Problem 1 A machine produces rope at a mean rate of mean of 4 feet per minute with standard deviation of 5 inches. Assume that the amounts produced in different minutes are independent and identically distributed, approximate the probability that the machine will produce at least 250 feet in one hour.

$$S.D: (\sqrt{60} \cdot 5)/12 = 3.23 \text{ feet}$$

$$P(S_n \geq 250) = P\left(S_n^* \geq \frac{250 - 4 \cdot 60}{3.23}\right)$$

$$= P(S_n^* \geq 3.10)$$

$$= 1 - \Phi(3.10)$$

$$= 0.001$$

Problem 2 Assume that the distribution of the number of defects on any given bolt of cloth is the Poisson distribution with mean 5, and the number of defects on each bolt is counted for a random sample of 125 bolts. Determine the probability that the average number of defects per bolt in the sample will be less than 5.5 defects per bolt.

$$S.D: \sqrt{5/125} = 0.2$$

$$P(\bar{P} < 5.5) = P\left(\bar{P}^* < \frac{5.5-5}{0.2}\right)$$

$$= P(\bar{P}^* < 2.5)$$

$$= \Phi(2.5)$$

$$= 0.9938$$

Problem 3 Suppose that the proportion of defective items in a large manufactured lot is 0.1. What is the smallest random sample of items that must be taken from the lot in order for the probability to be at least 0.99 that the proportion of defective items in the sample will be less than 0.13?

$p = 0.1 \qquad \text{S.D:} \sqrt{\frac{(0.1)(0.9)}{n}} = \frac{0.3}{\sqrt{n}}$

$P(\bar{P} < 0.13) = P\left(\bar{P}^* < \frac{0.13 - 0.1}{0.3/\sqrt{n}}\right) \geq 0.99$

$= \Phi(0.1\sqrt{n}) \geq 0.99$

$0.1\sqrt{n} \geq 2.327 \qquad n \geq 542$

Problem 4 Suppose that 16 digits are chosen at random with replacement from the set $0, \ldots, 9$. What is the probability that their average will lie between 4 and 6?

Let X be a random digit.

$E(X) = \frac{1}{10}(0+1+\cdots+8+9) = 4.5$

$E(X^2) = \frac{1}{10}(0^2+1^2+\cdots+9^2) = 28.5$

$Var(X) = 28.5 - 4.5^2 = 8.25$

$s.d(\bar{X}) = (8.25/16)^{1/2} = 0.7181$

$P(4 \leq \bar{X}^* \leq 6) = P\left(\frac{4-4.5}{0.7181} \leq \bar{X}^* \leq \frac{6-4.5}{0.7181}\right)$

$= \Phi(2.0888) - (1 - \Phi(0.6963))$

$= 0.9816 - 0.2431$

$= 0.7385$