Background

Given n independent random variables, denoted $X_1, X_2, \ldots X_n$, described by the PDFs $f_1(y), f_2(y), \ldots f_n(y)$, the PDF of their sum $X_1 + X_2 + \cdots X_n$ is given by the convolution of their PDFs $f_1(y) * f_2(y) * \cdots f_n(y)$.

In this case all sources are statistically identical but independent. Hence the overall PDF is $f_{EGC}(y) = f(y) * f(y) * \cdots f(y) = f^{(n-1)*}(y)$ (using the notation that $f^{2*} \equiv f(y) * f(y) * f(y)$).

From before, $f(y) = \sum_{\Delta_i} \omega_{i_{\Delta_i}} T(\Delta_i) \frac{[\cdots]}{\sqrt{2\pi}\sigma_X}$

This was found using the Gauss-Legendre method, which states that:

$$\int_a^b f(x)dx \simeq \frac{b-a}{2} \sum_{i=1}^{n} \omega_i f\left(\frac{b-a}{2} z_i + \frac{b+a}{2}\right)$$

EGC case assuming Gram-Charlier approximation

For l=2 antennae, we must solve the case where each antenna output is summed together, ie $f_{EGC}(y) = f(y) * f(y)$.

Using the Gauss-Legendre method to break the convolution integral into a sum,

$$\begin{aligned} f_{EGC}(y) &= \int\limits_{-\infty}^{\infty} f(x)f(y-x)dx \\ &\simeq \int\limits_{-1}^{5} f(x)f(y-x)dx \\ &\simeq \sum_{i=1}^{n_f} 3\omega_i\ f(3z_i+2)\ f(y-3z_i-2) \end{aligned}$$

For the l=3 antennae, the solution is now $f^{2*} = f(y) * f(y) * f(y)$.

$$
\begin{aligned}
f_{EGC}(y) &= \int_{-\infty}^{\infty} f(y-x) f^*(x) dx \\
&\simeq \int_{-1}^{5} f(y-x) f^*(x) dx \\
&\simeq \sum_{j=1}^{n_f} 3\omega_j \, f(y-3z_i-2) \, f^*(3z_i+2) \\
&\simeq \sum_{j=1}^{n_f} 3\omega_j \, f(y-3z_i-2) \, \sum_{i=1}^{n_f} 3\omega_i \, f(3z_i+2) \, f(3z_j-3z_i) \\
&\simeq \sum_{i=1}^{n_f} \sum_{j=1}^{n_f} 3^2\omega_i\omega_j \, f(3z_j-3z_i) \, f(3z_i+2) \, f(y-3z_j-2)
\end{aligned}
$$

Doing this a few more times it appears that the PDF of the output of a l=m+1 antenna EGC system is given by

$$
f_{EGC}(y) = f^{m*}(y) \simeq \sum_{a_1=1}^{n_f} \sum_{a_2=1}^{n_f} \cdots \sum_{a_m=1}^{n_f} 3^m \omega_{a_1}\omega_{a_2} \cdots \omega_{a_m}
$$
$$
f(3z_{a_m} - 3z_{a_{m-1}}) \, f(3z_{a_{m-1}} - 3z_{a_{m-2}}) \cdots f(3z_{a_2} - 3z_{a_1}) \, f(3z_{a_1} + 2) \, f(y - 3z_{a_m} - 2)
$$