Self-test Questions on Prerequisites

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Abstract

The following is a collection of solutions of the self-test questions for Real Variables at the Courant Institute.

Question.

Solution. Two sets are said to be equipotent provided there is a bijective map from one to another. Hence, to show that the sets (0,1] and [0,1] are equipotent, it suffices to construct a bijective map from (0,1] to [0,1].

Question.

Solution. Let $\{a_n\}$ be a sequence of extended real numbers, X be a set of cluster points of $\{a_n\}$. We first show that $\limsup\{a_n\}$ is a cluster point of $\{a_n\}$. First, the limit infimum of $\{a_n\}$ can be written as

$$\lim \sup\{a_n\} = \lim_{n \to \infty} [\sup\{a_k \mid k \ge n\}].$$