Sensitivity to Timing Errors in EGC and MRC Techniques

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***Abstract*—The effect of imperfect timing is analyzed in equal gain combining (EGC) and maximal ratio combining (MRC) techniques over Rayleigh and Nakagami-m fading channels with binary phase-shift keying modulation. In the case of EGC, the bit-error probability is derived, while in the case of MRC, error rate bounds are presented. Theoretical results are justified by computer simulation. Numerical results demonstrate that both EGC and MRC are fairly sensitive to timing errors, and comparatively, MRC is more sensitive.**

***Index Terms*—Diversity combining, error-rate analysis, imperfect fading estimates.**

I. Introduction

In MAXIMAL ratio combining (MRC) and equal gain combining (EGC), the signals on different channels are processed in such a way that the signal-to-noise ratio (SNR) for the combined signal is

$$\gamma = \begin{cases} \sum_{k=1}^{L} \gamma_k, & \text{MRC} \\ \frac{1}{L}\left[\sum_{k=1}^{L} \sqrt{\gamma_k}\right]^2, & \text{EGC} \end{cases} \tag{1}$$

where γ_k is the SNR on the kth diversity channel and L is the number of diversity branches. These expressions are valid only under perfect conditions. In particular, perfect estimates of fading parameters and perfect estimates of timing information on every branch are required. In MRC, both the amplitude and the phase, while in EGC, the phase, of all fading components are required at the receiver. Hence, it is important to investigate the sensitivity of the diversity combining techniques to system impairments. Several studies in the literature have reported the sensitivity to various estimation errors under different conditions [1], [8]–[12]. The effect of phase errors with binary phase-shift keying (BPSK) and quaternary phase-shift keying (QPSK) signals has been presented in a Rayleigh channel without diversity in [8] and with EGC in [9]. Few other studies [10]–[12] deal with the sensitivity to phase errors in code-division multiple-access (CDMA) systems in fading channels. In [13], the combined effect of phase and amplitude estimates has been considered with quadrature amplitude modulation (QAM) in a Rayleigh channel without diversity. A review of some of the other studies are presented in [1]. In this study, we analyze the sensitivity of EGC and MRC techniques to timing errors. We consider BPSK signaling with coherent demodulation in a Nakagami-m fading channel, which corresponds to a Rayleigh channel when $m = 1$. We analyze the system with EGC theoretically, and present theoretical bounds with MRC. We also verify the theoretical results by computer simulation. This study differs from [12] primarily because: 1) it considers Nakagami-m fading as opposed to Rayleigh fading considered in [12]; 2) it considers a Tikhonov distribution for the timing error, which is known to be a better model in practice [15], as opposed to a uniform distribution considered in [12]; and 3) in the case of MRC, it presents performance bounds in integral form as opposed to approximations presented in series form.

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II. System Model

Fig. 1 shows the conceptual low-pass equivalent model of EGC and MRC with imperfect timing. The quantities $\epsilon_1 T, \epsilon_2 T, \ldots, \epsilon_L T$ represent the associated timing errors of the individual diversity branches, respectively (ϵ_k represents the normalized timing error on the kth branch normalized to the bit duration T). It is assumed here that $\epsilon_1, \epsilon_2, \ldots, \epsilon_L$ are independent and identically distributed (i.i.d.) with a Tikhonov probability density function (pdf) [15]

$$f_\epsilon(\epsilon_k) = \frac{exp\left[\dfrac{(\cos(2\pi\epsilon_k))}{(2\pi\sigma_\epsilon)^2}\right]}{I_0\left(\dfrac{1}{(2\pi\sigma_\epsilon)^2}\right)}, \qquad |\epsilon_k| \le 0.5 \tag{2}$$

where σ_ϵ is the standard deviation of ϵ_k, and $I_0(\cdot)$ is the zeroth-order modified Bessel function of the first kind. Using the complex envelope notation, the transmitted BPSK signal is

$$s(t) = \sqrt{\frac{2E}{T}} \sum_i a_i u_T(t - iT) \tag{3}$$

where E is the bit energy, $a_i \in \{+1, -1\}$ is the ith bit, and $u_T(t)$ is the unit magnitude rectangular pulse over $[0, T)$. Assuming a flat and slow fading channel with additive white Gaussian noise (AWGN), the received signal on any kth diversity branch in presence of the timing error ϵ_k can be written as

$$r_k(t) = s(t - \epsilon_k T)\xi_k(t) + n_k(t), \qquad k = 1, 2, \ldots, L \tag{4}$$

where $\xi_k(t) = d_k(t)e^{j\alpha_k(t)}$ is the complex fading process with unity variance, magnitude $d_k(t)$, and phase $\alpha_k(t)$, $n_k(t)$ is the channel noise which is modeled by a complex zero-mean Gaussian process with power spectral density N_0, and L is the total number of diversity branches. It is assumed that the fading processes and the noise components on different diversity branches are all mutually independent. Denoting $\xi_k(t)$, $d_k(t)$, and $\alpha_k(t)$ over a duration of T by ξ_k, d_k, and α_k, respectively (which is justified under slow fading), the received signal $r_k(t)$ during any ith interval, $iT \le t < (i+1)T$, can be written as

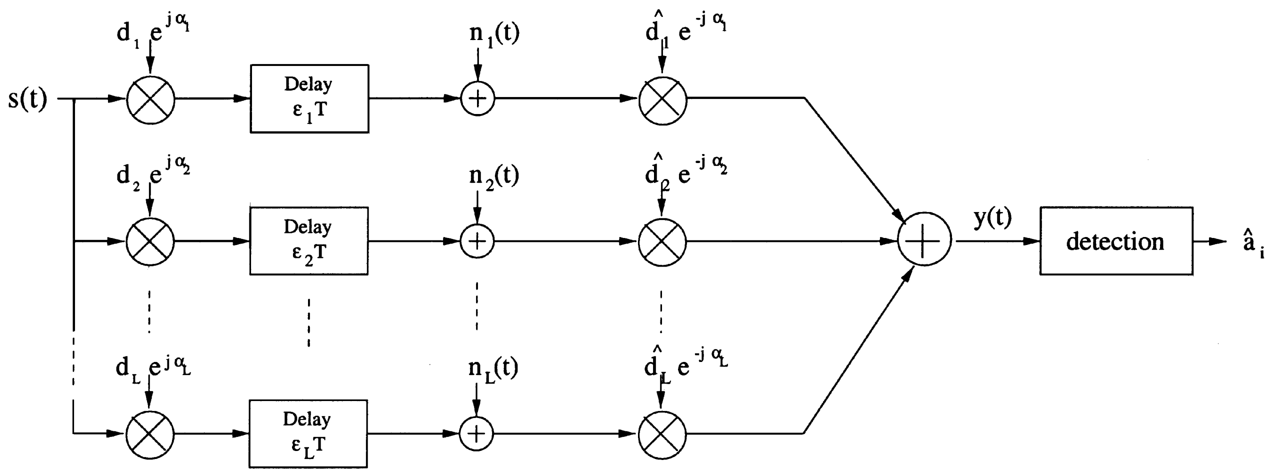

Fig. 1. EGC and MRC with imperfect timing.

shown in (5) at the bottom of the page. For MRC, $\hat{d}_k = d_k$, while for EGC, $\hat{d}_k = 1$. For Nakagami-m fading, the pdf of d_k, $f_d(d_k)$, is [1]

$$f_d(d_k) = \frac{2}{\Gamma(m)} d_k^{2m-1} (m)^m e^{-m d_k^2} \tag{6}$$

where $\Gamma(m)$ is the standard Gamma function, which equals $(m-1)!$ for integer $m > 0$.

III. EGC With Timing Errors

In EGC, with perfect knowledge of phases α_k, the input to the detector is (see Fig. 1)

$$y(t) = \sum_{k=1}^{L} r_k(t) e^{-j\alpha_k}. \tag{7}$$

Considering a correlator-type BPSK demodulator, the decision variable of the ith symbol can be obtained from $y(t)$ as

$$\begin{aligned} D_i &= \sqrt{\frac{1}{N_0}} \Re \left\{ \int_{iT}^{(i+1)T} \sqrt{\frac{1}{T}} y(t) dt \right\} \\ &= a_i \sqrt{\frac{2E}{N_0}} \sum_{k=1}^{L} d_k c_k + \sum_{k=1}^{L} w_k \end{aligned} \tag{8}$$

where $\Re\{\cdot\}$ denotes the real part of the argument

$$c_k = \begin{cases} 1 & \text{if } a_i = a_{\text{adj}} \\ (1 - 2|\epsilon_k|) & \text{if } a_i \neq a_{\text{adj}} \end{cases} \tag{9}$$

and w_k follows from the real part of the noise component in (5) and is zero-mean Gaussian with unit variance. It is seen from (5) and (8) that timing errors introduce intersymbol interference (ISI) in the decision variable. It follows from (5) and (8) that the effect of the timing error on D_i from any kth branch depends on the effective adjacent symbol a_{adj}, which for the symbol a_i is a_{i+1} if $\epsilon_k < 0$, and a_{i-1} if $\epsilon_k > 0$. It is noticed from (5), (8), and (9) that if $a_i = a_{\text{adj}}$, ϵ_k has no effect on D_i, and if $a_i \neq a_{\text{adj}}$, the effect of ϵ_k is to reduce the signal component from the kth branch by a factor $(1 - 2|\epsilon_k|)$. The term $1/\sqrt{N_0}$ in D_i which does not affect the decision has been used simply to express it in terms of E/N_0. It is also seen from (8) that the instantaneous SNR of the decision variable is

$$\gamma = \frac{2E\left[\sum_{k=1}^{L} d_k c_k\right]^2}{L N_0} = \frac{1}{L}\left[\sum_{k=1}^{L} \sqrt{\gamma_k} c_k\right]^2 \tag{10}$$

where $\gamma_k = 2E d_k^2 / N_0$ is the SNR on the kth branch without a timing error.

In this letter, the bit-error probability (BEP) is calculated by extending the analysis of EGC with perfect conditions presented in [5] using the characteristic function (CHF) of the decision variable. In [5], the BEP has been found as

$$P_{\text{e}} = \frac{1}{2} - \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\text{Im}\{\phi_D(\omega)\}}{\omega} d\omega \tag{11}$$

where $\phi_D(\omega)$ is the CHF of the decision variable D_i in (8) and $\text{Im}\{\cdot\}$ represents the imaginary part of the argument. Since D_i

$$r_k(t) = \begin{cases} \begin{cases} a_i \sqrt{\frac{2E}{T}} d_k e^{j\alpha_k} + n_k(t), & iT \le t < (i+1)T + \epsilon_k T \\ a_{i+1} \sqrt{\frac{2E}{T}} d_k e^{j\alpha_k} + n_k(t), & (i+1)T + \epsilon_k T \le t < (i+1)T \end{cases} & \epsilon_k < 0 \\ \begin{cases} a_{i-1} \sqrt{\frac{2E}{T}} d_k e^{j\alpha_k} + n_k(t), & iT \le t < iT + \epsilon_k T \\ a_i \sqrt{\frac{2E}{T}} d_k e^{j\alpha_k} + n_k(t), & iT + \epsilon_k T \le t < (i+1)T \end{cases} & \epsilon_k > 0 \end{cases} \tag{5}$$

depends on the values of c_k's, its CHF $\phi_D(\omega)$ is calculated by conditioning on the number of terms l in (8) for which $c_k = 1$. For any given l, assuming independent diversity branches, $\phi_D(\omega)$ follows from (8) as

$$\phi_{D|_l}(\omega) = \overline{e^{j\omega D|_l}} = \phi_1^l\left(\sqrt{\frac{2E}{N_0}}\omega\right)\phi_2^{(L-l)}\left(\sqrt{\frac{2E}{N_0}}\omega\right)\phi_{w_k}^L(\omega) \quad (12)$$

the overline denotes the averaging operation, $\phi_1(\omega)$ is the CHF of d_k

$$\phi_1(\omega) = \int_{-\infty}^{\infty} e^{jd_k\omega} f_d(d_k)dd_k \quad (13)$$

which can be expressed as shown in (14) at the bottom of the page [3], [6], $\phi_2(\omega)$ is the CHF of $d_k(1-2|\epsilon_k|)$ which is

$$\phi_2(\omega) = \overline{e^{j\omega d_k(1-2|\epsilon_k|)}} = 2\int_0^{0.5}\phi_1\left[\omega(1-2|\epsilon_k|)\right] f_\epsilon(\epsilon_k)d\epsilon_k \quad (15)$$

and $\phi_{w_k}(\omega)$ is the CHF of w_k which follows from the Gaussian distribution as [5], [7]

$$\phi_{w_k}(\omega) = \overline{e^{jw_k\omega}} = \int_{-\infty}^{\infty} e^{jw_k\omega} f(w_k)dw_k = e^{-\frac{\omega^2}{4}}. \quad (16)$$

In the above equations, ${}_1F_1(a,b,z)$ is the confluent hypergeometric function [16]. Combining (11), (12), and (16), the BEP can be expressed as

$$P_e|_l = \frac{1}{2} - \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\mathrm{Im}\left[\phi_1^l\left(\sqrt{\frac{2E}{N_0}}\omega\right)\phi_2^{(L-l)}\left(\sqrt{\frac{2E}{N_0}}\omega\right)\right]}{\left[\omega exp\left(\frac{L\omega^2}{4}\right)\right]}d\omega. \quad (17)$$

The overall BEP is calculated by averaging $P_e|_l$ over l which requires averaging over all combinations of symbols (a_{i-1}, a_i, a_{i+1}). Noticing that: 1) $c_k = 1$ for all k when $a_{i-1} = a_i = a_{i+1}$ (which occurs with probability 1/4); 2) $c_k = (1-2|\epsilon_k|)$ for all k when $a_{i-1} = a_{i+1} \neq a_i$ (which occurs with probability 1/4); and 3) in all other cases, $a_{i-1} = a_i \neq a_{i+1}$ or $a_{i-1} \neq a_i = a_{i+1}$ (which occurs with probability 1/2), the number of branches with $c_k = 1$ can vary between 0 and L, and the overall BEP can be expressed using the binomial expansion as

$$\mathrm{P}_e = \frac{1}{4}(P_e|_0 + P_e|_L) + \frac{1}{2^{(L+1)}}\sum_{l=0}^{L}\binom{L}{l}P_e|_l$$

$$= \frac{1}{4}(P_e|_0 + P_e|_L) + \frac{1}{2^{L+1}}\left(2^{L-1} - \frac{1}{2\pi}\times\int_{-\infty}^{\infty}\frac{\mathrm{Im}\left[\phi_1\left(\sqrt{\frac{2E}{N_0}}\omega\right)+\phi_2\left(\sqrt{\frac{2E}{N_0}}\omega\right)\right]^L}{\left[\omega exp\left(\frac{L\omega^2}{4}\right)\right]}d\omega\right) \quad (18)$$

which can be computed using integral routines in Matlab. For the special case when $L = 1$ with Rayleigh fading, (18) can be simplified using standard integrals [16] as

$$P_e = \frac{1}{2} - \frac{1}{4}\sqrt{\frac{2}{2+\frac{N_0}{E}}} - \frac{\sqrt{\frac{E}{N_0}}}{2I_0\left(\frac{1}{(2\pi\sigma_\epsilon)^2}\right)} \times\int_0^{0.5}\frac{(1-2\epsilon)exp\left[\frac{\cos(2\pi\epsilon)}{(2\pi\sigma_\epsilon)^2}\right]}{\sqrt{\frac{(1-2\epsilon)^2E}{N_0+1}}}d\epsilon. \quad (19)$$

IV. MRC With Imperfect Timing

We analyze the effect of timing errors in MRC assuming that both amplitude and phase estimates of fading components are available at the receiver. The decision variable with MRC in presence of timing errors during any ith interval, similar to (8), follows from Fig. 1 as

$$D_i = a_i\sqrt{\frac{2E}{N_0}}\sum_{k=1}^{L}d_k^2c_k + \sum_{k=1}^{L}d_kw_k \quad (20)$$

Further, the SNR of D_i can be expressed as

$$\gamma = \frac{2E\left[\sum_{k=1}^{L}d_k^2c_k\right]^2}{N_0\sum_{k=1}^{L}d_k^2} \quad (21)$$

which reduces to (1) for the case with perfect timing (i.e., $c_k = 1$ for all k).

We obtain upper and lower bounds of γ as

$$\gamma^{(1)} = \sum_{k=1}^{L}\gamma_k^{(1)} \leq \gamma \leq \gamma^{(2)} = \sum_{k=1}^{L}\gamma_k^{(2)} \leq \gamma^{(3)} = \sum_{k=1}^{L}\gamma_k \quad (22)$$

where

$$\gamma_k^{(1)} = \gamma_k e_k, \gamma_k^{(2)} = \gamma_k c_k^2$$
$$e_k = \begin{cases}[1-4|\epsilon_k|], & 0, \leq |\epsilon_k| \leq 0.25\\ 0, & 0.25 < |\epsilon_k| \leq 0.5.\end{cases} \quad (23)$$

$$\phi_1(\omega) = \begin{cases} {}_1F_1\left(1;\frac{1}{2};\frac{-\omega^2}{4}\right) + j\omega\sqrt{\frac{\pi}{4}}exp\left(-\frac{\omega^2}{4}\right), & m = 1\\ {}_1F_1\left(m;\frac{1}{2};\frac{-\omega^2}{4m}\right) + j\omega\frac{\Gamma\left(m+\frac{1}{2}\right)}{\Gamma(m)}\sqrt{\frac{1}{m}}{}_1F_1\left(m+\frac{1}{2};\frac{3}{2};-\frac{\omega^2}{4m}\right), & m \neq 1\end{cases} \quad (14)$$

$\gamma^{(2)}$ is obtained from (21) using Schwartz inequality, while $\gamma^{(3)}$ is the case without timing errors. $\gamma^{(1)}$ is obtained in three steps: 1) writing $c_k = 1 - p_k$, where p_k has values 0 and $2|\epsilon_k|$; 2) taking the square and ignoring the term $(\sum_{k=1}^{L} d_k^2 p_k)^2$; and 3) observing that no term in the sum can be negative. Thus, the error probability is bounded by

$$P_{e3} \leq P_{e2} \leq P_e \leq P_{e1}, P_{ex} = \overline{P_e(\gamma^{(x)})}. \quad (24)$$

Following the analysis in [1], [2] for $\gamma = \sum_{k=1}^{L} \gamma_k$ with independent γ_k

$$\overline{P_e(\gamma)} = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \prod_{k=1}^{L} M_{\gamma_k}\left(-\frac{1}{\sin^2\phi}\right) d\phi. \quad (25)$$

Thus, conditioning on the number of terms l for which $e_k = c_k = 1$, we get

$$P_{ex|l} = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} M_\gamma^l \left(-\frac{1}{\sin^2\phi}\right) M_{\gamma^{(x)}}^{L-l}\left(-\frac{1}{\sin^2\phi}\right) d\phi \quad (26)$$

where

$$M_{\gamma^{(1)}}(s) = q + 2\int_0^{0.25} M_\gamma\left[s(1-4\epsilon_k)\right] f(\epsilon_k) d\epsilon_k,$$

$$q = 2\int_{0.25}^{0.5} f(\epsilon_k) d\epsilon_k \quad (27)$$

$$M_{\gamma^{(2)}}(s) = 2\int_0^{0.5} M_\gamma[s(1-2\epsilon_k)^2] f(\epsilon_k) d\epsilon_k \quad (28)$$

and for Nakagami-m fading

$$M_\gamma(s) = \left(1 - \frac{2sE}{mN_0}\right)^{-m}. \quad (29)$$

We have already established that with probability 0.25, $\epsilon_k = 0$ for all k, with probability 0.25, $\epsilon_k = 1$ for all k, and with probability 0.5, $\epsilon_k = 0$ for some values of k and 1 for the others. Thus, for $x = 1$ and 2

$$\begin{aligned} \mathrm{P}_{ex} &= \frac{1}{4}(P_{ex|0} + P_{ex|L}) + \frac{1}{2^{L-1}} \sum_{l=0}^{L} \binom{L}{l} P_{ex|l} \\ &= \frac{1}{4}(P_{ex|0} + P_{ex|L}) + \frac{1}{\pi 2^{L-1}} \int_0^{\frac{\pi}{2}} \\ &\quad \times \left[M_\gamma\left(-\frac{1}{\sin^2\phi}\right) + M_{\gamma^{(x)}}\left(-\frac{1}{\sin^2\phi}\right)\right]^L d\phi \quad (30) \end{aligned}$$

where we have again used the binomial expansion. For $x = 3$

$$P_{e3} = P_{e3|L} = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} M_\gamma^L\left(-\frac{1}{\sin^2\phi}\right) d\phi. \quad (31)$$

Hence, P_e with MRC can be upper and lower bounded using (27), (28), and (30) for any value of L. It is mentioned here that the upper bound in (24) has a limitation as it has a floor due to q, whereas, the actual error probability monotonically decreases with increasing SNR. Specifically, the error floor associated with the upper bound follows from (24), (27), and (30) as $1/8[1 + 2^{-(L-1)}]q^L$. Hence, the upper bound in (27) and (30) can be effectively used for error probabilities higher than the above floor.

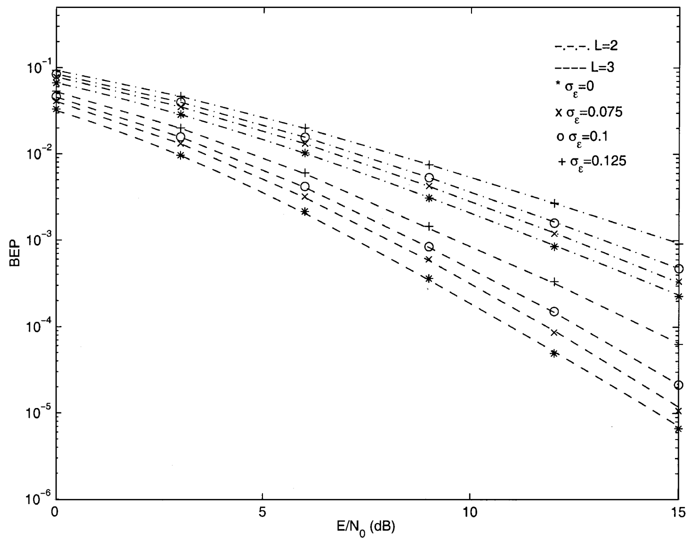

Fig. 2. BEP variation of EGC in presence of timing errors in a Rayleigh channel when $L = 2$ and 3.

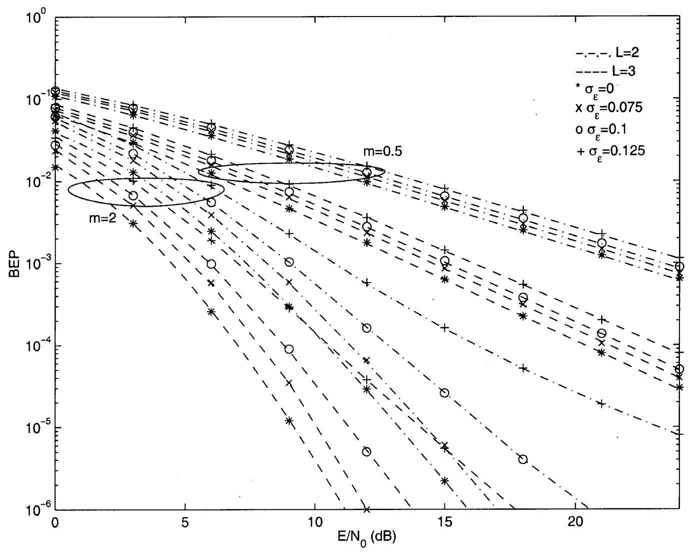

Fig. 3. BEP variation of EGC in presence of timing errors in a Nakagami-m channel with $m = 0.5$ and 2 when $L = 2$ and 3.

V. Numerical Results and Discussion

Fig. 2 shows the BEP variations of EGC at different values of σ_ϵ over a Rayleigh channel. In Fig. 2 (and also in Fig. 3), the lines represent the theoretical results while the specially marked points represent simulated results. Similarly, Fig. 3 shows the BEP variations with EGC over a Nakagami-m channel with $m = 2$ and $m = 0.5$. It is seen that the theoretical results match well with the simulations. It is further seen that BEP is fairly sensitive to timing errors, and as one can expect, the sensitivity increases with increasing L.

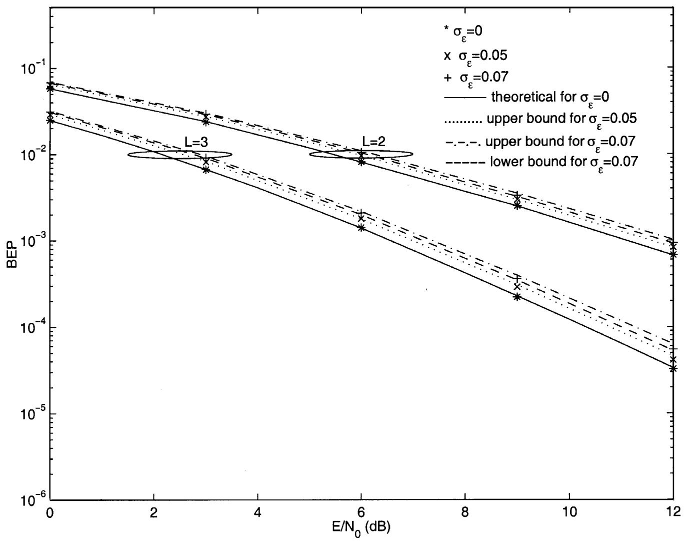

Fig. 4. BEP variation of MRC in presence of timing errors in a Rayleigh channel when $L = 2$ and 3.

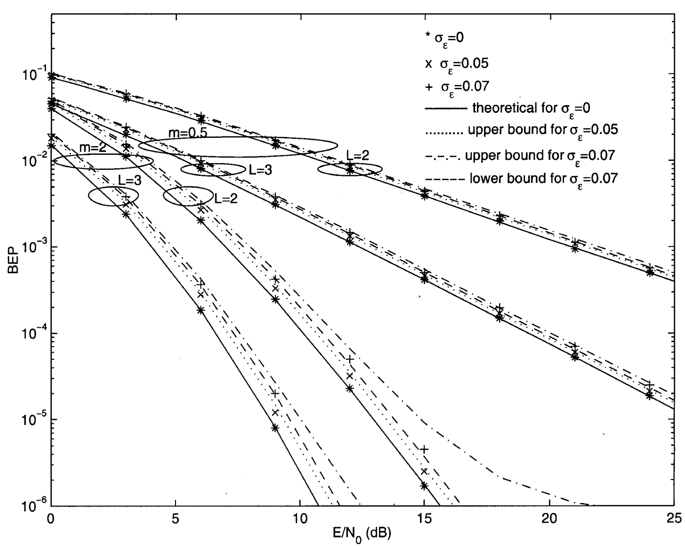

Fig. 5. BEP variation of MRC in presence of timing errors in a Nakagami-m channel with $m = 0.5$ and 2 when $L = 2$ and 3.

Figs. 4 and 5 show similar BEP variations with MRC in presence of timing errors. Simulated results, the theoretical variations with perfect estimates, variations of the upper bound according to (27) and (30), and selected cases of lower bound according to (28) and (30) are plotted separately. It is seen that the bounds match well with the simulated results, particularly at lower values of σ_ϵ. Fig. 5 also demonstrates the effect of the error floor of the upper bound when $m = 2$, $L = 2$, and $\sigma_\epsilon = 0.07$. Comparing with Figs. 2 and 3, it is seen from Figs. 4 and 5 that MRC is more sensitive to timing errors than EGC. Comparing different channels, it is seen that the sensitivity to timing errors increases with increasing m both in EGC and MRC.

VI. Conclusion

The effect of imperfect symbol timing has been analyzed in EGC and MRC techniques over Rayleigh and Nakagami-m fading channels. BPSK modulation with coherent demodulation has been considered. Timing errors on different diversity branches have been assumed to be i.i.d. according to a Tikhonov distribution. In the case of EGC, the effect of timing errors has been theoretically analyzed and justified by computer simulation. In the case of MRC, error-rate bounds have been derived and justified by simulation. It has been numerically demonstrated that both EGC and MRC are fairly sensitive to timing errors, and comparatively, MRC is more sensitive.

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