Intro to Macroeconmics: Assignment I

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Abstract

This document contains the solutions to the problem set I.

1 Solutions to the problems

Question 1. Review Questions.

Solution. (3) "A model is, basically, a simplification of a particular part or feature of the world. Modeling is essential and inseparable part of any scientific activity. Models illustrate the essence of the real object it is designed to resemble. Maybe without noticing it, we also use models when thinking about a problem; there is always an implicit model." (pg.10 from the lecture II slides)

(4) Simplification is an important part of modelling because it help us "to dispense with irrelevant details and to focus on the important connections." (pg.10 from the lecture II slides)

(5) Economics use math, because math "gives precision and structure to our thinking." (pg.17 from the lecture II slides)

Question 3. Utility function.

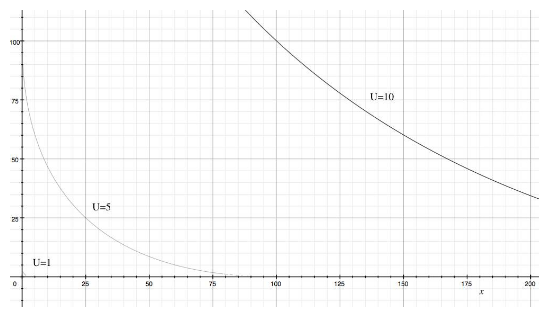
Solution. (1) We claim that preferences are strictly monotonic with respect to z and s. Taking the partials with respect to z and s, we obtain

$$
\begin{aligned}
\frac{\partial U}{\partial z} &= \frac{\alpha}{2} z^{-\frac{1}{2}}, \\
\frac{\partial U}{\partial s} &= \frac{1-\alpha}{2} s^{-\frac{1}{2}}.
\end{aligned}
$$

As $\alpha \in (0,1)$, we see that both partials are strictly positive for all positive values of z and s, which is the domain of interest in this case. Hence, we have shown that the preferences are strictly monotonic with respect to z and s. Note that strict monotonicity implies monotonicity as well.

(2) The following figure contains the graphs of indifference curves for the corresponding utility levels $\bar{U} = 1, 5, 10$ with $\alpha = \dfrac{1}{2}$.

Figure 1: Indifference curves



(3) Yes, the marginal utility is decreasing. Evaluating the second-order partials from the first-order partials obtained in part (1), we get

$$\begin{aligned} \frac{\partial^2 U}{\partial z^2} &= -\frac{\alpha}{4} z^{-\frac{3}{2}}, \\ \frac{\partial^2 U}{\partial s^2} &= -\frac{1-\alpha}{4} s^{-\frac{3}{2}}. \end{aligned}$$

As $\alpha \in (0,1)$, we see that for any positive values of z and s, we have

$$\begin{aligned} \frac{\partial^2 U}{\partial z^2} &< 0, \\ \frac{\partial^2 U}{\partial s^2} &< 0. \end{aligned}$$

Therefore, we have shown that the marginal utility is decreasing with respect to both variables. It is, in fact, strictly decreasing.

(4) First, note that we have the following formula for the computation Marginal Rate of Substitution of z for s at bundle (z, s):

$$MRS_{\text{z for s}}(z,s) = \frac{\frac{\partial U}{\partial z}}{\frac{\partial U}{\partial s}}.$$

Substituting the first-order partials computed previously, we obtain

$$MRS_{\text{z for s}}(z,s) = \frac{\frac{a}{2} z^{-\frac{1}{2}}}{\frac{1-\alpha}{2} s^{-\frac{1}{2}}}.$$

Substituting $\alpha = \dfrac{1}{2}$ and $(z, s) = (1, 4)$ to the above equation and simplifying, we get

$$MRS_{\text{z for s}}(1,4) = \frac{1^{-\frac{1}{2}}}{4^{-\frac{1}{2}}} = 2.$$

This means that the consumer under consideration is ready to give up twice as much soda in exchange for pizza while maintaining the same level of utility.

(5) We solve the given maximization problem of the consumer by using the substitution method. First of all, notice that we can factor out the constant factor $\frac{1}{2}$, as the constant uniformly scales the objective down across the domain and does not affect where the maximum objective is attained. Hence, the maximization problem can be reduced to the following equivalent problem:

$$\max_{\{z,s\}} z^{\frac{1}{2}} + s^{\frac{1}{2}}$$
$$\text{s.t.} \quad : \; p_z \cdot z + p_s \cdot s = m$$

Solving for s in the constraint equation, we obtain

$$s \quad = \quad \frac{m - p_z \cdot z}{p_s}. \tag{1}$$

Substituting the above equation into the objective, we now have the following unconstrained maximization problem in terms of only the z variable:

$$\max_{\{z\}} z^{\frac{1}{2}} + (\frac{m - p_z \cdot z}{p_s})^{\frac{1}{2}}$$

Taking the first-order derivative with respect to s, we get

$$\frac{\partial U}{\partial z} \quad = \quad \frac{1}{2}\{z^{-\frac{1}{2}} - \frac{p_z}{p_s} \cdot (\frac{m - p_z \cdot z}{p_s})^{\frac{-1}{2}}\}.$$

As we only need to check the first order conditions for the optimal solution, we have

$$z^{*-\frac{1}{2}} - \frac{p_z}{p_s} \cdot (\frac{m - p_z \cdot z^*}{p_s})^{\frac{-1}{2}} \quad = \quad 0,$$

where z^* denotes the solution for the z variable in the maximization problem. Moving the first term to the RHS, squaring both sides and expressing the power of -1 explicitly, we get

$$\frac{1}{z^*} \quad = \quad \frac{p_z^2}{p_s^2} \cdot \frac{p_s}{m - p_z \cdot z^*}.$$

Canceling the p_s factor on the RHS and multiplying $z^* \cdot p_s \cdot (m - p_z \cdot z^*)$ to both sides yields

$$z^* \cdot p_z^2 \quad = \quad p_s \cdot (m - p_z \cdot z^*).$$

Re-arranging the terms, we get

$$p_z \cdot (p_z + p_s) \cdot z^* = p_s \cdot m.$$

Isolating the z^* variable gives

$$z^* \quad = \quad \frac{p_s}{p_s + p_z} \cdot \frac{m}{p_z}.$$

Substituting z^* into the equation (1), we can also express s^* as follows:

$$s^* \quad = \quad \frac{m - p_z \cdot \frac{p_s}{p_s + p_z} \cdot \frac{m}{p_z}}{p_s}.$$

Factoring out $\frac{m}{p_z}$ and simplifying the subtraction at the denominator yields

$$s^* \quad = \quad \frac{p_z}{p_s + p_z} \cdot \frac{m}{p_s},$$

which completes the task of finding the explicit solution to the given maximization problem.

(6) Pizza is a normal good because by definition a normal good is good whose demand increases with income. It can be said that with more income the price of a good decreases. Thus, if p_z was to go down we an see that the quantity demand of Pizza (Z) would go up. Thus, by the definition of a normal good we can see that pizza is a normal good.

(7) The demand for pizza is always downward sloped because an increase in p_z causes a decrease in the quantity demanded of pizzas. This can be seen in the Z* equation. Also due to ceteris paribus the curve slopes downwards because lower prices mean a greater quantity demanded. Since we are keeping other variables fixed while we focus on pizza.