Market Complexity with Asymmetric Firms and Bounded Rational Consumers

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Abstract

I present a model of the interaction between a population of identical consumers who finds some markets complex and two rational firms maximizing their payoff functions. The firms know how the consumers behave in different markets: All consumers choose the best product according to their preferences when the market is easy, and purchase a product their default firm offers when it is hard. Both firms in the duopoly have a certain share of consumers having them as their default, potentially representing the market power or the salience of the firm. I analyze two cases: symmetric firms having equal power, and asymmetric firms with different market powers and some but not all markets are hard. Two special structures are imposed on the hard markets: In the *complex product* setting, a market is hard for the consumer if and only if it includes at least one complex product. The second structure is *limited attention* setting, where I assume the market is hard for the consumer whenever it contains more than a certain number of products. I investigate the equilibrium behavior under these settings and discuss the market implications of public policies such as educating the consumers. I then generalize the model by introducing imperfect information about the cognitive characteristics of the consumers and later competition. I conclude by discussing the relation to the literature, and the effects of changing the determinants of complexity.

1 Introduction

Deciding what to choose in a market is usually hard due to the potential complexity of the choice problem. A market can be considered complex for several reasons: If it requires the consumer to remember too many products, then it is complex because of the memory constraints. If the consumer is overwhelmed because of the high number of alternatives, then it is complex because of the limited attention capacity of the consumer.[1] If it affects the mood of the consumer by reminding her something about the past, then the problem can be emotionally hard to solve, and the consumer might want to finish the purchase as soon as possible.[2] Lack of expertise, time constraints or the importance of the problem can be other factors in determining the complexity of the market.[3] In some cases, even including a single product which involves

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[1]Iyengar and Lepper [17].

[2]For a detailed discussion, I refer the reader to Bettman and Payne [1].

[3]Maheswaran et al. [20] argues that heuristic processing would be effective when the task is not highly important to the consumer, while systematic processing would dominate in the case of a highly important problem.

a complicated description can make the market complex. For example, a consumer with a low financial literacy can find a duopoly of two banks each offering a single portfolio complex. If any of these factors make the problem hard to solve for the consumer, then she can use a heuristic to reduce the effort spent on the problem, balancing the trade-off between accuracy and effort.[4]

In light of these examples, I model two firms competing in a market consisting of a measure 1 of identical consumers. A market $\mathbf{M}$ is a pair of menus (M_1, M_2) s.t. $M_i \subseteq X$ for $i \in \{1,2\}$, where X is the finite set of all alternatives. A market $\mathbf{M}$ is either *hard* or *easy*. The set of hard and easy problems, denoted with $\mathbf{H}$ and $\mathbf{E}$ respectively, is a partition of the set of all possible markets. In the general case, I assume that if $\mathbf{M} \in \mathbf{H}$, then any $\mathbf{M}' \supseteq \mathbf{M}$ is also in $\mathbf{H}$.[5] Because the set of all markets is assumed to be a partition, this implies any market included in an easy market is easy, so $\mathbf{E}$ is closed under subsets. Furthermore, I assume that the identity of firm is inessential for the complexity of the market, meaning that $\mathbf{M} = (M_1, M_2) \in \mathbf{H}$ if and only if $\mathbf{M}' = (M_2, M_1) \in \mathbf{H}$. Finally, I assume that if a market consisting of singletons offered by each firm is in $\mathbf{H}$, then at least one firm offers a complex product (which I will later define more precisely). I analyze the cases in which firms are symmetric (meaning they have equal market power), the benchmark cases in which every market is complex and no market is complex for the consumers, and asymmetric firms with different market powers and some but not all markets are complex. Later, in order to get a clearer picture, I investigate two specific structures on $\mathbf{H}$: The first one is called *complex product* setting, and the second one is called *limited attention*. I assume that each firm in the duopoly has a market power of α_i s.t. $\alpha_1 + \alpha_2 = 1$. If the market is complex, then consumers go to her default firm, which is equal to a measure of α_i for firm i. Otherwise, the consumer chooses the $\succ$-maximum product in the market, where $\succ$ is a complete asymmetric preference relation interpreted as the true preference relation of the consumer.

In *complex product* setting, I assume a market is complex if and only if at least one firm offers a complex product. A simple example of a complex product is a product with a complicated description or a product with an unfamiliar measurement unit. An example is financial products: Celerier and Vallee [5] shows that the complexity of financial products increase in EU during the period of 2002-10, and they measure the complexity of a single product by analyzing its features. In *limited attention* setting, the consumer has scarce attention, represented by an integer threshold $m \geq 2$. If the number of products available in the market is strictly above m, then the market is complex for the consumer. Finding an example is easier in this setting: Most of the online shopping sites are overwhelmed with thousands of options. A consumer typically looks only at a limited number of options showing first in the list.[6]

On the firm side, I assume that the firms in the market are *rational* in the traditional sense, and they maximize their payoffs. Firms offer menus that are subsets of X to the consumers for which they pay the fixed cost of the menu, and if the consumer chooses a product available in the offer, then firms gain revenue. If more than one firm provides the chosen product and the market is easy, then I assume that they split the revenue equally, that is, an equal share of consumers buy from each firm that provides the chosen product. In the market, the consumer

[4]This is compatible with the effort-reduction framework developed by Shah and Oppenheimer [27], in which heuristics main function is to reduce the effort associated with a problem.

[5]A market $\mathbf{M}$ is a subset of another market $\mathbf{M}'$ when $M_i \subseteq M_i'$ for $i \in \{1,2\}$. Note that $\subseteq$ and $\supseteq$ defined this way are possibly incomplete binary relations.

[6]Gerasimou [12] characterizes a choice procedure in which when the consumer is overwhelmed by the number of alternatives (which I model in limited attention setting), she defers to choose something and therefore her choice is equal to $\emptyset$. Instead, I assume that the individual is decisive, but uses a heuristic to choose something.

considers all offers together without considering each firm's offer separately. This implies that if one of the firms offers a hard menu, then the offered alternatives in the market is automatically hard, which will be important in the results. Thus, the complexity in the market does not depend necessarily on the coordination of the firms, a firm can make the market complex unilaterally.[7]

The rational consumer benchmark corresponds to the case in which all markets are easy for the consumer. When this is the case, the competition forces both firms to offer only the best quality product in equilibrium. In the opposite case when all markets are hard, the consumers go to their default firm in all markets, so both firms offer the worst quality product as the only choice option. The rational consumer benchmark result also holds when the firms have equal market power and the cost of the best quality product is bounded above by a certain threshold.

The model becomes more interesting when the firms have asymmetric power. Assume wlog that firm 1 is the market leader, meaning that it has a greater default share. First of all, I show that under some conditions, firms play mixed strategies in equilibrium. Then, I analyze the equilibrium play of the game in the specific settings for hard markets. In complex product setting, I show that there is a mixed strategy equilibrium in which firms play asymmetrically provided that the weaker firm is weak enough. In equilibrium, three products are offered: The complex product only offered by the market leader, the best quality product by both, and the worst quality product only by the weaker firm. The equilibrium profit of the market leader is the minimum in the rational benchmark. The comparison between all markets hard case and asymmetric setting depends on the values of the primitives, but the leader gets a strictly higher payoff under the latter iff its excess market power relative to the symmetric case is higher than the excess cost of the least costly complex product relative to the worst quality product. On the other hand, the weaker firm achieves its highest equilibrium payoff when all markets are hard, and it is possible for this firm to have a higher payoff in the rational benchmark compared to the asymmetric case. For instance, if the cost difference between the best and worst quality products are sufficiently high, then for any market share this holds true for the weaker firm. Finally, I show that provided the market share of the weaker firm is bounded above by a certain number, then the market leader gains a strictly higher payoff in equilibrium. This bound is not hard to satisfy: For example, if the weaker firm has a market share of $\frac{1}{3}$, then this is strictly lower than the bound.

In limited attention setting, the equilibrium has a unique form provided that the maximum cost difference is sufficiently low. The market leader offers a menu of size exactly equal to the attention limit of the consumer consisting of the cheapest, and hence worst quality products. In addition to this, it offers a menu of smaller size containing the best quality product. The weaker firm offers two menus: The singleton menu consisting of the best quality product, and a menu that complements the smaller menu offered by the leader in the sense that the number of products it contains makes all markets hard. Consider a special case for ease of interpretation: Every product has the same cost k. Then, in equilibrium the market leader gets a payoff of its default share minus the cost of m products where m is the consumer's attention limit. Provided that the leader has a sufficient excess power compared to the competitor, the equilibrium payoff of the leader is maximized in the asymmetric case. On the contrary, in the simple case of equal cost, the weaker firm gets the least payoff in the asymmetric case. The industry profit is inversely proportional to the unit cost and the attention limit, while it increases if the smaller

[7] Chioveanu and Zhou [6] is an example in which the complexity of the market depends on the coordination between firms. Spiegler [29] discusses the importance of this in detail. Note that even though a single firm can make the market complex, it might not be profitable to do so because of high costs. Therefore, in equilibrium I can see a "non-cooperative collusive behavior".

menu the leader offers gets larger in size.

In both special cases, there is a common observation: The market leader offers a menu which makes all markets hard. Indeed, this seems intuitive when appropriate conditions are satisfied and a firm can unilaterally make a market complex. After generalizing the model to include different types of consumers in the population (meaning it is possible to have different hard and easy markets for different consumers), I show in Section 4 that if there is a menu s.t. it makes all markets (resulting from the competitor's play) hard for a sufficiently high share of consumers, then the market leader offers this menu. I introduce competition into this setting by allowing more than two firms, in which case the pure strategy equilibrium of the game takes a simple form: Firms with a market power less than the symmetric case with $N \geq 2$ firms only offer the best quality product in order to protect their default share, because otherwise more powerful firms can capture the whole market. Firms with a greater power offer more products, but any product offered by a less powerful firm is offered by a more powerful one. Thus, the menus offered by firms are nested, and the product differentiation in the market is determined by the offer of the most powerful firm. In the simple case of duopoly where firms have the least information about the consumers attention limit (uniform distribution) and zero cost of production, I demonstrate the existence of the pure strategy equilibrium in which the market leader offers all products to make the market complex for every type, while the weaker firm only offers the best quality product.

Finally, I demonstrate two important points. The first one is related to the ability of firms making the market complex unilaterally. What happens if the structure of hard markets enable the firms also make the market simple unilaterally? I show that in this case it is possible to see a non-cooperative collusion to make the market complex. In fact, competition is harmful for the consumers in this case: The best product offered in the market is the $(m+1)^{th}$ worst quality product, which implies that the best quality product might never be offered. The second one is related to the incentive of making the market complex: If the markets are complex to the consumer because of her indecisiveness related to her preferences, then I show that there is a pure strategy equilibrium in which both firms only offer the best quality product, so the rational consumer benchmark is retained.

The organization of the paper is as follows: In Section 2, I discuss the related literature. In Section 3, I present the model and discuss the case of symmetric and asymmetric firms. I further investigate the model with asymmetric firms in Section 3.2.1 (complex product setting) and in Section 3.2.2 (limited attention) by imposing more structure on the set of hard markets. Section 4 generalizes the binary nature of easy and hard markets by introducing different cognitive types, and presents a sufficient condition for the market leader to complicate the market. Then, in Section 5 I introduce competition using the extended framework in Section 4. I conclude in Section 6 by further discussing our relation to the literature, and two alternative ways in which consumers find markets complex.

2 Related Literature

This paper is most closely related to the recent literature of modelling bounded rationality in IO. Spiegler [28] provides a textbook treatment of different modelling tools. There are various papers investigating the effect of the decision problem's complexity on the consumer. Spiegler [29] summarizes several different approaches. Carlin [4] models complexity as a separate variable chosen by the firm, and the consumers have different cognitive abilities. He shows that competition has an adverse effect, as the number of firms in the market increases, the complexity level chosen by a firm also increases. Chioveanu and Zhou [6] use frames to model the complexity. In particular, depending on the choice of the frames by firms, a fraction of consumers gets confused and confused consumers purchase from one of the two firms with $\frac{1}{2}$ probability. Gabaix and Laibson [10] looks at a model in which the consumer confusion increases when firms choose to shroud an attribute to hide the details of the pricing scheme. Eliaz and Spiegler [8, 9] uses the attention function to model an individual who can switch from her default firm in the case the competitor offers a better alternative and this attracts her attention. A choice-theoretic analysis of the default option is done in Masatlioglu and Ok [21]. Piccione and Spiegler [23] allows the firms to choose formats of presentation in addition to the prices of the products they offer. The format choices of the firms affect the consumer's ability of comparison, and hence their choice. They mainly investigate the interaction between the price and format decisions of the firms. Gerasimou and Papi [13] analyzes duopolistic competition when consumers are overwhelmed by being offered a large number of alternatives. Rubinstein [25] analyzes the behavior of a single firm when consumers differ in their ability to process information.

Naturally, the literature on heuristics is related to our paper. I mainly benefit from the conceptual discussions in the literature in defining our heuristic. The first connection of our model with the literature is through the assumption that people use heuristics when the problem is hard, while they can solve the problem according to their preferences when it is easy. This is supported by the existing literature, such as Kahneman and Tversky [30], Gigerenzer and Todd [16], Shah and Oppenheimer [27], and Payne [22]. The heuristic I use, *choosing the default*, is well-known in the literature, for a discussion please see Gigerenzer [14].

Finally, I see an inspiration from the area of computational complexity theory, which is a branch of computer science. I discuss two specific structures on the set of hard problems, but a main premise is that there can be various ways of classifying problems as hard and easy, and this provides us a natural field to investigate the consumer's decision procedures. In particular, I think that when I try to learn about the decision process of a consumer, I need to first look at the classification of decision problems for the particular consumer. Similarly, in a market setting, I need to take into account this classification when I investigate the equilibrium outcomes, as done in this paper. Bossaerts and Murawski [3] argues for a similar approach. The type of complexity discussed in the limited attention part can be considered as an example of *state complexity*, where the set of states for the consumer is the set of alternatives the consumer is offered. Salant and Spenkuch [26] refers to the complexity in the complex product setting as *object complexity*.

3 A Model

I model a market situation in which two firms facing a measure 1 of identical consumers play a simultaneous move complete information game. Note that the consumers are identical in two dimensions: preferences, and the cognitive type. Let X denote the set of all alternatives with cardinality $n \geq 2$ and $\mathbb{X}$ the set of all nonempty subsets of X. A menu is an element in $\mathbb{X}$, and firms play the game by simultaneously choosing menus. A mixed strategy σ is a probability distribution over menus, so $\sigma \in \triangle(\mathbb{X})$. I denote the support of σ as $supp(\sigma)$. A *market* is defined as a pair of menus offered by each firm, so it is an element of $\mathbb{X} \times \mathbb{X}$, and I denote a generic element as $\mathbf{M}$. Let $\mathbf{M} = (M_1, M_2)$ where M_i is the menu offered by firm i.[8] A product x is offered in a market $\mathbf{M}$ iff $x \in M_1 \cup M_2$.[9] I assume that there are two types of markets, a market $\mathbf{M}$ is either *hard* or *easy* for the consumer. The set of hard markets is denoted as $\mathbf{H}$, while the set of easy markets is denoted as $\mathbf{E}$. Finally, I say x is a complex product if any market $\mathbf{M}$ in which x is offered is hard, while substituting x with another product which is not complex makes the market $\mathbf{E}$ provided that there is no other complex product. I assume the following properties hold:

- **P1**: $\{\mathbf{E}, \mathbf{H}\}$ is a partition of $\mathbb{X} \times \mathbb{X}$.
- **P2**: $\mathbf{H}$ is closed under $\subseteq$, and hence $\mathbf{E}$ is closed under $\supseteq$.
- **P3**: $(M_1, M_2) \in \mathbf{H}$ iff $(M_2, M_1) \in \mathbf{H}$.
- **P4**: If $(\{x\}, \{y\}) \in \mathbf{H}$ for some $x, y \in X$, then either x or y is a complex product.

I am going to look at two special cases of such a partition:

Complex Products:
A complex product is a product s.t. whenever it is included in a menu, any market in which this menu is offered is hard. Products with a complicated description or pricing are primary examples.[10]

Limited Attention:
Assume that an individual can only pay attention to m products. The complexity of a menu depends on the attention capacity of the consumer, m, and the total number of products offered in the market, $|M_1| + |M_2|$. Thus, a market is hard iff $|M_1| + |M_2| > m$. This setting models the well-known *choice overload* phenomenon investigated in Iyengar and Lepper [17].

I assume that the consumer chooses the default firm when the market is complex for her, and otherwise selects the best product according to her true preferences among the alternatives available in the market. Thus, all consumers are endowed with a complete asymmetric preference relation $\succ$ which I interpret to be the true preference relation. The share of consumers that purchase a product from a firm depends on the market power (or salience/visibility) of that firm.[11] In particular, I assume firm i is the default firm for α_i share of consumers, and $\alpha_{-i} = 1 - \alpha_i$. I view this parameter as a proxy for the market power or visibility of the firm. Without loss of generality, let $\alpha_1 \geq \alpha_2$, so firm 1 is more powerful compared to firm 2. When $(M_1, M_2) \in \mathbf{H}$, I assume that the consumer is choosing a product offered by firm i with a certain probability. The interpretation is as follows: When the consumer is confused in a market because it is hard, she goes to her default firm and chooses one of the products offered by this firm. When $(M_1, M_2) \in \mathbf{E}$, the consumer chooses the $\succ$-maximal product in $M_1 \cup M_2$. If only one firm offers the product chosen by the consumer, then that firm captures the whole market and gets a share of 1, while other firm gets a share of 0. If both firms offer the chosen product when $(M_1, M_2) \in \mathbf{E}$, then I assume that they equally share the market.

[8] As usual, I denote the competitor of firm $i \in \{1, 2\}$ as $-i$. So, M_{-i} denotes the menu offered by the competitor of i.

[9] I sometimes abuse the notation and say $x \in \mathbf{M}$ in this case.

[10] Gaudeul and Sugden [11] provide a model in which common standards are self-sustaining in equilibrium, where a deviation from a common standard makes the decision problem complex for the consumer.

[11] Alternatively, the consumer can defer the choice and chooses nothing, which is extensively analyzed in Gerasimou [12]. Another option is to keep buying the same good you Ire previously buying, which can be captured in our framework. For a discussion related to default architecture, please refer to Spiegler [29].

Each product x is sold for a fixed price 1, and firm i incur the fixed cost of the menu M_i independent of the choice of the consumers, denoted $k(M_i)$, where $k : \mathbb{X} \to [0,1]$. I denote $k(\{x\})$ as k_x. The cost function k is assumed to have an additive structure, that is, $k(M) = \sum_{x \in M} k_x$ for any $M \in \mathbb{X}$. Throughout the paper, I assume that $k_x \geq k_y$ whenever $x \succ y$. Let $b(M)$ denote the $\succ$-best product in menu M, while $w(M)$ denotes the worst one. The maximum cost difference between products, which is equal to $k_{b(X)} - k_{w(X)}$, is denoted simply as k^*. Firms are expected profit maximizers: Firm i's payoff in market (M_1, M_2) is equal to $\alpha_i - k(M_i)$ if $(M_1, M_2) \in \mathbf{H}$. Otherwise, i.e. when $(M_1, M_2) \in \mathbf{E}$, if i is the sole supplier of $\succ$-best product, then its payoff is equal to $1 - k(M_i)$, if both firms supply it $\frac{1}{2} - k(M_i)$, and $-k(M_i)$ otherwise.

Take a particular menu $M \in \mathbb{X}$ and assume that it is offered by one of the firms. By **P3**, the identity of the firm which offers M is inessential. Consider a market in which M is offered. The set of menus for which the market is hard conditional on M being offered can be defined as $\mathbf{H}(M) = \{M' \in \mathbb{X} : (M, M') \in \mathbf{H}\}$. Since $\{\mathbf{E}, \mathbf{H}\}$ is a partition of $\mathbb{X} \times \mathbb{X}$, the menus for which the market in which M is offered is easy can be defined as $\mathbf{E}(M) = \mathbb{X} \setminus \mathbf{H}(M)$. Note that if $\mathbf{H}(M) = \mathbf{H}(M \cup S)$, then $\mathbf{H}(\hat{M}) = \mathbf{H}(M)$ for all $M \subseteq \hat{M} \subseteq M \cup S$. Let $\mathbf{h}(M)$ denote the least costly menu (or menus) in $\mathbf{H}(M)$. For simplicity, I am going to assume that there is a unique element of $\mathbf{h}(M)$ for each $M \in \mathbb{X}$. Let us start by making a preliminary observation about the benchmark cases, which relates the discussion to the standard literature.

Proposition 3.1.

- *If* $\mathbf{H} = \emptyset \times \emptyset$ *and* $k^* < \frac{1}{2}$, *then the unique equilibrium of the game is* $(\{b(X)\}, \{b(X)\})$.
- *If* $\mathbf{H} = \mathbb{X} \times \mathbb{X}$, *then the unique equilibrium of the game is* $(\{w(X)\}, \{w(X)\})$.

Proof.

- Let $\sigma = (\sigma_1, \sigma_2)$ be a mixed strategy profile. For any $M_i \in supp(\sigma_i)$, the payoff of firm i is equal to $\sum_{M_{-i} \in supp(\sigma_{-i})} \sigma_{-i}(M_{-i}) s_i(M_i, M_{-i}) - k(M_i)$ where $s_i(M_i, M_{-i}) = 1$ if only i provides $b(M_1 \cup M_2)$, $s_i(M_i, M_{-i}) = \frac{1}{2}$ if both provide and 0 otherwise. Observe that no firm offers a menu with multiple products, so $|M_i| = 1$ for all $M_i \in supp(\sigma_i)$. Let w_i be the $\succ$-worst product offered in $supp(\sigma_i)$. It is easy to show that $w_1 = w_2 = w$ for some $w \in X$ and the payoff to $\{w\}$ is equal to $\frac{1}{2}\sigma_{-i}(\{w\}) - k_w$. If firm i deviates to $\{b(X)\}$, the payoff to this menu is equal to $1 - \frac{1}{2}\sigma_{-i}(\{b(X)\}) - k_{b(X)}$ and the change in the payoff is equal to $1 - \frac{1}{2}[\sigma_{-i}(\{b(X)\}) + \sigma_{-i}(\{w\})] - [k_{b(X)} - k_w]$, which is strictly positive because $k_{b(X)} - k_{w(X)} < \frac{1}{2}$ and $\sigma_{-i}(\{b(X)\}) + \sigma_{-i}(\{w\}) \leq 1$. Therefore, the deviation is profitable. Since w is arbitrary, this implies in the unique equilibrium firms offer $\{b(X)\}$.
- The payoff of firm i becomes $\alpha_i - k(M_i)$ when $\mathbf{H} = \mathbb{X} \times \mathbb{X}$. No firm offers a menu with multiple products also in this case, so $|M_i| = 1$ for all $M_i \in supp(\sigma_i)$. Since α_i is independent of the identity of the product, both firms offer the cheapest product, $w(X)$.

$\square$

In the rational benchmark (i.e. $\mathbf{H} = \emptyset \times \emptyset$), we retain the classical result that competition forces firms to offer the best quality product. On the other hand, when $\mathbb{H} = \mathbb{X} \times \mathbf{X}$, firms can only sell to their default share. This implies that there is no point of product differentiation or competing by offering a better quality product, and both firms simply offer the worst-quality products.

I divide the following analysis into two parts: Symmetric firms where both firms has a market power of $\frac{1}{2}$, and asymmetric firms with firm 1 being the market leader s.t. $\alpha_1 > \alpha_2$.

3.1 Symmetric Firms

The first question I ask is how the firms would behave if they have equal market power, i.e. $\alpha_1 = \alpha_2 = \frac{1}{2}$. Under mild assumptions on the production costs, the game admits a unique pure strategy equilibrium $(\{b(X)\},\{b(X)\})$, although there can be possibly a mixed strategy equilibrium.

Proposition 3.2. *If $k_{b(X)} < min\{\frac{1}{2}, k(\mathbf{h}(\{b(X)\}))\}$, then $(\{b(X)\},\{b(X)\})$ is the unique pure strategy equilibrium.*

Proof. First, I show that there is no pure strategy equilibrium if $\mathbf{M} \in \mathbf{H}$. Firm i gets a payoff of $\frac{1}{2} - k(M_i)$. But then firm i with $b(M_1 \cup M_2) \in M_i$ can profitably deviate to $\{b(M_1 \cup M_2)\}$, which implies that $|M_1| = |M_2| = 1$. Let $M_i = \{x_i\}$ for some $x_i \in X$. Assume that $b(X) \neq b(M_i)$ for some i. In this case, a deviation by firm i to $\{b(X)\}$ is profitable since $k_{b(X)} < \frac{1}{2}$. This only leaves $(\{b(X)\},\{b(X)\})$ as a possible equilibrium, which is a contradiction because of **P3** and Assumption 2 that $b(X)$ is not a complex product.

Now let $\mathbf{M} = (M_1, M_2) \in \mathbf{E}$ and assume wlog $b(M_1 \cup M_2) \in M_i$ for some $i \in \{1,2\}$. The payoff of firm i is equal to $1 - k(M_i)$ if $b(M_1 \cup M_2) \in M_i \setminus M_{-i}$ for only i, and $\frac{1}{2} - k(M_i)$ otherwise. Consider the first one, and note that this cannot be an equilibrium because the competitor of i can profitably deviate to $\{b(M_i)\}$. This deviation is profitable because otherwise $-i$ gets a negative payoff. Now consider the latter. Because $(M_1, M_2) \in \mathbf{E}$, firm i can profitably deviate to $\{b(M_i)\}$. Furthermore, it is straightforward to show that $|M_1| = |M_2| = 1$ in equilibrium. Thus, $(M_1, M_2) \in \mathbf{E}$ is an equilibrium only if $M_1 = M_2 = \{x\}$ for some $x \in X$. Because $k_y < \frac{1}{2}$ for all $y \in X$, firm i can profitably deviate by offering $\{y\}$ s.t. $y \succ x$. This implies that for $(M_1, M_2) \in \mathbf{E}$ to be an equilibrium, $M_1 = M_2 = \{b(X)\}$. So, if $(M_1, M_2) \in \mathbf{E}$ and this is an equilibrium, then I need to have $M_1 = M_2 = \{b(X)\}$.

To show $(\{b(X)\},\{b(X)\})$ is the unique pure strategy equilibrium, I need to show that there is no profitable deviation by a firm to a mixed strategy. Let σ_i denote a mixed strategy played by firm i. The payoff to $M \in supp(\sigma_i)$ is equal to:

$$\frac{1}{2}\sum_{\{M:(M,\{b(X)\})\in\mathbf{H}\}} \sigma_{-i}(M) + \frac{1}{2}\sum_{\{M:(M,\{b(X)\})\in\mathbf{E}\ \&\ b(X)\in M_i\}} \sigma_{-i}(M) - k(M_i)$$

Note that if $b(X) \in M$, then the payoff is strictly less than the payoff i receives from $(\{b(X)\}, \{b(X)\})$, which is equal to $\frac{1}{2} - k_{b(X)}$. So, $b(X) \notin M$ for all $M \in supp(\sigma_i)$, in which case the payoff to M reduces to:

$$\frac{1}{2}\sum_{\{M:(M,\{b(X)\})\in\mathbf{H}\}} \sigma_{-i}(M) - k(M)$$

It is easy to show that firm i can profitably deviate to $\mathbf{h}(\{b(X)\})$, so the support of σ_i is equal to $\{\mathbf{h}(\{b(X)\})\}$. This is also strictly less than $\frac{1}{2} - k_{b(X)}$ because $k(\mathbf{h}(\{b(X)\})) > k_{b(X)}$, which shows that there is no profitable deviation. □

Note that the assumption required about costs is stronger than the one need for the rational consumer benchmark, which implies that this assumption is also sufficient for the rational consumer result. In fact, $k_{b(X)} < \frac{1}{2}$ is a sufficient condition for both results.

3.2 Asymmetric Firms

From now on, I will maintain the following assumptions unless otherwise specified:

Assumption 1. $\alpha_1 > \alpha_2$.

Assumption 2. $\mathbf{H}, \mathbf{E} \neq \emptyset \times \emptyset$.

Assumption 3. *$b(X)$ and $w(X)$ are not complex products.*

Next, I ask whether there is a pure strategy equilibrium of this game.

Proposition 3.3. *If $k(\mathbf{h}(\{b(X)\}) - k_{b(X)} < \alpha_1 - \frac{1}{2}$, then there is no pure strategy equilibrium.*

Proof. Let $\mathbf{M} = (M_1, M_2) \in \mathbf{E}$ and assume wlog $b(M_1 \cup M_2) \in M_i$ for some $i \in \{1,2\}$. The payoff of firm i is equal to $1 - k(M_i)$ if $b(M_1 \cup M_2) \in M_i \setminus M_{-i}$ for only i, and $\frac{1}{2} - k(M_i)$ otherwise. First, consider the former, and note that this cannot be an equilibrium because the competitor of i can profitably deviate to $\{b(M_i)\}$. This deviation is profitable because otherwise $-i$ gets a negative payoff. Now consider the latter. Because $(M_1, M_2) \in \mathbf{E}$, firm i can profitably deviate to $\{b(M_i)\}$. Furthermore, it is straightforward to show that $|M_1| = |M_2| = 1$ in equilibrium. Thus, $(M_1, M_2) \in \mathbf{E}$ is an equilibrium only if $M_1 = M_2 = \{x\}$ for some $x \in X$. Because $k(X) < \frac{1}{2}$, firm i can profitably deviate by offering $\{y\}$ s.t. $y \succ x$. This implies that for $(M_1, M_2) \in \mathbf{E}$ to be an equilibrium, $M_1 = M_2 = \{b(X)\}$. Consider a deviation by firm 1 to $\mathbf{h}(\{b(X)\})$. After this deviation, firm 1 gets a payoff of $\alpha_1 - k(\mathbf{h}(\{b(X)\}))$. Thus, the change in payoff is equal to $(\alpha_1 - \frac{1}{2}) - (k(\mathbf{h}(\{b(X)\}) - k_{b(X)})$, which is strictly positive by assumption.

Assume that $\mathbf{M} = (M_1, M_2) \in \mathbf{H}$. Firm i gets a payoff of $\alpha_i - k(M_i)$. If there is y s.t. $\{y\} \succ^* M_1$ and $(M_1, \{y\}) \in \mathbf{E}$, then firm 2 can profitably deviate to $\{y\}$. The same is true for firm 1. Assume there is no such y, that is, $(M_i, \{y\}) \in \mathbf{H}$ for any $\{y\} \succ^* M_i$. This implies that if $(M_1, M_2) \in \mathbf{H}$ is an equilibrium, then either $|M_i| = 1$ and $M_i \succ^* M_{-i}$ or M_i is a subset of the lower-contour set of $b(M_{-i})$ for i. Both M_i's cannot be of the former type. So, either both of them are of the latter type or each one is of different type. I can directly eliminate the latter if firm 2 is offering the singleton x_2 s.t. $\{x_2\} \succ^* M_1$, because it can profitably deviate to $\{b(M_1)\}$ and increase its share to $\frac{1}{2}$ while decreasing costs. If firm 1 is the firm offering $\{x_1\} \succ^* M_2$, then firm 2 can deviate to any $\{y\}$ s.t. $(\{x_1\}, \{y\}) \in \mathbf{E}$ and $y \succsim x_1$. Such a y exists because $b(X)$ is not a complex product. If $(\{x_1\}, \{b(X)\}) \in \mathbf{H}$, then this implies x_1 is a complex product, which implies further that a deviation from M_2 to $\{w(X)\}$ is profitable. If I already have $M_2 = \{w(X)\}$, then firm 1 can profitably deviate to $\{b(X)\}$ because both $b(X)$ and $w(X)$ are not complex, so $\{b(X), w(X)\} \in \mathbf{E}$. Now consider the case in which both are subsets of the lower contour sets of each other's best product. This is possible only if $b(M_1) = b(M_2)$. Firm 2 can profitably deviate to $\{b(M_2)\}$ unless $|M_2| = 1$. If $|M_2| = 1$ and furthermore $|M_1| = 1$, then this implies $M_1 = \{b(M_2)\}$ because of the lower-contour set condition. But then since this is a hard market, $b(M_1)$ should be a complex product. By the structure of $\mathbf{H}$, any market including $b(M_1)$ should be complex, so a firm can profitably deviate to $\{w(X)\}$ again unless $b(M_1) = w(X)$, which cannot be true because by assumption $w(X)$ cannot be a complex product. So, let $|M_1| > 1$ and consider a deviation by firm 1 to $\{b(X)\}$ if $b(X) \notin M_1$. Such a deviation is profitable because $(\{b(X)\}, \{b(M_1)\}) \in \mathbf{E}$. To wit, note that otherwise by the structure of $\mathbf{H}$ either $b(X)$ or $b(M_1)$ is a complex product. The former cannot be true by assumption, while the latter is ruled out because otherwise $|M_1|$ should be equal to 1 in that case. Therefore, let us consider the case $b(X) \in M_1$, and therefore $M_2 = \{b(X)\}$. Firm 2 can profitably deviate to $\{x\}$ s.t. $x \neq b(X)$ if $(M_1, \{x\}) \in \mathbf{H}$, because $k_x < k_{b(X)}$. This is not possible only if $(M_1, \{x\}) \in \mathbf{E}$ for any $x \neq b(X)$. This implies that $b(X)$ is a complex product, a contradiction.

$\square$

To better understand the implications of the model, I investigate two special structures of $\mathbf{H}$. Until otherwise specified, I imagine that the excess market power of firm 1 w.r.t to the symmetric case is above the cost of offering whole set of alternatives. Given the previous result, I focus on the mixed strategy equilibrium of the game.

3.2.1 Complex Products

I start by analyzing the implications of this choice procedure in the complex product setting. Let $X = Y \times \{s,c\}$ where the Y denotes the set of products. s is a simple description while c is a complex one. So, each product produced by a firm can be presented in two formats: For any $y \in Y$, (y,s) is the simply presented version of product y, while (y,c) is the complex version. Assume that the cost of a description format is zero, so cost of $x \in X$ is equal to k_{y_x} if the first coordinate of x is equal to y_x. Take any complex product x. Given the choice procedure of the consumers, firm i can insure a payoff of $\alpha_i - k_x$ by offering $\{x\}$. Furthermore, since there exists a least costly complex product, firms would prefer to use this complex product to complicate the market. Therefore, without loss of generality the attention can be focused on the simpler case when there is only one complex product, d, and any product in $X \setminus \{d\}$ is not complex. I assume that n, the number of products in X, is odd. Let us order the products in $X \setminus \{d\}$ according to their costs, so $X \setminus \{d\} = \{x_1, x_2, ..., x_{n-1}\}$ where $x_1 = w(X)$ and $x_{n-1} = b(X)$ because $w(X)$ and $b(X)$ are not complex. Observe that $w(X)$ not being complex automatically implies that $d \succ w(X)$. I assume furthermore that $k_x \neq k_y$ for any $x, y \in X$, so $k_d > k_{w(X)}$.

I first show a proposition about the equilibrium behavior of the market leader. In particular, I show that in every equilibrium there is a hard market and firm 1 always offers the complex product.

Proposition 3.4. *In any equilibrium strategy profile (σ_1, σ_2), $\{d\} \in supp(\sigma_1)$ and hence the equilibrium payoff of 1 is equal to $\alpha_1 - k_d$.*

Proof. Let us first define some concepts I use in the proof. The induced preference relation over menus, denoted $\succsim^*$, is defined as follows: $M \succsim^* M'$ iff $max(M,\succ) \succ max(M',\succ)$ or $max(M,\succ) = max(M',\succ)$. Therefore, $M \sim^* M'$ iff $max(M,\succ) = max(M',\succ)$. For any M' s.t. $(M,M') \in \mathbf{E}$ and $M' \succ^* M$, firm that offers M cannot sell any product. If $M \sim^* M'$, both firms share the market equally. Let us define the probability that firm i offers an indifferent menu to M (not necessarily in $supp(\sigma)$) s.t. the market is easy as:

$$\beta(M,\sigma) = \sum_{\{M':M' \sim^* M \ \& \ (M,M') \in \mathbf{E}\}} \sigma(M')$$

When it leads to no confusion, I suppress σ, and just use $\beta(M)$. Similarly, I define the probability of offering a $\succ^*$-better menu as $\beta^*(M)$, and the probability offering a $\succ^*$-worse menu as $\beta_*(M)$.

Lemma 1. $|M| = 1$ *for all* $M \in supp(\sigma_i)$ *and* $i \in \{1,2\}$.

Proof. This follows directly for any menu $M \in supp(\sigma_i)$ s.t. $(M,M') \in \mathbf{E}$ for all $M' \in supp(\sigma_{-i})$. If this is not the case and not all markets in which i offers M is hard, then this means that M does not contain a complex product. This implies that in any market $(M,M') \in \mathbf{H}$, the menu offered by $-i$, M', contains a complex product. By the choice procedure of the consumers, whenever the market is hard, the consumers go to their. The menu offered by the default firm is irrelevant to the consumer's choice, and therefore the firm offering M can profitably deviate to $\{max(M,\succ)\}$. Finally, if $(M,M') \in \mathbf{H}$ for all $M' \in supp(\sigma_{-i})$ and therefore M contains a complex product, the payoff received from this menu is equal to $\alpha_i - k(M)$. The firm has a profitable deviation to the singleton menu consisting only of the complex product.[12] $\square$

The following lemma will show that at least one firm offers a menu containing a complex product, which is equal to $\{d\}$. Therefore, there are two cases to be considered: Either one firm offers $\{d\}$ or both firms offer it. In the latter case, both firms get a payoff of $\alpha_i - k_d$ in equilibrium. If only firm i offers $\{d\}$, then i receives a payoff equal to $\alpha_i - k_d$, while i's competitor receives at least $\alpha_{-i} - k_d$, because otherwise a deviation to $\{d\}$ from one of the menus offered in the support would be profitable.

Lemma 2. $(supp(\sigma_1) \times supp(\sigma_2)) \cap \mathbf{H} \neq \emptyset$.

Proof. Assume to the contrary $(supp(\sigma_1) \times supp(\sigma_2)) \cap \mathbf{H} = \emptyset$. I know that in this case any menu offered in equilibrium is a singleton. Let $\{w_i\}$ be the $\succsim^*$-worst among these menus. Since these menus are singletons, there is a unique $\succ$-worst product which is $\succsim^*$-worst menu in the support of a firm's strategy. The payoff of firm i from this menu is equal to $\frac{1}{2}\beta_*(\{w_i\},\sigma_{-i}) - k_{w_i}$, and a deviation to $\{b(X)\}$ leads to a change in payoff of $1 - \frac{1}{2}\{\beta_*(\{w_i\},\sigma_{-i}) + \sigma_{-i}(\{b(X)\})\} - \{k_{b(X)} - k_{w_i}\}$, which is profitable because $k(X) < \alpha - \frac{1}{2} \leq \frac{1}{2}$ and $\beta_*(\{w_i\},\sigma_{-i}) + \sigma_{-i}(\{b(X)\})) \leq 1$. This implies that I need to have $w_i = \{b(X)\}$ for $i \in \{1,2\}$, so $supp(\sigma_i) = \{\{b(X)\}\}$. Note that if this is the case, both firms receive a payoff of $\frac{1}{2} - k_{b(X)}$, because $b(X)$ cannot a complex product by assumption. But firm 1 can profitably deviate from $\{b(X)\}$ to $\{d\}$ because $k(X) < \alpha - \frac{1}{2}$. Therefore, for any $(M_1,M_2) \in (supp(\sigma_1) \times supp(\sigma_2)) \cap \mathbf{H}$, $M_i = \{d\}$ for at least one $i \in \{1,2\}$.

$\square$

I conclude by showing that firm 1, the market leader, always offers $\{d\}$. Assume to the contrary 2 offers $\{d\}$ but not 1. This implies that firm 2 gets $\alpha_2 - k_d$ in equilibrium. Consider a deviation to $\{b(X)\}$ by firm 2. Since 1 does not offer $\{d\}$, the payoff of player 2 from offering $\{b(X)\}$ is equal to $1 - \frac{1}{2}\sigma_1(\{b(X)\}) - k_{b(X)}$ because any menu offered in the support should be a singleton by the first lemma and therefore $\beta(\{b(X)\},\sigma_1) = \sigma_1(\{b(X)\})$. Therefore, the change in payoff is equal to:

$$1 - \frac{1}{2}\sigma_1(\{b(X)\}) - \alpha_2 - \{k_{b(X)} - k_d\}$$

which is equal to $\alpha_1 - \frac{1}{2}\sigma_1(\{b(X)\}) - \{k_{b(X)} - k_d\}$. By the assumption $\alpha_1 - \frac{1}{2} > k(X)$, this change is strictly positive, which shows the deviation is profitable and $\{d\} \notin supp(\sigma_2)$, a contradiction.

Since I showed that $(supp(\sigma_1) \times supp(\sigma_2)) \cap \mathbf{H} \neq \emptyset$, $\{d\} \in supp(\sigma_i)$ for some $i \in \{1,2\}$. By the previous lemma, I can conclude that $\{d\} \in supp(\sigma_1)$. $\square$

I showed that firm 1 always complicates the market, and hence it gets in equilibrium its max-min payoff of $\alpha_1 - k_d$. In the following proposition, I show that there is an asymmetric equilibrium in which only the market leader complicates the market, while the less powerful firm gains a payoff which is at least $\alpha_2 - k_d$ and does not complicate the market.

Proposition 3.5. *Assume that $\alpha_2 \leq \frac{k_d - k_{w(X)}}{2k^*}$. There is an asymmetric equilibrium (σ_1,σ_2) with $supp(\sigma_1) = \{\{d\},\{b(X)\}\}$ and $supp(\sigma_2) = \{\{b(X)\},\{w(X)\}\}$ described as follows:*

[12] An important thing to notice here is the following fact: If both firms offer the same complex product $\{d\}$, then this is considered to be a hard market, and therefore firm i captures a share of α_i.

- $\sigma_1(\{d\}) = 1 - 2k^*$ *and* $\sigma_1(b(X)) = 2k^*$.
- $\sigma_2(b(X)) = 2(\alpha_2 - [k_{b(X)} - k_d])$ *and* $\sigma_2(w(X)) = (\alpha_1 - \alpha_2) + 2(k_{b(X)} - k_d)$.

In equilibrium, firm 1 *gets a payoff of* $\alpha_1 - k_d$*, while firm* 2 *gets* $\alpha_2 + (\alpha_1 - \alpha_2)k^*$.

Proof. First, I am going to construct the mixed strategy profile I claimed to be an equilibrium, and then show it is indeed an equilibrium. Let $supp(\sigma_1^*) = \{\{d\}, \{b(X)\}\}$ and $supp(\sigma_1^*) = \{\{b(X)\}, \{w(X)\}\}$, so $\{d\}$ is only offered by firm 1. Let $\{x\}$ be any menu offered in $supp(\sigma_i)$ s.t. $x \neq d$. The following is the payoff firm i gets by offering it:

$$\beta_*(\{x\}, \sigma_{-i}) + \frac{1}{2}[\sigma_{-i}(\{x\}) + \sum_{\{\{z\} \in supp(\sigma_{-i}) : (\{z\}, \{x\}) \in \mathbf{H}\}} \sigma_{-i}(\{z\})] - k_x$$

Since $\{d\}$ is only offered by firm 1, $\sum_{\{\{z\} \in supp(\sigma_{-i}) : (\{z\}, \{x\}) \in \mathbf{H}\}} \sigma_{-i}(\{z\}) = 0$ if $i = 1$ and equal to $\sigma_1(\{d\})$ otherwise. Firm 1's payoff from $\{d\}$ is equal to $\alpha_1 - k_d$, which should be equal to the payoff it gets from $\{b(X)\}$:

$$\alpha_1 - k_d = \frac{1}{2}(1 + \sigma_2(w(X))) - k_{b(X)}$$

which implies that $\sigma_2(w(X)) = (\alpha_1 - \alpha_2) + 2(k_{b(X)} - k_d)$, which also gives $\sigma_2(b(X))$.

Player 2's payoff from offering $\{w(X)\}$ should be equal to its payoff from offering $\{b(X)\}$:

$$\alpha_2 \sigma_1(\{d\}) - k_{w(X)} = \alpha_2 \sigma_1(\{d\}) + \frac{1}{2}(1 - \sigma_1(\{d\})) - k_{b(X)}$$

which implies $\sigma_1(\{d\}) = 1 - 2k^*$ and $\sigma_1(b(X)) = 2k^*$.

Now I am going to show that this constitutes an equilibrium. To achieve this, I need to check the following for firm 2: It cannot profitably deviate by offering $\{d\}$ or any $\{x\}$ s.t. $x \neq d$ and $b(X) \succ x \succ w(X)$. If firm 2 offers $\{d\}$, then its payoff will be equal to $\alpha_2 - k_d$. On the other hand, by playing σ_2, it receives a payoff of $\alpha_2 \sigma_1(\{d\}) - k_{w(X)}$ which is equal to:

$$\alpha_2(1 - 2k^*) - k_{w(X)}$$

The deviation is not profitable because of the assumption that $\alpha_2 \leq \frac{k_d - k_{w(X)}}{2k^*}$. This shows that firm 2 will not deviate to $\{d\}$. For the other possibility, note that the payoff to any $\{x\}$ s.t. $x \neq d$ and $b(X) \succ x \succ w(X)$ will result in a payoff of $\alpha_2 \sigma_1(\{d\}) - k_x$, which is strictly less than the payoff of 2 from σ_2 because $x \succ w(X)$ and hence $k_x > k_{w(X)}$. Observe that these imply there is no profitable deviation to a mixed strategy, showing that firm 2 cannot profitably deviate from σ_2 provided firm 1 plays σ_1.

On the other hand, for firm 1 I need to show that a deviation to $\{x\}$ s.t. $x \neq d$ and $b(X) \succ x$ is not profitable. If $x = w(X)$, the payoff 1 would receive is given by $\frac{1}{2}\sigma_2(\{w(X)\}) - k_x$, which is equal to:

$$(\alpha_1 - k_{b(X)}) + (k_d - \frac{1}{2})$$

Because $k_{b(X)} > k_d$, $\alpha_1 - k_{b(X)} < \alpha_1 - k_d$. Also, since $k(X) \leq \alpha_2$, I have $k_d < \frac{1}{2}$, and therefore the payoff 1 receives from σ_1, $\alpha_1 - k_d$, is greater than or equal to the payoff from deviation. If $x \succ w(X)$, then the payoff from the deviation becomes $\sigma_2(\{w(X)\}) - k_x$, equal to:

$$2\alpha_1 - 2(k_{b(X)} - k_d) - 1 - k_x$$

Assume to the contrary the deviation is profitable, so:

$$\alpha_1 - k_d < 2\alpha_1 - 2(k_{b(X)} - k_d) - 1 - k_x$$

which implies that $\alpha_2 < (k_d - k_x) - 2(k_{b(X)} - k_d)$, a contradiction to $k(X) \leq \alpha_2$. This also shows that firm 1 does not have a profitable deviation to a mixed strategy, the proof is concluded.

□

I already showed in Proposition 3.4 that the market leader complicates the market in any equilibrium, so the payoff it gets is equal to $\alpha_1 - k_d$. On the other hand, in the specific equilibrium I demonstrated, firm 2 does not complicate the market, and gets at least the payoff it gets if it would complicate. In fact, it gets an equilibrium payoff equal to $\alpha_2 + (\alpha_1 - \alpha_2)k^*$, which is strictly higher than its max-min payoff. Observe that this payoff is increasing both its own market power and the excessive market power, and also the maximum cost difference between products. The industry profit is equal to $1 + (\alpha_1 - \alpha_2)k^* - k_d$. This is equal to $1 - 2k_{b(X)}$ both in the symmetric case and the rational benchmark. On the other hand, I have $1 - 2k_{w(X)}$ when all markets are hard. Thus, the total industry profit is greater under the asymmetric case with $\mathbf{H}, \mathbf{E} \neq \emptyset$ compared to the rational benchmark. The comparison between the former and the case when all markets hard depend on the costs: For instance, if $k_d \leq 2k_{w(X)}$, then the industry profit is higher in the asymmetric case. For single firms, the comparison is similar. Consider the market leader: Because $k_d \leq k_{b(X)}$ and $\alpha_1 > \frac{1}{2}$, the profit of the market leader is greater under asymmetric case compared to the rational benchmark, and similarly the profit under all markets hard case. The comparison between all markets hard and asymmetric case depends on the comparison between the excess market power of the leader (compared to symmetric case) and the cost difference between the complex and worst product. If $\alpha_1 - \frac{1}{2} > k_d - k_{w(X)}$, then the profit of the leader is maximized in the asymmetric case. On the other hand, the weaker firm gains its maximum payoff when all markets are hard. The comparison between other cases depend on the relative market power and costs. In particular, if $\frac{1}{2} + \alpha_2(1 - k_{w(X)}) < \alpha_1(2k_{b(X)} - k_{w(X)})$, then the profit of the weaker firm is strictly greater in the rational benchmark. For example, if $k_{b(X)} = \frac{3}{4}$ and $k_{w(X)} = 0$, then this is satisfied for all α_2. Finally, I see that the market leader gains a strictly higher payoff in equilibrium provided that $\alpha_2 < \frac{1-k^*}{3-2k^*}$. For instance, if firm 2 has a market share of $\frac{1}{3}$, this condition is satisfied.

The equilibrium level of complexity is equal to the probability that 1 offers $\{d\}$, so $1 - 2k^*$. This does not depend directly on the cost of the complex product, and it is inversely related to the maximum cost difference. I assumed for simplicity that there is only one complex product, because the firms would prefer to complicate by using the least costly complex product. Educating the consumers can be interpreted as a decrease in the number of complex products for the consumers. Given I made the simplification of a single complex product, this is equivalent to an increase in k_d. Thus, educating the consumers does not change the complexity level in equilibrium. However, it increases consumer welfare by increasing the probability that consumers face an easy market in which $b(X)$ is offered.

As the cost of the complex product increases, the probability that firm 2 offers $\{w(X)\}$ decreases, which implies that the probability it offers $\{b(X)\}$ increases. For the intuition of this, first observe that firm 2 best responds to firm 1 complicating the market by offering $\{w(X)\}$, and an increase in the price of the complex product implies that the probability firm 1 complicates the market decreases. This increases the probability that firm 2 can capture the market by offering $\{b(X)\}$. Also, note that the probability firm 2 offers the worst product directly depends on $\alpha_1 - \alpha_2$, the excess market power firm 1 has compared to firm 2. As this difference increases, the probability of firm 2 offering $\{w(X)\}$ increases, because the likelihood (according to firm 2) that firm 1 complicates the market and benefits from its excess power increases. If the market power of firm 2 increases, this implies a decrease in the market power of 1 (because $\alpha_1 + \alpha_2 = 1$), and hence an increase in the probability that firm 2 offers $b(X)$.

On the other hand, as the cost of $b(X)$ increases, the probability that the firm 2 offers $\{b(X)\}$ decreases. This implies that firm 1 can capture all consumers by offering $\{b(X)\}$, and hence the probability that firm 1 offers $\{b(X)\}$ increases. This means that as it becomes more costly to capture the market for firm 2, firm 1 does not use its Iapon of complicating the market. In general, note that how a firm behaves depends on the expectations of the behavior of the competitor. More precisely, if firm 2 expects the firm will use the Iapon of complicating the market (which can happen according to firm 2 when it has either excess market power or the cost of complicating the market decreases), then the probability that it offers $w(X)$ increases. Otherwise, firm 2 tries to capture the full market by offering $b(X)$.

Note that the probability that a market consumers face is hard for her is equal to $\sigma_1(\{d\}) = 1 - 2k^*$. On the other hand, the probability that the consumers end up buying the best product, $b(X)$, is equal to $\sigma_1(\{b(X)\}) = 2k^*$. The Welfare of the consumers depend on what she ends up buying when she goes to her default firm. Assuming that the consumers buy any product offered by her default firm with strictly positive probability, this implies that the likelihood that the consumers end up buying the worst product on X, $w(X)$, is strictly positive.

3.2.2 Limited Attention

In this section, I discuss the case of a consumer with limited attention. I assume that a problem is complex for the consumer if and only if the number of alternatives in the market exceeds her attention capacity, denoted m. Thus, if (M_1, M_2) is the market the consumers face, then $(M_1, M_2) \in \mathbf{H}$ if and only if $|M_1| + |M_2| > m$.

Recall that the consumer goes to her default firm and buys a product offered by this firm when the market is complex. Because every product is being sold for the same price, and the firm should incur the cost of any product produced, what the default firm offers to the consumer is inessential. Therefore, the firm would rather prefer to offer the least costly products when the market is hard. Let us denote the least costly k products in $M \in \mathbb{X}$ as $L_k(M)$. Thus, each menu M offered by a firm in the equilibrium is of the following form:

$$M = L_{|M|-1}(X \setminus \{max(M, \succ)\}) \cup \{max(M, \succ)\}$$

The following proposition provides a characterization of the equilibrium profile (σ_1, σ_2) under certain restrictions. For simplicity, let us denote $k(L_m(X))$ as $k(L_m)$ without referring to X.

Proposition 3.6. *Assume that $k(L_s(X)) > k_{b(X)}$ for some $s < m$ and let s^* be the smallest among these. There is $\hat{k} \in (0,1)$ s.t. if $k^* \leq \hat{k}$, then the equilibrium has the following form:*

- *$supp(\sigma_1) = \{M_1, L_m(X)\}$ s.t. $b(X) \in M_1$ and $|M_1| < m + 1 - s^*$.*
- *$supp(\sigma_2) = \{\{b(X)\}, L_{m+1-|M_1|}(X)\}$.*

The probability that firms offer these menus are given below:

$$\begin{aligned} \sigma_1(L_m(X)) &= 1 - 2[k(L_{m+1-|M_1|}) - k_{b(X)}] \\ \sigma_2(\{b(X)\}) &= \frac{2(k(L_m) - k(M_1))}{\alpha_1 - \alpha_2} \end{aligned} \tag{1}$$

with the rest of the probabilities induced from above equations.

Proof.

First, let us define $\gamma(\sigma_i|M)$ as the probability that the final market consumers face is hard when I fix what firm i offers to menu M. Let us denote the $\succsim^*$-worst menu M in $supp(\sigma_i)$ s.t. $\gamma(\sigma_{-i}|M) < 1$ as W_{σ_i}.

Step 1: $W_{\sigma_1} \sim^* W_{\sigma_2} \sim^* \{b(X)\}$

Assume to the contrary $W_{\sigma_1} \nsim^* W_{\sigma_2}$, and $W_{\sigma_{-i}} \succ^* W_{\sigma_i}$ for some $i \in \{1,2\}$. Firm i gets a strictly positive payoff from W_{σ_i} only if there is an $M \in supp(\sigma_{-i})$ s.t. $(W_{\sigma_i}, M) \in \mathbf{H}$. Thus, the payoff of firm i from offering W_{σ_i} is equal to:

$$\alpha_i \gamma(\sigma_{-i}|W_{\sigma_i}) - k(W_{\sigma_i})$$

Because $W_{\sigma_{-i}} \succ^* W_{\sigma_i}$, $b(X) \notin W_{\sigma_i}$. Consider a deviation to $W'_{\sigma_i} = (W_{\sigma_i} \setminus \{max(W_{\sigma_i}, \succ)\}) \cup \{b(X)\}$. The payoff to this menu is given by:

$$1 - \frac{1}{2}\beta(W'_{\sigma_i}, \sigma_{-i}) - \alpha_{-i}\gamma(\sigma_{-i}|W'_{\sigma_i}) - k_{b(X)}$$

The change in payoff is equal to

$$1 - \frac{1}{2}\beta(W'_{\sigma_i}, \sigma_{-i}) - \gamma(\sigma_{-i}|W'_{\sigma_i}) - \{k_{b(X)} - k_{max(W_{\sigma_i}, \succ)}\}$$

because $\gamma(\sigma_{-i}|W_{\sigma_i}) = \gamma(\sigma_{-i}|W'_{\sigma_i})$ and $\alpha_i\gamma(\sigma_{-i}|W_{\sigma_i}) + \alpha_{-i}\gamma(\sigma_{-i}|W'_{\sigma_i}) = \gamma(\sigma_{-i}|W'_{\sigma_i})$. Furthermore, since $b(X) \in W'_{\sigma_i}$, $\beta(W'_{\sigma_i}, \sigma_{-i}) = 1 - \gamma(\sigma_{-i}|W'_{\sigma_i}) - \beta_*(W'_{\sigma_i}, \sigma_{-i})$ and therefore $\beta(W'_{\sigma_i}, \sigma_{-i}) \leq 1 - \gamma(\sigma_{-i}|W'_{\sigma_i})$. This implies the change in payoff is greater than or equal to:

$$\frac{1}{2}(1 - \gamma(\sigma_{-i}|W'_{\sigma_i})) - \{k_{b(X)} - k_{max(W_{\sigma_i}, \succ)}\}$$

Let us choose $\hat{k} = \frac{1}{2}(1 - \gamma(\sigma_{-i}|W'_{\sigma_i}))$, in which case the deviation is profitable. This shows that $W_{\sigma_1} \sim^* W_{\sigma_2}$.

When this is the case, the payoff firm i receives from W_{σ_i} is equal to $\frac{1}{2}\beta(W_{\sigma_i}, \sigma_{-i}) + \alpha_i\gamma(\sigma_{-i}|W_{\sigma_i}) - k(W_{\sigma_i})$. Assuming $b(X) \notin W_{\sigma_i}$, a deviation to W'_{σ_i} will lead to the following change in payoff:

$$1 - \gamma(\sigma_{-i}|W'_{\sigma_i}) - \frac{1}{2}\{\beta(W'_{\sigma_i}, \sigma_{-i}) + \beta(W_{\sigma_i}, \sigma_{-i})\} - \{k_{b(X)} - k_{max(W_{\sigma_i}, \succ)}\}$$

Because $\beta(W'_{\sigma_i}, \sigma_{-i}) + \beta(W_{\sigma_i}, \sigma_{-i}) \leq 1 - \gamma(\sigma_{-i}|W'_{\sigma_i})$ and $k_{b(X)} - k_{max(W_{\sigma_i}, \succ)} \leq \hat{k}$, the above expression is greater than or equal to:

$$\frac{1}{2}(1 - \gamma(\sigma_{-i}|W'_{\sigma_i})) - \hat{k}$$

and hence the deviation is profitable if one chooses the same k^* as above. Since this argument holds for any $max(W_{\sigma_i}, \succ) \neq b(X)$, so I have $W_{\sigma_1} \sim^* W_{\sigma_2} \sim^* \{b(X)\}$.

Step 2: $supp(\sigma_2) = \{\{b(X)\}, L_{m+1-min_{M \in supp(\sigma_1)}|M|}(X)\}$

Because $b(X)$ is the $\succ$-best product in X, I have $M \sim^* M'$ for any $M, M' \in supp(\sigma_i)$ s.t. $\gamma(\sigma_{-i}|M), \gamma(\sigma_{-i}, M') < 1$ and $i \in \{1,2\}$. Because $M = L_{|M|-1}(X \setminus \{max(M, \succ)\}) \cup \{max(M, \succ)\}$, $|M| \neq |M'|$ for any $M, M' \in supp(\sigma_i)$, and I have either $M \subset M'$ or $M' \subset M$ for any distinct menus M and M' in the support. Consider any menu M offered by firm i s.t. $\gamma(\sigma_{-i}|M) < 1$. The payoff to M for firm i is equal to:

$$\frac{1}{2} + (\alpha_i - \frac{1}{2})\gamma(\sigma_{-i}|M) - k(M)$$

which decreases as the size of M increases for firm 2 since $\alpha_2 \leq \frac{1}{2}$. Therefore, the unique such M that can be offered by firm 2 is $\{b(X)\}$. Because this is the unique M s.t. $\gamma(\sigma_1|M) < 1$, the only other menu that can be in the support is $L_{m+1-min_{M \in supp(\sigma_1)}|M|}(X)$. For firm 1, if two different menus M and M' s.t. $M \sim^* M' \sim^* \{b(X)\}$ and $\gamma(\sigma_2|M), \gamma(\sigma_2|M') < 1$ offered, then the following should be satisfied assuming $M \subset M'$ wlog:

$$\gamma(\sigma_2|M') - \gamma(\sigma_2|M) = \frac{k(M' \setminus M)}{\alpha_i - \frac{1}{2}} \tag{2}$$

If $supp(\sigma_2) = \{\{b(X)\}\}$, then $\gamma(\sigma_2|M) = 1$ if and only if $M = L_m(X)$, and 0 otherwise. Since $\gamma(\sigma_2|M), \gamma(\sigma_2|M') < 1$ and $\alpha_1 > \frac{1}{2}$, both are equal to 0 and I need to have $k(M' \setminus M) = 0$, a contradiction to $M \subset M'$. Therefore, there should be a unique M s.t. $\gamma(\sigma_2|M) = 0$, which would be equal to $\{b(X)\}$ because $b(X) \in M$ and $\gamma(\sigma_2|M) = 0$ is equivalent to market being easy (which would therefore imply to get a positive payoff, firm 1 needs to offer $\{b(X)\}$ and unless $M = \{b(X)\}$, firm can profitably deviate to $\{b(X)\}$). These imply that $supp(\sigma_1) \subseteq \{\{b(X)\}, L_m(X)\}$. If this is satisfied with equality, then I need to have $\alpha_1 - \frac{1}{2} = k(L_m(X)) - k_{b(X)}$, a contradiction because $\alpha_1 - \frac{1}{2} > k(X)$. This implies that firm 1 either offers $\{b(X)\}$ or $L_m(X)$. The former does not constitute an equilibrium, because a firm can profitably deviate to $L_m(X)$. For the latter, observe that firm 2 can profitably deviate by offering $\{w(X)\}$ instead of $\{b(X)\}$. This shows that the support of σ_2 is either equal to $\{L_{m+1-min_{M \in supp(\sigma_1)}|M|}(X)\}$ or $\{\{b(X)\}, L_{m+1-min_{M \in supp(\sigma_1)}|M|}(X)\}$. Similarly, I can show that firm 2 does not offer only $L_{m+1-min_{M \in supp(\sigma_1)}|M|}(X)$ because one can choose an appropriate k^* s.t. deviation to $\{b(X)\}$ is profitable. So, I conclude that $supp(\sigma_2) = \{\{b(X)\}, L_{m+1-min_{M \in supp(\sigma_1)}|M|}(X)\}$.

Step 3: $supp(\sigma_1) = \{M_1, L_m(X)\}$

Given this, a menu $M \in supp(\sigma_1)$ either has a complexity level of $\sigma_2(L_{m+1-min_{M \in supp(\sigma_1)}|M|}(X))$ or 1. If the latter is true for M, then $M = L_m(X)$. Using equation 2, one can show that firm 1 only offers a single menu M_1 s.t. $\gamma(\sigma_2|M_1) < 1$, hence $\gamma(\sigma_2|M_1) = \sigma_2(L_{m+1-|M_1|}(X))$. Therefore, $supp(\sigma_1) \subseteq \{M_1, L_m(X)\}$ where $b(X) \in M_1$ and $|M_1| < m$. Similar to the previous case, I can show that there is no pure strategy equilibrium, so $supp(\sigma_1) = \{M_1, L_m(X)\}$.

Step 4: Deriving (σ_1, σ_2)

Let us choose M_1 s.t. $|M_1| < m + 1 - s^*$ where s^* is the smallest s with $k(L_s(X)) > k_{b(X)}$. By indifference condition, I need to have:

$$\alpha_1 - k(L_m(X)) = \frac{1}{2} + (\alpha_1 - \frac{1}{2})\sigma_2(L_{m+1-|M_1|}(X)) - k(M_1)$$

which implies that:

$$\sigma_2(L_{m+1-|M_1|}(X)) = 1 - \frac{k(L_m(X)) - k(M_1)}{\alpha_1 - \frac{1}{2}}$$

The probability that firm 1 offers M_1 can be derived similarly using indifference conditions. Note that $\sigma_2(L_{m+1-|M_1|}(X)) < 1$ because $k(L_m(X)) - k(M_1) = k(L_{m+1-|M_1|}(X)) - k_{b(X)})$ and $|M_1| \leq m + 1 - s^*$ implies $k(L_{m+1-|M_1|}(X)) \geq k(L_{s^*}(X)) > k_{b(X)}$.

□

This result implies several properties. In equilibrium, firm 1 gets a payoff of $\alpha_1 - k(L_m(X))$, while firm 2 gets $\alpha_2 - k(L_{m+1-|M_1|}(X))$. Thus, firm i guarantees a payoff which is positively related to its market power α_i and negatively to the cost of the complicators they use. Both firms offer a menu s.t. $\gamma(\sigma_{-i}|M_i) = 1$. So, each firm offers a menu s.t. fixing this menu, any market constituted with what the competitor offers is complex. The cost of these menus affect the equilibrium payoff of firms negatively.

Moreover, note that for each menu firm 2 offers, firm 1 offers a menu with a greater cardinality. This suggests that the market leader works harder in order to make the market complex, because this will assure a payoff correlated with its market power. The only easy market the consumers face is $(M_1, \{b(X)\})$, so the complexity level at equilibrium is given by the $1 - \sigma_1(M_1)\sigma_2(\{b(X)\})$, which is equal to:

$$1 - \frac{4(k(L_{m+1-|M_1|}) - k_{b(X)})(k(L_m) - k(M_1))}{\alpha_1 - \alpha_2} \tag{3}$$

The probability that the market is complex increases as the excess market power of firm 1 increases. The equilibrium complexity level inversely depends on the m, the attention limit of the consumer. An increase in the cost of $b(X)$ increases the complexity, while an increase in the cost of the least costly products, which are used to complicate the market, decreases the complexity level at equilibrium.

Observe that these properties are tightly linked to our assumptions, which are of two types: Assumptions about the comparison of the cost of complicators and the cost of $\succ$-best product, and also an assumption about the market power of firm 1. For the former, I assume that $k(L_s(X)) > k_{b(X)}$ for some s, i.e. complicating the market is not so cheap. Because k_x increases w.r.t $\succ$, this implies that the cost of the least costly m-products is above the cost of all single products. This intuitively suggests that if a firm is not so powerful, then the firm would rather produce the $\succ$-best product instead of complicating the market.

To grasp the intuition better, I can simplify by assuming that each product bears the same cost of k. Note that when this is the case, all of the assumptions of the previous result holds. Thus, I have the following corollary:

Corollary. *If $k_x = k$ for all $x \in X$, then the unique equilibrium of the game is as follows:*

$$\begin{aligned} \sigma_1(L_m(X)) &= 1 - 2(m - |M_1|)k \\ \sigma_2(\{b(X)\}) &= \frac{2(m - |M_1|)k}{\alpha_1 - \alpha_2} \end{aligned} \tag{4}$$

with the rest of the probabilities induced from above equations. The equilibrium complexity level can be derived easily from Equation 3:

$$1 - \frac{4(m - |M_1|)^2 k^2}{\alpha_1 - \alpha_2}$$

Educating the consumers in this setting amounts to increasing the attention limit m, which implies that the equilibrium complexity level decreases.

4 Cognitive Types and Enforceable Complexity

All special cases I investigated had a common property: A firm can unilaterally enforce a complex market. In the complex product setting, this is done by using the least costly complex product, while in the limited attention this can be done using by offering m least costly products. Furthermore, each consumer in the population has the same cognitive type. I can generalize the discussion as follows: First, I am going to relax the assumption the consumers are of the same cognitive type. Then, I will introduce a property which will in general show that the firm with the higher market power will complicate the market.

Let Θ be the finite set of cognitive types with a probability mass function μ.[13] Assume $|\Theta| = a$ for some finite bounded integer a and $\Theta = \{\theta_1, \ldots, \theta_a\}$. $\mu(\theta_s)$ is the share of consumers with type θ_s. For each type $\theta_s \in \Theta$, there is a partition $\{\mathbf{E}_s, \mathbf{H}_s\}$ of $\mathbb{X}$. I say θ_s is *more sophisticated* than θ_t if $\mathbf{H}_s \subseteq \mathbf{H}_t$. Θ is completely ordered if I can compare each type according to this relation. The share of consumers who find the market $\mathbf{M}$ hard is given by:

$$G_\mu(\mathbf{M}) = \sum_{\Theta} \mu(\theta) \mathbb{1}(\mathbf{M} \in \mathbf{H}_\theta)$$

Next, I provide examples which relate our previous discussion to the extended framework:

Example 1. (**Complex Products as Cognitive Type**) Let μ be a discrete probability mass function on $\{\theta_1, ..., \theta_n\}$ where $n = |X|$. Assume that θ_s corresponds to the case when there are s complex products for the consumer, and this includes all $s-1$ complex products of type θ_{s-1}. In this case, Θ is completely ordered. Our investigation in Section 3.2.1 can be seen as a special case of this by assuming $\mu(\theta_1) = 1$ and d is the complex product of this consumer. Alternatively, I can associate each type with a unique complex product x_s in X. In fact, for any $\theta_i, \theta_j \in \Theta$, $\mathbf{H}_i \setminus \mathbf{H}_j = \{\mathbf{M} : x_i \in \mathbf{M} \;\; \& \;\; x_j \notin \mathbf{M}\}$ and vice versa, which are both nonempty. This also generalizes the setting I discussed in Section 3.2.1, but in this case Θ is not completely ordered.

Example 2. (**Attention Parameter as Cognitive Type**) μ is a discrete probability mass function on $\{\theta_1, ..., \theta_n\}$ where θ_m corresponds to an attention limit of m. Any market which is hard for a type with attention parameter m is also hard for a type with attention parameter less than m. Therefore, Θ is completely ordered. A recent paper by Dardanoni et al.(20) uses aggregate choices observed in a population to infer the cognitive capacities of the consumers in the population.

Definition 1. *Let σ be any mixed strategy. Enforceable complexity holds if there is a menu M^* s.t.:*

$$\gamma_\sigma(M^*) = \sum_{M \in supp(\sigma)} \sigma(M) G_\mu((M^*, M)) \geq \gamma^*$$

[13] An alternative interpretation is as follows. An important limitation of our model is the binary nature of hard and easy markets. In general, different markets can induce different level of complexities, as exemplified in Piccione and Spiegler [23] and Chiovenau and Zhou [6]. still, I can interpret the extended framework introduced in this section as a non-binary structure on complexity. Instead of interpreting each type θ as a cognitive type of the consumer, let us interpret it as a dimension of complexity. For instance, θ can refer to the number of alternatives in a market, while θ' can refer to the level of quality difference between products offered in the market. Given this interpretation of types, $\mathbf{H}_\theta$ is the set of hard markets for the consumer of type θ, based solely on the cardinality dimension. Then, $\mu(\theta)$ represents the share or contribution of θ dimension on the overall complexity of the market for the consumer. The induced $G_\mu(\mathbf{M})$ on is the complexity level of the market $\mathbf{M}$.

In both special cases I investigated, this is satisfied with $\gamma^* = 1$. In the complex product setting, $\gamma_\sigma(\{d\}) = 1$, while in limited attention $\gamma_\sigma(L_m) = 1$. Note that γ_σ is increasing as one moves to supersets, therefore satisfying this property implies $\gamma_\sigma(X) \geq \gamma^*$. On the other hand, because offering X is more costly, a firm would prefer to offer a smaller menu with a sufficiently high γ_σ. I showed in Section 3.2 that if the cost of the menu $\mathbf{h}(\{b(X)\})$ is not large compared to the cost of best product $b(X)$, then there is no pure strategy equilibrium of the game. Indeed, the threshold is the excessive market power of the market leader w.r.t the symmetric case. I can generalize this using the enforceable complexity property:

Proposition 4.1. *Assume that enforceable complexity holds with $\gamma^* > \frac{2(k(M^*)+k^*)}{\alpha_1-\alpha_2}$. Then, firm 1 always complicates the market in equilibrium.*

Proof. I am going to show that $(supp(\sigma_1) \times supp(\sigma_2)) \cap \mathbf{H} \neq \emptyset$. Assume to the contrary $(supp(\sigma_1) \times supp(\sigma_2)) \cap \mathbf{H} = \emptyset$. When this is the case, it is simple to show that any menu offered in equilibrium is a singleton. Let $\{w_i\}$ be the $\succsim^*$-worst among these menus. Since these menus are singletons, there is a unique $\succ$-worst product which is $\succsim^*$-worst menu in the support of a firm's strategy. The payoff of firm i from this menu is equal to $\frac{1}{2}\beta_*(\{w_i\}, \sigma_{-i}) - k_{w_i}$. It is easy to show that both firms offer the same worst product, so $w_1 = w_2 = w$. Consider a deviation by firm 1 from $\{w\}$ to $M^* \cup \{b(X)\}$. The payoff of firm 1 from this deviation is greater than or equal to:

$$(\alpha_1 - \frac{1}{2})\gamma^* + \frac{1}{2} - k(M^*) - k_{b(X)}$$

by enforceable complexity. This makes the change in payoff greater than or equal to:

$$(\alpha_1 - \frac{1}{2})\gamma^* + \frac{1}{2}(1 - \sigma_2(\{w\})) - (k(M^*) + k_{b(X)} - k_w)$$

This expression reaches its lowest value when $w = w(X)$. By assumption $\gamma^* > \frac{2(k(M^*)+k^*)}{\alpha_1-\alpha_2}$, and I can rewrite the term on the right hand side as $\frac{k(M^*)+k^*}{\alpha_1-\frac{1}{2}}$, so the above expression is strictly greater than 0. $\square$

In the special cases I investigated, $\gamma^* = 1$ and because of the assumption that $k(X) < \alpha_1 - \frac{1}{2}$, the enforceable comparability condition is satisfied with the desired threshold.

5 Introducing Competition

I assumed throughout the paper that there are only 2 firms in the market. Now, in this section, I relax this assumption. In particular, I assume that there are $N \geq 2$ firms. I also keep open the possibility that different consumers are different in their cognition. I assume that Θ is completely ordered and associate θ_1 with the lowest type, and θ_a with the highest type, so $\mathbf{H}_{\theta_1} = \mathbb{X}^N$, and $\mathbf{E}_{\theta_a} = \mathbb{X}^N$. The consumer chooses the default when the market is complex, and as usual each firm has a default share of α_i s.t. $\sum_{i\in\{1,\ldots,N\}} \alpha_i = 1$. Wlog, I assume that $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_N$.

Let us denote the resulting market if a single firm adds a product x not offered by the same firm in $\mathbf{M}$ as $\mathbf{M} \cup \{x\}$, while $\mathbf{M} \setminus \{x\}$ denotes the resulting market when a single firm i drops x s.t. $x \in M_i$. I am going to use the following additional assumptions:

(A1) $\frac{G_\mu(\mathbf{M}\cup\{x\})+G_\mu(\mathbf{M}\setminus\{x\})}{2} > G_\mu(\mathbf{M})$ for any $x \in M_i \cap (X \setminus M_j)$ s.t. $i \neq j$.

(A2) $\mu(\theta_a) > \frac{N}{N-1}k^*$.

(A3) $n \geq N^2 + N - 1$.

The first assumption puts a condition on the distribution of types. Note that the condition can be equivalently written as $G_\mu(\mathbf{M} \cup \{x\}) - G_\mu(\mathbf{M}) > G_\mu(\mathbf{M}) - G_\mu(\mathbf{M} \setminus \{x\})$. Thus, the measure of types that would start finding the market complex with the addition of a single product to the market $\mathbf{M}$ is higher than the measure of types who would start finding the market easy when a single firm already offering this product stops offering it. The second condition says that the share of the most sophisticated consumers is above a certain threshold that depends on the number of firms in the market and the maximum cost difference between the products. Finally, the last condition says that the number of products is sufficiently high compared to the number of firms. Next, I provide a proposition which lays out the structure of the market provided that there exists a pure strategy equilibrium. The existence of it will be demonstrated later in this section in a simple duopoly.

Proposition 5.1. *Assume that there is a pure strategy equilibrium* $\mathbf{M} = (M_1, \ldots, M_h, \ldots, M_N)$. *Under assumptions* 1-3*, the equilibrium is asymmetric and* $M_i \supseteq M_{i+1}$ *for all* $i \in \{1, \ldots, N-1\}$ *s.t.* $M_i = \{b(X)\}$ *for all* $i > h$.

Proof.
Step 1: No symmetric pure strategy equilibrium.
First, I am going to show that there is no symmetric pure strategy equilibrium. Let $\mathbf{M} = (M, \ldots, M)$ be the profile of menus offered by firms in the market. Note that firm i receives a payoff of:

$$\frac{1}{N} + (\alpha_i - \frac{1}{N})G_\mu(\mathbf{M}) - k(M)$$

Observe that there is at least one firm j s.t. $\alpha_j \leq \frac{1}{N}$. Because I assumed wlog that firm N has the lowest market power, I have $\alpha_N \leq \frac{1}{N}$. Firm N offers a singleton product in equilibrium, since otherwise it can profitably deviate from M to $\{max(M, \succ)\}$. Therefore, if there is any symmetric pure strategy equilibrium, then all firms offer the same single product, say $\{x\}$. This implies the following payoff for firm i:

$$\frac{1}{N} + (\alpha_i - \frac{1}{N})G_\mu((\{x\}, \ldots, \{x\})) - k_x$$

Because increasing the variety increases the complexity, a firm j with $\alpha_j > \frac{1}{N}$ would find it profitable to deviate from $\{x\}$ to $\{y\}$ s.t. $x \succ y$. To see, note that if G_μ stays the same, then the deviation is profitable because the cost goes down. Otherwise, both the cost goes down and the revenue increases since $\alpha_j > \frac{1}{N}$. This shows that for this to be an equilibrium, I need to have $x = w(X)$. In this case, firm i with $\alpha_i \leq \frac{1}{N}$ can profitably deviate to the $\succ$-worst product in $X \setminus \{w(X)\}$, say z. Because N^{th} firm is the firm with the lowest market power, consider a deviation implemented by this firm. The change in payoff when firm N deviates as described is equal to:

$$(\frac{1}{N} - \alpha_N)G_\mu((\{w(X)\}, \ldots, \{w(X)\}\})) + (\alpha_N - 1)G_\mu((\{w(X)\}, \ldots, \{z\}\})) + (1 - \frac{1}{N}) - (k_z - k_{w(X)})$$

Because firm N increases variety, $G_\mu((\{w(X)\}, \ldots, \{z\}\})) \geq G_\mu((\{w(X)\}, \ldots, \{w(X)\}\}))$, which implies the change in payoff is greater than or equal to:

$$(1 - \frac{1}{N})[1 - G_\mu((\{w(X)\}, \ldots, \{w(X)\}\}))] - (k_z - k_{w(X)})$$

I assume that $\mu(\theta_a) > \frac{N}{N-1}k(X)$, which implies that $G_\mu((\{w(X)\},\ldots,\{w(X)\}\})) < 1-\frac{N}{N-1}k^*$ and therefore $1-G_\mu((\{w(X)\},\ldots,\{w(X)\}\})) > \frac{N}{N-1}k^*$. This implies that the above expression is strictly greater than:

$$\frac{N-1}{N}\frac{N}{N-1}k^* - (k_z - k_{w(X)})$$

which is strictly positive. Therefore, the deviation is profitable, which shows there is no symmetric pure strategy equilibrium.

Step 2: $b(X) \in \bigcap \mathbf{M}$ for any pure strategy equilibrium profile $\mathbf{M}$.
Let $\mathbf{M}$ be an equilibrium profile. Assume to the contrary $b(X) \notin \mathbf{M}$. Take any firm i s.t. $s_i(\mathbf{M}) = 0$. Observe that if this does not hold, then $s_i(\mathbf{M}) = \frac{1}{N}$ for all $i \in \{1,\ldots,N\}$. First, let us look at the former case. I claim that such a firm has a profitable deviation from M_i to $M_i \cup \{b(X)\}$. Let us denote the resulting market as $\mathbf{M}'$. Note that in this case $s_i(\mathbf{M}') = 1$ because $b(X) \notin \mathbf{M}$. The change in payoff is equal to:

$$\alpha_i[G_\mu(\mathbf{M}') - G_\mu(\mathbf{M})] + (1-G_\mu(\mathbf{M}')) - s_i(\mathbf{M})[1-G_\mu(\mathbf{M})] - (k_{b(X)} - k(M_i))$$

Observe that $k^* > k_{b(X)} - k(M_i)$, which implies with the fact that $s_i(\mathbf{M}) = 0$ the above expression is greater than or equal to:

$$\alpha_i[G_\mu(\mathbf{M}') - G_\mu(\mathbf{M})] + (1-G_\mu(\mathbf{M}')) - k^*$$

Since for the highest type no market is complex, $\max\{G_\mu(\mathbf{M}), G_\mu(\mathbf{M}')\} \le 1-\mu(\theta_a)$. Thus, this expression is greater than or equal to:

$$\alpha_i[G_\mu(\mathbf{M}') - G_\mu(\mathbf{M})] + \mu(\theta_a) - k^*$$

By assumption $\mu(\theta_a) > k^*$, and because $\mathbf{M} \subseteq \mathbf{M}'$, I have $G_\mu(\mathbf{M}') \ge G_\mu(\mathbf{M})$. This shows that the above term is strictly positive and therefore the deviation is profitable.

Now consider the latter case: $s_i(M) = \frac{1}{N}$ for all i. In particular, I consider firm N because it has the lowest market power, so $\alpha_N \le \frac{1}{N}$. Assume to the contrary $b(X) \notin \mathbf{M}$. If firm N deviates to $\{b(X)\}$, the change in its payoff will be equal to:

$$(1-\frac{1}{N}) + (\frac{1}{N} - \alpha_N)G_\mu(\mathbf{M}) - \alpha_{-i}G_\mu(\mathbf{M}') - \{k_{b(X)} - k(M_i)\}$$

Because $k_{b(X)} - k(M_i) \le k^*$, $\alpha_{-i} \ge 1-\frac{1}{N}$, and $G_\mu(\mathbf{M}) \ge G_\mu(\mathbf{M}')$, the above expression is greater than or equal to the following:

$$(\frac{N-1}{N})(1-G_\mu(\mathbf{M}')) - k^*$$

Since the highest value $G_\mu(\mathbf{M}')$ can take is $1-\mu(\theta_a)$ and $\mu(\theta_a) > \frac{N}{N-1}k^*$, this final expression is strictly positive. The conclusion follows.

Step 3: $M_i = \{b(X)\}$ for all $i > h$.
Assume to the contrary $|M_i| > 1$ for some $i > h$. If i deviates to $\{b(X)\}$ and I denote the resulting market $\mathbf{M}'$, the resulting change in payoff is equal to:

$$(\alpha_i - \frac{1}{N})[G_\mu(\mathbf{M}') - G_\mu(\mathbf{M})] + k(M_i \setminus \{b(X)\})$$

Because $\alpha_i \le \frac{1}{N}$ and $G_\mu(\mathbf{M}') \le G_\mu(\mathbf{M})$, the deviation is profitable, which concludes the proof.

Step 4: $M_i \supseteq M_{i+1}$ for all i.

Assume to the contrary i offers a product x not offered by a firm j s.t. $\alpha_j \geq \alpha_i$. Given the market is induced by an equilibrium profile, this is possible only if dropping x is not profitable for firm i while adding x is not profitable for j. For the former, the condition is read as follows:

$$G_\mu(\mathbf{M}) - G_\mu(\mathbf{M} \setminus \{x\}) \geq \frac{k_x}{\alpha_i - \frac{1}{N}}$$

while the latter implies:

$$G_\mu(\mathbf{M} \cup \{x\}) - G_\mu(\mathbf{M}) \leq \frac{k_x}{\alpha_j - \frac{1}{N}}$$

which implies $G_\mu(\mathbf{M} \cup \{x\}) - G_\mu(\mathbf{M}) \leq G_\mu(\mathbf{M}) - G_\mu(\mathbf{M} \cup \{x\})$ because $\frac{k_x}{\alpha_i - \frac{1}{N}} \geq \frac{k_x}{\alpha_j - \frac{1}{N}}$. This is a contradiction to **A1**.

□

The product differentiation in the market increases as different firms offer different products. On the other hand, it decreases as the total number of products in the market decreases. So, a natural measure for product differentiation in the market is the number of different products offered in $\mathbf{M}$ divided by the total number of products offered in $\mathbf{M}$, which I denote by $\partial(\mathbf{M})$. The lower bound for product differentiation in the market follows as a corollary to the previous proposition.

Corollary. $\partial(\mathbf{M}) \geq \frac{1}{h} + \frac{|M_1|}{N-h}$.

Proof. Note that $|\bigcup \mathbf{M}| = |M_1|$. On the other hand, total number of products offered is equal to $\sum_{i \in \{1,\dots,N\}} |M_i| = \sum_{i \leq h} |M_i| + (N-h)$. Therefore, $\partial(\mathbf{M}) = \frac{|M_1|}{\sum_{i \leq h} |M_i| + (N-h)} \geq \frac{|M_1|}{h|M_1| + (N-h)} = \frac{1}{h} + \frac{|M_1|}{N-h}$. □

The product differentiation in the market is bounded below by $\frac{1}{h} + \frac{|M_1|}{N-h}$. Obviously, if $|M_1|$ increases or N decreases, this lower bound increases according to this measure. The effect of h requires a more careful analysis. Assume for a moment all parameters are allowed to be continuous, so I can use the classical first-order and second-order conditions. Using the second-order condition, I can show that product differentiation is convex in h. Without going into calculations (which is standard), one can show that the optimal h, denoted as h^*, is equal to $h^* = \frac{N}{\sqrt{|M_1|} + 1}$. Hence, fixing N and $|M_1|$, the minimal lower bound is achieved when the number of firms with a market power of at least $\frac{1}{N}$ is equal to h^*.

If the costs of adding a particular product x is sufficiently low for a firm with a market power above $\frac{1}{N}$, and if x further complicates the market by making it complex for additional types, then all firms with a power above $\frac{1}{N}$ would offer this product. Next, I state this as a corollary of the previous proposition.

Corollary. *Assume that for any* $\mathbf{M}$ *there is* $x \notin \bigcap_{i \leq h} M_i$ *s.t.* $G_\mu(\mathbf{M} \cup \{x\}) > G_\mu(\mathbf{M})$. *If* $k_{b(X)}$ *is sufficiently low, then* $M_i = X$ *for any* $i \leq h$.

Proof. I showed in the Step 4 of the previous proposition that adding x is not profitable if and only if:

$$G_\mu(\mathbf{M} \cup \{x\}) - G_\mu(\mathbf{M}) \leq \frac{k_x}{\alpha_j - \frac{1}{N}}$$

It is clear that for a sufficiently low $k_{b(X)}$, because I have $G_\mu(\mathbf{M} \cup \{x\}) > G_\mu(\mathbf{M})$, this condition would be violated for any such x and firm $i \leq h$. $\square$

Thus, provided that there exists a pure strategy equilibrium and the production costs are sufficiently low, then the consumers face the following market in equilibrium:

$$(X, \dots, X, \{b(X)\}, \dots, \{b(X)\})$$

Note that this is an extreme case: The costs could be a little bit higher and the firms offering the product would be the firms with a sufficiently high market power.

In the following, I demonstrate a case in which the unique equilibrium played in a duopoly is of this structure. In other words, the more powerful firm offers X, while the less powerful one offers $\{b(X)\}$ in equilibrium. So, I show that under this simplified structure a pure strategy equilibrium exists.

Assume that $N = 2$ and μ is a discrete probability mass function on $\{\theta_1, ..., \theta_n\}$ where θ_m corresponds to an attention limit of m, and the set of hard problems are defined as before.[14] Let μ be the discrete uniform distribution, so $\mu(m) = \frac{1}{n}$ for all $m \in \{1, ..., n\}$.[15]Note that in this setting $G_\mu(\mathbf{M} \cup \{x\}) > G_\mu(\mathbf{M})$ for any x, because offering an additional product increases the total number of products in the market and hence it makes the market complex for an additional type. Provided that there is a pure strategy equilibrium and production is costless (so $k^* = 0$), I know that the only candidate for the equilibrium in pure strategies is $(X, \{b(X)\})$. The following proposition shows that this is indeed the unique equilibrium in the context of this example.

Proposition 5.2. *Assume that $N = 2$ and production is costless. In the setting of limited attention with a uniform distribution over cognitive types, the unique equilibrium of the game is $(X, \{b(X)\})$.*

Proof. I know that $(X, \{b(X)\})$ is the unique candidate for a pure strategy equilibrium. Therefore, I need to show that no firm has a profitable deviation to a mixed strategy. Let σ_i denote a generic mixed strategy that firm i can deviate to. Because $b(X)$ is offered by the competitor, the payoff of a menu $M \in \sigma_i$ s.t. $b(X) \notin M$ is equal to $\alpha_i \frac{|M|}{N}$ and $G_\mu(\mathbf{M}') = \frac{|M|}{N}$. This cannot be a profitable deviation for firm i, because its payoff from the strategy profile $(X, \{b(X)\})$ is equal to α_i. So, any $M \in supp(\sigma_i)$ should contain $b(X)$. If $i = 2$, because 1 offers X, the payoff of 2 is equal to α_2 in any case. Therefore, there is no profitable deviation for firm 2. Assume $i = 1$. The payoff of 1 is given by:

$$\alpha_1 + (\frac{1}{2} - \alpha_1)(\frac{|M| + 1}{N})$$

which is always smaller than or equal to α_1 because $\alpha_1 \geq \frac{1}{2}$. Therefore, firm 1 also has no profitable deviation, which concludes the proof.

$\square$

[14]Note that X is the only hard problem if $m = n - 1$ and there is no hard problem if $m = n$.

[15]A reason to choose uniform distribution beyond its simplicity is that it is the most uninformative distribution for the firm. Gerasimou and Papi [13] examines a similar setting under the heading of *uniformly distributed overload* using a specific choice procedure called *choice with deferral* for which you can refer to Gerasimou [12].

Thus, if production is costless, then the market leader will prefer to complicate the market for every type. On the other hand, the less powerful firm will only offer $\{b(X)\}$. Intuitively, I can ask why the second firm offers $b(X)$. For instance, both firms would get the same payoff if $(X,\{x\})$ is played in equilibrium where $x \neq b(X)$. still, observe that firm 1 can profitably deviate to some $\{y\}$ s.t. $y \succ b(X)$ and capture the whole market. Therefore, firm 2 offers $b(X)$ in order to hold a share of α_2, because otherwise it will lose all of this and gets no share. At the same time, the market leader firm 1 tries to get a share of α_1, which is at least $\frac{1}{2}$, by complicating the market. If the market is not complex for every type, then this means that firm 1 foregoes a share of $\alpha_1 - \frac{1}{2}$ for every type who does not find the market complex. Since production is costless[16], firm 1 can try to get this foregone share by complicating the menu until it offers X. In the end, both firms end up getting their default probabilities as payoff in this equilibrium, so the payoff profile is (α_1, α_2).

6 Further Discussion

6.1 Relation to the Consideration Function

Eliaz and Spiegler [8] models the use of attention grabbers strategically. They define a beating relation on the set of menus: A menu M beats another menu M' if M contains an attention grabber that attracts the consumer to menu M, and furthermore M is preferred to M'. Thus, there are two primitives: a function governing the consideration of the consumer, f, and preferences of the consumer over menus, $\triangleright$. So, formally: M beats M' if there exists $x \in M$ s.t. $f(x,M') = 1$ and $M \triangleright M'$. Let us denote this beating relation as b.

To simplify comparison of the consideration function with our setting, I am going to express our assumptions using a functional notation. Let g denote the complexity function s.t. $g : \mathbb{X} \times \mathbb{X} \to \{0,1\}$ s.t. $g(M,M') = 1$ if and only if $(M,M') \in \mathbf{E}$. First of all, note that the domain of two functions differ from each other, function g is more general than function f. From firms point of view, a menu M "beats" another menu M' in the sense of Eliaz and Spiegler if $g(M,M') = 1$ and $M \succsim^* M'$ in our setting. When the market is hard, there is no beating relation, all consumers purchase from their default firm. Let us denote the beating relation in our setting as b^*.

First of all, note that I define the preferences over menus using max-max preferences because the consumer is able to choose the $\succ$-best element when the market is easy. This preferences is also used in Eliaz and Spiegler [8] when they discuss a special case of their model. It is easy to note that max-max preferences is a special case of the preference relation they define on the menus, $\triangleright$. Furthermore, $g(M,M') = 1$ implies by definition of easy markets $g(\{x\},M') = 1$ for all $x \in M$. Such a consumer is interpreted as a rational consumer in Eliaz and Spiegler. However, note that in our model this does not guarantee that $g(M,M') = 1$. For instance, the consumer can pay attention to single product s in any case, but if M contains too many of such products then this market can be complex for her. On the other hand, the complex product setting satisfies this property, because $\mathbf{E}$ is further closed under union. So, I can conclude the following:

Observation 1. *If* $\mathbf{E}$ *is closed under union, then* b^* *is a special case of* b.

[16] Note that a weaker assumption would be sufficient.

6.2 Cooperative Complexity

Our main goal in this section is to show that competition may adversely affect the welfare of the consumers. Specifically, I am going to show that under plausible assumptions on the market power of the firms, there is a pure strategy equilibrium in which no firm offers $b(X)$. I maintain the assumptions I imposed in the asymmetric case with N firms, but I assume for simplicity there is no cognitive differentiation.

To demonstrate this, I am going to use a specific type of $\mathbf{H}$. The equilibrium I propose for this case is an example of behavior which can be called a "non-cooperative collusion". Let us first define the structure I talk about, which is very similar to limited attention, but only differs because it counts the number of *different* products.

$$\tilde{\mathbf{H}}_m = \{\mathbf{M} \in \mathbb{X} \times \mathbb{X} : |\bigcup \mathbf{M}| > m\}$$

So, a market is complex for the consumer if and only if it contains at least $m+1$ different products. Observe that this structure satisfies the 3 properties I imposed on the set of hard markets: It is straightforward to show the first two properties, and the third property is trivially satisfied when $m \geq 2$, as I assume below in the proposition. In the next proposition, I denote the k^{th} worst product in X w.r.t $\succsim$ as $w^k(X)$.

Proposition 6.1. *Assume that $h \geq m \geq 2$, $k(X) < \alpha_1 - \frac{1}{2}$ and $\alpha_N \geq \frac{1}{2} - (k_{w^{m+1}(X)} - k_{w(X)})$. The pure strategy profile $(\{w^{m+1}(X)\}, \ldots, \{w^2(X)\}, \{w(X)\})$ is an equilibrium provided that the set of complex markets for the consumers is given by $\tilde{\mathbf{H}}_m$.*

Proof. Let $\mathbf{M} = (\{w^{m+1}(X)\}, \ldots, \{w^2(X)\}, \{w(X)\})$. Clearly, $\mathbf{M} \in \tilde{\mathbf{H}}_m$. I need to show that there is no profitable deviation from this strategy profile. First, consider a deviation in pure strategies. There are two types of such deviations: A firm i can make the market easy or keep it complex. For the latter, it is easy to see that there is no profitable deviation. Observe that in such a deviation the motivation for a deviator is to reduce costs, because the revenue it gets is fixed and equal to its market share. $i > m$ already offers the cheapest product. For $i \leq m$ the cheapest product that they can deviate is $\succsim$-next best product to $w^{m+1}(X)$, because otherwise the market becomes easy. Since k is increasing w.r.t $\succsim$, such a deviation is not profitable. Now consider a deviation so that the market becomes easy. From such a deviation, the deviator gets a positive revenue only if it sells the $\succsim$-best product in the market. Therefore, any deviator $i \neq 1$ should deviate to some x s.t. $x \succsim w^{m+1}(X)$. But note that if it deviates to $x \neq w^{m+1}(X)$, then the product differentiation in the market increases and hence the market is not easy. Therefore, the only possible profitable deviation for firm $i \neq 1$ is to $\{w^{m+1}(X)\}$. On the other hand, firm 1 can only profitably deviate to $\{w^m(X)\}$, which follows from a similar argument. Both deviations are ruled out by the assumptions, and hence there is no profitable deviation in pure strategies. Now consider a deviation in mixed strategies. Say firm i deviates to σ_i and let $M \in supp(\sigma_i)$. If $|M \cup (\bigcup \mathbf{M}_{-i})| \leq m$ and the $\succsim$-best product offered in this new market is not in M, then the deviator gets a negative payoff from the mixed strategy profile, which cannot constitute a profitable deviation. If the $\succsim$-best product offered in the market is in M, then the least costly way to implement this deviation is by offering $\{w^{m+1}(X)\}$ for $i \neq 1$, while it is by offering $\{w^m(X)\}$ for $i = 1$. The latter is ruled out because $k(X) < \alpha_1 - \frac{1}{2}$. The former is ruled out by the assumption that $\alpha_N \geq \frac{1}{2} - (k_{w^{m+1}(X)} - k_{w(X)})$. So, if there is a deviation to a mixed strategy, then for any $M \in supp(\sigma_i)$ I have $|M \cup (\bigcup \mathbf{M}_{-i})| > m$. The payoff i receives from M is equal to $\alpha_i - k(M)$, which is less than the payoff i receives under the pure strategy profile $\mathbf{M}$ because i already offers the cheapest menu that makes the market complex. This concludes the proof. $\square$

This proposition shows that it is possible to have a pure strategy equilibrium in which no firm offers $b(X)$. In fact, even though the consumers are offered different products, the welfare of a consumer depends on her default firm. So, offering different products does not necessarily imply an increase in the consumer welfare, because when the market is hard she cannot consider all and choose the $\succsim$-best among them. Thus, if firms play this equilibrium, increasing the competition in the market may adversely affect the consumer. Note that firms play non-cooperatively, but the market outcome in this equilibrium resembles a cooperative outcome: Firms with small market power (less than $\frac{1}{N}$) offer the cheapest single product, thinking that there are enough number of firms with high power who will benefit from complicating the market. On the other hand, the firms with high power behave as if they made an agreement to offer different products (in fact, maximal differentiation), and each of them offers the least costly different product relative to its market power. So, firms with high market power can be thought as a cartel who made an agreement as such I described, and each member outside the cartel best-responds to this by offering the cheapest product, since they know they cannot make the market easy.

Finally, note that what differentiates this structure on the set of hard markets is the following: Each firm can unilaterally make the market simple. This implies that firms need to cooperate if they want to complicate the market, because unilaterally making the market complex can be negated in this setting.

6.3 Preference-Induced Complexity

Throughout the paper, the set of hard markets is induced from the properties of the markets and cognitive limitations of the consumer. For example, in the complex product setting, some products in X has the property of being complex for the consumer, while in limited attention, the complexity of the market depends on the number of alternatives offered in the market and consumer's attention capacity. On the other hand, **indecisiveness** can be another reason for the complexity of the market. This type of complexity is different from what I discussed extensively in the paper: First, it does not necessarily satisfy the property of being closed under supersets as $\mathbf{H}$ does throughout the paper. Second, it is not necessarily due to the *intrinsic complexity* of the market. In particular, consider the limited attention case. Even without knowing the attention parameter of the consumer, if you know that paying attention to more elements is harder, then you can say that a market consisting of more alternatives is weakly more complex than another one containing less. When complexity is preference-induced, this is not necessarily true: A market with a very few elements can be very complex for the consumer.

Remark. *This does not exclude the possibility that preference-induced complexity arises from intrinsic-complexity of the market. Letting $X = \{0,1\}^k$ where k is the number of possible attributes a product can have, a consumer can find a market hard if there are too many attributes she needs to consider to select the best product among them. However, I can also claim that these attributes and the valuation of each attribute in turn depends on the preferences of the consumer. In particular, the selection of attributes considered can be based on the preferences over attributes.*[17]

Below, I provide three examples of possible constructions of $\mathbf{H}$. For simplicity, I assume that $N = 2$. In all examples, I induce a binary relation comparing markets, $\succsim^*$, and the associated

[17] This is one of the core ideas of rational inattention literature, although there the consumer also considers the cognitive cost of internalizing the relevant information (for a recent review, see Mackowweak et al. [19]).

indecisiveness relation, $||^*$. Recall that I previously defined $\succsim^*$ using max-max preferences, and therefore $\succsim^*$ was complete. However, in the examples below, $\succsim^*$ induced is not necessarily complete.

Example 3. $M \succsim_1^* M'$ iff $w(M) \succsim b(M')$.

Example 4. $M \succsim_2^* M'$ iff $M \supseteq M'$.

Example 5. $M \succsim_3^* M'$ iff $x \succ y$ for all $x \in M$ and $y \in M'$.

The set of hard markets given a firm offers M is then defined as $\mathbf{H}_i(M) = \{M' : M||_i^* M'\}$ where $i \in \{1,2\}$. Note that if $M \succsim_2^* M'$, then $M \succsim_3^* M'$, and this further implies $M \succsim_1^* M'$. Thus, the following inclusion holds: $\mathbf{H}_1 \subseteq \mathbf{H}_3 \subseteq \mathbf{H}_2$.

Let us focus on the first type of hard markets, $\mathbf{H}_1$. I showed in the main body that under a simple condition which relates the market power of firms (excess power in the case of the market leader) and the total cost of the set of all products, there is no pure strategy equilibrium of the game. I demonstrate below that the case is completely different when the complexity is preference-induced as in $\mathbf{H}_1$ for the consumer under the same assumption.

Proposition 6.2. *Assume that the set of hard markets for the consumer is $\mathbf{H}_1$. If $k_{b(X)} < \frac{1}{2}$, then the pure strategy profile $(\{b(X)\},\{b(X)\})$ is an equilibrium of the game.*

Proof.

- **Step 1:** If there is a pure strategy equilibrium of the game, then it is $(\{b(X)\},\{b(X)\})$.

  Consider any pure strategy profile (M_1,M_2) and denote this market as $\mathbf{M}$. If $w(M_1) \succ b(M_2)$, then $\mathbf{M} \in \mathbf{E}$ and the payoff profile is given by $(1-k(M_1),-k(M_2))$, from which firm 2 can profitably deviate to $\{b(M_1)\}$ because by assumption $k_{b(M_1)} < \frac{1}{2}$. I can apply the same argument if $w(M_2) \succ b(M_1)$. Therefore, I need to have either $\mathbf{M} = (\{b(X)\},\{b(X)\})$ or $\mathbf{M} \in \mathbf{H}$ in equilibrium.

  First, assume that the latter holds, so $\mathbf{M} \in \mathbf{H}$. It is straightforward to show that no firm offers more than two products. Because $\mathbf{M} \in \mathbf{H}$, I need to have $b(M_1),b(M_2) \succsim w(M_1),w(M_2)$. If $b(X) \neq b(M_1),b(M_2)$, then firm i can profitably deviate to $\{b(X)\}$ because $(1-\alpha_i)-(k_{b(X)}-k(M_i)) = \alpha_{-i}-(k_{b(X)}-k(M_i))$ and this is strictly greater than 0 by the assumption. Thus, at least one firm offers $b(X)$. Observe that if only one firm offers $b(X)$, say i, then $|M_i| = 2$ because otherwise $\mathbf{M} \in \mathbf{E}$. If both offer $b(X)$, then $b(M_1) = b(M_2) = b(X)$ and $|M_1| = |M_2| = 2$, again because otherwise $\mathbf{M} \in \mathbf{E}$. So, if $\mathbf{M} \in \mathbf{H}$ and $\mathbf{M}$ is an equilibrium, then there are 2 possibilities:

  1. $M_i = \{b(X),w_i\}$ for some $w_i \neq b(X)$ and $i \in \{1,2\}$.
  2. $M_i = \{b(X),w_i\}$ and $M_{-i} = \{z_{-i}\}$ s.t. $b(X) \succ z_{-i} \succ w_i$.

  In both cases, the payoff profile is equal to $(\alpha_1-k(M_1),\alpha_2-k(M_2))$. Consider a deviation to $\{b(X)\}$ by firm 2. If the first case holds, then the market becomes easy and the change in payoff of firm 2 is equal to $(\frac{1}{2}-\alpha_2)+k_{w_2}$, which is strictly positive because $\alpha_2 \leq \frac{1}{2}$. If the second case holds and $i = 2$, the deviation is profitable because firm 2 captures the whole market by making the market easy. Finally, if the latter case holds and $i = 1$, then the change in payoff is given by $(\frac{1}{2}-\alpha_2)-(k_{b(X)}-k_z)$. This is also profitable by assumption. This concludes the proof because the only remaining possibility is to have $(\{b(X)\},\{b(X)\})$.

- **Step 2:** There is no deviation to a mixed strategy equilibrium.

  The payoff profile under $(\{b(X)\},\{b(X)\})$ is equal to $(\frac{1}{2}-k_{b(X)},\frac{1}{2}-k_{b(X)})$. Assume to the contrary there is a profitable deviation by firm i to a mixed strategy σ_i, say $i=1$. For any $S \in supp(\sigma_1)$, $(S,\{b(X)\}) \in \mathbf{E}$, which implies that firm 1 gets a negative payoff by offering S unless $b(X) \in S$. This implies that $b(X) \in S$ for all $S \in supp(\sigma_1)$. This only increases the cost incurred by firm 1, so the deviation is not profitable. The argument is the same when $i=2$, which concludes the proof.

$\square$

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