# Putnam: Assignment II

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## Abstract

Analysis problems

## 1 Solutions to the problems

**Question.**

**Solution.** We wish to find the minimum value of the given expression:

$$\frac{(x+\frac{1}{x})^6-(x^6+\frac{1}{x^6})-2}{(x+\frac{1}{x})^3+(x^3+\frac{1}{x^3})},$$

where $x>0$. Observe that the following identity which follows from the difference of squares:

$$\begin{aligned}((x+\frac{1}{x})^3+(x^3+\frac{1}{x^3}))((x+\frac{1}{x})^3-(x^3+\frac{1}{x^3})) &= (x+\frac{1}{x})^6-(x^3+\frac{1}{x^3})^2 \\ &= (x+\frac{1}{x})^6-(x^6+\frac{1}{x^6})-2.\end{aligned}$$

Dividing the both side of the identity by $(x+\frac{1}{x})^3+(x^3+\frac{1}{x^3})$, we see that the given expression is equivalent to:

$$(x+\frac{1}{x})^3-(x^3+\frac{1}{x^3}),$$

which can be further simplified to $3(x+\frac{1}{x})$. As $x>0$, by the AM-GM inequality we see that the minimum happens at $x=1$, thereby showing that the minimum of the given expression with $x>0$ is 3. $\square$

**Question.**

**Solution.** We wish to show that $\prod_{i=1}^{n}\frac{a_i}{(a_i+b_i)}$