Gram-Charlier Probability Density Function

$$f_{X|\Delta}(y) = \frac{1}{\sqrt{2\pi}\sigma_X} \exp\left(-\frac{(y-\omega_0 g_0)^2}{2\sigma_X^2}\right)\left[1+\sum_{m=2}^{\infty}\frac{\alpha_{2m}}{(2m)!\sigma_X^{2m}}\, H_{2m}\left(\frac{y-\omega_0 g_0}{\sigma_X}\right)\right]$$

$$\sigma_X^2 = \sigma_\mu^2 + \frac{1}{3}\left(L^2-1\right)\sum_{i=2}^{m-2}\binom{2m-1}{wi}\kappa_{2(m-i)}\alpha_{2i} \quad m \geq 2$$

$$\kappa_m = -j^m T_{m-1}\frac{L^m-1}{2^m-1}\sum_{k=1}^{\infty}\left(g_{-k}^m + g_k^m\right) \quad m \geq 2$$

$$T_m = \left.\frac{d^m}{dt^m} tan(t)\right|_{t=0}$$

$$H_m(y) = (-1)^m e^{y^2/2}\frac{d^m}{dy^m}e^{-y^2/2}$$

or

$$H_m(y) = \sum_{i=0}^{m/2}(-1)^i\frac{m!}{(m-2i)!}\frac{1}{2^i i!}y^{m-2i}$$

Derivation of the Gram-Charlier Integral

Starting with the Gram-Charlier definition described earlier,

$$f_{X|\Delta}(y) = \frac{1}{\sigma_X}\phi\left(\frac{y-\mu_X}{\sigma}\right)+\sum_{m=2}^{M}\frac{\alpha_{2m}}{(2m)!\sigma_X^{2m}}\left[\frac{1}{\sigma_X}\,\phi\left(\frac{y-\mu_X}{\sigma}\right)\,H_{2m}\left(\frac{y-\mu_X}{\sigma_X}\right)\right]$$

and incorporating the given identites,

$$\int_{-\infty}^{x}\frac{1}{\sigma}\,\phi\left(\frac{y-\mu}{\sigma}\right)dy = \frac{1}{2}\left(1+\operatorname{erf}\left(\frac{x-\mu}{\sqrt{2}\sigma}\right)\right)$$

$$\int_{-\infty}^{x}\frac{1}{\sigma}\,\phi\left(\frac{y-\mu}{\sigma}\right)H_m\left(\frac{y-\mu}{\sigma}\right)dy = -\phi\left(\frac{x-\mu}{\sigma}\right)H_{m-1}\left(\frac{x-\mu}{\sigma}\right)$$

the solution for the integral of the Gram-Charlier series was found to be:

$$\int_{-\infty}^{x} f_{X|\Delta}(y) = \frac{1}{2}\left(1+\operatorname{erf}\left(\frac{x-\mu_X}{\sqrt{2}\sigma_X}\right)\right)-\sum_{m=2}^{M}\frac{\alpha_{2m}}{(2m)!\sigma_x^{2m}}\phi\left(\frac{x-\mu_X}{\sigma_X}\right)H_{2m-1}\left(\frac{x-\mu_X}{\sigma_X}\right)$$

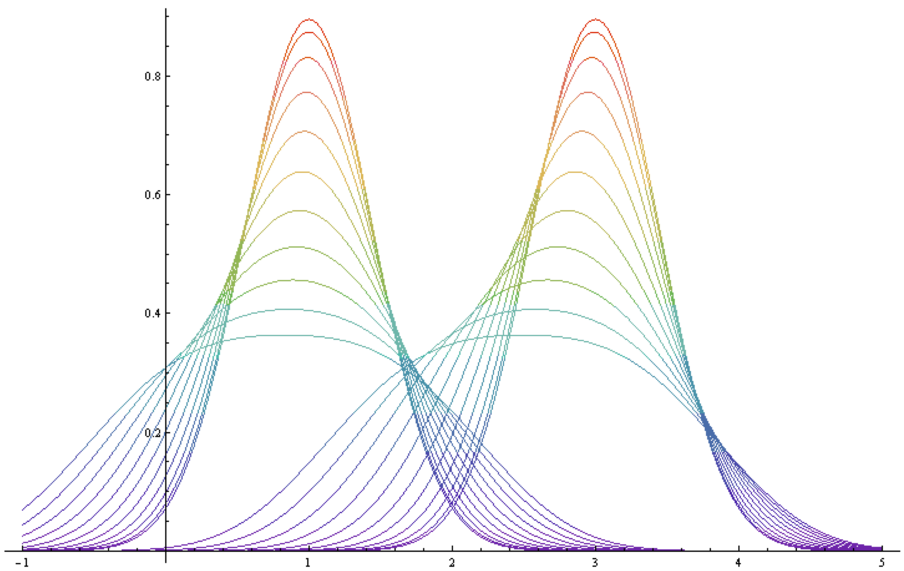

Figure 1: Received symbol pdf vs timing offset

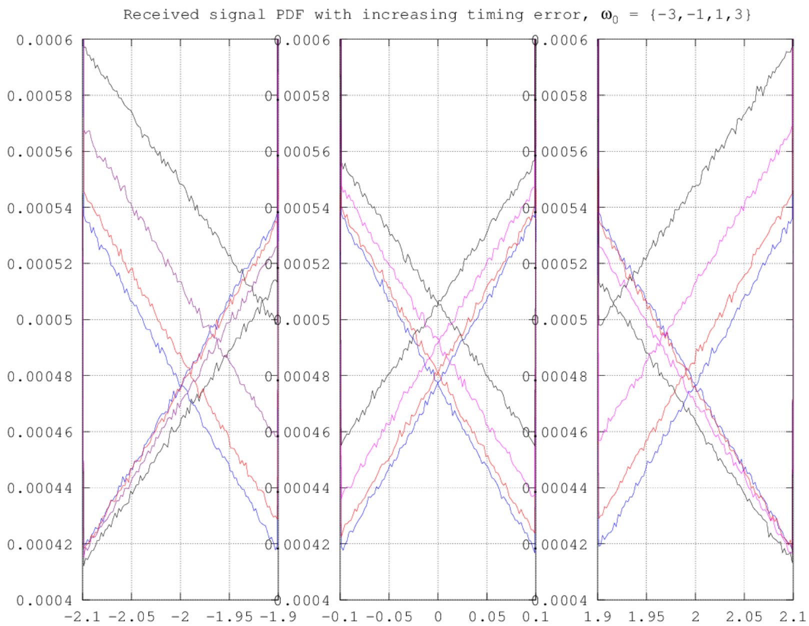

Figure 2: Fixed timing offset PDF intersections

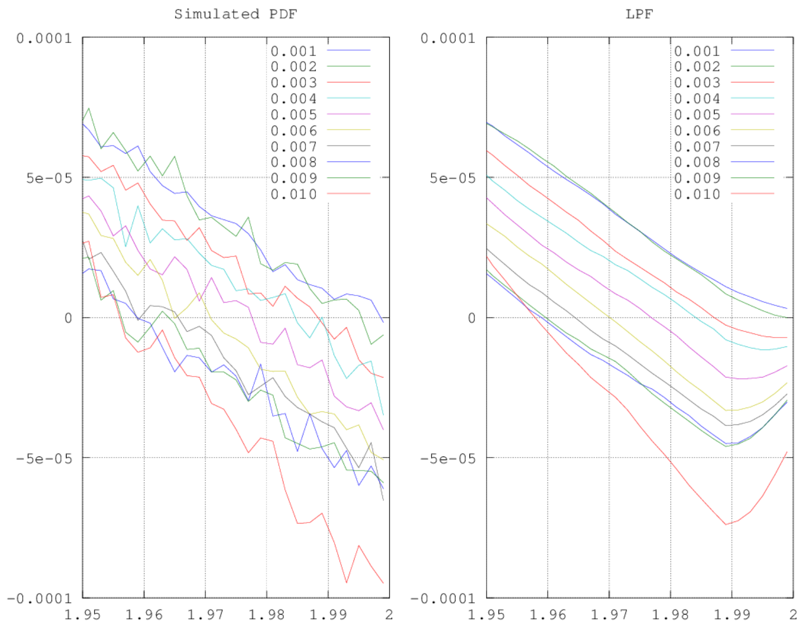

Figure 3: PDF difference vs timing offset variance (non-fading)

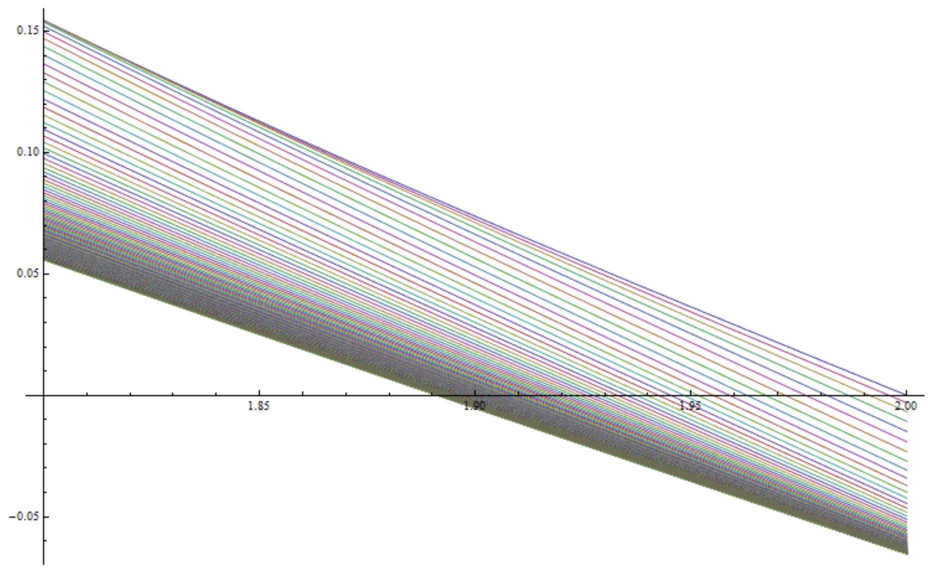

Figure 4: PDF difference vs timing offset variance

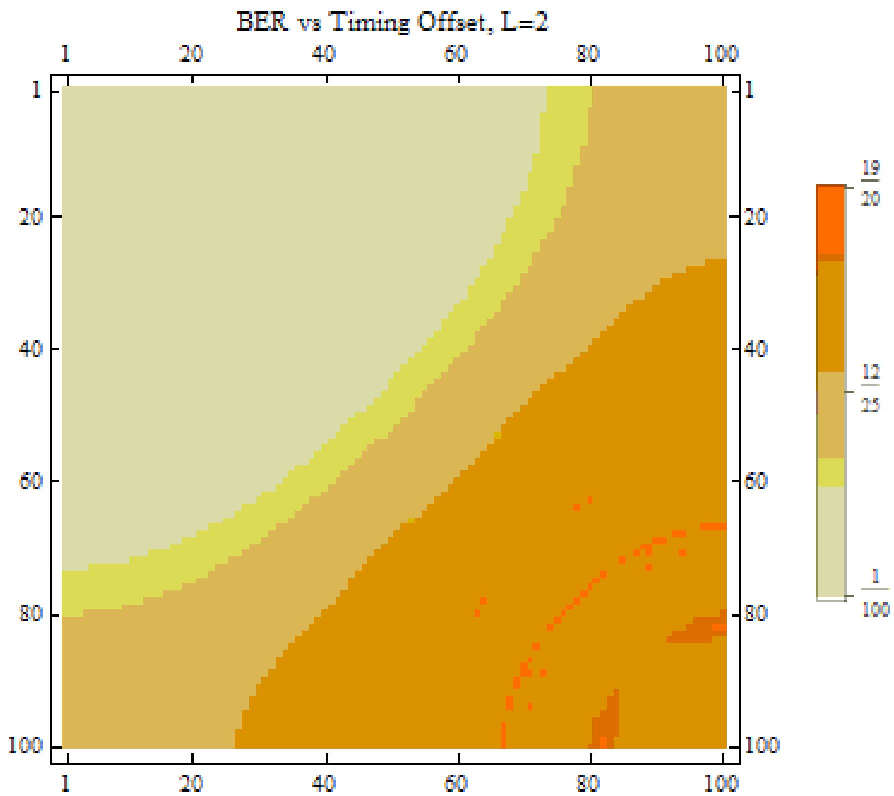

Figure 5: SER vs Timing Offset

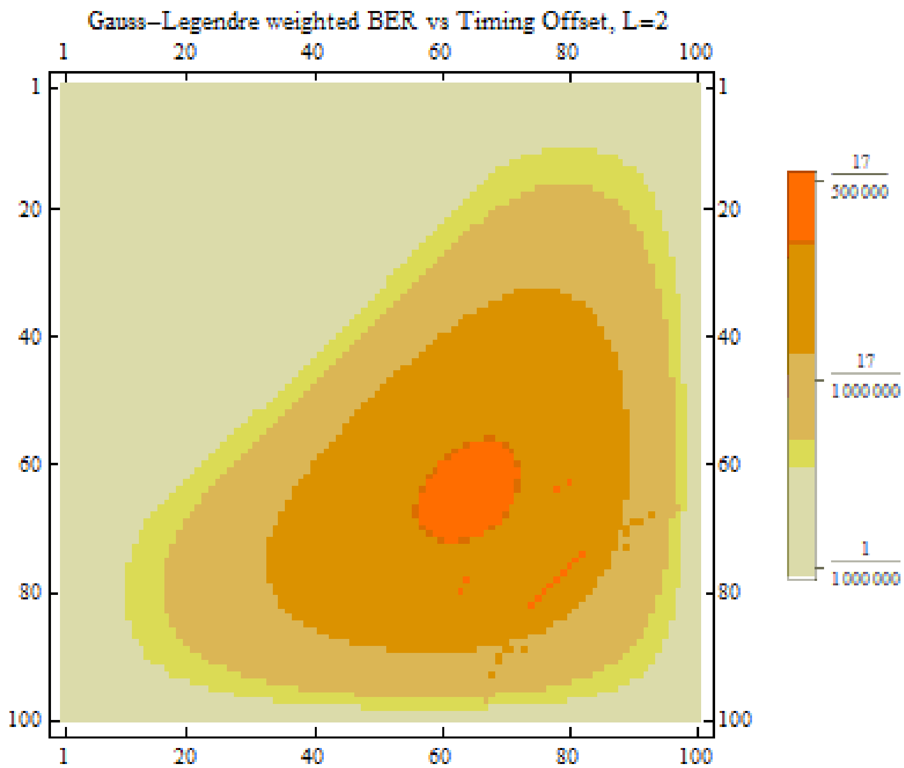

Figure 6: SER vs Timing Offset after Tikhonov weighting

Tikh var	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009	0.01
Prev ER	0.111	0.115	0.119	0.124	0.129	0.133	0.139	0.148	0.154	0.159
New ER	0.0751	0.0801	0.0844	0.0900	0.0951	0.0994	0.106	0.115	0.122	0.126
% drop	33.	30.	29.	27.	26.	25.	24.	22.	21.	20.
MRC ER	6.00×10^{-8}	0.0000298	0.000354	0.00121	0.00320	0.00565	0.00943	0.0146	0.0193	0.0265
MRC ER	8.00×10^{-8}	0.0000159	0.000218	0.000970	0.00246	0.00470	0.00818	0.0129	0.0179	0.0242
% drop	−33.	47.	38.	20.	23.	17.	13.	12.	7.	9.

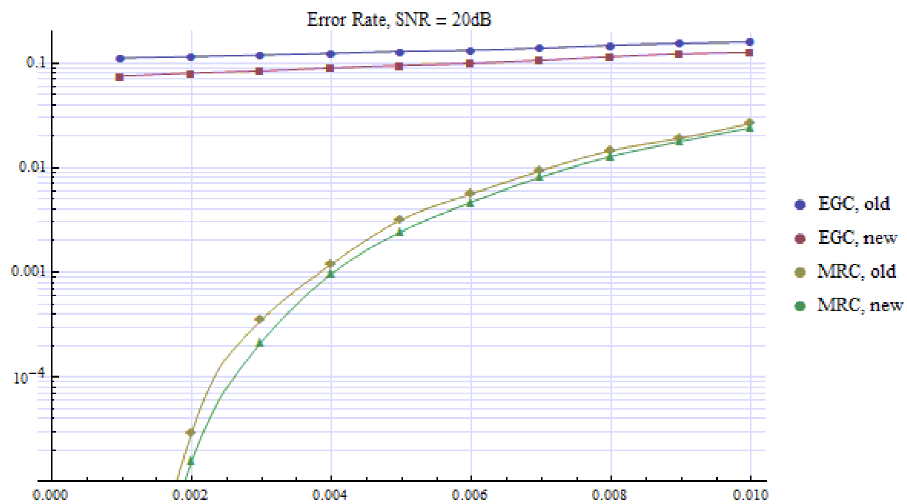

Figure 7: EGC, MRC SER vs timing offset variance (20dB SNR)

Tikh var	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009	0.01
Prev ER	0.111	0.115	0.119	0.124	0.129	0.133	0.139	0.148	0.154	0.159
New ER	0.0751	0.0801	0.0844	0.0900	0.0951	0.0994	0.106	0.115	0.122	0.126
% drop	33.	30.	29.	27.	26.	25.	24.	22.	21.	20.
MRC ER	6.00×10^{-8}	0.0000298	0.000354	0.00121	0.00320	0.00565	0.00943	0.0146	0.0193	0.0265
MRC ER	8.00×10^{-8}	0.0000159	0.000218	0.000970	0.00246	0.00470	0.00818	0.0129	0.0179	0.0242
% drop	−33.	47.	38.	20.	23.	17.	13.	12.	7.	9.

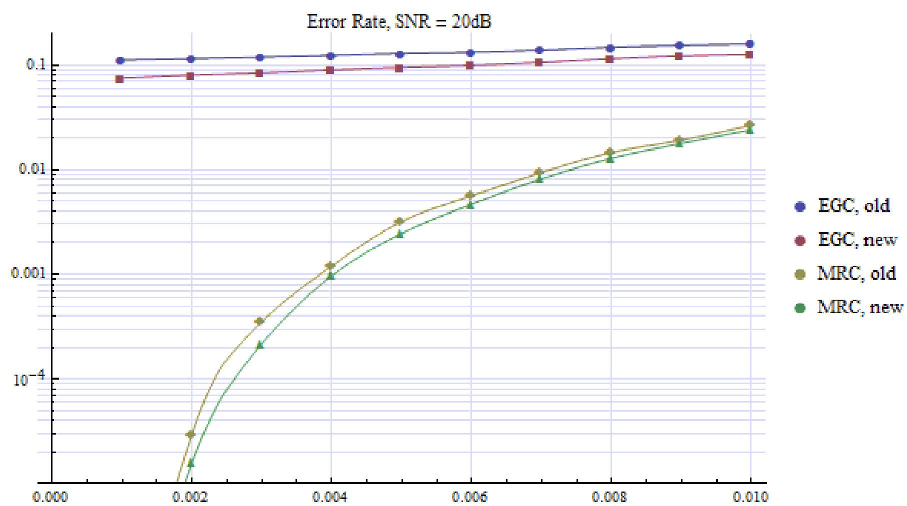

Figure 8: EGC, MRC SER vs timing offset variance (28dB SNR)

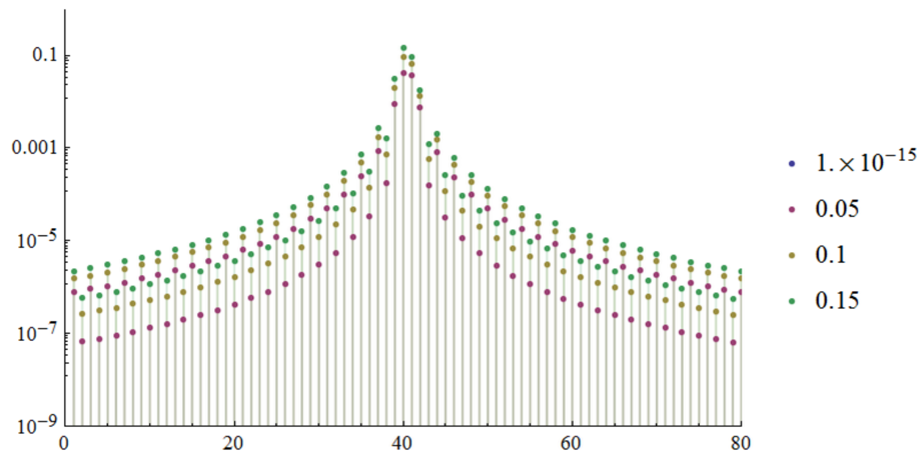

Figure 9: ISI weighting with timing error

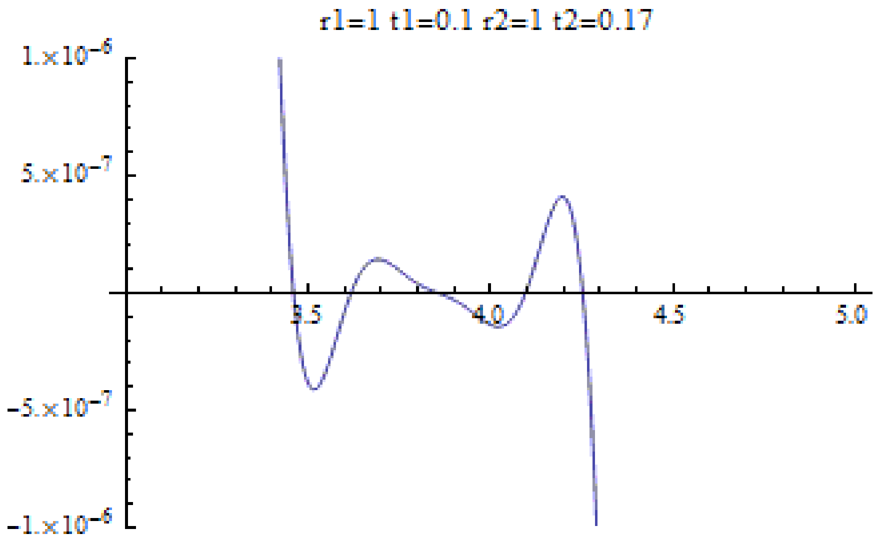

Figure 10: Gram-Charlier approximation ripple

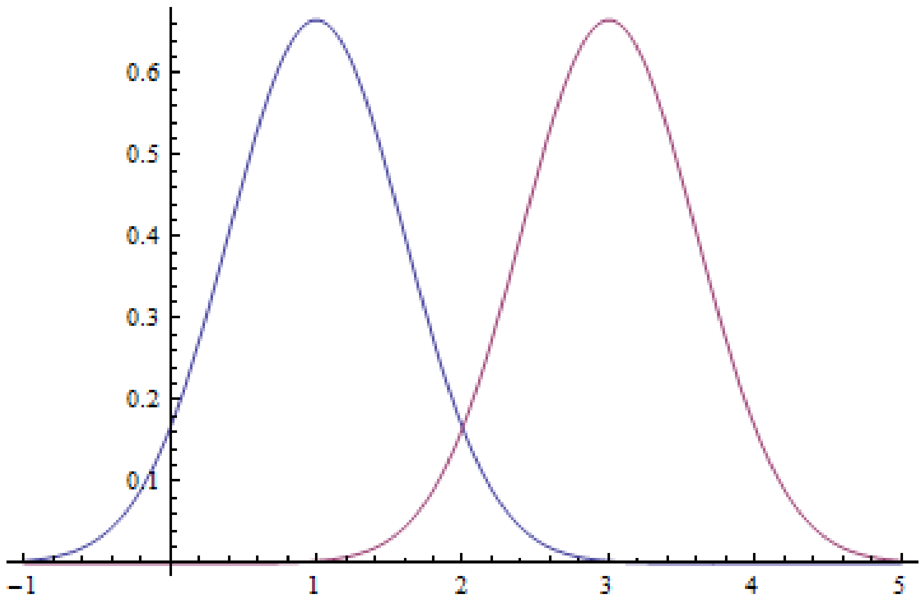

Figure 11: Seminar: Gaussian, synchronised case

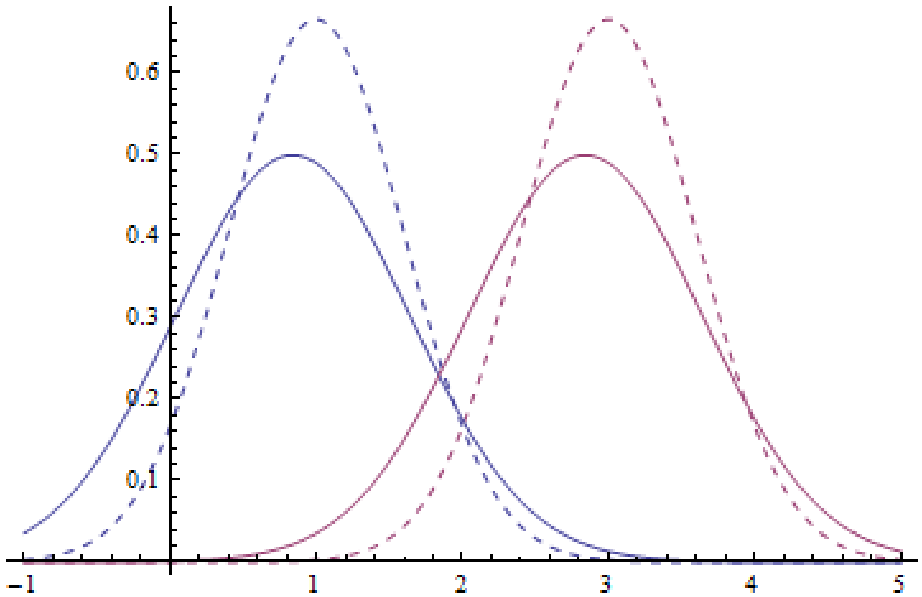

Figure 12: Seminar: Gaussian, timing offset case

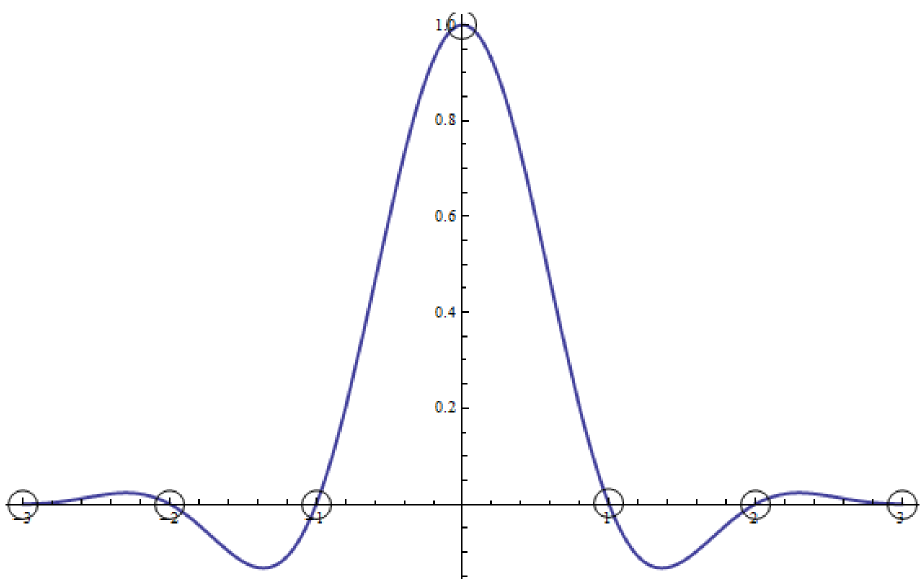

Figure 13: Seminar: Root-raised Cosine response, synchronised case

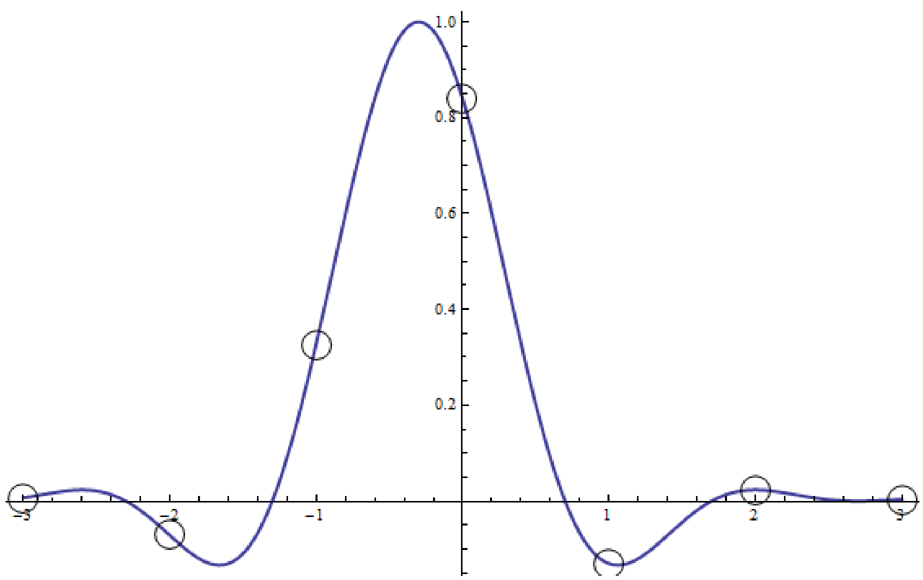

Figure 14: Seminar: Root-raised Cosine response, timing offset case