
Intro to Macroeconomics: Problem Set IV

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Abstract

This document contains the solutions to the problem set III.

1 Solutions to the problems

Question 1. RC Economy.

Solution. (1) We are given the RC economy such that RC preferences over C and L is given by

$$U(C, L) = -\frac{1}{3}C^{-3} - \frac{1}{2}L^2,$$

which can be re-written in terms of C and O as

$$U(C, O) = -\frac{1}{3}C^{-3} - \frac{1}{2}(1 - O)^2,$$

and the production technology is given by

$$F(K, L) = AK^{\frac{1}{3}}L^{\frac{2}{3}}.$$

With K fixed, we want to find the solution to the RC problem. We first compute the tangency condition of $MRS_{O \text{ for } C} = MPL$. To compute MRS on the LHS, we first take the partials with respect to first and second variable, obtaining

$$\begin{aligned} U_1(C, O) &= C^{-4} \\ U_2(C, O) &= 1 - O \\ &= L. \end{aligned}$$

Hence, we have that

$$\begin{aligned} MRS_{O \text{ for } C} &= \frac{U_2(C, O)}{U_1(C, O)} \\ &= LC^4. \end{aligned}$$

Computing the MPL, we obtain

$$\begin{aligned} MPL &= F_L(K, L) \\ &= \frac{2}{3}AK^{\frac{1}{3}}L^{-\frac{1}{3}}. \end{aligned}$$

Therefore, via substitution, we have the tangency condition as

$$LC^4 = \frac{2}{3}AK^{\frac{1}{3}}L^{-\frac{1}{3}}.$$

With the constraint is given by $C = F(K, L)$, by substitution we have the explicit constraint as

$$C = AK^{\frac{1}{3}}L^{\frac{2}{3}}.$$

Substituting the above equation into the tangency condition and re-arranging, we have

$$L = \frac{2}{3} \frac{AK^{\frac{1}{3}} L^{-\frac{1}{3}}}{A^{\frac{4}{3}} K^{\frac{4}{3}} L^{\frac{8}{3}}}.$$

Simplifying gives

$$L = \frac{2}{3} A^{-3} K^{-1} L^{-3}.$$

Moving L on the RHS to LHS and explicitly solving for it, we get

$$L^* = \left(\frac{2}{3} A^{-3} K^{-1}\right)^{\frac{1}{4}}.$$

So, the consumption is,

$$\begin{aligned} C^* &= AK^{\frac{1}{3}} \left(\frac{2}{3} A^{-3} K^{-1}\right)^{\frac{1}{4}} \\ &= \left(\frac{2}{3}\right)^{\frac{1}{4}} A^{\frac{1}{4}} K^{\frac{1}{12}} \\ &= \left(\frac{2}{3} AK^{\frac{1}{3}}\right)^{\frac{1}{4}}. \end{aligned}$$

Hence, we have computed the solution to RC's problem.

(2) L is not increasing in A . It is, in fact, decreasing as we have A^{-3} in the computed solution. L is an inferior good. Hence, the income effect is C goes up and O goes down, therefore L goes up. The opportunity cost of leisure became relatively more expansive, so RC substitutes toward C .

(3) The effect of a natural disaster, which translates to reduction of the quantity K , will decrease the consumption, but will increase the labor. This comes from a direct relationship we have from the computed solutions above.