ME 543

COMPUTATIONAL FLUID DYNAMICS

Assignment 3

Name : Dushyant Pratap Singh

Roll NO : 244103101

[Computational Mechanics]

Question 1

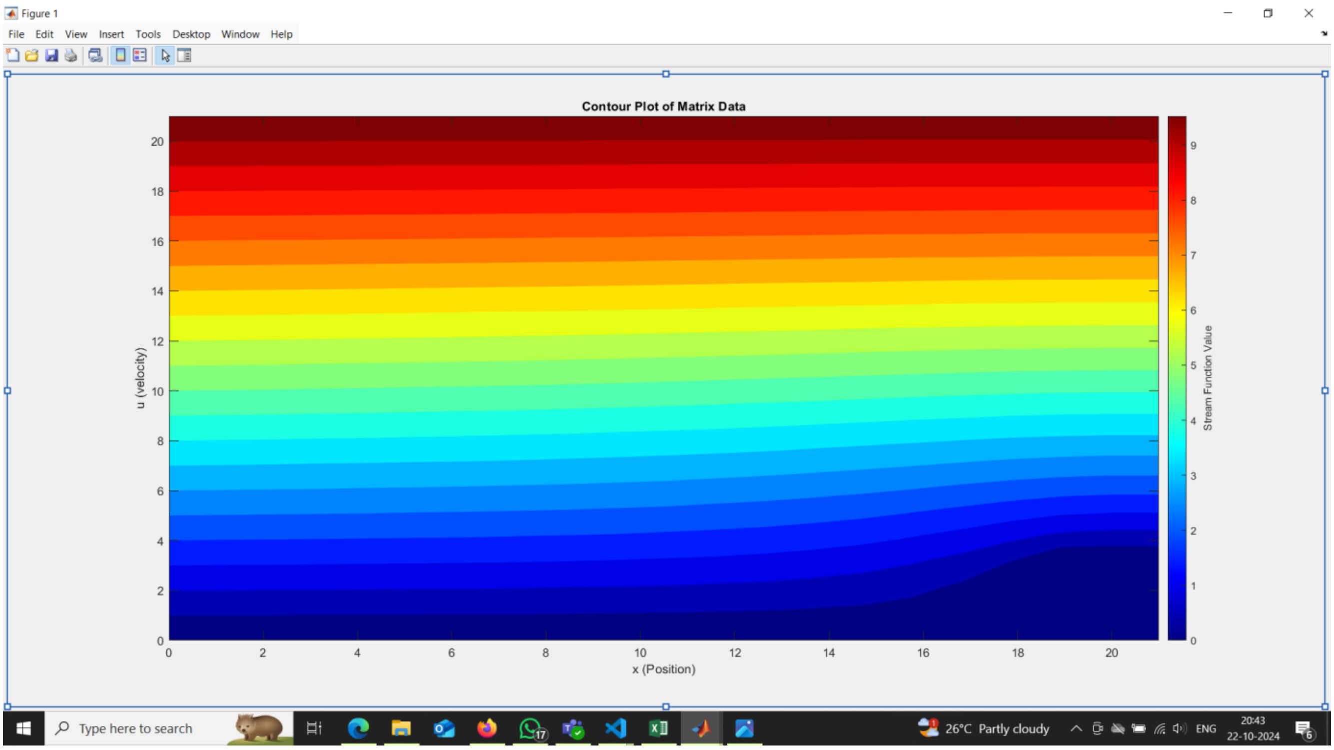
Figure 1
File Edit View Insert Tools Desktop Window Help
Contour Plot of Matrix Data
u (velocity)
x (Position)
Stream Function Value
0
2
4
6
8
10
12
14
16
18
20
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26°C Partly cloudy
ENG
20:43
22-10-2024

Question 2

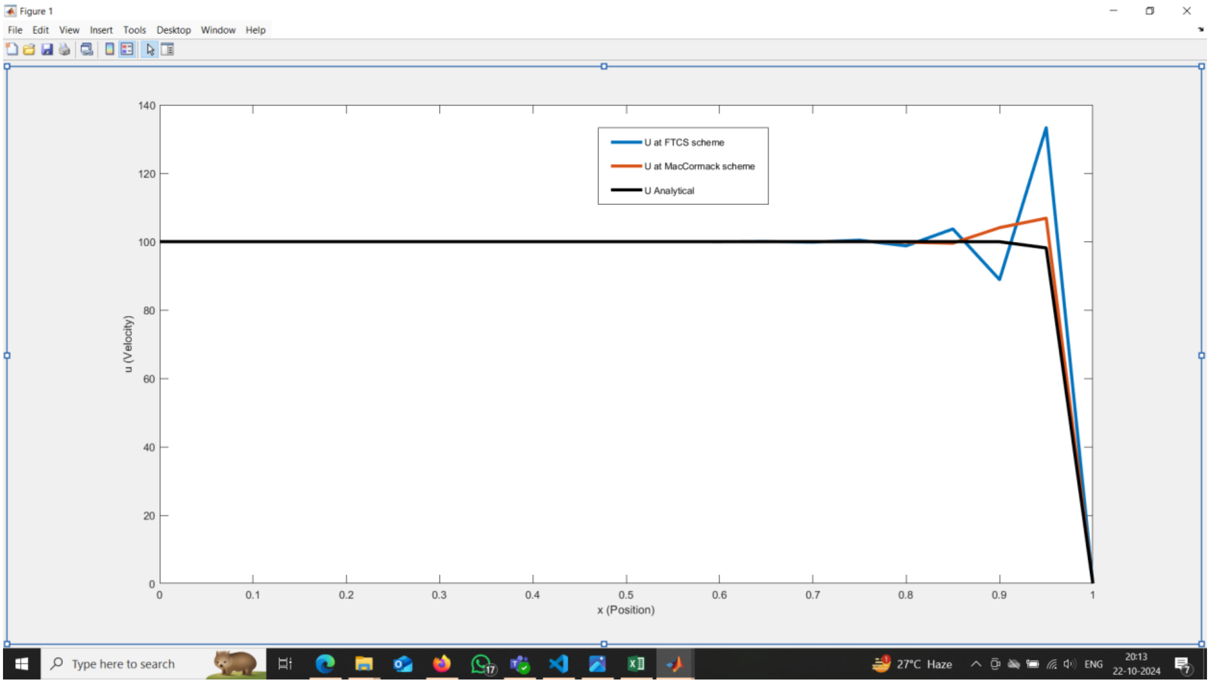

Figure 1
File Edit View Insert Tools Desktop Window Help
U at FTCS scheme
U at MacCormack scheme
U Analytical
u (Velocity)
x (Position)
140
120
100
80
60
40
20
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
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27°C Haze
ENG
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22-10-2024

Que. 2 Berger's equation.

$$\boxed{\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = \mu\frac{\partial^2 u}{\partial x^2}}$$

(a) FTCS:

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = \mu\frac{\partial^2 u}{\partial x^2}$$

$$\frac{U_j^{n+1} - U_j^n}{\Delta t} + c\left(\frac{U_{j+1}^n - U_{j-1}^n}{2\Delta x}\right) = \mu\left(\frac{U_{j+1}^n - 2U_j^n + U_{j-1}^n}{\Delta x^2}\right)$$

$$U_j^{n+1} - U_j^n + \frac{c(\Delta t)}{2(\Delta x)}\left(U_{j+1}^n - U_{j-1}^n\right) = \frac{\mu(\Delta t)}{\Delta x^2}\left(U_{j+1}^n - 2U_j^n + U_{j-1}^n\right)$$

$$\frac{c(\Delta t)}{\Delta x} = C_0 \quad , \quad \frac{\mu(\Delta t)}{(\Delta x)^2} = \gamma$$

$$U_j^{n+1} - U_j^n + \frac{C_0}{2}\left(U_{j+1}^n - U_{j-1}^n\right) = \gamma\left(U_{j+1}^n - 2U_j^n + U_{j-1}^n\right)$$

$$\boxed{U_j^{n+1} = U_j^n - \frac{C_0}{2}\left(U_{j+1}^n - U_{j-1}^n\right) + \gamma\left(U_{j+1}^n - 2U_j^n + U_{j-1}^n\right)}$$

Initial Condition: $U(x,0) = 0 \qquad 0 \le x \le 1$

Boundary condition: $U(0,t) = 100$

$U(1,t) = 0$

$$\left.\begin{array}{l}\gamma = 0.10 \\ C_0 = 0.40\end{array}\right] \text{(given)}$$

Mesh size = 21 grid point

(b) **MacCormack Scheme :-**

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \mu\frac{\partial^2 u}{\partial x^2}$$

Predictor : (forward differencing)

$$u_j^{\overline{n+1}} = u_j^n - c\Delta t\left(\frac{u_{j+1}^n - u_j^n}{\Delta x}\right) + \mu\Delta t\left(\frac{u_{j+1}^n - 2u_j^n + u_{j-1}^n}{\Delta x^2}\right)$$

$$= u_j^n - \frac{c\Delta t}{\Delta x}\left(u_{j+1}^n - u_j^n\right) + \frac{\mu\Delta t}{(\Delta x)^2}\left(u_{j+1}^n - 2u_j^n + u_{j-1}^n\right)$$

$$= u_j^n - C_o\left(u_{j+1}^n - u_j^n\right) + r\left(u_{j+1}^n - 2u_j^n + u_{j-1}^n\right)$$

Corrector: (Backward differencing)

$$u_j^{n+1} = \frac{1}{2}\left[u_j^n + u_j^{\overline{n+1}} - \frac{c\Delta t}{\Delta x}\left(u_{j-1}^{\overline{n+1}} - u_j^{\overline{n+1}}\right) + \mu\Delta t\left(\frac{u_{j+1}^{\overline{n+1}} - 2u_j^{\overline{n+1}} + u_{j-1}^{\overline{n+1}}}{\Delta x^2}\right)\right.$$

$$= \frac{1}{2}\left[u_j^n + u_j^{\overline{n+1}} - C_o\left(u_{j-1}^{\overline{n+1}} - u_j^{\overline{n+1}}\right) + r\left(u_{j+1}^{\overline{n+1}} - 2u_j^{\overline{n+1}} + u_{j-1}^{\overline{n+1}}\right)\right.$$

Analytical Solution

$$U = U_0 \left\{ \frac{1 - \exp\left[Re_L \left(\frac{x}{L} - 1 \right) \right]}{1 - \exp(-R_L)} \right\}$$

where $Re_L = \frac{CL}{\mu}$

We know $L = 1$, $\quad r = \frac{\mu (\Delta t)}{\Delta x^2} \Rightarrow \mu = \frac{r(\Delta x)^2}{\Delta t}$, $\quad C_0 = \frac{C(\Delta t)}{\Delta x}$

putting value of μ in Re_L

$$Re_L = \frac{C L \, \Delta t}{r (\Delta x)^2} = \frac{\overset{1}{L} \; \overbrace{\frac{C(\Delta t)}{(\Delta x)}}^{C_0}}{r(\Delta x)}$$

$$Re_L = \frac{C_0}{r(\Delta x)}$$

putting value of Re_L in U

$$U = \overset{100}{U_0} \left\{ \frac{1 - \exp\left[\frac{C_0}{r(\Delta x)} (x - 1) \right]}{1 - \exp\left(- \frac{C_0}{r(\Delta x)} \right)} \right\}$$

$$\boxed{U = 100 \left\{ \frac{1 - \exp\left[\frac{C_0}{r(\Delta x)} (x - 1) \right]}{1 - \exp\left(- \frac{C_0}{r \Delta x} \right)} \right\}}$$

Question 3

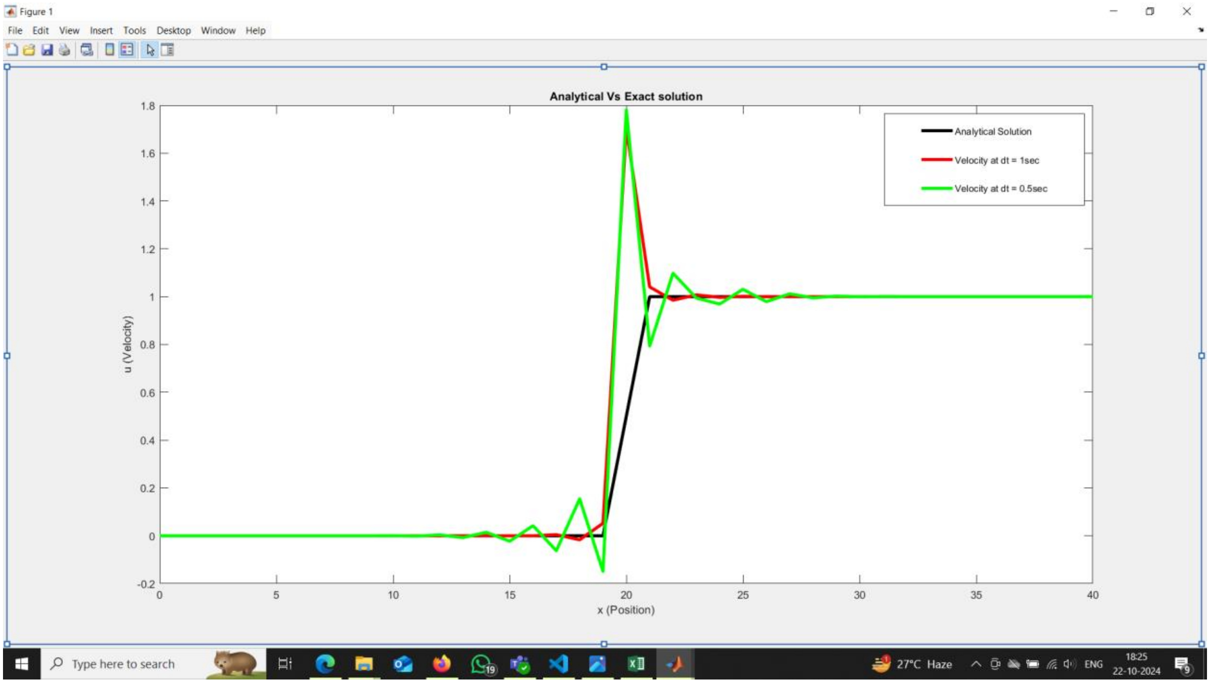

Figure 1
File Edit View Insert Tools Desktop Window Help
Analytical Vs Exact solution
Analytical Solution
Velocity at dt = 1sec
Velocity at dt = 0.5sec
u (Velocity)
x (Position)
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0
5
10
15
20
25
30
35
40
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27°C Haze
ENG
18:25
22-10-2024

1 Ques③ $U_t + \frac{1}{2}\left(\frac{1}{2} - U\right) U_x = 0.001\, U_{xx}$

$$\frac{\partial U}{\partial t} + \frac{1}{2}\left(\frac{1}{2} - U\right)\frac{\partial U}{\partial x} = 0.001\,\frac{\partial^2 U}{\partial x^2}$$

Initial Condition: $U = \frac{1}{2}\left\{1 + \tanh\left[250(x-20)\right]\right\}$

$$x \le x \le 40$$

Drichet Boundary Condition

$$U(0) = 0.0$$
$$U(L) = 1.0$$

MacCormack Method:

Predictor:

$$U_j^{\overline{n+1}} = U_i^n - \Delta t\left[\left(\frac{1}{2} - U_i^n\right)\left(\frac{U_{i+1}^n - U_i^n}{\Delta x}\right) + \Delta t\,(0.001)\left(\frac{U_{i+1}^n - 2U_i^n + U_{i+1}^n}{\Delta x^2}\right)\right.$$

Corrector:

$$U_i^{n+1} = \frac{1}{2}\left[U_i^n + U_i^{\overline{n+1}} - \Delta t\left[\left(\frac{1}{2} - U_i^{\overline{n+1}}\right)\left(\frac{U_{i+1}^{\overline{n+1}} - U_i^{\overline{n+1}}}{\Delta x}\right)\right.\right.$$
$$+ \Delta t\,(0.001)\left(\frac{U_{i+1}^{\overline{n+1}} - 2U_i^{\overline{n+1}} + U_{i-1}^{\overline{n+1}}}{\Delta x^2}\right)$$