Quantum Physics

2025

The Theory/Framework Of *Almost* Everything *Today*

But Most Likely NOT of Tomorrow

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Course Overview

Course Structure And Goals

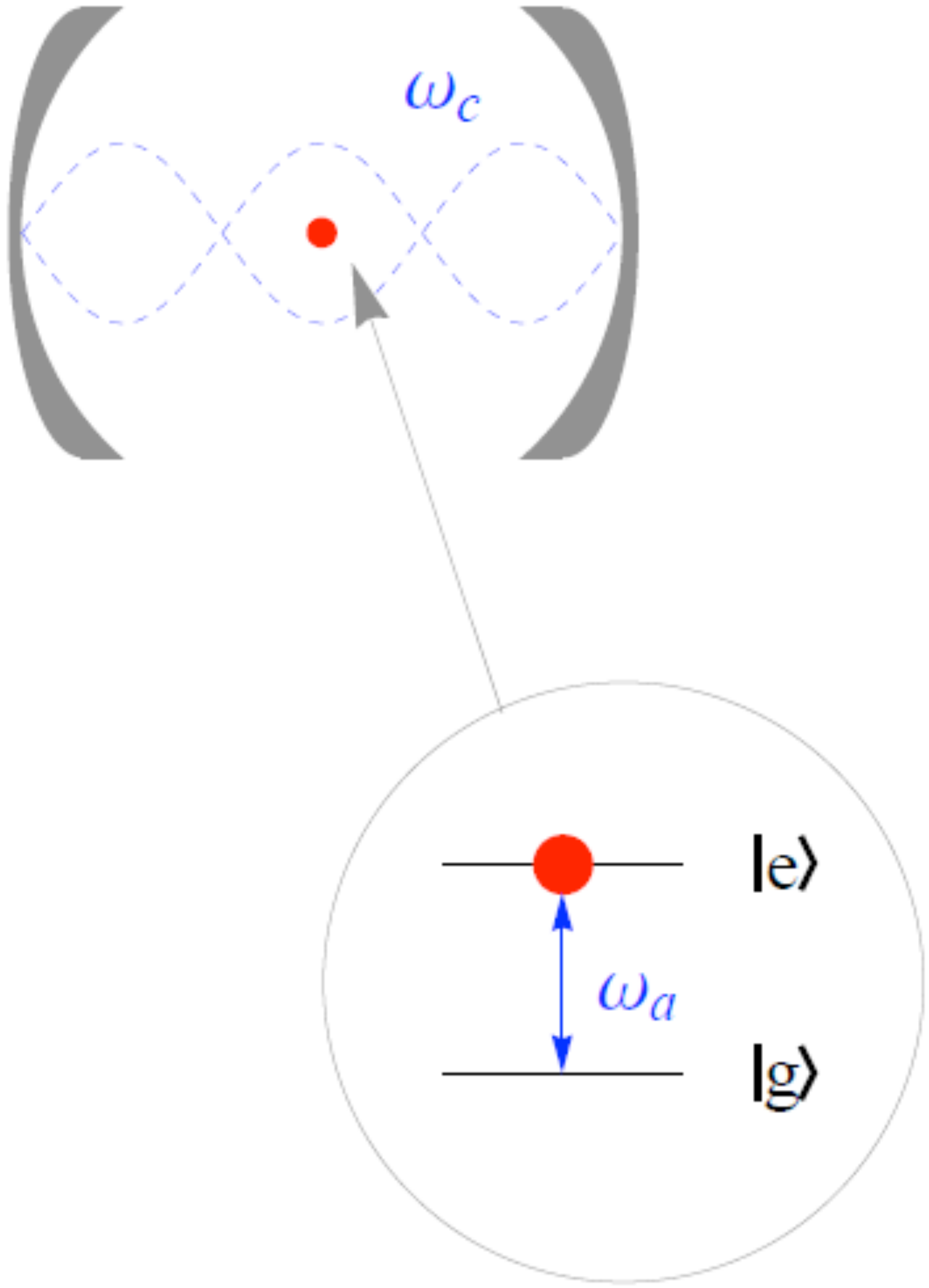
- **Part 1** : Mathematical Concepts And Tools.
- **Part 2** : Classical Physics.
- **Part 3** : Quantum Physics.

- Learn the language of quantum physics.
- Enhance the knowledge of classical physics.
- Develop modern quantum thinking.

We will focus on this one today.

Jaynes–Cummings Model

Simple. Interesting. Useful. Fully Quantum.



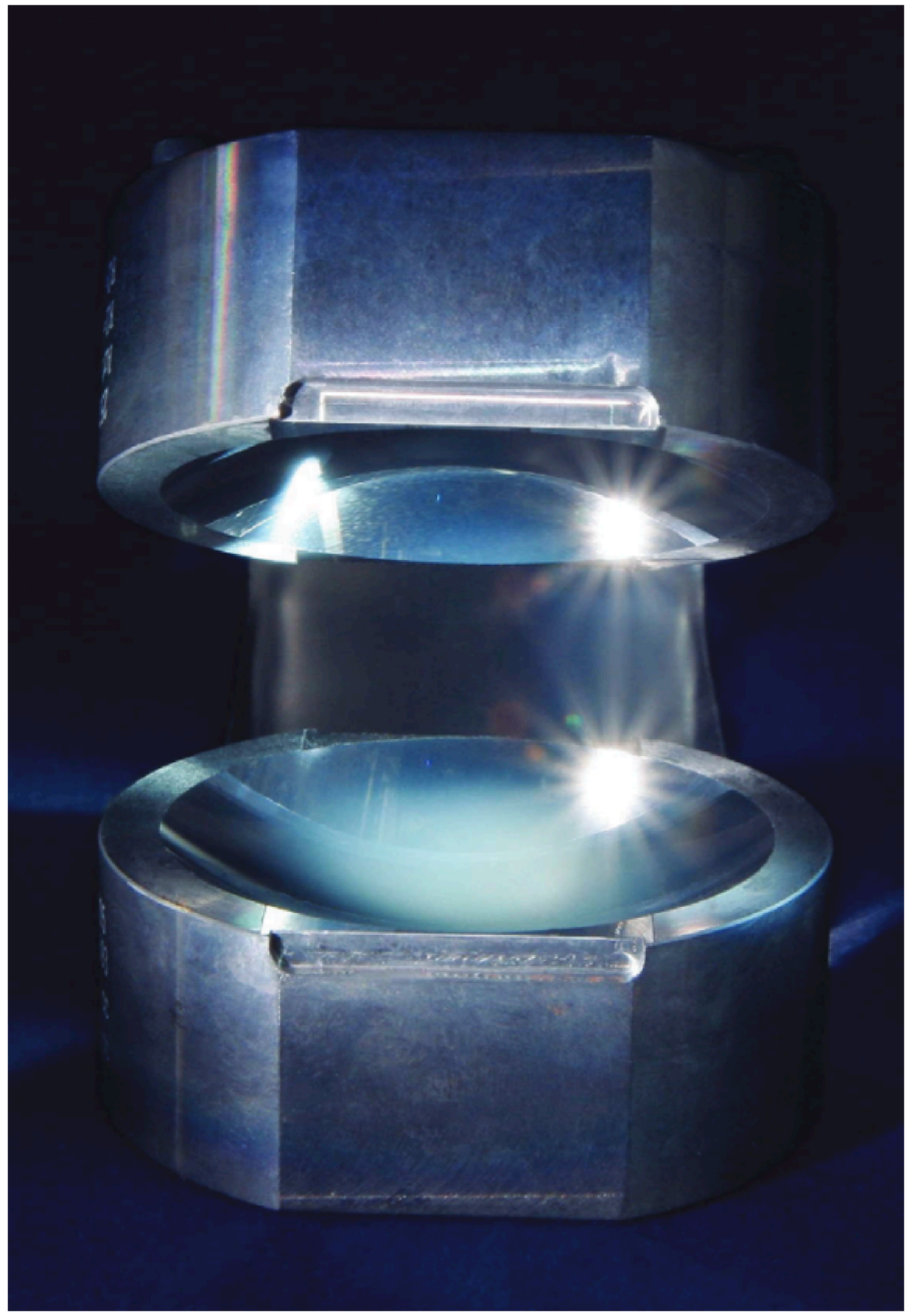
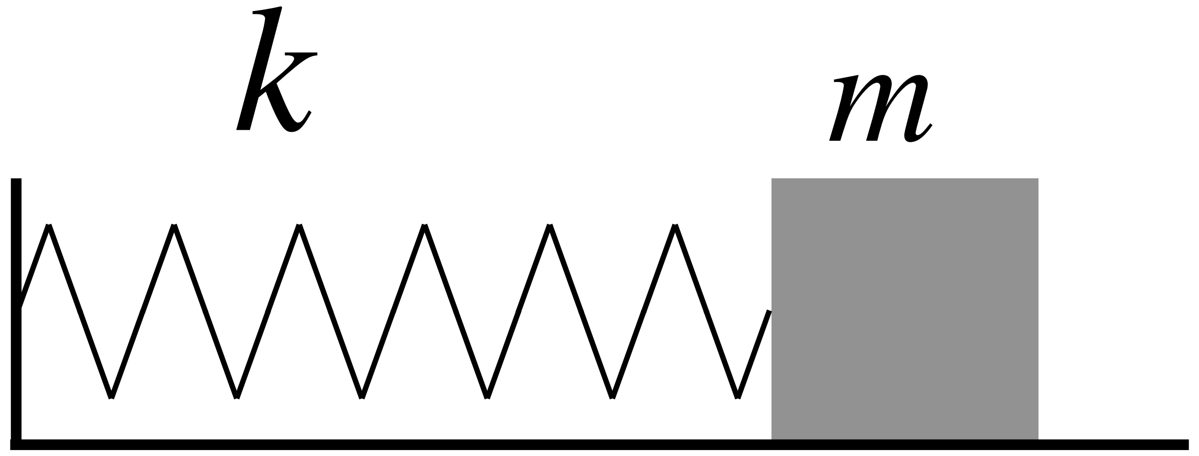


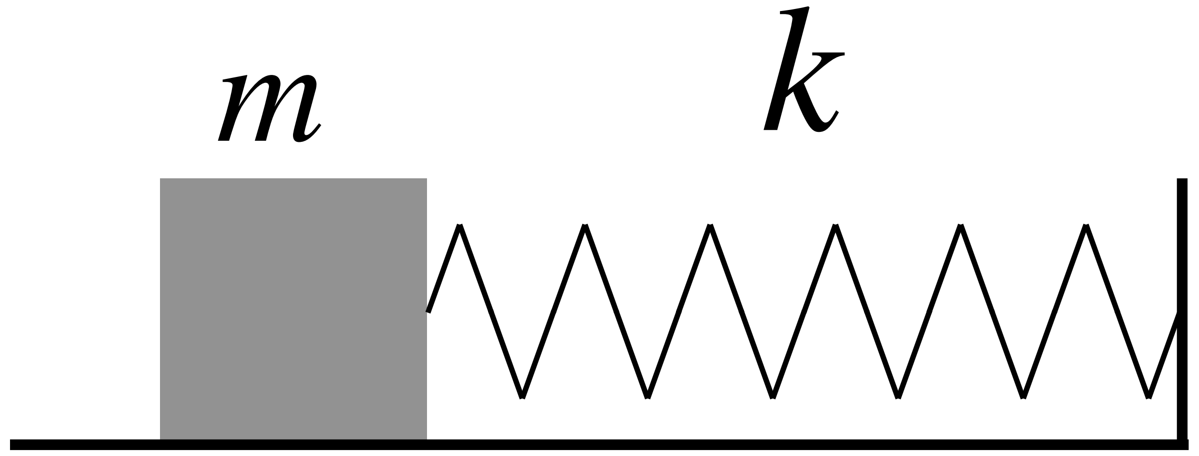
FIGURE 8. The ENS photon box (Photograph by Michel Brune). The mirrors have a diameter of 5 cm and are 2.7 cm apart (for a clear view, they are more separated in this picture than in the actual experimental set-up).

- The model describes *interaction* between a simple "atom" (two-level quantum system) and "electromagnetic field" (single mode of EMF in cavity).

Jaynes–Cummings Model

Mechanical Analog (Almost)





$$H_a = \frac{p_1^2}{2m} + \frac{kx_1^2}{2}$$

$$|\xi_1\rangle \sim (x_1, p_1)$$

- Two independent oscillators

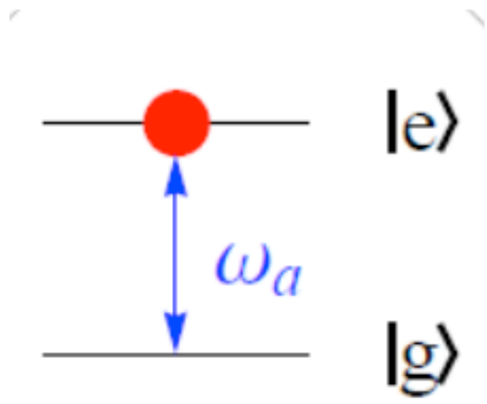
$$H_b = \frac{p_2^2}{2m} + \frac{kx_2^2}{2}$$

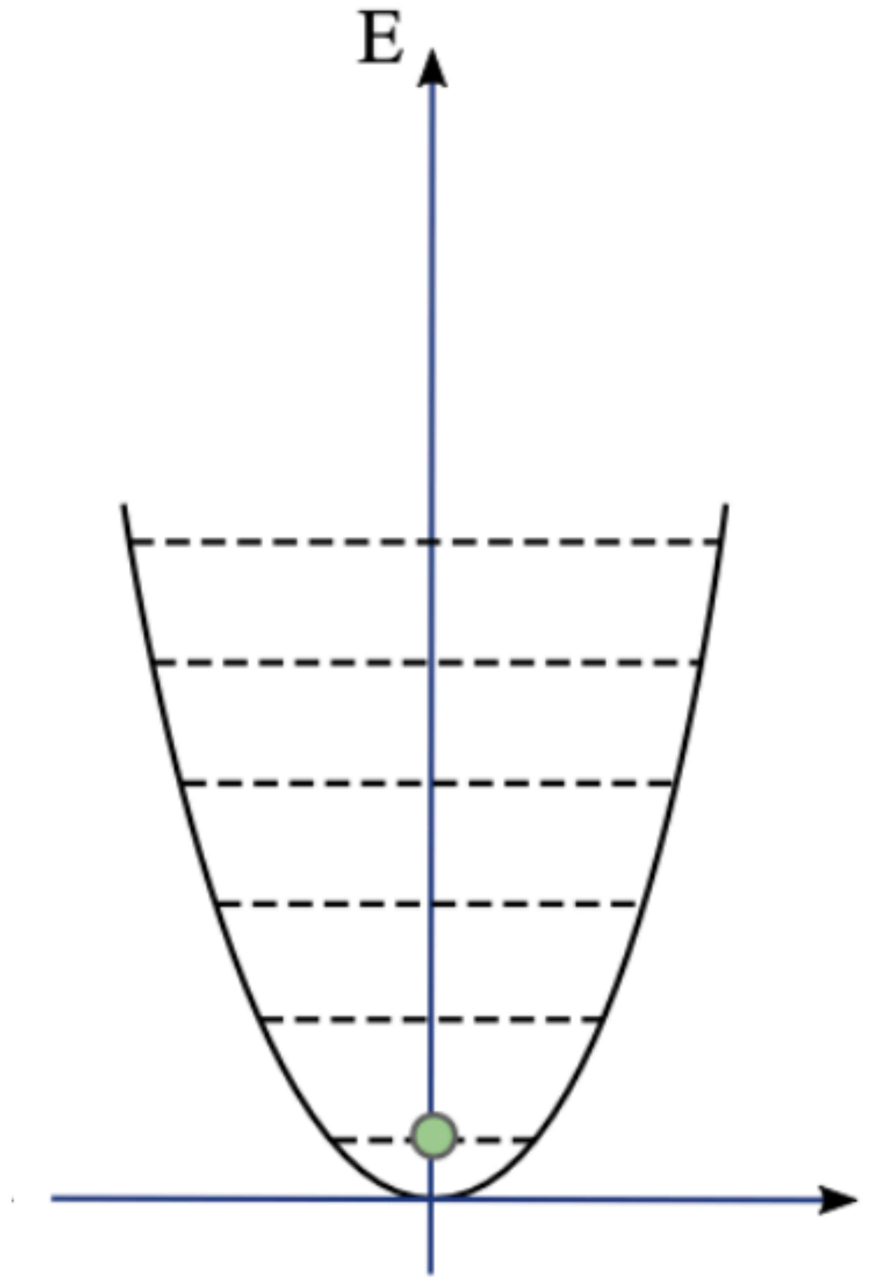
$$|\xi_2\rangle \sim (x_2, p_2)$$

$$|\xi\rangle = |\xi_1\rangle \oplus |\xi_2\rangle \sim (x_1, p_1, x_2, p_2)$$

Jaynes–Cummings Model

Composite System: Atom + Field Mode





$$\hat{H}_a = E_g |g\rangle\langle g| + E_e |e\rangle\langle e|$$

$$|\Psi\rangle = c_g |g\rangle + c_e |e\rangle$$

$$i\hbar\partial_t |\Psi\rangle = \hat{H}_a |\Psi\rangle$$

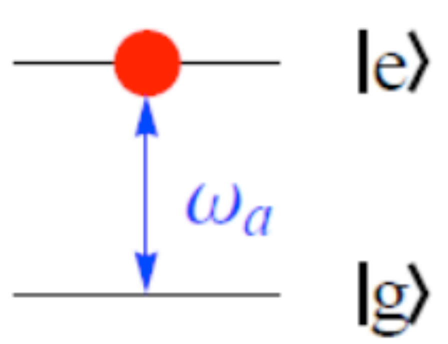
$$\hat{H}_m = E_0 |0\rangle\langle 0| + \hbar\omega_m \int_n n |n\rangle\langle n|$$

$$|\Phi\rangle = \int_k b_k |k\rangle$$

$$i\hbar\partial_t |\Phi\rangle = \hat{H}_m |\Phi\rangle$$

Raising-Lowering Operators

For Atom and Mode



$$\boxed{\begin{aligned}\hat{\sigma}_+ &= |e\rangle\langle g| \\ \hat{\sigma}_- &= |g\rangle\langle e|\end{aligned}} \longrightarrow \begin{aligned}\hat{\sigma}_+\hat{\sigma}_- &= |e\rangle\langle e| \\ \hat{\sigma}_-\hat{\sigma}_+ &= |g\rangle\langle g|\end{aligned}$$

$$\boxed{\begin{aligned}\hat{I} = \int_k |k\rangle\langle k| \\ k = 0,1,2,\ldots\end{aligned}}$$

$$\hat{H}_a = E_g|g\rangle\langle g| + E_e|e\rangle\langle e|$$

$$\hat{I} = |g\rangle\langle g| + |e\rangle\langle e| = \hat{\sigma}_+\hat{\sigma}_- + \hat{\sigma}_-\hat{\sigma}_+$$

$$|\Psi\rangle = c_g|g\rangle + c_e|e\rangle$$

$$\boxed{\hat{H}_a = E_g\hat{I} + \hbar\omega_a\hat{\sigma}_+\hat{\sigma}_-}$$

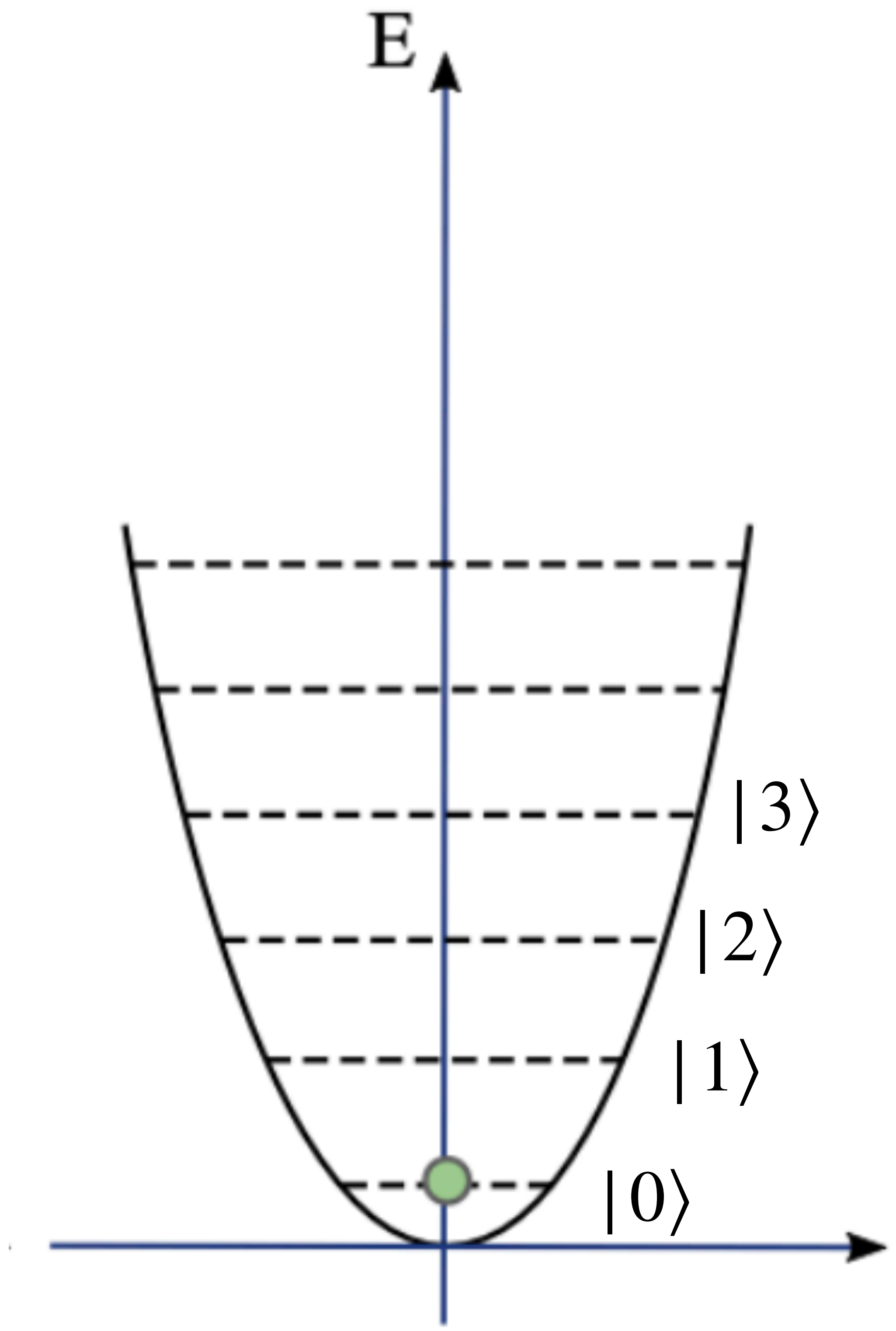
Show it!

$$i\hbar\partial_t|\Psi\rangle = \hat{H}_a|\Psi\rangle$$

$$\hat{H}_a = \hbar\omega_a(e_0\hat{I} + \hat{\sigma}_+\hat{\sigma}_-) \quad e_0 = E_g/\hbar\omega$$

Raising-Lowering Operators

For Atom and Mode



$$\hat{a}_+|k\rangle = |k+1\rangle$$

$$\hat{a}_-|k\rangle = |k-1\rangle$$

$$\hat{a}_-|0\rangle = |0\rangle$$

$$\hat{a}_+ = \int_k |k+1\rangle\langle k|$$

$$\hat{a}_- = |0\rangle\langle 0| + \int_n |n+1\rangle\langle n|$$

Show it!

$$\hat{H}_m = \int_k E_k |k\rangle\langle k| \qquad \hat{I} = \int_k |k\rangle\langle k|$$

$$\hat{H}_m = E_0|0\rangle\langle 0| + \hbar\omega_m \int_n n|n\rangle\langle n|$$

$k = 0,1,2,\ldots \qquad n = 1,2,3,\ldots \longleftarrow$ Number of excitation quanta

$$\boxed{\hat{H}_m = E_0\hat{I} + \hbar\omega_m \hat{a}_+\hat{a}_-}$$???

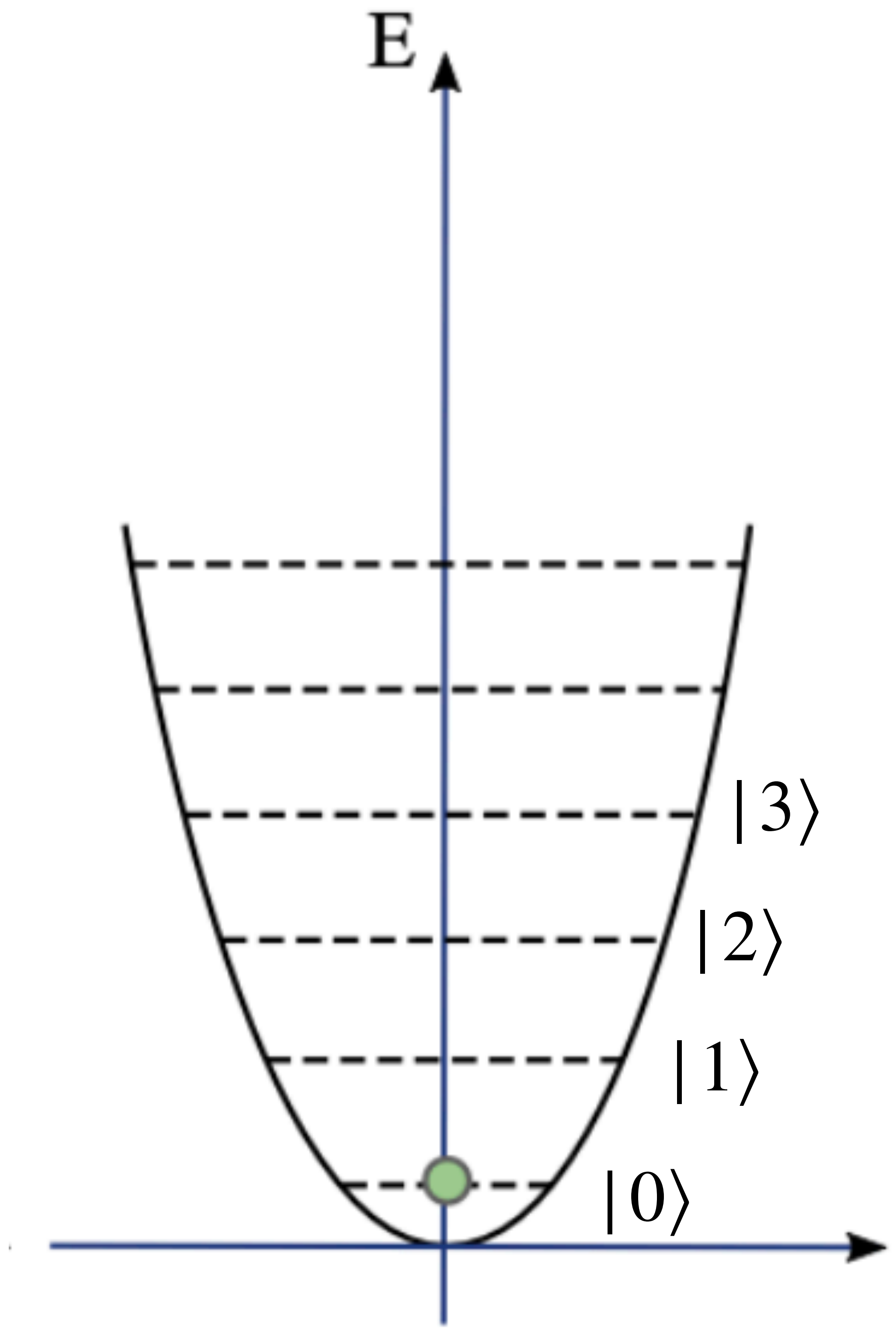
$$\hat{H}_a = E_g\hat{I} + \hbar\omega_a \hat{\sigma}_+\hat{\sigma}_-$$

How to fix?

$$\hat{a}_+\hat{a}_- \neq \hat{n}$$

Raising-Lowering Operators

For Atom and Mode



$$\hat{H}_m = E_0\hat{I} + \hbar\omega_m\hat{a}^\dagger\hat{a}$$

$$\hat{H}_m = \hbar\omega_m(e_0\hat{I} + \hat{a}^\dagger\hat{a})$$

$$\hat{a}\hat{a}^\dagger - \hat{a}^\dagger\hat{a} = \hat{I}$$

$$\hat{a}^\dagger\hat{a} = \hat{n}$$

$$\hat{a}^\dagger|k\rangle = \sqrt{k+1}\,|k+1\rangle$$

$$\hat{a}|k\rangle = \sqrt{k}\,|k-1\rangle$$

$$\hat{H}_m = E_0\hat{I} + \hbar\omega_m\hat{a}^\dagger\hat{a}$$

$$\updownarrow$$

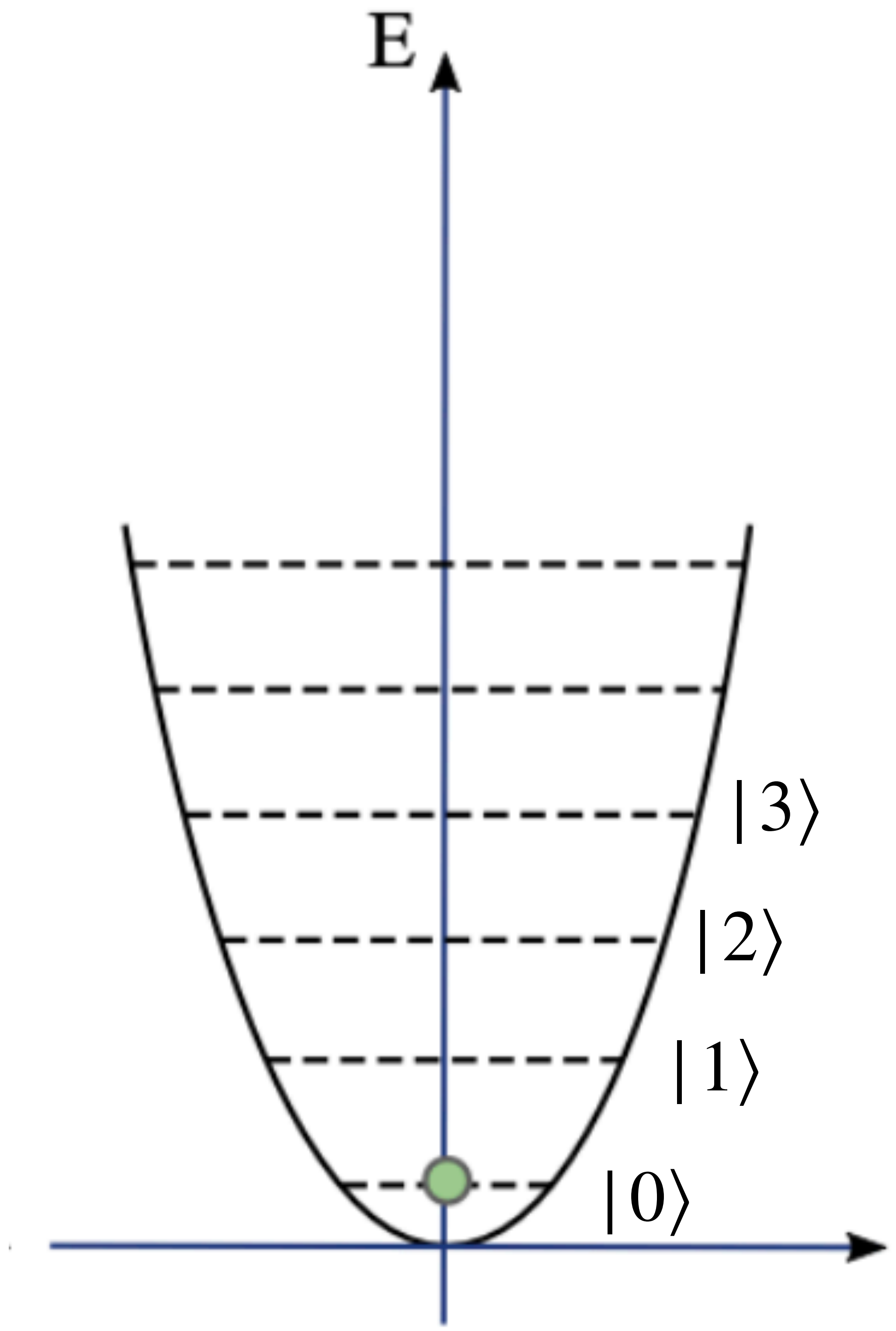
$$\hat{H}_m = \frac{\hat{p}^2}{2m} + \frac{k\hat{x}^2}{2}$$

$$\boxed{E_0 = \hbar\omega_m/2}$$

Big, interesting, important, and not so difficult problem!

Raising-Lowering Operators

For Atom and Mode

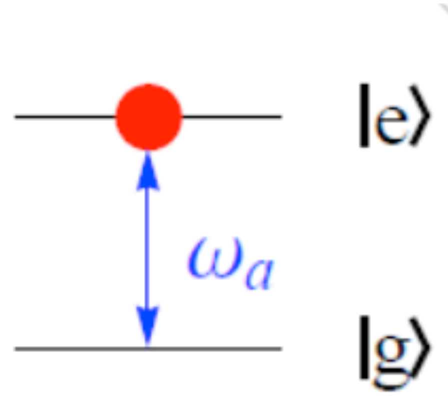


$$\hat{H}_m = E_0\hat{I} + \hbar\omega_m\hat{a}^\dagger\hat{a}$$

Field emits,
atom absorbs

$$\hat{H}_{int} = \hat{\sigma}_+ \otimes \hat{a} + \hat{\sigma}_- \otimes \hat{a}^\dagger$$

Atom emits,
field absorbs



$$\hat{H}_a = E_g\hat{I} + \hbar\omega_a\hat{\sigma}_+\hat{\sigma}_-$$

Self-Test

Answer These Questions 1hr After Class

1. How is mode different from motion of individual body?
2. What physical systems may have modes?
3. What are two types of “devices” used to contain modes? What is the difference between them.
4. What are raising and lowering operators?

Homework Problems

Jaynes-Cummings Model

1. Show that the raising operator $\hat{a}^\dagger$ can be written as follows: $\hat{a}^\dagger = \int_k \sqrt{k+1}\,|k+1\rangle\langle k|\,, k = 0,1,2,\ldots$
2. Evaluate $\left(\hat{\sigma}_- \otimes \hat{a}^\dagger\right)\ |g\rangle\langle n|$.
3. Show that $\hat{H}_m = E_0\hat{I} + \hbar\omega_m\hat{n}$ follows from $\hat{H}_m = \int_k E_k|k\rangle\langle k|$ and $E_n = E_0 + n\hbar\omega_m$. Use the important relation (easy to prove): $\hat{I} = \int_k |k\rangle\langle k|$.
4. Study the Jupyter Notebook with Jaynes-Cummings Model.
5. Show that for harmonic oscillator $\hat{x} = \sqrt{\dfrac{\hbar}{2m\omega}}\left(\hat{a}^\dagger + \hat{a}\right)$ and $\hat{p} = i\sqrt{\dfrac{\hbar m\omega}{2}}\left(\hat{a}^\dagger - \hat{a}\right)$.
6. Using the fact that $\hat{a}\hat{a}^\dagger - \hat{a}^\dagger\hat{a} = \hat{I},$ calculate $\hat{x}\hat{p} - \hat{p}\hat{x}$.

Homework Problems

Hint

$$\hat{H}|n\rangle = E_n|n\rangle = (E_0 + n\hbar\omega_m)|n\rangle \longrightarrow (\hat{H} - E_0\hat{I})|n\rangle = n\hbar\omega_m|n\rangle \longrightarrow (\hat{H} - E_0\hat{I})/\hbar\omega_m|n\rangle = n|n\rangle$$

$$(\hat{H} - E_0\hat{I})/\hbar\omega_m = \hat{n}$$

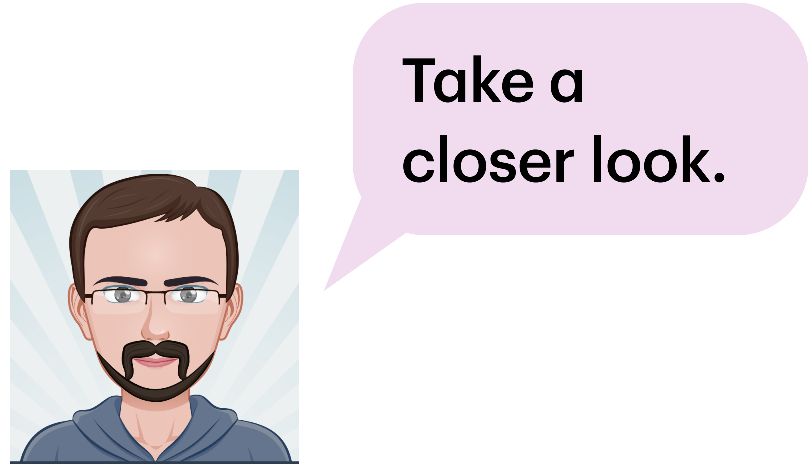
$$\hat{H} = E_0|0\rangle\langle 0| + \int_n E_n|n\rangle\langle n| = E_0|0\rangle\langle 0| + \int_n (E_0 + n\hbar\omega)|n\rangle\langle n| = E_0 \int_{m=0} |m\rangle\langle m| + \hbar\omega \int n|n\rangle\langle n|$$

$$\hat{I} = \int_{m=0} |m\rangle\langle m|$$

$$\hat{H} = E_0\hat{I} + \hbar\omega \int n|n\rangle\langle n| \longrightarrow \hat{n} = (\hat{H} - E_0\hat{I})/\hbar\omega_m = \int_{n=1} n|n\rangle\langle n| = \int_k k|k\rangle\langle k|$$

$$\boxed{\hat{H} = E_0\hat{I} + \hbar\omega\hat{n}}$$

Homework Problems



Hint

$$\hat{\sigma}_+|0\rangle = 1|1\rangle \qquad \hat{\sigma}_+|k\rangle = \sqrt{k+1}\,|k+1\rangle$$

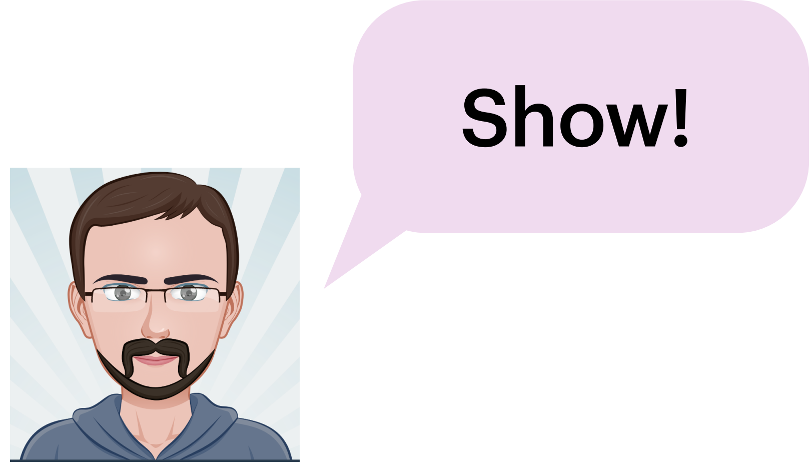
$$\hat{\sigma}_-|1\rangle = 1|0\rangle \qquad \hat{\sigma}_-|k\rangle = \sqrt{k}\,|k-1\rangle$$

$$\hat{\sigma}_+\hat{\sigma}_-|k\rangle = \sqrt{k}\hat{\sigma}_+|k-1\rangle = \sqrt{k}\sqrt{k}\,|k\rangle = k|k\rangle$$

$$\hat{a}^\dagger|k\rangle = \sqrt{k+1}\,|k+1\rangle$$

$$\hat{a}|k\rangle = \sqrt{k}\,|k-1\rangle$$

$$\hat{a}^\dagger\hat{a}|k\rangle = k|k\rangle \qquad \hat{a}^\dagger\hat{a} = \hat{n}$$



$$\hat{a}\hat{a}^\dagger|k\rangle = (k+1)|k\rangle$$

Homework Problems

Hint

$$\hat{H}_m = E_0\hat{I} + \hbar\omega_m\hat{a}^\dagger\hat{a} \qquad \hat{a}\hat{a}^\dagger - \hat{a}^\dagger\hat{a} = \hat{I}$$

$$\hat{H}_m = \frac{\hat{p}^2}{2m} + \frac{k\hat{x}^2}{2} \qquad \hat{H}_m = E_0\hat{a}\hat{a}^\dagger + (\hbar\omega_m - E_0)\hat{a}^\dagger\hat{a}$$

$$\hat{x} = A\hat{a} + B\hat{a}^\dagger \qquad \hat{x}^2 = \,?$$

$$\hat{p} = C\hat{a} + D\hat{a}^\dagger \qquad \hat{p}^2 = \,?$$

Quantum Theory

In a Nutshell

II. POSTULATES FOR QUANTUM MECHANICS

In this paper, all state vectors are supposed to be normalized, and mixed states are represented by density operators, i.e., positive operators with unit trace. Let A be an observable with a nondegenerate purely discrete spectrum. Let $\phi_1, \phi_2, \ldots$ be a complete orthonormal sequence of eigenvectors of A and $a_1, a_2, \ldots$ the corresponding eigenvalues; by assumption, all different from each other.

According to the standard formulation of quantum mechanics, on the result of a measurement of the observable A the following postulates are posed:

(A1) *If the system is in the state ψ at the time of measurement, the eigenvalue a_n is obtained as the outcome of measurement with the probability $|\langle\phi_n|\psi\rangle|^2$.*

(A2) *If the outcome of measurement is the eigenvalue a_n, the system is left in the corresponding eigenstate ϕ_n at the time just after measurement.*

The postulate (A1) is called the *statistical formula*, and (A2) the *measurement axiom*. The state change $\psi \mapsto \phi_n$ described by the measurement axiom is called the *state reduction*.

You will understand this paragraph in the end of the course.