Linear Algebra I: Problem Set I

Youngduck Choi
CIMS
New York University
yc1104@nyu.edu

Abstract

This work contains the solutions to the problem set I of Linear Algebra I 2015 at Courant Institute of Mathematical Sciences.

Question 1.

Solution. We show that the three vectors $u+v+w$, $v+w$, and w are linearly independent. Assume that

$$a_1(u+v+w) + a_2(v+w) + a_3(w) \quad = \quad 0.$$

This is equivalent to

$$a_1 u + (a_1 + a_2)v + a_3(u+v+w) \quad = \quad 0.$$

This is again equivalent to

$$a_1 u + a_2 v + a_3 w \quad = \quad 0.$$

Since the set $\{u, v, w\}$ are linearly independent, we have that $a_1 = 0$ Hence, $u + v$ $\square$

Question 2.

Solution.

Question 3.

Solution.

Question 4.

Solution. Let W_1 and W_2 be subspaces of V. First, assume that $W_1 \subseteq W_2$. Then, we have $W_1 \cup W_2 = W_2$. Since W_2 is a subspace, we have that $W_1 \cup W_2$ is a subspace. By symmetry, we also have if $W_2 \subseteq W_1$, then $W_1 \cup W_2$ is a subspace.

Now, assume that $W_1 \cup W_2$ is a subspace. Suppose that $W_1 \setminus W_2 \neq \emptyset$. Let $x \in W_1$ and $y \in W_1 \setminus W_2$.

Hence, we have shown that $W_1 \cup W_2$ is a subspace iff $W_1 \subseteq W_2$ or $W_2 \subseteq W_1$.

Question 5.

Solution.

Question 6.
Solution. Assume that

Question 7.
Solution.

Question 8.
Solution.