Self-test Questions on Prerequisites

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Abstract

The following is a collection of solutions of the self-test questions for Real Variables at the Courant Institute.

Question.

Solution. Two sets are said to be equipotent provided there is a bijective map from one to another. Hence, to show that the sets $(0, 1]$ and $[0, 1]$ are equipotent, it suffices to construct a bijective map from $(0, 1]$ to $[0, 1]$.

Question.

Solution. Let $\{a_n\}$ be a sequence of real numbers, X be a set of cluster points of $\{a_n\}$. We first show that $\limsup\{a_n\}$ is a cluster point of $\{a_n\}$. First, we simply denote $\limsup\{a_n\}$ as s. It can be written as

$$s = \lim_{n\to\infty} [\, \sup\{a_k \mid k \geq n\} \,].$$

Let x be any cluster point of $\{a_n\}$. By the definition of a cluster point, we have a subsequence $\{a_{n_k}\}$ such that converges to x. Then, for any $\epsilon > 0$, we have N such that for $n_k \geq N$, $x - a_{n_k} < \epsilon$ holds. Hence, $s \geq x$. We have shown that $\limsup\{a_n\}$ is the largest cluster point.

Question.

Solution.