Probability distribution under optimum synchronization

Example: 4-PAM. The received signal with the receiver synchronised is given by:

$$X(t) = \omega_0(t) + n(t)$$

where $n(t)$ is the Additive White Gaussian Noise (AWGN). $r_i(t)$ therefore has PDF:

$$f_X(y) = \frac{1}{\sqrt{2\pi}\sigma} exp\left(-\frac{(y-\omega_0)^2}{2\sigma^2}\right)$$

The decision region boundaries are defined as the points where there is equal probability of adjacent symbols, ie:

$$f_{X,\omega_0}(y) = f_{X,\omega_1}(y)$$

In reality, we could consider an additional two sources of error:

- Fading errors due to multiple propagation paths.
- Synchronisation (timing) errors in the receiver.

The former can be modelled with a Nagakami distribution, the latter with a Tikhonov distribution. The ultimate goal is to incorporate all of these statistical models in an analysis of their effects and ways of compensating for them.

First steps

Initially, it's a good idea to ignore fading, and just study the channel under AWGN. Timing error can be set as constant and the response over a range of timing error values evaluated.