2. Let $X_1, \ldots, X_n \sim \text{Bernoulli}(p)$ and let $Y_1, \ldots, Y_m \sim \text{Bernoulli}(q)$. Find the plug-in estimator and estimated standard error for p. Find an approximate 90 per cent confidence interval for p. Find the plug-in estimator and estimated standard error for $p-q$. Find an approximate 90 per cent confidence interval for $p-q$.

$x_1 \ldots x_n$ Bernoulli (p)

$y_1 \ldots y_m$ Bernoulli (q)

$$\hat{p} = \frac{1}{n} \sum \text{Success}_i = \frac{1}{n} \sum_{i=1}^{n} x_i = \overline{x_n}$$

c.

plug in estimator for $p-q$:

$$\hat{p} - \hat{q} = \overline{x_n} - \overline{y_m}$$

$$\text{s.e.} = \sqrt{\text{Var}(\hat{p}-\hat{q})}$$

$$= \sqrt{\text{var}(\hat{p}) + \text{var}(\hat{q})}$$

$$= \sqrt{\overline{x_n}(1-\overline{x_n}) + \overline{y_m}(1-\overline{y_m})}$$

d. 90% CI for $p-q$.

$$\left((\overline{x_n} - \overline{y_m}) - 1.645\sqrt{\overline{x_n}(1-\overline{x_n}) + \overline{y_m}(1-\overline{y_m})},\right.$$

$$\left.(\overline{x_n} - \overline{y_n}) + 1.645\sqrt{\overline{x_n}(1-\overline{x_n}) + \overline{y_m}(1-\overline{y_m})}\right)$$

4. Let $X_1, \ldots, X_n \sim F$ and let $\widehat{F}_n(x)$ be the empirical distribution function. For a fixed x, use the central limit theorem to find the limiting distribution of $\widehat{F}_n(x)$.

$$\hat{F}_n(x) = \frac{\sum_{i=1}^{n} I(X_i \le x)}{n}$$

$$I(X_i \le x) = \begin{cases} 1 & X_i \le x \\ 0 & X_i > x \end{cases}$$

Let $Y_i = I(X_j \le x)$

CLT: $E(Y_i) = \mu_Y \quad Var(Y_i) = \sigma_Y^2 < \infty$

$n \to \infty$

$\sqrt{n}(\bar{Y}_n - \mu_Y) \to N(0, \sigma_Y^2)$

As $n \to \infty$

$\bar{Y}_n \to \mu_Y$

$I(X_j \le x) \to F(x)$ for every x

$$\hat{F}_n(x) \to \frac{\sum_{i=1}^{n} F(x)}{n} = F(x) \quad \text{so} \quad \hat{F}_n(x) \to F(x)$$

$F(x)$ is the limiting distribution.