

Fourth Year Project Logbook

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Week 1

30/09/13 - Exploring simple case with PAM modulation

I received the *PAM.pdf* file outlining the case where a signal is sent through a channel with AWGN and received with a timing error at the receiver. I

read through the file several times to get an understanding of the underlying equations.

Leaving the Gram-Charlier series aside for the moment, I started getting to grips with Mathematica and implementing the transmission system model:

$$X = \omega_0 g_0 + \sum_{k=1}^{40} (\omega_{-k} g_k + \omega_k g_k) + \nu$$

where $g_k = g((\Delta + k)T)$, $g(t) = (u_T * h_l * u_R)(t) \times \cos(\theta)$ and ν is a zero-mean Gaussian random variate with $\sigma_\nu^2 = N_0 \varepsilon_R$.

I learnt the basics of the interface, and began implementing the filter and channel impulse responses (I.R.). I need to double-check the definition of the Root-Raised Cosine (RRC) Filter, as the impulse response wasn't as expected.

Later, I found the [correct form for the RRC](#) and double-checked it using Octave. The equation used is listed below. A plot showed that this equation is invalid at $t = \left[-\frac{T_s}{4\beta}, 0, \frac{T_s}{4\beta}\right]$, so I plan to find its limit at these points using Mathematica to obtain the complete solution.

$$h_{RRC}(t) = \frac{2\beta}{\pi\sqrt{T_s}} \frac{\cos\left[(1+\beta)\frac{\pi t}{T_s}\right] + \frac{\sin\left[(1-\beta)\frac{\pi t}{T_s}\right]}{\frac{4\beta t}{T_s}}}{1 - \left(\frac{4\beta t}{T_s}\right)^2}$$

01/10/13 - Implementing Raised Cosine functions

I implemented the function above in Mathematica, and using the `Limit` function found the value of the function at the following undetermined points:

$$h_{RRC}(t) = \begin{cases} \frac{4\beta + \pi(1-\beta)}{2\pi\sqrt{T_s}} & t = 0 \\ \frac{\beta}{2\pi\sqrt{T_s}} \left(\pi \sin\left[\frac{(1+\beta)\pi}{4\beta}\right] - 2\cos\left[\frac{(1+\beta)\pi}{4\beta}\right] \right) & t = \pm \frac{T_s}{4\beta} \\ \frac{2\beta}{\pi\sqrt{T_s}} \frac{\cos\left[(1+\beta)\frac{\pi t}{T_s}\right] + \frac{\sin\left[(1-\beta)\frac{\pi t}{T_s}\right]}{\frac{4\beta t}{T_s}}}{1 - \left(\frac{4\beta t}{T_s}\right)^2} & \text{otherwise} \end{cases}$$

I also implemented the Raised Cosine function for the channel function, using the impulse response below¹. I was unable however to convolve the receiver and

¹Proakis, "Digital Communications"

transmitter filter functions using the `Convolve` function, even when I limited the impulse response using a `UnitBox`.

$$h_{RC}(t) = \frac{\text{sinc}\left(\frac{\pi t}{T}\right) \cos\left(\beta \frac{\pi t}{T}\right)}{1 - \left(2\beta \frac{t}{T}\right)^2}$$

I looked into Mathematica's treatment of the Gaussian distribution, and figured out how to generate random noise vectors following a Gaussian distribution, as well as how to generate a list of random binary symbols.

After discussing the convolution issue with David, he suggested that the channel should be initially modelled as ideal and therefore the overall channel and filter I.R. $g(t)$ can be defined as a Raised Cosine function, as defined above. I should therefore be ready to implement the simple ISI model tomorrow.

02/10/13 - Wrapping Up the Initial PAM Model

I pulled together the Raised Cosine function and random number generator to implement the given simplified function for the PAM receiver output, given below. Playing around with the settings, I was able to show how the g_k function increases with the timing error. I decided to study the Mathematica environment a little more before carrying on with any programming.

$$X = \omega_0 g_0 + \sum_{k=1}^{40} (\omega_{-k} g_{-k} + \omega_k g_k) + \nu$$

03/10/13 - Delving deeper into Mathematica

I devoted some time into looking through Michael Quinlan's notebooks and better understanding the workings of the `Table` functions and the various plotting options. Fortunately my notebook was corrupted so I was able to rewrite it and understand the model a bit more. I need to figure out what variance value the noise PDF should take on, as the noise appears to be overwhelming the timing error effects. Translating the resulting PDF's into patterns is another question that needs some thought.

Week 1 Summary

Week 1 was mostly spent becoming acquainted with Mathematica and getting a feel for the equations underlying PAM transmissions. A simple model of a PAM receiver was constructed.

Goals for Week 2

- The PAM model will need to be extended to calculate the optimum decision region boundary from the estimated PDF.
- A better setup will be required to perform large-scale simulations within an acceptable time period. We will look into applying for an account on the Boole cluster.

Week 2

07/10/13 - Matrix manipulations

I decided to spend another day learning about the Mathematica environment, in particular matrix manipulation and generation. I looked into the **Apply**, **Map** and **Partition** functions and wrote some examples to figure out how to convert mathematical problems to Mathematica notation using matrices. I hope to convert the code to use matrices tomorrow to hopefully simplify and speed things up.

I also implemented David's equation for properly calculating the AWGN function variance from SNR², from last Friday's meeting.

08/10/13 - Fixed I.R. and Kernel Density Estimation

The first job was to rewrite the code to make use of the simple dot operator to calculate all the ISI components³. With the new code I was able to carry out many more runs and get much more detailed output. In addition, when I was rewriting the code I noticed a typo in the Raised Cosine I.R. that was degrading performance in the perfectly synchronised case. With both of these changes made, I decided to use Kernel Density Estimation to see what effects the timing offset has.

Using offsets of 10^{-15} , 0.05, 0.1 & 0.15, the following values of $g_k, k \in \{-40 \dots -1, 1 \dots 40\}$ were calculated.

²See *davenotes.pdf*

³The ISI components are now calculated using:

$$\left[\sum_{k=0}^{k=40} (g_k \omega_k^1 + g_{-k} \omega_{-k}^1) \cdots \sum_{k=0}^{k=40} (g_k \omega_k^m + g_{-k} \omega_{-k}^m) \right] = [g_{-40} \cdots g_{-1} g_1 \cdots g_{40}] \bullet \begin{bmatrix} \omega_{-40}^1 & \cdots & \omega_{-40}^m \\ \vdots & & \vdots \\ \omega_{-1}^1 & \cdots & \omega_{-1}^m \\ \omega_1^1 & \cdots & \omega_1^m \\ \vdots & & \vdots \\ \omega_{40}^1 & \cdots & \omega_{40}^m \end{bmatrix}$$

where ω_k^j is the k 'th ISI with the j 'th timing offset.

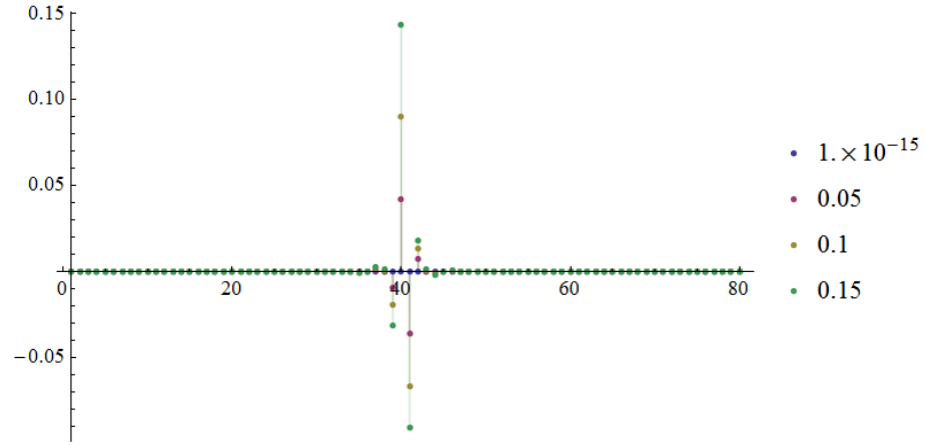


Figure 1: g_k linear plot

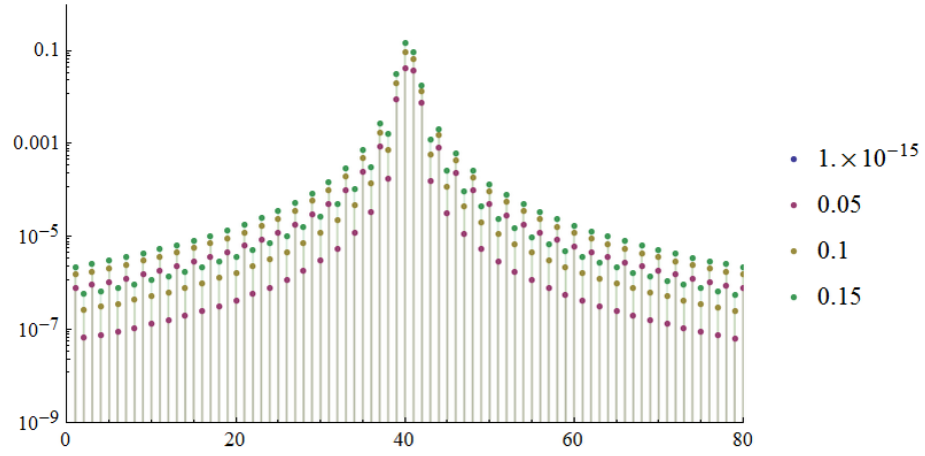


Figure 2: g_k log plot

Using `SmoothKernelDistribution` to perform Kernel Density Estimation with 1 million points produced the following estimated PDFs for both possible transmitted values. As the timing error increases, we note that the PDF spreads out, but the mean remains steady.

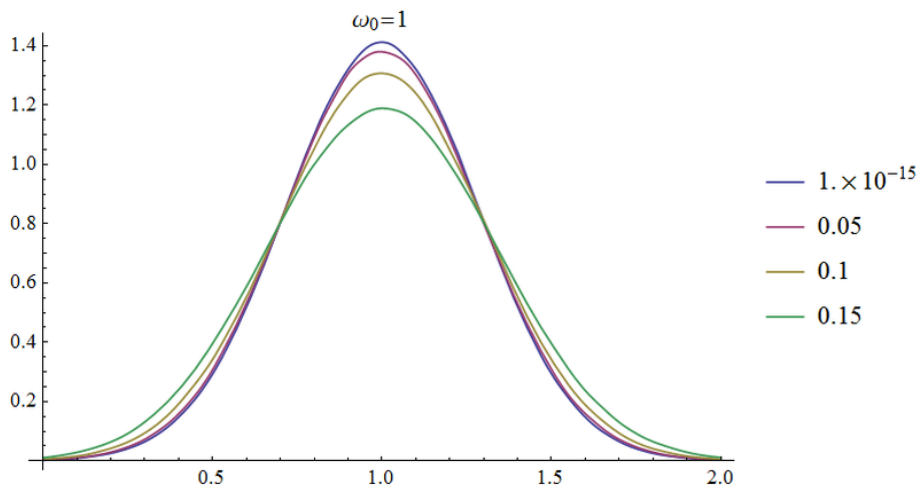


Figure 3: kernel density estimation $\omega_0 = 1$

09/10/13 - Setting up Digital Comms Lab PC

With Ger's help, I set up an account on **Digital Comms Lab 1** & **Digital Comms Lab 2** and got the internet working. Mathematica 8 is installed and working on both machines, we will have to consider whether an upgrade to Mathematica 9 would be useful or not. Git and VNC or similar have to be installed next. A request was made to the Boole cluster for access for this machine, however the email given (bcrisupport@bcricri.ucc.ie) was invalid.

10/10/13 - Probing the Elec Eng network

After finding out the Boole cluster was no more, I used today to examine what hardware I had available to me. I got access from Ger to the public UEPC004 server, and from there I am able to access machines on the elec eng network. I set up a *Remote Desktop Protocol* link to **Digital Comms Lab 1** through this server, allowing me to control the machine from any location. I am also able to log remotely into EDA lab machines, and run Mathematica 6 on those machines.⁴

⁴The GUI does not work when using `ssh` to access the EDA Lab machines, but using the command `math` to start and operate Mathematica kernels does.

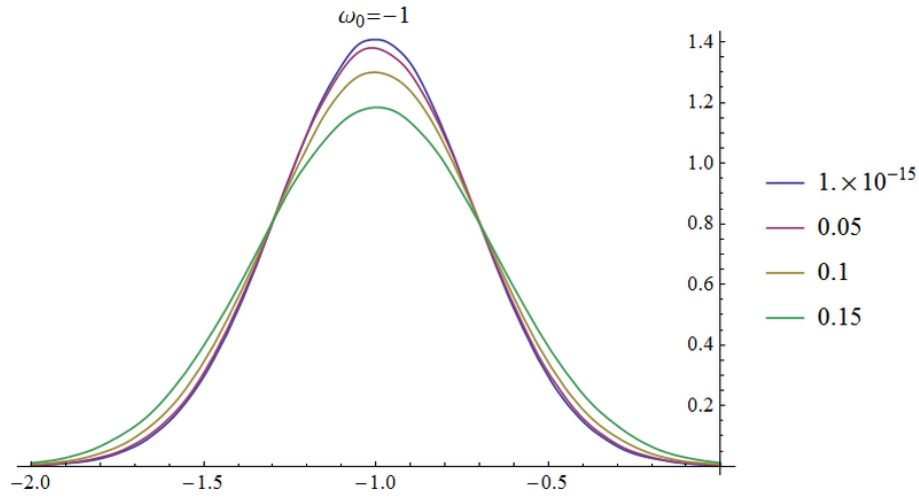


Figure 4: kernel density estimation $\omega_0 = -1$

Ger has been known to tweak machines in response to personal requests, so if asked nicely he may let me use two or three of these machines concurrently.

Given these resources, I feel there are three ways I could continue:

- I could upgrade to the latest version of Mathematica on all machines, and set up a Mathematica cluster with **Digital Comms Lab 1** as the front end and the EDA Lab PCs as remote nodes. With this setup, all machines would act as one (as in a traditional cluster). This would be the easiest to use, but would require considerable work to set up.
- I could use the **MathLink** interface to achieve a similar, lower-level version of the former, with the EDA Lab machines as independent, remote slaves and **Digital Comms Lab 1** sending commands to these slaves and collating the replies. This setup is distributed computing with a star topology, and would be easier to setup. The downside is that the code needs to manually divide the task between each of the nodes, and needs to be well designed to minimise network delays.
- I could simply run the code in parallel on each of the machines available to me, dumping the results to text files, and collate the data at the end. This would require no setup, and code written on any machine would only require porting to another version of Mathematica. Additionally this seems like it would deal best with hiccups such as machines going down and it does not depend on a connection between the machines. The downside is there would be some overhead with collecting the results afterwards.

Week 2 Summary

I fixed the code written last week and began setting up my simulation environment.

Goals for Week 3

- Work out a setup that will allow me to carry out large-scale simulations.
- Adapt the previous code to run in parallel and produce useful machine-readable output.

Week 3

14/10/13 - Running longer scripts on the EDA machines

Today I spent some time figuring out how to build and run scripts on the EDA machines. I found that defining a module in a text file and copying the Mathematica code into the module allows the code to be called with input arguments, and writing the output to a text file and placing the module in a loop allows each pass to be recorded for later parsing⁵. After running the code overnight, this system appears to work, and is scaleable over multiple machines. The main disadvantage is the size of these files (7.7MB per 400,000 values), so I must either figure out how to transfer them over the network or see whether reducing the precision of the output values will reduce the file sizes.

15/10/13 - Reducing output size

Given the 15GB of samples produced the night before was far too much to pull off the machine, I copied 20 million of the samples and plotted them to make sure the script had worked in practice⁶. I then looked into how I could reduce the size of the output produced, and decided to replace the `SmoothKernelDistribution` function (which came in after Mathematica 6.0 and therefore couldn't be used on the EDA machines) with a fine-grained histogram function⁷. This allowed me to add the probabilities generated in each sweep to those generated before and keep the output to a handful of 1kB files. I ran the simulation overnight to check it.

⁵Using the `Get` and `Put` methods. The `DumpSave` method is supposed to be more efficient, but was added after Mathematica 6.

⁶For the record, I could only use a fraction of them, as loading all 20 million samples crashed the machine for over an hour.

⁷I am assuming that both approach the true PDF as $N \rightarrow \infty$

16/10/13 - Moving onto 4-PAM

Checking the output from the night before, I get a similar PDF plot as with the `SmoothKernelDistribution` function. I therefore modified the code to examine all 3 decision region boundaries in a 4-PAM system and ran the simulation for 100 million samples per condition. The resulting distributions shown below show increased probability of error with timing error, as expected, but decision region boundaries in this case remain the same.

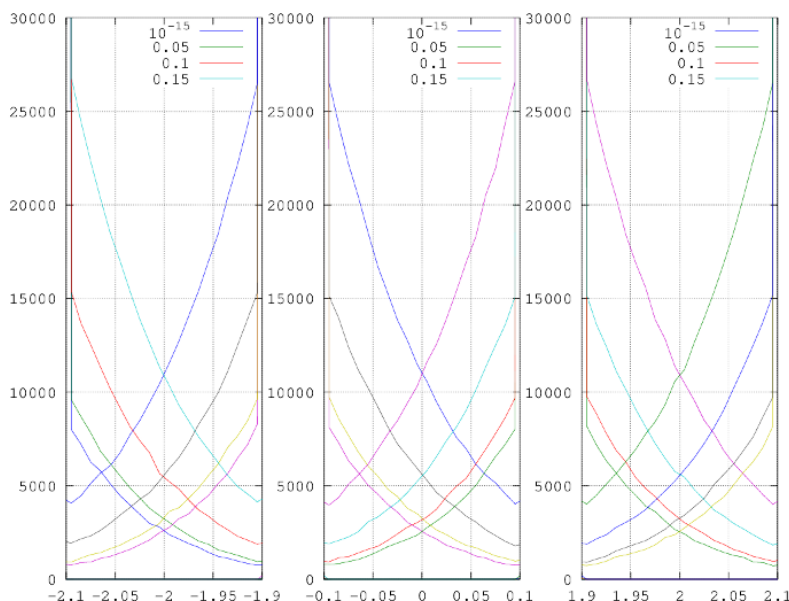


Figure 5: PDF for 4-PAM, $\omega_0 \in -3, -1, 1, 2, 10^8$ samples

I could imagine finding a value for the probability of error and moving onto PSK systems as the next steps in the process.

Week 3 Summary

Code was written that could be executed in parallel on multiple machines, and this was demonstrated in practice. The code was extended to the 4-PAM case, and showed no change in optimum decision region boundaries. Upon later consultation with Dave, it seems this is because the decision region boundaries shift due to a change in the g_0 term, and not the appearance of ISI components due to the g_k terms; the latter was believed to be the expected cause, and so the g_0 term was assumed to be 1 in the code.

Goals for Week 4

- Re-run the simulations to see if implementing the change in g_0 with timing error changes the location of the optimum decision region boundaries.
- If so, it would be interesting to see if the Gram-Charlier approximation produces the same boundary locations.

Week 4

21/10/13 - Implementing the Gram-Charlier series

Over the weekend, I implemented the g_0 term fix discussed in our Friday weekly meeting and re-ran the simulation, this time across two machines. Results showed that the Decision Region Boundary is displaced towards the origin as the timing offset increases.

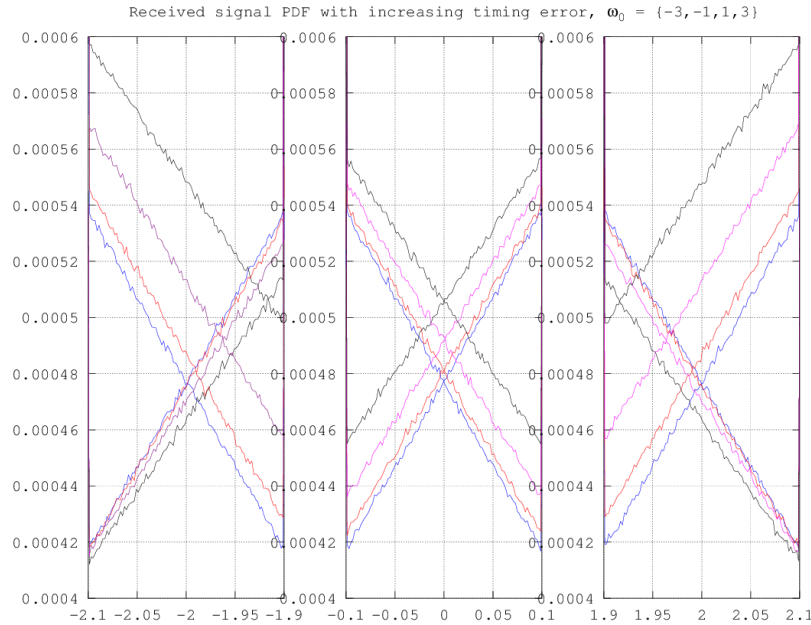


Figure 6: PDF for 4-PAM, $\omega_0 \in -3, -1, 1, 2$

I spent Monday carrying out two tasks:

1. I re-wrote Dave's Gram-Charlier equations⁸ for Mathematica, and should

⁸See *davenotes.pdf*

be ready to try them out tomorrow.

2. I modified the PAM simulation with a coarser-grained histogram, but more timing offset values, in order to see how the decision variate varies with timing offset. The results should be available in the morning.

22/10/13 - More Gram-Charlier series

The simulation results showed that the decision region boundaries did decrease with timing error, however the histogram was not fine-grained enough to accurately determine the exact boundary locations, so the simulation was re-run with more bins.

I fixed some bugs in my implementation of the Gram-Charlier series and was able to generate a few plots, which were very similar to those generated by the simulator, albeit with half the amplitude. A goal for tomorrow is to generate the plots with identical timing offsets to the simulation and compare both plots.

23/10/13 - Proper Gram-Charlier plots

The simulation results had been appended to the previous set of results by accident, so the whole thing had to be run again for tomorrow. On a more positive note, I noticed a missing power in my implementation of the Gram-Charlier series, and the plots are now a lot closer to those generated previously.

Week 4 Summary

I implemented the Gram-Charlier series and was able to compare the results from the simulation and the Gram-Charlier series. These are close, but not exact, so we will have to look closely at where the differences may be coming from.

Goals for Week 5

- I will make use of the long weekend to run some extra-long simulations and compare these to the Gram-Charlier series.

Week 5

29/10/13 - Comparing Gram-Charlier to Simulation

The simulations ended, and I was able to compare simulated and gram-charlier PDF plots. I extracted a rough estimate of the decision region boundaries given

by both methods and compared them to the corresponding values of $2g(\Delta)$, and found very close correlation.

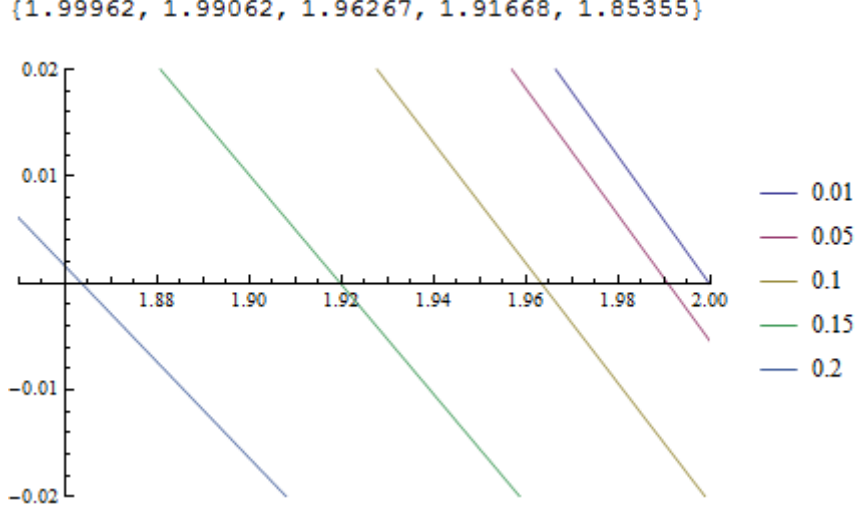


Figure 7: Gram Charlier approximation of $P(\omega_0 = 1, R) - P(\omega_0 = 3, R)$

30/10/13 - Applying the Tikhonov Distribution

I was able to implement the Tikhonov Distribution using the equation provided in *PAMTikhonov.pdf*:

$$F_{\Delta}(y) = \frac{\text{Exp} \left[\frac{\cos(2\pi y)}{(2\pi\sigma_{\Delta})^2} \right]}{I_0 \left(\frac{1}{(2\pi\sigma_{\Delta})^2} \right)} \text{ where } -\frac{1}{2} \leq y \leq \frac{1}{2}$$

Given these timing error probabilities and the optimum decision region boundaries for each timing error, I calculated the overall optimum decision region boundary for each timing error probability distribution using

$$B_{\text{OPT}} \sim \sum_{\Delta} P(\Delta) B_{\text{OPT}, \Delta}$$

It is important to note that with increasing variance, the probability density function places more weight on larger timing errors outside the range simulated, so these results are less accurate for higher variances.

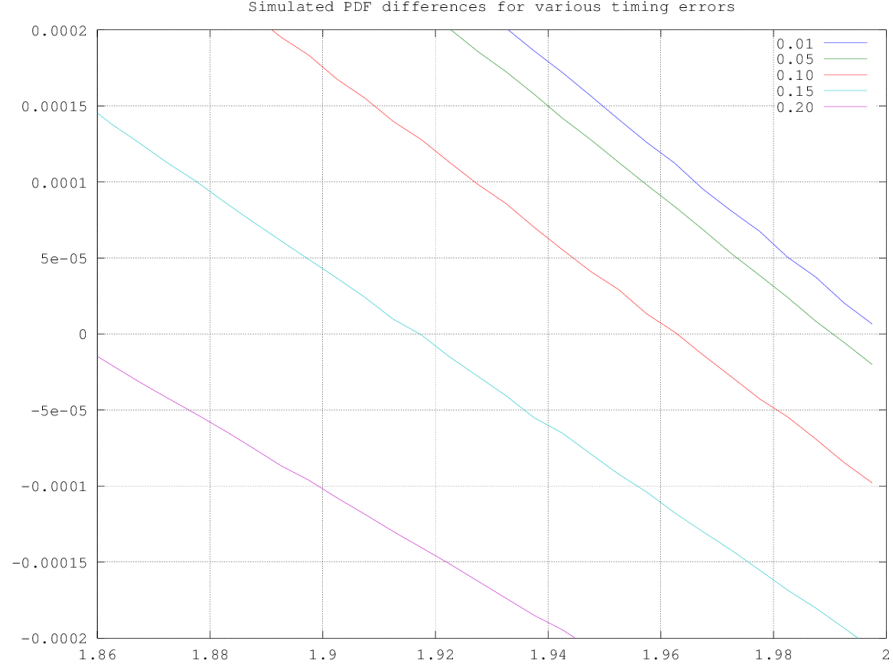


Figure 8: Simulation of $P(\omega_0 = 1, R) - P(\omega_0 = 3, R)$, $N=3 \times 10^6$

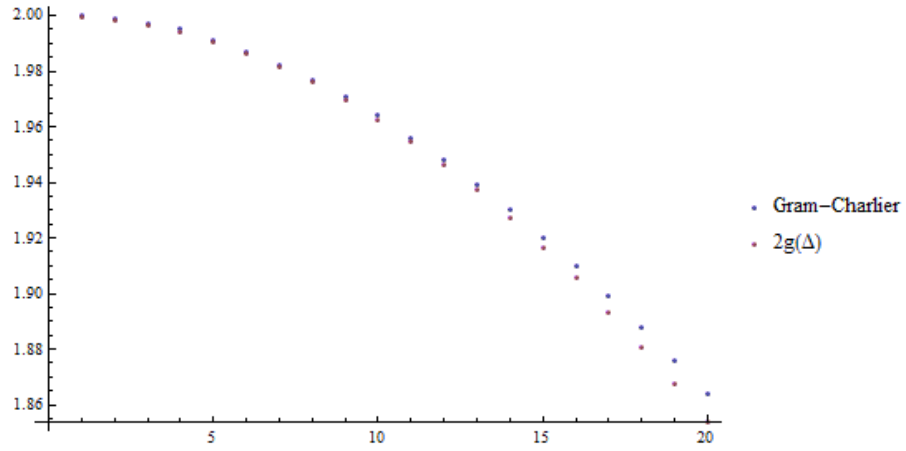


Figure 9: Comparison of Gram-Charlier Decision Region Boundaries and $2g(\Delta)$ estimation ($0.01 \geq \Delta \geq 0.2$)

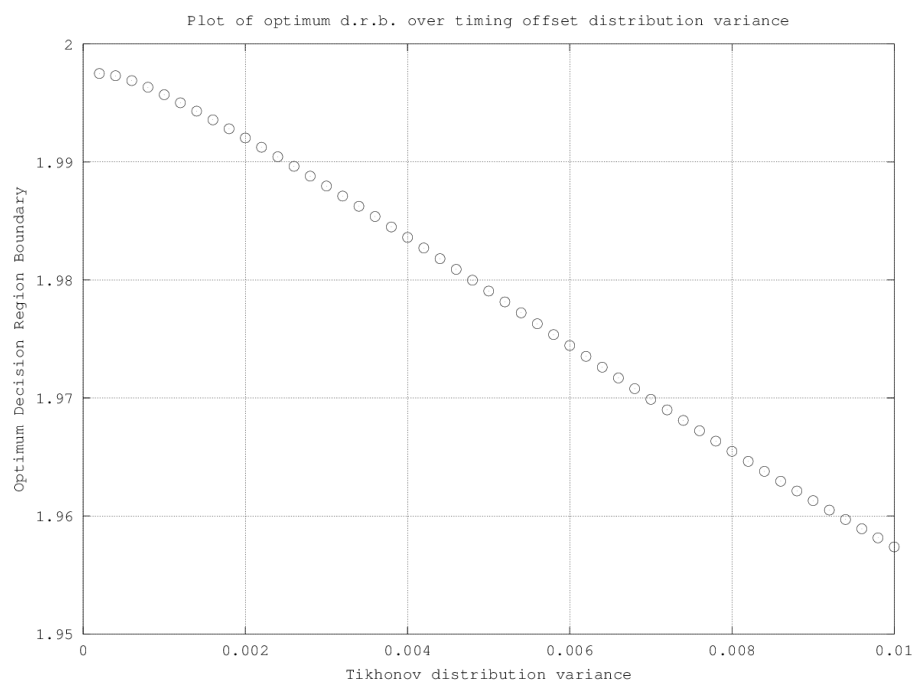


Figure 10: Optimum Decision Region Boundary for various timing error probability distributions

Week 5 Summary

In week 5, I calculated the optimum decision region boundary for a range of timing offsets, through simulation and the Gram-Charlier approximation. I demonstrated a close correlation between these boundaries and the $2g_k$ term. A slight difference between the Gram-Charlier approximation was found to be due to a typo in its implementation. I applied the Tikhonov distribution to the calculated optimum decision region boundaries for each timing offset, in order to calculate an optimum decision region boundary for a given Tikhonov distribution of timing offsets

Goals for Week 6

- On the simulation side, a key goal for week 6 is to randomly generate timing offsets according to the Tikhonov distribution and apply these to the simulation as timing offsets, in order to verify correlation with the Gram-Charlier and $2g_k$ approximations.
- A typo in the Gram-Charlier implementation has been found and corrected, and it would be interesting to see if this approximation matches $2g_k$.

Week 6

04/11/13 - Fixing errors

Dave took a look at my code and spotted errors which I fixed. The fixed Gram-Charlier implementation was found to match $2g_k$ very closely. The fixed simulation was left to run overnight; unfortunately Mathematica 6.0 running on the Unix machines was unable to run it, so the number of points had to be reduced.

05/11/13 - Corrected simulation results

The produced PDFs were too inaccurate to properly calculate the zero crossing points, so the simulation will have to be run over several days.

Week 6 Summary

A simulation was constructed that generated timing error offsets according to a Tikhonov distribution of predetermined variance, and used to produce received symbol PDFs. The simulation was found to run very slowly, and could only be run on Mathematica 9. Ger has been asked whether it would be possible to upgrade the Unix machines to this version and he will look into it.

Goals for Week 7

- Continue running the simulation, trying to speed it up if at all possible.

Week 7

11/11/13 - Returning to the UNIX machines

The UNIX machines were upgraded to Mathematica 9 over the weekend, so I was able to port the code in order to run off these. In addition, Dave suggested that I look into parralising the code. Since these were dual-core machines I was able to make use of Mathematica's `ParallelTable` function to reduce run times a little. Th simulation will have to run over several days, however, as the expected deviation in optimum decision region boundary is very small.

19/11/13 - Day 9 of Week 7

After several days of running the simulations, we found the optimum decision region boundaries given by the simulations, in red, converged to roughly those predicted by averaging the optimum decision region boundary of a timing offset over the tikhonov distribution of timing offsets, in blue, given by the equation:

$$B_{\text{OPT}} \sim \sum_{\Delta} P(\Delta) B_{\text{OPT}, \Delta}$$

Week 7 Summary

Simulations supported the theory that the optimum decision region boundary in the presence of statistically distributed receiver timing errors will decrease from the expected value. Additionally, it was shown through simulation that the new optimum decision region boundary can be aproximated, assuming a known distribution of these timing errors, by averaging the optimum decision region boundary given each timing offset over the distribution of timing offsets.

Goals for Week 8

- Verify that the Gram-Charlier series provides an adequate approximation to the received symbol PDF in the presence of timing errors.
- Provide numerical values for the probability of error P_e in the presence of a distribution of timing errors.

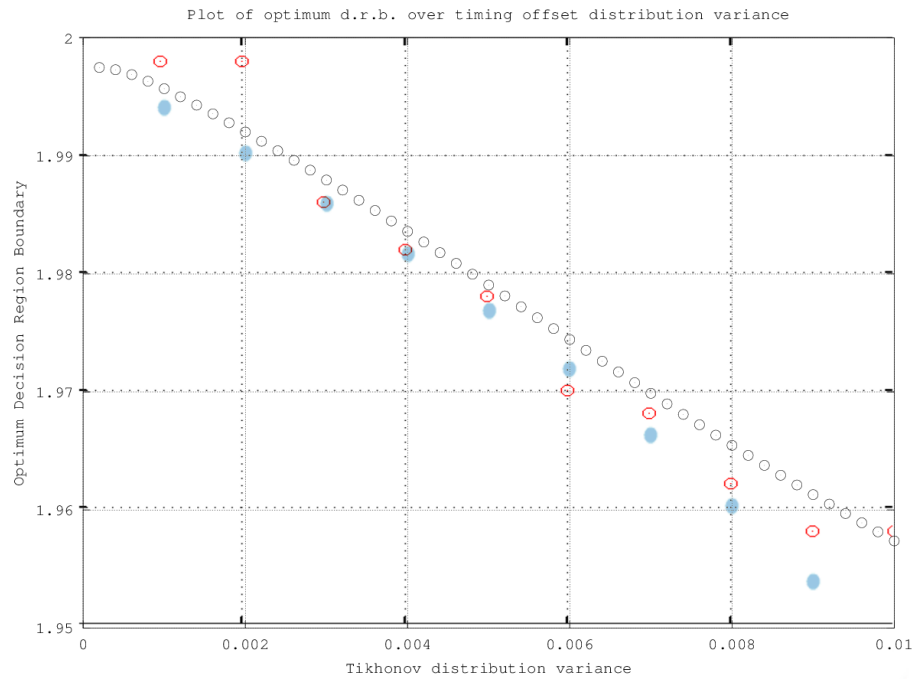


Figure 11: Optimum Decision Region Boundary for various timing error probability distributions

Week 8