

# Comparing Strategies of Random Sample Consensus Algorithms for Small Unmanned Aerial Vehicles Using Structure From Motion

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# Agenda

- Introduction
- SFM Architecture & Pipeline
- Original RANSAC Algorithm
- RANSAC Variant Algorithms
- Conclusion

# Introduction

Broad Impact



Small AUTONOMOUS UAVs

**COLLABORATIVE SWARM –  
To DETERMINE SAME SIDE  
OF BUILDING OR OBJECT**

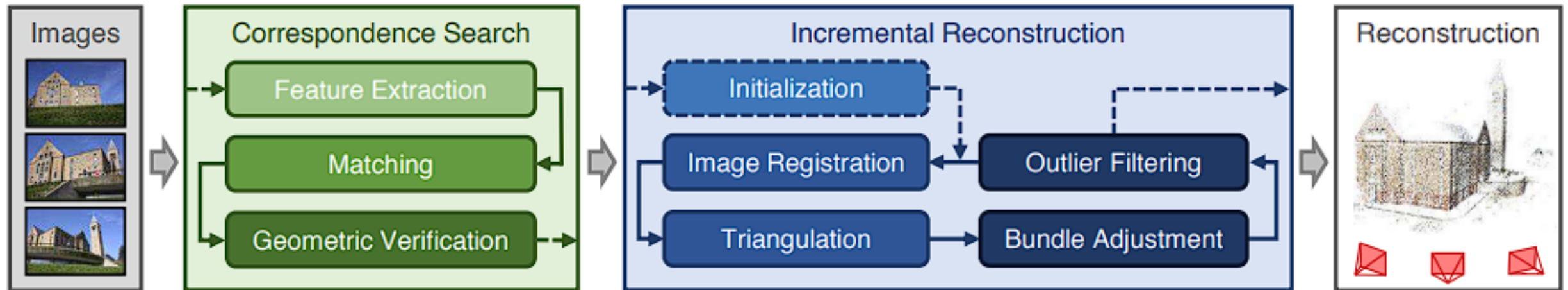
**STRUCTURE FROM MOTION  
VS. SLAM**

Our Work

**EXPLORE EFFICIENCY OF  
RANSAC Algorithms  
WITHIN THE SFM PIPELINE**



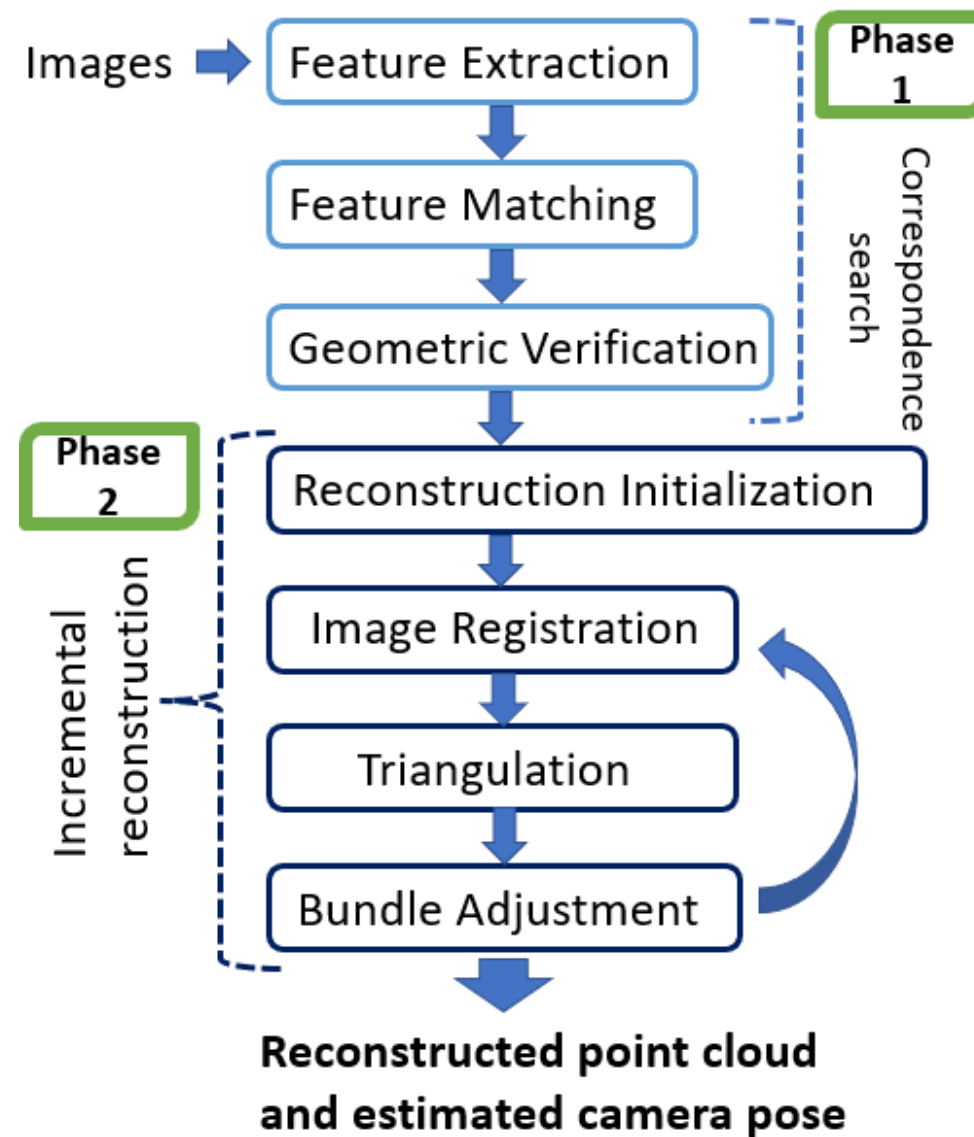
# Structure From Motion Architecture



J. L. Schonberger and J. M. Frahm, "Structure-from-motion revisited," in Proc. of the IEEE Conf. on Computer vision and Pattern Recognition, pp. 4104-4113. 2016.

# The Incremental Structure From Motion Pipeline

S. Bianco, G. Ciocca and D. Marelli, "Evaluating the Performance of Structure from Motion Pipelines," J. of Imaging, vol. 8, 2018.



# RANSAC Algorithm

- Random Sample Consensus
- Fishler and Bolles (1981)
- One of the most cited papers in computer vision

M. A. Fischler and R. C. Bolles, "Random sample consensus: a paradigm for model fitting with applications to image analysis and automated cartography," *Commun. of the ACM*, vol. 24, no. 6, pp. 381-395, 1981.

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The original RANSAC algorithm

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Let  $D$  be the set of all datapoints where  $|D| = P$ .

Let  $S_i$  be a set of datapoints where  $S_i \subset D$ ,

$|S_i| = N$ , and  $P \geq N > 0$ .

Let  $t$  be the maximum number of iterations.

Let  $i$  be the number of the current iteration.

Let  $max$  be the number of inliers.

$M_i$  is a model (hypothesis).

```
1:  $i, max \leftarrow 0$ 
2: while ( $i < t$ ) do
    // Hypothesis generation
3:    $S_i \leftarrow$  Randomly select minimal subset of  $D$ 
4:    $U_i \leftarrow$  Potential consensus set of  $S_i$ 
5:    $M_i \leftarrow$  Generate a model from  $S_i$ 
    // Hypothesis evaluation
6:   Calculate error from estimated model
7:    $I_i \leftarrow$  The number of detected inliers
8:   if  $I_i > max$  then
9:      $max \leftarrow I_i$ 
10:     $U_i^* \leftarrow$  Redefined consensus set
11:     $M_i^* \leftarrow$  Fitted model to  $U_i^*$ 
12:   end if
13:    $i \leftarrow i + 1$ 
14: end while
```

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# RANSAC

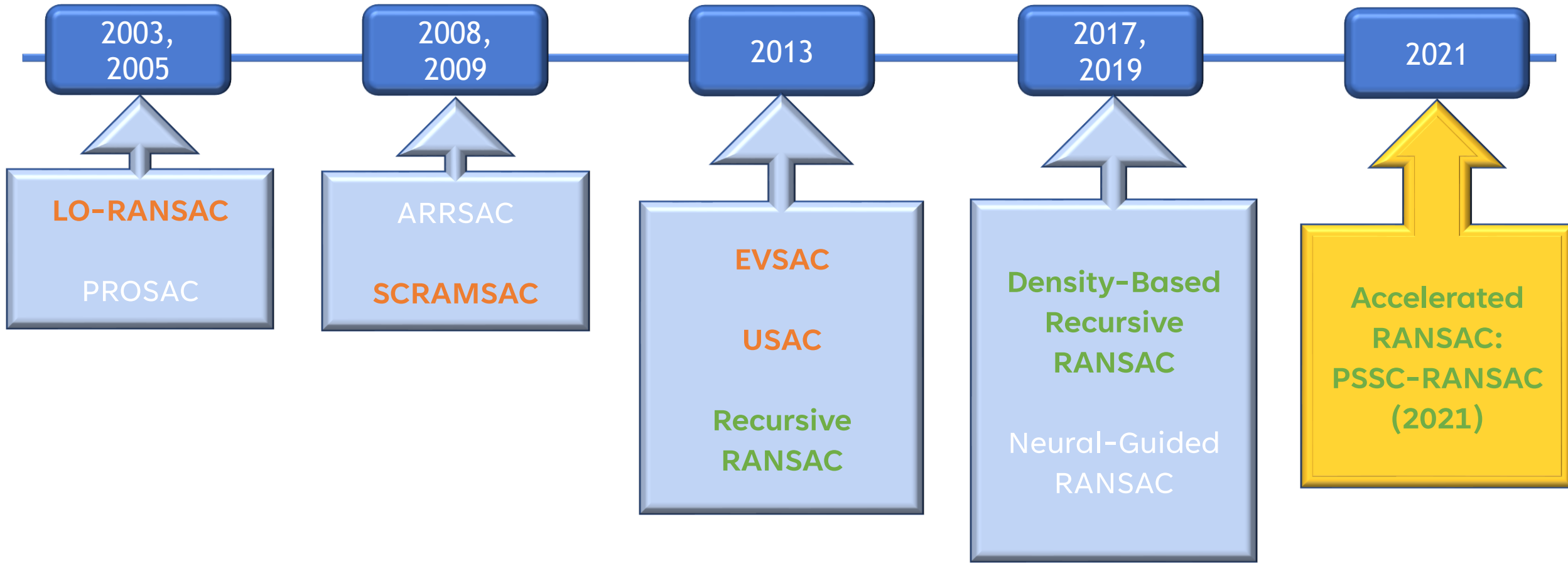
Quality of Datapoint :

- "Bad" quality = outliers
- "Good" quality = inlier

- An iterative method estimating parameters of a mathematical model from a set of observed data that likely contains outliers
- A global parameter estimation that robustly finds model parameters from a set of datapoints
- RANSAC can be used to detect outliers.
- If outlier percentage is higher than inlier's, RANSAC becomes slower.



# 10 RANSAC VARIANT ALGORITHMS

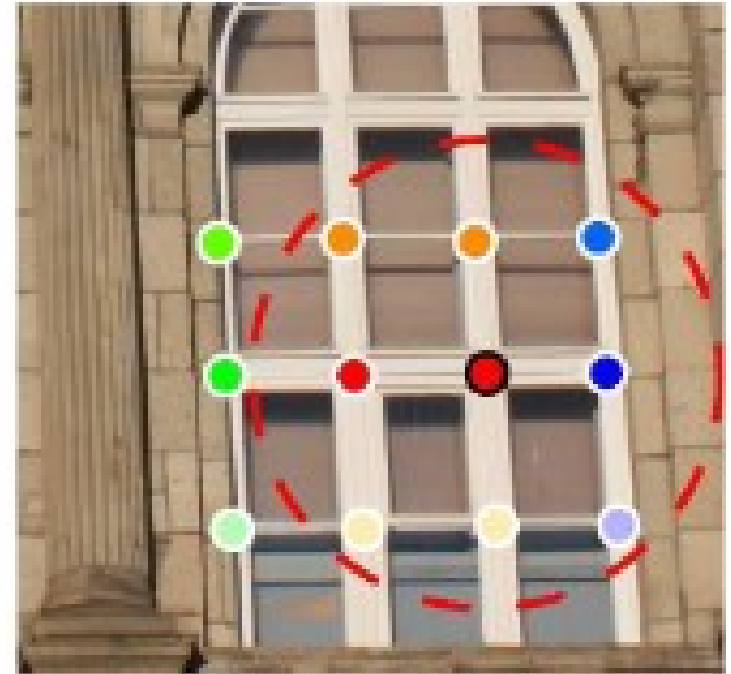


# Locally Optimized RANSAC (LO-RANSAC)

- Local optimization using *inner* RANSAC
- Modeling inside a model  
(Iterative Least squares → Local Optimization)
- Accuracy Improved and may stop the algorithm earlier  
(faster performance)

# Spatially Consistent Random Sample Consensus (SCRAMSAC)

- Spatial Consistency Check is a descriptor
  - It considers the matching quality in a larger spatial neighborhood
  - It measures the fraction of neighboring features in a circular region around a feature and check if it surpasses a certain threshold or not
- Results in a reduced set of higher-quality correspondences



T. Sattler, B. Leibe and L. Kobbelt, "SCRAMSAC: Improving RANSAC's efficiency with a spatial consistency filter," 2009 IEEE 12th Int. Conf. on Computer Vision, pp. 2090-2097, 2009

# EVSAC: Accelerating Hypotheses Generation by Modeling Matching Scores with Extreme Value Theory

- A nearest-neighbor matcher based on *Extreme Value Theory* is used to calculate the confidence if a datapoint is an inlier
- This probabilistic framework accelerates accurate hypothesis generation in RANSAC by taking the datapoints with the greatest confidence first

# USAC: A Universal Framework for Random Sample Consensus

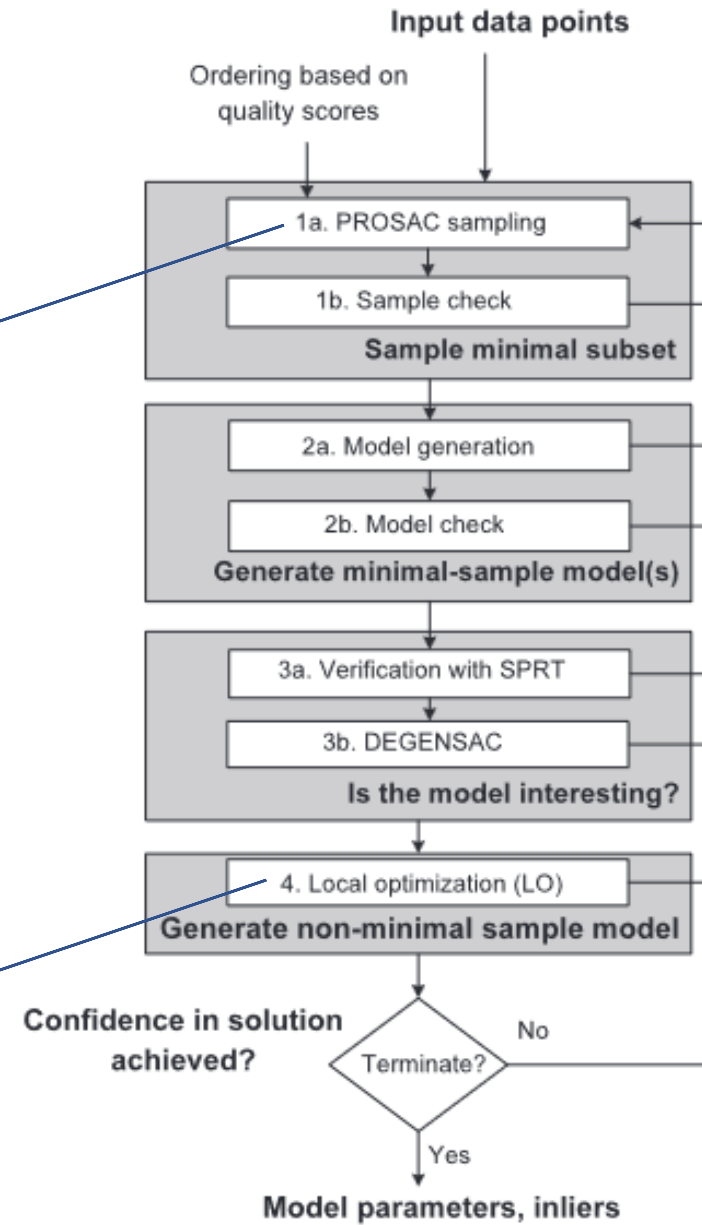
Raguram	Chum	Pollefeys	Matas	Frahm
<ul style="list-style-type: none"><li>• ARRSAC (2008)</li></ul>	<ul style="list-style-type: none"><li>• LO-RANSAC (2003)</li><li>• PROSAC (2005)</li></ul>	<ul style="list-style-type: none"><li>• ARRSAC</li></ul>	<ul style="list-style-type: none"><li>• LO-RANSAC</li><li>• PROSAC</li></ul>	<ul style="list-style-type: none"><li>• ARRSAC</li><li>• Structure from motion revisited</li></ul>

- USAC-1.0: A general-purpose C++ software library that implements USAC framework to be used as a stand-alone tool or a benchmark

# USAC 1.0 Implementation

- PROSAC (Progressive Random Sample Consensus) : A quality function to weigh the selection procedure
- Datapoints are sorted in terms of quality
- Non-uniform sampling based on ordering datapoints by quality scores
- 41% faster than RANSAC

For more accurate result



# Prior Sampling & Sample Check-RANSAC (PSSC-RANSAC)

- Pre-Sampling Technique
- Image is broken into N-Blocks
- Evaluated for Texture Magnitude:  
Comprised of Contrast, Coarseness, Roughness

Difficult to implement in simulation environment

# Only 4 RANSAC algorithms were comparable

Name of RANSAC variant algorithm	# of image pairs	Average # of datapoints	Proposed RANSAC variant algorithm's average inlier ratio	RANSAC's average inlier ratio
LO-RANSAC [24]	2	127	$0.175 \pm 0.000$	$0.146 \pm 0.000$
SCRAMSAC [25]	6	369	$0.605 \pm 0.067$	$0.362 \pm 0.046$
EVSAC [15]	4	945.75	$0.069 \pm 0.003$	$0.047 \pm 0.001$
USAC [20]	5	1172	$0.256 \pm 0.042$	$0.229 \pm 0.030$

Y. L. Bond, E. Osornio, S. Ledwell and A. C. Cruz, "Comparing Strategies for Small Unmanned Aerial Vehicles Using Structure From Motion," in Third Annual Computer Science Conference for California State University (CSU) Undergraduates, Seaside, Northridge, CA, 2023



# Conclusion

**Difficult** to compare all the variant RANSAC algorithms experiment results

**Why** PSSC-RANSAC for small Unmanned Aerial Vehicles?

It addresses datasets with large outlier ratios and a concern for the computational speed on airborne platforms.

**Wait**, don't "Prior Sampling" and "Sample Check" sound familiar now after learning all the RANSAC variant algorithms' strategies?



# Summary Table of Strategies in RANSAC Algorithms

Strategies	Names of the RANSAC Algorithms
Ranking-based Sampling	EVSAC (greatest confidence) PROSAC (quality score) USAC (PROSAC-based sampling) <b>Prior Sampling &amp; Sample Check - RANSAC</b> (Texture Magnitude Evaluation)
Spatial Coherence	SCRAMSAC (Spatial Coherence Check) Density-based RANSAC
Local Optimization and Early termination	LO-RANSAC USAC (LO-RANSAC is employed for Model Refinement)



# Questions?



**Thank you.**

# LO-RANSAC – Modeling inside a model

LEBEDA, MATAS, CHUM: FIXING THE LOCALLY OPTIMIZED RANSAC

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in standard RANSAC,  $K$  is a function of the user-defined desired probability  $\eta$  of finding the optimal solution and the number of inliers of the best model.

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## Algorithm 1 LO-RANSAC [6].

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```

1: for  $k = 1 \rightarrow K(|\mathcal{I}^*|, \eta)$  do
2:    $\mathcal{S}_k \leftarrow$  randomly drawn minimal sample
3:    $M_k \leftarrow$  model estimated from sample  $\mathcal{S}_k$ 
4:    $\mathcal{I}_k \leftarrow \text{find\_inliers}(M_k, \theta)$ 
5:   if  $|\mathcal{I}_k| > |\mathcal{I}_s^*|$  then
6:      $M_s^* \leftarrow M_k; \mathcal{I}_s^* \leftarrow \mathcal{I}_k$ 
7:      $M_{LO}, \mathcal{I}_{LO} \leftarrow$  run Local Optimization (Alg. 2)
8:     if  $|\mathcal{I}_{LO}| > |\mathcal{I}^*|$  then
9:        $M^* \leftarrow M_{LO}; \mathcal{I}^* \leftarrow \mathcal{I}_{LO}$ 
10:      update  $K$ 
11:    end if
12:  end if
13: end for
14: return  $M^*$ 

```

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Table 1: Notation.

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LEBEDA, MATAS, CHUM: FIXING THE LOCALLY OPTIMIZED RANSAC

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## Algorithm 2 Local Optimization step.

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**Input:**  $M_s^*, m_\theta, \text{reps}$

```

1:  $M_{m_\theta} \leftarrow$  model estimated by LSq on  $\text{find\_inliers}(M_s^*, m_\theta \cdot \theta)$ 
2:  $\mathcal{I}_{base} \leftarrow \text{find\_inliers}(M_{m_\theta}, \theta)$ 
3: for  $r = 1 \rightarrow \text{reps}$  do
4:    $\mathcal{S}_{is} \leftarrow$  sample of size  $s_{is}$  randomly drawn from  $\mathcal{I}_{base}$ 
5:    $M_{is} \leftarrow$  model estimated from  $\mathcal{S}_{is}$  by LSq
6:    $M_r \leftarrow$  Iterative Least Squares ( $M_{is}, m_\theta, \text{iters}$ ) (Alg. 3)
7: end for
8: return the best of  $M_s^*$ , all  $M_{is}$ , all  $M_r$ , with its inliers

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## Algorithm 3 Iterative Least Squares.

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**Input:**  $M_{is}, m_\theta, \text{iters}$

```

1:  $M' \leftarrow$  model estimated by LSq on  $\text{find\_inliers}(M_{is}, \theta)$ 
2:  $\theta' \leftarrow m_\theta \cdot \theta$ 
3: for  $i = 1 \rightarrow \text{iters}$  do
4:    $\mathcal{I}' \leftarrow \text{find\_inliers}(M', \theta')$ 
5:    $w' \leftarrow$  computed weights of  $\mathcal{I}'$  (depend on model)
6:    $M' \leftarrow$  model estimated by LSq on  $\mathcal{I}'$  weighted by  $w'$ 
7:    $\theta' \leftarrow \theta' - \Delta_\theta$ 
8: end for
9: return the best  $M'$ 

```

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<http://www.bmva.org/bmvc/2012/BMVC/paper095/paper095.pdf>



# Spatially Consistent Random Sample Consensus (SCRAMSAC) – SC Check



Figure 1: Local features often lead to incorrect correspondences due to similar structures (colored dots). By taking into account a larger spatial neighborhood, they can however be disambiguated (red circle). We use this idea to develop a faster and more robust RANSAC procedure.

**Spatial Consistency Check.** Given  $(\mathcal{I}_1, \mathcal{I}_2)$  and  $C$ , we define the neighborhood set  $N(c)$  of a correspondence  $c = (f_j^1, f_k^2) \in C$  as

$$N(c) = \{(f^1, f^2) \in C \mid f^1 \in N_{\mathcal{I}_1}(f_j^1) \wedge f^2 \in N_{\mathcal{I}_2}(f_k^2)\} \quad (3)$$

and we accept a correspondence as *spatially consistent* iff

$$|N(f_j^1)| = |\{(f^1, f^2) \in C \mid f^1 \in N_{\mathcal{I}_1}(f_j^1)\}| > 0 \wedge \frac{|N(c)|}{|N(f_j^1)|} \geq \theta$$

with a threshold  $\theta \in [0, 1]$ . This results in a reduced set  $C_{\text{red}} \subseteq C$  of spatially consistent correspondences.

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## Algorithm 1 RANSAC with SCC (SCRAMSAC)

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### 1. Computation of the reduced set

$C_{\text{red}} \leftarrow SCC(C), N = |C_{\text{red}}|$

### 2. RANSAC application

$k = 0, \varepsilon_{\text{red}} = m/N, I^* = 0$

**while**  $\eta = (1 - \varepsilon_{\text{red}}^m)^k \geq \eta_0$  **do**

Sample  $m$  random correspondences from  $C_{\text{red}}$ .

Compute model  $\Phi$  from samples.

Compute number  $I$  of inliers for  $\Phi$  on  $C_{\text{red}}$ .

**if**  $I > I^*$  **then**

$I^* = I, \varepsilon_{\text{red}} = I^*/N$ , store  $\Phi$ .

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## Spatial Consistent Random Sample Consensus (SCRAMSAC) compared to SIFT and PROSAC

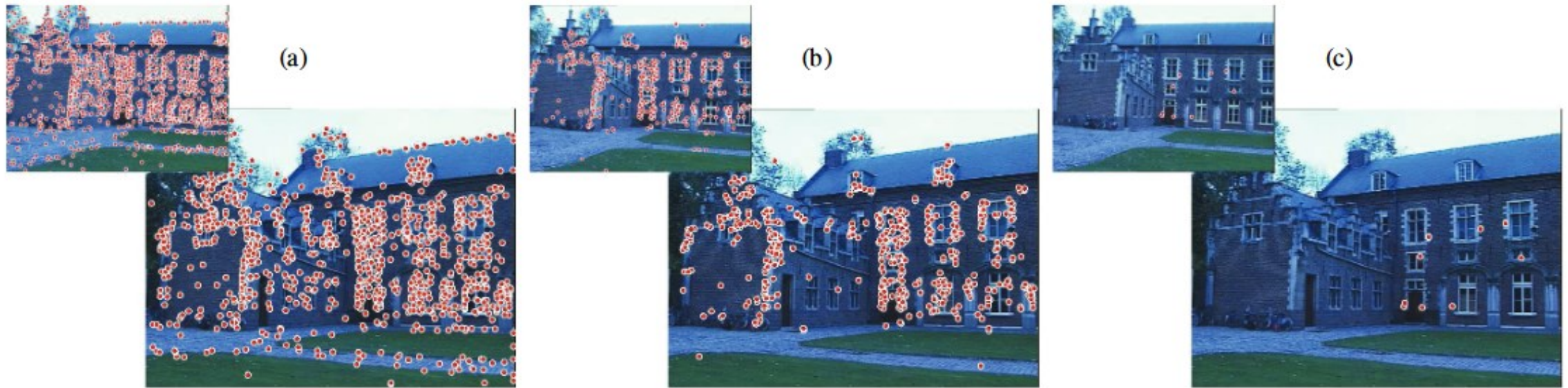


Figure 3: (a) Initial correspondences computed by SIFT matching (50% inliers). (b) Remaining correspondences after one application of the SCC (80% inliers). (c) Correspondences used by PROSAC for hypothesis generation. The SCC results in a reduced set of higher-quality correspondences, while still providing a sufficient coverage to ensure robust model estimation.

T. Sattler, B. Leibe and L. Kobbelt, "SCRAMSAC: Improving RANSAC's efficiency with a spatial consistency filter," 2009 IEEE 12th Int. Conf. on Computer Vision, pp. 2090-2097, 2009



# What is a Sequential Probability Ratio Test?

A **sequential probability ratio test (SPRT)** is a hypothesis test for [sequential samples](#).

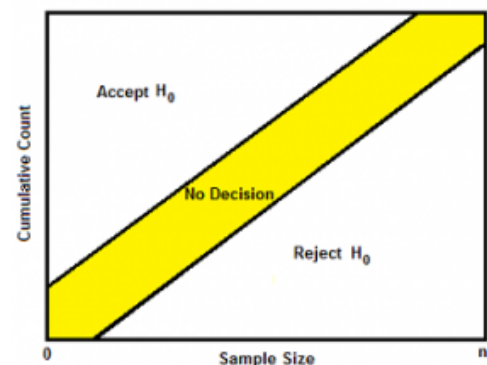
Sequential sampling works in a very non-traditional way; instead of a fixed [sample size](#), you choose one item (or a few) at a time, and then test your hypothesis. You can either:

- [Reject the null hypothesis](#) ( $H_0$ ) in favor of the [alternate hypothesis](#) ( $H_1$ ) and stop,
- Keep the null hypothesis and stop,
- Fail to reach either conclusion and continue sampling.

If you fail to reach a conclusion, you repeat the sampling and then the hypothesis test. You keep on repeating this process until you have a sound conclusion, so *you don't know the how big your sample will be until you're finished testing*.

## About the SPRT

Sequential analysis hypothesis testing generally **enables a researcher to come to a conclusion with a minimum amount of data**. With Wald's SPRT, the amount of data points required to come to a conclusion can be defined by a [random variable](#), called the **sample number  $N_s$** . The boundary of the decision region depends on the [expected value](#) of this random variable, called the **Average Sample Number (ASN)**. The ASN for the SPRT is lower than all other sequential tests and is usually lower than traditional, fixed-size sampling methods.



SPRT is based on the [likelihood ratio](#) statistic  $\lambda^n$ . Likelihood ratio tests are extremely difficult to perform by hand, and so software is necessary. However, you do need to specify some conditions, including two constants, A and B (where  $A > B$ ). These define the conditions under which the null hypothesis will be rejected or not:

- Accept  $H_0$ :  $\lambda^{(n)} \leq B$
- No conclusion ([resample](#)):  $B < \lambda^{(n)} < A$
- Reject the null hypothesis in favor of  $H_1$ :  $\lambda^{(n)} \geq A$

These three conditions are represented as **decision regions** (accept/no decision/reject) in the above image.

A and B are relatively simple to calculate with the following formulas:

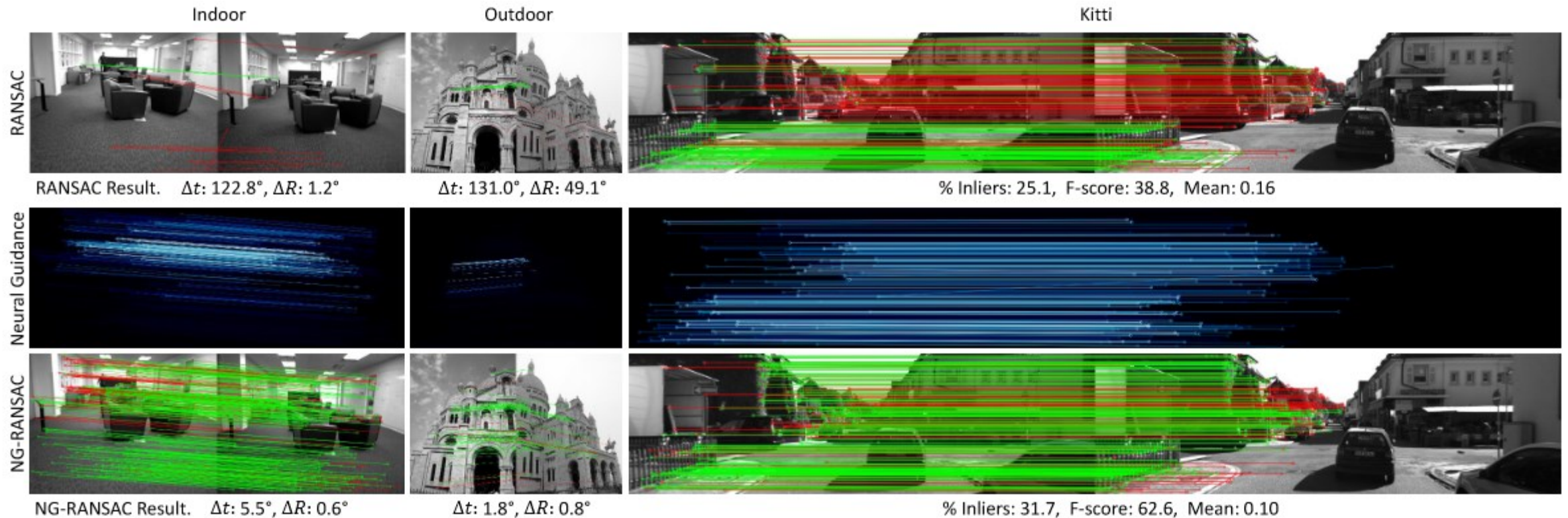
$$A = \frac{1 - \beta}{\alpha} \quad B = \frac{\beta}{1 - \alpha}$$

# SPRT

<https://www.statisticshowto.com/sequential-probability-ratio-test/>



# Neural-Guided RANSAC



**Figure 3. Qualitative Results.** We compare fitted models for RANSAC and NG-RANSAC. For the indoor and outdoor image pairs, we fit essential matrices, and for the Kitti image pair we fit the fundamental matrix. We draw final model inliers in green if they adhere to the ground truth model, and red otherwise. We also measure the quality of each estimate, see the main text for details on the metrics.

E. Brachmann and C. Rother, "Neural-Guided RANSAC: Learning Where to Sample Model Hypotheses," *2019 IEEE/CVF Int. Conf. on Computer Vision (ICCV)*, pp. 4321-4330, 2019



# Extreme Value Theory-Based RANSAC: EVSAC

Inspired by BEEM's prior search method

Requires image feature correspondences

$K$  nearest neighbor matching scores sorted in an ascending order for every  $i$ -th correspondence

Computing distributions for correct and incorrect matches

Gamma distribution

GEV distribution

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## Algorithm 1 EVSAC

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**Require:**  $\{\mathbf{x} \leftrightarrow \mathbf{x}'\}_{i=1}^n$  and  $\{s_{i,1:k}\}_{i=1}^n$

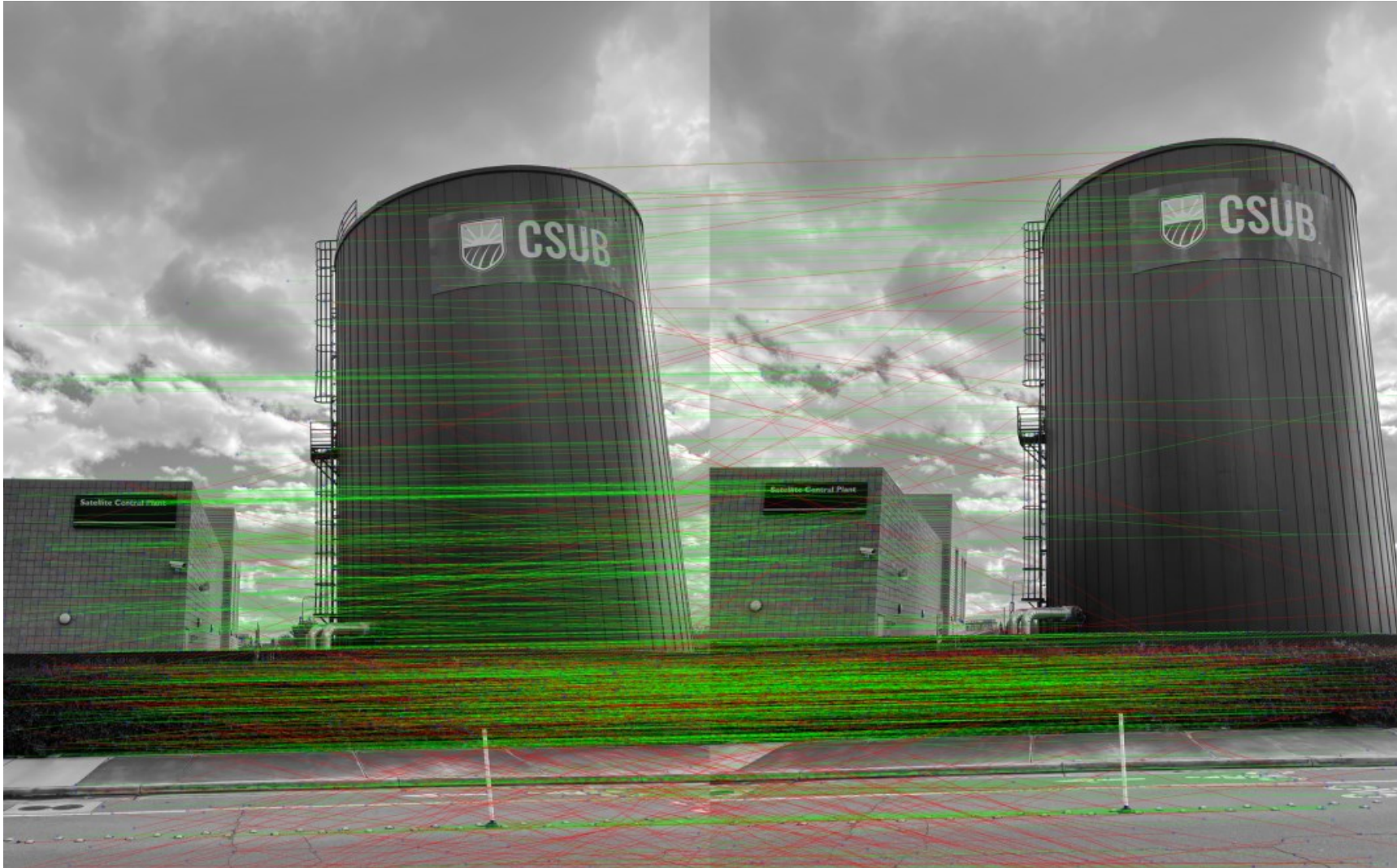
**Ensure:**  $\{w_i\}_{i=1}^n$  and  $\{p_i\}_{i=1}^n$

- 1:  $\mathbf{v} \leftarrow \text{Predict}(\{s_{i,1:k}\}_{i=1}^n)$
  - 2:  $(\alpha, \beta) \leftarrow \text{FitGamma}(\{s_{i,(1)} \text{ such that } v_i = 1\})$
  - 3:  $(\mu, \sigma, \xi) \leftarrow \text{FitGEV}(\{s_{i,(2)}\})$
  - 4: Calculate the empirical cdf using  $s_{i,j^*}$
  - 5: Find  $\varepsilon$  by solving (5)
  - 6: Calculate posterior-weights  $p_i$  using Eq. (4)
  - 7: Calculate weights  $w_i$  using Eq. (6)
  - 8: Use the weights  $w_i$  for generating hypotheses
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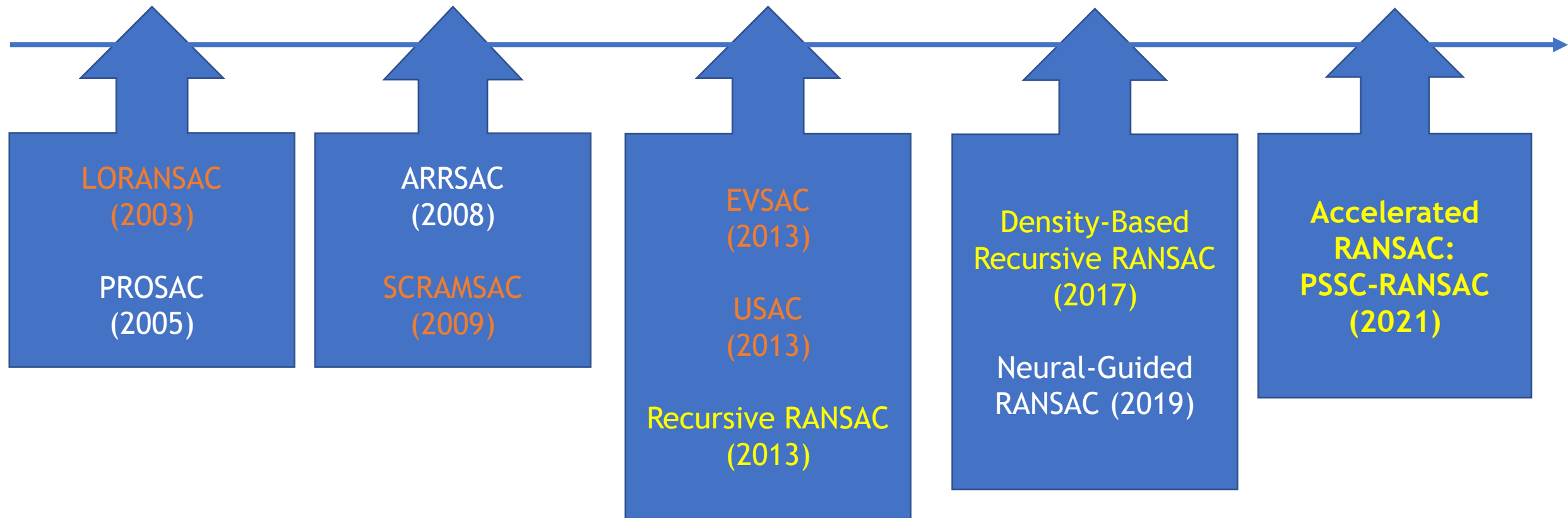
Balanced Exploration and Exploitation Model search:  
<https://pubmed.ncbi.nlm.nih.gov/18550905/>

V. Fragoso, P. Sen, S. Rodriguez and M. Turk, "EVSAC: Accelerating Hypotheses Generation by Modeling Matching Scores with Extreme Value Theory," *2013 IEEE Int. Conf. on Computer Vision*, pp. 2472-2479, 2013.

# Example of Homography: <https://github.com/dastratakos/Homography-Estimation>



# Our quick journey to 10 RANSAC VARIANT ALGORITHMS is completed. Thank you very much.



Y. L. Bond, E. Osornio, S. Ledwell and A. C. Cruz, "Comparing Strategies for Small Unmanned Aerial Vehicles Using Structure From Motion," in Third Annual Computer Science Conference for California State University (CSU) Undergraduates, Seaside, Northridge, CA, 2023



# Thank You

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