Intro to Macroeconmics: Assignment I

Youngduck Choi, Noah Gentile, Yuliya Takh
New York University

Abstract

1 Solutions to the problems

Question 1. Review Questions.

Solution. (3) "A model is, basically, a simplification of a particular part or feature of the world. Modeling is essential and inseparable part of any scientific activity. Models illustrate the essence of the real object it is designed to resemble. Maybe without noticing it, we also use models when thinking about a problem; there is always an implicit model." (pg.10 from the lecture II slides)

(4) Simplification is an important part of modelling because it help us "to dispense with irrelevant details and to focus on the important connections." (pg.10 from the lecture II slides)

(5) Economics use math, because math "gives precision and structure to our thinking." (pg.17 from the lecture II slides)

Question 3. Utility function.

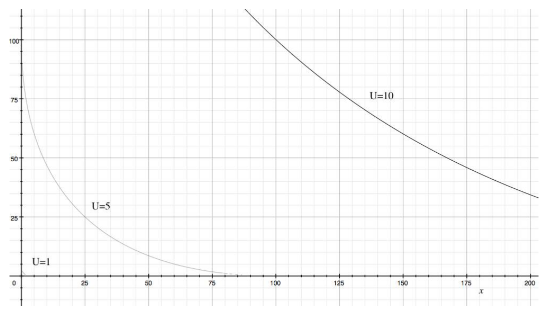
Solution. (1) We claim that preferences are strictly monotonic with respect to z and s. Taking the partials with respect to z and s, we obtain

$$\begin{aligned} \frac{\partial U}{\partial z} &= \frac{\alpha}{2} z^{-\frac{1}{2}}, \\ \frac{\partial U}{\partial s} &= \frac{1-\alpha}{2} s^{-\frac{1}{2}}. \end{aligned}$$

As $\alpha \in (0, 1)$, we see that both partials are strictly positive for all positive values of z and s, which is the domain of interest in this case. Hence, we have shown that the preferences are strictly monotonic with respect to z and s. Note that strict monotonicity implies monotonicity as well.

(2) The following figure contains the graphs of indifference curves for the corresponding utility levels $\bar{U} = 1, 5, 10$ with $\alpha = \dfrac{1}{2}$.

Figure 1: Indifference curves



(3) Yes, the marginal utility is decreasing. Evaluating the second-order partials from the first-order partials obtained in part (1), we get

$$\begin{aligned} \frac{\partial^2 U}{\partial z^2} &= -\frac{\alpha}{4} z^{-\frac{3}{2}}, \\ \frac{\partial^2 U}{\partial s^2} &= -\frac{1-\alpha}{4} s^{-\frac{3}{2}}. \end{aligned}$$

As $\alpha \in (0, 1)$, we see that for any positive values of z and s, we have

$$\begin{aligned} \frac{\partial^2 U}{\partial z^2} &< 0, \\ \frac{\partial^2 U}{\partial s^2} &< 0. \end{aligned}$$

Therefore, we have shown that the marginal utility is decreasing with respect to both variables. It is, in fact, strictly decreasing.

(4) First, note that we have the following formula for the computation Marginal Rate of Substitution of z for s at bundle (z, s):

$$MRS_{\text{z for s}}(z, s) = \frac{\frac{\partial U}{\partial z}}{\frac{\partial U}{\partial s}}.$$

Substituting the first-order partials computed previously, we obtain

$$MRS_{\text{z for s}}(z, s) = \frac{\frac{a}{2} z^{-\frac{1}{2}}}{\frac{1-\alpha}{2} s^{-\frac{1}{2}}}.$$

Substituting $\alpha = \dfrac{1}{2}$ and $(z, s) = (1, 4)$ to the above equation and simplifying, we get

$$MRS_{\text{z for s}}(1, 4) = \frac{1^{-\frac{1}{2}}}{4^{-\frac{1}{2}}} = 2.$$

This means that the consumer under consideration is ready to give up twice as much soda in exchange for pizza while maintaining the same level of utility.

(5) We solve the given maximization problem of the consumer by using the substitution method. Solving for z in the constraint equation, we obtain

$$z = \frac{m - p_s \cdot s}{p_z}.$$

Substituting the above equation into the objective, we end up with the following maximization problem:

$$\max_{\{s\}} \frac{1}{2}(\frac{m - p_s \cdot s}{p_z})^{\frac{1}{2}} + \frac{1}{2}s^{\frac{1}{2}}.$$

Taking the first-order derivative with respect to s, we get

$$\frac{\partial U}{\partial s} = \frac{1}{4}\{-p_s \cdot (\frac{m - p_s \cdot s}{p_z})^{\frac{-1}{2}} + s^{\frac{-1}{2}}\}$$

(6)

(7)