Complex Analysis I:
Problem Set VI

Youngduck Choi
CILVR Lab
New York University
yc1104@nyu.edu

Abstract

This work contains the solutions to the problem set VI of Complex Analysis I 2015 at Courant Institute of Mathematical Sciences.

Question 1. 237-2.

Solution. (a) We have

$$\begin{aligned}
\frac{1}{z+z^2} &= \frac{1}{z}\frac{1}{1+z} \\
&= \frac{1}{z}(1 - z + z^2 \dots) \\
&= \frac{1}{z} - 1 + z \dots
\end{aligned}$$

for $0 < |z| < 1$. The coefficient of $\frac{1}{z}$ term is 1. Hence, the residue is 1.

(b) We have

$$\begin{aligned}
z\cos(\frac{1}{z}) &= z(1 - \frac{1}{2!}\frac{1}{z^2} + \frac{1}{4!}\frac{1}{z^4} \dots) \\
&= z - \frac{1}{2!}\frac{1}{z} + \frac{1}{4!}\frac{1}{z^3} \dots
\end{aligned}$$

for $|z| < \infty$. The coefficient of $\frac{1}{z}$ term is 0. Hence, the residue is 0.

(c) We have

$$\begin{aligned}
\frac{z - \sin(z)}{z} &= \frac{1}{z} \cdot \frac{1}{\sin(z)} \\
&= \frac{1}{z}(z - (z - \frac{z^3}{3!} + \frac{z^5}{5!} \dots))
\end{aligned}$$

for $0 < |z| < \infty$. The coefficient of $\frac{1}{z}$ term is 0. Hence, the residue is 0.

(d) We have

$$\frac{\cot(z)}{z^4} = \frac{1}{z^4} \cdot \frac{\cos(z)}{\sin(z)}$$

By dividing the series representation of $\cos$ by $\sin$, we obtain

$$\frac{\cos(z)}{\sin(z)} = \frac{1}{z} - \frac{z}{3} - \frac{z^3}{45} \dots$$

for $0 < |z| < \pi$. It follows that

$$\frac{\cot(z)}{z^4} = \frac{1}{z^5} - \frac{1}{3}\frac{1}{z^3} - \frac{1}{45} \cdot \frac{1}{z} \dots$$

for $0 < |z| < \pi$.

Question 1. Brown p.147-2.
Solution.

Question 1. Brown p.147-2.
Solution.

Question 1. Brown p.147-2.
Solution.

Question 1. Brown p.147-2.
Solution.