Quantum Physics

2024

The Theory/Framework Of *Almost* Everything *Today*

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Phase Difference

Part A

$$|a\rangle = a_1|e_1\rangle + a_2|e_2\rangle$$

Analogy with vectors, but why?

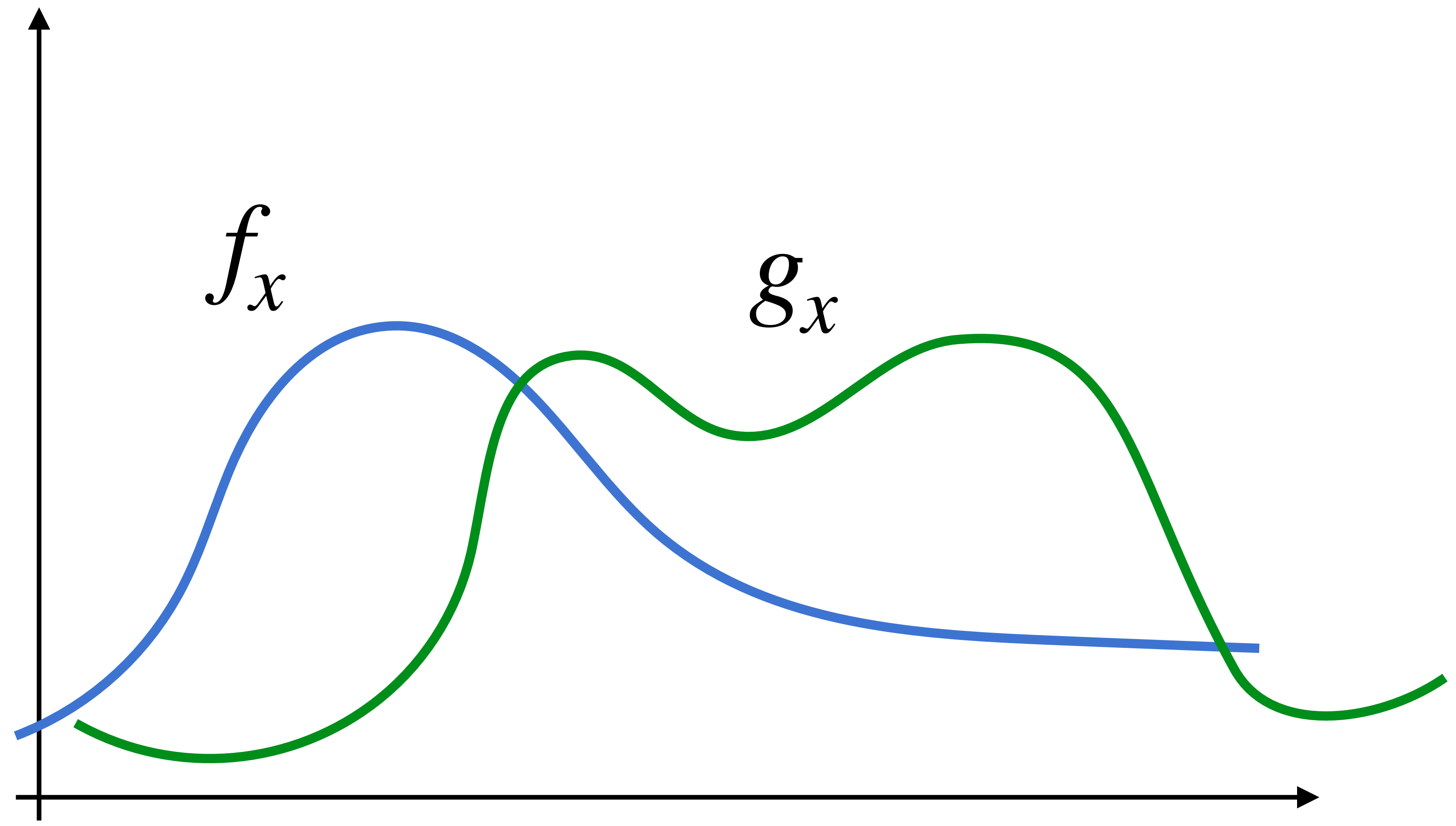
Simplest (linear) first guess:

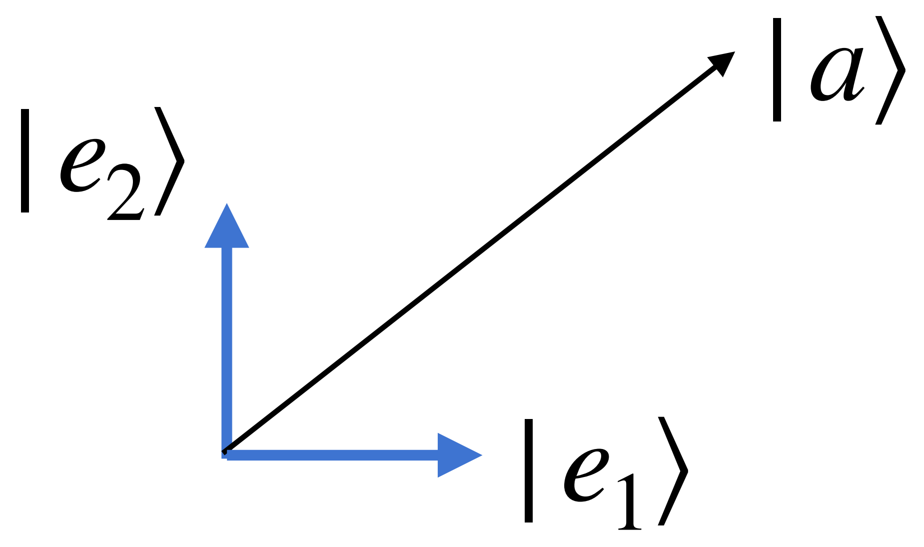
$$\boxed{|\Psi_t\rangle = p_1|\Phi_1\rangle + p_2|\Phi_2\rangle + \cdots + p_k|\Phi_k\rangle}$$

$$p_1 + p_2 + \cdots + p_k = 1$$

Functions Are Vectors

Not All, But Some Important Classes





$f + g$ Can be added to create a new function. Can multiplied by a number.

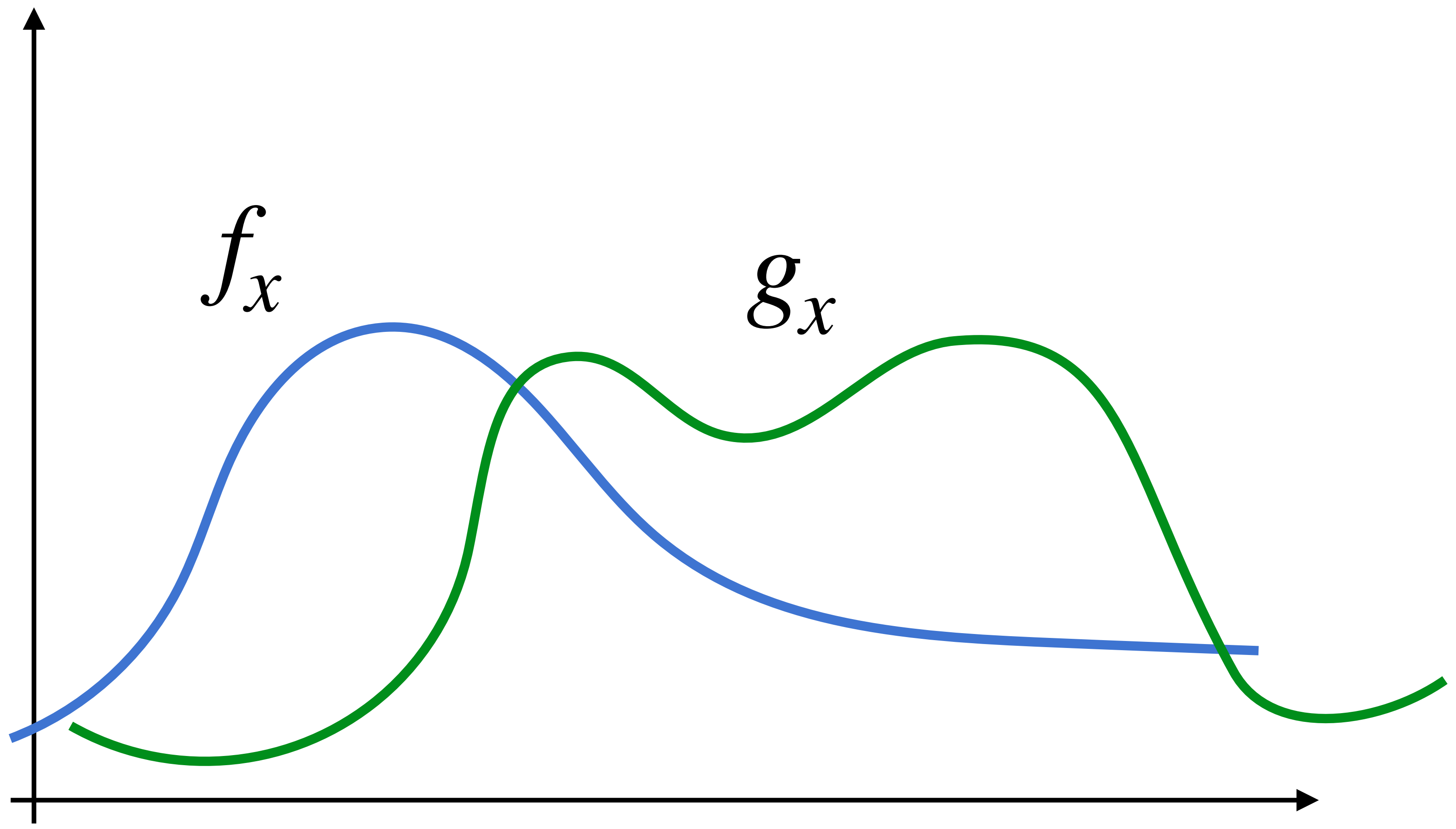
$3f$

Are there bases for functions?

Yes!

Functions Are Vectors

Two Most Often Used Bases



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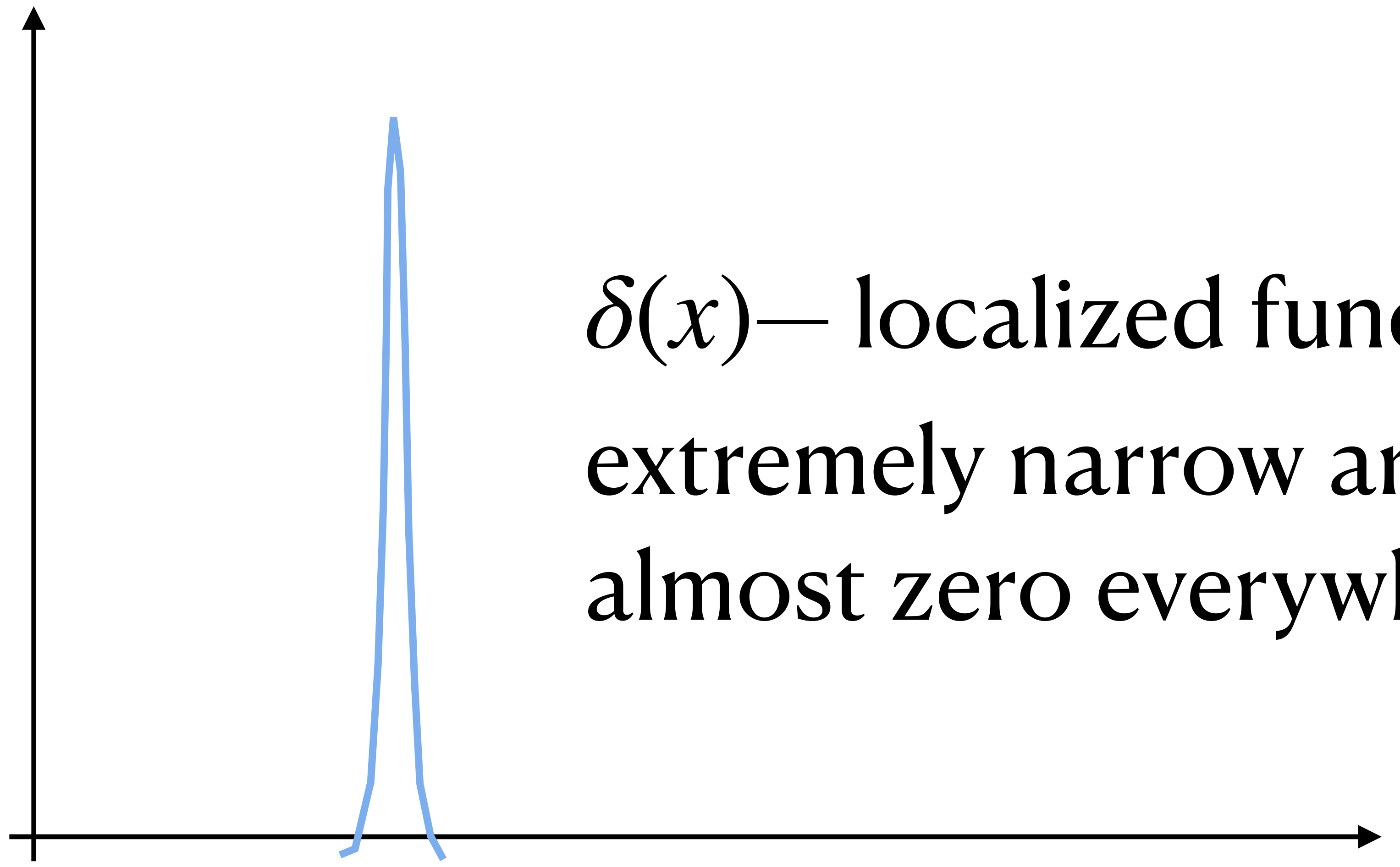
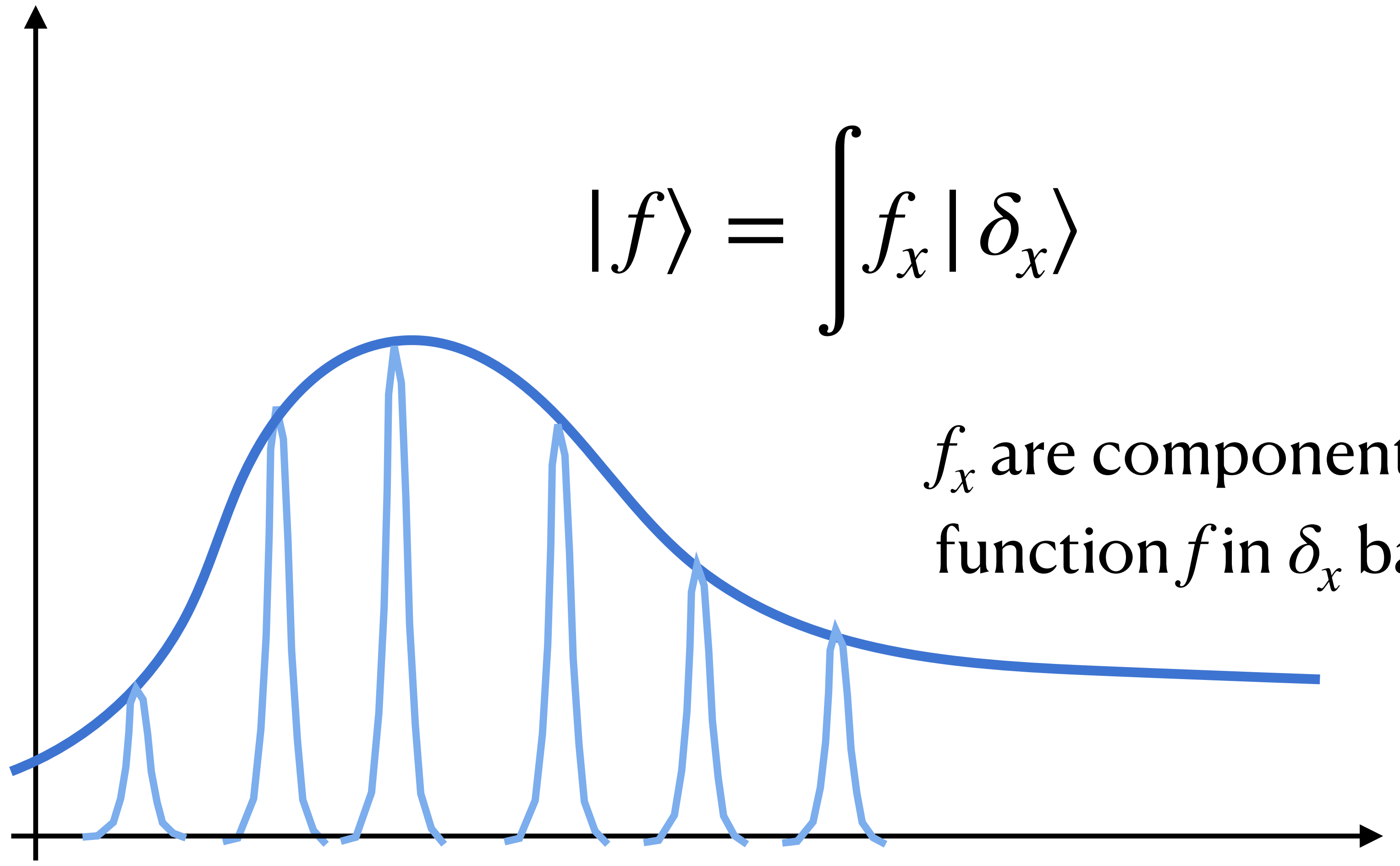


$\delta(x)$— localized functions — extremely narrow around x, almost zero everywhere else.

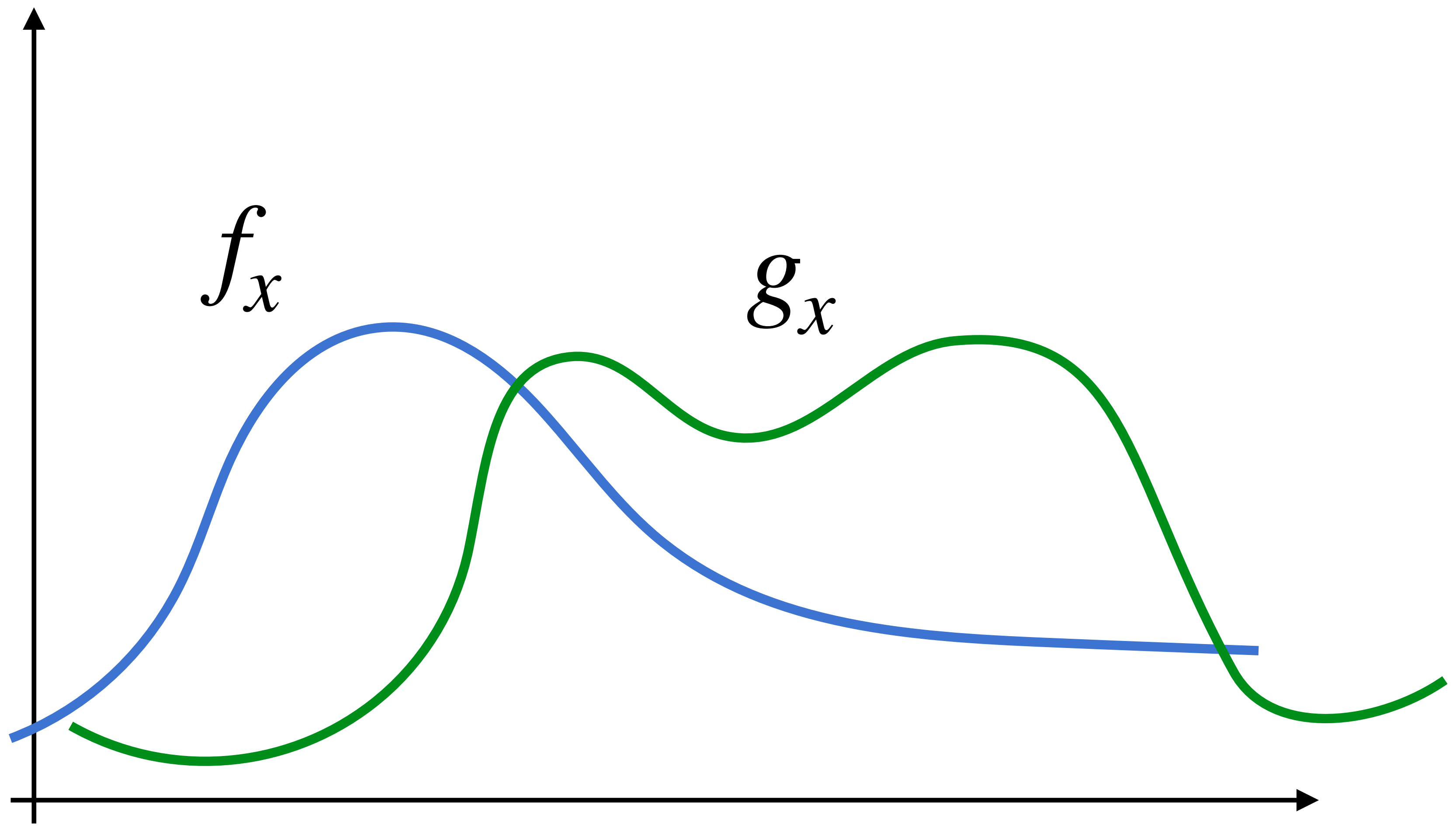


$$|f\rangle = \int f_x \,|\, \delta_x\rangle$$

f_x are components of the function f in δ_x basis.

Functions Are Vectors

Two Most Often Used Bases



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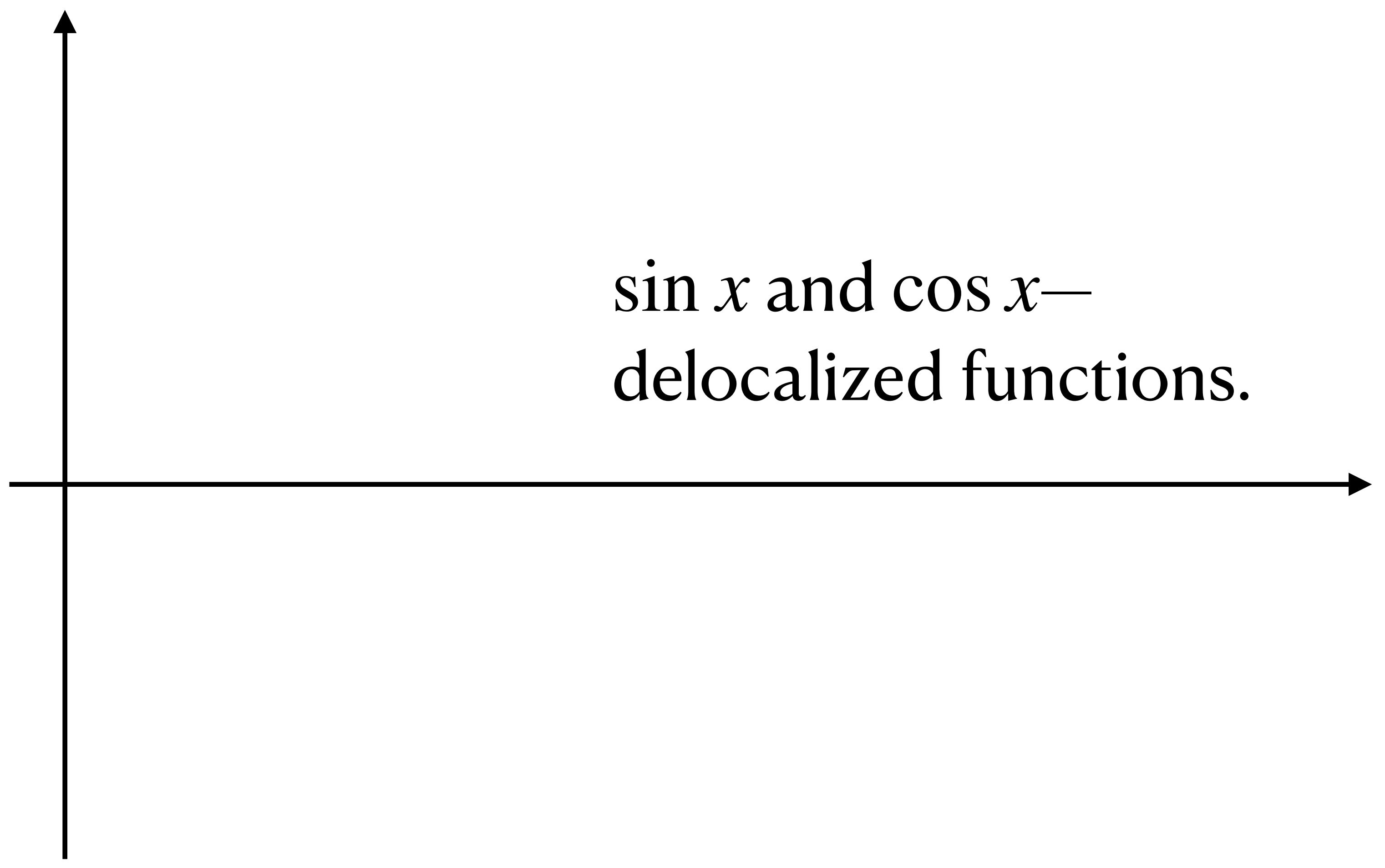


$$f_x = \int e^{\hat{J}px} f_p$$

$$|f\rangle = \int f_p \,|p\rangle$$

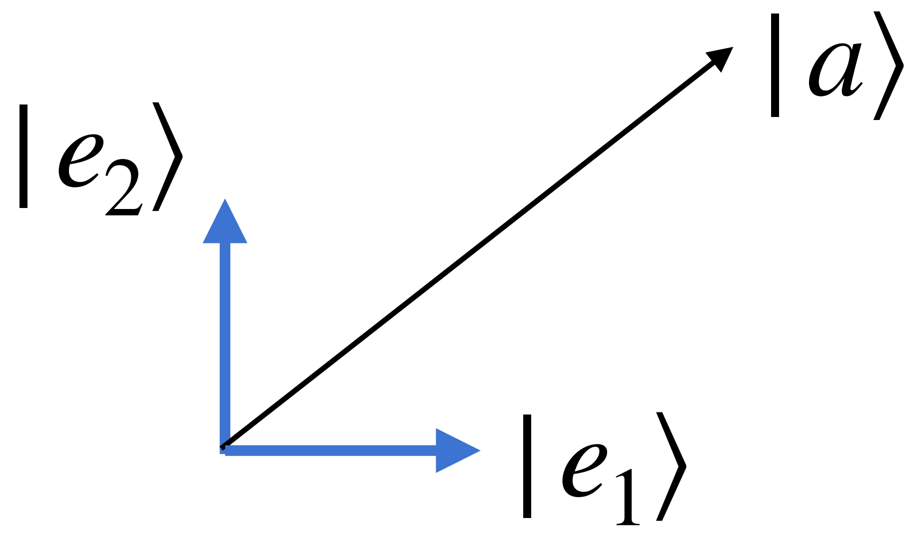
Fourier transform.

Many other “bases” and transforms are possible and used in mathematics.

f_p are components of the function f in sin cos basis.

Linear Algebra

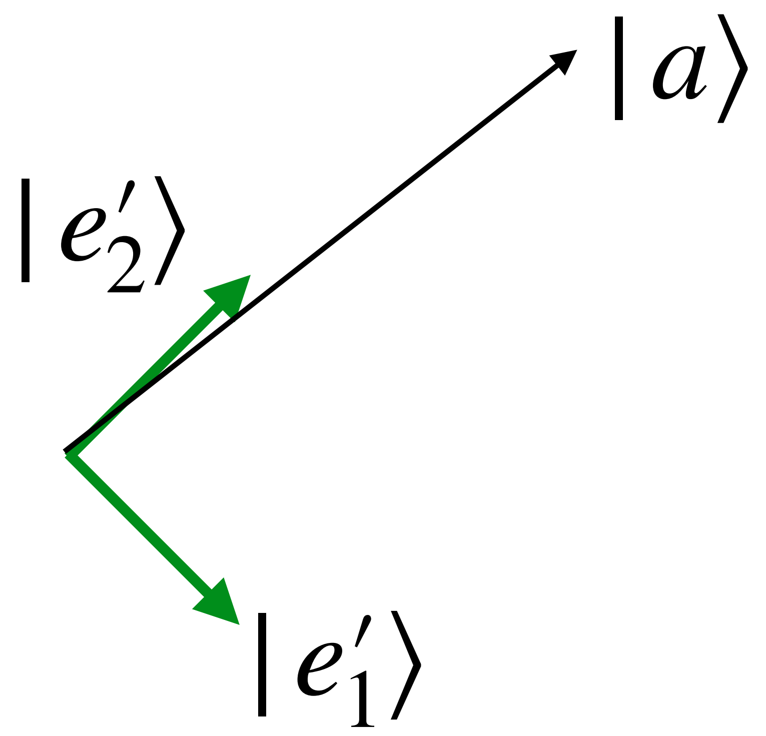
Algebra of Vectors and (simple) Linear Operators



$$|a\rangle$$

$$|a\rangle = a_1|e_1\rangle + a_2|e_2\rangle \qquad\qquad |a\rangle = a_1'|e_1'\rangle + a_2'|e_2'\rangle$$

Representation of *the same* vector $|a\rangle$ in different bases

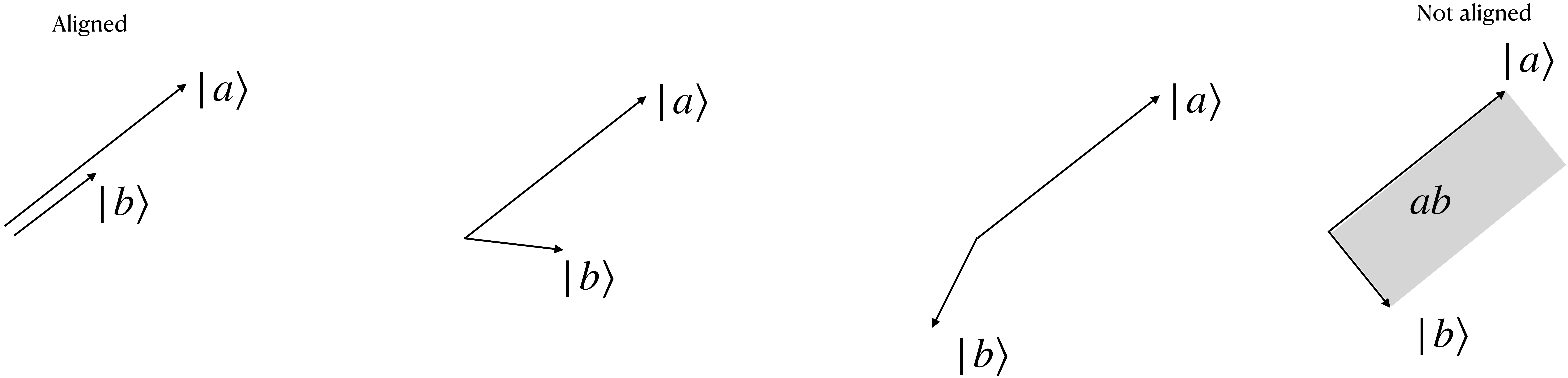


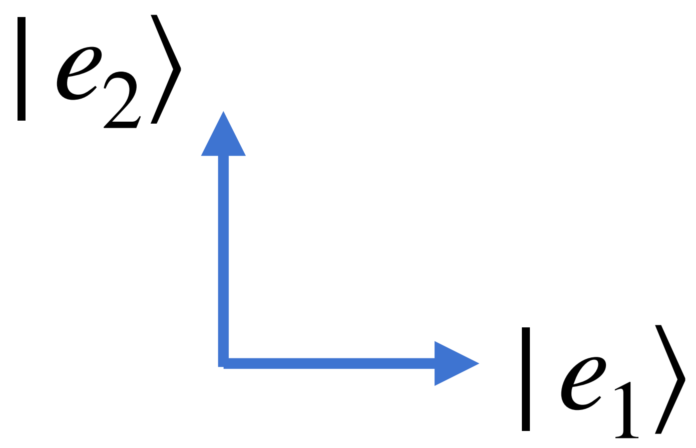
$$|a\rangle \longrightarrow (a_1, a_2)$$

Abstract vector object → Concrete numbers

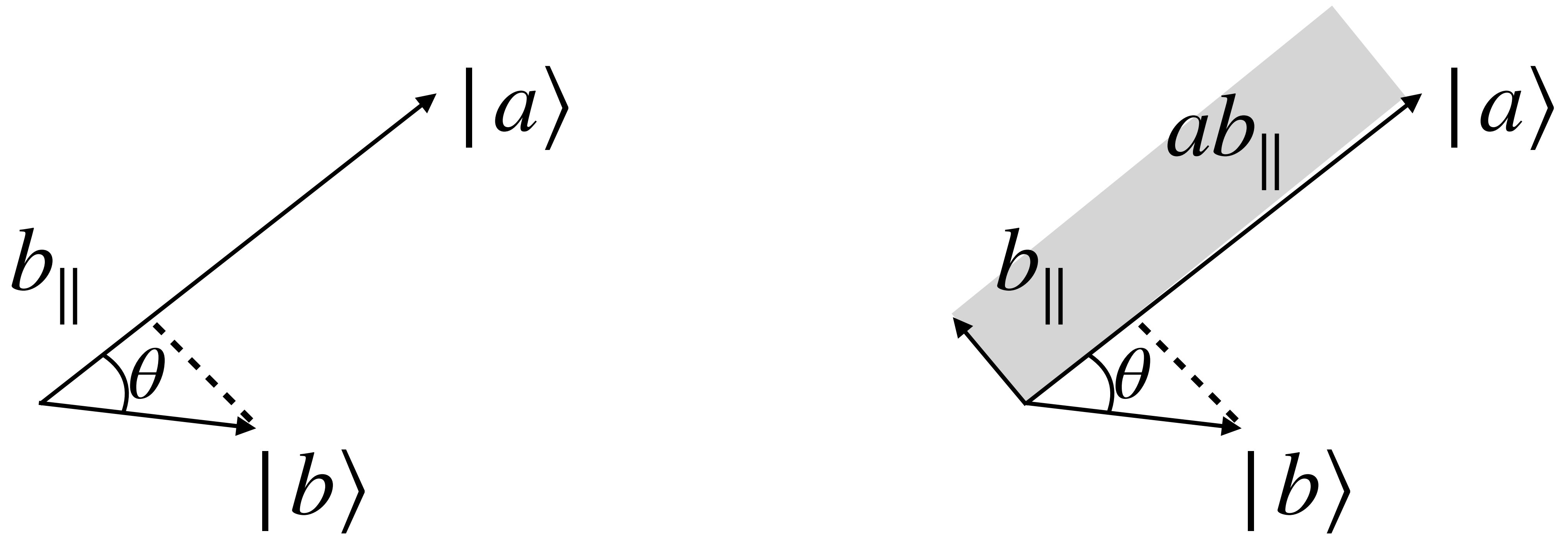
Comparing Vectors

The Measure of Alignment





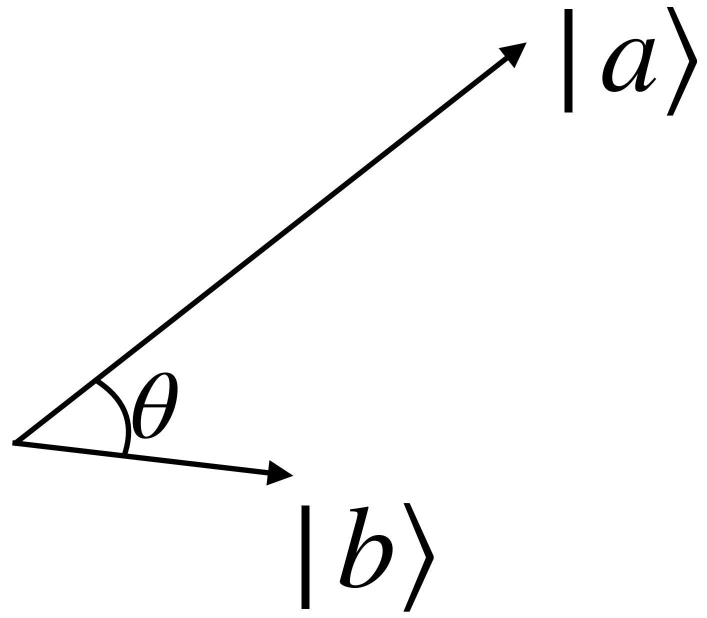
$\hat{\bullet}\,|e_1\rangle\,|e_2\rangle = |e_1\rangle \bullet |e_2\rangle = 0$



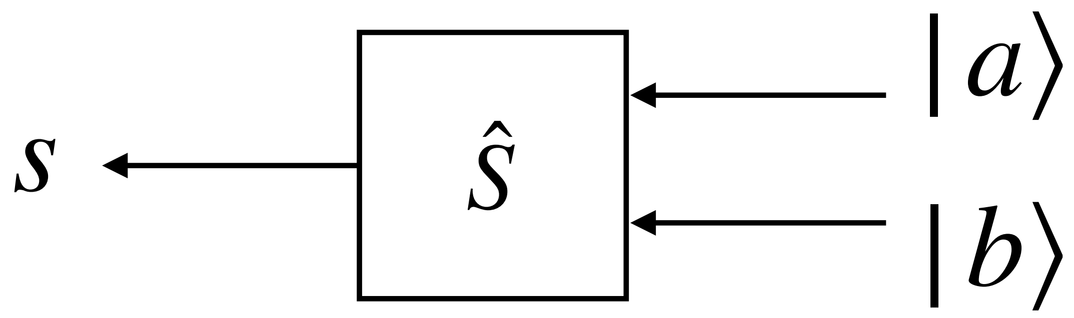
$$\boxed{ab_{\parallel} = ab\cos\theta}$$

Scalar Product

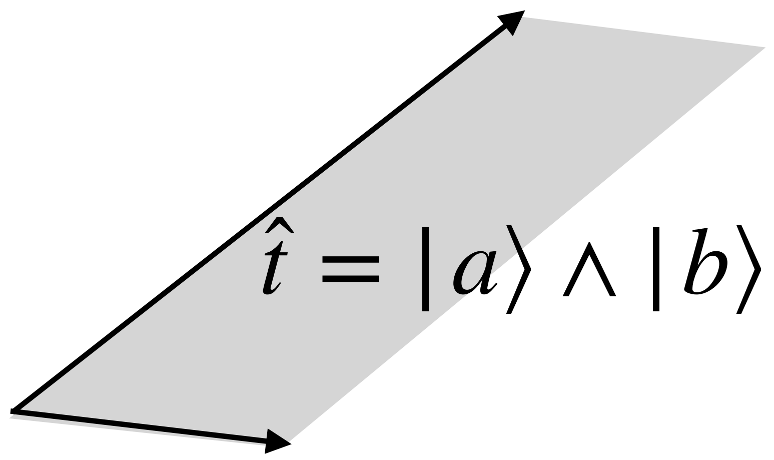
Simplest Numeric Measure Of Alignment

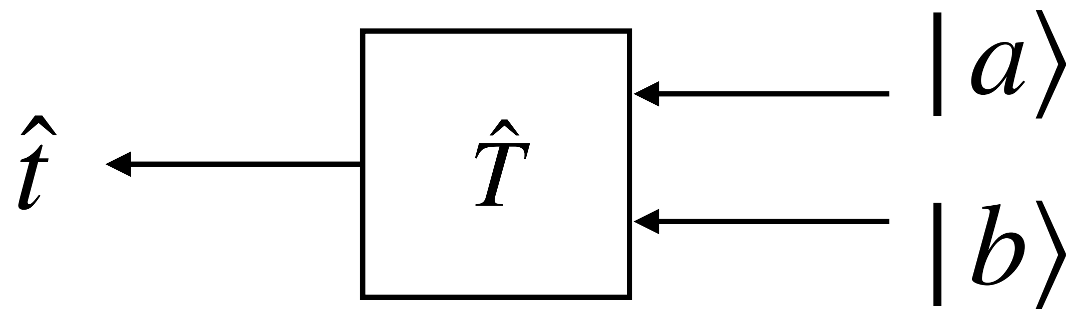


$$|a\rangle \bullet |b\rangle = ab\cos\theta$$



S — Scalar (number) product



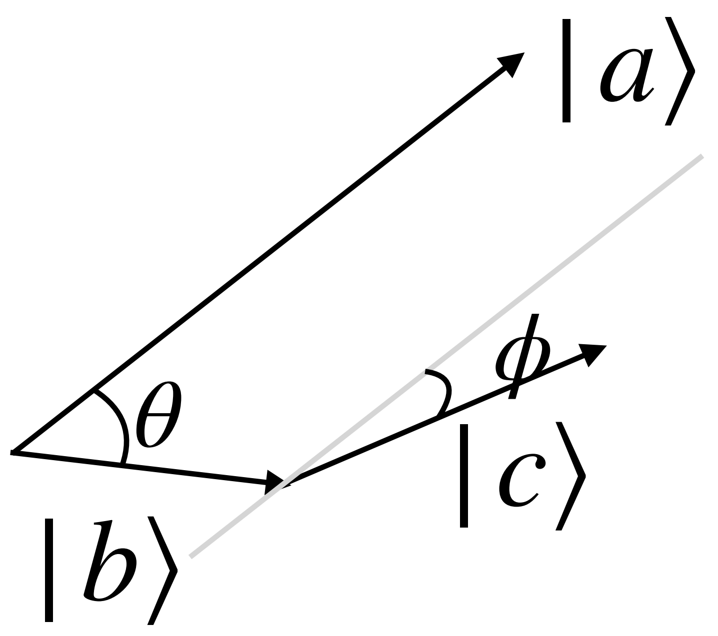


t — tensor (wedge) product

$$t \sim ab\sin\theta$$

Scalar Product

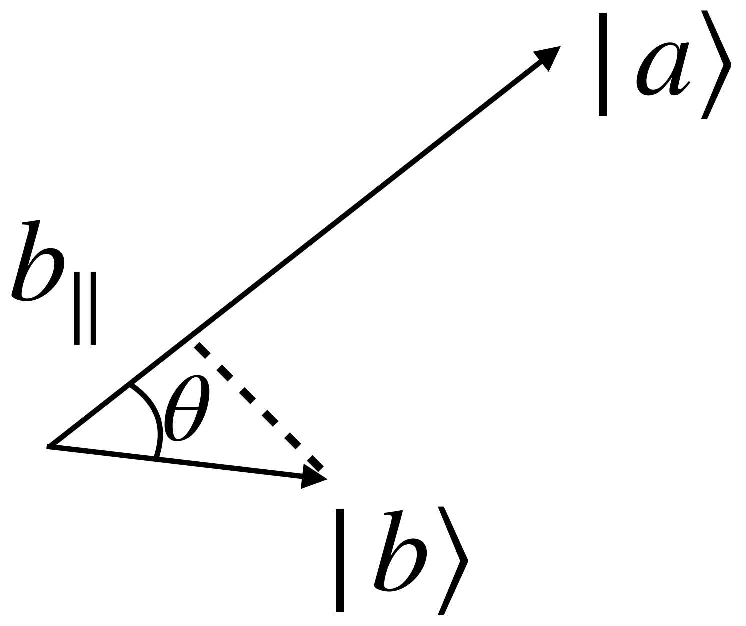
Basic Properties



$$|a\rangle \bullet |b\rangle = ab\cos\theta = |b\rangle \bullet |a\rangle$$

<u>Symmetric —</u> order of the arguments does not matter.

Sort of like $x * y$, thus “product”



$$|a\rangle \bullet (|b\rangle + |c\rangle) = |a\rangle \bullet |b\rangle + |a\rangle \bullet |c\rangle$$

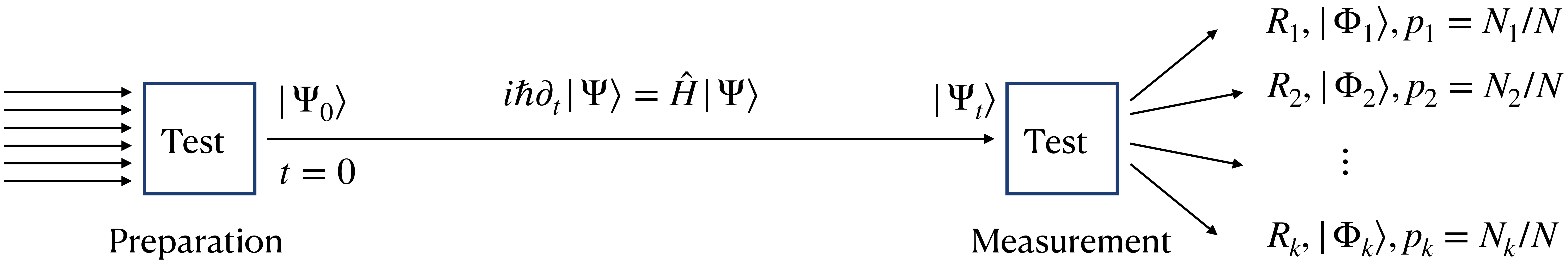
<u>Bilinear —</u> linear in each argument.

Sort of like $x * y$, thus “product”

Exercise: Prove to yourself that this is tru.

Preparation-Evolution-Measurement

State Vector Description



Possible outcomes.

$|\Psi\rangle$ precisely predicts all p_k.

Deterministic+Reversible

Random+Irreversible

R_1 — result from the (classical) measuring device

$|\Phi_1\rangle$ — state of the quantum system after the measurement with result R_1

p_1 — probability (relative frequency N_1/N) for obtaining the result R_1

Simplest (linear) first guess: $\boxed{|\Psi_t\rangle = p_1|\Phi_1\rangle + p_2|\Phi_2\rangle + \cdots + p_k|\Phi_k\rangle}$ $\quad p_1 + p_2 + \cdots + p_k = 1$

Inner Product

Improving Superposition Formula

State Vector

And Its "Length" — Measure of Complete Knowledge

$$|\Psi\rangle = p_1|\Phi_1\rangle + p_2|\Phi_2\rangle + \cdots + p_k|\Phi_k\rangle \qquad p_1 + p_2 + \cdots + p_k = 1$$

$$|\Psi\rangle \bullet |\Psi\rangle = 1\,(100\%)$$ Every state vector contains complete (100 %) knowledge of the system.

$$|\Phi_1\rangle \bullet |\Phi_2\rangle = 0\,(0\%)$$ States for different results are incompatible, they share no "knowledge"/information.

$$|\Psi\rangle \bullet |\Psi\rangle = (p_1|\Phi_1\rangle + p_2|\Phi_2\rangle + \cdots + p_k|\Phi_k\rangle) \bullet ((p_1|\Phi_1\rangle + p_2|\Phi_2\rangle + \cdots + p_k|\Phi_k\rangle) = p_1^2 + p_2^2 + \cdots + p_k^2$$

Easy fix:

$$|\Psi\rangle = \sqrt{p_1}|\Phi_1\rangle + \sqrt{p_2}|\Phi_2\rangle + \cdots + \sqrt{p_k}|\Phi_k\rangle$$

$$|\Psi\rangle = c_1|\Phi_1\rangle + c_2|\Phi_2\rangle + \cdots + c_k|\Phi_k\rangle$$ Where $c_n^2 = p_n$ Coefficient/component squared is the probability of the result R_n

Exercise: Calculate $|\Phi_i\rangle \bullet |\Phi_j\rangle$

State Vectors

And Their Comparison

$$|1\rangle \longrightarrow |\Phi_1\rangle \longrightarrow |\Phi_3\rangle \longrightarrow |\Phi_2\rangle \longrightarrow |2\rangle$$

$|1\rangle$

$|\Phi_1\rangle = x|1\rangle + 0.2|2\rangle$

$|\Phi_3\rangle = y|1\rangle + y|2\rangle$

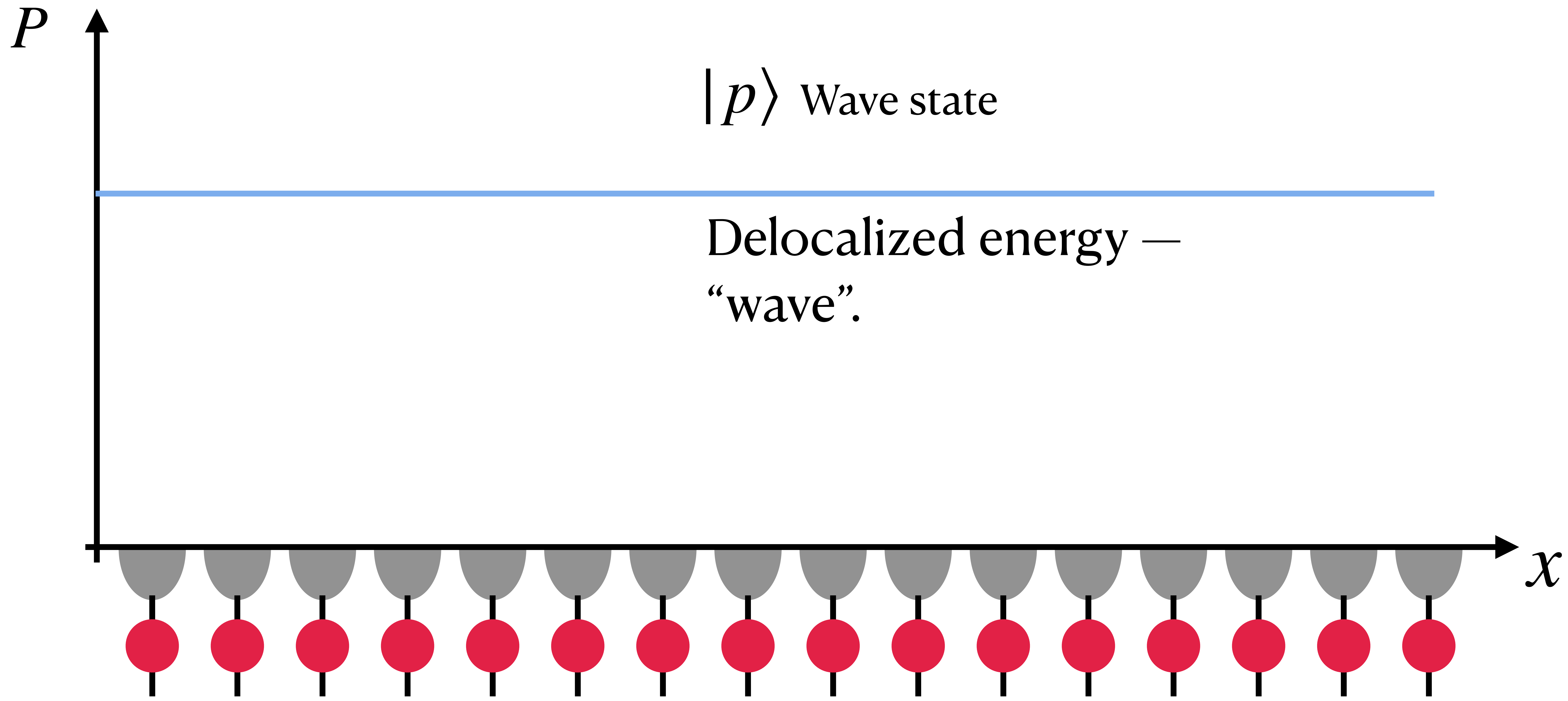
$|\Phi_2\rangle = 0.2|1\rangle + z|2\rangle$

$|2\rangle$

Position Measurement

Using Simple Detectors

“Special” (for humans) State Functions $\Psi(x)$.

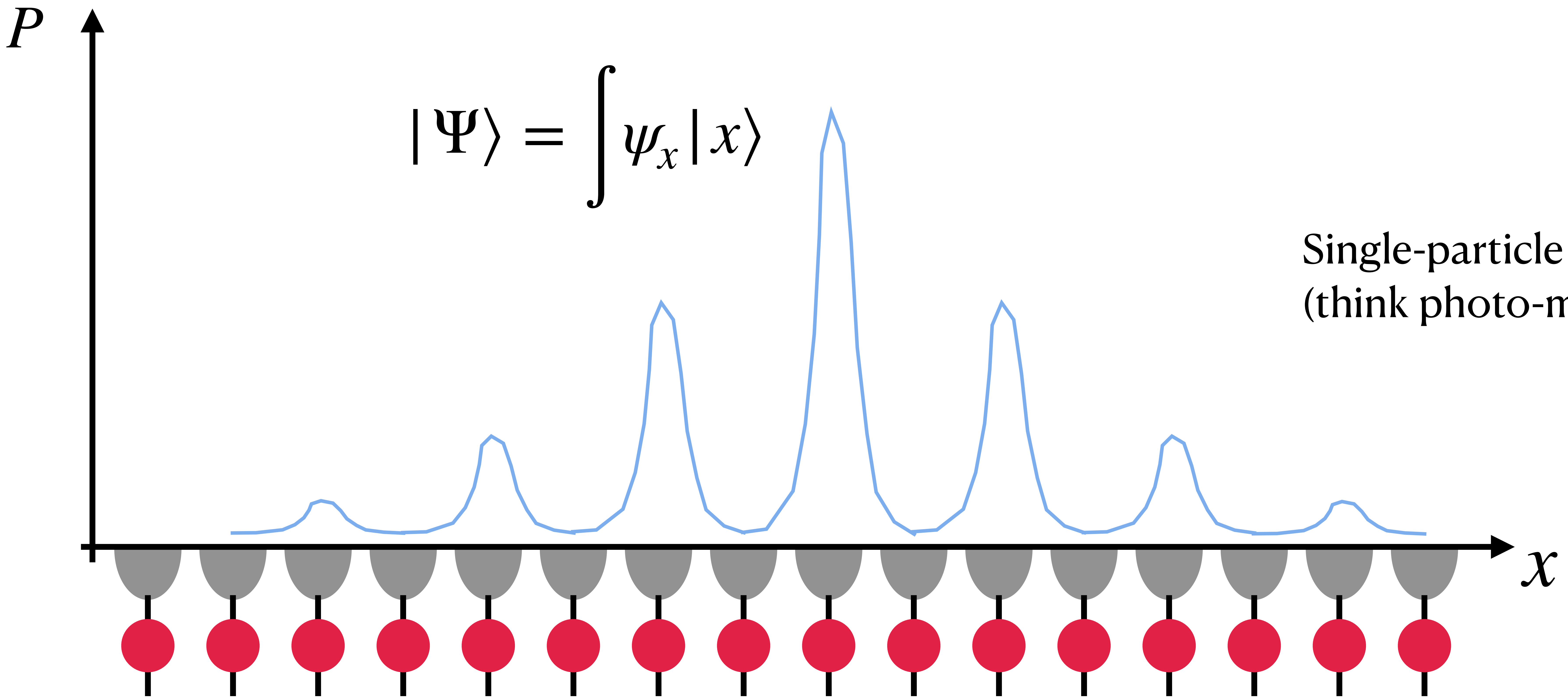


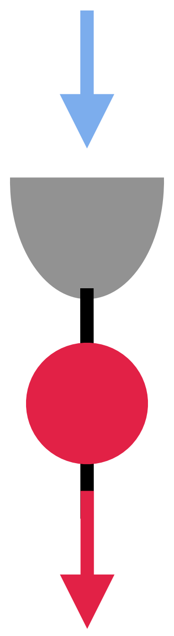
Position Measurement

Using Simple Detectors

State Functions $\Psi(x)$. (NOT ~~Wave functions~~)

$$|\Psi\rangle = \int \psi_x |x\rangle$$



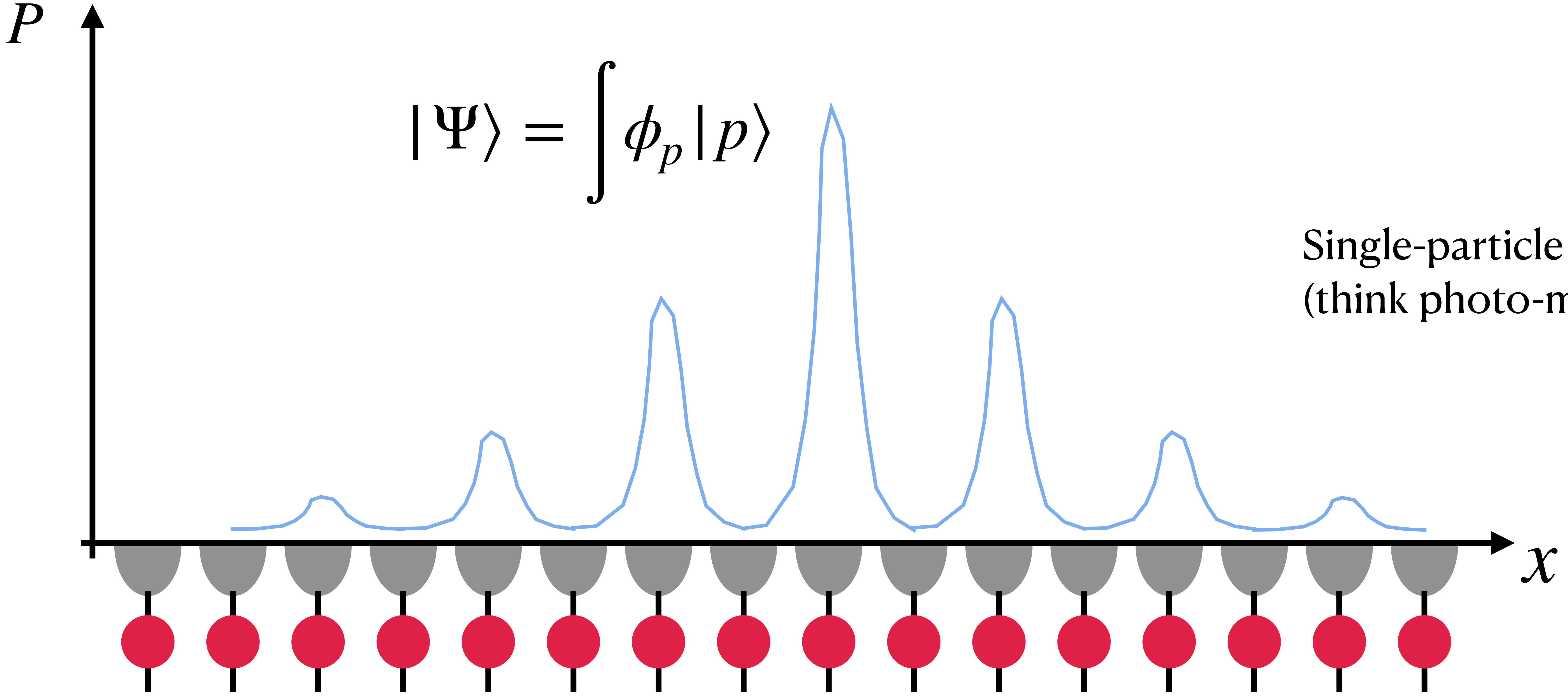


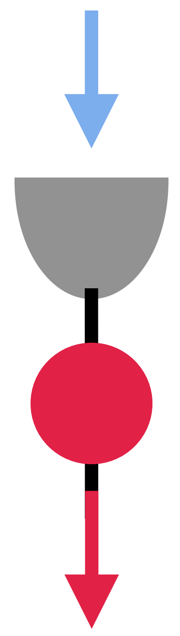
Single-particle detector
(think photo-multiplier tube)

Position Measurement

Using Simple Detectors

State Functions $\Psi(x)$**. (NOT ~~Wave functions~~)**





Single-particle detector
(think photo-multiplier tube)

How Do We Use All This?

State Vectors And Operators In Specific Representation

$$i\hbar\partial_t|\Psi\rangle = \hat{H}|\Psi\rangle$$

$|x\rangle$

$$|\Psi\rangle \to \Psi(x)$$

$$\hat{H} = -\frac{\hbar^2}{2m}\partial_x^2 + V(x)$$

$$i\hbar\Psi(x) = -\frac{\hbar^2}{2m}\partial_x^2\Psi(x) + V(x)\Psi(x)$$

Position representation.

Standard, canonical approach.

$|p\rangle$

$$|\Psi\rangle \to \Psi(p)$$

$$\hat{H} = \frac{p^2}{2m} + V(\partial_p)$$

$$i\hbar\Psi(p) = \frac{p^2}{2m}\Psi(p) + V(\partial_p)\Psi(p)$$

Momentum representation.

Often used. Less often discussed.

But many problems can also be solved or reasoned about without specific representation, purely in terms of vectors and operators.

Self-Test

Answer These Questions 1hr After Class

1. What is the role of the concept of “state” in physics?
2. How is state represented mathematically in Newtonian and Hamiltonian mechanics?
3. How do we obtain the information about a system in physics?
4. What is the difference between a quantum system and a measuring apparatus?
5. What does a measuring device do to a system? What does a system do to the device?
6. What is one way to represent the results of the multiple measurements?
7. What are two basic types of states?
8. Why are the ideas of “probability field” and “wave function” not good?

Homework Problems

Mathematical Concepts and Notation Day 3

- Suppose a system in a such a state that $\hat{H}|\Psi_0\rangle = E|\Psi_0\rangle$ — that is, the measured energy is E with 1.0 probability (with certainty). Write down the Schrödinger equation for this case and show that the state changes in time as follows $|\Psi_t\rangle = e^{-iEt/\hbar}|\Psi_0\rangle$.
- Suppose a harmonic oscillator is in such a state that $|\Phi\rangle = 0.7|1\rangle + 0.3|2\rangle$. What is the average energy of harmonic oscillator?
- In the previous problem, do you think it can be that $\hat{H}|\Phi\rangle = E|\Phi\rangle$ for some energy E?
- **Advanced**: If the state vector $|\Psi\rangle$ allows different *representations*: $|\Psi\rangle = \int \psi_x |x\rangle$ and $|\Psi\rangle = \int \phi_p |p\rangle$, can you write the relationship between the functions (components) ϕ_p and ψ_x?
- Watch the video about quantum properties of light (previously recommended/assigned). Learn about photo-multiplying tube (PMT).

Quantum Theory

In a Nutshell

II. POSTULATES FOR QUANTUM MECHANICS

In this paper, all state vectors are supposed to be normalized, and mixed states are represented by density operators, i.e., positive operators with unit trace. Let A be an observable with a nondegenerate purely discrete spectrum. Let $\phi_1, \phi_2, \ldots$ be a complete orthonormal sequence of eigenvectors of A and $a_1, a_2, \ldots$ the corresponding eigenvalues; by assumption, all different from each other.

According to the standard formulation of quantum mechanics, on the result of a measurement of the observable A the following postulates are posed:

(A1) *If the system is in the state ψ at the time of measurement, the eigenvalue a_n is obtained as the outcome of measurement with the probability $|\langle\phi_n|\psi\rangle|^2$.*

(A2) *If the outcome of measurement is the eigenvalue a_n, the system is left in the corresponding eigenstate ϕ_n at the time just after measurement.*

The postulate (A1) is called the *statistical formula*, and (A2) the *measurement axiom*. The state change $\psi \mapsto \phi_n$ described by the measurement axiom is called the *state reduction*.

You will understand this paragraph in the end of the course.