Quantum Physics

2024

The Theory/Framework Of *Almost* Everything *Today*

Yury Deshko

Phase Difference

Part B

Superposition approach to expressing the results of measurements

$$\boxed{|\Psi\rangle = c_1|\Phi_1\rangle + c_2|\Phi_2\rangle + \cdots + c_k|\Phi_k\rangle}$$

$$c_1^2 + c_2^2 + \cdots + c_k^2 = 1$$

$c_k^2 = p_k$ Probability intensity

c_k Probability **amplitude**

Phase Difference

Part B

$$|\Psi\rangle = c_1|\Phi_1\rangle + c_2|\Phi_2\rangle + \cdots + c_k|\Phi_k\rangle$$

$$|\Phi_i\rangle \bullet |\Phi_j\rangle = 0$$

Mutually exclusive results/ states.

WRONG/VERY BAD LANGUAGE: Quantum system is *simultaneously* in all mutually exclusive states ("dead" and "alive" at the same time).

Schrödinger Explanation of QM in 1935

Are Variables Really Blurred?

One can even set up quite ridiculous cases. A cat is penned up in a steel chamber, along with the following diabolical device (which must be secured against direct interference by the cat): in a Geiger counter there is a tiny bit of radioactive substance, *so* small, that *perhaps* in the course of one hour one of the atoms decays, but also, with equal probability, perhaps none; if it happens, the counter tube discharges and through a relay releases a hammer which shatters a small flask of hydrocyanic acid. If one has left this entire system to itself for an hour, one would say that the cat still lives *if* meanwhile no atom has decayed. The first atomic decay would have poisoned it. The ψ-function of the entire system would express this by having in it the living and the dead cat (pardon the expression) mixed or smeared out in equal parts.

It is typical of these cases that an indeterminacy originally restricted to the atomic domain becomes transformed into macroscopic indeterminacy, which can then be *resolved* by direct observation. That prevents us from so naively accepting as valid a "blurred model" for representing reality. In itself it would not embody anything unclear or contradictory. There is a difference between a shaky or out-of-focus photograph and a snapshot of clouds and fog banks.

NOTE: The situation needs more careful analysis, since involves complicated systems with many parts. We only learned about simple 1-particle systems so far.

$$|\Psi\rangle = c_1|A_iC_a\rangle + c_2|A_dC_d\rangle$$

Here A_i means atom intact and C_a cat alive, while A_d means atom decayed and C_d cat dead.

Trying to talk about what happens/exists in between the preparation and measurement leads to "language misuse" and statements like "both alive and dead."

$$|\Psi\rangle = c_1|A_iC_a\rangle + c_2|A_dC_d\rangle$$

This is an example of "entangled" state, we need better tools to discuss this topic.

Phase Difference

Part B

Superposition approach to expressing the results of measurements

$$|\Psi\rangle = c_1|\Phi_1\rangle + c_2|\Phi_2\rangle + \cdots + c_k|\Phi_k\rangle$$

Play important role in **interference** phenomena!

$$c_1^2 + c_2^2 + \cdots + c_k^2 = 1$$

$c_k^2 = p_k$ Probability intensity

c_k Probability **amplitude**

John Archibald Wheeler

1911-2008

American theoretical physicist.

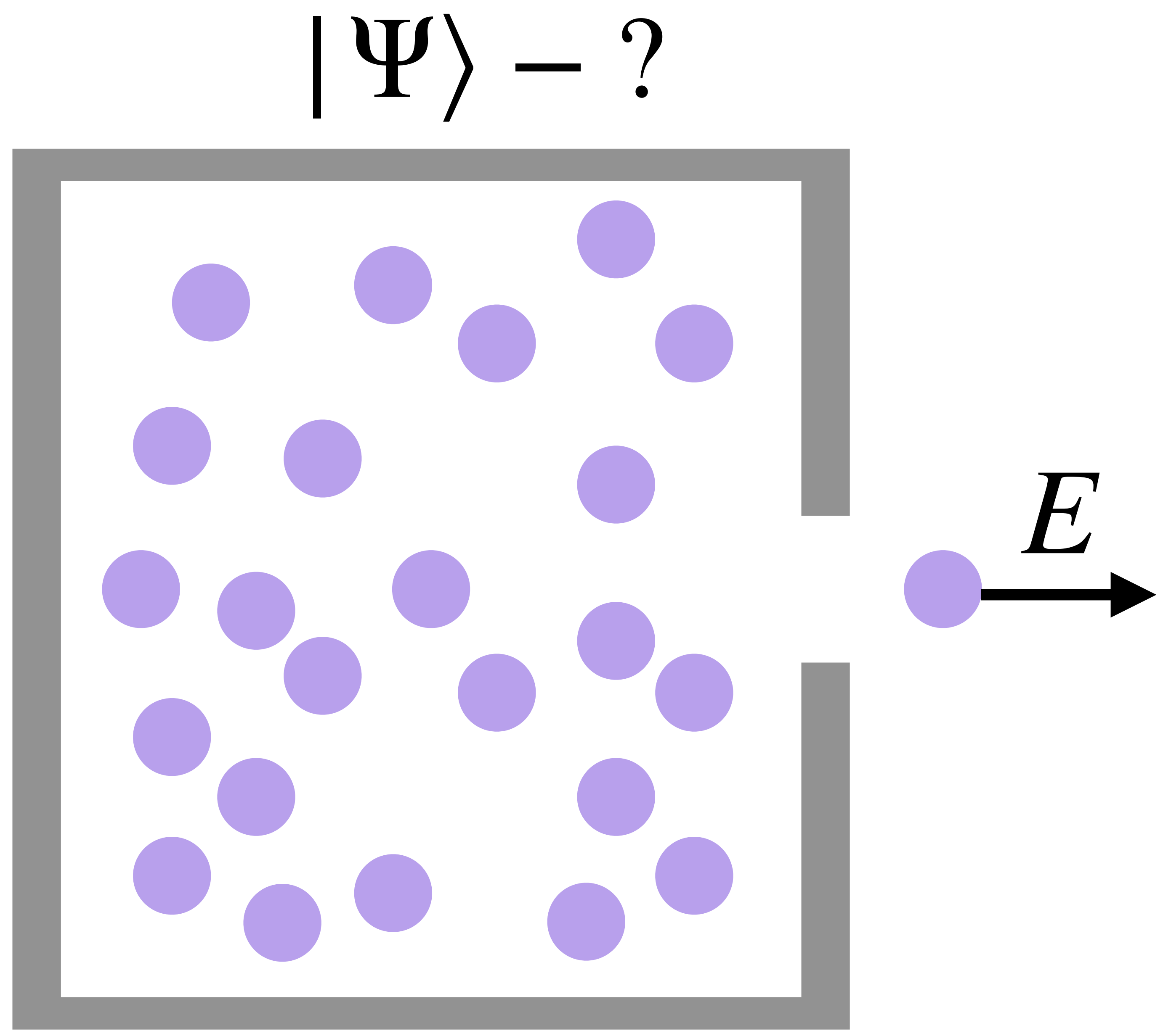
General relativity, “black hole”, “wormhole”

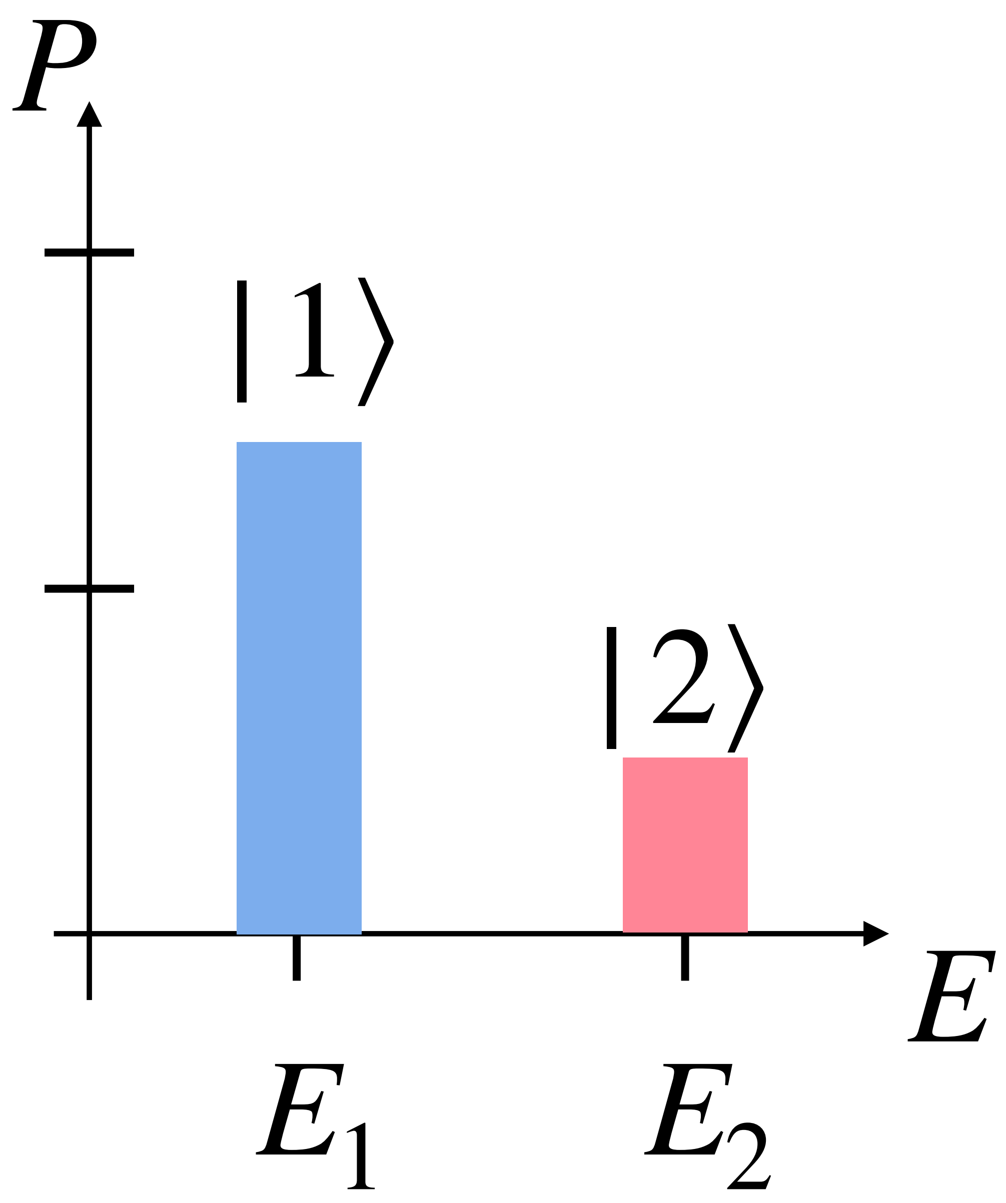
origin of probability amplitude and phase, and phase we are to learn to consider as the definer of all fields, of all geometry, of all spacetime -- and in the end, therefore of all particles and all physics. Physics, on this view, is the child of meaning even

“Mystery and the message of the quantum”

Quantum States

Pure vs Mixed





Qubit = quantum system with 2 distinct states available for manipulation.

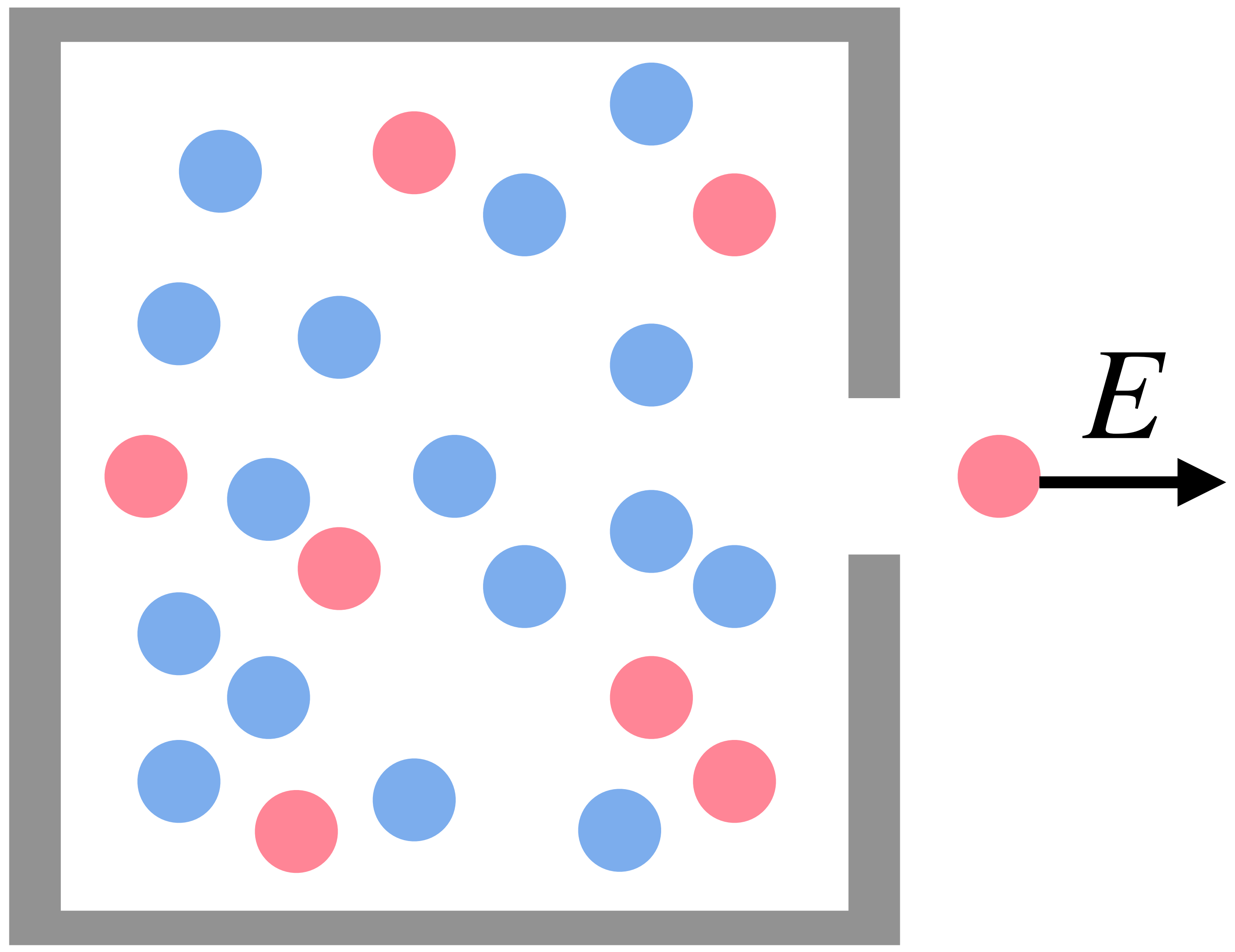
Measuring energy of a large number of qubits (e.g., harmonic oscillators with limited energy supply) reveals their state $|\Psi\rangle$.

"Sorting" Experiment

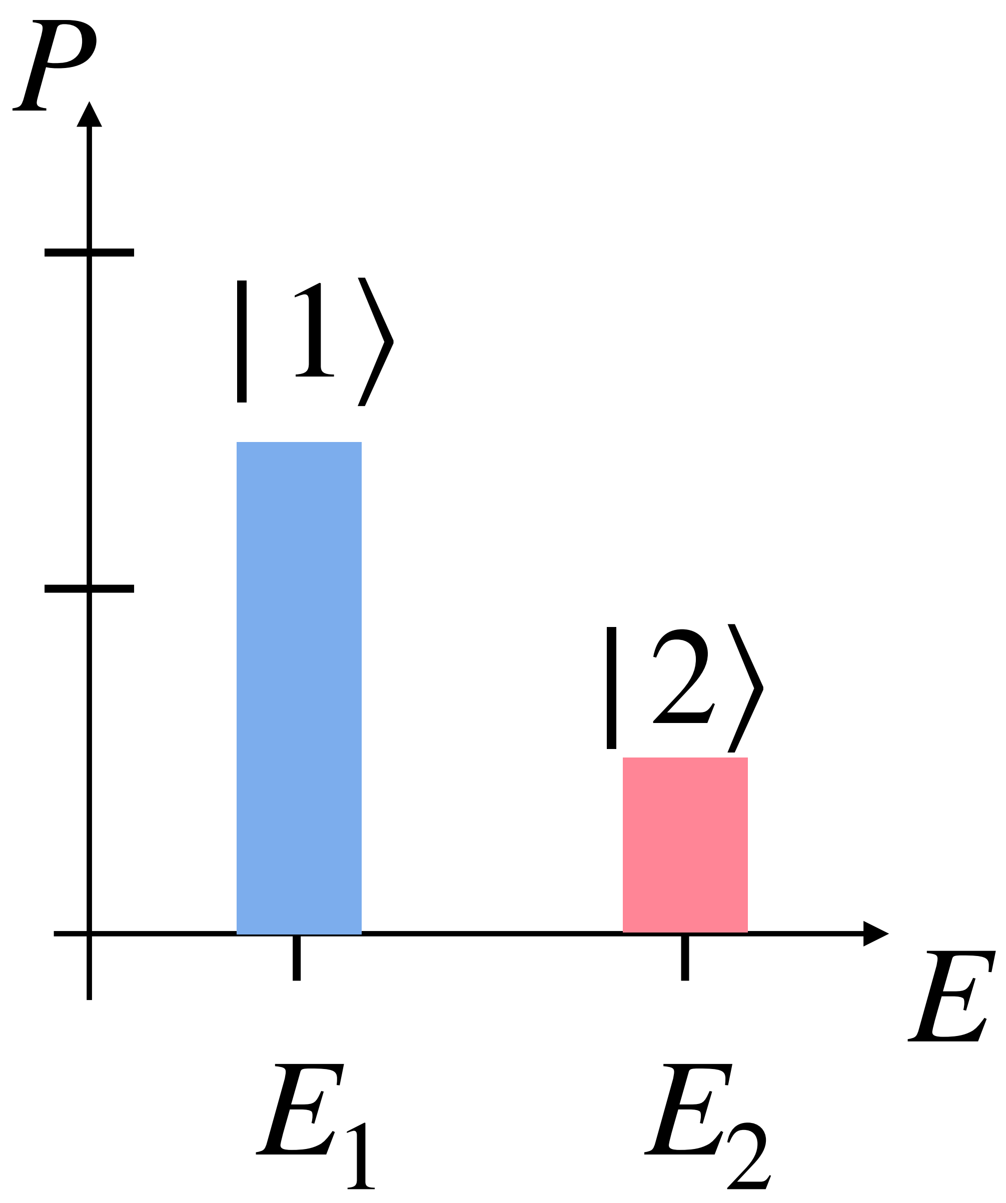
$$|\Psi\rangle = c_1|1\rangle + c_2|2\rangle$$

Quantum States

Simple, Intuitive, Naive View



mixture



We might have two distinct groups, one in $|1\rangle$ and the other one in $|2\rangle$ – *mixture*.

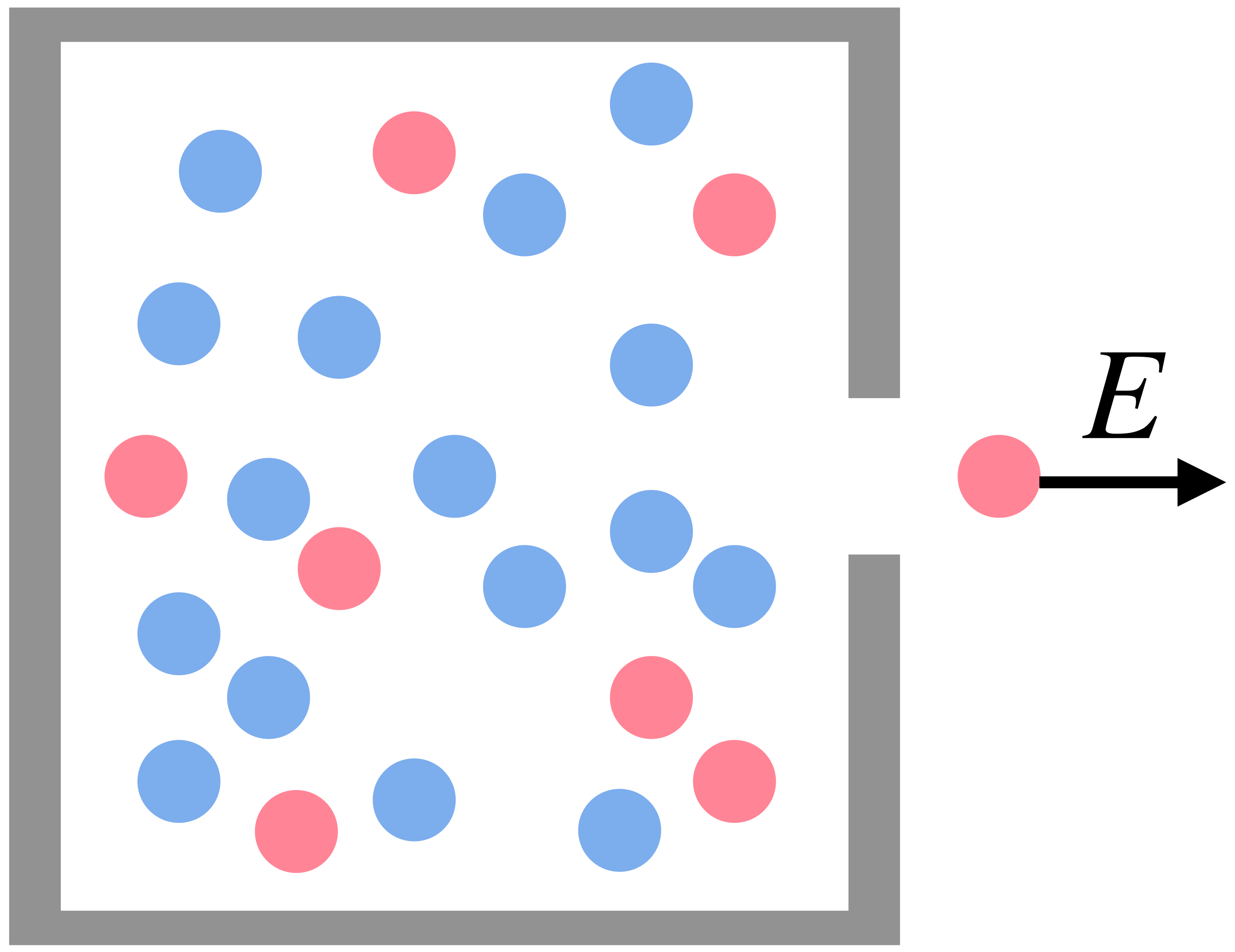
This happens in both classical (e.g. gas molecules) and in quantum physics.
But in quantum physics this <u>does not always work</u>. (*Interference* phenomena!)

"Sorting" Experiment

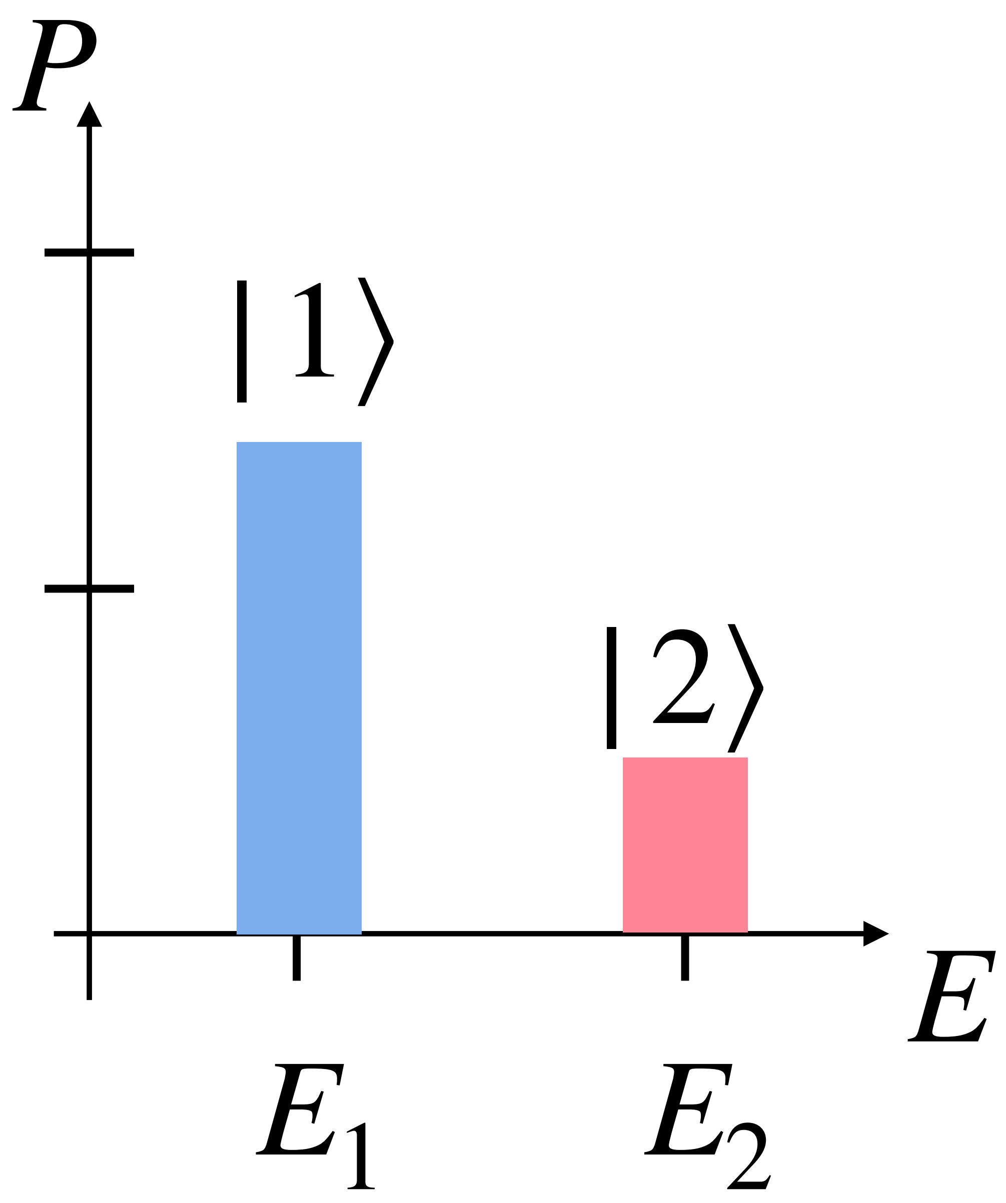
$$|\Psi\rangle \neq c_1|1\rangle + c_2|2\rangle$$

Quantum States

Simple, Intuitive, Naive View



mixture



We might have two distinct groups, one in $|1\rangle$ and the other one in $|2\rangle$ – *mixture*.

This happens in both classical (e.g. gas molecules) and in quantum physics.
But in quantum physics this <u>does not always work</u>. (*Interference* phenomena!)

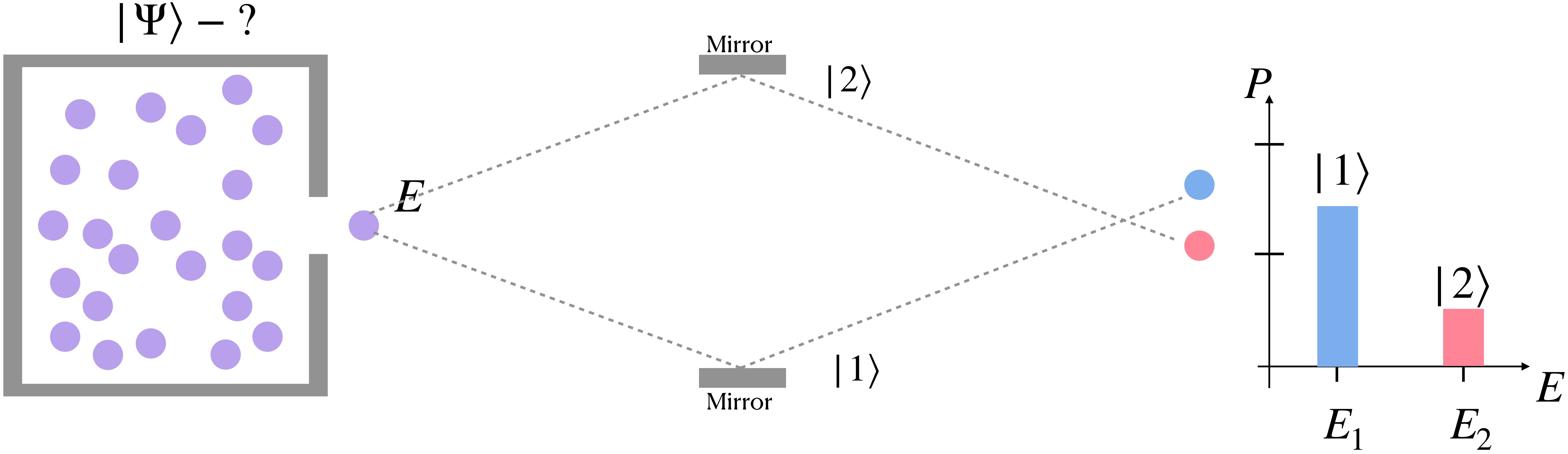
"Sorting" Experiment

$$|\Psi\rangle \neq c_1|1\rangle + c_2|2\rangle$$

From pure to mixed = loss of phase = decoherence

Quantum States

Pure vs Mixed: Interference Reveals The Difference



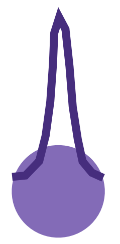
Measuring energy of a large number of qubits (e.g., harmonic oscillators with limited energy supply) reveals their state $|\Psi\rangle$.

Interference Experiment

$$|\Psi\rangle = c_1|1\rangle + c_2|2\rangle$$

Two Basic Quantum Systems

Electromagnetic and Electron-Positron Fields



0

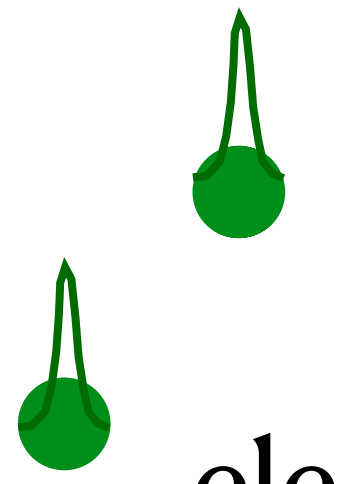
photon

EMF

Interact = Influence each other.

Can produce each other.

positron



$+e$

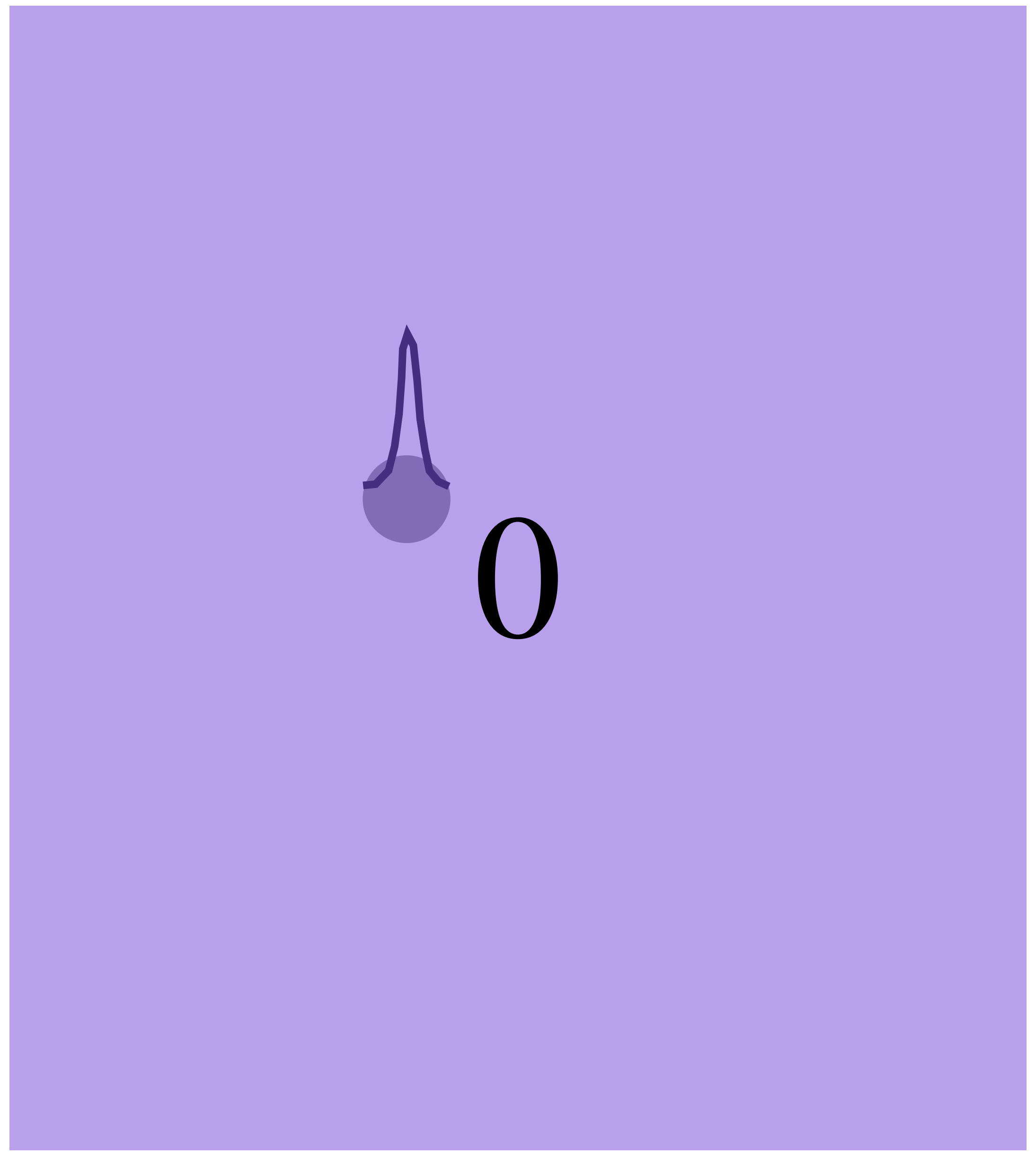
electron

$-e$

EPF

Two Basic Quantum Systems

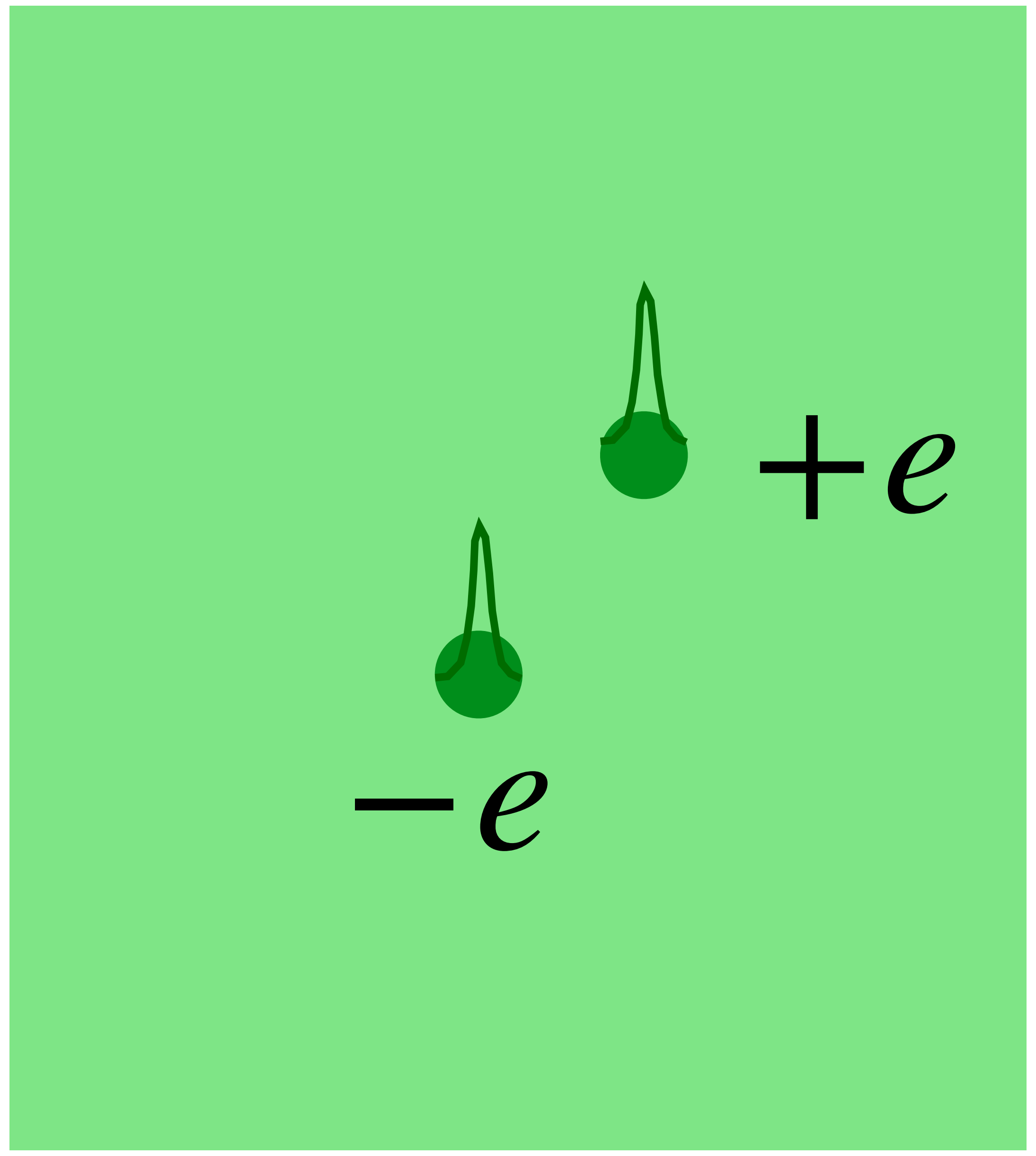
Electromagnetic and Electron-Positron <u>Fields</u>



EMF

Interact = Influence each other.

Can produce each other.



EPF

Extended “objects”/entities.

Exist ALWAYS and EVERYWHERE

Two Basic Quantum Systems

Electromagnetic and Electron-Positron Fields

Vacuum

$$|\Psi\rangle = |0\rangle|0\rangle\cdots|0\rangle$$

EMF

Interact = Influence each other.

Can produce each other.

$$\longleftrightarrow$$

Vacuum

$$|\Phi\rangle = |0\rangle|0\rangle\cdots|0\rangle$$

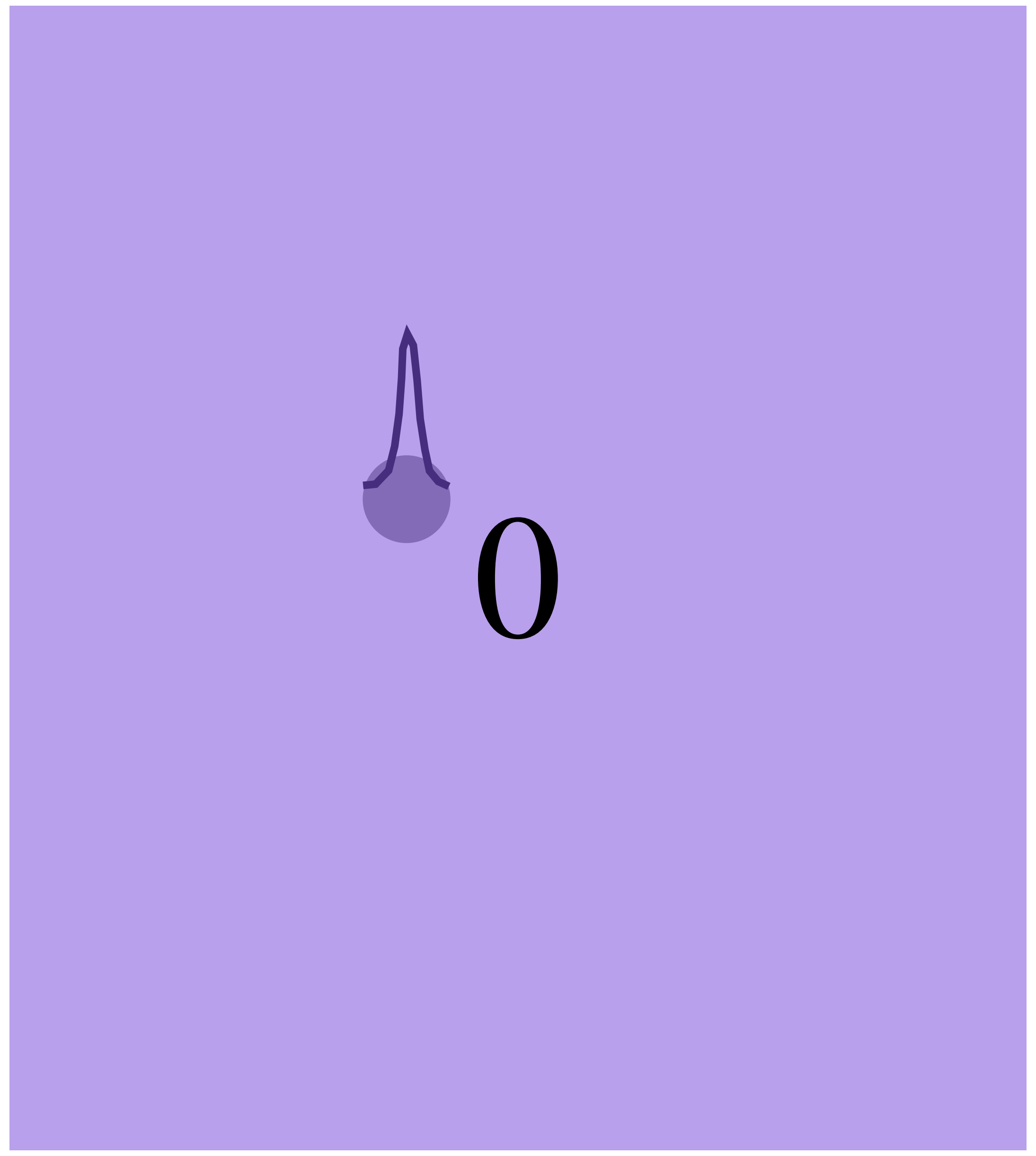
EPF

Extended "objects"/entities.

Exist ALWAYS and EVERYWHERE

Two Basic Quantum Systems

Electromagnetic and Electron-Positron Fields



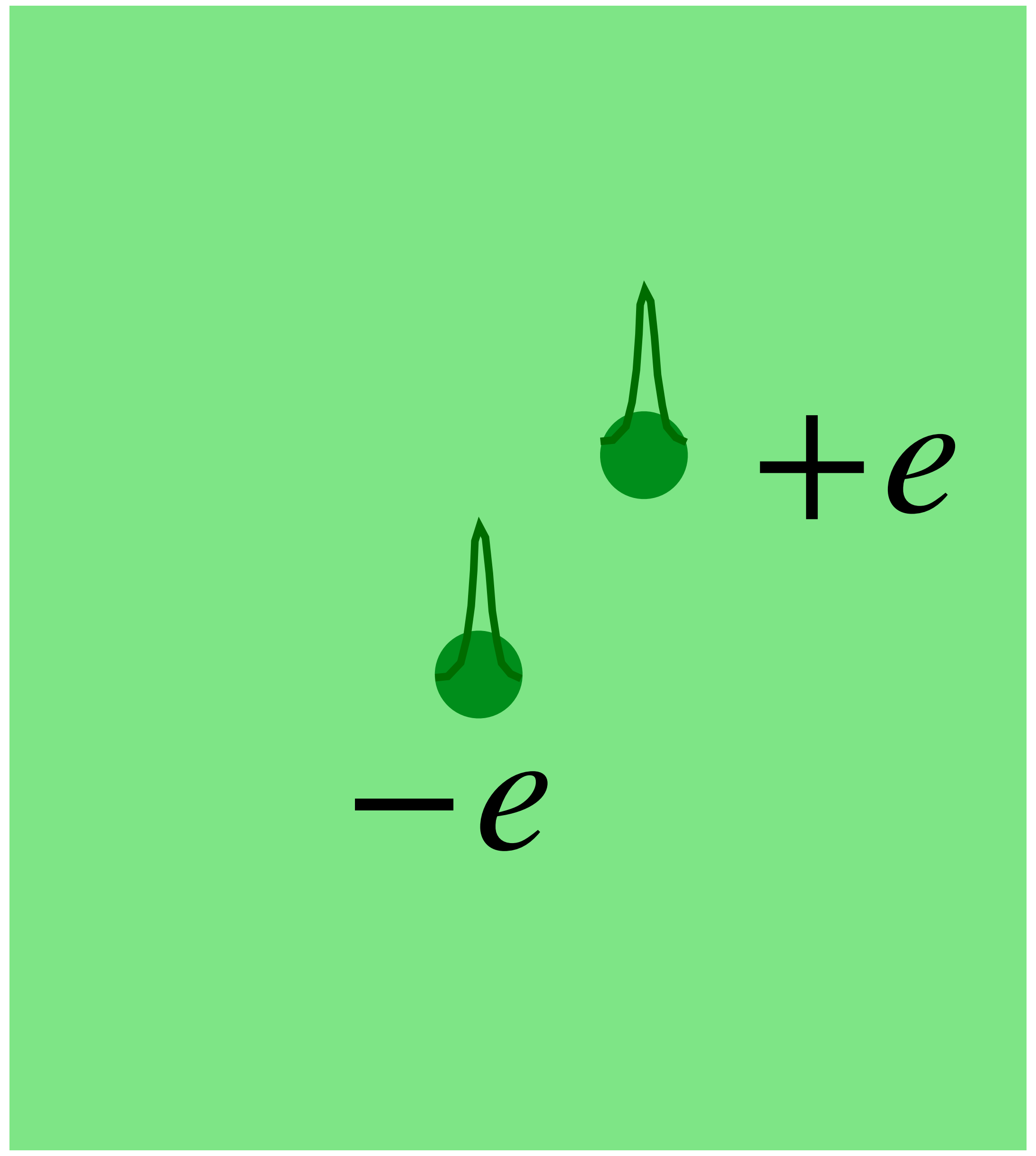
EMF

Interact = Influence each other.

Can produce each other.

Broad field:

Light-matter interaction



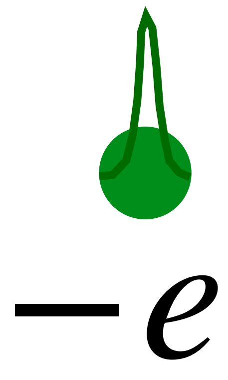
EPF

Extended "objects"/entities.

Exist ALWAYS and EVERYWHERE

Moving Particle

Isolated Quantum System



Has a fixed, well defined energy E

$$i\hbar\partial_t|\psi\rangle = \hat{H}|\psi\rangle$$

$$\hat{H}|\psi\rangle = E|\psi\rangle$$

$$E^2 = p^2 + m^2c^4$$
$$E = pv$$

Relativistic relationship (recall HW problem with Hamiltonian $H^2 = p^2 + m^2$

From HW problem: $|\psi_t\rangle = e^{-iEt/\hbar}|\psi_0\rangle$

$$|\psi_t\rangle = e^{-ipvt/\hbar}|\psi_0\rangle$$

$$|\psi_x\rangle = e^{-\boxed{ipx/\hbar}}|\psi_0\rangle$$

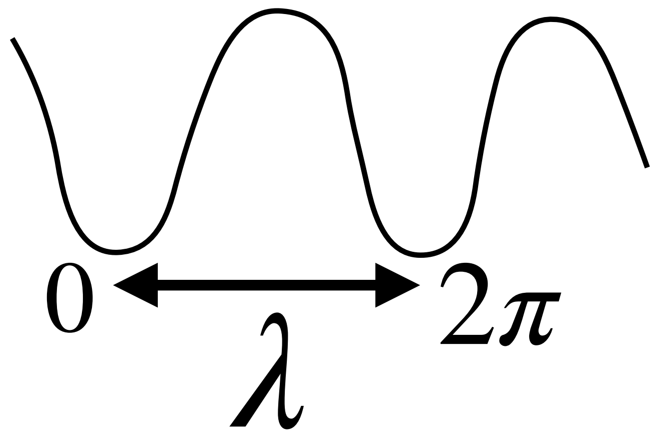
"Particles" accumulate phase as they "move".

$$x = vt$$

Moving Particle

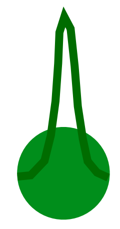
Louis De Broglie Idea

$$p\lambda = h$$



$$|\psi_x\rangle = e^{-ipx/\hbar}|\psi_0\rangle$$

De Broglie wavelength is NOT related to the particle size, and depends on the momentum/energy!



$-e$

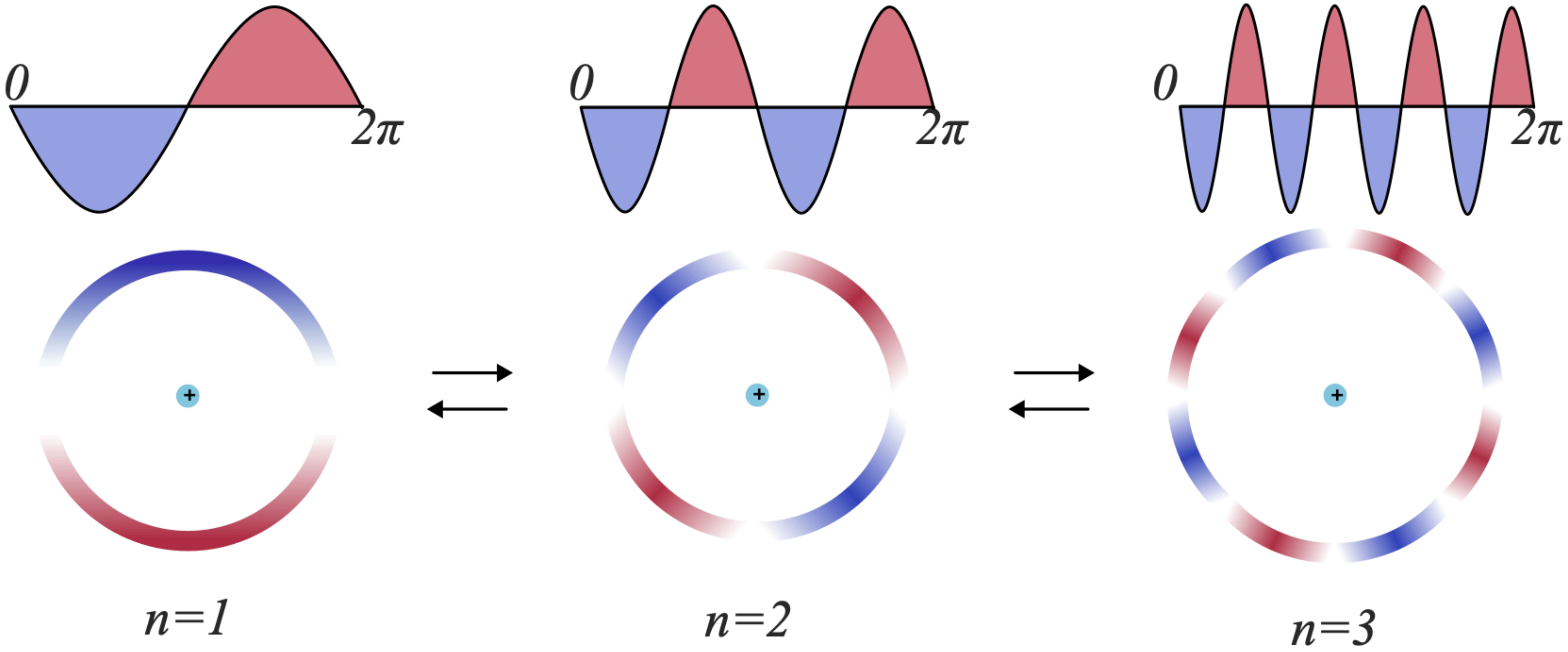
“Particles” accumulate phase as they “move”.

This periodic behavior is associated with “waves”

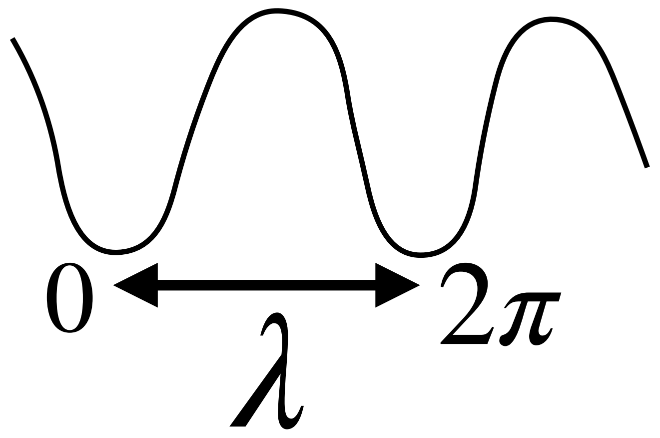
Phase rules interference patterns.

Moving Particle

Louis De Broglie Idea Applied To Hydrogen



$$p\lambda = h$$

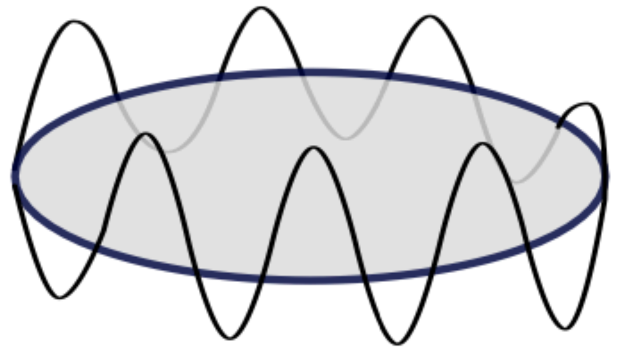


$$n\lambda = 2\pi r$$

$$nh/p = 2\pi r$$

$$n\hbar = pr$$

Bohr quantization rule



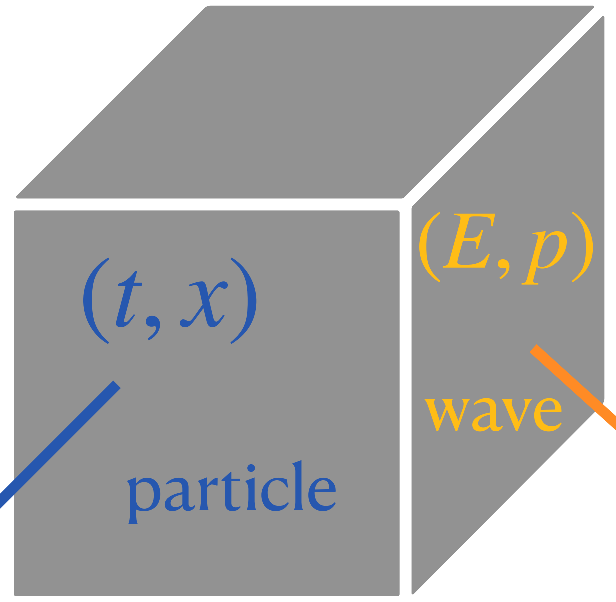
Standing waves —

confined field.

Complementarity

Niels Bohr Big Idea

Complementarity: any given application of classical concepts precludes the simultaneous use of other classical concepts which in a different connection are equally necessary for the elucidation of the phenomena. BOHR (1934), p. 10



Spacetime description

(t, x), localized, "particles"

Momentum and energy,

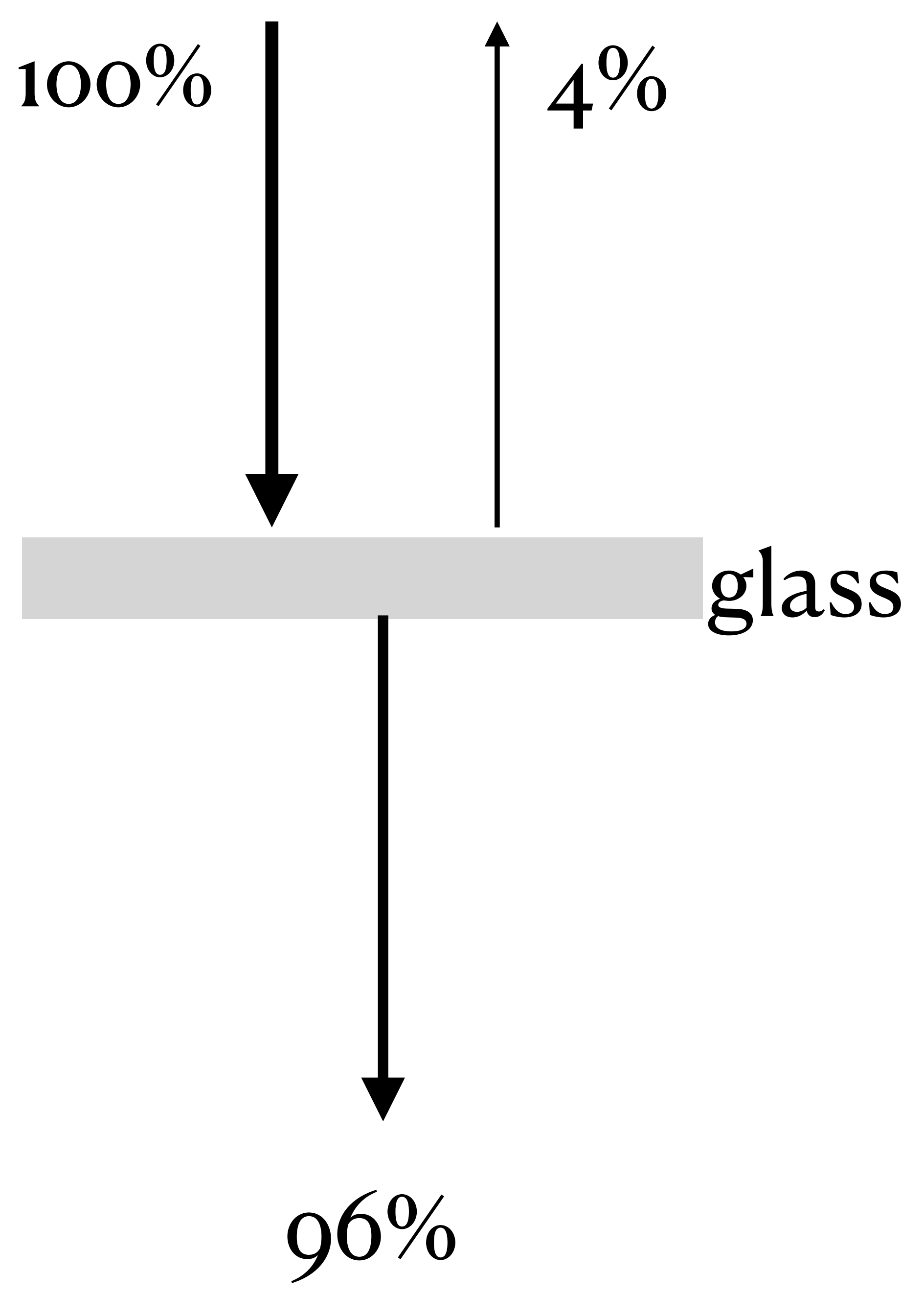
(E, p), delocalized, "waves"

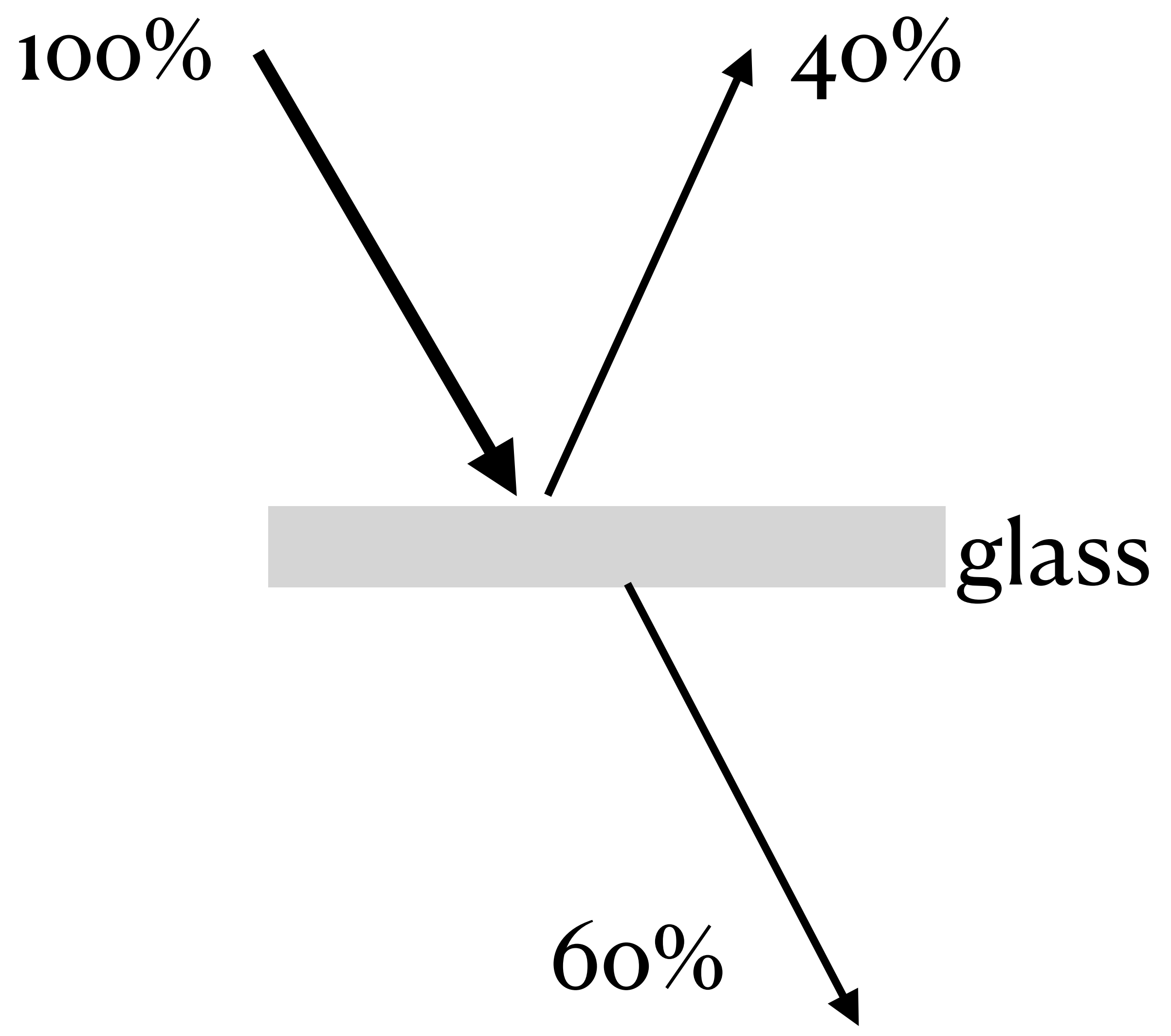
Fourier transform

Particle view and wave view are not mutually exclusive, but mutually complementary concepts. Experimental arrangements are exclusive.

Beam Splitter

Basic Interference Device

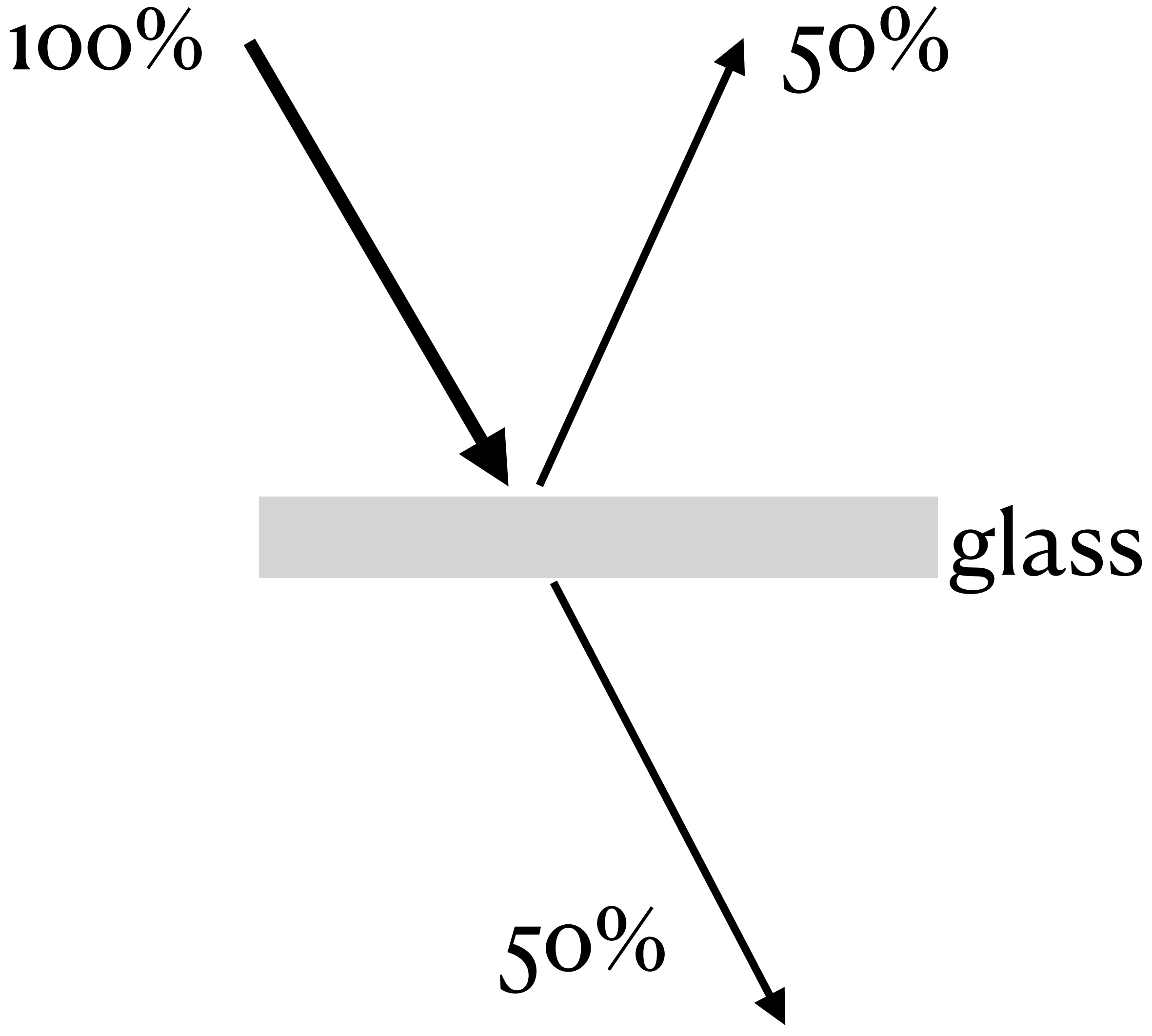


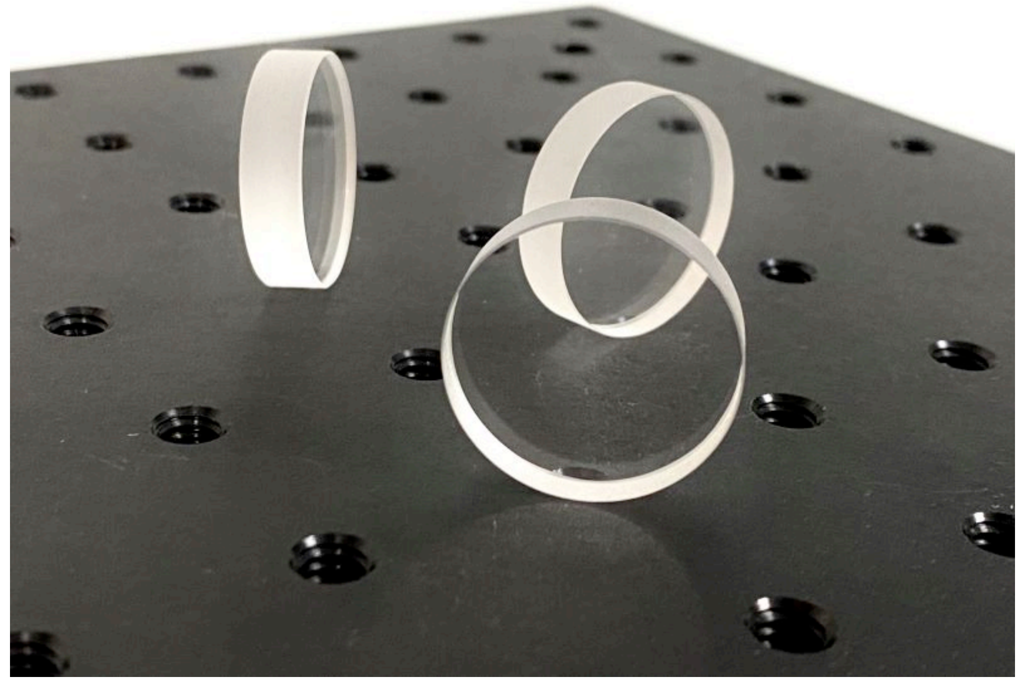


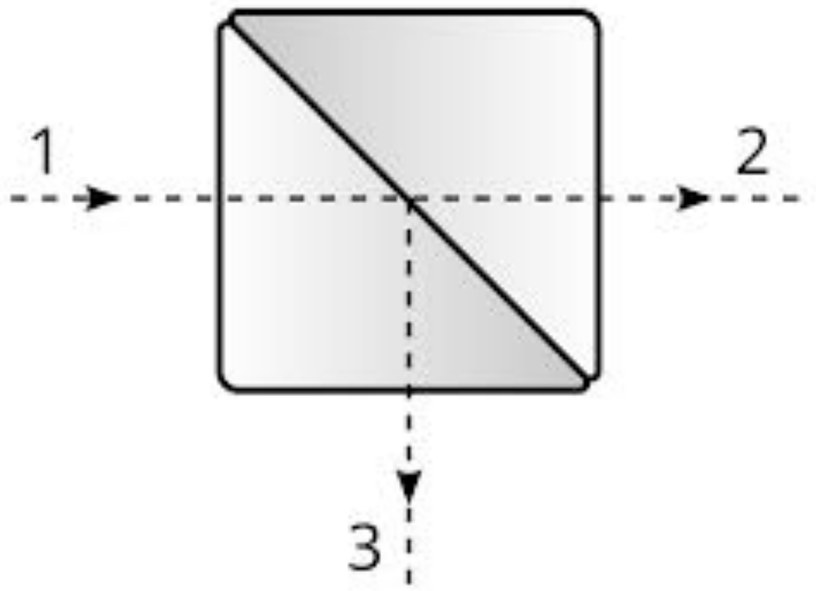
Transparent materials, like glass, transmit and reflect different amount of light, based on the material property (refractive index) and the angle of incidence.

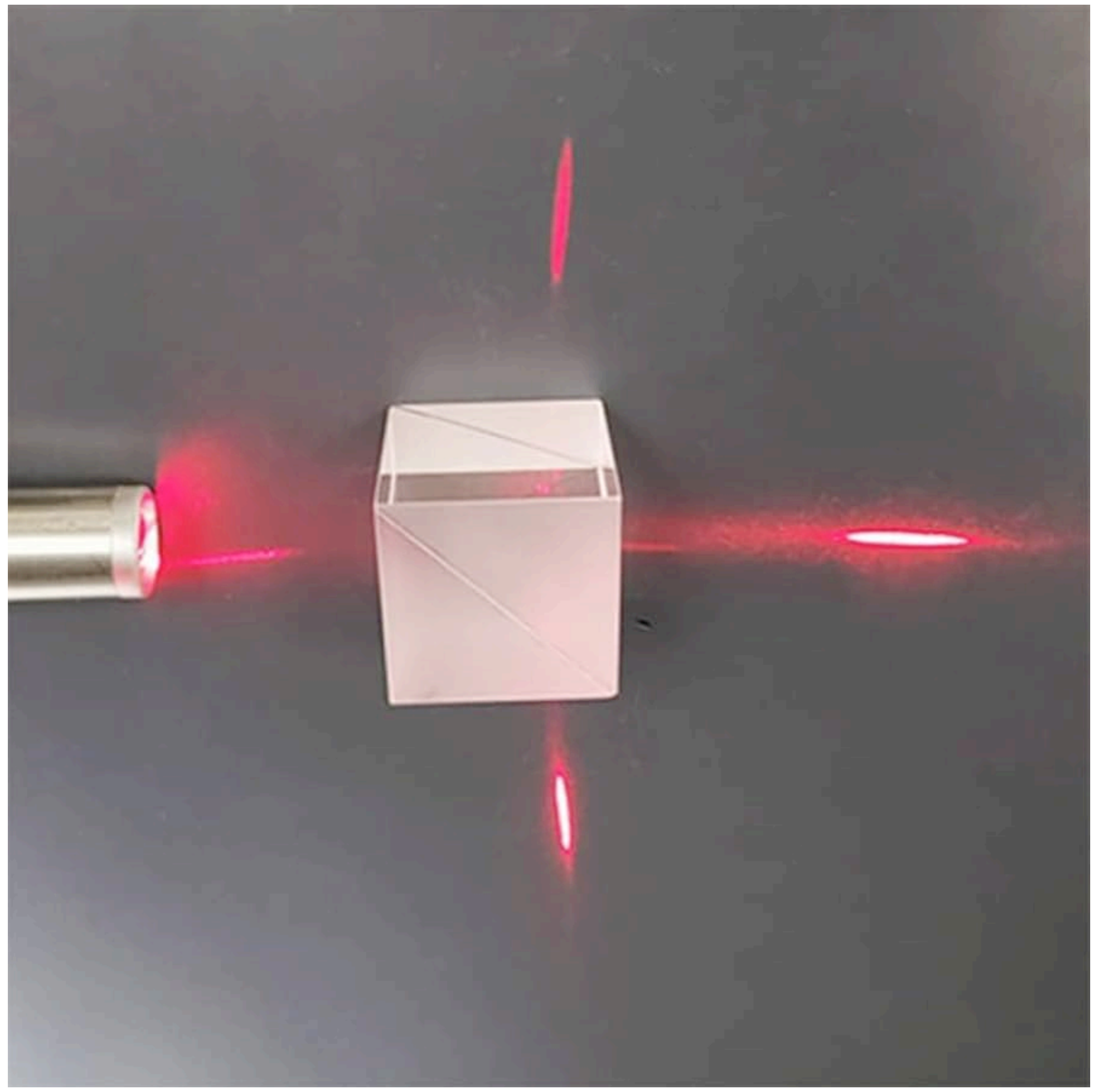
Beam Splitter

50/50





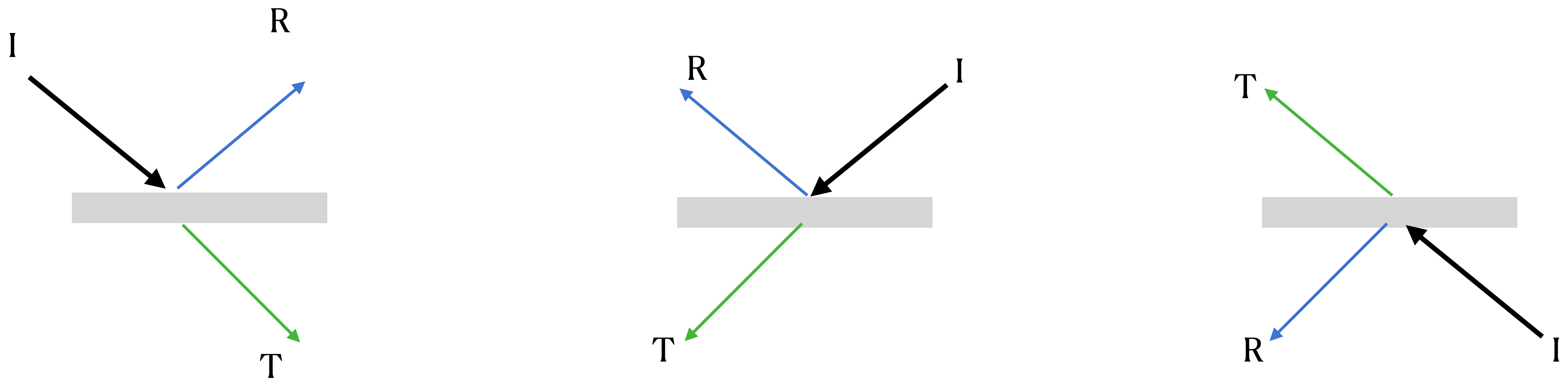




50/50 splitters are important in optical experiments. Plates and cubes are used.

Beam Splitter

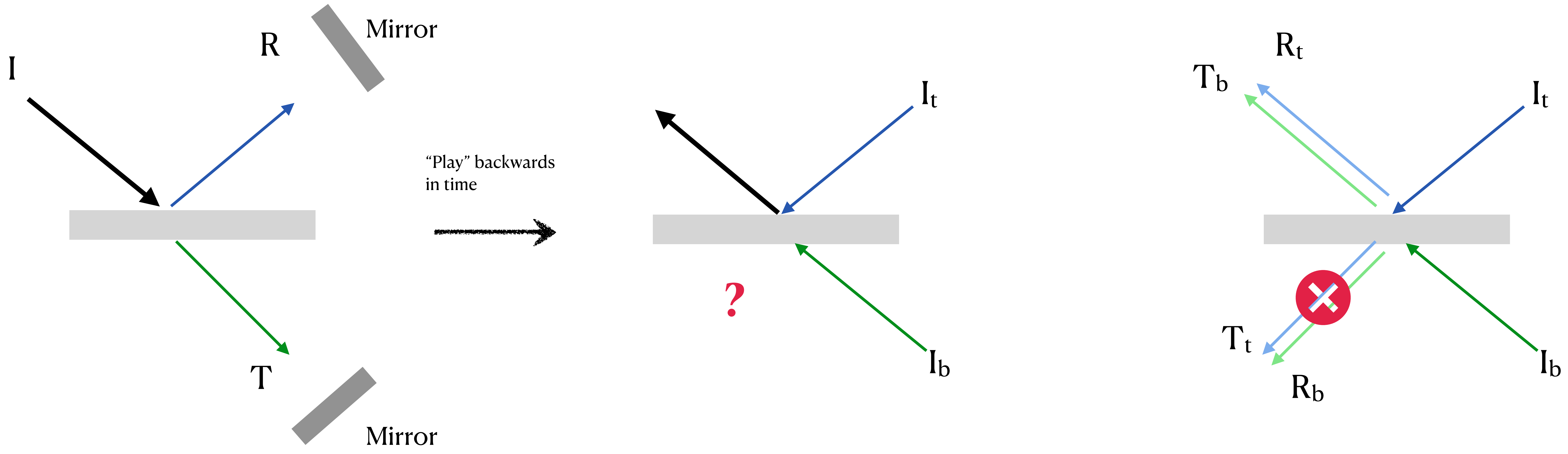
Sending Signal From Different Sides



Basic physical theories, like electromagnetism (and therefore optics) and quantum physics behave the same regardless of the direction of time. “Movies” can be played backwards to show equally valid physical process. No “arrow of time” or preferred time direction in basic equations of physics.

Beam Splitter

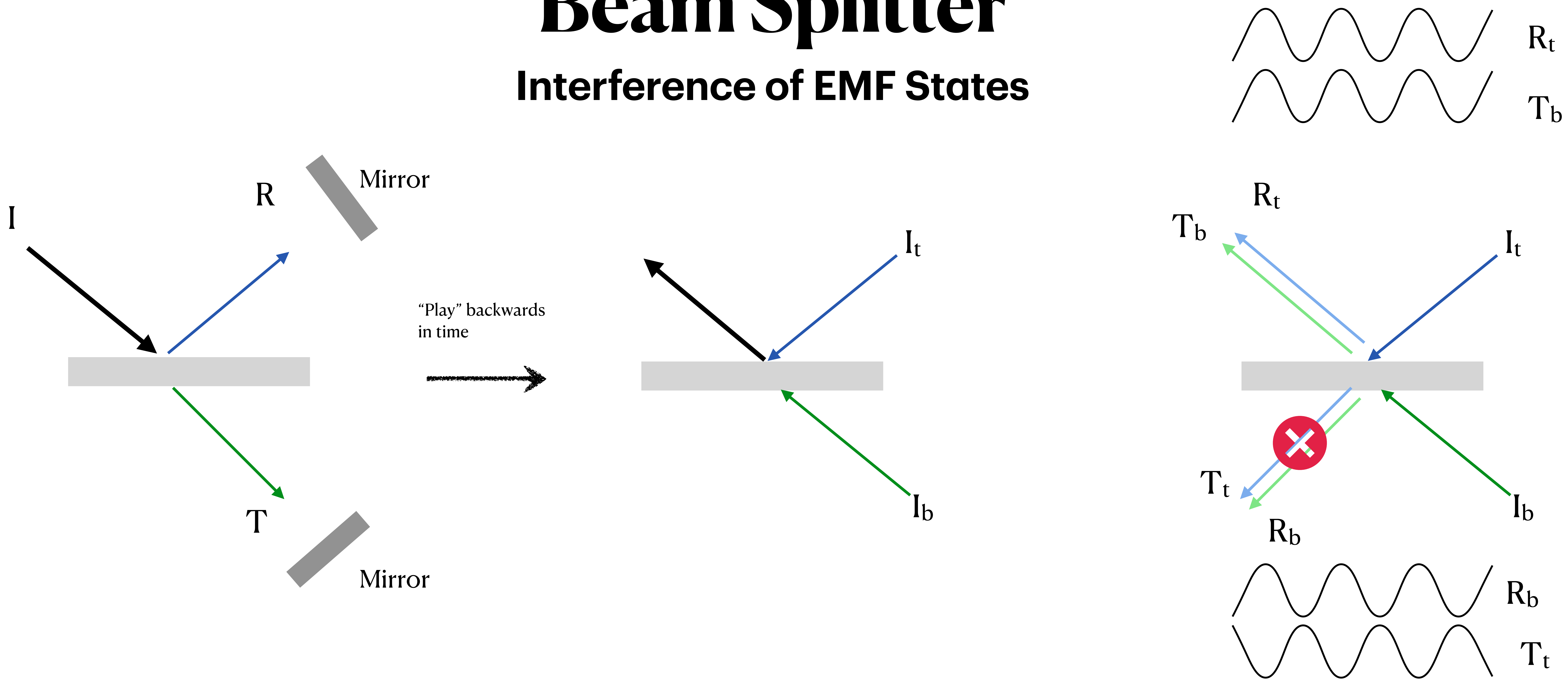
“Playing” Process Backwards



Basic physical theories, like electromagnetism (and therefore optics) and quantum physics behave the same regardless of the direction of time. “Movies” can be played backwards to show equally valid physical process. No “arrow of time” or preferred time direction in basic equations of physics.

Beam Splitter

Interference of EMF States

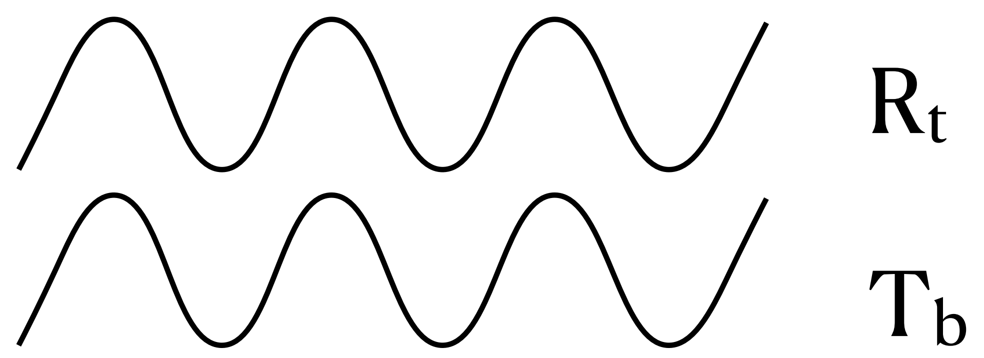


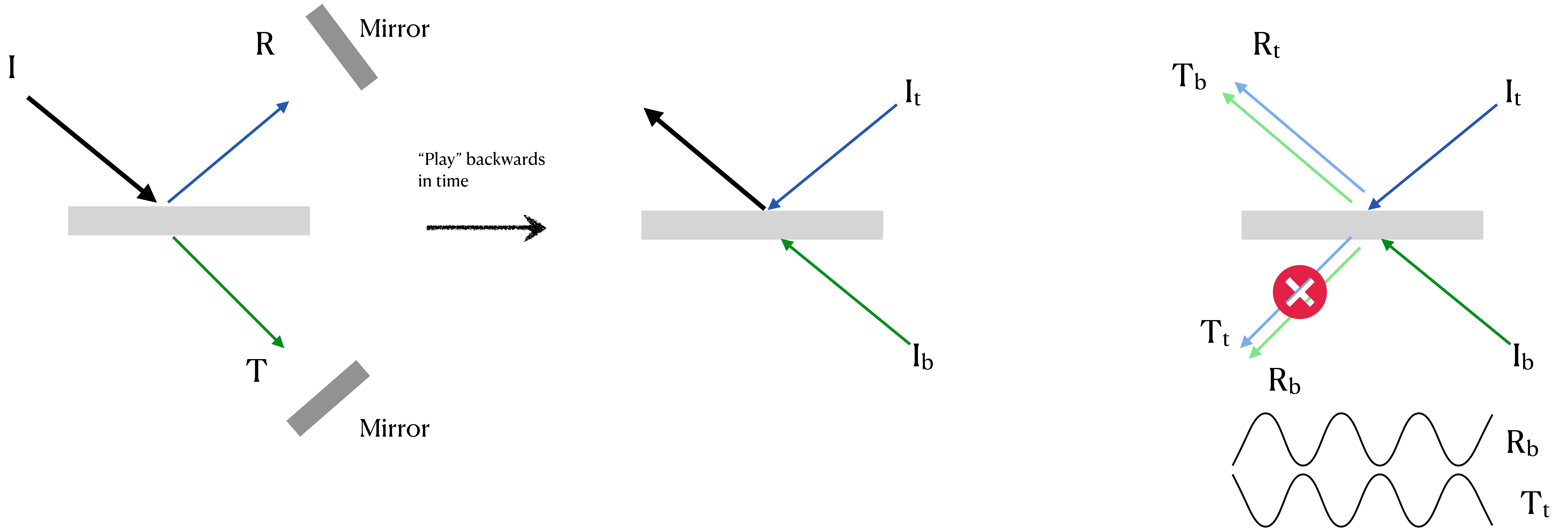
Light (electromagnetic wave) not only has *energy*, but also ***phase***. Phase varies in time and in space and leads to *interference* (***constructive*** and ***destructive***).

Beam Splitter

Interference of EMF States

Hong-Ou-Mandel (HOM) interference when single photons are incident.

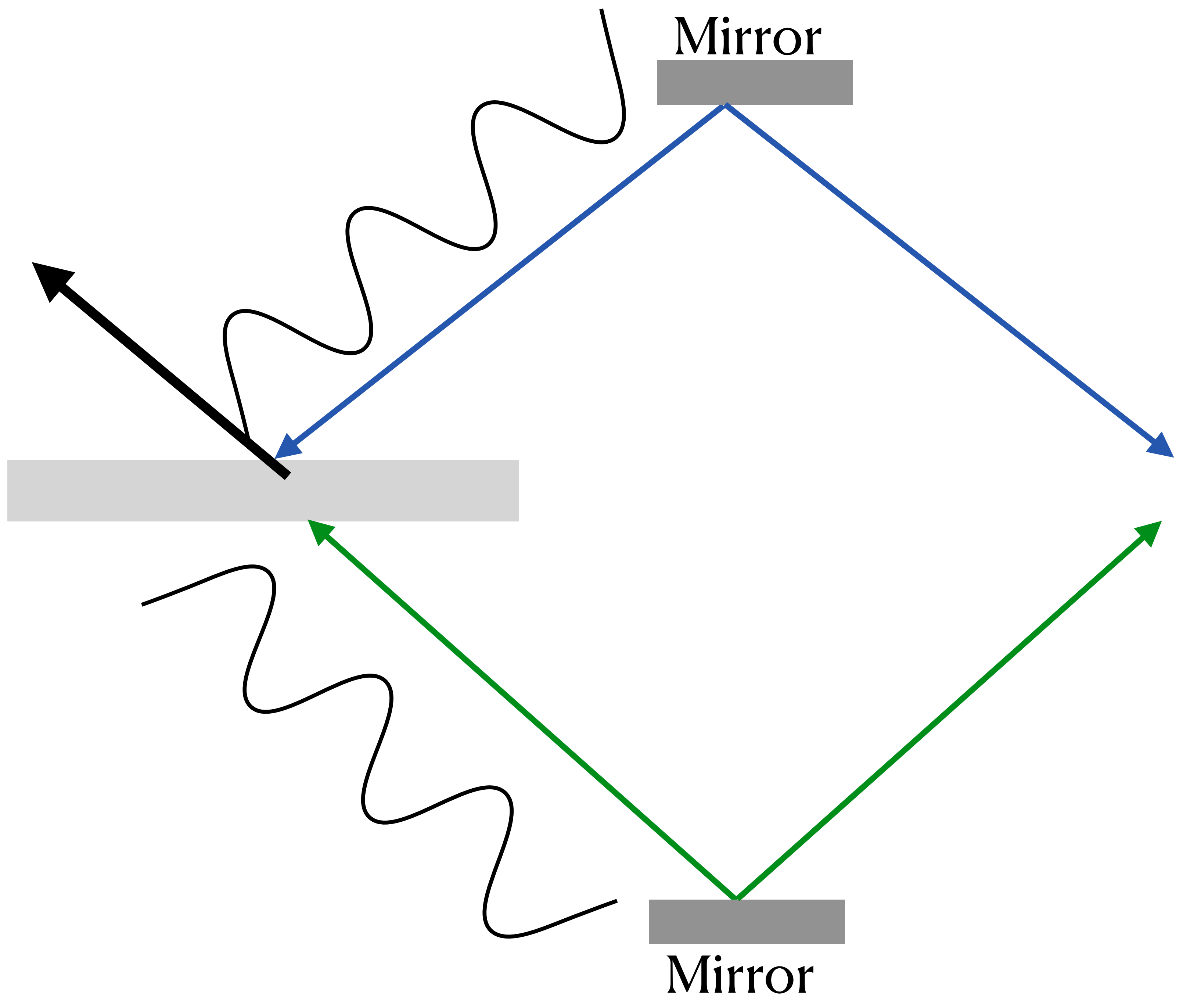




Light (electromagnetic wave) not only has *energy*, but also ***phase***. Phase varies in time and in space and leads to *interference* (***constructive*** and ***destructive***).

Beam Splitter

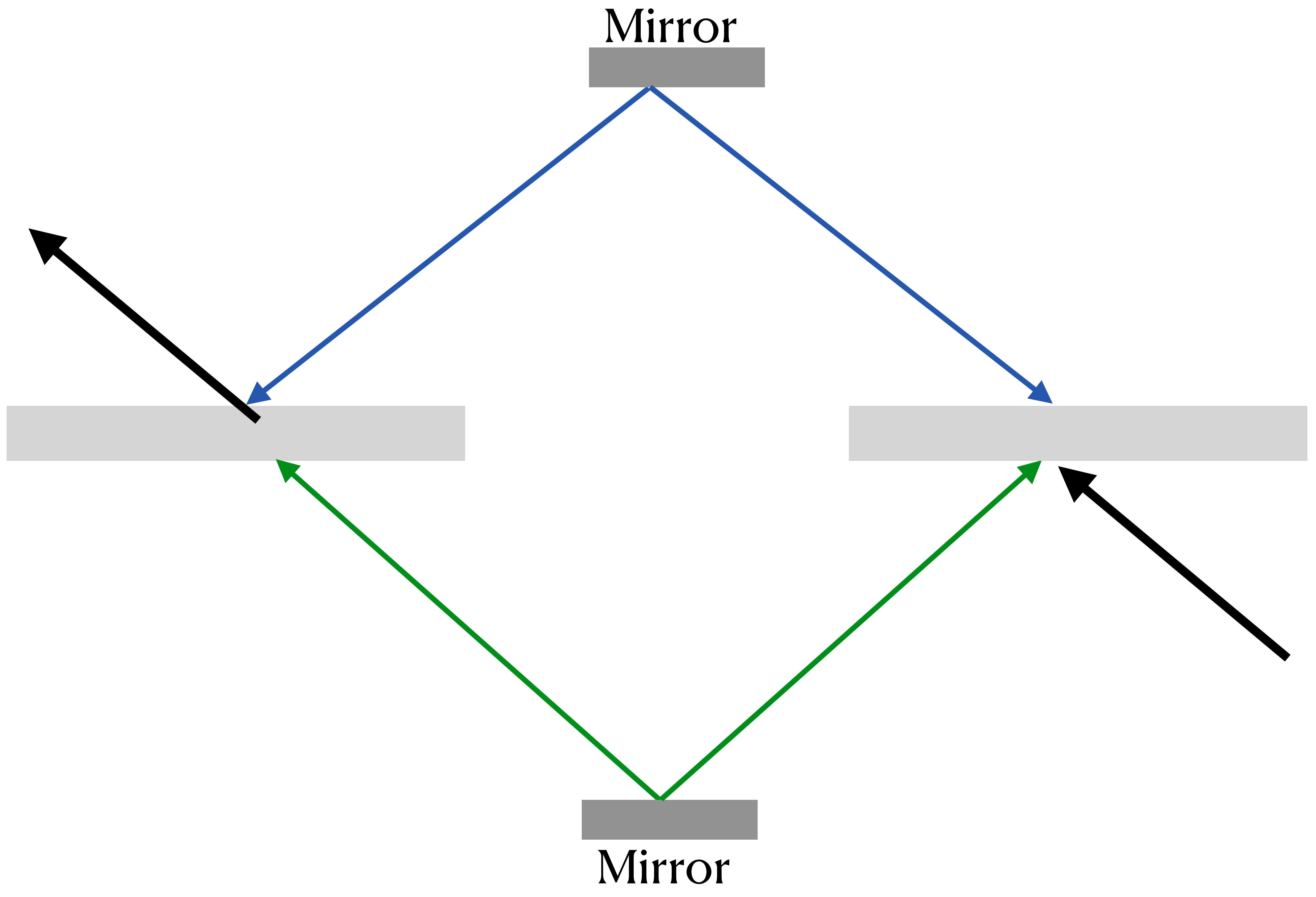
Interference of EMF States

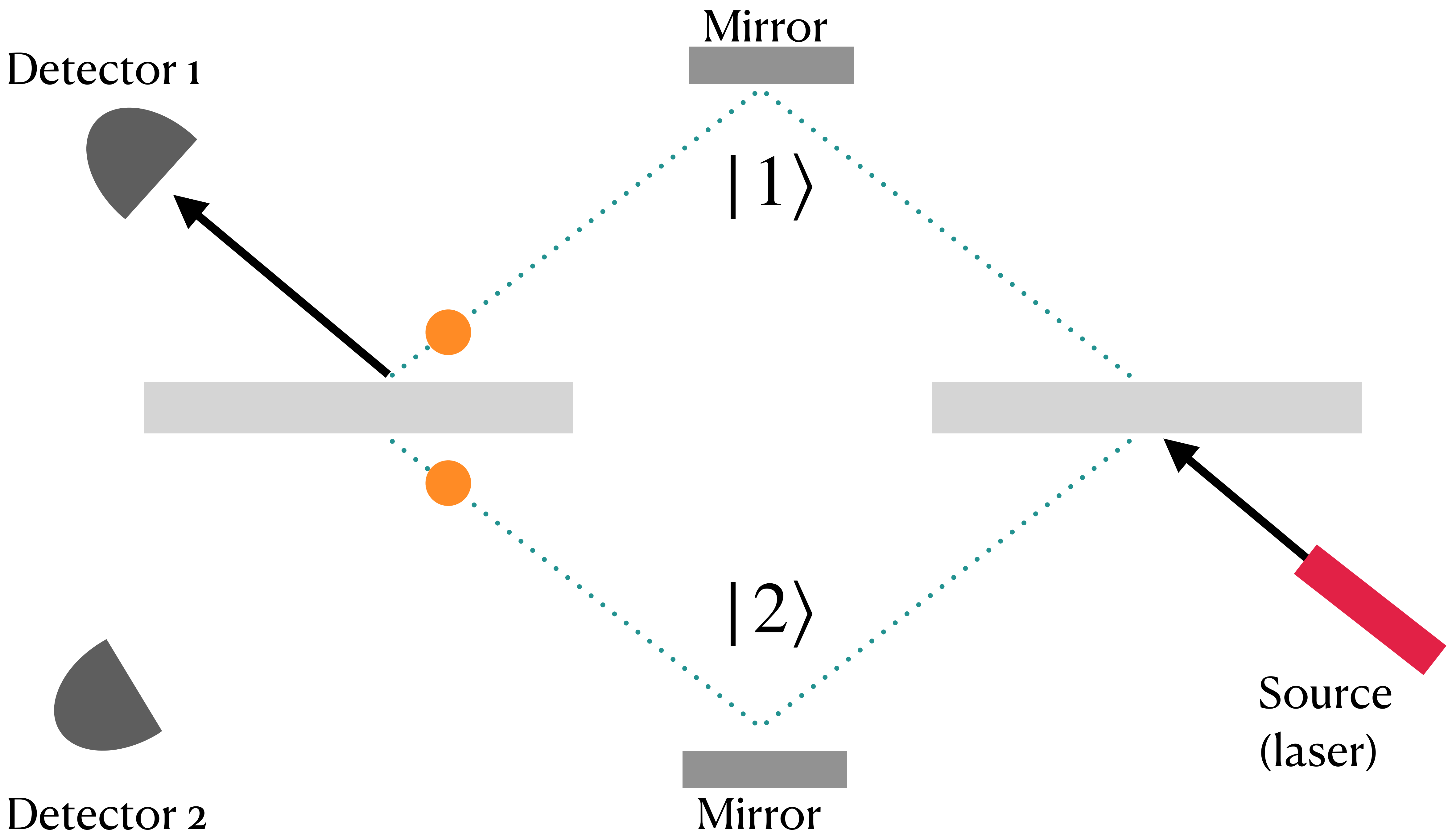


Interference of large number of EMF-excitations (photons) — simple classical picture.

Beam Splitter

Interference of EMF States

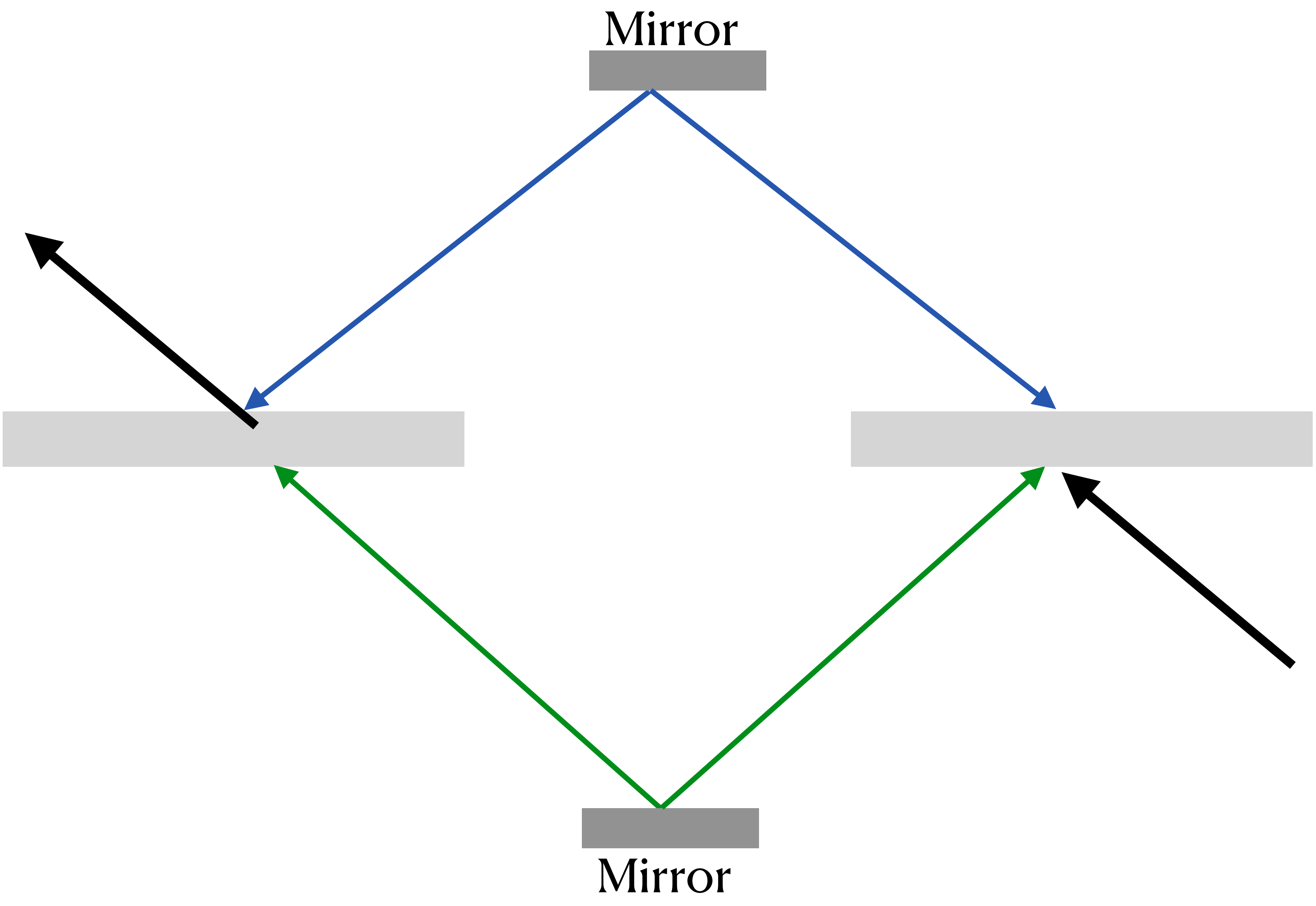


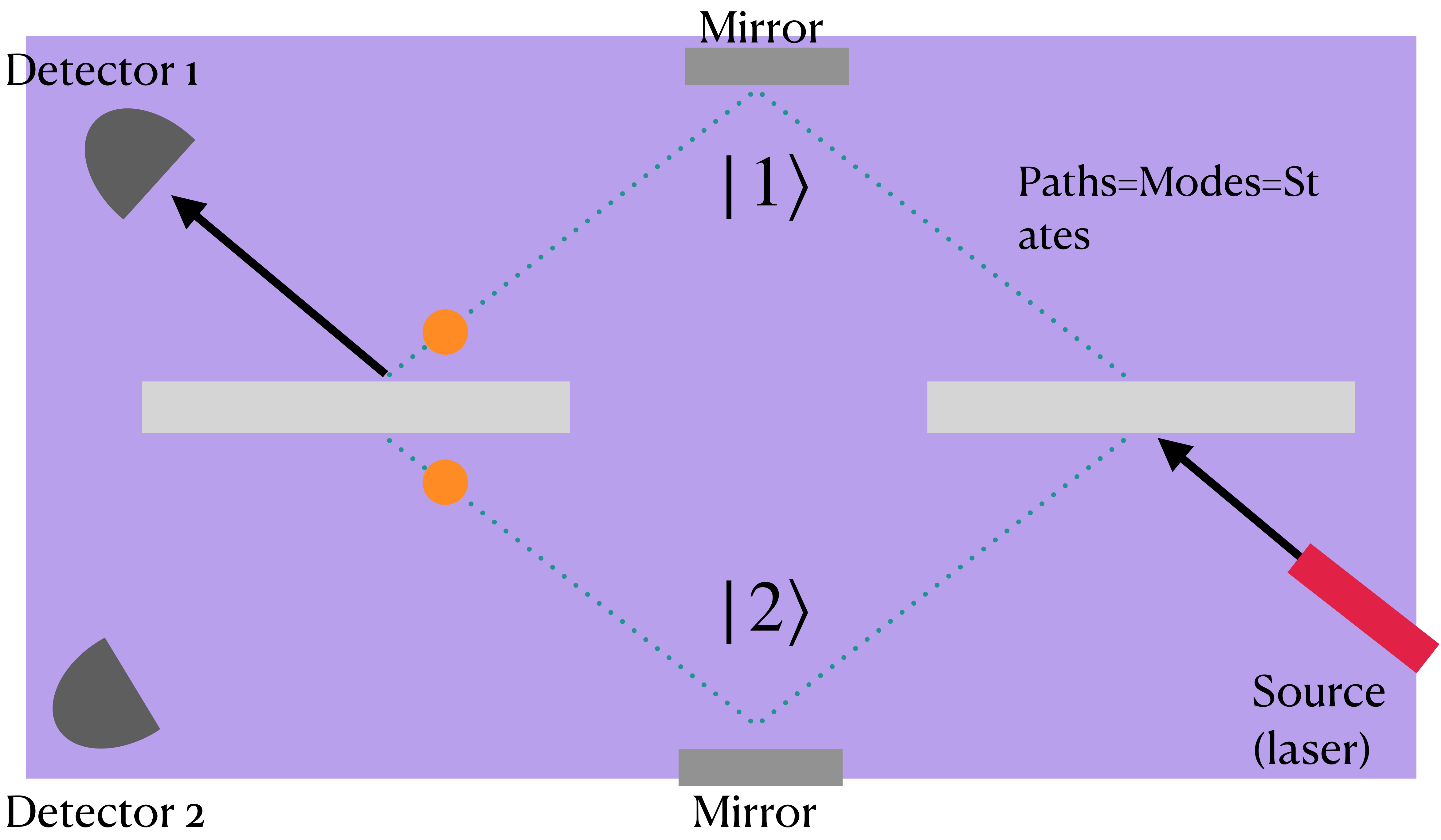


But what happens when detection rate is 1 excitation/photon per interferometer length?

Beam Splitter

Interference of EMF States

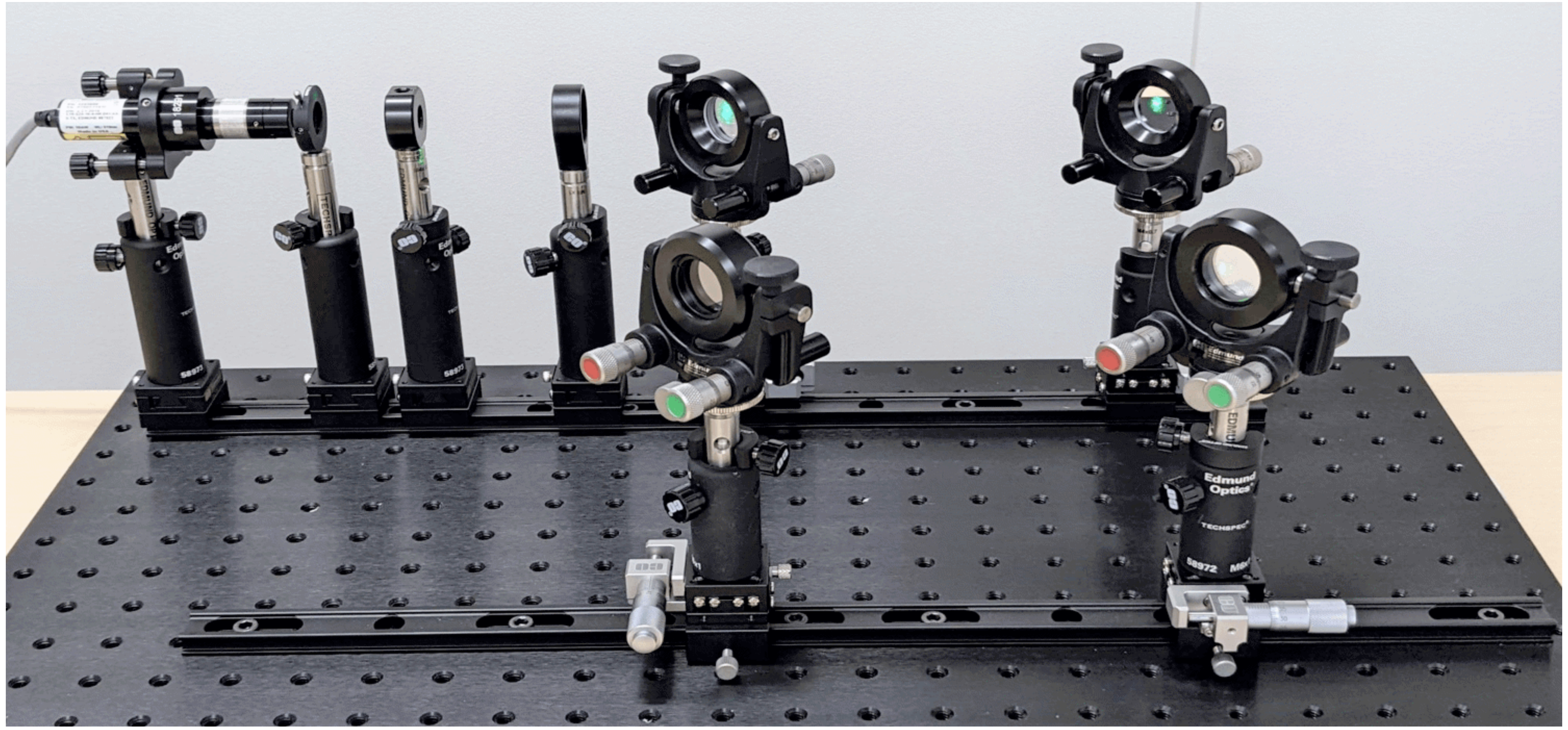


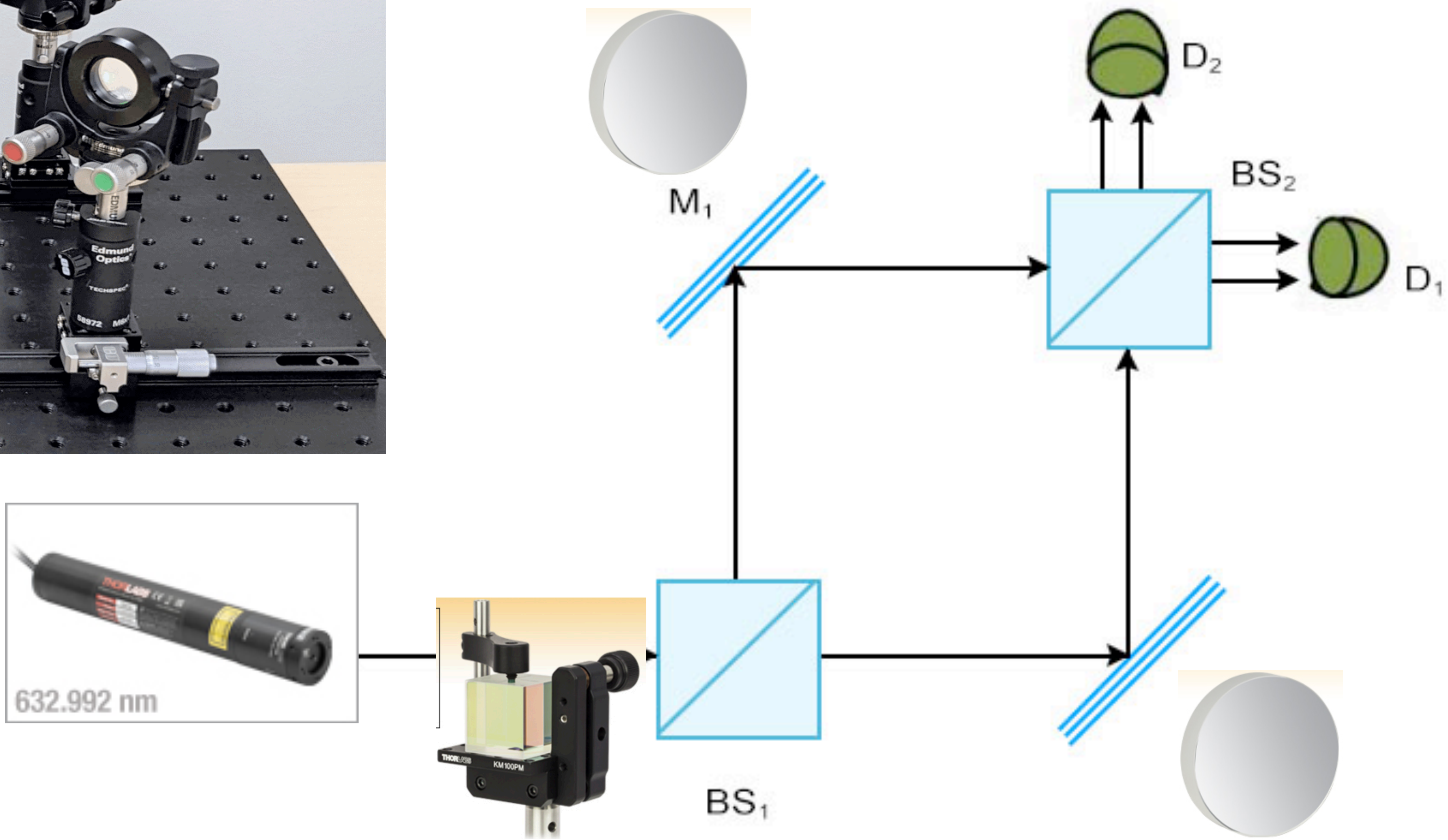


But what happens when detection rate is 1 excitation/photon per interferometer length?
Tracking localized photon like a particle — does not make sense according to quantum physics!

Interference

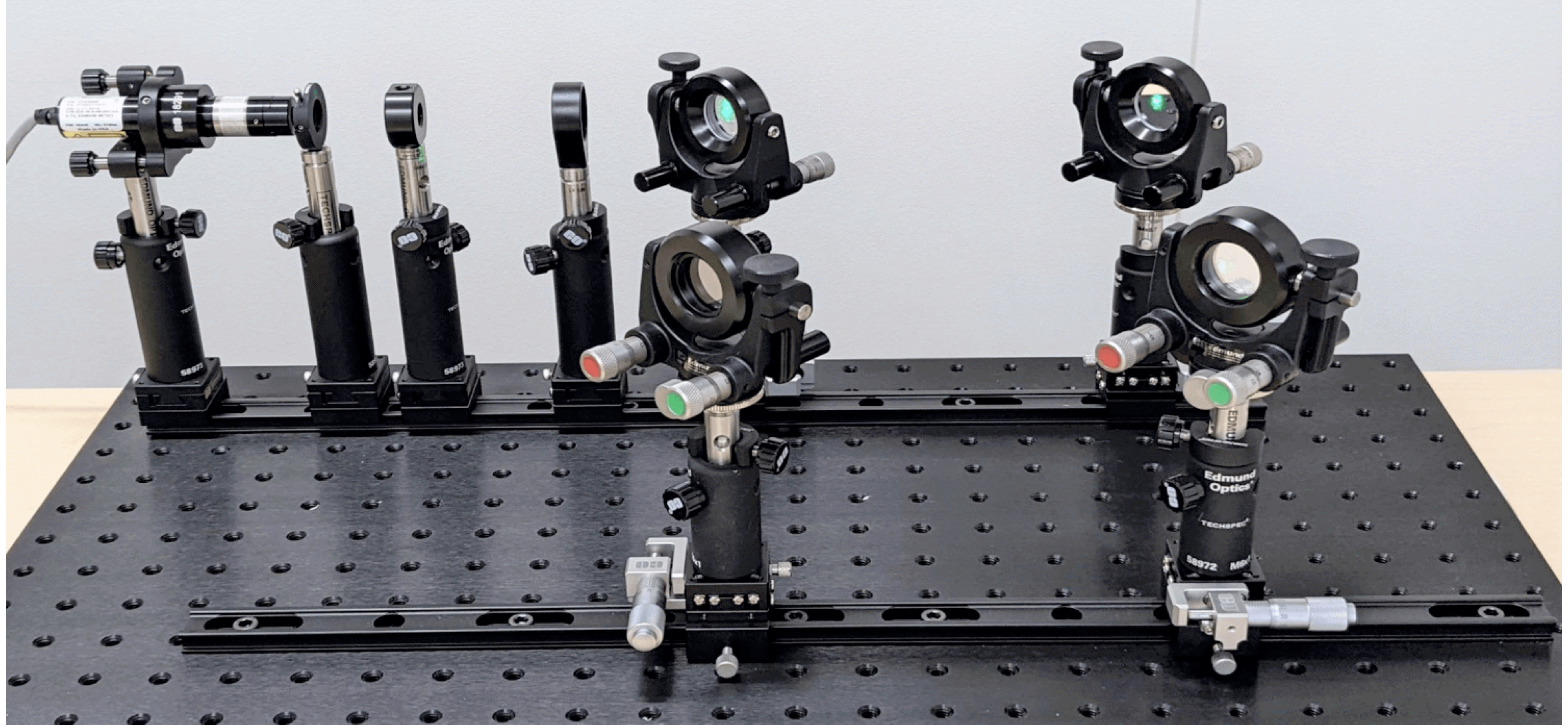
Mach-Zehnder Interferometer

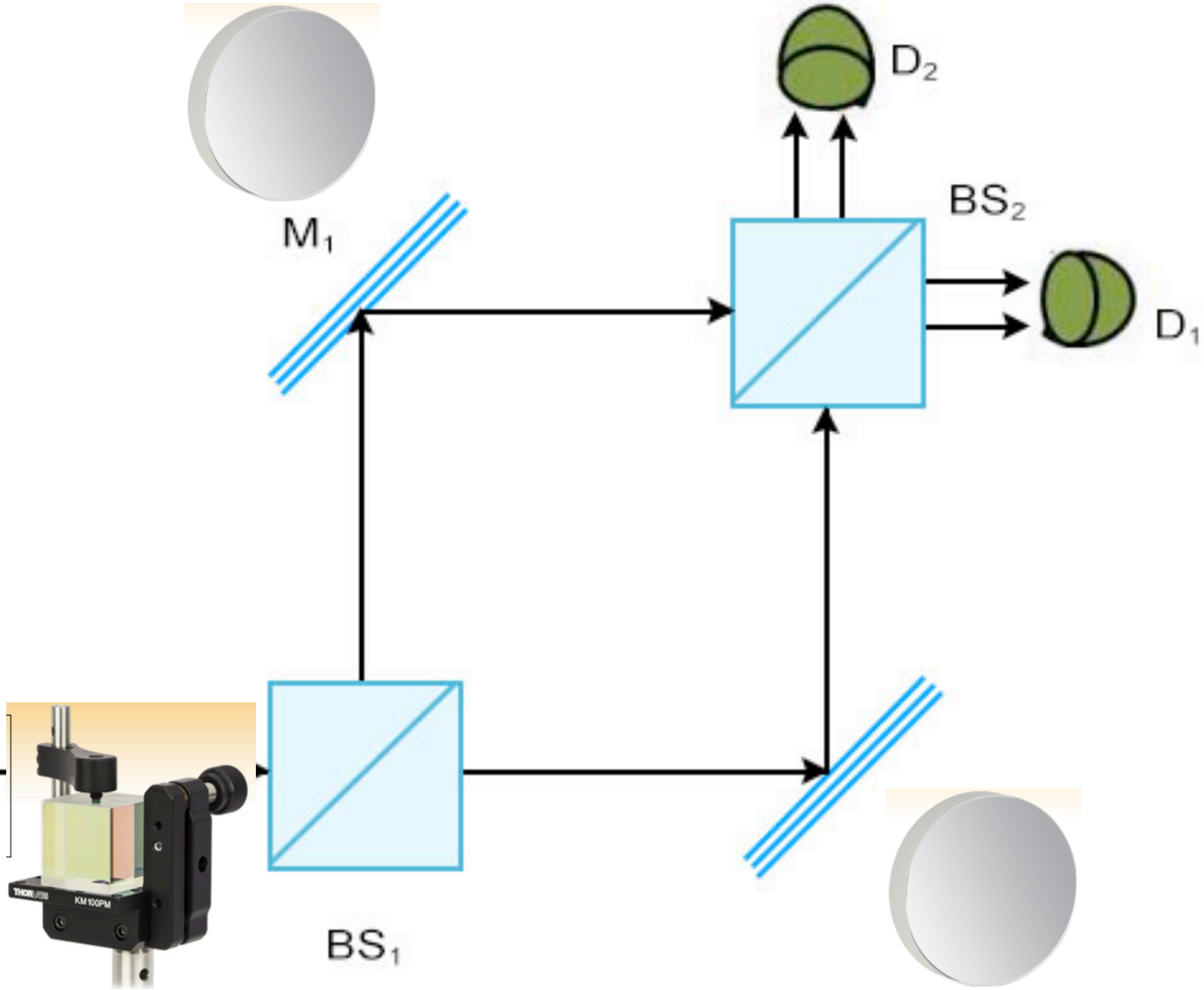




Interference

Mach-Zehnder Interferometer



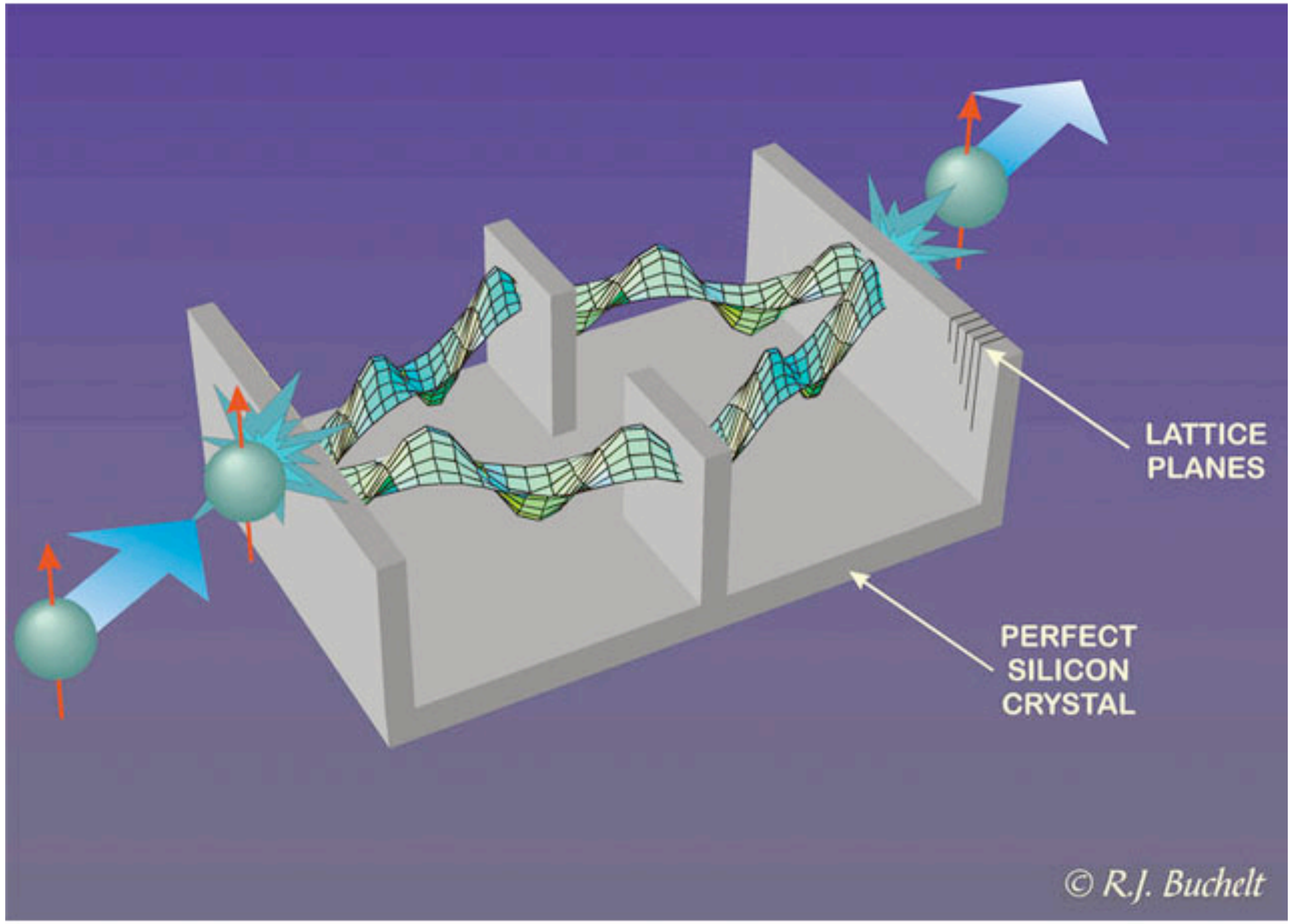


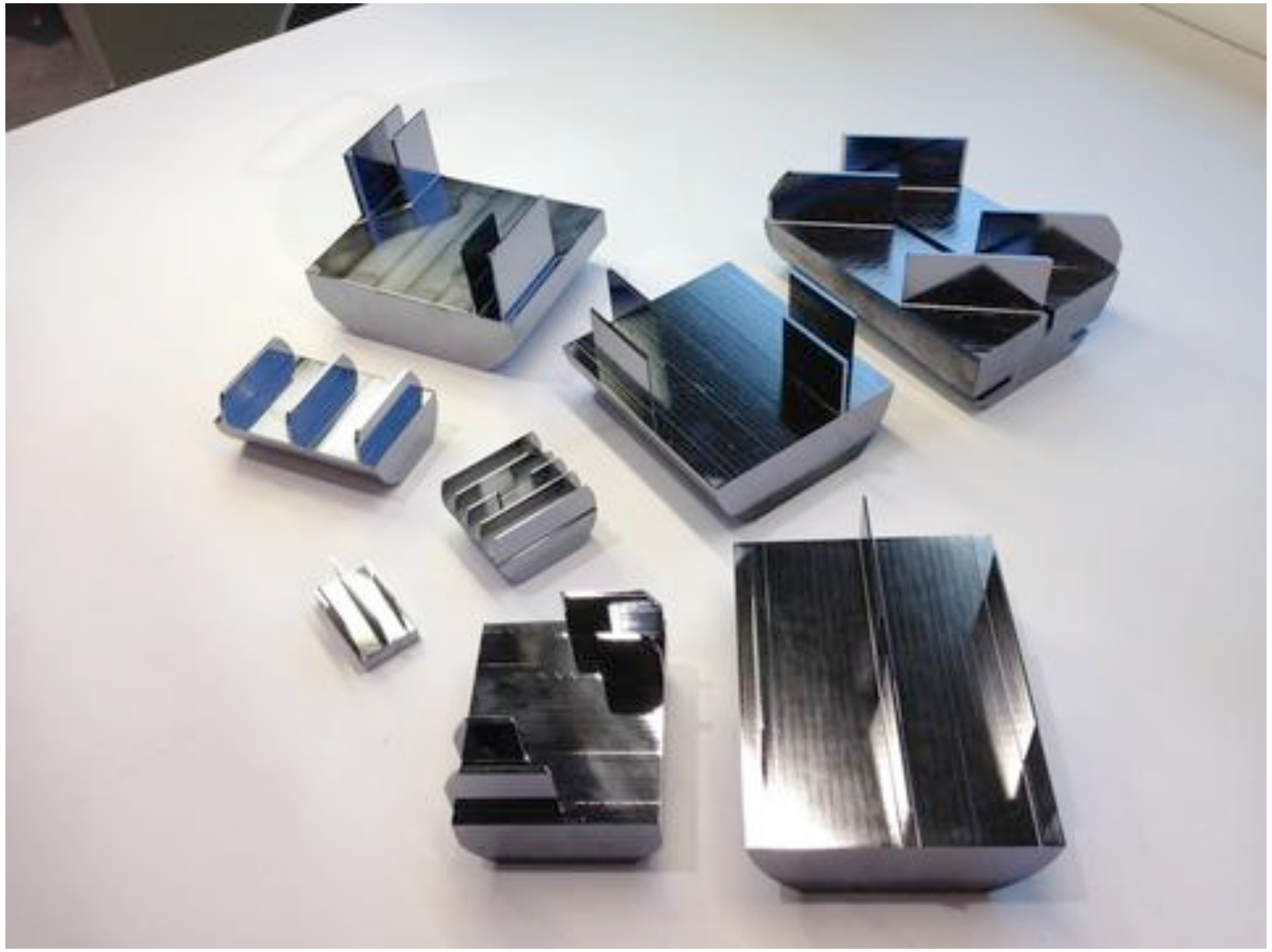
It is wrong to say that the photon travels one route and it is equally wrong to say it travels both routes. It is wrong to speak of "what it is doing" between the point of entry and the point of registration.

J.A. Wheeler, *"Mystery and the message of the quantum"*

Interference

Mach-Zehnder Interferometer

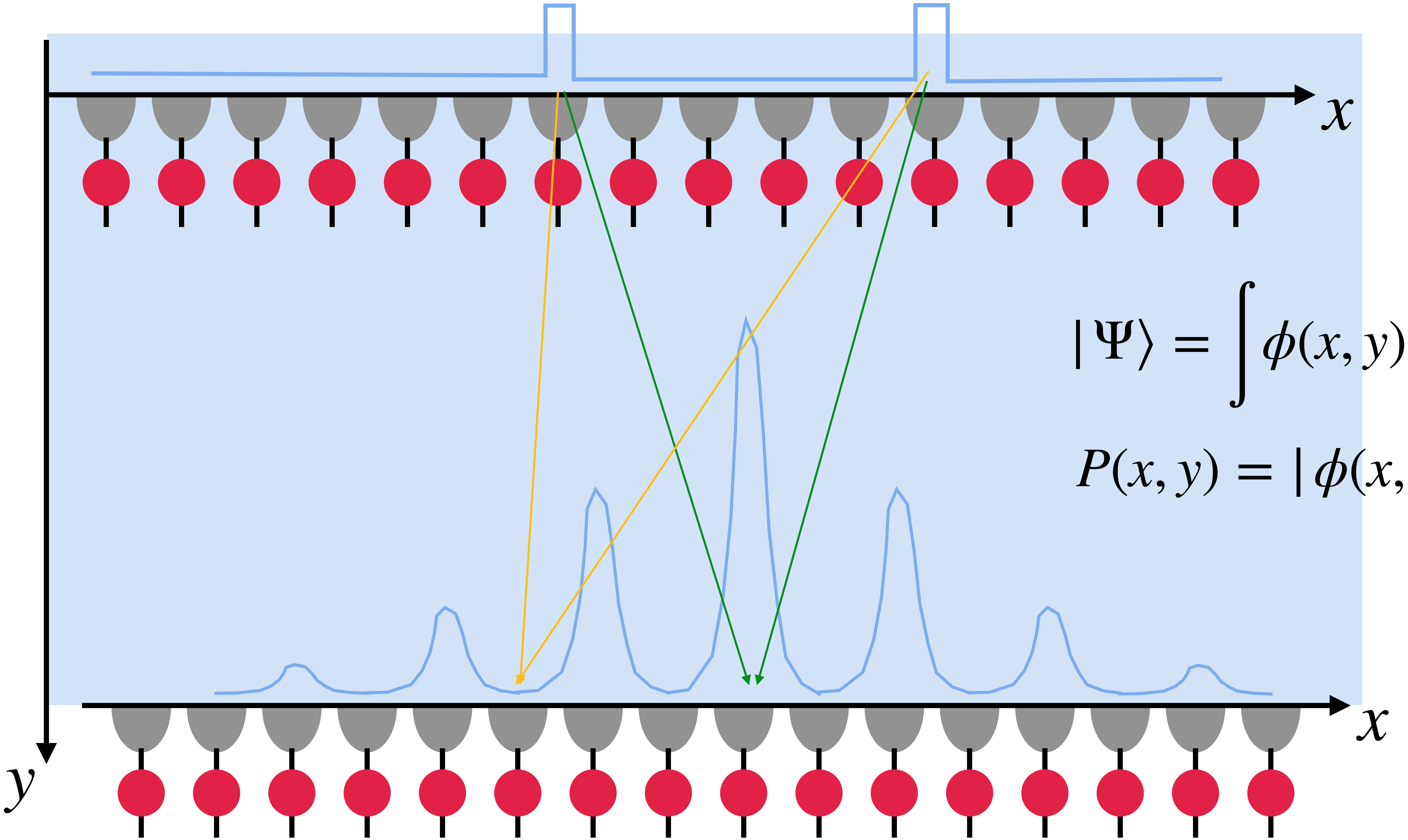




https://www.nist.gov/laboratories/tools-instruments/neutron-interferometry-and-optics-facilities

Two-Slit Experiment

Field-Based Understanding



$$|\Psi\rangle = \int \phi(x,y)\,|x\rangle$$

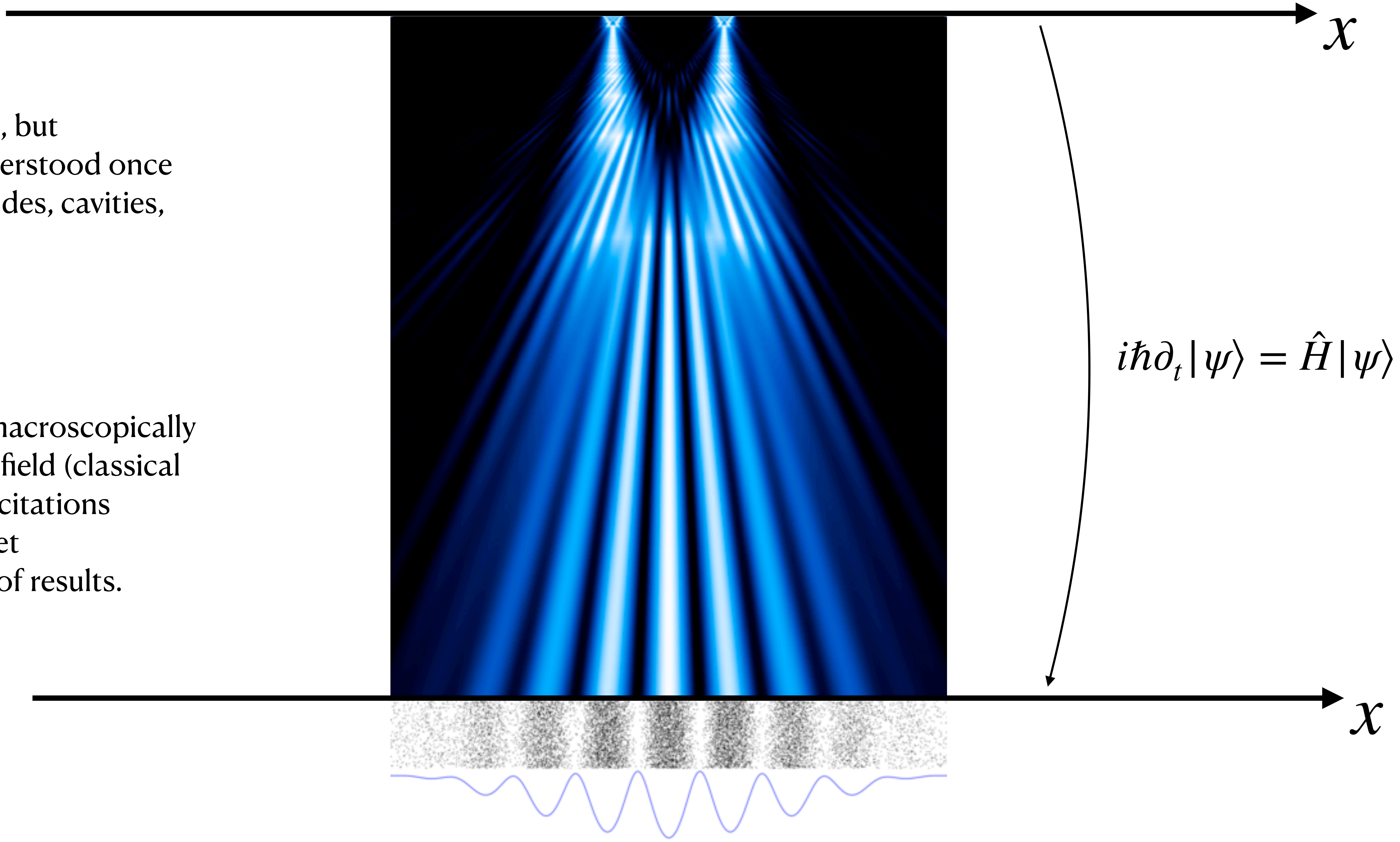
$$P(x,y) = |\phi(x,y)|^2$$

Two-Slit Experiment

Field-Based Understanding

Double slit is interesting, but complicated. Better understood once you learn about waveguides, cavities, and modes.

This picture works for macroscopically many excitations of the field (classical waves) and for single excitations averaged over time to get macroscopical number of results.



Self-Test

Answer These Questions 1hr After Class

1. What is superposition? Give an example of superposition in classical physics (e.g. electrostatics).
2. What is the meaning/role of the coefficients c_k in state vector decomposition (superposition) $|\Psi\rangle = c_1|\Phi_1\rangle + c_2|\Phi_2\rangle + \cdots + c_k|\Phi_k\rangle$?
3. Does the superposition mean that quantum system is in all states $|\Psi_k\rangle$ at once? ("Dead" and "alive")
4. What is the importance of the phase part of the coefficients c_k?
5. What is coherence?
6. What is the idea of De Broglie? How is it related to Bohr's quantization of angular momentum.
7. What is Mach-Zehnder interferometer?
8. What is complementarity?

Homework Problems

State Expansion And Scalar Product

1. Watch MIT video on superposition: https://youtu.be/CR-eOhdxbes
2. Watch videos on Mach-Zehnder interferometer: https://youtu.be/HTHcJjtm6TU
3. Play around with simulation: https://lab.quantumflytrap.com/lab/mach-zehnder?mode=beam
4. **Quantum Dot**: Use the idea of De Broglie to find possible states of a particle with mass m confined to a region of size L.
5. **Bloch Sphere***: Quantum system with only two quantum states is called *qubit*. Let's denote the state of a qubit as $|1\rangle$ and $|2\rangle$. An arbitrary state of a qubit can be represented in terms of the basis states as follows: $|\Psi\rangle = c_1|1\rangle + c_2|2\rangle$. Use normalization condition $|\Psi\rangle \bullet |\Psi\rangle = 1$, show graphically all possible quantum states by plotting them in Cartesian coordinates (x, y, z).
6. Read the paper on Double Slit Experiment with electrons. (Classroom)

Quantum Theory

In a Nutshell

II. POSTULATES FOR QUANTUM MECHANICS

In this paper, all state vectors are supposed to be normalized, and mixed states are represented by density operators, i.e., positive operators with unit trace. Let A be an observable with a nondegenerate purely discrete spectrum. Let $\phi_1, \phi_2, \ldots$ be a complete orthonormal sequence of eigenvectors of A and $a_1, a_2, \ldots$ the corresponding eigenvalues; by assumption, all different from each other.

According to the standard formulation of quantum mechanics, on the result of a measurement of the observable A the following postulates are posed:

(A1) *If the system is in the state ψ at the time of measurement, the eigenvalue a_n is obtained as the outcome of measurement with the probability $|\langle\phi_n|\psi\rangle|^2$.*

(A2) *If the outcome of measurement is the eigenvalue a_n, the system is left in the corresponding eigenstate ϕ_n at the time just after measurement.*

The postulate (A1) is called the *statistical formula*, and (A2) the *measurement axiom*. The state change $\psi \mapsto \phi_n$ described by the measurement axiom is called the *state reduction*.

You will understand this paragraph in the end of the course.