董云达的代表作系列一

假设 f 在欧氏空间中是二次连续可微的。考虑无约束极小化问题

$$\min \quad f(x).$$

相应的拟Newton法为

$$x^{k+1} = x^k - \alpha_k W_k \nabla f(x^k),$$

其中 W_k 由某个对称秩二校正公式自动生成，而 α_k 作为步长由下面 Wolfe 条件确定

$$\begin{aligned} &f(x+\alpha d) \le f(x) + c_1 \alpha \nabla f(x)^T d, \\ &\nabla f(x+\alpha d)^T d \ge c_2 \nabla f(x)^T d, \end{aligned}$$

其中 $0 < c_1 < c_2 < 1$.

在2007-2010年间，董云达提出了 Dong 条件

$$c_2 \nabla f(x)^T d \le \nabla f(x+\alpha d)^T d \le c_1 \nabla f(x)^T d,$$

其中 $0 < c_1 < c_2 < 1$.

国际小同行的引用与评议（原始材料）如下：“与 Wolfe 条件形成鲜明对比的是，Dong 条件仅仅依赖梯度的估值...”

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iteration reads

$$x^{k+1} = x^k - a^k \mathcal{H}^{-1}(x^k) \nabla f(x^k)$$

with the Hessian $\mathcal{H}$ of f and a damping factor $a^k \in (0, 1]$, cf. Chapter 9 in Boyd's book [60]. Instead of inverting $\mathcal{H}(x^k)$, the update can be performed by

$$x^{k+1} = x^k + a^k \Delta x$$

where Δx is an approximate solution of

$$\mathcal{H}(x^k) \Delta x = -\nabla f(x^k). \tag{5.10}$$

The damping factor a^k is determined by a back-tracking procedure. In this paper, we use the stopping criteria of Dong [63]

$$c_2 \langle \nabla f(x^k), \Delta x \rangle_V \le \langle \nabla f(x^k + a^k \Delta x), \Delta x \rangle_V \le c_1 \langle \nabla f(x^k), \Delta x \rangle_V$$

with $0 < c_1 < c_2 < 1$. In contrast to the Wolfe conditions [64], Dong's criteria rely solely on gradient evaluations. This is beneficial, as evaluating f requires either the primal or dual condensed incremental potential, cf. Table 1, which is generally not available in FFT-based homogenization. Both w and w^* carry no physical meaning as they

注：Dong 条件下的拟Newton法 已成为优化领域的最基础性成果之一。