### Answers to Exercise 1:

For two consecutive experiments, the total number of possible outcomes is calculated using the addition principle:

$$\text{Total outcomes} = \sum_{i=1}^{m} n_i$$

where $m$ is the number of outcomes for the first experiment and $n_i$ is the number of outcomes for the second experiment corresponding to each outcome of the first.

### Answers to Exercise 2:

To find the possible initials consisting of 2 letters:

$$\text{Total initials} = 26 \times 26.$$

To guarantee at least two students share the same initials, at least $26^2 + 1$ students are needed in a class.

### Answers to Exercise 3:

For the 10 volumes of an encyclopedia arranged randomly:

- Total arrangements without restrictions:
$$10!.$$
- If volumes 7, 6, and 5 must be placed next to each other in that specific order, treat them as a single unit:
$$8!.$$

### Answers to Exercise 4:

1. Total 3-letter words with no restrictions:
$$26^3.$$
2. Total with at least 2 consonants:
$$\text{Total} = (3 \text{ consonants}) \text{ or } (2 \text{ consonants, } 1 \text{ vowel}).$$
$$21^3 + 3 \cdot 21^2 \cdot 5$$
3. Exactly 2 consonants and 1 vowel:
$$21^2 \cdot 5 \cdot 3,$$
select 2 consonants, 1 vowel, and arrange them (ccv or cvc or vcc).
4. At most 1 vowel:
$$\text{Total} = \text{no vowel or } (2 \text{ consonants, } 1 \text{ vowel}) = (3 \text{ consonants}) \text{ or } (2 \text{ consonants, } 1 \text{ vowel}).$$
$$21^3 + 3 \cdot 21^2 \cdot 5$$
Same as question 2.

### Answers to Exercise 5:

1. Total ways to visit 7 family members on the same day:
$$7!.$$
2. If 4 are visited on Eid and 3 the next day:
$$(7)_4 \cdot 3! = 7!.$$

### Answers to Exercise 6:

1. Total football teams from 22 people:
$$\binom{22}{11}.$$
2. Teams formed with 3 goalkeepers including at least 1:
$$\text{Choose 1 goalkeeper and 10 from 19 others} \Rightarrow 3 \cdot \binom{19}{10}.$$

### Answers to Exercise 7:

A company with 2 managers, 3 administrators, and 11 workers:

1. Total ways without restrictions:
$$\binom{16}{4}.$$
2. 1 manager, 2 administrators, and 1 worker:
$$\binom{2}{1} \cdot \binom{3}{2} \cdot \binom{11}{1}.$$
3. Workers and administrators cannot be together:
   - Solution 1: Choose them from ($admins \cup managers$) or ($workers \cup managers$):
   $$\binom{3+2}{4} + \binom{11+2}{4} = \binom{5}{4} + \binom{13}{4}$$
   - Solution 2: Consider all the cases:
   $$\left.\begin{array}{rc} 4W: & \binom{11}{4} \\ 3W,1M: & \binom{11}{3}\binom{2}{1} \\ 2W,2M: & \binom{11}{2}\binom{2}{2} \\ 3A,1M: & \binom{3}{3}\binom{2}{1} \\ 2A,2M: & \binom{3}{2}\binom{2}{2} \end{array}\right\} \overset{A.P.}{\Longrightarrow} Total = \binom{11}{4} + \binom{11}{3}\binom{2}{1} + \binom{11}{2}\binom{2}{2} + \binom{3}{3}\binom{2}{1} + \binom{3}{2}\binom{2}{2}$$
   - Solution 3: We consider all the total number - the number where admins and workers meet (3W,1A),(2W,2A),(2W,1A,1M),(1W,3A),(1W,2A,M),(1W,1A,2M):
   $$\binom{16}{4} - \left(\binom{2}{2}\binom{3}{1}\binom{11}{1} + \binom{2}{1}\binom{3}{2}\binom{11}{1} + \binom{3}{3}\binom{11}{1} + \binom{2}{1}\binom{3}{1}\binom{11}{2} + \binom{3}{2}\binom{11}{2} + \binom{3}{1}\binom{11}{3}\right).$$

### Answers to Exercise 8:

To divide a class of 100 students into groups of sizes 15, 18, 20, 21, and 26, we can use the binomial coefficient approach:

$$\text{Total choices} = \binom{100}{15} \times \binom{85}{18} \times \binom{67}{20} \times \binom{47}{21}$$

The last group will consist of the remaining 26 students, which is represented as $\binom{26}{26} = 1$.

Thus, the total number of choices for this division is given by:

$$\text{Total choices} = \binom{100}{15} \times \binom{85}{18} \times \binom{67}{20} \times \binom{47}{21} = \frac{100!}{15! \cdot 18! \cdot 20! \cdot 21! \cdot 26!}$$

**Answers to Exercise 9:**

In a committee of 10 people, a chairperson, a secretary and a treasurer should be selected

1. Knowing that accumulation is not allowed, in how many ways can these offices be allocated if

   i. There are no restrictions?
      Answer: $(10)_3 = \binom{10}{3}3! = 720$

   ii. $A$ and $B$ cannot be together?

Answer 1: $\bar{A}\bar{B} \vee \bar{A}B \vee A\bar{B}$

$$\binom{8}{3}3! + \binom{1}{1}\binom{8}{2}3! + \binom{1}{1}\binom{8}{2}3!$$

Answer 2: $\bar{A} \vee A\bar{B}$

$$\binom{9}{3}3! + \binom{1}{1}\binom{8}{2}3!$$

Answer 3: $Total - AB$

$$\binom{10}{3}3! - \binom{2}{2}\binom{8}{1}3!$$

   iii. $C$ and $D$ are together or not at all?
      Answer: $CD + \bar{C}\bar{D}$:

$$\binom{2}{2}\binom{8}{1}3! + \binom{8}{3}3! = 384$$

   iv. $F$ must have a task?
      Answer:

$$\binom{2}{2}\binom{8}{1}3! + \binom{8}{3}3! = 216$$

   v. $F$ only accepts the task of chairperson?
      Answer: $F$ president or not all

$$\binom{9}{2}2! + \binom{9}{3}3!$$

2. Knowing that accumulation is allowed, in how many ways can these offices be allocated if there are no restrictions?
   Answer:

$$10^3$$

**Answers to Exercise 10:**

- If the student has to answer 5 out of 8 questions:

$$\binom{8}{5} = 56$$

- If the student has to answer at least 3 out of the first 5 questions: So he exactly 3 or 4 or 5 out of the first 5 questions and 2 from the remaining 3:

$$\binom{5}{3} \times \binom{3}{2} + \binom{5}{4} \times \binom{3}{1} + \binom{5}{5} = 46$$

**Answers to Exercise 11:**

- For the word "ALGER":

$$5! = 120$$

- For the word "ANNABA":

$$\frac{6!}{3! \times 2!} = 60$$

- For the word "TIZIOUZOU":

$$\frac{9!}{(2!)^4}$$

- For the word "MISSISSIPPI":

$$\frac{11!}{4! \times 4! \times (2!)} = 34,650$$

**Answers to Exercise 12:**

In how many ways can 8 people be seated?

- In a row,

1. **If there are no restrictions:** The total arrangements of 8 people is given by:

$$8!$$

2. **Two persons A and B stay together:** Treat A and B as a single unit. This creates 7 units (AB and 6 others):

$$7! \times 2!.$$

3. **There are 4 adults and 4 children, each adult has only children as neighbors and vice versa:** Arrange adults and children alternatively:

$$\text{Arrangement: } ACACACAC \text{ or } CACACACA.$$

Total arrangements:

$$4! \times 4! \times 2.$$

4. **The number of adults is 6, they must stay together:** Treat the 6 adults as a single unit. This creates 3 units (ADULTS and 2 children):

$$3! \times 6!.$$

5. **There are 4 children, each accompanied by his mother who stays next to him:** Treat each mother-child pair as a single unit. This creates 4 units (2! permutations of each pair):

$$4! \cdot (2!)^4.$$

- Around a table

1. **If there are no restrictions:**

$$7!$$

3. **There are 4 adults and 4 children, each adult has only children as neighbors and vice versa:**

$$3! \times 4!$$

**Answers to Exercise 13:**

Group the athletes by nationality:

- Algerian block (4 athletes)
- Libyan block (3 athletes)
- Tunisian block (2 athletes)

$$3! \times 4! \times 3! \times 2!$$

## Answers to Exercise 14:

## Answers to Exercise 15:

## Answers to Exercise 16:

## Answers to Exercise 17:

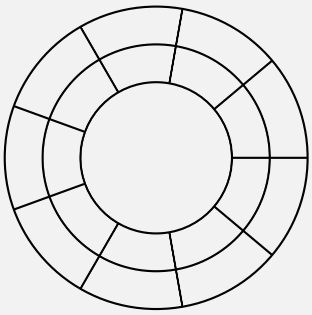
## Answers to Exercise 18:

## Answers to Exercise 19:

Consider $n$ fathers, each accompanied by their child, attending a school meeting. The meeting room has exactly $2n$ seats. Solve the following problems for the seating arrangements:

- The seats are arranged in two parallel linear rows, each containing $n$ seats. Answer the following:
  1. Without any restrictions.
  2. If all fathers must sit in the same row.
  3. If each father has only children as neighbors and vice versa.
  4. If no father sits in the same row as his child, but they must sit in the same position in their respective rows.


- The seats are arranged in two circles. Answer 1.), 2.) and 4.)



## Answers to Exercise 20:

## Answers to Exercise 33:

1. The recurrence relation is given by:

$$D_n^p = nD_n^{p-1} + D_{n-1}^{p-1}, \quad (p \geq n+1).$$

To justify this, consider the distribution of $p$ balls into $n$ indistinguishable boxes such that no box is empty. Let us take one ball from the set of $p$ balls. This ball can either:

- **Case 1: Go into an Existing Box**
  - First, distribute the remaining $p-1$ balls into $n$ boxes such that no box is empty. The number of

ways to do this is $D_n^{p-1}$.
- Next, place the chosen ball into any of the $n$ existing boxes. Since there are $n$ boxes, there are $n$ ways to choose the box.
- By the multiplication principle, the total number of ways for this case is:

$$nD_n^{p-1}.$$

- **Case 2: Create a New Box**
  - Let the chosen ball create a new box. Now, distribute the remaining $p-1$ balls into $n-1$ boxes such that no box is empty. The number of ways to do this is $D_{n-1}^{p-1}$.
  - Thus, the total number of ways for this case is:

$$D_{n-1}^{p-1}.$$

Adding the contributions from both cases, the total number of distributions is:

$$D_n^p = nD_n^{p-1} + D_{n-1}^{p-1}.$$

2. (a) $D_1^p = 1$: All $p$ balls must go into the single box.
   $D_n^n = 1$: Each of the $n$ boxes receives exactly one ball.

   (b) $D_2^p$: To distribute $p$ balls into 2 indistinguishable boxes such that no box is empty:
   If the boxes were distinguishable, there are $2^p$ ways since each ball has two choices.
   Subtract the cases where one box is empty (2 cases): $2^p - 2$.
   Since the boxes are indistinguishable, divide by 2 to remove permutations of boxes:

$$D_2^p = \frac{2^p - 2}{2} = 2^{p-1} - 1.$$

   Alternatively, we can compute $D_2^p$ by considering the number of ways to divide $p$ balls into two indistinguishable boxes such that no box is empty:

   - Choose $k$ balls to place in box 1, where $k$ satisfies $1 \leq k \leq p-1$ (ensuring neither box is empty).
   - For each $k$, there are $\binom{p}{k}$ ways to choose which balls go into box 1.
   - Sum over all valid $k$:

$$\sum_{k=1}^{p-1} \binom{p}{k} = 2^p - 2.$$

   - Since the boxes are indistinguishable, divide by 2 to remove duplicate configurations:

$$D_2^p = \frac{2^p - 2}{2} = 2^{p-1} - 1.$$

   $D_3^p$ :

   (c) $D_{p-1}^p$:
   $D_{p-2}^p$:
   $D_{p-3}^p$: