Complex Analysis I: Problem Set VIII

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Abstract

This work contains the solutions to the problem set VIII of Complex Analysis I 2015 at Courant Institute of Mathematical Sciences.

Question 1. 273-12.
Solution.

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Question 287-1.
Solution. Observe that the following equality holds, by the linearity of integration:

$$\int_0^{2\pi} \frac{d\theta}{5 + 4\sin\theta} = \frac{1}{5}\int_0^{2\pi} \frac{d\theta}{1 + \frac{4}{5}\sin\theta}$$

From the example 1 from pg.285 in the section 92, it follows that

$$\begin{aligned}\int_0^{2\pi} \frac{d\theta}{5 + 4\sin\theta} &= \frac{1}{5}\int_0^{2\pi} \frac{d\theta}{1 + \frac{4}{5}\sin\theta} \\ &= \frac{1}{5}\frac{2\pi}{\sqrt{1 - \frac{4}{5}^2}} \\ &= \frac{1}{5}\frac{2\pi}{\frac{3}{5}} \\ &= \frac{2}{3}\pi.\end{aligned}$$

□

Question 293-6.
Solution. (a) Inside the circle $|z| = 1$, write

$$f(z) = -5z^4 \text{ and } g(z) = z^6 + z^3 - 2z.$$

Then, observe that when $|z| = 1$,

$$|f(z)| = 5|z|^4 = 5 \text{ and } |g(z)| \leq |z|^6 + |z|^3 + 2|z| = 4.$$

The conditions of Rouche's theorem are thus satisfied. Consequently, since $f(z)$ has 4 zeroes, counting multiplicities, inside the circle $|z| = 1$, $f(z)+g(z)$ has 4 zeroes. Therefore, the polynomial $z^6 - 5z^4 + z^3 - 2z$ has 4 zeroes inside the circle $|z| = 1$. $\square$

(b) Inside the cirlce

(c) Inside the circle $|z| = 1$, write

$$f(z) = -4z^3 \text{ and } g(z) = z^7 + z - 1.$$

Then, observe that when $|z| = 1$,

$$|f(z)| = 4|z|^3 = 4 \text{ and } g(z) \leq |z|^7 + |z| - 1 = 1.$$

The conditions of Rouche's theorem are thus satisfied. Consequently, since $f(z)$ has 3 zeroes, counting multiplicities, inside the circle $|z| = 1$, $f(z)+g(z)$ has 3 zeroes. Therefore, the polynomial $z^7 - 4z^3 + z - 1$ has 3 zeroes inside the circle $|z| = 1$. $\square$

Question 293-8.

Solution.