Simplified Noisy Reference Loss Evaluation for Digital Communication in the Presence of Slow Fading and Carrier Phase Error

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***Abstract*—Using a Maclaurin series expansion of the conditional (on the carrier phase error) bit error probability of binary and quaternary phase-shift keying (BPSK and QPSK, respectively) and then averaging this result over both the statistics of the carrier phase error and the channel fading, closed-form expressions for the average bit error probability are derived in a form that lend themselves toward obtaining simple formulas for the associated noisy reference loss. Numerical evaluation of this loss based on the use of these formulas is shown to provide excellent accuracy when compared with the exact evaluation, which requires two-fold numerical integration. Although the method is specifically applied to BPSK and QPSK, it is easily extended to other forms of modulation whose conditional bit error probability is known in closed form.**

***Index Terms*—Carrier synchronization, digital communicating, fading channels, noisy reference loss.**

I. Introduction

THE PROBLEM of imperfect carrier synchronization and its deleterious effect on the performance of digital communication systems has long been of interest, dating back to the early 1960s, when Tikhonov [1], [19] and Viterbi [2] first modeled the probability density function (pdf) of the phase error in a first-order phase-locked loop (PLL). Using this model (which was later shown to also be appropriate for second-order loops [3]), Lindsey in 1966 [4] analyzed the error probability performance of such nonideal coherent detection[1] of binary phase-shift keying (BPSK) in additive white Gaussian noise (AWGN), assuming that the phase error process varies slowly relative to the data rate. Since that time many others [8]–[12] have studied this problem both for BPSK and other modulations, such as quaternary phase-shift keying (QPSK), arriving at results primarily in the form of upper and lower bounds and approximations for evaluating bit error probability (BEP).

The extension of the above to the fading channel began with the work of Weber [13], who studied the performance of PLLs in the presence of slow fading channels such as those characterized by Rayleigh, Nakagami-n (Rice), and log-normal distributions. Later, many authors [5], [6], [14] considered the effects of imperfect carrier tracking on RAKE reception [both maximal ratio combining (MRC) and equal-gain combining (EGC)] of BPSK and QPSK in the presence of frequency-selective multipath fading. For that purpose, exact, approximate, and numerical methods, e.g., Gauss–Chebyshev quadrature based on the moment generating function (MGF) of the test statistic [15], were used in these papers to evaluate the average BEP.

In this paper, we consider the single-channel (no diversity) problem and based on the approximate technique discussed in [9] for unbalanced QPSK, we obtain closed-form expressions for average BEP of BPSK and QPSK for several slow fading channel statistical models. Using these results, we derive, as an example, simple-to-evaluate formulas for the noisy reference loss of BPSK and QPSK in slow Rayleigh fading. The excellent accuracy of these closed-form results over a large range of average signal-to-noise ratios (SNRs) is demonstrated by comparison with exact results that require two-fold numerical integration for their evaluation.

II. Average Probability of Error BPSK in the Presence of Slow Fading and Carrier Phase Error

Consider the case of nonideal coherent detection of BPSK where the word “nonideal” refers to the fact that the phase $\hat{\theta}$ of the local carrier reference provided at the receiver is not perfectly matched to the phase θ of the received carrier. Letting $\phi = \theta - \hat{\theta}$ denote the phase error, then it is well known that for the AWGN channel, the conditional (on the phase error) BEP of BPSK is given by

$$P_b(E|\phi) = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\,\cos\phi\right) \tag{1}$$

where $\mathrm{erfc}(\cdot)$ is the complementary error function and E_b/N_0 is the energy-per-bit to noise power spectral density ratio. Regardless of whether the local reference is provided by a PLL in the discrete carrier case or a Costas or decision feedback data-stripping loop, e.g., a data-aided loop, in the suppressed carrier case, the pdf of the phase error is typically modeled by a Tikhonov distribution [1], [19]. For the simplicity of our discussion, here we shall assume the former, which is tantamount to assuming that a pilot tone (unmodulated carrier) is transmitted and available at the receiver for carrier synchronization purposes. As such,

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[1]Some authors [5], [6] refer to coherent detection with imperfect carrier synchronization as “partially coherent detection,” which more properly should be reserved for the situation where *partial* information of the carrier phase is available at the receiver and is exploited in designing the structure of the receiver (see [7, ch. 6]).

a PLL is appropriate, and the pdf of ϕ is given by[2]

$$p_\phi(\phi) = \frac{\exp(\rho_c \cos \phi)}{2\pi I_0(\rho_c)}, \qquad -\pi \le \phi \le \pi \tag{2}$$

where

- $I_n(\cdot)$ — nth-order modified Bessel function of the first kind;
- $\rho_c \triangleq P_c/N_0 B_L$ — loop SNR with P_c denoting the power in the discrete carrier (pilot);
- B_L — single-sided loop bandwidth.

To exactly compute the average BEP on the AWGN one would simply average the conditional BEP of (1) over the pdf in (2). For a given available total power P_t, a fixed fraction of it $\eta = P_c/P_t$ is allocated to the pilot and the remaining fraction $1-\eta = (P_t - P_c)/P_t = P_d/P_t$ is available for data detection. Since the power available for detection P_d is related to the bit energy and bit time by $P_d = E_b/T_b$, then the loop SNR can be linearly related to E_b/N_0 by

$$\begin{aligned}\rho_c &= \frac{P_c}{N_0 B_L}\\ &= \frac{P_c/P_t}{(P_d/P_t)N_0}\,\frac{P_d T_b}{B_L T_b}\\ &= \frac{\eta}{1-\eta}\,\frac{1}{B_L T_b}\,\frac{E_b}{N_0}\\ &\triangleq C\,\frac{E_b}{N_0}\end{aligned} \tag{3}$$

where C is a constant of proportionality, which, when expressed in dB, represents the amount by which the loop SNR (in dB) exceeds the energy-per-bit to noise power spectral density ratio (in dB). Note that a plot of error probability versus E_b/N_0 for the scenario described above wherein the ratio of carrier-to-data power is fixed does not exhibit an irreducible error probability since ρ_c increases (linearly) as E_b/N_0 increases.

For the slow fading channel and under the further assumption that the fading bandwidth is much smaller than the loop bandwidth,[3] the results of (1) and (2) together with (3) would be modified by replacing E_b/N_0 by the instantaneous fading SNR $\gamma = \alpha^2 E_b/N_0$, where α is the fading amplitude. The exact average BEP is then evaluated as

$$P_b(E) = \int_0^\infty \int_{-\pi}^{\pi} P_b(E|\phi,\gamma) p_\phi(\phi|\gamma) p_\gamma(\gamma)\, d\phi\, d\gamma \tag{4}$$

where $p_\gamma(\gamma)$ is the pdf of γ. For example, if α is Rayleigh distributed, or equivalently, γ has an exponential pdf, then in the presence of slow Rayleigh fading and carrier phase error, the average BEP is given by[4]

$$\begin{aligned}P_b(E) = \int_0^\infty \int_{-\pi}^{\pi} \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\gamma}\cos\phi\right) \frac{\exp(C\gamma\cos\phi)}{2\pi I_0(C\gamma)}\,\frac{1}{\overline{\gamma}}\\ \cdot \exp\left(-\frac{\gamma}{\overline{\gamma}}\right) d\phi\, d\gamma\end{aligned} \tag{5}$$

where $\overline{\gamma} = \Omega E_b/N_0$ is the average fading SNR with $\Omega = \overline{\alpha^2}$ the average fading power.

A plot of $P_b(E)$ versus $\overline{\gamma}$ obtained from (5) for fixed C relative to a plot of BEP for ideal coherent detection in AWGN, i.e., $P_b(E) = 1/2\,\mathrm{erfc}(\sqrt{E_b/N_0})$, reveals the amount of degradation in this performance measure caused by the combination of fading and carrier phase error. Alternatively, this degradation is specified in terms of the amount of additional SNR that must be provided at a fixed value of BEP and is referred to as *noisy reference loss*. To compute noisy reference loss, one must translate the vertical degradation (increase in BEP at a fixed SNR) obtained from the plot of $P_b(E)$ versus SNR to a horizontal degradation (increase in required SNR at a fixed BEP). A simple method to accomplish this goal was discussed in [9] for the AWGN channel and relied upon first simplifying (approximating) the evaluation of $P_b(E)$ by expanding $P_b(E|\phi)$ in a Maclaurin series in ϕ prior to averaging over $p_\phi(\phi)$ and then maintaining only the first two terms. This approach will now be extended to the fading channel to arrive at a similar simple rule-of-thumb formula for evaluating noisy reference loss.

For BPSK modulation, adapting the expansion of (1) into a Maclaurin series to the fading channel and averaging over the pdf of ϕ, we obtain analogous to [9, (26)]

$$\begin{aligned}P_b(E|\gamma) = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\gamma}\right) + \frac{1}{2}\sqrt{\frac{\gamma}{\pi}}\,\exp(-\gamma)\sigma_\phi^2\\ + \text{higher order terms.}\end{aligned} \tag{6}$$

For the AWGN case, if one makes the assumption of large-loop SNR, then the loop is said to perform in its linear region of operation. The Tikhonov pdf of (2) can be approximated by a Gaussian pdf for which $\sigma_\phi^2 \cong 1/\rho_c = [C(E_b/N_0)]^{-1}$. For the fading channel, we shall (as was done in [5]) make the analogous assumption that $\sigma_\phi^2 \cong (C\gamma)^{-1}$. Strictly speaking, the assumption of linear region behavior for the fading channel case is not totally valid since here the SNR γ is a random variable (RV) that ranges from zero to infinity. However, for large $\overline{\gamma}$, the pdf of γ will be skewed to the right, and thus the contribution to the average BEP from small values of γ (where the linear region assumption would be violated) will be likewise small. The true degree of validity of this linear region approximation over the entire range of γ can only be justified by comparison with the exact results computed based upon using (5) for evaluation of average BEP and will be demonstrated shortly.

[2]Extension to the case of suppressed carrier transmission is straightforward in view of the analogous statistical properties of Costas and data-aided loops given in [16] and is not treated here. By analogous statistical properties, we mean that the pdf can still be modeled as a Tikhonov distribution; however, because of the squaring loss associated with such loops, the effective loop SNR (which includes the squaring loss) will not be a linear related to E_b/N_0 as in (3). Thus, although the technique to be described is, in principle, still applicable, the ability to obtain closed-form results for the SNR loss due to the noisy reference depends on the particular fading statistics assumed.

[3]This is the same assumption as that made by Eng and Milstein in [5]. In the mobile communication application where Doppler spread is a consideration, the implication of this assumption is that the loop bandwidth should be at least on the order of five times the Doppler spread. This, together with the fixed carrier-to-data power ratio assumed, has a direct impact on the values of the constant C that characterize this application. Since the theory is most accurate when C is large, the numerical results that follow are more typical of the low-mobility (small Doppler spread) case.

[4]Eng and Milstein [5] also consider an approximation to (5) obtained by replacing the average of $P_b(E|\phi,\gamma)$ over ϕ by $1/2\,\mathrm{erfc}(\sqrt{\gamma}\,\overline{\cos\phi}) = 1/2\,\mathrm{erfc}(\sqrt{\gamma}\,I_1(\rho_c)/I_0(\rho_c)) = 1/2\,\mathrm{erfc}(\sqrt{\gamma}\,I_1(C\gamma)/I_0(C\gamma))$, thus eliminating one of the integrations.

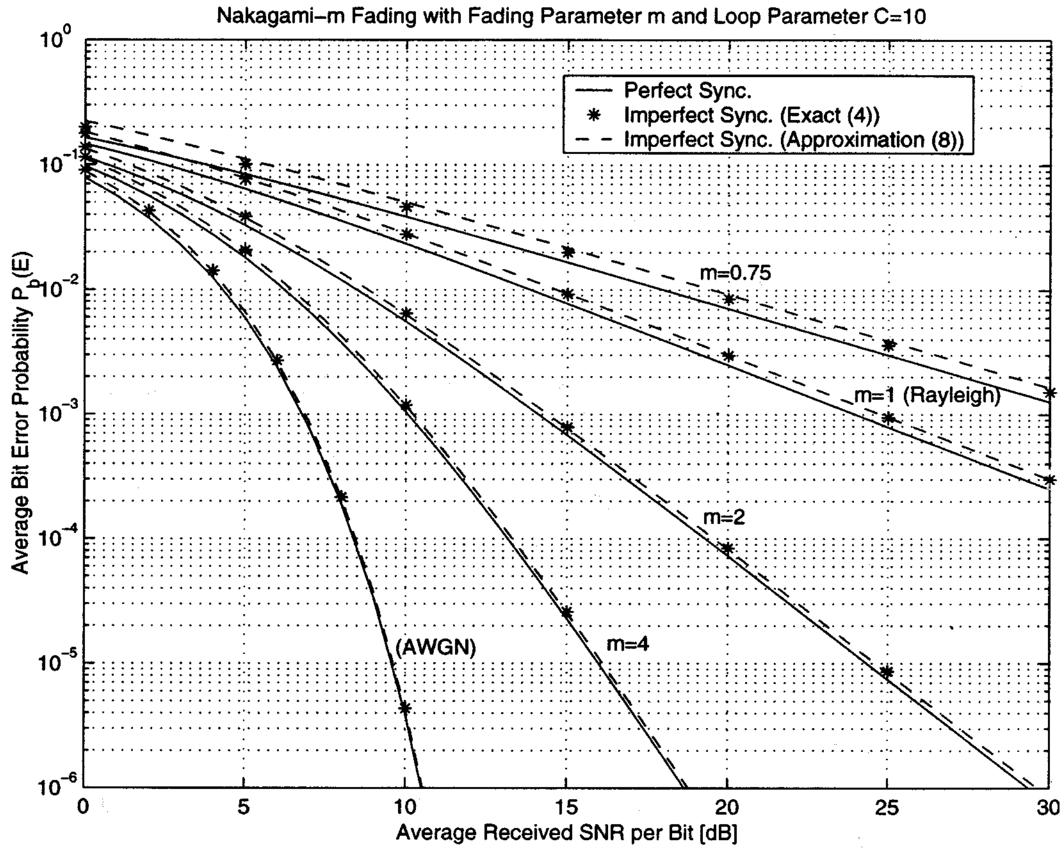

Fig. 1. Combined effect of Nakagami-m fading and imperfect synchronization on the average bit error probability of BPSK.

In the meantime, proceeding with the above inverse linear relation between σ_ϕ^2 and γ and neglecting higher order terms, we obtain from (6)

$$P_b(E|\gamma) \cong \frac{1}{2}\,\mathrm{erfc}\,(\sqrt{\gamma}) + \frac{1}{2C\sqrt{\pi\gamma}}\;\exp(-\gamma) \tag{7}$$

where the first term represents the ideal (perfect carrier phase reference) performance. Averaging (7) over the pdf of γ gives the desired average BEP, which is tabulated below in closed form (with the help of tabulated integrals in [17]) for several different fading channel models whose pdfs can be found in standard texts on the subject, e.g., [18, ch. 2]:

A. Nakagami-m Fading

$$\begin{aligned} P_b(E) \cong& \frac{1}{2}\sqrt{\frac{\overline{\gamma}}{\pi}}\,\frac{m^m}{(m+\overline{\gamma})^{m+1/2}}\,\frac{\Gamma\left(m+\frac{1}{2}\right)}{\Gamma(m+1)^2} \\ &\cdot {}_2F_1\left(1,\, m+\frac{1}{2},\, m+1;\, \frac{m}{m+\overline{\gamma}}\right) \\ &+ \frac{\Gamma\left(m-\frac{1}{2}\right)}{2\sqrt{\pi\overline{\gamma}}\,C\Gamma(m)}\,\frac{m^m}{(m+\overline{\gamma})^{m-1/2}}, \qquad m \geq \frac{1}{2} \end{aligned} \tag{8}$$

where $\Gamma(\cdot)$ is the gamma function and ${}_2F_1(\cdot,\cdot,\cdot;\cdot)$ is the Gaussian hypergeometric function and m is a parameter that characterizes the severity of the fading process, e.g., $m = 1/2$ for a one-sided Gaussian fading, $m = 1$ for Rayleigh fading, and $m = \infty$ corresponds to the no fading case.

B. Nakagami-n Fading (Rice -$K = n^2$)

$$\begin{aligned} P_b(E) \cong& Q_1(a,\, b) - \frac{1}{2}\left(1+\sqrt{\frac{p}{1+p}}\right)\exp\left(-\frac{a^2+b^2}{2}\right) \\ &\cdot I_0(ab) + \frac{(1+n^2)}{2C\sqrt{\overline{\gamma}^2+(1+n^2)\overline{\gamma}}} \\ &\cdot \exp\left(-n^2\,\frac{2\overline{\gamma}+1+n^2}{2(\overline{\gamma}+1+n^2)}\right) I_0\left(\frac{n^2(1+n^2)}{2(\overline{\gamma}+1+n^2)}\right) \end{aligned} \tag{9}$$

where $Q_1(\cdot,\,\cdot)$ is the first-order Marcum Q-function and

$$\begin{aligned} a =& n\left[\frac{1+2p}{2(1+p)} - \sqrt{\frac{p}{1+p}}\right]^{1/2} \\ b =& n\left[\frac{1+2p}{2(1+p)} + \sqrt{\frac{p}{1+p}}\right]^{1/2} \end{aligned} \tag{10}$$

with $p = \overline{\gamma}/(n^2+1)$. The n parameter, which ranges from zero to ∞, corresponds to the square root of the ratio of specular to diffuse power of the fading process.

C. Rayleigh Fading (Nakagami-m Fading with and Nakagami-n with $n = 0$)

$$P_b(E) \cong \frac{1}{2}\left[1-\sqrt{\frac{\overline{\gamma}}{1+\overline{\gamma}}}\right] + \frac{1}{2C\sqrt{\overline{\gamma}(1+\overline{\gamma})}}. \tag{11}$$

Illustrated in Figs. 1 and 2 are the average BEP performances for Nakagami-m and Nakagami-n (Rice) fading with a loop SNR of

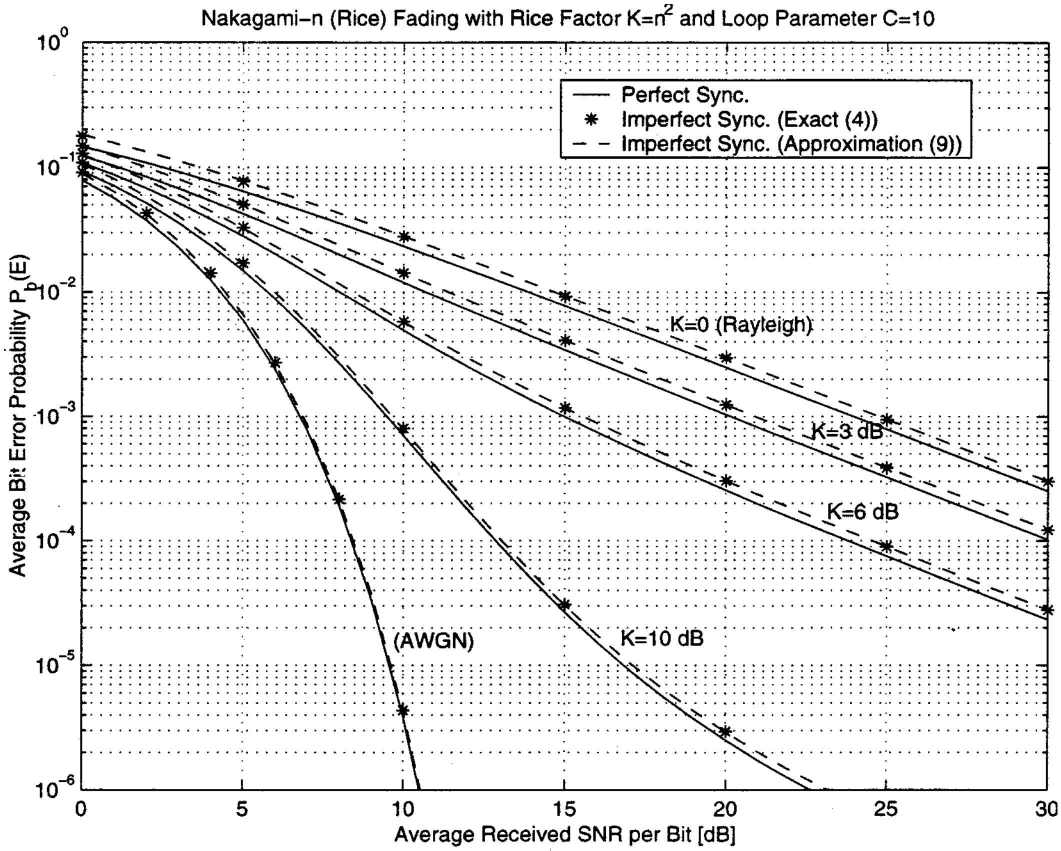

Fig. 2. Combined effect of Nakagami-n (Rice) fading and imperfect synchronization on the average bit error probability of BPSK.

10 dB above E_b/N_0. In each figure, the exact BEP as computed from (4) together with the appropriate fading pdf is plotted along with the approximate result obtained from (8) or (9). We observe that over a wide range of values for the m and K fading parameters, the approximate and exact results are virtually in perfect agreement. Further numerical experiments show that these approximate expressions can be used safely over a wide range of fading conditions as long as the loop parameter C is above approximately 8 dB (i.e., loop SNR about 8 dB above E_b/N_0).

III. Noisy Performance Loss of BPSK in the Presence of Slow Fading and Carrier Phase Error

Because of the linear relation between E_b/N_0 and loop SNR in (3), a plot of $P_b(E)$ on a logarithmic scale versus average SNR in dB would asymptotically be parallel to the analogous curve for ideal coherent detection. Thus, the noisy reference loss at a given BEP can be evaluated by dividing the degradation in BEP from the ideal by the slope of the BEP performance curve. In mathematical terms, the noisy reference loss L (in dB) is evaluated as

$$L = \frac{\log_{10} \dfrac{P_b(E)}{P_{bl}(E)}}{-\dfrac{d \log_{10} P_{bl}(E)}{d\Gamma}}, \qquad \Gamma = 10 \log_{10} \overline{\gamma} \tag{12}$$

where $P_{bl}(E)$ denotes the ideal BEP as determined from the first term in the Maclaurin series expansion of $P_b(E)$.

As an example of the evaluation of (12), consider the case of Rayleigh fading where from (11)

$$\begin{aligned} \log_{10} P_{bl}(E) &= \log_{10} \frac{1}{2}\left[1 - \sqrt{\frac{\overline{\gamma}}{1+\overline{\gamma}}}\right] \\ &= \log_{10} \frac{1}{2}\left[1 - \frac{1}{\sqrt{10^{-\Gamma/10}+1}}\right]. \end{aligned} \tag{13}$$

Differentiating (13) with respect to Γ and then substituting back in terms of $\overline{\gamma}$ gives

$$\frac{d \log_{10} P_{bl}(E)}{d\Gamma} = -\frac{1}{20\overline{\gamma}\left(1+\dfrac{1}{\overline{\gamma}}\right)\left(\sqrt{1+\dfrac{1}{\overline{\gamma}}}-1\right)}. \tag{14}$$

Finally, since from (11)

$$\frac{P_b(E)}{P_{bl}(E)} = 1 + \frac{1}{C\sqrt{\overline{\gamma}}\,\left(\sqrt{1+\overline{\gamma}} - \sqrt{\overline{\gamma}}\right)} \tag{15}$$

the noisy reference loss (in dB) is from (12) given by the desired simple-to-evaluate formula

$$\begin{aligned} L &= 20\overline{\gamma}\left(1+\frac{1}{\overline{\gamma}}\right)\left(\sqrt{1+\frac{1}{\overline{\gamma}}}-1\right) \\ &\quad \cdot \log_{10}\left[1 + \frac{1}{C\sqrt{\overline{\gamma}}\left(\sqrt{1+\overline{\gamma}}-\sqrt{\overline{\gamma}}\right)}\right] \end{aligned} \tag{16}$$

where from the first term of (11), $\overline{\gamma}$ is related to the ideal BEP at which one desires to evaluate this loss by

$$\overline{\gamma} = \frac{(1-2P_{bl}(E))^2}{1-(1-2P_{bl}(E))^2}. \tag{17}$$

TABLE I
NOISY REFERENCE LOSS L (IN dB) FOR BPSK OVER RAYLEIGH FADING CHANNELS

Average BEP, $P_{bI}(E)$	Loop Parameter C=10 (10 dB)	Loop Parameter C=20 (13 dB)
10^{-2}	0.824	0.431
10^{-3}	0.795	0.415
10^{-4}	0.792	0.414
10^{-5}	0.791	0.413

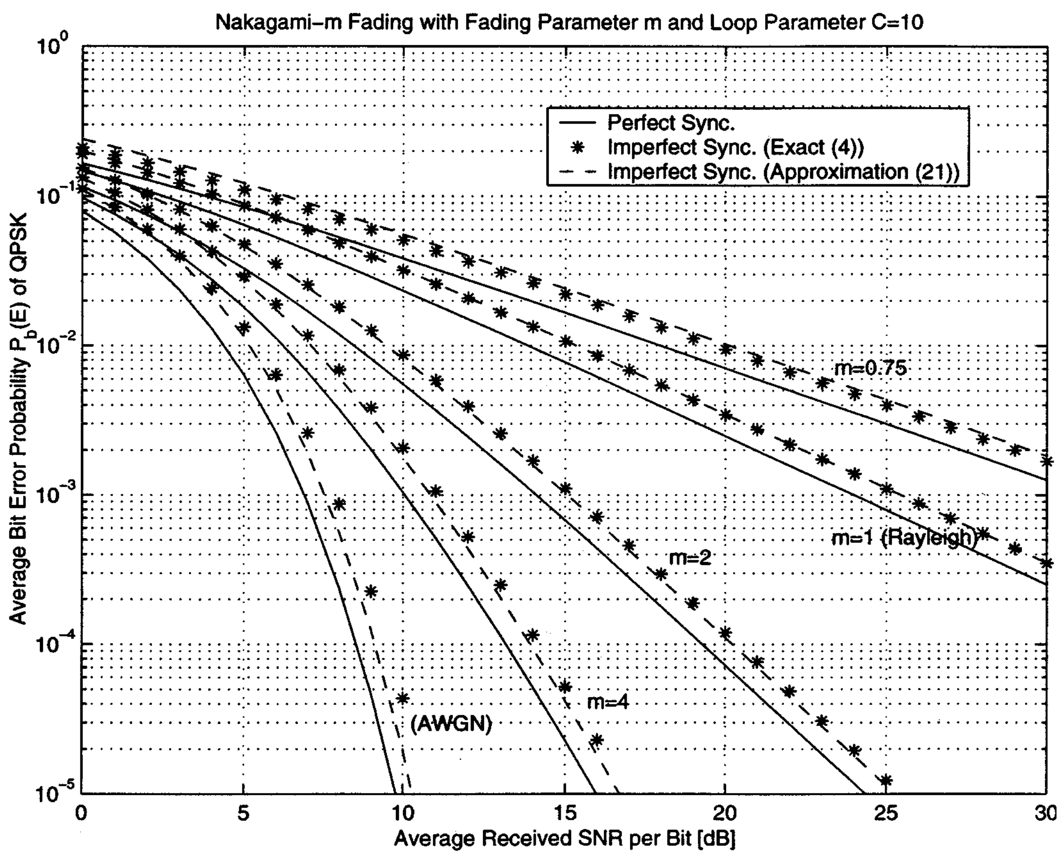

Fig. 3. Combined effect of Nakagami-m fading and imperfect synchronization on the average bit error probability of QPSK.

For sufficiently large $\overline{\gamma}$, the noisy reference loss of (16) asymptotically becomes

$$L = 10\log_{10}\left(1+\frac{2}{C}\right) \tag{18}$$

independent of $\overline{\gamma}$. This behavior is exhibited by the numerical results of Fig. 1 for $m = 1$ and Fig. 2 for $K = 0$. Table I shows for values of $C = 10$ and 20, the evaluation of the noisy reference loss for BPSK as given by (17) as a function of the average BEP at which one desires to operate and also confirms the convergence of this loss to the asymptotic result given in (18). As would be expected, the noisy reference loss decreases as BEP decreases since for smaller BEP, E_b/N_0 is larger, and hence from the linear relationship in (3), ρ_c is proportionally larger, resulting in a smaller loss.

IV. EXTENSION TO QPSK

The above results are easily extended to QPSK by using the appropriate Maclaurin series expansion of the conditional BEP for that modulation, namely

$$\begin{aligned} P_b(E|\phi) = &\frac{1}{4}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\,(\cos\phi+\sin\phi)\right)\\ &+\frac{1}{4}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\,(\cos\phi-\sin\phi)\right) \end{aligned} \tag{19}$$

which when adapted to the fading channel becomes (analogous to [9, (26)])

$$\begin{aligned} P_b(E|\gamma) = &\frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\gamma}\right)+\frac{1}{2}\sqrt{\frac{\gamma}{\pi}}\,(1+2\gamma)\,\exp(-\gamma)\sigma_\phi^2\\ &+\text{higher order terms.} \end{aligned} \tag{20}$$

TABLE II
NOISY REFERECE LOSS L (IN dB) FOR QPSK OVER RAYLEIGH FADING CHANNELS

Average BEP, $P_{bI}(E)$	Loop Parameter C=10 (10 dB)	Loop Parameter C=20 (13 dB)
10^{-2}	1.494	0.8088
10^{-3}	1.4664	0.7935
10^{-4}	1.4616	0.7919
10^{-5}	1.4613	0.7918

Then, the exact average BEP is computed by substituting γ for E_b/N_0 in (19) and averaging over the fading pdf as in (4), whereas the approximation to the average BEP would be obtained by averaging (20) over this same pdf substituting $(C\gamma)^{-1}$ for σ_ϕ^2. With regard to the latter, the following equations, expressed in terms of (8) and (11), are obtained.

A. Nakagami-m Fading

$$P_b(E)|_{\mathrm{QPSK}} \cong P_b(E)|_{\mathrm{BPSK}} + \frac{1}{C}\sqrt{\frac{\overline{\gamma}}{\pi}}\,\frac{m^m}{(m+\overline{\gamma})^{m+1/2}} \cdot \frac{\Gamma\left(m+\frac{1}{2}\right)}{\Gamma(m)}, \qquad m > \frac{1}{2}. \tag{21}$$

B. Rayleigh Fading (Nakagami-m Fading with $m = 1$)

$$P_b(E)|_{\mathrm{QPSK}} \cong P_b(E)|_{\mathrm{BPSK}} + \frac{1}{2C}\sqrt{\overline{\gamma}}\,\frac{1}{(1+\overline{\gamma})^{3/2}}. \tag{22}$$

Unfortunately, the authors were unable to obtain a closed-form result for the Rician case. The results in (21) and (22) clearly identify the added BER degradation attributed to QPSK relative to BPSK. Fig. 3 illustrates the analogous curve to Fig. 1, now for the QPSK case. As would be expected, because of the crosstalk between the I and Q channels, the noisy reference loss is larger for QPSK than it is for BPSK; however, the relative accuracy of the approximate evaluation is still quite good. Thus, we can still use the result in (20) to come up with a simple expression for the noisy reference loss in the QPSK case. Following the same procedure as for BPSK, we eventually arrive at the noisy reference loss formula (in dB)

$$L = 20\overline{\gamma}\left(1+\frac{1}{\overline{\gamma}}\right)\left(\sqrt{1+\frac{1}{\overline{\gamma}}}-1\right) \cdot \log_{10}\left[1+\frac{1+2\overline{\gamma}}{C\sqrt{\overline{\gamma}}\left(\sqrt{1+\overline{\gamma}}-\sqrt{\overline{\gamma}}\right)(1+\overline{\gamma})}\right] \tag{23}$$

with $\overline{\gamma}$ still given by (17). For sufficiently large $\overline{\gamma}$, the noisy reference loss of (23) asymptotically becomes

$$L = 10\,\log_{10}\left(1+\frac{4}{C}\right) \tag{24}$$

which is again independent of $\overline{\gamma}$. Again, Table II shows for values of $C = 10$ and 20 the evaluation of the noisy reference loss for QPSK, as given by (20), as a function of the average BEP at which one desires to operate and also confirms the convergence of this loss to the asymptotic result given in (24).

V. CONCLUSION

Using a Maclaurin series expansion of the conditional (on the carrier phase error) bit error probability of BPSK and QPSK and then averaging this result over both the statistics of the carrier phase error and the channel fading, it is possible to arrive at closed-form expressions for the average bit error probability in a form that lends itself toward obtaining simple formulas for the associated noisy reference loss. Numerical evaluation of this loss based on the use of these formulas provides excellent accuracy when compared with the exact evaluation, which requires two-fold numerical integration.

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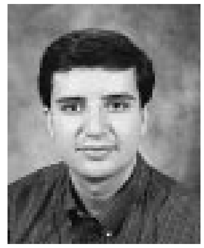

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