Basic Probability: Problem Set I

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Abstract

This work contains a collection of solutions for selected problems of the Basic Probability course of Fall 2015.

Question 1.dd.

Solution. (i) Let $X = \{(a, b)|a \in \mathbb{Q}, b \in \mathbb{Q}\}$. In particular, $X = \mathbb{Q} \times \mathbb{Q}$. Notice that by the Cantor diagonalization argument, there exists a bijective map $\phi : \mathbb{Q} \times \mathbb{Q} \to \mathbb{N} \times \mathbb{N}$. Now, consider a map $\psi : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ such that $\psi(m, n) = (m + n)^2 + n$. With some algebra, we can show that if $\psi(m, n) = \psi(m', n')$, then $m = m'$ and $n = n'$. Hence, ψ is injective, and $\mathbb{N} \times \mathbb{N}$ is equipotent to $\psi(\mathbb{N} \times \mathbb{N})$, which is a subset of the countable set $\mathbb{N}$. Therefore, we have shown that $\mathbb{N} \times \mathbb{N}$ is countable, and consequently X is countable, due to the existence of the bijective map ϕ.

(ii) Let $X = \{B((x, y), r)|x = y\}$. Consider the set Y such that $Y = \{B((x, y), 1)|x = y\}$. Y is in fact equipotent with $\mathbb{R}$, as you can arbitrarily pick the center from $\mathbb{R}$ and that determines the circle in Y. Hence, Y is uncountable. Furthermore, notice that $Y \subset X$. Hence, X is uncountable.

(iii)

Question 2-(i). σ-field.

Solution. Let $\{\mathcal{G}_\lambda\}_{\lambda \in \Lambda}$ be a collection of σ-fields of the space Ω. We wish to show that $\cap_{\lambda \in \Lambda} \mathbb{G}_\lambda$ is a σ-field of Ω. As $\emptyset$, $\Omega \in \mathcal{G}$ for all $\lambda \in \Lambda$, we have that

$$\emptyset, \;\; \Omega \in \cap_{\lambda \in \Lambda} \mathcal{G}_\lambda,$$

thereby satisfying one basic property of σ-field. It remains to show that a union of countable collection of subsets in

Question 4.

Solution.