ME 543

COMPUTATIONAL FLUID DYNAMICS

LID DRIVEN CAVITY

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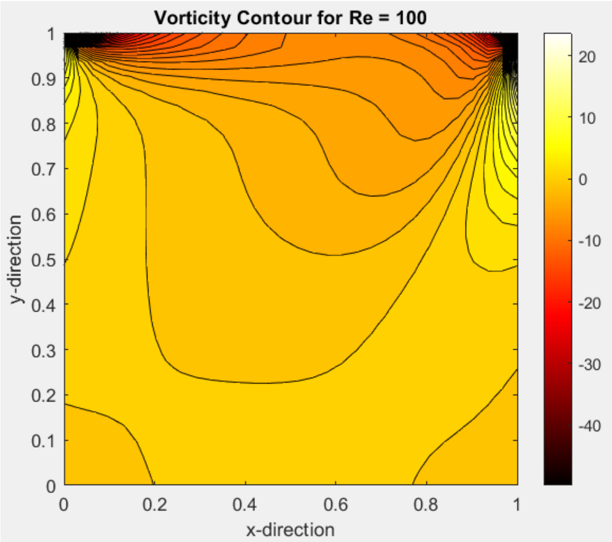
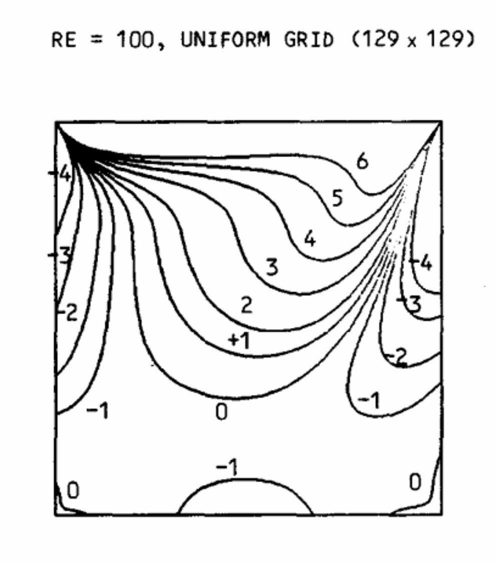
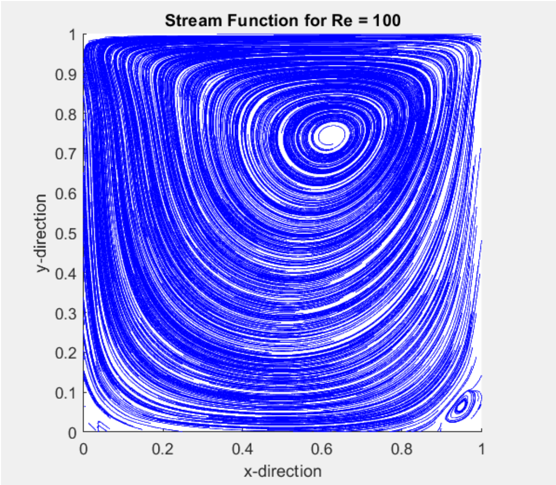
[Computational Mechanics]

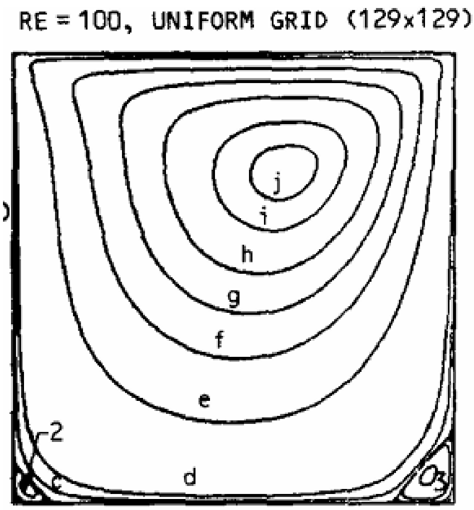
PROBLEM 1

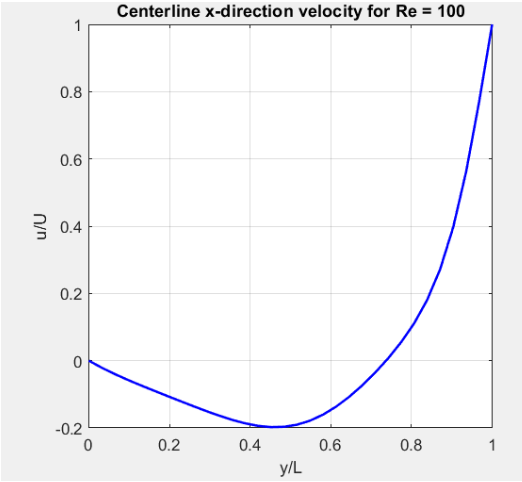
For Re = 100



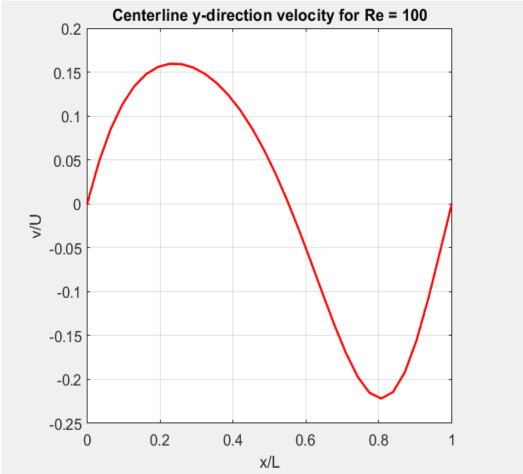




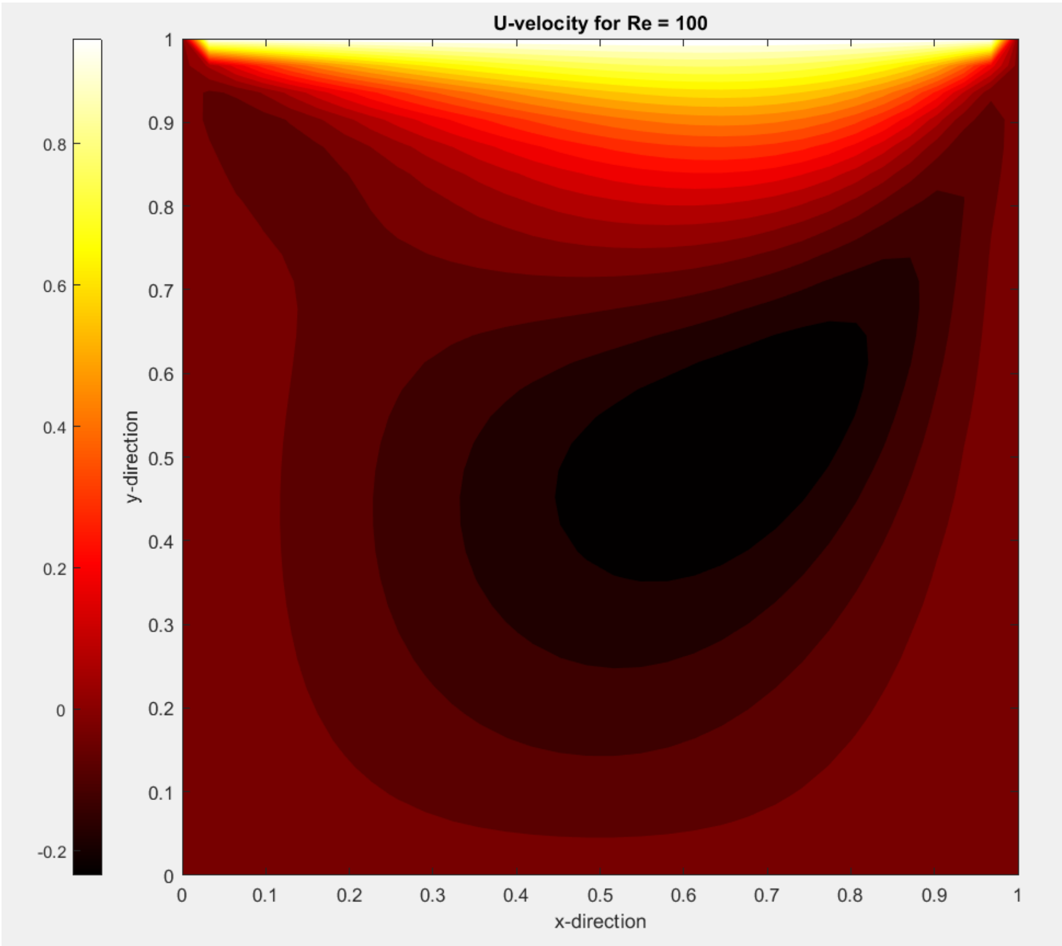




Centerline x-direction velocity for Re = 100
u/U
y/L

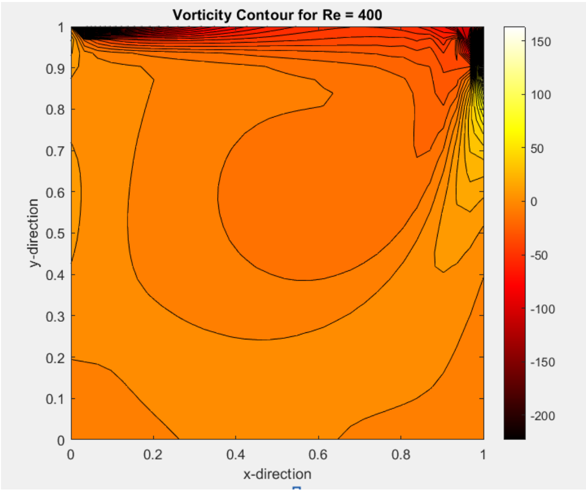


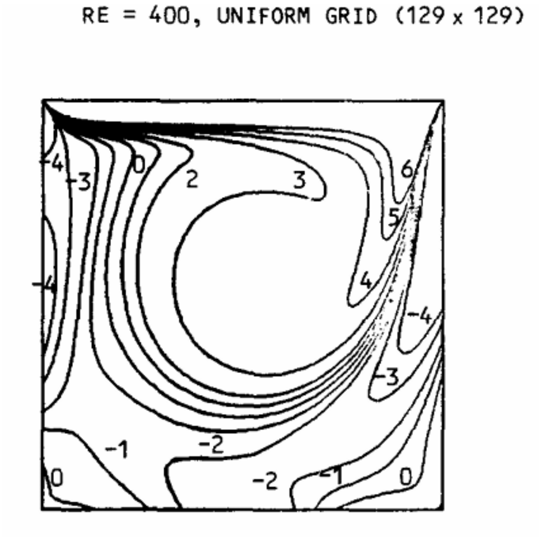
Centerline y-direction velocity for Re = 100
v/U
x/L

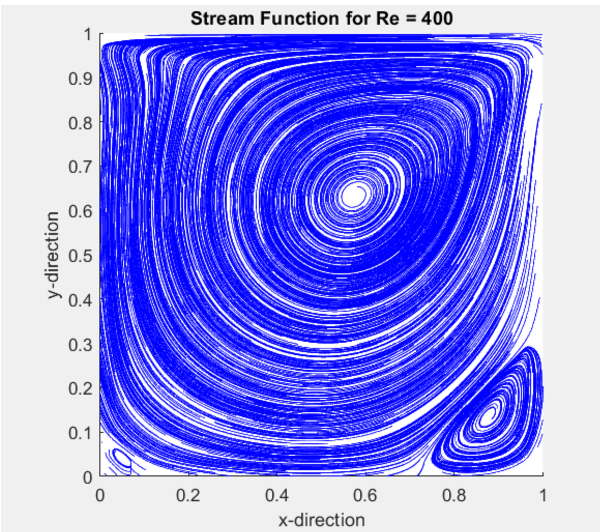


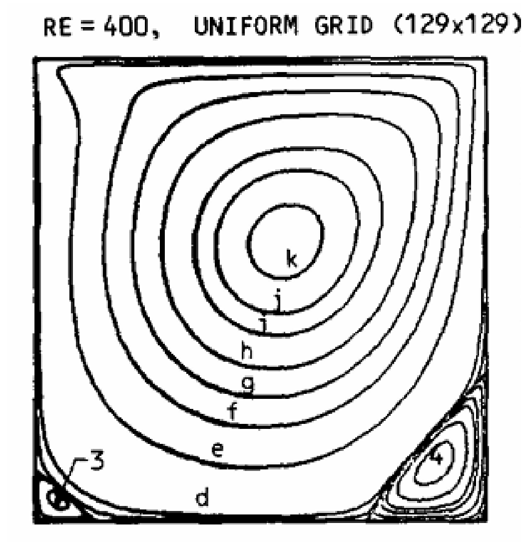
U-velocity for Re = 100
y-direction
x-direction

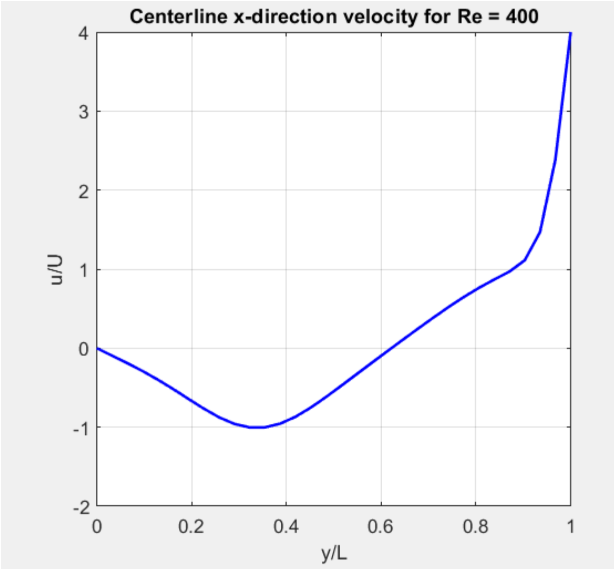
For Re = 400

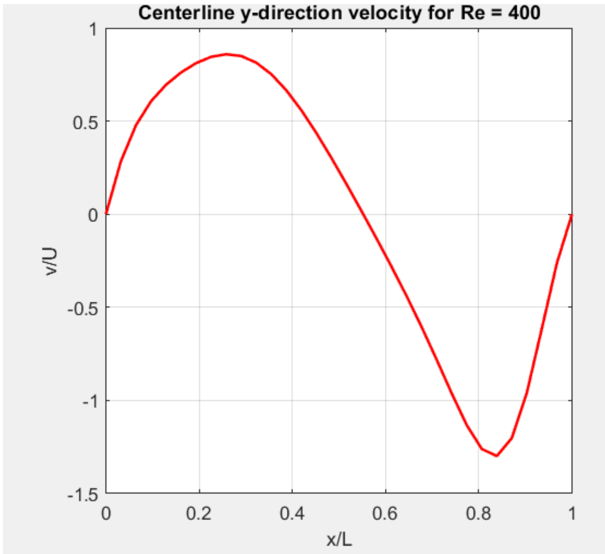

Vorticity Contour for Re = 400
y-direction
x-direction
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0.2
0.4
0.6
0.8
150
100
50
-50
-100
-150
-200

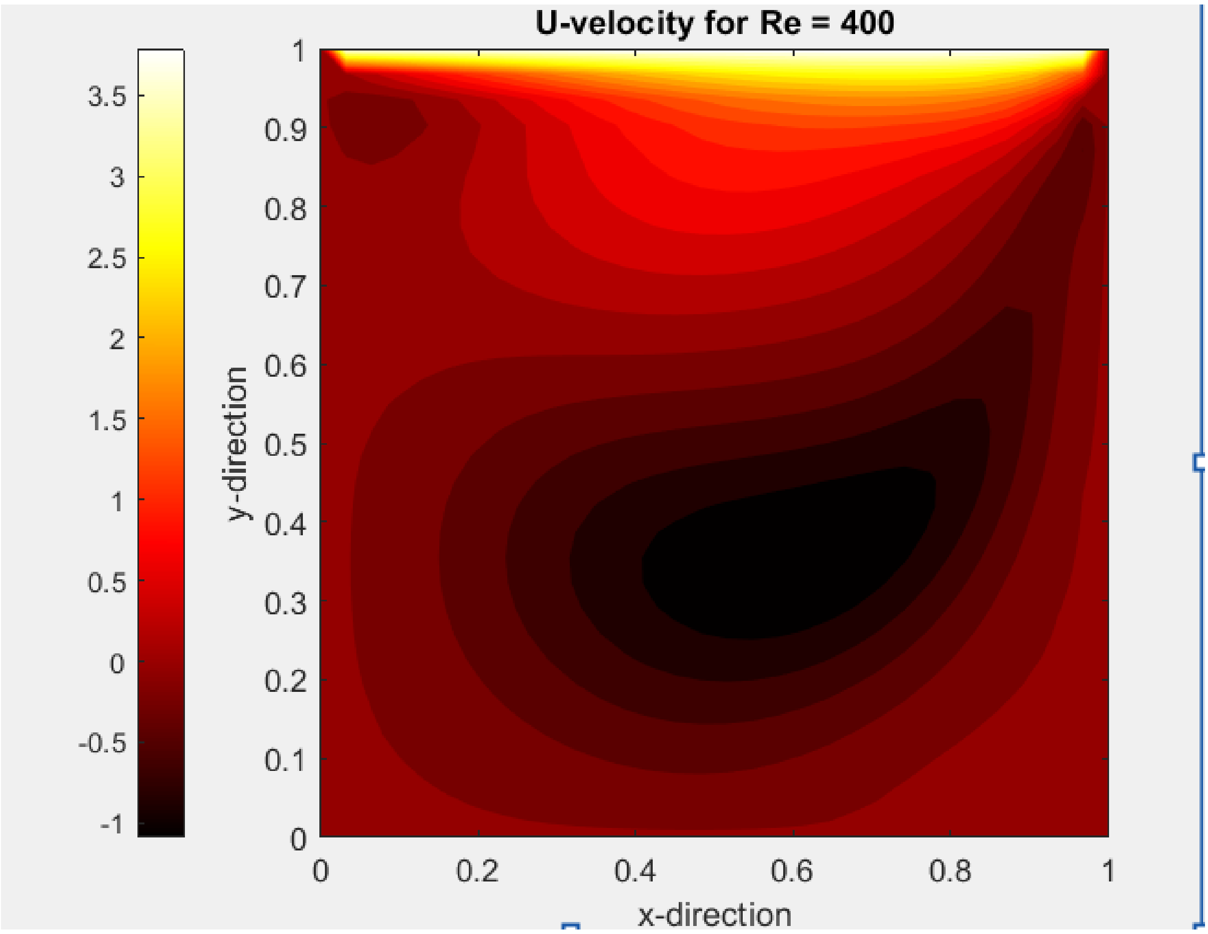

RE = 400, UNIFORM GRID (129 x 129)
-4
-3
0
2
3
6
5
4
-4
-3
-1
-2
-2
-1
0
0


Stream Function for Re = 400
y-direction
x-direction
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0.2
0.4
0.6
0.8


RE = 400, UNIFORM GRID (129x129)
k
j
i
h
g
f
e
d
-3
4


Centerline x-direction velocity for Re = 400
u/U
y/L


Centerline y-direction velocity for Re = 400
v/U
x/L


U-velocity for Re = 400
y-direction
x-direction

Problem-1

Governing Equations :

1. Continuity equation !

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

2. Navier-Stokes Equations (in 2D)

- x-momentum

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

- y-momentum

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial y} + \frac{\mu}{\rho}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

Boundary conditions

- Top wall (lid) : $u = U$, $v = 0$
- Bottom & side walls : $u = 0, v = 0$ (no slip cond^n)

3. Poisson equation for the Stream Function

$$\nabla^2 \psi = -\omega$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -\omega$$

~~4. Vorticity Transport eq^n~~

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = \frac{\psi_{i+1,j} - 2\psi_{i,j} - \psi_{i-1,j}}{\Delta x^2} + \frac{\psi_{i,j+1} - 2\psi_{i,j} + \psi_{i,j-1}}{\Delta y^2}$$

4. Vorticity Transport equation

- Time Derivative

$$\frac{\partial \omega}{\partial t} = \frac{\omega_{i,j}^{n+1} - \omega_{i,j}^{n}}{\Delta t}$$

• Convective Terms:

$$u\frac{\partial \omega}{\partial x} \approx \frac{u_{i,j}}{2h}\left(\omega_{i+1,j} - \omega_{i-1,j}\right)$$

$$v\frac{\partial \omega}{\partial y} \approx \frac{v_{i,j}}{2h}\left(\omega_{i,j+1} - \omega_{i,j-1}\right)$$

• Diffusion Term:

$$\mu \nabla^2 \omega = \mu\left(\frac{\omega_{i+1,j} - 2\omega_{i,j} + \omega_{i-1,j}}{\Delta x^2} + \frac{\omega_{i,j+1} - 2\omega_{i,j} + \omega_{i,j-1}}{\Delta y^2}\right)$$

Combining all terms, the descritized vorticity transport equation is:

$$\omega_{i,j}^{n+1} = \omega_{i,j}^{n} - \Delta t\left[\frac{u_{i,j}}{2\Delta x}\left(\omega_{i+1,j} - \omega_{i-1,j}\right) + \frac{v_{i,j}}{2\Delta y}\left(\omega_{i,j+1} - \omega_{i,j-1}\right)\right]$$

$$+ \Delta t \mu\left[\frac{\omega_{i+1,j} - 2\omega_{i,j} + \omega_{i-1,j}}{\Delta x^2} + \frac{\omega_{i,j+1} - 2\omega_{i,j} + \omega_{i,j-1}}{\Delta y^2}\right]$$

Boundary condition

1. Top wall (Lid): $\psi = 0$, $\omega = -\frac{2U}{h}$

   $u = U$, $v = 0$

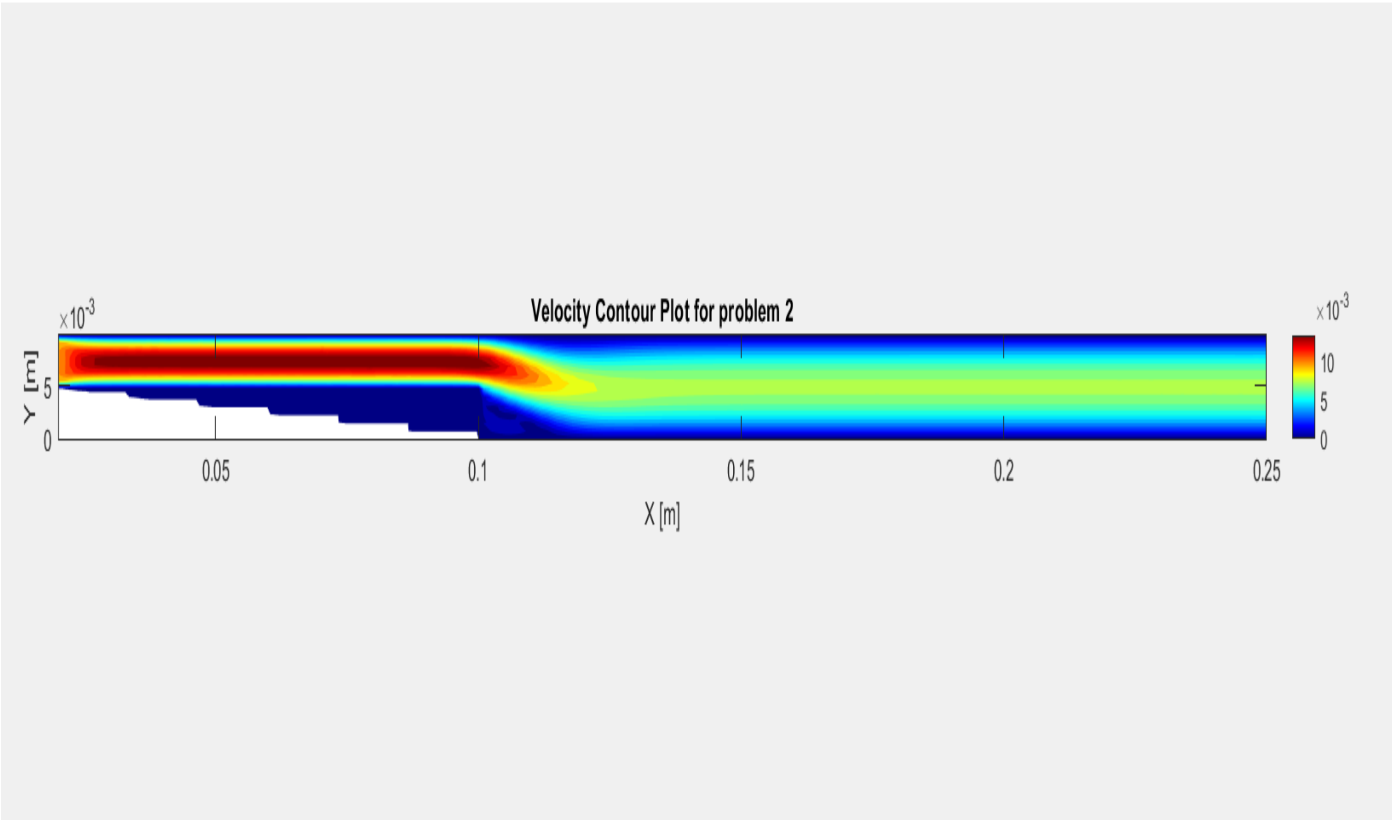
2. Bottom wall: $\psi = 0$, $\omega = -\frac{2}{h^2}\psi$

   $u = 0$, $v = 0$

3. Side walls: $\psi = 0$, $\omega = -\frac{2}{h^2}\psi$

   $u = 0$, $v = 0$

PROBLEM 2



Velocity Contour Plot for problem 2
×10^-3
Y [m]
5
0
0.05
0.1
0.15
0.2
0.25
X [m]
×10^-3
10
5
0

PROBLEM - 2

Governing eq^n

1. Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

2. N.S. equations

- x-momentum

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dP}{\partial x} + \frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

- y-momentum

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial y} + \frac{\mu}{\rho}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

Backward-Facing Step Geometry

Step height $S = 0.4712$

Channel height $H = 0.9712$

Downstream length ; 15

$Re = \frac{\rho U_\infty D}{\mu} = 100$; $D = 2h = 1$, $U_\infty = 1$

Discretized equations

1. Continuity Equation

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$$

$$\frac{U_{i+1,j} - U_{i-1,j}}{2\Delta x} + \frac{U_{i,j+1} - U_{i,j-1}}{2\Delta y} = 0$$

2. x-momentum

$$\frac{U_{i,j}(U_{i+1,j} - U_{i-1,j})}{2\Delta x} + \frac{V_{i,j}(U_{i,j+1} - U_{i,j-1})}{2\Delta y} = \frac{-1}{\rho}\frac{P_{i+1,j} - P_{i-1,j}}{2\Delta x}$$
$$+ \frac{\mu}{\rho}\left(\frac{U_{i+1,j} - 2U_{i,j} + U_{i-1,j}}{\Delta x^2} + \frac{U_{i,j+1} - 2U_{i,j} + U_{i,j-1}}{\Delta y^2}\right)$$

Similarily y-momentum eq^n discretised as

$$\frac{U_{i,j}(V_{i+1,j}-V_{i-1,j})}{2\Delta x} + V_{i,j}\frac{(V_{i,j+1}-V_{i,j-1})}{2\Delta y} = -\frac{1}{\rho}\frac{P_{i,j+1}-P_{i,j-1}}{2\Delta y}$$

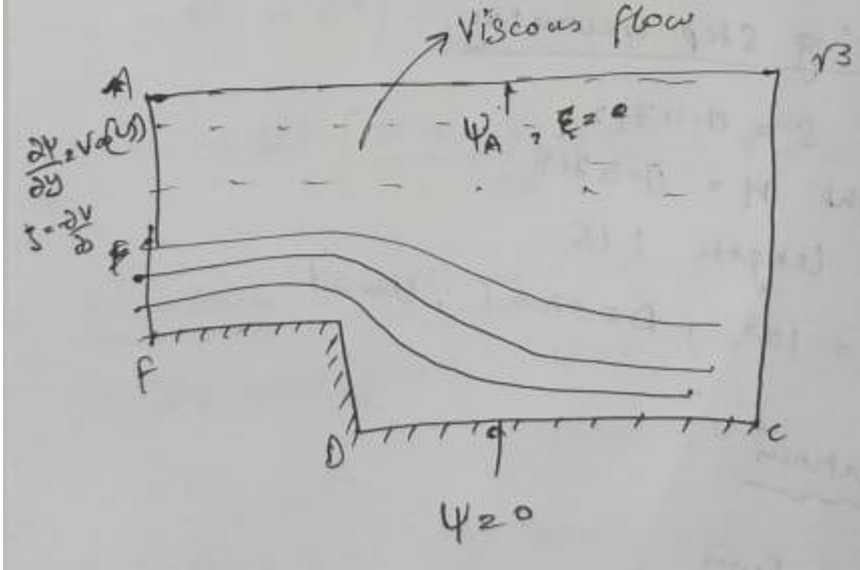
$$+\frac{\mu}{\rho}\left(\frac{V_{i+1,j}-2V_{i,j}+V_{i-1,j}}{\Delta x^2}+\frac{V_{i,j+1}-2V_{i,j}+V_{i,j-1}}{\Delta y^2}\right)$$

Vorticity (ω)

$$\omega_{ij} = \frac{V_{i+1,j}-V_{i-1,j}}{2\Delta x} - \frac{U_{i,j+1}-U_{i,j-1}}{2\Delta y}$$

Stream function ψ

$$\frac{\psi_{i+1,j}-2\psi_{i,j}+\psi_{i-1,j}}{\Delta x^2} + \frac{\psi_{i,j+1}-2\psi_{i,j}+\psi_{i,j-1}}{\Delta y^2} = -\omega_{ij}$$



$\psi = 0$

Boundary cond^n $\frac{\partial U}{\partial y} = 0$, $V = 0$ (Symmetry)

$U = U_\infty$ and $V = 0$ (free stream)

fully developed $\frac{\partial}{\partial x} = 0$