Frequent Itemset Mining & Association Rules

Association Rule Discovery

Supermarket shelf management – Market-basket model:

- **Goal:** Identify items that are bought together by sufficiently many customers
- **Approach:** Process the sales data collected with barcode scanners to find dependencies among items
- **A classic rule:**
  - If one buys diaper and milk, then he is likely to buy beer
  - Don't be surprised if you find six-packs next to diapers!

TID	*Items*
1	Bread, Coke, Milk
2	Beer, Bread
3	Beer, Coke, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Coke, Diaper, Milk

Rules Discovered:

{Milk} --> {Coke}

{Diaper, Milk} --> {Beer}

The Market-Basket Model

- A large set of ***items***
  - e.g., things sold in a supermarket
- A large set of ***baskets***, each is a small subset of items
  - e.g., the things one customer buys on one day

TID	*Items*
1	Bread, Coke, Milk
2	Beer, Bread
3	Beer, Coke, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Coke, Diaper, Milk

- A general many-many mapping (association) between two kinds of things
  - But we ask about connections among "items," not "baskets"

Association Rules: Approach

- **Given a set of baskets**
- Want to discover **association rules**
  - People who bought {x,y,z} tend to buy {v,w}
    - Amazon!
- **2 step approach:**
  - **1)** Find frequent **itemsets**
  - **2)** Generate **association rules**

Input:

TID	*Items*
1	Bread, Coke, Milk
2	Beer, Bread
3	Beer, Coke, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Coke, Diaper, Milk

Output:

Rules Discovered:

{Milk} --> {Coke}
{Diaper, Milk} --> {Beer}

Applications – (1)

- **Items** = products; **baskets** = sets of products someone bought in one trip to the store
- **Real market baskets**: chain stores keep TBs of data about what customers buy together
  - Tells how typical customers navigate stores, lets them position tempting items
  - Suggests tie-in "tricks", e.g., run sale on diapers and raise the price of beer
- High **support** needed, or no $$'s
- Amazon's people who bought X also bought Y

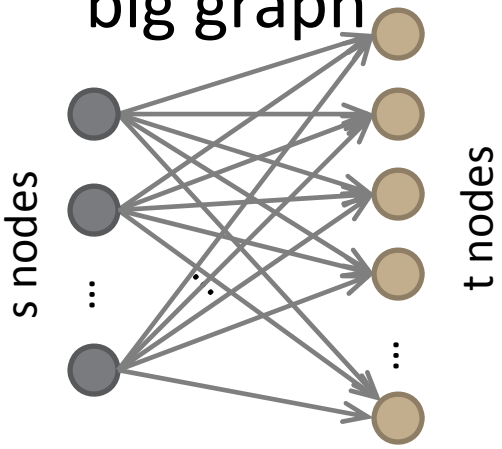
Applications – (2)

- **Baskets** = sentences; **items** = documents containing those sentences
  - Items that appear together too often could represent plagiarism
  - Notice items do not have to be "in" baskets

- **Baskets** = patients; **items** = drugs & side-effects
  - Has been used to detect combinations of drugs that result in particular side-effects
  - **But requires extension**: absence of an item needs to be observed as well as presence.

Applications – (3)

- **Finding communities in graphs (e.g., web)**
- **Baskets** = nodes; **items** = outgoing neighbors
  - Searching for complete bipartite subgraphs $K_{s,t}$ of a big graph



A dense 2-layer graph

Use this to define topics:
What the same people on the left talk about on the right

- **How?**
  - View each node i as a bucket B_i of nodes i it points to
  - $K_{s,t}$ = a set Y of size t that occurs in s buckets B_i
  - Looking for $K_{s,t}$ → set of support s and look at layer t – all frequent sets of size t

OUTLINE

First: Define

- **Frequent Itemsets**
- **Association rules:**
  - Confidence, Support, Interestingness

Then: Algorithms for finding frequent itemsets

- **Finding frequent pairs**
- **Apriori algorithm**
- **PCY algorithm + 2 refinements**

Frequent Itemsets

- **Simplest question:** Find sets of items that appear together "frequently" in baskets
- ***Support*** for itemset I: number of baskets containing all items in I
  - Often expressed as a fraction of the total number of baskets
- Given a ***support threshold*** s, then sets of items that appear in at least s baskets are called ***frequent itemsets***

TID	*Items*
1	Bread, Coke, Milk
2	Beer, Bread
3	Beer, Coke, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Coke, Diaper, Milk

Support of {Beer, Bread} = 2

Example: Frequent Itemsets

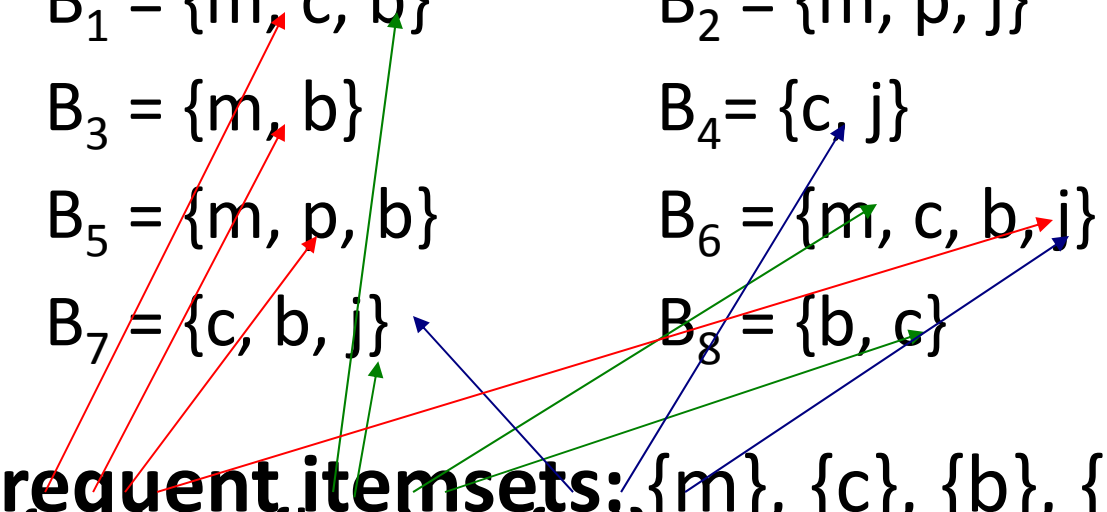
- **Items** = {milk, coke, pepsi, beer, juice}
- **Minimum support** = 3 baskets

B_1 = {m, c, b} B_2 = {m, p, j}

B_3 = {m, b} B_4= {c, j}

B_5 = {m, p, b} B_6 = {m, c, b, j}

B_7 = {c, b, j} B_8 = {b, c}



- **Frequent itemsets:** {m}, {c}, {b}, {j}, {m,b} , {b,c} , {c,j}.

Association Rules

- **Association Rules:**
  If-then rules about the contents of baskets

- $\{i_1, i_2, \ldots, i_k\} \rightarrow j$ means: "if a basket contains all of $i_1, \ldots, i_k$ then it is ***likely*** to contain j"
- ***Confidence*** of this association rule is the probability of j given $I = \{i_1, \ldots, i_k\}$

$$\text{conf}(I \rightarrow j) = \frac{\Pr[I \cup j]}{\Pr[I]} = \frac{\text{support}(I \cup j)}{\text{support}(I)}$$

Interesting Association Rules

- **Not all high-confidence rules are interesting**
  - The rule $X \rightarrow milk$ may have high confidence for many itemsets X, because milk is just purchased very often (independent of X)
- **Interest** of an association rule $I \rightarrow j$: difference between its confidence and the fraction of baskets that contain j

$$\text{Interest}(I \rightarrow j) = \text{conf}(I \rightarrow j) - \Pr[j]$$

- Interesting rules are those with high positive or negative interest values

Example: Confidence and Interest

B_1 = {m, c, b} B_2 = {m, p, j}

B_3 = {m, b} B_4= {c, j}

B_5 = {m, p, b} B_6 = {m, c, b, j}

B_7 = {c, b, j} B_8 = {b, c}

- **Association rule: {m, b} →c**
  - **Confidence** = 2/4 = 0.5
  - **Interest** = |0.5 – 5/8| = 1/8
    - Item *c* appears in 5/8 of the baskets
    - Rule is not very interesting!

Finding Association Rules

- **Problem:** Find all association rules with support $\geq s$ and confidence $\geq c$
  - **Note:** Support of an association rule is the support of the set of items on the left side
- **Hard part:** Finding the frequent itemsets!
  - If $\{i_1, i_2, \ldots, i_k\} \rightarrow j$ has high support and confidence, then both $\{i_1, i_2, \ldots, i_k\}$ and $\{i_1, i_2, \ldots, i_k, j\}$ will be "frequent"

Mining Association Rules

- **Step 1:** Find all frequent itemsets I
  - (we will explain this next)
- **Step 2:** **Rule generation**
  - For every subset A of I, generate a rule $A \rightarrow I \setminus A$
    - Since I is frequent, A is also frequent
    - **Variant 1:** Single pass to compute the rule confidence
      - conf(AB$\rightarrow$CD) = supp(ABCD)/supp(AB)
    - **Variant 2:**
      - **Observation:** If ABC$\rightarrow$D is below confidence, so is AB$\rightarrow$CD
      - Can generate "bigger" rules from smaller ones!
  - **Output the rules above the confidence threshold**

Example

B_1 = {m, c, b} B_2 = {m, p, j}

B_3 = {m, c, b, n} B_4= {c, j}

B_5 = {m, p, b} B_6 = {m, c, b, j}

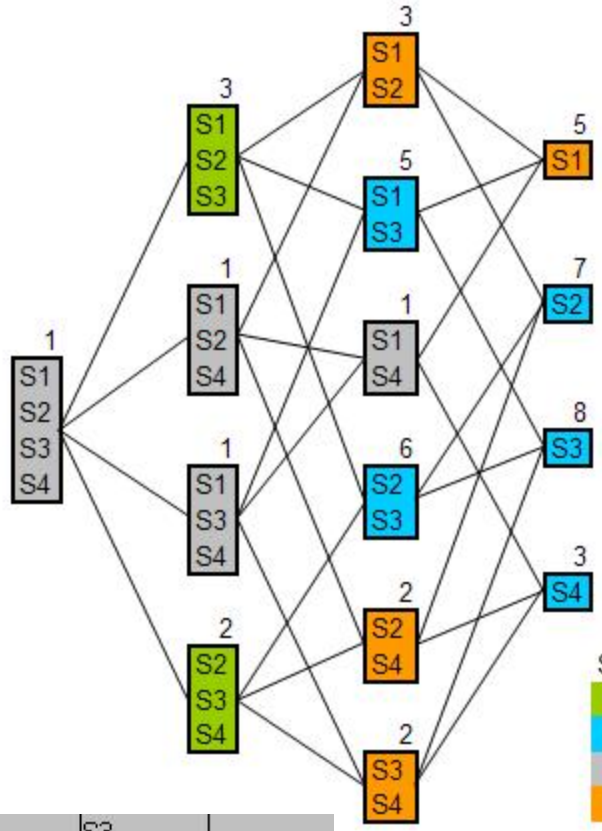
B_7 = {c, b, j} B_8 = {b, c}

- Min support $s=3$, confidence $c=0.75$
- **1) Frequent itemsets:**
  - {b,m} {b,c} {c,m} {c,j} {m,c,b}
- **2) Generate rules:**
  - b→m: c=4/6 b→c: c=5/6 b,c→m: c=3/5
  - m→b: c=4/5 … b,m→c: c=3/4

Compacting the Output

1. ***Maximal Frequent itemsets*:**
   no immediate superset is frequent

2. ***Closed itemsets*:**
   no immediate superset has the same count (> 0).
   – Stores not only frequent information, but exact counts



dataminingarticles.com

- A frequent itemset is one that occurs in at least a user-specific percentage of the database. That percentage is called support.
- An itemset is closed if none of its immediate supersets has the same support as the itemset.
- An itemset is maximal frequent if none of its immediate supersets is frequent.

Support = 2

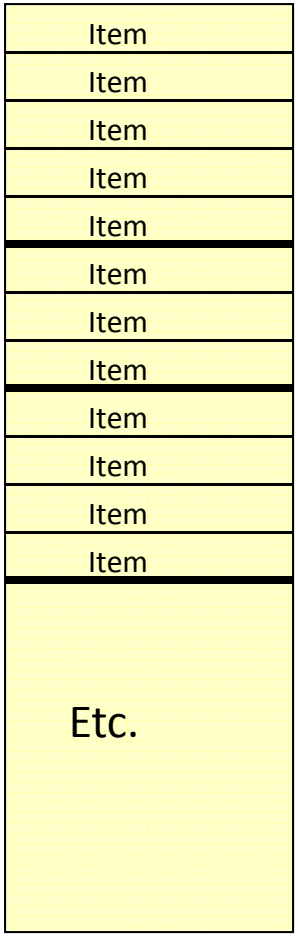
- Maximal Frequent Itemset
- Closed Frequent Itemset
- Non-Frequent Itemset
- Non-Closed Frequent Itemsets

C1	S1		S3	
C2		S2		
C3				S4
C4		S2	S3	S4
C5		S2	S3	
C6		S2	S3	
C7	S1	S2	S3	S4
C8	S1		S3	
C9	S1	S2	S3	
C10	S1	S2	S3	

Finding Frequent Itemsets

Itemsets: Computation Model

- Back to finding frequent itemsets
- Typically, data is kept in flat files rather than in a database system:
  - Stored on disk
  - Stored basket-by-basket
  - Baskets are **small** but we have many baskets and many items
    - Expand baskets into pairs, triples, etc. as you read baskets
    - Use ***k*** nested loops to generate all sets of size ***k***



Items are positive integers, and boundaries between baskets are –1.

Computation Model

- The true cost of mining disk-resident data is usually the number of disk I/O's
- In practice, association-rule algorithms read the data in *passes* – all baskets read in turn
- We measure the cost by the **number of passes** an algorithm makes over the data

Main-Memory Bottleneck

- For many frequent-itemset algorithms, **main-memory** is the critical resource
  - As we read baskets, we need to count something, e.g., occurrences of pairs of items
  - The number of different things we can count is limited by main memory
  - Swapping counts in/out is a disaster (why?)

Finding Frequent Pairs

- **The hardest problem often turns out to be finding the frequent pairs of items $\{i_1, i_2\}$**
  - **Why?** Often frequent pairs are common, frequent triples are rare
    - **Why?** Probability of being frequent drops exponentially with size; number of sets grows more slowly with size.
- Concentrate on pairs, then extend to larger sets

- **The approach:**
  - We always need to generate all the itemsets
  - But we would only like to count/keep track only of those that at the end turn out to be frequent

Naïve Algorithm

- **Naïve approach to finding frequent pairs**
- Read file once, counting in main memory the occurrences of each pair:
  - From each basket of *n* items, generate its *n(n-1)/2* pairs by two nested loops
- **Fails if (#items)^2 exceeds main memory**
  - **Remember:** #items can be 100K (Wal-Mart) or 10B (Web pages)
    - Suppose 10^5 items, counts are 4-byte integers.
    - Number of pairs of items: $10^5(10^5-1)/2 = 5*10^9$
    - Therefore, $2*10^{10}$ (20 gigabytes) of memory needed

Counting Pairs in Memory

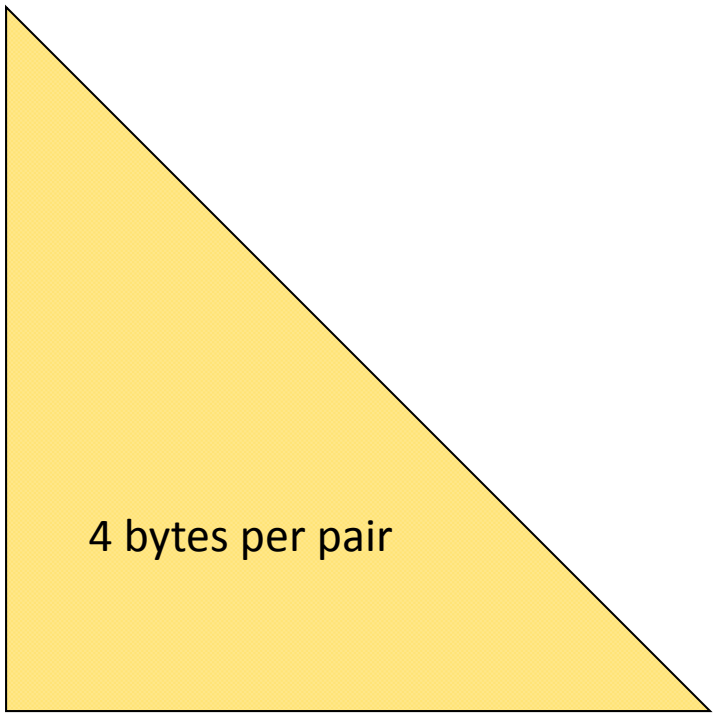
Two Approaches:

- **Approach 1:** Count all pairs using a matrix
- **Approach 2:** Keep a table of triples [*i*, *j*, *c*] = "the count of the pair of items {*i*, *j*} is *c*."
  - If integers and item ids are 4 bytes, we need approximately 12 bytes for pairs with count > 0
  - Plus some additional overhead for the hashtable

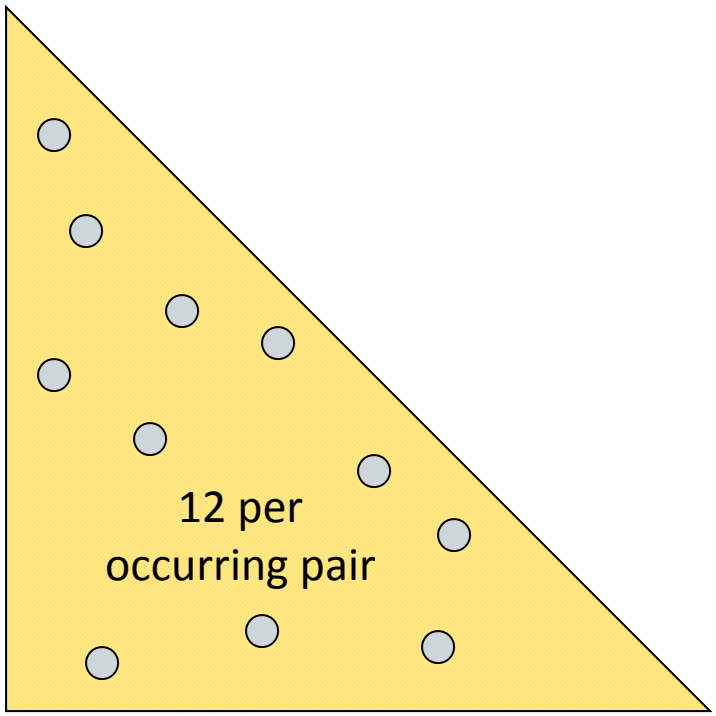
Note:

- **Approach 1** only requires 4 bytes per pair
- **Approach 2** uses 12 bytes per pair (but only for pairs with count > 0)

Comparing the 2 Approaches



Triangular Matrix



Triples

Triangular Matrix Approach

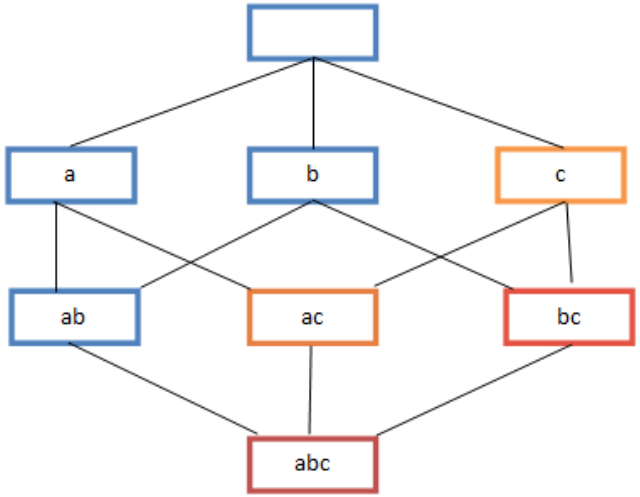
Triangular Matrix Approach

- n = total number items
- Count pair of items $\{i, j\}$ only if $i<j$

- Keep pair counts in lexicographic order:
  - $\{1,2\}, \{1,3\},\ldots, \{1,n\}, \{2,3\}, \{2,4\},\ldots,\{2,n\}, \{3,4\},\ldots$
- Pair $\{i, j\}$ is at position $(i-1)(n- i/2) + j - l$
- Total number of pairs $n(n-1)/2$; total bytes= $2n^2$
- **Triangular Matrix** requires 4 bytes per pair
- **Approach 2** uses 12 *bytes* per pair *(but only for pairs with count > 0)*
  - Beats triangular matrix if less than 1/3 of possible pairs actually occur

A-Priori Algorithm

A-Priori Algorithm – (1)

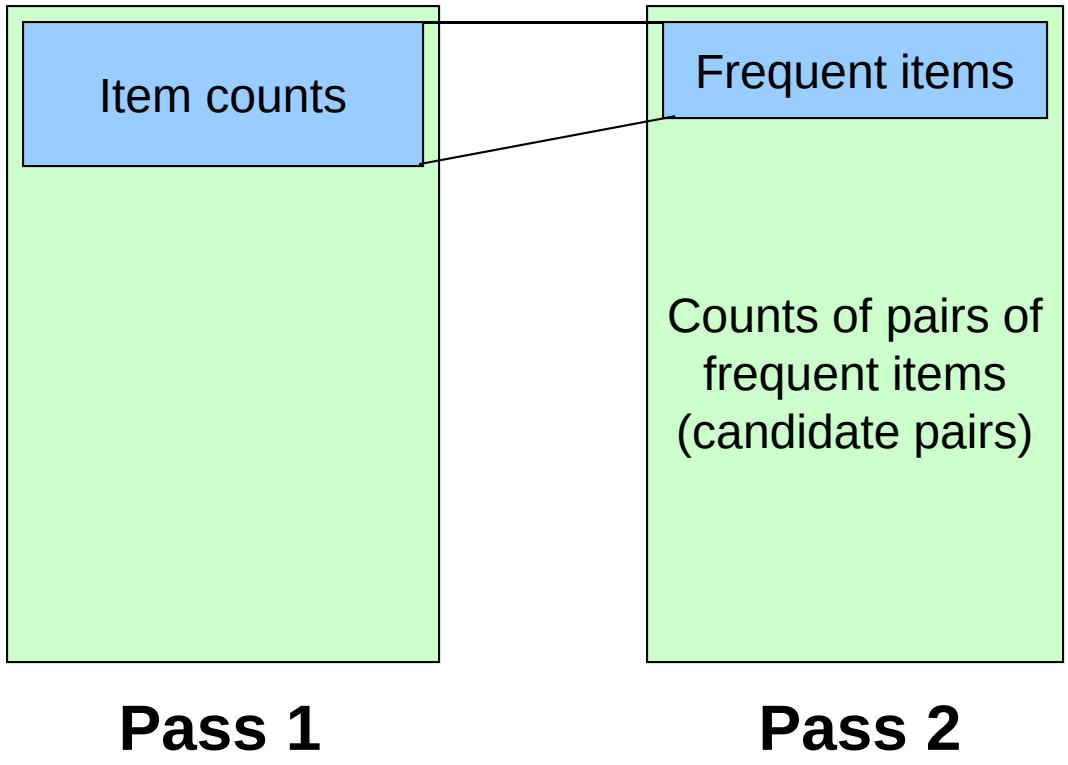


- A two-pass approach called *a-priori* limits the need for main memory
- **Key idea: *monotonicity***
  - If a set of items I appears at least s times, so does every subset J of I.
- **Contrapositive for pairs:**
  If item i does not appear in s baskets, then no pair including i can appear in s baskets

A-Priori Algorithm – (2)

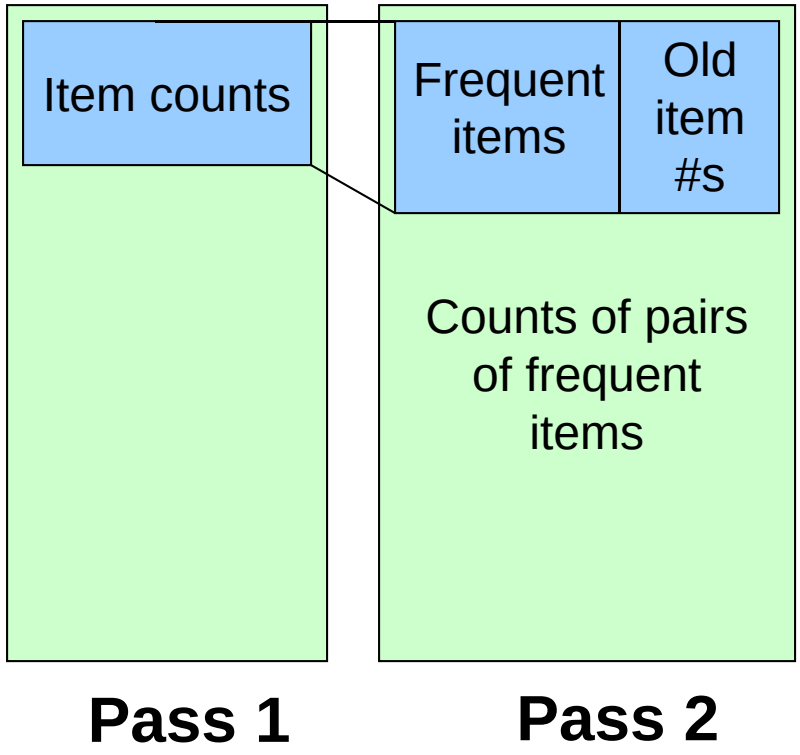
- **Pass 1:** Read baskets and count in main memory the occurrences of each individual item
  - Requires only memory proportional to #items
- **Items that appear at least s times are the frequent items**
- **Pass 2:** Read baskets again and count in main memory only those pairs where both elements are frequent (from Pass 1)
  - Requires memory proportional to square of **frequent** items only (for counts)
  - Plus a list of the frequent items (so you know what must be counted)

Main-Memory: Picture of A-Priori



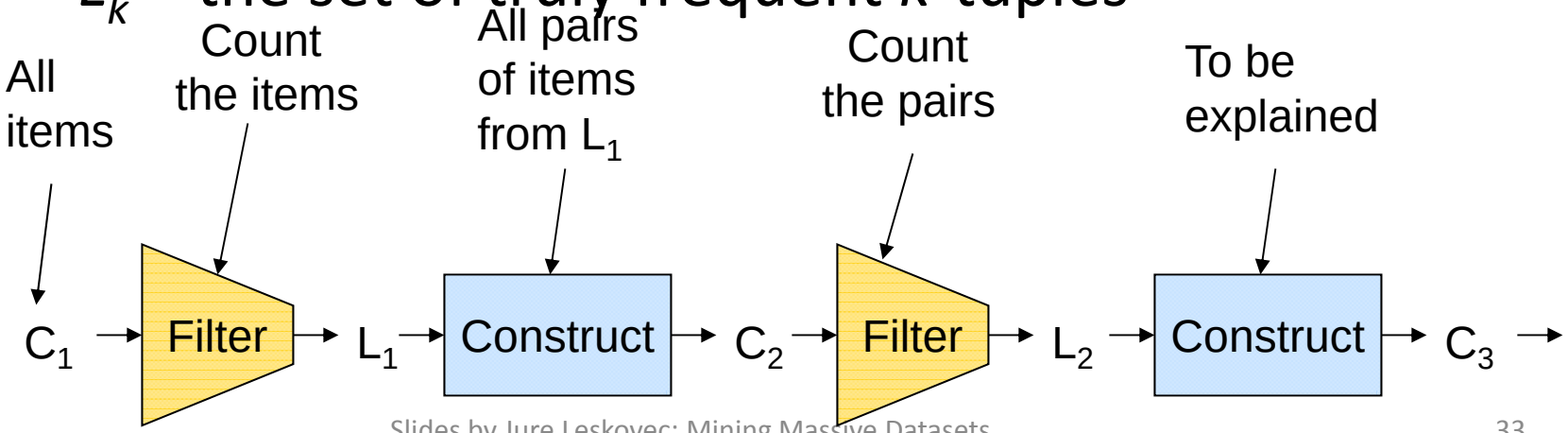
Detail for A-Priori

- You can use the triangular matrix method with ***n*** = number of frequent items
  - May save space compared with storing triples
- **Trick:** re-number frequent items 1,2,… and keep a table relating new numbers to original item numbers



Frequent Triples, Etc.

- **For each *k*, we construct two sets of *k-tuples*** (sets of size *k*):
  - C_k = ***candidate k-tuples*** = those that might be frequent sets (support $\geq$ s) based on information from the pass for *k*–1
  - L_k = the set of truly frequent *k*-tuples



Example

- **Hypothetical steps of the A-Priori algorithm**
  - C_1 = { {b} {c} {j} {m} {n} {p} }
  - Count the support of itemsets in C_1
  - Prune non-frequent: L_1 = { b, c, j, m }
  - Generate C_2 = { {b,c} {b,j} {b,m} {c,j} {c,m} {j,m} }
  - Count the support of itemsets in C_2
  - Prune non-frequent: L_2 = { {b,m} {b,c} {c,m} {c,j} }
  - Generate C_3 = { {b,c,m} {b,c,j} {b,m,j} {c,m,j} }
  - Count the support of itemsets in C_3
  - Prune non-frequent: L_3 = { {b,c,m} }

A-Priori for All Frequent Itemsets

- One pass for each *k* (itemset size)
- Needs room in main memory to count each candidate *k*–tuple
- For typical market-basket data and reasonable support (e.g., 1%), *k* = 2 requires the most memory

PCY (Park-Chen-Yu) Algorithm

PCY (Park-Chen-Yu) Algorithm

- **Observation:**
  In pass 1 of a-priori, most memory is idle
  - We store only individual item counts
  - **Can we use the idle memory to reduce memory required in pass 2?**
- **Pass 1 of PCY:** In addition to item counts, maintain a hash table with as many buckets as fit in memory
  - Keep a count for each bucket into which **pairs** of items are hashed
    - Just the count, not the pairs that hash to the bucket!

PCY Algorithm – First Pass

```
FOR (each basket) :
    FOR (each item in the basket) :
        add 1 to item's count;
    FOR (each pair of items) :
        hash the pair to a bucket;
        add 1 to the count for that bucket;
```

New in PCY

- Pairs of items need to be generated from the input file; they are not present in the file
- We are not just interested in the presence of a pair, but we need to see whether it is present at least s (support) times

Observations about Buckets

- **If a bucket contains a frequent pair, then the bucket is surely frequent**
  - But we cannot use the hash to eliminate any member of this bucket
- Even without any frequent pair, a bucket can still be frequent
- **But, for a bucket with total count less than s, none of its pairs can be frequent**
  - Pairs that hash to this bucket can be eliminated as candidates (even if the pair consists of 2 frequent items)
- **Pass 2:**
  Only count pairs that hash to frequent buckets

PCY Algorithm – Between Passes

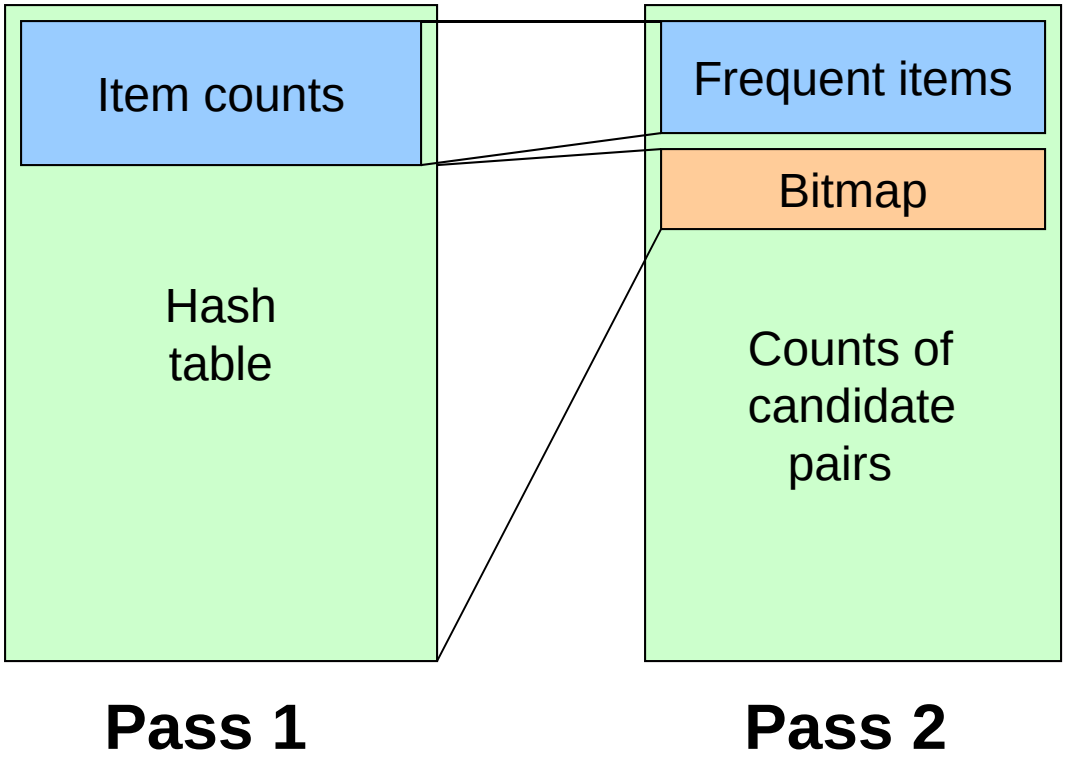
- **Replace the buckets by a bit-vector:**
  - 1 means the bucket count exceeded the support s (a *frequent bucket*); 0 means it did not
- 4-byte integers are replaced by bits, so the bit-vector requires 1/32 of memory
- Also, decide which items are frequent and list them for the second pass

PCY Algorithm – Pass 2

- Count all pairs $\{i, j\}$ that meet the conditions for being a **candidate pair**:
  1. Both i and j are frequent items
  2. The pair $\{i, j\}$ hashes to a bucket whose bit in the bit vector is 1 (i.e., frequent bucket)

- **Both conditions are necessary for the pair to have a chance of being frequent**

Main-Memory: Picture of PCY



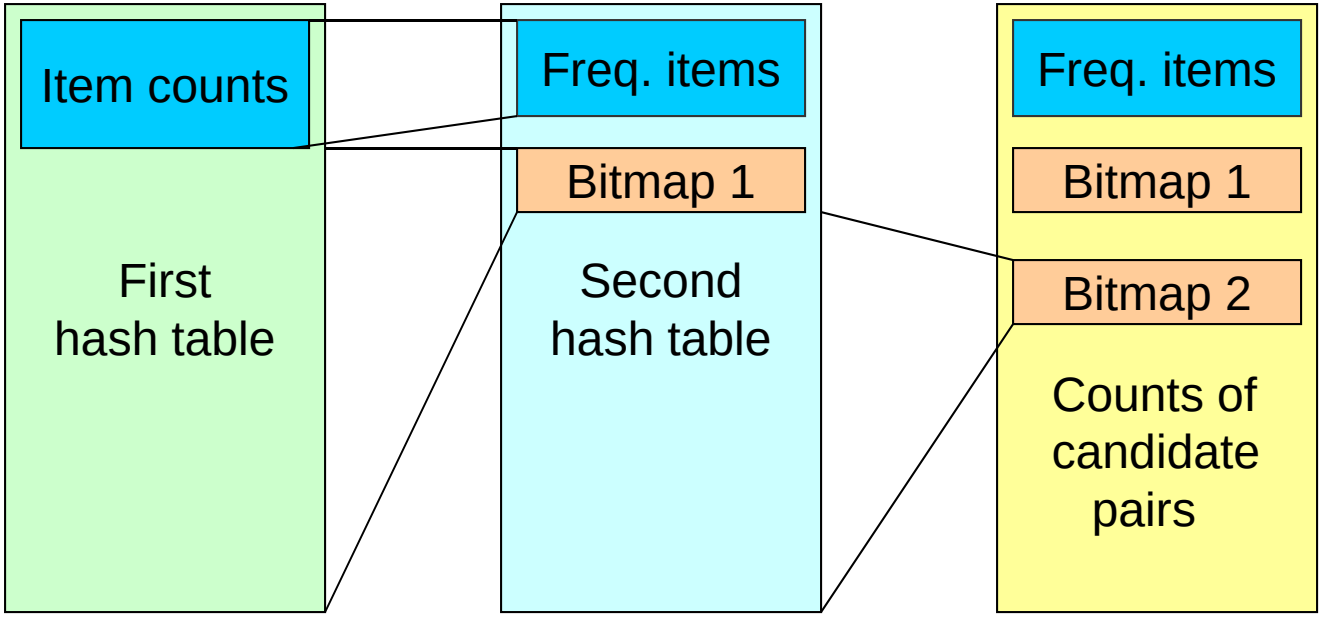
Main-Memory Details

- **Buckets require a few bytes each:**
  - **Note:** we don't have to count past s
  - #buckets is O(main-memory size)
- On second pass, a table of (item, item, count) triples is essential (we cannot use triangular matrix approach, why?)
  - Thus, hash table must eliminate approx. 2/3 of the candidate pairs for PCY to beat a-priori.

Refinement: Multistage Algorithm

- **Limit the number of candidates to be counted**
  - **Remember:** memory is the bottleneck
  - Still need to generate all the itemsets but we only want to count/keep track of the ones that are frequent
- **Key idea:** After Pass 1 of PCY, rehash only those pairs that **qualify** for Pass 2 of PCY
  - i and j are frequent, and
  - {i,j} hashes to a frequent bucket from Pass 1
- On middle pass, fewer pairs contribute to buckets, so fewer ***false positives***
- **Requires 3 passes over the data**

Main-Memory: Multistage



Pass 1

Count items
Hash pairs {i,j}

Pass 2

Hash pairs {i,j}
into Hash2 iff:
i,j are frequent,
{i,j} hashes to
freq. bucket in B1

Pass 3

Count pairs {i,j} iff:
i,j are frequent,
{i,j} hashes to
freq. bucket in B1
{i,j} hashes to
freq. bucket in B2

Multistage – Pass 3

- **Count only those pairs $\{i, j\}$ that satisfy these candidate pair conditions:**
  1. Both i and j are frequent items
  2. Using the first hash function, the pair hashes to a bucket whose bit in the first bit-vector is 1.
  3. Using the second hash function, the pair hashes to a bucket whose bit in the second bit-vector is 1.

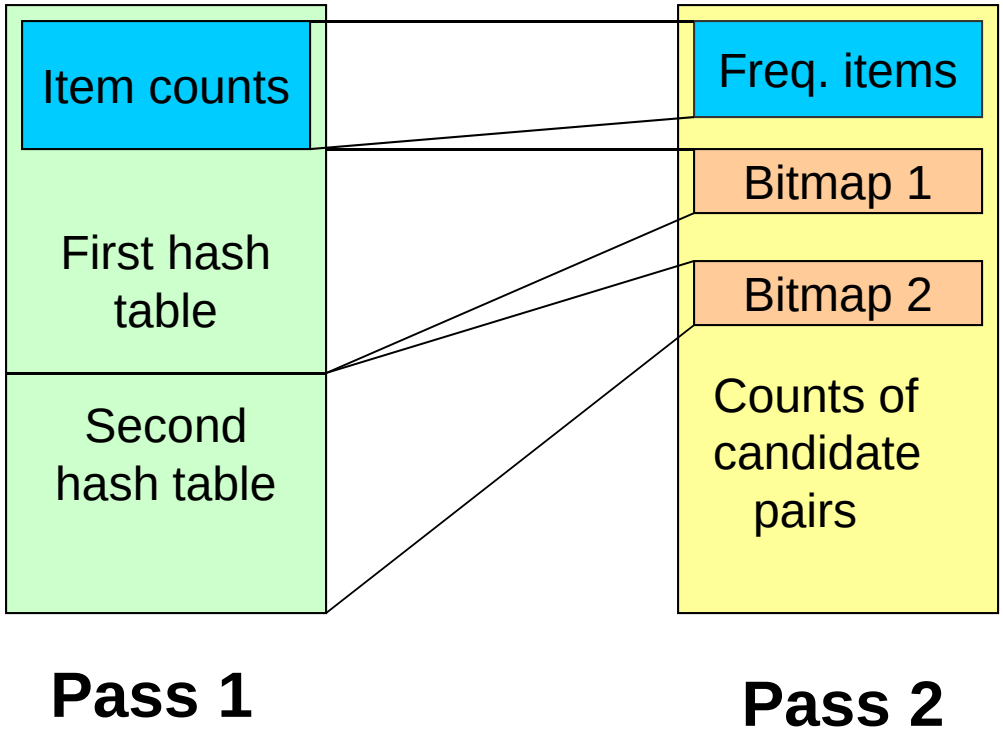
Important Points

1. The two hash functions have to be independent.
2. We need to check both hashes on the third pass:
   - If not, we would wind up counting pairs of frequent items that hashed first to an infrequent bucket but happened to hash second to a frequent bucket

Refinement: Multihash

- **Key idea:** Use several independent hash tables on the first pass
- **Risk:** Halving the number of buckets doubles the average count
  - We have to be sure most buckets will still not reach count s
- If so, we can get a benefit like multistage, but in only 2 passes

Main-Memory: Multihash



Extensions

- Either multistage or multihash can use more than two hash functions
- In multistage, there is a point of diminishing returns, since the bit-vectors eventually consume all of main memory
- For multihash, the bit-vectors occupy exactly what one PCY bitmap does, but too many hash functions makes all counts $\geq s$

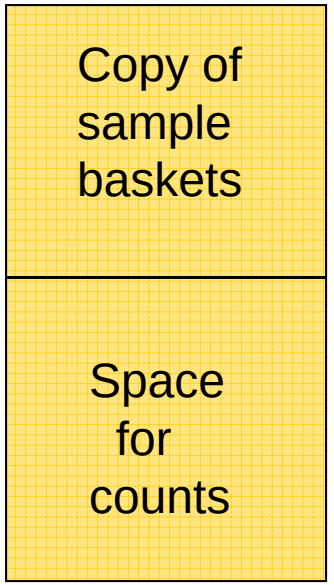
Frequent Itemsets in $\leq$ 2 Passes

Frequent Itemsets in ≤ 2 Passes

- A-Priori, PCY, etc., take *k* passes to find frequent itemsets of size *k*
- **Can we use fewer passes?**
- Use 2 or fewer passes for all sizes, but may miss some frequent itemsets
  - Random sampling
  - SON (Savasere, Omiecinski, and Navathe)
  - Toivonen (see textbook)

Random Sampling (1)

- Take a random sample of the market baskets
- Run a-priori or one of its improvements in main memory
  - So we don't pay for disk I/O each time we increase the size of itemsets
  - Reduce support threshold proportionally to match the sample size



Random Sampling (2)

- Optionally, verify that the candidate pairs are truly frequent in the entire data set by a second pass (avoid false positives)

- But you don't catch sets frequent in the whole but not in the sample
  - Smaller threshold, e.g., *s*/125, helps catch more truly frequent itemsets
    - But requires more space

SON Algorithm – (1)

- Repeatedly read small subsets of the baskets into main memory and run an in-memory algorithm to find all frequent itemsets
  - Note: we are not sampling, but processing the entire file in memory-sized chunks

- An itemset becomes a candidate if it is found to be frequent in *any* one or more subsets of the baskets.

SON Algorithm – (2)

- On a second pass, count all the candidate itemsets and determine which are frequent in the entire set

- Key "monotonicity" idea: An itemset cannot be frequent in the entire set of baskets unless it is frequent in at least one subset.

SON – Distributed Version

- SON lends itself to distributed data mining
- Baskets distributed among many nodes
  - Compute frequent itemsets at each node
  - Distribute candidates to all nodes
  - Accumulate the counts of all candidates.

SON: Map/Reduce

- Phase 1: Find candidate itemsets
  - Map?
  - Reduce?
- Phase 2: Find true frequent itemsets
  - Map?
  - Reduce?