Intro to Macroeconmics: Problem Set IV

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Abstract

This document contains the solutions to the problem set IV.

1 Solutions to the problems

Question 1.

Solution. (a) We are given the demand function as follow:

$$\begin{aligned} C_1^A &= \frac{1+\rho_A}{2+\rho_A}(Y_1^A + \frac{Y_2^A}{1+r}), \\ C_1^B &= \frac{1+\rho_B}{2+\rho_B}(Y_1^B + \frac{Y_2^B}{1+r}), \\ C_2^A &= \frac{1+r}{2+\rho_A}(Y_1^A + \frac{Y_2^A}{1+r}), \\ C_2^B &= \frac{1+r}{2+\rho_B}(Y_1^B + \frac{Y_2^B}{1+r}). \end{aligned}$$

We have that $S = Y_1 - C_1$. Therefore, we can compute Ss as

$$\begin{aligned} S_A &= Y_1^A - C_1^A, \\ S_B &= Y_1^B - C_1^B, \end{aligned}$$

where C_1^A and C_1^B are values which can be computed from the previous equality.

(b) We are given a general solution for the equilbrium interest rate as follows:

$$r^* = \frac{\frac{1+\rho_A}{2+\rho_A}{Y_2}^A + \frac{1+\rho_B}{2+\rho_B}{Y_2}^B}{\frac{1}{2+\rho_A}{Y_1}^A + \frac{1}{2+\rho_B}{Y_1}^B} - 1.$$

As we have $\rho_A = \rho_B = \rho$ assumption, we can further simplify the above solution as

$$r^* = (1+\rho)\frac{({Y_2}^A + {Y_2}^B)}{({Y_1}^A + {Y_1}^B)} - 1.$$

(c) Yes, r^* is a function of total/aggregate endowments, as we can see from the above equation. The real interest rate would not change if the endowments were redistributed. Because the total is still the same.

(d) We first compute the saving of Anne. First note that we can do further derivation on S_A and obtain

$$S^* = \frac{1}{2+\rho}(Y_1 - \frac{1+\rho}{1+r^*}Y_2).$$

Substituting the r^* into the above equation, we have

$$S_A^* = \frac{1}{2+\rho}(Y_1^A - \frac{1+\rho}{(1+\rho)\frac{(Y_2{}^A+Y_2{}^B)}{(Y_1{}^A+Y_1{}^B)}}Y_2^A).$$

Simplifying further gives,

$$S_A^* \quad = \quad \frac{1}{2+\rho}(Y_1^A - \frac{(Y_1{}^A+Y_1{}^B)}{(Y_2{}^A+Y_2{}^B)}Y_2^A).$$

The information given is not complete to deduce whether or not Anne will be saving or borrowing in the new equilibrium. We need to know the relation between Y_1 and Y_2, to see that second term in the saving computation is either positive or negative. The information is not given. If we take $Y_1^A = Y$ as an implicit assumption, Anne will be borrwing as Y_1^B will be higher, causing the second term in the saving computation to be negative.

(e) This is the same situation as the previous part. We can use the relation that $S = Y - C$ and compute the C as $C = Y - S$, using the above equation. We realize that this is a negative linear relation with a Y being a fixed y-intercept. Hence, Anne's consumption will increase with Bob's endowment increasing (saving will decrease with increase in Y_B^1 which will mean that the consumption increases with a fixed endowment for Ann.

Question 2.

Solution. **(a)** We are given a general solution for the equilbrium interest rate as follows:

$$r^* = \frac{\frac{1+\rho_A}{2+\rho_A}{Y_2}^A + \frac{1+\rho_B}{2+\rho_B}{Y_2}^B}{\frac{1}{2+\rho_A}{Y_1}^A + \frac{1}{2+\rho_B}{Y_1}^B} - 1.$$

Substituting Y for all instances of endowment variables and simplifying, we obtain

$$r^* = \frac{\frac{1+\rho_A}{2+\rho_A} + \frac{1+\rho_B}{2+\rho_B}}{\frac{1}{2+\rho_A} + \frac{1}{2+\rho_B}} - 1.$$

Multiplying both numerator and denominator yields

$$r^* = \frac{(1+\rho_A)(2+\rho_B) + (1+\rho_B)(2+\rho_A)}{2+\rho_B+2+\rho_A} - 1$$

Expanding the denominator and simplying the numerator, we get

$$r^* = \frac{4+3\rho_A+3\rho_B+2\rho_A\rho_B}{4+\rho_B+\rho_A} - 1.$$

Combining the -1 term into the fraction, and factoring out 2 we get

$$r^* = 2\frac{\rho_A+\rho_B+\rho_A\rho_B}{4+\rho_B+\rho_A},$$

as desired.

(b) Recall that the consumer actually saves if $\dfrac{Y_1}{Y_2} > \dfrac{1+\rho}{1+r}$. Therefore, as $\rho_A > r^*$ and $Y_1 = Y_2 = Y$, we have $1 < \dfrac{1+\rho}{1+r^*}$. Hence, with the equilibrum interest rate Anne is a borrower in the first period.

Question 3.

Solution. **(a)** We are given a general solution for the equilbrium interest rate as follows:

$$r^* = \frac{\frac{1+\rho_A}{2+\rho_A}{Y_2}^A + \frac{1+\rho_B}{2+\rho_B}{Y_2}^B}{\frac{1}{2+\rho_A}{Y_1}^A + \frac{1}{2+\rho_B}{Y_1}^B} - 1.$$

Substituting the given conditions into the equation gives,

$$r^* = \frac{\frac{1+\rho}{2+\rho}Y}{\frac{1}{2+\rho}Y} - 1.$$

Further simplification yields

$$r^* = \rho.$$

(b) As $r^* = \rho$, by substituting to the derived demand function, we can see that Anne's consumption path is constant over time (i.e. ${C_1}^A = {C_2}^B$). Bob's consumption path is also constant over time, as they share the same impatience.