Tutorial Week 2

0. **EXAMPLE:** Show $\sim(\sim(p \vee q \vee p) \wedge p) \equiv T$

$$
\begin{aligned}
& \sim(\sim(p \vee q \vee p) \wedge p) & \\
\equiv\ & ((p \vee q \vee p) \vee \sim p) & \text{by DeMorgan's Law} \\
\equiv\ & p \vee q \vee p \vee \sim p & \text{by Associative Law} \\
\equiv\ & p \vee p \vee \sim p \vee q & \text{by Commutative Law} \\
\equiv\ & p \vee \sim p \vee q & \text{by Idempotent Law} \\
\equiv\ & T \vee q & \text{by Negation Law} \\
\equiv\ & T & \text{by Universal Bound Law}
\end{aligned}
$$

1. Show $p \vee q \equiv \sim(\sim p \wedge ((\sim q \wedge \sim p) \vee (\sim q \wedge p)))$

2. Translate the following Truth Table to a propositional logic statement: Can you simplify it?

Truth Table			
p	q	r	output
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

   (**hints** : the brute force approach needs three variables to describe each individual time the output is 1. is there a way to use less than three variables to cover more than 1 case where the output is 1?)

3. Prove the following equivalence: $(\sim p \vee s) \wedge (p \rightarrow (s \rightarrow \sim p)) \equiv \sim p$

4. Prove that $(p \vee q) \equiv (\sim q \rightarrow p) \vee \sim(\sim p \wedge \sim q) \vee ((p \wedge \sim p) \wedge (q \vee \sim q))$