Individual Choice Under Social Influence

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Abstract

Individual decision makers (DMs) are often influenced by their social environment when making choices. In this paper, we propose a simple choice-theoretic model to take into account the impact of social groups on individual decision making. First, we define a notion of socially acceptable alternatives by a particular social group from an arbitrary menu, which is analogous to indifference sets in consumer theory. We then proceed to study two potential scenarios in which a DM's choices are distorted by social influence: While social groups serve as tie-breakers in the first case, in the second case they become more influential in shaping DM's preferences in conformity to its own social preferences. Thus, the two scenarios we discuss could be seen as two extreme cases of social influence on individual choices. In each case, characterization theorems are provided to identify choice correspondences that satisfy our models. Some of our axioms are extensions of classic axioms, so the characterizations theorems also show what choice behaviors these axioms describe when adapted to the context of social influence on individual decision making.

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“The first thing you must realize is that power is collective. The individual only has power in so far as he ceases to be an individual” - O’Brien in 1984, by George Orwell

1 Introduction

We rarely make decisions in isolation; that is, our choice environment consists of many factors that influence our decision-making process. In this paper we focus on one such factor in particular: social groups a decision maker (DM henceforth) either naturally belongs to (such as family or country of residence) or voluntarily affiliates with (such as political parties or religious groups). These groups usually exert significant influence over DMs and their choices. Such examples are almost ubiquitous: Legislation involves a small group of elites making decisions and each citizen is to live within the bounds of government’s legal requirements. During presidential election, a candidate usually makes statements acceptable to the members of the political party to which the candidate belongs, given its ideology and stance on important social and political issues. Even in the more mundane daily life, such examples cannot be easily ignored: high school graduates consult their parents for advice on the choices of colleges and studies; PhD students seek their advisors’ counsel concerning the direction of their research projects; food-lovers follow their friends’ recommendations to choose what restaurant to dine in. The point is clear: we rarely make decisions in complete social isolation. Because we live in the presence of other DMs, we are consciously or subconsciously influenced by them.

However, in spite of the progress made in social psychology and experimental economics and recent attempts to incorporate social influence into choice theoretic models, a formal approach to identify a rational DM under the influence of multiple social groups is still needed. The models presented in this paper are meant to be regarded as one possible way to investigate individual choice under social influence and thus a first attempt at providing some theoretical foundation for future endeavors.

We begin by providing a reasonable definition of acceptable alternatives by a social group in Section 3. Two distinct yet related strands of literature in social choice theory are foundational to our model: Arrovian social choice theory and tournament solutions. The former is interested in aggregating a collection of individual preferences into one single standard social preference so that a group can make decisions concerning everybody in the group. However, as Arrow (1951) points out, even some minimal and reasonable requirements may render it impossible to find such a social preference. Furthermore, even if we find such a social preference, one concern is that such social preference may not be standard, i.e. complete and transitive. Therefore, more needs to be done to clarify how a social group may choose based on a nonstandard social preference. That is where the tournament literature comes in: various tournament solutions have been proposed to resolve decisions with cyclical preferences. In particular, assume the social preference is observable from pairwise majority voting. Tournament solutions are procedures that tell us what choices are made from an arbitrary menu given the social preferences. In proposing a concept for socially acceptable alternatives, we employ the tournament approach by taking the nonstandard social preference as given and building on a simple assumption.

Our main results can be found in Section 4, where we study two plausible ways in which social groups exert influence on the DM. There are two primitives for our models: DM’s choices and the set of socially acceptable alternatives from an arbitrary menu under the influence of a particular group. Let $\mathscr{G} = \{G_1, \dots, G_n\}$ denote the set of social groups with which the DM is affiliated. The set of socially acceptable alternatives in a menu S under the influence of G

is represented by a correspondence $N : \mathscr{X} \times \mathscr{G} \to \mathscr{X}$ s.t. $N(S,G) \subseteq S$. We assume that this social constraint correspondence is observable and obtained through the choice procedure in Section 3. Similarly, $c : \mathscr{X} \times \mathscr{G} \to \mathscr{X}$ is a correspondence that represents the DM's choices, where $c(S,G) \subseteq S$ is the set of alternatives chosen in S under the influence of group G.

Concerning the observability assumption on $N(S,G)$, we concede that N could be unobservable in many scenarios and identifying unobservable social norms could be a meaningful exercise. However, we choose to keep $N(S,G)$ as observable; and deduce it from group G's social preference in Section 3. We argue that in some cases, it is reasonable to assert that social constraints are at least roughly observable. The most prominent examples of such observable constraint sets are of political, religious or legal nature: In an election, candidates often make statements in accordance with the overall ideology endorsed by the political party; followers of religions have specific sets of commands to obey, such as dietary restrictions and moral requirements; the laws of a particular nation constrain the possible actions of its residents. Thus, our model could account for the choice behavior of, say, a DM who is simultaneously affiliated with a political party and a religious group.

There are many ways in which individual preferences are impacted by the social environment. The two cases studied in Section 4 can be considered two simple extremes of social influence on individual decision making. In both cases, the DM has a collection of preferences he uses when making choices, each of which is associated with a particular social group the DM is affiliated with. In the first model we present, these preferences are consistent in the sense that they do not strictly contradict with each other but may differ in the case of incomparability. The DM uses a particular preference under the social influence of a particular group, and therefore social influence serves as a tiebreaker in the case of incomparability. In other words, when the DM is unsure about two alternatives, the group comes in and helps the DM decide. However, in the second model, social influence shapes DM's underlying preferences. More specifically, the DM still has a collection of preferences. However, each preference is completely conformed to the social group's preference. Mathematically, the individual preference under group G is a subrelation of the social preference of group G. Therefore, instead of serving as a tiebreaker only, social influence now shapes the individual preference to be in conformity to it.

The paper is organized in the following manner: Section 2 introduces concepts that are fundamental in choice theoretic literature and notations useful for understanding our models. We propose one way to formulate socially acceptable alternatives in Section 3 and discuss its relation to the literature on consideration sets in choice theory. We characterize two cases where individual DMs are influenced by their social groups in Section 4. Section 5 discusses the uniqueness properties of our model representation and a condition on socially acceptable alternatives that can generalize the characterization theorems. We conclude the paper with a literature review in Section 6.

2 Nomenclature

In this section, we introduce some basic choice theoretic notations. We denote the set of all alternatives as X and assume that it is finite. A menu is simply a collection of alternatives from X a DM must choose from; that is, a menu is a subset of X. We write the collection of all possible menus as $\mathscr{X}$. In the rest of the paper, unless otherwise specified, we suppose $\mathscr{X}$ to include all nonempty subsets of X, i.e. $\mathscr{X} = 2^X \setminus \{\emptyset\}$. A typical menu from $\mathscr{X}$ will be denoted as S or T.

A binary relation on X is a subset of $X \times X$. Let $R \subseteq X \times X$ be a binary relation; if $(x,y) \in R$, we write xRy instead of $(x,y) \in R$ for simplicity. Given a menu $S \subseteq X$ and an alternative $x \in X$, we say ARx if aRx for all $a \in A$ and xRA if xRa for all $a \in A$. The symmetric part of R is $\{(x,y) \in X \times X : xRy \text{ and } yRx\}$ and the asymmetric part of R is the complement of the symmetric part of R. We also write the symmetric part of R (or $\succsim$) as $R^{=}$ (or $\sim$) and asymmetric part as $R^{>}$ (or $\succ$). A binary relation R restricted to menu S, denoted as $R|_S$, is defined as $R \cap (S \times S)$. We call a reflexive and transitive binary relation a preorder. If a preorder is also antisymmetric, it is called a partial order. A complete partial order is referred to as a linear order. The transitive closure of a binary relation R, written as $\text{tran}(R)$ is the smallest transitive binary relation that contains R. It is straightforward to show that $x \,\text{tran}(R) y$ if and only if there exists $\{x_1, \ldots, x_n\}$ such that $xRx_1 \ldots Rx_nRy$. Given two binary relations R and $\tilde{R}$, the latter extends the former if and only if $R \subseteq \tilde{R}$ and $R^{>} \subseteq \tilde{R}^{>}$. If furthermore $\tilde{R}$ is complete, the latter is said to be a completion of R. $\tilde{R}$ is said to be R-transitive if and only if either $x\tilde{R}yRz$ or $xRy\tilde{R}z$ implies $x\tilde{R}z$. Ok and Nishimura (2021) calls such an ordered pair $(R, \tilde{R})$ a *preference structure*.

We define the maximum and the maximal of a binary relation in a menu. Given a binary relation R on S, we say x is a R-maximum in S, denoted as $\max(S,R)$, if xRy for all $y \in S$ and x is R-maximal in S, denoted as $MAX(S,R)$, if there is no $y \in S$ such that $yR^{>}x$. It is straightforward to show that every finite preordered set (X,R) has a R-maximal element, and furthermore if R is a linear order, then R-maximum also exists.

A DM's choices are usually modeled by a choice correspondence $c : \mathscr{X} \to \mathscr{X}$ such that $c(S) \subseteq S$ for all S.[1] Because we model a DM influenced by social groups, we define an extension of the usual choice correspondence which we call an *extended choice correspondence*.[2] Recall that $\mathscr{G}$ is the set of social groups a DM is affiliated with, then $c : \mathscr{X} \times \mathscr{G} \to \mathscr{X}$ is an *extended choice correspondence* if $c(S,G) \subseteq S$ for all $(S,G) \in \mathscr{X} \times \mathscr{G}$.

3 Socially Acceptable Alternatives

Our goal in this section is to provide a plausible notion for the set of *socially acceptable alternatives* by an arbitrary group G from an arbitrary menu $S \in \mathscr{X}$, which will be further used in Section 4. As noted in the Introduction, we assume a social group G is endowed with a nonstandard social preference, denoted as $\succsim_G$, which is not necessarily complete nor transitive. However, an assumption we do maintain is that there exists $X_G \subseteq X$ such that $\succsim_G|_{X_G}$ is complete and x, y are $\succsim_G$-incomparable if either x or y is not in X_G. The interpretation of this assumption is as follows: Each social group has its particular constraining sphere. For example, religious groups typically set constraints upon moral or spiritual matters, whereas political parties care about political issues. However, social groups are far less restrictive outside their respective "spheres of influence". Therefore, we focus our analysis on X_G and simply say at the end of our analysis that the socially acceptable alternatives include whatever is accepted by the group within X_G and everything not in X_G.

The key to formulate socially acceptable alternatives is to begin with the "indifference relation" among alternatives. To wit, for a group of DMs, we posit that behaviorally it can treat two alternatives, x and y, as indifferent in two ways: First, the group is genuinely indifferent

[1] Since $\mathscr{X}$ is assumed not to include the empty set, we cannot have $c(S) = \emptyset$ for any $S \in \mathscr{X}$.

[2] If one views G as a frame, then extended choice correspondence is the "correspondence" version of an extended choice function defined in Rubinstein and Salant (2008).

between alternatives x and y, in other words, $x \sim_G y$. We refer to this case as "strict indifference". Secondly, several alternatives form a cycle according to $\succsim_G$, which means that the group cannot decisively choose x or y or any other alternative in the "cycle". Consequently, we call such a case "cyclic indifference". The best-known example of the latter is the *Condorcet's paradox* under majority voting that involves three voters and three alternatives. And clearly, strict indifference is a special case of cyclic indifference.

Motivated from the Condorcet's paradox example, it stands to reason that a group does not restrict alternatives that are in the same cycle and therefore the "larger" the cycle is, the more socially acceptable alternatives there are and therefore the less socially constrained DMs associated with the group are. Thus, we begin with the following definition, formally establishing what we mean by "cycles":

Definition 1. Given $\succsim_G$ and a menu $S \subseteq X_G$, we call a collection of alternatives in $\{x_1, \dots, x_n\} \subseteq S$ a $\boldsymbol{\succsim_G}$**-cycle** if $x_1 \succsim_G \dots x_n \succsim_G x_1$. A **maximal** $\boldsymbol{\succsim_G}$**-cycle** in set S is a set of alternatives $\{x_1, \dots, x_n\} \subseteq S$ such that $x_1, \dots x_n$ form a $\succsim_G$-cycle and there is no larger $\succsim_G$-cycle that contains $\{x_1, \dots, x_n\}$ in S. We denote the collection of all maximal $\succsim_G$-cycles in S as

$$A(S,G) := \{\bigcirc \subseteq S : \bigcirc \text{ is a maximal } \succsim_G \text{ -cycle in } S\}$$

Proposition 9 in Black (1950) states that if two cycles are intersecting, then there must exist another cycle that contains these two cycles. Since we consider maximal $\succsim_G$-cycles, then none of these cycles can intersect with each other. Therefore, we obtain the following proposition as a corollary of Black's proposition:

Proposition 3.1. *Given a complete binary relation $\succsim_G$, $A(S,G)$ is a partition of S for all $S \subseteq X_G$.*

Proposition 3.1 provides us with a way to partition any menu into maximal $\succsim_G$-cycles. Behaviorally, these $\succsim_G$-cycles are analogous to indifference sets in consumer theory: The collection of indifference sets partitions the set of bundles of goods, and all bundles in the same indifference set are equally appealing for the consumer. In the same way, the collection of maximal $\succsim_G$-cycles partitions menus and if any one alternative in a maximal $\succsim_G$-cycle is shown to be socially acceptable to the group, then it is only reasonable that all the alternatives in the same maximal $\succsim_G$-cycle are socially acceptable.

In consumer theory, if a consumer has a complete and transitive preference over all the bundles of goods, then his preference naturally induces an order on the indifference sets. Formally, if $\succsim$ is the DM's underlying preference and I_1 and I_2 are two indifference sets, then we can easily define $\trianglerighteq$ as $I_1 \trianglerighteq I_2$ if $x \succsim y$ for any two bundles $x \in I_1$ and $y \in I_2$. Now we define the following analogous concept on the set of maximal $\succsim_G$-cycles for each menu S.

Definition 2. Fix a menu $S \subseteq X_G$ and consider two maximal $\succsim_G$-cycles in S, denoted as $\bigcirc_1$ and $\bigcirc_2$. We say that $\bigcirc_1 \trianglerighteq_S^G \bigcirc_2$ if for any two alternatives $x \in \bigcirc_1$ and $y \in \bigcirc_2$, we have $x \succsim_G y$.

We know that if the consumer's underlying preference $\succsim$ is complete and transitive, then the induced order on indifference sets is a linear order. We now prove an analogous proposition here: the induced preference by $\succsim_G$ on $A(S,G)$ is complete, antisymmetric, and transitive. Even though $\succsim_G$ itself is not necessarily transitive, the structure of $A(S,G)$ helps us obtain both antisymmetry and transitivity.

Proposition 3.2. *$\trianglerighteq_S^G$ is a linear order on $A(S,G)$ for all $S \subseteq X_G$ and $G \in \mathscr{G}$.*

In consumer theory, the consumer chooses any bundle in the most preferred indifference set. In the same way, given any menu S that is completely contained in group G's sphere of influence X_G, the socially acceptable alternatives should simply be $\max(A(S,G),\trianglerighteq_S^G)$. Formally, we define a constraint correspondence $\tilde{N}$ for all $S \subseteq X_G$ and $G \in \mathscr{G}$ as

$$\tilde{N}(S,G) = \max(A(S,G),\trianglerighteq_S^G)$$

As we mentioned at the beginning of the section, if a menu includes any alternatives outside group G's sphere of influence, we assume that the group is lenient and does not impose any restriction on the DM. Therefore, we simply define the social constraint function N for all $S \subseteq X$ and $G \in \mathscr{G}$ as

$$N(S,G) = \tilde{N}(S \cap X_G, G) \cup (S \setminus X_G)$$

Remark 3.1. *We remark that the set of socially acceptable alternatives we constructed above is equivalent to the top-cycle social choice procedure first proposed in Schwartz (1972). Formally, we have $\tilde{N}(S,G) = \bigcirc(S,\succsim_G)$ where $\bigcirc(S,\succsim_G)$ is the top-cycle of menu S under the complete binary relation $\succsim_G$. In this remark, we clarify our contribution to the tournament literature on top-cycle.*

First, top-cycle is so named because every alternative in the top-cycle strictly dominates those that are not (hence "top"[3]) and all alternatives in the top-cycle actually forms a $\succsim_G$-cycle (hence "cycle"). However, we began with a notion of cyclic indifference first, then construct an order on the indifference cycles, and finally deduce that the group deems the most preferred cycle as socially acceptable. Hence, there is a distinction in our motivation.

Secondly, Theorem 1.3.13 and Proposition 1.3.16 in Laslier (1997) state that every irreducible tournament admits a unique scaling decomposition and top-cycle is the first component in this decomposition [4]. Our contribution to the understanding of top-cycles is that we identified exactly what the unique scaling decomposition is and how to obtain it. We also showed that the ordering on the scaling decomposition is not only transitive but also complete and antisymmetric.

We conclude this section by briefly discussing the relationship between the set of socially acceptable alternatives and two well-known consideration sets in the choice theoretic literature. A consideration set function is a mapping $\phi : \mathscr{X} \to \mathscr{X}$ such that $\phi(S) \subseteq S$ for all $S \in \mathscr{X}$. Masatlioglu, Nakajima, and Ozbay (2012) defined a type of consideration set called attention filter: ϕ is an attention filter if and only if $\phi(S) = \phi(S \setminus \{x\})$ for any $x \in S \setminus \phi(S)$. Then we have the following proposition:

Proposition 3.3. *$N(S,G)$ is an attention filter for all $(S,G) \in \mathscr{X} \times \mathscr{G}$.*

Another type of consideration set defined by Lleras et al. (2017) is called a competition filter. In particular, a consideration set function is a competition filter if for all $S \subseteq T$, $x \in \phi(T)$ only if $x \in \phi(S)$. This definition captures the intuition that if the DM pays attention to an alternative in a larger menu, then he must also consider it in a smaller menu. However, this property does not necessarily hold for our socially acceptable alternative set, which we show in the following example.

[3]Top-cycles are also the maximum element in a menu S with respect to the transitive closure of $\succsim_G|_S$, so the top-cycle is optimal or top in this sense as well.

[4]A menu is reducible with respect to $\succsim_G$ if there exists a subset S of X such that if for an arbitrary $y \in X \setminus S$ $x \succ_G y$, then $x' \succ_G y$ for all $x' \in S$. A decomposition of S with respect to $\succsim_G$ is a collection of subsets that are reduced through $\succsim_G$. A scaling decomposition refers to a decomposition that has a transitive ordering on it. Therefore, as one can see, there is no sense of indifference or cyclicity in this definition. Our approach of beginning with maximal cycles and treating them as indifference sets is indeed novel.

Example 3.1. Let $X = \{x,y,z\}$ such that $x \succ_G y \succ_G z \succ_G x$. Observe that $N(X,G) = \bigcirc(X, \succsim_G) = X$, while $N(\{x,y\},G) = \bigcirc(\{x,y\}, \succsim_G) = x$. This violates the competition filter property.

4 Individual Choice Under Social Influence: Two Scenarios

Consider a DM affiliated with a finite collection of social groups denoted as $\mathscr{G} = \{G_1, \ldots, G_n\}$ and each group has a complete but not necessarily transitive preference $\succsim_G$ over its sphere of influence X_G. We denote $\{\succsim_G: G \in \mathscr{G}\}$ as $\succsim_{\mathscr{G}}$. For any x,y such that either x or y is not in X_G, x and y are $\succsim_G$-incomparable. Furthermore, since these groups are interpreted as religious, cultural or political groups, their spheres of influence could be largely regarded as non-intersecting. In other words, we further assume $X_G \cap X_{G'} = \emptyset$ for distinct $G, G' \in \mathscr{G}$. Group G's acceptable alternatives from a given menu S are written as $N(S,G)$, which has been defined in the previous section. We simply inherit and apply the same notion of socially acceptable alternatives to this section. In other words, unless otherwise stated, $N(S,G) = \tilde{N}(S,G) \cup (S \setminus X_G)$ for all S. The choice correspondence in this model setup is an extended choice correspondence $c : \mathscr{X} \times \mathscr{G} \to \mathscr{X}$, and we assume that the DM's choice from menu S under G's influence is observable for all menus S and groups G. [5]

In this section, we study two plausible ways in which individual DM's choices may be influenced or distorted by their social environment: In the first scenario, the DM has a collection of underlying preferences, one associated with each group. While these preferences are largely consistent with each other, they differ at indecisiveness, i.e. when two alternatives are incomparable. Therefore, social groups serves the function of tie-breaking. The second scenario sees the group having a much stronger influence on the DM: it does not merely break ties but also actively shapes the individual's underlying preferences to be conformed to its own social preferences. We use P_G to denote the binary relation of the DM associated with the group G, and define $P_{\mathscr{G}} = \{P_G : G \in \mathscr{G}\}$. For the rest of this section, we formalize these two models and provide characterizations to them.

4.1 Consistent Individual Preferences

First, we formalize a notion of consistent individual preferences. Since the DM has a collection of underlying preferences each associated with a social group, consistent preferences should not significantly contradict with each other. In particular, if one preference dictates x is weakly preferred to y then y should not be strictly preferred to x according to any other preference. The following definition formalizes this intuition:

Definition 3. A set $P_{\mathscr{G}}$ of binary relations on X is said to be **consistent** if xP_Gy for some $P_G \in P_{\mathscr{G}}$, then $y\hat{P}_G^{>}x$ is false for all $\hat{P}_G \in P_{\mathscr{G}}$.

In other words, a set of preferences is consistent if they do not contradict with each other and only differ at incomparable cases. Consider the following examples of consistent sets of binary relations:

[5] $\mathscr{G}$ can be viewed as the description of the *identity* of the DM, while N serves as the prescription mapping of the corresponding part of the identity that is effective in the choice problem DM faces. We assume that each part of the identity forbids a subset of actions, rather than prescribing a unique alternative for each part of the identity in different menus. It is easy to see that our approach includes the latter if it allows a unique action in each menu. For a classic work discussing the importance of identity in the context of economic theory, please refer to Akerlof and Kranton (2000).

Example 4.1 (Complete binary relations)**.** If P_G is complete, then it is straightforward to show that $P_{\mathscr{G}}$ is consistent if and only if $P_{\mathscr{G}} = \{P_G\}$.

Example 4.2 (Super-relations of binary relations)**.** A binary relation P_{G_i} is said to be a super-relation of P_{G_j} if $P_{G_j} \subseteq P_{G_i}$ and $P^{>}_{G_j} \subseteq P^{>}_{G_i}$. $P_{\mathscr{G}}$ forms a consistent set of binary relations if $P_{G_{i+1}}$ is a super-relation of P_{G_i} for all $i \in \{1, \dots, |\mathscr{G}| - 1\}$. Example 4.1 is a special case of this one.

Example 4.3 (Preferences on Partitions)**.** Let $\{S_1, \dots, S_n\}$ be a partition of X and assume that $P_{G_i} \cap (X \times X) \subseteq S_i \times S_i$ for all $G_i \in \mathscr{G}$. Then $P_{\mathscr{G}}$ is a consistent set of binary relations.

We shall now consider two properties of the choice correspondence c and the social constraint set N. The first axiom simply requires the DM to follow the social constraint or norm, whereas the second axiom is an adaptation of a standard axiom with a flavor of social influence.

Axiom 1 (Obedience)**.** For all $S \in X$ and $G \in \mathscr{G}$, $c(S,G) \subseteq N(S,G)$.

Axiom 1 is fairly straightforward: According to *Obedience*, a DM can only choose what is socially acceptable from a particular menu. Obedience to social norm could appear to be a very strong assumption, because introspection seems to tell us that we disobey social norm all the time. However, it has been well-documented in the behavioral and experimental economics literature that effective norm enforcement mechanisms, such as collective punishment and in-group communication, can enhance obedience to social norms[6]. So it is not far-fetched to say that most individuals perfectly obey some social norms, especially in a religious, political or cultural context.

Axiom 2 (WARP-**N**)**.** Consider any $S, T \in \mathscr{X}$ and $G, G' \in \mathscr{G}$ such that $x, y \in S \cap T$. If $x \in c(S,G)$, $y \in c(T,G')$ and $x, y \in N(S,G) \cap N(T,G')$, then $x \in c(T,G')$.

We want to model the choice of a DM under the simultaneous social influence of multiple groups which are "compatible" with each other. Given this compatibility, the DM should possess fairly "well-behaved" preferences and hence choices. Axiom 2 accomplishes precisely this goal to both preserve rationality and add social constraints to the model. Two alternatives are chosen in each other's presence under some social environment from possibly different menus. However, if we know further that they are also socially acceptable in both social environments in respective menus the other alternative is chosen, then it only makes sense for a well-behaved DM to choose both alternatives in both social environments. Therefore, if WARP describes the behavior of a rational or well-behaved DM, then our axiom WARP-**N** should also sufficiently capture the choice of a boundedly rational DM under social influence.

The following characterization theorem identifies all extended choice correspondences where the DM chooses the most preferred alternatives from the social constraint set according to the preference associated with the influential social group.

Theorem 4.1. *An extended choice correspondence c satisfies Obedience and WARP-**N** if and only if for every $G \in \mathscr{G}$, there exists a preorder P_G on X such that $c(S,G) = \max(N(S,G), P_G)$ for all menus $S \in \mathscr{X}$ and $P_{\mathscr{G}}$ is consistent.*

[6]See Camerer and Fehr (2004) for an excellent summary.

4.2 Compliant Individual Preferences

In the previous section we discussed the case where each social group serves two basic functions: it first restricts what can or cannot be chosen, or what is or is not socially acceptable for members of the group, and in addition it serves as a tiebreaker when the individual DM is indecisive between two alternatives in its sphere of influence. In this section we study a scenario where a social group exerts much stronger influence on its members: Instead of merely serving as a tie-breaking agent, a social group actively shapes its members' preferences in conformity to the group's preferences. To model this choice behavior more specifically, a proper definition of preference conformity or compliance must be proposed:

Definition 4. A binary relation P_G is compliant to another binary relation $\succsim_G$ if $\succsim_G$ is complete and $P_G \subseteq \succsim_G$. P_G is compliant to $\succsim_G$ on X_G if $P_G|_{X_G}$ is compliant to $\succsim_G|_{X_G}$. Finally, we say a set of preferences $P_{\mathscr{G}}$ is compliant to $\succsim_{\mathscr{G}}$ if P_{G_i} is compliant to $\succsim_{G_i}$ on X_{G_i} for all $i \in \{1,\dots,n\}$.

As suggested in its name, a set of preferences the individual DM has is compliant if every preference conforms to or does not contradict with the social preference on the set where the group exerts its influence. In addition to Obedience, we will use two axioms. The following is the first one we use in this context:

Axiom 3 (Weak Aizerman Axiom)**.** Given $G \in \mathscr{G}$ and $S \subseteq T \subseteq X_G$, if $c(T,G) \subseteq S$, then $c(S,G) \subseteq c(T,G)$.

Weak Aizermann Axiom states that if the choices of the DM from a larger menu is completely contained in a smaller menu, then the DM's choice from the smaller menu should only contain what has been chosen from the larger menu. In other words, nothing not chosen from the larger menu should become more appealing to the DM in the smaller menu. Notice that this axiom is a weakening of WARP, which would imply that if $S \subseteq T \subseteq X$ and $c(T,G) \subseteq S$, then $c(S,G) = c(T,G) \cap S = c(T,G)$. This axiom is also weaker than the classic Aizermann Axiom in the sense that it is restricted only on the subsets of X_G. The next axiom is built on the following definition:

Definition 5. Given two alternatives x and y and a group G, we write $xR(c_G)y$ if there exists a menu S such that $x,y \in S$, $x \in c(S,G)$ and $y \in N(S,G)$.

So, according to this definition x can be regarded as revealed preferred to y if we can find a menu in which x is chosen in the presence of y, and we furthermore know that y is socially acceptable. Now given this definition, consider the following adaptation of WARNI:

Axiom 4 (WARNI-**N**)**.** Given any $G \in \mathscr{G}$ and $S \in \mathscr{X}$, if $xR(c_G)y$ for all $y \in c(S,G)$ and $x \in N(S,G)$, then $x \in c(S,G)$.

WARNI-**N** is an adaptation of the classic WARNI axiom first proposed by Eliaz and Ok (2006): A choice correspondence c satisfies WARNI if $xR(c)y$ for all $y \in c(S)$, then $x \in c(S)$. WARNI is a necessary and sufficient condition to identify a DM who chooses by maximizing over an incomplete preference relation. Our axiom modifies the classic WARNI in the following way: if an alternative is revealed to be more preferred to every alternative chosen from the menu *and* the alternative is socially acceptable, then the alternative itself must also be chosen.

The combination of the above axioms amounts to the following characterization theorem that identifies all extended choice correspondences where DM chooses maximal alternatives from each menu based on a set of compliant underlying preferences.

Theorem 4.2. *An extended choice correspondence c satisfies Obedience, the Weak Aizerman Axiom, and WARNI-***N** *if and only if for all $G \in \mathscr{G}$ there exists a preorder P_G on X such that $c(S,G) = MAX(N(S,G),P_G)$ for all menus $S \in \mathscr{X}$ and $P_{\mathscr{G}}$ is compliant to $\succsim_{\mathscr{G}}$.*

4.3 Extension: Complete Individual Preferences

We conclude our study of social influence on individual choice by considering the following extension: Instead of having multiple underlying preferences each associated with a particular social group, can the DM be stubborn in the sense that he has a single stable preference and chooses his most preferred alternative based on this preference from the set of socially acceptable alternatives? This unification of preferences proves to be difficult and requires a significant strengthening of the previous axioms. Just as the Strong Axiom of Revealed Preference (SARP) is a stronger version of WARP, we likewise strengthen WARP-**N** to SARP-**N**. As indicated by its name, SARP-**N** is a SARP-like axiom, adapted to our choice model with social influence.

Axiom 5 (SARP-**N**). Let $G_1,\dots,G_n \in \mathscr{G}$. For any $x_1,\dots,x_n \in X$ and $S_1,\dots,S_n \in \mathscr{X}$ such that $x_i \in c(S_i,G_i)$ for all i, if $x_i \in N(S_{i-1},G_{i-1}) \cap N(S_i,G_i)$ for all $i > 1$ and $x_1 \in N(S_n,G_n) \cap N(S_1,G_1)$, then $x_1 \in c(S_n,G_n)$.

With the help of SARP-**N**, we are able to unify the DM's underlying preferences in the following characterization theorem where we identify all such stubborn DMs and their underlying preference.

Theorem 4.3. *An extended choice correspondence c satisfies Obedience and SARP-**N** if and only if there exists a complete preorder $\succsim$ on X such that $c(S,G) = \max(N(S,G),\succsim)$ for all $S \in \mathscr{X}$.*

We highlight at the end our discussion that three of our axioms above are adaptations of standard axioms: WARP, WARNI, and SARP. However, in each case, the extension of the standard axiom helps us identify an interesting scenario where social groups influences individual decision making in a particular way, be it tie-breaking or compliance or mere restriction on social acceptance. Therefore, our discussion in a sense tells us what effect these standard axioms have in the context of social influence. However, these characterization theorems should not be regarded as trivial extensions of standard axioms and results. As one can see from the proofs, the existence of spheres of influence and incomparability between alternatives makes the characterization theorems nontrivial.

5 Further Discussion

5.1 Uniqueness of Representation

The underlying individual preferences from Theorems 4.1 and 4.2 are not uniquely represented. In other words, given a particular collection of sets of socially acceptable alternatives N, there may be multiple distinct preferences that induce the same extended choice correspondence. The lack of a unique representation could be seen from the fact that if an alternative is deemed socially unacceptable in every menu by a social group, then we cannot be sure how it is ranked by the DM in relation to other alternatives under the influence of the corresponding group, because the alternative can be ranked anywhere and the choice correspondence will be unaffected. However, the question remains whether different representations of the model have anything in common. In other words, if we fix an arbitrary extended choice correspondence c and let $\hat{P} = \{\hat{P^1},\dots,\hat{P^m}\}$ denote the collection of all sets of individual preferences that rationalize c according to the models specified either in Theorem 4.1 or 4.2, is there anything we can say about $\bigcap \hat{P}$? Recall each $\hat{P^i}$ in $\hat{P}$ is a set of individual preferences, one for each social group to which the individual belongs. Therefore, we use $\hat{P}^i_G$ to denote the individual preference under the social influence of group G from the i-th representation.

We begin with the consistent individual preferences in Section 4.1 and define a binary relation P_G as xP_Gy if and only if $x = y$ or there exists a menu T such that $x \in c(T,G)$ and $x,y \in N(T,G)$. That is, if x and y are both deemed socially acceptable by group G from menu T and x is chosen by the individual, then it can be deduced that the DM at least weakly prefers x over y. It is easy to see that any representation must include P_G. The following proposition claims something even stronger: not only is P_G in every representation, it is also the *only* part that is in every representation.

Proposition 5.1. *Fix an extended choice correspondence c and let $\hat{P} = \{\hat{P^1}, \ldots, \hat{P^m}\}$ be all representations that rationalizes c under the model specified in Section 4.1. Then $\cap_{i=1}^m \hat{P}^i_G = P_G$ for all $G \in \mathscr{G}$.*

Proof. $(\supseteq)$: Suppose xP_Gy. Then we know there exists a menu T such that $x \in c(T,G)$ and $x,y \in N(T,G)$. Since $\hat{P}^i_G$ also rationalizes c, it necessarily means $x \in \max(N(T,G),\hat{P}^i_G)$. Therefore, $y \in N(T,G)$ implies $x\hat{P}^i_Gy$ as well.
$(\subseteq)$: It is equivalent to show $\neg(P_G) \subseteq \neg(\cap P^1_G \cup \cdots \cap P^m_G) = \neg\hat{P}^1_G \cup \cdots \cup \neg P^m_G$. It is sufficient to show that if $\neg(xP_Gy)$ then there exists a representation $\hat{P}^i_G$ such that $\neg(x\hat{P}^i_Gy)$ as well. Now assume $(x,y) \notin P_G$ and further suppose $\hat{P}_G$ rationalizes c and $(x,y) \in \hat{P}_G$. Then it suffices to check $\max(N(S,G),\hat{P}_G) = \max(N(S,G),\hat{P}_G \setminus (x,y))$. If either x or y is not in $N(S,G)$, then it is not necessary that $x\hat{P}y$, so $\hat{P}_G \setminus (x,y)$ also rationalizes c. $\neg(xP_Gy)$ means there is no menu T such that $x \in c(T,G)$ and $x,y \in N(T,G)$. Therefore, if x and y are both in $N(S,G)$, then it must be that $x \notin c(S,G)$, and so for every $z \in c(S,G)$, $z\hat{P}_Gx$ and $z\hat{P}_Gy$. Thus, it is not necessary that $(x,y) \in \hat{P}_G$, and so $\hat{P}_G \setminus (x,y)$ also rationalizes c. □

Now for the case of compliance, consider the following preference relation: $xP^>_Gy$ if and only if $x,y \in N(xy,G)$ and $\{x\} = c(xy,G)$. That is, if x and y are both deemed socially acceptable in the binary menu, then x being the lone choice must imply that the DM strictly prefers x over y. Though we cannot obtain a proposition as strong as Proposition 5.1, it remains true that every representation must include $tran(P^>_G)$.

Proposition 5.2. *Fix an extended choice correspondence c and let $\hat{P} = \{\hat{P^1}, \ldots, \hat{P^m}\}$ be all representations that rationalizes c under the model specified in Section 4.2. Then $tran(P^>_G) \subseteq \cap_{i=1}^m \hat{P}^i_G$ for all $G \in \mathscr{G}$.*

Proof. Take any $\hat{P}_G$ that rationalizes c. If $x\ tran(P^>_G)y$, then it means $xP^>_Gx_1 \ldots P^>_Gx_nP^>_Gy$ for some $x_1,\ldots x_n$. But because $\hat{P}_G$ also rationalizes c, it is necessary that $x\hat{P}^>_Gx_1 \ldots \hat{P}^>_Gx_n\hat{P}^>_Gy$. And because $\hat{P}_G$ is a preorder, we can conclude $x\hat{P}_Gy$. □

5.2 Group Idempotence

We previously noted in Section 3 that our construction of socially acceptable alternatives is mathematically identical to top-cycle and then proceeded to characterize models built on this construction of N. Recall that $N(S,G) = \tilde{N}(S \cap X_G,G) \cup (S \setminus X_G)$ where $\tilde{N}(S \cap X_G,G) = \bigcirc(S,\succsim_G)$. Now we ask the question: Can we relax this assumption and simply find a general property on N such that our characterization theorems still hold true? We propose the following property:

Definition 6 (Group Idempotence)**.** A correspondence N is said to be *group idempotent* if and only if

$$N(N(S,G) \cup N(T,G), G) = N(S,G) \cup N(T,G)$$

for any $S, T \in \mathscr{X}$ and $G \in \mathscr{G}$ such that $N(S,G) \cap N(T,G) \neq \emptyset$.

The property is so named because when $S = T$, group idempotence implies idempotence, i.e. $c(c(S,G)) = c(S,G)$. First, we verify that our construction in Section 3 satisfies this property –

Claim 5.1. *N defined in Section 3 is group idempotent.*

Proof. Recall that $N(S,G) = \tilde{N}(S \cap X_G, G) \cup (S \setminus X_G)$, and $\tilde{N}(S \cap X_G, G) = \bigcirc(S \cap X_G, \succsim_G)$. Note that $N(N(S,G) \cup N(T,G), G) = \tilde{N}([N(S,G) \cup N(T,G)] \cap X_G, G) \cup ([N(S,G) \cup N(T,G)] \setminus X_G) = \bigcirc(\bigcirc(S \cap X_G, \succsim_G) \cup \bigcirc(T \cap X_G, \succsim_G), \succsim_G) \cup [(S \cup T) \setminus X_G]$. forms a $\succsim_G$-cycle, say $\bigcirc_1$, and $N(T,G)$ another $\succsim_G$-cycle, $\bigcirc_2$. It can be shown that $\bigcirc(S \cap X_G, \succsim_G) \cup \bigcirc(T \cap X_G, \succsim_G)$ forms a $\succsim_G$-cycle itself. Therefore, $\bigcirc(\bigcirc(S \cap X_G, \succsim_G) \cup \bigcirc(T \cap X_G, \succsim_G), \succsim_G) = \bigcirc(S \cap X_G, \succsim_G) \cup \bigcirc(T \cap X_G, \succsim_G)$. So, we have:

$$\begin{aligned} N(N(S,G) \cup N(T,G), G) &= \bigcirc(S \cap X_G, \succsim_G) \cup \bigcirc(T \cap X_G, \succsim_G) \cup [(S \cup T) \setminus X_G] \\ &= [\bigcirc(S \cap X_G, \succsim_G) \cup (S \setminus X_G)] \cup [\bigcirc(T \cap X_G, \succsim_G) \cup (T \setminus X_G)] \\ &= N(S,G) \cup N(T,G) \end{aligned}$$

where the final equality follows since $\tilde{N} = \bigcirc$. $\square$

We note that it is precisely this property of the top-cycle choice procedure that was used in the proofs of the characterization theorem. Thus, our characterization theorem remains valid for any construction of socially acceptable alternatives that satisfies group idempotence. Finally, to demonstrate the applicability of idempotence, we provide examples to show that other social choice procedures satisfy this property in addition to the top-cycle procedure.

Example 5.1 (Top-Cycle)**.** As shown in the proof of Claim 5.1, the top-cycle procedure satisfies group idempotence.

Example 5.2 (Arrow's Framework and Condorcet)**.** In the Arrovian framework, each member i of a group G is endowed with a linear order as her individual preference, say P_i. A social choice function F associates a unique linear order aggregated from $\{P_i\}_{i \in G}$. We can interpret $F(P_1, \ldots, P_{|G|})$ as the preference relation of group G, and in line with the famous impossibility result it is possible to have $F(P_1, \ldots, P_{|G|}) = P_i$ for some $i \in G$. Suppose that G chooses its most preferred alternative from a menu, so $N(S,G) = \max(S, F(P_1, \ldots, P_{|G|}))$. In this case, N satisfies group idempotence. In fact, any social choice procedure that yields a unique socially acceptable alternative from every menu satisfies the group idempotence property, which is satisfied for instance by all social choice functions satisfying *Condorcet criterion*.

Example 5.3 (Approval Voting)**.** In the method of "approval voting", each voter can vote for multiple candidates on the ballots, and the candidate with the most amount of vote is the winner of the election. It is easy to show that approval voting satisfies group idempotence[7].

[7]To wit, let $v(x)$ be the number of voters who approve x. Then if $x \in N(S,G)$, then $v(x) \geq v(y)$ for all $y \in N(S,G)$. If $y \in N(S,G) \cap N(T,G)$, then $v(y) \geq v(z)$ for all $z \in N(T,G)$. So we must have $v(x) \geq v(z)$ for all $N(S,G) \cup N(t,g)$, which means $x \in N(N(S,G) \cup N(T,G))$.

6 Literature Review

The goal of our paper is to provide a benchmark model that incorporates social influence into the decision-making of individuals. The claim that people are influenced by the social environment and groups they are members of is hardly disputed. For a reference on this issue, see Cialdini and Goldstein (2004). Furthermore, our paper is related to the social choice theory literature, more specifically the definition and properties of tournament solutions. This line of literature mainly focuses on social groups whose members are assumed to have well-defined preferences where majority voting procedure may induce a complete but non-transitive social preferences called tournaments. The objective of the tournament literature is to propose reasonable solutions to model how such social groups make decisions. Many such solutions have been proposed and refined. Earlier attempts include top cycle by Schwartz (1972), uncovered set by Miller (1980), Banks set by Banks (1985), and the minimal covering set by Dutta (1988). These solutions are summarized by Laslier (1997). More recent studies on tournament solutions, such as minimal extending set by Brandt, Harrenstein and Seedig (2017), and unsurpassed set by Han and Van Deeman (2019), further refine these classic concepts.

Though much of decision theory literature abstracted away from social influence on individual decision-making, some recent papers did make the attempt to incorporate social influence into their decision models. Borah and Kops (2021) use the concept of an attention filter first proposed in Masatlioglu, Nakajima, and Ozbay (2012) where the alternatives that the DM pays attention to are the ones sufficiently favored by one of the DM's *reference groups*. Similarly, Chambers, Cuhadaroglu, and Masatlioglu (2019) incorporate social influence to the Luce model by focusing on two DMs. To do so, they modify the Luce model with a parameter called *degree of influence of j on i*, denoted by α_i. Then, given an alternative x in menu S, the probability of DM i choosing x in S is $\frac{w(x)+\alpha_i p_j(x,S)}{\sum_{y\in S} w(y)+\alpha_i p_j(y,S)}$. In this way, DM i's choice is not only determined by his own weight function but also influenced by DM j's choice from menu S and the degree of influence she exerts on DM i. Lastly, in Fershtman and Segal (2018), each DM is endowed with two utilities, one representing her core preference and the other her behavioral preferences. Social influence upon a particular DM i is modeled by a social influence function that takes i's own core preference and all other DMs' behavioral preferences. This social influence function then determines DM i's behavioral preference, and so the model amounts to solving a fixed point problem for each individual's preference.

However, the paper most related to ours is Cuhadaroglu (2017), in which the DM takes advice from other DMs. Given a menu, the DM first deletes dominated alternatives according to her own preference and then applies other DMs' preferences to make final decision. Other DMs each have their own "areas of expertise" and the DM uses the preference of the expert to eliminate more alternatives in the menu. Our paper differs in two ways: First, social influence comes from a social group as opposed to an individual rational DM. Furthermore, the social constraint set in our model is exogenously given, whereas in Cuhadaroglu (2017), the individual choices are simultaneously and endogenously determined. In other words, we focus on only one direction of social influence, from the social group to the individual members, and abstract away from how individual preferences may contribute to social preferences. Adding the impact of individual group member's preferences on social preference into our model could be a worthwhile endeavor for future research.

Appendix A Proofs

A.1 Independence of Axioms

For all theorems in the main body, we used the axiom of *Obedience*. It is straightforward to show the independence of *Obedience* from the remaining axiom(s) for each theorem. In Theorem 4.1 and Theorem 4.3, there is one additional axiom, and therefore independence is a trivial issue for these results. Therefore, we only need to show the independence of axioms for Theorem 4.2. Since showing the independence of *Obedience* is straightforward, we only demonstrate the independence of *Weak Aizerman* and *WARNI-N*. To show a particular axiom is independent from the others, we show that when other axioms are satisfied, we can provide an example in which this axiom is not satisfied.

WARNI-N is independent
Consider the following example with $\mathscr{G} = \{G\}$ and $X = \{x, y, z\}$:

$$c(\{x,y\},G) = \{x,y\} = c(X,G)$$

$$c(\{x,z\},G) = \{z\} = c(\{y,z\},G)$$

and let $N(X,G) = X$, $N(\{x,y\},G) = \{x,y\}$, $N(\{x,z\},G) = \{x,z\}$, $N(\{y,z\},G) = \{z\}$.

Weak Aizerman is independent
Consider the following example with $\mathscr{G} = \{G\}$ and $X = \{x, y, z\}$:

$$c(\{x,y\},G) = \{x,y\} \quad c(\{x,z\},G) = \{x\} = c(X,G)$$

$$c(\{y,z\},G) = \{y\}$$

and let $N(X,G) = X$, $N(\{x,y\},G) = \{x,y\}$ $N(\{x,z\},G) = \{x\}$, $N(\{y,z\},G) = \{y\}$.

As you will see in the proof of Theorem 4.2, Weak Aizerman axiom is used to show that revealed preference of the individual in a group is transitive. The independence proof of Weak Aizerman demonstrates an example in which the transitivity is violated. To see, observe that $c(\{x,y\},G) = \{x,y\}$ implies that the individual is either indifferent or indecisive because she cannot compare between x and y. Furthermore, knowing that $N(X,G) = X$ and $c(X,G) = \{x\}$ reveals that there is an alternative that strictly dominates y and z, which are not necessarily the same. We know from our previous observation that the individual does not strictly prefer x over y, so it should be the case that z is strictly preferred to y. Note that this is compatible with $c(\{y,z\},G) = \{y\}$, because $N(\{y,z\},G) = \{y\}$. But then x should be strictly preferred to z, and hence by transitivity x is strictly preferred to y, a contradiction to $c(\{x,y\},G) = \{x,y\}$. Weak Aizerman axiom fixes this problem, because $\{x\} = c(X,G) \subset c(\{x,y\},G)$ implies that $c(\{x,y\},G) = \{x\}$, which avoids the violation of transitivity.

A.2 Proofs of Results

Proof of Proposition 3.2
We fix an arbitrary $S \subseteq X$ and $G \in \mathscr{G}$, and begin with the completeness of $\trianglerighteq^G_S$.

Claim A.1. *$\trianglerighteq^G_S$ is complete.*

Proof. Take any two maximal $\succsim_G$-cycles $\bigcirc_1, \bigcirc_2 \in A(S,G)$. The only possible case for $\bigcirc_1$ and $\bigcirc_2$ to be $\trianglerighteq^G_S$-incomparable is when there are $x_1, x_2 \in \bigcirc_1$ and $y_1, y_2 \in \bigcirc_2$ such that $x_1 \succsim_G y_1$ but $y_2 \succ_G x_2$ or $x_1 \succ_G y_1$ but $y_2 \succsim_G x_2$. Without loss of generality, suppose the former is true, because the proof for the latter can be obtained analogously.

Again, for clarity of exposition, we write $\bigcirc_1 = \{a_1, \ldots, a_m\}$ and $\bigcirc_2 = \{b_1, \ldots, b_n\}$, where $a_1 \succsim_G \cdots \succsim_G a_m \succsim_G a_1$ and $b_1 \succsim_G \cdots \succsim_G b_n \succsim_G b_1$. We further assume by the cyclicity of $\bigcirc_1$ and $\bigcirc_2$ that $x_1 = a_1$ and $b_1 = y_1$ and we also denote $x_2 = a_j$ and $y_2 = b_i$. Now consider a_2 and b_n. Suppose $b_n \succ_G a_2$. Then

$\bigcirc_1 \cup \bigcirc_2 = \{a_1, b_1, b_3 \ldots, b_n, a_2, \ldots, a_m\}$ form a larger cycle where $a_1 \succsim_G b_1 \succsim_G b_2 \succsim_G \cdots \succsim_G b_n \succsim_G a_2 \succsim_G \cdots \succsim_G a_m$, which contradicts the maximality of $\bigcirc_1$ and $\bigcirc_2$. Therefore, we must have $a_2 \succsim_G b_n$. We apply the same logic and conclude that $a_{j-1} \succsim_G b_{n-j+2}$. Now we consider the following $\succsim_G$-cycle: $a_1 \succsim_G a_2 \succsim_G \ldots a_{j-1} \succsim_G b_{n-j+2} \succsim_G \cdots \succsim_G b_i = y_2 \succ_G x_2 = a_j \succsim_G a_{j+1} \succsim_G \cdots \succsim_G a_m$. Clearly this is a $\succsim_G$-cycle and it strictly contains $\bigcirc_1$, contradicting the maximality of $\bigcirc_1$. Therefore, it is impossible to have $x_1, x_2 \in \bigcirc_1$ and $y_1, y_2 \in \bigcirc_2$ where $x_1 \succsim_G y_1$ and $y_2 \succ_G x_2$. Thus, $\trianglerighteq_S^G$ is complete. □

Claim A.2. *$\trianglerighteq_S^G$ is antisymmetric.*

Proof. If $\bigcirc_1 \trianglerighteq_S^G \bigcirc_2$ and $\bigcirc_2 \trianglerighteq_S^G \bigcirc_1$ and $\bigcirc_1 \neq \bigcirc_2$, this means for any $x \in \bigcirc_1$ and $y \in \bigcirc_2$, we must have $x \sim_G y$. In other words, all alternatives in $\bigcirc_1$ and $\bigcirc_2$ are indifferent to one another according to $\succsim_G$. Therefore, $\bigcirc_1 \cup \bigcirc_2$ forms a larger $\succsim_G$-cycle, contradicting the maximality of $\bigcirc_1$ and $\bigcirc_2$. Thus, $\trianglerighteq_S^G$ is asymmetric. □

Claim A.3. *$\trianglerighteq_S^G$ is transitive.*

Proof. Let $\bigcirc_1, \bigcirc_2, \bigcirc_3 \in A(S,G)$ such that $\bigcirc_1 \trianglerighteq_S^G \bigcirc_2 \trianglerighteq_S^G \bigcirc_3$. We need to show $\bigcirc_1 \trianglerighteq_S^G \bigcirc_3$. Suppose this is not the case, then by completeness and antisymmetry of $\trianglerighteq_S^G$, we have $\bigcirc_3 \triangleright_S^G \bigcirc_1$. Write $\bigcirc_1$ as $\{a_1, \ldots, a_m\}$. Since $\bigcirc_1 \trianglerighteq_S^G \bigcirc_2 \trianglerighteq_S^G \bigcirc_3$, pick arbitrarily $b \in \bigcirc_2$ and $c \in \bigcirc_3$ and we know $a_1 \succsim_G b \succsim_G c \succsim_G a_m$. Then we have the following $\succsim_G$-cycle: $a_1 \succsim_G b \succsim_G c \succsim_G a_n \succsim_G a_{n-1} \succsim_G \cdots \succsim_G a_1$. Clearly, this $\succsim_G$-cycle strictly contains $\bigcirc_1$, which contradicts the maximality of $\bigcirc_1$. This means $\neg(\bigcirc_3 \trianglerighteq_S^G \bigcirc_1)$. By completeness of $\trianglerighteq_S^G$, we must have$\bigcirc_1 \trianglerighteq_S^G \bigcirc_3$, which concludes the proof. □

Combine Claim A.1, A.2, and A.3 and we complete the proof of Proposition 2.

Proof of Proposition 3.3

Fixing G, $N(S,G)$ is clearly a consideration set for all $S \in \mathscr{X}$. Take any $x \in S \setminus N(S,G)$. Recall that $N(S,G) = \tilde{N}(S \cap X_G, G) \cup (S \setminus X_G)$ for all S. We noted that $\tilde{N}(S,G) = \bigcirc(S, \succsim_G)$ where $\bigcirc(S, \succsim_G)$ is the top-cycle of menu S under the complete binary relation $\succsim_G$. Therefore, $x \notin S \setminus X_G$ by definition and furthermore $x \notin \bigcirc(S \cap X_G, \succsim_G)$. Let us show that top-cycle is itself an attention filter, which is sufficient to show the claim. Since $x \notin \bigcirc(S \cap X_G, \succsim_G)$, $\bigcirc(S \cap X_G, \succsim_G) \succsim_G (S \cap X_G) \setminus \{x\}$. Furthermore, there is no $T \subset \bigcirc(S \cap X_G, \succsim_G) \subseteq (S \cap X_G) \setminus \{x\}$ and $T \succ_G (S \cap X_G) \setminus (T \cup \{x\})$, because this would contradict $\bigcirc(S \cap X_G, \succsim_G)$ being a top-cycle in $S \cap X_G$. This implies that $\bigcirc(S \cap X_G, \succsim_G) = \bigcirc((S \cap X_G) \setminus \{x\}, \succsim_G)$, so top-cycle is an attention filter. This concludes the proof.

Proof of Theorem 4.1

*The Necessity of WARP-**N***

Since $x, y \in N(S,G) \cap N(T,G')$, $x \in c(S,G) = \max(N(S,G), P_G)$ implies xP_Gy. Applying the same logic to (T,G'), we obtain $yP_{G'}x$. By the consistency of P, we conclude that $x(P_G)^=y$ and $x(P_{G'})^=y$. Assume to the contrary $x \notin \max(N(T,G'), P_{G'}) = c(T,G')$, which implies there is $z \in N(T,G')$ such that $zP_{G'}^{>}x$. By transitivity of $P_{G'}$, then we have $zP_G^{>}y$, a contradiction to $y \in c(T,G')$. Therefore, $x \in \max(N(T,G'), P_{G'}) = c(T,G')$. Similarly, we can conclude that $y \in \max(N(S,G), P_G) = c(S,G)$.

Now we turn to the sufficiency of Obedience and WARP-**N**. Our first step is to show that given the two axioms, there exists a reflexive and transitive binary relation P_G for every G such that $c(S,G) = \max(N(S,G), P_G)$. Consider alternatives x and y. We cannot infer that x is preferred to y by the DM simply because x is chosen in the presence of y, since the DM may indeed prefer y over x but y is not chosen because it is not allowed, that is $y \notin N(S,G)$, and the DM cannot choose y by Obedience. However, we can only be sure that the individual DM prefers x over y when there exists a menu S where the individual chooses x over y ***and*** they are both allowed in the constraint set, so both of them are socially acceptable. To formally establish this intuition, we say xP_Gy if and only if $x = y$ or there exists $T \subseteq X$ such that $x \in c(T,G)$ and $\{x,y\} \subseteq N(T,G)$ (Recall that N is assumed to be observable). We put all such preferences together to form $P := \{P_G : G \in \mathscr{G}\}$.

Claim A.4. *P_G is reflexive and transitive.*

Proof. The reflexivity of P_G comes directly from the definition of P_G by the axiom of Obedience. Now to prove the transitivity of P_G, we take three distinct alternatives $x,y,z \in X$ where xP_GyP_Gz. By definition of P_G, there exists two menus S and T such that $x \in c(S,G)$, $y \in c(T,G)$, $\{x,y\} \subseteq N(S,G)$ and $\{y,z\} \subseteq N(T,G)$. Since $N(N(S,G) \cup N(T,G),G) = N(S,G) \cup N(T,G)$ by definition of $\tilde{N}$ and N[8], $\{x,y,z\} \subseteq N(N(S,G) \cup N(T,G),G)$. There are two cases to consider: Either $c(N(S,G) \cup N(T,G),G) \cap N(S,G) \neq \emptyset$ or $c(N(S,G) \cup N(T,G),G) \subseteq N(T,G)$. Assume to the contrary $x \notin c(N(S,G) \cup N(T,G),G)$. If the former holds, then there is some $t \in N(S,G) \cap \setminus \{x\}$ such that $t \in c(N(S,G) \cup N(T,G),G)$. So, we have $x,t \in N(S,G)$, $x \in c(S,G)$ and $t \in c(N(S,G) \cup N(T,G),G)$, which implies by WARP-**N** that $x \in c(N(S,G) \cup N(T,G),G)$, a contradiction. Now assume the latter holds. Take any $t' \in N(T,G)$ such that $t' \in c(N(S,G) \cup N(T,G),G)$. Then, we have $y,t' \in N(T,G)$, $y \in c(T,G)$ and $t' \in c(N(S,G) \cup N(T,G),G)$, which again by WARP-**N** implies that $y \in c(N(S,G) \cup N(T,G),G)$, and thus $c(N(S,G) \cup N(T,G),G) \cap N(S,G) \neq \emptyset$, a contradiction. $\square$

Claim A.5. *P is consistent, i.e. if xP_Gy for some $G \in \mathscr{G}$, then either $xP_{G'}y$ or x,y are $P_{G'}$-incomparable for all $G' \in \mathscr{G}$.*

Proof. Take any two alternatives x,y such that xP_Gy. It suffices to show that if x,y are $P_{G'}$-comparable, then $xP_{G'}y$ for any $G' \in \mathscr{G}$. Suppose not, in other words, $yP^{>}_{G'}x$ for some $G' \in \mathscr{G}$. This means there exist $T \subseteq X$ such that $y \in c(T,G')$ and $\{x,y\} \subseteq N(T,G')$. Since xP_Gy, there is a menu $S \subseteq X$ such that $x \in c(S,G)$ and $x,y \subseteq N(S,G)$. Then by WARP-**N**, we have $x \in c(T,G')$ and thus $xP_{G'}y$, contradicting with $yP^{>}_{G'}x$. Therefore, we conclude the proof. $\square$

Now to conclude the proof, we only need to show that P rationalizes the DM's choice correspondence c.

Claim A.6. *$c(S,G) = \max(N(S,G),P_G)$ for all $S \subseteq X$ and $G \in \mathscr{G}$.*

Proof. ($\subseteq$) If $x \in c(S,G)$, by Obedience we know that $x \in N(S,G)$. Take any $y \in \max(N(S,G),P_G)$, by definition $y \in N(S,G)$. Since $x,y \in N(S,G)$ and $x \in c(S,G)$, xP_Gy, and arbitrariness of y implies that xP_Gy for any $y \in \max(N(S,G),P_G)$, which shows that $x \in \max(N(S,G),P_G)$.

($\supseteq$) Take any $x \in \max(N(S,G),P_G)$. Suppose $x \notin c(S,G)$. Then take an arbitrary $y \in c(S,G) \subseteq \max(N(S,G),P_G)$ by the ($\subseteq$) part of the proof. Then $\{x,y\} \subseteq N(S,G)$ and $x(P_G)^{=}y$. Since $x(P_G)^{=}y$, there exists $T \subseteq X$ such that $x \in c(T,G)$ and $\{x,y\} \subseteq N(T,G)$. By WARP-**N**, we conclude $x \in c(S,G)$, contradicting with the assumption that $x \notin c(S,G)$. Thus, $\max(N(S,G),P_G) \subseteq c(S,G)$ for all $S \subseteq X$. $\square$

Proof of Theorem 4.2
The Necessity of Weak Aizermann Axiom
Take an arbitrary $G \in \mathscr{G}$ and $S \subseteq T \subseteq X_G$ such that $c(T,G) \subseteq S$. Let $x \in c(S,G) = MAX(N(S,G),P_G) = MAX(\tilde{N}(S \cap X_G,G),P_G)$. Suppose on the contrary that $x \notin c(T,G)$. Then there exists $y \in \tilde{N}(T \cap X_G,G)$ such that $yP^{>}_{G}x$. Then we have the following claim:

Claim A.7. *$y \notin S$.*

Proof. If $y \in S \setminus X_G$, i.e. $y \in N(S,G)$, then $yP^{>}_{G}x$ implies $x \notin MAX(N(S,G),P_G) = c(S,G)$, which contradicts with the assumption. On the other hand, if $y \in S \cap X_G$, then either $y \in \tilde{N}(S \cap X_G,G)$ or $y \notin \tilde{N}(S \cap X_G,G)$. If the former is true, then by the same logic as above, $x \notin c(S,G)$. If the latter is true, then by the definition of $\tilde{N}$, we have $x \succ_G y$, which then contradicts with the compliance of P_G. Therefore, $y \notin S$. $\square$

Therefore, by the above claim, $y \in \tilde{N}(T \cap X_G,G) \setminus S$. We define $x^{\uparrow} = \{z \in \tilde{N}(T \cap X_G) \setminus S : zP^{>}_{G}x\}$. $x^{\uparrow}$ is nonempty because y is in it. Let $m \in MAX(x^{\uparrow},P_G)$.

[8]To show $N(N(S,G) \cup N(T,G),G) = N(S,G) \cup N(T,G)$ whenever $N(S,G) \cup N(T,G) \neq \emptyset$, first notice $\subseteq$ direction is trivial. Now for the $\supseteq$ direction, take an arbitrary $x \in N(S,G)$. There are two cases to consider: When $x \in S \setminus X_G$, we have $x \in N(S,G) \cup N(T,G) \setminus X_G$; therefore, $x \in N(N(S,G) \cup N(T,G),G)$. When $x \in S \cap X_G$, then $x \in \tilde{N}(S \cap X_G,G)$. By the definition of $\tilde{N}$, we have $x \in \tilde{N}(\tilde{N}(S \cap X_G,G) \cup \tilde{N}(T \cap X_G,G),G) \subseteq N(N(S,G) \cup N(T,G),G)$.

Claim A.8. $m \in c(T,G)$.

Proof. We prove by contradiction. Suppose there exists an alternative t in $\tilde{N}(T,G)$ such that it dominates m according to P_G. For all $t \in \tilde{N}(T,G) \setminus S$, if $tP_G^{>}m$, then $tP_G^{>}x$. Then $t \in x^{\uparrow}$, which contradicts with the maximality of m in $x^{\uparrow}$. For all $t \in (\tilde{N}(T,G) \cap S) \setminus \tilde{N}(S,G)$, since $x \in \tilde{N}(S,\succsim_G)$, we must have $x \succ_G t$. Now if $tP_G^{>}m$, then $tP_G^{>}x$, which together with $x \succ_G t$ contradicts with the conformity of P. Lastly, if $t \in \tilde{N}(S,G)$, $tP_G^{>}m$ again implies $tP_G^{>}x$, which means $x \notin MAX(\tilde{N}(S,G),P_G) = c(S,G)$. Therefore there is no $t \in \tilde{N}(T,G)$ such that $tP_G^{>}m$ and hence $m \in MAX(\tilde{N}(T,G),P_G) = c(T,G)$. ☐

However, by the claim above, we now just found an alternative $m \in c(T,G)$ that is not in S, contradicting with the assumption that $c(T,G) \subseteq S$. Therefore, $x \in c(T,G)$.

*The Necessity of WARNI-***N**

Take an arbitrary alternative x such that $xR(c_G)y$ for all $y \in c(S,G)$ and $x \in N(S,G)$. Suppose there exists an alternative $y \in N(S,G)$ such that $yP_G^{>}x$. If $y \in X_G \cap S$ and $x \in \tilde{N}(S,G)$, then by the conformity of P_G, we have $y \succ_G x$ and so for all $T \subseteq X$ that includes x and y and $x \in c(T,G)$, $y \in \tilde{N}(T,G)$, which means x cannot be in $c(T,G)$, contradicting with $x \in c(T,G)$. If $y \in S \setminus X_G$, then for all T that includes x and y and $x \in c(T,G)$, y must also be in $N(S,G)$, because $y \in T \setminus X_G$. Then x cannot be in $c(T,G)$, contradicting again. Lastly, if $x \notin X_G$, then for any T that includes x and y and $y \in N(T,G)$, x cannot be chosen from T. Therefore, as long as such y exists that is more preferred to x, $xR(c_G)y$ cannot be true, contradicting with the assumption. Therefore, $x \in c(T,G) = MAX(N(T,G),P_G)$.

Now we show the sufficiency of the three axioms. First we define the individual DM's preference in affiliation with group G. We say that an alternative x c_G-dominates y if $y \notin c(S,G)$ for all S that includes x, and x c_G-dominates y on X_G if $y \notin c(S,G)$ for all $S \subseteq X_G$ that includes x. Now we construct the individual underlying preference P_G in the following way: If both x and y are in X_G, then xP_Gy if x c_G-dominates y on X_G. However, if either x or y or both are not in X_G, then xP_Gy if and only if $x \in c(xy,G)$.

We first show that P_G is transitive. Take three distinct alternatives x, y and z such that xP_GyP_Gz. Suppose $x,y,z \in X_G$. Then we take any $S \subseteq X_G$ where $x,z \in S$. If $y \in S$, then yP_Gz implies $z \notin c(S,G)$. If $y \notin S$, then consider $S \cup \{y\}$. Then we have $y,z \notin c(S \cup \{y\},G)$. In other words, $c(S \cup \{y\},G) \subseteq S$. By Weak Aizerman Axiom, we have $c(S,G) \subseteq c(S \cup \{y\},G)$. Thus, $z \notin c(S,G)$. Therefore, x c_G-dominates z on X_G and so $xP_G^{>}z$. Now if any of the three alternatives is not in X_G, say $z \notin X_G$. Then we have $x \in c(xy,G)$ by c_G-domination, and $y \in c(yz,G)$ by construction. It is trivial to show that WARNI-**N** implies the following version of α axiom: For any two menus S,T such that $S \subseteq T \subseteq X$, if $x \in c(T,G) \cap S$ and $x \in N(S,G)$, then $x \in c(S)$, because by construction of N, we have $xR(c_G)y$ for all $y \in c(S,G)$ and WARNI-**N** implies $x \in c(S,G)$. Therefore, $x \in c(xy,G)$ and $y \in c(yz,G)$ jointly imply that $x \in c(xyz,G)$, which then further suggests $x \in c(xz,G)$ by WARNI-**N**. Therefore, xP_Gz.

Now we show that $P := \{P_G : G \in \mathscr{G}\}$ is compliant to $\{\succsim_G : G \in \mathscr{G}\}$. It is equivalent to show that $\succsim_G$ is a completion of P_G on X_G for all $G \in \mathscr{G}$. If $xP_G^{>}y$, then we have $\{x\} = c(xy,G)$. Indeed, $x,y \in X_G$ implies x c-dominates y, so $\{x\} = c(xy,G)$ and if either x or y is not in X_G, then x alone is chosen in the binary comparison between x and y by definition. So $xP_G^{>}y$ implies $\{x\} = c(xy,G)$, which further suggests $x \succsim_G y$. Given that P_G is a partial order, if $xP_G^{=}y$, then $x = y$ and so $x \sim_G y$. Thus $\succsim_G$ is a completion of P_G.

We now conclude the proof by showing P_G rationalizes $c(\cdot,G)$ for all $G \in \mathscr{G}$, i.e. $c(S,G) = MAX(N(S,\succsim_G),P_G)$.

($\subseteq$): Take $x \in c(S,G)$. By Obedience, $x \in N(S,G)$. Suppose there exists $y \in MAX(N(S,G),P_G)$ such that $yP_G^{>}x$. Then by definition of P_G, $x \notin c(S,G)$. Indeed, if $x,y \in X_G$, then y c_G-dominates x on X_G, so $x \notin c(S,G)$; if $y \in X_G$ and $x \notin X_G$, then $\{y\} = c(xy,G)$, but WARNI-**N** would imply $x \in c(xy,G)$; the other two cases can be proved using the same logic. And $x \notin c(S,G)$ would be a contradiction to our assumption.

($\supseteq$): Take $x \in MAX(N(S,G),P_G)$. Then $x \in N(S,G)$. Suppose $x \notin c(S,G)$. Then we take any $y \in c(S,G)$. We must have $xR(c_G)y$; otherwise we have $yP_G^{>}x$. To wit, if $y \in X_G$, then x is never chosen in the presence of y, so $yP_G^{\geq}x$; if $y \notin X_G$, then y must be the only one chosen in the binary comparison between x and y, so again $yP_G^{\geq}x$. However, $yP_G^{\geq}x$ contradicting with the maximality of x in $N(S,G)$. Therefore, $xR(c_G)y$ for all $y \in c(S,G)$. Then by WARNI-**N**, we have $x \in c(S,G)$.

Thus, $c(S,G) = MAX(N(S,G), P_G)$ for all $S \subseteq X$ and $G \in \mathscr{G}$. And the proof is complete.

Proof of Theorem 4.3

The Necessity of SARP-N

Consider x_1 and x_2: By assumption, $x_2 \in N(S_1,G_1)$ and $x_1 \in c(S_1,G_1) = \max(N(S_1,G_1),\succsim)$, which implies that $x_1 \succsim x_2$. Now apply the same logic to the subsequent x_i's and we have $x_i \succsim x_{i+1}$ for all i. If we further assume $x_1 \in N(S_n,G_n)$, then $x_n \succsim x_1$. Therefore, $x_1 \succsim \cdots \succsim x_n \succsim x_1$. By the transitivity of $\succsim$, we have $x_1 \sim \cdots \sim x_n$. Thus, $x_1 \in N(S_n,G_n)$ and $x_1 \sim x_n \in \max(N(S_n,G_n),\succsim)$ imply $x_1 \in \max(N(S_n,G_n),\succsim) = c(S,G_n)$.

In the proof of Theorem 4.1, we constructed a set of consistent preferences. Now we simply need to find a way to unify these preference relations into a single reflexive and transitive relation, interpreted as the individual preference which is independent from his social association, which we construct in the following way: we first take the union of all P_G and then find its transitive closure. In other words, we consider $\text{tran}(P^*)$ where $P^* = \cup_{P_G \in P} P_G$ where P is defined in the previous proof. Observe that WARP-**N** is a special case of SARP-**N**, which implies that we can show each P_G is a preorder and P is consistent. Therefore, $\text{tran}(P)$ is a preorder. We already showed that $c(S,G) = \max(N(S,G),P_G)$ in the previous proof, so to conclude the proof, we only need to show the following claim:

Claim A.9. $\max(N(S,G),P_G) = \max(N(S,G), tran(P))$ *for all* $G \in \mathscr{G}$ *and* $S \subseteq X$.

Proof. ($\subseteq$) This is trivial because $P_G \subseteq \text{tran}(P)$.

($\supseteq$) We prove the contraposition of the statement. In other words, take any $y \notin \max(N(S,G),P_G)$ and we need to show $y \notin \max(N(S,G),\text{tran}(P))$. If $y \notin N(S,G)$, then the claim is trivial, so assume $y \in N(S,G)$. Take any $x \in \max(N(S,G),P_G) = c(S,G)$, then we know $xP_G^{\geq}y$. For y to be in $\max(N(S,G),\text{tran}(P))$, for any $x \in N(S,G)$ there must exist $\{y_1,\ldots,y_{n-1}\} \subseteq X$ and a set of groups $\{G_1,\ldots,G_n\}$ such that

$$y = y_1 P_{G_1} y_2 \ldots P_{G_{n-1}} y_n P_{G_n} x P_G^{\geq} y = y_1$$

Then by the definition of $\mathbf{P_{G_i}}$, we can find a collection of menus $\{S_1,\ldots,S_n\}$ such that $y_i \in c(S_i,G_i)$ for all i and $y_i, y_{i+1} \in N(S_i,G_i)$ for all $i \geq 1$. In addition, we also have $x,y \in N(S,G)$. Therefore, we can apply SARP-**N** to conclude that $y \in c(S,G)$. However, this would imply that yP_Gx, contradicting the assumption that $xP_G^{\geq}y$. Therefore, $x \notin \max(N(S,G),\text{tran}(P))$. This means $y \notin \max(N(S,G),P_G)$ implies $y \notin \max(N(S,G),\text{tran}(P))$. The contraposition concludes the proof. □

Now we apply Szpilrajn's Theorem to extend the preorder $\text{tran}(P)$ to a complete and transitive binary relation $\succsim$, and hence the proof is complete.

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