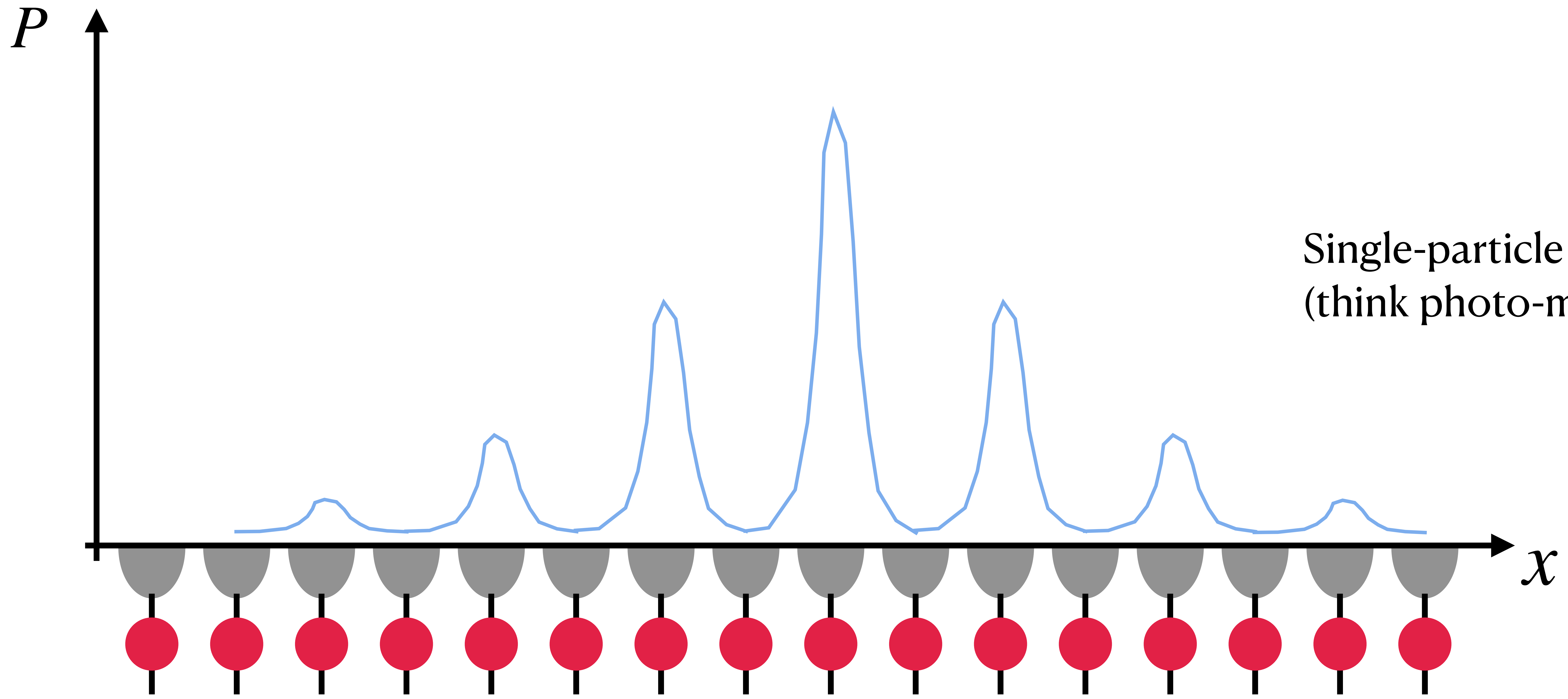


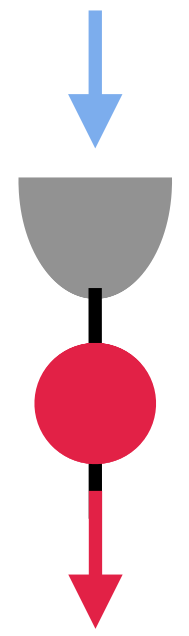

These are measured by scattering charged particles from atoms.

# Position Measurement

## Using Simple Detectors

**State Functions $\Psi(x)$. (NOT ~~Wave functions~~)**



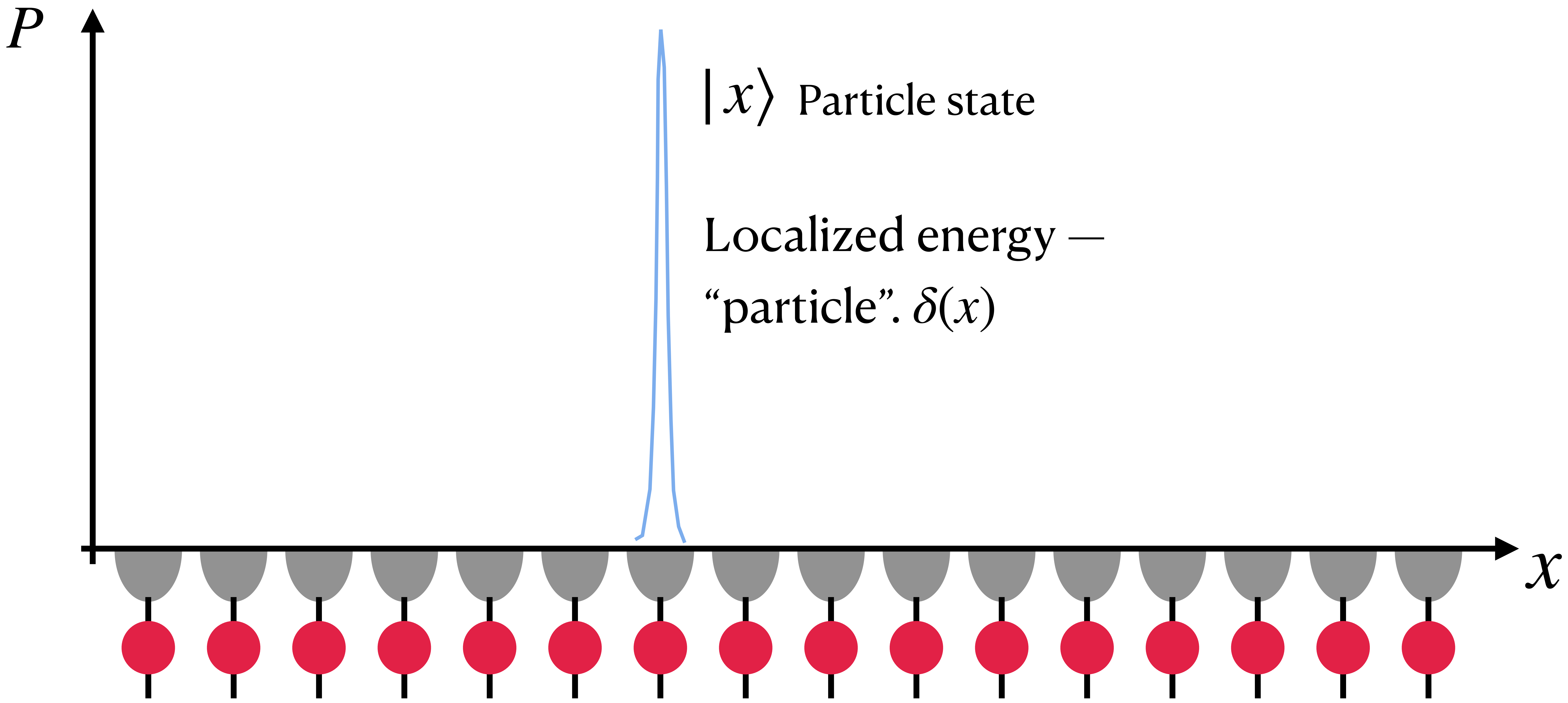



Single-particle detector
(think photo-multiplier tube)

# Position Measurement

## Using Simple Detectors

**“Special” (for humans) State Functions $\Psi(x)$.**



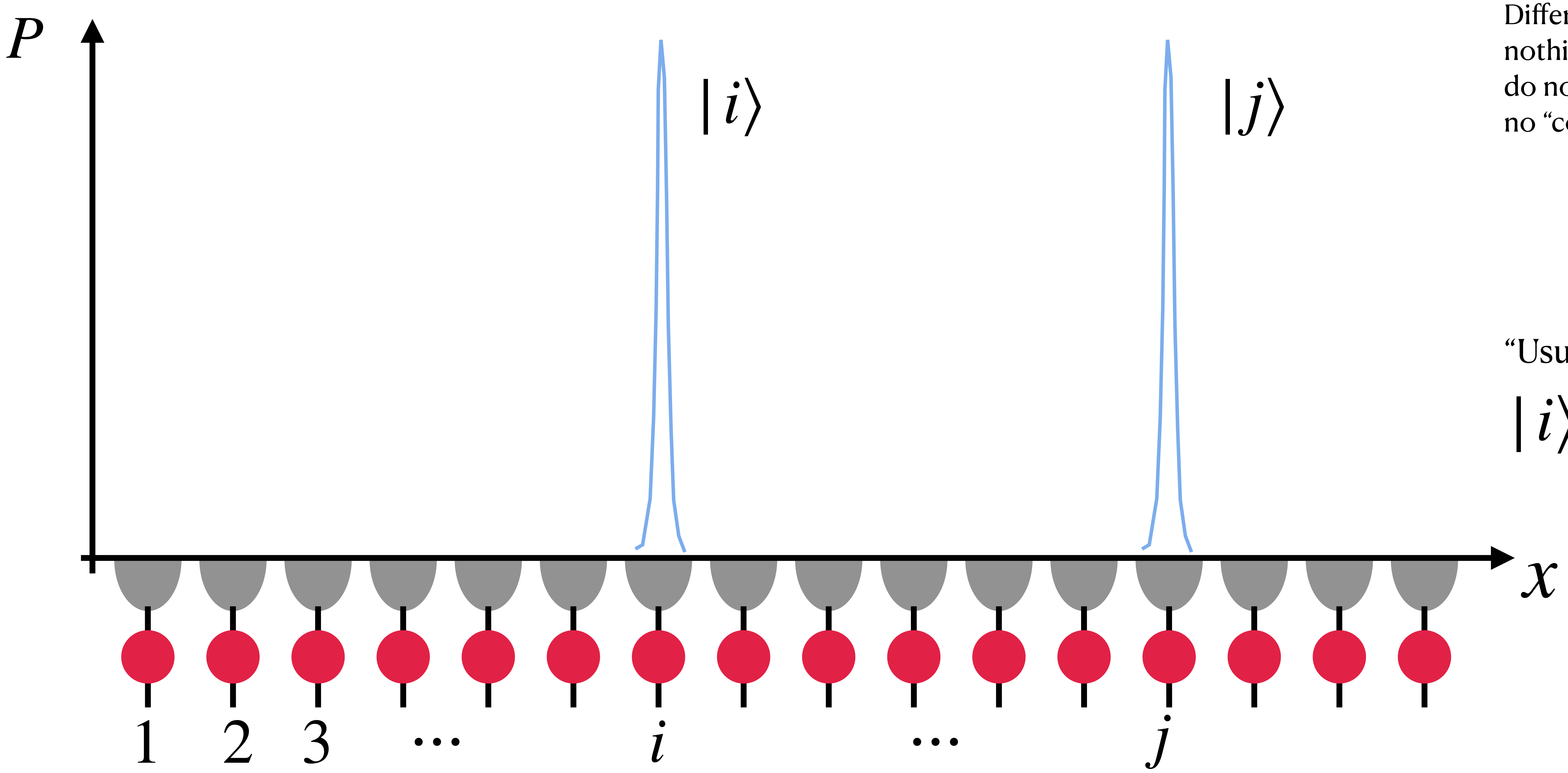

$|x\rangle$ Particle state

Localized energy — “particle”. $\delta(x)$

# Position Measurement

## Using Simple Detectors

### Different Results Do NOT Overlap




$$\widehat{olp}\ |i\rangle\, |j\rangle = 0$$

Different states have nothing in common. They do not overlap and Share no "common information"

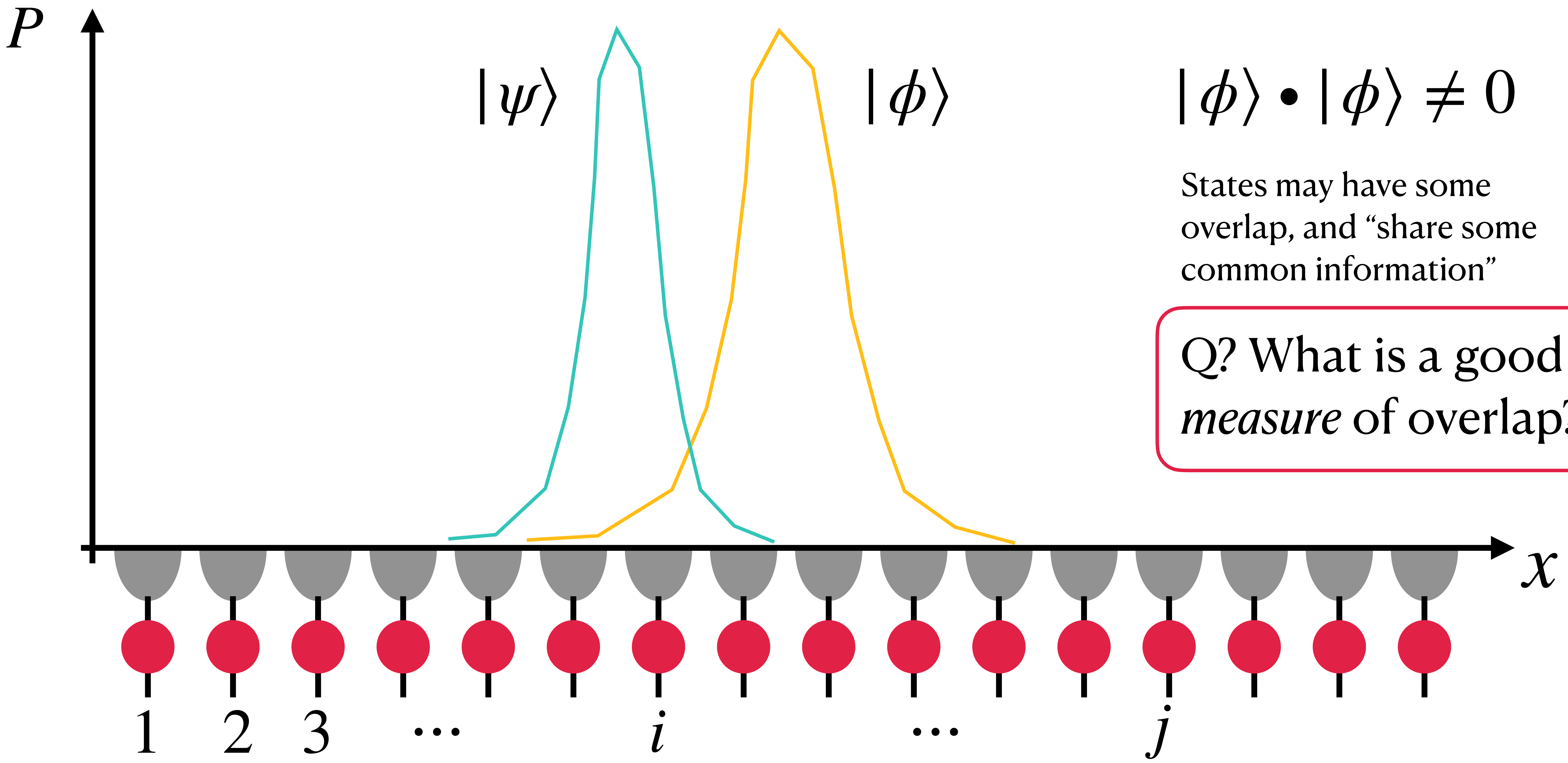
"Usual" infix notation

$$|i\rangle \bullet |j\rangle = 0$$

# Position Measurement

## Using Simple Detectors

### Different Results Do NOT Overlap




$|\phi\rangle \bullet |\phi\rangle \neq 0$

States may have some overlap, and "share some common information"

Q? What is a good *measure* of overlap?

# State Vectors

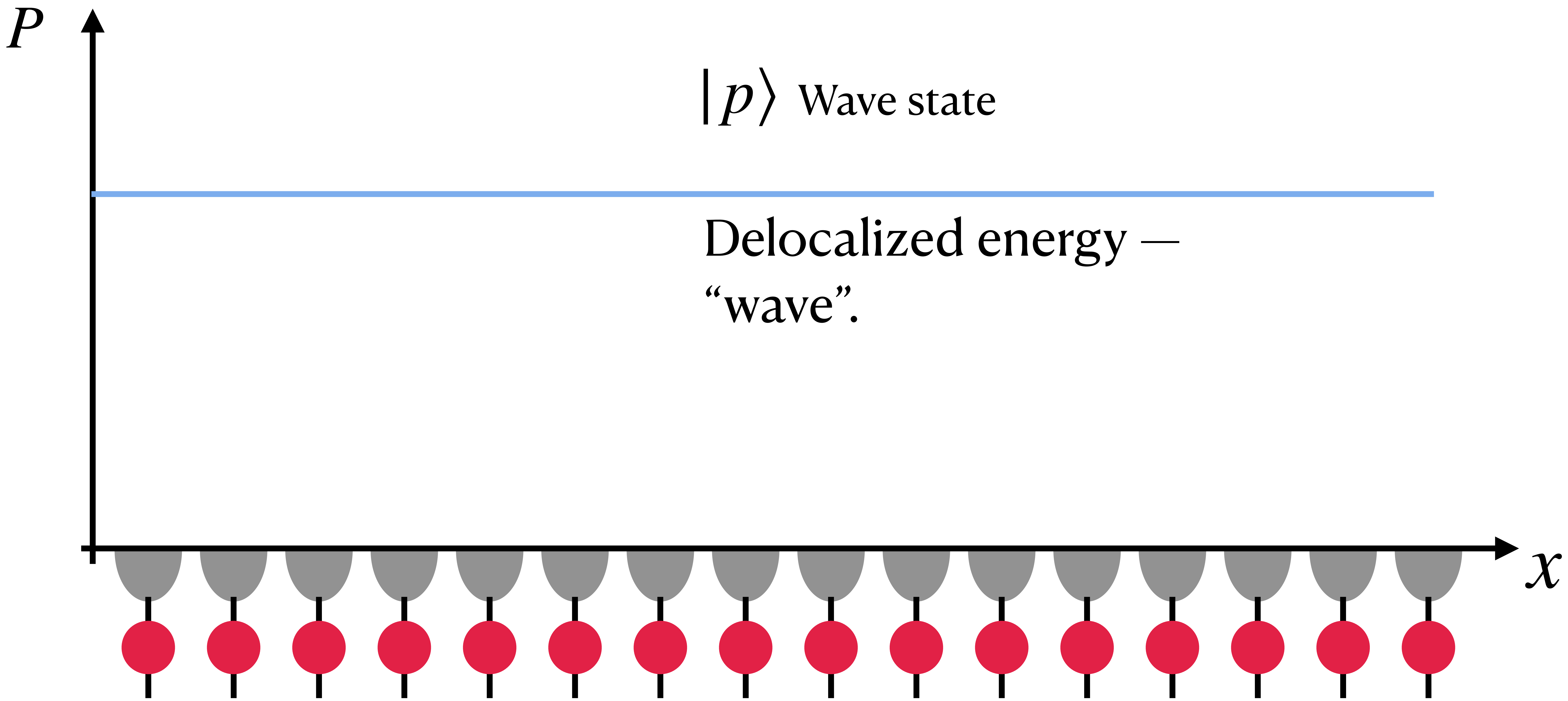
## And Their Comparison

$$|1\rangle \longrightarrow |\Phi_1\rangle \longrightarrow |\Phi_3\rangle \longrightarrow |\Phi_2\rangle \longrightarrow |2\rangle$$

# Position Measurement

## Using Simple Detectors

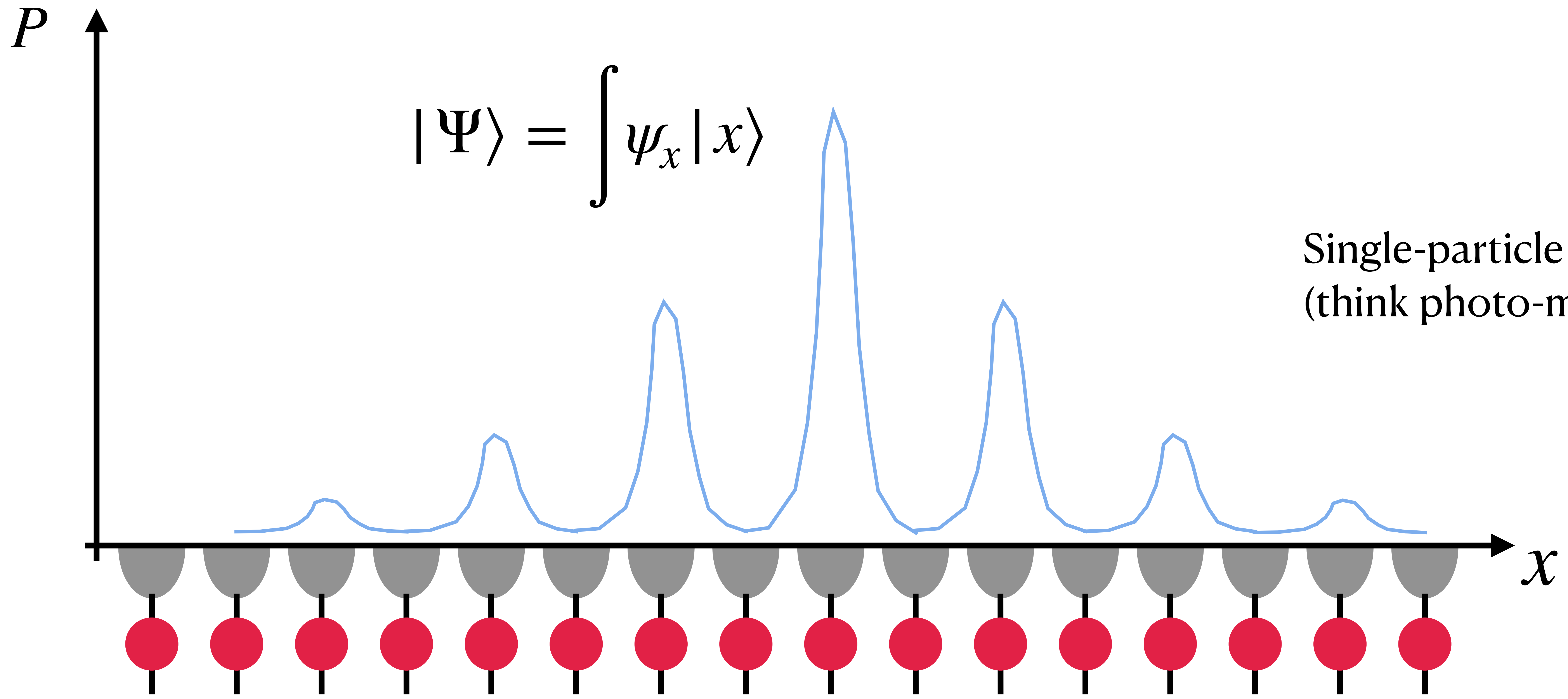
**“Special” (for humans) State Functions $\Psi(x)$.**

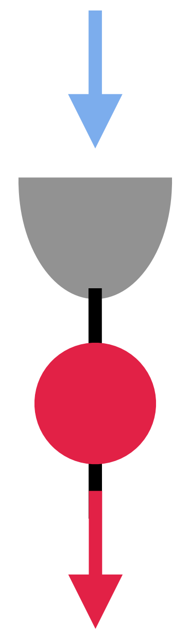


# Position Measurement

## Using Simple Detectors

**State Functions $\Psi(x)$. (NOT ~~Wave functions~~)**



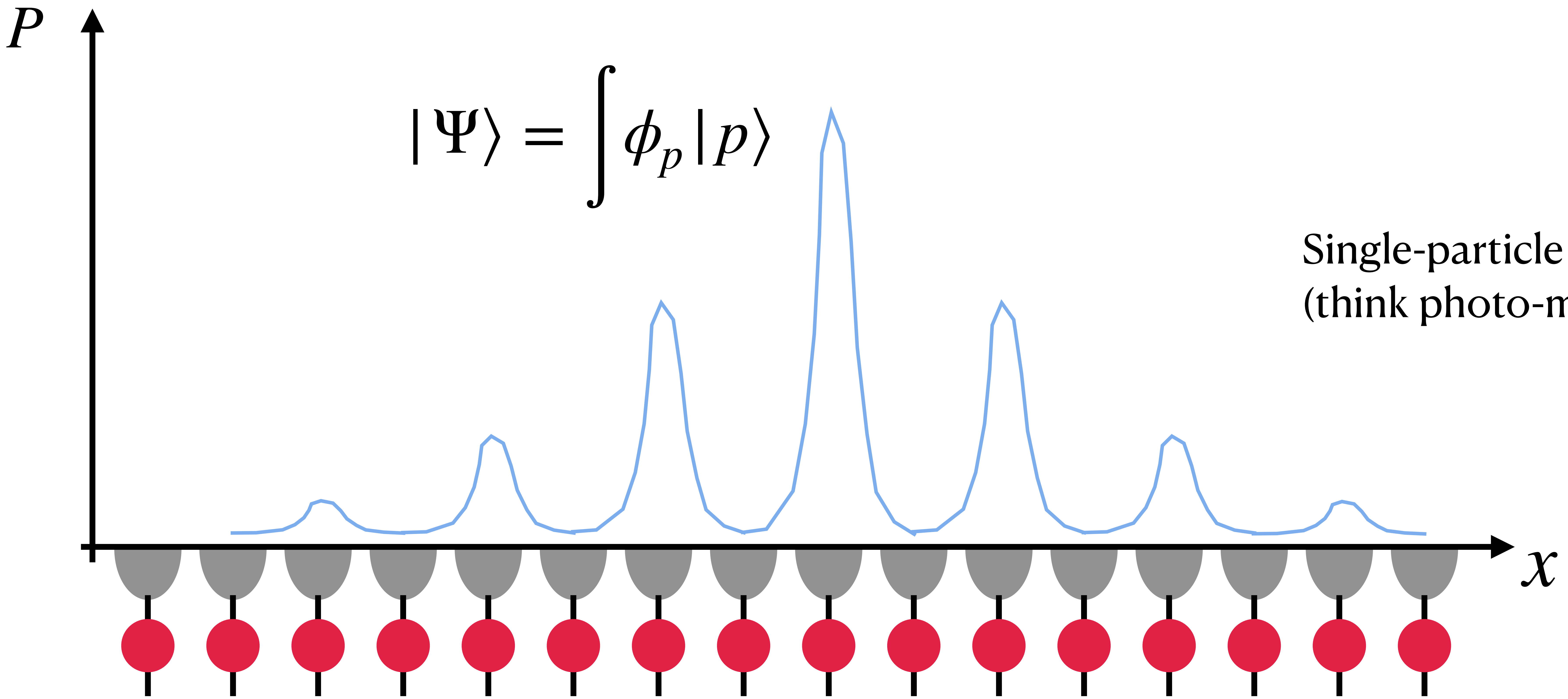
Single-particle detector
(think photo-multiplier tube)

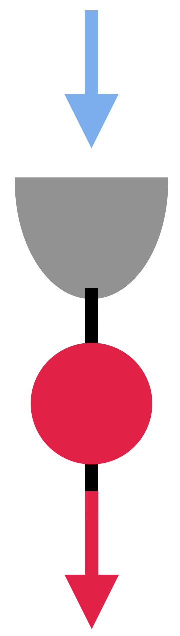
# Position Measurement

## Using Simple Detectors

**State Functions $\Psi(x)$. (NOT ~~Wave functions~~)**

$$|\Psi\rangle = \int \phi_p |p\rangle$$



Single-particle detector
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# What $|\Psi\rangle$ IS

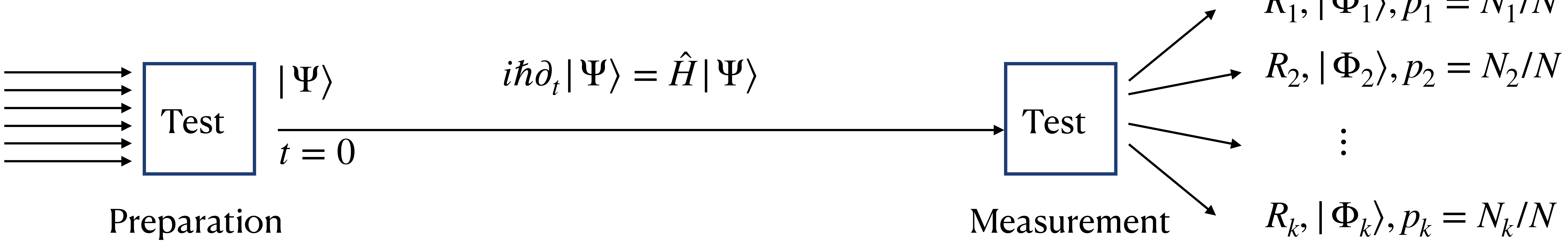
## Operational Viewpoint

State is a mathematical /conceptual **tool** that 1) captures the information about how the system was prepared (what happened) and 2) what the future for any measurement will be (what will happen).

It says nothing about "now", what is happening now to an isolated system between the tests. This is the job of Schrödinger equation.

In certain sense, $|\Psi\rangle$ is the *"experimenter's notebook."*

$$\boxed{|\Psi\rangle = p_1|\Phi_1\rangle + p_2|\Phi_2\rangle + \cdots p_k|\Phi_k\rangle}$$



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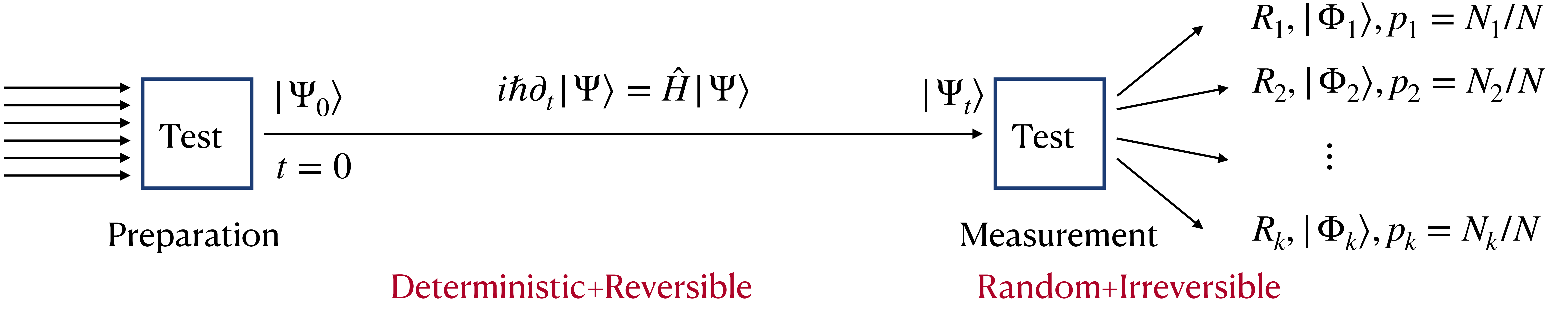


Possible outcomes.

$|\Psi\rangle$ precisely predicts all $p_k$.

# What $|\Psi\rangle$ is NOT

# State Vector

## Is NOT a Field

It **is not** a “physical object" that is spread out in space, propagates through space, or scatters from matter. It **is not** like EMF.

It **is not** a wave. “Wave function” **is not** a good phrase.

# State Vector

**Can Only Be Measured on Many Identical Systems**

A single measurement **does not** reveal $|\Psi\rangle$

# State Vector

**Does not always apply**

In many important cases **we can't** write $|\Psi\rangle$ for a system (composite, "multi-part" system with entangled parts.)

$$|\Psi_{2photons}\rangle \neq |\Psi_{1photon}\rangle \otimes |\Psi_{1photon}\rangle$$

We might have complete knowledge about a **total** system, but NOT about each "**part**".

# Summation

## Bernoulli Notation

... Atque si porrò ad altiores gradatim potestates pergere, levique negotio sequentem adornare laterculum licet :

*Summae Potestatum*

$\int n = \frac{1}{2}nn + \frac{1}{2}n$
$\int nn = \frac{1}{3}n^3 + \frac{1}{2}nn + \frac{1}{6}n$
$\int n^3 = \frac{1}{4}n^4 + \frac{1}{2}n^3 + \frac{1}{4}nn$
$\int n^4 = \frac{1}{5}n^5 + \frac{1}{2}n^4 + \frac{1}{3}n^3 - \frac{1}{30}n$
$\int n^5 = \frac{1}{6}n^6 + \frac{1}{2}n^5 + \frac{5}{12}n^4 - \frac{1}{12}nn$
$\int n^6 = \frac{1}{7}n^7 + \frac{1}{2}n^6 + \frac{1}{2}n^5 - \frac{1}{6}n^3 + \frac{1}{42}n$
$\int n^7 = \frac{1}{8}n^8 + \frac{1}{2}n^7 + \frac{7}{12}n^6 - \frac{7}{24}n^4 + \frac{1}{12}nn$
$\int n^8 = \frac{1}{9}n^9 + \frac{1}{2}n^8 + \frac{2}{3}n^7 - \frac{7}{15}n^5 + \frac{2}{9}n^3 - \frac{1}{30}n$
$\int n^9 = \frac{1}{10}n^{10} + \frac{1}{2}n^9 + \frac{3}{4}n^8 - \frac{7}{10}n^6 + \frac{1}{2}n^4 - \frac{1}{12}nn$
$\int n^{10} = \frac{1}{11}n^{11} + \frac{1}{2}n^{10} + \frac{5}{6}n^9 - 1n^7 + 1n^5 - \frac{1}{2}n^3 + \frac{5}{66}n$

Quin imò qui legem progressionis inibi attentuis ensperexit, eundem etiam continuare poterit absque his ratiociniorum ambabimus : Sumtâ enim c pro potestatis cujuslibet exponente, fit summa omnium $n^c$ seu

$$\int n^c = \frac{1}{c+1}n^{c+1} + \frac{1}{2}n^c + \frac{c}{2}An^{c-1} + \frac{c \cdot c-1 \cdot c-2}{2 \cdot 3 \cdot 4}Bn^{c-3}$$
$$+\frac{c \cdot c-1 \cdot c-2 \cdot c-3 \cdot c-4}{2 \cdot 3 \cdot 4 \cdot 5 \cdot 6}Cn^{c-5}$$
$$+\frac{c \cdot c-1 \cdot c-2 \cdot c-3 \cdot c-4 \cdot c-5 \cdot c-6}{2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8}Dn^{c-7} \cdots \text{\& ita deinceps,}$$

exponentem potestatis ipsius n continué minuendo binario, quosque perveniatur ad $n$ vel $nn$. Literae capitales A, B, C, D & c. ordine denotant coëfficientes ultimorum terminorum pro $\int nn$, $\int n^4$, $\int n^6$, $\int n^8$ , & c. nempe

$$A = \frac{1}{6}, B = -\frac{1}{30}, C = \frac{1}{42}, D = -\frac{1}{30} .$$

$$|\Psi\rangle = \int \psi_x |x\rangle$$

$$|\Psi\rangle = \int \psi_i |i\rangle = \psi_1 |1\rangle + \psi_2 |2\rangle + \cdots$$

$$|\Psi\rangle = \int \psi_x |x\rangle = \int (\delta x f_x) |x\rangle$$

$$\frac{1}{1-q} = \int_i q^i \qquad \frac{1}{1-q} = \sum_{i=0}^{i=\infty} q^i$$

# Single-Particle Detection

## Avalanche Devices

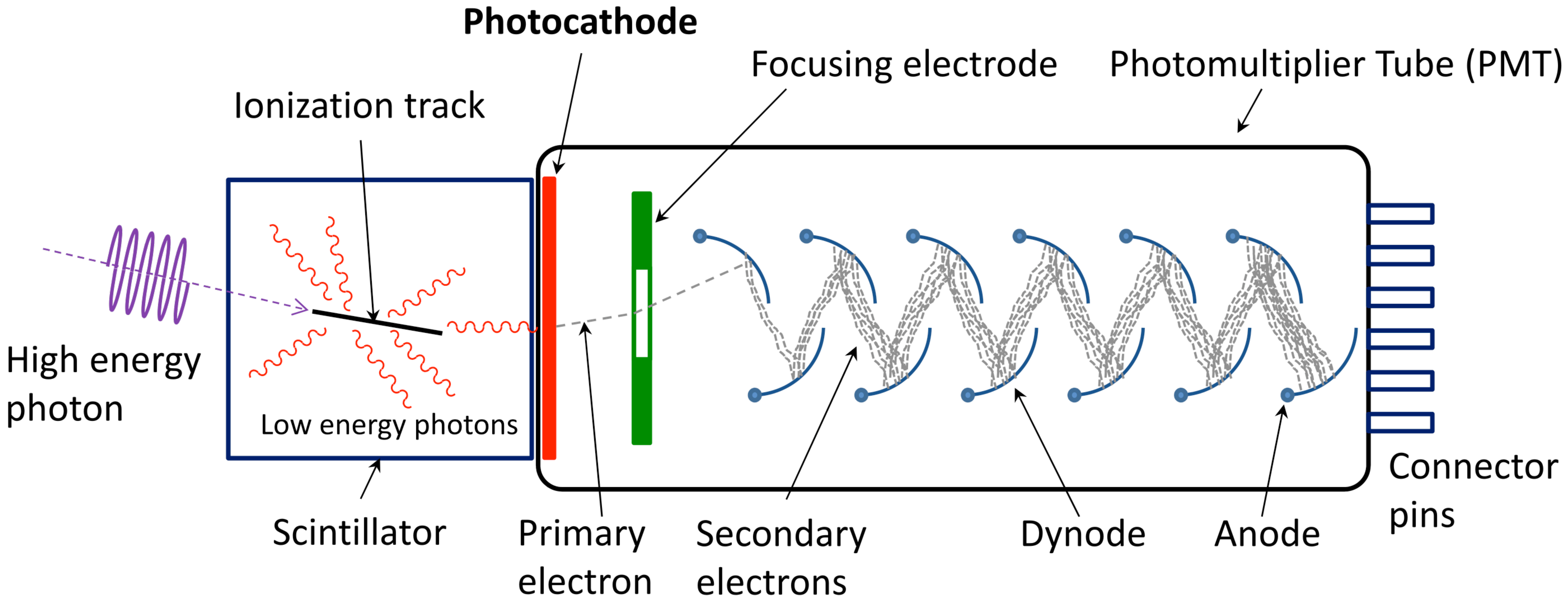


Photo-multiplier tube

# Single-Particle Detection

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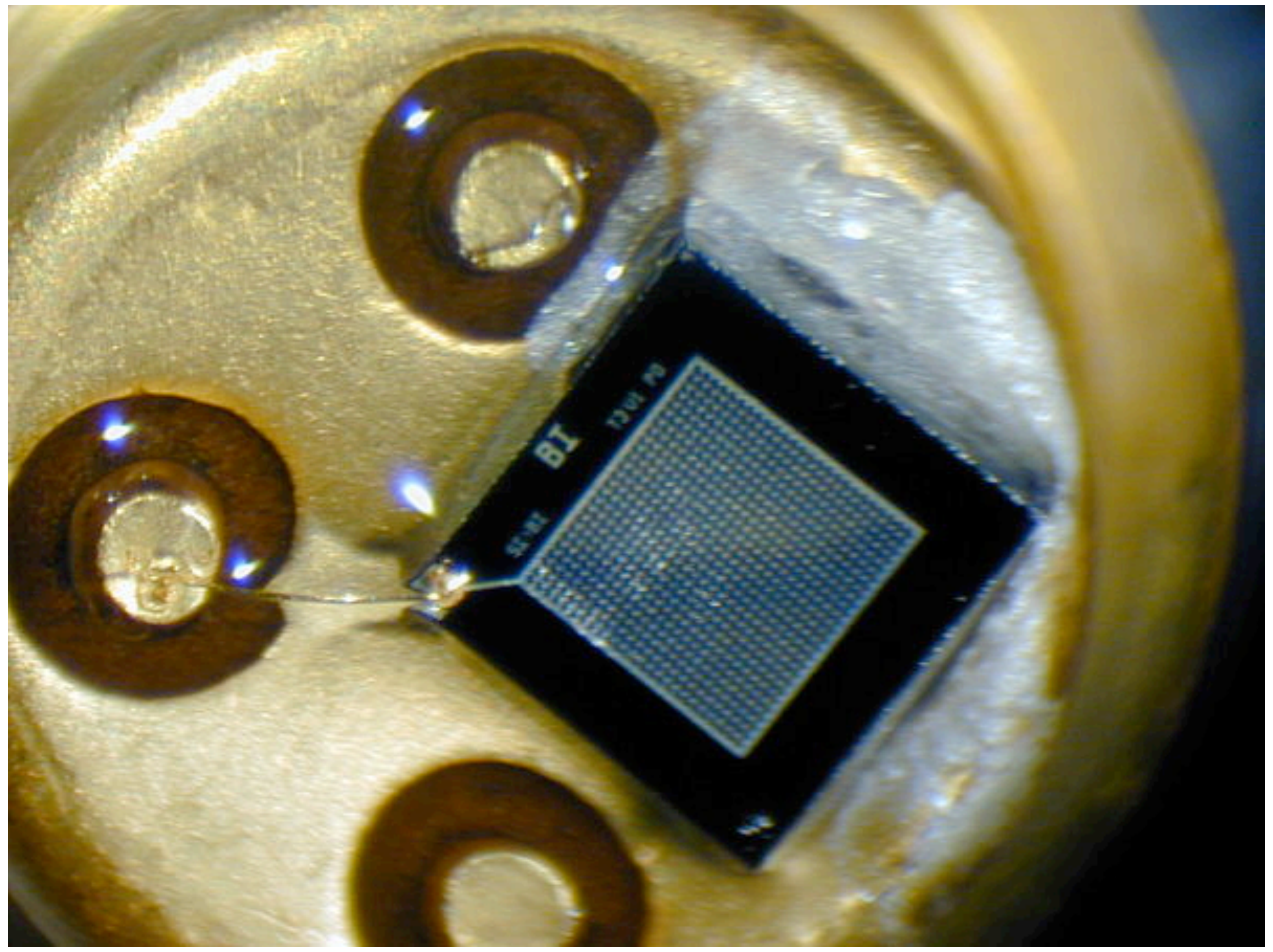

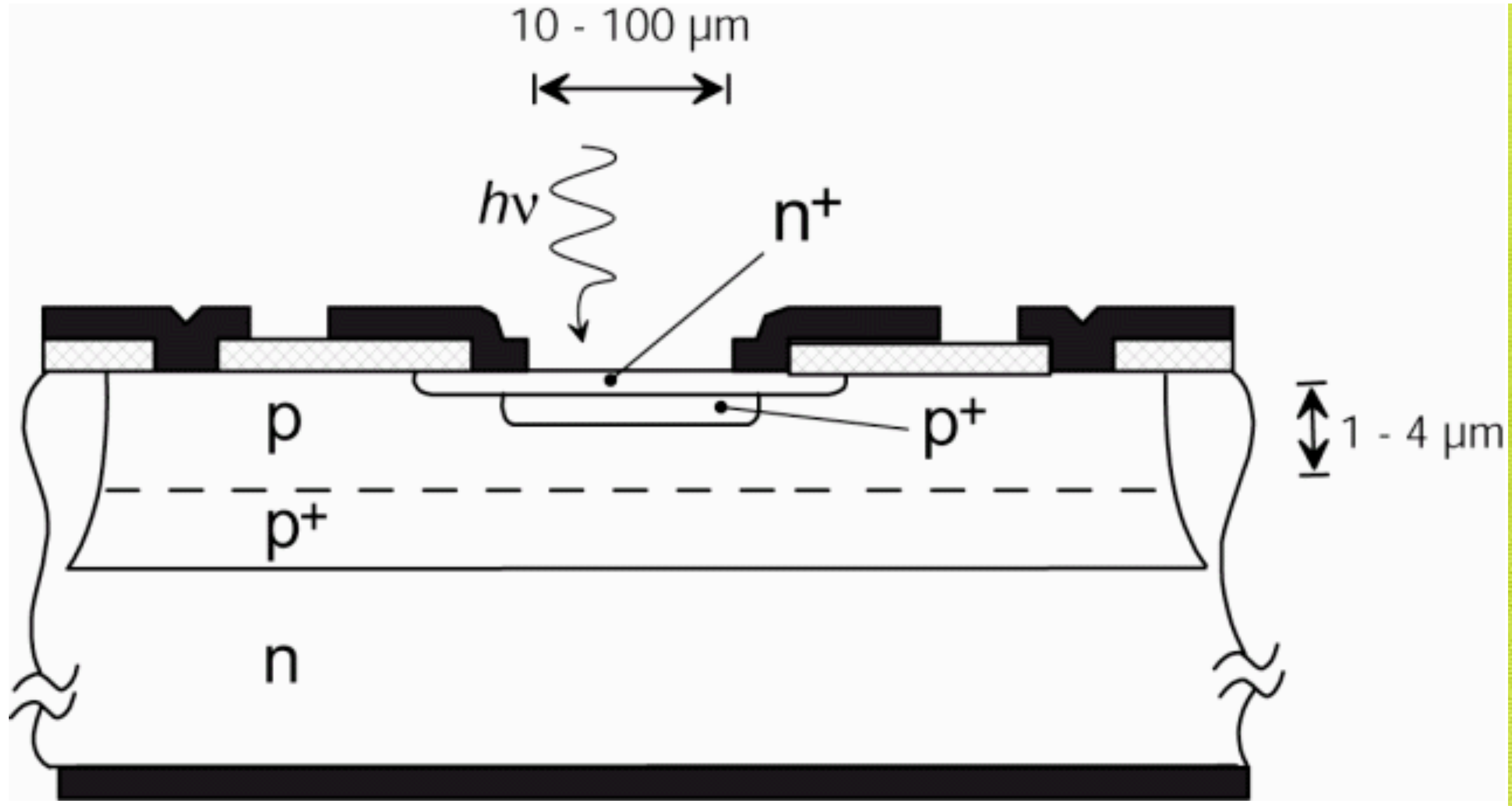



Silicon-based avalanche photodiode

# Single-Particle Detection

## Avalanche Devices

Read about Rutherford-Geiger experiment

# Self-Test

## Answer These Questions 1hr After Class

1. What is the role of the concept of “state” in physics?
2. How is state represented mathematically in Newtonian and Hamiltonian mechanics?
3. How do we obtain the information about a system in physics?
4. What is the difference between a quantum system and a measuring apparatus?
5. What does a measuring device do to a system? What does a system do to the device?
6. What is one way to represent the results of the multiple measurements?
7. What are two basic types of states?
8. Why are the ideas of “probability field” and “wave function” not good?

# Homework Problems

## Mathematical Concepts and Notation Day 3

- Suppose a system in a such a state that $\hat{H}|\Psi_0\rangle = E|\Psi_0\rangle$ — that is, the measured energy is $E$ with 1.0 probability (with certainty). Write down the Schrödinger equation for this case and show that the state changes in time as follows $|\Psi_t\rangle = e^{-iEt/\hbar}|\Psi_0\rangle$.
- Suppose a harmonic oscillator is in such a state that $|\Phi\rangle = 0.7|1\rangle + 0.3|2\rangle$. What is the average energy of harmonic oscillator?
- In the previous problem, do you think it can be that $\hat{H}|\Phi\rangle = E|\Phi\rangle$ for some energy $E$?
- **Advanced**: If the state vector $|\Psi\rangle$ allows different *representations*: $|\Psi\rangle = \int \psi_x |x\rangle$ and $|\Psi\rangle = \int \phi_p |p\rangle$, can you write the relationship between the functions (components) $\phi_p$ and $\psi_x$?
- Watch the video about quantum properties of light (previously recommended/assigned). Learn about photo-multiplying tube (PMT).

# Quantum Theory

## In a Nutshell

### II. POSTULATES FOR QUANTUM MECHANICS

In this paper, all state vectors are supposed to be normalized, and mixed states are represented by density operators, i.e., positive operators with unit trace. Let $A$ be an observable with a nondegenerate purely discrete spectrum. Let $\phi_1, \phi_2, \ldots$ be a complete orthonormal sequence of eigenvectors of $A$ and $a_1, a_2, \ldots$ the corresponding eigenvalues; by assumption, all different from each other.

According to the standard formulation of quantum mechanics, on the result of a measurement of the observable $A$ the following postulates are posed:

(A1) *If the system is in the state $\psi$ at the time of measurement, the eigenvalue $a_n$ is obtained as the outcome of measurement with the probability $|\langle\phi_n|\psi\rangle|^2$.*

(A2) *If the outcome of measurement is the eigenvalue $a_n$, the system is left in the corresponding eigenstate $\phi_n$ at the time just after measurement.*

The postulate (A1) is called the *statistical formula*, and (A2) the *measurement axiom*. The state change $\psi \mapsto \phi_n$ described by the measurement axiom is called the *state reduction*.

You will understand this paragraph in the end of the course.