Quantum Physics

2024

The Theory/Framework Of *Almost* Everything *Today*

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Course Overview

Course Structure And Goals

- Part 1 : Mathematical Concepts And Tools
- Part 2 : Classical Physics
- Part 3 : Quantum Physics

We want to understand SchrEq

Operator

$$i\hbar\partial_t|\Psi\rangle = \hat{H}|\Psi\rangle$$

yector, but can be also made an operator

Quantum of action

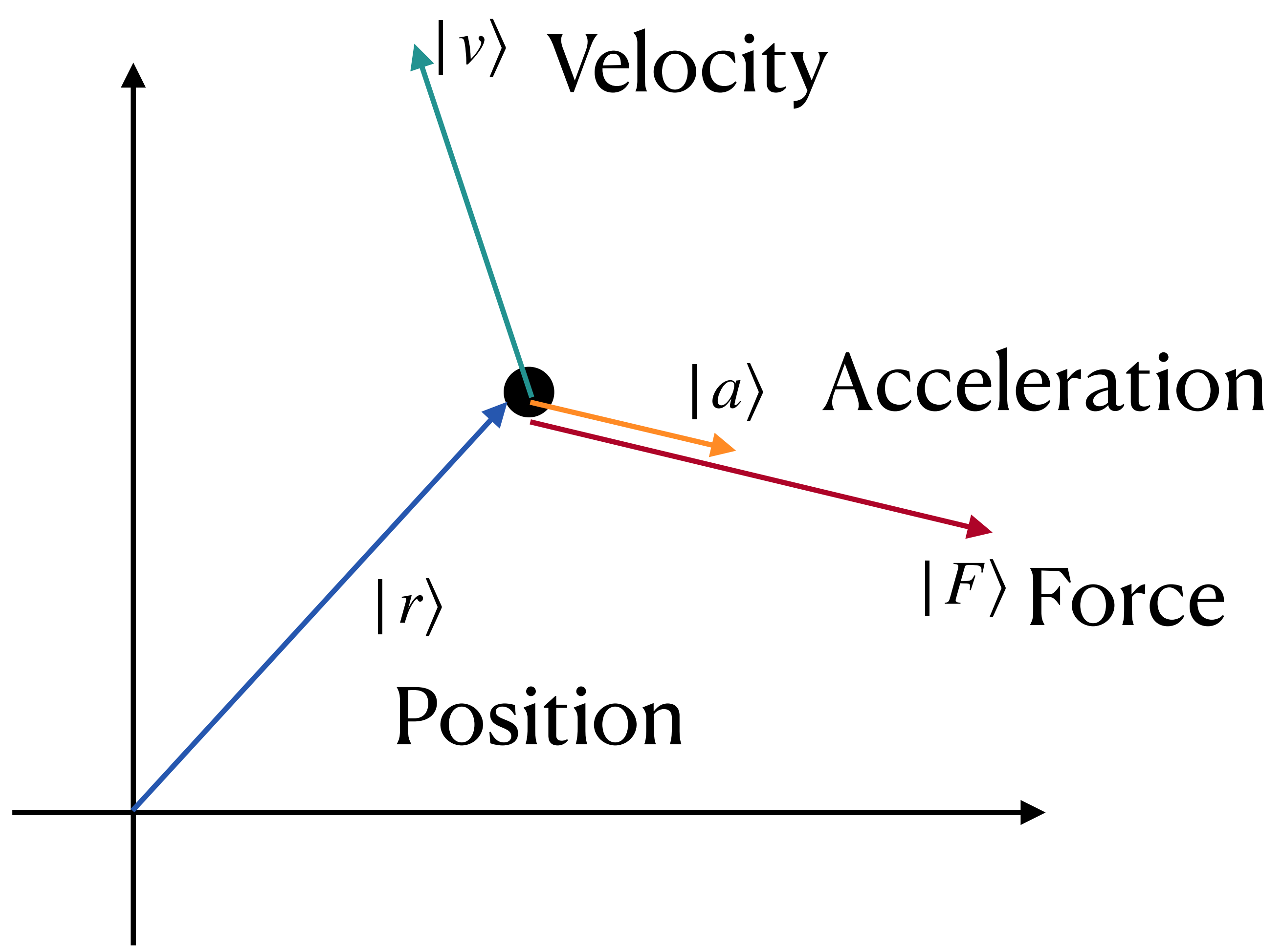
Rate of change with respect to time

Hamiltonian

Today we will understand $\hbar$ better.

Physical Quantities

A large array of concepts and quantities



Energy

Mass

Viscosity

Resistance

Momentum

Temperature

Entropy

Charge

Volume

Density

Pressure

Polarizability

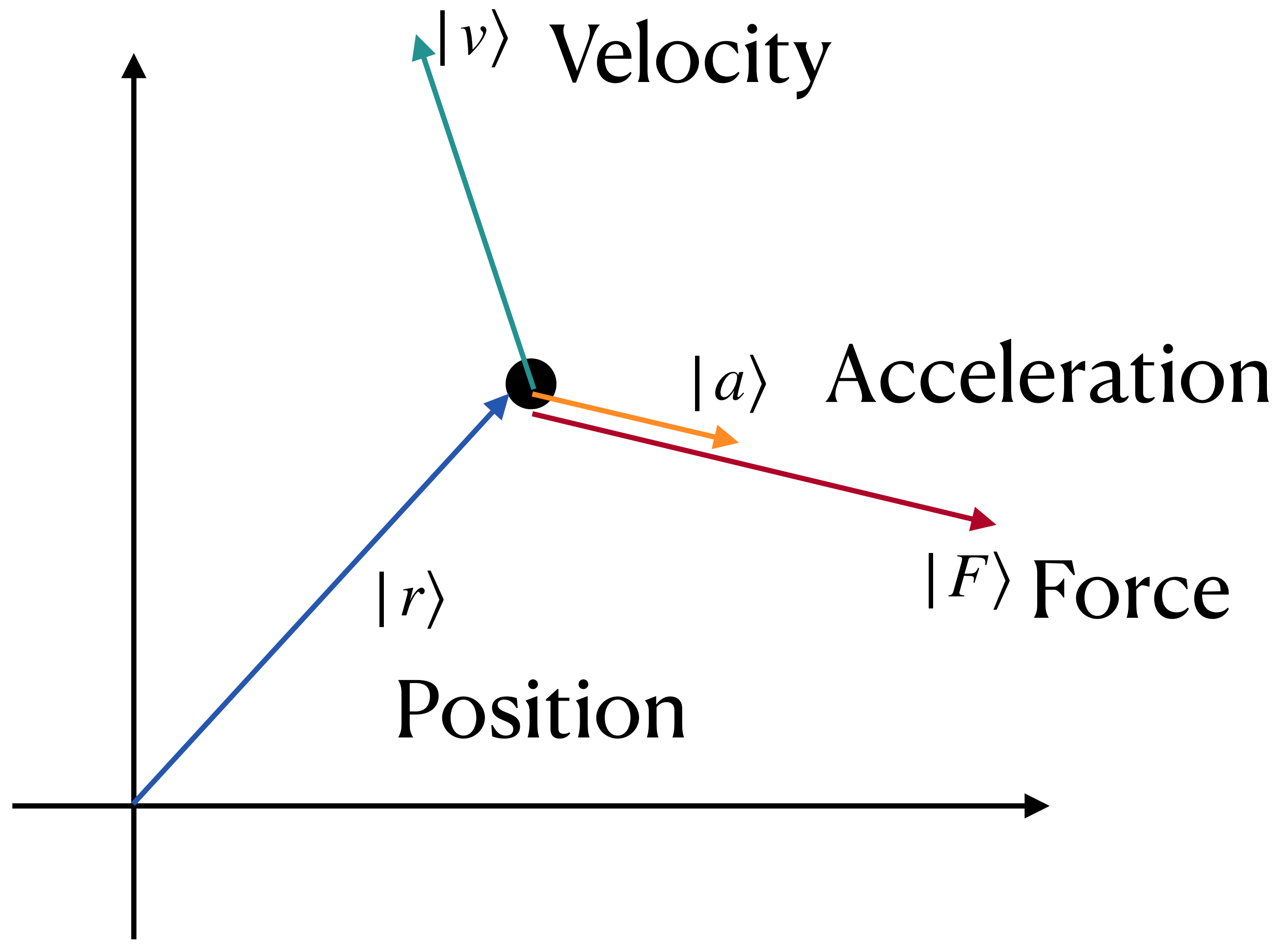
Luminosity

Power

Action

Physical Quantities

A large array of concepts and quantities



Energy

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__Entropy__

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__Action__

Action

Fundamental And Quantized Physical Quantity

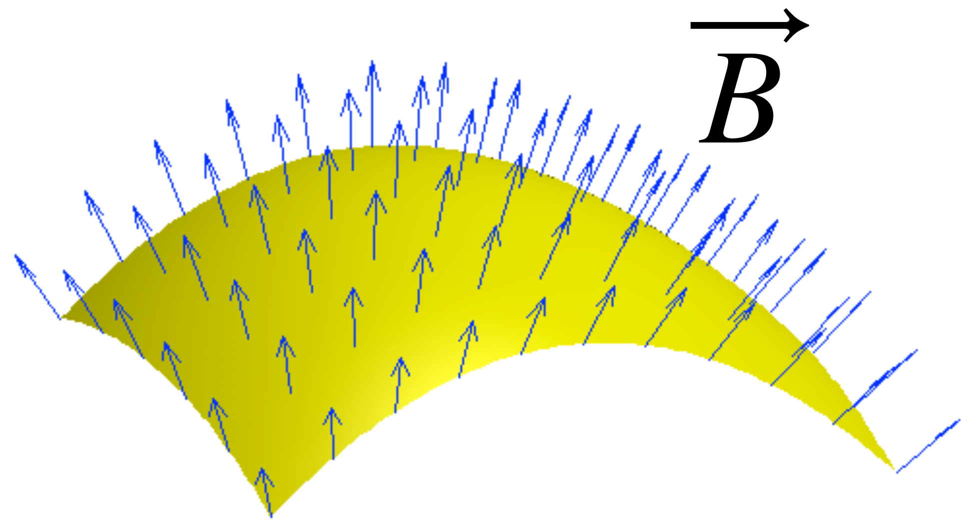
- We are mostly familiar and comfortable with basic physical quantities: velocity, acceleration, energy, temperature, resistance, and some others.
- There are important physical quantities which are not fully appreciated or understood: entropy and **action**.
- Action plays a role in mechanics, electricity, magnetism, spectroscopy.
- Action, like electric charge, is **quantized:** There exists the smallest value for the "transfer" of action — $h = 6.626 \times 10^{-34}\,(J \cdot s)$.
- Action was known since Lagrangian mechanics, but its quantum was discovered by Planck in 1900 when studying the problem of heat.

Action

Fundamental And Quantized Physical Quantity

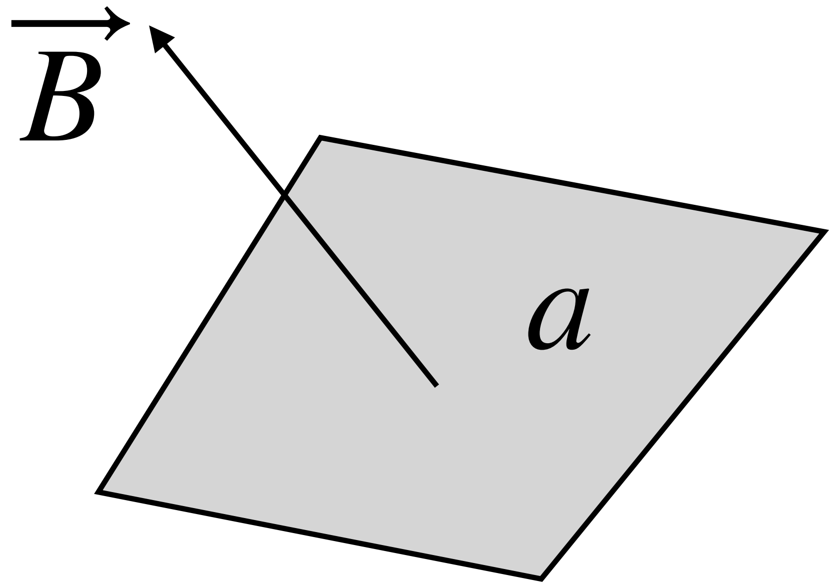
- $h = 6.626 \times 10^{-34}\,(J \cdot s)$
- Action has units of (Energy*time): $\Delta E \Delta t$
- But so do other combinations: $\Delta x \Delta p$, ΔL — angular momentum, $e\Delta\Phi$ where $\Delta\Phi$ is the flux of magnetic field, $e^2/\Delta G$ where ΔG is the change of conductance (ease of current flow).

$$\Phi_0 = \frac{h}{2e}$$ Quantum of magnetic flux



Like a flow of water through a given area

$$\Phi = Ba$$



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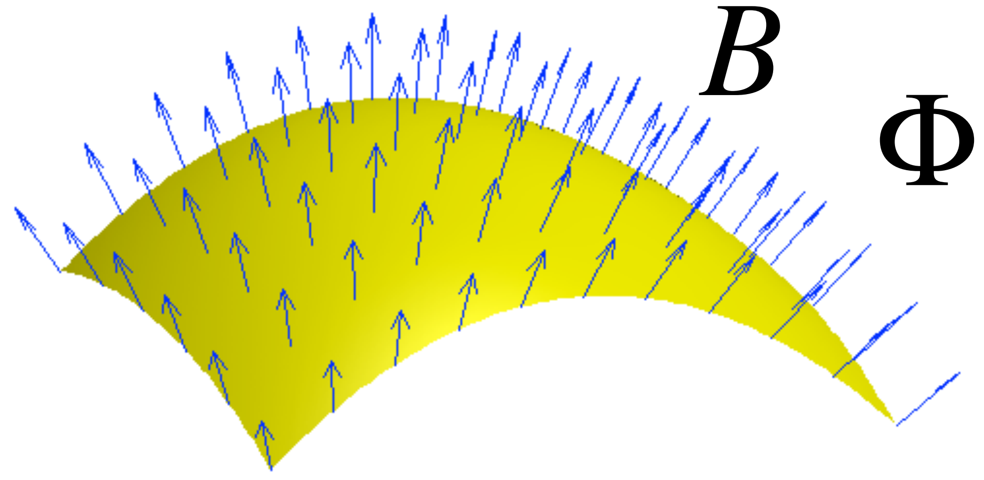
Quantum of magnetic flux

$$G_0 = \frac{2e^2}{h}$$

Quantum of conductance

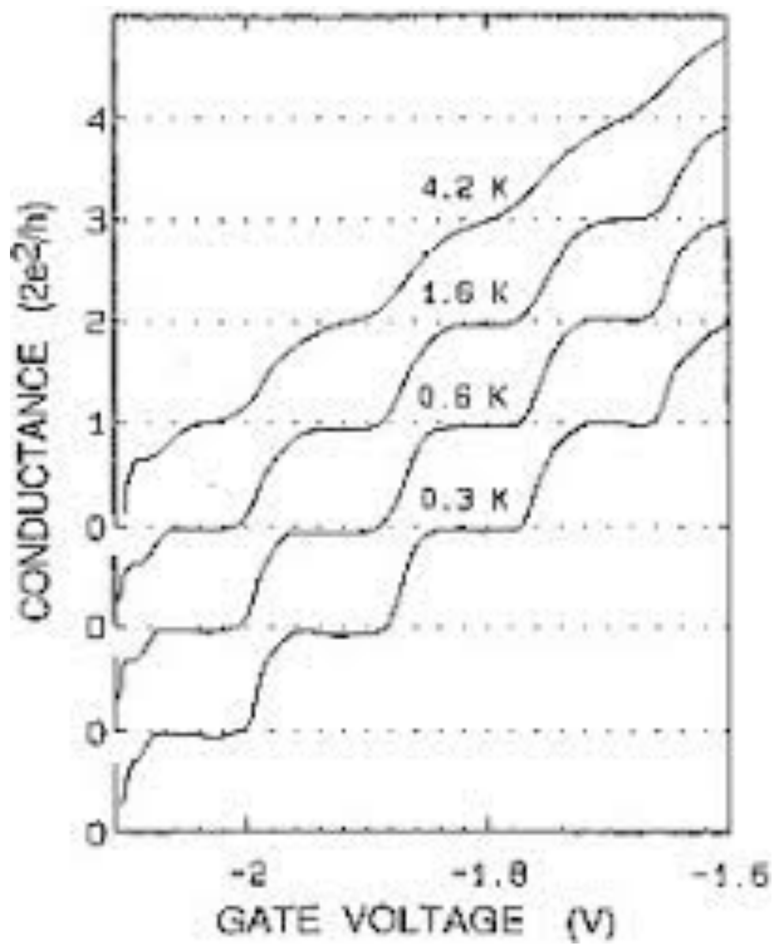


$\overrightarrow{B}$

$$\Phi = \int B\delta a$$

$$I = GV$$

$$G = \frac{1}{R}$$



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- $h = 6.626 \times 10^{-34} \, (J \cdot s)$
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- Action is one measure of "*quantumness*": Processes involving 1-1000 quanta of action are quantum. 1000-1000000 — Semi-classical, 1000000+ — Classical.

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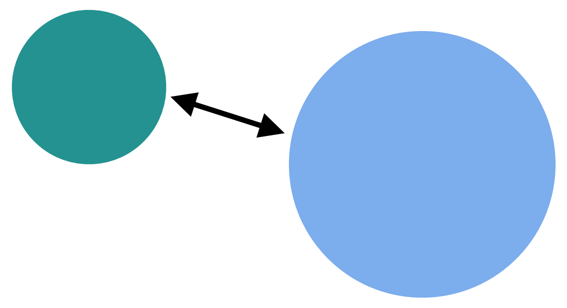

Rule of thumb:

Consider two bodies interacting during time Δt and exchanging energy ΔE. We need quantum physics if $\Delta A = \Delta E \Delta t \leq 10^3\, h$

Action

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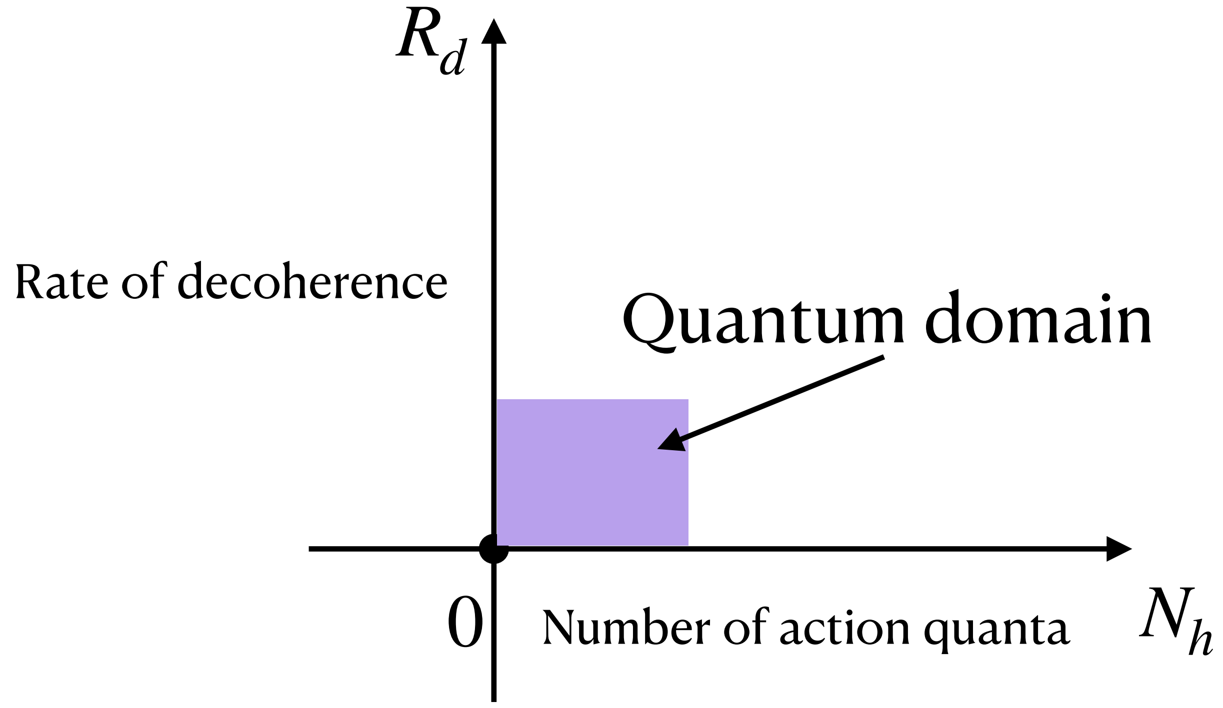


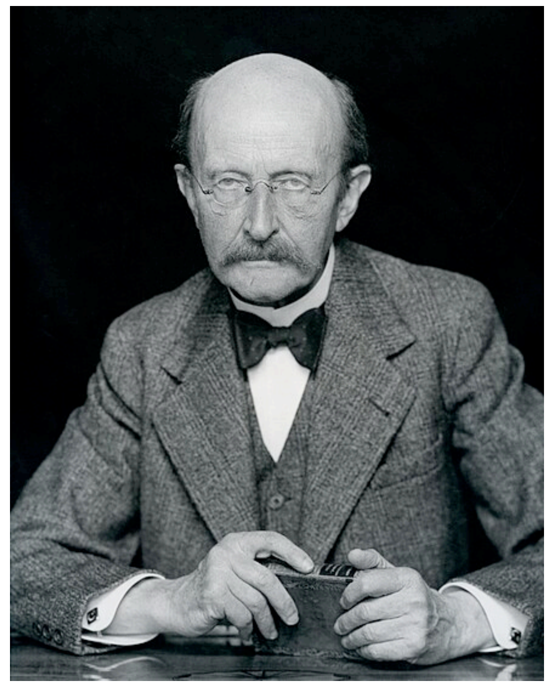
There are at lest three approaches:

- Quantum
- Semi-classical
- Classical

Action

Fundamental And Quantized Physical Quantity

Planck:



My futile attempts to fit the **elementary quantum of action** somehow into the *classical theory* continued for a number of years [actually until 1915] and they cost me a great deal of effort. Many of my colleagues saw in this something bordering on a tragedy. But I feel differently about it, for the thorough enlightenment I thus received was all the more valuable. I now know for a fact that **elementary quantum of action** played a far more significant part in physics than I had originally been inclined to suspect, and this recognition made me see clearly the need for the introduction of *totally new methods of analysis and reasoning* in the treatment of atomic problems.

Bohr:

A new epoch in physical science was inaugurated, however, by Planck's discovery of the **elementary quantum of action,** which revealed a feature of *wholeness* inherent in atomic processes, *going far beyond* the ancient idea of the limited divisibility of matter.

Atomics Physics and Human Knowledge, Quantum Physics and Philosopy, Complimentarity and Causality.

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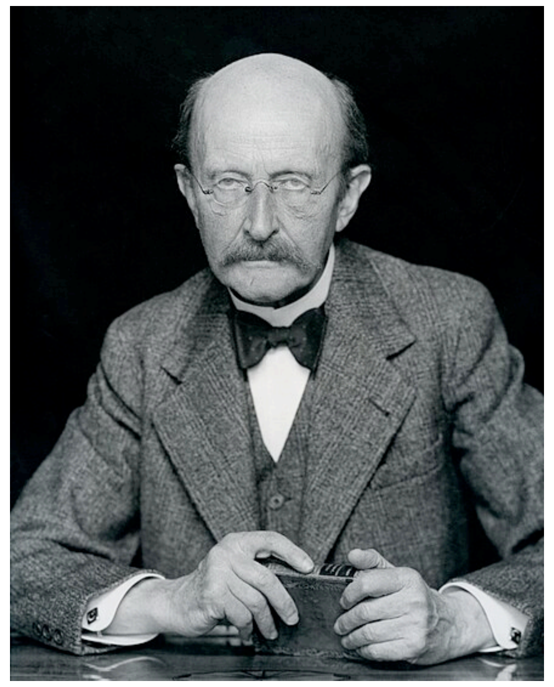

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Used quantization hypothesis for oscillator energy $\Delta E = h\nu = \hbar\omega$ to explain spectra of hot bodies.

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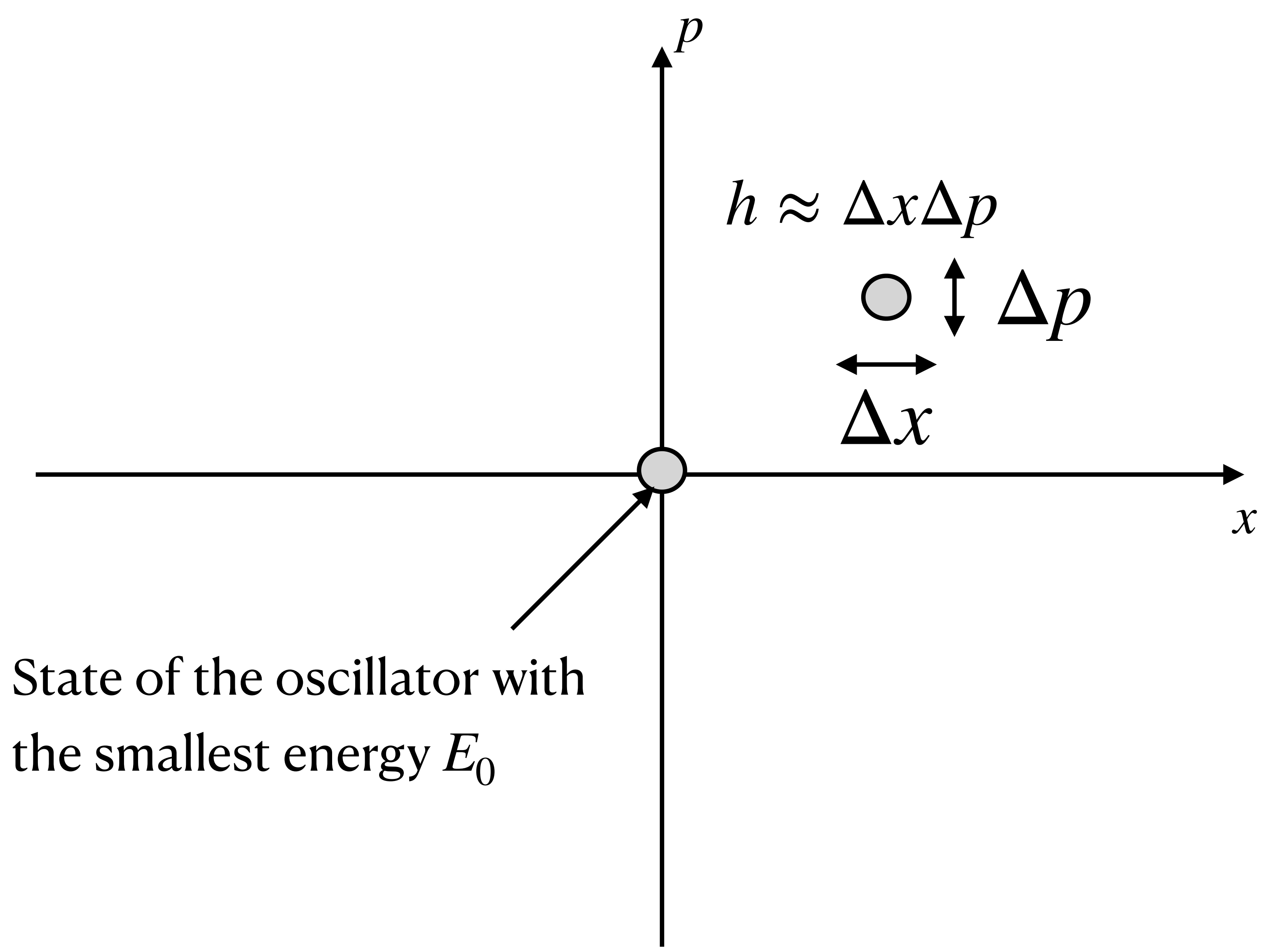
Used quantization hypothesis for electron's angular moment $mvr = n\hbar$ to explain spectra of hydrogen atoms.

Phase Space

Smallest Area — State is not a point



Area is phase space has values $h, 2h, 3h, \cdots$

Smallest area is $h \approx \Delta x \Delta p$

State of a system can not be precisely determined

$$|\xi\rangle = (x \pm \Delta x, p \pm \Delta p)$$

This has far reaching consequences: Absolutely precise trajectory is an idealization. It looses its meaning for electrons in atom.

Exercise: Estimate $\Delta x \Delta p$ for the lowest energy state of hydrogen atom.

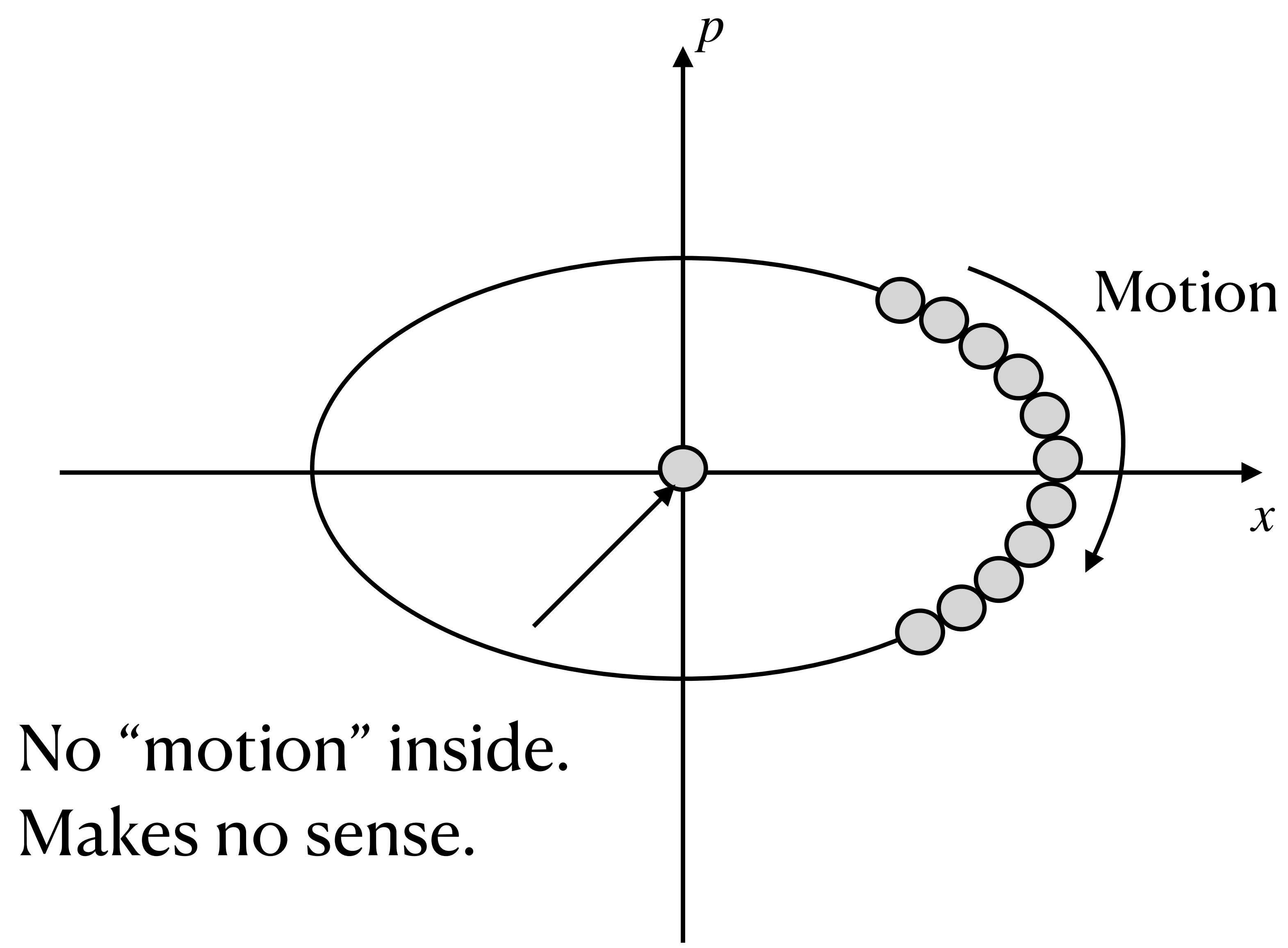
Does area have sharp boundaries? How to properly calculate it?

$h \approx \Delta x \Delta p$?

Answer: Use "spread" σ_x and σ_p for a given type of motion!

Phase Space

Smallest Area — State is not a point



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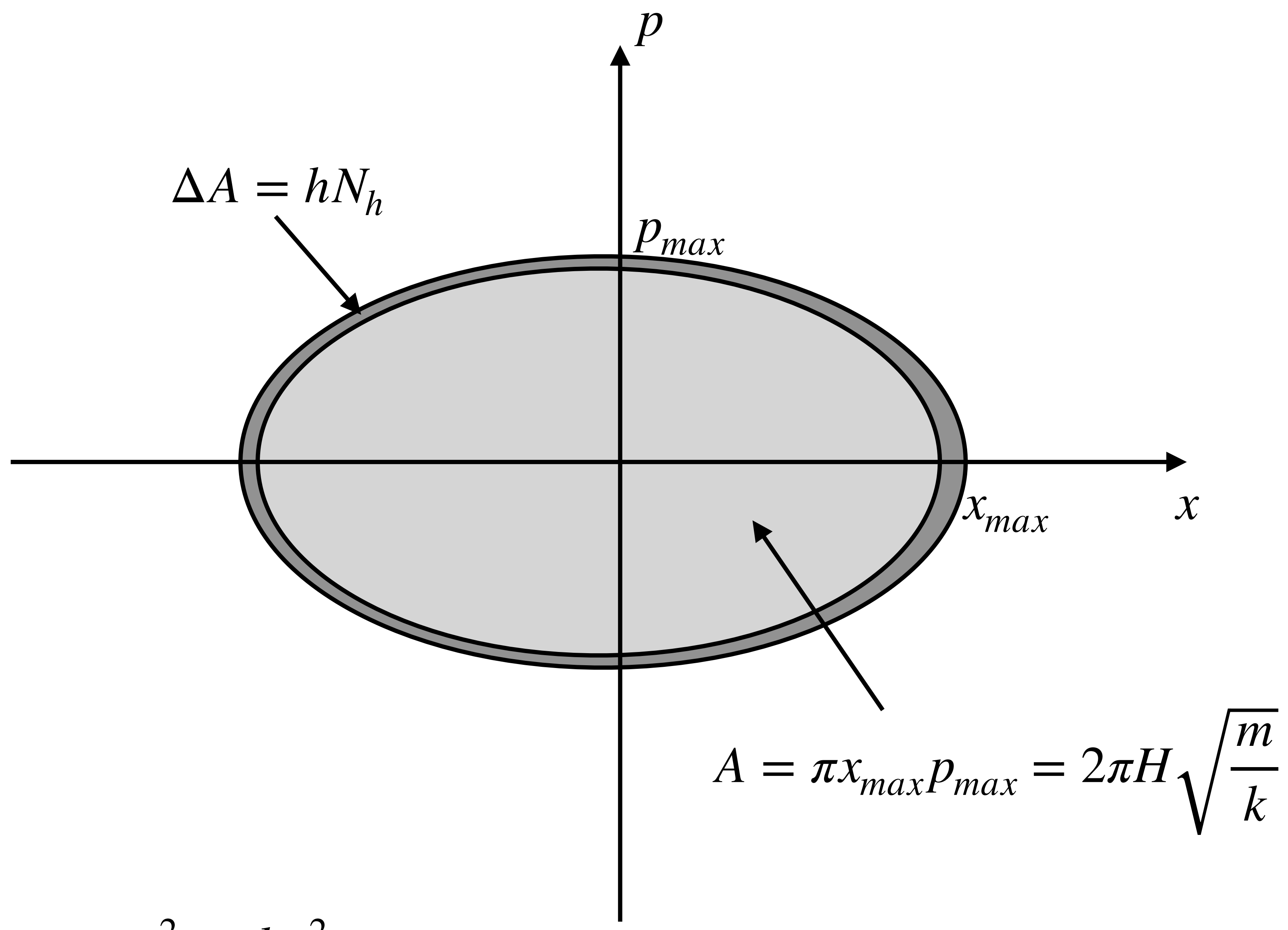
Phase Space

Quantization of Area — Quantization of Oscillator Energy



$$H = \frac{p^2}{2m} + \frac{kx^2}{2}$$

$$H = \frac{p_{max}^2}{2m} \qquad p_{max} = \sqrt{2mH}$$

$$H = \frac{kx_{max}^2}{2} \qquad x_{max} = \sqrt{2H/k}$$

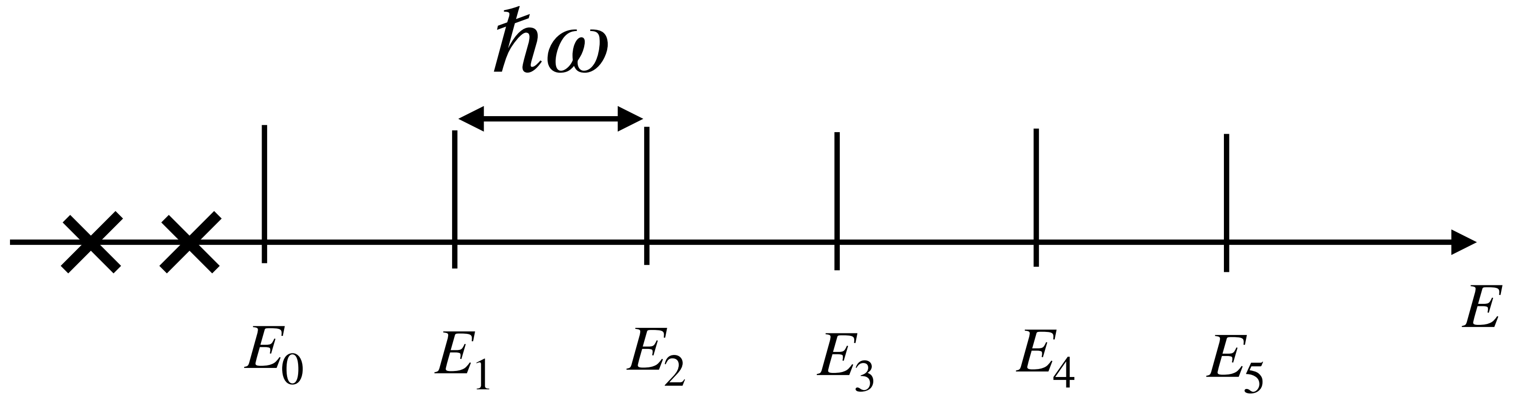
Area in phase space has units of action. Action is quantized. Area is quantized. Area is proportional to the total energy of the oscillator. Energy of the oscillator is quantized. It can change only in fixed steps.

$$H = \frac{A}{2\pi}\sqrt{\frac{k}{m}} \qquad \Delta H = \frac{\Delta A}{2\pi}\sqrt{\frac{k}{m}} = N_h \hbar \sqrt{\frac{k}{m}} \qquad \hbar = \frac{h}{2\pi}$$

$$E = E_0 + \Delta H = E_0 + N_h \hbar \omega$$ Energy of an oscillator

$$\omega = \sqrt{\frac{k}{m}}$$

$E_0 > 0$ Smallest energy. Can't be zero.



Energy levels of an oscillator

Self-Test

Answer These Questions 1hr After Class

1. Which fundamental physical quantities are always quantized?
2. Who discovered the elementary quantum of action?
3. What are the units for action?
4. Which physical quantities have the units of action?
5. How can the action be used to determine quantum regime?
6. In what areas of physics does the quantum of action play a role?
7. What effect does the action has on the structure of phase space?

Homework Problems

Homework 6

- Consider a body of mass m being accelerated by some constant force F. The acceleration is then $a = F/m$. In time t the body will acquire speed $v = at$ and kinetic energy $E_k = mv^2/2$. The action in this scenario has the magnitude on the order of $A = E_k t \approx mv^3/a$. Action is small for light and slow objects, and for very large accelerations. Estimate how many quanta of action are involved in the process described above for a speck of dust with $m = 1\ (microgram)$, accelerated up to $v = 1 (mm/s)$, with acceleration $a = 100g$. Is this a quantum process?

- **Landau Levels**: In a uniform magnetic field $\overrightarrow{B}$ an electron moving perpendicular to the field lines with the speed v will experience a force with magnitude $F = qvB$. The force is directed perpendicular to both the velocity and the magnetic field. As the result, the electron will move in a circle. 1) Show that the angular frequency of this circular motion is $\omega = qB/m$. 2) Use Bohr's quantization of angular momentum $L = mvr = n\hbar$ to show that flux of magnetic field through the area $\Phi = B\pi r^2$ is a multiple of the flux quantum $\Phi_0 = \dfrac{h}{2e}$. 3) Show that radius of the circle can only have discrete set of values.

Quantum Theory

In a Nutshell

II. POSTULATES FOR QUANTUM MECHANICS

In this paper, all state vectors are supposed to be normalized, and mixed states are represented by density operators, i.e., positive operators with unit trace. Let A be an observable with a nondegenerate purely discrete spectrum. Let $\phi_1, \phi_2, \ldots$ be a complete orthonormal sequence of eigenvectors of A and $a_1, a_2, \ldots$ the corresponding eigenvalues; by assumption, all different from each other.

According to the standard formulation of quantum mechanics, on the result of a measurement of the observable A the following postulates are posed:

(A1) *If the system is in the state ψ at the time of measurement, the eigenvalue a_n is obtained as the outcome of measurement with the probability $|\langle\phi_n|\psi\rangle|^2$.*

(A2) *If the outcome of measurement is the eigenvalue a_n, the system is left in the corresponding eigenstate ϕ_n at the time just after measurement.*

The postulate (A1) is called the *statistical formula*, and (A2) the *measurement axiom*. The state change $\psi \mapsto \phi_n$ described by the measurement axiom is called the *state reduction*.

You will understand this paragraph in the end of the course.