Fourth Year Project Logbook

Week 1

30/09/13 - Exploring simple case with PAM modulation

I received the *PAM.pdf* file outlining the case where a signal is sent through a channel with AWGN and received with a timing error at the receiver. I read through the file several times to get an understanding of the underlying equations.

Leaving the Gram-Chandler series aside for the moment, I started getting to grips with Mathematica and implementing the transmission system model:

$$X = \omega_0 g_0 + \sum_{k=1}^{40} (\omega_{-k} g_k + \omega_k g_k) + \nu$$

where $g_k = g((\Delta + k)T)$, $g(t) = (u_T * h_l * u_R)(t) \times cos(\theta)$ and ν is a zero-mean Gaussian random variate with $\sigma_\nu^2 = N_0 \varepsilon_R$.

I learnt the basics of the interface, and began inplementing the filter and channel impulse responses (I.R.). I need to double-check the definition of the Root-Raised Cosine (RRC) Filter, as the impulse response wasn't as expected.

Later, I found the correct form for the RRC and double-checked it using Octave. The equation used is listed below. A plot showed that this equation is invalid at $t = \left[-\frac{T_s}{4\beta}, 0, \frac{T_s}{4\beta}\right]$, so I plan to find its limit at these points using Mathematica to obtain the complete solution.

$$h_{RRC}(t) = \frac{2\beta}{\pi\sqrt{T_s}} \frac{cos\left[(1+\beta)\frac{\pi t}{T_s}\right] + \frac{sin\left[(1-\beta)\frac{\pi t}{T_s}\right]}{\frac{4\beta t}{T_s}}}{1 - \left(\frac{4\beta t}{T_s}\right)^2}$$

01/10/13 - Implementing Raised Cosine functions

I implemented the function above in Mathematica, and using the `Limit` function found the value of the function at the following undetermined points:

$$h_{RRC}(t) = \begin{cases} \dfrac{4\beta + \pi(1-\beta)}{2\pi\sqrt{T_s}} & t = 0 \\ \dfrac{\beta}{2\pi\sqrt{T_s}}\left(\pi sin\left[\frac{(1+\beta)\pi}{4\beta}\right] - 2cos\left[\frac{(1+\beta)\pi}{4\beta}\right]\right) & t = \pm\frac{T_s}{4\beta} \\ \dfrac{2\beta}{\pi\sqrt{T_s}} \dfrac{cos\left[(1+\beta)\frac{\pi t}{T_s}\right] + \dfrac{sin\left[(1-\beta)\frac{\pi t}{T_s}\right]}{\frac{4\beta t}{T_s}}}{1-\left(\frac{4\beta t}{T_s}\right)^2} & \text{otherwise} \end{cases}$$

I also implemented the Raised Cosine function for the channel function, using the impulse response below[1]. I was unable however to convolve the receiver and transmitter filter functions using the `Convolve` function, even when I limited the impulse response using a `UnitBox`.

$$h_{RC}(t) = \frac{sinc\left(\frac{\pi t}{T}\right) cos\left(\beta\frac{\pi t}{T}\right)}{1-\left(2\beta\frac{t}{T}\right)^2}$$

I looked into Mathematica's treatment of the Gaussian distribution, and figured out how to generate random noise vectors following a Gaussian distribution, as well as how to generate a list of random binary symbols.

After discussing the convolution issue with David, he suggested that the channel should be initially modelled as ideal and therefore the overall channel and filter I.R. $g(t)$ can be defined as a Raised Cosine function, as defined above. I should therefore be ready to implement the simple ISI model tomorrow.

02/10/13 - Wrapping Up the Initial PAM Model

I pulled together the Raised Cosine function and random number generator to impiment the given simplified function for the PAM receiver output, given below. Playing around with the settings, I was able to show how the g_k function increases with the timing error. I decided to study the Mathematica environment a little more before carrying on with any programming.

$$X = \omega_0 g_0 + \sum_{k=1}^{40}(\omega_{-k}g_{-k} + \omega_k g_k) + \nu$$

[1]Proakis, "Digital Communications"