Quantum Physics

2024

The Theory/Framework Of ***Almost*** Everything ***Today***

Yury Deshko

$$|\Psi\rangle$$

Part A

$$|\Psi\rangle$$

State Vector

$$|\Psi\rangle$$

State Vector

Knowledge <u>independent</u> of specific <u>representation</u>

Course Overview

Course Structure And Goals

- Part 1 : Mathematical Concepts And Tools
- Part 2 : Classical Physics
- Part 3 : Quantum Physics

$$i\hbar\frac{\delta|\Psi\rangle}{\delta t} = \hat{H}|\Psi\rangle$$

<u>State:</u> Complete *information* about the system. Full *knowledge* of it.

$$|\Psi\rangle \begin{cases} \to x \\ \to p \\ \to E \\ \to L \end{cases}$$

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$|S\rangle = (x, v)$ Newton

$|\xi\rangle = (x, p)$ Hamilton

$|\Psi\rangle = ?$ Schrödinger

$|\Psi\rangle$ is a mathematical tool used as a "container" for complete information/knowledge about a system.

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$$i\hbar\frac{\delta|\Psi\rangle}{\delta t} = \hat{H}|\Psi\rangle$$

In "real" mechanics we need $\rho(x, p)$

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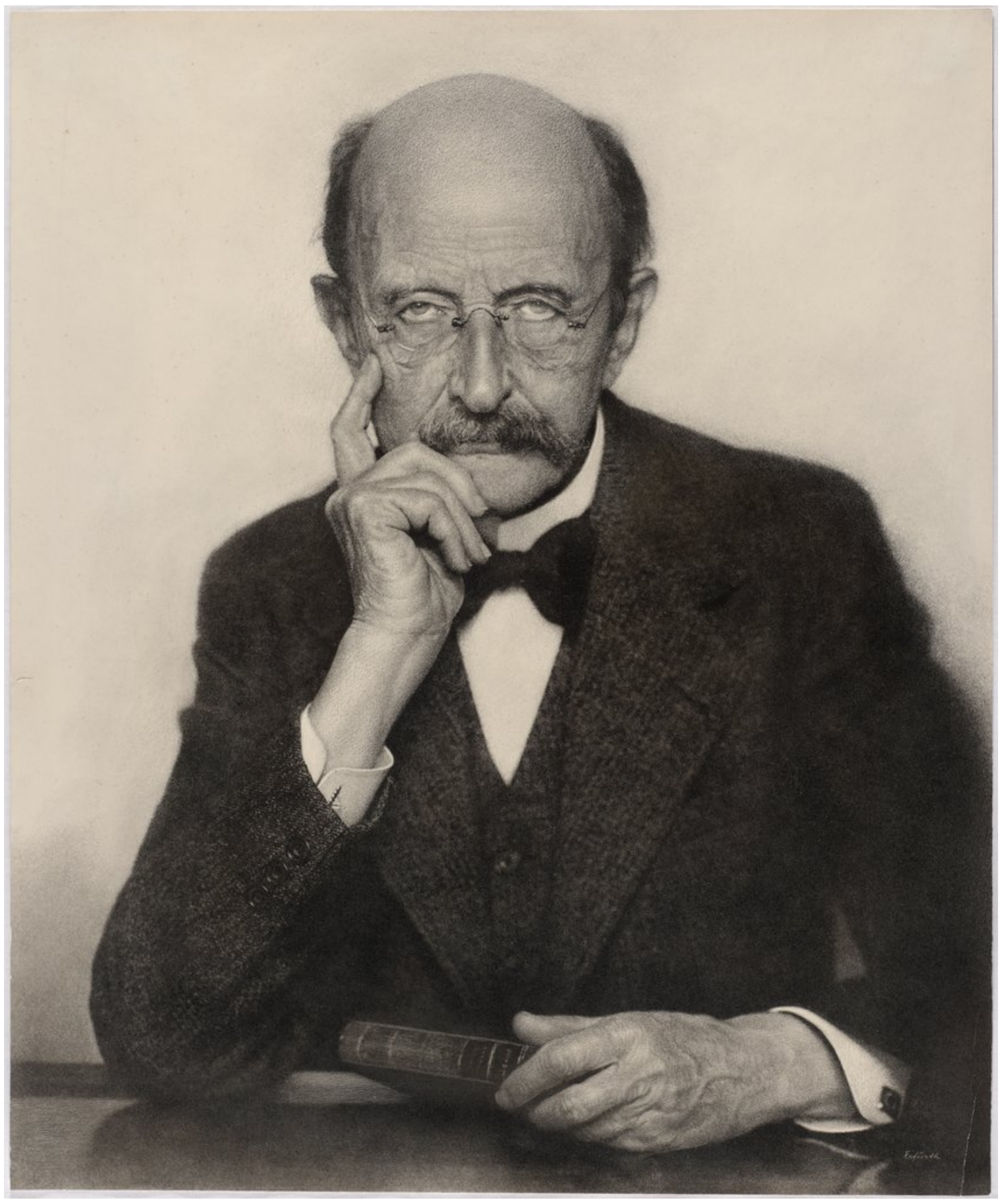
Information

How do we gain knowledge about a system

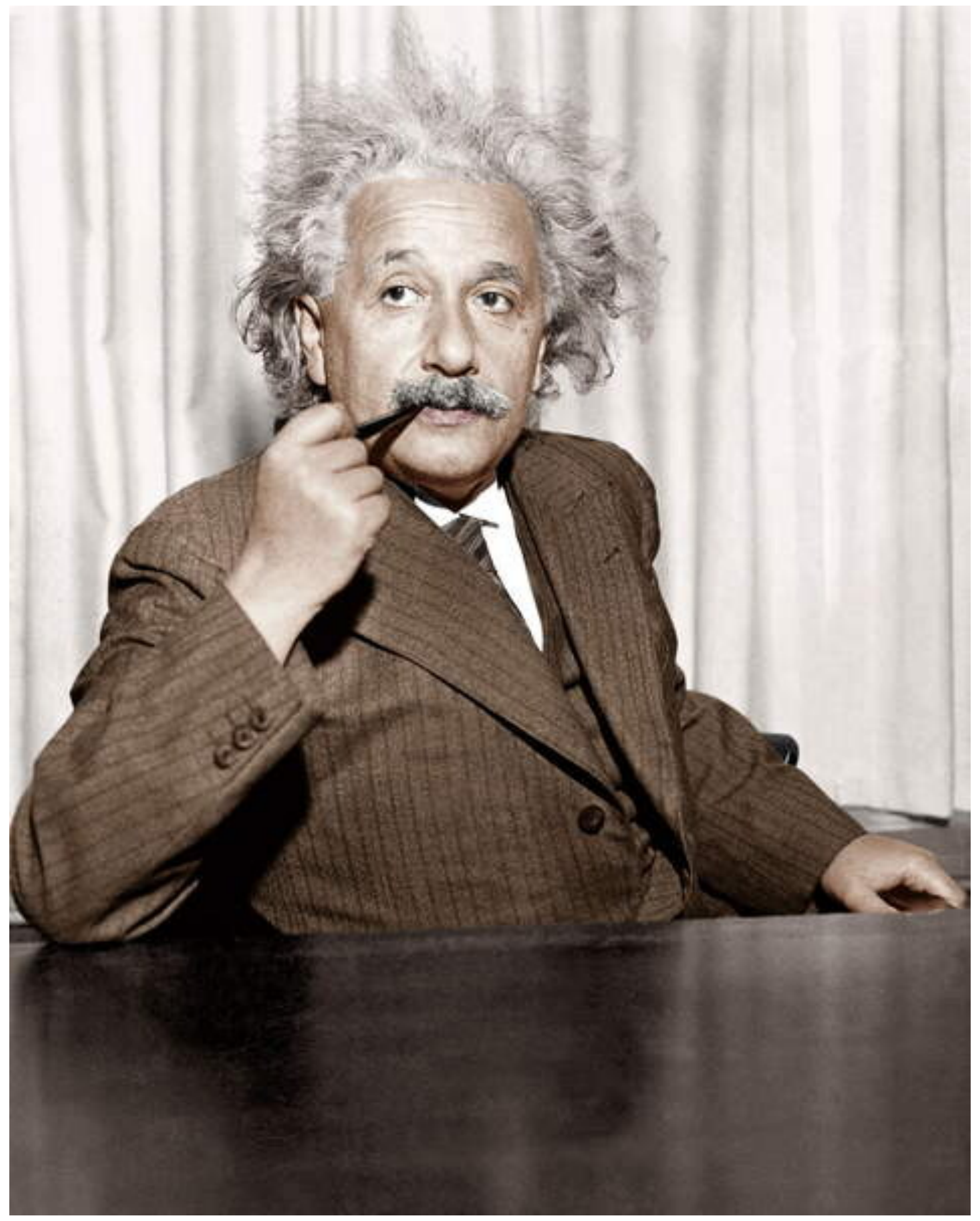


Experiments are the only means of **knowledge** at our disposal. The rest is poetry, imagination.

As quoted in “Advances in Biochemical Psychopharmacology”, Vol. 25 (1980), p. 3

Information

How do we gain knowledge about a system



The correctness of the theory is judged by the degree of agreement between the conclusions of the theory and human experience. This experience, which alone enables us to make inferences about reality, in physics takes the form of **experiment** and **measurement**.

in *"Can Quantum-Mechanical Description of Physical Reality Be Considered Complete"*

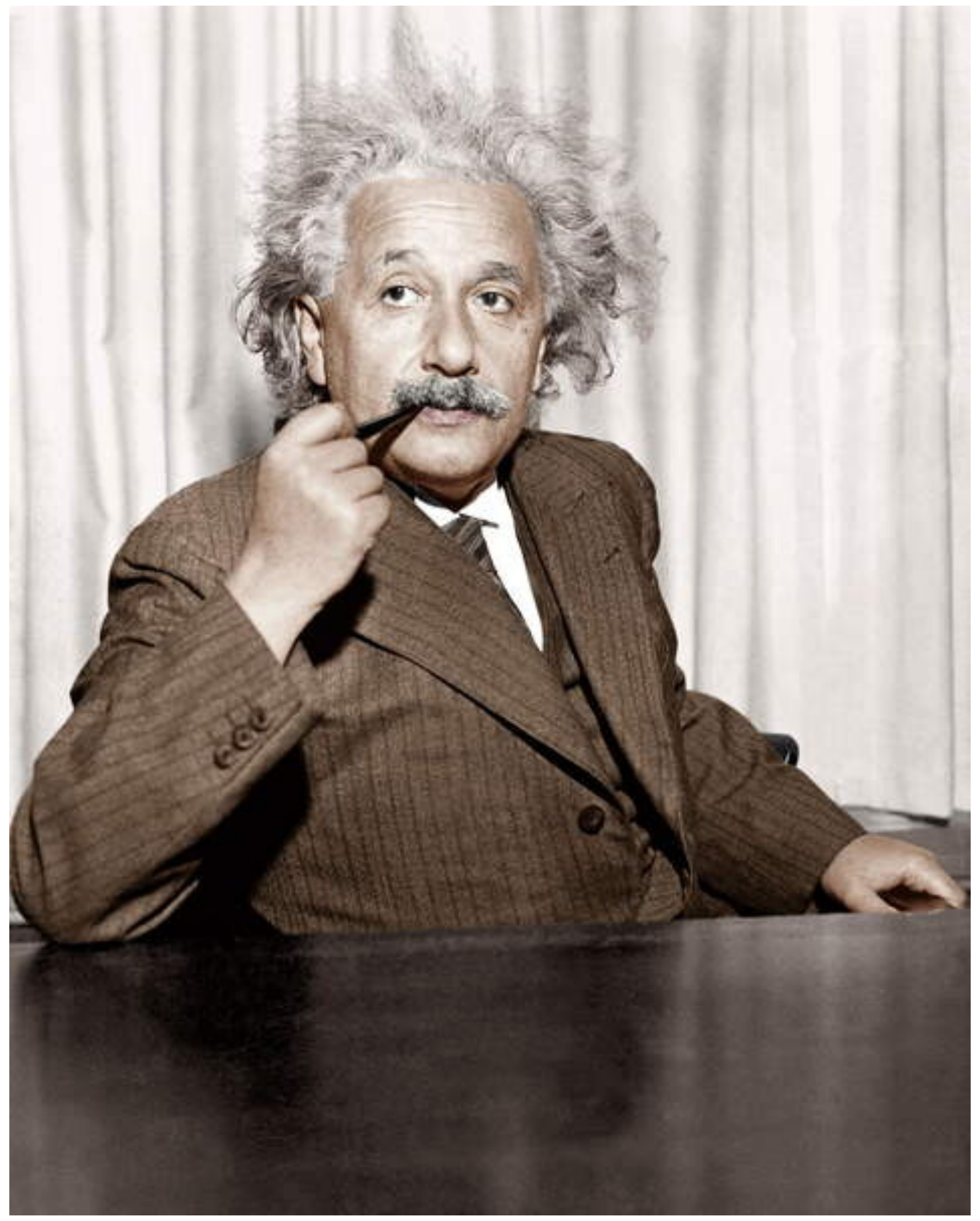
Information

How do we gain knowledge about a system

At this stage it becomes important to remember that science is concerned only with **observable** things and that we can **observe** an object only by letting it **interact** with some outside influence. in *“Principles of Quantum Mechanics, 1, The Need for Quantum Theory, p 3”*

Information

How do we gain knowledge about a system



The fundamental concept of the theory is the concept of **state**, which is supposed to be completely characterized by the <u>wave function</u> P, which is a function of the variables chosen to describe the particle's behavior. Corresponding to each physically observable quantity A there is an operator, which may be designated by the same letter.

in *"Can Quantum-Mechanical Description of Physical Reality Be Considered Complete"*

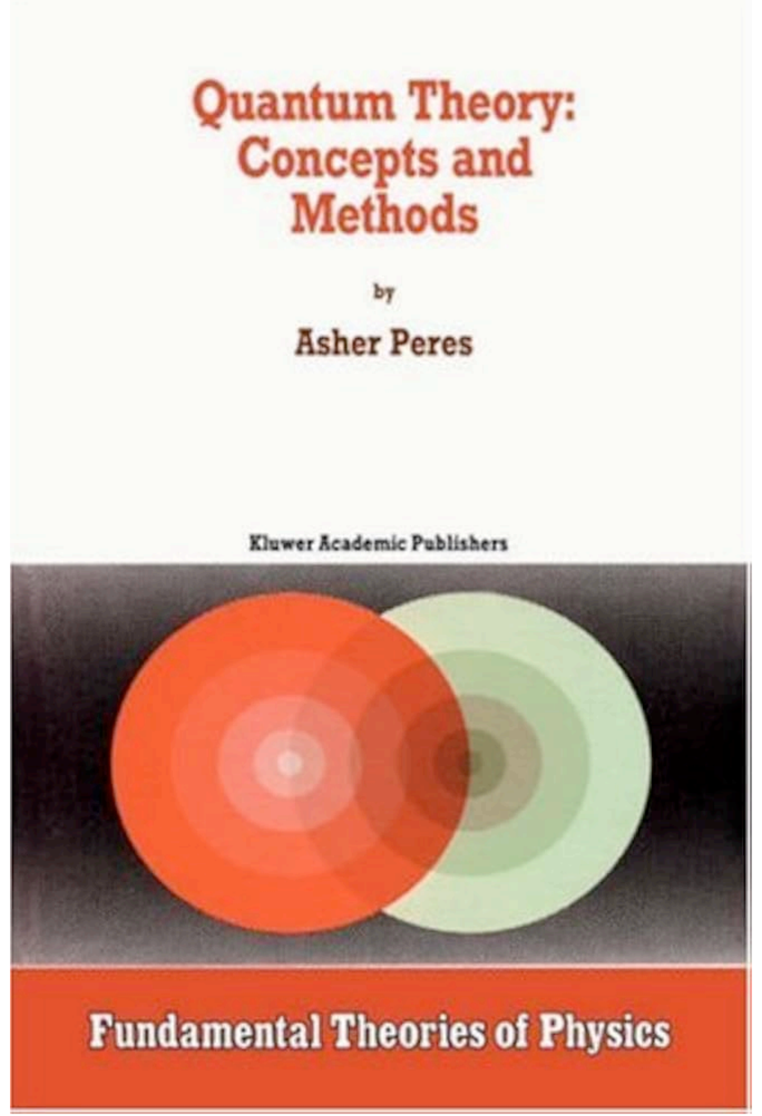
Information

How do we gain knowledge about a system

In strict sense quantum theory is a set of rules allowing the computations of probabilities for the outcomes of tests which follow specified preparations.

The essence of quantum theory is to provide a **mathematical representation of states** (that is, of preparation procedures), together with **rules** for computing the probabilities of the various outcomes of any test.

In "Quantum Theory: Concepts and Methods"



Information

How do we gain knowledge about a system

Reality resists imitation through a model. So one lets go of naive realism and leans directly on the indubitable proposition that actually (for the physicist) after all is said and done there is only observation, measurement. Then all our physical thinking thenceforth has as sole basis and as sole object the results of measurements which can in principle be carried out, for we must now explicitly not relate our thinking any longer to any other kind of reality or to a model. All numbers arising in our physical calculations must be interpreted as measurement results.

Schrödinger in *"The present situation in quantum mechanics."*

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Schrödinger in *“The present situation in quantum mechanics.”*

We will talk about measurement more later. It is an extremely important issue, with technical and philosophical aspects.

Information

How do we gain knowledge about a system

Erwin with his psi can do
Calculations quite a few.
But one thing has not been seen:
Just what does psi really mean?

Erich Hückel, translated by Felix Bloch and quoted in Traditions et tendances nouvelles des études romanes au Danemark (1988) by Ebbe Spang-Hanssen and Michael Herslund, p. 207; also in The Pioneers of NMR and Magnetic Resonance in Medicine : The Story of MRI (1996) by James Mattson and Merrill Simon, p. 278

Measurement

Of a value of a property of a quantum system

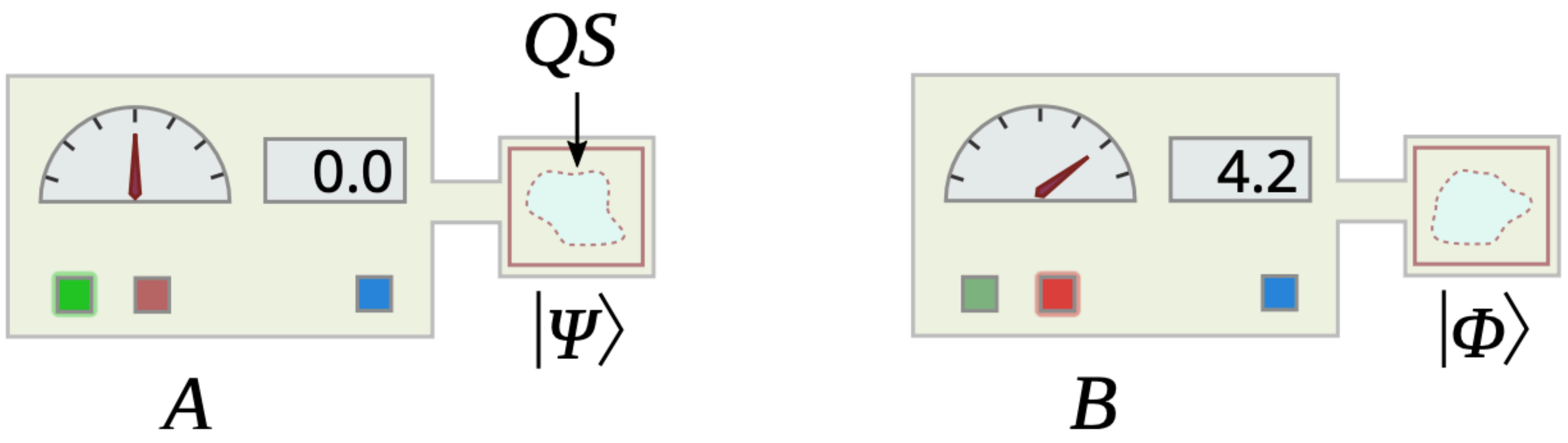


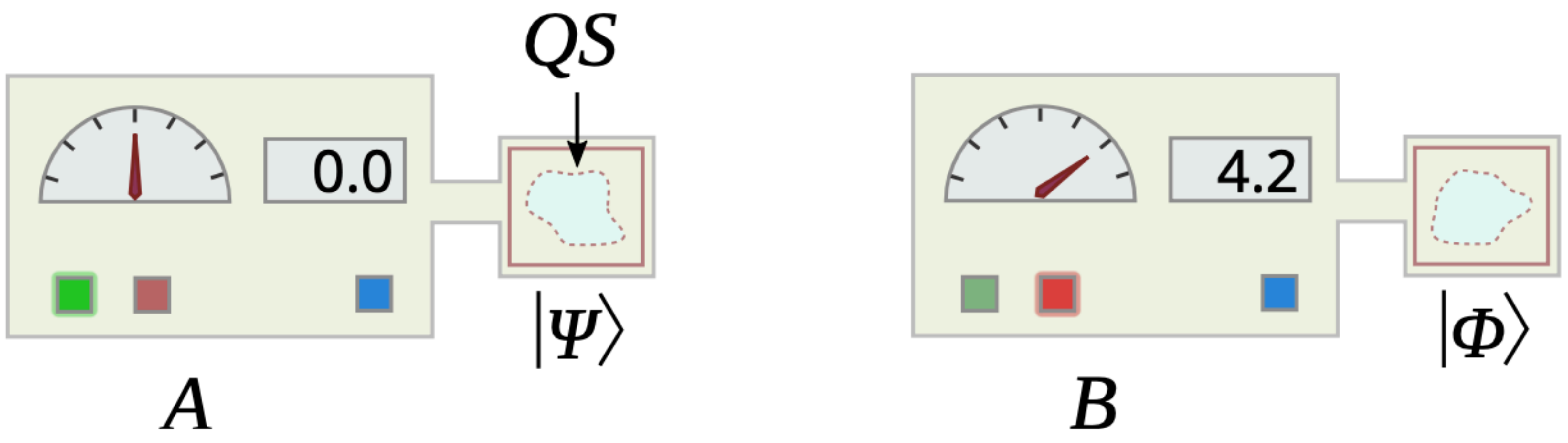
Figure 3.2: (a) Initial state of classical apparatus and quantum system (QS). (b) After the measurements, the apparatus and the quantum system might change their state.

$$A\,,\ |\Psi\rangle \quad \xrightarrow{\widehat{M}} \quad B\,,\ |\Phi\rangle\,;$$

Quantum physics does not work without classical physics. We use macroscopic classical apparatus, and macroscopic classical language.

Measurement

Apparatus and system are "aligned"



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$$A\,,\,|\Psi\rangle \quad \xrightarrow{\widehat{M}} \quad A\,,\,|\Psi\rangle \quad \xrightarrow{\widehat{M}} \quad A\,,\,|\Psi\rangle \quad \xrightarrow{\widehat{M}} \quad \ldots$$

$$\widehat{M}|\Psi\rangle = A|\Psi\rangle\,.$$

Eigen-problem. Eigen-value.

Eigen-vector. Eigen-state.

The state $|\Psi\rangle$ is called "proper"/own/eigen-state of the measurement operator $\hat{M}$.

Eigen - "own" in German.

Measurement

Apparatus and system are not "aligned"

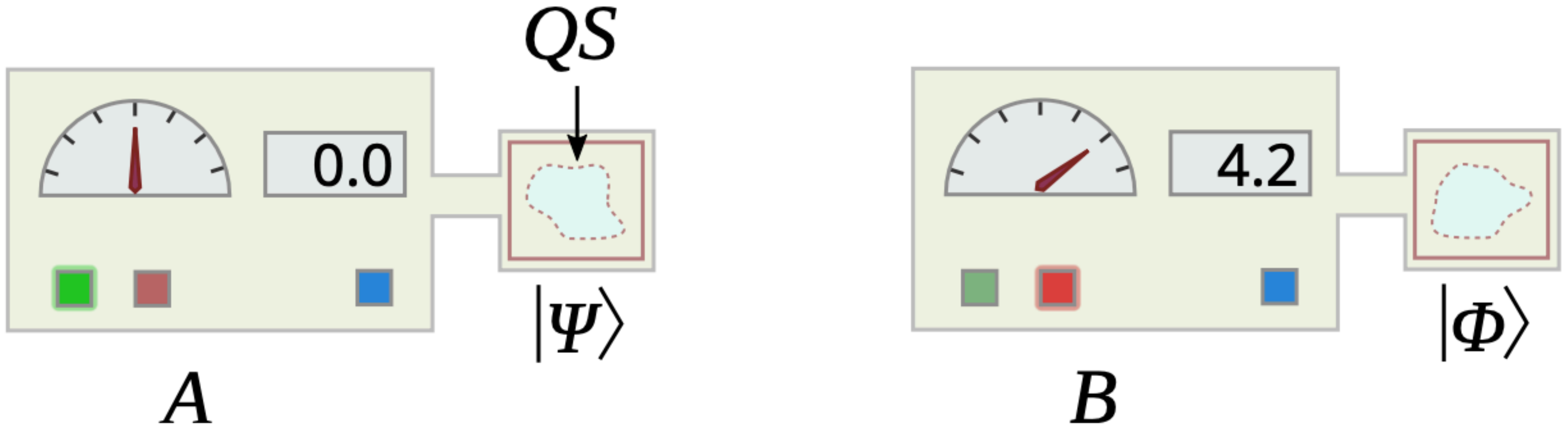


Figure 3.2: (a) Initial state of classical apparatus and quantum system (QS). (b) After the measurements, the apparatus and the quantum system might change their state.

$$A\,,\,|\Psi\rangle \quad \xrightarrow{\widehat{M}} \quad \begin{cases} p_1,|\Phi_1\rangle,V_1 \\ p_2,|\Phi_2\rangle,V_2 \\ \dots \\ p_n,|\Phi_n\rangle,V_n \end{cases}$$

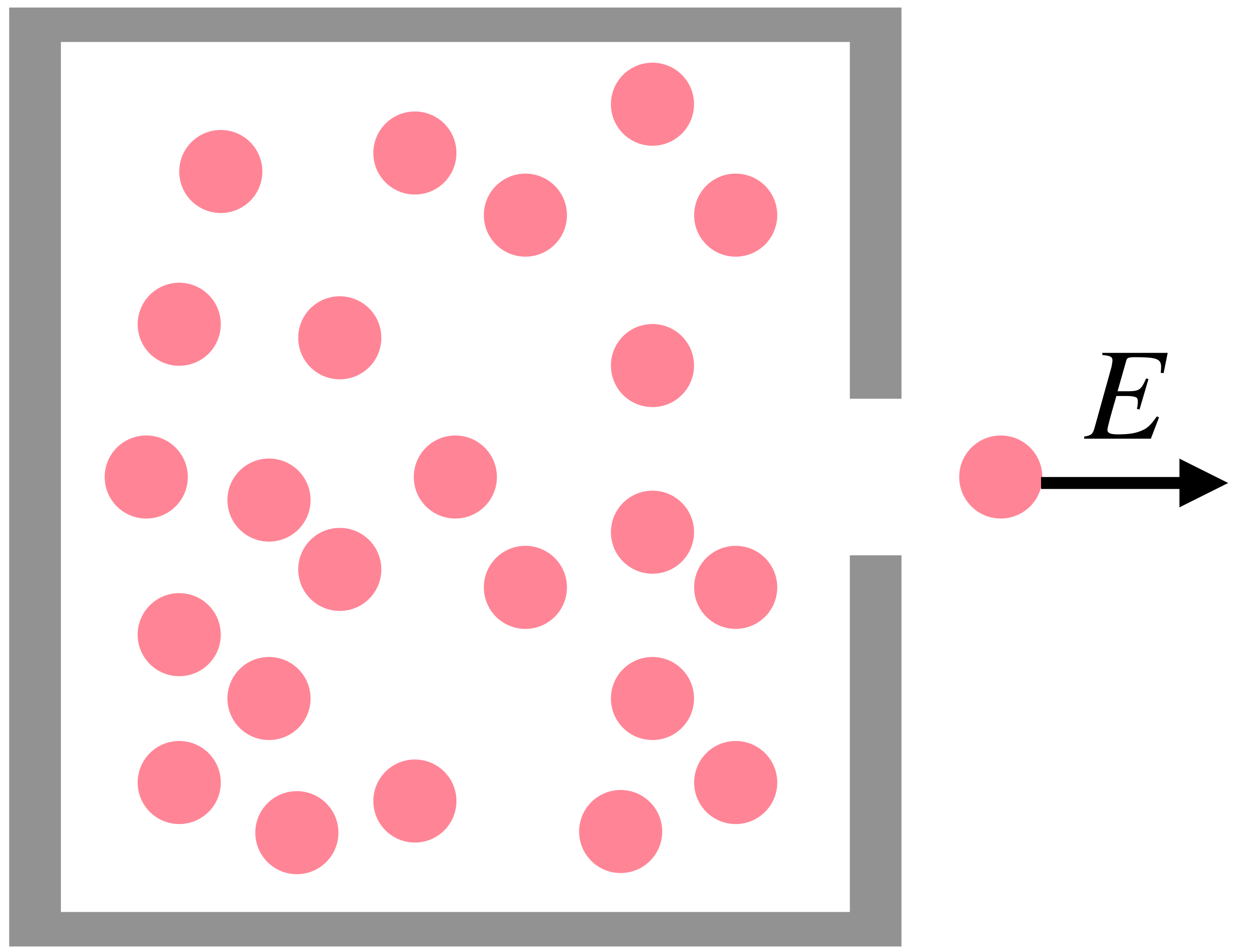
$$p_1 = \frac{N_1}{N}\,.$$

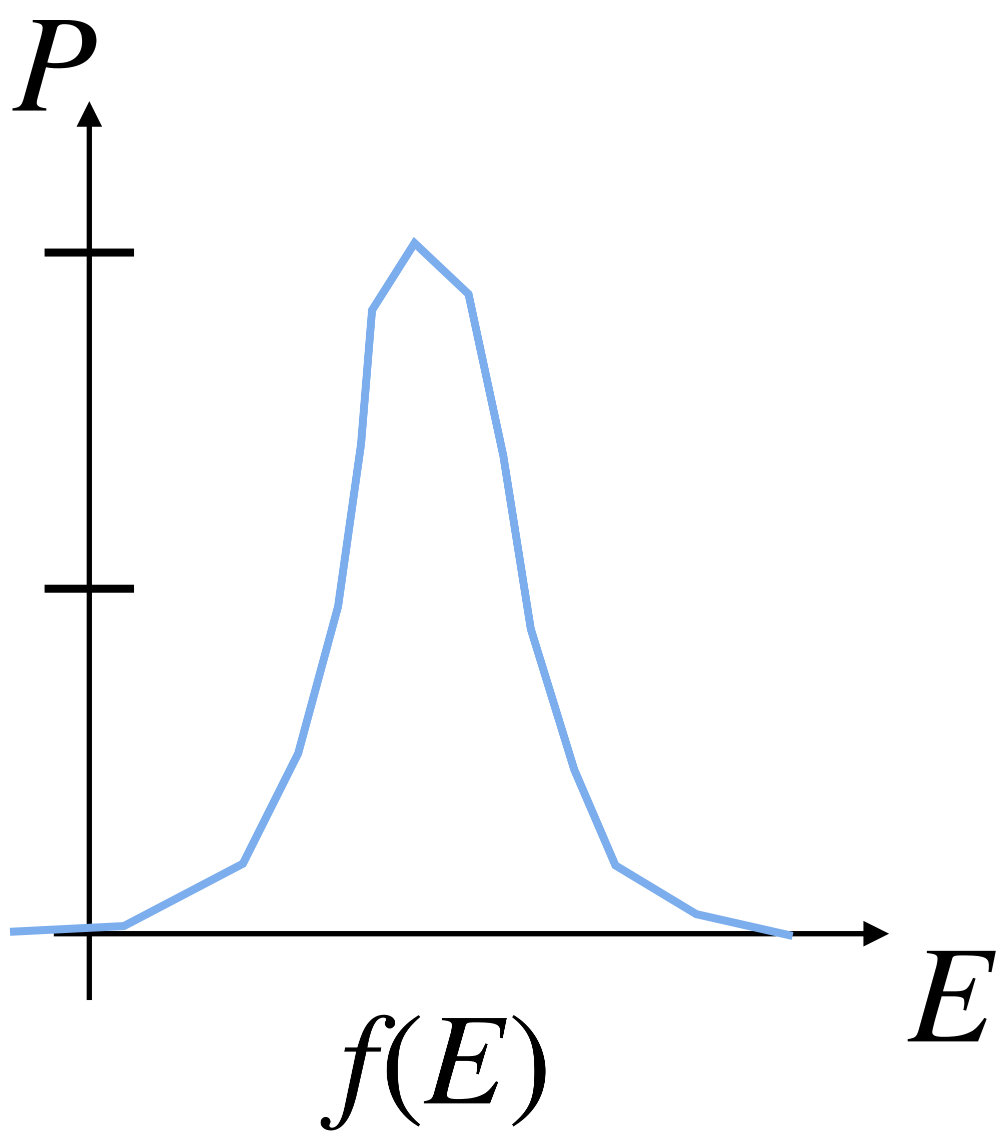
$$|\Psi\rangle = F(|\Phi_1\rangle,|\Phi_1\rangle,\dots,|\Phi_1\rangle,p_1,p_2,\dots,p_n)\,,$$

$$|\Psi\rangle = p_1|\Phi_1\rangle + p_2|\Phi_2\rangle + \dots p_n|\Phi_n\rangle\,.$$

Measurement

Learning From "Ensemble"





$E_{avg} \sim T$

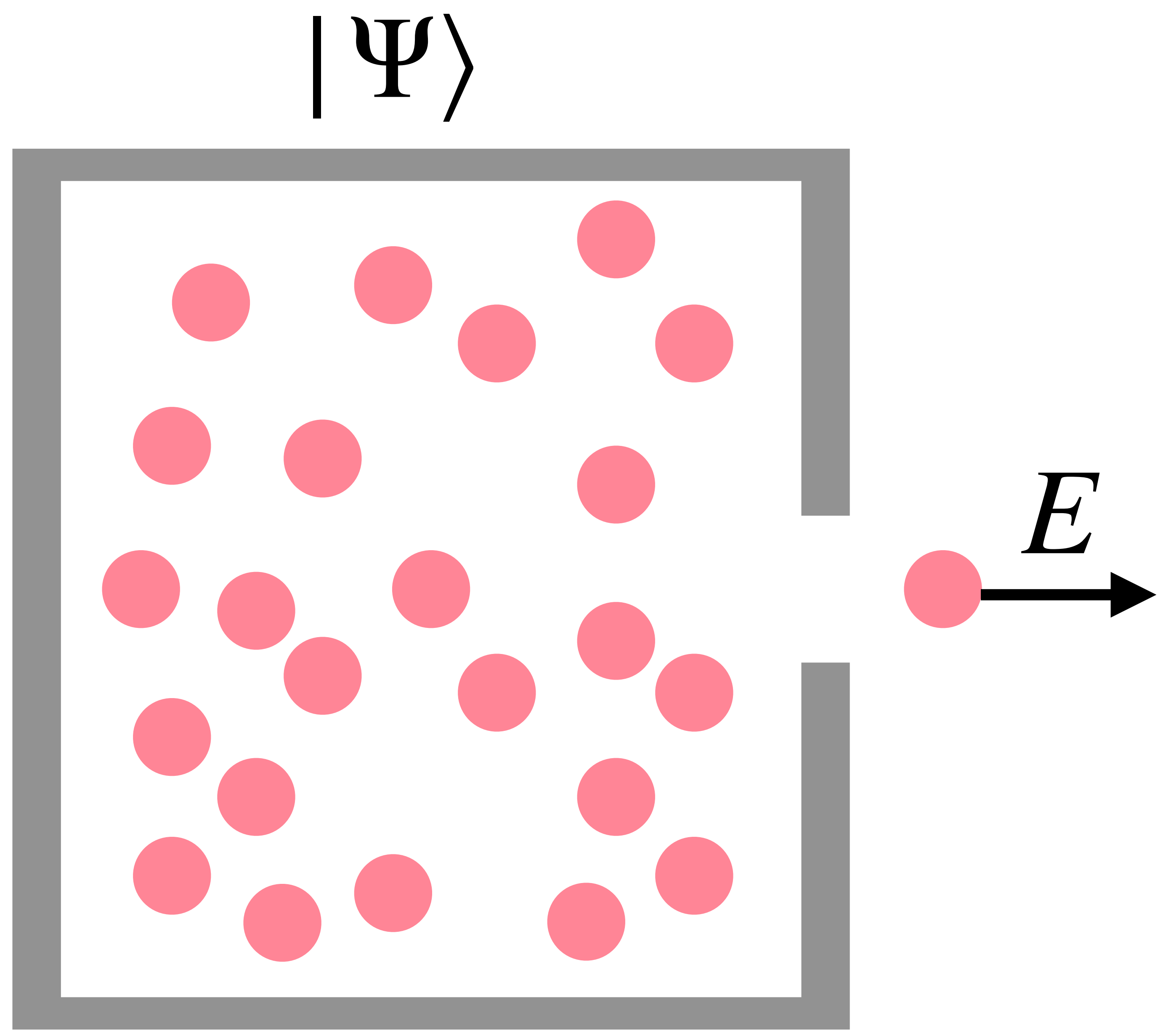
Statistical physics and kinetic theory of gases uses this idea.

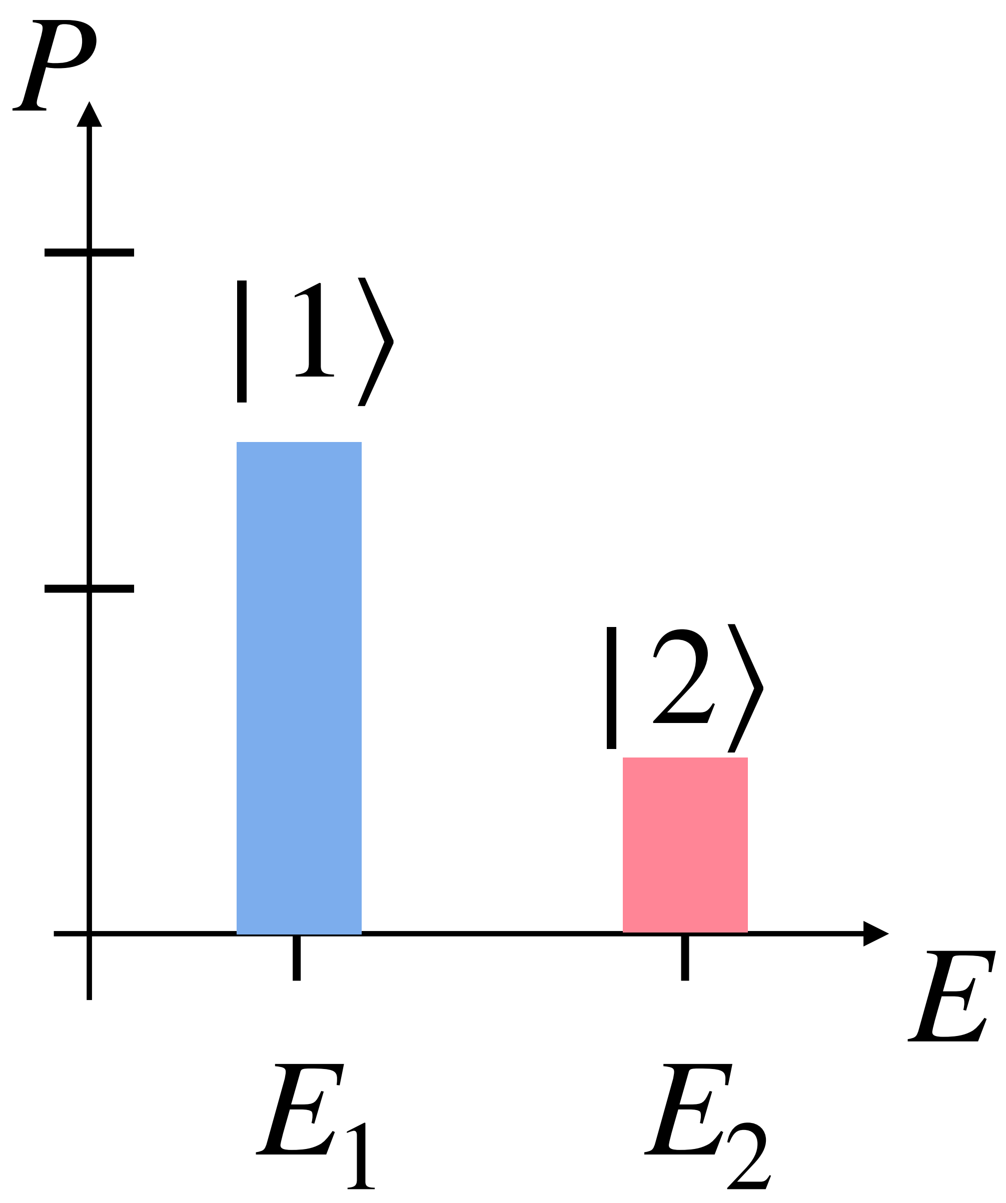
$f(E)$ describe the whole "group" of particles, all in similar/identical conditions (temperature?)

Question: Why is the distribution? What is the source of randomness?

Measurement

Learning From "Ensemble"





Qubit = quantum system with 2 distinct states available for manipulation.

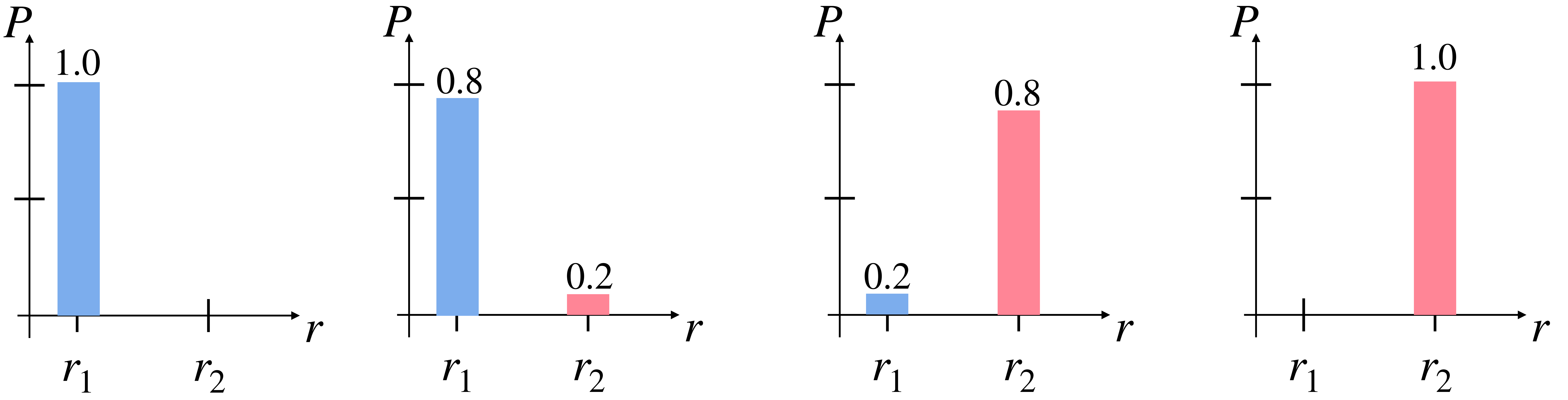
Qutrit = 3 states.

Qudit = d states

Measuring energy of a large number of qubits (e.g., harmonic oscillators with limited energy supply) reveals their state $|\Psi\rangle$.

Probability

And its representation



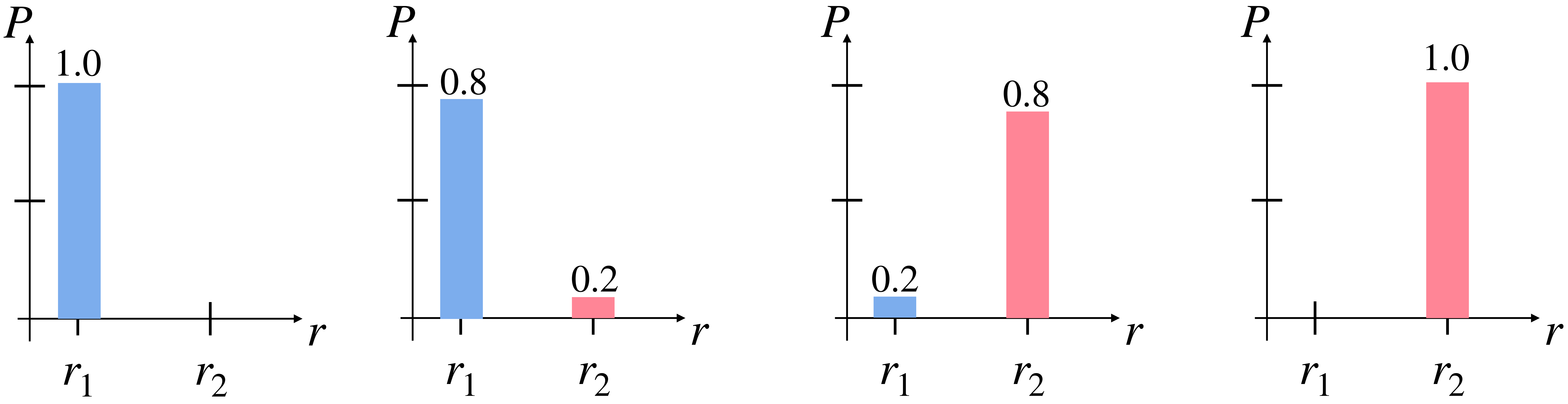
$|\Psi_1\rangle = |1\rangle$ $\quad$ $|\Phi_1\rangle$ $\quad$ $|\Phi_2\rangle$ $\quad$ $|\Psi_2\rangle = |2\rangle$

Probability

And its representation

Reasonable first try.

$|\Phi_1\rangle = 0.8|1\rangle + 0.2|2\rangle$ $\qquad$ $|\Phi_2\rangle = 0.2|1\rangle + 0.8|2\rangle$



$|\Psi_1\rangle = |1\rangle$ $\qquad$ $|\Phi_1\rangle$ $\qquad$ $|\Phi_2\rangle$ $\qquad$ $|\Psi_2\rangle = |2\rangle$

Probability

And its representation

$$|\Phi_3\rangle = 0.5\,|1\rangle + 0.5\,|2\rangle$$

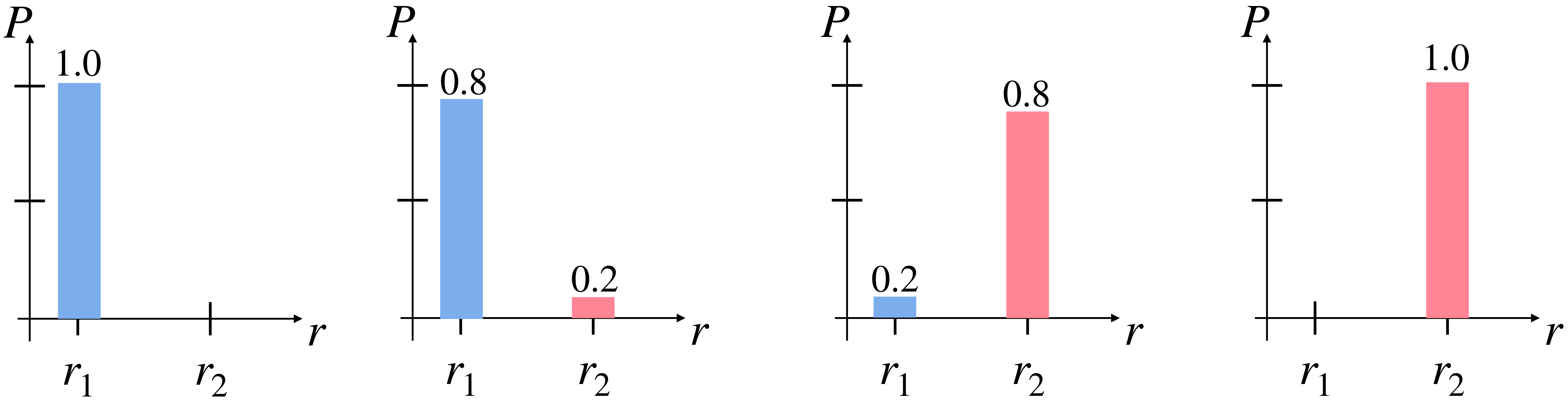
$$|\Phi_3\rangle \leftrightarrow |\Phi_1\rangle$$

$$|\Phi_3\rangle \leftrightarrow |\Phi_2\rangle$$

$$|\Phi_1\rangle \leftrightarrow |\Phi_2\rangle$$

Reasonable first try.

$|\Phi_1\rangle = 0.8\,|1\rangle + 0.2\,|2\rangle$ $\qquad$ $|\Phi_2\rangle = 0.2\,|1\rangle + 0.8\,|2\rangle$

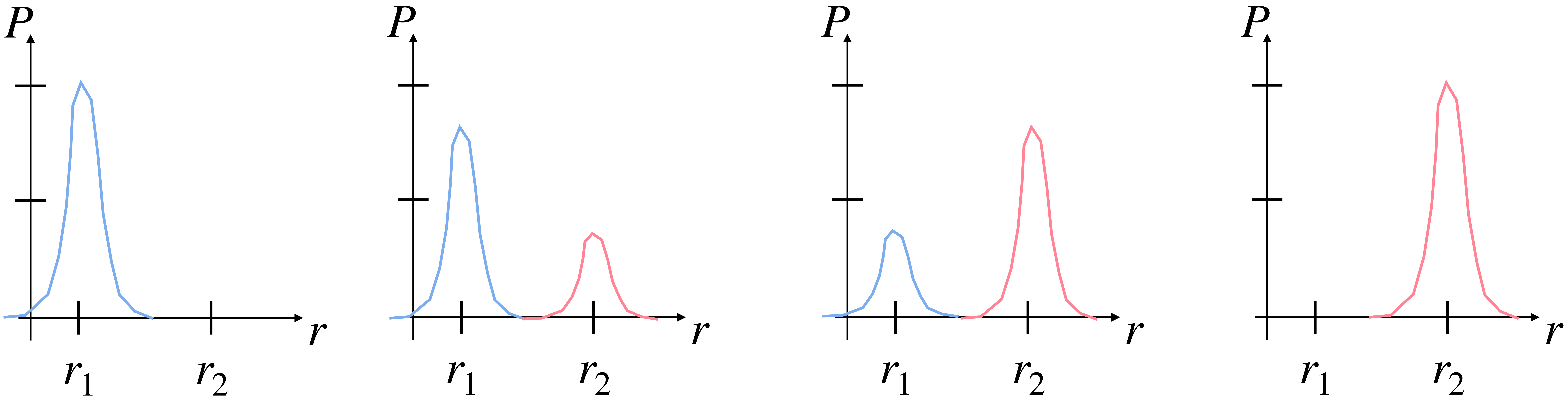


$|\Psi_1\rangle = |1\rangle$ $\qquad$ $|\Phi_1\rangle$ $\qquad$ $|\Phi_2\rangle$ $\qquad$ $|\Psi_2\rangle = |2\rangle$

Probability

And its representation. Continuous Case

State Functions $f(r)$. (NOT ~~Wave functions~~)

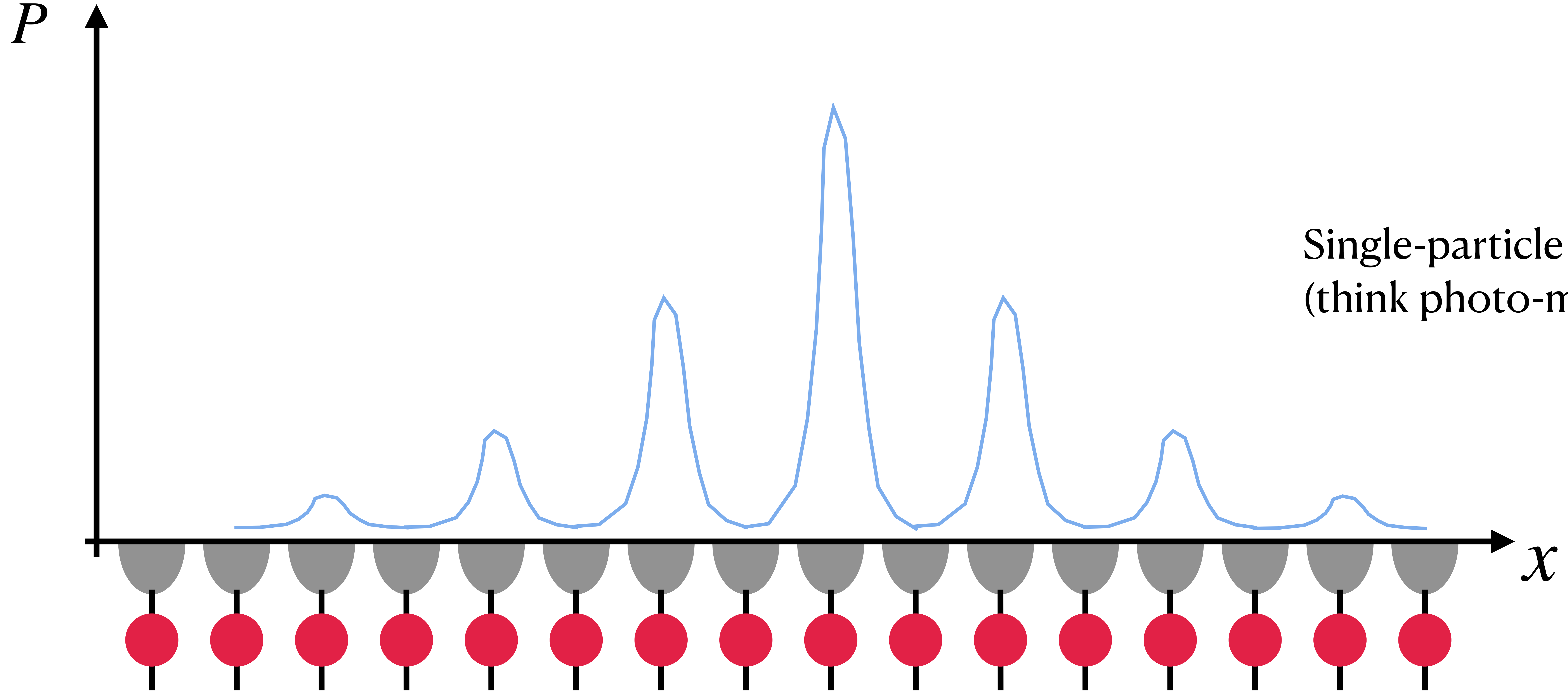


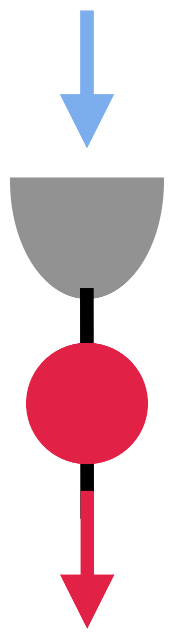
$\Psi_1(r)$ $\Phi_1(r)$ $\Phi_2(r)$ $\Psi_2(r)$

Position Measurement

Using Simple Detectors

State Functions $\Psi(x)$. (NOT ~~Wave functions~~)



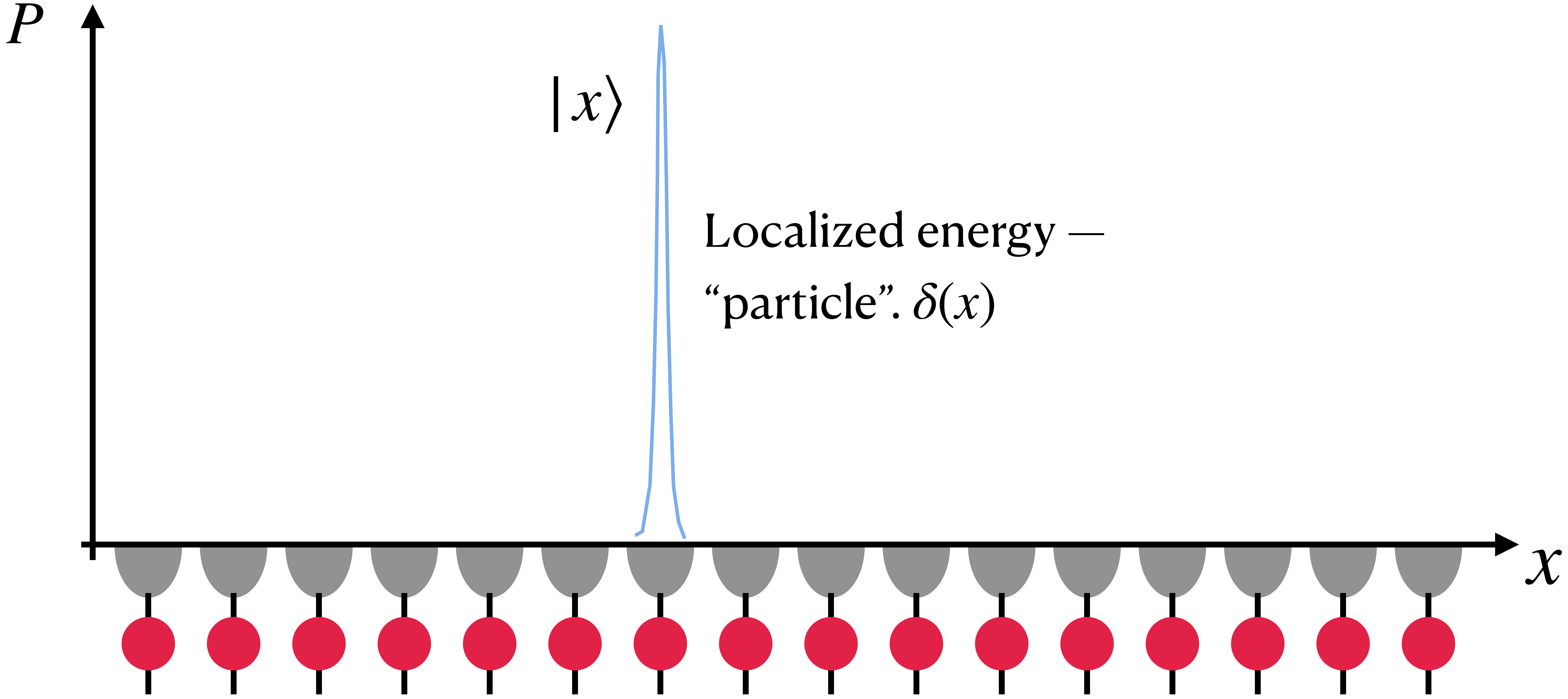


Single-particle detector
(think photo-multiplier tube)

Position Measurement

Using Simple Detectors

“Special” (for humans) State Functions $\Psi(x)$.



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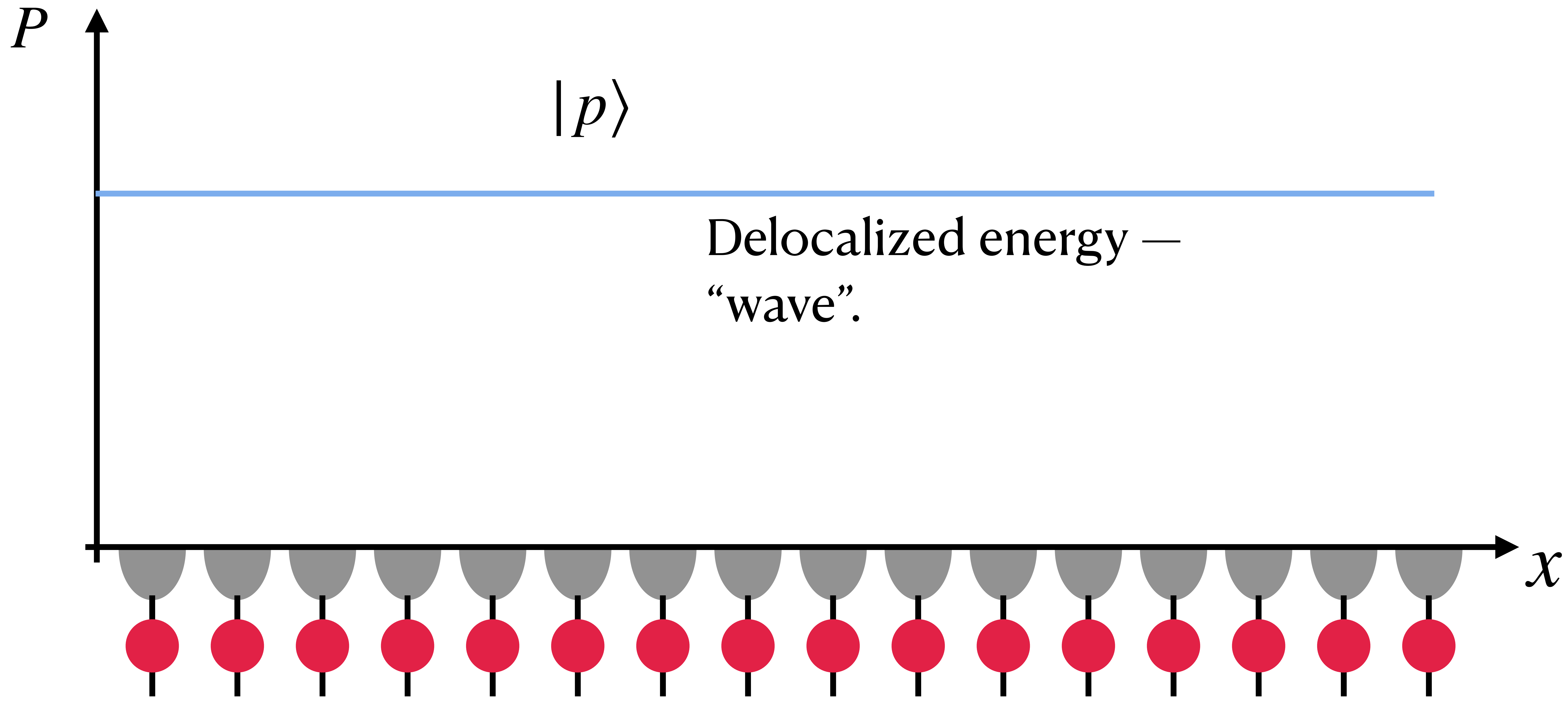
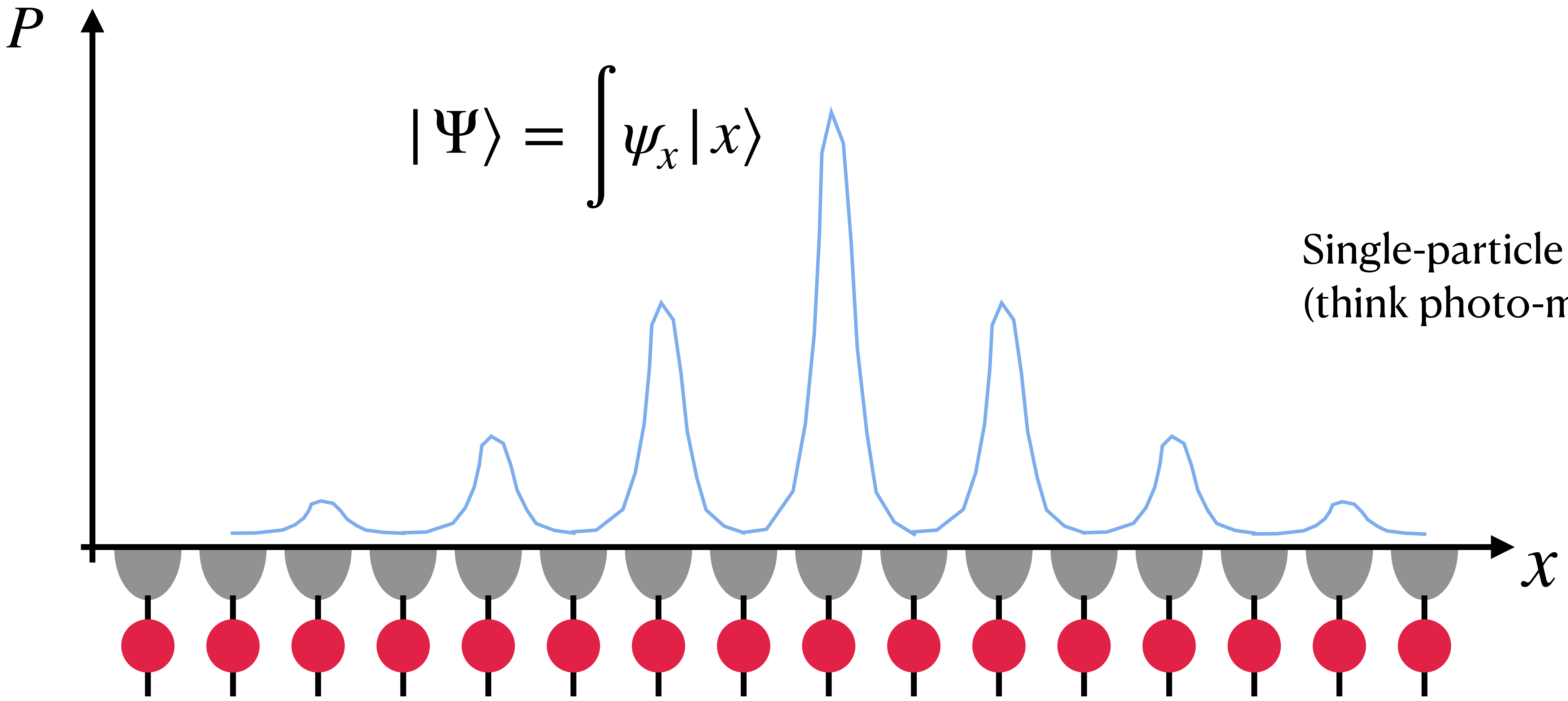


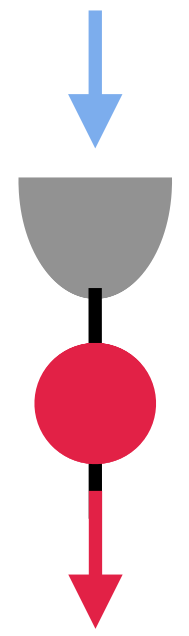
Position Measurement

Using Simple Detectors

State Functions $\Psi(x)$. (NOT ~~Wave functions~~)

$$|\Psi\rangle = \int \psi_x |x\rangle$$





Single-particle detector
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Position Measurement

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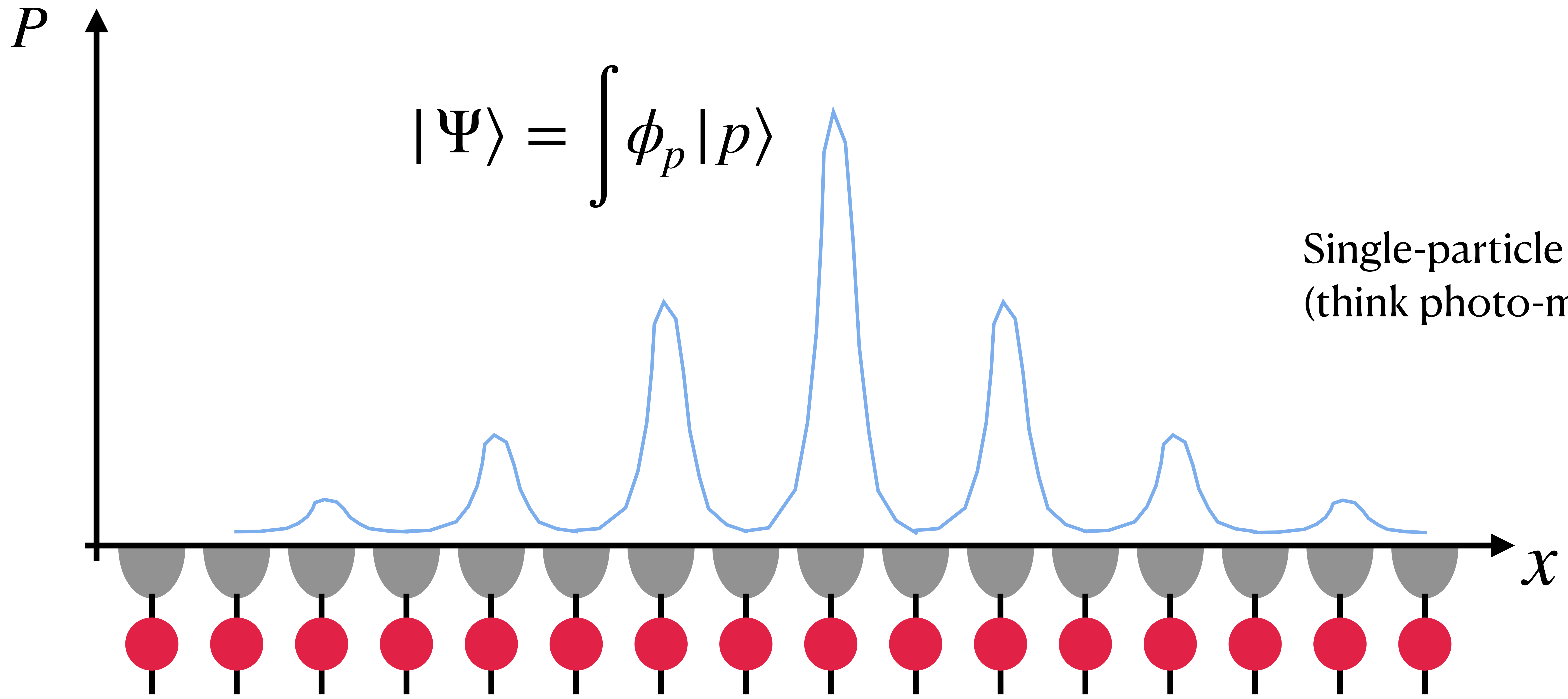

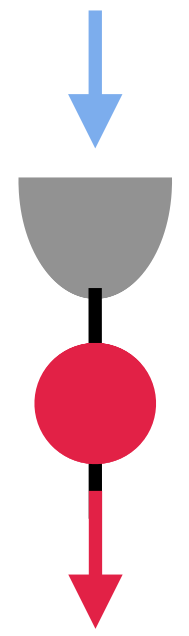

Single-particle detector
(think photo-multiplier tube)

What $|\Psi\rangle$ is NOT

State Vector

Is NOT a Field

It **is not** a “physical object" that is spread out in space, propagates through space, or scatters from matter. It **is not** like EMF.

It **is not** a wave. “Wave function” **is not** a good phrase.

State Vector

Can Only Be Measured on Many Identical Systems

A single measurement **does not** reveal $|\Psi\rangle$

State Vector

Does not always apply

In many important cases **we can't** write $|\Psi\rangle$ for a system (composite, "multi-part" system with entangled parts.)

Self-Test

Answer These Questions 1hr After Class

1. What is the role of the concept of "state" in physics?
2. How is state represented mathematically in Newtonian and Hamiltonian mechanics?
3. How do we obtain the information about a system in physics?
4. What is the difference between a quantum system and a measuring apparatus?
5. What does a measuring device do to a system? What does a system do to the device?
6. What is one way to represent the results of the multiple measurements?
7. What are two basic types of states?
8. Why are the ideas of "probability field" and "wave function" not good?

Homework Problems

Mathematical Concepts and Notation Day 3

- Suppose a system in a such a state that $\hat{H}|\Psi_0\rangle = E|\Psi_0\rangle$ — that is, the measured energy is E with 1.0 probability (with certainty). Write down the Schrödinger equation for this case and show that the state changes in time as follows $|\Psi_t\rangle = e^{-iEt/\hbar}|\Psi_0\rangle$.
- Suppose a harmonic oscillator is in such a state that $|\Phi\rangle = 0.7|1\rangle + 0.3|2\rangle$. What is the average energy of harmonic oscillator?
- In the previous problem, do you think it can be that $\hat{H}|\Phi\rangle = E|\Phi\rangle$ for some energy E?
- **Advanced**: If the state vector $|\Psi\rangle$ allows different *representations*: $|\Psi\rangle = \int \psi_x |x\rangle$ and $|\Psi\rangle = \int \phi_p |p\rangle$, can you write the relationship between the functions (components) ϕ_p and ψ_x?
- Watch the video about quantum properties of light (previously recommended/assigned). Learn about photo-multiplying tube (PMT).

Quantum Theory

In a Nutshell

II. POSTULATES FOR QUANTUM MECHANICS

In this paper, all state vectors are supposed to be normalized, and mixed states are represented by density operators, i.e., positive operators with unit trace. Let A be an observable with a nondegenerate purely discrete spectrum. Let $\phi_1, \phi_2, \ldots$ be a complete orthonormal sequence of eigenvectors of A and $a_1, a_2, \ldots$ the corresponding eigenvalues; by assumption, all different from each other.

According to the standard formulation of quantum mechanics, on the result of a measurement of the observable A the following postulates are posed:

(A1) *If the system is in the state ψ at the time of measurement, the eigenvalue a_n is obtained as the outcome of measurement with the probability $|\langle\phi_n|\psi\rangle|^2$.*

(A2) *If the outcome of measurement is the eigenvalue a_n, the system is left in the corresponding eigenstate ϕ_n at the time just after measurement.*

The postulate (A1) is called the *statistical formula*, and (A2) the *measurement axiom*. The state change $\psi \mapsto \phi_n$ described by the measurement axiom is called the *state reduction*.

You will understand this paragraph in the end of the course.