
Exercise 2 – Expressions & Integers

Informatik I für Mathematiker und Physiker (HS 2015)
Yeara Kozlov

Agenda

- ◆ Judge system
- ◆ Integer division and modulo
- ◆ Expressions
- ◆ Binary representation
- ◆ Two's Complement Representation *
- ◆ HW #2

Agenda

- ◆ Judge system
- ◆ Integer division and modulo
- ◆ Expressions
- ◆ Binary representation
- ◆ Two's Complement Representation *
- ◆ HW #2

Judge

- Missing usernames - are you registered?
- Compiler errors
- Questions about last week's homework?

Agenda

- ◆ Judge system
- ◆ **Integer division and modulo**
- ◆ Expressions
- ◆ Binary representation
- ◆ Two's Complement Representation *
- ◆ HW #2

Integer types

- Integers store whole numbers
- Memory size may vary, because they depend on the implementation and Hardware.

Data-type	Function	
int	Stores integer numbers	
unsigned int (uint)	Stores non-negative integer numbers	

Integer Division /

- Dividing between two int or unsigned int variables discards the remainder (decimal digits)
- The resulting type of such a division is always an integer
- Rational parts of the division will be truncated
- Examples:

`5 / 2 → 2 //The .5 is truncated`

`1 / 2 → 0 //Same reason here`

(Integer) Modulo %

- The remainder of a division operation can be computed using the modulo operator
- $a \% b$ returns the remainder of a divided by b .
- The modulo operation is defined only for integers.
- Examples:

$$5 \% 2 \rightarrow 1$$

$$4 \% 4 \rightarrow 0$$

$$2 \% 3 \rightarrow 2$$

Integer Division and Modulo

- Use modulo to find if a number is even or odd.
 - ◆ how?
- ◆ Or if it's a leap year
 - ◆ how?
- The original number can be expressed in terms of these operators:

$$(15 \ \% \ 4) * 4 + 15 \ \% \ 4 \rightarrow 15$$

Division and Modulo - Example

- Schreibe ein Programm, welches zwei ganze Zahlen **a** und **b** einliest, dann den Quotienten **a/b als gemischten Ausdruck berechnet** und zurückgibt. Zum Beispiel gibt es für $a = 17$ und $b = 6$ den gemischten Ausdruck $2 \frac{5}{6}$ zurück.
- Write a program which reads in two integers **a** and **b**, then calculates the quotient as **a / b mixed expression** and outputs it. For example, if $a = 17$ and $b = 6$, the output should be $2 \frac{5}{6}$.

Division and Modulo - Example

```
// Program: division.cpp

#include <iostream>

int main()
{
    // input

    // computation

    // output

    return 0;
}
```

Agenda

- ◆ Judge system
- ◆ Integer division and modulo
- ◆ **Expressions**
- ◆ Binary representation
- ◆ Two's Complement Representation *
- ◆ HW #2

Expressions and arithmetic operations

- There are 5 types of different arithmetic operators: $+$, $-$, $*$, $/$ and $\%$
- Order of evaluation:
 - Brackets before operators
 - Division, multiplication and modulo before minus and plus
- If two operators have the same precedence level, expression is usually evaluated *left to right*.
- Use brackets!
- http://en.cppreference.com/w/cpp/language/operator_precedence

Arithmetic operations

- Examples

$5 * 4u + 1;$ //21

$5 * (4u + 1);$ //25

$2 * 3 + 3 * 4 / 2;$ //12

$9 \% 4 + 1;$ //2

$9 \% (4 + 1);$ //4

$5 * 3 \% 4 * 2;$ //6

$5 * (3 \% 4) * 2;$ //30

Order of evaluation - Exceptions

- What happens here?

`b = ++a + a++;`

- The order of some expressions is not guaranteed!
- When in doubt be explicit.

Expressions - lvalue and rvalue

- An expression is a program statement that yields a value.
- All expressions are either *rvalues* or *lvalues*.
- *lvalues* persist behind a single operation.
- *rvalues* persist only for a single operation - no allocated storage.
- The operator = assigns the rvalue to the lvalue.
 - lvalue = rvalue

Expressions - lvalue and rvalue

- Arithmetic operations can be used to combine rvalues:
 - $[\text{rvalue} \times] \text{rvalue} \rightarrow \text{rvalue}$
 - $\text{lvalue} = [[\text{rvalue} \times] \text{rvalue}]$
- lvalues can be used on the right hand side, but rvalues cannot be placed to the left of an assignment.

Class Exercise

- Are the following expressions correct?
- If so, what are the types (l-value, r-value)
- What are their values?

$1 * (2 * 3)$ //R-value, 6

$(a=1)$ //L-value, 1

$(1$ //Parenthesis not closed

$(a * 3) = (b * 5)$ //Left of '=' must be an L-value

$127 \% 7 * 67 \% 17 \% 1$ //R-value, 0 (since $x \% 1$ always 0)

Class Exercise

```
127 % 7 * 67 % 17 % k;  
(((127 % 7) * 67) % 17) % k;  
((1 * 67) % 17) % k;  
(67 % 17) % k;  
16 % k;
```

```
k = 1: final answer is 0;
```

Agenda

- ◆ Judge system
- ◆ Integer division and modulo
- ◆ Expressions
- ◆ **Binary representation**
- ◆ Two's Complement Representation *
- ◆ HW #2

Binary Representation

- The memory is binary:

5 -> 101 4 -> 100 12 -> 1100

- Larger Numbers need many bits:

32,767 -> 01111111 11111111



16 bits = 2 bytes

Binary Representation

- Conversion into binary:

61	$2 * 30 + 1$
30	$2 * 15 + 0$
15	$2 * 7 + 1$
7	$2 * 3 + 1$
3	$2 * 1 + 1$
1	$2 * 0 + 1$
	111101

- And vice versa:

1	1	1	1	0	1	
32	16	8	4	2	1	61

Binary Representation

- Tables for 4bit integers:

bin	uint	int
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7

bin	uint	int
1000	8	-8
1001	9	-7
1010	10	-6
1011	11	-5
1100	12	-4
1101	13	-3
1110	14	-2
1111	15	-1

Hexadecimal Representation

- ... and how to count them.

hexadecimal:	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f	10	11	12	13
decimal:	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19

hexadecimal:	14	15	16	17	18	19	1a	1b	1c	1d	1e	1f	20	21	22	23	24	25	26	...
decimal:	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	...

Binary Representation Example

- Schreibe ein Programm `threebin.cpp`, welches eine Dezimalzahl $n \neq 0$ vom Standard-Input einliest und die letzten drei bits von n s Binärdarstellung ausgibt. Wenn die Binärdarstellung weniger als drei Bits umfasst, fülle vorne mit Nullen auf.
- Write a program `threebin.cpp` that reads a (decimal) number $n \neq 0$ from standard input and outputs the last three bits of n 's binary representation. Fill up with leading zeros in case the binary representation has less than three bits.

Binary Representation Example

- Getting the last three digits:

61	$2 * 30 + 1$	$n \% 2$
30	$2 * 15 + 0$	$n / 2 \% 2$
15	$2 * 7 + 1$	$n / 4 \% 2$

- Let's program this...

Binary Representation Example

```
// Program: threebin.cpp

#include <iostream>

int main()
{
    // input
    std::cout << "Last three binary digits of n =? ";
    unsigned int n;
    std::cin >> n;

    // computation and output
    std::cout << "The digits are "
              << n / 2 / 2 % 2 << n / 2 % 2 << n % 2 << ".\n";

    return 0;
}
```

Agenda

- ◆ Judge system
- ◆ Integer division and modulo
- ◆ Expressions
- ◆ Binary representation
- ◆ **Two's Complement Representation ***
- ◆ HW #2

Two's Complement Representation

- Motivation: $1 + (-1) = 0$

- in 4 bits:

$$\begin{array}{r} 1111 \\ +0001 \\ \hline (1)0000 \end{array}$$

- The (1) gets deleted - overflow
 - called 2's complement!
- Conversion to negative values:

$$+1 \xrightarrow{1.)} 0001 \xrightarrow{2.)} 1110 \xrightarrow{3.)} 1111$$

Two's Complement Representation

- 4 bits allow to store:
 - unsigned int 0 -> 15
 - int -8 -> 7

bin	uint	bin	uint
0000	0	1000	8
0001	1	1001	9
0010	2	1010	10
0011	3	1011	11
0100	4	1100	12
0101	5	1101	13
0110	6	1110	14
0111	7	1111	15

Two's Complement Representation

- How do you represent 9 with a 4 bit int?

bin	uint	int	bin	uint	int
0000	0	0	1000	8	-8
0001	1	1	1001	9	-7
0010	2	2	1010	10	-6
0011	3	3	1011	11	-5
0100	4	4	1100	12	-4
0101	5	5	1101	13	-3
0110	6	6	1110	14	-2
0111	7	7	1111	15	-1

unsigned int range overflow

For unsigned int (4 bit)

1111

$1111+1 = 10000 \ \% (2^4) \rightarrow 0000$ (0 dec)

$10000+1 = 10001 \ \% (2^4) \rightarrow 0001$ (1 dec)

$10001+1 = 10002 \ \% (2^4) \rightarrow 0010$ (2 dec)

Agenda

- ◆ Judge system
- ◆ Integer division and modulo
- ◆ Expressions
- ◆ Binary representation
- ◆ Two's Complement Representation *
- ◆ **HW #2**

Homework #2 Submission

- Exercise 2
- Submit answers to Q 1-2 to:
 - ◆ yeara.kozlov@inf.ethz.ch
 - ◆ Subject: [IFMP15] Ex 2
 - ◆ (Automatic filter!)
- Homework cycles - one week for submission, one week for grading.